LEWIS

JPRS: 9170

1 May 1961

THE STABILITY OF OPERATION OF A LIQUID FUEL ROCKET ENGINE

By K. I. Artomonov

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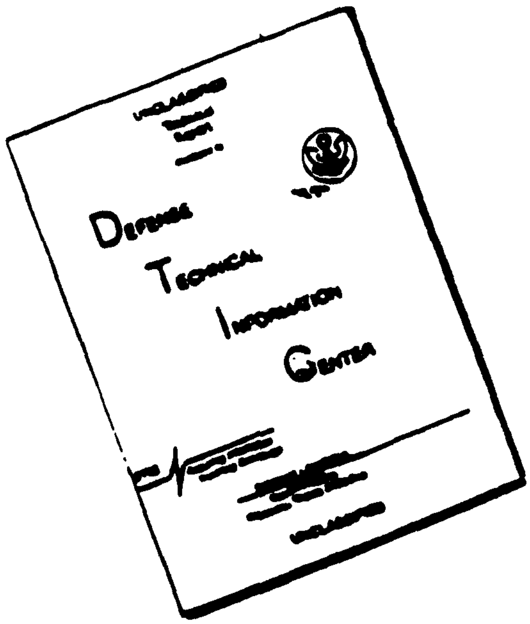

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JPRS: 9170

CSO : 6177-N

THE STABILITY OF OPERATION OF A LIQUID FUEL ROCKET ENGINE

[The following is a translation of "Ustoychivost' raboty zhidkostnogo raketnogo dvigatelya" (English version above) by K.I. Artomonov in Iz AN SSSR, Otdeleniye Tekh. Nauk, Mekhanika i mashinostroyeniye, No 1, Jan-Feb. 1961, pages 64-69.]

The liquid rocket engine (LRE) is considered as an oscillatory system with delayed feedback; transient processes in one of the elements of the engine (the fuel lines) are described by means of a wave equation. The lag and the wave processes induce an alternation of stable and unstable regimes, as the thrust of the LRE is reduced. The oscillation frequencies are then close to the natural frequencies of the fuel pipeline. If the lag is a large one, it is only possible to have relatively low-frequency oscillatory regimes.

1. The LRE as a self-oscillating system.

The self-excitation of oscillations in an LRE is to a large extent determined by the character of the process of converting liquid fuel components to gaseous combustion products. Before being converted to gaseous combustion products the oxidizer and the fuel must go through a series of preparatory processes (atomization of the liquid jets, heating and evaporation of the droplets, mixing of the components, chemical reactions, etc.). The volume of fuel, passing through this stage, has practically no effect on the gaseous products, the inflow of which also determines the chamber pressure. Given sufficiently rapid variations in fuel consumption this means that at any given moment the chamber pressure is determined by the consumption of fuel during a certain preceding interval of time. In other words, the connection between the flow of fuel through the atomiser and the chamber pressure is characterised by a delay. In existing LRE fuel feed systems this connection is also two-way: fluctuations in the chamber pressure also affect the rate of flow of fuel into the chamber. Thus, with respect to the nature of the working process in the combustion chamber and the characteristics of the fuel feed system the LRE is a system with delayed feedback.

One of the elements of the propulsion plant - the fuel lines - ought to be considered an element with distributed parameters, since the transit time for the pressure wave through the pipeline and the period of the oscillations are commensurable.

The lag and the wave processes determine a series of properties of the dynamics of an LRE which we shall discuss below with reference to the simplest type of propulsion plant.

2. Dynamic equations of an LRE

In order to derive the stability conditions, we shall write out the equations describing non-steady processes in an LRE. As is usual, in investigating stability in the small we shall use linearised equations.

Equation of the conversion process.

We shall characterise the process of converting liquid fuel to gas by means of the conversion curve $\varphi(t)$ - ratio of the mass of the gas formed at a moment t from the combustion of a given volume of fuel to the initial mass of the fuel introduced at the moment t = 0.

Evidently, the conversion curve climbs smoothly from zero and tends asymptotically to unity (Fig. 1). This curve is only an averaged characteristic of the processes at work; its form is determined both by the law of conversion of the individual particle (the drop) of fuel and by differences in the conditions associated with the conversion of the different particles (e.g. due to the non-uniform size of the drops). We shall assume that the conversion curve is determined only by steady-state parameters and is not affected by small fluctuations.

If we introduce the rate of conversion $\psi = d\varphi/dt$, then the mass of gas converted per unit of time $M_r(t)$ may be expressed as:

$$M_r(t) = \int_0^\infty M_\phi(t - t_1)\,\psi(t_1)\,dt_1 \qquad (2.1)$$

where $M_\Phi(t)$ is the mass rate of inflow of fuel through the spray head.

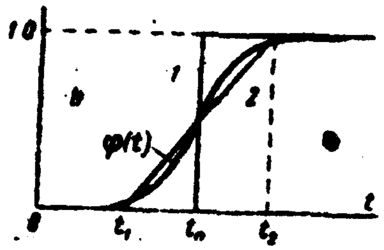


Fig. 1

We shall introduce the small dimensionless deviations

$$m_r = \frac{M_r - M_{r0}}{M_{r0}}, \qquad m_\Phi = \frac{M_\Phi - M_{\Phi 0}}{M_{\Phi 0}}$$

The index 0 denotes stationary values of the functions. Taking into account that

$$\int_0^\infty \psi(t_1)\,dt_1 = 1$$

we get instead of (2.1)

$$m_r(t) = \int_0^\infty m_\Phi(t - t_1)\,\psi(t_1)\,dt_1 \qquad (2.2)$$

Equation of combustion chamber

We shall consider that the gas pressure is uniform throughout the chamber at all times. This is correct for oscillations, the period of which is much greater than the time the pressure wave takes to traverse the length of the chamber.

The equation of conservation of mass of the gas in the chamber has the form

$$\frac{dQ_K}{dt} + M_c = M_r \qquad (2.3)$$

where Q_K is the mass of gas in the chamber and M_c is the mass rate of flow of gases through the supersonic nozzle.

If during the oscillation the relationship between the fuel components does not change, it is possible to consider the chamber processes isoentropic, i.e.

$$p_к \rho_к^{-\gamma} = \text{const} \tag{2.4}$$

To equations (2.4) and (2.5) we must add the equation of state of an ideal gas:

$$p_к = \rho_к R T_к$$

and expressions for Q_K and M_c

$$Q_к = \rho_к V_к, \quad M_c = \frac{p_к F_{кр}}{g\beta}; \quad \beta = \frac{\sqrt{\gamma R T_к}}{\gamma} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma-1}{2(\gamma+1)}} \tag{2.5}$$

where p_K, ρ_K and T_K are respectively the pressure, density and temperature of the gases in the combustion chamber; R is the gas constant, γ the adiabatic coefficient, V_K the volume of the combustion chamber, F_{Kp} the area of the critical cross section of the nozzle and g the acceleration due to gravity.

Equations (2.3)-(2.5) give us the connection between m_r and the dimensionless pressure variation in the chamber

$$\eta_к = \frac{p_к - p_{к0}}{p_{к0}}$$

in the following form:

$$t_к \frac{d\eta_к}{dt} + \chi \eta_к = m_r \quad \left(t_к = \frac{Q_{к0}}{\gamma M_{r0}}, \ \chi = \frac{\gamma+1}{2\gamma}\right) \tag{2.6}$$

Here t_K is the time the gases remain in the combustion chamber divided by the adiabatic coefficient.

<u>Equation of fuel feed system</u>

It is assumed that the flow of liquid is not influenced by inertia forces due to the acceleration of the rocket. We shall consider a compressed gas fuel feed system consisting of a fuel tank, a homogeneous pipeline and a non-elastic spray head.

The motion of the liquid in the straight cylindrical pipe is determined by Zhukov's equations:

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial v}{\partial t}, \qquad -\frac{\partial p}{\partial t} = \rho a^2 \frac{\partial v}{\partial x} \tag{2.8}$$

In these equations ρ is the constant density of the liquid, a is the speed of sound in the liquid in an elastic pipe.

The end of the pipeline at the tank may be considered closed; during oscillations the pressure in this section is constant:

$$x = 0, \qquad \frac{\partial v}{\partial x} = 0 \tag{2.9}$$

Neglecting the inertia of the liquid in the spray head cavity, we get a second boundary condition from the expression for the flow of fuel through the spray head:

$$x = l; \qquad \rho v = \mu_\Phi \sqrt{p - p_\kappa} \tag{2.10}$$

Introducing the variables

$$m = \frac{\rho v - \rho u_0}{\rho v_0}, \qquad \eta = \frac{p - p_0}{p_{\kappa 0}}$$

and eliminating η from (2.8), we get for m the wave equation

$$\frac{\partial^2 m}{\partial t^2} - a^2 \frac{\partial^2 m}{\partial x^2} = 0 \tag{2.11}$$

with the boundary conditions:

$$\frac{\partial m}{\partial x} = 0 \quad \overset{\text{WHEN}}{\text{при}}\ x = 0, \qquad m = \frac{\eta - \eta_\kappa}{h_\Phi} \quad \overset{\text{WHEN}}{\cancel{\text{при}\ x}} = l \tag{2.12}$$

where

$$h_\Phi = \frac{2(p_0(l) - p_{\kappa 0})}{p_{\kappa 0}} = \frac{2\Delta p_\Phi}{p_{\kappa 0}}$$

The second condition in (2.12) is obtained by linearizing equation (2.10).

3. Characteristic equation of the system

We shall apply the Laplace transformation to the equations thus obtained, retaining the previous notation for the transformed functions. The operator for differentiation with respect to time will be designated q.

The wave equation (2.11) then goes over into an ordinary differential equation, the solution of which for m(q,l), equal to m_Φ, taking into account boundary conditions (2.12), is obtained in the form:

$$m_\Phi = -\frac{\eta_\kappa}{h_\Phi + h_B \operatorname{th} \frac{ql}{a}} \qquad \left(h_B = \frac{\rho v_0 a}{p_{\kappa 0}}\right) \tag{3.1}$$

Instead of (2.1) and (2.7) we get:

$$m_\Gamma = \psi(q)\, m_\Phi \tag{3.2}$$

$$\eta_\kappa = \frac{m_\Gamma}{\chi + q t_\kappa} \tag{3.3}$$

From eqs. (3.1)-(3.3) it is easy to obtain the characteristic equation of the system:

$$h_\Phi + h_B \operatorname{th} \frac{ql}{a} + \frac{\psi(q)}{\chi + q t_\kappa} = 0 \tag{3.4}$$

4. Limits of stable operation of an LRE

We obtain the equations for establishing the limits of the regions of stable operation of an LRE and for computing the oscillation frequency ω at the limits of stability by substituting in (3.4) $q = i\omega$ $(i = \sqrt{-1})$.

These equations can be reduced to the form

$$h_\phi^2 = \frac{S^2(\omega)}{\chi^2 + \omega^2 t_H^2} - h_H^2\,\mathrm{tg}^2\,\alpha \qquad \left(\alpha = \frac{\omega l}{a}\right) \tag{4.1}$$

$$\mathrm{tg}\,\theta = -\frac{h_\phi \omega t_H - \chi h_H\,\mathrm{tg}\,\alpha}{-\chi h_\phi + \omega t_H h_H\,\mathrm{tg}\,\alpha} \tag{4.2}$$

Here $\quad S^2(\omega) = \mathrm{Re}^2\psi(i\omega) + \mathrm{Im}^2\psi(i\omega), \quad \mathrm{tg}\,\theta = \frac{\mathrm{Im}\,\psi(i\omega)}{\mathrm{Re}\,\psi(i\omega)}$

The frequency functions $S(\omega)$ and $\theta(\omega)$ are the amplitude and phase spectra of the curve $\psi(t)$ respectively.

To continue the analysis it is necessary to assign a concrete form to the conversion curve. In many instances the $\varphi(t)$ curve can be approximated by means of a step function (curve 1 in Fig. 1):

$$\varphi = 0, \quad 0 < t < t_H; \quad \varphi = 1, \quad t \geqslant t_H$$

This means that the fuel is converted instantaneously into gas t_H seconds after admission into the combustion chamber. For such a curve:

$$S(\omega) = 1, \qquad \theta = -\omega t_H$$

Approximating $\varphi(t)$ with function 2 (Fig. 1) we get

$$S^2(\omega) = \left(\frac{\sin(\omega t_2/2)}{\omega t_2/2}\right)^2, \qquad \theta = -\omega\left(t_1 + \frac{t_2}{2}\right)$$

For curve 2, as for any symmetrical $\psi(t)$ curve, where t_H is the abscissa of the axis of symmetry.

Let us consider in greater detail the case of a step conversion curve. In this case, on introducing the quantity l/a as a time scale, we shall have instead of (4.1) and (4.2):

$$h_\Phi = \sqrt{\frac{1}{\chi^2 + \alpha^2\tau_H^2} - h_H^2\,\mathrm{tg}^2\,\alpha} \tag{4.3}$$

$$\operatorname{tg} \alpha \tau_n = \frac{h_\phi \alpha \tau_\kappa - \chi h_B \operatorname{tg} \alpha}{-\chi h_\phi + \alpha \tau_\kappa h_B \operatorname{tg} \alpha} \tag{4.4}$$

$$\operatorname{sign} \sin \alpha \tau_n = \operatorname{sign} (h_\phi \alpha \tau_\kappa - \chi h_B \operatorname{tg} \alpha)$$

$$\left(\chi = \frac{\gamma + 1}{2\gamma}, \quad \alpha = \frac{\omega l}{a}, \quad \tau_n = \frac{t_n l}{a}, \quad \tau_\kappa = \frac{t_\kappa l}{a} \right)$$

As follows from (4.3) and (4.4), the stability of the operating regime of an LRE is determined by the following four dimensionless parameters:

$$\tau = \frac{t_n l}{a}, \quad \tau_\kappa = \frac{t_\kappa l}{a}, \quad h_\phi = \frac{2 \Delta p_\phi}{p_{\kappa 0}}, \quad h_B = \frac{\rho v_0 a}{p_{\kappa 0}}$$

where τ_n is the ratio of the time lag to the time of passage of a sound wave through the length of the pipeline l/a; τ_k is the ratio of the time the gases stay in the chamber (reduced γ times) to l/a; h_ϕ is double the ratio of the pressure drop at the spray head to the chamber pressure and h_B is the impact pressure, due to instantaneous total retardation of the liquid, to the chamber pressure. $h = h_B / h_\phi = \rho v_0 a / 2 \Delta p_\phi$, would be a more graphic hydraulic parameter, but in the analysis of stability it is more convenient to use the parameter h_B .

Another parameter is the quantity $\chi = (\gamma + 1)/2\gamma$, however, it is sufficiently correct to put it equal to unity.

In controlling the thrust of an LRE by reducing the pressure in the fuel tanks, the parameters τ_k and h_B remain constant, the parameter h_ϕ is reduced in proportion to the pressure in the chamber (and the thrust); the variation in τ_n may be very diverse and depends on the type of fuel and the form of atomisation. It is possible to reckon that for the majority of engines the control time lag increases with the decrease in thrust. Thus, it is expedient to plot the limits of stability in the plane h_ϕ , τ_n.

Since h_ϕ is a real quantity, equation (4.3) gives the limitations on the oscillation frequency: the possible frequencies are either close to $\alpha = 0$ or close to $\alpha = n\pi \, (n = 1, 2, \ldots)$. For these frequencies the reactive resistance of the column of liquid (the quantity $h_B \tan \alpha$) is close to zero. It is clear that $\alpha = n\pi$ corresponds to the natural frequencies of the acoustical oscillations of the liquid in the pipeline, i.e. to the frequencies

$$f = n \frac{a}{2l} \text{ (c/s)}$$

The value n = 1 corresponds to the fundamental, n = 2,3,... to the second, third, etc. harmonics. For low frequencies the function

$$h_{\text{в}} \operatorname{tg} \alpha \approx \omega t_l \quad (t_l = \rho v_0 l / p_{\text{н0}})$$

and the inertia of the liquid are defined by a single time constant. With sufficient accuracy

$$\operatorname{tg} \alpha \approx \omega l / a \quad \overset{\textit{for}}{\text{при}} \ \alpha \leqslant {}^1/_6 \pi$$

accordingly, the compressibility of the liquid can be neglected up to

$$f \leqslant \frac{a}{12l} \text{ c/s.}$$

By determining h_ϕ for a given value of α from eq. (4.3), we can find the value of $\tau_\text{в}$ at the limit of stability from (4.4). The quantity $\alpha \tau_n$ is determined correct to $2k\pi$ $(k = 1, 2, \ldots)$, i.e. it is possible to have oscillations with the same dimensionless frequency α for several values of τ_n, differing from one another by $2\pi / \alpha$.

Fig. 2 shows the complete limit of stability in the plane of the parameters h_ϕ, $\tau_\text{в}$ for $h_\text{в} = 2$ and $\tau_\text{н} = 1$ and for the values n = 1,2 and k = 1,2. The region of stability lies to the right and is bounded by a smooth upper curve (low frequencies) and a saw-toothed curve, for which the oscillation frequencies are close to the natural frequencies of the pipeline. In this particular case the "teeth", corresponding to the values n = 1, k > 1 and n = 2, k > 2, do not change the region of stability.

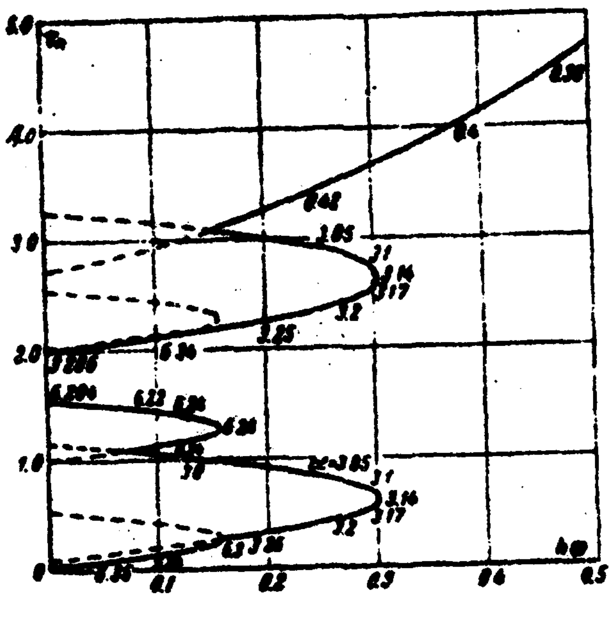
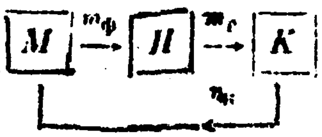


Fig. 2

In the case of this particular engine, when the pressure in the fuel tanks is reduced, the working point (h_ϕ, τ_n) is displaced upwards and to the left. Accordingly, as the thrust of the LRE is reduced, stable and unstable regimes may alternate. An increase in will not take the engine out of the region of stability, if the working point crosses the upper smooth boundary, which retreats to infinity as h_ϕ approaches 1 (more accurately χ).

The effect of other parameters on the region of stability can be conveniently traced by considering separately the upper boundary and the tooth with $k = 0$, $n = 1$. The effect of and h_s may be seen in Figs. 2 and 3. An increase in h_s leads to a narrowing of the teeth, without affecting their position and length, and sharply displaces the upper boundary, at the same time increasing the number of teeth (value of k). The effect of l/a is illustrated in the plane h_ϕ, t_n in Fig. 4. The effect of l/a on the operating stability of an LRE may be very diverse. An increase in l/a leads to a reduction in the region of stability due to elongation of the teeth and powerfully broadens it by dispacing the upper boundary. When l/a is increased, the points of the teeth ascend, but at large values of l/a it is necessary to take into account teeth corresponding to harmonics of the fundamental. It is interesting to note that, as l/a varies, for $n = 1$ the points of the teeth are displaced along a curve corresponding to $l = 0$ (broken line in Fig. 4). This case ($l = 0$, step conversion curve) has already been discussed by M.S.Patanson.

As the preceding discussion shows, it is possible to represent an LRE by means of the following block diagram:



where the links M, P and K denote respectively the fuel supply system, the conversion process and the combustion chamber as a gas capacity. The characteristic equation of such a system can be written out at once from the known transfer functions of the links:

$$K(q)\,M(q)\,\Pi(q) = 1$$

For the propulsion unit discussed above equations (3.1), (3.2) and (3.3) give

$$M(q) = -\frac{1}{h_\phi + h_s\, th\,(ql \cdot a)}$$

$$\Pi(q) = \psi(q), \quad K(q) = \frac{1}{\chi + q t_k}$$

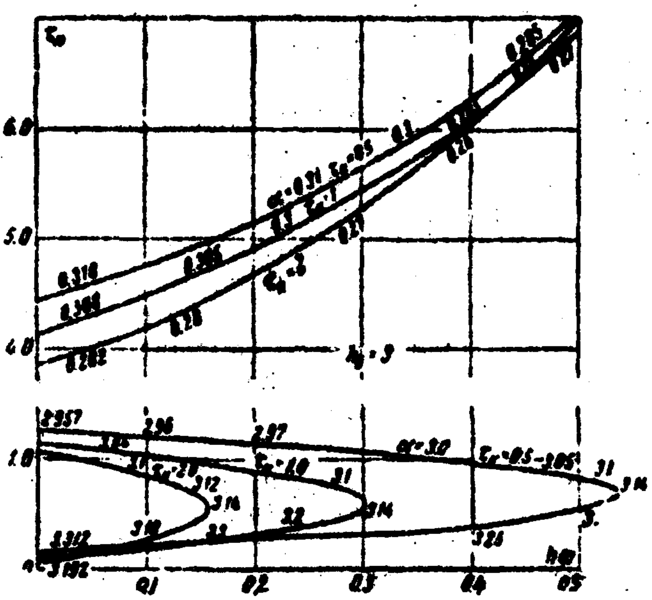


Fig. 3

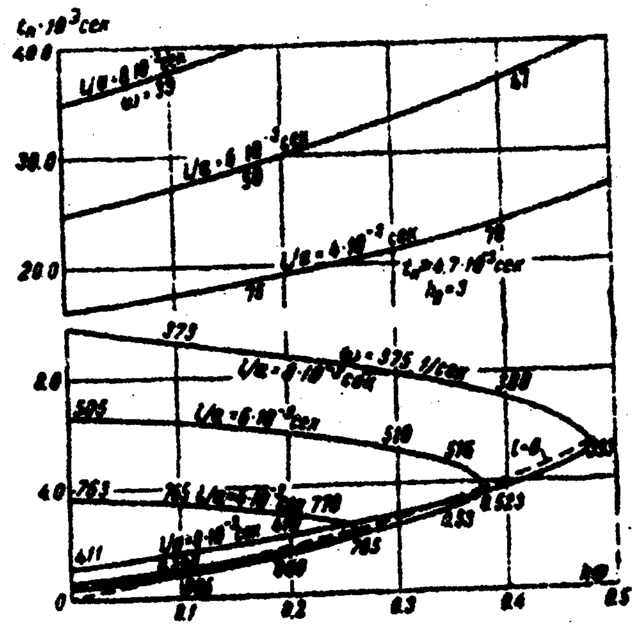


Fig. 4
(cek means sec.)