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# Measurement of Aircraft Speed and Altitude 

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## PREFACE

The problem of devising instrument systems for the accurate measurement of the speed and altitude of aircraft has been the subject of a great many research invest: :. cions during the past 50 years. The greater part of this research has been-performed by a variety of organizations in Great Britain, Germany, and the United States. In the United States, investigations have been conducted by government agencies (National Aeronautics and Space Administration (NASA), its predecessor, the National Advisory Committee on Aeronautics (NACA), the Federal Aviation Administration (FAA), the National Bureau of Standards (NBS), the U.S. Air Force, and the U.S. Navy), by aeronautical schools in the universities, and by aircraft manufacturers, instrument manufacturers, and air carriers. Studies relating to one area of the altitudemeasuring problem (the vertical separation of aircraft) have been promoted by international orgaizations such as the International Civil Aviation Organization (ICAO) and L.ie International Air Transport Association (IATA).

The results of this research have been published in several hundred reports, each of which deals with only one, or a few, of the many facets of the speed- and altitude-measuring problem. In this text, the information in these reports has been combined and is presented in a condensed, organized form. In the presentation of the material on some of the topics, only enough data have been included to define a concept or illustrate a point. For a more detailed discussion of these subjects, the readex is referred to the reference reports which are listed at the end of each chapter.

The scales of the instruments described in this text and all of the test data derived from their calibration and operational use are in U.S. Customary Units. Accordingly, it appeared inappropriats ir this text to adhere to the prevailing practice of giving test values in the International System of Units (SI) as well as in the U.S. Customary system. For those readers having a need to convert any of the data to metric units, a table of conversion factors and metric equivalents is included in appendix A. Also included in appendix $A$ are tables of airspeed and altitude in SI Units.

In writing this book, I received considerable help and support from many of my former associates at NASA Latigley Research Center. I would like to acknowledge this assistance and to thank, in particular, the following:

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## SYMBOLS AND ABBREVIATIONS

| a | speed of sound |
| :---: | :---: |
| $\mathbf{a}_{\mathbf{v}}$ | vertical cceleration |
| $a_{x}$ | longitudinal acceleration |
| $\mathrm{a}_{\mathbf{z}}$ | normal acceleration |
| b | wing span of airplane |
| $b^{\prime}$ | wing span of airplane image on camera film |
| c | wing chord |
| C | total volume of instrument chambers |
| $c_{L}$ | lift coefficient |
| CL | confidence level |
| d, D | diameter |
| $E$ | elevation of airport |
| f | compressibility factor; focal length of camera lens |
| $g$ | acceleration of gravity |
| h | height of aircraft above camera |
| H | pressure altitude, geopotential feet |
| $\mathrm{H}^{\prime}$ | indicated (or measured) pressure altitude (barometric scale set to QFE) |
| $\mathrm{H}_{\mathbf{i}}$ | indicated altitude (barometric scale set to QNH ) |
| $\Delta H$ | altitude error, $\mathrm{H}^{\prime}$ - H |
| K | recovery factor of temperature probe |
| $l$ | length of aircraft |
| $2^{\prime \prime}$ | length of aircraft image on camera film |
| L | length of pressure tubing |
| M | free-stream Mach number |
| $M^{\prime}$ | indicated (or measuriei) Mach number |

```
\begin{tabular}{|c|c|}
\hline \(\Delta M\) & Mach number error, \(M^{\prime}\) - M \\
\hline \(\mathrm{N}_{\mathrm{Re}}\) & Reynolds number, \(\rho \frac{V \ell}{\mu}\), where \(\ell\) is a linear dimension \\
\hline P & free-stream static pressure \\
\hline \(\mathrm{p}^{\prime}\) & measured static pressure \\
\hline \(\Delta p\) & static-pressure error or position error, p' - p; pressure drop in tubing \\
\hline \(\delta \mathrm{p}\) & static-pressure increment \\
\hline \(\mathbf{P a}_{\mathbf{a}}\) & pressure at altitude \\
\hline \(\mathbf{P}_{\mathbf{c}}\) & cabin or compartment pressure \\
\hline \(p_{i}\) & pressure inside instrument \\
\hline \(\mathrm{P}_{2}\) & local static pressure \\
\hline \(\Delta p_{l}\) & pressure error due to leak \\
\hline \(P_{t}\) & free-stream total pressure for subsonic flow and total pressure behind normal shock wave for supersonic flow \\
\hline \(p_{t}^{\prime}\) & measured total pressure \\
\hline \(\Delta p_{t}\) & total pressure error, \(p_{t}^{\prime}-p_{t}\); total pressure loss through normal shock wave \\
\hline \(\mathrm{P}_{\mathrm{T}}\) & test pressure \\
\hline q & dynamic pressure \\
\hline \(q_{c}\) & free-stream impact pressure \\
\hline \(\mathrm{q}_{\mathrm{c}}^{\prime}\) & measured impact pressure \\
\hline QFE & stardard altimeter setting (barometric scale set to \(29.92 \mathrm{in} . \mathrm{Hg}\) ) \\
\hline QNE & barometric scale setting for altimeter to indicate zero at airport elevation \\
\hline QNH & barometric scale setting for altimeter to indicate elevation of airport \\
\hline R & gas constant for air, \(f t-1 \mathrm{~b} / \mathrm{slug}-\mathrm{O}_{\mathrm{R}}\) \\
\hline \(\overline{\mathbf{R}}\) & gas constant for air, \(f t-1 \mathrm{~b} / \mathrm{lb}-\mathrm{mol}-{ }^{\circ} \mathrm{R}\) \\
\hline \(\mathbf{R}^{*}\) & universal gas constant \\
\hline
\end{tabular}
```

$u$ horizontal component of induced velocity
$v$ vertical velocity
Ve equivalent airspeed
$-V_{i} \quad$ indicated airspeed (corrected for instrument scale error)
$\Delta z \quad$ vertical displacement of aircraft image from center line of film Erame
$\lambda$ pressure lag constant
$\lambda_{l} \quad$ pressure lag of leak
$\mu$
p
$\bar{\rho}$
$\sigma$
$T$
$\phi$
Subscripts:
1 initial
a altitude: actual
c critical; computed; camera
2 local: leak
m - measured; midpoint
o sea level
s standard
Abbreviations:

AAEE Aeroplane and Armament Experimental Establishment (British)
AFCRC Air Force Cambridge Research Center
afmic Air Force Missile Test Center
ANA Air Force-Navy Aeronautical
A.R.C. Aeronautical Research Comittee (British)

FAA Federal Aviation Administration
NACA National Advisory Committee for Aeronautics (predecessor to NASA)
NAES Naval Air Experimental Station
NASA National Aeronautics and Space Administration
NBS National Bureau of Standards
NOAA National Oceanic and Atmospheric Administration
R.A.E. Royal Aircraft Establishment (British)
WADC Wright Air Development Center (USAF)
NACA and NASA Reports:
ARR Advanced Restricted Report
RM
5P Special Publication
TM Technical Memorandum
TN Technical Note
2P Technical Paper
TR or Rep. Technical Report
WR Wartime Report

## CHAPTER I

## INTRODUCTION

Accurate measurements of speed and altitude are essential to the safe ard efficient operation of aircraft. Accurate speed measurements, for example, are needed to avoid loss of control at low speeds (stall condition) and to prevent exceedance of the aerodynamic and structural limitations of the aircraft at high sperds, whereas accurate altitude measurements are needed to insure clearance of terrain obstacles and to maintain prescribed vertical separation minina along the alrways.

The instruments that are used to measure speed and altitude include the altimeter, the airspeed indicator, the true-airspeed indicator, the Machmeter. and the rate-of-climb (or vertical-speed) indicator. All these instruments are actuated by pressures, while one, the true-airspeed indicaior, is actuated by air temperature as well.

Two basic pressures, static pressure and total pressure, are used to ac=uate the instruments. The static premsure is the atmospheric pressure at the flight level of the aircraft, while the total pressure is the sun of the stazic pressure and the impact pressure, which is the pressure developed by the for-ard speed of the aircraft. The relation of the three pressures can thus be expressed by the following squation:

$$
\begin{equation*}
p_{t}=p+q_{c} \tag{i,1}
\end{equation*}
$$

where $p_{t}$ is the total pressure, $p$ the static pressure, and $q_{c}$ the impazt pressure.

The static pressure is used to actuate both the altimeter and the rate-afclimb indicator. Although this pressure varies from day to day, the decrease in static pressure with height is generally corit: .uous at any one time and plac:. Accordingly, a pressure-height relation based on average atmospheric conditions has been adopted as a standard (see "standard atmosphere" in chapter III). Measuremer,ts of static pressure are then used to provide indications of heient in terms of pressure altitude (chapter XII) and indications of vertical sjees in terms of rate of change in the pressure altitude.

For the three forward-speed indicators, impact pressure is derived i: $\equiv$ differential pressure from measurements of total pressure and static pressur: in accordance with equation (1.1). The airspeed indicator $i=$ actiated sol:z $\because$ F. impact pressure and is calibrated to indicate true airspeed $3 t$ sea-level density in the standard atmosphere; at altitude, however, the indicated airsmed is lower than the true airspeed (chapter III). The true-airspeed indicator. or: the other hand, combines the measurement of impact pressure with measuremen: $n$ : static pressure and temperature to ind -ate true airspeed indevendent of al=itude. The Machmeter (named for the Austrian physicist. Ernst :ach) combine =
measurements of impact pressure and static pressure to provide indications of true airspeed as a fraction or multiple of the speed of sound (sonic speed).

The airspeed indicator, true-airspeed indicator, and Machmeter measure speed with respect to the air mass. Since the air mass can move with respect to the ground, the measurement of ground speed, the speed of basic importance to air navigation, must be derived from inputs from ground navigational aids.

The pressures and temperatures that actuate the instruments are derived from pressure and temperature sensors located at positions on the aircraft which are remote from the instruments. The problem of designing and locating the sensors for the accurate measurement of pressure and temperature is complicated by many factors. As a consequence, the pressures and temperatures registered by the sensors can be in error by amounts which, in some cases, produce sizable errors in the indications of the instruments. The indications of an instrument can also be in error iecause of imperfections in the instrument itself. Additional errors may be introduced because of a time lag in the transmission of the pressures to the instruments whenever the pressure at the pressure source is changing rapidly, as in the case of high-speed climbs or dives.

In the following chapter, a typical instrument system is described, and the various errors associated with the system are defined. In succeeding chapters, the errors relating to the design of the total- and static-pressure sensors and to the location of the sensors on an aircraft are discussed, and the flight calibration methods for determining the pressure errors are described. Information is then presented on ways of applying corrections for these errors and on methods of keeping the other errors within acceptable limits.

## CHAPTER II

## INSTRUMENT SYSTEMS AND ERRORS

The five types of instruments which are used to measure speed and altitude and the "pressurp" and temperature' sensors which sctuate-the-instruments-were-............... described in chapter I. This chapter describes a typical instrument system (instruments and sensors) and the errors associated with the various parts of the system.

As noted in the first chapter, the two basic pressures that are exployed in the measurement of speed and altitude are total pressure and static pressure. Total pressure is sensed by an opening in a forward-facing tube called a totalpressure tube or pitot tube (named for the French physicist, Henri Pitot). The static pressure is sensed by orifices in the side of another type of tube, called a static-pressure tube, or by a set of holes in the side of an aircraft fuselage, called fuselage vents or static ports. Since the pitot tube and the staticpressure tube can be combined into a single tube, two types of pressure-measuring installations are possible: a pitot-static tube installation or a pitot tube in combination with a fuselage vent system. Diagrams of a pitot tube, a staticpressure tube, a pitot-static tube, and a pitot-tube/fuselage-vent installation are shown in figure 2.1.

The pressures that are sensed by the pitot tube and the static-pressure tube (oir fuselage vents) are conveyed through tubing to pressure-sensing elements which are generally in the form of capsules, diaphragms, or bellows. All of these types of sensing elements are used in the electrical instrument systems to be described in chapter XI. The capsule-type sensing element is used in simpler, mechanical instruments described in this chapter.

The pressure capsules are formed by joining together two corrugated diaphragms which are about 2 in . in diameter. Two types of capsule are used in aircraft instruments: one for measuring absolute pressure and the other for measuring differential pressure. The absolute-pressurn (or aneroid) capsule is evacuated and sealed, while the differential-pressure capsule has an opening that is connected to a pressure source. As indicated in figure 2.2, the absolute-pressure capsule reacts to the pressure inside the instrument case, while the differential-pressure capsule reacts to the difference between the pressure inside the capsule and the pressure in the instrument case. Thus, for both types of capsule, the instrument case is used as a pressure chamber to form one element of the pressure-measuring system.

Also shown in figure 2.2 are the dixections of the deflection of the capsules for a given pressure change. These deflections, which are very small, are amplified through a system of gears and levers (gear train) to rotate a pointer in front of the scale on the dial of the instrument.

The routing of the pressure tubing from a total-pressure tube, staticpressure tiabe, and temperature probe to a set of the five types of instruments is shown in figure 2.3. The static-pressure tube is connected to all the instruments, whereas the total-pressure tube is connected only to those instru-
ments that measure forward speed. The temperature probe, which is connected to the true-airspeed indicator, is a type used with liquid-pressure themometers. The pressure tubing from the total-pressure and static-pressure tubes is generally about 0.2 to 0.3 in . in inside diameter, whereas the capillary tubing from the temperature probe is about 0.01 to 0.02 in .

The pressure-sensing element of the altimeter (fig. 2.3) is an aneroid capsule that expands_as the static_pressure_inside_the instrument case decreases with increasing altitude. (See fig. 2.2(a).)

In the rate-of-climb indicator, the static-pressure tube is connected to a differential-pressure capsule and to a capillary tube that opens into the instrument case. With a change in static pressure, the simultaneous flow of air into, or out of, the capsule and the capillary tube is adjusted (by the size of the capillary leak) so that the capsule deflects in terms of a race of change of pressure, which is calibrated to yield a measure of vertical speed.

The pressure-sensing element of the airspeed indicator is a differentialpressure capsule that expands as the total pressure increases. Since the pressure inside the case is the static pressure, the instrument performs a mechanical subtraction of total and static pressures to yield a measure of impact pressure in accordance with equation (1.1). (See fig. 2.2(b).)

The Machmeter contains both an aneroid capsule and a differential-pressure capsule to provide measures of static pressure and impact pressure. The deflections of the two capsules are coupled to yield, mechanically, the ratio of impact pressure to static pressure $\left(q_{c} / p\right)$ which, as discussed in the next chapter, is a function of Mach number.

The true-airspeed indicator contains (1) two differential-pressure capsules to provide measures of impact pressure and air temperature and (2) an aneroid capsule to provide a measure of static pressure. Since the true airspeed is a function of dynamic pressure, derived from the measured impact pressure and static pressure as discussed in chapter $V$, and the air density, derived from static pressure and temperature, the deflections of the three capsules can be coupled to yield a measure of true airspeed.

Also shown in figure 2.3 are the pressures ( $p_{t}^{\prime}$ and $p^{\prime}$ ) sensed by the total- and static-pressure tubes and the temperature ( $T^{\prime}$ ) sensed by the temperature probe. For any one flight condition, the differences between $p_{t}^{\prime}$ and the free-stream total pressure $P_{t}$ and between $T^{\prime}$ and the free-air temperature $T$ depend primarily on the design characteristics of the pitot tube and the temperature probe. The difference between $p^{\prime}$ and the free-stream static pressure $p$ depends on both the design of the static-pressure tube and on the location of the tube in the pressure field surrounding the aircraft (chapter $V$ ).

The difference between $p_{t}^{\prime}$ and $p_{t}$. called the total-pressure error $\lambda_{p_{t}}$. is defined by

$$
\begin{equation*}
\Delta p_{t}=p_{t}^{\prime}-p_{t} \tag{2.1}
\end{equation*}
$$

Similarly, the difference between $p^{\prime}$ and $p$, the static-prcssure error $\Delta p$, is defined by

$$
\begin{equation*}
\Delta p=p^{\prime}-p \tag{2.2}
\end{equation*}
$$

The difference between $T^{\prime}$ and $T$, the temperature error $\Delta T$, is defined by

$$
\begin{equation*}
\Delta T=T^{\prime}-T \tag{2.3}
\end{equation*}
$$

As noted in the previous chapter, the indications of the instruments may be affected by errors due to the time lag in the transmission of the pressures and to imperfections in the instrument mechanism. The ervors associated with the instrument mechanism depend on (1) the elastic properties of the pressure capsule (scale error, hysteresis, and drift) and (2) the effects of temperature, acceleration, and friction on the linkage mechanism. The scale error is the difference, for a given applied pressure, between the value indicated by the instrument and the correct value corresponding to the applied pressure. From the foregoing discussion, the overall error of an instrument system is a combination of

## 1. Total- and static-pressure errors of the pitot-static installation and the temperature error of the temperature probe

2. Errors due to time lag in the transmission of the pressures
3. Errors relating to the operation of the instrument mechanism

The magnitude and nature of the errors vary widely, so that different means are used to minimize different errors. The total-pressure, static-pressure, and temperature errors, for example, are systematic; that is, for a given flight condition, the errors are essentially repeatable and hence can be determined by calibration. The static-pressure error can be quite large, whereas the totalpressure error is generally negligible (chapters IV and VII). The magnitude of the temperature error, expressed in terms of a recovery factor, is discussed in chapter III.

The errors due to pressure lag are transitory and vary with the rate of climb or descent of the aircraft. For a given rate of change of altitude, the magnitude of the lag error depends primarily on the length and diameter of the pressure tubing and on the volume of the instruments connected to the tubing. Accordingly, the lag errors of a particular pressure system are kept within acceptable limits by proper design of the system (chapter X ).

Of the various instrument errors, the scale error is systematic, while the other errors are generallv random. The scale error is usually the largest of the instrument errors and can be determined by laboratory calibration. The remaining erzors are kept within acceptable limits by careful design, construction, and adjustment of the instrument mechanism.

The instrument errors and the errors of the pitot-static installation are required to meet specified tolerances (allowable errors). The tolerances for the instrument errors can be combined to yield an "instrument ersor," and this error can be combined with the tolerance for the static-pressure erzor to yield an "instrument system error" (chapter XII). Mathematical procedures for combining the tolerances for the irstrument errors and the static-pressure error are described in references 1 through 4.

Since the scale error of the instrument and the static-pressure error of the installation can be determined by calibration, corrections for these two errors can be applied. With mechanical instrument systems, corrections for these errors are applied by means of correction charts, or cards, that are supplied to the pilot. With electrical instruments, the corrections are applied automatically by some form of computer (chapter XI). For systems in which corrections for the two errors are applied, the instrument system error is usually much lower than the error derived from a sumation of the instrument and staticpressure error tolerances. The laboratory procedures for determining the scale error are described in chapter XI and the flight procedures for determining the static-pressure error are described in chapter IX. Since the procedures for determining the scale error are well established, this text emphasizes flight procedures by which static-pressure installations are calibrated.

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(a) Pitot tube.

(b) Static-pressure tube.

(c) Pitot-static tube.

(d) Pitot-tube/fuselage-vent installation.

Figure 2.1.- Diagrams of pressure tubes and a pitot-tube/fuselage-vent installation.

(a) Aneroid capsule. For a decrease in static pressure inside the instrument case, the capsule deflects in the direction indicated by the large arrow.

(b) Differential-pressure capsule. For an increase in toial pressure inside the capsule, the capsule deflects in the direction indicated by the large arrow.

Figure 2.2.- Aircraft instruments with the two types of pressure capsule.


Figure 2.3.- Diagram of routing of pressure tubing from pressure and temperature sensors to five types of instruments measuring altitude and speed.

## CHAPTER III

STANDARD ATMOSPHERE AND EQUATIONS FOR AIRSPEED,
MACH NUMBER, AND TRUE AIRSPEED
As noted in chapter 1 , the pressure altimeter is calibrated in accordance with the pressure-height relation in the standard atmosphere. In the first section of this chapter, the equations and the atmospheric properties on which the standard atmosphere is based are presented. In succeeding sections, the equations relating (1) impact pressure to airspeed, (2) impact pressure and static pressure to Mach number, and (3) impact pressure, static pressure, and temperature to true airspeed are described. These equations are of fundamental importance to both the laboratory calibrations of the instruments and the deduction of flight parameters from measured pressures and temperature.

In the following sections, reference is made to tables of airspeed and altitude in U.S. Customary Units (appendix A). As noted in the Preface, tables of the same quantities in the International (metric) System of Units (SI) are also included in appendix $A$.

## Standard Atmosphere

The so-called standard atmosphere is a representation of the atmosphere based on average conditions at a latitude of $45^{\circ}$ north. A number of standard atmospheres have been developed through the years (ref.. . through 13). Each new standard has differed from the previous standard because of the adoption of revised values of some of the physical constants on which the atmospheres are based or because of the acquisition of new information on some of the atmospheric properties (particularly at the higher altitudes). All the atmospheres are based on mean values of pressure, temperature, density, and he acceleration cf gravity at sea level and on a mean value of the variation of temperature with height.

In the construction of a standard atmosphere on the basis of these mean values, assumptions are made that

1. The air is a dry, perfect gas that obeys the laws of Charles and soyle,

$$
\begin{equation*}
\rho=\rho_{0} \frac{\mathrm{pT}_{0}}{\mathrm{P}_{0}{ }^{T}} \tag{3.1}
\end{equation*}
$$

and thus the perfect gas law,

$$
\begin{equation*}
\rho=\frac{\mathrm{PW}_{\mathrm{m}}}{R^{*} T}=\frac{P}{R T} \tag{3.2}
\end{equation*}
$$

2. The atmosphere is in hydrostatic equilibrium, so that the relation between the pressure $p$ and the geometric height $z$ can be expressed by the equations,

$$
\begin{equation*}
d p=-g \rho d z=-\bar{\rho} d z \tag{3.3}
\end{equation*}
$$

$1 p=-g \frac{p}{R T} d z=-\frac{p}{R T} d z$
where $\rho$ (or $\bar{\rho}$ ) is the density, $p$ the pressure, $T$ the temperature, $g$ the acceleration of gravity, $W_{m}$ the mean molecular weight of air, $R^{*}$ the universal gas constant, and $R$ (or $\bar{R}$ ) the gas constant for air. The two symbols given for density and the gas constant for air denote differences in units which are found in some of the reference reports. For the symbols given in this text, the unit of $\rho$ is slugs per cubic foot and the unit of $\bar{\rho}$ is pounds per cubic foot. The value of $R$ is $1716.5 \mathrm{ft}-1 \mathrm{~b} / \mathrm{slug}-O_{R}$ and the value of $\bar{R}$ is $53.352 \mathrm{ft}-1 \mathrm{~b} /\left(\mathrm{lb}\right.$ mol) ${ }^{\circ} \mathrm{R}$.

The earlier atmospheres (refs. 1 through 5) were based on the assumption that the acceleration of gravity remained constant at its sea-level value $g_{0}$. For the later atmospheres (refs. 6 through 13), the decrease of $g$ with heicht was taken into account by the formation of a new height parameter called geopotential altitude $H$. The relation between $H$ and $Z$ is given by

$$
\begin{equation*}
\mathrm{dz}=\frac{g_{0}}{g} \mathrm{dH} \tag{3.5}
\end{equation*}
$$

The value of $4 / H$ varies uniformly from 1.0 at sea level to 1.0048 at 100000 ft : The relation between $P$ and $H$ is given by the following equations:

$$
\begin{equation*}
d p=-g_{O} \rho d H=-\frac{g_{O}}{g} \bar{\rho} d H \tag{3.6}
\end{equation*}
$$

or

$$
\begin{equation*}
d p=-g_{0} \frac{p}{R T} d H=-\frac{g_{0}}{g} \frac{p}{R T} d H \tag{3.7}
\end{equation*}
$$

Pressure-altitude tables for the calibration of altimeters in terms of ceopotential feet are given in references 6 through 13. All these tables are the same for altitudes up to 65800 ft , and the tables of references 11 through 13 are the same for altitudes up to 100000 ft . The tables of reference 11 (the U.S. Standard Atmosphere, 1962) have been selected for presentation in this =ext because the pressures and altitudes are given in both U.S. Customary Units (tie system of units used in this text) and SI Units. The pressure-altitude tables of references 12 and 13 are in SI Units.

Thu sea-level values of pressure, temperature, density, and the acceleration cf gravity for the atmosphere of reference 11 are as follows:

$$
P_{0}=29.9213 \mathrm{in} . \mathrm{Hg} \text { or } 2116.22 \mathrm{lb} / \mathrm{ft}^{2}
$$

$$
t_{0}=59.0^{\circ} \mathrm{F} \text { or } 15.0^{\circ} \mathrm{C}
$$

$$
T_{0}=518.67^{\circ} \mathrm{R} \text { or } 288.15^{\circ} \mathrm{K}
$$

$$
\rho_{0}=0.0023769 \text { slug/ft }{ }^{3}
$$

$$
\bar{\rho}_{0}=0.076474 \mathrm{lb} / f t^{3}
$$

$$
\mathrm{g}_{0}=32.1741 \mathrm{ft} / \mathrm{sec}^{2}
$$

The temperature gradient or lapse rate $\mathrm{dT} / \mathrm{dH}$ is $-0.00356616^{\circ} \mathrm{F}$ per geopotential foot from sea level to 3609 C geopotential feet. From this altitude to 65800 ft , the temperature is constant at $-69.7^{\circ} \mathrm{F}$ and then increases to $-50.836^{\circ} \mathrm{F}$ at 100000 ft .

Tables of pressure, density, temperature, coefficient of viscosity, speed of sound, and the acceleration of gravity are given in appendix $A$ for geopotential altitudes up to 100000 ft :

In table Al, values of pressure are given in inches of mercury ( $0^{\circ} \mathrm{C}$ ) (to correspond with the scales of mercury-in-glass barometers used for calibration of altimeters): in table A2, the values are given in pounds per square foot.

In table A3, values of air density are given in pounds per cubic foot. Values in units of slugs per cubic foot can be derived by dividing the values of table $A 3$ by the acceleration of gravity.

In tables A4 and A5, values of free-air temperature are given in degrees Fahrenheit and Celsius. Values of absolute temperature in degrees Rankine and Kelvin can be derived by means of the following equations:

$$
\begin{align*}
& T\left({ }^{\circ} K\right)=t\left({ }^{\circ} F\right)+459.67  \tag{3.8}\\
& T\left({ }^{\circ} K\right)=t\left({ }^{\circ} \mathrm{C}\right)+273.15 \tag{3.9}
\end{align*}
$$

In table A6, values of the coefficient of viscosity are given in pound-seconds per square foot. Values in pounds per foot-second the unit used in ref. 11) can be derived by multiplying the values in table ab by the acceleration of gravity.

In table A7, values of the speed of solund are given in miles per hour and knots.

In table A8, values of the acceleration of gravity are give. in feet per second squared.

## Airspeed Equations

In-incompressible flow, the pressure developed by the forward motion of a bocy is called the dynamic pressure $g$ : which is related to the true airspeed $V$ by the equation,
*

$$
\begin{equation*}
q=\frac{1}{2} \rho v^{2} \tag{3.10}
\end{equation*}
$$

where $\rho$ is the density of the air and $V$ is the speed of the body relative to the air. Air, however, is compressible, and when airspeed is measured with a pitot-static tube, the air is compressed as it is brought to a stop in the pitot tube. As a consequence of this compression, the measured impact pressure $q_{c}$ (eq. (1.1)) is higher than the dynamic pressure of equation (3.10). The effects of compressibility can be taken into account by determining the relation between the true airspeed $V$ and the impact pressure $q_{c}$ by means of the following equacions:

1. The equation for the total pressure (eq. (1.1)),

$$
\begin{equation*}
p_{t}=q_{c}+p \tag{1.1}
\end{equation*}
$$

2. The equation for the speed of sound a di air,

$$
\begin{equation*}
a=\sqrt{\frac{Y p}{\rho}} \tag{3.11}
\end{equation*}
$$

where $Y$ is the ratio of the specific heats of air.
3. Bernoulli's formula for total pressure in compressible flow,

$$
\begin{equation*}
p_{t}=p\left(1+\frac{\gamma-1}{2 \gamma} \frac{p}{p} v^{2}\right)^{\frac{\gamma}{\gamma-1}} \tag{3.12}
\end{equation*}
$$

4. The formula for total pressure behind a normal shock wave (for $v \geqq a$ ).

$$
\begin{equation*}
p_{t}=\frac{1+\gamma}{2 \gamma} \rho v^{2}\left[\frac{\frac{(\gamma+1)^{2}}{\gamma} \frac{\rho}{p} v^{2}}{\frac{4 \rho}{p} v^{2}-2(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \tag{3.13}
\end{equation*}
$$

$$
(v \geqq a)
$$

With the substitution of equation (1,1) in equation (3.12) and equations (1.1) and (3.11) in equation (3.13), $V$ can be expressed in terms of $q_{c}$ by the following equa¿ions:

$$
\begin{equation*}
q_{c}=p\left[\left(1+\frac{\gamma-1}{2 \gamma} \frac{p}{p} v^{2}\right)^{\frac{\gamma}{\gamma-1}}-1\right] \tag{3.14}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{C}=\frac{1+\gamma}{2}\left(\frac{v}{a}\right)^{2} p\left[\frac{(y+1)^{2}}{4 \gamma-2(\gamma-1)\left(\frac{a}{v}\right)^{2}}\right]^{\frac{1}{\gamma-1}}-p \quad(v \geqq a) \tag{2.15}
\end{equation*}
$$

For the calibration of airspeed indicators, the concept of zalibrated airspeed $V_{c}$ is introduced and, by definition, $V_{c}$ is made equal to $V$ at sea level for standard sea-level conditions. Thus, by substituting the standard sea-level values of $p, \rho$, and $a$ in equations (3.14) and (3.15), $v_{c}$ can be related to $q_{c}$ by the following equations:

$$
\begin{equation*}
q_{c}=p_{0}\left[\left(1+\frac{\gamma-1}{2 \gamma} \frac{\rho_{0}}{p_{0}} v_{c}\right)^{\frac{\gamma}{\gamma-1}}-1\right] \quad\left(v_{c} \leqq a_{0}\right) \tag{3.16}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{c}=\frac{1+\gamma}{2}\left(\frac{v_{c}}{a_{0}}\right)^{2} p_{0}\left[\frac{(\gamma+i)^{2}}{4 \gamma-2(\gamma-1)\left(\frac{a_{0}}{v_{c}}\right)^{2}}\right]^{\frac{1}{\gamma-1}}-p_{0} \quad\left(v_{c} \geqq a_{0}\right) \tag{3.17}
\end{equation*}
$$

Airspeed indicators are calibrated in accordance with equation (3.16) for subsonic speeds ( $V_{c} \leqq a_{0}$ ) and equation (3.17) for supersonic speeds ( $V_{c} \geqq a_{0}$ ). The sea-level values of pressure, density, and speed of sound used in these equations are those given in reference 11, namely,

$$
\begin{aligned}
& P_{0}=2116.22 \mathrm{lb} / f \mathrm{t}^{2} \\
& \rho_{0}=0.0023769 \mathrm{slug} / \mathrm{ft}^{3} \\
& a_{0}=1116.45 \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

Tie value that has been adopted for $\gamma$ is 1.4. Note, however, that at high altitudes, the value of $Y$ may vary slightly from 1.4 (refs. 11 and 14 ).

For subsonic speeds, the true airspeed $V$ can be deduced fro: the calibrated airspeed $V_{c}$ and the air density $\rho$ by means of the following equation which is derived by dividing equation (3.14) by equation (3.16):

$$
v=v_{c} \frac{f}{f_{0}} \sqrt{\frac{\rho_{0}}{\rho}}
$$

where $f$ is a compressibility factor defined $b_{i}$

$$
\begin{equation*}
f=\sqrt{\frac{Y}{y-1} \frac{p}{q_{c}}\left[\left(\frac{q_{c}}{p}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]} \tag{3.19}
\end{equation*}
$$

Values of $f$ and $f_{0}$ (the compressibility factor for standard sea-level conditions) are given in figure 3.1 for values of $q_{c} / p$ up $=0.893$ (the ratio for $M=1.0$ for which $V=a$ ). The value of $p$ for use in equation (3.18) can be determined from equation (3.1) and measured values of static pressure and air zemperature.

In aircraft structural design, use is made of an airspeed that equates the dynamic pressure at altitude $\left(q=\frac{1}{2} \rho V^{2}\right)$ to the dynamic pressure at sea level for standard sea-level density $\left(q=\frac{1}{2} \rho_{o} v_{e}^{2}\right)$. This airspeed $v_{e}$ is called the equivalent airspeed and is related to $V$ by the following esuation):

$$
\begin{equation*}
v_{\mathrm{e}}=v \sqrt{\frac{\rho}{\rho_{0}}} \tag{3.20}
\end{equation*}
$$

Another airspeed term, indicated airsgeed, is generally defined as the indication of an airspeed indicator uncorrected for instrument error and the error of the pitot-static installation. In this text, however, the indi=ated airspeed $v_{i}$ is defined as the airspeed indication correctec for instrument scaie error (chapter II). Thus, since the calibrated airspeed $V_{c}$ is the indication of an airspeed indicator corrected for both instraent scale error and static-pressure error, the difference between $\vartheta_{i}$ and $\because=$ is a measure of the static-pressure error.

To sumarize the relations between $V_{i}, V_{c}$, and $V$ in simple terms, $V_{i}$ is the indication of an airspeed indicator corrected for instrument scale error, $v_{c}$ is $\gamma_{i}$ corrected for static-pressure error, anc $V$ is the true airspeed, which is equal to $V_{c}$ at sea level.

Tables relating calibrated airspeed to impact sressure are presented in references $4,10,12,15,16$, and 17 . The tables of reference 10 are given in this text because they are based on a revised value of the na:tical mile adopted in 1959 and because the units of $v_{c}$ and $q_{c}$ are in U.S. Cistomary units.

Values of impact pressure $q_{c}$ for calibrated airspeeds $v_{c}$ (or $q_{c}^{\prime}$ for indicated airspeeds $V_{i}$ ) up to 1100 mph and 1000 knots are given in tables A9 through al2 of appendix A. The values in miles per hour are based on a statute mile equal to 5280 ft , and the values in knots are based on the 1959 value of the nautical mile ( 6076.12 ft ).

## Mäch Wümber Equations

As noted in chapter $I$, the Mach number $M$ is the ratio of the true airspeed $V$ to the speed of sound $a$ in the ambient air; that is,

$$
\begin{equation*}
\mathrm{m}=\mathrm{v} / \mathrm{a} \tag{3.21}
\end{equation*}
$$

By substituting in this expression the equation for the speed of sound given in equation (3.11), $M$ can be related to $V$ by the following equation:

$$
\begin{equation*}
v=M \sqrt{\frac{Y P}{p}} \tag{3.22}
\end{equation*}
$$

The Mach number may then be expressed in terms of $p_{t}$ by substituting equation (3.22) in equations (3.12) and (3.13), which then become

$$
\begin{equation*}
p_{t}=p\left(1+\frac{\gamma-1}{2} n^{2}\right)^{\frac{\gamma}{\gamma-1}} . \tag{3.23}
\end{equation*}
$$

and

$$
p_{t}=\frac{1+\gamma}{2} M^{2} p\left[\frac{(1+\gamma)^{2} M^{2}}{4 \gamma M^{2}-2(\gamma-1)}\right]^{\frac{1}{\gamma-1}}
$$

With the additional substitution of equation (1.1) in equatines (3.23) and (3.24), $M$ can be expressed as a function of $q_{c} / p$ as follows:

$$
\frac{q_{c}}{p}=\left(1+\frac{\gamma-1}{2} n^{2}\right)^{\frac{\gamma}{\gamma-1}}-1
$$

and

$$
\frac{q_{c}}{p}=\frac{1+\gamma}{2} n^{2}\left[\frac{(1+\gamma)^{2} m^{2}}{4 \gamma m^{2}-2(\gamma-1)}\right]^{\frac{1}{\gamma-1}}-1
$$

Machmeters are calibrated in accordance with equation (3.25) for subsonic speeds (M§1) and equation (3.26) for supersonic speeds ( $\mathrm{m} \geqq 1$ ).

In table A26 of appendix $A$, values of $q_{c} / p$ for given values of $M$ (or values of $q^{\prime} / p^{\prime}$ for given values of $M^{\prime}$ ) are tabulated for Mach numbers up ts $5 . C$ (from ref. 4).

## True-Airspeed Equations

As noted earlier, true airspeed can be derived from calibrated airspeed in the subsonic range by mears of equation (3.18). The true airspeed can also be determined, at both subsonic and supersonic speeds, from its relation to Mach number and the speed of sound in equation (3.21). For this case, $M$ is determined from equations (3.25) and (3.26) and a is determined by combining equations (3.1) and ;3.11) which yields the following equation relating a to the temperature of the ambient air:

$$
\begin{equation*}
a=\sqrt{\gamma \frac{P_{0}}{\rho_{0}} \frac{T}{T_{0}}} \tag{3.27}
\end{equation*}
$$

where $T$ is the absolute temperature in degrees Rankine or Kelvin. For $P_{0}$ in slugs per cubic foot and $P_{0}$ in pounds per square foot, the value of $a$ is in feet per second.

For values in terms of miles per hour or knots, the speed of sound can ise calculated from any of the following equations derived from equation (3.27):

1. If $a$ is in miles per hour an: $T$ is in degrees Rankine, $a=33.424 \sqrt{T}$
2. If $a$ is in knots and $T$ is in degrees Rankine,
$a=29.045 \sqrt{T}$
3. If $a$ is in miles per hour and $T$ is in degrees Kelvin, $a=44.844 \sqrt{T}$
4. If $a$ is in knots and $T$ is in degrees Kelvin,

$$
a=38.968 \sqrt{T}
$$

The value of $T$ required for the calculation of $a$ is the temperature of the free stream. While some aircraft temperature probes register free-stream temperature directly, the temperature registerad by other types of probes is higher than the stream value because of the adiabatic heating effect of the airflow on the sensor. The exter.t to which the probe measures the adiabatic heating effect is stated in terms of a recovery factor, which ranges from zero (no adiabatic heating) to 1.0 (full adiabatic +emperature rise). The recovery factor of a temperature probe can be determined from calibration tests in a wind tunnel. An electrical-type temperature probe having a recovery factor near unity ( 0.99 ) is shown in figure 3.2 (from ref. 18).

If the recovery factor of the probe is 1.0 or if the probe is located in a region where the local velocity of the air is equal to the free-stream velocity, the free-air temperature $T$ can be calculated from the following equation:

$$
\begin{equation*}
T=\frac{T^{\prime}}{1+\frac{Y-1}{2} \mathrm{KM}^{2}} \tag{3.28}
\end{equation*}
$$

where $T^{\prime}$ is the measured (or total) temperature and $X$ is the recovery factor of the probe. For the more general case in which the recovery factor is less than 1.0 and the probe is located in a region where the local velocity differs from the free-stream value, the free-air temperature can be calculated from the following:

$$
\begin{equation*}
T=\left(\frac{T^{\prime}}{1+\frac{Y-1}{2} K_{i}^{2}}\right)\left(\frac{1+\frac{Y-1}{2} M_{2}^{2}}{1+\frac{Y-1}{2} M^{2}}\right) \tag{3.29}
\end{equation*}
$$

where $M_{2}$ is the local Mach number, which can be determined from measurements of the locel impact and static pressures in the region in which the probe is located.

Values of the speed of $\because-$ ind $a$ in miles per hour and knots are given in table A7 of appendix a for geopotential altitudes up to 100000 ft . The values of a are based on the values of $T$ in the standard atmosphere of reference 11 .

Values of true airspeed $V$ for calibrated airspeeds from 0 to 1000 knots and geopotential altitudes from 0 to 100000 ft are given in table Al3 of appendix $A$. The values of $V, V_{C}$, and $H$ in this table are based on the standard atmosphere of reference 11.

A chart showing the relations of calibrated airspeed, true airspeed, anc Mach number for altitudes up to 60000 ft and temperatures from $-100^{\circ} \mathrm{F}$ to $120^{\circ} \mathrm{F}$ is presented in figure 3.3 (from ref. 19).

## Conversion Factors

For applications requiring the conversion of the pressure units in tables Al and A2 and A9 through A12 of appendix A to other units, conversion factors for a variety of other pressure units are given in table A27 of appendix A. For conversion of U.S. Customary Units to SI Units, conversion factors and metric equivalents are given in table $0 \geqslant 8$ of appendix $A$ (ref. 20).

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Figure 3.1.- Compressibility factors.


Figure 3.2.- Electrical-type temperature probe having a recovery factor of 0.99. (Adapted from ref. 18.)


Figure 3.3.- Chart of calibrated airspeed, true airspeed, and Mach number. (Adapted from ref. 19.)

## TOTAL-PRESSURE MEASUREMENT

The equations for airspeed, Mach number, and true airspeed given in the previous chapter are all based on the measurement of impact pressure. As showf by equation (1.1), however, the impact pressure is derived from measured values of total pressure and static pressure. In this and the following chapters,
-therefore, the problems relating to the measurement of total pressure with pitot tubes and the measurement of static pressure with static-pressure tubes or fuselage vents are considered in some detail.

As noted in the next chapter, the static pressure at successive points along lines of airflow past a body can vary widely, whereas the total pressure along these lines of flow remains constant. For this reason, the measurement of total pressure is much less difficult than the measurement of free-stream static pressure. The measurement of total pressure is also easier because the problem of total-pressure tube design is less difficult than the design problem for static-pressure tubes.

The principal difficulty encountered in the measurement of total pressure relates to the change in the measured pressure when the pitot tube is inclined to the airflow. Since the magnitude of this change is largely dependent on the design, or configuration, of the pitot tube (which can take a wide variety of forms), the problem of measuring total pressure with tubes inclined to the flow is considered separately from the simpler case of tubes aligned with the flow.

Tubes Aligned With the Flow
When aligned with the flow in the subsonic speed range, almost any open-end tube registers total pressure correctly provided that the tube is located away from any boundary layer, wake, propeller slipstream, or engine exhaust. For operations at high subsonic speeds, the tube should be located away from any area of high curvature on the structure where shock waves form when the local speed becomes sonic. As all these locations can usually be avoided, there is generally little problem in measuring total pr: ssure at subsonic speeds when the tube is aligned with the flow. Locations which have proved satisfactory for pitot-tube installations include positions ahead of the fuselage, wind, or vertical fin for tubes mounted on short horizontal booms or positions along the fuselage or under the wing for tubes mounted on short struts. Examples of service-type pitot tubes designed for end-mounting end strut-mounting are shown in figure 4.1.

For operations in the supersonic speer range, the tube should be located ahead of shock waves emanating from ani part of the aircraft. The location that best meets this requirement is, obviously, a position ahead of the fuselage nose. When located ahead of the fuselage bow shock, however, the tube is still influenced by a shock, for a small normal shock wave forms ahead of the tube. The presence of this shock is important to the measurement of total pressure because the total pressure decreases through the shock, so that the
pressure measured by the tube is lower than the free-stream value ahead of the shock. The magnitude of the total-pressure loss through the shock $\Delta p_{t}$ as a fraction of the free-stream total fressure $p_{t}$ is given by the following expression derived by subtracting equation (3.24) from equation (3.23), dividi the resulting quantity by equatioll (3.23), and assigning the value of 1.4 to

$$
\frac{\Delta p_{t}}{P_{t}}=1-\frac{1.2 M^{2}\left(\frac{5.76 M^{2}}{5.6 M^{2}-0.8}\right)^{2.5}}{\left(1+0.2 M^{2}\right)^{3.5}}
$$

where $M$ is the free-stream Mach number. The variation of $\Delta p_{t} / p_{t}$ with Mach. number for the Mach range from 1.0 to 3.0 is shown in figure 4.2. For the tat ratory calibration of airspeed indicators and Machmeters in the supersonic spe range, the total-pressure loss through the shock is taken into account in the computation of the pressure tables by which the instruments are calibrated (see eqs. (3.17) and (3.26)).

## Tubes Inclined to the Flow

When a pitot tube is inclined to the flow, the total pressure begins to decrease at some angle of inclination. The angular range through which the tu: measures total pressure correctly is called the range of insansitivity to inclination. In this text, the range of insensitivity is defined as the angu: range through which the total-pressure error remains within 1 percent of the impact pressure. For a criterion based on a smaller total-pressure error, the range of insensitivity would, of course, be smaller than that quoted for the tubes to be described in this chapter.

The configurations of the total-pressure tubes to be described are of two general types: simple pitot tubes and pitot tubes enclosed in a cylindrical shield. For the simple pitot tubes, the range of insensitivity is shown to depend for the most part on the shape of the nose section of the tube and on the size of the impact opening relative to the frontal area of the tube.

Early designers Eavored tubes with hemispherical nose shapes and small impact openings. An ex unple of the use of small-bore, round-nosed tubes was the pitot-static tube designed by the German physicist, Ludwig Prandtl. The pitot part of this tube (fig. 6.5) was very sensitive to inclination, for the range of insensitivity was only $\pm 5^{\circ}$ (ref. 1). Of interest here is the fact tin. the sensitivity of the pitot tube to inclination was considered to be of littl. concern, because the static-pressure portion of the tube was equally sensitive to inclination in a compensating manner. As a result, the impact pressure measured by the tube remained unaffected by inclination through an angular range about $\pm 12^{\circ}$. As discussed in this chapter and in chapter VI, later designers have tried to reduce the sensitivity to inclination of both total- and staticpressure tubes.

In an investigation of a number of pitot-static tubes in 1935 (rti. 2; , tests of pitot tubes having cylindrical nose shapes disclosed a significant design feature, namely, that the range of insensitivity could be increased by increasing the size of the pitot opening. An extrapolation of test results indicated that maximum insensitivity to inclination should be achieved with a thin-wall tube.

Ir another investigation in 1935, G. Kiel, a German aerodynamicist, showed in reference 3 that the range of insensitivity could be extended considerably by placing the pitot tube inside a venturi-like shield, as shown in figure 4.3. In tests of this tube at low speeds, the range of insensitivity was found to be $\pm 43^{\circ}$. Later tests of the tube in a NASA wind tunnel confirmed this range of insensitivity, but showed that the tube could not be used at Mach numbers greater rhan 0.6 because of excessive vibrations caused by the airflow around the mounting strut.

The errors of simple tubes due to inclination can be avoided by equipping the tube with a pivcis and vanes to align the tube with the airstream (fig. 4,3). While swiveling tubes are satisfactory for flight-test work at subsonic speeds. they are impractical for service use on operational aircraft.

In an effort to devise fixed (as opposed to swiveling) total-pressure tubes that would be insensitive to inclination and suitable for use on both operational and flight-test aircraft, the NACA conducted a series of wind-tünnel tests on a variety of tube designs from 1951 to 1954 (refs. 4 through 9). The tests were conducted in five wind tunnels at Mach numbers ranging from 0.26 to 2,40 and at angles of inclination up to $67^{\circ}$. Diagrams of the tube configurations that were investigated are presented in figure 4.4. As indicated by the six series of tube designs, the configurations included shielded tubes based on the Kiel design and simple tubes with cylindrical, conical, and ogival nose shapes. For the simple tubes, the principal design variables, aside from the nose shape, were the shape of the entry to the impact opening (cylindrical, hemispherical, and conical) and the relative size of the impact opening on the face of the tube. The shielded tubes were all designed with vent holes along the aft portion of the tube to allow mounting at the end of a hori zontal boom. The variables tested with the shielded tubes included (1) the shape of the entry to the throat (conical and curved), (2) the relative size of the throat ( $\mathrm{D}_{2} / \mathrm{D}$ ), (3) the position of the pitot tube from the face of the shield (a/D), and (4) the area of the vents with respect to the frontal area of the shield ( $A_{v} / A_{0}$ ).

The results of the tests of a few of the tubes have been selected for this text to show the effects of some of the more significant design features. An assessment of all of the design variables is given in the sumary report of the investigation in reference 9 .

In the presentation of the results in the figures to follow, the angle of inclination of the tube is the angle in the vertical plane (angle of attack). For symmetrical tubes, the variation of the total essure error is the same in the horizontal plane (angle of yaw). For unsymmetrical tubes $A-6, A_{s}-10, A_{s}-11$, E-3, and E-4 of figure 4.4, however, the error variation at angles of attack and angles of yaw are different.

The effect of varying the size of the impact opening with i=spect to the frontal area of cylindrical tubes is shown by a comparison of the test results of tubes A-1 and A-2 at a Mach number of 0.26 (fig. 4.5). For the small-bore tube, the range of insensitivity is $\pm 11^{\circ}$, while for the thin-wall tube, it is $\pm 23^{\circ}$. These results confirm the data from reference 2 in showing the zange of insensitivity to increase with an increase in the size of the impact opening.

The test data on figure $4.6(a)$ show the effect of cutting the nose of the thin-wall tube at a slant angle of $10^{\circ}$... The range of insensitivity is increased (from the $\pm 23^{\circ}$ value for the thin-wall tube) to $32^{\circ}$ at positive angles of attack but decreased to $13^{\circ}$ at negative angles. The effect of the $10^{\circ}$ slant profile, therefore, is simply to shift the curve of figure 4.5 (b) $10^{\circ}$ along the angle-ofattack axis. At angles of yaw, the range of insensitivity is $\pm 23^{\circ}$, the same as that for the thin-wall tube. For some applications, the use of a slant-profile tube could be advantageous, since the angle-of-attack range through which an aircraft operates is greater at positive angles than at negative.

The effect of changing the shape of the internal entry of cyli-inical tubes can be shown from a comparison of the test data of tubes A-2, A-5, and A-7 through A-11. Changing the entry from a cylindrical shape (tube A-2) to a hemispherical shape (tube A-5) increased the range of insensitivity by about $3^{\circ}$. A change to a $50^{\circ}$ conical entry (tube A-11) showed no improvement over the value for the cylindrical entry. By decreasing the internal cone angle to $30^{\circ}$, however, the range of insensitivity increased to a vilue of $=27^{\circ}$ (tube A-9 in fig. $4.6(\mathrm{~b})$ ). Decreasing the cone angle to $20^{\circ}$ (tube $\mathrm{A}-8$ ) and to $10^{\circ}$ (tube A-7) produced no further extension in the range of insensitivity.

The $27^{\circ}$ range of insensitivity for the tube with the $30^{\circ}$ conical entry is $5^{\circ}$ lower than that for the thir-wall, slant-profile tube at positive angles of attack. However, because of the relative fragility of the slant-profile tube and the lack of space for the installation of deicing heating element, the tube with the $30^{\circ}$ conical entry would be a more practical tube for service operations.

Some effects of the external nose shape on the range of insensitivit; can be shown from a comparison of the data for the tubes having conical and ogival nose sections. For the tube with a $15^{\circ}$ conical nose (tube $\mathrm{B}-1$ ), the range of insensitivity is $\pm 21^{\circ}$ (fig. $4.7(\mathrm{a})$ ); for the $30^{\circ}$ nose (tube $C-1$ ), the range is $\pm 17.5^{\circ}$; and for the $45^{\circ}$ nose (tube $D-1$ ), it is $\pm 14^{\circ}$.

The ogival-nose tube in figure 4.7 (b) is a service-type tube which, in the production model, had a small wall thickness at the impact opening. To make the pitot configuration of this tube comparable with that of tubes $\mathrm{B}-1, \mathrm{C}-1$, and $D-1$, the impact opening was reamed to a sharp leading edge. As shown in figure 4.7 (b), the range of insensitivity of this modified tube was $\pm 16^{\circ}$, which. is about midway between that for the $30^{\circ}$ and $45^{\circ}$ conical-nose tubes.

The test data for a Kiel-type shielded tube having a vent area equal to ti: frontal area of the shield are shown on figure $4.8(a)$. The range of insensitivity of this tube is $\pm 41^{\circ}$, which is very nearly the same as that for the original Kiel design. These tests are significant, therefore, in showing that
a shielded tube can be vented along the walls of the shield, as opposed to the straight-through venting of the Kiel shield, without loss in performance.

The test data presented thus far were all obtained at a Mach number of 0.26 . When tubes $A-2, A-6, A-9$, and $B-1$ (figs. 4.5, 4.6, and 4.7) were tested at $M=1.62$, the range of insensitivity was greater than that at $M=0.26$ by as much as $4^{\circ}$ to $10^{\circ}$. In contrast, the range of insensitivity of shielded tube $\mathbf{A}_{\mathbf{s}}-3$ (fig. $4.8(\mathrm{a})$ ) was lower at $M=1.62$ by about $3^{\circ}$.

In tests of the shielded tubes with the crived entries, the entry with the highest degree of curvature (tube $A_{s}-12$ ) provided the greatest range of insensitivity. At $M=0.26$, for example, the range was $\pm 63^{\circ}$ (fig. 4.8 (b)). With increasing Mach number, the range of insensitivity decreased to about $58^{\circ}$ at $M=1 . C$ and to about $40^{\circ}$ at $M=1.61$ (fig. 4.9). Despite this loss in performance with increasing Mach number, however, the range of insensitivity of this shielded tube is still greater than that of any of the simple tubes at both subsonic and supersonic speeds.

In the foregoing discussion, only the aerodynamic aspects of the design of pitot tubes have been considered. For a tube intended for operational use, the nose configuration would have to allow for the installation of an electric heating element for deicing and drain holes for the removal of any water that may be ingested. In at least two cases, pitot configurations examined in the NASA investigation have been successfully incorporated in the design of servicetype pitot and pitot-static tubes; the configuration of tube A-9 is incorporated in the pitot tube shown in figure 4.10 and the configuration of tube $\mathrm{B}-4$ is incorporated in the pitot-static tive jescribed in reference 10 and shown in figure 6.14.

## References

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10. Pitot Static Tube TRJ-1/A, Electrically Heated. Mil. Specif. MIL-P-25757B (ASG), Jan. 26, 1960.

(b) Strut mounting.


Figure 4.2.- Total-pressure loss through a nomal shock wave.

(a) Shie:ded total-pressure tube designed by G. Kiel. (Adapted from ref. 3.)

(b) Swiveling total-pressure :ube.

Figure 4.3.- Shielded and swiveling total-pressure tukes.

(a) Series A - cylindrical nose.

Figure 4.4.- Diagrams of total-pressure tubes examined in NACA investigations. (Adapted from ref. 9.)


Figure 4.4.- Continued.


Figure 4.4.- Continced.

(c) Series B-15 conical nose.

Figure 4.4.- Continued.

(d) Series $C-30^{\circ}$ conical nose.

Figure 4.4.- Continued.



(e) Series D - $45^{\circ}$ conical nose.

Figure 4.4.- Continued.

(f) Sexies E - ogival nose.

Figure 4.4.- Concluded.
$\xrightarrow{\text { Flow }} \underset{\text { Tube } A-1}{ } \frac{1}{7}$ in.

(a) Small-bore cylindrical tube.


Figure 4.5.- Variation of total-pressure error with angle of attack fur cylindrical tubes with different size impact cpenings. $M=0.26$. (Adapted from ref. 4.)


Eigure 4.6.- Variation of total-pressure error with angle of attack for cylindrical tubes with impact openings of different shapes. $M=0.26$. (Adapted from ref. 4.)


(a) $15^{\circ}$ conical-nose tube.

(b) Ogival-nose tuoe.

Figure 4.7.- Variation of total-pressure error with angle of attack for tubes having conical- and ogival-nose shapes. $M=0.26$. (Adapted from ref. 4.$)$

(b) Shielded tube with curved entry to shield. (Adapted from ysf. 5.)

Figure 4.8.- Variation of total-pressure error with angle of atta=k for two shielded tubes. $\because=0.26$.



## CHAPIER V

## STATIC-PRESSURE MEASUREMENT

For a steady flow condition, the flow of the air over a body creates a pressure-field-in-which-the static pressures vary from point to point, while the total pressure at ail points remains the same. For this reason, the measurement of free-siream static pressure on an aircraft is much more complicated than the measurement of free-stream total pressure. The pressure field created by the airflow may change with the configuration of the aircraft and with Mach number and angle of attack. For a given aircraft configuration, therefore, the problem of designing a static-pressure-measuring system is primaxily one of finding a location where the static-pressure error varies by the least amount throughout tine operating range of the aircraft.

The variation of the pressures in the flow field can be described by Bernoulli's equation for the total pressure $P_{t}$ in incompressible flow:

$$
\begin{equation*}
p_{t}=p_{2}+\frac{1}{2} p v_{l}^{2}=\text { Constant } \tag{5.1}
\end{equation*}
$$

where $p_{2}$ is the local static pressure and $v_{l}$ is the local flow velocity. This equation states that the total pressure remains constant (at the freestream value) at all points along lines of flow, whereas the local static pressure varies inversely with the square of the local velociiy.

The variation of local static pressure expressed by equation (5.1) is illustrated by the diagram of the flow around a fuselagelike body in figure 5.1. The five lines of flow (streamlines) shown in this figure represent the paths of the individual particles of the air. At a great distance ahead of the body, the streamlines are parallel and the total pressure $p_{t}$, static pressure $p_{\text {, }}$ and velocity of the particles $V$ on each of the streamlines are the free-stream values. As the air particles move closer to the body, the streamlines begin to diverge and the velocities of the particles begin to increase as the air flows past the body. At some considerable distance behind the body, the streamiines return to parallel flow and the pressures and velocities return to their freestrean values.

Relative magnitudes of the local pressure and the local velocity at three points near the nose of the body are also shown in figure 5.1. At a position just aft of the nose, the local velocity is higher than the free-stream velocit, and the local static pressure is lower than the free-stream static pressure. At a position directly ahead of the nose, the local velocity is lower than the stream velocity, so that the local static pressure is higher than the stream value. At a point on the leadirg edge of the nose, where the air particles come to a stop, the local static pressure is equal to the feee-stream total pressure.

The flow pattern, or field, shown in figure 5.1 applies to incompressible flow or to compressible flow at very low speeds. For higher speeds in compressible flow, the flow field changes markedly, particularly at transonic and supersonic speeds.

In the subsonic speed range, the flow field extends in all directions from the aircraft. The difference between the local static pressure and the freestream static pressure is greatest. in the vicinity of the aircraft and decreases with distance from it. In the transonic speed range, the flow field is altered by shock waves that form along the lines of maximum curvature of the fuselage, wings, and tail surfaces. At supersonic speeds, the flow field is confined to the regions behind the shock wave that forms ahead of the nose of the fuselage (fuselage bow shock). As discussed in the next two chapters, the changes in the characteristics of the flow fields in the three speed ranges can produce large variations in the pressures measured by a static-pressure installation.

An orifice on a surface oriented parallel to the airstream has been universally used to measure static pressure on aircraft. The orifice may be located on the surface of the fuselage cr on a static-pressure tube attached to some part of the aircraft. For fuselage-vent installations, the orifices are usually installed in pairs (one on each side of the fuselage) and are generally located some distance aft of the nose of t.re fuselage. With the static-pressure tube, the orifices are ordinarily located well aft of the nose of the tube and may either encircle the tube or be oriented in unsymetrical arrangements described in the next chapter. On some early static-pressure tubes, the crifices were in the form of rectangular slots; on present-day tubes, the orifices are circular.

Like the total-pressure tubes described in the last chapter, the staticpressure tubes are designed with either a transverse strut for attachment to some part of the aircraft structure or with end fittings for mounting on a horizonta: boom. Since the diameter of the boom is generally larger than that of the tube, the aft end of the tube is enlarged to form a collar of the same diameter as the boom. As shown in the next chapter, the mounting struts and the collars of the tubes can have a marked influence on the pressures measured by the tubes. The tubes with strut supports have generally been attached either to the underside of the wing or, in pairs, to the sides of the fuselage. The tubes designed for end-mounting on booms have been installed on the nose of the fuselage, the outboard section of the wing, and the tip of the vertical fin. Examples of service-type pitot-static tubes designed for end-mounting and strutmounting are shown in figure 5.2.

A diagram showing four types of static-pressure-measuring installations (static-pressure tubes ahead of the fuselage nose, wing tip, and vertical fin and fuselage vents on the side of the fuselage) is presented in figure 5.3. Also shown are the local static pressures $p_{l}$ and the measured static pressures $p^{\prime}$ at t.ie four pressure sensors. For each installation, the difference between the measured pressure and the free-stream static pressure $p$ is derined by equation (2.2):

$$
\begin{equation*}
\Delta p=p^{\prime}-p \tag{2.2}
\end{equation*}
$$

where $\Delta p$ is the static-pressure error of the installation, or installation error: this error is also called the position error because the magnitude of the static-pressure error depends primarily on the position of the pressure sensor in the flow field of the aircraft.

For the fuselage-vent installation, the measured static pressure is essentially the same as the local static pressure at the vents. With the static-pressure-tube installations, on the other hand, the local static pressure is altered by the presence of the tube, because the tube creates a small flow field of its own. Since the flow of the air causes the pressures along the tube to vary in a manner similar to that described for flow about the aircraft, some part of the position error of a static-pressure-tube installation is due to the configuration of the tube (size, shape, and location of the orifices).

The errors of a static-pressure tube vary primarily with Mach number and angle of attack, while the position errors of a static-pressure installation vary primarily with Mach number and lift coefficient (a function of angle of attack). The errors of a static-pressure tube are determined by wind-tunnel tests, whereas the position errors of a static-pressure installation are determined by flight calibrations.

In steady, level flight, the lift coefficient $C_{L}$ is normally a linear function of angle of attack at speeds above the stall. For this condition, $C_{L}$ is defined by the following equation:

$$
\begin{equation*}
C_{L}=\frac{W}{q S} \tag{5.2}
\end{equation*}
$$

where $W$ is the weight of the aircraft, $S$ the area of the wing, and $q$. the dynamic pressure. Values of $q$ can be determined from measured values of the impact pressure $q_{C}$ and the static pressure $p$ and the following equation derived from equations (3.10) and (3.22):

$$
\begin{equation*}
q=\frac{\gamma p q^{2}}{2} \tag{5.3}
\end{equation*}
$$

where $M$ is determined from the ratio $q_{c} / p$ as discussed in chapter III.
In wind-tunnel calibrations of static-pressure tubes and flight calibrations of static-pressure installations, the static-pressure errors are usually presented as fractions of the static pressure, $\Delta p / p$, or as fractions of the impact pressure, $\Delta p / q_{c}$. For calibrations at high Mach numbers, the staticpressure error is often converted to an error in Mach number $\Delta M$ and expressed as a fraction of the Mach number, $\Delta M / M$. In this text, the static-pressure errors for all of the wind-tunnel and flight calibrations are presented in terms of $\Delta p / q_{c}$. For a comparison of a position error calibration in terms of $\Delta p / q_{C}, \Delta p / p$, and $\Delta M / M$, see figure 7.23.

Values of $\Delta p / q_{c}$ in be converted to values of $\Delta p / p$ by means of the $a_{c} / p$ values given in table a26 of appendix A. A grapk showing the relation of $\Delta p / p$ to $\Delta p / q_{c}$ for Mach numbers up to 2.0 is presented in figure 5.4.

Values of $\Delta p / q_{c}$ and $\Delta p / p$ can ipe converted to values of $\Delta M / M$ by means of the following equations from reference 1 :

$$
\begin{equation*}
\frac{\Delta p}{p}=-\frac{1.4 M^{2}}{1+0.2 M^{2}} \frac{\Delta M}{M} \tag{5.4}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\Delta p}{q_{C}}=-\left[\frac{1}{\left(1+0.2 M^{2}\right)^{3.5}-1}\right] \frac{1.4 M^{2}}{1+0.2 M^{2}} \frac{\Delta M}{M} \tag{5.5}
\end{equation*}
$$

for $M \leqq 1$, and

$$
\begin{equation*}
\frac{\Delta p}{p}=\left(\frac{4.0}{5.6 M^{2}-0.8}-2\right) \frac{\Delta M}{M} \tag{5.6}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\Delta p}{q_{c}}=\left(\frac{4.0}{5.6 M^{2}-0.8}-2\right) \frac{1}{1.2 M^{2}\left(\frac{5.76 M^{2}}{5.6 M^{2}-0.8}\right)^{2.5}-1} \frac{\Delta M}{M} \tag{5.7}
\end{equation*}
$$

for $M \geqq 1$. A graph of the relation between $\Delta p / p$ and $\Delta M / M$ and between $\Delta p / q_{c}$ and $\Delta M / M$ for mach numbers up to 5.0 is presented in figure 5.5.

The altitude error $\Delta H$, airspeed error $\Delta V_{C}$, and Mach number error in that are associated with the position error $\Delta p$ are defined by the following equations:

$$
\begin{equation*}
\Delta H=H^{\prime}-H \tag{5.8}
\end{equation*}
$$

where $H^{\prime}$ is the indicated altitude and $H$ is the pressure altitude,

$$
\begin{equation*}
\Delta v_{c}=v_{i}-v_{c} \tag{5.9}
\end{equation*}
$$

where $V_{i}$ is the indicated airspeed and $V_{c}$ is the calibrated airspeed,

$$
\begin{equation*}
\Delta M=M^{\prime}-M \tag{5.10}
\end{equation*}
$$

where $M^{\prime}$ is the indicated Mach number and $M$ is the free-stream Mach number.

To provide an indication of the errors in airspeed and altitude that result from a given static-pressure error, the altitude errors $\Delta H$ and the airspeed errors $\Delta V_{c}$ corresponding to a static-pressure error equal to 1 percent of the impact pressure $\left\langle\Delta p / q_{c}=0.01\right.$ ) are presented in figure 5.6 for mach numbers up to 1.0 and altitudes up to 40000 ft . The altitude errors corresponding to an error of 1 percent oi the static pressure ( $\Delta \mathrm{p} / \mathrm{p}=0.01$ ) are presented in figure 5.7 for altitudes up to 50000 ft , and the altitude errors corresponding to an error of 1 percent of the Mach number ( $\Delta M / M=0.01$ ) are presented-in-figare 5.8 for. Mach numbers-up-to 1.0 and altitudes up to 40000 ft . For positive static-pressure errors ( $\Delta \mathrm{p} / \mathrm{q}_{\mathrm{c}}$ or $\Delta \mathrm{p} / \mathrm{p}$ ), the signs of both $\Delta H$ and $\Delta V_{c}$ are negative; for positive values of $\Delta M / M$, the signs of $\Delta H$ and $\Delta V_{c}$ are positive.

In appendix $B$, sample calculations are given for the determination of $\Delta H$, $\Delta V_{C}$, and $\Delta M$ from a given value of $\Delta p$ and the indicated altitude $H^{\prime}$, the iraicated airspeed $V_{i}$, and the indicated Mach number $M$ '.

Reference

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rigure 5.1.- Diagram showing local pressures and velocities in vicinity of fuselagelike body.



Figure 5.4.- The relation between $\Delta p / p$ and $\Delta p / q_{C}$.


Figure 5.5.- The relation between $\Delta p / p$ and $\Delta M / M$ and between $\Delta p / q_{C}$ and $\Delta M / M$. (Adapted from ref. 1.)

(a) $\Delta H$ corresponding to $\Delta p / q_{C}=0.01$.

(b) $\Delta v_{c}$ corresponding to $\Delta p / q_{c}=0.01$.

Figure 5.6.- Altitude errors $\Delta H$ and airspeed errors $\Delta V_{c}$ corresponding to a static-pressure er of 1 percent of impact pressure $\left(\Delta p / q_{C}=0.01\right)$.


Figure 5.7.- Altitude errors $\Delta H$ corresponding to a static-pressure error of 1 percent of the static pressure $(\Delta p / p=0.01)$.


Fisure 5.8.- Altitude errors -H corresponcing so a Nach number error of 1 persent ( $\because M / \because=0.31$ ).

## CHAPTER VI

## STATIC-PRESSURE TUBES

As discussed in the previous chapter, the flow of the air past a staticpressure tube causes the pressures along the surface of the tube to vary from one point to another. These variations in static pressure can be described in terms of pressure distributıc.as along the tube (when the tube is aligned with the flow) and pressure distributions around the tube (when the tube is inclined to the flow).

The difference between the pressure sensed by the orifices and the freestream static pressure is the static-pressure error of the tube, sometimes casled the error of the isolated tube. In the following sections, the errors of tubes aligned with the flow are considered separately from the errors of tubes inclined to the flow. At the end of the chapter, data are presented on the effects of orifice size and shape on the pressure sensed by the tube.

Tubes Aligned with the Flow
Theoretical pressure distributions along cylindrical bodies (fig. 6.1, from ref. 1) are useful in understanding the problem of locating orifices along a static-pressure tube. For both the subsonic and supersonic $f 1$ vw conditions shown in the figure, the static-pressure errors are negative at a station just beyond 1 tube diameter from the nose of the tube. The pressures at this point on the tubs are, inerefore, below the free-stream pressure. With increasing distance from the nose, the pressures approach the free-stream pressure and should reach that value at a distance of about 5 tube diameters in subsonic flow and about 8 tube diameters in supersonic flow.

In using the theoretical data to design a tube to measure froe-stream pressure, many designers place the orifices a greater distance from the nose than that indicated by the theoretical distributions. A typical example is the l0-diameter location on the tube in figure 6.2. is shown by the calitration data, the statiz-pressure error is near zero throughout most of the subsonic speed range.

In the subsonic speed range, the pressure at the orifices can be influcnoud by the presence of a strut or collar downstream from the orifices. The cistec: of a strut is illustrated by figure 6.3 wich shows tie pressure distribuem: for incompressible, twodimensional flow ahead of a body of inti:ite lerac. transverse to the flow. For application to pressure measurements diti a ieat: pressure tube, this body cais be considered to refrestat the suiport :ation ot a tube. The curve on this figure shows that the pressurc errors anoac of $=0.6$
 diminish toward the free-stream value with increasing distance from fore sor. .
 upstrean from the body is called the blocking effest.

Wind-tunnel tests of the blocking effect of a strut at a number of distances behind a set of static-pressure orifices were reported in reference 2. The results of the tests (fig. 6.4) confirm the theoretical variation by showing the errors to be greatest for the shortest strut position ( $x / t=3.6$ ) and least for the longest $(x / t=10.5)$. The vise in the errors at Mach numbers above 0.5 shows that the blocking effect increases in the upper subsonic speed range.

Early designers of static-pressure tubes favored short tubes for strutmounting, because the blocking effect of the strut could be used to balance the negative errors incurred by locating the orifices near the nose of the tube. The outstanding example of this design concept was the Prandtl pitot-static tube (fig. 6.5) on which the orifices were located 3 tube diameters aft of the nose and 10 strut thicinesses ahead of the strut. This tube, and variations of the original design, has been the subject of many wind-tunnel investigations (refs. 3, 4, and 5, for example). In the most extensive of these tests (ref. 5), the error was essentially zero in the Mach range up to 0.5 (fig. 6.5), but increased at higher Mach numbers in the same manner as the errors of the tubes in figure 6.4.

In contrast to early tubes designed for strut-mounting, later tubes were designed for end-mounting on horizontal booms. These designs permitted the use of longer tubes on which the orifices could be located a greater distance from the nose. In addition, the collars at the rear of the tube could be so located that the blocking effect would be smaller than that of a strut. Thus, the positive and negative pressure errors at the orifices could both be made smaller than those of the strut-mounted tubes.

The blocking effect of a collar on the pressures at orifices at three locations abead of a collar was investigated in the tests of reference 2 . The results of the tests (fig. 6.6) show that even for orifices located as slose to the collar as 1.8 collar diameters, the errors are relatively small 11.5 percent $\mathrm{q}_{\mathrm{c}}$ ) and essentiaily constant for Mach numiers up to 0.8 .

A number of service-type tubes have been designed for end-mounting on booms. On one of the most widely used of these tubes (fig. 6.7), the orifices are located 5.5 tube diameters from the nose and 2.8 collar diameters ancad of the collar. As siown by figure 6.7, the static-pressure error of this tube is constant at about 0.5 percent $q_{C} u p$ to a Macin numbe: of about 0.3 .

Another end-mounted tube, designed for use on righ-speed research aircraí=, has the orifices located 9.1 tube diameters behind the nose and 5.3 collar diameters ahead of the collar. The calibration of this tube (fig. 5.8. som ref. 6) at both subsonic and supersonic specds shows an error of 1 persent of at $M=0.6$, a sharp rise in error at kach numbers around 0.7 , anc an abrupt decrease to error'; near zero at a sach number just bejond 1.0. The airyit fall of the error is due to she passage over the oritioes of a shock wave that froms ahead of the collar when the flow reaches sonic speed. A sinilar decrease :n static-pressure error at low supersonic speerls is experier iti iuselat-ade installations, as is discussed in some detail in :he nex:

## Tubes Inclined to the Flow

The pressures sensed by a static-pressure tube inclined to the flow depends not only on the location of the orifices along the tube but also on their spacing around the tube. When the orifices encircle the tube, the measured pressure decreases as the tube is inclined, and the static-pressure error reaches a value of -1 percent $q_{c}$ at angles of attack and yaw of about $5^{\circ}$.

The range of insensitivity of a tube at positive angles of attack can be extended by spacing the orifices around the tube in one nf two unsymetrical arrangements. The selection of the proper spacing can be illustrated by the pressure distribution around a circular cylinder at an angle of attack of $45^{\circ}$ and a Mach number of 0.2 (fig. 6.9, from ref. 7). This distribution shows the static-pressure error to be positive at the bottom of the tube $\left(\phi=0^{\circ}\right)$, negative on the top $\left(\phi=180^{\circ}\right)$, and zero at radial stations of about $35^{\circ}$. These data suggest that a tube could be made less sensitive to inclination at positive angles of attack by (1) locating two orifices ayproximately $\pm 35^{\circ}$ from the bottom of the tube or (2) locating a number of orifices ol. the top and bottom of the tube to achieve a balance of the positive and negative pressures ir these regions. Since the pressure distribution, and thus the radial position for zero pressure error, varies with angle of attack and Mach number, null-type (dual orifice) tubes have been designed with a number of orifice stations $\left( \pm 30^{\circ}\right.$ to $\pm 41.5^{\circ}$ ) in an attempt to pruduce a configuration that would be satisfactory through a range of angles of attack and Mach numbers.

In tests of a tube with orifices at the $\pm 30^{\circ}$ station (ref. 8), the range $o f$ insensitivity at positive angles of attack was found to be $20^{\circ}$ at $M=0.3$ and about $9^{\circ}$ at $M=0.65$ (fig. 6.10). Note that for the static-pressure tubes. the range of insensitivity is defined the angular range through which the static-pressure error remains with attack of $0^{\circ}$. This definition is d...nt from that given for the totalpressure tubes because the errors of static-pressure tubes at an angle of attack of $0^{\circ}$ are usually not zero. However, whenever corrections are applied for the errors of static-pressure-tube installations at or near an angle of attack of $0^{\circ}$, the definition of the range of insensitivity for static-pressure subes becomes the same as that for the total-pressure tubes, namely, the range through which tie error remains within 1 percent $q_{c}$.

In an investigation to determine the errors at a number of orifice stations (cef. 9), a cylindrical tube was tested with orifices located at the $=33^{\circ}$, $=33^{\circ}$. $\pm 36^{\circ}, \pm 37.5^{\circ}$, and $\pm 40^{\circ}$ stations. The tests were cc.aducted with the tube at an angle of attack of $12^{\circ}$ through a rach range f.om 0.4 to 1.2 . The resuliss of the tests (fig. 6.11) show the errors to be positive at the $=30^{\circ},=33^{\circ}$, and $=35^{\circ}$ stations and negative at the $=40^{\circ}$ station. For the $\pm 37.5^{\circ}$ station, tic $\in \mathbf{r}=0=3$ were near zero through the :fach sange up to 1.2 .

A top-and-bottom orifice arrangemert is :sed on she servizerge :upe show in figure 6.7. oith this arrangement, :-ur orifizes are siucren mitil:, radial angie of $\pm 20^{\circ}$ on the cop of the :ube ard sex orizizes witit. a ratiat angle of $+30^{\circ}$ on the jotsom. Tests of this fiee at a ract ominet if $\because$



In an attempt to extend the range of insensitivity of this tube at positive angles of attack, the orifice configuration was altered by progressively increasing the orifice area on the bottom of the tube. For the final configuration tested (fig. $6.12(b)$ ), the two orifices at the $\pm 30^{\circ}$ station on the bottom were enlarged from 0.043 in . in diameter to $0.052 \mathrm{in} .$, and an additional orifice, 0.052 in . in diameter, was drilled at the $0^{\circ}$ station just aft of the six orifices. With this configuration, the range of insensitivity was extended to $+45^{\circ}$ at $M=0.2$, but to only $+20^{\circ}$ at $-M=0.68$

The modified orifice configuration on the service tube in figure 6.12 (b) was incorporated in the design of the research-type tube in figure 6.8. In tests of this tube through a Mach range from 0.6 to 2.87 (fig 6.13), the range of insensitivity at positive angles $1 f$ attack was found to be about $15^{\circ}$ at both subsonic and supersonic speeds.

A service-type pitot-static tube exemplifying modern design trends is shown in figure 6.14 (ref. 11). For small errors at zero inclination, the orifices are located 13 tube diameters aft of the nose and 3.6 collar diamerers ahead of the collar $(x /(D-d)=7.2)$. The radial position of the two orifices is $\pm 37.5^{\circ}$ which, as shown by the data of figure 6.11, minimizes the erro: at positive angles of attack up to at least $12^{\circ}$. The pitot configuration is the same as that of tube B-4 (chapter IV) which is insensitive to inclination (to within 1 percent $q_{C}$ ) at angles of attack and yaw of $\pm 21^{\circ}$.

## Orifice size and Shape

The influence of orifice diameter and edge shape on the pressures measured by a static-pressure tube can be seen in figure 6.15 (from ref. 12). The variation of the static-pressure error with orifice diameter for a squaie-edge orifice at Mach numbers of 0.4 and 0.8 is shown in $f: g u r e 6.15(a)$. These errors can be related to the orifice size of static-pressure tubes by noting that for tubes with multiple orifices, the orifice diameter is usually on the order of 0.04 in . and for dual orifice tubes, the orifice diameter is 0.06 to 0.03 in. The effect of orifice size is, of course, included with the other effects (axial and radial location of the orifices and blocking effects of strut or collar) that contribate to the error of a static-pressure tube.

The effect of varying the edge shape of a 0.032 -in.-diameter orifice for a Hach range from 0.4 to 0.8 is shown in figure 6.15 (b). The errors for the rounded and angled edge shapes are referenced to the error of the square-edge orifice (which can be found from fig. 6.15(a)). The data for tine various orifice configurations show the effect of edge shape to he relatively smail except for the orifice with the wide curred entry. with present-day tubes, it is considered good practice to drill orifices with clean, sharp edges, free from surrs, and to make certain that the orifices are not damaged or deformed in operational use.

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Figure 6.1.- Theoretical pressure distributions along cylindrical bodies. (Adapted from ref. 1.)


Figure 6.2.- Calibration of a static-pressure tuice aligneciwth the E:Ow. Support for this tube was located abou: 30 in. downstrian from :... orifices.


Figure 6.3.- Theoretical pressure distribution abead of a body of infinite length transverse to the flow. (Adapted from ref. 1.)


Figure 6.4.- Blocking effect of a transverse strut for a static-pressure tube alisned with the flow. (Adapred from ref. 2.)


Figure 6.5.- Calibration of Prandtl pitot-static tube aligned with the flow. (idapted from ref. 3.)


Figure 6.6.- Blocking effect of a collar for static-pressure tuive aligned with the flow. (Adapted from ref. 2.)


Figure 6.7.- Ealibration of a service-type pitot-static zube aligned with the flow.




Figure 5.9.- Pressure distribution around a cylinder at an angle of attack of $45^{\circ}$ and a Mach number of 0.2 . The two pressure distributions are for fiow conditions below and above the critical Reynolds number $N_{\text {Re, }}$ at which flow separation crecurs. (Adapted from ref. 7.)


Figure 6.10.- Calibration at angle: of attack of a static-pressure tube with orifices at radial stations of $\pm 30^{\circ}$. (Adapted from ref. 8.)


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\text { O Data for } \phi=37.5^{\circ}
$$


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(a) Original orifice configuraticn.

(b) Modified orifice configuration.

Figure 6.12.- Calibration of a service-type pitot-static tube at ancles of attack. (Adapted from ref. 10.)


9!

(b) Supersonic speed range.

تigure 6.13.~ Calibration of researin-type litot-static Eube at a:gles of attack. (inajeed Erom $5 \approx$. 5.)


## CHAPTER VII

## STATIC-pressume I:istallations

As noted in chapter $V$, the position error of a static-prossur: inctullatact varies with Mach number and lift coefficient. In the low subsonic :feed rance, where large changes in lift coefficient can occur over a small Mach number range, the error depends largely on lift coefficient. In the high subsinic speed range, the change in lift coefficient is usually yuite small, so that the error in this range depends manly on :ach number. The errors at the low tach numbers are determincd from alibration tests at low altitude:; wiereas the errors at the higher :Aach numbers are determined in calibrations at :ayh altitudes (because of the speed limitations of the aircraft at low altitide.s). Whent the low-altieudre calibration tests are conducted at heights near sea logel, time curves are lataled "sea-level calibration" on the calibration fhart:;

As the variations of the errors with lift coefficicnt and : $\because$ det. :umber differ markedly for different tymes of installations, the charactreistics are
 fuselige sose, the wing tip, and the vertical in and fueducu-vert inital:ations. For each installation, the varistions of the reror:s in fix low ata bidt. Math ranges are considered separately. For one of the itstallat:onf. :aror.r.

 ranges.
ill the calibrations to be presented apply to ievel-fligit, era:.f arditions. For the landing configuration, the calibratio: is getarally bisery.fit because of changes in the flow field that result from deflection ,: the : :af: and extension of the landing ciear.









circular nose, 4 percent $q_{C}$ for the elliptical nose, and 1 percent $q_{C}$ for the ogival nose.

The magnitude of the static-pressure error at three positions ahead of an airplane having an elliptical nose section is shown in figure 7.3. Also shown in the figure is the curve for the wind-tunnel model with the elligical nose in figure 7.2. The errors for the airplane installations were detemined at a low speed " $M=0.37$ ) and a low angle of attack $\left(C_{L}=0.3\right)$, a condition comparable with that of the wind-tunnel tests. As shown by the two curves, the variation of the error with orifice position ( $x / D$ ) is about the same fo: the two teste.

The variation of the error with Mach number at low subsonic speeds for each of the three boom lengths on the airplane in figure 7.3 is shown in figure 7.4. As this is the speed range in whish the effects of lift coetzizient (or angle of attack) predominate, the lift coefficients at the stall speed ( $C_{i}=1.2$ ) and at the maximum speed of the tests $\left(C_{L}=0.3\right)$ are noted in the figure. As shown by the three curves, the errors for nose-boom installations decrease with increasing lift coefficient.

The variation of the error of a nose-boom installation in the transonic speed range can be illustrated with caliorations of static-pressure probes aincad of a body of revolution (fig. 7.5, from ref. 2) having a profile like the $\{-1$ research airplane (fig. 7.6). The errors were determined at thede :ositions ahead of the bedy through a :lach range from 0.68 to 1.05 (fig. 7.5). For wach orifice position, the erzors increase rapidly in the upper subsoni= rance, reaci peak values at Mach numbers just beyond 1.0, and then decrease abriptly to values near zero. The initial increase in the error is caused by a shock that forms around the body at its maxizum diameter when the flow at that poire becomes sonic. This shock isolates the negative pressure region aiong the rear of the body, so that the pressures at the orifices are then deterained by the positive pressures along the nose section. When the free-strean flow becomes; sonic, a shock wave forms ahead of tie body (bow shock), and the error contiluris
 the orifices, the stati" pressure at the orifices becomes tiat of the Eree strean, because the pressure field of the body is then contined tin tio royion behind the snock. For all higher :ach numbers, the pressure aces ot the shock
 is that of the isolated tube.



 which is the cube error of the tree $A$ tute. In later tests of tro $\because=15$ rionatit

 numbers up te $2 . a 7$ (refs. 4 ant 3 ).



data on this figure also show a fairly consistent derrease in the error with increasing boom length, despite the variations in the shapes of the nose sections.

The variation with Mach number of the static-pressure error ahead of fuselages with nose inlets has been determined from both mudel tests (ref. 2) arin flight tests (ref. 7). The results of the two tests (figs. 7.8 and 7.9) shed

- the same general variation of the error in the transonic syeed range as for che $\mathrm{X}-1$ model in figure 7.5 and the $\mathrm{X}-1$ airplane in figure 7.6. The calibratiors of nose-boom installations on five other airplanes with nose inlets (fig. 7.in, from ref. 6) show the errors in the Mach range from 0.8 to 1.0 to rise shari:y in a manner similar to those for the airplanes on figure 7.7.


## Wing-Tip Installations

For a given position of static-pressure orifices ahead of a wing, the magnitude and variation of the error depend on the shape of the airfoil section, the maximum thickness of the airfoil, and the spanwise location of the boom. In ordez $=0$ lessen the influence of the pressure field of the fuselage, the change in the flow field about the wing die to flap deflection and landing-esar extension, and the effect of propeller slipstream or jet enyine exhaust, the static-pressure tube should be installed on the outboard span of the wing. Eor the installations to be described here, the booms were in all cases located iear the wing tip.

Tine magnitudes of the errors ahead of a wing tip are shown in figure $7 .: 1$ for six orifice locations expressed in terms of the maximum wing thickness $=$. The errors were measured with the airplane at $a$ low angle of attack $\left(C_{L}=0.2\right)$ at a :lach number of 0.30 (ref. 8). The test data show that the error is his: 0 .st at the position closest to the wing and it decreases rapidly to a ralue o ! percent $q_{C}$ at an orifice location of $x / t=10$. Beyond this point, surtior reduction in the error is minimal.

The distance $x / t=8$ for the wing in sigure 7.11 is the sare as the chord length of the wing at the spanwise location of the brom. fur a compat:son with the error at this location, the errors of l-chord installayions on nime other airplanes are ircluded in flgurs 7.11. The stavic-iressurs tube $\because$, all the installations was the salne (tuke $\dot{\text { i }}$ ) and the errors were all Dediar. $=$.
 wings differed, the static-pressure errors are all in the same rarro. Thus the shape of the airEoll section uprears to have little effect on six magat: did of the errors at a distance of 2 chord lerçt: (or greater) ainead os siec winc.

The variations of the errors in the lid tach ratge for each of sion six
 figure, the orifice locations are given in terms of the lucal wing ciorit $z$. For brom lengths of 1 :hord or greater, fie error is \%ery nearl gioneant $a$ :
 for all the boom lengtis become increasingly recation and reacil $\quad$ :alde o:
about -6 percent $q_{c}$ at the stall speed. For such large variations of the error over a small Mach range. the problem of applying rorrections for the errors would be quite difficult.

In order to show the relative decrease of the error with lift cocfinicient for comparable boom lengths of fuselage-nose and wing-tip installations, the calibration of the 1.50 brom of the airplane in figure 7.4 is compared in 6 igce 7.13 with that of a l-chord wing-tip boom on the same airplane. for both ne the installations, the static-pressure tube was the same (tube A) and the tests were conductad tinrough the same lift coefficient range. As shown by the two calibrations, the magnitude of the error of the fuselage nose installation is higher than that of the wing-tip installation, but the variation of the error with lift coefficient is considerably greater for the wing-tip installation. Thus, corrections for the errors of the nose-boom installation could be afthied more accurately, even though the magnitudes of the errors are nigher tian too: $\%$ of the wing-tip installation.

The variation of the errors of a wing-tip installation in the transonic speed raige can be described from the calibration of a l-chord installation on the $\mathrm{X}-1$ airplane (fig. 7.14. from ref. 3). It is apparent from this calibritic: that the variation of the error is the same as that for the fuselage-nose insta:lations up to the mach number at which the discontinuity due to shock passage occurs. At this point; however, the error falls to a large negative value and then, with increasing Mach number, begins to increase to positive values. The explanation for this behavior may best be illustrated by diagrans of the shock waves ahead of the airplane (fic. 7.15). At a sach number of 1.22 , the wing brio shock has passed the orifices, and thus has effectively isolated them from :he pressure field of the wing. The pressure at the orifices is then influenced b: the negative pressures aroind the rear portion of the fuselage nose, tioe affect of which extends outward frum the surface of the fuselage behind she rach cone. As the Mach number increases, the cone slants backward, and the orifices come under the influence of the positive pressures around the forward portion of the fuselage nose and behind the fuselage bow shock. it sone hither sach number, the fuselage bow shock traverses the orifices, which are then isnlated from the flow fields of both wing and fuselace. At this and hager mach sumber, :hut static-pressure error, like that for the faselage-nose installations, is the error of the $\cdots$ itself.
Unrtical-Fin Installitions

The factors that affect the measureme: 0 : stat: $=$ fressire shead of a vertical fin are similar to those for wincutip inistaliations. Zalitrafion: . a 0.55-chore rertical-fin installation at lat and aich siesoni= arods ar: ar
 a value that ss about 1 porcent lower than that for the j.5-chord wing-:it installation in Eigure 7.12. In the high subsonic vance, the mrror iforrane. with tazh numer in a manner similar to that for the wireg-tit, :ns:allation $1:$

 ormisecs.

## Fuselage-Vent Installations

For the purpose of selecting a location for static ports, the fuselact cat. . in a general way, be likened to a static-pressure tube. When the fuselage :s aligned with the flow, the pressure at a vent is determined by its locatio:. along the body, and when the fuselage is inclined to the flow, the pressur: : : dependent on the radial position of the orifine around the body. The presiare at any given point on the body may, of course, be modiried by the efferts , the wing or othe protuberance on the fuselage.

Because of the complex nature of the pressure distribution along rhe sisclage, it is difficult to predict, with any degree of certainty, those locat:ons where the statie-pressure errox is a minimum. It is customary, therefore. is make pressure-3istribution tests in a wind tunnel with a detailed replica $\operatorname{sit}$ t.-. aircraft and to shoose from the results a number of vent locations that affear promising. These locations are then calibrated on the full-scale airfraft ind the best locatica is choren for the operational installation.

In the midsabsonic speed range, the errors of the three stativ-pressur.tube installatios (fuselage nose, wing tip, and vertical finj are ir all :isa: positive. In coatrast, the errors of fuselage-vent installations ian er e:t:ie: positive or nega:ive. This fact is illustrated by the calibrations of the fuselage-vent systems on three tra:-sport airplanes fig. 7.i7, fron rizi. . .
 can vary with Math number in the same general wis\% as the error:s of the itut:-pressure-tube installations. For the installation on the turbojet trats:cy: shown in figure -18 (ref. 10), for example, the error rises in the dazin rane above 0.8 (due the blocking effect of the wing) in a maner similar 0 ocat for each of the static-pressure-tube installations.

With anothe: vent installation, for which the vents were located ust it:

 fuselage-vent system, however, the discontinut $\because$ in the calibration iours it i










 single large hol. (on the neder of $3 / \beta$ ia. if diantor or a



With the salt-shaker pattern, the measured pressures can be affected by deformations of the orifices as discussed in chapter VI. :or both types of ports, ter measured pressures can also be altered by changes in the contour of the fuselige skin in the vicinity of the port; such changes can result from damage caused =f ground handing, repairs to the skin, or aging of the aircraft.

The effects of simulated damage to the ports (in the form of protuberancis and changes in edge shape) and of skin waviness in the vicinity of the ports were determined in tests reported in reference 12 . The results of the tests (fig. $7.20(a))$ show that even relatively small deformations at the edge of the vent can produce sizable changes in the measured pressure. For a vent locate. $B$ close to a wave in the fuselage skin, the effects can also be appreciable (fig. $7.20(b)$ ). To avoid the possibility of the kind of skin waviness that can occur with thin skins and to provide a uniform vent configuration, some manufacturers install a thick plate having a machined surface that extends some $d=s$ tance around the vents. Such plates also provide a higher degree of consisterazy in the calibrations of a given type aircraft (ref. 10).

## Combined Calibrations at Low and High Altitudes

As mentioned earlier, the calibrations of installations at low and high altitudes usually are not joined (e.g.. fig. 7.16), because the low-altitude calibration is not carried to sufficiently high Mach numbers and the highaltitude calibration is not carried to sufficiently low Mach numbers. In ene case, howevor, the calibration of a wing-tip installation was extended down $t=$ the stall at a series of altitudes by means of a high-speed trailing bomb to described in chapter IK.

The calibrations at five altitudes are shown in figure 7.21 (fzom ref. 1-). For the sea-level calivration, the variation of the error with lift coetficie: = in the low Mach range is the characteristic variation expected of wing-tip installations. Of interest with this set of calibrations, however, is the fa== that the error variation at each of the altitudes above sea level is essentia: $\%$ the same. Of further interest is the fact that the calibrations all converge at a Mach number of about 0.75. At lach numbers beyond this point, where the errors are basically a function of lach number, the error variation for all:altitudes can be represented by a single curve.

In the lower Mach range where the error is primarily a furction of $1: 5$ coefficient (below $A=0.75$ for this installation), the lift sceffizion: su: a given value of the error should be the same at each altitude. For an error of -0.075 . For example, the lift coefficient at $A=1.9$ at sea level siould be the same as that at $: 1=2.3$ at $10000 £ t, A=2.7$ at $20200 \mathrm{f}=$ 。 $\mathrm{A}=\mathrm{z}$. Zomputations if the lift coefizicients at each altitude snow tha: the: are, in fact, approximately the same.

The primary deyendence of the static-pressure error on li: = sec: Eicier. :the lower :tach range has led a number of investigators to devise analyticai
 level calioration. in two mothods proposed by sritish investigators (rofs. $1 ;$

number as well as the lift coefficient at which the sea-level value was det $=$ mined. Other investigators have extrapolated the sea-level values on the $s=1 \mathrm{~m}$. assumption that the errors are dependent solely on lift oefficient. asen $c=$ these methods is limited, of course, to the Mach range below tiat at whica shocks form on the body.

An example of the application of the extrapolation method based only or th. iift wefficient dependence is shown in figure 7.22 (from ref. 16). In this example, the extrapolation of a sea-level calibration to 25000 ft is compar $=\mathrm{d}$ with the fight-test calibration at 25000 ft . The test data from which the sealevel and 25000 -foot calibrations were derived are discussed in chapecr iK. i.. indicated by the agreement between the measured and computed errurs at al-it $2 d e$. the simpler corputa"ional method would appear to be adequate for the predict=on of the errors $a=$ altitude.

## Calibration Presentations

The errors of the static-pressure installations described in this chapt $\leqslant$ r have in all cas:ss been expressed as fractions of the impact pressurc, as $\therefore$ f $\exists_{c}$. As noted in chapter $V$, howe:er, the static-pressure errors are sometimes ;resented as fractions of the static pressure, $\Delta \mathrm{p} / \mathrm{p}$, or the Mach number, $\mathrm{M} / \mathrm{l}$.

Comparabie values of $A \mathrm{p} / \mathrm{q}_{\mathrm{C}}, \mathrm{Ap} / \mathrm{p}$, and $\mathrm{A} / \mathrm{M}$ for a nypotiotical $\therefore$ variation based on calibrations of fuselage nose installations are shown in Eigure 7.23 for a lach number range from 0.2 (the stall speed) to 2.0. For :hi $\equiv$ example, the variations in terms of $\Delta p / \psi$ and $\Delta M / M$ were derived from tie $\Delta p / q_{c}$ variation by using figures 5.4 and 5.5.

In the tigh subsonic range (above $i \mathcal{A}=0.8$ ), the variation of the errors with Mach nurber for each of the three calibrations is roughly the same a:d =-e peak values of the errors are generally of the same magnitude. In the lod ssonic range, : owever, the variation of the error with lift coefzicrent, as sidew: by the decrease in the magnitude of the error from $A=0.4$ to 0.2 , is grtat- st for the $\therefore \mathrm{i} / \mathrm{q}_{\mathrm{c}}$ calibration and least for the ip/y calioriticn. In the su=tr-
 increase with increasing :Hach number.

Even though the position error of an installation in terns of $\% / \mathcal{A} \quad i=:$ supersonic range may be small, the altitude error corresponding ic the $\mathrm{i}=\mathrm{z}=$. pressure error can be quite large. For a value of $\therefore \mathrm{p}$; c , of 1 percent, for example, the aititude error at $M=2.0$ and an altitude of 40 :0\% Et is $35=\mathrm{E}$.

## Installation-Error Tolerances








The altitude errors zorrespondil. 7 to the civil ind military requiremerts for a Macn range up to 1.0 and for altitudes up to 40000 ft are presented in figure 7.24.

## Installation Design Consisierations

Erom a consideration of the variations of tie exrors of the four tyes static-pressure installations with lift coefficient and ach number, it snw: be evident that a primary consideration in the selection of an installation: a new aircraft is the Mach range through winct it is designed to operate.

If the operating range extends to supersonic speeds, the fuselage-nose installation is obviously the best choice, because the installation erfor a: supersonic speeds will be that of the tube itselz. The error of the tube a. supersonic speeds can be determined from wind-tunnel tests, so that the tlit calibration of the installation could be limited to the subsonic speed rang. The errors in the subsonic range might be zelatively large but the variati of the errors with Mach number and lift coefficient follows a consistent pa: for which corrections for the errors can be applied by means of air data computers to be described in chapter XI. The errors of fuselage-nose installat at subsonic speeds can also be minimized by use of specially designed conto: tubes to be discussed in the next chapter.

Aircraft designed for operations in the sutsonic speed rang.i ordinaril: cruise at Mach numbers below 0.9. For this llacin range, any of the other =h: installations - wing tip, vertical fin, and fuselage vent - should prove ia. factory. If the shape of the fuselage approximates that of a alreular ryil: satisfuctory locations - an usually be found in areas where the statia-presis. errors will be small and where the measured fressures will not be adversely affected by local shocks in the upper subsonic range. With the wing-tip an: vertical-fin installations, very salal (and consistent) errirs an be real:: when the boom length is about 0.5 chord length at the vertical in or 1 an: lengtin at the wing tip.

Nith all the installations, the pressure sensor should be desiurwed an: located to prevent obstruction of the static-pressure orifices or Euseluge by dejris, water ingestion, or ice. The distance of the pressure sour in : the rockpit should also be consiaered because long lengtis of pessure aibi san introduce pressure lag errors, a subject to be discu;sert in singrer $\because \because$. These consideratlons, Eogether with the many other fioctors that aust iu. af: into account in the design of pitot-static s?stems, are discusied w: $\therefore:=1:=$ able detail in refirence 13.

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Tube A


Fube E
Eigure 7.1.- Diagrams of static-pressure tukes used on aircraft installations.


Pigur: 7.2.- Static-press.re errors $=$ various iistances ainead 0 three bodies of revolution aligned with tie flcw at $\because=0.21$. (Adapted from rez. 1.)


Tube A
$D=$ Maximum effective fuselage diameter


Figure 7.3.- Static-pressure errors at three positions ahead of the fuselage nose of an airplane. (Adapted from ref. 8.)


Tube A


Figure 7.4.- Variation of static-pressure errors of fuselage-nose installations in low subsonic speed range. (Adapted from ref. 8.)



Figure 7.5.- Variation of static-pressure error ahead of a model of an airplane fuselage in the transonic speed range. (Adapted from ref. 2.)


Tube A


Figure 7.6.- Variation of static-pressure error of fuselage-nose installation in transonic speed range. (Adapted from ref. 3.)


Figure 7.7.- Calibrations of fuselage-nose installarions on five airplanes. (Adapted from ref. 6.)


Figure 7.8.- Variation of static-pressure error anead of model with nose inlet in transonic speed range. (Adapted from ref. 7.)


Figure 7.9.- Variation of static-pressure errors in transonic speed range of fuselage-nose installations on airplane with nose inlet. (Adapted from ref. 7.)


Tube A 1.82


Figure 7.10.- Calibrations of fuselage-nose installations on five airplanes with nose inlets. (Adapted from ref. 6.)


Tube A


Figure 7.11.- Static-pressure errors at various positions ahead of wing tips of ten airplanes. (Adapted from ref. 8.)



Figure 7.12.- Variation of static-pressure errors cf wing-tip installations in low subsonic speed range. (Adapted from 2ef. 8.)


Figure 7.13.- Variation of static-pressure errors of wing-tip and fuselage-nose installations of same boom length. (Adapted from ref. 8.)


Figure 7.14.- Variation of static-pressure error of wing-tip installation in transonic speed range. (Adapted from ref. 3.)


Figure 7.15.- Diagram showing position of shock
waves with respect to a wing-Eip installation in transonic speed range.


Figure 7.16.- Variation of static-pressure error of vertical-fin instaliation in lea and high subsonic spetd rance.


Figure 7.17.- Variation of static-pressure errors of fuselage-vent installations of three airplanes. (Adapted from ref. 9.)



Figure 7.18.- Variation of static-pressure error of a fuselage-vent installation in high subsonic speed range. (Adapted from ref. iv.)



Figure 7.19.- Variation of static-pressure error of a fuselage-vent installation in transonic speed range. (Adapted from ref. 11.)

(a) Effect of protuberances and indentations.


Figure 7.20.- Effect of protuberances and skin waviness on static pressures measured by a fuselage vent. (Adapted from ref. 12.)


Tube D


Figure 7.21.- Variation of static-pressure error of a wing-tip installation at five altitudes. (Adapted from ref. 13.)





Figure 7.24.- Altitude errors corresponding to allowable static-pressure errors of installations on civil and military aircraft. (Adapted from refs. 17 and 18.)
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## CHAPTER VIII

## AERODYNAMIC COMPENSATION OF POSITIION ERROR

For research-type static-pressure installations, corrections for the position errors are normally applied during the reduction of the test data after the flight. For service-type installations, corrections for the position errors are applied during the flight by means of correction cards or automatic computing systems (chapter II). With some service installations, however, the position errors are effectively canceled at the static-pressure source, so that the need for manual or automatic corrections is eliminated. This cancellation or reduction of the position errors at the static-fressure source is accomplished by applying the concept of aerodynamic compensation to be discussed in this chapter.

With fuselage-vent installations, the position errors of the original vent configuration are compensated by installing small ramps or projecting plates in the vicinity of the vents (ref. 1). These devices are designed to alter the local flow in such a way that the iocal static pressure at the vents is changed to a value more nearly equal to the static pressure of the free stream.

With static-pressure-tube installations, the conventional tube is replaced with a specially contoured tube, called a compensated tube, that is designed to nullify the position errors of the conventional tube installation. The shape of the compensated tube and the lonation of the orifices along the tube are so designed that the static-pressure errors of the tube are equal and opposite to the position errors of the conventional tube installation.

The concept of compensation of position error is illustrated in figure 8.1 by hypothetical calibrations of a fuselage-nose installation. The curve labeled "position error" represents the calibration of a conventional tube at a given position ahead of the iuselage nose, the curve labeled "compensated tuhe error" represents the variation of the static-pressure error of the isolated compensated tube, and the dashed line along the zero axis represents the calibration of the compensated tube when installed at the same position as the conventional tube.

In an investigation of compensated tubes designed to reduce the position errors of fuselage-nose installations in the subsonic speed range (ref. 2), the negative tube errors required to balance the positive position errors were created with a tube having a collar with a conical af ody and orifices at the base of the afterbody. In a more extensive investigat. n (ref. 3), the negative tube errors were developed with two types of tubes having ogival nose shapes. In one type, the orifices were located along the ogive near the nose, while in the other type they were located on a contoured contraction of the tube some distance behind the nose. With both types of tubes, the shape of the tube and the location of the orifices along the tube can be designed to compensate the position errors at a given position ahead of a fuselage having a given nose shape.

In the investigation of reference 3 , three compensated tubes (a long ogival tube, a short ogival tube, and a contoured contraction tube (fig. 8.2)) were tested on a body of revolution having an ogival nose shape. The calibration of the long ogival tube with its orifices 0.95 of the bcly diameter (0) abead of the body is shown in figure 8.3. The data for the curve labeled "position error" were obtained with a conventional (i.e., cylindrical) tube with orifices 10 tube diameters aft of the nose of the tube. The data obtained with the compensated tube (circular test points) show the position error to be effectively compensatf.d throughout the subsonic speed range. To determine how well the larger position errors at a shorter distance ahead of the body could be compensated, tests were conducted with the short ogival tube wich the orifices at a distance of 0.27 D ahead of the body. As indicated by the data from these tests (fig. 8.4), the position error for this location was also compensated throughout the subsonic speed range. In tests of the contoured contraction tube with orifices at a distance ahead of the body, comparable with that of the tube with the long ogival nose (fig. 8.5), the position error was compensated to the same extent throughout the subsonic speed range.

Since the tube errors of the compensated tubes are negative in the subsonic speed range, the position errors of the nose-boom installations in figures 8.3, 8.4, and 8.5 would be expected to become negative at the low supersonic speed at which the body bow shock traverses the orifices. In tests of the installations of figures 8.3 and 8.5 at low supersonic speeds, the position errors at a Mach number just beyond 1 were found to be -3 percent $q_{c}$ for the installation in figure 8.3 and -4 percent $q_{c}$ for the installation of figure 8.5.

However, for a tube having a shape similar to that of the long çival tube but with orifices nearer the nose (fig. 8.6 from ref. 4), the error is only -0.5 percent $q_{c}$ at the Mach number following shock passage ( $M \approx 1.01$ ). At $M=1.2$ the exror is still small, bit at $M=1.65$ the error is about. 1 percent $q_{C}$, a sizable error in terms of altitude error ( 550 ft , for example, at 40000 ft$)$.

In other tests in reference 3 , the nose of the long ogival tube was cut to form a pitot opening having a conical entry of $82^{\circ}$. Cutting the tip of the tube was found to change the error compensation by less than 0.3 percent $q_{c}$ at mach numbers up to 1.2.

In further tests of the long ogival tube, orifices were located at a radial station of $\pm 37.5^{\circ}$ to reduce the errors at positive angles of attack. The results of the tests of this tube (fig. 8.7) show the error to be essentially zero at angles of attack up to $15^{\circ}$ at a Mach number of 0.6 . Note that the errors on this figure are incremental errors from the error of the tube at an angle of attack of $0^{\circ}$.

Compensated static-pressure tubes similar to those tested in the investigation of reference 3 have been used on the fuselage-nose installations of $a=$ least three airplanes (refs. 4, 5, and 6). The calibration of an installation on an F-104 fighter is shown in figure $8.8(a)$, on a $B-70$ bonber in figure $\mathbf{3 . 8 ( b )}$. and on a British Harrier vtol airplane in figure 8.8(c). For each of the installations, the static-pressure errors with the compensated tubes are within about

1 percent $q_{\text {. }}$ throughout the subsonic speed range. The tubes used on these installations were pitot-static tubes with pitot openings similar to that of the tube in figure 8.6.

Although compensated tubes have been designed to minimize the errors of fuselage-nose installations at Mach numbers as high as 1.2 , the errors of these tubes would be expected to be larger than those of conventional tubes at higher supersonic speeds. As a means of achieving small errors at both subsonic and supersonic speeds, it was suggested in reference 3 that a tube could be designed

-     - that would combine the features of the compensated tube for subsonic operation and the conventional tube for supersonic operations. With this type tube, one set of orifices would be located on the ogival nose of a cylindrical tube and a second set of orifices at least 10 tube diameters aft of the nose. A tube of this type would, of course, require an automatic pressure switch which would be activated at the speed at which the shock passes over the rear set of orifices.


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Figure 8.1.- Illustration of concept of aerodynamic compensation of position error.

(a) Long ogival tube.

(b) Short ogival tube.

(c) Contoured contraction tube.

Figure 8.2.- Diagrams of compensated static-fressure tubes. (Adapted from ref. 3.)


Figure 8.3.- Calibration of long ogival tuie with orisices 0.95D ahead of body. (Adapted from =ef. 3.)


Figure ©.4.- Calioration of short ogival tube with orifices 0.27 D anead of body. (idafted from ref. 3.)


Figure 8.5.- Calibration of contoured contraction tube with orifices 0.79 D ahead of body. (idarted from ref. 3.)




Figure 8.7.- Variation of errors with angle of attack of compersated tubes with orifices encircling the tube and at a radial station of $\pm 37.5^{\circ}$. $M=0.6$. Errors on this figure are incremental errors from the errir at zero angle of attack. (Adapted from ref. 3.)

(a) F-104 airplane. (Adapted from ref. 4.)

(b) B-70 airplane. (Adapted from ref. 5.)

(c) Harrier airplane. (Adapted from ref. 6.)

Figure 8.8.- Calibrations of compensated static-pressure-tube installations on three airplanes.

CHAPTER IX

## FLIGHT CALIBRATION METHODS

The accuracy with which altitude, airspeed, and Mach number are determined from pitot-static measurements depends for the most part on the accuracy with which the position error of the static-pressure installation is established by a flight calibration of the installation...The accuracy. of airspeed and Mach number also depends on the accuracy of the total-pressure measurement, but as noted in chapter IV, the total-pressure error at low angles of attack is generally negligible. For flight tests in which accurate measurements of total pressure at high angles of attack are required, the total-pressure installation can be calibrated against a test installation (swiveling or shielded total-pressure tube) which is insensitive to angle of attack. Since the difference between the pressures of the two installations can be measured with a sensitive differential-pressure instrument, the errors of the aircraft total-pressure installation can be determined with a high degree of accuracy.

In contrast to the ease with which the total-pressure error can be determined, the position error of the static-pressure installation can be quite difficult to determine. This difficulty 15 reflected in the wide variety of calibration methods that have been devised for the determination of this error. These methods are first discussed in terms of the measuring principles that form the basis of the calibration techniques. Application of each of the methods is then described in terms of accuracies, operational limitations, and instrumentation requirements. In a final section, the calibration of an airplane installation by two of the methods is described in some detail.

Calibration Methods for Deriving Position Error
hs an introduction to the description of the various methods for determining the position error $\Delta \mathrm{p}$, the calibration techniquer are classified in terms of four parameters from which position error is derived: (1) free-stream static pressure $p$, (2) free-air temperature $T$, (3) true airspeed $V$, and (4) Mach number M. A listing of the calibration methods in accordance with this classification is as follows:

1. Free-stream static-pressure methods ( $\Delta \mathrm{p}$ derived from measurements of $p^{\prime}$ and $p$ )
(a) $p$ measured at reference pressure source

Trailing-bomb method Trailing-cone method Pacer-aircraft method
(b) $p$ derived from height of aircraft and measured pressure gradient

Tower method
Tracking-radar method
Radar-altimeter method
(c) $p$ at height of aircraft calculated from $p$ and $I$ at ground

Ground-camera method
(d) $p$ derived from change in height of airplane from initial height Tracking-radar/pressure-altimeter method Accelerometer method
2. Temperature method ( $\Delta \mathrm{p}$ derived from $T$ ' and pressure-temperature survey)

Recording-thermometer method
3. True-airspeed methods ( $\Delta \mathrm{p}$ derived from values of V )

Trailing-anemometer method
Speed-course method
4. Mach number methcds ( $\Delta \mathrm{p}$ derived frem values of $M^{\prime}$ and $M$ )

Sonic-speed method
Total-temperature method
Note that although the names given to most of the methods are based on specific measuring equipment, the measuring principles of some of the methods can be applied with other types of equipment.

For the free-stream static-pressure methods, $\Delta p$ is determined as the difference between the static pressure $p^{\prime}$ measured by the aircraft installation and the free-stream static pressure $p$ at the flight level of the aircraft. The four basic techniques for determining the value of $p$ at the flight level are illustrated by the diagrams in figure 9.1.

With the first of these techniques, $p$ is measured from a reference pressure source moving with the aircraft, but located where the effect of the pressure field of the aircraft is negligible. As shown in figure $9.1(a)$, the reference pressure source is either (1) a pressure sensor trailed below the a icraft (trailing bomb) or behind it (trailing cone) or (2) a calibrated static-pressure installation on another aircraft (pacer aircraft) flying alongzide the test. aircraft.

In the second technique (fig. $9.1(b)$ ), the vaiue of $p$ at the flight level 2 is obtained from an interpolation of the measured pressure gradient through the test altitude range. For the tower method, the pressure gradient is measured through a small heigit range near the ground, while for the
tracking-radar and radar-altimeter nethods, the gradient is determined through a wide height range at high altitudes.

In the third technique (fig. 9.1(c)), $p$ at the height $z$ of the aircraft is calculated from measurements of $F$ and $T$ at the ground and an assumed standard temperature gradient up to the flight level. To minimize the errors that might be introduced by the assumption of the standard temperature gradient, the height of the aircraft should be less than about 500 ft .
. With the fourth-technique (fig. $9.1(d)), p$ at the height $z$ of the aircraft is derived from (1) measurements of the change in height from an initial height. (2) measurements of $\mathrm{P}^{\prime}$ and $\mathrm{T}^{\prime}$ at the initial height and at an airspeed for which $\Delta p$ is known, and (3) either an assumption of a standard temperature gradient or an integration of equation (3.4). For the tracking-radar/ pressure-altimeter method, the height increment is determined from a tracking radar, whereas with the accelerometer method, the height increment is derived from measurements of the aircraft accelerations and attitude.

In the temperature method (recording thermometer), values of $\Delta p$ are determined from measurements of $p^{\prime}$ and values of $p$ derived from (1) measurements of $T^{\prime}$ and (2) a pressure-temperature survey of the test altitude range.

For the true-airspeed methods; values of $\Delta p$ are derived from measured values of $V, p^{\prime}, q_{c}^{\prime}$, and $T^{\prime}$. The values of $V$ are determined by two techniques: from measurements with a wind-driven anemometer suspended below the aircraft or by timed runs over a prescribed ground course.

With the Mach number methods, $\Delta \mathrm{p}$ is derived from values of $\Delta \mathrm{M}$, which are determined from measurements of $M^{\prime}$ and $M$. In the sonic-speed method, the values of $M$ are derived from measurements of $V$ and the speed of sound $a$, while in the total-temperature method, the values of $M$ are determined from measurements of $T^{\prime}$ and $T$ (derived from a temperature-height survey of the test altitude range).

Of the various methods outlined in the foregoing paragraphs, some can be applied only at low altitudes, while others can be applied only at high altitudes. For the low-altitude calibration methods, the maximum speed at which the tests can be conducted is restricted by the speed capability of the aircr: f at the test altitude or by some linitation in the calibration method. For the highaltitude methods, the speed range of the calioration is determined by the minimum and maximum Mach numbers at which the aircraft can be flown at the test altitude. Thus, for some airplanes, a complete calibration throughout the Mach range may require tests at a number of altitudes using more thar one calibration method.

With some of the methods, the tests must be conducted in steady, level flight, whereas with others, the tests can be conducted in dives and accelerated maneuvers as well as in level flight. In the first case, indicating instruments can be used for the measurement of the slight quantities, whereas in the second. recording instruments must be employed. Recording instruments provide measurements of the flight quantities against a time scale and, in addition, generally provide greater accuracy than indicating instruments.

In the following sections, the operational limitations (speed and altitude), instrumentations requiremerts, and accuracy (or precision) of each method are discussed in detail. As an aid in comparing the various calibration techniques, the characteristics of each method are sumarized in table 9.1. From an examination of this table, it is evident that the selection of a method for the calibration of an installation on a particular airplane requires consideration of a variety of factors, such as (1) the desired accuracy in the determination of $\Delta p$, (2) the speed and altitude range for which calibration data are required, and (3) the available instrumentation. In general, greater accuracy, and thus more complex instrumentation, is required for the calibrations of flight research installations than for the installations on service aircraft.

## Trailing-Bomb Method

With the trailing-bomb method, the static pressure measured by the aircraft installation is compared directly with the static pressure measured by orifices on a bomb-shaped body suspended on a long length of pressure tubing below the aircraft (refs. 1 and 2). With one type of bomb (fig. 9.2), the orifices are on the body of the bomb, while with another iype (fig. 9.3), they are in a staticpressure tube ahead of the bomb. The type of bomb shown in figure 9.2 is a weighted body ( 15 lb ), whereas the type shown in figure 9.3 has small wings set at a negative angle of incidence to keep the bomb below the aircraft. Both types are equipped with vanes on the afterbody to keep the orifices aligned with the airflow.

Since a trailing bomb, like static-pressure tube, may have static-pressure error, this error shculd be determined (by calibration in a wind tunnel) so tiat corrections for the error can be applied. For both of the bombs in figures 9.2 and 9.3. the static-pressure error is 0.5 percent $q_{c}$.

The length of tubing required to place the bomb in a region where the local static pressure approximates free-stream static pressure was shown in reference 1 to be about 2 times the wing span of the aircraft (fig. 9.1ia)). Since the bomb is below the aircraft, the static pressire at the bomb is higher than the static pressure at the flight level of the aircraft. However, as the decrease in pressure with height inside the suspension tubing is the same as that of the outside air, the pressure measured by the instrument in the aircraft is the pressure at the flight level.

The accuracy with which $\Delta p$ is determined with the trailing-bomb method depends on (1) the accuracy of the measurement of the difference between $p$ ' and the local pressure $p_{2}$ at the bomb and (2) how closely the value of $p_{2}$ approximates $p$. Sinct $\Delta p$ is very smail compared with $p^{\prime}$ and $p_{2}$, the difference between the two pressures is measured most precisely with a sensitive differential-pressure indicator or recorder.

With trailing bombs, calibrations can be conducted through a wide range of altitudes and through a speed range from the stall speed to the maximum speed at which the bomb can be towed. This limiting speed is determined by the speed at which the suspension tubing develops unstable oscillations (ref. 3). For the
bomb in figure 9.2, instability of the suspension tubing is encountered at a Mach number of about 0.4. The bomb in figure 9.3, on the other hand, has deen towed successfully at Mach numbers as high as 0.85 (at an altitude of 38000 ft ).

The accuracy of the trailing-bomb method with the equipment used in the tests of reference 4 varied from about $\pm 2.0$ percent $q_{c}$ at 60 knots ( $M=0.1$ ) to about $\pm 0.2$ percent $q_{C}$ at 220 knots ( $M=0.35$ ).

## Trailing-Cone method

With the trailing-cone method (ref. 5), the static pressure measured ty the aircraft installation is compared with the pressure measured by a set of orifices near the end of a long length of pressure tubing trailed behind the aircraft (figs. 9.1(a) and 9.4). A lightweight drag cone is attached to the end of the tube to feep the tubing taut.

The accuracy with which free-stream static pressure is measured with a trailing onne system depends on the configuration of the cone system (size and shape of the cone ard position of the orifices ahead of the cone (ref. 6)), on the distance of the cone behind the aircraft, iad on the type of the aircraft (size, configuration, aki propulsion system). Because of the uncertainties associated with eact of these variables, trailing-cone systems have not been considered suitable for the basic calibration of an aircraft static-pressure installation. However, since the difference between the pressures of the cone system and the aircraft installation can be measured with good precision (i.e.., repeatability), a calibrated cone system is useful as a secondary standard for production line testing. In practice, a cone system at a given trail length behind a particular airplane is calibrated by methods such as the tower or tracking-radar methods for which values of the free-stream static pressure are determined with a higher degree of certainty. The calibrated cone system is then used for the periodic recalibration of the installation on that airplane or for the original calibrations on airplanes of the same model iref. 7).

With trailing-cone systems, calibrations can be conducted through a wide range of altitude and from relatively low speeds (defined by the minimum speed at which the pressure tubing trails straight back) to speeds as high as $M=1.5$ (ref. 8).

In unpublished tests of a variety of cone systems, conducted by NASA Langley Research Cencer, the precision of the measurement of $\Delta p$ was found to be $\pm 0.2$ percent $q_{C}$ at $M=0.7$ to 0.88 .

## Pacer-Aircraft Method

With the pacer-aircraft metiod, a measure of the free-stream static pressure is derived from the calibrated static-pressure installation of a pacer aircraft flying alongside the test aircraft being calibrated (refs. a and 10).

The difference $\Delta H$ between the altimeter indication $H^{\circ}$ in the test aircraft and the corrected altimeter indication $H$ in the pacer aircraft is found from equation (5.8):

$$
\begin{equation*}
\Delta H=H^{\prime}-H \tag{5.8}
\end{equation*}
$$

where $\Delta H$ is the altitude error. The pressures $p$, and $p$ corresponding to the values of $H^{\prime}$ and $H$ can be found in table $A 2$ of appendix $A$. The difference ietween $p^{\prime}$ and $p$ is then the position error $\Delta p$ for the test aircraft. The value of $\quad \Delta \mathrm{p}$. can also be found from the value of $\Delta \mathrm{H}$ and equation (3.6). An example of the determination of $\Delta p$ by the two procedures is given in part II of appendix B.

Since the value of $\Delta p$ (a small quantity) is determined as the difference between two large quantities ( $p$ ' and $p$ ), the altimeters in the two aircraft should be precision instruments which, to minimize hysteresis errors, should be calibrated only to the altitudes at which the tests are to be conducted. The precision with which $\Delta p$ is determined, however, depends not only on the accuracy of the two altimeters, but also on the degree to which the two aircraft. maintain formation flight. At very low speeds, the precision of the measurements generally deteriorates because of an inability to maintain formation flight. At high speeds, on the other hand, where speed and position control are more precise, the value of $\Delta p$ can be determined with good precision ( $\pm 0.2$ percent $M$ for $M$ ip to 1.0 and altitudes up to 35000 ft (ref. 10)). The corresponding precision in terms of $\Delta p / q_{c}$ is about $\pm 0.7$ percent at $M=0.5$ and about $\pm 0.2$ percent at $M=1.0$.

For best results with the pacer-aircraft method, the speed capability of the pacer aircraft should be very nearly that of the test aircraft. The speed range of the calibration tests is limited to speeds well above the stall of either aircraft and to the maximum level-flight speed of either aircraft.

In a variation of the pacer-aircraft method, a reference aircraft is flown at constant altitude at a low airspead for which the position error is known (refs. 11 and 12). The test aircraft is then flown past the reference aircraft in a series of level-flight, constant-speed runs. The indications of the altimeters in the two aircraft are noted at the instant the test aircraft flies past, and the position error of the test aircraft is determined from the difference between the indications of the two altimeters.

The reference-aircraft method differs from the pacer-aircraft method in that the installation in the reference aircraft requires a calibration at only one airspeed, and the speed range of the calibration of the test aircraft is no: limited to the speed capabil:ty of the reference aircraft.

The accuracy of this method is generally lower than that of the paceraircraft method because of the difficulty in synchronizing the altimeter indications in the two aircraft and because the height of the test aircraft at the time of the fly-by may differ from that of the reference aircraft.

## Tower Method

For calibrations with the tower method, the aircraft is flown at reastant speed and constant altitude past the top of a tall tower (ref. 11). For each test run, the position error $\Delta p$ is determined as the difference between (1) the static pressure $p^{\prime}$ as measured by the cockpit altimeter at the instant the aircraft passes the tower and (2) the free-stream static pressure $p$ at the height of the aircraft determined by interpolation of measured values of $p$ at a number of points along the tower height (fig. 9.1(b)).

A movie camera mounted with the axis of the lens aligned with the horizontal is often used to determine the airplane height. With this technique, the height increment $\Delta z$ of the airplane with respect to the lens axis is computed from the equation:

$$
\begin{equation*}
\Delta z=\frac{l}{l^{\prime}} \Delta z \tag{9.1}
\end{equation*}
$$

where $l$ is the length of the aircraft, $l^{\prime}$ is the length of its image, and $\Delta z$ is the displacement of the image from the center line of the film frame. The aircraft height 2 is then determined from the elevation of the camera and the height increment $\Delta z$.

It may be noted that precise measures of $\Delta z$ are more important in determining $\Delta p$ in terms of $\Delta p / q_{c}$ than in terms of $\Delta p / p$, For an error of 1 ft in $\Delta z$, for example, the error in $\Delta p / p$ would be only 0.004 percent, whereas the error in $\Delta \mathrm{p} / \mathrm{q}_{\mathrm{c}}$ would be 1 percent at 50 knots, 0.2 percent at 100 knots, and 0.1 percent at 150 knots. The reference point on the aircraft for the $\Delta Z$ measurements should be the vertical position of the altimeter in the aircraft.

For accurate measurements of $p^{\prime}$, the cockpit altimeter should be a precision instrument, and to minimize hysteresis errors, the laboratory calibration of the instrument should be limited to an altitude range only slightly greater than the tower height. Since the altimeter is used to measure pressure rather than altitude, it is convenient to calibiate the instrument as a pressure gage, that is, in terms of pressure versus altimeter indication.

The accuracy of the tower method depends primarily on the accuracy of the pressure measurements $p^{\prime}$ and $p$, since the height measurements (aircraft and pressure gradient) can be measured with good accuracy. To retain the advantage of the limited-range calibration of the altimeter in the laboratory, the height of the aircraft during the calibration tests should at all times be restricted to the same limited altitude range.

The speed range fcr calibrations by the tower method is limited to airspeeds well above the stall speed and up to the maximum level-flight speed of the aircraft at the tower height.

In tests (unpublished) of the tower method at the NASA Langley Research Center, the accuracy of the measurement of $\Delta p$ was found to range from $\pm 1.0$ percent $q_{C}$ at 90 knots $(M=0.15)$ to $\pm 0.2$ percent $q_{C}$ at 190 knots $(M=0.3)$.

## Tracking-Radar Method

With this high-altitude calibration method, the position error $\Delta_{y}$ is determined as the difference between the measured static pressure $p^{\prime}$ a.ld the free-stream static pressure $p$ which is determined from measurements of the height of the aircraft by the tracking radar and from a pressure-heigh: survey of the test altitude range (ref. 13:.

The pressure-height survey is conducted prior to the calibration tests in one of two ways: (1) by tracking a radiosonde (transmitting pressure measurements! as it ascends through the test altitude range or (2) tracking the aircraft through the test altitude range while flying at a low indicated airspeed for which the position error $\Delta p$ is known from a calibration by a low-altitude method (fig. 9.1(b)). With the aircraft tracking procedure, the value of $p$ at each height is determined from equation (2.2) expressed here as

$$
\begin{equation*}
p=p^{\prime}-\Delta p \tag{9.2}
\end{equation*}
$$

where $p^{\prime}$ is the sta!.ic pressure measured by the aircraft installation and $\Delta p$ is the position error of the installation at the airspeed of the ascent.

For the higher speeds of the calibration test runs, the height of the aircraft, is measured cceitinuously by the tracking radar. The positior error $\Delta p$ at the test airspeed is then determined from equation (2.2) here restated as

$$
\begin{equation*}
\Delta p=p^{\prime}-p \tag{2.2}
\end{equation*}
$$

where $p^{\prime}$ is the pressure of the aircraft installation during the test run and $p$ is the free-stream static pressure at the height of the aircraft determined from the pressure-height survey. Because the pressure-height relation may change during the period of the tests, it is advisable to repeat the survey at the conclusion of the test runs.

With the tracking-radar metiod, calibrations can be conducted in dives as well as in level flight. The accuracy of the method, as determined by calibration tests to be described later in the chapter, is about $\pm 0.2$ percent $q_{c}$ at $i=0.5$ and $\pm 0.1$ percent $q_{c}$ at $M=0.88$.

It may be noted that this calibration method has also been used with other types of ground-tracking equipment such as the radar-phototheodolite of references 14 and 15 and the phototheodolite of reference 16.

## Radar-Altimeter Method

Gith t:us high-altitude method, the position errcs of the aircraft installation is derired from the height of the aircraft measared with an onboard cadar altim:in $=-\boldsymbol{d}$ from a pressure-height survey of the test altitude range (ref. 17). The pressure-ht'ght survey is conducted by flying the aircraft at a low, constant
airspeed for which the position error is known from a calibration by one of the low-altitude methods.

Because of the height-measuring characteristics of the raciar altimeter, the calibration tests are restricted to level-flight runs and to test areas over a level ground reference plane, such as a large body of water.

The accuracy of the method at a Mach number of 0.8 and an altitude of 30000 ft is about $\pm 1$ percent $\mathrm{q}_{\mathrm{C}}$ (ref. 17).

## Ground-Camera Method

For calibrations with this method, the aircraft is flown in a series of constant-speed, level-flight runs over a camera located on the ground (ref. 13). For each tesi run, the position error $\Delta p$ is determined as the difference between (1) the static pressure $p^{\prime}$ measured by the aircraft installation when the aircraft is directly above the camera and (2) the free-stream static pressure $p$ computed from the measured height of the aircraft, measured values of $p$ and $T$ at the camera station, and the assumption of a standard temperature gradient. The height of the aircraft above the camera is calculated on the basis of the dimensions of the aircraft and its film image and the focal length of the camera lens (fig. 9.1(c)).

The calibration tests with the camera method are limited so speeds well above the stall and up to the maximum levei-flight airspeed of the aircraft at the heigint of the tests. Since the application of the method requires the assumption of a standard temperature gradient, accurate measurements of the froe-stream static pressure can be realized at heights no gr .eer than about 500 ft .

The accuracy of the method, as determined in calibration tests to be described later in the chapter, is about $\pm 0.2$ percent $q_{C}$ at 200 knots $(M=0.3)$ and $\pm 0.1$ percent $q_{c}$ at 320 knots $(M=0.5)$.

In another method for determining the height of an aircraft with a camera, a movie camera is installed in the aircraft with the camera lens sesing downard (ref. 18). The camera photographs reference marks on a runwe? as the aircraft flies at a constant speed and altitude along the rusway. Its height above the runway is then cetermined from the geometry of the camera lens system as in the ground-camera niethod.

With another calibration technique for measuring aircraf: heights near the ground, the height is determined from $\pi$-asurements of eleratisn angles with a theodolite (ref. 19). With two theodolites located an equal sistance on each side of a ground course, the height of an aircraft flying at conssant altitude along the ground course is determined from the intersection of the two lines of sight to the aircraft. The theodolite used in the tests of reference 14 was a simple angle-measuring device called a sighting stand.

## Tracking-Radar/Pressure-Altimeter Method

For calibration tests with this high-altitude method, the dircraft is first stabilized at a selected height and at a low airspeed for which the position error $\Delta p$ is known from a calibration by one of the lowaltitude methods. The aircraft is then accelerated at a constant altitude (constar.t p') indicated by the cockpit altimeter (ref. 10). During the calibration test run, the variation of $\Delta p$ with airspeed causes the pilot to vary the height of the aircraft in order to maintain constant $p^{\prime}$. At any given airspeed, therefore, the change in height corresponds to a change in free-stream static pressure from which the position error $\Delta p$ can be determined from the following equation:

$$
\begin{equation*}
\Delta p=p_{1}^{\prime}-\left(p_{1}-\delta p\right) \tag{9.3}
\end{equation*}
$$

where $p_{1}^{\prime}$ is the initial (and constant) value of the static pressure measured by aircraft installation, $P_{1}$ is the free-stream static pressure at the initial height, and $\delta p$ is the change in free-stream static pressure corresponding to the change in beight (fig. $9.1(\mathrm{~d})$ ).

The initial height $z_{1}$ of the aircraft and the change in height $\Delta z$ from the initial height are determined from continuous measurements with a tracking radar. The free-stream values of $p, g_{C}$, and $T$ at the initial height are determined from the initial indicated values $p^{\prime}, q_{C}^{\prime}$, and $T^{\prime}$ corrected for the known position error $\Delta p_{1}$ at the initial airspeed. The pressure increment $\delta p$ corresponding to a height increment $\Delta z$ is computed from equation (3.3), expressed here as

$$
\begin{equation*}
\delta p=-g \rho_{1} \Delta z \tag{9.4}
\end{equation*}
$$

where $\rho_{1}$ is the density at the initial height and is calculated from equation (3.1), expressed here as

$$
\begin{equation*}
\rho_{1}=\rho_{0} \frac{P_{1} T_{0}}{P_{0} T_{1}} \tag{9.5}
\end{equation*}
$$

where $P_{0}$ and $T_{0}$ are the standard sea-level values.
Since $p_{i}^{\prime}=p_{1}+\Delta p_{1}, p_{i}^{i}$ can be substituted in equation (9.3) to yield

$$
\begin{equation*}
\Delta p=\Delta p_{1}+\delta p \tag{9.n}
\end{equation*}
$$

Since the values of $p$ euring the calibration test run are based on a constant value of $\rho$ determined at the initial height, the accuracy in the determination of $\delta$ p varies with $\Delta z$. Whenever $j z$ is too great for accurate deteminations of $\delta p$ from a single initial height, successive sets of initial conditions can be established at various points during the flight.

The accuracy of this method, as determined in the tesis =eported in reference 10, varies from about $\pm 0.01 \mathrm{M}$ at $M=0.5$ to about $\pm 0.02 \mathrm{M}$ at $\mathrm{M}=3.0$. The corresponding errors in terms of $\Delta p / q_{c}$ are $\pm 3.5$ percent and $\pm 0.1$ percent.

## Accelerometer Method

… In the accelerometer method (ref. 20), the value of $\Delta \mathrm{p}$ is deternined from the seasured static pressure $p^{\prime}$ and the free-stream static pressure $p$ calculated from the value of $p$ at an initial reference height. The value of $p$ at the reference height is established by flying the aircraft at a constant, low airspeed for which the position error $\Delta p$ is known from a calibration by a low-altitude method. The change in $p$ from its initial value is derived from the change in height from the initial height which is calculated from measurements of the accelerations and pitch attitude of the aircraft (fig. 9.1(d)).

The application of the method is restricted to vertical-plane maneuvers from the initial stabilized condition. During the maneuver, the va=iation of $p$ with height $z$ is obtained from equation (3.4):

$$
\begin{equation*}
d p=-\frac{p}{R T} d z \tag{3.4}
\end{equation*}
$$

The value of $T$ can be derived approximately from the measured temperature $T^{\prime}$ and equation (3.28). Since the value or $M$ in this equation is not known, the value of $T$ at any given airspeed in the test run ca.. be stated in terms of $M^{\prime}$ as follows:

$$
\begin{equation*}
T=\frac{T^{\prime}}{1+0.2 \mathrm{KM}^{\circ} 2} \tag{9.7}
\end{equation*}
$$

where $K$ is the recovery factor of the temperature probe and $\gamma$ in equation (3.28) is 1.4. Since the use of $M^{\prime}$ in equation (3.28) results in a small error in the value of $p$ in equation (3.4); two or more approximations may be necessary.

The integration of equation (3.4) results in : $:=$ following :quation:

$$
\begin{equation*}
\left(\frac{p}{p_{1}}\right)^{n}=1-n \int_{Z_{1}}^{Z}\left(\frac{p}{p_{1}}\right)^{n} \frac{d Z}{\bar{R} T} \tag{9.8}
\end{equation*}
$$

where the subscript 1 refers to initial conditions.
Substitution of $p$ ' for $p$ in the right side of equation (9.8) and further substitution of equation (9.7) for $T$ results in

$$
\begin{equation*}
\left(\frac{p}{p_{1}}\right)^{n}=1-n \int_{z_{1}}^{z}\left(\frac{p^{\prime}}{p_{1}}\right)\left(\frac{1+0.2 K M^{2}}{R T^{\prime}}\right) d z \tag{9.9}
\end{equation*}
$$

The values of $n$ may be selected so that only one ayproximation is required for the detemination of $p$ (appendix $A$ of ref. 20). For a value of $k$ sear unity and for subsonic and low supersonic speeds, a value of $n$ of $\frac{\gamma-1}{\gamma}$ or 0.286 gives satisfactory results.

The change in height $d z$ in equation (9.9) may be determined f:om the vertical velocity computed from (1) values $C f P^{\prime}$ and $T$ for an initial condition where $\Delta p$ is known and (2) the vertical acceleration computed from measurenents of normal and longitudinal acceierations and pitch attitude angles:

$$
\begin{equation*}
d z=\left(v_{1}+\int_{t_{1}}^{t} a_{v} d t\right) d t \tag{9.10}
\end{equation*}
$$

where $t$ is time and the initial vertical velocity $v_{1}$ is

$$
\begin{equation*}
v_{1}=\frac{-\bar{R} T_{1}}{P_{1}}\left(\frac{d p}{d t}\right)_{1} \tag{9.11}
\end{equation*}
$$

and

$$
\begin{equation*}
a_{v}=a_{z} \cos \theta-a_{x} \sin \theta-g \tag{9.12}
\end{equation*}
$$

where $a_{y}$ is the vertical acceleration, $a_{z}$ is the normal acceleration, $a_{x}$ is the longitudinal acceleration, and $\theta$ is the pitch attitude angle of the aircraft.

For any given instant during the calibration test run, the difference between the value of $p$ determined from equation (9.9) and the measured value of $p^{\prime}$ is the position error $\Delta p$ of the aircraft installation at that instant.

The application of the accelerometer method requires the continuous measurement of $p^{\prime}, q_{C}^{\prime}, T^{\prime}, a_{z}, a_{x}$, and $\theta$ against a time scale. The pressures, temperatures, and accelerations should be measured with research-type recording instruments. For the measurement of $T^{\prime}$, the recovery factor $K$ of tie temperature probe should be very nearly 1.0. The attitude angle $G$ can be measured with a horizon camera, a Sun camera, or an attitude gyroscope. A detailed discussion of the problems associated with the use of each of the three attitudeargle measuring instruments is given in reference 20.

The accuracy of the method depends primarily on the accuracy in tiae determination of $\theta$ and the accuracy of the acceleration measurements. In a flight evaluation of the accuracy of the method (ref. 20), the position error $1 p$ of an aircraft installation was determined with an accuracy of about $\pm 0.5$ percent $q_{c}$ in shailow dives from an altitude of 31000 ft at Mach numbers from 0.6 to 0.8 .

With the restriction that maneuvers during the test runs be conducted in a vertical plane, calibration data can be obtained with the aircraft in level flight, climbs, dives, push-downs, pull-outs, or any combination of these maneuvers. The test maneuver should cover as short a time interval as practical (less than 2 minutes) in order to avoid an accumulation of errors in the measurements.

## Recording-Thermometer Method ${ }^{-}$

With this high-altitude method, values of $\Delta p$ are deiermined from values of $p^{\prime}$ measured by the aircraft installation and values of the free-stream static pressure $p$ derived from a pressure-temperature survey of the test altitude range (ref. 21).

The $p / T$ relation is dei.symined by flying the aircraft at a low airspeed for which the value of $\Delta p$ of the static-pressure installation is known from a calibration by a low-altitude calibration method. The value of $T$ at the survey airspeed is determined from measurements of $T$ ' and equation (3.28) with $\gamma=1.4$ :

$$
\begin{equation*}
T=\frac{T^{\bullet}}{1+0.2 K M^{2}} \tag{9.13}
\end{equation*}
$$

where $K$ is the recovery factor of the temperature probe and $M$ is derived from values of $q_{c}^{\prime}$ and $p^{\prime}$ (both corrected for the value of $\Delta p$ at the survey speed). As noted in chapter III, the use of equation (3.28) requires that $K$ be near unity.

For the calibration test runs, continuous recordings are made of $p^{\prime}, q_{c}^{\prime}$, and $T^{\prime}$. Then, at any given instant during the test run, the value of $p$ can be obtained as a function of $T$ from the measured value of $\mathrm{F}^{\prime}$. equation (9.13), and equations (3.23) and (3.24), expressed here (with $\gamma=1.4$ ) as

$$
\frac{p_{t}}{p}=\left(1+0.2 M^{2}\right)^{3.5}
$$

and

$$
\begin{equation*}
\frac{P_{t}}{p}=1.2 M^{2}\left(\frac{5.76 M^{2}}{5.6 M^{2}-0.8}\right)^{2.5} \tag{9.14}
\end{equation*}
$$

where $p_{t}$ is derived from zeasured values of $p^{\prime}$ and $q_{c}^{\prime}$. Combining equations (9.13) and (9.14) and eliminating $M$ yields the following equarions:

$$
p=\frac{p_{t}}{\left[1+\frac{1}{K}\left(\frac{T^{0}}{T}-1\right)\right]^{3.5}}
$$

and

Equation (9.15) is an expression of anotrer $p / T$ curve which, when compared with the $p / T$ survey plot, yields an intersection that defines the values of $p$ and $T$ for the iest condition.

The accuracy of the recording thermometer method depends, for the most part, on the variation of the free-air temperature $T$ with time and distance (both vertical and horizontal; , on the value of the recovery factor $k$, and on the ascuracy with which $K$ is known.

The effects of atmospheric temperature variations can be minimized by conducting the calibration tests on days when the thermal currents at the test altitudes are very small or at altitudes where the thermal currents are negligibie (generally above 35000 ft ). The effects of air temperature variations can also be reduced by repeating the $p / T$ surveys at various times during the calibration tests. Since there is no temperature gradient at altitudes above 35000 ft , the accuracy of this calibration method improves appreciably at these altitudes. at altitudes below 35000 ft . for example, an error of $1^{\circ} \mathrm{F}$ in the measurement of $T^{\prime}$ at $:=0.8$ corresponds to an error in $M$ of about 0.02. Above 35000 ft , the error in $M$ for a temperature error of $1^{\circ} \mathrm{F}$ would be $1 / 3$ of this value.

For altitudes below 35000 ft , an error of 0.01 in the value of K (for K of unity) corresponds to an error in $M$ of about 0.01 at $M=0.8$. For higier altitudes, the error in $M$ is appreciably lower.

With pressure recorders having an accuracy of 0.25 percent of full scale, the combined error in the measurement of $p^{\prime}$ and $q_{c}^{\prime}$ produces an error in $:$ of about 0.004 at $M=0.8$ and 30000 ft (ref. 21).

The accuracy of the method at $M=0.8$ and an altitude of 30000 ft based on the errors given for $\mathrm{I}^{\prime}, \mathrm{K}, \mathrm{P}^{\prime}$, and $q^{\prime}$, is estimated to be about $\pm 2.3$ percent $M$. The corresponding error in $\Delta p / q_{c}$ is about $\pm 4.5$ percent.

## Trailing-Anenometer Method

With this calibration method, the position error $A p$ of the aircraf $=$ installation is derived from measured values of true airspeed $v$, impact
pressure $q_{C}^{\prime}$, static pressure $P^{\prime}$, and air temperature $T^{\prime}$. The true airspeed is measured with a wind-driven anemometer suspended on a long cable below the aircraft (ref. 22).

For speeds below $M=0.2$, the effects of compressibility are sufficiently small that $q_{c}$ can be approximated (within 1 percent) by $q$. Therefore, from equation (3.10),

$$
\begin{equation*}
q_{c}=q=\frac{1}{2} \rho v^{2} \quad(m \leq 0.2) \tag{9.16}
\end{equation*}
$$

In equation (1.1), $p_{t}$ can usually be considered correct, so that

$$
\begin{equation*}
q_{c}^{\prime}=p_{t}-p^{\prime} \tag{9.17}
\end{equation*}
$$

From equation (2.2),

$$
\begin{equation*}
p^{\prime}=p+\Delta p \tag{9.18}
\end{equation*}
$$

By combining equations (9.17) and (9.28).

$$
\begin{equation*}
q_{c}^{0}=p_{t}-(p+\Delta p) \tag{9.19}
\end{equation*}
$$

Then, since $q_{c}=p_{t}-p$,

$$
\begin{equation*}
q_{c}=q_{c}^{\prime}+\Delta p \tag{9.20}
\end{equation*}
$$

Equation (9.16) can then be written as

$$
q_{c}^{\prime}+\Delta p=\frac{1}{2} \rho v^{2}
$$

With the substitution of equation (3.2),

$$
\begin{equation*}
p=\frac{P}{R T} \tag{3.2}
\end{equation*}
$$

for $p$ in equation (9.21),

$$
\begin{equation*}
q_{c}^{0}+\Delta p=\frac{p V^{2}}{2 R T} \quad(1!\leqq 0.2) \tag{9.22}
\end{equation*}
$$

With the further substitution of $p^{\prime}-\Delta p$ for $p$ (eq. (9.2)) and $T$ for $T$ (since, for $M \leqq 0.2, T^{\prime}=T$ ), equation (9.22) becomes

$$
q_{c}^{\prime}+\Delta p=\frac{\left(p^{\prime}-\Delta p\right) v^{2}}{2 R T^{\prime}}
$$

The position error $\Delta p$ can then le found from the following equation:

$$
\Delta \mathrm{p}=\frac{\frac{\mathrm{p}^{\prime} \mathrm{V}^{2}}{2 R T^{\prime}}-q_{c}^{\prime}}{1+\frac{v^{2}}{2 R T^{\prime}}}
$$

The anemometer assembly of reference 22 consists of (1) a small six-blaced, low-inertia propeller that activates a self-generating tachometer, (2) a lowdrag housing with tail fins to keep the body aligned with the airstream, and (3) a support cable that transmits the tachometer signals to a magnetic tape recorder in the aircraft (fig. 9.5).

The rotational speed of the anemometer propeller is proportional to true airspeed. Accurate measurements of true airspeed are realized, however, only when the anemoter is trailed in a region where che local velocity is tiat of the free stream, that is, where the velocity induced by the flow around the aircraft is zero (or nearly so). An example of an induced velocity field below an airplane is presented in figure 9.6 as contours of constant velocity ratios $u / V$, where $u$ is the horizontal component of induced velocity. The verical and horizontal distances below the airplane are given in terms of the fractions $2 / b$ and $x / b$, where $b$ is the wing span. Also shown in the figure are anemometer positions (with a 100-ft cable length) for the aixplane at a low speed with flaps down and at a high speed with flaps up. For both anemometer positions, the irduced velocity is essentially zero and, since $v_{l}=v-u$, the local velocizy, is very nearly the free-stream velocity.

The usable speed range of the anemometer system of figure 9.5 is from 7 knots to about 165 knots (the speed at which the suspension cable develops unstable oscillations). Because of the $M=0.2$ limitation of this metiod, however, the maximum speed of the calibration tests is restricted to airspeeds of about 130 knots at altitudes near sea level.

In tests of the anemometer of figure 9.5 with impact pressure recorders of widely differing sensitivities, the accuracy of the calibration tes $s$ with the most sensitive recorder was $\pm 0.5$ knot at 40 knots, while that with re least sensitive recorder was $\pm 3.0$ knots at 50 knots. The effect of this single element of the instrumentation on the accurac: of the test results illustra=es the fact that the stated accuracy of a calibration method is dependent not only on the inherent accuracy of the calibration technique, but also on the a=curacies of each of the component instruments. For an insight into the contribution of the various component errors for the anemometer tests of reference 22 , the reader is referred to table I of that zeport.

For the anemometer system having an accuracy of $\pm 0.5$ knot at 40 knots $(M=0.08)$, the accuracy at 100 knots $(M=0.16)$ was also $\pm 0.5$ knot. The corresponding accuracies in terms of $\Delta \mathrm{p} / \mathrm{q}_{C}$ are $\pm 2.5$ percent and $\pm 1$ percent.

## Speed-Course Method

The measured quantities and equations for the measurement of $\Delta p$ by the speed-course method are the same as those for the trailing-anemometer method. with the-speed-course method, however, the true airspeed is derived from measurements of the ground speed of the aircraft and the wind speed at the flight level (ref. 23).

The ground speed is determined by measuring the time for the aircraft to fly, in a constant indicated airspeed and altitude, between landmarks a known distance apart. The wind speed at the flight level can be measured by a windspeed indicator or the effects of the winds can be effectively canceled by flying a triangular course or by flying in opposite directions along a straightline course. For best results, the tests should be conducted when the wird speed is near zero, such as the period just after sunrise or before sunset.

The values of $q_{C}^{\prime}, p^{\prime}$, and $T^{\prime}$ needed for the solution of equation (9.24) can be derived from measurements with an airspeed indicator, pressure altimeter, and indicating thermometer. From values of the indicated airspeed $v_{i}$, the value of $q_{c}^{\prime}$ can be calculated from the equation,

$$
\begin{equation*}
q_{c}^{\prime}=\frac{1}{2} \rho_{o} v_{i}^{2} \tag{9.25}
\end{equation*}
$$

where the unit of $\rho_{0}$ is slugs per cubic foot and the unit of $v_{i}$ is feet per second.

The application of the speed-course method is limited to airspeeds well above the stall speed and up to maximum speeds defined by the $M=0.2$ limitation referred to in the preceding discussion, namely, about 130 knots at altitudes near sea level.

The accuracy of the method is largely dependent on the accuracy of the time measurements of the speed run, the constancy of the wind speed, and the constancy of the airspeed throughout the speed run.

Sonic-Speed Method
With the sonic-speed method (ref. 15), the position error $\Delta p$ is derived from the Mach number error $\Delta M$ which is defined as

$$
\begin{equation*}
\Delta M=M^{\prime} \sim M \tag{5.10}
\end{equation*}
$$

where $A$ is the free-stream Mach number and $M^{\prime}$ is the indicated Nach number which is derived from measurements of $q_{c}^{\prime}$ and $p^{\prime}$.

$$
\begin{aligned}
& \text { ORIGIN: } \\
& \text { OF POCR: JA }
\end{aligned}
$$

The value of $M$ is derived from equation (3.21):

$$
\begin{equation*}
M=v / a \tag{3.21}
\end{equation*}
$$

where $V$ is the true airspeed of the aircraft and $a$ is the speed ois sound at the level of the test runs. The true airspeed $v$ is determined from the ground speed of the aircraft and the wind speed ac the flight levei, and the speed of sound $a$ is derived fiom the free-air temperature $T$ at the flight level and equation (3.27).

For the calibration tests, the aircraft is flow in a series of constantspeed, level-flight runs during which the ground speed and the height of the aircreft are measured with a tracking radar. Prior to the test runs, the variations of wind speed and free-air temperature with height are determined by tracking a rawinsonde through the test altitude range.

The values of $\Delta M$ determined by this method can be converted to values of $\Delta p / p$ or $\Delta p / q_{c}$ by means of equations (5.4) through (5.7).

The accuracy of the method depends on the accuracy of the rawinsonde thermometer and the accuracy of the ground-tracking equipment in measuring the speed and height of the aircraft and the rawinsonde.

In calibration tests with the sonic-speed method using a radarphotothecdolite for ground tracking (ref. 15), the accuracy in the measurement of the ground speed of the airplane was found to be 50 to $75 \mathrm{ft} / \mathrm{sec}$. The accuracy of the measurement of wind speed was found to depend on the height and elevation angle of the rawinsonde from the tracking station; at a height of $50 \mathrm{co0} \mathrm{ft}$ and an elevation angle of $20^{\circ}$, the accuracy of the wind-speed measurement was 1.8 knots. The accuracy of the measurements of the height of the airplane and the rawinsonde was about 100 ft , and the accuracy of the temperature measured by the rawinsonde thermometer was about $1^{\circ} \mathrm{C}$.

In an analysis based on the foregoing accuracies, the accuracy in the measurement of Mach number was estimated, in reference 15, to be about 0.06M at $M=1.0$ and altitudes between 50000 and 80000 ft . The corresponding error in $\Delta p / q_{c}$ at $M=1.0$ is about 8 percent.

## Total-Temperature Method

With the total-temperature method (ref. 24), the position error $\Delta p$ is derived from $\Delta M=M^{\prime}-M$, where $M^{\prime}$ is determined from $q_{C}^{\prime}$ and $P^{\prime}$ and $M$ is calculated from equation (3.28) with $\gamma=1.4$, here expressed as

$$
\begin{equation*}
M=\sqrt{\frac{1}{0.2 K}\left(\frac{T^{\prime}}{T}-1\right)} \tag{9.26}
\end{equation*}
$$

where $T$ is the free-air temperature, $T^{\prime}$ the measured (or total) tempe:atre, and $K$ the recovery factor of the temperature probe. As noted in chapter III,
equation (3.28) is valid only when $K=1$ or when the probe is located in a region where the local velocity $V_{l}$ is equal to the free-stream velocity $v$. Since $V_{2}$ in the regions near the aircraft where a probe might be located is usually different from $V$, the application of this method requires, essentially, that the recovery factor of the probe be 1.

The calibration tests are conducted by flying the aircraft in a series of speed runs during which the height of the aircraft is measured with groundtracking equipment and $\mathbf{T}^{\prime}, q_{C}^{\prime}$, and $p^{\prime}$ are measured with recording instruments. The value of $T$ at the height of the test run is derived from a

- temperature-height survey which is made prior to the calibration tests by tracking a radiosonde (transmitting temperature measurements) through the test altitude range.

As in the case of the son:c-speed method, the values of $\Delta M$ derived from $M^{0}$ and $M$ can be converted to values of $\Delta p / p$ or $\Delta p / q_{c}$ by use of equations (5.4) through (5.7).

The accuracy of the calibration method depends, for the most part, on the accuracies in the measurement of $T^{\prime}$ and $T$.

In one series of calibration tests using the total-temperature method (ref. 24), the overall accuracy in the measurement of $T$ (including accuracies of radiosonde thermometer and ground-tracking equipment) was estimated to be $\pm 2.5^{\circ} \mathrm{F}$. The accuracy of the measurement of $T^{\prime}$ by the recording themmoter was about $\pm 1^{\circ} \mathrm{F}$. For these two accuracies in the temperature measurements, the accuracy of the value of $M$ was estimated to be about $\pm 0.02 \mathrm{M}$.

Ir a later series c.f tests (ref. 10), the accuracy of the determinition of $M$ was found to range from $\pm 0.01 \mathrm{M}$ at $M=1.5(30000 \mathrm{ft})$ to $\pm 0.04 \mathrm{M}$ at $M=3.0$ ( 60000 to 70000 ft ). The corresponding errors in terms $0=\Delta \mathrm{p} / \mathrm{q}_{\mathrm{C}}$ are $\pm 0.5$ percent and $\pm 2.0$ percent.

## Calibrations by Ground-Camera and Tracking-Radar Methods

In this section, a series of tests desigred to determine the accuracies that can be realized with the ground-ramera and tracking-radar iethods is desc:-ded. These two methods were selected for accuracy tests (ref. 13) because (1) the ground-camera method like the tower method provides accurate determinations of the free-stream static pressure at heights near the ground, while at the same time allowing greater flexibility in the choice of test heights and locations, and (2) the tracking-radar method, using the aircraft tracking procedure for measuring static pressure in the pressure-height survey, pzovides the most direct means of deriving precise measures of free-stream static pressure at high altitudes.

The tests of the two calibratica methods were conducted using a large turbojet transport as the test vehicle. The calibration tests with the groundcamera method were conducted at heights of about 500 ft and those with the tracking-radar method at altitudes of about 2500 ft .

Test instrumentation.- The pressure-measuring instruments used for poth calibration methods consisted of an airspeed-altitude recorder and a recording statoscope (fig. 9.7). The airspeed-altitude recorde= was connected to the service pitot-r,tatic installation of the airplane and the recording statoscope to the stat:c-pressure source (fuselage vents) of that installation.

The recording statoscope is a sensitive d:fferential-pressure instrument which, for these tests, measured the difference between the pressures from the fuselage-vent system and a constant reference pressure in a thermostatically controlled chamber. Since the reference pressure in the chamber could be fixed at any selected height, the difference between the static press re at that height and the static pressure at other heights cculd be measured more precisely with the statoscope than with the recording altimeter.

The pressures measured by both the recording statoscope and the airspeedaltitude recorder were recorded as traces along a moving photographic film. Each of the recorders was equipped with an event-marking device for synchronizing the measured pressures with the heights of the airpiane measiured with the ground camera or tracking radar.

The instrumentation for the ground-camera method consisted of a 5 by 5 in. single-exposure camera having a 7 -in. focal length, a mercury-in-glass thermometer, a precision altimeter, and a radio transmitter (fig. 9.8). The camera was mounted with its optical axis aligned with the vertical and was equipped with 3 sighting device to aid in photocraphing the airplane when it was directly overhead. By transmitting a radio signal the instant he actuated the camera, the photographer synchronizec the records oi the instrumanis $n$ the airplane with the photoyraph of the airplane. ht the time of each test run, the atmospheric pressure and temperature at the camera station were measured with the altimeter and the thermometer.

The precision-tra=hing radar was used for the gromnd-radar methe 3 (fig. 9.9). This $:$ edar provided measurements of elevation angle and slant range from which the geometiic height of the airplare could be complted. The elevation angle and slant range were recorded on a magnetic tape which. was synchronized with the reccrds af the airborne isstruments by radio signals.

Ground-camera tests.- With the airplanc at rest on the ground Fior to the test runs, the statoscope chamber was sealed and the pressure in the shamber recorded. The airplane was then flown over the camera at an altitude of about 500 ft at a succession of test airspeeds. When the airplane returned to the ground, the pressure in the st: ioscope was recosisd acain to measure any difference from the initial recording.

The pressure recorded by che statoscupe when the airplane is above the camera is the sum of (i) tia difference letween the static pressure at the ground level where the statoscope was sealed and the static pressure at the fligint level of the airplane and (2) the position error of the static-pressure installation.

As shown in figure 9.10, the flight level $z$ of the airplane is determined from the elevation $E_{c}$ of the camera station, the height $h_{c}$ of the camera lens al ove $E_{c}$, and the height $h$ of the airplane above the camera lens, measured at the level of the wing tips. For airplanes with wings tat flex upward in flight, the value of $h$ is adjusted by an amount $\Delta h$ to account for the deflection of the wing tips. The height $h$ is calculated from

$$
\begin{equation*}
h=\frac{b f}{b^{\prime}} \tag{9.27}
\end{equation*}
$$

where $b$ is the wing span of the airplane, $b$ '-the-span of the airplane image on the photographic film, and $f$ the focal length of the camera lens.

Since the reference height at which the statoscope is sealed is $z_{r}$, the difference between this height and the flight level is $z-z_{x}=\Delta z$. The decrease in the static pressure $\delta p_{c}$ through this height increment is computed fron equation (3.3) expressed here as

$$
\begin{equation*}
\delta p_{c}=-\bar{\rho}_{m} \Delta z \tag{9.28}
\end{equation*}
$$

where $\bar{D}_{m}$ is the density at tre midpoint between $Z_{r}$ and $Z$. The density at the midpoint is computed from the following equation:

$$
\begin{equation*}
\bar{\rho}_{m}=\bar{\rho}-\left(\bar{\rho}_{s}-\bar{\rho}_{s, m}\right) \tag{9.29}
\end{equation*}
$$

where $\bar{\rho}$ is the density at the cairera (determinea from measurements of $p$ and $T$ at that elevation), $\bar{\rho}_{s}$ is the standard density at the camera elevation, and $\bar{\rho}_{s, m}$ is the standard density at the midpoint.

The position error $\Delta p$ of the aircraft installation is then determined from

$$
\begin{equation*}
\Delta p=\delta p-\delta p_{c} \tag{9.30}
\end{equation*}
$$

where $\delta p$ is the pressure increment measured by the statoscope and $\delta p_{c}$ is the pressure increment computed from equation (9.28).

A sample calculation of the determination of $\Delta p$ by the ground-camera method is given in part $I$ of appendix $B$.

In the tests to determine the accuracy of the ground-camera method, four test runs were made at each of four airspeeds ( $150,200,260$, and 320 knots) during one flight and at two airspeeds during a second flight. Since the weight of the airplane varied by as much as 15 percent during a flight, the weight for each test run was computed (from indications of the fuel consumed) so that the static-pressure errors at each test speed could be compared directly on the basis of lift coefficient.

The results of the tests are presented in figure 9.11 in terms of the var: ation of the position error of the aircraft installation with lift coefficient. The standard deviation $\sigma$ of these data, determined from measurements of the displacement of the data points from the faired curve, is about $0.3 \mathrm{lb} / \mathrm{ft}^{2}$, which corresponds to an altitude error of about 4 ft at sea level. For this value of $\sigma$, the maximum probable error (defined as 3 times the standard deviatiou and having a probability of 99.7 percent) is about $2 \mathrm{lb} / \mathrm{ft} \mathrm{t}^{2}$, or about 12 ft at sea level. The corresponding error (lo) in terms of $\Delta \mathrm{p} / \mathrm{q}_{c}$ is $\pm 0.2$ percent at 200 knots $(M=0.3)$ and $\pm 0.2$ percent at 320 knots ( $M=0.5$ ).

The confidence with which the mean value of the data was determined is given by the following equation for a confidence level cL of 99 percent:

$$
C L_{99}=5.84 \frac{\sigma}{\sqrt{n-1}}
$$

where $n$ is the number of measurements for a given test condition. For the value of $c$ of 4 ft and for four measurements at each of the test airspeeds, the confidence level of ihe data is 10 ft . Thus, for a given position error i: terms of an altitude exror, the accuracy of the value of the altitude error, for a confidence level of 99 percent, is $\pm 10 \mathrm{ft}$.

Tracking-radar tests.- For the pressure-neight survey required of the tracking-radar method, the airplane was flown in a series of level-flight runs at each of three altitudes ( 24000,25000 , and 26000 ft ) through an area abo 10 miles in diameter. For each survey run, the geometric height of the airpla: was measured by the radar. Prior to the first survey run, the statoscope was sealed at an altitude of 24000 ft with the airplane at an indicated airspeed 200 knots. With the airplane remaining at 200 knots, survey runs were then ma at six locations at each of the three test altitudes. For each survey run, th. value of tie pressure measured by the statoscope was corrected for the positic error at the $200-\mathrm{knot}$ speed determined by the ground-camera tests. These corrected pressures thus provided a measure of free-stream static pressure a: eac measured geometric height.

After the initial pressure-height survey, four calibration test runs were made at each of three airspeeds (235, 320, and 370 knots) at an altitude of about 25000 ft . Immediately after the last test run, a second pressure-heig: survey was made at the same airspeed and altitudes as in the initial survey

Figure 9.12 is a plot of the initial pressure-height survey and of the seciond survey 72 min later. For each calibration test run, the free-stream static pressure was determined from the geometric height of the airplane, the time of the run after the initial survey, and an interpolation of the two surveys for the pressure at that time. Note that the pressure and height scales the figure are broken to provide expanded scales for the two measurements. F the evaluation of the data of the tests, the surveys were plotted on a much larger chart to form continuous curves throughout the height range.

The results of the high-altitude calibration tests are presented if. Iigure 9.13 in terms of the variation of the position error of the aircraft installation with lift coefficient. For these data, the standard deviation is about $0.34 \mathrm{lb} / \mathrm{ft}^{2}$ with a corresponding altitude error of about 10 ft at an altitude of 25000 ft . The maximum probable error, therefore, is about $1 \mathrm{lb} / \mathrm{ft}^{2}$ or about 30 ft at 25000 ft . The corresponding error (10) in terms of $\Delta p / q_{c}$ is $\pm 0.2$ percent at 235 knots ( $M=0.5$ ) and $\pm 0.1$ percent at 370 knots ( $H=0.88$ ). The confidence level of the mean of the data (for $C L=99$ percent) is $\pm 34 \mathrm{ft}$.

The variation of the static-pressure errors of figures 9.11 and 9.13 as a 1...function of ... M rather than $C_{L}$ was shown previously in figure 7.22.

Since the flight manual for the test airplane gives che position errors of the fuselage-vent system in terms of altitude errors, the position errors in figures 9.11 and 9.13 have been converted to altitude errors and plotted in figure 9.14. For sea-level calibrations, the flight-manual values and the calibration with the ground-camera method are essentially the same. At an altitude of 25000 ft , the flight-manual values and the tracking-radar calibration differ by less than 50 ft for airspeeds up to 350 knots.

In the description of the tracking-radar method given in this chapter, some details relating to the experimental procadure and the test data evaluation have been omitted. For a complete discussion of the application of this rethod, the reader is referred to reference 13.

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TABLE 9.1.- FLIGHT CALIBRATION METHODS FOR DETFRMINANG

| Calibration method | Operational limits |  |  | Method accuracy or precision ${ }^{\text {a }}$ (approximate 10 values) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Speed zestrictions |  | Accuracy, percent $\mathcal{L}_{c}$ | Precision, percent $q_{c}$ |
|  |  | Minimum | Maximum |  |  |
| Trailing bomb | Low/'uigh | $\left.\right\|^{\text {stall }} \begin{aligned} & \text { speed } \end{aligned}$ | $\left[\begin{array}{l} v_{m}=0.4 \\ 500.85 \end{array}\right.$ | $\begin{aligned} & \pm 2.0 \quad(M=0.1) \\ & \pm 0.2(M=0.35) \end{aligned}$ |  |
| Trailing cone | Low/hich | Kin. Ers3 | $e_{M}=1.5$ |  | $(n=0.7 \text { to } 0.88)$ |
| Pacer a.s- aft | Low/i:igh | Min. LES | Max. LFS |  | $\begin{aligned} & \pm 0.7 \quad(M=0.5) \\ & \pm 0.2 \quad(M=1.0) \end{aligned}$ |
| Tower | Very low | Mitit LES | Max. LFS | $\begin{cases} \pm 1.0 & (M=0.25) \\ \pm 0.2(K=0.30)\end{cases}$ |  |
| Tracking radar | High | Min. LES | Max. dive speed | $\begin{aligned} & \pm 0.2(M=0.5) \\ & \pm 0.1 \quad(M=0.88) \end{aligned}$ |  |
| Radar altimeter | High | Min. LFS | Max. LFS | \pm 1.0 ( $M=0.8)$ |  |
| Ground camera | Very low | Min. LES | Hax. LF'S | $\begin{array}{ll}  \pm 0.2 & (M=0.3) \\ \pm 0.1 & (M=0.5) \end{array}$ |  |
| $\begin{aligned} & \text { Tracking-radar/ } \\ & \text { pressure- } \\ & \text { altimeter } \end{aligned}$ | High | Min. LES | Max. LFS | $\begin{aligned} & \pm 3.5 \quad(\mathrm{M}=0.5) \\ & \pm 0.1 \quad(\mathrm{~K}=3.0 ? \end{aligned}$ |  |
| Accelerometer | High | Min. LFS | Max. dive speed ${ }^{9}$ | $(M=0.6 \text { to } 0.8)$ |  |
| Recording therwometer | High | Min. LFS | Max. dive speed | $\pm 4.5$ ( $M=0.8$ ) |  |
| Trailing anemometer | Low | Stall speed | $h_{M 4}=0.2$ | $\begin{array}{ll}  \pm 2.5 & (M=0.03) \\ \pm 1.0 & (M=0.16) \end{array}$ |  |
| Speed cours: | Low | Min. LPS | $h_{M}=0.2$ |  |  |
| Sonic speed | High | Min. LFS | Max. LFS | \pm 8.0 (M $=1.0)$ |  |
| Total - emperature | High | Min. LFS | Max. dive speed | $\begin{aligned} & \pm 0.5(M=1.5) \\ & \pm 2.0(M=3.0) \end{aligned}$ |  |

See page 148 fo: footnotes.

FOSITION ERPOR OF STATIC-PRESSURE INSTALLATION

| Calibration method requirements |  |  |  |  |  | Refs. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ```Initial reference pressure. p. obtainad from -``` | Survey of atwosphere | Measurements |  | Instruments (b) |  |  |
|  |  | Aircraft | Ground | Aircraft | Ground |  |
| --- | --- | $q_{C}^{\prime}, p^{\prime}, p$ | -- | ASI, Alt, DPI | --- | 1,2,3,4 |
| -- | -- | $q_{C}^{*}, p^{\prime}, p$ | - | ASI, Alt, DPI | - | 5,6,7,8 |
| $\cdots$ | --- | $q_{C}^{\prime} \cdot{ }^{\prime}$ | $\cdots$ | ASI, Alt | --- | 9,10 |
| --- | Pressureheight | $q_{C}^{\prime}, P^{\prime}$ | $z_{c}, \Delta z$ | ASI, Alt | Camera in tower | 11 |
| Low-speed calibration ${ }^{\text {f }}$ | Pressureheight | $q_{6} \cdot^{\prime} p^{\prime}$ | 2 | IPR, APR | $\left\lvert\, \begin{gathered} \text { Tracking } \\ \text { radar } \end{gathered}\right.$ | 13 |
| Low-speed calibration | Pressureheight | $q_{c}^{*} \cdot p^{\prime}, z$ | -- | ASI, Alt, Radar alt. | --- | 17 |
| -- | --- | $q_{c}^{\prime} \cdot p^{\prime}$ | P.T.Z | ASI, Alt | Camera, Alt or barograph, IT | 13 |
| Low-speed calibration | - | $q_{C}{ }^{\prime} \cdot P^{\prime}, T^{\prime}$ | 2 | Alt, IPR, APR, RT | Tracking radar | 10 |
| Low-speed calibration | -- | $\left\{\begin{array}{l} q_{c}^{\prime} \cdot p^{\prime}, T^{\prime} \\ a_{x}, a_{z}, \theta \end{array}\right.$ | $\cdots$ | IPR, APR, RT, RA, AAR | -- | 20 |
| Low-speed calibration | Pressuretemperature | $q_{C}^{\prime}, P^{\prime}, T^{\prime}$ | - | IPR, APR, RT | --- | 21 |
| --- | --- | $\begin{aligned} & q_{c}^{\prime}, p^{\prime} \\ & T^{\prime}, v \end{aligned}$ | --- | IPR, $A P R, R T$, Trailing anemometer | --- | 22 |
| --- | --- | $q_{C^{\prime}}^{\prime} P^{\prime}, T^{\prime}$ | ${ }^{1} \mathrm{~V}_{\mathrm{g}} \cdot \mathrm{T}$ | ASI, Alt, IT | Stop watch | 23 |
| -** | Temperatureheight. Wind speed | $q_{c}, p^{\prime}$ | $v_{g} .2$ | IPR, APR | Tracking radar, Rawinsonde | 25 |
| --- | Temperatureheight | $q_{c}^{*} P^{\prime} \cdot T^{\prime}$ | 2 | IPR, 次 ${ }^{\text {PR, }} \mathrm{RT}$ | $\begin{aligned} & \text { Tracking } \\ & \text { radar. } \\ & \text { Radiosonde } \end{aligned}$ | 10.24 |

avalues quoted have been achieved. With difierent instrumentation and experimental techniques, the accuracy or precision obtained may vary Erom these values.
bThe following abbreviations are used in this column:
AAR attitude-angle recorder
Alt altimeter
APR absolute-pressure recorder
ASI airspeed indicator
DPI differential-pressure instrument
IPR impact-pressure recorder
IT indicating thermometer
RA recording accelerometer
RT recording thermometer
CMaximum speed at which bomb can be trailed without unstable oscillations in surpension cable.
dLFS level flight speed
$\varepsilon_{M}=1.5$ is the highest speed at which tests have been conducted (ref. 8).
${ }^{\text {f }}$ Low-speed calibration is necessary if radiosonde is not used to make pressure-height survey.
$\mathrm{g}_{\text {Maneuvers }}$ must be conducted in vertical plane.
$h_{M}=2.0$ limitation determined by a requirement that $q_{C}=q$.
$i_{v_{g}} \quad$ ground speed of aircraft

(a) $p$ measured at reference pressure source below, behind, or alongsice aircraft.

(b) $P$ derived from measurement of height of aireraft and pressure gradient at test aititude range.

Figure 9.1.- Four techniques for determining free-stream static pressure $p$ at flight level of aircraft.

(c) $p$ at height of aircraft calculated from $p$ and $T$ at ground and assumption of standard temperature gradient.

(d) $p$ at height of aircraft derived Erom change in height from an initial heigit.

Figure 9.1.- Concluded.


Figure 9.2.- Trailing bomb. Weight $=15 \mathrm{lb}$. (Adapted from ref. 1.)

(b) Photograph.

Figure 9.z.- Concluded.


Figure 9.3.- Trailing bomb with wings at negative angle of incidence. (Adapted from ref. 2.)


Figure 9.5.- Trailing anemometer.


Eigure 9.6.- Anemometer trail positions for two flight conditions superimposed on induced velocity field below airplane. $z$ is vertical distance, $x$ is horizontal distance, and $b$ is wing span. (Adapted from ret. 22.)



Figure 9.8.- Ground-based equipnent used Eor calibrations
at low altitudes. (Adapted from ref. I3.)


(12
© 25000 ft (flight 1)
$\Delta \quad 25000 \mathrm{ft}$ (flight 2)


Figure 9.13.- Calibration of fuselage-vent system at an altitude of 25000 ft. (Adapted from ref. 13.)
Ground-camera and tracking-radar methods
rigure 9.14.- Comparison of flight-manual calibration with calibrations determined by
(Adapted from ref. 13.) ground-camera and tracking-radar methods.


## ERRORS DUE TO PGESSURE-SYSTEM LAG AND LEAKS

As noted in chapter II, the pressure at an instrument car be different from the pressure at the pressure source because of a time lag in the transmission of pressures. The pressur; at the instrument can also differ from trat at the pressure source when there is a leak in the pressure system. For both cases,

- the instrument indications will be in error by an amount corresponding to the pressure drop in the system. In this chapter, analytical and experimental methods ior determining the errors due to pressure-system lag and leaks are discussed. Sample calculations of an estimation of the lag and leak errors of a given pressure system are given in part II of appendix $B$.


## System Las

When the pressure at the pressure source is changing rapidly, as in the case of high-speed dives or climbs, air flows into, or out of, the pressure source (pitot tube, static-pressure tube, or fuselage vents). Under these conditions, the pressure at the instruments lags behind the pressare at the source because of (1) the time for the pressure change to propagate along the tubing (acoustic lag) and (2) the pressure drop associated with the flow through the tubing (pressure lag). In the following sections, mathematical expressions for both forms of lag are described.

Acoustic lag.- As noted in reference 1, the speed of the pressure propagation along the pressure tubing is the speed of sound. The magnitude of the acoustic lag thus depends only on the speed of sound $a$ and the length of the tubing $L$ as expressed in the following equation:

$$
\begin{equation*}
\tau=\mathrm{L} / \mathrm{a} \tag{10.1}
\end{equation*}
$$

where $\tau$ is the acoustic lag time. Since the speed of scund at the lower altitudes is on the order of $1000 \mathrm{Et} / \mathrm{sec}$, errors due to acoustic lag are of concern only for pressure systems having very long lengths of pressure tubing. For the tubing lengths of the instrumer:t systens in service aircraft, errors associated with acoustic lag are of no significance.

Pressure lag.- When air in tubing between a pressure source and an instrument is flowing, the pressure at the instrument is different from the pressure at the source, and the indication of the instrument is in error by an amount equivalent to the pressure drop between the two ends of the tubing. For a rate of pressure change $d p / d t$ at the pressure source, the pressure drop $\Delta p$ and the lag of the pressure sys:em are related by the following equation:

$$
\begin{equation*}
\dot{A} p=\therefore \frac{d p}{d t} \tag{10.2}
\end{equation*}
$$

where $\lambda$ is the lag constant of the system defined by the following equation from reference 2:

$$
\begin{equation*}
\lambda=\frac{128 \mu L C}{\pi d^{4} p} \tag{10.3}
\end{equation*}
$$

where $L$ and $d$ are the length and internal diameter of the tubing, $C$ is th. total volume of the instrument chambers, $P$ is the pressure, and $u$ is the coefficient of viscosity of air. This equation assumes laminar flow in the tubing and applies rigorously only to straight tubing of constant diameter.

Once the value of $\lambda$ of an instrument system is known, the errors in airspeed and altitude associated with any given rate of climio or descent of the air craft can be determined frum equation (10.2) and the appropriate pressure table. in appendix A .

The condition of laminar flow required by equation (10.2) is met when the pressure drop $\Delta p$ along the tubing remains lower than that given by the following equation fros reference 2:

$$
\Delta p=-\frac{32 \mu^{2} L N_{R e}}{\rho d^{3}}
$$

where $N_{R e}$ is the Reynolds number. Since airflow in a straight tube remains laminar for $N_{R e}$ no greater than about 2000, the limiting pressure drcp fc: laminar flow at sea level can be expressed as

$$
\frac{\Delta p}{L}=\frac{6.5 \times 10^{-3}}{d^{3}}
$$

where $\Delta p / L$ is in pounds per square $f t$ per $f t$ and $d$ is the internal diameter of the tubing in inches. At altitude, the limiting pressure drop for lamirar flow is given by

$$
\frac{\Delta p}{L}=\frac{p_{0}}{p_{a}}\left(\frac{\mu_{a}}{\mu_{0}}\right)^{2}\left(\frac{6.5 \times 10^{-3}}{d^{3}}\right)
$$

where the subscripts $O$ and a refer to sea level and altitude. In table 10 . the limiting pressure drops for laminar flow at sea level and 30000 fr are gi: for four tubing diameters.

For relatively simple pressure systems with few bends and tees in the tubing, the lag zonstant can usually be calculated with satisfactory accuracy from equation (19.2) and a knowledge of the geometry of the system. For more complex pressure systems, and especially for those research installations in which lag is an important factor, the lag constant of the system zan be dearmined experimentally by one of the three test procedures described in efer-
ence ... The computational procedures for correcting measured pressures for pressire-lag errors are also given in reference 1.

For pitot-static pressure systems, the lag characteristics of mechanical instrument systems differ markedly from those of systems incorporating electrical pressure transducers. With $t^{2}$ ? mechanical instruments, for example, the lag of the pitot system is very much smaller than that of the static-pressure system because of the great difference in the volumes at the ends of the two pressure lines. The volume at the end of the pitot line is very small (the volume of the differential-pressure capsule), whereas the volume at the end of the staticpressure line is the combined volume of all instrument chambers connected to the line (fig. 2.3). Thus, for those instruments corisected to both the pitot and static-pressure lines, the errors in the indications due to lag are determined primarily by the lag in the static-pressure system.

For the measurement of airspeed (or impact pressure) in research investigations, the lags of the pitot and static-pressure systems are sometimes "balanced" in an attempt to eliminate the airspeed error due to the difference in the lag of the two systems. This balancing of the lag of the two systems is accomplished by adding tubing to the pitot system until the lag of that system equals the lag of the static-pressure system. However, while balancing the pressure lines can often eliminate airspeed errors in rate-of-climb testing, airspeed errors in dive testing can be larger than those that were present before balancing (ref. 1).

With systems employins electrical pressure transducers (figs. 11.13 and 11.14), the lag in the pl:ご and static-pressure lines is essentially the same because the volumes at the ends of the two lines are very nearly equal. Since the volumes of the transducers are also very small and since the length of tubing between the transducer and the pressure source is generally short, the lag of this type system is usually so small that it is of no concern.

Means of reducing lag.- In the design of a pressure system incorporating mechanical instruments, the principal means of reducing the acoustic lag and pressure lag are related to the size of the tubing and the instrument volume. For example, the acoustic lag (eq. (10.1)) can be minimized by simply keeping the pressure tubing line reasonably short, while the pressure lag (eq. (10.2)) can be reduced by reducing tubing length, increasing tubing diameter, or reducing instrument volume. For installations requiring more than one set of instruments, the volume at the end of each pres re line can be reduced by installing a separate pressure source for each set o. instruments. For a system with a given instrimenc volume, the lag can generally be reduced by increasing the diameter of the tubing. However, if the tubing is connected to a staticpressure tube, eny increase in the tubing diameter should be related to the number and size of the orifices, because usually the total area of the orifices should be about the same as the cross-sectional area of the tubing. Finally. for any pressure system, the pressure lag can be reduced by minimizing the number of bends and cennections in the tubing system. For a more exiensive discussion of the influence of the various design farameters on the lag of pressuremeasuring systems, the reader is referred to reference 3.

With systems employing electrical pressure transducers, both forms of are small because of the snall volume of the pressure chambers and the shc: lengths of tubing ordinarily used with this type system.

## System Leaks

The pressure at the instrument can be different from that at the pres source if there is a l'ak in the system and if the pressure outside the $s$ : is different from that inside. A leak within the cockpit of a pressurized cabin, for example, can alter the pressure inside the instrument when the craft is at a nigh altitude. On the other hand, a leak in a part of the $s$ in an unpressurized area might have little effect. The magnitude of the $f$ sure error due to a leak, therefore, depends not only on the size of the: but also on the pressure drop across the leak.

To minimize pressure errors resulting from leaks, the civil and milit. agencies require leak tests of individual instruments (for case leaks) and the complete instrument system installed in the aircraft. The tiests of $t$.-static-pressure system are conducted by applying suction to the static-pre. source until the pressure in the system reaches a specified pressure altit With che pressure held constant, the effects of any leaks appear as rates change in airspeed and altitude indicated by the cockpit instruments. Tes the pitot system are conducted in the same manner, except that pressure is applied to the pitot tube.

A number of different leak tolerances for the systems have been spec: from time to time, by the civil and military agencies. The most stringen: these tolerances requires the leak rate for the static-pressure system to more than $100 \mathrm{ft} / \mathrm{min}$ (indicated by the altimeter) when the system pressure sponds to the maximum pressure altitude for which the aircraft is certific For the pitot system, the tolerance is $1 \mathrm{knot} / \mathrm{min}$ (indicated by the airsf. indicator) when the system pressure equals the impact pressure correspond: the maximum speed of the aircraft.

The errors in airspeed and altitude that result from a leak of a give and a given pressure differential across the leak can be determined from leak rate (i.e., the rate of pressure change dg/dt) determined from a gr: test of the system, (2) the lag constant $\lambda$ computed from equacion (10.3: (3) the lag constant $i_{2}$ of the leak. The value of $\lambda_{\text {? }}$ can be calculat: from the following equation:

$$
\therefore=\left(\frac{p_{T, 0}-p_{T, a}}{d p / d t}\right)\left(\frac{p_{T, 0}+p_{T, a}}{p_{c}+p_{a}}\right)
$$

where

| PT,O $\quad$ ambient pressure during ground test |  |
| :--- | :--- |
| $P_{\text {T,a }}$ | test pressure in system during ground test |

dp/dt rate of pressure change due to leak measured in ground test Pa pressure at pitot or static-pressure source at flight alritude Pc compartment or cabin pressure at flight altitude

The pressure error $\Delta p_{2}$ due to the leak can then be computed from

$$
\begin{equation*}
\Delta p_{l}=p_{i}-p_{a}=\frac{\lambda}{\lambda_{2}+\lambda}\left(p_{c}-p_{a}\right) \tag{10.8}
\end{equation*}
$$

where $p_{i}$ is the pressure inside the instrument. From the value of $\Delta p_{l}$, the corresponding errors in airspeed and altitude can be determined from the tables in appendix $A$.

The errors in the instrument indications that result from a leak in the pressure system can also be determined experimentally in flight. In tests reported in reference 4 , for example, a calibrated leak device, capable of introducing five different size leaks into a pressure system, was connected to the static-pressure line in the cockpit of a transport airplane. The altitude error produced by each leak was then determined at a number of altitudes and for different cabin pressures. After the flight tests, ground tests were conducted to measure the leak rate of each leak in terms of altitude change per minute. The ground and flight tests thus provided a means of directly relating the altitude error and leak rate of a given size leak. The results of these tests showed that for leaks producing altitude errors as small as 10 ft , the leak rate was much larger than the $100 \mathrm{ft} / \mathrm{min}$ rate specified for the leak tolerance discussed earlier. In other words, the altimeter errors of systems complying with this leak tolerance would be essentially negligible.

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TABLE 10.1.- LIMITING PRESSURE DROP PER FOOT FOR LAMINAR FLOW IN TUBING

| Tubing diameter, <br> in. |  | Limiting ip/L, <br> $\left(1 \mathrm{~b} / \mathrm{ft}^{2}\right) / \mathrm{ft}, \mathrm{at}$, |  |
| :---: | :---: | :---: | :---: |
| Outside | Inside | Sea level | 30 cooft |
| $1 / 8$ | 0.060 | 30.1 | 69.4 |
| $3 / 16$ | .114 | 4.4 | 10.0 |
| $1 / 4$ | .188 | 1.0 | 2.3 |
| $5 / 16$ | .250 | .4 | 1.0 |

## AIRCRAFT INSTRUMENT ERRORS

Aircraft instruments are required to meet specified standards of accuracy. These accuracies are expressed in terms of error tolerances (allowable errors) which may be stated as a percent of the measured quantity, as a percent of the full-scale range of the instrument, or as a series of individual tolerances for given values of the measured quantities.

The specified accuracies of the instruments vary depending on the type of instrument and on the state of the art at the time the instrument was developed. The accuracy of the "precision" mechanical altimeter, for examele, is greater than that of the older "sensitive" altimeter. Similarly, the accuracics of electrical instruments are greater than those of the mechanical types, and of the twc electrical instrument systems, the electronic pressure-transducer system is sominhat more accurate than the servoed instrument systems.

Until recent years, mechanical instruments were used in ail types of aircraft; they are still widely used in general aviation aircraft and in older civil transport and military aircraft. Servoed instzumen. sys:ems, a later development, have been used for some years in turbojet transport and military jet aircraft, while electronic pressure-transducer systems, an even later development, are now keing used in some turbojet transport anc military jet aircraft.

The Federal Aviation Administration specisies the accurac: of instruments used in civil aircraft, wile the $\because . S$. Air Force, Army, and Na:y specify the accuracy of instruments used in military aircraft. For the instruments discussed in this chapter, the accuracies have, for the most part, been extracted from instrument standards specified by the Air Force.

Sechanical Irstruments
As noted in chapter II, the scale error (i.e.. the differonce between an instrument indication and the correct value) is generally the jargest of the various instrument errors. Thus, the deternination of this ervor is the primary concern of the laboratory testing of the instrumer:s.

When it has been determined that the seale errors of a particular i:striment conform to the specified tolerances, the instrument is censidered accoctable for operational use However, since the seale error is s:stematic (repeatable), many aircraft operators require that zorrection for the error be applied in order to achieve an accurac: greater than the sfitified accuracy.

In this section, the speci三ied tolerances for the errors of each thee of instrment are presented and she :aboratory east procedures Ez the ealibration. of the instruments are outlined.

scale errors (circ:lar test points in fig. 11.2) are the values used in the preparation of correction charts or for the scale-error corrections in air data computers.

The scale-error tolerances for two types of sensitive altimeters (refs. 1 and 2) and two types of precision altimeters (refs. 3, 4, and 5) are presented in table 11.1. Also tabulated are the $h_{1}$ teresis tolerances at two test alti-

- tudes and the aftereffect wherance at sea-level pressure. Note that the calibration standards for these instruments do not require tests for the irift and recovery errors. A comparison of the scale-error tolerances for the four altimeters provides an indication of the improved accurac; that has beer. achieved through the years.

Determination of the hysteresis at two test points, specified by standard test procedures, defines only a part of the hysteresis cycle. in tests :o determine the complete hysteresis cycles of tincee types of altimeter (ref. 6), a number of type C-12, C-13, and :1A-1 altimeters were calibrated throughoir t:e hysteresis cycle. The calibrations of representative instruments of ach altimeter type are presented in figure 11.3. In table 11.2, values of hysteresis errors (at the standard test points) for all the inseruments are compared mita tie hysteresis tolerances. Also tabulated are the afturefect errors and inlerances. These results are of interest in shiwing the iysteresis and afteresfort errors of the precision-type altimeter to be very much lowirg than the secis.ud tolerances.

 sentative instrument of each altimeter tye are show in figure 11.4. These data show the major part of the 6-hour drift occurs sithi:: a short perixi after the start of the test.

 is 50 to 650 knots and the scale-ermer toleranees tinnugh ti:3 sine.t ange ar: given in table 11.1 (Erom ref. 7).



 tables A9 and All of appendix $A$.





 in figure 11.6 . immersed in a temperature－controlled bath，and tire pressure inside the instru－ munt rase is adjusted to a specified value of pressure altitude（meatiured alath a barometer）．Piessures corresponding to given values of calibrat．ad ailoreed， measured with a manometer，are then applied to the pitot ier：ot tie fhotramoth．

As the tables of the scale－error tolerances for the tede－atrarod and－
 values are 1 dited in table 11.4 to indicate the sgecified accuracy of che instrument．


 numbers are required to met the following dolerances（ro：．3）：

$$
\begin{aligned}
& \text {-J.JつめM for } 32 \text { test :onnts } \\
& \text { :u.jlon Eur } 7 \text { tust posiats }
\end{aligned}
$$







 Erom the static－pressure suurce into the instrument chamier．This device pro－




 1：のdi＝st．r．s．







 table 11.0 （：ER：ref． 1 ）．





With the mechanical instrument system, the pressure-sensing element (caprule) is located in the instrument, the instrument indications are not corrected for scale error or the position error of the atatic-pressure installation, and the flight information is presented on dial-pointer displays !single or multiple pointer, drum-pointer, or counter-pointer).

With the servoed instrument system, the pressure-sensing element (capsule) is located in a computer (central air data computer (ref. 1l)) which can correct

- for both the scale error of the capsule and the position error of che staticpressure installation. The output signals of the computer thus refresent corrected flight quantities (pressure altitude, calibrat ad airspeed, eic.). These computer-corrected signals are transmitted to the instriment where she flight information is presented on dial-pointer displays (including the ccinter-drumpointer display in fig. 11.10) or on vertically moving scale displä́s such as those in figure 11.11.

With electronic pressure-transducer systems, the pressure-sensing element (diaphragm or bellows) is located in the electrical pressure tras.sducer. The signals generated in the transducer are linearized in a micruprocessor (computer) which can also apply corrections for the position error of the sta:ic-cressure installation. These corrected signals can then be presented on dial-pointer displays, vertical scale displays, LED (light emitting diode) displays, or CRT (cathode ray tube) displays.

As noted previously, the accuracy of servoed instrument systens is grader than that of mechanical instruments and the accuracy of electronic iressuretransducer systems is generally greater than that of servoed inser:ment oistems. In the following sections, the accuracies of a servoed instrument sistem and of two types of electronic pressure-transducer systems are discussed.

Servoed instrument system.- The servoed instrumerit system is a form of servomechanism incorporating feedback between the compurer and tie instrument (fig. 11.12). In the computer, a synchrotel is actuated by the de:iections of a capsule, while in the instrument, the pointer or other tipe disfiaj:i actuated (through a gear train) by a servomotor that is controiled by ingnals ger:erated by the differences in the electrical fields of the synchrotel i: the romputer and another synchrotel in the instrument. Additional syrishriedis in the computer are controlled by two-dimensional cams to generate the screecional signals for the scale error of the capsule and the position error cite staticpressuis installation.

The accuracy of a servoed instrumerit system is determaned $b y$ (1) :in basic accuracy of the computer (which incluces the accuracy of tie sajo-rintor iorrecticn). (2) the accuracy of the position-error correction, and (i) tou accuracy with which the corrected signals from the comfuter are transm:.tect and displayed in the instrument.

The basic accuracy of an air data computer stated in turms of the rerror tolerances for cach of the flight quantities is as follows:

Altitude
Airspeed
True airspeed
Mach number
Vertical speed $\pm 2$ percent of the indicated value

The accuracy with which the fosision erzor is corrected in tine arr gata comper varies depending on the slope of the calibration curve. For positic::error calibrations with low slopes, the accuracy of the position-error screcetion is greater than for calibrations with steep slopes.

The accuracy with whici the romputer-zenerated signals are transmited and displayed on the various servoed instruments (refs. 12 through 15) is yi?en ki the following specified error colerances:

| Altimeter | $\pm 15 \mathrm{ft}$ |
| :--- | :--- |
| Airspeed indicator | $=1 \mathrm{knot}$ |
| True-airspeed indicatcr | $=1 \mathrm{knot}$ |
| :fachmeter | $\pm 0.001 \mathrm{M}$ |
| Vertisal-speed indicator | $\pm 2$ percent of indicated value |

For installations incorporating servoed systems, a mechanical counterfart of each servoed instrument is installed on the instrument panel for emergency use whenever the servoed system becomes inoperative becauce of electrical powity failure. With one type of altimeter (a servopneurntic typ: in which the =aptile: is located in the instrument), the mechanical transmission is activatur $z \because$ a monitoring circuit whenever the servoed system becomes inoperative.

Electronic pressure-tzansducer systems.- An electrical prossure trar.: ducer is a small pressure-sensing device that produces electrical signals propertioral to the deflection of a capsule, diaphraym, bellows, or wther pressure-icrsing element (ref. 16). Jependiag on the characteristics of the rransducer elemer:,
 voltage).

In the digical transducer deseribed in zeference i7, the prossurn-sconsine element is a single bellows in the absolute-pressure transducer and twn ertos:ig bellows in the differential-pressure transducer (fio. 11.13). The =ran:s : element in these units is a quartz crystal oscillatiog jonm ainioi : : ir: $\because$ an $:$ its resonant frequency threugh piezoelectric excitation. Tio ,ariat: resonant frequency witi, load applied by the bellows froridy a aifitai , ita, ut signal that is porportional to the applied pressurc. whon fore rut: at :ara:

either a cockpit display or a magnetic tape recorder (in fl:ght-sest applı=ations). The repeatability of the transducer is $\pm 0.005$ fercent it the full-scale pressure range, while the accuracy of the transducer spstem is ibout $=0.05$ percent of full scale. If corrections for the position error of the static-pressire installation are applied, the additional error for this corfecticn deiends un the slope of the position-error calibration curve, as in ther case of Eerveed systems.

For analog transducers, the pressure-sensing eleant is a ilat, ©ircuiar diaphragm that divides the transducer assembly into too chamiers (fig. 11.:1). The transducer element most commonly used in, this rype of transiocer is e:ther a variable-capacitance or a variable-reluctance device. These ar.a sther trarsducer elements (strain gage, variable-resistance devion, etc.) ire lescrited $1:=$ reference 16.

Analog transducers are used primarily in flight-iast recoreing systens, :cy whi. the output signals of the transducers are recorced on hacrevic tape :itior in analog form (frequency modulation) or in digital form (analoc-to-bigita: cor.version). For analog recording, the output signal is processed in a signal co:crui wint and a vcltage-controlled oscillator, whereas for jigi:al recurdi:y =:-: signal is proressed in a signal control unit anc a pulse code modilater. se accuracy of analog eecording systems is about 11 percent of the Ell-scale pressure range, while the accuracy of analog-to-digital reccrdier systems :s about $\pm 0.4$ percent of full scale.

## Ascuracy of Calibration Equ:pmeat

The accuracy with whith instrument errors are detergined detends fundimentally on the accurac/ of the calibration test apparatas and the alibratica ter: technique. With high-grade barometars and manometers and skilled operatori, i=
 (ref. 18). For routine calibrations, however, the aczarac; is probabl $\because$ : :c bet...r then 0.005 in . Hg at sea-level pressure and 0.003 in . Hg at isessires corrosp,nding tu .ititudes on the order $c: 70000$ ft. The altitude urrors cort:sfonding to these pressure accuracies are 5 ft at oea level ana 70000 ft .

For the terts of referense 6, two different rypes of baromeders were seed :u measure scale crrurs and drift errors. The barometer tor tion siniterror -..jt. was equipped with an automatic system for measuring tion hei,it: : the mur: ir: column, whereas the barometer for the drift tests had an ateoma: = meinanisa : maintaining the pressure in the system at a selectod : alae. $\mathrm{N}_{1}$ tit the firs:



 increments corresponding to pressure securactes $)^{\circ}$..: := are ifiven in figure 11.15.

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table 11.1.- ERROR tolerances for four types of altameters ${ }^{3}$

$$
[\text { From refs. } 1 \text { to } 5]
$$

| Test-point altitude, ft | Sensitive altimeters |  | Precision altimeters |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{b}_{\text {Type }} \mathrm{c}-12$ | ${ }^{\text {brype }}$ c-13 | $\mathrm{b}_{\text {Type }}$ :A-1 |  |
| Scale-error tolerance, $f t$ |  |  |  |  |
| 0 | $\pm 50$ | $\pm 50$ | $\pm 30$ | $=30$ |
| 5000 | $\pm 150$ | $\pm 100$ | $\pm 55$ | :55 |
| 10000 | $\pm 175$ | $\pm 150$ | :80 | :80 |
| 15000 | $\pm 235$ | +200 | $\pm 105$ | -105 |
| 20000 | $\pm 300$ | :200 | $\pm 130$ | $\pm 130$ |
| 25000 | $\pm 375$ | -300 | $\pm 155$ | -155 |
| 30000 | $\pm 450$ | $\pm 300$ | $\pm 180$ | -130 |
| 35000 | $\pm 525$ | $\pm 300$ | $\pm 205$ | -205 |
| 40000 | $\pm 600$ |  | $\pm 300$ | $=230$ |
| 45000 | $\pm 675$ |  | $\pm 400$ | :255 |
| 50000 | $\pm 750$ |  | +500 | $=230$ |
| 60000 |  |  | $\pm 800$ | :800 |
| 70000 |  |  | $\pm 1200$ | $=1200$ |
| 80000 |  |  | $\pm 1530$ | $=1500$ |
| Hysteresis tolerance, ft |  |  |  |  |
| 16000 | -- | $=70$ | ----- | --- |
| 18000 | - | $\pm 70$ | ----- | ----- |
| 20000 | $\pm 150$ | ---- | $=100$ | $\pm 100$ |
| 25000 | $\pm 150$ | ---- | $=100$ | :10Q |
| Aftereffec: tolerance, ft |  |  |  |  |
| 0 | $\pm 60$ | $: 50$ | \% 50 | -50 |

a abbreviated list of test points.
Du.s. Air Force types.
table 11.2.- hystereis and aftereffect of
three types of altimeters
[From ref. 6]

| Altimeter type | Minimum | Maximum | Average | Tolerance |
| :---: | :---: | :---: | :---: | :---: |
| Hysteresis, ft |  |  |  |  |
| C-12 | 80 | 160 | 112 | 153 |
| C-13 | 60 | 110 | 87 | 79 |
| : $\mathrm{S}_{1}$ 1 | 10 | 45 | 25 | 100 |
| Aftereffect, ft |  |  |  |  |
| C-12 | 25 | 60 | 41 | 50 |
| C-13 | 25 | 55 | 33 | 50 |
| YA-1 | 5 | 20 | 10 | 50 |

TABLE 11.3.- SCALE-ERROR TOLERANCES OF AIRSPEED INDICATOR ${ }^{\text {a }}$
[From ref. 7]

| Calibrated airspeed, <br> knots | Tolerance, <br> knots |
| :---: | :---: |
| 50 | $=4.0$ |
| 80 | $\pm 2.0$ |
| 150 | $\pm 2.5$ |
| 250 | $\pm 3.0$ |
| 300 | $\pm 40$ |
| 550 | $\pm 5.0$ |
| 650 | $\pm 5.0$ |

a $_{\text {Abbreviated }}$ list of test points.

TABLE 11.4.- SCALE-ERROR TOLERANCES OF TRUE-AIRSPEED INDICATOR ${ }^{\text {a }}$
[From ref. 8]

| $\begin{gathered} \text { Altitude, } \\ \mathrm{ft} \end{gathered}$ | Calibrated airspeed, knots | True airspeed, knots, for bulb temperature of - |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $-60^{\circ} \mathrm{C}$ | $-40^{\circ} \mathrm{C}$ | $0^{\circ} \mathrm{C}$ | $40^{\circ} \mathrm{C}$ |
| 0 | 100 | - | - | -- | $104 \pm 7$ |
|  | 450 | $373 \pm 8$ | $390 \pm 8$ | $423=9$ | --- |
| 5000 | 100 | -- | - | $160 \div 7$ | $114 \pm 7$ |
|  | 450 | $403 \pm 8$ | $421 \pm 8$ | --- | --- |
| 10000 | 100 | $103 \pm 7$ | $168 \pm$ ? | $117=7$ | -- |
|  | 450 | $434 \pm 9$ | --- | --- | --- |
| 15000 | 100 |  | $119 \pm 7$ | $129 \pm 7$ | --- |
|  | 400 | $424 \pm 9$ | $444 \pm 7$ | --- | - |
| 20000 | 109 | $126 \pm 7$ | $132: 7$ | --- | - |
|  | 350 | $410 \pm 8$ | $429 \pm 7$ | - | --- |
| 35000 | 100 | $17 i \pm 6$ | $182 \pm 6$ | --- | --- |
|  | 250 | --- | $423 \pm 9$ | - | -- |

a Abbreviated list of test points.

TABLE 11.5.- SCALE-ERROR TOLERAVCES FOR THE :HAC!METER
[From ref. 9]
(a) Tolerances

| Tolerance | No. of test <br> Mach numbers |
| :---: | :---: |
| $\pm 0.008 \mathrm{M}$ | 32 |
| $\pm .010 M$ |  |
| $\pm .015 M$ | 7 |
| Total.. | 4 |

(b) Test Mach numbers Eor scale-error calibration ${ }^{\text {a }}$

| Altitude. $f t$ | Calibrated airspeed, mph | Test <br> Mach number |
| :---: | :---: | :---: |
| 0 | 400 | 0.526 |
|  | 1100 | 1.445 |
| 5000 | 400 | . 573 |
|  | 1000 | 1.413 |
| 10000 | 400 | . 625 |
|  | 900 | 1.378 |
| 15000 | 300 | . 518 |
|  | 900 | 1.498 |
| 20.000 | 300 | . 570 |
|  | 800 | 1.443 |
| 35000 | 200 | . 528 |
|  | 600 | 1.430 |
| 50000 | 200 | . 752 |
|  | 450 | 1.475 |

TABLE 11.6.- SCALE-ERROR TOLERAICES EOR THE
RATE-OF-CLIMB INDICATOR
[From ref. 10]

| altituas, <br> ft | Test altitude <br> rate of change, <br> $\mathrm{ft} / \mathrm{min}$ | Tolerance, <br> Et/min |
| :---: | :---: | :---: |
| 1000 to 1500 | 500 | 100 |
| 1000 to 2000 | 1000 | $\pm 200$ |
| -000 to 4000 | 2000 | $\pm 300$ |
| 2000 to 4000 | 3000 | $\pm 300$ |
| 2000 to 4000 | 4000 | $\pm 400$ |
| 2000 to 4000 | 5000 | $\pm 500$ |
| 15000 to 17000 | 2000 | $\pm 400$ |
| 15000 to 17000 | 4000 | $\pm 300$ |
| 28000 to 30000 | 2000 | $\pm 400$ |
| 28000 to 30000 | 4000 |  |


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(b) Drum-Feinter display.

Figure 11.1.- Pressure aitimeters with difserent altitude displays. (Courtesy of follsman Instryment Co.)

Figure 1l.2.- Illustration of scale-error calibration of a pressure altimeter. Also shown are the errors due to hysteresis, drift, aftereffect, and recovery.


Figure 11.3.- Scale errors and hy:teresis of thren tyes of altincters. (Adapted from rof. 6.)

(a) Type C-13.

(b) Type C-12.

(c) Type MA-i.

Figure 11.4.- Drift errors of three types of altimeters. (Adapted from ref. 5. )



Figure 11.7.- Machmeter. (Courtesy of Kollsman Ins:rument Co.)


Figure 11.8.- Rate-of-climb indicator. $\begin{gathered}\text { L-79-362 } \\ \text { (Ccurtes; }\end{gathered}$ of Kollsman Instrument Co.)
Display

$$
\begin{gathered}
\stackrel{\rightharpoonup}{0} \\
\stackrel{0}{0} \\
\stackrel{y}{3} \\
\square
\end{gathered}
$$


Pressurc tublng 7


L-79-363
Figure 11.10.- Counter-drum-pointer servoed altimeter. (Courtesy of Harowe Systems, Inc.)

(a) Altitude/vertical-speci indicator. (b) Airspeed/Mach-number indicator. $\quad$ L-79-364
Figure 11.11.- Servoed instruments with vertical-scale displays.


(a) Absolute-pressure transducer.

(b) Differential-pressure transducer.

Figure 11.12.- Quartz crystal digital pressure transducer. ( (ou tesy of Paroscientific, Inc.)

(a) Absolute-pressure transducer.

(b) Differential-pressure transducer.

Figure 11.14.- Analog pressure transducers.

Altitude, ft

(Adapted from ref. 6.)


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## CHAPTER XII

## GPERATIONAL ASPECTS OF ALTIMETRY

In the description of the altimeter test procedures in chapter XI, it was noted that altimeters are calibrated with the barometric subdial scale set at 29.92 in . Hg, the sea-level pressure in the standard atmosphere. If the barometric subdial is also set at $29.92 \mathrm{in} . \mathrm{Hg}$ for operational use, the altimeter indicates pressure altitude above sea level. This pressure altitude differs from the geometric height whenever the sea-level pressure or temperature gradient of the atmosphere differs from the standard value. To account for these variations in pressure and temperature, the barometric subdial can be adjusted so that the altimeter indicates either the elevation of the airport or zero height at the airport elevation. Thus, in service operations, the barometric subdial may be set at ons of three settings, which are assigned the following 2 signals in the Aeronatical Code:

QFE barometric subdial set at 29.92 in. Hg
QNH barometric subdial setting for altimeter to indicate elevation of airport

QNE barometric subdial setting for altimeter to indicate zero at the airport

The QNH settings are used by all aircraft for take-off and landing and for the vertical separation of aircraft at altitudes below 18000 ft (ref. 1). The QNE settings are used by scme airline operators during landing approaches to provide a cross-check with another altimeter set to quH. The QFe settings are used by all aircraft for vertical separation at altitudes above 18000 ft .

In practice, the pilot adjusts the barometric scale prior to take-off until the altimeter indicates the elevation of the airport (QNH value). Before landing at his destination, he resets the barometric scale to the existing $Q \mathrm{NH}$ value for that area so that the altimeter indicates the elevation of that airport when the aircraft lands. The current QNH settings are measured at the airport weather stations and are reported to the pilots by radio.

## Barometric Scale Settings

The mechanisms that rotate the barometric scale and the pointers of the altimeter are linked together so that adjusting the barometric scale rotates the pointer. The correspondence between the two scales is the same as the pressu:e-height relation in the standard atmosphere.

The interaction between the barometric scale and the altimeter pointer can be illustrated with the two hypothetrcal atmospheric conditions shown in figure 12.1. The curve to the right in both charts represents the pressure-heigit relation in the standard atmosphere. Since the barometric scale and the altitude scale of the altimeter have the same relation, an identical curve,
representing the two altimetor scales, can be "hought to ite on top of the atmospheric curve. Thus the abscissa of the charts can we labeled barometric subdial scale as well as atmospheric pressure, and the ordinate can be lateled altimeter scale as well as geometric height.

The curve to the left in figure $12.1(1)$ represcats in otmsipheric condition in which the temperature yradient is stondard and the :ien-luvel prosisure: is $28.75 \mathrm{in} . \mathrm{Hg}$. For this condition, the altineter indicates 1100 ft if th . barometric scale is set at 29.92 in . Hg. When the :esle is adusted to 28.75 in . Hg, the altimeter scale curve is moved down until it intur:sert:; 28.75 in . Hg on the zero-height axis. The altimeter pointer will then indirate zoro, and the altimeter will indicate geometric hoight throughout the altitude range.

The curve to the left in firure $12.1(\mathrm{~b})$ depicts an atmuspheric condition in which the sea-level pressure is standard and the temperature yradient is beiow standard. For this condition, the altimeter indicates zero height it sea level when the barometric scale is set at 29.92 in . Hy (the exisitimy :ici-livent pre:isure). At heights above sea level, however, the altimeter indications ate higher than the geometric heights. For example, it the altimeter i:; taken :o a height of 15000 ft where the existing pressure $\mathrm{i}: \mathbf{1 4 . 8 2 \mathrm { in } \text { . Hy, the altimetor } . ~}$ will indicate 18200 tt (as shown by the intersection oi this pros:arry with the altimeter scale curve).
 the existing sea-level pressure. When the aigort elevation is an appociable height above sea level. however, the oni value differs from the :iod-level presisure whenever the temperature gradent differs from that in the :itandard atmusphere. This difference can be illu:ithated by the cxample shown in fiume la... for the case shown, the airport eliovation is 5000 ft , the :iow-levol prosisure t : 29.92 in. Hg, and the temperature gradient is below standard. Whon an altimet,:
 28.30 in . Hg (as shown by the intersection of the altimetor :aill curve with the zero-height axis). For this: casc, thorofore, the barometric :uhtial intat









 cround :station.





and the maximum probable error was $\pm 207 \mathrm{ft}$. The signs of the bias values of the two error distributions were in directions that could be accounted for by pressure-system lag and instrument friction lag.

The QNH setting is also used on cross-country flights where altitude information is needed for terrain clearance in mountainous areas and for the vertical separation of aircraft below 18000 ft . On such flights, the pilots are required to continually reset the barometric scales to the QNH values reported by stations along the route.

Even with altimeters set to the latest reported $Q \mathrm{NH}$ settings, however, the vertical separation between two aircraft may be less than the prescribed minimum. The separation may be reduced, for example, when two aircraft approach each other from airports reporting different QNi! settings. The separation may also be reduced if there is a change in the atmospheric conditions after an altimeter has been set to a $Q \mathrm{NH}$ value. The effects of atmospheric changes depend on the distance between the QNH reporting stations and on the variation of the atmospheric pressure with time. In an analysis of these effects in reference 3 , the following conditions were assumed: a distance of 130 miles between stations, a pressure variation of 4 millibars per hour, and a time lapse of $1 / 2$ hour from the time of the QNH report. At the miapoint between the stations, the altitude error under these conditions was estimated to $b \geqslant 200 \mathrm{ft}$. As noted in the study, nowever, even this value might be too conservative, for errors of as much as 500 ft have been reported at the boundaries of 2 NH reporting stations in some areas of Europe.

To avoid the uncertainties in the indications of altimeters set to 2 NH for high-altitude and transoceanic flights, the altimeters of all aircraft operating above 18000 ft are set to the QFE value ( $29.92 \mathrm{il}: . \mathrm{Hg}$ ). With this setting, the altimeters in the aircraft above any given point on the Earth are referenced to the same pressure. If the reference pressure changes, the flight level of each of the aircraft moves up or down by the same amount, so that the reiative separation remains the same lassuming that the temperature gradient of the air is standard). If the temperature gradient varies from the standard, the distance between the flight levels decreases when the gradient is belord standard and increases when the gradient is above standard.

During flights over mountains, the cifference between th.e indicated altitude and the geometric height presents the greatest hazard when the atmospheric temperature is extremely low, for then the altimeter inc: cation is higher than the geometric height. To determine the altimeter errors, that might be encountered at extremely low temperatures, the ceometric heichts at civen fligh: levels were computed for the coldest day in the winter of 1961-62 at three airports in the northwestern United States. The temperature-height profiles Eor this day at the three airports are shown in figure 12.3 zogether with the =emperature variation in the standard atmostiere.

For each of the airport locations, the airsrafi wes considered to be floing at the minimun en route altitude specified by the civil requlations (200) Et above the highest peak in the region). The barometric scale was assumed to be set to the existing ginh value, so that the indicated altitudes we: measures of
origiNat ri. .
the presisure altitude above the airport. The geometric heiqht $z$ of the aircraft was computed from

$$
\begin{equation*}
z=E+\left(H_{i}-E\right) \frac{T_{m, a}}{T_{m, s}} \tag{12.1}
\end{equation*}
$$

where $E$ is the elevation of the airport, $H_{i}$ the indicated altitude, and $T_{m, a}$ and $T_{m, s}$ the uctual and standard mean temperatures of the air between the airport and the flight level. The results of these computations, listed in table 12.1, show the difference between the indicated altitude and the yeometria height, $H_{i}-Z$, to be as much as 950 ft .

The preceding discussion has considered only the effects of atmospheric variations on the indications of altimeters set to $Q N H$. The accuracy of the altitude indications, however, also depends on the accuracy with which the QNH value is measured at the ground station and on how closely the pilot adjusts the barometric scale to the reported value. The altitude perceived by the pilot in turn depends on his interpretation of the altitudn displayed on the instrument dial. With the three-pointer altitude display (chapter x ), piluts sometimes misread the displayed altitude by one or more thousands of feet. The drumpointer and counter-pointer displays, with digital readouts in l000-ft increments, were developed to overcome this kind of reading error.

## Flight Technical Error

The actual flight level of an aircraft during cruising flight usually differs from its assigned fligat level by an amount equal to the instrument system error (defined in chapter II). Because of difficulties in constantly maintaining level Elight (either because of the characteristics of the elevator control system or deficiencies in the autopilot and its altitude-hold, or height-lock, system), the aircraft may occasionally deviate from the flight level the filot is attempting to maintain. These occasional deviations from level flight are called flight technical error (ref. 3).

Efforts to collect stacistical intormation on the maynituse and irequency of the flight technical error were initiated by the International civil diviation Organization (ICAO) ia 1956. Additional investigations were eonductad by the British Ministry of Transpert and Civil Aviation (MTCA) in 1957, the $3 . S$. Civil Aeronautics Administration (CAA) in 1958, the National derchautics and Space Administration (NASA) in 1961-63, and the International Air Transport Association (IATA) in 1962, 1963, and 1965 (refs. 4 through 9).

In the initial ICAO study, and in the later CM and ITCA staisos, the filut of civil aircraft were asked to keep records of all excursions ot the arrerat from level flight as indicated b: the cockit altimeters. In the:e thren studies, pilct obseriations of aititude deviations were collected irnm a wide variety of arcraft in cruising iliqiat at altitudes up to as mot.

The pilots reports were correlated in terms of the manitubes wita doviatiows and the frequency of their oecurrence. The deviations: wire ramomly
distributed about the flight level and had values that would conform, approximately, to a normal distribution curve. The probability of the occurrence of a deviation of a given magnitude could, therefore, be calculated. The magnitude selected by ICAO was the maximum probable error, defined as the value equal to three times the standard deviation ( $\sigma$ ) of the data. This maximum probable error represents the altitude deviation that would be equaled or exceeded for 0.3 per-

- cent of the deviations. The data collected in the ICAO, CAA, and MTCA studies showed the flight technical error to increase with altitude and to have a 30 value of about 500 ft at an altitude of 40000 ft (ref. 3).

In the IATA investigations (refs. 5 and 6), pilot reports of altitude deriations were obtained in routine flights of commercial transports flying across the North Atlantic Ocean at altitudes above 29000 ft . The data from these flights were analyzed, as in the ICAO study, to yield a 30 value which was found to be 190 ft for these particular operations. The much lower value from these tests (compared with 500 ft found in the earlier studies) can be accounted for by the fact that the transports in the IATA tests were equipped with autopilots with altitude-hold systems, whereas the aircraft in the earlier tests were operated, for the most part, under manual control.

In the NASA investigations (refs. 8 and 9), the flight technical errors were determised trom an evaluation of the altitude traces ohtained from NASA recording altimeters. These recorders were installed in a variety of civil transports flying both domestic and transoceanic routes at altitudes up to 40000 ft . The altitude recordings were analyzed in terms of the altitude deviation beyond which the airplane would be expected to operate for 0.3 percent of the cruise time. Since this criterion provides an indication of the length of time the airplane was away fromits flight level, it represents a more meaningful measure of collision exposure than that provided by the 30 errors.

The results of the NASA analysis are presented in figure 12.4. The values of the altitude deviations are plotted at the middle of each 5000-ft altitude bracket within which the values were recorded. The deviations were all experienced when the airplanes were under autopilot altitude-hold control. With the exception of one airplane, the deviations in the altitude range below 25000 ft were within 160 ft . The deviations in the altitude range above 25000 ft were within 225 ft .

## Overall Altitude Errors

The overall altitude error is the deviation of an aircraft from its assigned altitude, that is, the sum of the altimeter-system error and the flight technical error (fig. 12.5). A number of attempts have been made to estimate the overall altitude errors of aircraft (refs. 3, 4, 6, and 10 to 13) to see whether these overall errors provide adequate clearance within the prescribed vertical separation minima ( 1000 ft for alritudes up to 29000 ft and 2000 f : for altitudes above 29000 ft (ref. l)). For the altitude range from 29000 to 40000 ft , assessments have also been made to see whether the overall altitude errors would permit a reduction in the separation minimum from 2000 to 1000 ft . As shown in the following discussion, the validity of these
assessments depends on the accuracy of the values assigned to the altimetersystem and flight technical errors and on the procedure by which these errors are combined.

In an early assessment of the errors of aircraft operating in the $29000-\mathrm{ft}$ to 40 000-ft range (ref. 10), the overall altitude error was determined by combining the altimeter-system and flight technical errors by statistical sumation. With this procedure for combining the errors, the maximum probable value ( 30 ) of the overall altitude error was determined as three times the square roct of the sum of the squares of the standard deviations of the individual errors. The value of the altimeter-system error was derived from a survey of the available data on the instrument and static-pressure errors of the aircraft in service at the time of the study. An analysis of these data showed the two errors to be normally distributed, to increase with altitude, and to have maximum probable values at an altitude of 40000 ft of 250 ft for the instrument error and 265 ft for the static-pressure error. The maximum probable value for the flight technical error was the 500-ft value determined in the studies discussed in the previous section, Grom these three valuee, the maximum probable overall altitude error was calculated to be $\sqrt[3]{\left(\frac{250}{3}\right)^{2}+\left(\frac{265}{3}\right)^{2}+\left(\frac{500}{3}\right)^{2}}$ or 618 ft . This 618-ft value was considered to represent the deviation that would be equaled or exceeded by 0.3 percent of the aircraft assigned to a flight level of $40000 \mathrm{f}=$. for aircraft flying adjacent flight levels, the overall altitude errors of the aircraft on the two levels were calculated by combining two of the 613-ft values by statistical summation. This calculation, $3 \sqrt{\left(\frac{6,3}{3}\right)^{2}+\left(\frac{618}{3}\right)^{2}}$, which can also be expressed as $618 \sqrt{2}$, yields a value of 874 ft , which was then considered to represent the loss in veitical sepazation that would be experienced by 0.3 percent of the aircraft assigned to the two flight levels. When this separationloss figure was increased $b_{j} 50 \mathrm{ft}$ to account for the vertical dimensions of the aircraft, the actual separation for an assigned separation of 1000 ft was 76 ft .

A more conservative approach to the vertical separation problem would require that the maximum probable overall altitude errcrs of the aircraft on adjacent flight levels be less than one-half of the vertical separation minimm, or 500 ft for an assigned separation of 1000 ft . This approach was taken by IATA in its assessment of tie altimeter and flight technical errors in reference 6. The altimeter-system errors for this study were determined experimentally during the same tests, discussed in the previous section, that the flic: $=$ technical errors of commercial transports were measured over the :Jorth Atlantia in the altitude range above 29000 ft . In these tests, the comined altimetersystem errors of two aircraEt were determined from a comparison of tie geometric and indicated altitudes of aircraft on adjacent flight levels. The indicated altitudes were measured with the cockpit altimeters, while the geomerric alititudes were measured with racar altimeters. The results of the tests snowed tie combined altimeter-system errors to have a normal distribution with a maximum probable value ( 35 ) of 510 Et. From this value for two aircraft, the maximun probable value for one aircraft was calculated to be $510 / \sqrt{2}$, or 360 E. Tiut overall altitude error for one aircraft was then determined as the statistica: sum of this $360-\mathrm{ft}$ value aris the maximum probable value of the fligit sechnien:
error (190 ft) which had also been measured in the IATA tests. The resulting error, $\sqrt[3]{\left(\frac{360}{3}\right)^{2}+\left(\frac{190}{3}\right)^{2}}$ or 408 ft , is thus 92 ft less than one-half the 1000-ft separation minimum.

While the vertical separation problem is a major part of the collision avoidance problem for aircraft flying at adjacent flight leveis, the longitudinal and lateral separations of the aircraft must also be taken into account in any assessment of collision risk. A mathematical model for estimating collision probabilities is described in references 14 and 15. An assessment of this model and of other methods of evaluating collision xisk is contained in reference 16.

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TABLE 12.1.- INDICATED ALTITUDES AND GEOMETRIC HEIGHTS FOR LOW-TEMPERATURE ATMOSPHERES AT THREE AIRPORTS

| QNH station | $\mathrm{a}_{\mathrm{H}_{\mathrm{i}}, \mathrm{ft}}$ | $\dot{z}_{\mathrm{Z}, \mathrm{ft}}$ | $\mathrm{H}_{\mathrm{i}}-\mathrm{z}, \mathrm{ft}$ |
| :---: | :---: | :---: | :---: |
| Seattle, Washington | 12000 | 11225 | 775 |
| Great Falls, Montana | 13000 | 12150 | 850 |
| Spokane, Washington | 14000 | 13050 | 950 |

antitude indicated by altimeter with barometic subdial set to Q NH .
beometric height computed from equation (12.1).

(a) Sea-level pressure below standard and temperature gradient standard.

(b) Sea-level pressure standard and temperature gradient below standard.

Figure 12.1.- Two hypothetical pressure-height variations in the atmosphere.

$\begin{array}{ll}E & \text { elevation of airport } \\ \text { QNH barometric scale setting at elevation } E \\ P_{0} & \text { pressure at sea level } \\ Z & \text { geometric height of airplane } \\ P_{a} & \text { pressure at height } Z \\ H_{i} & \text { height indicated by altimeter at } Z\end{array}$
Figure 12.2.- Pressure-neight variation 1 . an atmospiere in winich tie sea-ievel Eressure is standard and the temperature jradiant is below standard. Altimeter at elevation $E$ is set vo tioc zuly value at that elevation.

```
_-Great Falls, Montana
- - - Spokane,Washington
-_ Seattie, Washington
```



Figure 12.3.- Low-iemperature atmospheres at three airports in the northwestern United States.


Overall altitude error $\left\{\begin{array}{l}\text { Alcimeter system error } \\ \text { Fliglit Lechicical error }\end{array}\right.$
Pigure 12.5.- Uverall altitude error. (Adapted from ref. 13.)

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CHAPTER XIII

## OTHER ALTITUDE-MEASUNING METHODS

Thus far, the only altitude-measuring method that has been discussed is based on the measurement of atmospheric pressure and the pressure-height variation in the stanciard atmosphere. Because of the exponential decrease of pressure with heig'it in this atmosphere and the decreased accuracy of the pressure altimeter at altitudes above 50000 f.t, a variety of other methods have been investigated tor measuring altitude at high altitudes (refs. 1 and 2). A number of low-range altimeters have also been investigated for measuring height aiove the terrain during landing approaches. For a discussion of both the high-range and low-range methods, the various altimeters are grouped according to the following classification:

Measurement of height above the terrain
Radio and radar altimeters
Laser altimeter
Sonic altimeter
Capacitance altimeter
Measurement of altitude (pressure or dersity) above sea level
Density altimeter
Limited-range pressure altimeter
Hypsometer
Measurement of neight abive sea level
Cosmic-ray altimeter
Gravity meter
Magnetometer
0 all the altimeters in the foregoing list, only the radio and radar altimeters have betr developed for operational use in service aircraft. The limited-range pressure altimeter has been used ir: flicht tests oz an experimental airplane, while the hypsometer has been used in radiosondes, rocketsondes, and balloons. The remaining aitimeters have been develcped as experimental models to test the feasibility of the altitude-measuring principies.

## Radic and Radar Altimeters

:Heasurement of jeight by radio and radar aitimeters is accomplisied by transmitring a radio-frequincy wave from the aircraft to the ground and measuring sone characteristic of the reflected wave.
ixith radio altimeters, a continuous wave, moculated in either srequency or amplitude, is transmitted from the aircraft, and the retirn signal is comparsi with a sample of the instantaneous signal being transmitted. In the Erequanel
modulated type, the difference between the frequencies of the transmitted and received signals, which is a function of the modulation ratc and time, frovides a measure of the height. In the phase-comparison-type altimeter, the prase relation between the transmitted signal (which may be either erequency cr ampl:tude modulated) and the received signal provides a measure of the signal trans: time and, thus, of height.

The accuracy of radio altimeters is generally $\pm 2 \mathrm{ft}$ fir heights up to 40 ft and $\pm 2.5$ percent of the height for heigits above 40 ft . The heigh: ranc: is usually limitec to 3000 ft because the errors become excessive at greater heigits.

Wi ih the radar altimeter, the radiation is transmitted as a series of discrete pulses, and the distance between the aircraft and the ground is determined by measuring the time for the reflected wave to be received at the aircraft. Since the accuracy of the instrument depends on the width of the transmi=ted pulse and oin the accuracy of the time measurement, measuremerts at low heights require ultrashort pulses and extremely precise time measurements. For this reason, the lower limit of the range of radar altimeters is generally at least $50^{\circ} 0$ Et above the ground.

The accuracy of radar altimeters is $\pm(25 \mathrm{ft}+0.22 \mathrm{f}$ percert of the height $)$ and the height range is 500 ft to 60 c 0 ft . To provide neight measurements belcw 500 ft, some manufacturers have developed radio-radar altimeters in winic: the radio altimeter operates from 0 to 3000 ft and the radar altimeter Ercm 3000 ft to 60000 ft .

The accuracy and the maximum range of radar and radio altimeters depend not only on the characteristics of the instrument but also on the nature of the terrain below the aircraft. With the exception of very smooth and dense surfaces (such as calm lakes and paved runway surfaces), the reflection of the transry thed wave from the terrain is diffuse rather than spezil-r (mirror reflection). This diffused scattering of the wave results in a los in ミower of the reflected wave which, in combination with the fower lost by the ajsorption of wave energy by the terrain, linits the maximun altitude ca:jabili=y of the altimeter.

The accuracy of the heigit indica:ions can also be aftected when the transmitted signal is captured anc reflected by the terrain nearsst the aircrāt. Thus, when the aircraft is flying in tie vicinity of mouncain.i, the altimeter may measure the distance to some part of the nearcst hill.

Fadar and radio altimeters have a high order of accrrac'f and are valuable instruments for indications of terrai:. clearance. They wouls te unsuitajle sce the rertical separation of aircraft at high alcitudes, fowever, because =hey measure height above the terrain rather than above sea level. Eurthermcre, the accuracy of the radar at an altitiade of 50000 ft is rot siguificanti better than that of the best of the present-cay computer-corrected pressuye altimeters.

## Lāser Altimeter

A laser-cype a! :imeter has recontly been developed for measurirs height above the terraiis at altitudes up to 3000 ft !ref. 3;. The laser system consists of a pulsed laser transmitter and receiver and a timing device to measure the transit time of the pulse to the ground and back to the receiver.

The experimental model described in reference 3 has been flight-tested over various types of terrain (farmland, wooded areas, and open bedies of water) at altitudes up to 2000 ft . Recordings of the ground profiles indicated good signal return over well-defined terrain, but some uncertainty in the height measurements over wooded areas where the laser pulses did not always penetrate the foliage to the ground level. In addition, discontinuities in the Eecorded data occurred over surfaces with low diffuse reflectivity, such as asphalt paving.

## Sonic Altimeter

Sonic altimeters measure height above the terrain by transmitting a sound wave from the aircraft and measuring either (1) the time for the groundreflected signal to be received at the aircraft or (2) the phase shift of the reflected signal. Because of the relatively low speed of sound, altimeters utilizing sound transmission are limited to low altitudes and low speeds. For one pulse-type altimeter, the altitude limitation is 300 ft and the aircraftspeed limitation is 150 knots.

Tine reliability of sonic altimeters is very dependent on the character of the terrain below the aircraft. In flight tests of a pulse-type altimeter over a soft terrain such as grassland, for example, the phinter of the indicator fluctuated through a wide amplitude. Even over haz surfaces suci as a concrete runway, pointer fluctuations occurred at altitudes above 100 ft because of the weak signal return at those heights.

## Capacitance Altimeter

Since an aircraft and the Earth can act as the two plates of a condenser, the capacitance, which varies with the distance between the two plates, can be used as a means of measuring the height of the aircraft above the ground. In one appiication of this method (ref. 4), use was made of the principle that the capacitance between two insulated conductors is altered by the proximity $c=a$ third conductor. Thus, two insulated electrodes can be mounted some distance apart on an aircraft, so that the capacitance between the electrodes provides a measure of the distance between the aircraft and the ground. The change in capacitance with height is greatest when the aircraft is close to the ground and decreases rapidiy as the height of the aircraft increases.

In the development of the capacitance altimeter reported in reference 4 , flight tests were conducted with various types of electrodes instalied on the wing tips or on the underside of the fuselage of a variety of airaraf:. The results of tie tests showed that the altitude range over which reliable height indications could te obtained was generally less than 200 ft .

## Density Altimeter

A number of devices have been investigated for the measurement or air density on radiosondes, aircraft, and missiles. In one system, air from an ai=sampling sensor is brought into a chamber where the density of that air is determined by (1) measuring the breakdown fotential between two electrodes, (2) measuring the change in resistance of a heated wire resulting from the cooling action of the air, or (3) ionizing the air by means of a heated or radioactive cathode and then measuring the resulting ionic current. In another system, a beta- or ultraviolet-ray emitter on the forward part of the aircrát ionizes a portion of the air immediately ahead of the aircraft; the backscatter produced by the ionization of the air is then measured by a detector located near the emitter.

The altitude range of density-type altimeters begins at an altitude of about 50000 ft , because at lower altitudes, the measurements are adversely affected by the presence of water vapor in the air. The use of a ciensity altimeter as an operational instrument, therefore, would require an auxiliary pressure altimeter below 50000 ft . Furthermore, since the accuracy of the density altimeter $=$ that have been deselope, is no greater than that of the pressure altimeter, tie density altimeter offers no advantage over present-day operational syitems.

## Limited-Range Pressure Altimeter

With the limited-range pressure altimeter, the aneroid is a so-called collapsed, or nesting, capsule that is designed to start its deflection at some high altitude. In one design of this type of instrument, the lower limit of the operating range was 50000 ft . Thus, like the density altimeter, the ise of a limited-range pressure altimeter would requi- $s$ an auxiliary pressure altineter at the lower altitudes. The accuracy that can be achieved with the limitecrange pressure altimeter is greater than that of the pressure altimeter i:- ine range from 50000 to 80000 ft , but is no greater than the accuracy of the digital-type transducer system described in chapter XI.

## Hypsometer

The operation of the hypsometer is'based on the principle that the boiling point of a pure liquid is a function of the atmospheric pressure acting on the surface of the liquid (refs. 5, 6, and 7). The atmospheric pressure can tius be derived from measurements of the temperatur: just above the surface of a boilinc liquid. The attractive feature of this instrumert is that the boiling point $0=$ most liquids is approximately a logarithmic function of pressure and, finus, varies in an approximately linear manner with altitude.

- In its simplest form, the hypsometer consists of an insulated container which is open to the atmosphere, an evaporative liquid which boils at some reduced pressure, and a temperature-measuring element located in the vapor abo: $=$ the surface of the liquid. In a more advanced form, a condenser, surrouncied with a coolant, is attached to the liquid container in order to reflux tie rapcr back to the container. This type has the advantage that the level of the
evaporative sluid remains approximately constant and thereby insures moze ccrisistent measurements of the vapor temperature. It has the additional ejvantage of having a longer operating time for a given quantity of fluid because vapor is not lost as rapidly as with the simplified type.

The accuracy that can be achieved with a hypsometer depends on the degrse to which the vapor-liquid equilibrium is maintained, on the stability $c:$ the temperature-measuring element, and on the accuracy of the thermometer. Since the best accuracy that can be achieved is no greater than about 0.5 persent of the indicated altitude, the accuracy of hypsometer systems is considerably iower than that of the pressure altimeter.

## Cosmic-Ray Altimeter

Measurement of altitude by means of cosmic rays is possible because the intensity of the cosnic rays in the atmosphere increases in an approxizately linear manner with height through an altitude range from about $15000 \equiv$ to 100000 ft . Measurements below 15000 ft are unreliable because of the marked decrease in the variation of cosmic-ray inさensity with height near the Earth.

A cosmictray altimeter utilizing two groups of five Geiger counters to detect the concentration of the cosmic radiation is described in refersnce 0. The outputs of the Geiger counters, which provide a statistical measur^ of the radiation, are registered on a galvanometer which is calibrated in terns of altitude. In flight tests of a model of this instrument through an al:itude range up to 30000 ft , the altitude indications agreed with those of a pressure altimeter to within $\pm 500 \mathrm{ft}$ at altitudes above 15000 ft .

The use of cosmic rays for the measurement of altitude would be linited by the fact that the cosmic-ray intensity at a given height varies markeciv with latitude. i cosmic-ray altimeter would also be affected by the large raria*ions in cosmic radiation tiat accompany solar flares and magnetic storms.

## Gravity Meter

Measurement of gravity can be used as a means of deriving altitucie because the acceleration of gravity decreases with height in a linear manner (Eor ai=itudes up to 100000 ft ) and because the gravitational-height relation sessentially invariant (along a line above any given point on the Earti).

The change in the acceleration of gravity from sea level to 100 fioft in thr middle latitudes, however, is only about 0.01 g . With one airborre gravi: $-\bar{y}$ meter (ref. 9), the best accuracy that could be attained was about $10^{-3} 3$, wich is equivalert to a height error of about 100 ft .

Also, the accuracy of the height measurements would be feterninec so a large exten: by horizontal gravity gradients. The gradient betwe 2 n ti: equa=0: and the poles, for example, is about 0.005 g , or an equivalent heigit inarement of about 50300 ft . Horizontal gradients also occur because of gravi=i:ionai aromalies due to local variations in the donsity of the Earti. Over $\equiv$ :...e
regions of the Earth the gradients san re as $m u=$ h $a s 10^{-5} \mathrm{~g}$, or 100 ft , per male (ref. 10). Although the gradients cue to anomalies are attenuated wita height, the effects remain severe even at aporeciable alti=ides. The tests ofreserence 9, for example, showed that, in a level fligh: zun at 12500 ft cuer a mountainous area, a gravimeter recordeu 3 change $c=10^{-4} \mathrm{~g}$, or 1000 ft , over a distance of about 30 miles.

The measurements of a gravity meter are also Efected by accelerations resulting from (1) changes in the aircraft attitude. (2) airc:aft response to air turbulence, (3) maneuvers, and (4) airspeed wisi respect vo the Earth's rotation. The accelerations resulting from flight =hrough turbulent air and from vertical-plane maneuvers can, of course, be verf large with resfect is the 0.01 g increment corresponding to the $100000-\mathrm{ft}$ al: t tude range. The accelerations which result from the speed of the aircraft with respect :0 the Earin's rotation are in the form of centrifugal and Coriolis accelerations which, or some flight conditions, can be quite large (ref. 2).

## Magnetmeter

The magnetometer measures the total field intensity at any given poi:= within the Carth's magnetic field. Since the mago=ic field strength decreass with distance above the Earth, the magnetometer has been investigated as a possible means of measuring height (ref. 11).

The measurements of a magnetometer, however, muld be affected ig the variation of the vertical rate of ciange of intens:-y with latitude ( ${ }^{\text {a }}$ ( 0 the convergence of the lines of force at the poles). His change in intensity with height varies from about 6 gamas per 1000 ft at the equator to about 10 cammas per 1000 ft at the poles. Thus, for the 3 -gamma ascuracy of the magnetometer described in reference 11 , the error ir. the height measurement would rary f:om about 500 ft at the equator to about 300 ft at the goles.

The measurements of a magnetometer would als= $\approx \in$ affected oy ervã: : jariaー tions of the field intensity over certain fortions of the Earth. Periodic variations, which occur with the solar cycle, can je as much as 80 gamas at the equator while being negligible at the poles. Eperiodic variatiors, associated with aurora and magnetic storm activity, ca be quite severe. The effec= of the aurora can cause changes of as much as $100 \geqslant$ anmas at the poles while being negligible at the equator, whereas magnetic 三Eorm activity can accc:ant for fluctuations of as much as 200 gamas.

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TABLES OF AIRSPEED, ALTITUDE, AND MACH NUMBER
Some of the tables in this appendix present the independent variable in two parts: large increments in the left column and smaller increments along the top row. In table Al, for example, the pressure at 1100 ft is 28.7508 in . Hg.

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## APPENDIX A

TARLE AL. - STATIC PRESSURE $p$ (OR $p^{\prime}$ ) IN INCHES OF SERCURY ( $C^{\circ} C$ ) FOR VALUES OF PRESSURE ALTITUDE $H$ (OR INDICATED ALTITUDE H') IN GEOPOTENTIAL FEET

$$
\begin{array}{ll}
\text { [frce ref. A1] } & \text { ORIGNAL PAGE! } \\
& \text { OF POOR QUALII; }
\end{array}
$$

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | 0 | 100 | 200 | 300 | 430 | 500 | 600 | 700 | 800 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 000 | 31.0185 |  |  |  |  |  |  |  |  |  |
|  |  | 30.0295 | 30.1381 | 30.2471 | 30.3563 | 30.4659 | 30.5751 | 30.6859 | 30.7955 | 30.9073 |
| 0 | 29.9213 | 29.8133 | 29.7756 | 25.5983 | 29.4913 | 29.3846 | 29.2782 | 29.1721 | 29.0663 | 28.9608 |
| 1000 | 28.8557 | 28.7508 | 28.6463 | 28.5421 | 28.4382 | 28.3345 | 28.2312 | 28.1282 | 28.0255 | 27.9231 |
| 2000 | 27.8210 | 27.7193 | 27.6178 | 27.5166 | 27.4157 | 27.3151 | 27.214F | 27.1148 | 27.0152 | 26.9158 |
| 3000 | 26.8167 | 26.7179 | 26.6194 | 26.5211 | 26.4232 | 26.3256 | 26.228: | 26.1312 | $26.034 j$ | 25.9380 |
| 4000 | 25.8418 | 25.7460 | 25.6504 | 25.5551 | 25.4600 | 25.3653 | 25.271 .9 | 25.1767 | 25.0828 | 24.9892 |
| 5000 | 24.8959 | 24.8029 | 24.710: | 24.6177 | 24.5255 | 24.4336 | 24.3420 | 24.2506 | 24.1595 | 24.0687 |
| 6000 | 23.9782 | 23.8880 | 23.7980 | 23.7083 | 123.6189 | 23.5298 | 23.4409 | 23.3523 | 23.2640 | 23.1759 |
| 7000 | 23.0881 | 23.0006 | 22.9133 | 1220504 | 22.7397 | 22.6532 | 22.5n70 | 22.4811 | 22.3955 | 22.3101 |
| 8000 | 22.2250 | 22.1401 | 22.0555 | 21.9712 | 21.8871 | 21.8033 | 21.7197 | 22.6364 | 21.5534 | 21.4706 |
| 9000 | 21.3381 | 21.3059 | 21.2238 | 21.1421 | 21.0606 | 20.9794 | 20.8384 | 20.8177 | 20.7372 | 20.6569 |
| 10000 | 20.5770 | 20.4972 | 20.4178 | 2c. 3385 | 20.2596 | 20.1808 | 20.1324 | 20.0241 | 19.9461 | 19.8684 |
| 11000 | 19.7909 | 19.7137 | 19.6367 | 19.5599 | 19.4834 | 19.4071 | 19.3311 | 19.2553 | 19.1797 | 19.1044 |
| 12000 | 19.0294 | 18.9545 | 18.8799 | 18.8056 | 18.7315 | 18.6576 | 18.5839 | 18.5105 | 18.4374 | 18.3644 |
| 13000 | 18.2917 | 18.2192 | 12.1470 | 18.0759 | 18.0032 | 17.0317 | 17.8603 | 17.7893 | 17.7184 | 17.6478 |
| 14000 | 17.5774 | 17.5072 | 17.4373 | 17.3575 | 17.2981 | 17.2288 | 17.1597 | 17.0909 | 17.0223 | 16.9540 |
| 15000 | 16.8858 | 16.8179 | : 5.7502 | 16.6827 | 16.6154 | 16.5484 | 16.4816 | 16.4150 | 16.3486 | 16.2824 |
| 16000 | 16.2164 | 16.1507 | 16.0852 | 16.6.199 | 15.9548 | 15.8899 | 15.8252 | 15.76J\% | 15.6966 | 15.6325 |
| 17000 | 15.5687 | 15.50s 1 | 15.43i7 | 1:. 3785 | 15.3156 | 15.2528 | 15.1903 | 15.1279 | 15.0658 | 15.0038 |
| 18000 | 14.9421 | 14.8806 | 14.8193 | 14.7562 | 14.6973 | 14.6366 | 14.5761 | 14.5158 | 14.4557 | 14.3958 |
| 19000 | 14. 3361 | 14.2766 | 14.2173 | 14.1582 | 14.0993 | 14.0406 | 13.9821 | 13.9238 | 13.865; | 13.8078 |
| 20000 | 13.7501 | 13.6926 | 13.6353 | 13.5782 | 13.5212 | 23.4645 | 13.4079 | 13.3516 | 13.2954 | 13.2395 |
| 21000 | 13.1837 | 13.1281 | 13.0727 | 13.0175 | 12.9625 | 12.9676 | 12.8530 | 12.7985 | 12.7443 | 12.6902 |
| 22000 | 12.6363 | 12.5826 | 12.5291 | 12.4757 | 12.4226 | 12.3696 | 12. 3168 | 12.2642 | 12.2118 | 12.1595 |
| 23000 | 12.1075 | 12.0556 | 12.0039 | 11.9524 | 11.9010 | 11.8499 | 11.7989 | 11.7481 | 11.6974 | 11.6470 |
| 24000 | 11.5967 | 11.5466 | 11.4967 | 11.4469 | 11.3974 | 11.3480 | 11.2997 | 11.2497 | 11.2003 | 11.1521 |
| 25000 | 11.1035 | 11.0552 | 11.0070 | 10.9589 | 10.9111 | 10.8634 | 10.8159 | 10.7685 | 10.7213 | 10.6743 |
| 26000 | 10.6275 | 10.5808 | 10.5343 | 10.4879 | 10.4417 | 10.3957 | 10.3499 | 1c. 3042 | 10.2587 | 10.2133 |
| 27 00c | 10.1681 | 10.1230 | 10.0782 | 10.0335 | 9.98889 | 9.94450 | 9.90026 | 9.85619 | 9.81227 | 9.76851 |
| 28000 | 9.72491 | 9.68147 | 9.63818 | 9.59505 | 9.55208 | 9.50926 | 9.46660 | 9.42413 | 9.38174 | -.72955 |
| 29000 | 9.29750 | 9.25561 | 9.21388 | 9.17229 | 9.13086 | 9.08958 | 9.04845 | 9.0074 ? | 0.96665 | 8.92597 |
| 30000 | 8.88544 | 8.84506 | 8.80483 | 8.76475 | 8.72481 | 8.68502 | 3.64539 | 8.60589 | 8.56654 | 8.52734 |
| 31000 | 8.48829 | 9.44938 | 8.41060 | 8.37199 | 8.33351 | 8.29517 | 8.25698 | 8.21893 | 8.18122 | 9.14326 |
| 32000 | 8.10563 | 8.06815 | 8.03081 | 7.93300 | 7.95654 | 7.91961 | 7.88283 | 7.84619 | 7.80967 | 7.77339 |
| 33000 | 7.73707 | 7.75097 | 7.66501 | 7.62919 | 7.59350 | 7.55794 | 7.52253 | 7.48724 | 7.45239 | 7.41708 |
| 34000 | 7.38219 | 7.34744 | 7.31283 | 7.27834 | 7.24399 | 7.20977 | 7.17568 | 7.14172 | 7.107391 | 7.07417 |
| 35000 | 7.04062 |  | 6.97386 |  | 6.90762 |  | 6.84189 |  | 6.776e? |  |
| 36000 | 6.71195 |  | 6.64775 |  | 6.58415 |  | 6.52115 |  | 5.458781 |  |
| 37000 | 6.39699 |  | 6.33579 |  | 6.27518 |  | 6.21515 |  | 6.155691 |  |
| 38000 | 6.09680 |  | 6.03847 |  | 5.99871 |  | 5.92349 |  | 5.86582; |  |
| 39000 | 5.81070 |  | 5.75511 |  | 5.72005 |  | 5.64552 |  | 5.59151 : |  |
| 40000 | 5.53802 |  | 5.48504 |  | 5.43257 |  | ¢. 38.06ci |  | 5.32912 |  |
| 41000 | 5.27814 |  | 5.22765 |  | 5.17763 |  | 5.12810 |  | 5.37904. |  |
| 42000 | 5.03045 |  | 4.98233 |  | 4.92 .466 |  | 4.38746 |  | 4.845751 |  |
| 43000 | 4.79439 |  | 4.74852 |  | 4.73310 |  | 4.65819 |  | 4.51354' |  |
| 44000 | 4.569 .1 |  | 4.52569 |  | 4.48280 |  | 4.43951 |  | 4.33734. |  |
| 45000 | 4.35498 |  | 4.31332 |  | 4.27205 |  | 4.23113 |  | +.12:- |  |
| 46300 | 4.15061 |  | 4.11091 |  | 4.07158 |  | 4.33263 |  | 3. 3975 |  |
| 47000 | 3.9558: |  | 3.91800 |  | 3.88051 |  | 3.84339 |  | 3.3565: |  |
| 48300 | 3.77020 |  | 3.73414 |  | 3.6こ941 |  | 3.66303 |  | 3.5-33 |  |
| 49000 | 3.59328 |  | 3.55891 |  | 3.52:86 |  | 3.47114 |  | 3.45--4: |  |

TABLE Al.- Concluded

| н, $f t$ | 0 | 200 | 400 | 600 | 300 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50000 | 3. 42466 | 3.39190 | 3.35945 | 3.32731 | 3.29548 |
| S1 000 | 3.26395 | 3.23273 | 3.20180 | 3.17117 | 3.14983 |
| 52000 | 3.11079 | 3.08103 | 3.05155 | 3.02236 | 2.99344 |
| 53000 | 2.96481 | 2.93644 | 2.90835 | 2.88053 | 2.85297 |
| 54000 | 2.82568 | 2.79865 | 2.77187 | 2.74535 | 2.71909 |
| 55000 | 2.6.3 308 | 2.66731 | 2.64180 | 2.61652 | 2.59149 |
| 56000 | 2.56670 | 2.54215 | 2.51783 | 2.49374 | 2.46988 |
| 57000 | 2.44625 | 2.42285 | 2.39967 | 2.37672 | 2.35308 |
| 58000 | 2.33146 | 2.30916 | 2.23706 | 2.26519 | 2.24351 |
| 59000 | 2.22205 | 2.20079 | 2.17974 | 2.15889 | $2.138: 3$ |
| 60000 | 2.11778 | 2.09752 | 2.07745 | 2.05758 | 2.03789 |
| 61000 | 2.01840 | 1.99909 | 1.97996 | 1.96102 | 1.94226 |
| 62000 | 1.92368 | 1.90528 | 1.38705 | 1.86900 | 1.85112 |
| 63000 | 1.83341 | 1.81587 | 1.79850 | 1.78129 | 1.76425 |
| 64000 | 1.74737 | 1.73066 | 1.71410 | 1.69770 | 1.68146 |
| 65000 | 1.66538 | 1.64944 | 1.63366 | 1.61803 | 1.60256 |
| 66000 | 1.58723 | 1.57206 | 1.55703 | 1.54216 | 1.52742 |
| 67000 | 1.51284 | 1.49840 | 1.48410 | 1.46994 | 1.45591 |
| 68000 | 1.44203 | 1.42828 | 1.41467 | 1.40119 | 1.38784 |
| 69000 | 1.37463 | 1.36154 | 1.34858 | 1.33575 | 1. 32304 |
| 70000 | 1.31046 | 1.29800 | 1.28567 | 1.27345 | 1. 26135 |
| 71000 | 1.24938 | 1.23751 | 1.22577 | 1.21414 | 1.25262 |
| 72000 | 1.19122 | 1.17992 | 1.16874 | 1.15767 | 1.14670 |
| 73000 | 1.13584 | 1.12509 | 1.11444 | 1.10389 | 1.09345 |
| 74 000 | 1.08311 | 1.07287 | 1.06273 | 1.05269 | 1.04274 |
| 75000 | 1.03290 | 1.02314 | 1.01349 | 1.00392 | . 994453 |
| 76000 | . 985074 | . 975787 | . 966589 | . 957481 | . 948461 |
| 77000 | . 939529 | . 930682 | . 921922 | . 913248 | . 904656 |
| 78000 | .596148 | . 887722 | . 879377 | . 871114 | . 862931 |
| 79000 | . 854826 | . 86:799 | . 838851 | . 830979 | . 823183 |
| E0 000 | . 815462 | . 807816 | . 800243 | . 792744 | . 785317 |
| 81000 | . 777962 | . 770677 | . 763463 | . 756317 | . 749241 |
| 82000 | . 742233 | . 735293 | . 723419 | . 721612 | . 714870 |
| 83000 | . 708192 | . 701579 | . 695029 | . 688543 | . 682119 |
| 84000 | . 675756 | . 669454 | . 663213 | . 657031 | . 650910 |
| 85000 | . 644846 | . 638841 | . 532893 | . 627003 | . 621169 |
| 86000 | . 615390 | . 609667 | . 603999 | . 598385 | . 592824 |
| 87000 | . 587317 | . 581862 | . 576460 | . $57110^{\prime}$ | . 565803 |
| 88000 | . 560560 | . 555361 | . 550212 | . 54511 : | . 540060 |
| 89000 | . 535655 | . 530101 | . 525192 | . 520330 | . 515513 |
| 90000 | . 510745 | . 506021 | . 501342 | . $\$ 96707$ | . 492117 |
| 91000 | . 487570 | . 483066 | . 478605 | . 74187 | . 463813 |
| 92000 | . 465475 | . 461182 | . 456928 | . 452716 | . 470543 |
| 93000 | - 4.4410 | .440316 | . +36261 | . 32294 | . 422255 |
| 94000 | . 424324 | . $i 29421$ | . 415554 | . +12724 | .4-3き? |
| 95000 | . 405172 | . 421449 | . 237752 | . 37411 ) | . 3974 22 |
| 96000 | . 386908 | . 333358 | . $30-72$ | . 35.535 | . $3^{-} \cdot 6=$ |
| 77000 | . 369430 | . 366105 | . 352751 | . 353429 | . 35.138 |
| 78000 | . 352379 | . 347655 | . 335451 | . 343283 | . 3i314i |
| 79300 | . 337035 | . 333955 | . 33.304 | . 327882 | . 323358 |
| :20.000 | . 321922 |  |  |  |  |

Thble az－static pressure $p$（OR $p^{\prime}$ ）in pounds per square foot for jalues of
Pressure abtitude h（OR IMDICATED ALTITUDE H＇）IM GEOPGTEMTIAL FEET
［Dastoud from ref．A1］

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{E} \end{aligned}$ | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{rr} -1 & 000 \\ & -c \end{array}\right\|$ | 2193.82 | 2123.87 | 2131.55 | 2139.26 | 2146.99 | 2154.74 | 2162.50 | 2170.29 | 2178.12 | 2185.96 |
| 0 | 2116.22 | 2100.58 | 2100.96 | 2093． 38 | 2085.81 | 2078.26 | 2070.74 | 2053.23 | 2055.75 | 248.29 |
| 1000 | 2040.85 | 2633.43 | 2026.04 | 2018.67 | 2011.33 | 2003.99 | 1996.69 | 1989.40 | 1982．：4 | 1974.89 |
| 2000 | 1967.67 | 1960.48 | 1953.30 | 1946．14 | 1939.01 | 1931.29 | 1924.80 | 1917.73 | 1910.68 | 1903.65 |
| 3000 | 1896.64 | 1889.66 | 1882.69 | 1875.74 | 1868.81 | 1861.91 | 1855.03 | 1848.16 | 1841.32 | 1834.50 |
| 4050 | 1827.69 | 1820.92 | 1814.16 | 1807.42 | 1800.69 | 1793.90 | 1737.31 | 1780.65 | 1774.01 | 11767．39 |
| 5000 | 1760.79 | 1754.21 | 1747.65 | 1741.12 | 1734.60 | 1728.10 | 1721.62 | 1715.15 | 1708.71 | ［1702．29 |
| 6000 | 1695.89 | 1689.51 | 1683.14 | 1676.80 | 1670.48 | 1664.17 | 1657.89 | 1651.62 | 1645.37 | 1639.14 |
| 7000 | 1632.93 | 1626.75 | 1620．57 | 1614.42 | 1608.29 | 1602.17 | ：596．08 | 1590.00 | 1583.95 | ［1577．91 |
| 8000 | 1571.90 | 1565.89 | 1559.90 | 1553．94 | 1547.99 | 1542.06 | 1536.15 | 1530.26 | 1524.39 | 1518.53 |
| 9000 | 1512.70 | 1506.89 | 1501.08 | 1495.30 | － 899.54 | 1483.79 | 1478．06 | 1472.36 | 1466.66 | ． 1460.98 |
| 10000 | 1455.33 | 1449.69 | 1444.07 | 1438.46 | 1432.88 | 1427.31 | 1421.77 | 1416.23 | 1410.71 | 1：405．22 |
| 11000 | 1399.74 | 1394.28 | i 388.83 | 1383.40 | 1377.99 | 1372.59 | 1367.22 | 1361.85 | 1356.51 | 1351.18 |
| 12000 | 1345．88 | 1340.58 | 1335.30 | 1330.05 | 1324．81 | 1319.58 | 1314.37 | 1309.18 | 1304.01 | －1298．84 |
| 13000 | 1293.70 | 1288.57 | 1283.47 | 1278．3¢ | 1273.30 | 1268.24 | 1263.19 | 1258.17 | 1253.16 | －1248．16 |
| 14000 | 1243.18 | 1238.22 | 1233.27 | 1228.34 | 1223.43 | 11218.53 | 1213.64 | 1209.77 | 1203．92 | 1199.09 |
| 15000 | 1194.27 | 1189.47 | 1184.68 | 1179.90 | 1175.14 | 1179.41 | 1155.68 | 1160．97 | 1：36．27 | 1151.59 |
| 16000 | 1146.92 | 1142.28 | 1137.65 | 1133.03 | 1128.42 | 1123.83 | 1119.26 | 1114.70 | 1110.16 | 1105.63 |
| 17000 | 1101.11 | 1596.62 | 1092.13 | 1087.65 | 1083.21 | 1078.77 | 1074.35 | 1069.94 | 1065.55 | 1361．16 |
| 18000 | 1056.80 | 1052.45 | 1048.11 | 1043.79 | 1039.48 | 1035.19 | 1030.91 | 1026.65 | 1022.40 | 1318.16 |
| 19000 | 1013.94 | 1509.73 | 1005.54 | 1001.35 | 997.190 | 993.638 | 998.901 | 984.777 ！ | 98r． 668 | 376．573 |
| 20000 | 972.492 | 968.426 | 964.373 | 960.334 | 956.303 | 952.293 | 948.290 | 944.308 | 940.333 | 936．387 |
| 21000 | 932.433 | 928.501 | 924.582 | 920.678 ！ | 916.788 | 912.905 | 909.044 | 905.1891 | 901.356 | 897.530 |
| 2200 c | 893.117 | 889.919 | 886.136 | 882.3591 | 878.603 | 874.855 | 871.120 | 867.400 | 863.694 | 859.995 |
| 23000 | 656.317 | 852.647 | 848.990 | 845.348 | 841.713 | 838.098 | 814.491 | 830.898 | 827.313 | 823.748 |
| 24000 | 820.191 | 816.647 | 813.118 | 809．596 | 806.095 | 202.601 | 799.114 | 795.649 | 792.190 | 788.746 |
| 25000 | 755.308 | 781.892 | 778.483 | 775.081 | 771.701 | 768.327 | 764.968 | 761.615 | 758.277 | 754.953 |
| 26000 | 751.643 | 748.340 | 745.051 | 741.769 | 738.502 | 735.248 | 732.009 | 728.777 | 725.559 | 722.348 |
| 27000 | 719.151 | 715.9 ＇31 | 712.793 | 709.631 | 706.476 | 703.337 | 700.206 | 697.091 | 693.985 | 690.890 |
| 28000 | 687.8061 | 684． 734 | 681.672 | 678.621 | 675.582 | 672.554 | 669.537 | 666.531 ！ | 663.535 | 660.551 |
| 29000 | 657.577 | 654．014 | 651.663 | 648．721 | 645.791 | 642.871 | 639.962 | 637.064 | 534.177 | 631.300 |
| 30000 | 688.433 | 525.577 | 622.732 | 619．897！ | 617.073 | 614.258 | 611.456 | 609.662 | 605.879 | 603.156 |
| 31000 | 500.344 | 597.593 | 594.850 | $592.119^{1}$ | 589.397 | 586.686 | 583.985 | 581.294 | 578．612 | 575.942 |
| 32000 | 573.280 | 570.630 | 567.989 | 565．357 | 562.736 | 560.124 | 557．523 | 554.930 | 552.343. | 549.775 |
| 33000 | 547.214 | 544．660 | 542.117 | 539.584 | 537.059 | 534.544 | 532.046 | 529.544 ！ | 527.058 | 524．582 |
| 34000 | 522.114 | 519.657 | 517.209 | 514．769 | 512.340 | 509.920 | 507.509 | 505.107 | 502.714 ： | 500.332 |
| 35000 | 497.956 |  | 493.235 |  | 488.550 |  | 483.901 |  | 479.283 |  |
| 36000 | 474.711 |  | 470.170 ！ |  | 465.672 |  | 461.217 |  | 456.805 |  |
| 37000 | 452.435 |  | $448.106{ }^{\circ}$ |  | 443.820 |  | 439.574 |  | 435.369 |  |
| 38000 | 431.203 |  | 427.078 |  | 422.993 |  | 418.946 |  | 414.939 |  |
| 39000 | 410.969 |  | 407．037 |  | 403.143 |  | 399.236 |  | 395.456. |  |
| 40700 | 391．633 |  | 387．936！ |  | 384.225 |  | 380.5491 |  | 375.350 |  |
| 14200 | 373.323 |  | 369.732 ： |  | 366.194 |  | 362.691 |  | 359.221 |  |
| 12300 | 355.785 |  | 352.381 ！ |  | 349.010 |  | 345.571 |  | 342.364 |  |
| 143000 | 339.089 |  | 335.845 ｜ |  | 332.632 |  | 329.450 |  | 326．293 |  |
| ， 44300 | 323.177 |  | 320.084 |  | 317.023 |  | 313.990 |  | 322.3 ¢5 |  |
| 45300 | 339.0111 |  | 305.065 |  | 302.146 |  | 299．255 |  | 236.392 |  |
| 46300 | ＝33．5：7 |  | 290．749 |  | 287.967 |  | ：35．213｜ |  | こ92．364 |  |
| 47300 | ：79．761 |  | 277.105 |  | 274．454 |  | 271.828 |  | こと9．：こう |  |
| ： 48 Ocs | ： 6.652 |  | 264．102！ |  | 261.574 |  | －59．072 |  | 256.534 |  |
| 49300 | 25：139 |  | 251．708！ |  | 249.300 |  | －\＄6．715 |  | 295．55う |  |

tasle in2．－Conclured

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | 0 | 200 | i30 | 620 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50000 ： | 242.213 | 239.896 | 237.601 | 235.328 | 233．：77 |
| 51000 | 230.347 | 228.639 | 226.451 | 224.285 | 222.239 |
| 52000 | 220.014 | 2：7．910 | 215.825 | 213.760 | 211.215 |
| 53000 | 209.690 | 297.683 | 205.697 | 203.729 | 201．：30 |
| 54000 | 199.850 | 197.938 | 196．044 | 194.168 | 192.311 |
| 55000 | 190.471 | 138.649 | 196.844 | 185.057 | 183.296 |
| 56000 | 181.533 | 179.797 | 178.077 | 176.373 | 174.535 |
| 57000 | 173.014 | 171.359 | 169.720 | 168.096 | 160．488 |
| 58000 | 164.895 | 163.318 | 161.755 | 160.208 | 153.575 |
| 59000 | 157.157 | 153.654 | 154．165 | 152.690 | 151．229 |
| 60000 | 149.783 | 148.350 | 146.930 | 145.525 | 144 532 |
| 61000 | 142.754 | 141.388 | 140.035 | 138.696 | 137.369 |
| 62000 | 136.055 | 134.753 | 153.464 | 132.187 | 130.323 |
| 63000 | 129.670 | 129．430 | 12\％．201 | 125.984 | 12：．779 |
| 64000 | 123.585 | 122.403 | 121． 2 | 120.072 | 113.323 |
| 65000 | 117.785 | 116.659 | 115．543 | 114.437 | 113.343 |
| 66000 | 112.259 | 111.186 | 112.123 | 109.071 | 108.529 |
| 67000 | 106.997 | ：35．976 | 104.965 | 103.963 | 102.371 |
| 68000 | 101.989 | 101.017 | 120.054 | 99.1008 | 98.1566 |
| 69050 | 97.2224 | 96.2366 | 95.3803 | 94.4725 | 93.5736 |
| 70000 | 92.6839 | 91.8026 | 20.9306 | 94.0663 | 39.2105 |
| 71000 | 88.3639 | 37.5244 | ®う．6941 | 35.3715 | ミ5． 2567 |
| 72000 | 84.2505 | 33.4513 | 22．66こ5 | 6 6． 3776 | 31.1017 |
| 73000 | 20.3336 | 79.5733 | 73．82：1 | 70．0739 | －． 3356 |
| 74000 | 76.6043 | 75.8800 | 75．16：3 | 74.4528 | ． 7490 |
| 75000 | 73.0531 | 72.3628 | 71.6803 | 71.0034 | 73． 3339 |
| 76000 | 69.6705 | 69.0137 | 68.3632 | 67.7190 | 67.3810 |
| 77000 | 66.4493 | 65.8236 | 65.2040 | 64.5906 | 63.9829 |
| 78000 | 63.3811 | 62.7852 | 62.1950 | 61.6106 | 61.9318 |
| 79000 | 60.4586 | 59.8309 | 59.3287 | 53.7720 | 53.2206 |
| 30000 | 57.6745 | 57.1338 | 56.5981 | 56.0678 | 55．5425 |
| 81000 | 55.0223 | 54.5071 | 53.9959 | 53.4914 | 52.7910 |
| 82000 | 52.4953 | 52.0045 | 51.5123 | 51.0369 | 53.5601 |
| 83000 | 50.0877 | 49.6200 | 49.1553 | 48.6980 | ：6．2457 |
| 84000 | 47.7937 | 47.3479 | 96． 9055 | 46.4693 | ie． 3361 |
| 85000 | 45.6075 | 45.1828 | 44.7621 | 44.3455 | － 3.7329 |
| 86000 | 43.5242 | \＄3．1194 | 42．7136 | ＋2．3215 | 11． 7282 |
| 87000 | 41.5387 | 41.1529 | 40.7738 | \＄3．392\％ | ： 5.3175 |
| 88600 | 39.6463 | 39.2786 | 32．91：4 | 38.5537 | 32.1964 |
| 89000 | ：7，8425 | 37.4920 | 37．17：3 | 35.8009 | 36.8604 |
| 90000 | 36．1231 | 35.7889 | 35.4553 | 35.1302 | ：7． 3056 |
| 91000 | 34.4840 | 34.1654 | 53．8479 | 33.5374 | ？3．2299 |
| 92000 | 32.9213 | 32.6176 | 22.3153 | 32.0189 | 3：．7237 |
| 93000 | 31.4314 | 31.1419 | 30.65 1 | 32.5712 | $3 . .2396$ |
| 94000 | 30.0108 | 29.7348 | 29．4Eこ3 | $23.190 ;$ | こ．$\because 221$ |
| 95000 | 28.6563 | 28.3930 | 28．1：こ2 | ：－． 8739 | ＝－．512； |
| 96200 | 27.3645 | 27．1135 | 26．06：a | 25．6184 | －̇． 3744 |
| 972001 | 2.1326 | 25.9932 | 25．6ミこ0 | ここ．421： | こミ．1303 |
| 98 non | 24.9578 | 24.7294 | －4．5：32 | －7．2791 | \＃i． $250:$ |
| － 3000 | 23.8372 | 23.6194 | 23．7：36 | 23.1898 | 2.3751 |
| 100000 | 22.7683 |  |  |  |  |

## ORIGI：AL PAGE I： OF POOR QUALII



PRESSIT：AL：
［from ref．A1］

| $\mathrm{H} \text {, }$ $\mathfrak{f t}$ | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 320 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.076474 | 0.076251 | 0.076028 | 0.075805 | 0.0755831 | 0.67536 ？ | $0.075141^{12}$ | 20 | 0．：74：20 | ． 74 |
| 1000 | ． 074261 | ． 074043 | ． 073825 | ． 3736.97 | ． 073390 | ． 073174 | ． 072557 | ． 672742 | 07252： | 272312 |
| 2000 | ． 072098 | ． 071884 | ． 071671 | ． 071458 | ． 071246 | ． 071034 | ． 070823 | ． 070612 | ． 3784321 | ．576134 |
| 3000 | ． 069983 | ． 069774 | ． 069566 | ． 069358 | ．0f－50 | ． 268943 | ． 068737 | ． 368541 | ． 0683251 | ． 76812 n |
| 4000 | ． 007916 | ． 067712 | ． 067508 | ． 067325 | ． 067102 | ． 265900 | － 106596 | ． 066497 | ． $\mathbf{6 6 6} 261$ | ． 266.296 |
| 5000 | ． 065896 | ． 065696 | ． 065497 | ． 065290 | ． 065101 | ． 064903 | ． 764706 | ． 064509 | ． 5643131 | ． $66+117$ |
| 6000 | ． 063922 | ． 063727 | ． 063532 | ． 063339 | ． 063145 | ． 262552 | ． 062759 | ． 062567 | ． 262376 | ． 262194 ！ |
| 7000 | ． 061993 | ． 061803 | ． 061613 | ． 061424 | ． 061235 | ． 261036 | ． 6654858 | ． 06.56791 | ． 26.489 | 260296 |
| 9 000 | ． 060110 | ． 059924 | ． 359739 | ． 059554 | ． 059369 | ． 259185 ！ | ． 059501 | ． 258818 | ． 258685 | 354531 |
| 9000 | ． 058271 | ． 058089 | ． 057908 | ． 357727 | ．05754？ | ． 257367 | ．657189 | ． 057.091 | ． $25683 \%$ | ．2560．5\％ |
| 10000 | ． 056475 | ． 056297 | ．05612； | ． 055944 | ． 05576 a | ． 255593 | ． 255413 | ． 2552431 | c55969： | －54．355 |
| 11 coo | ． 054721 | ．054548 | ． 054376 | ． 754293 | 034032 | ． 253860 | ． 353683 | ． 253519 | ． 2533491 | 2531：＇s |
| 12000 | ． 053010 | ． 052841 | ． 052673 | ．0525\％5 | ． 052337 | ． 252170 | ． 352003 | ． 251837 | ． 55671 ， | ． 51515 |
| 13000 | ． 051340 | ． 051175 | ． 051021 | ． 050847 | ．050683！ | ． 250520 | ． 053357 | ．050195！ | ． 5 5－233 | ． 93947 |
| 14500 | ． 049710 | ． 049549 | ． 049389 | ． 046229 | ． 0490731 | － 248910 | ． 248752 | ．9485\％${ }^{\text {｜}}$ | ． 046435 ！ | － 9305 |
| 15000 | ． 048120 | ． 047964 | ． 0478 CO | ． 047651 | ． 047496 | ．0473：31 | ． 247185 | ． 245131 |  | ． 24670 |
| 16000 | ． 046570 | ． 046417 | ． 046264 | ． 086112 | ． 045961 | ． 245809 | ． 745658 | ． 245 S28 | ． 245357 | ．$\$ 55$ |
| 17000 | ． 045058 | ． 044909 | ． 044760 | ．044612 | ．044464 | ． 244316 | ． 344359 | ． 244322 | ． 243976 | ． 4372 ． |
| 18000 | ． 043584 | ． 043438 | ． 043293 | ． 043149 | ． 043005 | ． 042861 | ． 042717 | ． 041574 | ． $04: 431$ | － 22307 |
| 19000 | ． 042147 | ． 042005 | ． 041864 | ． 041723 | ． 041582 | ． 041442 | ． 041302 | ． 041163 | ． 041024 ！ | ． 043985 |
| 20000 | ． 040746 | ． 040608 | ． 040471 | ． 040333 | ． 040196 | ． 040060 | ． 039923 | ． 039787 | ． $33: 652$ | ． $3351: 1$ |
| 21000 | ． 039382 | ． 039247 | ． 039113 | ． 038979 | ． 038846 | ． 038713 | ． 038580 | ． 038448 | ．236116： | ． 338154 |
| 22000 | ． 038052 | ． 037921 | ．037751 | ． 037660 | ． 037530 | ． 037401 | ． 037272 | ．037143 | ． 237314 |  |
| 23000 | ． 036758 | ． 036630 | ． 036503 | ． 036376 | ． 036249 | ． 036123 | ． 635997 | ．035672！ | ． 335745 | ． 356. |
| 24000 | ． 035497 | ． 035373 | ． 035249 | ． 035125 | ． 035002 | ． 034879 | ． 034757 | ． 2346341 | －$\because 3451$. | $\therefore 3$ |
| 25000 | ． 034270 | ． 034149 | ． 034028 | ． 033938 | ． 033788 | ．033668 | ． 2335491 | ． 233332 | ？33：${ }^{\text {a }}$ | － $33: 3$ |
| 26000 | ． 033075 | ． 032957 | ． 032540 | ． 032723 | ． 032606 | ． 032490 | ． 232374 ： | ． 332259 | ？ 3 ：19－ | － 3 |
| 27000 | ． 031912 | ． 031798 | ． 031684 | ． 031570 | ． 031456 ： | ． 331343 ！ | ． 331233 ！ | ．312： | ． $3: 305$ | －， |
| 280001 | ． 030781 | ． 030670 | ． 030559 | ． 030488 | ． 030338 ： | ． $030227!$ | ． 332119 | ． 233088 | －$\because 3 m$ ， |  |
| 29 ここう | ． 029681 | ． 029573 | ． 329765 | ． 029357 | ．029250 | ． 029143 | ． 2290361 | ． 225929 |  | 20：－ |
| 30000 | ． 028611 | ． 0285061 | ． 028401 | ． 228296 | ． 028192 | ． 028 sab | ． 027364 | ．22：38： | $\because$ | $\therefore$－${ }^{\text {a }}$ |
| 31000 | ． 027571 | ． 0274691 | ． 027367 | ． 027265 | ． 627164 | ． 327062 | ． 326951 ； | ． 326951 | こ－－ | － |
| 32000 | ． 026561 | ． 026461 | ． 026362 | ． 026263 | ． 025164 － | ． $32606{ }^{\text {s }}$ ： | ． $2: 50$ | ． $25=72$ | ${ }^{2}$ | －7： |
| 33000 | ． 025578 | ． 025482 ！ | ． 025385 | ． 225289 | ． 225193 | ． 225094 ： | ． 24 ¢， 5 | ． 229 | －2\％： | ．7：－ |
| 34000 | ． 029624 | ． 024530 ！ | ． 024437 | ． 325343 | ． 224253 | ． 324157 | ．929－mix | ． 22073 | －－¢： | $\therefore$ ：－ |
| 35000 | ． 023697 |  | ． 023515 |  | ． 023334 |  | ．231154 |  | － 0 |  |
| 36000 | ． 022798 |  | ． 022598 |  | ． 022382 |  | ． 222146 |  | $\because$ 为 |  |
| 37000 | ． 221796 |  | ． 021538 |  | ． 221332 |  | ．221127 |  | － |  |
| 38000 | ． 220725 |  | ． 020527 |  | ． 020330 ． |  | ． 220136 |  | ＊－5： |  |
| 39000 | ． 319753 |  | ． 019564 |  | ｜．019375 |  | ． $3191>1$ |  | －：$\cdot$－ |  |
| 40000 | ． 218826 |  | ． 018646 |  | ． 013657 |  | ． 315291 |  | ．$:=:$ |  |
| 41 0cs | ． 217942 |  | ． 217771 |  | ． 017501 |  | － 17432 |  | ${ }^{-}$ |  |
| 42003 | ． 017100 ！ |  | ． 216937 |  | ． 016775 |  | ． 316814 |  | ： 5 ： |  |
| 43500 | ． 016298 |  | ． 216.42 |  | ． 215987 |  | ． 215335 |  | ：$\because$－ |  |
| 44000 | ． 215533 |  | ． 315194 |  | ． 215237 |  | ．$: 15.31$ |  | ：$: 9$ |  |
| 45 200！ | ． $21+304$ |  | ． 014662 ！ |  | ． 214522 |  | ．133n？ |  | ：$\%$－ |  |
| 46 000． | ． 314109 ： |  | ． 213374 |  | ． 213891 |  | －137－8 |  | ！ |  |
| 47 200！ | ． 313447 ： |  | ． 313319 ， |  | ． 213191 |  | ． 11.26 .5 |  | ： 0 |  |
| 463001 | ． $3129: 6 \mid$ |  | － $2: 694$ |  | ． $21: 572$ |  | 22：5 |  | ：．：$\square^{\text {？}}$ |  |
| 49000 | ． 212215 |  | － 3 ここヲ9 |  | ．211782 |  | $\cdots 11 \mathrm{rin}$ |  | － |  |

APPENDIX A

TABLE A3．－Eon－1uded

|  | H. fe | 2 | 200 | 400 | 600 | － 70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 000 | 0.011642 | 0.011530 | 0． 211429 | 0.211311 | 2． 111292 |
| 51 | Oro | ． 211035 | ． 115989 | ． 212884 | ． 215790 | $.31>677$ |
| 52 | 000 | ． 010575 | .010473 | .210373 | ． 310274 | ． 017176 |
| 53 | 000 | ． 210078 | ． 3099820 i | 9098865 | ． 0097715 | ． $279698:$ |
| 54 | 000 | ． 0096055 | ． 0035136 | ． 3044226 | ． 2.93324 | ． 2092431 |
| 55 | 000 | ． 2991547 | ． 0090671 | ． 0689804 | ． 288945 |  |
| 56 | 000 | ． 0087251 | ． 0086416 | ． 06685590 | ． 2084771 | ． 0083960 |
| 57 | 020 | ． 0083157 | ． 0002361 | ．O0A1573 | ． 0080793 | ．308，020 |
| 58 | 000 | ． 00075254 | ． 0073496 | ． 0077745 | ． 3077001 | ． 0076265 |
| 59 | 000 | ． 0075535 | ． 0074813 | ．0074097 | ． 2073388 | ． 3072686 |
| 60 | 000 | ． 0071991 | ．007：302 | ． 3070620 | ． 0069944 | ． $2: 60075$ |
| 61 | 000 | ． 0068612 | ． 0067956 | ． 2067306 | ． 2066662 | ． 206 ＇． 1 |
| 62 | 00\％ | ． 0065393 | ． 0364767 | ． 0364147 | .0063534 | ． 00 ¢n2926 |
| 63 | 000 | ． 3662324 | ． 2061728 | ． 0061137 | ． 2067552 | .0059973 |
| 64 | 000 | ． $\cos 9399$ | ． 0058831 | ． 0058268 | ． 257711 | ． 2057159 |
| 65 | 000 | ． 0056612 | ． 0056070 | ． 3055534 | ． 3035003 | ． 554462 |
| 66 | 000 | ． 0053926 | ． 0053396 | ． 654871 | ． 2552351 | ． 2551636 |
| 67 | 000 | ． 2051327 | ． 3350823 | ． 375.5323 | ． 3749829 | ． 2054340 |
| 68 | 000 | ． $0348 \mathrm{AS6}$ | ． 2348376 | ． 3247902 | － 1947412 | ． 7 9．46．967 |
| 69 | 000 | ． 20.86537 | ． 6386051 | ． 0545600 | ． .045154 | ．-49712 |
| 70 | 000 | ． 6044274 | ． 2243841 | ． 3043412 | ． 24.988 | ．0042－567 |
| 71 | 000 | ． 0042151 | ． 0241740 | ． 0641332 | ． 2040928 | ． 0085529 |
| 72 | 000 | ． 0040133 | ． 0039742 | ． 0039354 | ． 70.89973 | ． 0030590 |
| 73 | 000 | ． 0038214 | ． 0037842 | ． 0037473 | ． 10.37108 | ． 0536747 |
| 74 | 000 | ．0036385 | ． 0036035 | ． 0035685 | ． 5035338 | ． 003499.4 |
| 75 | 000 | ． 2034654 | ． 0034318 | ．CO33984 | ． 2033054 | ． 2033327 |
| 76 | 000 | ． 0033004 | ． 0032684 | ． 3532367 | ．$\because 33253$ | ． 031742 |
| 77 | 000 | ． 0031434 | ． 00311130 | ． 0.032828 | ． 1330532 | －心3フ234 |
| 78 | 000 | ． 0029942 | ． 0.329652 | ． 3029355 | ． 3029081 | － $0: 39 \mathrm{gan}$ |
| 79 | 000 | ． 0028521 | ． 2023246 | ． 2227973 | ．$: 2 \times 17703$ | ． 2 － 745 |
| 80 | 000 | ．2227171 | ．．325708 | ． N （2664 4 | － 22.20 .392 | 13－ |
| 31 | 000 | ．20256a5 | ． 0025 ¢ 36 |  | ．$\because 25144$ |  |
| 82 | 000 | －．324のか） | － $7 \pm 4 \pm 25$ | ．$\because 241$ ¢ | －こ！－\％ | －－－－ |
| 83 | 000 | ． 39234 t\％ | － 023.373 | ． $2 \mathrm{j}=3050$ | －$\because 228$－ | －reste． |
| 14 | c00 | ． 2222332 | －$\because$ こここ177 | ．$\because 21464$ | ． $2: 1754$ | － $1+155^{\text {－}}$ |
| 45 | 000 | ． 0521337 | 40.1134 | ． 6020432 | ．$\because=2731$ | － |
| 86 | 000 | ． 0.32136 | ．3－ 3 141 | ． 519.949 | －：.$^{-c_{\text {cm }}}$ | ：c．， |
| 07 | 200 | － 01930 | ． $2: 31+1+7$ | －$n 1+213$ | －1533． | － |
| ¢ ${ }^{\text {d }}$ | 100 | ．${ }^{1919874}$ | －${ }^{\prime \prime}+27$ | －－191：3 |  |  |
| 63 | ヘッ： | ．j）1\％号7 | ．31－441 | ． $32:=75$ | －31713 | －i\％ |
| 9 | － 23 | ． $2316: 56$ | －Minouzh |  | －1F311 | －m： |
| 11 | －10\％ | ．3515003 | ． 3015 ¢51 |  | －${ }^{-15551}$ | ．：\％． |
| $\pm 2$ | 77） | ． $21575 \%$ | －Su151：2 | ． 214 ＊${ }^{\text {a }}$ | －！inc | －＊＊ |
| 13 | 3n | ． $11459^{7}$ | － 315104 | ． 214.7 | － 16137 | －$:$ ： |
| 4 | － | － $21: \%$ | －．1：5：7 | － 2136,34 | －！18～\％ | －：：： 5 ！ |
| \％ | －，J | ． 32120.4 ， | ．．18： 11 | －：こ コワ | ．．：iess | ．；：- － |
| － | 」－ | － 212513 | － $2: 374$ | ．：12：${ }^{\text {a }}$ | －12：5y | ：．$: 5$ |
| － | －${ }^{\text {a }}$ | ．今12 je | －$\therefore 2.15$ | ．－1153 | －11＊＊2 | $\cdots$ |
| $\therefore$ | － 20.5 | ．． 114 ：3 | ． 1120.5 |  | －－．11：5： | －$: ~ 4{ }^{-}$ |
| ＊ | －10： | ． 31.483 | －：m4， | ．．． $\mathrm{ran}^{3}$ | ．$: 2.83$ ？ | －$\because$ a－ |
| 130 | 129 | ． 213434 |  | ， |  |  |

## APPENDIX A

table a4．－temperature t in degrees fahretheit for values of
pressure altitide h in geopotential feet

> [From ref. Al]

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 59.000 | 58.643 | 58.287 | 57.930 | 57.574 | 57.217 | 56.860 | 56.504 | 56.147 | 55.790 |
| 1000 | 55.434 | 55.077 | 54.721 | 54.364 | 54.007 | 53.651 | 53.294 | 52.938 | 52.581 | 52.224 |
| 2000 | 51.868 | 51.511 | 51.154 | 50.798 | 50.441 | 50.085 | 49.728 | 49． 71 | 49.015 | 48．65d |
| 3000 | 48.302 | 47.945 | 47.588 | 47.232 | 46.875 | 46.518 | 46.162 | 45.805 | 45.449 | 45.092 |
| 4000 | 44.735 | 44.379 | 44.022 | 43.666 | 43.309 | 42.952 | 42.596 | 42.239 | 41.882 | 41.526 |
| 5000 | 41.169 | 40.813 | 40.456 | 40.099 | 39.743 | 39.386 | 39.029 | 38.673 | 38.316 | 37.960 |
| 6000 | 37.603 | 37.246 | 36.890 | 36.533 | 36.177 | 35.820 | 35.463 | 35.107 | 34.750 | 34.393 |
| 7000 | 34.037 | 33.680 | 33.324 | 32.967 | 32.610 | 32.254 | 31.897 | 31.541 | 31.184 | 30.827 |
| 9000 | 30.471 | 30.1141 | 29.757 | 29.401 | 29.044 | 28.688 | 28.331 | 27.974 | 27.618 | 27.261 |
| 9000 | 26.905 | 26.548 | 26.191 | 25.835 | 25.478 | 25.121 | 24.765 | 24.408 | 24.052 | 23.595 |
| 10000 | 23.338 | 22.9821 | 22.625 | 22.259 | 21.912 | 21.555 | 21.199 | 20.842 | 20.485 | 20.129 |
| 11000 | 19.772 | 19.416 | 17.059 | 18.702 | 18.346 | 17.989 | 17.633 | 17.276 | 16.919 | 16.563 |
| 12000 | 16.206 | 15.849 | 15.493 | 15.136 | 14.780 | 14.423 | 14.066 | 13.710 | 13.353 | 12.997 |
| 13000 | 12.640 | 12.283 | 11.927 | 11.570 | 11.213 | 10.857 | 10.500 | 10.144 | 9.787 | 9.430 |
| 14000 | 9.074 | 8.717 | 8.361 | 8.004 | 7.647 | 7.291 | 6.934 | 6.577 | 6.221 | 5.864 |
| 15000 | 5.508 | 5.151 | 4.794 | 4.438 | 4.081 | 3.725 | 3.368 | 3.011 | 2.655 | 2.298 |
| 16000 | 1.941 | i． 585 | 1.223 | ． 872 | ． 515 | ． 158 | －． 198 | －． 555 | －． 911 | －1．268 |
| 17000 | －1．265 | －1．981 | －2．339 | －2．695 | －3．051 | －3．408 | －3．764 | －4．121 | －4．478 | －4．834 |
| 18000 | －5．191 | －5．547 | －5．904 | －6．261 | －6．617 | －6．974 | －7．331 | －7．687 | －8．044 | －8．400 |
| 19000 | －8．757 | －9．114 | －9．470 | －9．827 | －10．184 | －10．540 | －10．897 | －11．253 | $-11.615$ | －11．967 |
| 20000 | －12．323 | －12．680 | －13．035 | －13．393 | －13．750 | －14．106 | －14．463 | －14．820 | －15．176 | －15．533 |
| 21000 | －15．889 | －i6．246 | －16．603 | －16．959 | －17．316 | －17．672 | －18．029 | －18．386 | －18．742 | －19．999 |
| 22000 | －19．456 | －19．812 | －29．169 | －20．525 | －20．882 | －21．239 | －21．595 | －21．952 | －22．308 | －22．665 |
| 23000 | $-23.022$ | －23．378 | －23．735 | －24．092 | －24．448 | －24．805 | －25．161 | －25．518 | －25．875 | －26．231 |
| 24030 | $-26.588$ | －25．944 | －27．3C1 | －27．653 | －28．014 | －28．371 | －28．728 | －29．084 | －29．441 | －29．797 |
| 25000 | －30．154 | －30．511 | －30．867 | －31．224 | －31．580 | －31．937 | －32．294 | －32．650 | －33．007 | －33．364 |
| 26000 | －33．720 | －34．077 | －34．433 | －34．790 | $-35.147$ | －35．503 | －35．860 | －36．216 | －36．573 | $-35.730$ |
| $27{ }^{7} 000$ | －37．286 | －37．6431 | －38．023 | －38．356 | －38．713 | －39．067 | －39．426 | －39．783 | －40．139 | －42．496 |
| $\because 8000$ | －40．852 | －+1.209 | －41．565 | －41．922 | －42．279 | －42．636 | －42．992 | －43．349 | $-43.705$ | －4．4． 262 |
| 29000 | －44．419 | －44．775 | $-45.132$ | －45．488 | －45．845 | －46．202 | －46．558 | －46．715 | －47．272 | －47．523 |
| 35000 | －47．985 | －48．341 ${ }^{\prime}$ | －48．693 | －49．055 | $-49.412$ | －49．768 | －50．124 | －50．481 | －52．333 | －51．！${ }^{\text {2 }}$ |
| 31000 | －51．551｜ | －51．908： | －52．26 | －52．261 | －52．977 | －53．334 | －53．691 | －54．747！ | －54．434 | －54．-1 |
| 32300 | －55．117 | －55．474！ | －55．8ミ3 | －56．187 | －56．544 | －56．900 | －57．257 | －57．613 | －57． 370 | ごら．3ヵ－ |
| 33000 | －58．683 | －5t．040； | －59．397 | －59．753 | －50．11\％ | －50．466 | －60．823 | －51．182 | － 51.536 | －it．$=33$ |
| 34000 | －62．249 | －52．606 | －5，2．963 | －53．319 | －63．676 | －54．033 | －64． 339 | －54．746！ | －55．122 | －i，¢．iEt |
| 35200 | －65．816 |  | －ら5．5こ？ |  | －67．242 |  | －67． 355 |  | －ris． 669 |  |
| 36800 | －5：38．38！ |  |  |  |  |  |  |  |  |  |
| 36990 |  |  |  |  |  |  |  |  |  |  |
| ： 0 | －09．700 |  |  |  |  |  |  |  |  |  |
| 55400 |  |  |  |  |  |  |  |  |  |  |

## APPENDIX A

table a4.- Concluded

| $\begin{aligned} & H_{1} \\ & \mathrm{ft} \end{aligned}$ | 0 | 200 | 400 | 600 | 800 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 55000 |  |  |  |  | -69.599 |
| 66000 | -69.490 | -69.380 | -69.270 | -69.161 | -69.051 |
| 67000 | -68.941 | -68.831 | -68.722 | -68.612 | -68.520 |
| 68000 | -68.392 | -68.283 | -68.173 | -68.063 | -67.954 |
| 63000 | -67.844 | -67.734 | -57.624 | -67.515 | -67.405 |
| 70000 | -67.295 | -67.185 | -67.076 | -66.966 | -66.856 |
| 71000 | -66.747 | -56.637 | -66.527 | -66.417 | -56.308 |
| 72000 | -66.198 | -66.088 | -65.978 | -65.869 | -65.759 |
| 73000 | -65.649 | - 65.540 | -65.430 | -65.320 | -65.210 |
| 74000 | -65.101 | -64.991 | -64.881 | -64.771 | -64.662 |
| 75000 | -64.552 | -64.442 | -64.333 | -64.233 | -64.113 |
| 76000 | -64.003 | -63.894 | -63.784 | -63.674 | -63.564 |
| 77000 | -63.455 | -63.345 | -63.235 | -63.126 | -63.015 |
| 78000 | -62.906 | -62.796 | -62.687 | -62.577 | -62.467 |
| 79000 | -62.357 | -62.248 | -62.138 | -62.028 | -61.919 |
| 80000 | -61.809 | -61.699 | -61.589 | -61.480 | -61.370 |
| 81000 | -61.260 | -61.150 | -61.041 | -60.931 | -60.821 |
| 82000 | -60.712 | -60.602 | -60.492 | -60.382 | -60.273i |
| 83000 | -60.163 | -60.053 | -59.943 | -59.834 | -59.724 |
| 84000 | -59.614 | -59.505 | -59.395 | -59.285 | -59.175 |
| 85000 | -59.066 | -58.956 | -58.846 | -58.736 | -58.627 |
| 86000 | -58.517 | -58.407 | -58.298 | -58.188 | -58.078 |
| 87000 | -57.968 | -57.859 | -57.749 | -57.639 | -57.529 |
| 83000 | -57.420 | -57.310 | -57.200 | -57.090 | -56.981 |
| 89000 | -56.871 | -56.761 | -56.652 | -56.542 | -56.432 |
| 90000 | -56.322 | -56.213 | -56.103 | -55.993 | -55.883 |
| 91000 | -55.774 | -55.664 | -55.554 | -55.445 | -55.335 |
| 92000 | -55.225 | -55.115 | -55.006 | -54.856 | -54.786 |
| 93000 | -54.676 | -54.567 | -54.457 | -54.347 | -54.238 |
| 34000 | -54.128 | -54.018 | -53.908 | -53.793 | -53.689 |
| 95000 | -53.579 | -53.469 | -53.360 | -53.250 | -53.140 |
| 96000 | -53.031 | -52.921 | -52.811 | -52.701 | -52.592 |
| 97000 | -52.482 | -52.372 | -52.262 | -52.153 | -52.343 |
| 98000 | -51.933 | -51.824 | -51.714 | -51.604 | -51. 494 |
| 99000 | -51.385 | -51.275 | -51.165 | -51.055 | -50.946 |
| 100300 | -50.836 |  |  |  |  |

TABLE A5.- TEMPERATURE $t$ IN DEGREES CENTIGRADE FOR VALUES OF

PRESSURE ALTITUDE H IN GEOPGTENEIAL FEET
[From ref. Al]


APPENDIX A

TABLE A5.- Concluced

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | 0 | 200 | 400 | 600 | 800 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 65000 |  |  |  |  | -56.444 |
| 66000 | -56.383 | -56.322 | -56.261 | -56.200 | -56.139 |
| 67000 | -56.078 | -56.017 | -55.956 | -55.896 | -55.835 |
| 68000 | -55.774 | -55.713 | -55.652 | -55.591 | -55.530 |
| 69000 | -55.469 | -55.408 | -55.347 | -55.286 | -55.225 |
| 70000 | -55.164 | -55.103 | -55.042 | -54.981 | -54.920 |
| 71000 | -54.859 | -54.798 | -54.737 | -54.676 | -54.615 |
| 72000 | -54.554 | -54.493 | -54.432 | -54.372 | -54.311 |
| 73000 | -54.250 | -54.189 | -54.128 | -54.067 | -54.006 |
| 74000 | -53.945 | -53.884 | -53.823 | -53.762 | -53.701 |
| 75000 | -53.640 | -53.579 | -53.518 | -53.457 | -53.396 |
| 76000 | -53.335 | -53.274 | -53.213 | -53.152 | -53.091 |
| 77000 | -53.030 | -52.969 | -52.908 | -52.848 | -52.787 |
| 78000 | -52.726 | -52.665 | -52.604 | -52.543 | -52.482 |
| 75 000 | -52.421 | -52.360 | -52.299 | -52.238 | -52.177 |
| 80000 | -52.116 | -52.055 | -51.994 | -51.933 | -51.872 |
| 81000 | -51.811 | -51.750 | -51.689 | -51.628 | -51.567 |
| 82000 | -51.506 | - 51.445 | -51.384 | -51.324 | -51.263 |
| 83000 | -51.202 | -51.141 | -51.080 | -51.019 | -50.958 |
| 84000 | -50.897 | -50.836 | -50.775 | -50.714 | -50.653 |
| 85000 | -50.592 | -50.531 | -50.470 | -50.409 | -50.348 |
| 86000 | -50.287 | -50.226 | -50.165 | -50.104 | -50.04j |
| 87000 | -49.982 | -49.921 | -49.860 | -49.800 | -49.739 |
| 88000 | -49.678 | -49.617 | -49.556 | -49.455 | -49.4:4 |
| 89000 | -49.373 | -49.312 | -49.251 | -49.190 | -49.129 |
| 30000 | -49.068 | -49.007 | -48.946 | -48.885 | -48.324 |
| 91000 | -48.763 | - 68.702 | -48.641 | -48.5 30 | -48.519 |
| 92000 | -48.458 | -48.397 | -48.336 | -48.276 | -48.215 |
| 93000 | -48.154 | -48.093 | -48.032 | -47.971 | -47.910 |
| 94000 | -47.849 | -47.788 | -47.727 | -47.666 | -47.605 |
| 95000 | -47.544 | -47.483 | -47.422 | -47.361 | -47.300 |
| 96000 | -47.239 | -47.178 | -47.117 | -47.056 | -46. 395 |
| 97000 | -46.934 | -46.873 | -45.812 | -46.752 | -45.691 |
| 98000 | -46.630 | -46.569 | -46.508 | -46.447 | -46.385 |
| 99000 | -46.325 | -46.264 | -46.203 | -46.142 | -46. 281 |
| 100000 | -46.020 |  |  |  |  |

APPENDIX A

TABLE A6. - COEFFICIENT OF VISCOSITY $\mu$ in POUND-SECIDS PER SQUARE FOOT

FOR VALUES OF PRESSURE ALTITUDE H IN GEOPCTEMTIAL FEET
[From ref. Al]

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | $\underset{1 b-\sec / f t^{2}}{ }$ | $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | $\begin{gathered} \text { Ue } \\ 1 b-\sec / f t^{2} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 0 | $3.7372 \times 10^{-7}$ | 36090 |  |
| 1000 | 3.7173 | to | $2.9691 \times 10^{-7}$ |
| 2000 | 3.6971 | 65800 |  |
| 3000 | 3.6769 | 66000 | 2.9704 |
| 4000 | 3.6567 | 67000 | 2.9740 |
| 5000 | 3.6365 | 68000 | 2.9774 |
| 6000 | 3.6163 | 69000 | 2.9809 |
| 7000 | 3.5958 |  |  |
| 8000 | 3.5752 | 70000 | 2.9844 |
| 9000 | 3.5547 | 71000 | 2.9879 |
|  |  | 72000 | 2.9914 |
| 10000 | 3.5342 | 73000 | 2.9949 |
| 11000 | 3.5134 | 74000 | 2.9984 |
| 12000 | 3.4926 | 75000 | 3.0018 |
| 13000 | 3.4717 | 76000 | 3.0053 |
| 14000 | 3.4509 | 77000 | 3.0088 |
| 15000 | 3.4301 | 781000 | 3.0123 |
| 16000 | 3.4090 | 79000 | 3.0157 |
| 17000 | 3.3878 |  |  |
| 18000 | 3. 3667 | 80000 | 3.0192 |
| 19000 | 3.3452 | 81000 | 3.0227 |
|  |  | 82000 | 3.0261 |
| 20 900 | 3.3238 | 83000 | 3.0296 |
| 21000 | 3. 3027 | 84000 | 3.0331 |
| 22000 | 三. 2809 | 85000 | 3.0365 |
| 23000 | 3. 2.595 | 86000 | 3.0400 |
| 24000 | 3.2377 | 87000 | $3.04=5$ |
| 25000 | 3.2160 | 88000 | 3.0469 |
| 26000 | 3.1942 | 89000 | 3.0504 |
| 27000 | 3.1721 |  |  |
| 28000 | 3.1501 | 90000 | 3.0538 |
| 29000 | 3.1280 | 91000 | 3.0573 |
|  |  | 92000 | 3.00007 |
| 30000 | 3.1360 | 93000 | 3.0641 |
| 31000 | 3.0837 | 94000 | $3.067 E$ |
| 32 onn | 3.0614 | 95000 | 3.0710 |
| 33001 | 3.0389 | 35000 | 3.0744 |
| 34 00C | 3.0164 | 97000 | 3.0779 |
| 35000 | 2.9938 | 98000 | 3.0813 |
| 36000 | 2.9711 | 99000 | 3.6847 |
|  |  | 100000 | 3.0882 |

APPENDIX A

TABLE A7.- SPEED OF SOUND a IN MILES PER HOUR AND KNOTS YOR VALUES OF PRESSLRE ALTITUDE H IN GEOPOTENTIAL FEET
[From ref. Al]

| $\begin{gathered} \mathrm{H}_{1} \\ \mathrm{ft} \end{gathered}$ | $\begin{aligned} & \mathrm{a}, \\ & \text { uph } \end{aligned}$ | knots | $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | $\begin{gathered} \text { a. } \\ m p h \end{gathered}$ | a. knots |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 761.22 | 661.48 | 36090 |  |  |
| 1000 | 758.60 | 659.20 | to | 660.05 | 573.57 |
| 2000 | 755.97 | 656.92 | 65800 |  |  |
| 3000 | 753.33 | 654.62 | 66000 | 660.23 | 573.73 |
| 4000 | 750.67 | 652.32 | 67000 | 660.70 | 574.13 |
| 5000 | 748.01 | 650.01 | 68000 | 661.16 | 574.53 |
| 6000 | 745.35 | 647.69 | 69000 | 661.62 | 574.93 |
| 7000 | 742.67 | 64.5 .35 |  |  |  |
| 8000 | 739.98 | 643.03 | 70000 | 662.09 | 575.34 |
| 9000 | 737.29 | 640.68 | 71000 | 662.54 | 575.73 |
|  |  |  | 72000 | 663.01 | 576.14 |
| 10000 | 734.58 | 638.33 | 73000 | 663.47 | 576.54 |
| 11000 | 731.86 | 635.97 | 74000 | 663.93 | 576.94 |
| 12000 | 729.13 | 633.60 | 75000 | 664.39 | 577.34 |
| 13000 | 726.40 | 631.22 | 76000 | 664.85 | 577.74 |
| 14000 | 723.65 | 628.84 | 77000 | 665.32 | 578.15 |
| 15000 | 720.89 | 626.44 | 78000 | 665.77 | 578.54 |
| 16000 | 718.12 | 624.03 | 79000 | 666.24 | 578.95 |
| 17000 | 715.34 | 621.62 |  |  |  |
| 18000 | 712.55 | 619.19 | 80000 | 666.70 | 579.34 |
| 19000 | 709.75 | 616.76 | 81000 | 567.16 | 579.75 |
|  |  |  | 82000 | 667.62 | 580.14 |
| 20000 | 706.94 | 614.32 | 83000 | 668.07 | 580.54 |
| 21000 | 704.12 | 611.86 | 84000 | 668.53 | 580.94 |
| 22000 | 701.28 | 609.40 | 85000 | 668.99 | 581.34 |
| 23000 | 698.44 | 6.36 .93 | 86000 | 669.45 | 581.74 |
| 24000 | 695.58 | 604.44 | 87000 | 669.91 | 582.13 |
| 25000 | 692.71 | 601.95 | 88000 | 670.36 | 582.53 |
| 26000 | 689.83 | 599.44 | 89000 | 670.82 | 582.93 |
| 27 c00 | 686.93 | 596.93 |  |  |  |
| 28000 | 684.03 | 594.41 | 90000 | 671.29 | 583.32 |
| 29000 | 681.11 | 591.87 | 91000 | 671.73 | 583.72 |
|  |  |  | 92000 | 672.19 | 584.12 |
| 30000 | 673.13 | 589.32 | 93000 | 672.65 | 584.51 |
| 31000 | 675.24 | 586.76 | 94000 | 673.10 | 584.91 |
| 32000 | 672.28 | 584.20 | 95000 | 673.55 | 585.30 |
| 33000 | 667.31 | 581.61 | 96000 | 674.01 | 585.73 |
| 34000 | 665.33 | 579.02 | 97000 | 674.47 | 585.12 |
| 35000 | 663.33 | 576.42 | 98000 | 674.92 | 566.49 |
| 36000 | 663.32 | 573.80 | 99000 | 675.37 | 536.86 |
|  |  |  | 100200 | 575.e2 | 537.23 |

table ar.- acceleratios due to grayity q in feet per second squarfo
for values of pressure altitide h in geopotential feet
[From ref. Al]

| $\begin{aligned} & \mathbf{H}_{,} \\ & \mathrm{ft} \end{aligned}$ | $\mathrm{Et}^{\mathrm{g} / \sec ^{2}}$ | $\begin{aligned} & \mathrm{H}, \\ & \mathrm{ft} \end{aligned}$ | $\frac{9}{\mathrm{ft} / \mathrm{sec}^{2}}$ |
| :---: | :---: | :---: | :---: |
| 0 | 32.174 | 50000 | 32.020 |
| 1000 | 32.171 | 51000 | 32.017 |
| 2000 | 32.168 | 52000 | 32.014 |
| 3000 | 32.165 | 53000 | 32.011 |
| 4000 | 32.162 | 54000 | 32.008 |
| 5000 | 32.159 | 55000 | 32.005 |
| 6000 | 32.156 | 56000 | 32.001 |
| 7000 | 32.152 | 57000 | 31.998 |
| 8000 | 32.149 | 58000 | 31.995 |
| 9000 | 32.145 | 59000 | 31.992 |
| 10 000 | 32.143 | 60000 | 31.989 |
| 11000 | 32.140 | 61000 | 31.986 |
| 12000 | 32.137 | 62000 | 31.983 |
| 13000 | 32.134 | 63000 | 31.980 |
| 14000 | 32.131 | 64000 | 31.977 |
| 15000 | 32.128 | 65000 | 31.974 |
| 15000 | 32.125 | 66000 | 31.971 |
| 17000 | 32.122 | 67000 | 31.966 |
| 18000 | 32.119 | 68000 | 31.965 |
| 19000 | 22.115 | 69000 | 31.961 |
| 20000 | 32.112 | 70000 | 31.958 |
| 21000 | 32.109 | 71000 | 31.955 |
| 22000 | 32.106 | 72000 | 31.952 |
| 23000 | 32.103 | 73000 | 31.949 |
| 24000 | 32.100 | 74000 | 31.946 |
| 25000 | 32.097 | 75000 | 31.943 |
| 26000 | 32.094 | 76000 | 31.940 |
| 27000 | 32.091 | 77000 | 31.937 |
| 28000 | 32.088 | 78000 | 21.934 |
| 29000 | 32.085 | 79000 | 31.931 |
| 30000 | 32.08. | 80000 | 31.929 |
| 31000 | 32.078 | 31000 | 31.925 |
| 32000 | 32.075 | 82000 | 31.922 |
| 33000 | 32.072 | 83000 | 31.918 |
| 34000 | 32.069 | 84000 | 31.915 |
| 35000 | 32.066 | 85000 | 31.912 |
| 36000 | 32.063 | 86000 | 31.909 |
| 37000 | 32.060 | 87000 | 31.906 |
| 38000 | 32.057 | 88000 | 31.903 |
| 39000 | 32.054 | 89000 | 31.900 |
| 40 -00 | 32.051 | 90000 | 31.897 |
| 41000 | 32.046 | 91000 | 31.894 |
| 42000 | 32.045 | 200 | 31.891 |
| 43000 | 32. 041 | - 300 | 21.888 |
| \$4000 | 32.038 | 74000 | 31.885 |
| 45000 | 32.335 | 75390 | 31.882 |
| 46000 | 22.032 | 96700 | 31.378 |
| 47000 | 32.029 | 97200 | 31.875 |
| 48000 | 32. 226 | 78 9\% | 31.872 |
| 49000 | 32.023 | 39000 | 31.869 |
|  |  | :90 902 | 31.256 |



[Fron res. A2]


## AFPENDIX A

ABLE A9．－Concluded

| $\begin{aligned} & \mathrm{V}_{\mathrm{C}^{\prime}} \\ & \text { mph } \end{aligned}$ | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  | $\rightarrow$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 600 | 15.1613 | 15.2195 | 15.2778 | 15.3363 | 15.3950 | 15.4538 | 15.51281 | 15．5719 | 5.5312 |  |
| 610 | 15.7502 | 15.8099 | 15.8698 | 15.9298 | 15.9900 | 16.0503 | 16.110812 | 16.1715 | 6.23231 | 6.2933 |
| 620 | 16.3544 | 16.4156 | 10.4771 | 16.5387 | 16.6004 | 16.6623 | 16.7244 | 16.7866 | 16.8490 ｜ | 6.9116 |
| 630 | 16.9743 | 17.0371 | 17.1001 | 17.1633 | 17.2267 | 17.2902 | 27.35381 | 17.4176 | 27.4816 | 7.5458 |
| 640 | 17.6101 | 17.6746 | 17.7392 | 17.8040 | 17.8690 | 17.5341 | 17.99941 | 12.0649 | 18.1305 | 8．1963 |
| 650 | 18.2622 | 18.3234 | 18.3946 | 18.4611 | 18.5277 | 15.5945 | 18.66151 | 18.7286 | 18．7959 | 8.3634 |
| 660 | 18.9310 | 18.9988 | 19.0668 | 19 | 19.2032 | 19.2717 | 19．3404 1 | 19.4092 | 4782 | 5474 |
| 670 | 19.5167 | 19.6863 | 19.7560 | 19.8258 | 19.8959 | 19.9661 | 20.0365 | 20.1070 | 20.1778 | 0.2497 |
| 680 | 20.3198 | 20.3911 | 20.4625 | 20.5341 | 20.6059 | 20.6779 | $20.7501 ; 2$ | 20.8224 | 20.8949 | 20.7676 |
| 690 | 21.0405 | 21.1136 | 21.1868 | 21.2602 | 21.3338 | 21.4075 | 21．4816｜2 | 21.5557 | 21.6300 | 21.7046 |
| 70 | 21.7793 | 21.8541 | 22． 9292 | 22.0045 | 22.0799 | 22.1555 | 22.23132 | 22.3073 | 22.3835 | 22.4549 |
| 710 | 22.5364 | 22.6132 | 22.6901 | 22.7672 | 22.8446 | 22.9220 | 22.99972 | 23.6776 | 23.1557 | 23.2339 |
| 720 | 23.3124 | 23.3911 | 23.4699 | 23.5489 | 23.6281 | 23.7076 | 23.7872 | 23.8670 | 23.9470 | 24． 3272 |
| 730 | 24.1076 | 24.1881 | 24.2689 | 24.3499 | 24.4311 | 24.5125 | 24.594012 | 24.6758 | 24.7578 | 24.8399 |
| 740 | 24.9223 | 25.0049 | 25.0877 | 25.1706 | 25.2538 | 25.3372 | 25．4207 2 | 25.5345 | 25.5885 | 25.6727 |
| 750 | 25.7571 | 25.8417 | 25.9265 | 26.0115 | 26.0967 | 26.1821 | 26．2677 2 | 26．3535 | 26.4396 | 26.5258 |
| 760 | 26.6122 | 26.6989 | $26.7 \times 61$ | 26.8731 | 26.9604 | 27.0479 | 27．1356：2］ | 27.2235 | 7.3116 | 7.4000 |
| 77 | 27.4885 | 27.5772 | 27.6661 | 27.7553 | 27.8446 | 27.9341 | 28.023912 | 28.1138 | 28．2040 | 943 |
| 78 | 28.3848 | 28.4756 | 285665 | 23.6577 | 28.7490 | 28.8405 | 28．9323！29 | ！29．0242 | 29.1163 | 86 |
| 790 | 29.3011 | 29.3939 | 29.4868 | 29.5799 | 29.6732 | 29.7667 | 29．8603｜2 | 29.9542 | 30.0483 | 25 |
| 8 | 30.2370 | 30.3 | 30.4 | 30.5214 | 35.6166 | 30.7 | 30.807613 | 30．9034 | 3． 9994 | S |
| 810 | 31.1918 | 31.2884 | 31.3 | 31.4820 | 31.5791 | 31.6763 | 31.7738 i | ； 31.8714 ； | 31.9692 | 672 |
| 820 | 32.1654 | 32.2638 | 32．3624 | 32．4611｜ | 32.5600 | $32.65 ¢, 1$ | 32.7584 | 32.8579 | 32.9575 | 3.3574 |
| 830 | 33.1574 | 33.2576 | 33.3579 | 33.4585 | 33.5592 | 33.6601 | $33.7612 \mid 3$ | ［33．8625 | 33.9630 | 34.2655 |
| 8 | 34.1 | －4． 2693 | 34.3715 | 34.4738 | 34.5763 | 34.6790 | 34.7819 | 34．8849 | 34.9581 | 35.3195 |
| 850 | 35.1951 | 35.2988 | 35.4627 | 35.5068 | 35.6111 | 35.7155 | 35.8201 | ； 35.9249 ！ | 36.0299 | 36.1350 |
| 96 | 36．24C3 | 36.3458 | 36.4514 | 36 | 36.6632 | 36.7694 | 36．8757 ${ }^{3}$ | 36．9822＇ | 889 | 957 |
| 870 | 37.3027 | 37.4099 | 37.5173 | 37.6248 | 37.7325 | 37.8404 | 37.9484 | 38.056 | 649 | 35 |
| 880 | 38.3822 | 38.4910 | 38.6001 | 38. | 38.8187 | 38.9282 | 39.0379 | 39.1478 | 78 | 89 |
| 890 | 39.4784 | 39．5959 | 39.6996 | 39.8105 | 39.9215 | 40.0327 | 40.1441 | 40.2556 | 40.3673 |  |
| 900 |  | 140.7034 | 40.8157 | 40.9283 | 41.0409 | 41.1538 | 41 |  |  | 63 |
| 910 | 41．7204 | 14.8342 | 41.9482 | 42.0624 | 42.1767 | 42.2911 | 42.4057 | 42．5205 | 42 | 506 |
| 920 | 42．8659 | 42.9813 | 43.0969 | 43.2126 | －3．3286 | 43.4446 | 43.5609 | 43.6772 | 43.7 | 05 |
| 930 | 44.3274 | 44.1444 | 44.2616 | 44.3790 | 4．4．4965 | 44.6141 | 44.7322 | 44.84 | 44.9681 | 45． 2364 |
| 970 | 45.20 | 45.3235 | 45.4422 | 45.4612 | 45. | 45.790 | 45.9189 | 146. | 46.1581 | 46．2751 |
| 950 | 46. | 46.5183 | 46．638 | ｜46．7591｜ | 46.8798 |  | 47.1216 |  | ． 3640 | 47．7354 |
| 960 | 47.60 | 47.7288 | 47.3506 | 47.9727 | 43.0949 | 48.2173 | 48.3398 | ＋ | 9 |  |
| 9 | 48.3 | 48.9547 | 49.3782 | 49.2013 |  |  | ． 5735 |  | 49 | 67 |
| 980 | 50．3713！ | 50.1762 | 150．3211 | 50.4663 | 5． 7.5726 | 50.6970 | $50.222 t$ | ：9．3434！ | ＇51． | ：． 2003 |
| 990 | 5i． 3265 | 51.4529 | ｜51．5794 | 51.7061 | 51.8329 | 151．9598 | $52.0872 \mid 5$ | ｜52．2142； | 52.3417 | 52．7592 |
| 1000 | 52.5970 | 52.7248 | 52.8529 | 52.98121 | 53 |  | 53.3665 | 53.4953 | 53.6242 | 53.7533 |
| 1010 | 53.8825 | 54.5119 | 54 | 54 | 54. |  | 54．6611 | 5 | 54．9223 | 5．． 524 |
| 1020 | 5.183 | 55．3140 | 55.4450 | 55.5 | ， | ． 55.8320 | 5 |  | － | 5 5 ． 3665 |
| 1030 | 56.4 | ！56．6311 | 56.7536 | 156．896 | －7．029 | 5．162． | 3．．195－ |  | 5519 | 5－．亏ヲ5； |
| 1040 | 57.3292 | 57.7530 | 153．2习73｜ | 52 | ミ3． 3655 | 58．7：79 | 58.6345 | 5，．7693 | 5.3742 | ミう． 3332 |
| 105 | 59.1744 | 59.3098 |  | 59．58 | 53．7167 | 59．35こ6 | 59.9837 | － |  |  |
|  | 6 c | 160. |  | ：60 | 161．0625 | 61． 2200. | 61．3575 | 62．495 | 2． 533 |  |
| 1070 | 61．309 | 62.347 | 62.1758 |  |  |  | ．52．341： | E2． | ミ． 31 |  |
| 1080 | 63.2985 | 63.4322 | 63．578！ | 53．7131 | （53．8 |  | 1 | ｜ 6 |  | － |
| 1090 | 64.7023 | 64.8435 | 64．38491 |  |  |  | $171$ | $y$ |  |  |
| 1100 | 66.1208 |  |  |  |  |  |  |  |  |  |



［rroa res．inj］

| \％－ | 0 | 1 | － | 1 | 4 | 5 | 5 | 7 | 9 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.0021161 | ｜0．010369 $\mid$ | 0.222355 | 0.040631, | 0.0636981 | 0.0518441 | $0.125058{ }^{1}$ | 0.163584 | 9．207： |
| 10 | ． 255427 | 109603 | ．368310， | ． 431708 | ． $501332^{\circ}$ | ． 753391 | ． 554752 | ． 739185 | ． 4280761 | ．32：－ |
| $20!$ | 1． 62277 | 1． 2731 | 1．23778｜ | 1.35290 | 1.47283 | 2．59796 | 1． 22853 | 1．86416 | 2.20497 | $\therefore .15$ |
| 33 | －． 30139 | 2.45777 | 12.61361 | 2.78536 | 2.956 .8 | 3．13344 | 3.31484 | 3.50149 | 3.59385 | 3.8 .11 |
| 431 | 4.67293 | \＄． 37058 | 4.51347 | 4.73059 | 4.95122 | c．19143 | 5.41413 | 5.65242 | 5.97557 | 6.144 |
| 3） | 6.39817 | 6.65720 | 6.92586 | 7．18942 | 7.46411 | 7．74345 | 9． 22804 | H． 31758 | 8.61237 | 8．912：． |
| らЈ： | 9.21782 | 9．52763 | 9.94358 | 10.1640 | 13.4903 | 12．3211 | 11.1575 | 11.4903 | 11.3447 | 12.134 ． |
| 73 ： | 12.5536 | 12.3151 | 13.2826 | 13.6547 | 14.9328 | 14．4i52 | 14.3032 | 15.1241 | 15.5942 | 15.998 |
| 3） | 16.4068 | 16.3210 | 17．23：6 | 17.6643 | 18.2934 | 19．52as | 28．9685 | 19.4135 | 19.8637 | －3．31\％ |
| 30 | 20．7832 | 21.2460 | 21.7170 | 22.1940 | 22.6753 | 23.1520 | 23.6547 | 24.1520 | 24.5549 | 25．102． |
| 1301 | 25.6756 | 26.1933 | 26.7172 | 27.2465 | 27.7802 | 28.3192 | 20.8641 | 29.4141 | 23.3692 | 39.5296 |
| 1101 | 31.0953 | 31.6664 | 32.2423 | 12.8238 | 33.4104 | 24.0032 | 34.6001 | 35.2026 | 35.8106 | 36.4245 |
| 120 | 37.0423 | 37.6663 | 38.2957 | 38.9308 | 39.3699 | 40.2153 | 49.8654 | \＄1．5212 | 42.2821 | 42.8485 |
| 133！ | 43.5202 | 44.4976 | 44.8803 | 45.5683 | ＋6．2609 | 45.9577 | 47.6633 | 40.3725 | 49.2867 | 49．0c．7： |
| 1：31 | 50.5325 | 51.2626 | 51.9986 | 52.7434 | 53.4876 | 54.2395 | 54.9967 | 55.7692 | 56.5284 | 5．． 322 ： |
| 150 | 52.0815 | 50.3662 | 59.6564 | 60.4521 | 51.2524 | 6.2 .2591 | 62.8707 | 63.6876 | 64.5125 | 55.3386 |
| 103 ！ | 66.1722 | 6，7．3111 | 57.8557 | 06.7051 | 69.5609 | 75.42 .3 | 71．288： | 74.1603 | 73.7372 | 73．9：～ |
| 175 | 74.8663 | 75．7224 | 76.6016 | 77.5062 | 75.4162 | 73.3321 | 60.2531 | －1．1858 | 22．113） | 9］ |
| 1931 | 83.9943 | 34.3441 | 85.8979 | 36.3532 | 97.6241 | 95． 70.6 | 89．772； | 70.7550 | ＋1．74：$=$ | 72．714 |
| ： $7 \boldsymbol{3}$ | 33.7355 | 94.7401 | 95.7508 | 36.7566 | 97.7885 | 33.3162 | 99.8486 | 1：0．887 | 122．731 | ：22．391 |
| 2\％）． | 104．336 | 105.297 | 1．．2．164 | 107.236 | 119.315 | 1：3．398 | 117.488 | 211．58： | 1：2．0त3 | 113.97 |
| 2：3 | 114．303 | 116.221 | 11.145 | 118.274 | 117.409 | 122.549 | 121.696 | ：12．848 | 2：4．596 | 1：5．2\％： |
| $22)$ | 126．337 | 12？ 515 | 120．6\％ | 123.893 | 131575 | 132．274 | 133.479 | 134.688 | ：35．324 | 13：．：25 |
| ： 11 | 133.353 | 133.587 | 145.826 | 142.771 | 14：．322 |  | 145.941 | 1；7．139 | 143．383 | 147．6．63 |
| －${ }^{\text {）}}$ | 153．75\％ | 152．242 | 153．439 | 154.843 | 154．153 | 5－．46） | 158.730 | ：20．117 | ：41．451 | このこ．：7\％ |
| © 5 | 164．135 | 165.947 | 166．6－4 | 168． 228 | 267．577 | 17－952 | 172.333 | 173．72： | 275．：14 | ：76．51： |
| 25 | 177.315 | 173．132 | $130.74^{-}$ | 182.172 | 143.599 | 195． $3: 5$ | 186．476 | ：57． 3 ：4 | ：49．3－5 | ：92．23？ |
| －7： | 1\％2．304 | 193．775 | 195.254 | 195.738 | 198.228 | 197．723 | 221.23 | －22．737 | こ： 5.51 | $\therefore 2.73$ |
| －0） | 25．303 | 208.634 ， | 210.374 | 211.919 | 213.472 | 215.233 | 216.535 | 216.155 | ：37．04： | E2．124 |
| 232 | 222.915 | 22：511 | 226.114 | 227.722 | 229.337 | 233.957 | 232.505 | 234.219 | 235．3．－6 | 237．5－5 |
| 327 | 239.157 | 240.917 | 242.481 | 244.133 | 245.831 | 257．516 | 249.237 | 250． 724 | ：52．699 | 254．313 |
| 15 | 256.235 | 257.757 | 259.487 | $\therefore 61.222$ | 262.965 | 254.714 | 256．363 | 259． 231 | 259． 399 | 271.374 |
| 323 | 273.356 | 275.343 | 277.137 | 2.7 .339 | 280.746 | 292.550 | 284.381 | 294． 228 | ：98．241 | 289．39？ |
| 332 | 291．729 | $\pm 73.582$ | 295.443 | 297.110 | 299.183 | $33: .363$ | 322.350 | 334.844 | 3：6． 343 | $328 . e 52$ |
| 32 | 31：． 565 | 312.435 | 314.412 | 3i6．j：4 | 314.297 | 125．234 | 322．：39 | 374.143 | ？2f．1：7 | ：－8． 232 |
| 351 | 337.373 | i32．551 | 334.256 | 336．054 | 334.566 | 34：． 382 | 132．1：5 | 24．114 | jac．：${ }^{\text {a }}$ | 345.211 |
| 152 | 357.263 | 352.320 | 354．384 | 356.455 | 358.533 | 365．617 | 162．739 | 354．89 | 345． $2: 4$ | 34，9．224 |
| 1\％ | 171．146 | 373．：73 | 375．406 | 377.249 | 373.675 | $381.25 \%$ | 384.313 | 136． 182 | iAR．3EA | 39\％．5： |
| 35 | 392．73： | 134．932 | 197．：34 | 399.347 | －$\because .566$ | 423.792 | 406.325 | 43.966 | ：$:$ ． $5: 4$ | －1： 50 |
| 19. | ＋15．212 | 417．32 | 419.577 | 421.364 | 42． 155 | 4 25.454 | 528．751 | \＄31．374 | 43．33 | ＋35．724 |
| is： | 439．759 | 443．：32 | ＋$\$ 2.753$ | ＋45．113 | 447．47 | 4＊） 4 ¢ | 452．：こe | 454.615 | 4¢7．11 | 459．4： |
| $\pm:$ | i51．d24 | ＋54．：72 | 466．056 | 459.199 | 471.532 | ＋7， 4 \％ | －76．742 | 4 （3．3）4 |  | ＋5］．75： |
| 425 | \＄85．336 | ＋7n．5： | 431．332 | 433．441 | ＋36．35－ | ¢9‥59］ | 5：2．413 | －－3． 352 | 5．6．73 | 5；7． 5 ？ |
| 4 l | 511.615 | 514．： 56 | $\cdots 9.764$ | 51．3．35 | 521.749 | ！ 25.545 | ¢2． 253 | 5：7．7－1 | 5： | ミこと．： |
| 44： | 537.675 | 542．31a | $5-374$ | 545.5317 | 54\％．31： | 55＇．7\％！ | 5s3．6－？ | ¢56．376 | 5ミ． 5 － |  |
| $\pm$－ | Sら4．513 | 567．．47 | 569． 976 | 5－．．－4 | 575．471 |  | －41． $2: 2$ | 593.75 | 三こう．$=$－4 |  |
| －4． | ミ．2．157 | 534．4． | 537．704 | －20．51： | 423.785 | $\therefore$ ： 2.0 － 6 | 4．1．13： |  | －： | $\therefore$－－－ |
| i： | ¢¢5．as： | 623．：${ }^{\text {a }}$ | 626.413 | 627.342 | 632．235 | －35．：45 | －：9．：3－ | －\＃：．$=$ ？ | －31．-7 |  |
| $\because:$ | 647．716 | ：52．792 | 655.277 | 653．879 | 9ini．al1 | －64．9 ${ }^{\text {a }}$ | －67．ã） | －79．-6 | $\div-3$ | －－－ |
| it： | 532． 259 | 593.122 | 636.191 | 589．：67 | 492．15\％ | 4，5．75； | － 4 ． $5 \leq a$ | 7－7．6．1 | －－．4．－3 | $\cdots$ |
| ：$:$ | 712.505 | 74．：3？ | 217．353 | ：20．535 | －23．71： | $\because 5.3 .95$ | －19．35 | 73． 30 | －：\％． | －1；．－． |
| 3： | 742． 749 | －45．2．55 | 743.431 | 1 752．5a7 | 755.75 ： | － 3 － 25 | －ヶ2．シン？ | 7 Ca | \＃．－． | －－．： |
| こ． | 725．725 | 77.354 | 2al． 391 | 795.737 | 799．793 | － $12.25{ }^{\text {－}}$ | 735．73\％ | －．t．： | －－2． | ＝－f． |
| ミ！ | ＋29．715 | －：2．357 | 515．：57 | 813.75 | 223．：53 | $\because 5 .-1$ | 43\％．an | $\square: 1 . c e 2$ |  | $74 \therefore$ ： |
| \％ | 44． 17 | －5i． $5: 1$ | 251．こ75 | 954．\％） | 759．24， | $\cdots 1.3$ | －45．．－3 | －67．7）： | －－2．： | －－ |
| ：5： | 479． 2.4 | 413．：${ }^{\text {a }}$ | 475．333 | 272．4．2 | 33.0 ； | －－－－ | $\therefore 1.505$ | －5．：74 | － $\mathrm{s}^{-3}$ | ＊．．i． |
| 三－； | 9：5．： 36 | $\because 9.6: 3$ | 323．54\％ | 727． 2 m 7 | 731．$:=7$ |  | －：4． | $\because$－．2？ | ¢\％． | －－－－ |
|  | －．．＇＊ |  | －$\rightarrow$ ．${ }^{\text {a }}$ | ＋5．：$=$ | －－．．． | $\bullet^{-} .^{--2}$ | ．－．．．． | －． | …－－ | ．－．．$\because$ |
| $\square$ | －9．．${ }^{\text {7 }} 7$ | －4．23 | 973．9＊9 | $\because: 32 .+1$ | ：：3\％．．a |  | ：－15．0－ | 1： $2 \rightarrow$ | ：$\because$ ？． |  |
| － | ： $1:$－ | ： 35. |  | $\therefore: 4.75$ |  |  | ： 55.5 .4 | 2\％： | ：－ | 4．： |

rable Alo.- Concluded



［From ref．：1］］

| knots | $0 \quad \cdots 1$ | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 6 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.000051 | 00001891 | 0.020428 | 0.000763 | 3．701：34 | $0.00172 \pm$ | 1．002346 | $\because 33767$ | 3．3）3－5 |
| ：0 | ．00478： | .005790 | ．006e71！ | ． 038085 | ． 009383 | －）こご幺 | ． 212253 | ． 313333 | ， 5500 | こごく¢ |
| 20 | ．©19153！ | ． 021118 | ．0231711 | ． 025331 | ． $2: 7581$ | ．029まうこ | ． $33237 \%$ | ． $3347 \%$ | ． $\mathrm{S}-\mathrm{E}$ ¢ | －92－－ |
| 30 | ．043109： | .046031 | ．047：\％7 | ． 052165 | ． 055375 ； | ． 055 ¢－6 | ． 262 cs | ． 265530. | －？ | $\because 2867$ |
| 40 | ． 076655 | ． 080548 | ． 084528 | ． 0886061 | ． 992777 | ．39：： 7 | ． 101412 | ． 125370 | ．$: 13$ | 1：5：93 |
| 50 | ． 119841 ！ | ． 124691 | ． 129640 | ． 134682 | ． 139822 ； | ． $145: 521$ | －150381 | ． 15515 | 1： $13: 7$ | ：96．955 |
| 00 | ． 172679 ： | ． 178492 | ． 184417 | ． 130422 ！ | ． 196526 ！ | ．20ごら2！ | ． 309037 | ． 215413 | 2：1929 | 28521 |
| 70 | ． 235205 ！ | ． 242000 | ． 248888 | ． 255866 | ． 262945 | ．277：20： | ．27205 | ． $834773^{\prime}$ | 292241 | －30ヶ11 |
| 30 | ． 307483 | ． 315247 | ． 323108 | ． 331067 | ． 3391191 | ． $34=-31$ | ． $35553 \%$ | ． 363887 | 172334 | らぢらの |
| 30 | .3895321 | ． 398282 | ．＋u7121 | ．416067 | ． 425139 i | ．434．501 | ． 243495 | ． 452325 | ． 46.2257 | ．77：7． 6 |
| ：30 | ． 481124 | ． 491160 | .501993 | ．5109．31 | .520953 | ． $531:-3$ | ． 541317 | ． $55163{ }^{\prime}$ | ． 520688 | ミロこ5 |
| ：10 | ． 583225 | ． 593949 | ．60．：783 | ．6157081 | ．626．40！ | ．63： $555^{\text {？}}$ | ． 4 49 ${ }^{\text {a }}$ | ． 560413 | 6．6：840 | cos375 |
| 220 | ．69：996 | ． 706731 | ． 718562 | ． 7304831 | ． 742511 ： |  | ．76588： | ．77921： | ．721651． | －94： |
| ： 30 | ． 816826 i | ．829561 | ． 342403 | ． $855344^{\prime}$ | ． 368384 |  | ．89474 | ． 0.78125 ！ | ． 221578 ： | ．3512．2 |
| －40 | ． 948779 | ． 902531 | ． 976394 | ． $3 \cdot 92364$ ！ | 1．20442 | 1． 215 E | 1.33286 | ：． 24723 | ． 56170 | 1．${ }^{\text {aras }}$ |
| －50 | 1.69097 | 1.10575 | 1.22063 | 1．13563 | 2.15072 | 2．16E－1 | 1．1012： | 1．19663 | ：．2：213 | 1． 2 － |
| －0 | 1． 24347 | 1.25929 | 1.27521 | 1.29125 | 1.3 .7738 | A－35ご | 1.23308 | 1． $55: 1$ | $\therefore 2-27 A$ | 1． $3=303$ |
| $: 70$ | 1．43645 | 1.42327 | 1.44925 | 1.45733 | 1．9745： | 1．40： | 1． 5.921 | ：5こちこ： | $\therefore 5 \div 43$ | 1．5゙こ：5 |
| ：30 | 1.57987 | 1.5978 v | 2.61534 | ：．63398 | 1．65ここ3 | 1．n．${ }^{\text {a }}$ | 1．6917\％ | 1．73゙ゥ | $\therefore$－$\because=32$ | 1．7： 5 ： |
| －90 | 1.75400 | 1.78300 | 1．80212 | 1.82133 | 1．84006 |  | 1．873： | 1．9923： | $\therefore: \geq 505$ |  |
| －0 | 1.95891 | 1.77900 | 1.99920 | 2.31351 | 2． $739 \rightarrow 2$ | $2.36: 5$ | $2.88: 8$ | $\therefore .1 .173$ | $\therefore 1=269$ | こ．intrm |
| －10 | 2.16473 | 2.18593 | 2.20722 | 2．22964 | 2.25016 | 2．27：－7 | 2．2935i | 2．31：30 | $\therefore 3 \equiv 735$ | 2． 5 5－io |
| ：20 | 2.38162 | 2.40392 | 2.42634 | ：．443d7 | 2.47151 | －．474：6 | 2.51713 | 2．54211 | $\therefore 56327$ | 2． 5 56：1 |
| $=30$ | 2.60972 | 2.63315 | 2.65670 | 2.68036 | 2.70412 | 2．7E： | ？．75202 | $\therefore 77014$ | $\therefore .8037$ | 2． $2=\square^{-1}$ |
| $-40$ | 2.84916 | 2.87375 | 2.89845 | 2.92325 | 2.94818 | 2．973： | 2.99838 | 2． 2365 | 3． $5: 704$ | 3．77855 |
| －50 | 3.10015 | 3.22590 | 3.15176 | 3.17773 | 3.22381 | $3.23: 53$ | 3.25636 | 3．23291 | ：．3：937 | 3.334 .5 |
| ：60 | $3.36 \cdot 89$ | 3.38377 | 3.41680 | 3.49376 | 3．4712： | $3.43 \div 8$ | 3.52615 | 3． 55378 | こ．さこ154 | 2.59091 |
| $: 70$ | 3.63781 | 3.66553 | 3.69377 | 3.72212 | 3．75060 | $3.77=1$ | 3.80792 | 3.9367 P |  | 3．0ヶニ3 |
| －30 | 3.92404 | 3.35337 | 3.78283 | 4.31241 | 7． $3: 1212$ | 4．${ }^{-9}$ | 4.1018 | $\div 23297$ | －．$=216$ | 4．：3こ5？ |
| －70 | 4.22273 | 4．25351 | ｜ 4.28421 | \＄． 31503 | \＄． 34597 | 4.30 | 4．403： | 4.43957 | －\％-192 | $4.5: 20$ |
|  |  |  |  |  |  |  |  |  |  |  |
| $\pm 30$ | 4．5343 | 4.59513 | 4.59809 | 4.63017 | ¢． 66238 | 4．6x | 4．72710 | 4．75973 | 4．－325 | 4.205 |
| 310 | 4．85633： | 4．29176 | 4.92459 | －． 35326 | 4.39156 | 5．：25：${ }^{\text {a }}$ | 5．0580 |  |  | 三． ¢ $^{\text {a }}$ |
| ：20 | 5.17523 | $5.2277!$ | 5．264：5 | 5．23892 | 5．33374 | 5． $35=05$ | 5．4）3t三 | 5．tis mit | ミ．－-32 | －ミッヅ |
| ：30 | 5.54533 | 5.59114 | 3.61699 | 5.55320 | 5.63914 | 5．72：52 | 5．76．153 | ミ．アコア3 | ミ．$-5=-6$ | 5． 27137 |
| ：4C | 5．9298？ | 5．94592 | 5.98315 | ¢．こここ51 | m． 35801 | ¢． 78505 | 6．133： | $\therefore .: 135$ | －．2．339 | －．－ites |
| 250 | 6.28595 | $6.32: 38$ | 5． 36278 | 5．40：73 | 6.44001 | 6．4．－${ }^{-1}$ | 5．ड1ヵッ | $\therefore$ 気E11 | －． 5.755 | ¢．．．うご |
| ：50 | $6.676 n 7$ | 6.71 infi | 6． 756.4 | 0.73690 | 6．33719 | 6．35－54 | 6．1102： | 2． $3 \leq 5 \times 7$ | $\div$－ 081 | …）n＝ |
| ：？0 | 7．23137 | 7．12320 | －1era | 7.20631 | －． 4974 | －25：-1 | 7.331 .94 | 7．30．412 | －．i：0：3 | －＊54\％ |
| iso | 7．5．0151 | 7.54427 | $\bigcirc .55717$ | 7.63021 | 7．6．342 | 7．71－－ | 7．759：－ | －，ij3：1 | －．－i7？ | －． $0: 5$ |
| ： 70 | ？ 3 557\％ | 7．94： | 3．3こ③＊ | H． 6894 | $\therefore$ ¢13e？ | ＋．15：7 | $3.2334^{\circ}$ | $\therefore . \therefore \div=-3$ |  | A． |
| ＋30 | 3． 38437 | 9．437．5 | 3．47568 | $8.529-4$ | 4．ミion？ | 3．6： $5: 5$ | 7．651e | F．-9.850 | こ，ここちら） | シ．$=:$ ： |
| ；10 | 3．8495 | 8． 87637 | ；3．3i＊34 |  | ว． $370^{\circ}$ |  | ＊． $355=$ | 2．15： | 3．23244 | － |
| ：20 | 9．3：335 | 9．37744 | 19．42759 | 3．：7671 | 2．526．3 | 3．5：－ | －．-2.55 ： | 2，$\therefore$ ¢ $5: 5$ | $\therefore . \mathrm{E} 537$ | － |
| $\therefore 10$ | 1．32531 | ч．8＊－i4） | 4.92738 | 7.97791 | 17．．23＊ | $19.05 \div$ | ＇：2．13：4 | $1^{-1.542}$ | ：．． 3 －5 | 1ご気車 |
| i4 | 11）．3384 | 10． 3.95 | i13．tis3 | ：0．4253 |  | 10．6\％ | ：．ev30 | ：－－， | $\therefore-\therefore 33$ | ：，＝： |
| －5 |  | 10． $3:$ | 1）．9こ53 | ：1． 295 | 2：．0．03 | ：1．： | 12．： | ：．－－ | －． 3 | －：：ミ－ |
| ij） | 111．713 | 1i．i＊＊： | 11．5こ：${ }^{\text {a }}$ | 11．${ }^{\text {a }}$ ： |  | ．．．＂ | 1：7－\％ | 1：．－＊i | －．．．－ | ：．$\times$ ． |
| ：7＇ | 11：．－ | 12．．＂3 | ：．． 213 | ここ．がら | －：こ－＊ | ． 2 ！ |  | ！－3 \％ |  | ：$:$ ： |
| － 56 | 112．55才3 | 12．617\％ | 12.5467 | ：．－${ }^{\text {？}}$－ | $\therefore$－50 | シミ： | 1 | ：，－${ }^{\text {a }}$ | $\therefore$ ○ 6 | ？${ }^{\text {？}}$ |
| ； | 13．：5？ | 13．－：38 | ：2．：－3ヵ） | ：3．34： | ここ．がこの | －．．＊ン： | 13．${ }^{-1}$ | 1：＝－＋3 |  | ：$:$ ：： |

## APPENDIX A

TABLE All.- Conciuand


## APPENDIX A



[:robet xul


## tazle al2.- Cencluded



## APPENDIX A

TABLE AI3.- TRUE AIRSPEED $V$ IN KNOTS FOR VALUES OF CALIBRATED airspeed $V_{c}$ in knots and values of pressure altitude h

IN GEDFOTENTIML FEET
[computation of $v$ based on standard temperature at each altitude]


## APPENDIX A

(jklin! ill PAGE OF PARIR QCALITY

TAASE A14.- STATIC PRESSURE $P$ (OR P'I IN wILLIMETERS OF MERCURY (OB C) fOR VAIUES OF PRESSURE ALTITUDE H (OR INDICATED ALTITUDE H') IN GEOPOTENTIAL METERS
[From ref. Al]

| H, | 0 | 100 | 200 | 300 | 400 | 500 | 500 | 700 | 800 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r}-1 \\ \\ \\ \\ \hline 000\end{array}$ | 854.538 | 769.054 | 778. 195 | 787.424 | 796.741 | 606.147 | 815.614 | 825.230 | 834.908 | 64:.677 |
| 0 | 760.000 | 751.032 | 742.151 | 733.354 | 724.643 | 716.015 | 707.470 | 699.009 | 690.629 | 682.331 |
| 1000 | 674.114 | 665.978 | 657.921 | 649.943 | 642.043 | 634.222 | 626.478 | 618.810 | 611.219 | 603.703 |
| 2000 | 596.263 | 598.997 | 581.604 | 574.385 | 567.239 | 560.165 | 553.162 | 546.231 | 539.370 | 532.579 |
| 3000 | 525.857 | 519.204 | 512.620 | 506.103 | 499.654 | 493.271 | 486.954 | 480.703 | 474.518 | 468.394 |
| 4000 | 462.339 | 456.346 | 450.416 | 444.548 | 428.742 | 432.998 | 427.314 | 421.692 | 416.129 | 1410.626 |
| 5000 | 405. 182 | 399.797 | 35.4.470 | 389.200 | 383.988 | 378.832 | 373.732 | 364.688 | 363.700 | :358.766 |
| 6000 | 353.886 | 349.061 | 344. 239 | 339.569 | 334.903 | 330.288 | 325.725 | 321.213 | 316.752 | 1312.342 |
| 7000 | 307.981 | 303.669 | 299.407 | 295.193 | 291.027 | 286.969 | 282.838 | 278.814 | 274.837 | 1270.906 |
| 8000 | 267.020 | 263.180 | 259.384 | 255.633 | 251.926 | 248.263 | 244.643 | 241.066 | 237.531 | 1234.038 |
| 9000 | 230.587 | 227.177 | 223.309 | 220.481 | 217.193 | 213.944 | 210.736 | 207.566 | 204.435 | 201. 343 |
| 10000 | 198.288 | 135.271 | 192.291 | 189.349 | 164.442 | 183.573 | 180.738 | 177.940 | 175.177 | 172.448 |
| 11000 | 169.754 | 167.098 | 164.484 | 161.911 | 159.377 | 156.884 | 154.430 | 152.013 | 149.635 | 147.294 |
| 12000 | 144.990 | 142.721 | 146.488 | 138.290 | 136.127 | 133.997 | 131.902 | 129.837 | 127.806 | 1125.806 |
| 13000 | 123.838 | 121.900 | 119.993 | 118.116 | 116.268 | 114.449 | 112.658 | 110.896 | 109.161 | 107.453 |
| 14000 | 105.772 | 104.117 | 102.488 | 100.885 | 99.3064 | 97.7527 | 96.2234 | 94.7179 | 93.2361 | 91.7774 |
| 15000 | 90.3415 | 88.9281 | 87.5366 | 86.1672 | 84.8191 | 33.4921 | 82.1859 | 80.9001 | 79.6344 | 78.3885 |
| 16000 | 77.1621 | 75.9549 | 74.7665 | 73.5968 | 72.4454 | 71.3119 | 70.1963 | 69.0980 | 63.5170 | 66.9528 |
| 17000 | 65.9053 | 64.8742 | 63.8593 | 62.8602 | 61.8767 | 60.9087 | 59.9547 | 57.0177 | 58.0944 | 57.1855 |
| 18 c00 | 56.2908 | 55.4101 | 54.5432 | 53.6899 | 52.8499 | 52.0335 | 51.2091 | 50.4080 | 49.6193 | 48.3430 |
| 19000 | 48.0788 | 17.3267 | 16.5862 | 45.3574 | 45.1399 | 44.4337 | 43.7385 | 43.0542 | 42.3806 | 41.7176 |
| 20000 | 41.0649 | 40.4226 | 39.7906 | 35.1688 | 38.5570 | 37.9550 | 37.3627 | 36.7799 | 36.2064 | 35.6421 |
| 21000 | 35.0865 | 34.5406 | 34.0031 | 33.4741 | 32.9536 | 32.4414 | 31.9375 | 31.4415 | 30.9536 | 30.4733 |
| 22000 | 30.0008 | 29.5358 | 29.0782 | 28.6279 | 28.1848 | 27.7487 | 27.3196 | 26.8973 | 26.4817 | 26.9727 |
| 23000 | 25.6703 | 25.2742 | 24.8844 | 24.5008 | 24.1232 | 23.7517 | 23.3861 | 23.0262 | <2.6723 | 22.3235 |
| $24 \cdot 300$ | 21.9804 | 21.6428 | 21.3105 | 20.9835 | 20.6616 | 20.3448 | 20.0330 | 19.7261 | 19.4243 | 19.1248 |
| 25000 | 18.8341 | 18.5461 | 18.2627 | 17.9637 | 17.7090 | 17.4387 | 17.1726 | 16.9107 | 16.6530 | 16.1992 |
| 26000 | 16.1495 | 15.9036 | 15.6616 | 15.4234 | 15.1889 | 14.9581 | 14.7309 | 14.5072 | 14.2871 | 15.0794 |
| 27000 | 13.857 | 13.6470 | 13.4403 | 13.2367 | 13.0364 | 12.8392 | 12.5450 | 12.4539 | 12. 2657 | 12.2535 |
| 28000 | 11.8981 | 11.7186 | 11.5418 | 11.3678 | 11.1965 | 11.0279 | 10.8618 | 13.6984 | 12.5375 | 12.3790 |
| 29000 | 10.223: | 10.0694 | 9.91825 | 9.76939 | 9.62281 | 9.47851 | 9.33643 | 9.19654 | $\rightarrow .25081$ | $5.32: 21$ |
| 30000 | 8. 7893 |  |  |  |  |  |  |  |  |  |


（OR INDICATED ALTITUEE H＇）IM GEOPOTEMEIAL NEFES
rifrom ref．Al］

| H． | 0 | 100 | 200 | 300 | 400 | 500 | 6 | $6 x$ |  | 703 | 80 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| －1000 | 113929. | 102532. | 103751. | 104981. | 106223. | 107477. | 108 | － | 110 | 022. | 111312. | 112614. |
| 0 | 101325. | 100129. | $94 \times 45.3$ | 97 ：72．5 | $96611 .:$ | 95460.8 | 943 | 3：．6 |  | 193.5 | 92 －76． 1 | 7 970. ： |
| 1000 | 89874.5 | 88789.7 | 87715.5 | 86 651．9 | 65 598．： | 84556.0 | 835 | 5.5 .5 | 82 | 501.3 | 81 \＄99．2 | 3） 487. |
| 2000 | 79495.2 | 78513.1 | 77 540．9 | $76: 78.4$ | 76625.6 | 74682.5 | 73 | －8．9 | ：2 | 824.3 | $71=50.0$ | 71204.6 |
| 3000 | 70108.5 | 69221.5 | 68343.7 | 67474.0 | 66615.3 | 65764.0 | 649 | 9：1．9 | 54 | 088.5 | $63=53.8$ | 62447.7 |
| $+000$ | 61640.2 | 60841.1 | 60050.5 | 59268.1 | 30484 | 57 728．3 | 56 | －0．6 | 56 | 223.7 | $55 \$ 79.3$ | 54 135．－ |
| 5000 | 54019.9 | 53301.9 | 52591.6 | 51889.1 | 51194.1 | 50506.8 | 49 | e．4．9 | 49 | 154.4 | $48 \div 9.3$ | 47 431．5 |
| 6000 | 47181.0 | 46537.6 | 45901.4 | 45272.2 | 44650.0 | 44034.6 | 43 |  | 42 | 824．7 | $42: 30.2$ | $\because$ E42．： |
| 7000 | 41060.7 | 40485.9 | 39917.6 | 39355.8 | 38800.4 | 38251.4 | 37 | －馬． 7 | 37 | 172．： | $36-51.9$ | ［6117．E |
| a 000 | 35599.8 | 35087.8 ｜ | 134581.7 | 34081.6 | 33587.4 | 33099.0 | 12 | ¢－-4 | 32 | 134．4 | 31 －4．2 | ？1 202.5 |
| 9000 | 30742.4 | 30287.8 ｜ | ｜ 29838.7 | 29395.0 | 28956.6 | 28523.6 | 28 ： | ¢ | 27 | 673.2 | $27-55.8$ | －6． 43.5 |
| 15000 | 26436.2 | 26034.0 | 25636.7 | 25 244．4 | 24357.0 | 24474.3 | 24 | 23.5 | 23 | 721．4 | 23155.0 | ： 22991.2 |
| 11000 | 22612.5 | 22277.9 | 21929.4 | 21586.3 | 21248.6 | 20916.1 | 20 | \％ 9 ¢ | 23 | 2t6．3 | 19 －49．7 | ： 27637.6 |
| 12000 | 17350.4 | 19027.9 | 18730.2 | 18437.2 | 16148.8 | 17864.8 | 17 | 545． 3 | 15 | 310．： | $17: 39.4$ | i is 772．a |
| 13000 | 1； 510.4 | 16252.1 | 15397.8 | 15747.5 | 15501.1 | 15258.6 | 15. | ：$\because: 9$ | E． | 784.7 | 14553.6 | ：4 325． |
| 14000 | $1+101.8$ | 13881.1 | 13664.0 | 13450.2 | 13239.8 | 13032.6 | 12 | Eis． 7 | 12 | 628．： | 12 43.5 | ：2：36．： |
| 15000 | 12044.5 | 12856.2 | 11670.6 | 11.488 .0 | 11308.3 | 11131.4 | 10 | cot．2 | 19 | 785.3 | 12 E： 0 | $\because 250.4$ |
| 16000 | 10287.4 | 10126.5 | 9968.05 | 9812.101 | 9658.39 | 9507.48 |  | 35－73 | 7 | 212．19 | 9 ： 60.14 ： | $\vdots 726 . \quad$ ： |
| 17000 | 8786.66 | 9649.19 ！ | （ 8513.87 | （ 380.67 | 8249.55 | 8 120.49 |  | －6！．44 | 7 | 868．${ }^{\text {\％}}$ | $7-55.28$ i | －․a |
| 13000 | 7504.82 ； | 7387.41 i | i 7271.83 | 7158.06 | 7046.07 | 6 \％ 73.83 |  | E－－32i | 5 | 720．s： | 50.5 .36, | 1－511．－＊ |
| 17000 | 6409.99 | 6309.70 ． | ． $6210.98 \mid$ | 6 113．81 | 6018.16 | 5924.01 |  | 7：$: 32$ ！ | 5 | 740．： | 5 ¢52．この． | S Stic．？ |
| 23000 | 5474.87 | 5389.24 | 5304.981 | 5222.08 | 5140.51 | 1 563.25 |  | 二゙心－28 | 4 | 903．5： | 4 9．7．： | ；751．${ }^{\text {－}}$ |
| 21000 | 4677.971 | 4605.04 ： | 4533.37 | 4 462．85， | 4393.45 ！ | ！ 425.17 |  | Ex－98； | 4 | 1 11. | ＋ 25.8 .8 | ＋ $62 .{ }^{-5}$ |
| ＇ 22000 | 3999.73 ！ | 3937.78. | 3376.78 ！ | 3116.74 | 3757.66 | 3699.53 |  | － $5 . .31$ | ： | 566．： | 3 ミ3こ．ら1 | 3：76．．．－ |
| 23 － 201 | 3422.42 | 369.61 | 3 317．65； | 3266.50 | 3216.17 | 3166.17 |  | $\because$－ 88 |  | 363．： | 3 ここ．5 | －－i6 |
| 120000 | 2930.481 | 2 d85．47 | －41．17 | 2797.56 | 2 754．65 | $=712.41$ |  | $\cdots-34$ | － | 5：9．－： | こ ミ59．64 | － 553. |
| 25000 | 2511.21 | 2 472．6： | ： 3 34．32 | 2397.62 | 2361.31 | $=324.97$ |  | －-7.50 |  | 254．：－ | こ ここ？．21 | ：－4．：－ |
| 260001 | 2153. ．${ }^{\text {2 }}$ | 2120.11 | －88．04． | $=256.28$ | $=025.22$ | 1294.25 |  | －：3．36 |  | 734．： | 1 \％-7.7 | －75．－2 |
| 27000 | 1847.55 | 1819.45 | 1 －71．89． | 764.75 | 173 A .04 | 1711.75 |  | － 86 | ： | 66）， 3 | 1.35 .23 | ：-1 |
| －3 000 | 1586.29 | 1562.35 | 1538.78 ！ | 1515.59 | 1492.75 | ： $479 . \geq 6$ | 1 | \％+1.13 | ： | 426.5 | 1－5．4．7 | ：吅3．－ |
| ：7000 | $1362.76 \text { ! }$ | $1342.48$ | $132.32$ | （1 302．44 | $1.82 .94 i$ | 1363.79 |  | －3．76 |  | ここの．： | ：ご，－ | ：：دヵ． |
| 32001 | 1171.561 |  |  |  |  |  |  |  |  |  |  |  |

TABLE A16．－DENSITY $D$ IN KILOCRAMS PER CUBIC METER TOR YALUES $=:$
PRESSURE ALTITUDE $H$ IN GEOPOTEMTIAL METERS
［From zef．A1］

| H. | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 830 | 900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1.2250 | 1.2133 | 1.2017 | 1.1901 | 1.1786 | 1.1673 | 1.1560 | 1.1448 | ：． 1336 | ． 1226 |
| 1000 | 1.1116 | 1.1008 | 1.0900 | 1.0793 | 1.0686 | 1.0581 | 1.0476 | 1.0372 | 11.0269 | 1.0156 |
| 2000 | 1．0065 | ． 99641 | ． 98641 | ． 97648 | ． 96663 | ． 95886 | ． 94716 | ． 93.54 | ． 92799 | ． 91852 |
| 3000 | ． 90912 | ． 89980 | ． 89055 | ． 88137 | ． 87226 | ． 86323 | ． 85427 | ． 84538 | ． 83656 | ． 82781 |
| 4000 | ． 81913 | ． 81052 | ． 80198 | ． 79351 | ． 78511 | ． 77677 | ． 76851 | ． 76031 | ． 75218 | 7：411 |
| 5000 | ． 73612 | ． 72818 | ． 70232 | ． 71251 | ． 70478 | ． 69711 | ． 68950 | ． 68195 | ． 67447 | ． 66705 |
| 6000 | ． 65970 | ． 65230 | ． 64517 | ． 63800 | ． 63089 | ． 62384 | ． 61636 | ． 60993 | ． 60306 | ． 57625 |
| 7000 | ． 58950 | ．5826i | ． 57618 | ． 56960 | ． 56308 | ． 55662 | ． 55022 | ． 54387 | ． 53758 | ． 53135 |
| 8000 | ． 52517 | ． 51904 | ． 51297 | ． 50696 | ． 50100 | ． 49509 | ． 48924 | ． 48343 | ． 57769 | ．47179 |
| 9000 | ． 46635 | ． 46076 | ． 45522 | ． 44973 | ． 44429 | ． 43890 | ． 43350 | ． 42827 | ． 42304 | ．41：85 |
| 10000 | ． 41271 | ． 40761 | ． 40257 | ． 39757 | ． 3926 | ． 38772 | ． 3829 | ． 37806 | ． 37330 | 36859 |
| 11000 | ． 36392 | ． 35822 | ． 35262 | ． 34710 | ． 34167 | ． 33633 | ． 33106 | ． 32589 | ． 32679 | ． 31577 |
| 12000 | ． 31083 | ． 30576 | ． 30118 | ． 29647 | ． 29183 | ． 28726 | ． 28277 | ． 27834 | ． 27399 | ． 25970 |
| 13000 | ． 26548 | ． 26133 | ． 25724 | ． 25322 | ． 24925 | ． 24535 | ． 24152 | ． 23774 | ． 23402 | ． 4036 |
| 14000 | ． 22675 | ． 22331 | ． 21971 | ． 21628 | ． 21289 | ． 20956 | ． 20628 | ． 203.16 | ． 19988 | ． 19675 |
| 15000 | ． 1936 | ． 1906 | ． 18766 | ． 18472 | ． 18183 | ． 17899 | ． 17619 | ． 17343 | ． 17072 | ． 16805 |
| 16000 | ． 16542 | ． 16283 | ． 16028 | ． 15778 | ． 15531 | － i 5288 | ． 15049 | ． 14813 | ．1458： | ． $1: 353$ |
| 17000 | ． 14129 | ． 13908 | ． 13690 | ． 13476 | ． 13265 | ． 13058 | ． $1: 253$ | ． 12685 | ． 12954 | ． $12: 59$ |
| 18000 | ． 120 | ． 11879 | ． 11693 | ． 12510 | ． 11330 | ． 11153 | ． 10978 | ． 10806 | ． 10637 | 10571 |
| 19000 | ． 10307 | ． 20146 | ． 099871 | ． 098309 | ． 096771 | ． 295257 | ． 093766 | ．09229 $=$ | ． 090855 | ． 350.34 |
| 20000 | ． 028035 | ． 086618 | ． 885224 | ． 083854 | ． 082506 | ． 781180 | ． 079977 | ． 078598 | ． 777333 | $\bigcirc-6293$ |
| 21000 | ． 074873 | ． 073674 | ． 072494 | ． 071333 | ． 070192 | ． 369069 | ． 067365 | ．366？${ }^{\text {－}}$ | ． 365211 | －-5.61 |
| 22000 | ． 063 | ． 062711 | ． 261711 | ． 060 | ． 059760 | ． 05880 | ．05：873 | ． $25695 \%$ | ． 3565.47 | 55： 56 |
| 23000 | ． 054280 | ． $053+18$ | ． 052570 | ． 051737 | ．05c916 | ． 050109 | ． 244315 | ． 348535 | ． 347556 | $\therefore \div 211$ |
| 24000 | ． 046267 | ． 045336 | ． 044816 | ． 044109 | ． 043412 | ． 342727 | ．04：354 | ． 34139 | ． 2 90－39 | \％ |
| 25000 | ． 039466 | ． 038345 | ． 038234 | ． 037633 | ． 037741 | ． 335459 | ． $33 \leq 387$ | －．3532： | ． $334 \sim$－ | －34こ， |
| 26000 | ． 033688 | ． 033160 | ． 032641 | ． 032130 | ． 031628 | ． 031133 | ． 032646 | － 3216 | ． 72045 | ． $0=0253$ |
| 27000 | ． 028777 | ． 023328 | ． 027886 | ． 027452 | ． 0271524 | ． 226604 | ． $3: 5$ | －こミー－ | ． $25:-1$ | －$=3 \rightarrow 7$ |
| 28000 | ． 024597 | ．023217 | ．023841 | ． 023471 | ． 023107 | ． 322749 | －$=: 376$ | － $22=35$ | ． $221: 18$ | －ミ－， |
| 29000 | ． 021042 | ．025717！ | ． 020397 | ． 020082 | ． 019771 | ． 019466 | ． 023166 | ． 31 ．8： | 18580 | ：－294 |
| 30000 | ． 018012 | ． $21 \sim 35$ | 4 n 2 | ．017193 | ． 1616729 | ． $31660^{\circ}$ | ． 19.413 | ． $31516:$ | ． 315313 | －｀ことこう |

AFEMTILIX A

TABLE A17．～TEMPERATURE ：IA DEGREES CENTIGRRLE FOR VALUES OF
Pfessure altitide h in geopotential metirs
［rom ref．A1］

| H, m | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 320 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 15.000 | 14.350 | 13.700 | 13.050 | 12.400 | 12.750 | 11.100 | 10.450 | 9.800 | 9．150 |
| 1000 | 8.500 | 7.850 | 7.200 | 6.550 | 5.900 | 5.250 | 4.600 | 3.950 | 3.300 | 2.650 |
| 2000 | 2.000 | 1.350 | ． 700 | ． 050 | －．600 | －1．250 | －1．900 | －2．550 | －3．200 | －3．859 |
| 3000 | －4．500 | －5．150 | －5．80C | －6．450 | －7．100 | －7．750 | －8．400 | －9．050 | －9．700 | $-10.353$ |
| 4000 | －11．000 | －11．65n | －12．300 | －12．950 | －13．600 | －14．250 | －14．900 | －15．550 | $-16.200 \mid$ | $-16.850$ |
| 5000 | －17．500 | －18．150 | －18．800 | －19．450 | －20．100 | －20．750 | －21．400 | －22．050 | －22．700 | －23．350i |
| 6000 | －24．000 | －24：650 | －25．300 | －25．950 | －26．600 | －27．250 | －27．900 | －28．550 | －29．200 | －29．850 |
| 7000 | $-30.500$ | －31．150 | －31．800 | －32．450 | －33．1c0 | －33．750 | －34．405 | 3． 5.250 | 35.700 | －36． $5 \pm 01$ |
| 8000 | －37．000 | －37．650 | －38．300 | －38．950 | －39．600 | －4．2． 250 | －40．900 | 1.550 | －42．200 | －42．850 |
| 9000 | －43．500 | －44．15C | －44．800 | －45．450 | －46．100 | －46．750 | －47．400 | －48．050 | －48．750 | －49．350 |
| 10000 | －50．300 | －50．650 | －51．300 | －51．955 | －5z．690 | －53．250 | －53．900 | －54．550 | －55 | 501 |
| 11000 | －56．500 | －56．500 | －56．500 | －56．50c | －56．500 | －56．500 | －56．500 | $-50.500$ | －56． 560 | －56． $520!$ |
| 12000 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －55．5う0 |
| 13000 | －56．500 | －56 500 | －56．500 | －56．50i | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56． 520 |
| 14000 | －56．500 | $-56.500$ | －56．500 | －56．500 | －56．500 | $-56.500$ | －56．500 | －56．500 | －56．530 | －56． 5.30 |
| 15000 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | $-56 . ミ 93$ |
| 16000 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | $-56.500$ | －56．500 | －56．500 | －56．500 | －56．$=20$ |
| 17 COO | －56．500 | －j6．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．530 | －5ミ．ミ 30 |
| 18000 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．50 | －56．50 | 56．ミ 30 |
| 19000 | －56．500 | $-56.500$ | －56．500 | －56．500 | －56．500 | －56．500 | －56．500 | －56．50 | 56 | －56． 520 |
| 20000 | －56．500 | －56．400 | －56．300 | －56．200 | －56．100 | －56．000 | －55．900 | －55．80 | －55．700 | －55．$=90$ |
| 21000 | －55．500 | －55．400 | －55．300 | －55． 200 | －55．100 | －55．000 | －54．900 | －54．800 | －54． | －5＊－ |
| 22000 | －54．500 | －54．400 | －54．300 | －54．230 | －54．100 | －54．000 | －53．900 | －53．30 | －53．730 | －5ミ．ここ0 |
| 23000 | －53．500 | －53．400 | －53．300 | －53．23 | －53．200 | －53．000 | －52．900 | －52．800 | －52．790 | －5こ．-30 |
| 24000 | －52．500 | －52．400 | －52．300 | －52．220 | － | －52．000 | －51．900 | －51． | －51．700 | $\therefore . . \div 5$ |
| 25000 | －51．500 | －51．400 | －51．330 | －51．200 | －51．100 | －51．000 | －50．900 | －50．800 | －50．710 | －5？．$=29$ |
| 26 0u0 | －50．500 | －50．400 | －50．300 | －50．230 | －50．100 | $-=0.000$ | －49．900 | －49．800 | 49． 20 | －3．$\because 30$ |
| 27000 | －49．503 | －49．400 | －49．300 | －49． 20 | －49．100 | －49．000 | －48．900 | －48．800 | 48.770 | －シャッここ |
| 28000 | －48．500 | －48．400 | －42．300 | －48．$=00$ | －48．100 | －48．000 | －47．900 | －4． 8.80 | －47．7．20 | －－－ |
| 29000 | －47．500 | －47．400 | －47．330 | －47． 200 | －47．100 | －\＄7．000 | －46．900 | －46．800 | －46．－70 | －－＋ |
| $30 \quad 000$ | －46．500 | －46．400 | －46． 320 | －46．200 | －46．100 | －46．000 | －45．300 | －45．972 |  | －こミ，$\div こ ゙$ |

table al8.- COEFFICIENT OF VISCOSITY $u$ IN PASCAL-SECCNDS zOR
values of pressure altitude h in geopotential meters
[From ref. Al]

| H, m | $\begin{gathered} \text { H, } \\ \mathrm{Pa}-\mathrm{sec} \end{gathered}$ | $\mathrm{H} \text {, }$ | $\begin{gathered} \text { H, } \\ \mathrm{Pa}-\mathrm{sec} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 0 | $1.7894 \times 10^{-5}$ | 15000 | $1.4216 \times 10^{-5}$ |
| 500 | 1.7737 | 15500 | 1.4216 |
| 1000 | 1.7578 | 16000 | 1.4216 |
| 1500 | 1.7419 | 16500 | 1.4216 |
| 2000 | 1.7260 | 1; 100 | 1.4216 |
| 2500 | 1.7099 | 17500 | 1.4216 |
| 3000 | 1.6937 | 18000 | 1.4216 |
| 3500 | $1.6775^{-}$ | 18500 | 1.4216 |
| 4000 | 1.6611 | 19000 | 1.4216 |
| 4500 | 1.6447 | 19500 | 1.4216 |
| 5000 | 1.6281 | 20000 | 1.4216 |
| 5500 | 1.6115 | 20500 | 1.4244 |
| 6000 | 1.5947 | 21000 | 1.4271 |
| 6500 | 1.5779 | 21500 | 1.4298 |
| 7000 | 1.5610 | 22000 | 1.4326 |
| 7500 | 1.5439 | 22500 | 1.4353 |
| 8000 | 1.5268 | 23000 | 1.4381 |
| 8500 | 1.5095 | 23500 | 1.4408 |
| 9000 | 1.4922 | 24000 | 1.4435 |
| 9500 | 1.4747 | 24500 | 1.4462 |
| 10000 | 1.4571 | 25000 | 1.4490 |
| 10500 | 1.4394 | 25500 | 1.4517 |
| 11000 | 1.4216 | 26000 | 1.4544 |
| 11500 | 1.4216 | 26500 | 1.4571 |
| 12000 | 1.4216 | 27000 | 1.4598 |
| 12500 | 1.4216 | 27500 | 1. 4625 |
| 13000 | 1.4216 | 28000 | 1.4652 |
| 13500 | 1.4216 | 28500 | 1.4679 |
| 14000 | 1.4216 | 29000 | 1.4706 |
| 1450 C | 1.4216 | 29500 | 1.4733 |
|  |  | 30000 | 1.4760 |

APPENDIX A

TABLE A19.- SPEED OF SOUND a IN KILOMETERS PER HOUR AND KNCTS
FOR values of pressure altitude h in geopotential meters
[From ref. Al]

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{~m} \end{aligned}$ | $\underset{\mathrm{km} / \mathrm{hr}}{\mathrm{a}}$ | knots | H, | $\begin{gathered} a_{1}, h r \\ k m / h r \end{gathered}$ | $\underset{\text { knots }}{\text { a, }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1225.06 | 661.48 | 15000 | 1062.25 | 573.57 |
| 500 | 1218.13 | 657.74 | 15500 | 1062.25 | 573.57 |
| 1000 | - 211.15 | 653.98 | 16000 | 2062.25 | 573.57 |
| 1500 | 1204.15 | 650.19 | 16500 | 1062.25 | 573.57 |
| 2000 | 1197.10 | 646.38 | 17000 | 1062.25 | 573.57 |
| 2500 | 1190.01 | 642.56 | 17500 | 1062.25 | 573.57 |
| 3000 | 1182.88 | 638.70 | 18000 | 1062.25 | 573.57 |
| 3500 | 1175.70 | 634.83 | 18500 | 1062.25 | 573.57 |
| 4000 | 1168.48 | 630.93 | 19000 | 1062.25 | 573.57 |
| 4500 | 1161.22 | 627.01 | 19500 | 1062.25 | 573.57 |
| 5000 | 1153.90 | 623.06 | 20000 | 1062.25 | 573.57 |
| 5500 | 1146.55 | 619.09 | 20500 | 1063.48 | 574.23 |
| 6000 | 1139.14 | 615.09 | 21000 | 1064.9.4 | 575.02 |
| 6500 | 1131.69 | 611.06 | 21500 | 1065.92 | 575.55 |
| 7000 | 1124.18 | 607.01 | 22000 | 1067.14 | 576.21 |
| 7500 | 1116.63 | 602.93 | 22500 | 1068.36 | 576.87 |
| 8000 | 1109.03 | 598.83 | 23000 | 1069.58 | 577.53 |
| 8500 | 1101.37 | 594.69 | 23500 | 1070.79 | 578.18 |
| 9000 | 1093.65 | 590.53 | 24000 | 1072.01 | 578.84 |
| 9500 | 1085.89 | 586.33 | 24500 | 1073.22 | 579.50 |
| 10000 | 1078.07 | 582.11 | 25000 | 1074.44 | 580.15 |
| 10500 | 1070.19 | 577.85 | 25500 | 1075.65 | 580.80 |
| 11000 | 1062.25 | 573.57 | 26000 | 1075.86 | 581.46 |
| 11500 | 1062.25 | 573.57 | 26500 | 1073.07 | 582.11 |
| 12000 | 1062.25 | 573.57 | 27000 | 1079.27 | 582.76 |
| 12500 | 1062.25 | 573.57 | 27 50n | 1080.48 | 553.41 |
| 13000 | 1062.25 | 573.57 | 28000 | 1081.68 | 584.06 |
| 13500 | 1062.25 | 573.57 | 28500 | 1082.89 | 584.71 |
| 14000 | 1062.25 | 573.57 | 29000 | 1084.09 | 585.36 |
| 14500 | 1062.25 | 573.57 | 29500 | 1085.29 | 386.01 |
|  |  |  | 30000 | 1086.49 | 536.66 |

APPENDIX A
table a20.- ACCELERATION DUE TO GRAVITY g IN METERS PER SECOND SQUARED for values of pressure alititude h in geopotential meters
[From ref. Al]

| $\begin{aligned} & \mathrm{H}, \\ & \mathrm{~m} \end{aligned}$ | $\begin{gathered} \mathrm{g} \\ \mathrm{~m} / \sec ^{2} \end{gathered}$ | $\begin{aligned} & \mathbf{H}, \\ & \mathbf{m} \end{aligned}$ | $\underset{\mathrm{m} / \mathrm{sec}^{2}}{\mathrm{~g}}$ |
| :---: | :---: | :---: | :---: |
| 0 | 9.8066 | 15000 | 9.7604 |
| 500 | 9.8051 | 15500 | 9.7589 |
| 1000 | 9.8036 | 16000 | 9.7573 |
| 1500 | 9.8020 | 16 5ن0 | 9.7558 |
| 2000 | 9.8005 | 17000 | 9.7543 |
| 2500 | 9.7989 | 17500 | 9.7525 |
| 3000 | 9.7974 | 18000 | 9.7512 |
| 3500 | 9.7959 | 18500 | 9.7496 |
| 4000 | 9.7943 | 19000 | 9.7481 |
| 4500 | 9.7928 | 19500 | 9.7466 |
| 5000 | 9.7912 | 20000 | 9.7450 |
| 5500 | 9.7897 | 20500 | 9.7435 |
| 6000 | 9.7881 | 21000 | 9.7420 |
| 6500 | 9.7866 | 21500 | 9.7404 |
| 7000 | 9.7851 | 22000 | 9.7389 |
| 7500 | 9.7835 | 22500 | 9.7373 |
| 8000 | 9.7820 | 23000 | 9.7358 |
| 8500 | 9.7804 | 23500 | 9.7343 |
| 9000 | 9.7789 | 24000 | 9.7327 |
| 9500 | 9.7774 | 24500 | 9.7312 |
| 10000 | 9.7758 | 25000 | 9.7297 |
| 10500 | 9.7743 | 25500 | 9.7281 |
| 11000 | 9.7727 | 26000 | 9.7266 |
| 11500 | 9.7712 | 26500 | $9.7 こ 50$ |
| 12000 | 9.7697 | 27000 | 9.7235 |
| 12500 | 9.7681 | 27500 | 9.7220 |
| 13000 | 9.7665 | 28000 | 9.7204 |
| 13500 | 9.7650 | 28500 | 9.7189 |
| 14000 | 9.-635 | 29000 | 9.7174 |
| 14500 | 9.7620 | 29500 | 9.7158 |
|  |  | 30000 | 9.7143 |



［Serived from $x=2 . \lambda 2$ ］

|  | $\because$ | 1 | ： | 1 | 4 | 5 | 6 | 7 | $\underline{3}$ | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $\checkmark$ | ） | $1.001:$ | 3.003 | 0.006 | J． 009 | 2.213 | 3.017 | 3.223 | 3.327 |
| 10 | ． 035 | ． 343 | ． $251^{i}$ | ． 060 | ． 069 | ．080： | ． 591 | ． 102 | 115 | 128 |
| 20 | ． 142 | ． 156 | ． 1721 | ． 189 | ．204／ | ．222j | ．240 | ．258） | ．278 | ． 296 |
| 30 | ． 319 | ． 341 | ． 363 | ． 386 ， | ．410！ | ． 134 ！ | ． 4601 | ． 485 | ． 512 | 539 |
| 45 | ． 567 | ． 596 | ．625， | ．656 | ．687！ | ． 718 ： | ． 750 ： | 783 | 3：7 | ． 351 |
| 50 | ． 887 | ． 922 | ．9591 | ． 996 ｜ | 1.0341 | 1.073 | 1.112. | 1.152 | 1．193 | 1235 |
| 60 | 1.277 | 1．320 | 1.364 | 1．408！ | 1.4531 | 1．499： | 1.545 | 1.592 | 1.540 | 1.689 |
| 70 | 1．738！ | 1.788 | 1.839 | ：8991． | 1.9431 | 1.996 ： | 2.549 | 2.104 | 2.159 | $\therefore 215$ |
| 80 | 2．271 | 2.328 | 2.386 | $2.345!$ | 2．524， | 2.564 | $2.625{ }^{\text { }}$ | 2.686 | 2.749 | 2.812 |
| 90 | 2.875 | 2.940 | 3.005 | 3.0751 | 9．137 | 3.204. | 3.2721 | 3.341 | 3.410 | 3.485 |
| 100 | 3.551 | ．622 | 1.694 | 3.7671 | 3.841. | 3．915＇， | $3.990^{\circ}$ | 4.066 | 4.143 | 4.220 ； |
| 110 | 4.2981 | 4.377 | 7.456 | 4．536， | 4．617： | 4.698 ： | 4.781 | 4.864 | 4.347 | 5.032 |
| 120 | 5.117 ！ | 5.203 ！ | 5.289 | 5.377 ！ | 5.465 ： | 5.553 | $5.643 ;$ | 5.733 | 5.824 | 5.915 |
| 130 | 6.008 | 6.104 | 6.194 | 6.289 | 6.384 | 6.480 ： | 6.577 ， | 6.674 | 6.772 | 6.871 |
| 140 | 6.971 | 7.071 | 7.172 | 7.2741 | 7.376 | $7.479{ }^{\circ}$ | $7.583 i$ | 7.688 | 7.793 | 7.899 |
| 150 | 8.006 | 8.113 | 6.222 | 9.331 ！ | 8.4401 | 8.551 | 8.662 | 8.774 | 8.886 | 9.000 |
| 160 | 9.114 | 9.228 | 7．344 | 9．4601 | 9.5771 | 9.695 | 9．913！ | 9.932 | 10.052 | 10.173 |
| 170 | 10.294 | 13.416 | 10.539 | 13.6621 | $10.787^{\prime}$ | 10.912 | 11.0371 | 11.164 | 11.291 ； | 11.419 |
| 180 | 11.547 | 11.677 | 14．807 | 11．938： | 12.063 | 12.202 | 12．335： | 12.468 | 12.503 | 12.738 |
| 190 | 12.874 | 13.011 | 13.148 | 13.286 | 13.425 | 13．565． | 13.705 | 13.346 | 13.388 | 14.131 |
| 200 | 14.274 | 14．413 | 1\＄．563 | 14.709 | 14.855 | 15.002 | $15.150^{\circ}$ | 15.298 | 15.447 | 15．59\％ |
| 210 | 15.748 | 15.899 t | 16.052 | $15.205 i$ | 16.358 | 16．5131 | 16.668 | 16.824 | $16.780^{\circ}$ | 17.138 |
| 220 | $1 / .2961$ | 17．455 | 17.614 | 17.775 | 17.936 | 18.098 | 18.260 | 18.424 | 18.553 ； | 18.753 |
| 230 | 18.9181 | 19.084 | 19.251 | 19．419！ | 19.588 | 19.757 | 19．927， | 20．038， | 20.270 | 22.442 |
| 240 | 23.615 | 20.789 | 20.963 | 21．1391 | 21.315 | $21.492^{\prime}$ | 21.669 | 21.847 | 22.227 | 22．206 |
| こ5： | $\therefore 2.3871$ | 22.569 | $\therefore 2.750$ | 2：．933 | 23．11： | 23.301 | 23.486. | 23.672 | 23．357， | 24.346 |
| 263 | 24．234： | 24.423 | 24.6131 | 24.803. | 24.994 | 25.186 | 25.379 | 25.572 | 25．76： | 25．762： |
| 275 | $26.157{ }^{\prime}$ | 26.354 | 26.551 | 26.749 | 26.948 | 27.147 | 27.348 | 27.547 | 27．751 | 27.753 |
| 283 | 28.156 | 28.361 | 28.565 | 26.771. | 28.978 | 29.185 ； | 29.393 | 29.621 | 29.811 | 30.021 ！ |
| 290 | 30.232 | $30.444{ }^{\circ}$ | 30．657 | 30．870 | 31.084 | 11.299 | 31.515 ： | 31.731 | 31.948 | 32.166 |
| 303 | 32.385 | 32.604 | 32.825 | 23.046 | 33.268. | 33.490 i | 33.7141 | 33.938 | 34.163 | 34.388 |
| 310 | 34.615 | 34.842 | 35.070 | 35.299 ｜ | 35.529. | 35.759 | 35．990： | 36.222 | 36.455 | 36.688 |
| 329 | 36.923 | 37.158 | 37.394 | 37.6301 | 37.868 | 38.106 | 38，345＇ | 38.585 | 36.325 | 39.067 |
| 332 | 39.309 | 39.552 | 39.795 | 4．3．0401 | 40.285 | 40.531 | 40．778： | 41.026 | 41.274 ！ | 41.524 ！ |
| 342 | 41.774 | 42.0251 | 42．276！ | 42.529 | 12.782 | 43.036 | 43．291． | 43.546 | 43.833 | 44.360 ： |
| 352 | 44.318 | 44．577； | 44．8361 | 45.097 | 45.358 | 45.620. | 45.883 | 46.145 | 46．411 | \＄ 6.6 .6 |
| 362 | 46.942 | 47.2081 | 47．476 | 47.744 ！ | 48．71\％ | 48.284 | 48．555 | 48.826 | \＄9．079 | 49．3：2 |
| $37=$ | 49.646 | 49.921 | 50.196 | 53．3i， | 54．756 | 51.228. | 51.177 | 51.567 | 5：．367 | 52.147 |
| $38 \%$ | 52．431 | 52．714 | ㄱ． 9.998 | 53．2Eこ＇ | 51.562 | 53．854 | 54．：41 | 54.459 | 54．71＂ | 35． 57 |
| 392 | 55．277！ | 55.588 | $\therefore 5.4$ ¢\％ | 56．1＇3） | Sti．is： | 50．${ }^{\text {a }}$ 1： | 57．：5n | 57． $5:$ | S－．0．4 | 57．0．471 |
| 405 | 58.245 | 58.545 | 53．845＇ | 59．146 | 5＊．4＊2 | 59.751 | 53.754 | 60.353 | 62.864 | 6．3．1 |
| 42 | 61．27 | 61.585 | \％1．893 | 52.203 | 62.513 | 62.824 | 63.136 | 55.443 | 53.752 | 64．3551 |
| 42 y | 64．391： | 64.707 | 65.324 | ¢5． 272 | 65.600 | 65.789 | 6in． 730 | 66．6．1 | 66.343 | 6． 2.266 |
| 13） | 67.537 | 67.913 | 58.239 | 63.565 | 68.93. | 69．223． | 69．548 | $69.0^{-9}$ | 72.238 | －2． 539 |
| $\ddagger$－ | 70．671． | 71.304 | ${ }^{*} 1.538$ | 71.872 | 72．22a | 72．54．4 | 72.891 | $73.2: 3$ | 73.553 | 7.398 |
| $45:$ | 7.938 | －4．533 | 74．312． | －a．ze5 | 75.639 | 75.954 | 76.320 | 74．6：7 | 76.734 |  |
| 46： | 77.691 | 78.051 | 78． 392 | 79．744 | 79.19 | 73.450 | 77.235 | 80.153 | 52．$\geq 16$ | 37．8～ |
| 47 － | 91.231 | 81．575 | 31． 319 | 32．313： | ：82．6？1 | ＋3． 333 | 33.376 | 83．750 | 34．1：5 | 04． 291 |
| 73： | 84．757； | 5＊．225 | 35．593． | －5， 762 | 86．33？ | 36.704 | 27.25 | 97． 849 | 97.522 | 28．196 |
| 4 C | 39．5：2： | त内．3：3 | 87． 325 ． | 79.723 | 90.382 | 70.462 | 90.343 | $\rightarrow 1.2: 4$ | 91，¢ご | 91.973 |
| 5： | 22．375 | 32.355 | 93．136， | 23．63； | 93.921 | 94． 310 | 34.690 | 35.293 | 35.481 | 75.824 |
| 三： | 24．－5： | $2=.001$ | 77． 361 | －． 452 | 9\％．647 | 29.24 7 | 98．64－ | 97.45 | 77．447 | 37.848 |
| ミ－： | ：3．－ | $: 2.65$ | ：21． 26 | ： 1.46 | 101．37 | 122.27 | 122.63 | 103．99 | 123．59 | 123．91 |
| $5 \cdot$ | $\cdots$ | ＊ | ：$-:=$ | ： －$^{\circ}$ | ，i＝．${ }^{\text {a }}$ | ：．．．： | －＊．．＊ | － | 1．${ }^{\text {－}}$ | 198．5？ |
| 54 | 1：B．： | ：9．41 | 1：7．33 | 1．3．76 | 11？．19 | $: 10.60$ | ：12． 3 | ：1：． 6 | ：1：．$=2$ | 1：2．32 |
| E | 1：2．5 | ： 3.3 | ：13．61 | ：$\%$ ．$=4$ | 11：．73 | ：14． 31 | ：15．： 5 | ： 25.9 | 1：6．：2 | ： 0.64 |
| \％． | ：${ }^{-}$．${ }^{\text {S }}$ |  | ：-7.04 | ： \％．4：$^{\text {a }}$ | 1：3．36 | ：17．31 | 113.5 | ： $2 . .2)$ | 1ここ． 25 | ：：．${ }^{\text {a }}$ |
|  | ： 2.5 ： | ：．：．+ | ：2．．＊4 | ：2－a ${ }^{\text {a }}$ | 1：3．：5 | ：21．72 | 1：4．25 | ：29． | 1ここ，${ }^{\text {a }}$ | ：25．－3 |
| ＂－＊ | ：こr．$\rightarrow$ | ：こと．- ＋ |  | ：－－in | ：2－． | 129．： | ： 0 －${ }^{\text {as }}$ | ：29．32 | 12？ 3 | 1：－25 |
| $\therefore$ |  |  | －31．～ヵs | ： $2 .: 1$ ： | ：？． | ：33． 7 | 13i．： 5 | ：34．$:$ | ：34．： | ： 5 － 5 |

TABLE R21. - Eontinder

| $V_{C}$ | 0 |  | 2 |  | 4 | $5 \quad 1$ | 6 ) | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 600 | 135.45 | 135.93 | 136.41 | 136.89 | 137.37 | 137.86 | 138.34 | 138.83 | 139.31: | 139.80 |
| 610 | 140.29 | 140.77 | 141.26 ! | 141.75 | 142.24 | 142.74 | 143.23: | 143.73 | 144.22: | 144.72 |
| 620 | 145.22 | 145.72 | 146.22 । | 146.72 | 147.22 | 147.72 | 148.22 | 148.73 | 149.23 , | 149.74 |
| 630 | 150.25 | 150.76! | 151.26 | 251.78 | 152.29 | 152.80 | 153.31 | 153.83 | 154.34 | 154.86 |
| 640 | 155.38 | 155.90 | 156.42 | 156.94 | 157.46 | 157.98 | 158.51 | 159.03 | 159.56 | :60.06 |
| 650 | 160.61 | 161.14 | 161.67 | 162,20 | 162.73 | 163.27 | 163.80 | 164.34 | 164.87 | 165.41 |
| 660 | 165.95 | 166.49 | 167.03 | 167.57 | 168.11 | 168.66 | 169.20 | 169.75 | 170.29 , | 170.84 |
| 670 | 171.39 | 171.94 | 172.49 | 173.04 | 173.57 | 174.15 | 174.70 | 175.26 | 175.82 | 176.37 |
| 680 | 176.93 | 177.49 | 178.06 | 178.62 | 179.18 | 179.75 | 180.31 | 180.88 | 181.45 | 182.02 |
| 690 | 182.59 | 183.16 | 183.73 | 184.30 | 184.88 | 185.45 | 186.03 ; | 186.61 | 187.18 | 187.76 |
| 700 | 188.35 | 188.93 | 189.51 | 190.09 | 190.68 | 191.27 | 191.85 | 192.44 | 193.03 | 193.62 |
| 710 | 194.21 | 194.81 | 195.40 | 196.00 | 196.59 | 197.19 | 197.79 | 198.39 | 190.99 ! | 139.59 |
| 720 | 200.19 | 200.79 | 271.40 | 202.01 | 202.61 | 203.22 | 203.83 | 204.44 | 205.05 | 205.66 |
| 730 | 206.28 | 206.89 | 207.51 | 208.13 | 208.75 | 209.16 | 209.99 | 210.61 | 221.23 ; | 211.85 |
| 740 | 212.48 | 213.11 | 213.73 | 214.36 | 214. 79 | 215.62 | 216.25 | 216.89 | 217.52 ; | 218.16 |
| 750 | 218.79 | 219.43 | 270.07 | 220.71 | 221.35 | 221.99 | 222.64 | 223.29 | 223.93 | 234.57 |
| 760 | 225.22 | 225.87 | 226.52 | 227.17 | 227.82 | 228.48 | 229.13 | 229.79 | 230.45 | 431.10 |
| 770 | 231.76 | 232.48 | 233.09 | 233.75 | 234.42 | 235.09 | 235.75 | 236.42 | 237.08 | 237.75 |
| 780 | 238.43 | 239.10 | 239.77 | 240.45 | 241. 12 | 241.80 | 242.48 | 243.16 | 243.84 | 244.52 |
| 790 | 1245.21 | 245.69 | 246.58 ; | 247.26 | 247.95 | 248.64 | 249.33 | 250.02 | 250.72 | 251.41 |
| 800 | 252.10 | 252.80 | 253.50 | 254.20 | 254.90 | 255.60 | 256.30 | 257.01 | 257.71 | 258.42 |
| 810 | 259.12 | 259.83 | 260.54 | 261.25 | 261.97 | 262.68 | 263.40 | 264.11 | 264.83 | 265.55 |
| 820 | 266.27 | 266.99 | 267.71 | 268.43 | 269.16 | 269.89 | 270.61 | 271.34 | 272.07 | 272.80 |
| 830 | 273.53 | 274.27 | 275.00 | 275.74 | 276.48 | 277.22 | 277.96 | 278.73 | 279.44 | 280.18 |
| 840 | 280.93 | 281.67 | 282.42 | 283.17 | 283.92 | 284.67 | 265.92 | こロ* 17 | 296.93 | 287.69 |
| 850 | 288.45 | 289.21 | 289.97 | 290.73 | 291.49 | 292.26 | 293.02 | 293.79 | 29.4 .56 | 245.34 |
| 860 | 296.10 | 296.83 | 297.64 | 298.41 | 299.19 | 299.97 | 300.75 | 331.53 | 302.11 | 303.01 |
| 872 | 303.87 | 304.60 | 305.44 | 306.23 | 307.02 | 307.81 | 308.60 | 309.40 | 310.19 | 310.98 |
| 882 | 311.78 | 312.53 | 313.38 | 314.18 | 314.98 | 315.79 | $3: 6.59$ | 317.40 | 318.20 | 319.31 |
| 890 | 319.82 | 320.64 | 321.45 | 322.26 | 323.08 | 323.90 | 324.71 | 225.53 | 326.35 | 327.18 |
| 900 | 328.00 | 328.83 | 329.65 | 330.48 | 331.12 | 332.14 | 332.37 | 333.81 | 334.64 | 335.48 |
| 910 | 336.31 | 337.15 | 337.99 | 338.84 | 339.68 | 340.52 | 341.37 | 342.22 | 343.06 | 343.91 |
| 920 | 344.77 | 345.62 | 346.47 | 347.33 | 348.18 | 349.54 | 349.90 | 350.76 | 351.53 | 352.49 |
| 930 | 353.36 | 354.22 | 355.09 | 35;.96 | 356.83 | 357.70 | 358.50 | 359.45 | 360.33 | 361.21 |
| 940 | 362.09 | 362,97 | 363.85 | 364.73 | 365.62 | 366.51 | 367.39 | 368.78 | 369.18 | 370.07 |
| 950 | 370.96 | 371.86 | 372.76 | 373.65 | 374.55 | 375.45 | 376.36 | 377.26 | 378.17 | 379.07 |
| 960 | 379.98 | 380.69 | 381.80 | 382.72 | 383.63 | 384.55 | 385.45 | 386.38 | 387.30 | 388.22 |
| 970 | 389.15 | 390.07 | 391.00 | 391.93 | 392.85 | 393.79 | 394.72 | 355.65 | 396.59 | 397.52 |
| 980 | 398.46 | 399.40 | 400.34 | 401.29 | 402.23 | 403.18 | 404.12 | 405.07 | 406.02 | 406.97 |
| 990 | 407.93 | 408.88 | 409.84 | 410.80 | 411.45 | 412.72 | 413.68 | 414.64 | 415.6 : | 416.57 |
| 1000 | 417.54 | 418.51 | 419.48 | 420.46 | 421.43 | 422.41 | 423.39 | 424.37 | 425.35 | 426.33 |
| 1015 | 427.31 | 428.30 | 429.29 | 430.27 | 431.36 | $432.2 i$ | 473.25 | 434.24 | 435.24 | 436.24 |
| 1020 | 437.24 | 438.24 | 439.24 | 440.25 | 441.25 | 442.26 | 443.27 | 444.28 | 445.2? | 446.31 |
| 1030 | 447.32 | 448.34 | 449.36 | 450.73 | 451.40 | 452.42 | 453.45 | 454.48 | 455.51 | 456.54 |
| 1045 | 457.57 | 458.60 | 459.64 | 460.67 | 461.71 | 462.75 | 463.79 | 464.8 | 465.88 | 466.93 |
| 1050 | 467.97 | , 469.62 | 470.07 | 471.13 | 472.18 | 473.24 | 474.29 | 475.35 | 476.42 | 477.48 |
| 1060 | 478.54 | $479.61{ }^{\circ}$ | 480.68 | 481.75 | 482.82 | 483.39 | 484.96 | 486.04 | 487.12 | 468.2? |
| 1070 | 489.28 | 490.36 | 491.44 | 492.53 | 493.62 | 494.72 | 495.80 | 496.89 | 497.79 | 499.08 |
| 1080 | 500.18 | 501.28 | 502.35 | 503.49 | 504.59 | 505.68 | 506.80 | 507.91 | 509.03 | 510.14 |
| 1090 | 511.25 | 512.37 | 513.49 | 514.61 | 515.73 | 516.86 | 517.98 | 519.11 | 520.24 | 521.37 |
| 1100 | ; 522.50 | 523.64 | 524.77 | 525.91 | 527.05 | 528.19 | 529.33 | 530.48 | 531.62 | 532.77 |
| 1110 | - 533.92 | 535.7\% | 536.23 | 537.38 | 538.54 | 539.70 | 540.86 | ¢ 42.02 | 543.19 | 544.35 |
| 1120 | ; 545.52 | 5\%0.69 | 547.86 | 549.03 | 550.21 | 551.38 | 552.56 | 553.74 | 554.93 | 556.11 |
| 1130 | 557.30 | 558.48 | 559.67 | 560.86 | 562.06 | Se 3.25 | 564.45 | 564.65 | 556.65 | 568.05 |
| 21:3 | 5Ar. 25 | 570.: | 571.67 | 572.38 | 574.09 | 575.30 | 576.52 | 577.73 | 572.75 | 550.17 |
| 1150 | 582.19 | 5́a2.52 | 583.84 | 585. 27 | 536.30 | 587.53 | 588.77 | 590.36 | 591.24 | 592.48 |
| 1:30 | 593.72 | 594.76 | 596. 21 | 597.46 | 598.71 | 599.35 | 601.21 | 602.46 | 603.22 | 504.38 |
| 1179 | 1626.24 | 607.5.) | 608.76 | 610.03 | 611.30 | 612.57 | 513.84 | 515.11 | 616.39 | 517.67 |
| 1289 | 1618.95 | 1620.22 | 621.5: | 622.80 | 624. 28 | 6:5. 37 | 626.66 | 527.76 | 629.25 | 530.55 |
| 1130 | 631.85 | 1633.:5 | 634.43 | 635.75 | 637.06 | 638.37 | 639.68 | 647.79 | 642.31 | 543.62 |

## APPENDIX A

table azl．－Concluied

| $\begin{gathered} \because_{c}, \\ \mathrm{k} / \mathrm{hr} / \mathrm{hr} \end{gathered}$ |  | 1 | 2 | 3 | 4 | 5 |  | 7 | 8 | ； |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1200 | 644.94 | 645.26 | 647.59 | 648.91 | 650.24 | $6 \equiv 1.56$ |  | 654.23 | 655.56 | 656.90 |
| 1210 | 658.24 | 659.58 | 660.92 | 662.26 | 663.61 | 554.96 | 666．：1： | 667.66 | $669 . \mathrm{C2}$ | 673．37 |
| 1220 | 671.73 | 675.09 | 674.46 | 675.82 | 577.19 | 678.56 | 679．引3i | 681.30 | 682.68 | 684.05 |
| 1230 | 685.43 | 686.81 | 688.20 | 689.58 | 690.97 | 692.36 | 693.75 | 295.14 | 696.54 | 697.04 |
| 1240 | 699.33 | 700.74 | 702.14 | 703.54 | 734.75 | 736.36 | 707．－－ | 709.19 | 710.60 | 712.02 |
| 1250 | 713.44 | 714.86 | 716.28 | 717.71 | $7: 9.13$ | 720.56 | 721． | 723.43 | 724.80 | 726.30 |
| 1260 | 727.73 | 729.18 | 730.62 | 732.07 | 733.51 | 734． 70 | 736.71 | 737.86 | 739.32 | 740.77 |
| 1270 | 742.21 | 743.69 | 745.15 | 746.67 | 743.08 | 749.55 | 75180 | i34．4 |  | 755.44 |
| 1280 | 756.92 | 758.40 | 759.88 | 761.36 | 767 | 154.33 | 765．Ez： | 767.31 | 768.80 | 7：0．30 |
| 1290 | 771.79 | 773.29 | 774.79 | 77－． 4 y | 777.79 | 779.30 | 780．E！ | 782.32 | 723.83 | 785.34 |
| 1300 | 786.85 | 788.37 | 789．89 | 791.41 | 792.93 | 734.45 | 795． | 797.51 | 799.04 | 800.57 |
| 1310 | 802.10 | $80^{3} .03$ | 805.17 | 806.71 | 808.25 | 9\％？． 79 | 811． $3^{\prime}$ | 312.88 | 814.43 | 815.98 |
| 1320 | 817.53 | 519.08 | 820.63 | 822.19 | 823.75 | 825.31 | 826．E．1 | 828.43 | 830.00 | 831.57 |
| 1330 | 833．1； | 834.71 | 836.28 | 837.85 | 839.43 | 351.01 | 942．E3 | 844.17 | 845.75 | 847.33 |
| 1340 | 848.92 | 850.51 | 852.10 | 853.69 | 855.28 | 856.88 | 658．431 | 860.07 | 861.68 | 863.28 |
| 1350 | 204．88 | 866.49 | 868.09 | 869.70 | 811．31 | 872.93 | 874．5s | 876.16 | 877.78 | 879.40 |
| 1360 | 981.12 | 882.64 | 884.26 | 885.89 | 887.52 | 889.15 | 890． 5 | 892.42 | 894.05 | 895.69 |
| 1370 | 397.33 | 658.97 | 900.61 | 902.25 | 903．¢ 0 | 925.54 | 907． | 908.84 | 910.53 | 912.15 |
| 1383 | 913.80 | 915.46 | 917.12 | 918.78 | 920．75 | 922.11 | 923 | Y25．44 | 927.11 | 929.78 |
| 1390 | 930.45 | 932.13 | 933.80 | 935.48 | 937．26 | 950.84 | 90ふ． | 942.21 | 943.89 | 945.58 |
| 1400 | 947.27 | 948.96 | 950.66 | 952.35 | 954.25 | 955.74 | 957．${ }^{\text {a }}$ | 959.14 | 960.85 | 962.55 |
| 1410 | 364.26 | 965.96 | 967.67 | 969.38 | 971．ここ | 9：2．81 | 974．E： | 976.24 | 977.96 | 979.68 |
| 14.3 | 381.41 | 983.13 | 984.86 | 986.58 | 983． 5 | $9 シ$－ 04 | 991． | 99.51 | 995.24 | 936.98 |
| 1430 | 398.72 | 1000.46 | 1002.20 | 1003．94 | 1065．67 | 1067.42 | 1009．： | 1010.94 | 1312.67 | 1014．44 |
| 1440 | 1016.20 | 1017.95 | 1019.71 | 1021.47 | 1023． 23 | 1025.05 | 1026． iE i | 1028.53 | 1030．29 | 1032.06 |
| 1450 | 1333.83 | 1035．61 | 1037.38 | 1039.16 | 1040.93 | 1042.71 | 1044.43 | 1046.28 | 1048.06 | 1049.85 |
| 1460 | 1051.63 | 105？．42 | 1055.21 | 1057.00 | 1059.80 | 1060．59 | 1062． 3 | 1064.19 | 1065．99 | 1057.79 |
| 1470 | 1069.59 | 1071.40 | ： 973.20 | 1075.01 | 1076.82 | 1c73．63 | 1080．4i | 1082.26 | 1084.07 | 1085.89 |
| 1480 | 1387.71 | 1089.53 | 1091.35 | 1093.17 | 1095.00 | 1155.83 | 1098．6 | 1100.48 | 1102.32 | 1104.15 |
| 1490 | 1105.98 | 1107.82 | 1109.66 | 1111.50 | 1113.34 | $11: 5.18$ | 1117．ご： | 1118.87 | 1120.72 | 1122.57 |
| 1500 | 1124.42 | 1126.27 | 1128.12 | 1129.98 | 11 ＇2． 83 | 1133.69 | 1．35．5ミ | 37.41 |  |  |
| 1510 | 1143.00 | 11：4．87 | 1146.74 | 1148.61 | 1150.46 | 1152.35 | 1154.25 | 1256.11 | 1157.93 | 1159.38 |
| 1520 | 1161.75 | i163．63 | 1165 | ：57．40 | 1169.29 | 1171.18 | 1173．： | 1i74．9 | 1176.8 | 1178.75 |
| 1530 | 1180.64 | 1182.54 | 1184.44 | 1186．34 | 1183．2： | $11=5.15$ | 1192．こ三 | 1193.96 | 1195.87 | 197.78 |
| 1540 | 1199.69 | 1201.61 | 1203.52 | 1205.44 | 120？． 5 | 125．28 | 1211．2 | 1213.12 | 1215.0 | 2：5．97 |
| 1550 | 1218.90 | 1220.33 | 1222.76 | 1224．69 | 1225．$=2$ | 12こう． 56 | 1230．i： | 1232.73 | 1234.37 | 1236．31 |
| 1500 | 1238.25 | 1240．20 | 1242．14 | 1244.09 | 1246．ご | 124－． 99 |  | 1251.89 | 1253.35 | 1255.30 |
| 1570 | 1257.76 | 1259.72 | 1261．63 | 1263.64 | 1265． 5 | 1267． 57 | 1269．ミ： | ：271．50 | 12：3．土 | 1275．44 |
| 1580 | 1277.42 | 1279.39 | 1281.37 | 1283.34 | 1285． $3=$ | 12E7．30 | 1289．こ＝ | ：291．27 | 1293.25 | 24 |
| 1570 | 1：237．22 | 1299.21 | 1301.20 | $11303.20$ | $13 C^{=} .12$ | $135.13$ | 1300．： | $1311.13$ | 1213.15 |  |
| 1600 | 11317.18 | 1310.10 | 122：．iy | $1323.20 \mid$ | 1325．21 | $13: 5$ | 1329．2三 | 1331．24 | 1333.25 | 1335.27 |
| 1610 | 11337．29 | 1339．31 | 1341.33 | 134335 | 1345．3－ | 135\％．70 | 1349．4： | 1351.45 | 1353．48 | 1355.51 |
| 1620 | 1357.54 | 1359.57 | 1361.61 | 1363.65 | 1365．引う | $136-.72$ | 136\％．：－ | 1371.81 | 1373.85 | 137 |
| 1630 | 13：7．94 | 1379.99 | 1382.04 | 1384.09 | 1386．1ミ， | 13 ¢́そ．こ0 | 11390．$=$ | ：372． 30 | 133.4 .37 | 1375.73 |
| 1640 | 1398.49 | $1+00.56$ | 1402.62 | 1：04．C3 | 1406．ミ三 | 14： 3.32 | 1412．E | ： 712.77 | 1415．04 | 1417．12 |
| 1650 | $1: 19.19$ | 1421.27 | 1423.35 | 1425．43 | 142－ミ： | 14： 3.57 ！ | 11431．$=$ | －i33．76 | 1435．85 | 14こ7．3i |
| 1660 | 1740.03 | 1442.12 | 1444.22 | 11446．31 | 114．4．： | 14E：．51 | 1452．6： | ：$: 54.71$ | $1+56.31$ | 1456． 32 |
| 12570 | 1351.32 | 1463.13 | 1465.23 | 1467.35 | 146ミ． if $^{\text {¢ }}$ | 147：． 57 | 1473．F三 | －75． 30 | 1477．92 | 14Eつ． 3 |
| 1680 | 1：22．：5 | 1．84．23 | 1486.401 | 11488.52 | 1422．シミ | 149：．75 | 1734．3： | ：37． 34 | 1439．17 | 15：1．37 |
| 1590 | 1533．72 | 1505.37 | 1507.71 | ｜1509．65 | 15：2．＊こ | 1512．13 | 15 | 18．72！ | 1529．5E |  |
| 1700 | 1524．86 |  |  |  |  |  |  |  |  |  |



［ourivod from sex．A2］

| $\begin{gathered} \text { ve. } \\ \operatorname{kgh} \end{gathered}$ | $\bigcirc$ | ： | こ | 3 | 4 | 5 |  | $?$ | － | － |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 3.35 | 2.13 | 0．43． | 2.76 | ：． 14 i | 1．7． | ．． 34 | 亏． | －－ |
| 10 | 4.73 | 5.72 i | 5.81 | ア．941 | 1． 26 | 1．．631， | ：．： | ：$\overline{5}$ ． 6 | ：$:$ ： | $\cdots \cdot$ |
| 20 | 18.911 | 20.841 | －2．88 | ＜5．701 | 27.23 | 2＇s． 54. | ：1．ts | 34．46． | ： 2 | $\bullet \cdot{ }^{\prime}$ ． |
| 10 | 42.54 | 45.431 | ＊＊．4つ． | 5i．48i | 54．64； | 57.91 | 51．26！ | 6．4．7． | －i．． 4 |  |
| 43 | 75.64 | 79．42！ | 33． 39 | 73．42． | 42.53 | 354． 74 | 100．041 | 1：4．44 | －\％．${ }^{\text {！}}$ | ：$:$ ：${ }^{\text {f．}}$ ． |
| 50 | 118．20 | ：22．38： | $: 2.85$ | 132.32 | 137.98 | 143． 4 | ：30． | －5． | $\because \cdot$－ | ＊．． |
| 63 | 170． 44 | ：75．97i | ：71．73． | 137．70； | 1＊3．72 | 109．521 | －35． 2 | ：$\times$ ． 1 | －- | ．－${ }^{\text {c．，}}$ |
| 70 | 431， 37 | 238.44 ， | 44.21 | 25：．08． | 259.4 | 24.5 .94 | ＜3．2\％ | －－．．3） | －${ }^{-}$－ | ， |
| 80 | 352．79 | 329.42 | ： 3.14 | 325．75 | 33：．sh | 361．671 | 4\％．97 | ヶヵカ．： |  | － 3 ． |
| $\rightarrow 0$ i | 383.33 | 3＋1．73 | \＄30．5a： | 497．35 | 416.21 | －6．7．77 | 4 36.4 | if：．${ }^{\text {？}}$ | ：4．．． | －$\because$ ： 4 |
| 100 | 473.40 | 482.93. | －72．5s | 532．25 | 512.33 | \＄22．31 | £．2．．． | 8．4．．． | －． |  |
| 110 | 571.01 | 583.50 | 574.281 | 6－8．76 | 515．5］ | 4，25．$\because$ ： | 637． 37 | rie．t： | $\pm$ ¢ ．． | － 3.4 |
| $1: 0$ ！ | 682.191 | $593.64{ }^{\prime}$ | 725.17 | 715．51 | 729．5s | 74：．37 | －6．． 3 ． | －6．6． | －－4． | －－． |
| 130 | 800.36 | 313.36. | \＃55．87 | 236．t6 | 351.26. | ¢4． 3.35 | －－6．4？ | －4．9．－． | － | $\because \cdot$ |
| 240 | 929．341 | 242.71 ． | F56．17 | ＇46：+73 | 743．17 | 797．14 | 1 11．， | 1 44．9 | ：：$\cdot$ |  |
| 15J | ： 267.4 | $\therefore$ ： 81.7 | ：36．1 | $1112 \%$ | ： 125.3 | － 34.9 | $1: 54 .=$ | ：．． | ：－－ |  |
| 10\％ | 1215.21 | －32． 3 | －－45． | 1 くbi． | ：$=76.6$ | ： 29.5 | ！：$¢$ ？ | ： 2.4. | ： |  |
| ： 30 | 1372.4 | －： 48.7 | ： $5: 5 . \mathrm{S}$ | 14．2 |  | －： 4.4 | ：$\because:=$ | ：：－－ |  |  |
| 160 | 1 537． 5 | － 555.3 | －574． | 15 51－5 | －\％99 | ：© At．${ }^{\text {a }}$ | ： 973 | ．．．．． | ．．． | ．．－ |
| ： 30 | 1710.4 | － 734.6 | － 5 こ．${ }^{\text {a }}$ | 1 ：7．．4 | ： 3.3 \％ | ：＝0．．s | －＝．2．． | ：-4 ． | $\cdots:$ | －－： |
| $200!$ | 1303.1 | $: 222.3$ | ： 941.5 | 1 \％${ }^{2}$ ． 5 | ： 9 in .5 |  | － 19.5 | －．：$\cdot:$ | ！${ }^{\text {c }}$ | － |
| 213 | 2 299．5 | － 123.8 | － 243.3 | 2165.4 | $\therefore 183.7$ | 2.01 .5 | 20.2 | －．7：． | ．${ }^{*}$ | ．－8 |
| 220 | 2305.9 | $=327.1$ | $2 \pm \pm 8.3$ | 2357.3 | 2371.2 | －＋1．．$=$ | $\pm 4: 4.5$ | －$\ddagger$ | ：+ |  |
| $=30$ | 2522.2 | $=344.4$ | 2506.7 | 2 Ses： | 2611.5 | 2 $0: 5.1$ |  | $\cdots$ ． | －．- | －； |
| 240 | 7748.4 | $=171.6$ | $2-34.3$ | 2 Ait． | 2841.7 | － 765.3 | 2 $=24$. | －$\because$ ． | ．．．． | － |
| 250 | 2984.7 | ？ 208.7 | ； 23.1 | 3 35－．5 | ： 26.2 ． | 3：－4．t． | ：： 11. | ：：${ }^{\text {c }}$ | － |  |
| 260 | 3231.0 | $\div 256.2$ | $?=32.4$ | $33 \cdot 4.7$ | 3 332． 3 | ．； | ！： 42.8 | ：$: 7$ | $\pm$ | $\therefore$ |
| 270 | 3437.4 | ； 513.6 | ： 539.3 | ） 565.2 |  |  | ！－Et． | ：• - | ．．．－ |  |
| 230 | 3 751．7 | 三 76．．： | 2 \％こヶ． 4 | 3－ミ．う | ＝ 2 －3．4 |  | $\therefore$－${ }^{\text {a }}$ | － | $\cdots:$ |  |
| 230 | ＊ 230.6 | － 5 á． | － 537.2 | $41: 5.6$ | ＋145 |  |  | －．． |  | －－ |
| 303 | 4317.6 | － 145.3 | 4 －-6.5 | 4 ：St． | － $4.5 .=$ | $\div$ | － 50.4. | ＊－ \％$^{-}$ |  | $\because$ |
| 310 | 4624.9 | ＋ 645.2 | 4.75 .5 | 4 ： $4 .$. | ＋$=3.4$ | － $\mathrm{id}^{-5}$ | －－AT． | －－－． | －－ | ；－$\cdot$ |
| 320 | － 322.6 | －$\$ 54.5$ |  | 5 小兄． | －Sin． | \％ $4 .$. \％ | ¢ ： | \％．$\%$ | － | － |
| 335 | 524.7 | ¢－ 3.1 | 5 シくらtt | ¢ 3 ：n．． | －3？．．． | \％tis． | － ：$_{6}{ }^{-}$ | ＝ 5 ＋．${ }^{-}$ | －．．－ | $\cdot$ |
| 370 ． | 5569.4 | \＃552．8 | \％ 535.4 | S 69. | ¥ ：\％． | $\because$ ご | \％－： | $\because$ ： | $\cdots$ ． | －： |
| 350 ， | 5908.6 | 5243.1 | 5 ¢77． | c 32．． | ＊－${ }^{\text {－}}$ | $\therefore$ | \％． | ．． E ．： | －． |  |
| 360 | 5259.4 | $\div 293.3$ | 4 ： 23.6 | 6 \％ 5 － | － 5.1 .8 | $4:$ | i＂： | －．． | $\because$ | $\stackrel{\text {－}}{ }$ |
| $37 \%$ | 5 5it 3 | － 655.5 | 5－9く， | －－．．． | －－6．． | － | －－3．i | －．－． | －$\because$ ． | ．．． |
| 35： | $599 \%$ こ | ：27． | － 0.5. | －： $\mathrm{E}^{-}$ | －：：．－ | ， | －．－．． | － | －．． |  |
| 3\％ | 7372.4 | i：${ }^{\text {a }}$ ： | － $45 . .$. | i－．．： | $\therefore 7$ | $\cdots$ | $\cdots \cdots$ | ． 7 ： | ＂${ }^{\text {－}}$ |  |
| せう | ＞-65.4 | － 3.5 .1 | －－i¢．t | －－－－ | －－2． | －＇0．＊ | － | －： | －－ | － |
| 4： | － 553.6 | －ここ： 6 | 7．こう， | －－： | －－． | ：${ }^{-\cdots}$ | －$\because$ ： | ： | － | ： |
| ちこ | 3 5̇4．t | Ecic． | $\rightarrow-6.2$ | －$\because:$ | $\rightarrow$－- － | \％ | －－： | －－－． | $\because$ | －．． |
| 43. | \％－ 11. | ＊： 5 ¢ 4 | ＊．t＊ | ，：i．．． | －-7.7 | ．－－． | －．－． | －． | ＊． | ．． |
| －\％ | ； $3 \pm \frac{\text {－}}{}$ | ，4E3．${ }^{\text {d }}$ | －：${ }^{-}$ | ：－ | $\cdots$ | ．．${ }^{\text {－}}$ ． |  | － | ．．． | －． |
| $\pm$－ | ，－9， | － 5 ：．： | －4＊ | 4 |  | ． |  | ．－ | ， |  |
| －： | $\therefore$ ： 5 ¢ | －$\quad$ ： | i： | －－ | 4 | $\bullet$ | － | ．－－ | ： |  |
| $i^{-}$ | ミ． | ：－－－ | $\cdots$ | $\rightarrow$ ： |  | ：－ |  |  |  | － |
| i $=$ ？ |  | $\therefore$ ： | $\therefore i:$ ： | i－： | － | ：• | ．$\cdot$ | － | － |  |
| $4 \rightarrow$ ？ | $\therefore$－ 3 | $\because-5$ ， | ：－ | － | ： | ．－ | ．．． | $\cdots$ | － |  |
| $\therefore \because$ | ：．：$: 5$ | ．－：＊ | －．－ | － | －． | －． | ．$\cdot$ | ．${ }^{-}$ |  |  |
| ：： | ：－ 3 | $\therefore$－${ }^{-}$ | $\cdots$ | $\cdots$ | － 5 | ＊＊ |  | ． |  |  |
| シ－ | ：－－5 | ．．：$:$ ， | $\dagger^{-}$ | ．${ }^{\text {－}}$ |  | － | ． | ：－ |  |  |
| ： | ： 3 －．＂ | ．+ ： | ．$\%$ ： | ；$\quad$ i | －．． | － | ： | －＊ | － |  |
| \＃ 4 ． | ：4 iri | －$\because$ ． | － | － | －$\cdot-$ | $\because$ | $\div$ | ： | － |  |
| ： | $\because \quad \because$ | $\therefore \quad-9$ | － \％$^{-}$ | ： | $\cdots$ |  | － | － |  |  |
| $\because$ | ．$\cdot$ ． | $\therefore$ |  | － | －${ }^{-}$ | － | $\cdots$ |  | $\because$ |  |
|  | －－． | －．＊＊ | $\cdots$ | －－ | － 3 | － | －$\cdot$ |  | ． | － |
| － | $\because-:$ | $\because-*$ | －－$\cdot$ | $\cdots$ |  |  | － |  |  |  |

APPENDIX A
table ar2.- Corizanued


TABLE in22.- Cuncluded



[Derined froen ref. A $\mathbf{E}^{\text {! }}$


## APPENDIX A

TABLE in3．－Concluded

| $\begin{gathered} \text { Ve, }^{\prime}! \\ \text { knots } \end{gathered}$ |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ส | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 500 | 349.90 | 351.50 | 353.10 | 354.70 | 356.32 | 357.93 | 359.55 | 361.18 ！ | 362．41 | 364.74 |
| 510 | 366.08 | 367.73 | 369.38 | 371.03 | 372.69 | 374.35 | 376.02 | 377.70 ！ | 379.381 | 381.76 |
| 520 | 382.75 | 384.45 | 386.15 | 367.85 | 389．56！ | 391．28 | 393.00 | 394．72： | 396.45 | 3．98．： 9 |
| 530 | 399.93 | 401.67 | 403.42 | 405．18 | 406.94 | 408.71 | 410.19 | 412.26 | －14．04 | i15．ri3 |
| 540 | 417.62 | 419．42 ${ }^{\text {j }}$ | 421.22 | ＋23．03 | 424.84 | 426.66 | 428.49 | 430.321 | －32． 15 | i33．9， |
| 550 | 435.84 | 437.69 ： | 439.55 | 441.41 | 443.28 | 445.15 | 447.03 | 448.91 | －50．80 | $\div 52$ |
| 560 | 454.60 | 456.51 | 458.42 | 460.34 | $4<2.26$ | 464.19 | 466.12 | 468.00 | \＄70．91 | ：71． 6 |
| 570 | 473.91 | 475.88 | 477.84 | 479.82 | 481.80 | 483.78 | 485.77 | 447.77 | \＄89．77 | 91.75 |
| 580 | 493.79 | 495.81 | 497.54 | 499.87 | 501.90 ！ | 503.95 | 506.00 | 508.25 | 510.11 | 512．：\％ |
| 590 | 514.25 | 516.33 | 518.41 | 520.50 | 522.60 | 524.73 | 526.81 | 528.92 | 531.34 | 533．：7 |
| 600 | 535.30 | 537.44 |  | 54.14 | 543.89 | 545，．06 | 548.22 | 550．40｜ | 552．56 | 534．77 |
| 610 | 556.96 | 559.16 | 561.37 | 563.58 | 565.80 | 563.02 | 570.25 | 572.491 | 574．741 | 576.39 |
| 620 | 579.24 | 581．511 | 583.78 | 586.05 | 588.33 | 590.62 | 592．92： | 535.221 | 597．53 | $599 .-4$ |
| 630 | 602.16 | 604．49． | 606.32 | 6．J9． 1 ml | 611.51 | 613.801 | 616．22！ | 618.54 | E29． 5 \％ | －2， 24 |
| 640 | 625.73 | 628．12 | 630.53 | 632.93 | 635.35 | 637.77 | 640.19 | 642.631 | －45． 77 | 647.52 |
| 650 | 649.97 | 652.43 | 654.90 | 657.37 | 654.86 | 2．．3．4 | 654.84 | 667．34 | $\therefore 69.85$ | 72． 37 |
| 660 | 674.90 | 677.42 | 679.97 | 582.51 | 685.06 | 687.62 | 690.18 | 692.75 | － 95.33 | －97．．2 |
| 670 | 709.51 | 733．11！ | 705．：2 | 75.33 | 710.95 | 713.58 | 716.21 | 718．05 | 7－1．5 | 2．4．9 |
| 680 | 726.82 | 729．48！ | 732.16 | 754.84 | 737.53 | 740.22 | 742.92 ！ | 7．45．61i | 74.4 | －51． |
| 690 | 753.79 | 755.52 | 759.26 | 762．01 | 764.76 | 767.52 | 77\％．28： | 773．9 | －ir．d3 | －7\％．．． |
| 700 | 781 | ， 2.21 | 787 | 739．82 | 192.64 | 45 | 794．29 | 411． 2.21 | 13．47 | 716.01 |
| 710 | 809.67 | 812.53 | 815.39 | 813.26 | 321.1 | 82：．03 | 826．92： | －${ }^{\text {a }}$ | －32．72 | 35 |
| 720 | 838.54 | 841.47 | 844． 39 | 847.33 | 859． $2 \cdot 7$ | 953．21 | 856.16 | 859．1－ | －6\％． 2 A | 165． 35 |
| 730 | 862．13 | 971.01 | 874.00 | 876.99 | 879.99 | 883.00 | 886.01 | 889 |  | 635．d |
| 740 | 898.11 | 901.15 | 904.20 | 907.25 | 910.31 | 37 | 916.44 | ＊19．52 | 92－6．0） | ＋25． $2+$ |
| 750 | 928.78 | 931．88！ | 934.99 | 938.10 | 941.21 | 944.33 | 947.16 | 950.53 | －53．73 | 456 |
| 760 | 960.03 | 93.18 ！ | 966.35 | 969.51 | 972.69 | 975.86 | 979.05 | 982.24 | 385.431 | 4 |
| 770 | 991.84 | $975.06!$ | 998.27 | 1501.50 | 1004.72 | 1007.96 | 1011．23 | 01： | ；17．7．＇1 | 139.6 |
| 780 | 1024．22＇． | 1027.49 | 1030.76 | 1034.24 | 1037.32 | 104．3．61 | 1043.91 | 104 | 50.52 | 153．33 |
| 90 | 1.5 | 1二61． 47 | 1063.30 ！ | 1.67 .13 | 1070.47 | 1073.82 | 107\％．17 | $1 う 8$ | 83．34 | 185．．5 |
| 400 | 1090 | 10 |  |  |  | 2． 50 |  |  |  | ：－ |
| 813 | 1124.64 | 1128.07 | 1131.50 | 1134 | 11134.3 | 11：1．85 | 1145. | 1140 | ：52．．： | 1155－${ }^{-1}$ |
| 320 | 1159.19 |  |  |  |  | 1120， 66 | 118：． | 11A3． | ： $7 . .1$ ！ | ［．2．－3 |
| 23） | 1194.27 | 1137.81 | 1201 | 12－） |  | 122．． | 1215．57 |  |  |  |
| 245 | 1229.87 | 1213.46 | 2 37 | こ24．25； | 124 | 12：7．${ }^{\text {a }}$ | 51．74 | 1．55．： | －5a． | ： |
| 55. | $1266 . J 0$ | ！－¢ч．6¢ | 1－35．－d |  | $1-8$ | 1－24．．＇s | 128：．．． |  |  |  |
| 55 | 13 J .04 | ！：¢ ． 3 | 131．．：3 | 1－1 | 1317.4 | 13.1 .15 | 112：57 | ：3＿－ |  |  |
| 37 | ：339．79 | 1：－3．53 | 134\％．－＊ |  | 1354. | 12 | 130．3： | －300 |  |  |
| 732 | 1377．45 | 1：31．24 | $13 \times 5.5$ | $1 \geq 5 ด ้-51$ | 113？2．6， | $13,6, .47$ | 14：3．29 | 1：3． | ：$\because 7$ | ＋11．${ }^{-7}$ |
| 3＇3： | i＋15．62 | ifit．is | 1：23．i1 | 1：3．：－ | 1＋31 13 |  | 13：3．7\％ | － |  |  |
| 1）： | 1454． 2 ： | こ：3ッ．1． | 19．－．${ }^{\text {a }}$ | － | 140）． | 14.0 |  |  |  |  |
| 12： | ：433．463 | ： 5 －${ }^{\text {¢ }}$ | $15:$. |  | ： 2 ， | 15： | 5： |  |  |  |
| 92 | ：5is． | ： 57 | 15：${ }^{\text {a }}$ ：11 | ： 5 | $\rightarrow$ | E：：． 1 ¢ | － | －．：． |  |  |
| 93： | ：57，．．is | ： 07.32 | ：5－． | ：$=-5.81$ | ：－4．it | ： | － |  |  |  |
| 14： | －0：3．t | i－1．4． | ！ここ．： | － | 1．3．．． 3 | 1．：i．is | 1，：－． |  |  |  |
| 35. | ：以5．7 | 1－59．： | i．t． 3 ． | － 17.51 | ： $1 \rightarrow$－${ }^{-}$ | ！．${ }^{\text {－}} \rightarrow$－ |  |  |  |  |
| －： | 1600． 7 | $1^{-}$ | 17 | ＇3．－${ }^{\prime}$ | 1．1 |  |  |  |  |  |
| 7 | ： 2 ？n．38 | ：－： 3.5 | 1：9． |  |  |  |  |  |  |  |
| $\cdots:$ | ：3n2．il | $1-5$. | 1 |  |  |  |  |  |  |  |
| － | －3＊－4． | ： | 1－： |  | ．－：．．－． |  |  |  |  |  |
| ：： | ： 6.5 |  |  |  |  |  |  |  |  |  |

## APPENDIX A

 CALIARATED AIKSPEED $V_{C}$ (OR INUICATED AIRSPEE $\gamma_{1}$ ) IN K:\&
[Darined irom ref. A 2 ]



APPENDIX A
table a $25 .-$ TRUE alrspeed $V$ in knots for values of cailbrated AIRSPEED $V_{c}$ in kNOTS and values of pressure altimude h

IN GEOPOTENTIAL METERS
[computation of $v$ based on standard temperature at each altitude]


## APPENDIX A

table az6．－ratio of impact pressure to static pressure qe／p（or que／pi）for
VALUES OF mach number $m$（CR indicated mach mumer m＇）

## ［From ref．A3］

ORIGNAL PAGE： OF POOR QLUALITY

| M | 0 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 | 0.009 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.100 | 0.00702 | 0.00716 | 0.00730 | 0.00745 | 0.00759 | 0.00774 | 0.00789 | 0.00804 | 0.00819 | 0.0 cü 34 |
| ． 110 | ． 00850 | ． 00865 | ． 00881 | ． 00897 | ． 00913 | ． 00929 | ． 00945 | ． 00962 | ． 00987 | ． 00933 |
| ． 120 | ． 01012 | ． 01029 | ． 01046 | ． 01063 | ． 01080 | ． 01098 | ． 01116 | ． 01134 | ． 01152 | ． 01170 |
| .130 | ． 01188 | ． 01206 | ． 01225 | ． 01244 | ． 01263 | ． 01282 | ． 01301 | ． 01320 | ． 01339 | ． 01359 |
| ． 140 | ． 01379 | ． 01399 | ． 01419 | ． 01439 | ． 01459 | ． 01480 | ． 01500 | ． 01521 | ． 01542 | ． 01563 |
| ． 150 | ． 01584 | ． 01605 | ． 01627 | ． 01648 | ． 01670 | ． 01692 | ． $01 \% 14$ | ． 01736 | ． 01758 | ． 01731 |
| ． 160 | ． 01804 | ． 01826 | ． 01844 | ． 01872 | ． 01895 | ． 01919 | ． 61942 | ． 01966 | ． 01990 | ． 02014 |
| ． 170 | ． 02038 | ． 02062 | ． 02086 | ． 02111 | ． 02135 | ． 02160 | ． 02185 | ． 02210 | ． 02236 | ． 02261 |
| ． 180 | ． 02286 | ． 02312 | ． 02338 | ． 02364 | ． 02390 | ． 02416 | ． 02443 | ． 02469 | ． 02496 | ． 02523 |
| ． 190 | ． 02550 | ． 02577 | ． 02604 | ． 02632 | ． 02659 | ． 02687 | ． 02715 | ． 02743 | .32771 | ． 22800 |
| ． 200 | ． 02828 | ． 02857 | ． 02886 | ． 02914 | ． 02944 | ． 02973 | ． 03002 | ． 03032 | ． 23061 | ． 03091 |
| ． 210 | ． 03121 | ． 03151 | ． 03182 | ． 03212 | ． 03243 | ． 03273 | ． 03304 | ． 03335 | ． 03366 | ． 03339 |
| ． 220 | ． 0.429 | ． 03461 | ． 03493 | ． 03525 | ． 03557 | ． 03589 | ． 03621 | ． 03654 | ． 33686 | ． 0378 |
| ． 230 | ． 03752 | ． 03785 | ． 03819 | ． 03852 | ． 03886 | ． 03919 | ． 03953 | ． 03987 | ． 04022 | ． 34055 |
| ． 240 | ． 24090 | ． 04125 | ． 04160 | ． 04195 | ． 04230 | ． 04265 | ． 04301 | ． 04336 | ． 34372 | ． 344 |
| ． 250 | ． 04444 | ． 04480 | ． 04516 | ． 04553 | ． 04589 | ． 04626 | ． 04663 | ． 04700 | ． 24738 | ． 0475 |
| ． 260 | ． 04813 | ． 04850 | ． 04888 | ． 04926 | ． 04964 | ． 05003 | ． 05041 | ． 05080 | ． 25119 | ． 25153 |
| ． 270 | ． 05197 | ． 05236 | ． 05275 | ． 05315 | ． 05355 | ． 05395 | ． 05435 | ． 05475 | ． 35515 | ． 05556 |
| ． 280 | ． 05596 | ． 05637 | ． 05678 | ． 05719 | ． 05761 | ． 05802 | ． 05844 | ． 55886 | ． 35927 | ．$n$ ¢ |
| ． 290 | ． 06012 | ． 06054 | .06097 | ． 06140 | ． 26182 | ． 106225 | ． 06569 | ． 35312 | ． 36356 | ． 76.30 |
| .300 | ． 06443 | ． 06487 | ． 06531 | ． 06575 | ． 06620 | ． 06665 | ． 06709 | ． 06754 | 36799 | ． 3 ¢395 |
| ． 310 | ． 06889 | ． 06936 | ． 06982 | ． 07027 | ． 07074 | ． 07120 | ． 07166 | ． 07213 | ． 37759 | ． 073.36 |
| ． 320 | ． 07353 | ． 07401 | ． 07448 | ． 07496 | ． 07543 | ． 07591 | ． 07639 | ． 07687 | ． 37736 | ． 07784 |
| ． 330 | ． 07833 | ． 07882 | ． 07931 | ． 07980 | ． 08029 | ． 08079 | ． 08128 | ． 08178 | ． 08228 | ． 08278 |
| ． 340 | ． 08329 | ． 08379 | ． 08430 | ． 08481 | ． 08531 | ． 08583 | ． 08634 | ．OBGES | ． 38737 | ． 23799 |
| ． 350 | ． 08841 | ． 08893 | ． 08945 | ． 08998 | ． 09050 | ． 09103 | ． 09156 | ． 09209 | ． 392 h | ． 39316 |
| ． 360 | ． 09370 | ．09424 | ． 09478 | ． 09532 | ． 09586 | ． 09641 | ． 09695 | ．09750 | ． 29875 | ．：2067 |
| ． 370 | ． 09916 | ． 09971 | ． 20027 | ． 10083 | ． 10139 | ． 10195 | ． 10251 | ． 13308 | ．17364i | ．1：721 |
| ． 380 | ． 10478 | ． 10535 | .10593 | ． 19650 | ． 10708 | ． 10766 | ． 10821 | ． 1.1585 | ．1901 | ．1 ．．． |
| ． 390 | ． 11058 | ．11117 | ． 11176 | ． 11235 | ． 11295 | ． 11354 | ． 11414 | ． 11474 | ． 11534 | ． 11535 |
| ． 400 | ． 11655 | ． 11716 | ． 21777 | ． 11438 | ． 11899 | ． 11960 | ． 12922 | ． $1=084$ | ．12146 | ．：．－ |
| ． 410 | ． 12270 | ．12332 | ． 12395 | ．1245a | ． 12521 | ． 12584 | ． 126.7 | ． 12711 | －1：ワ－ | ．1．－3\％ |
| ． 420 | ． 12902 | ． 12366 | ． 13031 | ． 13095 | ．131ヶO | ． 13225 | ．13294 | ． 13355 | ．13421 | $1:$ |
| ． 430 | ． 13552 | ． 13518 | ． 13 mas | ． 13751 | ． 1381 ה | ． 13894 | ．13751 | ．14013 | ．1．380． | $\cdots$ |
| ． 4.40 | ．14221 | ．1：289 | ． $1+357$ | ． 14425 | ．14433 | ． 14562 | ．1453 | －：¢nが | ．1：\％o | ：- |
| ． 453 | ． 14907 | ．1：975 | ． 3547 | ． 5117 | ． 15147 | ．15：＇ | ． 15329 | ． 15309 | ．154． | 1：3！ |
| ． 463 | ． 150 is： | ． 15534 | ． 15755 | ． 150.27 | ．156．${ }^{\text {P }}$ | ． 15.2 | ． 1 nind4 | ：n117 | ．1－1） | ： |
| .470 | ． 163 3 ${ }^{\text {an }}$ | ．Induy | ． u ¢́a 3 | ． 1.0557 | ． 16632 | ． 167 | ．14779 | ． 20 ¢5： | ． $\ln 925$ |  |
| ．4＊2 | ．575？ | ．1．154 | ．172： | ． 17305 | ．1－3ril | ．17： 5 | ．17533 | ．17， | ．${ }^{-}$－－ | ． |
| .490 | ．17a．） | ．1731？ | ．15 195 | ．18．72 | ．1815） | ． 2 ALS | ．1337 | ： $23+5$ | $\therefore \therefore-4 \div 3$ | ：－＝： |
| ．5no |  | －コロー？ | ．1354\％ | － 53359 | 1833） | ．1＂17 |  | ． $1+1 \cdot \times$ | ：$\because \cdot$ | ：• |
| ．51． | ．1．4．－ | ．135：3 | $\therefore$ ： 584 | －368\％ | 1．73i4 | ．1－43： | ．1．4．2 | ．$\because \cdot 0 \cdot 4$ |  |  |
| ． 520 | ．$\therefore こ=4$ | ．$\because 320$ | ． 4.9 | ．$\underbrace{7}$ | － 2050 | ．$\because$ 万nm： | $\ldots-4 i$ | －－． | $4!$ |  |
| ．i3） | ． ！$^{\text {a }}$ ？ | ． $21: \cdots$ | $\therefore \therefore-53$ | －130 | ． 1125 | －151！ | ．－1： | －－\％ | ：${ }^{-}$ |  |
| －¢） | ． $11+\mathrm{ir}$ | －－ 31 | ．$\therefore$ ： 17 | ．$\because 2:$ ； | ．$\because \because: 3$ |  | $\because:$ | － |  |  |
| ．55． | $\therefore 2+5$ | ． Cl 1 i | ． 309 | $\therefore 31.8$ | ．-314 | $\therefore 3-7$ | $\therefore 3 \cdot 6$ | $\ldots \because=$ | ．$: 1 \cdot$ |  |
| ．5．） | ．237：－ | － $3+1$. | －3：1） | ． 曲 $^{\text {a }}$ | $\therefore \therefore$ | $\therefore \rightarrow$－ | ．．7． | ． －$^{\text {a }}$ | ． | ． |
| －3： | － 2 dos： | ．－77： | $\ldots 2+34$ | ． 4 a2． | － 2 ¢－n | $\because: ~: ~$ | $\therefore \because:$ | － 3 ： | $\ldots{ }^{-}:$ |  |
| ？ | ．－350m | ． | $\ldots 5^{\circ}$ | ．Sinis | $\therefore$－ |  | ．${ }^{\text {：}}$ ． | －－ |  | － |
|  | －事この． | ．$\therefore \cdot$. | $\ldots{ }^{--}$ |  |  |  |  |  |  |  |

## APPENDIX A

Table az6．－Continued

| $\cdots$ | 0 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.000 | 3.077 | 2．5\％ | 2． 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| c． 600 | 0.27550 | 0.27650 | 0.27751 | 0.27852 | 0.27952 | 0.28053 | c． 28154 | 0.28255 | 0． 23357 | 7．2645 |
| ． 610 | ． 28561 | ． 28663 | ． 28766 | ． 28869 | ． 28972 | ． 29075 | ． 29178 | ． 29282 | ． 27385 | 294） |
| ． 620 | ． 29594 | ． 29699 | ． 29804 | ． 29909 | ． 30014 | ． 30119 | ． 30225 | ． 30331 | ． 30437 | 5ii |
| ． 630 | ． 30650 | ． 30757 | ． 30864 | ． 30972 | ． 31079 | ． 31187 | ． 31295 | ． 31403 | ． 31512 | 3 |
| ． 640 | ． 31729 | ． 31839 | ． 31948 | ． 32058 | ． 32168 | ． 32278 | ． 32388 | ． 32499 | ． 32615 | 3272： |
| ． 650 | ． 32832 | ． 32944 | ． 33056 | ． 3168 | ． 33280 | ． 33393 | ． 33505 | ． 33618 | ． 33732 | ． 3384 |
| ． 660 | ． 33959 | ． 34073 | ． 34187 | ． 34301 | ． 34416 | ． 34531 | ． 34646 | ． 34762 | ． 34877 | 3499 |
| ． 670 | ． 35110 | ． 35226 | ． 35343 | ． 35460 | ． 35577 | ． 35694 | ． 35812 | ． 3593 | ． 36048 | ． 3616. |
| ． 680 | ． 36285 | ． 36404 | ． 36523 | 36642 | ． 36762 | ． 36882 | ． 37002 | ． 37122 | ． 37243 | 3736： |
| ． 690 | ． 37485 | ． 37606 | ． 37728 | ． 37850 | ． 37972 | ． 38094 | ． 38217 | ． 38340 | ． 38463 | ． 385 E ． |
| ． 700 | ． 38710 | ． 38834 | ． 38958 | ． 39083 | ． 39207 | ． 39332 | ． 39458 | ． 39583 | ． 39797 | 3983． |
| ． 710 | ． 39961 | ． 40088 | ． 40214 | ． 40341 | ． 40469 | ． 40596 | ． 40724 | ． 40852 | ． 40980 | 411 |
| ． 720 | ． 41238 | ． 41367 | ． 41496 | ． 41626 | ． 41756 | ． 41886 | ． 42017 | ． 42147 | ． 42278 | 4241 |
| ． 730 | ． 42541 | ． 42673 | ． 42805 | ． 42937 | ． 43070 | .43203 | .43336 | ． 43469 | ． 43603 | ．4373 |
| ． 740 | ． 43871 | ． 44005 | ． 44140 | ． 44275 | ． 14410 | ． 44546 | ． 44682 | ． 44818 | ． 44954 | ． 4515 ： |
| ． 750 | ． 45228 | ． 45365 | ． 45503 | ． 45640 | ． 45778 | ． 45917 | ． 46055 | ． 4619 | ． 46333 | ．45i7 |
| ． 760 | ． 46612 | ． 46752 | ． 46893 | ． 47033 | ． 47174 | ． 47315 | ． 47457 | ． 47598 | ． 47743 | ＋794． |
| ． 770 | ． 48025 | ． 48168 | ． 88311 | ． 48454 | ． 48598 | ． 48742 | ． 48886 | ． 49030 | ． 49175 | $\rightarrow 3:$ |
| ． 780 | ． 49466 | ． 49611 | ． 49757 | ． 49903 | ． 50050 | ． 50197 | ． 503.44 | ． 50491 | ． 50639 | $5076^{-}$ |
| ． 790 | ． 50935 | ． 51084 | ． 51233 | ． 51382 | ． 51531 | ． 51681 | ． 51831 | ． 51981 | ． 52132 | 5229 |
| ． 800 | ． 52434 | ． 52586 | ． 52737 | ． 52689 | ． 53042 | ． 53195 | ． 53347 | ． 53501 | ． 53654 | $538 \%$ ． |
| ． 810 | ． 53962 | ． 54117 | ． 54272 | ． 54.427 | ． 54582 | ． 54738 | ． 54894 | ． 55050 | ． $53=07$ | 553 |
| ． 620 | ． 55521 | ． 55679 | ． 55836 | ．5：994 | ． 56153 | ． 50312 | ． 50.471 | ． 56630 | ． 56730 | 563 |
| ． 830 | －57110 | ． 57271 | ． 57432 | ． 57593 | ． 57754 | ． 57916 | ． 58078 | ． 5824 | ． 59404 | 3as． |
| ． 840 | ． 58730 | ． 58894 | ． 59058 | ． 59222 | ． 59387 | ． 59552 | ． 59717 | ． 5988 | ． 50049 | ． $502:$ |
| ． 850 | ． 60382 | ． 60549 | ． 60716 | ． 60884 | ． $6100^{\text {c }}$ ］ | ． 61220 | ． 61388 | ． 6155 | ． 61726 | 618\％． |
| ． 860 | ． 62066 | ． 62336 | ． 62406 | ． 62577 | ． 62748 | ． 62920 | ． 63091 | ． 63263 | ． 63436 | ． 636 |
| ． 870 | ． 63782 | ． 63955 | ． 64129 | ． 64303 | － 64477 | ． 64652 | ．64827 | ． 65003 | ． 65178 | 6535 |
| ． 880 | ． 65531 | ． 65708 | ． 65885 | ． 66062 | －60240 | ． 66418 | ． 66596 | ． 66775 | ． 66954 | 671： |
| ． 890 | ． 67314 | ． 67494 | ． 67674 | ． 67855 | ． 68036 | ． 68218 | ． 68399 | ． 68582 | ． 68764 | 689： |
| ． 900 | ． 69130 | ．69314 | ． 69498 | ． 69582 | ． 69867 | ． 70052 | ． 70237 | ． 70423 | ． 73609 | 775 |
| ． 910 | ． 70982 | ． 71169 | ． 71356 | ． 71394 | ． 71732 | ． 7192 | ． 72109 | ． $722 \pm$ | ． 2.4 e8 |  |
| ． 920 | ． 72868 | ． 73059 | ． 73250 | ． 73441 | ． 73633 | ． 73825 | ． 74017 | ． 74210 | 7403 | ミ． |
| ． 930 | ． 74790 | ． 74984 | ． 75179 | ． 53374 | ． 75569 | ． 75765 | ． 35961 | ． 7615 | ．7635： |  |
| ． 940 | ． 76749 | ． 76 | ． 77145 | ． 77343 | ． 77542 | ．777： | ． 77941 | 7814 | T＝3i2 | － |
| ． 950 | ． 78744 | ． 7894 | ． 79147 | ． 79350 | ． 795 | ． 77755 | 79059 | ． 8929 | － $330 \%$ | 3 |
| ． 960 | ．80：76 | ． 809 | ． 81187 | ． 81394 | ． 31600 | ． 61807 | ．32014 | ．822 | －2is： | 3－6 |
| ． 970 | ． 8284 | ． 8305 | ． 83266 | ． 83476 | ． 83686 | ． 83897 | ．84， 38 | ． $8+31$ | ． 3 4531 | ご・ |
| ． 980 | ． 84956 | ． 85169 | ． 85383 | ． 85597 | ． $3581 \times$ | ． 66025 | ． $862: 1$ | ． 3645 | －35．5： | －nm－ |
| ． 990 | ． 37125 | ．87322 | ． 87539 | ． 87757 | ． 37975 | ． 88194 | ．85412 | 3632 | －5052 | っつ－ |
| 1.000 | ． 39293 | ． 8951 | ． 89735 | ． 89957 | ．90180 | ． 90402 | ． 99625 | ． 37989 | ． 71373 | 312 |
| 1．310 | ． 31521 | ． 31746 | ． 91972 | ． 92198 | ． 72424 | ． 02551 | ． 9.373 | － 23.15 | ． 2333 \％ | ？ 3 |
| 1.220 | ． 73.90 | ． 24019 | － 94.29 | ． 34.478 | － 7.475 | ． 9.9 ：8 | ． 25 mm | 25：${ }^{\text {1 }}$ | 250： | －－ |
| 1.033 | ． 76297 | ． 9633.7 | ． 36563 | ． 96796 | ． 77.33 | ． 27265 | ． 2750 | ． 27335 | －－ | － |
| 1.040 | ． 38542 | ． 98679 | ． 28916 | ． 99153 | ． 99391 | ．$\rightarrow$－ | ． 3 ？${ }^{\text {ara }}$ | 1．9：30 | 3.5 |  |
| 1.350 | 1.30825 | ：． 11360 | 1.21306 | 1.21547 | 1.21783 | i．$\because 331$ | 1．ここここ？ | 1．こご5 | ここ5 |  |
| 1.363 | 1．3：2：45 | ：． 23489 | 1． 23734 | ：． 23.378 | －． $3.2-4$ | 1．Silat | ． 475 | ：．$\because+$ ： | 9 | $=:$ |
| 1.370 | 1．35702 | ：． 2534 \％ | 1.2137 | 1．－6445 | 1．Vonts | 1．गn¢ ${ }^{\text {a }}$ | －． 113 | －： |  |  |
| 1． 385 | $1.20: 34$ | ：． 28445 | 1．S3n97 | 1． $38 \rightarrow 47$ | 1．331 | 1． 2 $^{\text {a }}$ | ？ | － |  |  |
| 1.290 | 1．1こワこ | 1．1：7\％ | 1．1：232 | 1．11437 | 1．1173 | 1．11：4 | ：$: .15$ |  |  |  |

table a26．－Continued

| $\cdots$ | 0 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.308 | 0.009 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.100 | 1.13285 | 1.13543 | ：． 13801 | 1.14060 | 1.14320 | 1.14579 | 1.148391 | 1.15099 | 2.25360 | 1.25621 |
| 1.110 | 1.15882 | 1.16144 | 1.16406 | 1.16668 | 1.16930 | 1.17193 | 1.17457 | i． 17720 | 1.17984 | 1.18249 |
| 1.120 | 1.18513 | 1.18778 | 1.19044 | 1.19309 | 1.19575 | 1.19842 | 1.20108 | 1.20375 | 1.20643 | 1.20910 |
| 2.130 | 1.21178 | 1.21447 | 1.21715 | 1.21985 | 1.22254 | 1.22524 | 1.22794 | 1.23064 | 1.23335 | 1.23606 |
| 1.140 | 1.23877 | 1.24149 | 1.24421 | 1.24693 | 1.24966 | 1.25239 | 1.25512 | 1.25785 | 1.26059 | 1.26334 |
| 1.150 | 1.26608 | 1.26883 | 1.27159 | 1.27434 | 1.27710 | 1.27986 | 2.28263 | 1.28540 | 1.28817 | 1.29095 |
| 1.260 | 1.29372 | 1.29651 | 1.29929 | 1． 30208 | 1.30487 | ． 30767 | 1.31047 | 1.31327 | 1.31607 | 1.31888 |
| 1.170 | 1.32169 | 1.32450 | 1.32732 | 1.33014 | 1.33297 | 1.33579 | 1.33862 | 1.34146 | 1.34429 | 1.34713 |
| 1.180 | 1.34998 | 1.35282 | 1．35567 | 1.35852 | 1．36138 | 1．36424 | 2.36710 | 1.36997 | 1.37284 | 1.37571 |
| 1.290 | 1.37858 | 1.38146 | 1.38434 | 1.38722 | 1.39011 | 1.39300 | 1.39590 | 1.39879 | 1．70169 | 1.40460 |
| 1.200 | 1.40750 | 1.41041 | 1.41332 | 1.41624 | 1.41916 | 1.42208 | 1.42500 | 1.42793 | 1.43086 | 1.43380 |
| 1.210 | 1.43674 | 1.43968 | 1.44262 | 1.44557 | 1.44852 | 1.45147 | 1.45442 | 1.45738 | 1.46035 | 1.46331 |
| 1.220 | 1.46628 | 1.46925 | 1.47223 | 1.47520 | 1.47818 | 1.48117 | 1.48416 | 1.48715 | 1.49014 | 1.49313 |
| 1.230 | 1.49613 | 1.49914 | 1.50214 | 1.50515 | 1.50816 | 1.51118 | 1.51419 | 1.51721 | 1.52024 | 1.52326 |
| 1.240 | 1.52629 | 1.52933 | 1.53236 | 1.53540 | 1.53844 | 1.54149 | 1.54454 | 1.54759 | 1.55064 | 1.55370 |
| 1.250 | 1.55676 | 1.55982 | 1.56289 | 1.56596 | 1.56903 | 1.57210 | 1.57518 | 1.57826 | 1.58135 | 1.58444 |
| 1.260 | 1.58753 | 1.59062 | 1.59372 | 1.59682 | 1.59932 | 1.60302 | 1.60 F13 | 1.60924 | 1.61236 | 1.61548 |
| 1.270 | 1.61860 | 1.62172 | 1.62485 | 1.62797 | 1.63111 | 1.63424 | 1.63738 | 1.64352 | 1．6436： | －．64681 |
| 1.280 | 1.64996 | 1.65321 | 1.65627 | 1.65943 | 1.66260 | 1.66576 | 1.66893 | 1.67210 | 1.67527 | ． 67545 |
| 1.290 | 1.68163 | 1.68481 | 1.68800 | 1.69119 | 1.69438 | 1.69753 | 1.70077 | 1.70397 | 1.70718 | 1.71038 |
| 1.300 | 1.71359 | 1.71681 | 1.72002 | 1.72324 | 1.72646 | 1.72969 | 1.73291 | 1.73614 | 1.73938 | 1.74261 |
| 1.310 | 1.74585 | 1.74909 | 1.75234 | 1.75559 | 1.75884 | 1.76209 | 1.76535 | 1.75861 | 1.77187 | 1.77513 |
| 1.320 | 1.77840 | 1.78167 | 1.78495 | 1.78823 | 1.79151 | 1.79479 | 1.79807 | 1.83136 | 1.30465 | 1.82795 |
| 1.330 | 1.81125 | 1.81455 | 1.81785 | 1.82116 | 1.82447 | 1.92778 | 1.83109 | 1.83441 | 1.33773 | 1.84105 |
| 1.340 | 1.84438 | 1.84771 | 1.85104 | 1.85438 | 1.85772 | 1.86106 | 1.86440 | 1.86775 | 1.87110 | 1.87445 |
| 2.350 | 1.87781 | 1.88116 | 1.85452 | 1.88789 | 1.89126 | －． 89463 | 1.89800 | 1.90137 | 1.90475 | 1.90813 |
| 1.360 | 1.91152 | 1.91491 | 1.91830 | 1.92169 | 1.92508 | 1.92848 | 1.93186 | 1.93529 | 1.93870 | 1.94211 |
| 1.370 | 1.94552 | 1.94893 | 1.95235 | 1.95577 | 1.95520 | 1.96263 | 1.36606 | 1.96949 | 1.97293 | 1.97636 |
| 2.380 | 1.97981 | 1.98325 | 1.98670 | 1.99015 | 1.99360 | 1.99706 | 2.00052 | 2.00398 | 2.00744 | 2.01091 |
| 1.390 | 2.01438 | 2.01785 | 2.02133 | 2.02481 | 2.02829 | 2.03177 | 2.03526 | 2.03875 | 2.04224 | 2.04574 |
| 1.400 | 2.04924 | 2.05274 | 2.05624 | 2.05975 | 2.06376 | 2.06677 | 2.07029 | 2.07380 | 2.07733 | 2.38085 |
| 1.410 | 2.08438 | 2.08791 | 2.09144 | 2.09497 | 2.09851 | 2.10205 | 2.10560 | 2.13914 | 2.11269 | 2．11524 |
| 1.420 | 2.11980 | 2.12336 | 2.12692 | 2.13049 | $2 . .3405$ | 2.13762 | 2.14119 | 2.14476 | 2.21834 | 2.15132 |
| 1.430 | 2.15551 | 2.15909 | 2.16268 | 2.16627 | 2． 2.9987 | 2.17346 | 2.17706 | 2.13067 | 2.18427 | $\therefore 137 \mathrm{ce}$ |
| 1.440 | 2.19149 | 2． 19511 | 2.19872 | 2.20234 | 2.23597 | 2.20959 | 2.21322 | 2．：1685 | ：．22348 | 2．22412 |
| 1.450 | 2.22776 | 2.23140 | 2.23505 | 2.23869 | 2.24234 | 2.24630 | 2.24965 | 2.25331 | こ． 25697 | 2．こここ64 |
| 1.460 | 2.26431 | 2.26798 | 2.27165 | 2.27532 | 2.27900 | 2.28268 | 2.28637 | $\therefore .29005$ | －． 29374 | －．27i4 |
| 1.470 | 2.30113 | 2． 30483 | 2.30853 | 2．31223 | 2.31594 | 2.31965 | 2.32336 | 2.32737 | 2.33079 | $2.33: 51$ |
| 1.480 | 2.33823 | 2．34196 | 2.34569 | 2.34942 | 2.35315 | 2.35639 | $2.366^{63}$ | 2.35437 | 2．35812 | $2.3: 207$ |
| 1.490 | 2.37562 | 2.37937 | 2.38313 | 2.38688 | 2.39065 | 2．374．1 | 2.39816 | 2.75195 | 2.80572 | $\therefore$－ |
| 1.500 | 2.41347 | 2.41706 | 2.42064 | 2.42463 | 2． 428.42 | 2.43221 | 2．：3600 | 2.43989 | 2．4436． | 2．ifis |
| 1.510 | 2.45121 | 2.45502 | 2.45883 | 2.46264 | 2.46646 | 2.47028 | 2．：7．113 | 2.77 .73 | $\therefore 8176$ | 2.7355 |
| 1.520 | 2.48942 | 2.47326 | 2.49710 | 2.50094 | 2.59478 | 2.50863 | 2.51243 | こ．ड1533 | こ．5こ119 | こ．Eごら |
| 1.530 | 2.52791 | 2.53177 | ？．53564 | 2.53951 | 2.54338 | 2.54725 | 2.55113 | －．35301 | こ．55 369 | ごテ－ |
| 1.540 | 2.36667 | 2.57656 | 2.57445 | 2.57835 | 2.53225 | 2.58615 | 2.59005 | $2.5 \pm 396$ | こ．59737 | こ．¢：1～ |
| 1.550 | 2.60570 | 2.50962 | 2.61354 | 2.51747 | 2.52139 | 2.62532 | 2．529こ5 | －．03314 | －．n3713 | こ．〇： |
| ：． 560 | 2.64501 | 2.5489 | 2.55230 | 2.05536 | 2.50381 | 2．56477 | 2.56873 | こ．－7253 | こn乐为＝ | こ．$\%$ ¢2 |
| 2.570 | 2.68459 | 2.68855 | 2．－3254 | 2.53652 | 2．：905） | 2.71047 | 2．7084 | －－in | $\therefore$－1645 | $こ . こ .5$ |
| 11.580 | 2.72445 | 2．72845 | $2.73-45$ | 2.736 | 2.2046 | －iva | $\therefore$－ $\mathrm{Tin4}$ | $\therefore$－5－5： | －こ．ハ5： | 55 |
| $1: .570$ | 2.76457 | 2.76850 | $2 .-7-53$ | 2. | 2.73070 | －74 | 二．ios：a | $\therefore$－ 354 |  | $\therefore \div \cdot$－ |

table a26．－Continued

| M | 0 | 0.001 | 2.002 | 3.303 | 004 |  | 226 | 0.007 | 08 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.600 | 2.80497 | 2.80903 | 2.31308 | 2.61 | 2.82121 | 2.825 | 2. | 2.83341 | 49 |  |
| 1.610 | 2.84564 | 2.84 | 2.35381 | 2.85 | 2.86199 | 2.86698 | 2.870 | 2.87427 | 2.87837 | 2．a32： |
| 1．620 | 2.36658 | 2．35069 | 2.39480 | 2.39892 | 2.30374 | 2.90716 | 2.91128 | 2.91540 | 91953 | $2.323{ }^{\circ}$ |
| 1.630 | 2.92780 | 2.93193 | 2.73607 | 2.9 | 2．9443 | 2.94850 | 2.95265 | 2.95681 | 2.96096 | $5 i$ |
| 1.640 | 2.96928 | 2.97344 | 2.97761 | 2.98178 | 2.98595 | 2.99012 | 2.99430 | 2.99848 | 3.00266 | $3.006 \%$ |
| 1.650 | 3.01103 | 3.015 | 3.31 | 3.02361 | 3． C 278 | 3.03201 | 3.03621 | 3． 24042 | 3.04463 | 3． $248=$ |
| 1.660 | 3.05305 | 3.05727 | 3.36149 | 3.26571 | 3.06794 | 3.07417 | 3.07396 | 08263 | ． 0868 | 91 |
| 1.670 | 3.09535 | 3.099 | 3.10384 | 3.158 | 3.1123 | 3.11659 | 3.12 CB | 3.12511 | 3.1293 | 3.1336 |
| 1.680 | 3.13791 | 3.14218 | 3.14645 | 3.15073 | 3.15501 | 3.15929 | 3.16357 | 6786 | 7215 | 76： |
| 1.690 | 3.18074 | 3.18503 | 3.18933 | 3. | 3.19794 | 3.20225 | 3.20656 | 3.21083 | 3.21519 | 3．219 |
| 1.700 | 3.22383 | 3.2281 | 3.2 | 3.236 | $3.2: 115$ | 3. | 3.24982 | 3. | 59 |  |
| 1.710 | 3.26720 | 3.27155 | 3.27590 | 3.28026 | 3．28462 | 3.28898 | 3.29335 | 3.2977 | 3.36208 | 3.306 |
| 1.720 | 3.31083 | 3.315 | 3.31 | 3.323 | 3.3233 | 3．3327 | 3.33714 | 3.341 | 3.34593 | O |
| 1.730 | 3．35473 | 3.35914 | 3.36355 | 3.36196 | 3.37237 | 3．376：9 | 3.38120 | 3． 38562 | 3.39005 | $3.394 i$ |
| 1.740 | 3.39890 | 3.4033 | 3.40 | 3. | 3．4．665 | 3.421 .99 | 3．42553 | 3.42998 | 3.43443 | 36 |
| 1.750 | 3.44334 | 3.44780 | 3.45225 | $3.45 ¢ 72$ | 3.46119 | 3.4656 | 3.47023 | 4746 | 3.47908 | 2 |
| 1.760 | 3.48804 | 3.49 | 3.49701 | 3.50150 | 3.55600 | 3.51049 | 3．51：9 | ． 51949 | 3.52400 | 528： |
| 1.770 | 3.53301 | 3.5 | 3.34204 | 3.54655 | 3.55107 | 3.55 E60 |  | 56465 | 3.56913 | ．573－ |
| 1.780 | 3.57825 | 3.58278 | 3．5673？ | 3.59187 | 3.59642 | 3.60696 | 3.50552 | 3.61007 | 3.61463 | 10： |
| 1.790 | 3.62375 | $\begin{array}{r} 3.6283 i \\ 0 \end{array}$ | 3.63288 | 3.63745 | 3．64202 | 3.64500 | 3.65118 | 3.65576 | 3.66034 | $3.56 i$ |
| 1.800 | 3.66952 | 3．674）：1 | 3.6787 | 3.68 | 3.68790 | 69253 |  |  |  |  |
| 1.810 | 3.71555 | $3720{ }^{\circ}$ | 3.72479 | 3.72941 | 3.73404 | 3.73867 | 3.7 | 4793 | 525 | 57. |
| 1.820 | 3.76185 | 3．76\％ | 3.77114 | 9．77579 | 3.75044 | 3.78510 | 3.78 | 3.73442 | 3.79908 | 3．3ワ3－ |
| 1.830 | 3.80841 | 3．81：08 | 3.31776 | 3.32243 | 3．82711 | 3.83179 | 3－335．48 | 3.34117 | ． 84535 | 5 |
| 1.840 | 3.85524 | 3.85994 | 3. | 3.36934 | 3．87405 | 3.8 | 3. | 3.88813 | 3.8929 | 7. |
| 1.850 | 3.9023 | 3.90706 | 3.91 | 3.9165 | 3．92125 | 3.92598 | 3. | 3．93546 | 3.94020 | 3． 344 |
| 1.860 | 3.9 | 3.95445 | 3. | 3.9 | 3. | 3．973．77 | 3. | 3.98300 | 3.987 | 3． 392 ＝ |
| 1.870 | 3.99 | 4.00210 | 4.0068 | 4.31166 | 4.01644 | 4.02123 | ¢． 0 | 0308： | 4.03561 | Ci |
| 1.886 | 4.04521 | 4.05001 | 4.35482 | 4.35963 | 4.06444 | 4.06925 | －．07 | 4.07889 | 4.38371 | 288＝ |
| 1.890 | 4.09 | 4.09819 | 4.10302 | 4．${ }^{\text {d }} 3786$ | 4.11270 | 4.1175 | 4.12238 | 4.12722 | 4.13207 | 30 |
| 1.900 | 4.1417 | 4.14663 | 4.15 | 4.15635 | 4．15122 | 16ć | 4．17505 | 175 | $4.1307 \%$ | 4．135 |
| 1.910 | 4.19046 | 4.19534 | 4．200？3 | 4.25511 | 4．2：こ20 | 4．21733 | － | － 2 －46 | 4.22959 | 4．23： |
| 1.920 | 4.2394 | 4．24431 | 4.24922 | 4.2541 | 4.25305 | 4.26397 | －．24．3？ | 27382 | 4.27875 | \％ 3 |
| 1.930 | 4.28851 | 4．29355 | 4.29848 | 4．37342 | $4.3: 337$ | 4.31331 | －． 31526 | ．3232： | 4．32517 | i． 333 |
| 1.940 | 4.33808 | $4.3+304$ | 4.34801 | 4.35298 | 4． 35795 | 4． 36292 | $\therefore 35709$ | － 7.35 I | 4．377EE | －． 33 |
| 1.950 | 4.38782 | 4.37281 | 4.39730 | 4.73279 | 4．4：779 | 4．41278 | － .11773 | 4.42272 | 4．4：73） | 4．732 |
| 1.960 | 4.43782 | 4.44235 | 4.4478 | 4.45287 | 4.45739 | 4.46201 | 46 | －？ | 4．478： | －．733 |
| 1.970 | 4．488＝a | 4.49312 | 4.49816 | 4.53321 | 4．5ン826 | 4.51331 | 4． 51336 | 4．$=3$ 3：2 | 4．528i | 4．53： |
| 1.980 | ＋． 53860 | 4.54367 | 4．54874 | E338 | 4.55369 | 4.56336 | － 56438 | 57413 | 4．5792 | － $5 \pm$. |
| 1.990 | 4.5893 | 4.59448 | －．59953 | ¢．$\%$ ¢ 46 | 4．єうこ78 | 4. | ；． 61307 | 4.62515 | 53 | －¢ 3 \％ |
| 2.000 | 4.64 | 64556 | －．6506a | ． 355 | $4.65: 93$ | $4.660: 6$ | 9．6：123 | 4.57633 | 4．0514 | 9\％ |
| 2.010 | 4.59175 | 4.57670 | ： 772205 | －272 | 4．7：235 | 4．71こミ1 |  | －．72723 | 4．732？ | 4．－3－ |
| 2.020 | 4．74333 | 4.74850 | ． 75368 | － 7 ： 365 | 4.75 .73 | 4.7692 | ：773\％ | －．7735 | 4.7845 |  |
| 2.030 | 4.79517 | 4.30037 | ：． 30557 | －． 32777 | $4.6: 533$ | 4．52：：9 | －．35（4） | 4.831 al | 336e？ |  |
| 2.240 | 4.54727 | 4.35289 | ＋．35772 | 4．シラ295 | 4.5 －51d | 4.373 .2 | 4.37505 | 4. | 4.3091 | ¢．${ }^{\circ}$ |
| 2.25 | 4.39963 | i． 37488 | －．31．14 | 4.75539 | － 9.955 | 4． 72531 | －． $3311 \%$ | i． 363. | 4．3417： | －3．3． |
| 2.363 | 4.75225 | 4． 35753 | －． 90.281 | 4.0309 | 4．${ }^{\text {－}}$－ 3 | 4.97867 | －．383？ |  | ＋．गد＋54 |  |
| －．370 | 5． 20514 | 5． 11745 | 三． 21575 | $5 .: 2106$ | 5．：2：37 | 5．33：-3 | ミ． 3500 | 5． 34232 | 5． 24 －64 |  |
| 2.085 | 5.35829 | 5．3n湤 | E． 26.395 | ․：－429 |  | －－－ | － | 2．）2505 | 5．2．1 |  |
| 2． 2 | 5.11170 | 5．11736 |  | 778 | $5 .: 3:: 4$ | E．17－51 |  | 5．149： | 5．15．92 | ： |

TABLE A26．－Continued

| $\cdots$ | 0 | 3.001 | 1.002 | 3.203 | 2.004 | ）． 275 | 006 | 0.007 | 0.358 | $\bigcirc .209$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.100 | 5.16538 | 5.17076 | 5，17614 | 5.28153 | 5.18692 | 5.13231 | 5.19770 | 5.20317 | 5.20350 | 5.2139 |
| 2.110 | 5.21931 | 5.22472 | 5.23013 | 5.23554 | 5.24036 | 5.24637 | 5.25180 | 5.25722 | 5.26265 | 5.2680 |
| 2.120 | 5.27351 | 5.27894 | 5．28438 | 3.28981 | 5.29526 | 5.37070 | 5.30615 | 5.31160 | 5.31705 | 5.322501 |
| 2.130 | 5.32796 | 5.33342 | 5.33889 | 5.34435 | 5.34982 | 5.35529 | 5.36076 | 5.36624 | 5.37172 | 5.37720 |
| 2.140 | 5.38268 | 5.39817 | 5.39366 | 5.39915 | 5.40464 | 5.41014 | 5.41564 | 5.42114 | 5.82664 | 5.43215 |
| 2.150 | 5.43766 | 5.14317 | 5.44869 | 5.45421 | 5.45973 | 5.46525 | 5.47077 | 5.47630 | 5.48153 | 5.1873 |
| 2.160 | 5.49290 | 5.49844 | 5.50398 | 5.50953 | 5． 5.1507 | 5.52062 | 5.52617 | 5.53173 | 5.53728 | 5.34 |
| 2.170 | 5.54841 | 5.55397 | 5.55954 | 5.36511 | 5.57068 | 5.57625 | 5.58183 | 5.58741 | 5.59300 | 5.5985 |
| 2.180 | 5.60417 | 5.60976 | 5.61535 | 5.62095 | 5．62655 | 5.63215 | 5.63775 | 5.64336 | 5.64897 | 5.55450 |
| 2.190 | 5.65019 | 5.66581 | 5.67133 | 5.67705 | 5.68268 | 5.63830 | 5.69393 | 5.69957 | $5.700<0$ | 5. |
| 2.200 | 5.71648 | 5.72212 | 5.72777 | 5.73342 | 5.73907 | 5.74472 | 5.75038 | 5.75604 | 5.76170 | 5.76735 |
| 2.210 | 5.77303 | 5.77870 | 5.76437 | 5.79004 | 5.79572 | 5.80140 | 5.80708 | 5.81276 | 5.81845 | 5.32414 |
| 2.220 | 5.82933 | 5.83553 | 5.84123 | 5.84693 | 5.85263 | 5.35834 | 5.86404 | 5.86976 | 5.87547 | 三．38115 |
| 2.230 | 5.88690 | 5.89262 | 5.89835 | 5.90407 | 5.90980 | 5.91554 | 5.92127 | 5.92701 | 5.93275 | 5.938 .9 |
| 2.240 | 5.94423 | 5.94998 | 5.95573 | 5.96148 | 5.96724 | 5.97299 | 5.97875 | 5.98452 | 5.99028 | E． 39665 |
| 2.250 | 6.00182 | 6.00760 | 6.01337 | 6．0191； | 6． 22493 | 6.3307 | 6.03650 | 6.04229 | 6.24808 | E． 35385 |
| 2.260 | 6.05367 | 6.06547 | 0.07127 | 6.97758 | 6.78289 | 6． 23870 | 5.09451 | 6.10032 | 6.10614 | 6．11135 |
| 2.270 | 6.11778 | 6.12361 | 5.12944 | 5.13517 | 6.14110 | 6.14694 | 6.15273 | 6.15862 | $6.16: 46$ | 4.17031 |
| 2.280 | 6.17616 | 6.18201 | 6.18786 | 5．19：72 | 6.19558 | 5.22544 | 6.21130 | 6.21717 | 6.22 | E． |
| 2.290 | 6.23479 | E． 24066 | 5.24654 | 6.25243 | 6． 25831 | 6.26420 | 6.27009 | 5.27598 | 6.28122 | $=.28773$ |
| 2.300 | 6.29368 | ＝． 29958 | 7． 30549 | 6.31140 | 6.31731 | 6.32322 | 5.32914 | 6.33506 | $6.34=98$ | ． |
| 2.310 | 6.35233 | 5.35876 | －． 36469 | $5 .: 7063$ | 6.37657 | 6． 38251 | 6.38845 | 6.39439 | 6.40334 |  |
| 2.320 | 6.41225 | 6.41820 | 5.42416 | 5.33012 | 6.43608 | 6.44205 | 6．14822 | 6.45399 | 6.45296 | 594 |
| 2.330 | 6.47192 | 6.47790 | 6.48388 | 6.48987 | 6.49586 | 6.50185 | 6.50785 | 6.51384 | 6.513 | 52565 |
| 2.340 | 6.53185 | 6.53786 | 6.54387 | 6.54988 | 6.55590 | 6.36192 | 6.56794 | 6.57396 | 6.5 | ．536C1 |
| 2.350 | 6.59205 | 6.59808 | 6.60412 | ¢． 61015 | 6.61620 | 6.62224 | 6.62827 | 6.63434 | 6.64019 | ．646：4 |
| 2.360 | 6.65250 | 5.65856 | 5.66462 | ＇，．67069 | 6.67675 | 5.58282 | 6.58890 | 6.69497 | 6.70195 | 13 |
| 2.370 | 6.71321 | 6.71930 | 5.72539 | 6.73148 | 6.73757 | 6． 24367 | 6.74977 | 6.75587 | 6.76 | ． 768 cs |
| 2.380 | 6.77419 | 6.78030 | 6．78．41 | 5．79253 | 6.79865 | 6.30477 | 6.81090 | 6．8：702 | 6． | 29 |
| 2.390 | 6.83542 | E．84156 | 6.94770 | 6.35384 | 6.65999 | 6.36613 | 6.37229 | 6.87844 | $6.38 \div 59$ | \＃． 39075 |
| 2.700 | 6.39691 | ¢． 90308 | 5.90924 | 6.31541 | 5.92153 | 6.72776 | 5.33333 | 6.94011 |  |  |
| 2.410 | 6.95367 | ㅍ． 96486 | $5.9710^{5}$ | E．97724 | $6.983 \div$ | 5.38964 | 5.39554 | 7.30205 | 7．15826 |  |
| 2.420 | 7.02569 | －． 22.690 | 7.03311 | 7． 03934 | 7.04550 | 7．：5173 | 7.35371 | 7.96424 | 7． 57.28 |  |
| 2.432 | 7.08295 | 7.08920 | 7.075 | －． 10163 | －．13794 | 7.11413 | 7.12344 | 7.12670 | 7．13236 |  |
| 2.440 | 7． 14549 | －． 15175 | $\bigcirc .15312$ | ．1643 | 7．1725 | 7．：7585 | 7.18313 | 7．189： | 7.17577 |  |
| 2.455 | 7.25828 | ． 21457 | 7．22：87 | 7．22717 | 7.23347 | 7.33977 | 7．246．39 | 7．25237 | 7．こミラ：7 |  |
| 2.463 | 7.27133 | －． 27765 | 7.28197 | 7．29030 | 7.20563 | 7．33296 | 7.3092 | － | 7．35106 |  |
| 2.472 | 7.33464 | －．34099 | 7．34734 | $\bigcirc .35363$ | 7.36304 | 7． | 7．372－5 | 7．37212 | －． 3 E54 |  |
| 2.480 | 7.39821 | ． 42459 | －．4． 296 | ＋1734 | 7.92372 | ．－3313 | $7.436: 3$ | －．442a7 | －7i？-6 | ¢ |
| 2.493 | 7.46295 | 7.458 .4 | 7．：7384 | 7.73125 | 7.46755 | 7．434．6 | －，ここ： | －ミ¢ヶ03 | －5： 3 ？ |  |
| 2.500 | 7.52614 | －．53255 | 7． 33899 | 7.54541 | 7.55134 | －．55328 | 7．56i：I | 7.57115 | ミ－－5 |  |
| 2.510 | 7.59049 | 7.59694 | 7.5 .339 | ． 50984 | 7．61530 | － $522: 5$ | 7．62：22 | 7.635008 | － $5: 225$ | －くもこの |
| 2.522 | 7.65510 | －．65157 | $\rightarrow$－ $6380^{\circ}$ | －．57453 | 7.60101 | －．．a75？ | －．533こ9 | 7．วาก．48 | －．－－6？ | ． 7 |
| 2.533 | 7.71995 | $\bigcirc .72647$ | 7.73295 | $\bigcirc .73943$ | 7．7：533 | －5259 | －75a31 | 7.76553 | － 5 | －755 |
| 2．54： | 7．735：9 | $\cdots 3152$ | －．73315 | －．30463 | 7．3：122 | ． 3175 | －．32：2？ | 7．83754 | －．j3－39 | －3is |
| 2．55こ | 7．35：4a | －．35703 | $\therefore .25357$ | －．77：15 | ． $\mathrm{s}^{\text {－571 }}$ | － | 353E： | － 3064 | －． 2723 | つッシミミ |
| 2. | 7.31513 | －32：71 | 7.2929 | －． 33587 | ？． 3.246 | ． 3.4295 | ？5ミー： | －）f：23 | ．コニ゙ミa3 |  |
| こ．5？ | $7.352: 3$ | ． 38854 | － 77525 | 7． 3136 | 8．2．84 | ！$=^{\text {a }}$ | 3．32177 | －． 2832 | ミ．$\because ¢+4$ | － |
| $\therefore$ こ50 | 3．04アこう | 3． 25483 | 7．00：46 | \＃． 55812 | 8．P174 | 2． 2 1135 | 3．2a3： | 3．） 3 s | 2．1？ $3:$ | ：－${ }^{-}$ |
| 2．53－ | 3．1135 | －．：21：9 | 3.12794 | －．134n\％ | －．1：12＊ | E．14－33 | 3. | 3 | こ．入．こった | ミ |

## APPENDIX A

TABLE A26．－Eonc：nued

| $\cdots$ | C | 0.901 | 3.1002 | 0.003 | 0.004 | ． 205 | $0 .: 26$ | 0.307 | －．905 | 3. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.600 | 8.18131 | 8.18799 | a． 19468 ！ | 8.20136 | $8.20803^{\prime}$ | 6.21475 | 3．こここ＊4 | 8.228141 | 7．23：84 | 8．$=1$ |
| 2.610 | 8． 21325 | 8． 25.36 | 8．26167 | 3． 26638 | 8.2751 | E．：3182 | 8． 5.554 | 8.29527 | 3．315199！ | 5， 3 |
| 2.620 | 6． 31545 | 3．322：9 | 8．328921 | 3.33566 | 8．342： | ¢．34915 | 8． 5 5570 | 8． 36265 | 三． 269401 | 3.3 |
| 2.630 | 8.38 .91 | 8.38958 | 3．39644 | 8.40320 | 3．10997 | E．41674 | $\because .: こ 5521$ | 3．i？229 | 6．437， 7 | 3．4i |
| 2.640 | 8． 45.564 | 9.45742 | 3．46421 | 8.47100 | 8．471E： | E．73457 | 2．7：139 | 6.73319 | 5．5．5501 | 2．5： |
| 2.650 | 8.51362 | 8.52543 | 8． 53224 | 8．53306 | 8．5458a | 6.55270 | 2． 55353 | 8.56636 | E．57319 | $9.5 \%$ |
| 2.660 | 8．586B5 | 8.59359 | 8.60053 | 8.60738 | 8．614ここ？ | 3.52107 | 8．$\in=732$ | 8.63478 | 8.64263 | 8.64 |
| 2.670 | 8.65535 | $8.662: 2$ | 0.66908 | 3.67595 | $8.6328=$ | 三． 58970 | 8．6こちら7 | d． 23345 | 3.71034 | B．7： |
| 12.680 | 8． 72.11 | 8．731： | 8.737891 | 8．74：77 | Q．751eうi | ¢．75858 | 3． 5\％$^{\text {a }}$ | 8.75239 | 3.77931 | 8．7： |
| 2.690 | 8.79312 | 8.80034 | $3.3063{ }^{\circ}$ | 8.81388 | 8．8こごこ！ | 三．：273 3 | 3．$\because: \geq 56$ | 8.34159 | 3.84852 | 3．6： |
| 2.700 | 8．8e243 | 9.86 | 0．07629 | 8.88323 | B．8901＝ | －．33713， | 8．ミ．：39 | 8.31135 | 3．91－01i | 3．？ |
| 2.710 | 6.93193 | 8.93890 | 4.9458 ？ | 6.95284 | 8.959 E！ | $\pm .35680$ | 8． 3.378 | 8． $38575!$ | 4.98775 | 8． 3 |
| 3.720 | 9.00173 | 9.00872 | 9．01572 | 9.02271 | $9.029: 1$ | 3.33672 | 9．：33731 | 9.250731 | 3．05：751 | 9.6 |
| 2.730 | 9.07178 | 9.07880 | 9.08582 | 9.09284 | 9.09967 | －． 1.9690 | 9．： 3931 | 9.12097 | 3．123031 | $\therefore 1$ |
| 2.740 | 9172091 | 0.14913 | 7.15618 | 9.16323 | 9.17023 | 3.17734 | 9．：$=4 \div 0$ | $9.131: 6$ | 3．1935 ${ }^{\text {i }}$ | ＋．2 |
| 2.750 | 9.21266 | 9.21373 | 9.22680. | 9.23388 | 9.24095 | 3.24804 | $9 .: 5512$ | 9． 25221 | 9．26739 |  |
| 2.760 | 9.23543 | 9.29058 | 7．29760， | 9.30478 | $9.311 E 9$ | 3.31900 | 9.32511 | 9.33322 | 3．34331 | 2．3．0 |
| 2.770 | 9.35457 | 9.36159 | 7.36882 | 9.37595 | $9.383: 3$ | F． 39021 | $9.37: 35$ | 2．404：9 | 3．41153 | 9.7 ： |
| 12.780 | $9 .+2592$ | 9.43357 | 7.440221 | 9．44737 | 9.45453 | 3.75109 | 9.75385 | 9．47601 | 3．28j13： | 3.7 |
| 12.790 | 9.40752 | 9.50400 | 9．5118：＇ | 9．51905 | $9.5262 \div 1$ | －．53342 | 9．5：361 | 2．5473）！ | ？．55：33： | 0.5 |
| －2．809 | 9.5639 | 9.51259 | 9．58379： | 3.59 .9 | 9．598ここ | $\pm .50541$ | 9．7．－大 | 9.61284 | $3.62726 i$ | 3. |
| 2.810 | $9.6+151$ | 9．64373． | 2.65536 i | 9.66319 | $9.60 r_{5}$ | 2.57767 | 9．$=5: 90$ | 9．5．215 | 2.69939 | $3 .-$ |
| 12.820 | 9.73 .35 | 3.721 .4 | 2． 27840 ！ | 9.73565 | 3．742＝こ | ？．${ }^{-5} 518$ | 2． $5 \times 74$ | O．76．71 | 2.77198 | 7．－－ |
| 12.839 | 9．7シャ53 | 9.79331 | $\because .80109$ | $\rightarrow .83837$ | 9．315＝天 | 2.32294 | 9．4：324 | \％．33－53 | 2．74i83 | 3．－ |
| 2.843 | 9．6534 | 9．56¢7？ | 3.37414 | 9.32135 | 9.386 ¢ | \％．59597 | ？．$=: 3291$ | 0.429 | 7.917931 | $\pm$ ． |
| 12．352 | 9．35：53 | 4．13，9i | 3．94725： | 3.95458 | 7．751： | ․ 36926 | 7．$=-560$ | $\therefore .75395$ | 7.99129 | $\cdots$ |
| 12.860 | 10．：こe 3 | 10.01355 | 10.020711 | 10.02307 | $10.035 \div$ | $\therefore 2+280$ | 10．：5：17 | 12.25754 | 2． 6.42 | 2. |
| ！2．370 | 10．3－367 | 10.08735 | 10．09444 | 1.10183 | 10．109：1 | $\because 11561$ | 10．：：i00 | 10．131：0 | 1J．13ह5？ | 2． |
| 2.880 | 10.15351 | 10.16131 | 20．1684：＇ | 10.17584 | 10.18325 | ：．．19067 | 10．：？こう | 12.20551 | ： 2.21294 | 1．）． |
| 12.890 | 10．2：？ 30 | 10．235：31 | 10． 2426 \％ | 10.25010 | 10．25755 | 2． 26499 | 1）．$:-2+4$ | 1）．273月5 | $\because \therefore$ ： 734 | 10. |
| 12 | 10．3：225 | 10.30 | 1. |  | 10．332：3 | 33957 | 10．：-2.24 | 1. |  |  |
| 12.912 |  | 10.384 | 11.3919 | 10.37942 | 10．406こ1 | $\because \cdot .14+1$ |  | （ | －－43631 |  |
| 12．723 | 10．45： 72 | 10．453．4 | 1）． 16695 | 10．47446 | $10.481 \div 3$ | ．$i 8950$ | 10．7：－33 | $1: .5-: 55$ | 2．51．28 |  |
| －．93） | $10.5 こ 715$ | $10.53 i o ́ s$ | 10．542こ2 | 10.54977 | 10．55\％ 101 | － 56486 | 1？．：－こ．1 | 1：$: 5736$ | ：$\because 55351$ |  |
| （2．94） | 10．e：2aj | $1 \cdot .6131$ | 17.61770 | 10.62533 | $10.632=?$ | $\therefore$ ．¢：047 | $12.5 \div 395$ | 1こ，¢55n～ | 27．663：i | こ |
| 2．95， |  | $10.635 \pm 0$ | 13.69355 | 10.72125 | 12．7－s－i | ． 21634 | 1こ．ご） | 12．7：155 | ：－，！ |  |
| ！ 2.967 | 10．${ }^{\text {² }}$－ 33 | 10．75：39 | ：9．7696： | 11.77723 | 13．76： 5 | 792：7 | $10.3: 215$ | 1 －3：7？ | $\because$－＝ |  |
| ：こ． | $13 . シ 3 こ ち 4$ | 10．83525 | 13.34572 | 10.55355 | 12．66： | ． 3 ¢\％86 | 1～．－551 | ：こ，E®： |  |  |
| 2．983 | 10．3ワ\％15 | 10．91432 | に，ヲこご5 | 1． 3.7315 | $10.73^{-}=3$ | ． 34551 | 1，－5319 |  |  |  |
| ！2．900 | $10.553 \geqslant 3$ | 1.7 | ： 0 | $11.207: 1$ | $11.31 i^{-1}$ | ．3：241 | 11．${ }^{\text {a }} 12$ | 1：．33：23 | $\therefore \quad i$ |  |
|  | 11． 20 ？ | 111． 66 nco | 11．アフィ | 11．06： 3 | 11．）9ミこう | 53 | 1 | 12．125：4 |  |  |
| （3．31） |  | （11．145ン | ：1．133： | 1．．10152 | 1：100ここ | － | ： $2 \rightarrow$－ | －．： 225 |  |  |
| ；3．3：${ }^{\text {a }}$ | 11． 1531 | 11．：2：50 | 11．23135 | 11.83913 | 112．24\％シ） | －．5346A | 11．．－27 | 27225 | 11．5－3：4 | ． |
|  | 11．－－jos | 1．． | ：1． $2 \pm \pm$－ | 11．31：－1 | 11．32キシニ | ．．3326． | $11.37: 3$ | 11．3：321 | ：1．35＜5 ${ }^{1}$ | 1. |
| 1 3.37 | 11．：7：53 | 11．j755 | ：1．387： | 11.33515 | 11．70： 97 | －．iluna | 11．：23i5 | 11．42549 |  | $\therefore 2$ |
| 3． 3 ： | 11．7三： 2 | 11．as－9é | 11．405－： | 11．：－356 | 11．48：-2 | ．$\cdot 15929$ | 12．i371 | 11．555 | 11．51： | ： 2 |
| 3．063 | 11．5－E5） |  | ：．54． | ： $2.55=23$ | 11.50 | 5in9．4． | 12． 5 － | ミャッア |  | ： 1. |
| 3. | 11．．－－：5 | 11． 110.5 | ：．．．．こ： |  | 11 | 159\％ |  |  | ：1 |  |
| 3. | 11．0－55 | ：1．6－74 | ： 5.722 a | ：．7： 31 |  |  |  |  |  |  |
|  |  | 11．${ }^{-\cdots}$ | ：．． $31-$ ： |  |  |  |  |  |  |  |




spPEMTI:



OF Furl: (lCat!:
TABAL: A27.- CONVERSION FACTOHS FOK VAKIOUS mRESSURE UNITS
$[$ liom ret. $A 4]$


APPETUI $\because A$

THALE A28．－CONJERSION FACTORS，EqUIVALETTS．A：D ECNULAS FOR Z．S．G：STCMAYY


$$
\begin{aligned}
& \text { (a) Enverisin inctu: } \\
& {\left[\begin{array}{lll}
{[\operatorname{rom}} & : i t & i 5
\end{array}\right]}
\end{aligned}
$$

inenyti
1 foot（ft）
1 navidこal fiste
1 statuse mile
？inch（in．）
Speed
1 I：＇sec
1 £．$/ \mathrm{man}$
1 ：anle／hour（mph）
1 inot
iscebirration
1 ft／：se ${ }^{2}$

## Rass

1 ：s．ug
1 pound（．ts）

## Force

1 peund（！b）
Pressure
1 ＋b／ft ${ }^{\circ}$
1 inch ot merciry（1：1．Hy）
1m＋11：E．ar
aers．s：
$1 ; 1+4,:^{3}$
1 ： 5 待き
$\because 1: 1: 143$
$1::_{3}$
$1::^{3}$

－：ロ－5\％：：－

－ッ：usatis．．
－－
$\therefore \therefore 1$

1 1： 1









J．s．is mo：min ．．．．：in




## APPENDIX A

ORIGINAL Paj OF Puni QLALA:

## TABLE A28.- Continued

(b) Equivalents (primary constants and atnosiatric wrofar:iezs)

| Quantity | U.S. Customary Units | $\therefore 1$ 'inc: |
| :---: | :---: | :---: |
| Po | $\begin{aligned} & 2116.22 \mathrm{lb} / \mathrm{Et}^{2} \\ & 99.9213 \mathrm{in} . \mathrm{H} \cdot \end{aligned}$ | : 11325:2 |
| .$_{0}$ | $0.076474 \mathrm{lb} / \mathrm{Et}^{3}$ |  |
| $\therefore 0$ | 0.0023769 slug/fi ${ }^{3}$ | 1.2.50 i: $\mathrm{m}^{\text {? }}$ |
| $t_{0}$ | 59.0 ${ }^{\circ} \mathrm{F}$ | 15.31) $=$ |
| To | 514.67 | 26\%.15 |
| $\therefore 0$ | $\begin{aligned} & 1.2024 \cdot 10^{-5} 15 \text { it-s.e. } \\ & 3.7372 \cdot 10^{-7} 12-20 \cdot=2 \end{aligned}$ |  |
| ${ }^{9} 0$ |  |  |
| $a_{0}$ | $\begin{aligned} & 1116.45 \text { itisect } \\ & 761.22 \text { mf } \\ & 661.48 \text { knots } \end{aligned}$ |  |
| ${ }^{3} \mathrm{Wm}_{\mathrm{m}, \mathrm{O}}$ |  |  |
| $\mathrm{P}^{*}$ | 1545. $31 \div-1 \%$ (1\% mos) $\%$ |  |
| $\bar{i}$ | 53.:52: 5 -12:(1: moll) |  |
| $?$ | 1716.5 ft-1\%: $31.14-3$ | $\ldots$ - . ! ! $\because$ - \% |

TABLE A28.- Con:=:uded
(c) Formulas

| Fcrmuls for - | U.S. Customary :mits |  |
| :---: | :---: | :---: |
| 5 | .'9 |  |
| $\bar{R}$ | $r^{*} / W_{m, 0}$ |  |
| R | $\mathrm{K}^{*} \mathrm{~g} / \mathrm{W}_{\mathrm{m}, 0}$ | $\therefore * *$ |
|  |  | n-k: |
| Pa (pascal) |  | $i s / m^{2}=\mathrm{kg} / \mathrm{m}-\sec ^{2}$ |
| $J$ (joule) |  | $\therefore-m=m^{2}-k: \ldots, c^{2}$ |

AThe formulas for the gas constant:; $\overrightarrow{\mathrm{R}}$ and in $i:=\cdots$. Customary Linits also apuly to the metric (miks) svitum,.. . . ir

$R=287.05 \mathrm{n}^{2}-\mathrm{kg} /{ }^{\circ} \mathrm{K}-\mathrm{Kol}-\mathrm{sec}{ }^{2}$.

## SAMPLE CALCULATIONS

## Part I - Static-Pressure Errors and Eliuht 2uantitites

In this section, sample calculations are presented for the determination of - (1) the position error An by two of the flight calibratin: methods described in chapter $1 X,(2)$ values of calibrated airsped $\because C$ pressure aititude $: 3$, and Mach number $M$ from the indicated values of these quartithes and a uiven value of ip, (3) the lift coefficient $C_{L}$ from given vaiues of $i p$, the moasire d impact pressure $q_{c}^{\prime}$, ard the measired 三tatic p:essure $p^{\prime}$, and (f) true a.rspeed $V$ from given values of callbrated airspeed $V_{c}$, pressure altituthe $A$, and ambient temperature $t$.

## Determination of Position Error : $p$

Two calibration procedures, the pacer-aircraft method and the 'rrountcain?ra method, are used to iliustrate the determination of ip (i.e., $\mathbf{i}^{\prime}$ - : ). With the paces-aircraft method, the value of $p$ is derived from the calitrated installation on the pacer aircraft, while with the ground-eamera muthod, tie value of $p$ at che flight level is calculated from measurements ot $p$ u.d $T$ at the grotid and the assumption $\subset f$ a standard temperature yradient up to :ike flight level.

Pacer-aircraft metrod.- For the calculation $c: i p$ by this method, : 1 ; assumed that the alimeter indication in the test aircraft is 29 noo it at: fart the corrected altimeter indication in the pacer aircrait is 30 nou Et. :rmm

 of the test aircraft is then

$$
\begin{align*}
\therefore p & =p^{\prime}-p \\
& =639.962-628.433=11.529 \mathrm{lb} / \mathrm{ft}^{2}
\end{align*}
$$

 can also be durived from equation (3.6), irere express:-4 as

$$
\begin{equation*}
\therefore p=-\frac{g_{0}}{g} \bar{j}_{\mathrm{m}} \quad \therefore H \tag{31}
\end{equation*}
$$


 for an altitude increment of 400 ft :
 the value of $\therefore$ is then

$$
\therefore \mathrm{p}=\left(-0.028823:(-400)=11.529 \mathrm{lb} / \mathrm{ft}^{2}\right.
$$

## APPENDIX B

Ground-scmera method.- For the calculation of $\therefore$ p by this merhod, $1=$ assumed that (1) the pressure $p^{\prime}$ of the aircraft installation is measured an absolute-pressure recorder (in contrast to the statoscope used in the te des=ribed in chapter (X), and (2) that for the elevations in figure 9.10, $E_{c}=E_{r}$ and $h_{c}=h_{r}$.

It is further assumed that $h_{c}$ is 1000 ft , that the height of the ait: $\Delta Z$ above $h_{c}$ is 400 ft , and that the pressure measured by the absolutepressure recorder at the flight level is $1973 \mathrm{lb} / \mathrm{ft}^{2}$. The pressure $;$ and temperature $T$ at the ground (at $h_{c}$ ) are $20001 b / f^{2}$ and $500^{\circ}$ R. From table $A 2$ of appendix $A$, the standard pressure $P_{s}$ at 1000 f is $2040.85 \mathrm{l}:$ from table A4, the standard temperature $T_{s}$ at 1000 ft is $515.104^{\circ} \mathrm{K}$; and table A3, the standard density $\bar{\rho}_{s}$ at 1000 ft is $0.074261 \mathrm{lb} / \mathrm{ft}^{3}$ and the
 density $\bar{\jmath}$ at $h_{c}$ is

$$
\begin{aligned}
\bar{r} & =\bar{o}_{\mathbf{s}} \frac{\mathrm{pr}_{\mathbf{s}}}{\mathrm{P}_{\mathbf{S}}} \\
& =0.074261\left(\frac{200 \mathrm{C}}{2040.85}\right)\left(\frac{515.104}{500}\right)=0.0749731 b,^{\prime} \leqslant \mathrm{t}^{3}
\end{aligned}
$$

From equation (9.29), t.e density $\overline{\bar{r}}_{m}$ at the miduoint (1200 it) is

$$
\begin{aligned}
\overline{\bar{s}}_{m} & =\bar{i}-\left(\bar{\nu}_{s}-\overline{\bar{V}}_{s, m}\right) \\
& =0.074973-(0.074261-0.073825)=1) .07453716, t^{3}
\end{aligned}
$$

From equation (9.28), the pressure increment ipc corresponding to itwit: increment $\therefore$ iz

$$
\begin{aligned}
\therefore z_{c} & =-\bar{x}_{m} \therefore z \\
& =(-0.074537)(400)=-29.8 \mathrm{lz} / \mathrm{Et}^{2}
\end{aligned}
$$

 ground ( $\mathrm{S}_{\mathrm{c}}$ ), the value of p at $\mathrm{z}=1400$ ft is

$$
\begin{aligned}
: & =I_{h_{c}}-: p_{c} \\
& =2000-29.8:=197 i .21 b_{i}^{\prime} f t^{2}
\end{aligned}
$$

For the $\because a: j$ of $p$ of this example, the :osition etror $\because \quad \because \quad$ it installu=icn is then

$$
\begin{aligned}
\because & =p^{\prime}-p \\
& =1973-1970.2=2.8 \mathrm{lb}, \mathrm{t}^{2}
\end{aligned}
$$

APFENDIX B

Calculation of $V_{c}$ and $i b_{c}, H$ and $i H$, and $A$ and $A$
For these calculations, the indicated airspeed $V_{i}$, indicated altitude $\|^{\prime \prime}$. and indicated yach number $y^{\prime \prime}$ measured by the corkpit instrumerts are sorrested for the position ecror ip of the aircraft installation to yield val:des of $\%$.
 value of $A p$ are also calcilated.

It is assumed that $V_{i}$ is 300 knots, $H^{\prime}$ is 30000 Et . $\mathrm{Il}^{\prime}$ is 0.79 , and $\Delta p$ is 8 liofet $t^{2}$. From table Al2 of apyendix $A$, the impact pressure $G_{c}^{\prime}$ at 300 knots is $320.694 \mathrm{lb} / \mathrm{ft}^{2}$; and from table A2, the static pressure ?" at 30000 ft is $628.433 \mathrm{lb} / \mathrm{ft}^{2}$.

Calculation of $V_{c}$ and $\mathrm{IV}_{c}-$ From equation (9.20),

$$
\begin{align*}
q_{c} & =q_{c}^{\prime}+A_{p} & & \text { URHGNAL } P A G E ~  \tag{2.29}\\
& =320.694+8=328.694 \mathrm{lb} / \mathrm{ft}^{2} & & \text { OF POOR QCALIH }
\end{align*}
$$

Erom table Al2 of appendix $A$, the calibrated airspeed $\because$ sorevizunding to inis value of $q_{c}$ is 203.5 knots. From equation (5.9), the airspeed :rror is

$$
\begin{equation*}
\therefore v_{c}=v_{i}-v_{c} \tag{5.2}
\end{equation*}
$$

$=300-303.5=-3.5$ knuts

Calculation of $H$ and iH.- From equation (2.2).

$$
\begin{aligned}
p & =p^{\prime}-\therefore p \\
& =628.433-8=620.4331 b / 5 t^{2}
\end{aligned}
$$

$$
i \Xi \div)
$$

Erom table $A 2$ of appendix $A$, the altitude $A$ corresponding to tinis vilue : is 30281 ft . Erom equation (5.3), tie altitude error is

$$
\begin{equation*}
\therefore H=H^{\circ}-H \tag{5,-}
\end{equation*}
$$

$=30000-30.81=-281 \mathrm{ft}$




$$
\frac{q_{c}}{p}=\frac{320.694+8}{628.43 i-3}=0.5298
$$

## APPETDIX B

From table A26 of appendix A, the value of $: 1$ correspending to this $q_{c} / p$ is 0.804 . From equation (5.10), the Nach number error is

```
iM = M' - Y
    = 0.79-0.804 = -0.014
```

in the greceding examples, the signs of $\Delta V_{c}, \therefore 4$ and $\therefore$ are all nc: tive, when the sign of $\therefore \mathrm{ip}$ is positive. It is also trl: that when $\therefore \mathrm{p}$ is negative, $\Delta V_{C}, \Delta H$, and $\Delta M$ are positive.

In the preceding calculations, the ralues of $\therefore V_{n}, \therefore H$, and $\therefore \rightarrow$ have expressed in terms 0 Eerrors in the measured quantities. In many aircrate manuals, however, these errors are expressed in terms of corrections with s. opposite to those of the errors. An example of a flignt-manual correctior. for the airspeed and altitude errors of an airplane installation is present figure Bl.

## Calculation of $C_{L}$

As stated by squation (5.2), the lift coefficien: $C_{L}$ is expressed i: terms of the denamic pressure 4 , the aireraft weigit $N$, and the wing are. by the following equation:

$$
c_{L}=\frac{N}{4 S}
$$

From equation (5.3), the dynamic pressare $q$ is determined from values 0 : and $\because$ as follows:

$$
q=0.7 \mathrm{~m}^{2}
$$

Lor the following compration $o \mathcal{C}_{L}$, it is assumed that $\because_{i}=260$ knots,

 Erom, equation (9.20), =he value of ic is

$$
\begin{aligned}
& =237.341+6=243.84116, f t^{2}
\end{aligned}
$$

 Yolue of ! is

$$
\begin{aligned}
\because & -\because \\
& =785 \cdot 3: 8-5=779.308 \text { ib }:=
\end{aligned}
$$

The value of $q_{c} c^{\prime} p$ is then $\frac{243.84}{779.308}=0.3129$. From table $A 26$, the ?alue $c=A$ for this $q_{c} / p$ value is 0.636 , so that the vilue of $A^{2}$ is 0.4045 . From equation ( $B 5$ ), the vaiuc of $q$ is

$$
q=(0.7)(779.308)(0.4045)=220.7 \mathrm{lb} / \mathrm{ft}^{2}
$$

From cquation (5.2), the value of $c_{L}$ is then
ORIGiNal patied IE OF POOR QTAl.ITY

$$
c_{L}=\frac{172000}{(220.7)(2400)}=0.325
$$

Calculation of $v$
In this example, the true airspeed $V$ is ealculated for a calibrated airspead $V_{c}$ of 300 knots, a pressure altitude $H$ of 35000 ft , and an ambient temperature of $-60^{\circ} \mathrm{F}$. From table All the value of $\mathrm{q}_{\mathrm{c}}$ for 300 knots is $320.694 \mathrm{lb} / \mathrm{ft}^{2}$. From table a 2 the value of $p$ at 35000 ft is $497.956 \mathrm{li} / \mathrm{Et}^{2}$. The value of $q_{c} / p$ is then $\frac{220.694}{497.356}=0.64402$. From table A26 the value $c f(M$ correspending to $q_{c} / p=0.64+02$ is 0.87357 . From equation (3.27), the sieed of sound a in knots is

$$
\begin{equation*}
a=29.045 \sqrt{T} \tag{2.27}
\end{equation*}
$$

where the unit of $T$ is ${ }^{\circ}$. From table A 28 , the value of $T$ for $t=-0^{\circ} F$ is

$$
T=-60+459.67=399.67^{\circ} \mathrm{R}
$$

The value of a is then

$$
a=29.045 \sqrt{399.67}=(29.045)(19.992)=580.67 \text { knots }
$$

From equation (3.21),

$$
\because=\ddot{a}
$$

The value of $\because$ is tien

$$
\because=(0.87357)(580.67)=507.2 \text { knots }
$$

## APPENDIX 9

Part I: - Pressure Increments in the International System of inits
In tins sectizn, squations (3.3) and (3.4) are applied to determine seat pressure incr-ments in SI Uniss. With both equations, tie pressure increment $\therefore$ E E p a huigint ircrement $\therefore 2$ of 400 m is computed and compared with ralues
 $0: 7$ are the same as those in terms of $H$.

$$
\begin{gathered}
\text { Equation (3.3) is } \\
\operatorname{Sp}=-g-\therefore 2
\end{gathered}
$$

Fron table Alo, the value of 2 at 200 m is $1.2017 \mathrm{~kg} .^{3}{ }^{3}$. From table il of reforence Al of 3yeandix $A$, the ralue of 3 at 200 m is $9.8060 \mathrm{~m} / \mathrm{sec}^{2}$. The:. for $\quad \therefore 2=400 \mathrm{~m}$,

$$
\therefore p=(-9.3060)(1.2017)\left(4001=-4714 \mathrm{~kg} / \mathrm{m}-\sec ^{2}(\mathrm{~Pa})\right.
$$

Frc table Al5, the value of $\therefore \mathrm{P}$ as derived from the differential form of equation (3.3) is she sare, i.e., 96611 - $101325=-4714 \mathrm{~Pa}$.

Equation (3.4) can be written as

$$
\therefore c=-4 \frac{\square}{\therefore T} \therefore Z
$$

Erom table $I$ of reference al of appendix $A$, the value $s=y$ at $200 \mathrm{~m} i=$ $3.8060 \mathrm{~m}, \mathrm{sec}^{2}$. From table A15, the valun of $p$ at 200 m is 98945.3 pa ( $\mathrm{kg} / \mathrm{m}-\mathrm{sec}^{2}$ ). Erom table A 28 , the value of R is $0.28: 05 \times 10^{3} \mathrm{~J} / \mathrm{O}_{\mathrm{K}-\mathrm{kmol}}$. Erom table Al7, the value of $t$ at 200 m is $13.70^{\circ} \mathrm{C}$. From table A28. the value of $T$ is $13.70+273.15=285.85^{\circ} \%$. Then, for $\therefore 2=400 \mathrm{~m}$,

$$
\therefore=(-3.8060) \frac{98945.3}{(287.05)(286.85)}(400) \div-i 713 \therefore \varepsilon, \mathrm{Km}_{\mathrm{sec}}{ }^{2} \text { (pa) }
$$

Erom thble Ais, the value of $\therefore \mathrm{y}$ is esscntiaily the same, tiat is, $36611-101325=-4714$ Pa.

Tie other form of pruation (3.4) can be iriteon 3:

$$
\therefore p=-\frac{n}{\square 2} \therefore z
$$

## APPENDIX B

The values of $p, t$, and $T$ remain the same. From table $A 28$, the value of $\bar{R}$ is $29.271 \mathrm{~m}-\mathrm{kg} /{ }^{\mathrm{K}} \mathrm{K}-\mathrm{kmol}$. Then, for $\mathrm{dz}=400 \mathrm{~m}$,

$$
\Delta p=\frac{-98945.3}{(29.271)(286.85)}(400)=-4714 \mathrm{~kg} / \mathrm{m}-\sec ^{2}(\mathrm{~Pa})
$$

As in the previous cases, the value of $\Delta p$ from table Al5 is $\mathbf{- 4 7 1 4} \mathrm{Pa}$.

## Part III - Pressure-System Lag and Leaks

In tris section, sample calculations are presented for the determination of (1) the airspeed and altitude errors due to the pressure lag of a staticpressure system and (2) the altitude error resulting from a leak in that system.

## Calculation of Airspeed and Altitude Errors Due to Pressure Lag

In this example, the airspeed and altitude errors of a static-pressure system are determined for an indicated airspeed of 300 knots in a climb of $12000 \mathrm{ft} / \mathrm{min}$ at an altitude of 30000 ft . The system consists of four cockpit instruments (having a combined volume of $100 \mathrm{in}^{3}$ ) connected to a $50-\mathrm{ft}$ length of tubing $3 / 16 \mathrm{in}$. ( 0.188 in. ) in inside diameter (I.D.). From equation (10.3), the lag constant $\lambda$ is

$$
\lambda=\frac{128 u L C}{\pi d^{4} p}
$$

## 'INGinal page is


$F=0 m$ table $A 6$ of appendix $A$, the value of $\mu$ at 30666 ft is $3 . i J 6 \times 10^{-7} \mathrm{lb}-\mathrm{sec} / \mathrm{ft}^{2}$. From table A2, the value of p at 30000 ft is $628.433 \mathrm{lb} / \mathrm{ft}^{2}$. The value of C in cubic feet is 0.05787 , the value of d in feet is 0.01567 , and the value of $L$ is 50 ft . From equation (10.3), the lag constant $\lambda$ at 30000 ft is ther.

$$
\lambda=\frac{128\left(3.106 \times 10^{-7}\right)(50)(0.05787)}{3.1416(0.01567)^{4}(628.433)}=1.0 \mathrm{sec}
$$

From equation (10.2), the pressure drop $\Delta p$ is

$$
\begin{equation*}
\Delta p=\lambda \frac{d p}{d t} \tag{10.2}
\end{equation*}
$$

From table A2 of appendix A, a $100-\mathrm{ft}$ increment at 30000 ft corresponds to a pressure increment of $2.86 \mathrm{lb} / \mathrm{ft}^{2}$. Since the rate of climb is $12000 \mathrm{ft} / \mathrm{min}$
(or $200 \mathrm{ft} / \mathrm{scc}$ ), dp/dt is (2)(2.86) or $5.72\left(1 \mathrm{~b} / \mathrm{ft}^{2}\right) / \mathrm{sec}$. From the value of $\lambda$ of 1.0 sec , the value of $\Delta p$ is

$$
i p=(1.0)(5.72)=5.72 \mathrm{lb} / \mathrm{ft}^{2}
$$

From table A2 of appendix A, the altitude increment at 30000 ft corresponding to a pressure increment of $5.72 \mathrm{lb} / \mathrm{ft}^{2}$ is 200 ft . Thus, the altitude error for a rate of climb of $12000 \mathrm{ft} / \mathrm{min}$ at 30000 ft is 200 ft . From table Al2 of appendix $A$, the airspeed increment at 300 knots corresponding to a pressure increment of $5.72 \mathrm{lb} / \mathrm{ft}^{2}$ is 2.5 knots. Thus, the airspeed error for a rate of c].imb of $12000 \mathrm{f}=/ \mathrm{min}$ at 30000 ft is 2.5 knots.

To determine whether the conditions of this example meet the requirement for laminar flow as stated by equation (10.6), the pressure drop per foot must be determined. Since the pressure drop $\Delta p$ is $5.72 \mathrm{lb} / \mathrm{ft}^{2}$ and the length of tubing is 50 ft , the pressure drop per foot is 0.1 ( $1 \mathrm{~b} / \mathrm{ft} \mathrm{t}^{2}$ )/ft. From table 10.1, the limiting value of $\Delta p / L$ for laminar flow in 0.188-in. I.D. tubing at 30000 ft is $2.3\left(\mathrm{lb} / \mathrm{ft}^{2}\right) / \mathrm{ft}$. Thus, since the $\Delta \mathrm{p} / \mathrm{L}$ value of this example is only 5 percent of the limiting value, the flow can be considered laminar.

## Calculation of Altitude Error Due to a Leak

For this example, it is assumed that tine instrument system is the same as that used in the lag calculations (namely, four cockpit instruments connected to a $50-f t$ length of $3 / 16-i n$. I.D. tubing). It is also assumed (1) that in a ground test of the system at a test pressure corresponding to an altitude of 40000 ft , the system was determined to have a leak rate equivalent to a rate of change of altitude of $100 \mathrm{ft} / \mathrm{min}$ and (2) that the leak is located in the cockpit.

To determine the altitude error that would be caused by this leak, it is assumet that the aircraft is at an altitude of 30000 ft and that the cabin pressure corresponds to an altitude of 5000 ft. The pressures for this flight condition and the pressures involved in the ground test of the system are shown in the diagrams in figur. $B 2$.

From equation (10.7), the lag constant $\lambda_{l}$ of the leak is

$$
\begin{equation*}
\lambda_{l}=\left(\frac{P_{\mathrm{T}, \mathrm{o}}-\mathrm{P}_{\mathrm{T}, \mathrm{a}}}{\mathrm{dp} / \mathrm{dt}}\right)\left(\frac{\mathrm{P}_{\mathrm{T}, \mathrm{O}}+\mathrm{P}_{\mathrm{T}, \mathrm{a}}}{\mathrm{P}_{\mathrm{c}}+\mathrm{P}_{\mathrm{a}}}\right) \tag{10.7}
\end{equation*}
$$

From table Al of appendix A,

$$
\begin{aligned}
& P_{T, O} \text { at sea level is } 2116.22 \mathrm{lb} / \mathrm{ft}^{2} \\
& \mathrm{P}_{\mathrm{T}, \mathrm{a}} \text { at } 40000 \mathrm{ft} \text { is } 391.683 \mathrm{lb} / \mathrm{ft}^{2}
\end{aligned}
$$

APPENDIX B
$P_{a}$ at 30000 ft is $628.433 \mathrm{lb} / \mathrm{ft}^{2}$
$P_{c}$ at 5000 ft is $1760.79 \mathrm{lb} / \mathrm{ft}^{2}$
Also from table A2, the pressure increment corresponding to an altitude increment of 100 ft at 40000 ft is $1.88 \mathrm{lb} /: \mathrm{t}^{2}$. The pressure rate $\mathrm{dp} / \mathrm{dt}$ corresponding to a leak rate of $100 \mathrm{ft} / \mathrm{min}$ is thus $1.88\left(1 \mathrm{~b} / \mathrm{ft}^{2}\right) / \mathrm{min}$ or $0.0314\left(1 \mathrm{~b} / \mathrm{ft}^{2}\right) / \mathrm{sec}$. The lag constant of the leak is then

$$
\lambda_{l}=\left(\frac{2116.22-391.683}{0.0314}\right)\left(\frac{2116.22+391.683}{1760.79+628.433}\right)=57650 \mathrm{sec}
$$

From equarion (10.8), the pressure error $\Delta p_{l}$ due to the leak is

$$
\begin{equation*}
\Delta p_{l}=\frac{\lambda}{\lambda_{l}+\lambda}\left(p_{c}-p_{a}\right) \tag{10.8}
\end{equation*}
$$

For a system lag $\lambda$ of 1.0 sec at 30000 ft , the value of $\Delta \mathrm{p}_{2}$ is

$$
\Delta \mathrm{p}_{2}=\left(\frac{1.0}{57650+1.0}\right)(1760.79-628.433)=0.021 \mathrm{~b} / \mathrm{f} \mathrm{t}^{2}
$$

From table A2 of appendix A, the pressure increment corresponding to a $1-\mathrm{ft}$ increment at 30000 ft is $0.028 \mathrm{lb} / \mathrm{ft}^{2}$. Thus the altitude error corresponding to a $\Delta p_{l}$ of $0.02 \mathrm{lb} / \mathrm{ft}^{2}$ is less than 1 ft .



Figure Bl.- Flight-manual correction charts for the airspeed and altitude errors of the static-pressure installation of an airplane. These correction charts are used to determine the indicated airspeed and indicated altitude at which the airplane should $f l y$ to achieve a desired calibrated airspeed and pressure altitude.

APPENCIX B


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