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# A Note on the Regularity of Solutions of Infinite Dimensional Riccati Equations

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## ABSTRACT

This note is concerned with the regularity of solutions of algebraic Riccati equations arising from infinite dimensional LQR and LQG control problems. We show that distributed parameter systems described by certain parabolic partial differential equations often have a special structure that smoothes solutions of the corresponding Riccati equation. This analysis is motivated by the need to find specific representations for Riccati operators that can be used in the development of computational schemes for problems where the input and output operators are not Hilbert-Schmidt. This situation occurs in many boundary control problems and in certain distributed control problems associated with optimal sensor/actuator placement.

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## 1. INTRODUCTION

In [LCT], Lupi, Chen, and Turner considered a distributed parameter LQR problem for an Euler-Bernoulli beam model. Although this problem has been considered by several people over the past ten years, the approach in [LCT] is of interest in that they make no prior assumptions regarding the form of the controls/actuators in an effort to make decisions about where actuators and sensors are best placed. In particular, in [LCT] they assumed that the input operator was the identity. Miller and van Schoor [MvS] also considered the problem of constructing kernels for integral representations of feedback control laws obtained by solving LQR control problems for various beam models. They used these kernels to shape and design area averaging polyvinylidene fluoride sensors (a type of piezoelectric film). These sensors enable the real-time implementation of full state feedback for the infinite dimensional system governed by the Euler-Bernoulli equation (also, see [MvS]). As in [LCT], Miller and van Schoor assumed the existence of an integral representation for the feedback control law and then proceed to "approximate" these kernels by using finite element models. In both papers, fundamental mathematical questions concerning the existence of integral representations and the smoothness of the corresponding integral kernels are not considered. These issues are important in the development and analysis of rigorous numerical approximations. Also, the ability to accurately compute these kernels is an essential component in the study of actuator/sensor placement.

During the past ten years considerable effort has been devoted to the study of Riccati equations associated with LQR and LQG control of distributed parameter systems in Hilbert spaces. In the recent papers by Rosen ([R1], [R2]) and De Santis, Germani and Jetto [DGJ] it was shown that, under suitable assumptions on the system input, output and weighting operators, that the Riccati operator is Hilbert-Schmidt. This observation made it possible to develop an approximation theory in the space of Hilbert-Schmidt operators and, as noted in [DGJ], the smoothness action of Hilbert-Schmidt operators can be exploited to relax the hypothesis that approximation schemes converge to the dual semigroup. In addition, if the Riccati operator is Hilbert-Schmidt, then one has explicit representation theorems that can be used to analyze the convergence of numerical approximations.

The papers noted above represent two basic approaches to the problem. Rosen [R1] considered the problem for control systems where the generator of the semigroup was strongly coercive and developed a theory for this restricted class of systems. In [DGJ] the approach was to consider general dynamical systems (not necessarily analytic semigroups), and then require that certain system operators (input, weighting, etc.) be nuclear. The problem we consider in this short note lies between these two approaches, although it is more in the spirit of Rosen [R1]. We extend Rosen's results to control systems governed by parabolic equations without requiring that the other system operators be Hilbert-

Schmidt. Indeed, the results below apply to a large class of control problems with unbounded input operators. In order to focus the discussion we consider only the LQR problem. However, the ideas and methods extend to LQG and MinMax control problems.

## 2. THE PROBLEM DESCRIPTION

Consider the control system

$$(2.1) \quad \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$$

with controlled output

$$(2.2) \quad z(t) = Cx(t)$$

and cost function

$$J(u) = \frac{1}{2} \int_0^{+\infty} \{ \langle z(t), z(t) \rangle + \langle u(t), u(t) \rangle \} dt.$$

If  $Q = C^*C$ , then  $J$  becomes

$$(2.3) \quad J(u) = \frac{1}{2} \int_0^{+\infty} \{ \langle Qx(t), x(t) \rangle + \langle u(t), u(t) \rangle \} dt,$$

where the state weighting matrix  $Q$  is self-adjoint and non-negative definite. Although it is possible to consider more general systems or the case with  $Q$  unbounded and  $B$  bounded, we limit our discussion to systems satisfying the following standing hypothesis:

(H) *The spaces  $X$ ,  $Z$  and  $U$  are separable Hilbert spaces and;*

- (i) *The linear operator  $A$  is the generator of a  $C_0$ -semigroup  $S(t)$  on  $X$  and there exist  $M > 0$ ,  $\omega > 0$  such that  $\|S(t)\| \leq Me^{-\omega t}$ .*
- (ii) *The operator  $C: X \rightarrow Z$  is a bounded linear operator from  $X$  to  $Z$ .*
- (iii) *The (possibly unbounded) linear operator  $B$  maps  $U$  into  $X$  and  $B: U \rightarrow [Dom(A^*)]'$ . Moreover, there exists a gamma with  $0 \leq \gamma < 1$ , such that  $A^{-\gamma}B \in L(U, X)$ .*

- (iv) For each  $x_0 \in X$  there exists a control  $u(\cdot) \in L_2(0, \infty; U)$  such that the functional  $J(u)$  in (2.3) is finite.

When the optimal control exists, it is given in feedback form

$$(2.4) \quad u_{opt}(t) = -B^* P x_{opt}(t) = -K x_{opt}(t),$$

where  $P$  is the non-negative definite solution to the algebraic Riccati equation (ARE)

$$(2.5) \quad \langle Px, Ay \rangle_X + \langle Ax, Py \rangle_X - \langle B^* Px, B^* Py \rangle_U + \langle Cx, Cy \rangle_Z = 0$$

for all  $x, y$  in  $\text{Dom}(A)$ .

In [R1], [R2], [DGJ] and [GJP] it is assumed that  $C$  and  $B$  are bounded linear operators. Rosen [R1] assumes that  $A$  is strongly coercive,  $PB B^* P$  is Hilbert-Schmidt whenever  $P$  is Hilbert-Schmidt and that  $Q$  is Hilbert-Schmidt. On the other hand, De Santis, Germani and Jetto [DGJ] make no additional assumptions on  $A$ , but require that  $C$  be Hilbert-Schmidt and  $B$  be bounded from  $U$  to  $X$ . The assumption that  $C$  be Hilbert-Schmidt implies that the weighting operator  $Q = C^* C$  is nuclear. Hence, the assumption on  $Q$  in [DGJ] is stronger than the corresponding condition in [R1]. The two problems are not mutually exclusive and, as one might expect, there is no unified theory. In this paper we consider problems with  $B$  unbounded and  $Q$  not Hilbert-Schmidt. Unbounded  $B$  operators allow us to treat certain boundary control problems and problems with piezoelectric actuators/sensors. Also, the case where  $Q = B = I_X$  (the identity on  $X$ ) arises naturally in the solution of optimal sensor/actuator location problems (see [BK1], [KM] and [LCT]).

We consider here the case where  $A$  generates an analytic semigroup. The following theorem may be found in [LT].

**Theorem 2.1** *Assume that hypothesis (H) holds. If  $A$  generates an analytic semigroup, then there exists a self-adjoint, non-negative definite bounded linear operator  $P = P^*$  that solves the ARE (2.5). Moreover;*

- (a) For each  $\varepsilon > 0$ , the operator  $[A^*]^{1-\varepsilon} P$  belongs to  $L(X, X)$ .
- (b) If  $A$  is self-adjoint, normal or has a Riesz basis of eigenvectors, then  $\varepsilon$  can be taken to be 0.
- (c) The operator  $B^* P$  belongs to  $L(X, U)$ .

Observe that if  $A$  has a compact resolvent, then  $P$  is compact. In general it is not possible to conclude that  $P$  has additional smoothing properties unless more is known about the operators  $A$ ,  $B$  and  $C$ . Consider the following example:

**Example 2.2** Let  $X = Z = U = L_2(0,1)$  and set  $A = C = I$ . If  $B = \sqrt{3}I$ , then  $P = I$  is the unique positive-definite solution to the ARE (2.5). Hypothesis (H) holds and  $A$  generates an analytic semigroup. However,  $P$  is not compact.

In certain specific cases it is possible to obtain additional information about the regularity of the Riccati operator  $P$ . In the next section we consider a parabolic control problem similar to the one treated by Rosen (see [R1], [R2]) and use classical representation theory to show that  $P$  is Hilbert-Schmidt. This particular approach not only yields very precise information about the smoothness of  $P$ , it leads to a rather simple proof.

### 3. A PARABOLIC CONTROL PROBLEM

In order to keep the present paper short we shall limit our discussion to a 1D parabolic control problem. Although problems in higher dimensions can be treated in a similar fashion (subject to the Sobolev imbedding theorems), the analysis is more complex and will appear in a future paper.

We consider the operator  $A$  defined on the state space  $X = L_2(0,1)$  with domain

$$(3.1) \quad \text{Dom}(A) = H_0^1[0,1] \cap H^2[0,1],$$

and for  $\varphi \in \text{Dom}(A)$

$$(3.2) \quad [A\varphi](\xi) = \frac{d^2}{d\xi^2} \varphi(\xi).$$

In order to simplify the proof, we begin with the case where  $Z = X = L_2(0,1)$ ,  $C = Q = I_{L_2}$  and  $B:U \rightarrow [\text{Dom}(A')]'$  satisfies (H)-(iii). The extension to general  $C$  operators and unbounded input operators  $B$  satisfying hypothesis (H) is straightforward. We note that if  $B$  is bounded into  $X$ , then (H) is satisfied.

The controlled heat equation is (see [R1], [R2]) written as

$$(3.3) \quad \frac{\partial}{\partial t} w(t, \xi) = \frac{\partial^2}{\partial \xi^2} w(t, \xi) + [Bu(t)](\xi), \quad 0 < \xi < 1, \quad 0 < t,$$

with boundary conditions

$$(3.4) \quad w(t,0) = 0, \quad w(t,1) = 0, \quad 0 < t$$

and cost function

$$(3.5) \quad J(u) = \frac{1}{2} \int_0^{\infty} \int_0^1 \{ |w(t,\xi)|^2 + |u(t,\xi)|^2 \} d\xi dt.$$

This problem has the form (2.1)-(2.3) and hypothesis (H) holds. The operator A is self-adjoint and generates an analytic semigroup on X. Also, the finite cost condition (H)-(iv) holds for any B.

Note that Q is not Hilbert-Schmidt (it is not even compact). Therefore, the results in [R1] and [DGJ] do not apply to this problem. However, we shall show below that the Riccati operator P is Hilbert-Schmidt. Moreover, the special structure of the generator A can be exploited to obtain additional information about the functional gains. We shall need the following representation theorem which goes back to Fullerton in 1946 (see Theorem 6 on page 277 in [F1]).

**Theorem 3.1** *Let T be a bounded linear operator mapping  $L_2[0,1]$  into  $C^m[0,1]$ . Then there exists a function  $k(\xi, t)$  such that T has the representation*

$$(3.6) \quad [T\varphi](\xi) = \int_0^1 k(\xi, t)\varphi(t)dt,$$

where the kernel  $k(\xi, t)$  satisfies the following conditions;

- (i)  $k(\xi, t) \in L_2([0,1] \times [0,1])$ .
- (ii) For each  $\xi \in (0,1)$ , the mapping  $\xi \longrightarrow k(\xi, t)$  belongs to  $C^m[0,1]$  for almost all t and for  $\alpha \leq m$ ,

$$(3.7) \quad \frac{\partial^\alpha}{\partial \xi^\alpha} k(\xi, t) \in L_2([0,1] \times [0,1]).$$

- (iii) For each  $\varphi \in L_2[0,1]$ ,

$$(3.8) \quad \frac{\partial^\alpha}{\partial \xi^\alpha} \int_0^1 k(\xi, t)\varphi(t)dt = \int_0^1 \frac{\partial^\alpha}{\partial \xi^\alpha} k(\xi, t)\varphi(t)dt.$$

The following result establishes the existence of an integral representation for the Riccati operator and provides information about the smoothness of the kernel.

**Theorem 3.2** Assume  $C = Q = I_{L_2}$  and  $B:U \rightarrow [Dom(A^*)]'$  satisfies (H)-(iii). If  $P = P^*$  is the unique non-negative definite solution to the algebraic Riccati equation (ARE) defined by the system (3.1)-(3.5), then  $P$  is Hilbert-Schmidt. Moreover, there exists a function  $k(\xi, t)$  such that  $P$  has the representation

$$(3.9) \quad [P\varphi](\xi) = \int_0^1 k(\xi, t)\varphi(t)dt,$$

where the kernel  $k(\xi, t)$  satisfies the following conditions;

$$(1) \quad k(\xi, t) = k(t, \xi) \in C^1([0, 1] \times [0, 1]).$$

$$(2) \quad \text{For each } \varphi \in L_2[0, 1], \psi = P\varphi \in C^1[0, 1] \text{ and}$$

$$(3.10) \quad \frac{d}{d\xi}\psi(\xi) = \frac{\partial}{\partial\xi} \int_0^1 k(\xi, t)\varphi(t)dt = \int_0^1 \frac{\partial}{\partial\xi} k(\xi, t)\varphi(t)dt.$$

*Proof.* Let  $\tilde{A}$  be the extension of  $A$  defined by (3.1) - (3.2) to  $H^2[0, 1]$ . It follows from Theorem 2.1 that  $P$  and  $\tilde{A}P$  are bounded linear operators on  $L_2(0, 1)$ . Therefore, there exist constants  $c_1$  and  $c_2$  such that for all  $\varphi \in L_2[0, 1]$ ,  $P\varphi \in H^2[0, 1]$ ,  $\tilde{A}P\varphi \in L_2[0, 1]$  and

$$(3.11) \quad \|P\varphi\|_{L_2} \leq c_1 \|\varphi\|_{L_2} \quad \text{and} \quad \|\tilde{A}P\varphi\|_{L_2} \leq c_2 \|\varphi\|_{L_2}.$$

The space  $Dom(\tilde{A}) = H^2[0, 1]$  with graph norm  $\|\varphi\|_{Dom(\tilde{A})} = \|\varphi\|_{L_2} + \|\tilde{A}\varphi\|_{L_2}$  is equivalent to  $H^2[0, 1]$  (see [A]). It follows from the Sobolev imbedding theorem that  $Dom(\tilde{A}) = H^2[0, 1]$  is continuously imbedded in  $C^1[0, 1]$  and hence there exists a constant  $c_3$  such that for all  $\psi \in H^2[0, 1]$

$$\{\|\psi\|_{C^1} + \|\psi'\|_{C^0}\} \leq c_3 \|\psi\|_{Dom(\tilde{A})} = c_3 \{\|\psi\|_{L_2} + \|\tilde{A}\psi\|_{L_2}\}.$$

Let  $\varphi \in L_2[0, 1]$  and  $\psi = P\varphi \in H^2[0, 1]$ . Then,

$$\|P\varphi\|_{C^1} = \{\|\psi\|_{C^1} + \|\psi'\|_{C^0}\} \leq c_3 \{\|\psi\|_{L_2} + \|\tilde{A}\psi\|_{L_2}\} \leq c_3 \{\|P\varphi\|_{L_2} + \|\tilde{A}P\varphi\|_{L_2}\},$$

and it follows from (3.11) that



$$\|P\varphi\|_{C^1} \leq c_3 (\|P\varphi\|_{L_2} + \|\tilde{A}P\varphi\|_{L_2}) \leq c_3 \{c_1 + c_2\} \|\varphi\|_{L_2}.$$

Consequently,  $P$  is a bounded linear operator from  $L_2[0,1]$  into  $C^1[0,1]$ . Applying Theorem 3.1, yields the representation (3.9).

Since  $P = P^*$  the kernel  $k(\xi, t)$  satisfies  $k(\xi, t) = k(t, \xi)$ . It follows from part (ii) of Theorem 3.1 that  $k(\xi, t) = k(t, \xi) \in C^1([0,1] \times [0,1])$  and (3.10) is a consequence of part (iii) of Theorem 3.1. Finally, (see page 210 in [RS]) the operator  $P: L_2[0,1] \rightarrow L_2[0,1]$  is Hilbert-Schmidt since it has the representation (2.9) with  $k(\xi, t) \in L_2([0,1] \times [0,1])$ . ■

**Corollary 3.3** *Assume  $A$  is defined by (3.1)-(3.2),  $C = Q = I_{L_2}$  and  $B: U \rightarrow L_2[0,1]$  is bounded. If  $P = P^*$  is the unique non-negative definite solution to the algebraic Riccati equation (ARE) defined by the system (3.1)-(3.5), then the gain operator  $K = -B^*P$  is Hilbert-Schmidt.*

**Remark 3.4** Observe that if  $A$  is defined by (3.1)-(3.2), then the proof given above goes through without change for any bounded  $C: L_2[0,1] \rightarrow Y$  and  $B$  satisfying (H). In particular, there is no need to assume that  $C$  (or  $B$ ) is Hilbert-Schmidt (see [R1], [DGJ] and [GJP]) and  $B$  can be unbounded. Also, as noted above, similar results (weaker) are valid for 2D and 3D problems. Let  $\Omega$  denote a smooth bounded domain in  $\mathbb{R}^n$ ,  $n \leq 3$ , with boundary  $\Gamma$ . Although for each  $\delta > 0$  the imbedding  $H^2(\Omega) \rightarrow H^{3/2+\delta}(\Omega) \rightarrow C^0(\Omega)$  is valid, the imbedding  $H^{1+m}(\Omega) \rightarrow C^1(\Omega)$  holds only for  $m > \frac{n}{2}$ . Hence, one would expect less smoothness for  $n > 1$ . These issues will be addressed in a future paper.

#### 4. NUMERICAL RESULTS AND CONCLUDING REMARKS

In this section we present some numerical results to demonstrate the role that the operator  $B$  plays on the smoothness of the operator  $P$ . We conducted several experiments for the operators  $B = [-A]^\beta$  where  $\beta = -1/2, 0, 1/4, 1/2, 3/4$ , and 1. Note that if  $\beta < 1$ , then hypothesis (H) is satisfied. When  $\beta = 0$ ,  $B = I_{L_2}$  and  $B = [-A]^{(-1/2)}$  is compact. We selected this collection of  $B$  operators because as  $\beta \rightarrow 1$  the operator  $A^{-1}B = A^{-1}A^\beta = A^{\beta-1}$  in condition (H)-(iii) becomes "less smooth". Observe that if  $\beta = 1$ , then (H)-(iii) is not satisfied for any  $\gamma < 1$ .

We use standard linear finite elements to compute  $k^N(\xi, t) = k(\xi, t)$ , the " $N^{\text{th}}$  order approximation" of the kernel  $k(\xi, t)$ . Figure 4.1 and Figure 4.2 show the  $N = 4, 8, 16$  and

32 finite element approximations of  $k(\xi, t)$  for the cases  $B = [-A]^{-1/2}$  and  $B = I_{L_2}$ , respectively. Observe the fast convergence  $k^N(\xi, t) \xrightarrow{N \rightarrow \infty} k(\xi, t)$  and, as implied by Theorem 3.2, the kernel  $k(\xi, t)$  is smooth. Figure 4.3, Figure 4.4 and Figure 4.5 contain analogous plots for  $B = [-A]^\beta$  where  $\beta = 1/4, 1/2$  and  $3/4$ , respectively. It is interesting to note that as  $\beta \rightarrow 1$  the kernels  $k(\xi, t)$  become less smooth. Moreover, when  $\beta = 1$  Theorem 3.2 breaks down and  $k^N(\xi, t)$  appears to be converging to a "singular measure" concentrated on the line  $\xi = t$ . Similar results were observed for weakly damped hyperbolic problems in [BK1] and for beam equations in [LCT].

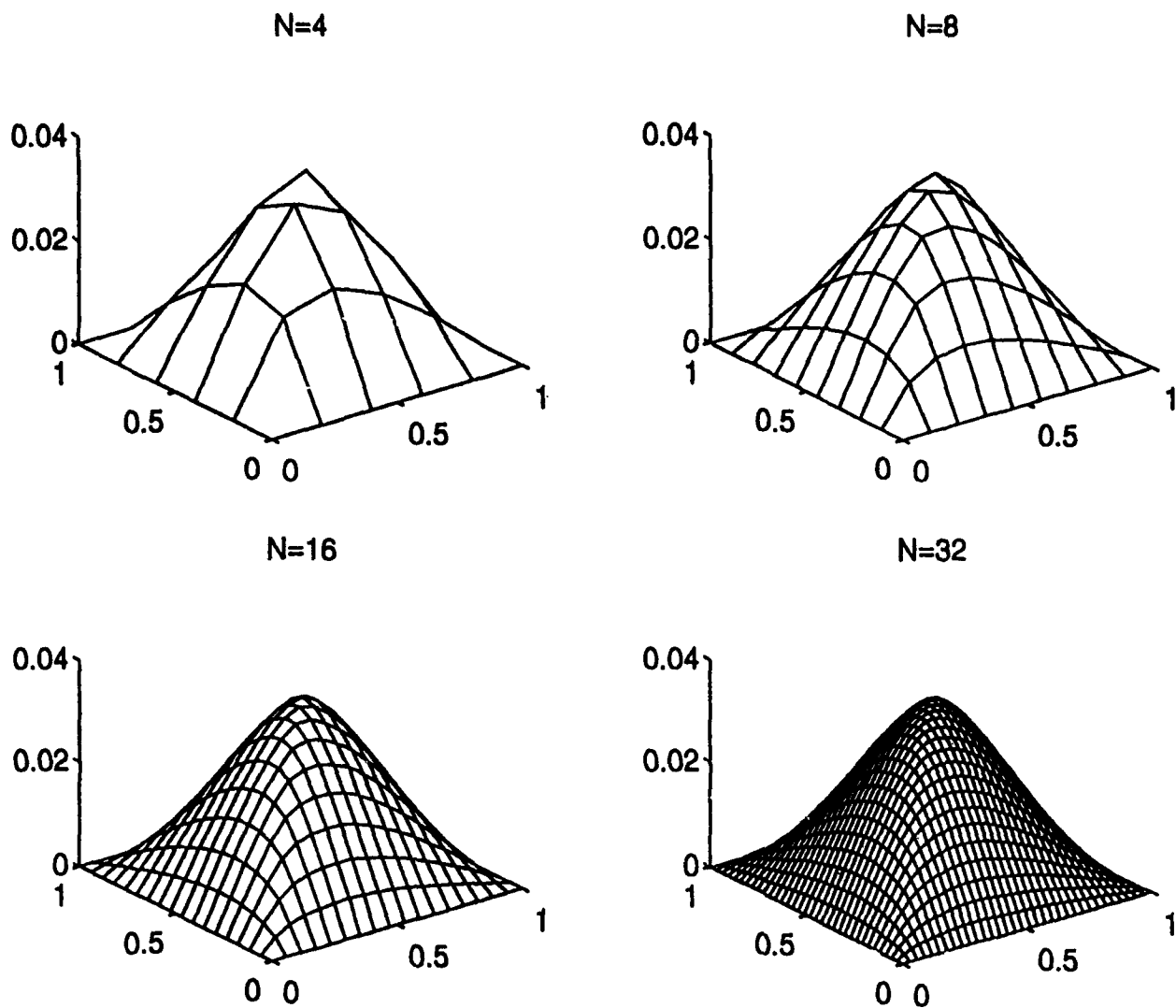
As noted in Corollary 3.4, when  $A$  is defined by (3.1)-(3.2),  $C = Q = I_{L_2}$  and  $B: U \rightarrow L_2[0,1]$  is bounded, then the gain operator  $K = -B^*P$  is Hilbert-Schmidt. In some cases (e.g.  $U = L_2[\Omega']$  where  $\Omega' \subseteq [0,1]$ ) it is possible to obtain explicit integral representations of  $K = -B^*P$ . The case where  $B$  is unbounded requires additional analysis. Finally, the authors wish to thank J.S. Gibson for Example 2.2 and several helpful comments.

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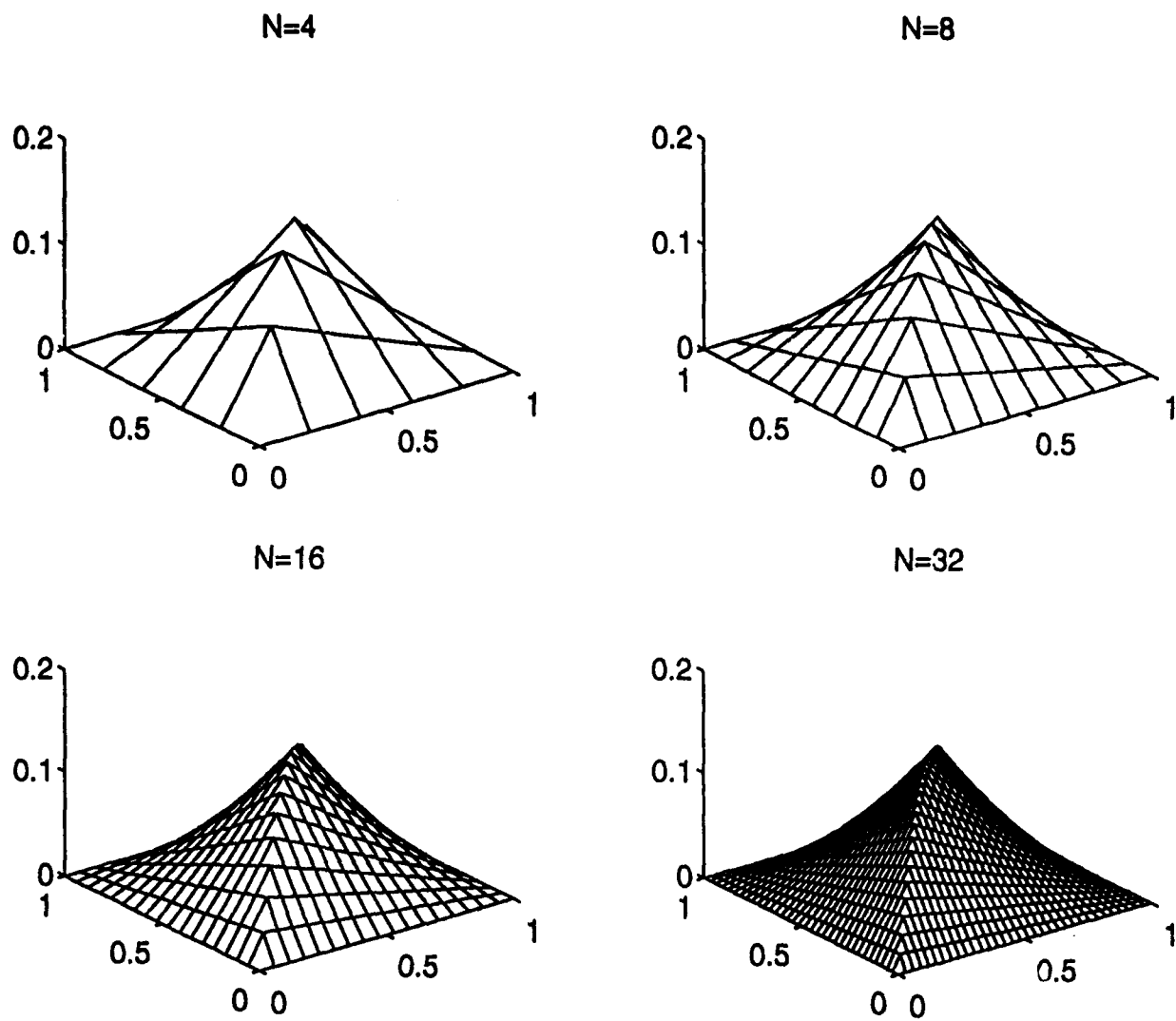
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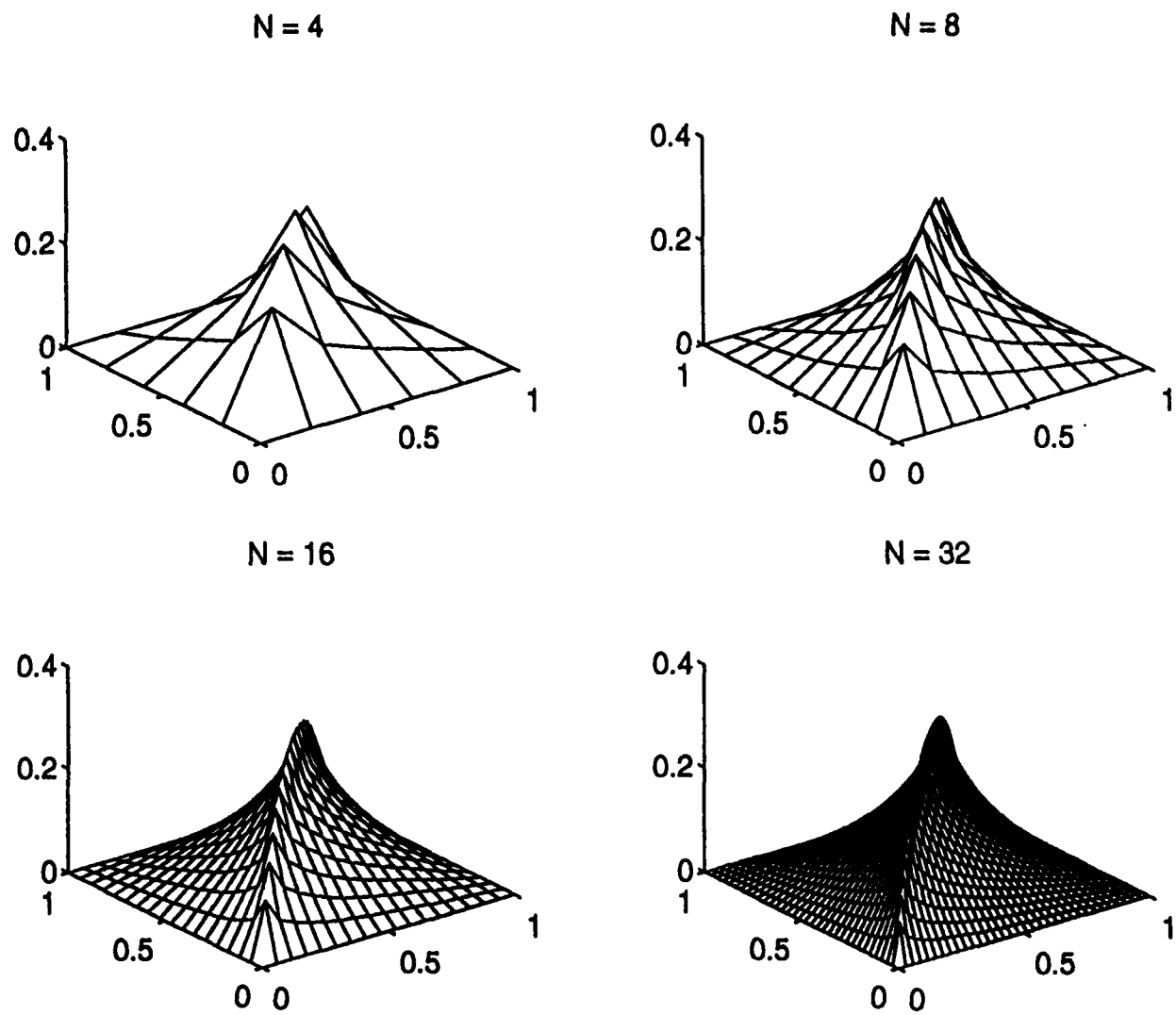
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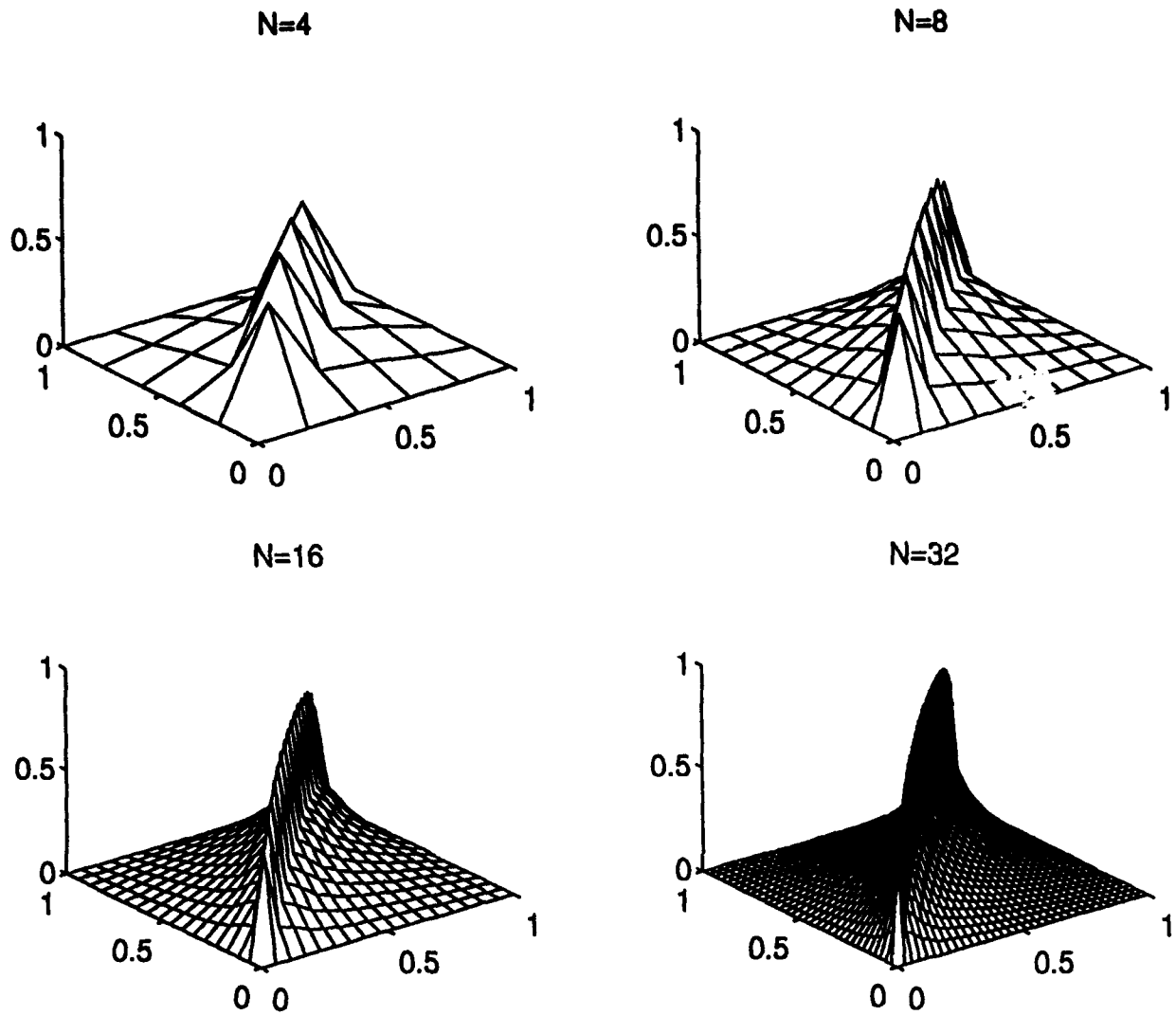
**Figure 4.1:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = [-A]^{-1/2}$



**Figure 4.2:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = I_{l_2}$

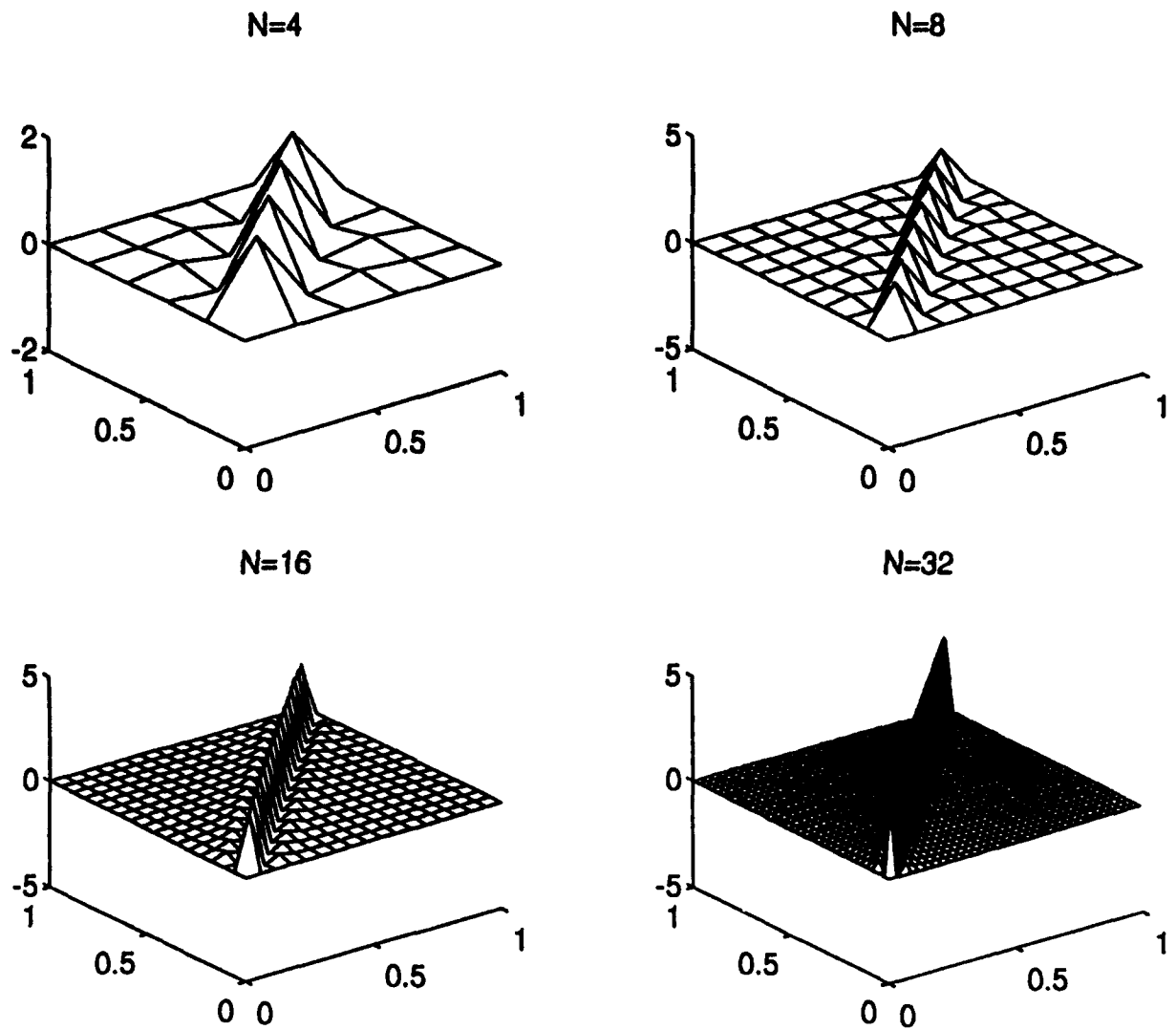


**Figure 4.3:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = [-A]^{1/4}$

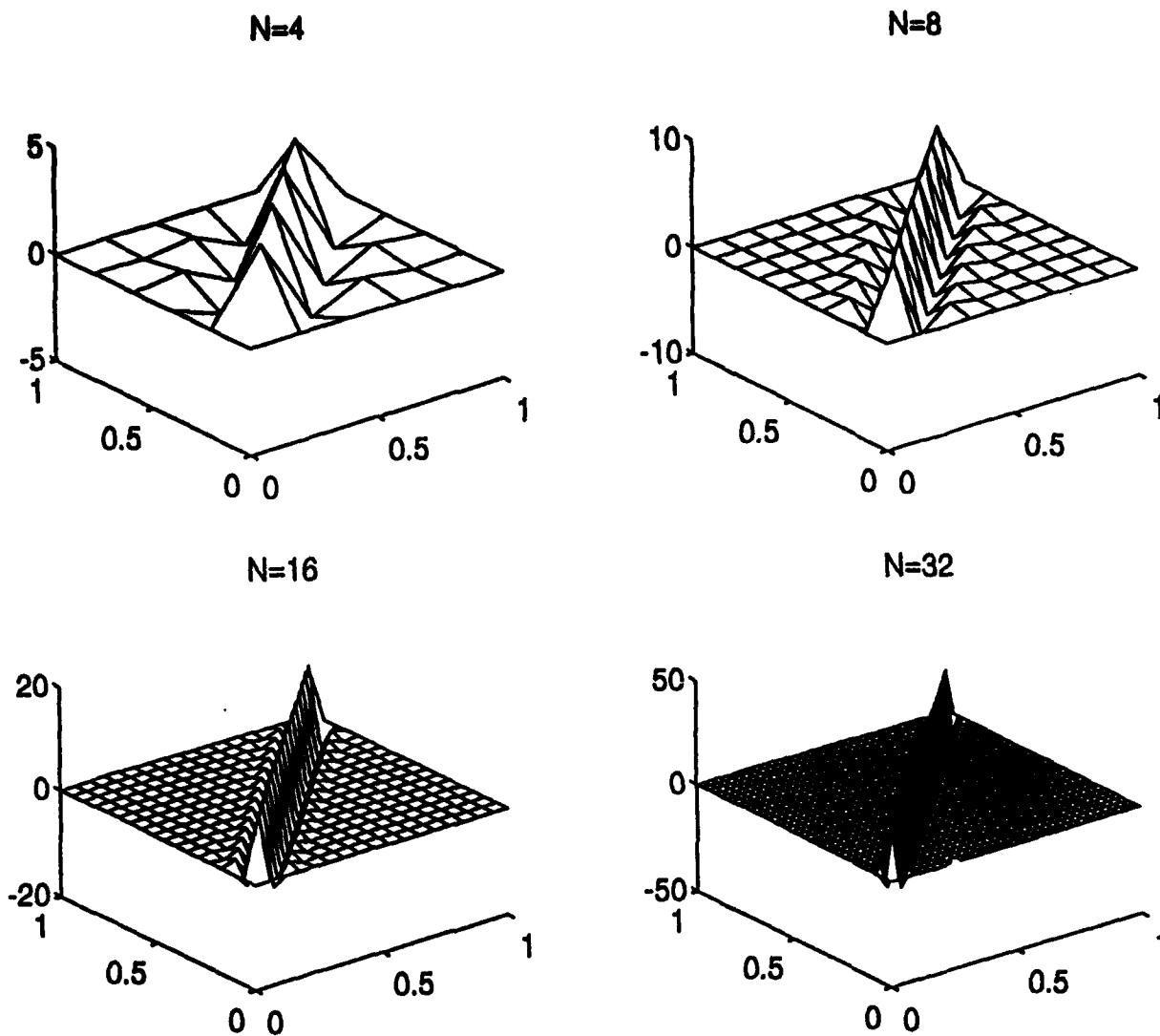


**Figure 4.4:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = [-A]^{1/2}$





**Figure 4.5:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = [-A]^{3/4}$



**Figure 4.6:** THE KERNELS  $k^N(\xi, t)$  FOR  $B = [-A]$

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