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WORKBOOK FOR PREDICTING PRESSURE WAVE AND FRAGMENT EFFECTS OF EXPLODING PROPELLANT TANKS AND GAS STORAGE VESSELS


SOUTHWEST RESEARCH INSTITUTE
San Antonio, Texas 78284

prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LEWIS RESEARCH CENTER
AEROSPACE SAFETY RESEARCH AND DATA INSTITUTE
CLEVELAND, OHIO 44135

R. D. Siewert, Project Manager

Contract NAS 3-19231
This workbook is intended to provide the designer and the safety engineer with the best available technology that they need to predict damage and hazards from explosions of propellant tanks and bursts of pressure vessels, both near and far from these explosion sources. The information is presented in the form of graphs, tables, and nomographs to allow easy calculation without resort to difficult mathematical manipulation or the use of extensive computer programs. When complex methods have been used to develop simple prediction aids, they are fully described in appendices.

Topics covered in various chapters are:

1) Estimation of explosive yield
2) Characteristics of pressure waves
3) Effects of pressure waves
4) Characteristics of fragments
5) Effects of fragments
6) Blast assessment and integrated effects.

Short chapters, with discussion of results, conclusions, and recommendations for further work are also included.
FOREWORD

Many staff members at Southwest Research Institute in addition to the authors contributed substantially to the work reported here. The authors gratefully acknowledge the special contributions of the following:

- Mr. M. A. Sissung, for organization and editing of the workbook,
- Miss C. A. Hajovsky, for extensive data reduction and curve plotting.

The technical support of our project manager at NASA-Lewis Research Center, Dr. Robert D. Siewert, contributed materially to the success of this work. In particular, he provided excellent technical guidance throughout the contract, and was also able to obtain for us the needed input data for all of the example cases presented in Chapter VI, and references to other related work.
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SUMMARY

This workbook is intended to provide the designer and the safety engineer with the best available technology that they need to predict damage and hazards from explosions of propellant tanks and bursts of pressure vessels, both near and far from these explosion sources. The information is presented in the form of graphs, tables, and nomographs to allow easy calculation without recourse to difficult mathematical manipulation or the use of extensive computer programs. When complex methods have been used to develop simple prediction aids, they are fully described in appendices.

Topics covered in various chapters are:

1. Estimation of explosive yield
2. Characteristics of pressure waves
3. Effects of pressure waves
4. Characteristics of fragments
5. Effects of fragments
6. Risk assessment and integrated effects.

Short chapters giving discussion of results, conclusions, and recommendations for further work are also included.
INTRODUCTION

Nature of the Hazards

The likelihood of accidental explosions, in the various activities that involve liquid propellants for space vehicles, can best be decreased by improvements in design practices and operating procedures. Over the years, the frequency of occurrence of accidental explosions in the space programs has decreased with advances in technology. Nevertheless, the possibility of space vehicle fuel/oxidizer explosions or pressure bursts will always exist, especially with reusable propulsion systems that must be more reliable compared with those in the "one-shot" space vehicle. Excessive cyclic stresses, wear of moving parts and the accumulation of contaminants are some of the factors that could contribute to component malfunctions or material failures during the lifetime use of such systems. These malfunctions or failures could, in turn, contribute to accidental explosions with risk of damage to facilities and hazards to people. Thus, it becomes important to predict the explosive yield and the effects of pressure wave and fragments in a quantitative manner.

It is the intent here to provide the designer and the safety engineer with the best available technology that they need to predict damage and hazards from explosions of propellant tanks and bursts of pressure vessels in the near and far fields of interest.

In a launch configuration within tankage in a rocket motor, liquid propellants and nonreacting gases are initially contained within vessels of various sizes, geometries, and strengths. Various modes of failure of these vessels are possible, from either internal or external stimuli. If the vessel is pressurized with static internal pressure, one possible mode of failure is simply fracture, instituted at a critical size flaw and propagated throughout the vessel. A similar kind of failure can occur if the vessel is accidentally immersed in a fire, and pressure increases internally because of vaporization of the internal propellant. Some launch vehicles have the liquid fuel and oxidizer separated by a common bulkhead. Accidental over-pressurization of one of these chambers can cause rupture of this bulkhead, and subsequent mixing and explosion of the propellant. External stimuli that can cause vessel failure include high-speed impact by foreign objects, accidental detonation of the warhead of a missile, dropping of a tank to the ground (as in toppling of a missile on the launch pad), as well as many other external sources. Vessel failure can result in an immediate release of energy or it can cause subsequent energy release because of mixing of propellant and oxidizer and subsequent ignition. Other modes of failure which have resulted or could result in violent explosions are fall-back immediately after launch due to loss of thrust, or low-level failure of the guidance system after launch resulting in impact.
into the ground at several hundred feet per second.

Failure of a vessel containing liquid propellants or compressed gas can result in various levels of energy release, ranging from negligible to the full heat value of the combined propellant and oxidizer, or full value of stored energy in the compressed gas. Toward the lower end of the scale of energy release might be the failure of a pressurized vessel due to ductile crack propagation. Here, the stored pressure energy within the compressed propellant or gas in an ullage volume above the propellant could split the vessel or generate a weak blast wave. In the intermediate range of energy releases could lie vessel failure by external stimulus and ignition, either very rapidly or at very late times, so that only small proportions of mixed propellant and oxidizer contribute to the energy release. In this intermediate range could also lie the rapid fracture of gas storage vessels after heating or very rapid crack propagation. At the upper end of the scale could be the explosion in a vessel wherein a premixed propellant and oxidizer detonate in much the same fashion as a high explosive, and explosions resulting after violent impact with the ground. In past studies of possible blast and fragmentation effects from vessel rupture, a critical problem has been to accurately assess the energy release as a result of the accident or incident. A common method of assessment of possible energy release or correlation of the results of experiments has been to assess the energy release on the basis of equivalent pounds of TNT. This method is used because a large body of experimental data and theoretical analyses exist for blast waves generated by TNT or other solid explosives (refs. 1 and 2). Although the comparison with TNT is convenient, the correlation is far from exact. Specific energies which can be released, i.e., energy per unit volume or mass of material, differ quite widely between TNT, various liquid propellants or mixtures of liquid propellants and oxidizers, and gases stored in pressure vessels. The characteristics of damaging blast waves from explosions which can occur in flight vehicle accidents can therefore be quite different from blast waves from TNT explosions. The accidental explosions usually generate waves with lower amplitudes (peak overpressures) and longer durations than equivalent energy TNT explosions, at least close to the explosion source. Reference 3 discusses in some detail the blast waves from accidental explosions of the classes covered in this handbook.

Dependent on the total energy release and the rate of this energy release, the sizes and shapes of fragments generated by bursting liquid propellant vessels and their appurtenances, and bursting gas vessels, cover a very wide spectrum. At one extreme is the case of a vessel bursting because of seam failure or crack propagation from a flaw wherein only one "fragment" is generated, the vessel itself. This fragment, from a very slow reaction, can be propelled by releasing the contents of the vessel. At the other extreme is the conversion of the vessel and parts near it into a cloud of small fragments by an explosion of the contents of a vessel at a
very rapid rate, similar to a TNT explosion (refs. 4 and 5). For most accidental vessel failures, the distribution of fragment masses and shapes undoubtedly lies between these two extremes. The modes of failure of the vessel may be dependent upon details of construction and the metallurgy of the vessel material. Some of the masses and shapes are dictated by the masses and shapes of attached or nearby appurtenances. In any event, assessment and prediction of these parameters is much more difficult than is true for the rather understood phenomenon of shell casing fragmentation.

Once the masses, shapes, and initial velocities of fragments from liquid propellant vessels or bursting gas vessels have been determined in some manner, then the trajectories of these fragments and their losses in velocity due to air drag or perforation or penetration of various materials must be computed. This problem is primarily one of exterior ballistics. It differs from conventional exterior ballistics studies of trajectories of projectiles, bombs, or missiles in that the body in flight is invariably very irregular in shape and can be tumbling violently. Exact trajectories cannot be determined then in the same sense that they can be for well-designed projectiles. Only approximate trajectories can be estimated, usually by assuming relatively simple geometric shapes, such as spheres, discs or cylinders, for which exterior ballistics data and techniques exist. But, in some fashion, one can predict the ranges and impact velocities for fragments which were initially projected in specified directions from the bursting vessel with specified initial velocities.

This problem is not complete until one can assess the effects of blast waves and fragments from the accident on various "targets." For a proper assessment of hazards, one should consider a wide variety of targets, including human beings, various classes of buildings, vehicles, and perhaps even aircraft. Problems of fragment damage are exceedingly complex, not only because of the inherent statistical nature of the characteristics of the impacting fragments but also because the terminal ballistic effects for large irregular objects impacting any of the targets described are not very well known. In most past studies of fragment damage from accidents, the investigators have been content to simply locate and approximate the size and mass of the fragments in impact areas and have ignored the important problem of the terminal ballistic effect of these fragments. Prediction of blast damage is also not simple, but much more work has been done and reasonable estimates can be made for most structures and for humans, provided the characteristics of the blast waves can be defined (refs. 6 and 7).

Means for Assessment of Risk

This handbook is directed toward estimation of blast and fragmentation effects of accidental explosions in flight vehicles. It is not intended to encompass the entire problem of risk assessment for launch and operation
of these vehicles. This more comprehensive task includes the estimation of the probability of occurrence of various types of failures or accidents, and must employ methods such as failure mode analysis or fault-tree analysis to obtain such probabilities. In this handbook, it is assumed that a specific explosive accident has occurred, and the hazardous effects of that accident are predicted. A brief discussion of methods of risk assessment is given in Chapter 6.

Scope and Significance of Material Presented

The material presented in the handbook is based on a previous study of fragmentation from bursting propellant vessels (ref. 8) and on the literature on characteristics and effects of blast waves and fragment impact. Methods are given for predicting the damage to facilities and hazards to people from exploding liquid propellant tanks or bursting gas storage bottles. Various chapters present material which allows estimation of explosive yield or energy for a variety of propellant explosions and gas vessel bursts, give predictions of characteristics of pressure waves from these explosions, and present techniques for making damage estimates for structures and facilities, and mortality or injury to people subjected to the blast waves. Other chapters include estimates of fragment initial velocities and the statistics of mass and shape, terminal velocities and impact conditions, and effects of such impact on facilities, structures and people. Throughout the workbook, presentations are made in the form of scaled graphs, equations, nomographs or tables which allow easy calculation without recourse to difficult mathematical manipulation or use of extensive computer programs. When such methods have been used to develop simple prediction aids, they are fully described in appendices.

It is believed that this workbook is the first to provide safety engineers with relatively simple yet comprehensive methods for estimating blast and fragment hazards for accidental explosions in liquid-propellant fueled flight vehicles. Some methods for estimating blast yield for classes of liquid propellant accidents are given in ref. 9, and ref. 10 discusses blast and fragmentation from such explosions. But, neither of these references allows estimation of fragment characteristics and effects for liquid propellant explosions, nor do they treat gas vessel bursts. Special features not seen elsewhere are the prediction of blast wave characteristics for gas vessel bursts, effects of fragment impact on structures and facilities, and extensive application of the pressure-impulse (P-I) damage concept to a wide variety of structures and to humans.
Intended Purpose and Limits of Use

The purpose of the workbook is to provide typical safety engineers, with training at the bachelor's degree level in some engineering specialty, with methods for rapid estimation of blast and fragment hazards from accidental explosions in flight vehicles. It should require only a desk or pocket calculator or slide rule to perform any of the needed calculations. There are, of course, a number of limits to the calculations and their applicability which the user should observe. Because almost all of the data he will use are graphical, these limits will often be self-evident from the extreme values on the graphs. In general, one should not extend or extrapolate these graphs, but should instead merely report that prediction is not possible if input parameters fall outside the range of the plot.

Factors of safety are included in the prediction methods in various ways. When curves are based on experiments, error bands are usually given. Use of average curves through the data will give most probable values for such loading parameters as blast overpressure and impulse; use of the upper limits of the error band will assure conservatism by encompassing all of the extreme values in the measured data rather than the most probable. Most of the fragment data must be presented statistically. The user is often given a choice of several regression lines through the data. Choice of such a line with a very high probability of, say, predicting that all fragments less than a certain mass will fall to earth within a given distance, will assure a high factor of safety in estimating exclusion distances for possible fragment damage. In estimating effects of blast and fragments, factors of safety are included by estimating different degrees of damage given blast envelopment or fragment impact. For structures, estimates can be made for lower limits to damage (threshold of no damage at all) through quite severe structural damage to buildings, vehicles, etc. For people, estimates can be made for threshold of ear damage through 1% chance of mortality to 99% chance of mortality. For estimation methods which are based on sparse data or analysis, we have large bands of uncertainty—the user should apply upper limits of these bands, if in doubt.

Applications to Areas Other Than Aerospace Rocket Launch and Research Facilities

This workbook was prepared primarily for use by safety engineers and site planners at aerospace rocket launch and research facilities. It emphasizes the blast and fragment hazards which could occur at such sites, and the prediction of their damaging effects. The prediction of blast and fragment hazards is specific for liquid propellant explosions and gas bottle bursts. The common use in explosive safety circles of conversion to TNT equivalency is nearly completely avoided, so the workbook cannot be easily used to predict hazards from detonations of condensed explosives such as TNT. On the other hand, the methods given in Chapters III, V and VI for
prediction of damaging effects are quite independent of the methods for estimating the hazards. Blast damage predictions could as easily be made for TNT or nuclear explosions as for propellant explosions, provided the blast wave characteristics were defined. Similarly, fragment damage predictions are independent of the sources of the fragments, and only depend on a knowledge of the impact conditions. Indeed, the treatment of damage effects is much more extensive than one would be apt to find in any other single document, and could find much wider use than for damage prediction near aerospace test and launch facilities.

Additional Areas of Research

The bases for the prediction methods given in this workbook range from a firm foundation of extensive testing and analysis, through analyses supported by limited testing or accident reporting, to some predictions which are quite speculative because of little corroborating evidence. Predictions in the latter case could often be improved by additional research.

Areas in which we feel there is a pressing need for additional study are:

1. Definition of fragmentation characteristics for bursting gas storage bottles. Existing data consist of only five tests for one bottle geometry and material, and one gas.

2. Definition of blast wave characteristics for burst of cylindrical gas storage vessels, either analytically or experimentally. Present methods are limited to essentially spherically symmetric cases.

3. Better definition of fragment impact effects on a variety of structures and facilities, for fragments typical of those occurring in aerospace vehicle explosions. Most fragment impact data or methods developed to date are related to high velocity, small mass penetrators which are not typical of accidentally produced fragments.

4. Extension of the present work to accidental explosions in thick-walled storage vessels typical of ground transport and storage vessels. The current work is directed toward explosions of lightweight hardware. Blast and fragmentation characteristics can be drastically different for heavier vessels.

In Chapter IX, we discuss these and other areas for further work in more detail.
LIST OF REFERENCES


CHAPTER I

ESTIMATES OF EXPLOSIVE YIELD

I-1 Explosive Yield as a Function of Propellant Type and Accident Conditions

I-1.1 General Discussion of Propellant Explosives

Accidents involving liquid propellant rockets, both during static firing on a test stand and during launch, have shown that liquid propellants can generate violent explosions. These explosions "drive" air blast waves, which can cause direct damage and can accelerate fragments or nearby objects. In fact, the specific energies of liquid rocket propellants, in stoichiometric mixtures, are significantly greater than for TNT (specific energy is energy per unit mass). The estimation of explosive yield, the energy released during the explosion, is a prerequisite to the determination of expected damage resulting from the explosion. Before delving into the method of calculating explosive yield, however, a general discussion of the characteristics of propellant explosions will help the reader understand the complexities involved in the determination of explosive yield.

One extremely important fundamental fact concerning liquid propellants is that their potential explosive yield is very high, but their actual yield is much lower. This situation occurs because the propellant and oxidizer are never intimately mixed in the proper proportions before ignition. The degree of confinement of propellant and oxidizer can also seriously affect the actual explosive yield of liquid propellants. For example, a liquid propellant mixture could conceivably explode inside a storage vessel or could leak out of a containment vessel and form a shallow pool of large lateral extent before detonation. Each case produces different values for explosive yield. Presently, there are at least four methods for estimating yield from liquid propellant explosions which, unfortunately, do not necessarily give the same predictions: One method is based on Project PYRO results, and two of the others are the "Seven Chart Approach" and the "Mathematical Model" of Farber and Deese. The fourth approach, which is really based on the previous three methods, was developed by Baker, et al., and is easy to use and readily adaptable to the calculation of explosive yield. For further information concerning the development of this method, Reference 1 is recommended.

From the test results reported in Reference 2 and 6 through 8, a number of observations can be made regarding blast yields from liquid
propellant explosions. (1)

(1) Yield is quite dependent on the particular fuel and oxidizer being mixed.

(2) The yield is very dependent on the mode of mixing of fuel and oxidizer, i.e., on the type of accident which is simulated. Maximum yields are experienced when intimate mixing is accomplished before ignition.

(3) On many of the liquid hydrogen/liquid oxygen (LH₂/LO₂) tests (regardless of investigators), spontaneous ignition occurred very early in the mixing process, resulting in very low percentage yields.

(4) Yield is very dependent on time of ignition, even ignoring the possibility of spontaneous ignition.

(5) Blast yield per unit mass of propellant decreases as total propellant mass increases.

(6) Variability in yields for supposedly identical tests was great, compared to variability in blast measurements of conventional explosives.

1-1.2 Scaled Curves for Explosive Yield for Various Propellants and Types of Accidents

If a blast source is placed on or near a reflecting surface, such as the ground, then the initial shock is very quickly reflected and the reflected wave merges with the incident wave so rapidly that a single, strengthened blast wave is formed. The characteristics of this single wave are often almost identical with the characteristics of blast waves in free-air experiments, except that the blast source appears to have greater energy than for free-air tests. The proportion of energy reflected from the ground is a function of how perfect a reflector it is, that is, how little energy is imparted to the ground in cratering, ground shock, and so forth. If the ground were a perfectly rigid surface, then the equivalent free-air energy driving the air blast wave would be \( E' = 2E \). The other extreme case is that of a perfect absorber, for which \( E' = E \). All actual tests will have equivalent free-air energies lying between these limits.

All of the PYRO experiments, on which the prediction curves in this section are based, were conducted on the ground surface, with no cratering. When the curves are used to predict blast yields for explosions occurring in flight or far enough above the ground that the imme-
diate reflection discussed does not occur, one must account for the absence of the "perfect" reflecting surface. This is done by dividing the blast yields calculated from curves in this chapter by a factor of two.

1-1.2.1 Terminology

In this document, three types of fuel and oxidizer combinations and three different modes of mixing will be considered. The three types of propellants are:

1. The hypergolic propellant which is in widest use. A fuel of 50% N₂H₄ - 50% UDMH and an oxidizer of N₂O₄ in a mass ratio of 1/2.

2. Liquid Oxygen-Hydrocarbon - This propellant uses kerosene (RP-1) as a fuel and liquid oxygen (LO₂) as the oxidizer in stoichiometric mass ratio of 1/2. 25.

3. Liquid Oxygen-Liquid Hydrogen - This propellant is an entirely cryogenic combination of liquid hydrogen (LH₂) fuel and liquid oxygen (LO₂) oxidizer in stoichiometric mass ratio of 1/5.

The three modes of mixing (failure modes) discussed are:

1. Confinement by Missile (CBM) - This type of accident consists of failure of an interior bulkhead separating fuel and oxidizer in a missile stage.

2. Confinement by Ground Surface (CBGS) - This type of accident includes impacts at various velocities (e.g., fall back on the launch pad) of the missile on the ground, with all tankage ruptured, and subsequent ignition.

3. High Velocity Impact (HVI) - This type of accident involves high velocity impact of a missile after launch.

1-1.2.2 Methods for Calculating Explosive Yield

Some parameters which become important in determining explosive yield are the type of propellant, the failure mode and in some cases, ignition time, impact velocity, and type of surface impacted. It is important to keep in mind, however, that blast yield as a percent or fraction of energy available decreases as total combined mass of propellant and oxidizer increases. Figure 1-1, which is a normalized plot for
Multipler Factors:

(1) Hypergolic - 240%
(2) $\text{LO}_2$/RP-1 - 125%
(3) $\text{LO}_2$/LH$_2$ - 370%

$1\text{b}_m = \text{kg} \times 2.2$

Figure 1-1. Estimated Terminal Yield as a Function of Combined Propellant and Oxidizer Mass (Ref. 9)
all propellants, depicts this relationship and should be used as an upper limit for percent explosive yield. Since this is a normalized plot, the percent yield for a particular propellant can be determined by obtaining the normalized fractional value from the graph and multiplying by the multiplier factor for the particular propellant under investigation. These multiplier factors, shown on the figure, are:

- **Hypergolic** (50% N₂H₄ - 50% UDMH fuel and N₂O₄ oxidizer in mass ratio of 1/2) - 240% (10)
- **Liquid oxygen-hydrocarbon** (RP-1 fuel LO₂ oxidizer in mass ratio of 1/2.25) - 125% (6)
- **Liquid oxygen-liquid hydrogen** (LH₂ fuel and LO₂ oxidizer in mass ratio of 1/5) - 370% (6)

Careful examination of Figure 1-1 and the multiplier factors indicate that explosive yield can be greater than 100 percent. Also, in certain cases, explosive yield will be greater than 100 percent when calculated by other methods. This anomaly occurs because explosive yield, as it is used here, is really terminal yield, or yield based on "TNT equivalence". Since the specific energies of the liquid propellants involved are greater than the specific energy of TNT, terminal (TNT equivalent) yield can be greater than 100 percent. Calculations were done in this manner to correlate with other methods discussed in subsequent paragraphs. Whenever the value of percent explosive (terminal) yield determined by these other methods exceeds the value of percent explosive (terminal) yield determined by using Figure 1-1, the value from Figure 1-1 is the correct choice.

1. Hypergolic materials, by definition, ignite spontaneously on contact, so it is not possible to obtain appreciable mixing before ignition unless the fuel and oxidizer are thrown violently together. Ignition time is therefore not an important determinant of blast yield for hypergolics, but impact velocity and degree of confinement after impact are important factors. If a CBM or CBGS failure mode is being considered, percent explosive yield can be acquired from Table 1-1. If a HVI failure mode is assumed, then percent explosive yield can be determined from Figure 1-2. The percent yield determined by any one of these methods must then be compared to the percent yield determined from the weight of the propellant (Figure 1-1). The smaller of the two is the correct choice.
Figure 1-2. Terminal Yield Vs Impact Velocity for Hypergolic HVI (Ref. 3)

(\text{ft/sec} = \text{m/s} \times 3.281)
TABLE 1-1. ESTIMATE OF TERMINAL YIELD FOR HYPERGOLIC CBM AND CBGS (REF. 3)

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Terminal Yield Range (%)</th>
<th>Estimated Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm rupture (CBM)</td>
<td>0.01 - 0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>Spill (CBGS)</td>
<td>0.02 - 0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Small explosive donor</td>
<td>0.8 - 1.2</td>
<td>2</td>
</tr>
<tr>
<td>Large explosive donor</td>
<td>3.4 - 3.7</td>
<td>5</td>
</tr>
<tr>
<td>Command destruct</td>
<td>0.3 - 0.35</td>
<td>0.5</td>
</tr>
<tr>
<td>310-ft drop (CBGS)</td>
<td>1.5</td>
<td>3</td>
</tr>
</tbody>
</table>

(2) Because liquid oxygen/hydrocarbon propellants are not hypergolic, considerable mixing can occur in various types of accidents, and time of ignition after onset of mixing is an important determinant of blast yield. For the case of mixing and an explosion within the missile tankage (CBM), percent explosive yield can be determined by assuming an ignition time and then examining Figure 1-3. For simulated fall-back on the launch pad (CBGS), impact velocity as well as ignition time are important parameters in estimating blast yield. A two-step approach has been developed to calculate blast yield. After assuming an impact velocity, maximum percent yield $Y_m$ can be determined for Equation (1-1):

$$Y_m = 5\% + \frac{(6.82\%)}{(m/s)} U_1, \quad 0 \leq U_1 \leq 16.8 \text{ m/s} \quad (1-1)$$

* A word of explanation will help clarify the meaning of the central solid line and shaded area of this graph and similar subsequent graphs. The shaded portion represents an area in which data from actual propellant blasts was found. The central solid line is an estimate of the most likely occurrence and, for most cases, is the recommended choice. Conservative estimates of explosive yield can be made by choosing the uppermost boundary of the shaded area. The vertical depth of the shaded area at any abscissa indicates the total range of data, and therefore the total uncertainty in the estimate.
Figure 1-3. Terminal Yield vs Ignition Time for I.O₂/8P-1 GNM
where \( Y_m \) is expressed in percent and \( U_i \) is in meters per second. Percent explosive yield can then be determined from \( Y_m \), an estimate of ignition time, and Figure 1-4. The determination of explosive yield for the HVI failure mode is somewhat simpler because there is little ignition delay and therefore only impact velocity affects yield. Thus, blast yield can be acquired by using Figure 1-5 directly. The percent yield determined by any one of these methods must then be compared to the percent yield determined from the weight of the propellant (Figure 1-1). The smaller of the two is the correct choice.

The determination of explosive yield, for the entirely cryogenic combination of liquid hydrogen (LH\(_2\)) fuel and liquid oxygen (LO\(_2\)) oxidizer is similar to that of liquid oxygen-hydrocarbon propellants. For the CMH case, it is necessary for one to assume an ignition time and then use Figure 1-6 to find explosive yield. For the CBCS case, an impact velocity is assumed and maximum percent yield \( Y_m \) can be determined from Equation (1-2):

\[
Y_m = 10\% + \frac{1}{10} \left( \frac{4.25}{U_i} \right) U_i \quad 0 \leq U_i \leq 24.4 \text{ m/s}
\]  

where \( Y_m \) is expressed in percent and \( U_i \) is in meters per second. Percent explosive yield can then be determined from \( Y_m \), an estimate of ignition time and Figure 1-7.

For high velocity impact (HVI) of this propellant, the blast yield is dependent only on the impact velocity and can be acquired from Figure 1-8 directly. The percent yield determined by any one of these methods must then be compared to the percent yield determined from the weight of the propellant (Figure 1-1). The smaller of the two is the correct choice.

Table 1-2 has been prepared to alleviate the necessity of re-reading the preceding presentation each time a value of explosive yield must be determined. To use the table, all one needs to do is identify the type of propellant and type of accident. Then the proper sequence in "Part 1" should be followed after making the necessary assumptions (e.g., ignition time or impact velocity and type of surface impacted) to arrive at a value for explosive yield. Explosive yield should then be determined by using the method depicted in "Part 2" which involves the use of Figure 1-1 and multiplier factors (see page 1-4). The smaller value for explosive yield
Figure 1-4. Normalized Terminal Yield Vs Ignition Time for LO$_2$/RP-1 CBGS
Figure 1-5. Terminal Yield vs Impact Velocity for LO₂/RP-1 HVI (Ref. 3)
Figure 1-6. Terminal Yield Vs Ignition Time for LO₂/LH₂ CBM
Figure 1-7. Normalized Terminal Yield Vs Ignition Time for LO$_2$/LH$_2$ CBGS
Figure 1-8. Terminal Yield vs Impact Velocity for L\textsubscript{O}_2/LH\textsubscript{2} HVI (Ref. 3)

(ft/sec = m/s \times 3.281)
determined in "Part 1" and "Part 2" is the correct value. This value can then be used to determine an effective weight of propellant, and pressure and impulse at scaled distances using the methods demonstrated in a subsequent chapter.

1-1.3 Examples for Determining Explosive Yield

Example 1:

Propellant-Hypergolic
Combined mass of propellant and oxidizer-10,000 kg (22,000 lbm)
Failure mode - CBM

Solution: Examine Table 1-2 for "Part 1" and "Part 2" solution sequences.

Part 1: Table 1-1 implies that for the CBM failure mode,

\[ Y = 0.01 \text{ - } 0.8\% \]

Using the higher portion for safety reasons,

\[ Y = 0.8\% \]

Part 2: Figure 1-1 implies that for a combined mass of propellant and oxidizer of 10,000 kg,

\[ Y = (24\%) (0.37) = 88.8\% \]

where 240% is the hypergolic multiplier factor,

\[ Y = 0.8\%, \text{ the smaller value, is the correct choice.} \]

Example 2:

Propellant-Hypergolic
Combined mass of propellant and oxidizer-1000 kg (2200 lbm)
Failure mode - HVI
Impact velocity (assumption) - 140 m/s (459 ft/sec)
Type of surface impacted - hard

Solution: Examine Table 1-2 for "Part 1" and "Part 2" solution sequences.

Part 1: Figure 1-2 implies that for an impact velocity of 140 m/s (459 ft/sec) onto a hard surface, \[ Y = 15\%. \]
TABLE 1-2. SEQUENCE FOR DETERMINATION
OF EXPLOSIVE YIELD*

<table>
<thead>
<tr>
<th>Type of Propellant &amp; Oxidizer</th>
<th>Type of Accident Failure Mode</th>
<th>Sequence†</th>
<th>Part 1‡</th>
<th>Part 2 (check)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypergolic (50% N₂H₄ - 50% UDMH/N₂O₄)</td>
<td>CBM</td>
<td>Table 1-1</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Table 1-1</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 1-2</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td>Liquid Oxygen - Hydrocarbon (LO₂/RP-1)</td>
<td>CBM</td>
<td>Figure 1-3</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Eq. (1-1)</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Figure 1-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 1-5</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td>Liquid Oxygen - Liquid Hydrogen (LO₂/LH₂)</td>
<td>CBM</td>
<td>Figure 1-6</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Eq. (1-2)</td>
<td>Figure 1-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Figure 1-7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 1-8</td>
<td>Figure 1-1</td>
<td></td>
</tr>
</tbody>
</table>

* For explosions occurring far above the ground (H/W₁/₃ > 10 m/kg₁/₃ where H is height above the ground), blast yields calculated from curves in this section should be divided by two.

† Correct choice is the smaller of Part 1 and Part 2.

‡ See footnote on page 1-7.
Part 2: Figure 1-1 implies that for a combined mass of propellant and oxidizer of 1000 kg (2200 lbm)

\[ Y = (240\%) \times (0.6) = 144\% \]

where 240\% is the hypergolic multiplier factor.

\[ Y = 15\%, \text{ the smaller value, is the correct choice.} \]

Example 3:

Propellant and oxidizer - LO$_2$/RP-1
Combined mass of propellant and oxidizer - 10,000 kg (22,000 lbm)
Failure mode - CBM
Ignition time (assumption) - 0.2 seconds

Solution: Examine Table 1-2 for "Part 1" and "Part 2" solution sequences.

Part 1: Figure 1-3 implies that for an ignition time of 0.2 seconds,

\[ Y = 52\% \]

Part 2: Figure 1-1 implies that for a combined mass of propellant and oxidizer of 10,000 kg (22,000 lbm)

\[ Y = (125\%) \times (0.37) = 46\% \]

where 125\% is the LO$_2$/RP-1 multiplier factor.

\[ Y = 46\%, \text{ the smaller value, is the correct choice.} \]

Example 4:

Propellant and oxidizer - LO$_2$/RP-1
Combined mass of propellant and oxidizer - 150,000 kg (330,000 lbm)
Failure mode - CBGS
Impact velocity (assumption) - 10 m/s (32.8 ft/sec)
Ignition time (assumption) - 0.5 seconds

Solution: Examine Table 1-2 for "Part 1" and "Part 2" solution sequences.

Part 1: Equation (1-1) implies that for an impact velocity of 10 m/s, (32.8 ft/sec)
\[ Y_m = 5\% + \left( \frac{6.82\%}{(m/s)} \right)(10 \text{ m/s}) \]

\[ Y_m = 5\% + 68.2\% \]

\[ Y_m = 73.2\% \]

Figure 1-4 implies that for an ignition time of 0.5 seconds

\[ \frac{Y \cdot 100}{Y_m} = 70 \]

or

\[ Y = \left( \frac{70}{100} \right) Y_m \]

\[ Y = \left( \frac{70}{100} \right)(73.2\%) = 51.2\% \]

**Part 2:** Figure 1-1 implies that for a combined mass of propellant and oxidizer of 150,000 kg (330,000 lb_m),

\[ Y = (125\%)(0.05) = 6.25\% \]

where 125\% is the \( \text{LO}_2 / \text{RP-1} \) multiplier factor.

\[ Y = 6.25\%, \text{ the smaller value, is the correct choice.} \]

**Example 5:**

Propellant and oxidizer - \( \text{LO}_2 / \text{LH}_2 \)
Combined mass of propellant and oxidizer - 10,000 kg (22,000 lb_m)
Failure mode - HVI
Impact velocity (assumption) - 40 m/s (131 ft/sec)
Type of surface impacted - hard

**Solution:** Examine Table 1-2 for "Part 1" and "Part 2" solution sequences

**Part 1:** Figure 1-4 implies that for an impact velocity of 40 m/s (131 ft/sec),

\[ Y = 30\% \]
Part Z: Figure 1-1 implies that for a combined mass of propellant and oxidizer of 10,000 kg (22,000 lb),

\[ Y = (370\%) \cdot (37) = 137\% \]

where 370% is the LO2/LH2 multiplier factor.

\[ Y = 30\%, \text{ the smaller value, is the correct choice.} \]

1-2 Explosive Yield as a Function of Fluid Type and Initial Conditions for Gas Vessel Bursts

1-2.1 General Discussion of Gas Vessel Explosions

When a pressurized gas-filled vessel bursts, a shock wave in many ways similar to that which results from a TNT explosion propagates from the source. The overpressure behind this shock wave may be quite large and capable of causing damage. The specific impulse associated with this shock wave is also important for the prediction of damage from a gas vessel burst. These two parameters vary with distance from the source.

In the analysis that was used for the overpressure and specific impulse calculations, the effects of the containing vessel and its fragments were ignored, that is, all of the energy of the gas in the vessel was put into the flow field, rather than into the fragments as kinetic energy. For a spherical vessel, the flow field was assumed spherically symmetric. Also, the surrounding atmosphere was assumed to be air.

To determine the overpressure and impulse, one must know the initial conditions of the gas in the vessel. The pressure \( p_1 \), temperature \( T_1 \), and ratio of specific heats of the gas \( \gamma_1 \) must be known.

The conditions of the atmosphere into which the shock wave propagates also must be known. These are the atmospheric pressure \( p_a \), the speed of sound \( a_1 \), and the ratio of specific heats \( \gamma_a \). The latter value will be a constant for all explosions in air.

1-2.2 Discussion of Energy

The energy contained in a pressurized gas vessel can be obtained by

\[ E = \left( \frac{p_1 - p_a}{\gamma_1 - 1} \right) v_1 \]
where

\[ V_i \] is the volume of the vessel before it bursts and 
\[ p_i \] indicates absolute pressures.

The overpressure and impulse are graphed versus a dimensionless scaled distance:

\[ R = \frac{r p_a^{1/3}}{E^{1/3}} \]

A scaled specific impulse is used:

\[ I = \frac{14 a}{p_a^{2/3} E^{1/3}} \]

Note that there is no need to calculate a TNT equivalent for gas vessel bursts.

1-2.3 Example Calculations

Example 1. Calculation of Energy

Let \( p_a = 1.013 \times 10^5 \) Pa (1 standard sea level atmosphere) 
(14.7 psi)
\[ p_1 = 41.013 \times 10^5 \] Pa (595 psi)
\[ \gamma_1 = 1.4 \) (diatomic gas)
\[ V_i = 1.0 \, \text{m}^3 \) (35.3 ft \( ^3 \))

\[ E = \frac{(p_1 - p_a)}{\gamma_1 - 1} \]
\[ V_i = \frac{(41.013 \times 10^5 \text{Pa} - 1.013 \times 10^5 \text{Pa})}{1.4 - 1} \, \text{m}^3 \]
\[ = 1.00 \times 10^7 \] J (1.34 \times 10^6 \text{ ft-lbf})
Example 2. Calculation of Scaled Distance

Using the same conditions as in the previous example,

\[ \overline{R} = \frac{r p_a^{1/3}}{E^{1/3}} = \frac{\gamma}{(p_i/p_a - 1) V_i} \left( \frac{4.013 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right)^{1/3} \left( \frac{1}{1.4 - 1} \right) \left( \frac{1}{1.00 \times 10^7} \right)^{1/3} \]

\[ = 0.216 \ r \ (r \ in \ m) \quad [0.0658 \ r \ (r \ in \ ft)] \]

Example 3. Calculation of Scaled Impulse

Let \( a_0 = 331 \text{ m/s} \) (speed of sound at standard sea level conditions) (1086 ft/sec).

For the conditions used in the previous examples,

\[ \overline{I} = \frac{1}{p_a^{2/3} E^{1/3}} = \frac{1 (331 \text{ m/s})}{(1.013 \times 10^5 \text{ Pa})^{2/3} (1.00 \times 10^7)^{1/3}} = 7.070 \times 10^{-4} \ I \ (\text{in} \ \text{Pa} \cdot \text{s}) \]
LIST OF REFERENCES


CHAPTER II
CHARACTERISTICS OF PRESSURE WAVES

2-1 General

Explosions from liquid rocket propellant accidents "drive" air blast waves, which can in turn cause direct damage and can accelerate fragments or nearby objects. The launch pads at the Air Force Eastern Test Range (ETR) have for a number of years been instrumented with air blast recorders to measure the overpressures generated during launch pad explosions, so some data are available on the intensities of the blast waves generated. Such measurements, and the common practice in safety circles of comparing explosive effects on the basis of blast waves generated by TNT, have led to expression of blast yields of propellant explosions in equivalent "pounds of TNT." (Although a direct conversion of pounds of TNT to energy can easily be made -- 1 lbm of TNT equals $1.4 \times 10^6$ ft-lb -- this is seldom done).

Liquid propellant explosions differ from TNT explosions in a number of ways, so that the concept of "TNT equivalence" quoted in pounds of TNT is far from exact. Some of the differences are described below.

(1) The specific energies of liquid propellants, in stoichiometric mixtures, are significantly greater than for TNT (specific energy is energy per unit mass).

(2) Although the potential explosive yield is very high for liquid propellants, the actual yield is much lower, because propellant and oxidizer are never intimately mixed in the proper proportions before ignition.

(3) Confinement of propellant and oxidizer, and subsequent effect on explosive yield, are very different for liquid propellants and TNT. Degree of confinement can seriously affect explosive yield of liquid propellants, but has only a secondary effect on detonation of TNT or any other solid explosive.

(4) The geometry of the liquid propellant mixture at time of ignition can be quite different than that of the spherical or hemispherical geometry of TNT usually used for generation of controlled blast waves. The sources of compiled data for blast waves from TNT or Pentolite invariably rely on
measurements of blasts from spheres or hemispheres of explosive. The liquid propellant mixture can, however, be a shallow pool of large lateral extent at time of detonation.

(5) The blast waves from liquid propellant explosions show different characteristics as a function of distance from the explosion than do waves from TNT explosions. This is undoubtedly simply a manifestation of some of the differences discussed previously, but it does change the "TNT equivalence" of a liquid-propellant explosion with distance from the explosion. Fletcher (Reference 1) discusses these differences and shows them graphically (see Figures 2-1 and 2-2). These differences are very evident in the results of the many blast experiments reported in Project PYRO (References 2-4). They have caused the coinage of the phrase "terminal yield", meaning the yield based on blast data taken at great enough distance from the explosion for the blast waves to be similar to those produced by TNT explosions. At closer distance, two different yields are usually reported; an overpressure yield based on equivalence of side-on peak overpressures, and an impulse yield based on equivalence of side-on positive impulses.

Accidents with bursting gas storage vessels also can generate damaging blast waves. The characteristics of the blasts from these and other accidental explosions are reviewed in Reference 5, and rather complete descriptions given of the theory of such "non-ideal" explosions. Again, these sources generate blast waves which can differ significantly from blast waves generated by condensed explosives such as TNT, with the differences being greatest close to the source. The trend is similar to that for propellant explosions, i.e., peak overpressures are less and impulses are greater than for "equivalent" TNT explosions. But, the potential maximum yield or blast energy from gas vessel bursts is much more apt to be realized than for liquid propellant explosions. The high pressure gas already contains the necessary energy and can be rapidly released without the prior mixing and ignition required for the propellants.

However, let us for the moment ignore the differences between accidental explosions and planned ones, and discuss instead the general characteristics of the blast waves generated by any explosion. References 6-8 are good general source references on air blast waves and their behavior.

As a blast wave passes through the air or interacts with and loads a structure or target, rapid variations in pressure, density, temperature and particle velocity occur. The properties of blast waves which are
Figure 2-1. Normalized Pressure and Impulse Yields from Explosion of $\text{N}_2\text{O}_4$/Aerozine 50 (Ref. 1)

Figure 2-2. Representative Shock Impulses Showing Coalescence of Shock Waves from Dissimilar Sources [Stages (a) Through (d)] (Ref. 1)
usually defined are related both to the properties which can be easily measured or observed and to properties which can be correlated with blast damage patterns. It is relatively easy to measure shock front arrival times and velocities and entire time histories of overpressures. Measurement of density variations and time histories of particle velocity are more difficult, and no reliable measurements of temperature variations exist.

Classically, the properties which are usually defined and measured are those of the undisturbed or side-on wave as it propagates through the air. Figure 2-3 shows graphically some of these properties in an ideal wave (Reference 6).

Prior to shock front arrival, the pressure is ambient pressure $p_o$. At arrival time $t_a$, the pressure rises quite abruptly (discontinuously, in an ideal wave) to a peak value $P^+ + p_o$. The pressure then decays to ambient in total time $t_a + T^+$, drops to a partial vacuum of amplitude $P^-$, and eventually returns to $p_o$ in total time $t_a + T^+ + T^-$. The quantity $P^+$ is usually termed the peak side-on overpressure, or merely the peak overpressure. The portion of the time history above initial ambient pressure is called the positive phase, of duration $T^+$. That portion below $p_o$, of amplitude $P^-$ and duration $T^-$ is called the negative phase. Positive and negative impulses, defined by

$$I_s^+ = \int_{t_a}^{t_a + T^+} [p(t) - p_o] dt$$

(2-1)
and

\[ I_s^- = \int_{t_a}^{t_f} [p_o - p(t)] dt \]  \hspace{1cm} (2-2)

respectively, are also significant blast wave parameters.

In most blast studies, the negative phase of the blast wave is ignored and only blast parameters associated with the positive phase are considered or reported. (The positive superscript is usually dropped). The ideal side-on parameters almost never represent the actual pressure loading applied to structures or targets following an explosion. So a number of other properties are defined to either more closely approximate real blast loads or to provide upper limits for such loads.

An upper limit to blast loads is obtained if one interposes an infinite, rigid wall in front of the wave and reflects the wave normally. All flow behind the wave is stopped, and pressures are considerably greater than side-on. The peak overpressure in normally reflected waves is usually designated \( P_r \). The integral of this pressure over the positive phase, defined similarly to Equation (2-1), is the reflected impulse \( I_r \). Durations of the positive phase of normally reflected waves are designated \( T_r \). The parameter \( I_r \) has been measured closer to high explosive blast sources than have most blast parameters.

In certain instances, damage estimates involve \( P_r \) and \( I_r \) instead of \( P_s \) and \( I_s \). This situation can occur when one is examining the effect of an air burst on ground structures. The ground, in this case, acts as the most significant reflecting surface. Reflected pressures and impulses are also used in analyzing face-on loading of windows and structures. Fortunately, reflected pressure and impulse can be calculated directly from side-on pressure and impulse. For values of \( \bar{P}_r \leq 3.5 \) where \( \bar{P}_r \) is \( P_r/p_o \) (\( p_o \) is atmospheric pressure),

\[ \bar{P}_r = 2 \bar{P}_s + \frac{(\gamma + 1) \bar{P}_s^2}{(\gamma - 1) \bar{P}_s + 2\gamma} \]  \hspace{1cm} (2-3)

where \( \bar{P}_s = P_s/p_o \) and \( \gamma \) is the ratio of specific heats which equals 1.4 for air. For values of \( \bar{P}_s > 3.5 \), \( \bar{P}_r \), and subsequently \( P_r \), can be determined from Figure 2-4.

Over the range 0.00141 \( \leq \bar{P}_s \leq 1.38 \), \( T_r \) can be calculated from

\[ T_r = \frac{a_o}{p_o} \left( \frac{1}{3} \right)^{1/3} \]
\[ I = \bar{I}_s \frac{P_r}{P_s} \]  
\[ \bar{I}_s = I_s a_o / p_o^{1/3} E^{1/3} \]  
\[ \bar{I}_r = 2\bar{I}_s \]

where \( \bar{I}_s \) is the scaled reflected overpressure, \( I_s \) is the incident overpressure, \( a_o \) is the speed of sound in free air, \( p_o \) is the atmospheric pressure, \( E \) is a constant, and \( \bar{I}_r \) is the scaled reflected overpressure.

For values of \( P_s < 0.00141 \),

A real target feels a very complex loading during the process of diffraction of the shock front around the target. Figure 2-5 shows schematically, in three stages, the interaction of a blast wave with an irregular object. As the wave strikes the object, a portion is reflected from the front face, and the remainder diffracts around the object. In the diffraction process, the incident wave front closes in behind the object, greatly weakened locally, and a pair of trailing vortices is formed. Rarefraction waves sweep across the front face, attenuating the initial reflected blast pressure. After passage of the front, the body is immersed in a time-varying flow field. Maximum pressure on the front face during this "drag" phase of loading is the stagnation pressure.

Figure 2-4. Scaled Reflected Overpressure Vs Scaled Side-on Overpressure
We are interested in the net transverse pressure on the object as a function of time. This loading, somewhat idealized, is shown in Figure 2-6 [details of the calculation are given by Glassstone (Reference 7)].

At time of arrival \( t_a \), the net transverse pressure rises linearly from zero to maximum of \( P_r \) in time \( (T_1 - t_a) \) (for a flat-faced object, this time is zero). Pressure then falls linearly to drag pressure in time \( (T_2 - T_1) \), and then decays more slowly to zero in time \( (T_3 - T_2) \). This time history of drag pressure \( q \) is a modified exponential, with a maximum given by

\[
C_D Q = C_D \cdot (1/2) \rho_s \cdot u_s^2
\]

(2-6)

where \( C_D \) is the steady-state drag coefficient for the object, \( Q \) is peak dynamic pressure, and \( \rho_s \) and \( u_s \) are peak density and particle velocity respectively for the blast wave. The characteristics of the diffraction phase of the loading can be determined if the peak side-on overpressure \( P_s \) or the shock velocity \( U \) is known, together with the shape and some characteristic dimension \( D \) of the object. The peak amplitude of the drag phase of the loading can be determined if the peak side-on overpressure \( P_s \) or the shock velocity \( U \) is known, together with the shape and some characteristic dimension \( D \) of the object. The peak amplitude of the drag phase, \( C_D Q \), can also be determined explicitly from \( P_s \) or \( u_s \).

Figure 2-5. Interaction of Blast Wave with Irregular Object
Because of the importance of the dynamic pressure \( q \) in drag or wind effects and target tumbling, it is often reported as a blast wave property. In some instances drag impulse \( I_d \), defined as

\[
I_d = \int_{t_a}^{t_a + T} q \, dt = \frac{1}{2} \int_{t_a}^{t_a + T} u^2 \, dt \quad (2-7)
\]

is also reported.

Although it is possible to define the potential or kinetic energy in blast waves, it is not customary in air blast technology to report or compute these properties. For underwater explosions, the use of "energy flux density" is more common (Reference 9). This quantity is given approximately by

\[
E_f = \frac{1}{\rho_o c_o} \int_{t_a}^{t_a + T} (p(t) - p_o)^2 \, dt \quad (2-8)
\]

Blast waves from real accidental explosions can differ in a number of ways from the essentially clean spherical waves considered in most theoretical treatments, and in many field or laboratory experiments. As an example, any explosion source which is not spherical in free air or
hemispherical in contact with a reflecting plane will generate a blast wave which is, at least in its early stages, non-spherical. The wave may well have an axis of symmetry, but requires definition in at least two space coordinates and time. Analytically, the treatment of non-spherical waves requires more mathematical complexity, and experimentally, measurement requires many more tests than for spherical waves.

The simplest type of non-spherical behavior probably results from elevation of a spherical explosion source above a reflecting plane (usually the ground). The resulting reflection process is described in Baker (Reference 6) and Glasstone (Reference 7). A structure or target on the ground feels a double shock if it is in the region of regular reflection close to the blast source, or a single strengthened shock if it is in the region of Mach reflection. Even this "simplest" case of non-spherical behavior is quite complex.

The second type of asphericity is that caused by sources which are not spherical. Most real blast sources are non-spherical, and can be of regular geometry such as cylindrical or block-shaped, or can be quite irregular in shape. Few analyses or experiments have been done for other than cylindrical geometry of solid explosive sources. For cylinders, the wave patterns are quite complex. The pressure-time histories exhibit multiple shocks, and decay in a quite different manner in the near field than do spherical waves. Fortunately, asymmetries smooth out as the blast wave progresses, and "far enough" from most real sources, the wave will become a spherical wave.

Other effects which can significantly alter blast wave properties are:

1. Effect of partial or total confinement.
2. Atmospheric propagation effects.
3. Absorption of energy by ground shock or cratering.
4. Transmission over irregular terrain.

These effects are often ignored or roughly approximated in safety studies because they are quite variable or can be adequately accounted for by use of simple safety factors or energy multipliers. They are discussed in some detail in References 5 and 6.
2-2 Pressure Waves From Propellant Explosions

2-2.1 Introduction

The characteristics of pressure waves, particularly peak side-on overpressure and specific impulse, are used extensively in developing damage estimates from propellant explosions. This portion of this chapter is therefore devoted to the calculation of pressures and specific impulses at varying distances from a propellant explosion based on methods given by Baker, et al.\(^\text{10}\).

The same terminology used in Chapter I, Section A for propellant types and failure modes is used in this chapter. Three different types of propellant-oxidizer combinations are considered. These are hypergolic (50% N\(_2\)H\(_4\) - 50% UDMH fuel and N\(_2\)O\(_4\) oxidizer in a mass ratio of 1/2), liquid oxygen-hydrocarbon [Kerosene (RP-1) fuel and liquid oxygen (LO\(_2\)) oxidizer in a mass ratio of 1/2, 25], and liquid oxygen-liquid hydrogen [cryogenic combination of liquid hydrogen (LH\(_2\)) fuel and liquid oxygen (LO\(_2\)) oxidizer in stoichiometric mass ratio of 1/5]. Three types of failure modes are considered; namely, confined by missile (CBM), confined by ground surface (CBGS), and high velocity impact (HVI). If needed, Chapter I should be consulted for a more complete explanation of types of propellants and failure modes.

2-2.2 Determination of Peak Side-On Overpressure and Specific Impulse

Throughout the PYRO (2-4) work, blast yield is expressed as percent yield, based on an average of pressures and impulses measured at the farthest distance from the source when compared to standard reference curves (Reference 11) for TNT surface bursts (terminal yield). Hopkinson's blast scaling is used when comparing blast data for tests with the same propellants and failure conditions, but different mass of propellant. So, the blast parameters \(P\) (peak side-on overpressure) and \(1/W^{1/3}\) (scaled impulse) are plotted as functions of \(R/W^{1/3}\) (scaled distance), after being normalized by the fractional yield. This procedure is equivalent to determining an effective mass of propellant for blast from:

\[
W = W_T \times \frac{Y}{100} \quad (2-9)
\]

where \(W_T\) is total mass of propellant and oxidizer, \(Y\) is terminal blast yield in percent and \(W\) is effective mass of propellant. Because the data are normalized by comparing to TNT blast data, the effective blast energy \(E\) can be obtained by multiplying \(W\) by the specific detonation energy of
TNT, $4.18 \times 10^6 \text{J/kg (1.4} \times 10^6 \text{ft lb/} \text{lb m)}$. Baker's(10) smoothed curves through the scaled PYRO blast data, and Equation (2-9) will be used to obtain blast wave properties for particular combinations of propellants and simulated accidents.

Table 2-1 contains the different propellant failure mode combinations under consideration and the figure numbers of the graphs (following Table 2-1) needed to determine peak side-on overpressure and scaled specific impulse as a function of scaled distance for each accident situation. The procedure for finding peak side-on overpressure and specific impulse are as follows:

1. Calculate terminal yield $Y$ using methods discussed in Chapter I, Section A.
2. Determine $W_e$, effective mass of propellant and oxidizer, from Equation (2-9).
3. Choose a specific standoff distance $R$ from the center of the anticipated blast and calculate scaled distance $R/W^{1/3}$.
4. Examine Table 1 and acquire the proper figure numbers for finding peak side-on overpressure $P$ and scaled impulse $I/W^{1/3}$ for the particular propellant/oxidizer and failure mode under consideration.
5. Determine $P$ from the appropriate Pressure versus Scaled Distance curve and the predetermined scaled distance.
6. Determine $I/W^{1/3}$ from the appropriate Scaled Positive Impulse versus Scaled Distance curve and the predetermined scaled distance.
7. Calculate specific impulse $I$ from scaled positive impulse $I/W^{1/3}$.

A word of explanation will help clarify the meaning of the central solid line and shaded area of the graphs. The shaded portion represents actual data from propellant blasts. The central solid line is an estimate of the most likely occurrence and, for most cases, is the recommended choice. The vertical distance between the two dashed lines at any abscissa is a measure of the data spread, or uncertainty in a prediction from the solid line.
That is

\[ I = \left( \frac{1}{w^{1/3}} \right) (w^{1/3}) \]  

\[ (2-10) \]

**Table 2-1. Guide to Selection of Proper Graphs for the Determination of Pressure and Specific Impulse**

<table>
<thead>
<tr>
<th>Type of Propellant &amp; Oxidizer</th>
<th>Type of Incident (Failure Mode)</th>
<th>Peak Side-On Overpressure (P)</th>
<th>Scaled Impulse (I/W^{1/3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypergolic (50% N_2H_4, 50% UDMII/N_2O_4)</td>
<td>CBM</td>
<td>Figure 2-7</td>
<td>Figure 2-8</td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Figure 2-8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 3-9</td>
<td>Figure 2-8</td>
</tr>
<tr>
<td>Liquid Oxygen-Hydrocarbon (LO_2/RP-1)</td>
<td>CBM</td>
<td>Figure 2-10</td>
<td>Figure 2-11</td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Figure 2-12</td>
<td>Figure 2-12</td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 2-12</td>
<td>Figure 2-13</td>
</tr>
<tr>
<td>Liquid Oxygen-Liquid Hydrogen (L.O_2/L.H_2)</td>
<td>CBM</td>
<td>Figure 2-14</td>
<td>Figure 2-15</td>
</tr>
<tr>
<td></td>
<td>CBGS</td>
<td>Figure 2-16</td>
<td>Figure 2-17</td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Figure 2-16</td>
<td>Figure 2-17</td>
</tr>
</tbody>
</table>
Figure 2-7. Pressure vs Scaled Distance. Hypergolic Propellant: CMB and CBGS Failure Modes.

\[ \text{psi} = \text{Pa} \times 1.450 \times 10^{-4} \]

\[ (\text{ft/lb})^{1/3} = \text{m/kg}^{1/3} \times 2.521 \]
Figure 2-8. Scaled Positive Impulse vs Scaled Distance. Hypergolic Propellant; CBM, CBGS and HVI Failure Modes.
Figure 2-9. Pressure vs Scaled Distance. Hypergolic Propellant; HVI Failure Mode.
Figure 2-10. Pressure vs Scaled Distance. LO₂/RP-1 Propellant; CBM Failure Mode.

(1 psi = 6.8947 × 10⁶ Pa)

(1 ft/ lb m = 0.001345 m³/kg² x 2.521)
Figure 2-11. Scaled Positive Impulse vs Scaled Distance.
LO₂/RP-1 Propellant; CBM Failure Mode.
Figure 2-12. Pressure vs Scaled Distance. LO\textsubscript{2}/RP-1 Propellant: CBGS and HVI Failure Modes.
Figure 2-13. Scaled Positive Impulse vs Scaled Distance.
LO₂/RP-1 Propellant: CBGS and HVI Failure Modes.
Figure 2-14. Pressure vs Scaled Distance. LO\textsubscript{2}/LI\textsubscript{2}
Propellant; CBM Failure Mode.
Figure 2-15. Scaled Positive Impulse vs Scaled Distance.
LO$_2$/LH$_2$ Propellant; CBM Failure Mode.

( psi-sec/lb$^{1/3}$ = Pa-sec/kg$^{1/3}$ X 1.114 X $10^{-4}$ )

( ft/lb$^{1/3}$ = m/kg$^{1/3}$ X 2.521 )

Scaled Positive Impulse $^{1/3}$, pascal-sec/kg$^{1/3}$

Scaled Distance $R/W^{1/3}$, $m/kg^{1/3}$
Figure 2-16. Pressure vs Scaled Distance. LO$_2$/LH$_2$
Propellant: CBCS and HVI Failure Modes.
Figure 2-17. Scaled Positive Impulse vs Scaled Distance.

LO₂/LH₂ Propellant: GBGS and HVI Failure Modes.
2-2.3 Examples for Determining Peak Side-On Overpressure and Specific Impulse

The problems which follow are continuations of some of the example problems started in Chapter I, Section A.

Example 1: (Continuation of Example 1 of Chapter I, Section A)

Propellant-Hypergolic
Combined mass of propellant and oxidizer - 10,000 kg (22,000 lb<sub>m</sub>)
Failure mode - CBM
Standoff distance R (assumption) - 50 m (164 ft)

Solution:

1. Terminal yield y = 0.8% (See Example 1 of Chapter I, Section A for calculations)

2. \[ W = W_T \times \frac{1}{100\%} \]
   \[ W = 10,000 \text{ kg} \times \frac{0.8\%}{100\%} \]
   \[ W = 80 \text{ kg (176 lb<sub>m</sub>)} \]

3. Scaled distance \[ R/W^{1/3} = \frac{50 \text{ m}}{(80 \text{ kg})^{1/3}} = 12 \text{ m/kg}^{1/3} \]

4. Table 2-1 indicates:
   Acquire P, peak pressure, from Figure 2-7.
   Acquire \( I/W^{1/3} \), scaled impulse, from Figure 2-8.

5. From Figure 2-7, \[ P = 8.2 \times 10^4 \text{ Pa (11.89 psi)} \]

6. From Figure 2-8, \( I/W^{1/3} = 27 \text{ Pa} \cdot \text{s/kg}^{1/3} \)

7. \[ I = \left( \frac{1}{W^{1/3}} \right) (W^{1/3}) = 116 \text{ Pa} \cdot \text{s (1.682 x 10^{-2} psi/sec)} \]

Example 2: (Continuation of Example 4 of Chapter I, Section A)

Propellant and oxidizer - LO<sub>2</sub>/RP-1
Combined mass of propellant and oxidizer - 150,000 kg (330,000 lb<sub>m</sub>)
Failure mode - CBGS
Impact velocity (assumption) - 10 m/s (32.8 ft/sec)
Ignition time (assumption) - 0.5 seconds
Standoff distance (assumption) - 100 m (328 ft)

Solution:

(1) Terminal yield \( y = 6.25\% \) (See Example 4 of Chapter I, Section A for calculations).

(2) \[ W = \frac{W_T \times \frac{y}{100\%}}{100\%} \]
\[ W = 150,000 \text{ kg} \times \frac{6.25\%}{100\%} \]
\[ W = 9375 \text{ kg (20,600 lb)} \]

(3) Scaled distance \( \frac{R}{W^{1/3}} = \frac{100 \text{ m}}{(9375)^{1/3}} = 4.7 \text{ m/kg}^{1/3} \)

(4) Table 2-1 indicates:
Acquire \( P \), peak pressure from Figure 2-12.
Acquire \( \frac{1}{W^{1/3}} \), scaled impulse from Figure 2-13.

(5) From Figure 2-12, \( P = 3.8 \times 10^4 \text{ Pa (5.5 psi)} \)

(6) From Figure 2-13, \( \frac{1}{W^{1/3}} = 55 \text{ Pa-s/kg}^{1/3} \)

(7) \( I = \left( \frac{1}{W^{1/3}} \right) (W^{1/3}) = 1160 \text{ Pa-s (0.168 psi/sec)} \)

Example 3: (Continuation of Example 5 of Chapter I, Section A)

Propellant and oxidizer - \( \text{LO}_2/\text{LH}_2 \)
Combined mass of propellant and oxidizer - 10,000 kg (22,000 lb) m
Failure mode - HVI
Impact velocity (assumption) - 40 m/s (131 ft/sec)
Type of surface impacted - hard
Standoff distance (assumption) - 100 m (328 ft)

Solution:

(1) Terminal yield \( y = 30\% \) (See Example 5 of Chapter I, Section A for calculations)
\[ W = W_T \times \frac{V}{100\%} \]
\[ W = 10,000 \text{ kg} \times \frac{30\%}{100\%} \]
\[ W = 3000 \text{ kg} \]

(3) Scaled distance \( \frac{R}{W^{1/3}} = 100 \text{ m} / (3000 \text{ kg})^{1/3} = 6.9 \text{ m/kg}^{1/3} \)

(4) Table 2-1 indicates:
- Acquire \( P_0 \), peak pressure, from Figure 2-16.
- Acquire \( I/W^{1/3} \), scaled impulse, from Figure 2-17.

(5) From Figure 2-16, \( P = 2.2 \times 10^4 \text{ Pa} \) (319 psi)

(6) From Figure 2-17, \( I/W^{1/3} = 45 \text{ Pa} \cdot \text{s/kg}^{1/3} \)

(7) \[ I = \frac{1}{W^{1/3}} \]
\[ (W^{1/3}) = 649 \text{ Pa} \cdot \text{s} \]
\[ (9.41 \times 10^2 \text{ psi/sec}) \]

2.3 Pressure Waves From Gas Vessel Bursts

Application to spherical vessels will be discussed first.

2.3.1 Overpressures for Various Gases and Initial Conditions

The overpressure versus distance relationship for a bursting gas vessel is strongly dependent upon the pressure, temperature, and ratio of specific heats of the gas in the vessel. For high pressures and temperatures, relative to the air outside the vessel, the overpressure behavior is much like that of a blast wave from a high explosive. On the \( P_s \) versus \( R \) graph (Figure 2-18), the curves for higher pressures and temperature are located near the high explosive curve. The curves for lower pressures and temperatures lie farther from the high explosive curve.

The procedure for relating overpressures and distance from the source of a gas vessel burst is the following: Determine the starting overpressure and distance. Locate this point on a \( P_s \) versus \( R \) graph (Figure 2-18). Follow the nearest curve on this graph for the overpressure versus distance behavior. Choose the \( R \) of interest, and read \( P_0 \) from the proper curve. Alternatively, one can choose a value for \( P_s \) and locate the corresponding \( R \). For \( R \) greater than about 2, Figure (2-19), \( P_s \) versus \( R \).
Figure 2-18. $p_\infty$ vs $R$ for Overpressure Calculations.
Figure 2-19. $\bar{P}_S$ vs $\bar{R}$ for Pentolite
for pentolite (a high explosive)\(^{(6)}\) can be used as an upper limit for \(P_s\).

For a given \(p_1/p_a\), \(T_1/T_a\), and \(\gamma_1\), \(P_{so}\), the nondimensional starting shock overpressure can be read from one of the graphs in Figure 2-20 and 2-21. For diatomic gases, such as air, \(O_2\), and \(H_2\), let \(\gamma_1\), equal 1.4. For monatomic gases such as \(He\), let \(\gamma_1\), equal 1.667. The nondimensional starting distance \(\bar{R}_o\) is

\[
\bar{R}_o = \frac{1}{\left[\frac{4 - 3}{1 - \left(\frac{p_1 - 1}{p_a - 1}\right) \gamma_1 - 1}\right]^{1/3}}
\]

Locate \(\bar{R}_o\) and \(P_{so}\) on the graph of \(P_s\) versus \(\bar{R}\) as in Figure 2-22. This is the starting point. Follow the nearest curve for the \(P_s\) versus \(\bar{R}\) behavior. The \(P_s\) versus \(\bar{R}\) curves in Figure 2-18 are accurate to about \(\pm 20\%\).

To determine the overpressure at a given distance, first compute \(\bar{R}\).

\[
\bar{R} = \frac{r}{\left[\frac{p_1 - 1}{p_a - 1} \gamma_1 - 1\right]^{1/3}}
\]

where \(r\) is the distance from the center of the vessel, and \(V_1\) is the volume of the vessel before it bursts.

Then, read \(P_s\) from the proper curve. Compute the overpressure:

\[
P_s = \frac{p_u - p_a}{p_a}
\]

where \(p_u - p_a\) is the overpressure.

To determine the distance at which a given overpressure will be observed, compute \(\bar{P}_s\) from the given \(p_s - p_a\):
Figure 2-21. Kernel Temperature vs Kernel Pressure
for Constant Values of $P'$, $y_1 = 1.667$
Figure 2-22. Location of Starting Point on Graph of $\bar{P}_s$ vs $\bar{R}$

\[ \bar{P}_s = \frac{P_s - P_a}{P_a} \quad (2-13) \]

Read $\bar{R}$ from the proper curve, and calculate $r$:

\[ \bar{R} = \left[ \frac{r}{\left( \frac{p_j}{p_a} - 1 \right) V_i} \right]^{1/3} \quad \text{then} \quad (2-14) \]

\[ r = \bar{R} \left[ \left( \frac{p_j}{p_a} - 1 \right) V_i \right]^{1/3} \quad (2-15) \]
2.3.2 Specific Impulses for Various Gases and Initial Conditions

For the burst of a pressure vessel, the \( I \) versus \( R \) relationship in Figure 2-23 or 2-24 should be used. For \( R \) in the range of \( 10^{-1} \) to \( 10^{0} \), the \( I \) versus \( R \) curve in Figure 2-24 is more convenient. This is an enlargement of part of Figure 2-23. These curves are accurate to about \( \pm 25\% \). For a given distance, \( R \) is calculated, and \( I \) is read from Figure 2-23 or 2-24. Then \( I \) is calculated. Alternatively, one can choose a maximum acceptable specific impulse and find the minimum distance at which the specific impulse is less than this value.

Example:

A spherical pressure vessel of radius \( 1.0 \text{ m} \) (3.3 ft) containing air \( (\gamma_1 = 1.4) \) bursts in a standard sea level atmosphere. The inside gas pressure is \( 1.013 \times 10^6 \text{ Pa} \) (147 psi) and the temperature is 300 K \( (80^\circ \text{F}) \). There are no reflecting surfaces nearby. Find the peak overpressure and specific impulse at a distance of 5.0 m (16.4 ft) from the source.

Solution for peak overpressure:

\( \bar{R}_o \) and \( \bar{R} \) for the distance of interest are calculated, \( \bar{P}_{so} \), the starting peak overpressure is obtained from Figure 2-20. The correct curve is located in Figure 2-18 and \( P_s \) is read from the graph for the \( R \) of interest.

\[
\bar{R}_o = \left[ \frac{4\pi}{3} \frac{p_1}{\gamma_1 - 1} \right]^{1/3} \left( \frac{P_s}{p_1} - 1 \right)
\]

\[
= \left[ \frac{4\pi}{3} \frac{1.013 \times 10^6 \text{ Pa}}{1.4 - 1} \right]^{1/3} = 0.2197
\]

2-33
Figure 2-23. $\bar{T}$ vs $\bar{R}$ for Pentolite and Gas Vessel Bursts
Figure 2-24. $I$ vs $R$ for Gas Vessel Bursts, Small $R$
For $p_1/p_a = 10$ and $T_1/T_a = 1$, $\bar{P}_{Sc} \approx 1.7$ (Figure 2-20). Looking at Figure 2-18, this point $(\bar{R}_o, \bar{P}_{So})$ falls near the third curve from the bottom. Following this curve, for $\bar{R} = 1.099$, $\bar{P}_s = 0.26$. Since $\bar{P}_s = P_s - P_a$, $P_s - P_a = \bar{P}_s P_a = (0.26)(1.013 \times 10^5 P_a) = 2.6 \times 10^4 Pa (3.77 psi)$.

Solution for specific impulse:

The $\bar{R}$ of interest has been calculated above. Read $\bar{I}$ for this $\bar{R}$ from Figure 2-23.

For $\bar{R} = 1.099$, $\bar{I} = 0.046$ (Figure 2-23).

Since $\bar{I} = \frac{I a}{p_a} \frac{2/3}{E a 1/3}$, \[ I = \frac{2/3}{E a 1/3} \frac{p_a}{a} \] (2-18)

\[ E = \left( \frac{P_1 - P_a}{\gamma_1 - 1} \right) \] \[ V_1 = \left( \frac{1.013 \times 10^6 P_a - 1.013 \times 10^6 P_a}{1.4 - 1} \right) \] (2-19)

\[ \times \frac{4 - 3}{3} (1.0 m)^3 = 0.55 \times 10^6 J \]
\[
I = \frac{(0.046)(1.013 \times 10^5 \text{Pa})^{2/3}(9.55 \times 10^6 \text{j})^{1/3}}{331 \text{ m/s}} = 64 \text{ Pa-s}
\]

\[
(9.28 \times 10^{-3} \text{ psi-sec})
\]

2-3.3 Cylindrical Vessel

For a cylindrical vessel, given the length \(L\) and the diameter \(D\), use its volume \(V_i = \frac{\pi D^2}{4} L\) in the equations above, performing the calculations as for a spherical vessel. After \(P_s\) and \(I\) have been determined, further corrections are necessary. For \(R\) less than about 0.3, the calculated overpressure should be multiplied by a factor of 4 or 5. For \(R\) near 1.0, the factor is 1.6. For \(R\) greater than about 3.5, multiply the calculated overpressure by about 1.4. For \(R\) less than about 0.3, the calculated specific impulse should be doubled. For \(R\) near 1.0, the factor is about 1.1. For \(R\) greater than about 1.6, no correction to the specific impulse is necessary. The difference between spherical and cylindrical vessel bursts is only known qualitatively. Therefore these corrections are very crude.

2-3.4 Ground Burst

The method described above is to be used for gas vessel bursts far from any reflecting surfaces. If there is a reflecting surface adjacent to the gas vessel, such as with a gas vessel on the ground, multiply \(V_i\), the volume of the vessel, by a factor of 2. Use this new \(V_i\) in the calculations. Also, once \(P_s\) has been calculated, increase it by 100% for \(R\) less than about 1, and by 10% for \(R\) greater than 1. After \(I\), has been calculated, increase it by 60% for \(R\) less than about 1. There is little effect for longer distances. Only qualitative effects are known, and therefore these corrections are very crude.
LIST OF REFERENCES


APPENDIX II A
GAS VESSEL BURST

2A-1. Nondimensional Parameters

A model analysis was performed for the pressurized gas-filled vessel burst. The following dimensionless parameters were obtained:

\[ \bar{T} = \frac{1}{2/3} \left( \frac{2}{1/3} \right) \text{specific impulse} \]  \hspace{1cm} (2A-1)

\[ \bar{P}_0 = \frac{P_0 - P_a}{P_a} \text{peak shock overpressure} \]  \hspace{1cm} (2A-2)

\[ \bar{R} = \frac{r_p}{\bar{P}_0} \text{distance} \]  \hspace{1cm} (2A-3)

2A-2. Source of Data

The data which were used in the shock overpressure and specific impulse calculations are generated numerically by a finite-difference computer program in which the one-dimensional (spherical coordinates) unsteady equations of conservation of mass, momentum, and energy in Lagrangian form are solved for a perfect gas. Artificial viscosity is used to smooth the shock waves. (2)

Previously, other investigators have used numerical methods to calculate the flow field variables after the burst of a pressurized sphere. Huang and Chou used the Hartree method of characteristics with Rankine-Hugoniot jump conditions across the shocks. (3) Boyer, et al., used a numerical program similar to the one used here and compared their results to experimental data obtained by breaking glass spheres pressurized with air, He, and SF₆. (4) Their values of nondimensional impulse versus distance turn well with those calculated here.

Table 2A-1 gives the initial conditions of the cases that were used to generate the data used in the analysis.
TABLE 2A-1. INITIAL CONDITIONS FOR PRESSURE SPHERE BURSTS

<table>
<thead>
<tr>
<th>Case</th>
<th>( p_1 ) (Pa)</th>
<th>( T_1 ) (K)</th>
<th>( \gamma_1 )</th>
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<tbody>
<tr>
<td>1</td>
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<td>1.4</td>
</tr>
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<tr>
<td>12</td>
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<td>5</td>
<td>1.4</td>
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<tr>
<td>18</td>
<td>22</td>
<td>1</td>
<td>1.4 (Ref. 4)</td>
</tr>
</tbody>
</table>

2A-2. One Pressure Calculation

The \( P_s \) versus \( R \) data were plotted for several sets of initial conditions, and it was found that the \( P_s \) versus \( R \) curves for high pressure bursts pass through the curves for lower pressures (see Figure 2A-1). (There is some crossing of the curves, but some of this is due to inaccuracy in the computer program.) Therefore, if a number of curves were generated for high vessel pressures and various temperatures, the \( P_s \) versus \( R \) behavior for bursts at lower pressures could be determined by finding the starting point \( (R_{start}, P_{start}) \) of a lower pressure burst and following the nearest \( P_s \) versus \( R \) curve. (1) The curves in Figure 2A-1 of the text were drawn based upon the curves in Figure 2A-1 of the appendix. Their uncertainty should not be assumed to be less than about ±20%.
Figure 2A-1. $\bar{P}_s$ vs $\bar{R}$
The starting distance $R_o$ corresponds to the surface of the vessel before it bursts, where $(r/r_o) = 1$.

\[
R = \left[ \frac{p_1 - 1}{\gamma_1 - 1} \right]^{1/3} V_i
\]

\[
\frac{r}{r_o} = \left[ \frac{p_1 - 1}{\gamma_1 - 1} \right]^{1/3}
\]

because $V_i = \frac{4\pi r_o^3}{3}$

Then $R_o = \left[ \frac{p_1 - 1}{\gamma_1 - 1} \right]^{1/3}$

\[
\bar{P}_{so} \text{ is obtained by use of the shock tube equation:}^6
\]

\[
\frac{p_1}{p_a} = \frac{p_s}{p_a} \left[ 1 - \frac{(\gamma - 1)(a_s/a_a)(p_s/p_a - 1)}{2\gamma_a/\gamma_s + (\gamma_a + 1)(p_s/p_a - 1)} \right]^{-2\gamma_a/\gamma_s - 1} \!
\]

\[\gamma_a = 1.4. \text{ For a given } \frac{p_1}{p_a}, \frac{T_1}{T_a}, \text{ and } \gamma_1, \text{ this equation must be solved iteratively for } \frac{p_s}{p_a}. \text{ Then } P_{so} = \frac{p_s}{p_a} - 1.\]
The $P_1$ versus $T_1$ versus $P_{so}$ graphs (Figures 2-20 and 2-21 in the text) were obtained from this equation.

2A-4 Impulse Calculation

Specific impulse is calculated as $I = \int (p - P_a) \, dt$, where $+$ denotes the positive phase of the pressure wave. See Figure 2A-2.

The $\bar{I}$ versus $\bar{R}$ data were plotted for several sets of initial conditions (Figure 2A-3). For $\bar{R}$ less than about 0.5, the behavior is not clear, and a maximum $\bar{I}$ was chosen for the $I$ versus $R$ relationship in Figure 2-23 of the text. For $\bar{R}$ greater than about 0.5, all of the curves lie within about 25% of the high explosive (pentolite) curve. The pentolite curve was therefore chosen as the best $I$ versus $R$ curve in this region.

2A-5 Effect of Cylindrical Geometry

This analysis can be used for bursts for cylindrically shaped pressure vessels if the volume $V_1$ is known. The cylinder is treated as an "equivalent sphere" with same energy as the given cylinder.

Besides the energy in the vessel, its orientation with respect to the target is important. Qualitative relationships between $P_{so}$ and $\bar{I}$ and the angle between the location of the target and the longitudinal axis of the vessel can be observed from the high explosive data in Reference 8. However, this angular orientation is usually unknown, and the "worse case" must be assumed. Then, the peak overpressure will be greater than that calculated for a spherical vessel for all $\bar{R}$. The specific impulse would also be greater in the near field.

2A-6 Effect of Reflecting Surface (Burst at Ground Level)

In this analysis, it was assumed that the vessel burst occurs far away from any reflecting surface. To apply this to a burst occurring on the ground, assume that twice as much energy is released, implying that the volume of the vessel is doubled. The reason for this is illustrated in Figure 2A-4. In (a), all of the energy is released above the reflecting ground surface. In (b) one-half of the energy is released above the reflecting ground surface, and one-half of the energy is released below. For a point $P$ located above the ground surface to experience the same blast wave, the energy in vessel $B$ must be twice the energy in vessel $A$. For the same pressure and ratio of specific heats of the gas, the volume of vessel $B$ must be twice the vessel $A$. The situation in (b) is chosen because the analysis requires a spherically symmetric flow field.
In addition to the effect upon the effective energy released, the presence of the ground surface must be accounted for in another way. Data in Reference 8 indicate that the peak overpressure should also be doubled near the source of the burst, and this factor should be decreased to unity in the far field. Those tests were conducted with high explosives, and, thus, only allow a qualitative description for pressure bursts, but it can be concluded that the overpressure would be greater for a sphere burst on the ground than would be expected from initial source energy considerations alone. The specific impulse would also be higher, at least in the near field.
Figure 2A-3. $\bar{T}$ vs $\bar{R}$
Figure 2A-4. Assumed Spherical Geometry

(a) Spherical vessel setting on ground surface

(b) Spherical vessel bisected by ground plane
## LIST OF REFERENCES


SYMBOLS

\( a \)  speed of sound
\( D \)  diameter of cylinder
\( E \)  energy in pressure vessel
\( I \)  specific impulse
\( \bar{I} \)  nondimensional specific impulse
\( L \)  length of cylinder
\( p \)  pressure
\( \bar{P}_s \)  nondimensional overpressure
\( \bar{P}_{so} \)  starting nondimensional overpressure
\( r \)  distance from center of vessel
\( R \)  radius of sphere
\( \bar{R} \)  nondimensional distance
\( r_0 \)  radius of spherical vessel
\( t \)  time
\( T \)  temperature
\( V_i \)  volume of vessel before burst
\( \gamma \)  ratio of specific heats
\( q \)  decay constant of pressure-time curve

SUBSCRIPTS

\( a \)  ambient conditions
\( s \)  behind shock wave
\( l \)  inside vessel before burst
CHAPTER III
EFFECTS OF PRESSURE WAVES

3-1 Damage Estimates to Structures

Previous sections have shown how to predict the peak free field side-on overpressure $P_s$ and side-on impulse $I_s$ at various locations around an accidental explosion. In this section, we show how to relate these two loading parameters to structural damage to buildings and vehicles in the vicinity. Before this discussion proceeds, the reader must decide what constitutes damage. For example, is breakage of glass and some damage to ceilings an acceptable or unacceptable level of damage? Or, can you accept minor structural damage with partitions distorted and joinery wrenched from fittings? On the other hand, might the target structure be dangerous to inhabit with the roof partially or totally collapsed, at least partial damage to one or more external walls, and some failed load-bearing structural members? Or, can one tolerate the building being 50% to 75% completely demolished? There is no one answer to what level of damage is acceptable. The engineer must decide for himself. If buildings are or can be inhabited by many people, the levels of damage should perhaps be low, while greater damage to individual dwellings could be tolerated.

Because different modes of response (or types of damage) must be considered, various solutions must be considered. The first solution deals with glass breakage. It yields a procedure for predicting the threshold of breakage of glass of various thickness and spans. The second solution is a curve fit to bomb damage data compiled by the British at the end of World War II. Although this curve fit was developed for a standard dwelling, it is also used for factories, main offices, and/or main engineering shops without introducing significant error. Three different empirical pressure versus impulse diagrams will be presented—the first is for minor structural damage involving wrenched joints and partitions, the second is for major structural damage with load bearing members at least partially destroyed, and the third is for 50% to 75% of the building demolished. These results yield general guidelines when accurate structural details are unknown. The third solution is for overturning a bus, truck, mobile home, missile on the launch pad, or other marginally stable target subject to toppling. The fourth and fifth solutions are for the initiation of yielding (the start of permanent deformation) in either beam or plate structural components. The beam elements can have
various support conditions. Plate elements can be either simply-supported or clamped. These generalized solutions can be applied whenever the response of a structure looks critical and structural details are known in sufficient detail to override the second empirical solution based upon bombs damaging British residences. These generalized beam and plate solutions can be applied to many types of structures: cranes, frames, powerline towers, and components of houses and buildings. Each of the solutions will now be presented.

3-1.1 Breakage of Glass

The threshold applied pressure $P_r$ for breaking glass can be determined from Equation (3-1).

$$
\frac{P_r}{\gamma Y^2} = \frac{1.0 + 3.08 \frac{X}{Y}}{8.68} + \frac{X}{Y^2} \left( \frac{0.79 + 0.11 \frac{X}{Y}}{0.79 + 0.11 \frac{X}{Y}} \right)^{1.5}
$$

where

- $\gamma$ = yield stress of glass 
- $h$ = thickness of glass 
- $X$ = short half span 
- $Y$ = long half span 
- $P_r$ = threshold applied maximum reflected pressure

(The equation is valid for any self-consistent set of dimensions). Equation (3-1) works for either sheet or plate glass. The yield stress $\gamma_y$ for glass, however, is not a simple material property, as in steel or other metals. The strength of glass is related to flaws which are both statistical in nature and a function of thickness. Although a complete theory of the kinematics of flaw behavior is not possible, the effective yield point for glass can be approximated by Equation (3-2).

Simple support boundary conditions imply restriction of displacements at the boundaries, but no restraint on rotations.
The parameter $P_r$ in Equation (3-1) is the peak applied load, and not the side-on free field overpressure $P_s$. For weak shocks with $P_s < 10$ kPa that strike a wall head-on, $P_r$ equals $2.0 P_s$. For all shocks with flow parallel to a wall, $P_r$ equals $P_s$. In general, the ratio $P_r/P_s$ is a complex function of both shock strength $P_s$ and the angle of incidence $\theta$. Figure 3-1 can be used to estimate $P_r$ from $P_s$ and $\theta$ under ambient sea level atmospheric conditions. An $\theta$ of $90^\circ$ in Figure 3-1 means that the flow is parallel to the wall. All pressures in Figure 3-1 and this analysis are overpressures and not absolute pressures.

To illustrate the use of this solution, assume that a building has glass windows with panes that are 1.0 by 1.5 by 0.01 meters (3.28 by 4.92 by 0.0328 ft). Equation (3-2) indicates that the yield stress $\sigma_y$ equals $10^{+8}$ Pa (1.43 x $10^4$ psi). Because the aspect ratio $X$ equals 0.6666, Equation (3-1) gives a scaled applied load of 0.296. Substituting for $\sigma_y$, $X$, and $h$ in the scaled applied load yields an applied pressure $P_r$ of 2960 Pa (0.424 psi). Because this is a weak applied load, glass will begin to break in walls facing an accident whenever $P_s$ exceeds $\frac{P_r}{2}$ or 1480 Pa (0.212 psi). If the building has no glass in the walls facing the accident, but has glass in side walls, breakage will begin whenever $P_s$ exceeds $P_r$ or at 2960 Pa (0.424 psi).

The derivations of Equation (3-1) and (3-2) are presented in Appendix 3A. Equation (3-1) is a special case of a plate equation from subsequent sections: a brittle, simply-supported plate in the quasi-static loading realm. Experimental test data from static tests in the literature are also presented in Appendix 3A to demonstrate the validity of this solution. Figure 3-1 comes from curve fits to high-pressure data described in Reference 1.

3.1.2 Empirical Blast Damage Curves for Buildings

Figure 3-2 shows three different isodamage lines plotted as curves of side-on, free-field impulse versus side-on maximum overpressure. The basis for these curves is British data from enemy bombing in World War II plus record of explosions dating from 1871. Although this relationship was developed for the average British dwelling
Figure 3-1. Reflected Overpressure Ratio as Function of Angle of Incidence for Various Side-on Overpressures
Figure 3-2. Pressure vs Impulse Diagram for Building Damage
house, it also works well for factories, main offices, and main engineering workshops. Reference 2 presents this relationship as a quantity-distance equation (charge weight versus standoff equation); however, we have modified the results so they can be presented as a pressure versus impulse diagram (a $P_i$ diagram). Notice that as the level of damage increases, the pressures and impulses required to create the increased damage also increase. For pressures and impulses less than those shown in Figure 3-2, the specified magnitude of damage should not be obtained. The three levels of damage shown in Figure 3-2 are for minor structural damage, major structural damage, and partial demolition.

By minor structural damage, we mean that glass has been broken, joints are wrenched, and partitions are out of some fittings. Major structural damage implies that the roof is partially or totally damaged, at least one external wall is partially damaged, and some load-bearing partitions or members have been destroyed. The term "partial demolition" implies that 50% to 75% of external brickwork or walls have been destroyed or rendered unsafe.

The British present an addition, a threshold for breakage of glass which we do not show in Figure 3-2. Most modern windows have larger spans and are often thinner than the standard window sizes associated with houses built in the 20's and 30's. The preceding analysis should be used if glass breakage constitutes serious damage. In addition, Jarrett presents a fifth contour for complete demolition. We do not present this contour because it is too extreme; a building suffering from partial demolition is uninhabitable and would have to be leveled.

Naturally, contours as presented in Figure 3-2 are approximations. These approximations suffice for large variations in structural types because the loads are also approximations. If a hardened structure or atypical building exists, damage can be better estimated by subdivision into its component plate and beam type elements. After a building has been subdivided into components, equations presented in subsequent sections can be applied to determine loads for initiating fracture in brittle structures and permanent deformation in ductile structures. These subsequent analysis procedures are more difficult to use than the graphical relationships shown in Figure 3-2; however, if they are properly applied, they can supplant Figure 3-2.

To illustrate the use of Figure 3-2, consider several illustrative examples. For example, suppose the free field blast conditions were $P_b$ equal to 10 kPa (1.45 psi), $i_b$ equal to 200 Pa·s (0.0290 psi/sec), then minor structural damage should be expected, but not major structural damage. On the other hand, if $P_b$ equaled 50 kPa (7.25 psi) and $i_b$ equaled 100 Pa·s (0.0415 psi/sec), partial demolition should be expected.
Figure 3D-2. (Concl’d)
To determine the momentum imparted to the body from the blast wave, it is necessary to consider both the diffracted phase of loading and the drag phase of loading.

Side-on overpressure is often expressed as a function of time by the modified Friedlander equation:

\[ p(t) = P_s \left( 1 - \frac{t}{T} \right) e^{-bt/T} \]

where

- \( P_s \) is peak side-on overpressure
- \( T \) is the duration of the positive phase of the blast wave
- \( b \) is a dimensionless time constant.

Integrating this equation gives specific impulse

\[ I_s = \frac{P_s T}{b} \left[ 1 - \left( \frac{1 - e^{-b}}{b} \right) \right] \]
Diffracted specific impulse $I_r$ can be approximated by

$$I_r = \frac{P_r \Delta t}{2}$$

(3D-3)

where

$P_r$ is peak reflected pressure

$\Delta t$ is length of time for the blast load to reach $P_r$ after initial interaction with the blast wave

or

$$I_r = \frac{P_r}{Z} \frac{d}{U}$$

(3D-4)

where

$d$ is effective diameter of human

$U$ is shock front velocity

Shock front velocity can be expressed as

$$\left(\frac{U}{a_0}\right)^2 = 1 + \frac{6}{7} \frac{P_s}{p_o}$$

(3D-5)

where

$a_0$ is the speed of sound

$P_s$ is peak side-on overpressure

$p_o$ is ambient atmospheric pressure

For shocks of intermediate to weak strengths ($P_s/p_o \leq 3.5$):

$$\frac{P_r}{P_s} = 2 \left(\frac{P_s}{p_o}\right) + \frac{3}{4} \left(\frac{P_s}{p_o}\right)^2$$

(3D-6)

Thus, from Equations (3D-4), (3D-5), and (3D-6),

3D-10
The time history of drag pressure, slightly modified from Glasstone, \(^{(3)}\) is

\[ q(t) = Q \left(1 - \frac{t}{T}\right)^2 e^{-bt/T} \]  \(\text{(3D-8)}\)

where

\[ Q \] is peak drag pressure.

Integrating Equation (3D-8) with respect to time over the time interval from 0 to \(T\), drag specific impulse \(I_q\) becomes:

\[ I_q = \left(\frac{QT}{b}\right) \left[1 - \frac{2}{b} + \frac{2}{b^2} (1 - e^{-b})\right] \]  \(\text{(3D-9)}\)

From Baker, \(^{(8)}\) peak drag pressure (for \(P_s/p_o \leq 3.5\)) can be determined by

\[ Q = \frac{5}{2} CD \frac{P_s^2}{(P_o + P_s)} \]  \(\text{(3D-10)}\)

where

\[ CD \] is the drag coefficient for the human body.

Solving Equation (3D-2) for \(T\) and inserting Equation (3D-10) into Equation (3D-9):

\[ I_q = \frac{5}{2} \left(\frac{P_s}{P_o}\right) CD I_s \left[1 - \frac{2}{b} + \frac{b}{e^{-b} + b - 1}\right] \]  \(\text{(3D-11)}\)
Specific impulse is the integral of pressure with respect to time, while impulse is the integral of force with respect to time, pressure being force per unit area. Total momentum imparted to the body is equivalent to total impulse $i_T$:

$$i_T = mV$$  \hspace{1cm} (3D-12)

where

- $m$ is the mass of the body
- $V$ is velocity

or

$$mV = (I_r + I_d) (5.5 d^2)$$  \hspace{1cm} (3D-13)

where

$(5.5 d^2)$ is the assumed cross-sectional area of the body.

Substituting Equations (3D-11) and (3D-17) into (3D-13) and rearranging terms produces

$$a_0 \frac{mV}{5.5 d^3 p_0} = \left[ \frac{P}{p_0} \right] + \left( \frac{3}{8} \left( \frac{P}{p_0} \right) \right)^2 \left( \frac{P}{p_0} \right) + \left( \frac{2.5}{7} \left( \frac{P}{p_0} \right) \right)$$  \hspace{1cm} (3D-14)

which is a nondimensional equation. If one assumes that the density of man is the same as the density of water $\rho_w$ and that the shape of man can be approximated by a cylinder with a length-to-diameter ratio of 5.5, then mass becomes

$$m = \rho_w \frac{\pi d^2}{4} (5.5 d)$$  \hspace{1cm} (3D-15)
Substituting Equation (3D-16) into (3D-14) and rearranging terms, one can obtain

\[
\frac{\pi w a_v}{4 p_o} = \left( \frac{P_s}{p_o} \right)^{\frac{3}{8}} \left( \frac{P_s^2}{p_o} \right)^{\frac{2}{7}} \left( \frac{P_s}{p_o} \right)^{(2.5)} \left( \frac{P_s}{p_o} \right) - \sqrt{1 + \left( \frac{6}{7} \right) \left( \frac{P_s}{p_o} \right)} + \left[ 7 + \left( \frac{P_s}{p_o} \right) \right]
\]

\[
\left( - \frac{2}{b} \right) + \frac{b}{e^{-b} + b - 1} \left( \frac{a_o C D I_s}{p_o} \right) \left( \frac{5.5 \pi}{4} \right)^{1/3} \frac{\rho_w}{m^{1/3}}
\]

Thus,

\[
\frac{\rho_w a_v}{p_o} = f \left( \frac{P_s}{p_o}, \frac{C D a_o I_s \rho_w}{p_o m^{1/3}} \right)
\]

Equation (3D-18) demonstrates a nondimensional functional dependence.

Dimensionally, for constant values of \( \rho_w, a_o, \) and \( p_o, \) Equation (3D-18) becomes

\[
V = f \left( \frac{P_s}{m^{1/3}} \right)
\]

Thus, after calculating the pressure and impulse combinations required to translate an individual of a certain mass at a specific potentially hazardous velocity, it is beneficial to plot the calculations on a graph of \( P_s \) versus \( (L_s/m^{1/3}) \). By doing this, pressure-impulse combinations needed to propel different body masses at the predetermined velocity can be acquired directly from the graph, assuming that the density and length-to-diameter ratios of the body are the same. Hence, eight graphs (2 damage conditions \( \times 4 \) altitudes), instead of the previously mentioned 32 graphs (2 damage conditions \( \times 4 \) masses \( \times 4 \) altitudes), are required to identify...
the pressure-impulse combinations which will produce the human translation velocities specified in Tables 3D-1 and 3D-2 for any mass human body within the limits of the aforementioned assumptions. The results are plotted in Figures 3-12 through 3-19.

It is a very simple matter to use the graphs to determine if there exists a potential threat due to whole body translation and subsequent decelerative impact on a hard surface. An example will demonstrate the procedure.

Suppose a propellant blast occurs at sea level \( p_0 = 101 \text{ kPa} \). The closest people to the blast are several adult males with an approximate mass of 70 kg (154 lbm). Using methods established in Chapters I and II for calculating pressure and impulse, it is determined that these humans are exposed to an incident overpressure of 1600 Pa (0.232 psi) and a specific impulse of 20 kPa·s (2.90 psi-sec). Scaled impulse \( \frac{I_s}{m^{1/3}} \) is then 4850 Pa·s/kg\(^{1/3}\). Examining Figures 3-12 and 3-16 indicates that the point (1600 Pa, 4850 Pa·s/kg\(^{1/3}\)) lies below the threshold curves for skull fracture and for whole body impact, respectively. Since the point is below the curves, no skull fracture or death is expected from whole body displacement. It should be kept in mind that the curves cannot be extended for impulses lower than those shown on the graph, and thus the graphs are somewhat limited since they do not appear to be asymptotic.
LIST OF REFERENCES


CHAPTER IV
CHARACTERISTICS OF FRAGMENTS

4-1 General

The fragments generated during accidental explosions of the types covered in this workbook can come from several sources. They can be pieces of the exploding vessels themselves, or pieces of wreckage from an impact which also results in an explosion, or nearby objects (appurtenances) accelerated by the blast waves from the explosion. The methods of prediction given in this chapter allow one first to estimate initial fragment velocities for various types of accident and geometry of vessel or explosion source. Next, predictions can be made of fragment ranges and impact conditions using initial velocities as inputs. Finally, fragment mass distributions can be predicted in a statistical sense for several classes of accidental explosion.

The various graphs and equations used to generate predictions of fragment characteristics are the result of exercise of a number of computer programs, and fits to experimental data. A number of appendices are included in the chapter to describe the computer programs, statistical analyses, and other supporting information.

4-2 Methods for Estimating Fragment Initial Velocities

4-2.1 Spherical Gas Vessels

The following is the deterministic technique for predicting initial fragment velocities for fragments emanating from containment vessels of spherical geometry. This technique requires that you know the external radius of the spherical confinement vessel, the thickness of the wall of the spherical confinement vessel, the density of the confinement vessel material, and the internal gas pressure at burst. The present figures allow this technique to be used for gases whose properties are similar to air and helium.

The first step in the procedure is to calculate a mass ratio consisting of the mass of a volume of gas equal to the internal volume of the sphere at standard temperature and pressure, divided by the mass of the confinement vessel. The equation for calculating this mass ratio is given in each of the examples in terms of the radius of the spherical confinement vessel, the thickness of the vessel walls, and the density of the gas and vessel material, respectively.
The second step in this procedure is to calculate a pressure ratio which is the pressure of the internal gas at the time the confinement vessel bursts divided by atmospheric pressure. Having obtained the mass ratio and the pressure ratio, a value for initial velocity ratio \( \bar{V} \) can be obtained from Figures 4-1 through 4-8, which constitutes Step 3 in the procedure. The figure used to look up the value of \( \bar{V} \) depends upon whether air or helium is considered and the radius of the confinement vessel. Since figures are not generated for all possible vessel radii, it is necessary to consult the figure with a radius value that is nearest to the radius of the confinement vessel considered. Since plots are not given in the figures for all possible mass ratios, it is also necessary to read values for \( \bar{V} \) from the plots for mass ratios having values most nearly above and below the mass ratio considered. The appropriate value for \( \bar{V} \) can then be obtained by the method of linear interpolation as in Step 4 of the examples. Once the velocity ratio value has been obtained for the specific case considered, it can be multiplied by the appropriate speed of sound at ambient conditions for the gas considered to obtain the initial fragment velocity.

Examples 1 through 3 illustrate how this procedure may be used to obtain initial fragment velocities from bursting spheres. Fragment velocities from bursting cylinders are discussed in the next section with examples. Example 1 is the calculation for a case in which some experimental data have been obtained to verify the results, as is Example 2. Discussion of experimental results which verify the code used to generate Figures 4-1 through 4-8 is given in Appendix 4A. Example 3 is taken from a proposed case where a Centaur pressure tank is overpressurized with helium.

The procedure illustrated here for estimating initial fragment velocities from bursting pressurized confinement vessels was made possible by the generation of Figures 4-1 through 4-8 from the computer code SPHER. This code and the theory behind it are discussed in the appendices. A similar code used to generate Figures 4-9 through 4-16 for cylindrical confinement vessels was generated from a code CYLIN. This code is also discussed in the appendices.

**Example 1. For Spheres**

**Assume:** A spherical containment vessel of glass

\[
R = 2.54 \, \text{cm} \quad (1 \, \text{in.}) \quad \text{internal radius of sphere} \\
t = 1 \, \text{mm} \quad (3.94 \times 10^{-2} \, \text{in.}) \quad \text{glass wall thickness}
\]
Figure 4-1. Fragment Velocities for Contained Air in a Sphere of Radius 0.0762 m
Figure 4-2. Fragment Velocities for Contained Air in a Sphere of Radius 0.254 m
Figure 4-3. Fragment Velocities for Contained Air in a Sphere of Radius 0.762 m
Figure 4-4. Fragment Velocities for Contained Air in a Sphere of Radius 2.54 m
Figure 4-5. Fragment Velocities for Contained He
in a Sphere of Radius 0.0762 m
Figure 4-6. Fragment Velocities for Contained Hz
in a Sphere of Radius 0.254 m
Figure 4-7. Fragment Velocities for Contained He in a Sphere of Radius 0.762 m
Figure 4-8. Fragment Velocities for Contained He in a Sphere of Radius 2.54 m
Burst from internal air pressure

\[ P_0 = 2.25 \times 10^6 \text{ Pa (326 psi)} \]

**Step 1.** Calculate mass ratio \( M_{os}/M_t \) from equation

\[
\frac{M_{os}}{M_t} = \frac{R_{air}^3}{[R^3 - (R-t)^3]} \frac{\rho_{glass}}{\rho_{air}}
\]

From assumptions \( R = 2.54 \text{ cm} \)

\( t = 0.1 \text{ cm} \)

From density tables \( \rho_{air} = 1.293 \times 10^{-3} \text{ g/cm}^3 \), at standard temperature and pressure.

\( \rho_{glass} = 2.60 \text{ g/cm}^3 \)

Therefore,

\[
\frac{M_{os}}{M_t} = \frac{(1.293 \times 10^{-3})(2.54)^3}{[(2.54)^3 - (2.54 - 0.1)^3]} = 4.38 \times 10^{-3}
\]

**Step 2.** Calculate pressure ratio \( \bar{p} \) from equation

\[
\bar{p} = \frac{P_0}{1.01 \times 10^5}
\]

From assumption \( P_0 = 2.25 \times 10^6 \text{ Pa} \)

Therefore,

\[
\bar{p} = \frac{2.25 \times 10^6}{1.01 \times 10^5} = 22.21
\]

**Step 3.** From Figure 4-2, for a vessel of radius closest to the vessel considered containing air, find \( \bar{V} \) for the \( M_{os}/M_t \) ratio
most nearly above and below the value of Step 1 for the $\bar{p}$ of Step 2. This is:

at $\bar{p} = 22.2$

$M_1 = (M_{os}/M_t) \text{ below } = \frac{1.40 \times 10^{-3}}{0.140}$, $\overline{V}_1 = 0.140$

$M_2 = (M_{os}/M_t) \text{ Step 1 } = \frac{4.38 \times 10^{-3}}{0.140}$, $\overline{V}_2 = ?$

$M_3 = (M_{os}/M_t) \text{ above } = \frac{4.67 \times 10^{-3}}{0.140}$, $\overline{V}_3 = 0.266$

**Step 4.** Find $\overline{V}_2$ by linear interpolation from the equation

$$\overline{V}_2 = (\overline{V}_3 - \overline{V}_1) \left( \frac{M_2 - M}{M_2 - M_p} \right) + \overline{V}_1 =$$

$$\overline{V}_2 = (0.266 - 0.140) \left( \frac{4.38 - 1.40}{4.67 - 1.40} \right) + 0.140 = 0.255$$

**Step 5.** Multiply $\overline{V}_2$ by the $a_{os}$ value given in Figure 4-2 to obtain the initial fragment velocity

$$V_i = \overline{V}_2 \cdot a_{os} = (0.255) (3.31 \times 10^2 \text{ m/s}) = 84.4 \text{ m/s}$$

The initial fragment velocity is 84.4 m/s (277 ft/sec).

**Note:** This example was run using code SPHER, the code used to generate Figure 4-2, for the specific data assumed. The results gave a $V_i = 80.2 \text{ m/s (263 ft/sec)}$ compared to our result of 84.4 m/s (277 ft/sec) obtained from the figure. This gives some indication of the interpolation error and error in reading numbers from the figure which can be expected.

This particular example was taken from the data of D. W. Boyer, et al., (1) who performed experiments in which glass spheres of various dimensions were burst under internal gas pressure. For this particular sphere dimension and gas pressure, the experimental data showed fragments had an average initial fragment velocity of 75.6 m/s (248 ft/s). Our value obtained from the tables is about 10% higher than this.
A comparison of other data from the tests by Boyer with fragment velocities obtained from code SPHER or the tables generated from code SPHER is given in Appendix 4A.

Example 2. For Spheres

Assume: A spherical containment vessel of glass

\[ R = 2.54 \text{ cm (1 in.) internal radius of sphere} \]
\[ t = 1 \text{ mm (3.94 x 10}^{-2} \text{ in.) glass wall thickness} \]

Burst from internal helium \((H^*)\) pressure

\[ P_0 = 2.25 \times 10^6 \text{ Pa (326 psi)} \]

Step 1. Calculate mass ratio \(M_{os}/M_t\) from equation

\[
\frac{M_{os}}{M_t} = \frac{R^3 \rho_{He}}{R^3 - (R - t)^3 \rho_{glass}}
\]

From assumptions:
\[ R = 2.54 \text{ cm} \]
\[ t = 0.1 \text{ cm} \]

From density tables:
\[ \rho_{He} = 0.1785 \times 10^{-3} \text{ gm/cm}^3 \text{ at STP} \]
\[ \rho_{glass} = 2.60 \text{ gm/cm}^3 \]

Therefore,

\[
\frac{M_{os}}{M_t} = \frac{(0.1785 \times 10^{-3})(2.54)^3}{(2.54)^3 - (2.54 - 0.1)^3(2.6)} = 6.05 \times 10^{-4}
\]

Step 2. \( \bar{p} = 22.2 \) (same as in Example 1)

Step 3. From Figure 4-5, for a vessel of radius closest to the vessel considered containing He, find \( \bar{V} \) for the \( M_{os}/M_t \) ratio most nearly above and below the value of Step 1 for the \( \bar{p} \) of Step 2.
at \( p = 22.2 \)

\[
M_1 = \frac{M_{os}}{M_t} \text{ below } = 1.88 \times 10^{-4}, \quad \overline{V_1} = 0.036
\]

\[
M_2 = \frac{M_{os}}{M_t} \text{ Step 1 } = 6.05 \times 10^{-4}, \quad \overline{V_2} = ?
\]

\[
M_3 = \frac{M_{os}}{M_t} \text{ above } = 6.26 \times 10^{-4}, \quad \overline{V_3} = 0.068
\]

**Step 4.** Find \( \overline{V_2} \) by linear interpolation from the equation

\[
\overline{V_2} = (\overline{V_3} - \overline{V_1}) \left( \frac{M_2 - M_1}{M_3 - M_1} \right) + \overline{V_1}
\]

\[
\overline{V_2} = (0.068 - 0.036) \left( \frac{6.05 - 1.88}{6.26 - 1.88} \right) + 0.036
\]

\[= 0.066\]

**Step 5.** Multiply \( \overline{V_2} \) by the \( a_{os} \) value in Figure 4.5 to obtain the initial fragment velocity

\[
V_i = \overline{V_2} \cdot a_{os} = (0.066) (9.63 \times 10^2 \text{ m/s}) = 64.0 \text{ m/s}
\]

The initial fragment velocity is 64 m/s (210 ft/sec)

**Example 3. For Spheres**

Assume: A Centaur pressurant tank

\[
V = 0.121 \text{ in}^3 (7361 \text{ in}^3) \text{ volume of tank}
\]

\[
t = 0.46 \text{ cm (0.181 in.) titanium wall thickness}
\]

Burst from internal He pressure

\[
P_o = 2.07 \times 10^7 \text{ Pa (3000 psi)}
\]

**Step 1.** Calculate mass ratio \( M_{os}/M_t \) from equation
\[ \frac{M_{01}}{M_t} = \frac{R^3 \rho_{\text{He}}}{[R^3 - (R - t)^3] \rho_{\text{titanium}}} \]

from assumptions

\[ V = \frac{4}{3} \pi R^3 = 0.121 \, \text{m}^3 \]

Therefore

\[ R = \left( \frac{(3)(0.121)}{4\pi} \right)^{1/3} \, \text{m} \]

\[ R = 0.307 \, \text{m} = 30.7 \, \text{cm} \]

\[ t = 0.46 \, \text{cm} \]

From density tables

\[ \rho_{\text{He}} = 0.1785 \times 10^{-3} \, \text{gm/cm}^3 \] at STP

\[ \rho_{\text{Ti}} = 4.5 \, \text{gm/cm}^3 \]

Therefore,

\[ \frac{M_{01}}{M_t} = \frac{(30.7)^3 (0.1785 \times 10^{-3})}{(30.7)^3 - (30.7 - 0.46)^3 (4.5)} = 8.96 \times 10^{-4} \]

Step 2. Calculate pressure ratio \( \bar{p} \) from equation

\[ \bar{p} = \frac{P_0}{1.01 \times 10^5} \]

from assumption

\[ P_0 = 2.07 \times 10^7 \, \text{Pa} \]

Therefore,
Step 3. From Figure 4-6, for a vessel of radius closest to the vessel considered containing He, find \( \overline{V} \) for the \( M_{os}/M_t \) ratio most nearly above and below the value of Step 1 for the \( \overline{p} \) value of Step 2. This is:

at \( \overline{p} = 205 \)

\[ M_1 = \frac{(M_{os} / M_t) \text{ below}}{8.96 \times 10^{-4}} = 6.95 \times 10^{-4}, \quad \overline{V}_1 = 0.30 \]

\[ M_2 = \frac{(M_{os} / M_t) \text{ Step 1}}{8.96 \times 10^{-4}} = 8.96 \times 10^{-4}, \quad \overline{V}_2 = ? \]

\[ M_3 = \frac{(M_{os} / M_t) \text{ above}}{2.32 \times 10^{-3}} = 2.32 \times 10^{-3}, \quad \overline{V}_3 = 0.49 \]

Step 4. Find \( \overline{V}_2 \) by linear interpolation from equation

\[ \overline{V}_2 = (\overline{V}_3 - \overline{V}_1) \left( \frac{m_2 - m_1}{m_3 - m_1} \right) + \overline{V}_1 \]

\[ V_2 = (0.49 - 0.30) \left( \frac{8.96 - 6.95}{2.32 - 6.95} \right) + 0.30 \]

\[ = 0.32 \text{ m/s} \]

Step 5. Multiply \( V_2 \) by the \( a_{os} \) value given in Figure 4-6 to obtain the initial fragment velocity.

\[ V_1 = \overline{V}_2 \cdot a_{os} = (0.32)(963 \text{ m/s}) = 311 \text{ m/s} \]

The initial fragment velocity is 311 m/s (1022 ft/s).

4-2.2 Cylindrical Gas Vessels

The following deterministic procedure may be used to estimate initial fragment velocities emanating from containment vessels of cylindrical configuration. Figures 4-9 through 4-16 may be used to estimate initial fragment velocities for a broad spectrum of burst pressures and confinement vessel dimensions. Figures 4-9 through 4-12
Figure 4-9. Fragment Velocities for Contained Air
in a Cylinder of Radius 0.0762 m
Figure 4-10. Fragment Velocities for Contained Air in a Cylinder of Radius 0.254 m
Figure 4-11. Fragment Velocities for Contained Air in a Cylinder of Radius 0.762 m
Figure 4-12. Fragment Velocities for Contained Air in a Cylinder of Radius 2.54 m
Figure 4-13. Fragment Velocities for Contained He in a Cylinder of Radius 0.0762 m
Figure 4.14. Fragment Velocities for Contained He in a Cylinder of Radius 0.254 m
Figure 4-15. Fragment Velocities for Contained Hg in a Cylinder of Radius 0.762 m
Figure 4.16. Fragment Velocities for Contained He in a Cylinder of Radius 2.54 m
may be used for estimating initial fragment velocities for a confined gas with properties similar to air. Figures 4-12 through 4-16 may be used for estimating initial fragment velocities for a confined gas whose properties are similar to helium. The code used to generate these figures (code CYLIN) is described in Appendix 4A. It should be pointed out that because of the two-dimensional nature of the analysis, the effects of the cylinder ends are not taken into account. The length of the cylinder thus does not affect results and is immaterial so long as the properties of the cylinder per unit length are known.

Example 1 below demonstrates how the initial velocity of fragments emanating from a bursting containment vessel may be obtained through the use of the figures. In this example, a steel cylinder approximately a foot in diameter and having wall thickness of approximately a quarter of an inch is assumed to burst at $10^4$ psi from the pressure of confined air. It should be noted that all calculations are done using standard SI units. It should also be noted that the tank mass is nondimensionalized in the figures by the use of a ratio with the mass of the confined gas at standard temperature and pressure. Example 2 is a calculation for the initial fragment velocities emanating from a cylinder similar to that of Example 1, which bursts under pressurization from helium gas.

The method for calculating fragment velocities shown in these examples for cylinders is the same as those for spheres with the exception of a different equation for calculating the mass ratio and the use of the different figures to obtain $\tilde{v}$ as a function of $\tilde{p}$ and the mass ratio.

**Examples 1. For Cylinders**

Assume:

A cylindrical confinement vessel

$t = 0.5 \text{ cm (} \sim 0.25 \text{ in.}) \text{ steel wall thickness}$

$2R = 0.30 \text{ m (} \sim 12.0 \text{ in.}) \text{ diameter of tank}$

Bursts from internal air pressure

$P_0 = 7.00 \times 10^7 \text{ Pa (} \sim 10^4 \text{ psi})$

**Step 1.** Calculate mass ratio $M_{3s} / M_t$ from equation

$$\frac{M_{3s}}{M_t} = \frac{R^2 \rho_{\text{air}}}{(R^2 - (R - t)^2) \rho_{\text{steel}}}$$
from assumptions

\[ R = 0.15 \text{ m} = 15 \text{ cm} \]
\[ \varepsilon = 0.5 \text{ cm} \]

from density tables

\[ \rho_{\text{air}} = 1.293 \times 10^{-3} \text{ g/cm}^3 \text{ at STP} \]
\[ \rho_{\text{steel}} = 7.8 \text{ gm/cm}^3 \]

Therefore,

\[ \frac{M_{\text{o}}}{M_t} = \frac{(1.293 \times 10^{-3})(15)^2}{(7.8)(15^2 - 14.5^2)} = 2.53 \times 10^{-3} \]

Step 2. Calculate pressure ratio \( p \) from equation

\[ p = \frac{P_o}{1.01 \times 10^5} \]

from assumptions

\[ P_o = 7.0 \times 10^7 \text{ Pa} \]

\[ p = \frac{7.0 \times 10^7}{1.01 \times 10^5} = 693 \]

Step 3. From Figure 4-10 for a vessel of radius closest to the vessel considered, find \( V \) for \( \frac{M_{\text{o}}}{M_t} \) ratio most nearly above and below the value of Step 1 for the \( p \) of Step 2. This is:

\[ p = 693 \]
Step 4. Find $\bar{V}_2$ by linear interpolation from equation

$$\bar{V}_2 = (\bar{V}_3 - \bar{V}_1) \left( \frac{M_2 - M_1}{M_3 - M_1} \right) + \bar{V}_1$$

$$\bar{V}_2 = (1.38 - 1.05) \left( \frac{2.53 - 1.73}{5.19 - 1.73} \right) + 1.05 = 1.13$$

Step 5. Multiply $\bar{V}_2$ by the $a_{os}$ value given in the figure to obtain the initial fragment velocity

$$V_i = \bar{V}_2 \cdot a_{os} = (1.13)(3.31 \times 10^2 \text{ m/s}) = 373 \text{ m/s}$$

The initial fragment velocity is 373 m/s (1220 ft/sec).

**Example 2. For Cylinders**

Assume same conditions as Example 1, but the vessel contains helium.

Step 1. Calculate mass ratio $M_{os}/M_t$ from equation

$$\frac{M_{os}}{M_t} = \frac{R^2 \rho_{\text{He}}}{[R^2 - (R - t)^2 \rho_{\text{steel}}]}$$

$\rho_{\text{helium}} = 0.1785 \times 10^{-3} \text{ gm/cm}^3$ at STP

Other values as in Example 1.
\[ \frac{M_{os}}{M_t} = \frac{(0.1785 \times 10^{-3})(15)^2}{(7.8)(15^2 - 14.5^2)} = 3.49 \times 10^{-4} \]

Step 2. Same as in Example 1, \( \bar{p} = 693 \).

Step 3. From Figure 4-14 for a tank of radius closest to the tank considered find \( V \) for \( M_{os}/M_t \) ratio most nearly above and below the value of Step 1 for the \( \bar{p} \) of Step 2. This is

\[ M_1 = \frac{(M_{os}/M_t) \text{ below}}{V_1} = 2.32 \times 10^{-4}, \quad \bar{V}_1 = 0.67 \]

\[ M_2 = \frac{(M_{os}/M_t) \text{ above}}{V_2} = 3.49 \times 10^{-4}, \quad \bar{V}_2 = ? \]

\[ M_3 = \frac{(M_{os}/M_t) \text{ above}}{V_3} = 6.95 \times 10^{-4}, \quad \bar{V}_3 = 0.41 \]

Step 4. Find \( \bar{V}_2 \) by linear interpolation from equation

\[ \bar{V}_2 = \bar{V}_3 - \bar{V}_1 \left( \frac{M_2 - M_1}{M_3 - M_1} \right) + \bar{V}_1 \]

\[ \bar{V}_2 = (0.67 - 0.41) \left( \frac{3.49 - 2.32}{6.95 - 2.32} \right) + 0.41 = 0.48 \]

Step 5. Multiply \( \bar{V}_2 \) by the \( a_{os} \) value given in the figure to obtain the initial fragment velocity

\[ V_1 = \bar{V}_2 a_{os} = (0.48)(963 \text{ m/s}) \]

\[ = 458 \text{ m/s} \]

The initial fragment velocity is 458 m/s (1500 ft/sec).

4-3 Estimate of Initial Velocities of Fragments from Spheres and Cylinders Bursting into Two Equal Halves

The method developed by Taylor and Price \((2)\) for calculating velocities of fragments from bursting spherical reservoirs was modified to provide velocity calculations for fragments from both cylindrical and spherical gas vessels. The development of the necessary equations, the numerical iteration method used to simultaneously solve the resulting
differential equations, the computer program and results of the analysis are all explained in considerable depth for the interested reader in Appendix 4C. Only assumptions and conclusions necessary for determining fragment velocities are included here.

The assumptions relevant to the calculation of fragment velocity in this section follow. More complete listing of assumptions are contained in Appendix 4C, but only essential elements are contained here. The pertinent assumptions upon which conclusions which follow are based are:

(1) The vessel with gas under pressure breaks into two equal halves along a plane perpendicular to the cylindrical axis, and the two container fragments are driven in opposite directions.

(2) The thickness of the containment vessel is uniform.

(3) The containment vessel has hemispherical end caps.

(4) Vessels are made of Titanium or Titanium alloys (material used for containment vessels in flight-weight vehicles) and has a length-to-diameter (L/D) ratio of 10, 6.

(5) Contained gases are either air, carbon dioxide (CO₂) or hydrogen (H₂).

The summarized calculations are presented in nondimensionalized units in order to condense and simplify the determination of fragment velocity. The pertinent nondimensional (unitless) relations are defined as follows:

(1) Nondimensional pressure

\[
\bar{P} = \text{initial pressure/ambient pressure (or } P/p_0)\]

where the change in atmospheric pressure \( p_0 \) with altitude is shown in Figure 4-17.

(2) Nondimensional thickness

\[
h/D = \text{cylinder thickness/cylinder diameter}\]

(3) Nondimensional length

\[
L/D = \text{total length/cylinder diameter}\]
Nondimensional velocity

\[ \bar{v} = \frac{V}{v_0} \]

where \( V \) is the velocity of the fragment and \( v_0 \) is sound speed.

The sound speeds \( v_0 \) for the gases mentioned are:

- Air - 344 m/s (13 550 in/sec)
- \( \text{CO}_2 \) - 258 m/s (10 150 in/sec)
- \( \text{H}_2 \) - 1270 m/s (50 000 in/sec)

Figures 4-18 through 4-20 contain plots of \( \bar{v} \) versus \( \bar{P} \) for air, carbon dioxide, and hydrogen gases, respectively, the \( L/D \) ratios being held constant at 10.0 and as many as three curves, one for each \( h/D \) ratio (0.001, 0.01, and 0.1), being plotted on a single figure. Nondimensional velocity and pressure combinations for intermediate values of \( h/D \) can be approximated from these figures. Figure 4-21 contains a plot of \( \bar{v} \) versus \( L/D \). It should be noted that, for high nondimensional pressures (\( \bar{P} \approx 4080 \)), \( \bar{v} \) is essentially independent for values of \( L/D \) from 1.0 (spheres) to 10.0.

The procedure for determining the velocity of fragments consisting of halves of spheres or cylinders made of Titanium or Titanium alloys and containing air, carbon dioxide, or hydrogen gas under pressure, is as follows:

1. Determination atmospheric pressure \( p_0 \) (see Figure 4-17).
2. Calculate nondimensional pressure \( \bar{P} \):
   \[ \bar{P} = \frac{P}{p_0} \]
3. Calculate nondimensional thickness \( h/D \).
4. Calculate nondimensional length \( L/D \).
Figure 4.18. Nondimensional Velocity Vs Nondimensional Pressure for Vessels Containing Air

- $M_{os}/M_t = 6.54 \times 10^{-2}$
- $M_{os}/M_t = 6.75 \times 10^{-3}$
- $M_{os}/M_t = 4.93 \times 10^{-4}$

$L = 2.85 \text{ m}$
$R = 0.1525 \text{ m}$
Figure 4-19. Nondimensional Velocity Vs Nondimensional Pressure for Vessels Containing CO$_2$
Figure 4-20. Nondimensional Velocity Vs Nondimensional Pressure for Vessels Containing H₂
Figure 4-21. Nondimensional Velocity Vs Length-to-Diameter (L/D) Ratio
For L/D equal to 10.0, choose the appropriate figure of Figures 4-13, 4-19 or 4-20 which relate to contained gases of air, carbon dioxide (CO₂) and hydrogen (H₂), respectively, and choose the curve with proper h/D value. Positions of curves for intermediate h/D values can be interpolated.

Knowing the gas, h/D and p, find \( \bar{V} \) from the figure and calculate velocity \( V \) (m/s):

\[
V = \bar{V} a_o
\]

where 
\[
a_o = \begin{align*}
344 \text{ m/s (air)} \\
258 \text{ m/s (CO}_2\text{)} \\
1270 \text{ m/s (H}_2\text{)}
\end{align*}
\]

Some examples will help clarify the procedure for calculating fragment velocity.

**Example 1:**

A pressure vessel made of a Titanium alloy is cylindrical in shape with hemispherical end caps. The vessel is 5 m (16.4 ft) long and 0.5 m (1.64 ft) outside diameter. Its thickness is uniform and is 0.5 cm (0.197 in).Contained gas is air at a pressure of \( 1.0135 \times 10^7 \) Pa (14.7 x 10³ psi). The vessel is located at sea level. If the vessel splits into two halves along a plane perpendicular to the cylindrical axis, what are the expected velocities of each of the two fragments?

**Solution:**

(1) From Figure 4-17,

\[
p_o = 1.0135 \times 10^5 \text{ Pa (14.7 psi)}
\]

(2) Nondimensional pressure \( \bar{P} \):

Extreme caution should be taken when L/D ratios differ from 10.0. Figure 4-21 is a graphical representation of \( \bar{V} \) versus L/D varying from 1.0 to 10.0. Accurate values of \( \bar{V} \) cannot be determined for values of L/D less than 10.0 and for gases, values of \( \bar{P} \) and h/D ratios which are not contained in Figure 4-21.
(3) Nondimensional thickness $L/D$:

$$
L/D = \frac{(0.5 \text{ cm}) \cdot (0.01 \text{ m/cm})}{0.5 \text{ m}} = 0.01
$$

(4) Nondimensional length $L/D$:

$$
L/D = \frac{5 \text{ m}}{0.5 \text{ m}} = 10.0
$$

(5) The center curve of Figure 4-18 is the appropriate curve for air, $L/D$ of 10.0, and $L/D$ of 0.01.

(6) From Figure 4-18 and $\bar{P}$ of 100, one finds that

$$
V \approx 0.36
$$

or

$$
V = (0.36)(344 \text{ m/s}) = 124 \text{ m/s} (407 \text{ ft/sec})
$$

Example 2:

Same as Example 1, except let $P$ be $3.45 \times 10^5 \text{ Pa}$ (48.5 psi) and length of the vessel be 1 m (3.28 ft).

Solution:

(1) $P_o = 1.0135 \times 10^6 \text{ Pa}$ (see Example 1)

(2) $\bar{P} = \frac{3.45 \times 10^5 \text{ Pa}}{1.0135 \times 10^5 \text{ Pa}} = 34$

(3) $h/D = 0.01$ (see Example 1)

(4) $L/D = \frac{1 \text{ m}}{0.5 \text{ m}} = 2$

(5) Since $L/D$ is not 10.0, Figures 4-18, 4-19, and 4-20 cannot be used. Normally, the fragment velocity could not
be calculated from the information in this chapter. However, Figure 4-21 applies for contained air, h/D of 0.01
and $P$ of 34, and can be used.

(6) From Figure 4-21:

$$V = 0.28$$

or

$$V = (0.28) (344 \text{ m/s}) = 96 \text{ m/s (315 ft/sec)}$$

Example 3:

Same as Example 1 except hydrogen is the contained gas.

Solution:

(1) $P_0 = 1.0135 \times 10^5 \text{ Pa (see Example 1)}$

(2) $P = 100$ (see Example 1)

(3) $h/D = 0.01$ (see Example 1)

(4) $L/D = 10.0$ (see Example 1)

(5) The center curve of Figure 4-20 is the appropriate curve for hydrogen, $L/D$ of 10.0, and $h/D$ of 0.01.

(6) From Figure 4-20 and $P$ of 100 one finds that

$$V = 0.07$$

or

$$V = (0.07) (1270 \text{ m/s}) = 89 \text{ m/s (292 ft/sec)}$$

4-4 Determination of Appurtenance Velocity

The method used here to calculate appurtenance velocity is an extension of work performed by Baker, et al. (3) Appendix 4D contains the development of the basic equations as well as a listing of the computer program used to generate nondimensional velocity curves as a function of nondimensional pressure and nondimensional impulse. The interested reader is encouraged to examine the appendix for a better understanding.
of the interaction of appurtenances with blast waves. In order to arrive
at values for appurtenance velocity, however, it is not necessary to refer
to the appendix since all of the curves and equations required to calculate
velocity are contained in this section. It is beneficial, however, to keep
in mind that the method used for calculating the velocity of an appurte-
nance assumes that the appurtenance behaves as a rigid body, that none
of the energy in the blast wave is absorbed in breaking the appurtenance
loose from its moorings or deforming it elastically or plastically, and
that gravity effects are ignored during the acceleration phase of the
motion.

The velocity of an appurtenance can be determined from a nondi-

dimensional velocity which depends on nondimensional pressure and non-
dimensional impulse. In functional format

\[
\frac{M V a}{P_o A(KH + X)} = \left( \frac{P_s}{P_o} \right) \frac{C D I_s a}{P_s (KH + X)}
\]

(4-1)

where

- \( M \) is the total mass of the appurtenance
- \( V \) is the velocity of the appurtenance
- \( a_o \) is the velocity of sound in air
- \( P_o \) is atmospheric pressure
- \( A \) is the mean presented area of the appurtenance
- \( K \) is a nondimensional constant which is 4 for appurtenances
  on the ground and 2 for appurtenances in the air
- \( H \) is the minimum transverse distance of the mean presented
  area
- \( X \) is the distance from the front of the object to the location
  of its largest cross-sectional area
- \( P_s \) is peak incident overpressure
The drag coefficient $C_D$ is......

1. static environmental variables ($a_0$, $p_0$)
2. blast wave variables ($P_g$, $I_g$)
3. appurtenance variables ($M$, $V$, $A$, $K$, $H$, $X$, $C_D$)

The static environmental variables, sound speed $a_0$ and atmospheric pressure $p_0$, vary with altitude or location above sea level. This altitude dependency is shown in Figures 4-22 and 4-23. The blast wave variables, peak incident overpressure $P_g$ and specific impulse $I_g$, at specific standoff distances (i.e., distance from center of the explosion to the center of the appurtenance) can be determined from Chapters I and II of this handbook. Appurtenance variables are all associated with the object which may be propelled after interaction with the blast wave. The method for determining the velocity $V$ will be discussed later. The choice of total mass $M$ depends on the volume and density of the object, and non-dimensional constant $K$ depends on its position. Representative values for drag coefficient $C_D$ can be acquired from Figure 4-24. The mean presented area $A$ of the appurtenance depends on its shape. It is the largest projected area of the appurtenance facing the approaching blast wave. The transverse distance $H$ is the minimum dimension of the largest cross-sectional area of the object facing the blast wave. The length $X$ is the distance from the point of the appurtenance which first interacts with the blast wave to the plane containing the largest cross-sectional area facing the approaching blast wave. For objects which have a flat face facing the blast wave, which is also the location of the plane with the largest cross-sectional area, $X$ equals zero ($X = 0$). Figure 4-25 helps explain the meaning of the various appurtenance variables.

Figure 4-26 is a graphical representation of Equation (1) for various values of nondimensional velocity. The procedure for calculating appurtenance velocity $V$ follows. Care, however, should be taken when interpolating between curves and extending curves. Estimates made by extending the curves to lower nondimensional impulses are especially hazardous.
Figure 4.22. Speed of Sound Vs Altitude

- Speed of Sound (m/s)
- Altitude (m)

- 316.4 m/s
- 332.7 m/s
- 340.2 m/s
- 340 m/s
- 350 m/s
- 360 m/s
- 370 m/s
- 380 m/s
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<tr>
<th>SHAPE</th>
<th>SKETCH</th>
<th>$C_D$</th>
</tr>
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<tr>
<td>Long Rectangular Member, edge-on</td>
<td><img src="image" alt="Flow diagram" /></td>
<td>1.55</td>
</tr>
<tr>
<td>Narrow strip, face-on</td>
<td><img src="image" alt="Flow diagram" /></td>
<td>1.68</td>
</tr>
</tbody>
</table>

Figure 4-24. Drag Coefficients, $C_D$, of Various Shapes
Figure 4-45. Pictorial Explanation of Appurtenance Variables
Figure 4-26. Nondimensional Appurtenance Velocity $\bar{V}$ as a Function of Nondimensional Pressure $\bar{P}$ and Nondimensional Impulse $I$
(1) Determine static environment variables, speed of sound $a_o \ (m/s)$ and atmospheric pressure $p_o \ (Pa)$, from Figures 4-22 and 4-23, respectively.

(2) Determine blast wave variables, peak incident overpressure $P_s \ (Pa)$ and specific impulse $I_s \ (Pa\cdot s)$, from Chapters I and II.

(3) Determine appurtenance variables:

(a) $M$ - total mass in kilograms
(b) $A$ - mean presented area in square meters
(c) $K$ - dimensionless constant which equals 4 for appurtenances on the ground and 2 for appurtenances in the air
(d) $H$ - minimum transverse distance of the mean presented area in meters
(e) $X$ - distance from the front of the appurtenance to the location of its largest cross-sectional area in meters
(f) $C_D$ - nondimensional drag coefficient of the appurtenance (see Figure 4-24)

(4) Calculate nondimensional pressure $\bar{P}$ where

$$\bar{P}_s = \frac{P_s}{P_o}$$

(5) Calculate nondimensional impulse $\bar{I}_s$ where

$$\bar{I}_s = \frac{C_D I_s a_o}{P_s (KH + X)}$$

(6) Find the location of the point $(\bar{I}_s, \bar{P}_s)$ on the graph in Figure 4-26 and estimate a value for nondimensional velocity $\bar{V}$ from the curves of constant nondimensional velocity.
(7) Calculate appurtenance velocity $V$ (in m/s) where

$$\bar{V} = \frac{MV_a}{pa_o A(KH + X)}$$

and thus

$$V = \frac{\bar{V} p_o A(KH + X)}{Ma_o}$$

Some examples will clarify the procedure.

Example 1:

Assume that an explosion occurs at sea level. A cubical cement block with a side facing the blast front is located at such a distance from the source of the explosion that it is exposed to an incident peak overpressure $P_s$ of $1.4 \times 10^5$ Pa (1.97 psi) and an incident specific impulse $I_s$ of $1.9 \times 10^4$ Pa.s (0.267 psi/sec). The density of the cement block is $1.792 \times 10^4$ kg/m$^3$ (0.647 lbm/in$^3$).

Solution:

(1) Static environmental variables:

$$a_o = 340 \text{ m/s}$$
$$p_o = 1.0135 \times 10^5 \text{ Pa}$$

(2) Blast wave variables:

$$P_s = 1.0 \times 10^5 \text{ Pa}$$
$$I_s = 1.9 \times 10^4 \text{ Pa.s}$$

(3) Appurtenance variables:

(a) $M = \text{density} \times \text{volume}$

$$= 1.792 \times 10^4 \text{ kg/m}^3 \times (2.5 \text{ m})^3$$

$$= 2.8 \times 10^5 \text{ kg}$$
(b) \[ A = (2.5 \text{ m})^2 \]

\[ A = 6.25 \text{ m}^2 \]

(c) Appurtenance is on the ground. Therefore,

\[ K = 4 \]

(d) \[ H = 2.5 \text{ m} \]

(e) \[ X = 0 \text{ m} \] since the portion of block facing the blast wave is flat.

(f) Drag coefficient from Figure 4-24

\[ C_D = 1.05 \]

(4) Nondimensional pressure

\[ \bar{P_s} = \frac{P_s}{P_0} = \frac{1.0 \times 10^5 \text{ Pa}}{1.0135 \times 10^5 \text{ Pa}} = 0.99 \]

(5) Nondimensional impulse

\[ \bar{I_s} = \frac{C_D I_s A_0}{P_s (KH + X)} = \frac{(1.05)(1.9 \times 10^4)(340)}{(1.0 \times 10^5)(4)(2.5 + 0)^2} = 6.8 \]

(6) From Figure 4-26, the point \((\bar{I_s}, \bar{P_s})\) or \((6.8, 0.99)\) is located very close to the \(\bar{V} = 5.0\) line

(7) Appurtenance velocity

\[ V = \frac{\bar{V} P_0 A (KH + X)}{M \bar{a}_0} \]

\[ V = \frac{(5)(1.0135 \times 10^5)(6.25)(4)(2.5 + 0)}{(2.8 \times 10^5)(340)} = 0.33 \text{ m/s (1.08 ft/sec)} \]
Example 2:

Assume an upright cylindrical pole weighing 10.0 kg (22 lbm) with radius of 0.01 m (0.0323 ft) and length of 4 m (13.1 ft) is subjected to the same blast parameters as mentioned in Example 1. The blast is incident on the curved portion of the pole.

Solution:

(1) Static environmental variables:

\[ a_0 = 340 \text{ m/s} \]
\[ p_0 = 1.0135 \times 10^5 \text{ Pa} \]

(2) Blast wave variables:

\[ P_s = 1.0 \times 10^5 \text{ Pa} \]
\[ I_s = 1.9 \times 10^4 \text{ Pa} \cdot \text{s} \]

(3) Appurtenance variables:

(a) \[ M = 10.0 \text{ kg} \]

(b) \[ A = 2 \times \text{radius} \times \text{length} \]
\[ A = (2)(0.01)(4) \]
\[ A = 0.08 \text{ m}^2 \]

(c) \[ K = 4 \]

(d) \[ H = 2r = (2)(0.01) = 0.02 \text{ m} \]

(e) \[ x = r = 0.01 \text{ m} \]

(f) \[ C_D = (\text{from Figure 4-24}) = 1.20 \]

(4) \[ \frac{P_s}{p_0} = \frac{1.0 \times 10^5}{1.0135 \times 10^5} \approx 0.99 \]
\[
\frac{I_g}{I_s} = \frac{C_D I_s^3}{P_g (KH + X)} = \frac{(1.20) (1.9 \times 10^4) (3+0)}{(1 \times 10^5) \times (4) (0.02) \times (0.01)}
\]

\[= 3.61\]

(6) Figure 4-26, the point \((I_g, T_g)\) or \((8.61, 0.99)\) is located above the \(V = 50.0\) line at a location where one might expect \(V = 200\).

(7) Appurtenance velocity

\[
V = \frac{V p_o A (KH + X)}{M a_o}
\]

\[V = \frac{(200) (1.0135 \times 10^5) (0.08) - (4) (0.02) + (0.01)}{(10) (340)}
\]

\[V = 42.9 \text{ m/s (141 ft/sec)}\]

4-5 Methods for Computing Fragment Ranges and Impact Conditions

Two methods have been developed to estimate the range and terminal velocities of fragments. They depend on the fragment shapes. In the first method, the fragment is assumed to have a disc shape with a diameter at least five times greater than the thickness of the fragment. For these fragments lift effects are taken into consideration. A second method may be used where fragments are "chunky" shaped. These fragments may be represented by a sphere or cube where no single linear dimension can be said to be very much greater than any other. For these fragments lift forces are neglected and only drag forces are considered.

To estimate fragment ranges, the following techniques may be used based on Figures 4-27 through 4-36 which have been generated by computer codes. The codes (FRISE and TRAJE) and their underlying analysis are discussed in Appendix 4E. To determine fragment terminal velocities, data must be input to the codes themselves.
Figure 4-27. Disc Fragment Range for $V_i = 100 \text{ m/s}, \text{AR} = 5$

All curves are to the right of the $\alpha = 90^\circ$ line

$V_i = 100 \text{ m/s}$

$\text{AR} = 5$

(1 ft = m x 3.281)
Figure 4-28. Disc Fragment Range for
\( V_i = 100 \, \text{m/s}, \ AR = 10 \)

\( (\text{ft} = \text{m} \times 3.281) \)
Figure 4.20. Disc Fragment Range for
$V_i = 150 \text{ m/s}, \text{AR} = 5$

$V_i = 150 \text{ m/s}$

$\text{AR} = 5$
Figure 4.30. Disc Fragment Range for
\( V_i = 150 \text{ m/s} \), \( \text{AR} = 10 \)

\( V_i = 150 \text{ m/s} \)
\( \text{AR} = 10 \)
Figure 4-31. Disc Fragment Range for
$V_i = 200 \text{ m/s}, \text{AR} = 5$

$V_i = 200 \text{ m/s}$

$\text{AR} = 5$
Figure 4-33. Disc Fragment Range for
$V_i = 250 \text{ m/s}, \text{ AR } = 5$
Figure 4-34. Disc Fragment Range for

\[ V_i = 250 \text{ m/s, } AR = 10 \]

All curves are to the left of the \( \alpha = 90^\circ \) line.

\( \text{ft} = \text{m} \times 3.281 \)
Figure 4-35.

Disc Fragment Range for $V_i = 300 \text{ m/s}, \ AR = 5$

$V_i = 300 \text{ m/s}$

$\ AR = 5$

$M/A = 224 \text{ kg/m}^2$

$a = 90^\circ$

$\text{INITIAL TRAJECTORY ANGLE, } \alpha_i (\text{degrees})$

$\text{FRAGMENT RANGE, } x (\text{meters})$

$(\text{ft} = \text{m} \times 3.281)$
All curves are to the left of the $\alpha = 90^\circ$ line.

Figure 4-3b. Disc Fragment Range for $V_i = 300 \text{ m/s}$, $AR = 10$
4-5.1 Lifting Fragments

Where fragments from bursting vessels are determined to be disc-shaped, the fragment could conceivably fly or lift in much the same manner as a Frisbee. If this occurs the fragment may go further than one without lift. A computer program (FRISB) was written to determine the trajectory of a flying disc generated by an explosion. Details of the FRISB code are shown in Appendix 4E.

Several independent variables govern the flight of a disc fragment. To determine the effects of these variables on the fragment trajectory, a number of sample cases were analyzed by the FRISB code. A list of the input variables for the sample cases is presented in Table 4-1. Plots of maximum range versus initial trajectory angle are shown for those values of the independent variables in Figures 4-27 through 4-36. These graphs may be used to predict the maximum range of fragments having similar properties.

The following information must be known about the fragment to calculate its range using the figures:

1. initial fragment velocity - \( V_i \) (m/s)
2. mass of the fragment - \( M \) (kg)
3. diameter of the fragment - \( d \) (m)
4. thickness of the fragment - \( t \) (m)
5. planform or top surface area of fragment - \( A \) (m\(^2\))
6. initial trajectory angle of the fragment - \( \alpha_i \) (rad)

The procedure for determining fragment range is:

Step 1. calculate the aspect ratio, \( AR = \frac{d}{t} \), for the fragment

Step 2. calculate the mass to area ratio \( M/A \) for the fragment

Step 3. locate the graph for the assumed value of \( V_0 \) and select the curve for the calculated value of the mass to area ratio, \( M/A \)

Step 4. locate the point on the curve for the initial trajectory angle, \( \alpha_i \), read the corresponding maximum range.

4-61
TABLE 4-1. PROGRAM FRISB TEST CASES

These cases were run for the following initial velocities:

- $V_i = 100 \text{ m/s}$
- $V_i = 150 \text{ m/s}$
- $V_i = 200 \text{ m/s}$
- $V_i = 250 \text{ m/s}$
- $V_i = 300 \text{ m/s}$

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<th>Fragment Aspect Ratio AR</th>
<th>Fragment Area $A - m^2$</th>
<th>Fragment Mass $M - kg$</th>
<th>Trajectory Angle $\theta_i - \text{deg.}$</th>
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Lift and Drag Coefficients

- $C_L = 0.32$
- $C_D = 0.85$
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<tr>
<th>Fragment Aspect Ratio AR</th>
<th>Fragment Area $A \text{ - m}^2$</th>
<th>Fragment Mass $M \text{ - kg}$</th>
<th>Fragment Initial Trajectory Angle $\beta_1 \text{ - deg.}$</th>
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</table>
Since there are an infinite number of possible combinations of the independent variables, the graphs presented must be limited to particular values. However, most of the curves are parallel for varying values of the independent variables M/A, AR and V, allowing estimations to be made for fragments that are not represented by the curves presented. The procedure for this extrapolation is explained in Examples 3 and 4.

**Example 1:**

Determine the maximum range of a disc shape fragment assuming the following properties: \( V_i = 100 \text{ m/s} \) (328 ft/sec), Mass = 1 kg (2.2 lbm), Area = 0.0139 \( \text{m}^2 \) (0.150 ft\(^2\)), \( d = 0.25 \text{ m} \) (0.820 ft), \( t = 0.05 \text{ m} \) (0.164 ft), and the initial trajectory of the fragment at \( t = 0 \) was \( \theta_i = 30^\circ \).

**Step 1.** First determine the value of the aspect ratio for the fragment \( AR = d/t = 0.25/0.05 = 5 \).

**Step 2.** Determine the value of the mass to area ratio for the fragment \( M/A = 1/0.0139 = 72 \).

**Step 3.** From Figure 4-27, which is the figure for \( V_i = 100 \text{ m/s} \) and \( AR = 5 \) (which are given), it can be seen that the maximum range of the fragment is 463 meters (1530 ft).
Example 2:

Determine the maximum range of a disc-shaped fragment assuming the following properties: \( V_i = 300 \text{ m/s (984 ft/sec)} \), Mass = 1 kg (2.2 lbm), Area = 0.0139 m\(^2\) (0.150 ft\(^2\)), \( d = 0.25 \text{ m (0.820 ft)} \), \( t = 0.05 \text{ m (0.169 ft)} \), and \( \alpha_i = 30^\circ \).

Step 1. Same as Example 1.

Step 2. Same as Example 1.

Step 3. From Figure 4-35 for \( V_i = 300 \text{ m/s} \) and \( AR = 5 \), the maximum range is 188 meters (617 ft). But this particular point lies to the left of the straight line marked, \( \alpha_i = 90^\circ \). This line indicates that all events to the left of this line resulted in the fragment attaining a completely vertical flight. At this point it was assumed that all lift and stability were lost and the fragment fell straight to the ground. All events to the right of the line indicate a "normal" flight. The \( \alpha_i = 90^\circ \) line is present only where the initial velocities and/or trajectory angles were sufficient to cause a vertical trajectory.

Example 3:

Determine the maximum range of a disc-shaped fragment assuming the following properties: \( V_i = 230 \text{ m/s (755 ft/sec)} \), Mass = 18.6 kg (40.9 lbm), \( A = 0.155 \text{ m}^2 \) (1.67 ft\(^2\)), \( d = 0.444 \text{ m (1.46 ft)} \), \( t = 0.0444 \text{ m (0.146 ft)} \), and \( \alpha_i = 30^\circ \).

Step 1. Calculate the aspect ratio for the fragment from the assumed data. \( AR = 0.444/0.0444 = 10 \).

Step 2. Calculate the mass to area ratio for the fragment from the assumed data. \( M/A = 18.6/0.155 = 120 \).

Step 3. Since no graph exists for \( V_i = 230 \text{ m/s} \), we must interpolate using the graphs for \( V_i = 200 \text{ and } V_i = 250 \text{ m/s} \), \( AR = 10 \).

A curve for \( M/A = 120 \) also does not exist so an interpolation procedure must also be used here. The curve for both graphs and all values of \( M/A \) are essentially parallel so it will only be necessary to determine one point at the correct \( M/A \) and \( V_0 \) to construct the curve for all values of \( \alpha_i \). At \( \alpha_i = 30^\circ \) for \( M/A = 98 \) and 141 for both \( V_i = 200 \text{ and} \)}
\( V_i = 250 \) (Figure 4-32 and 4-34) read the values of range.

\[
\begin{align*}
V_i &= 200 \text{ m/s} & V_i &= 250 \text{ m/s} \\
\gamma_i &= 30^\circ & \gamma_i &= 30^\circ \\
M/A &= 98 \quad \text{Range} = 283 \text{ m} & M/A &= 98 \quad \text{Range} = 255 \text{ m} \\
M/A &= 141 \quad \text{Range} = 440 \text{ m} & M/A &= 141 \quad \text{Range} = 400 \text{ m}
\end{align*}
\]

Step 4. For each velocity find the range for \( M/A = 120 \) by interpolating between \( M/A = 98 \) and \( M/A = 141 \). For \( V_i = 200 \text{ m/s} \):

\[
\begin{align*}
\text{Range Difference} &= 440 - 283 = 157 \text{ m} \\
\text{M/A Difference} &= 141 - 98 = 43 \text{ kg/m}^2 \\
\text{or} \quad \frac{157}{43} &= 3.65 \text{ m/(kg/m}^2\text{)}
\end{align*}
\]

The difference between value of nearest curve and required value of \( M/A \) is:

\[
141 - 120 = 21 \text{ kg/m}^2
\]

The amount of change in the range for this increment is:

\[
21 \text{ kg/m}^2 \times 3.65 \text{ m/(kg/m}^2\text{)} = 77 \text{ m}
\]

The range at \( M/A = 120 \) will therefore be

\[
440 \text{ m} - 77 = 363 \text{ m}
\]

Similarity for \( V_i = 250 \text{ m/s} \)

\[
\begin{align*}
440 - 265 &= 135 \\
141 - 98 &= 43 \\
135/43 &= 3.13 \text{ m (kg/m}^2\text{)} \\
(21) (3.13) &= 66 \text{ m} \\
440 - 66 &= 334 \text{ m}
\end{align*}
\]
With these values of range curves for $M/A = 120$ can be constructed by drawing the curves parallel to the curve for other values of $M/A$ and through these values of range at $\gamma_1 = 30^\circ$.

Step 5. With these values of range for $M/A = 120$, interpolate for the correct velocity of $V_1 = 230$ m/s.

Difference in range for $V_1 = 300$ m/s and $V_1 = 250$ m/s is $363 - 334 = 29$ m

The velocity difference is $250 - 200 = 50$ m/s.

The change in range per 1 m/sec change in velocity is $\frac{29\text{ m}}{50\text{ m/sec}} = 0.58\text{ m/(m/s)}$

The change in range from 200 to 230 m/s is:

$(30)(0.58) = 17$ m

Since the range is decreasing with increasing velocity, the range for $M/A = 120$, $V_1 = 230$ m/s will be:

$363 - 17 = 346$ m (1140 ft).

A curve can be constructed to give the values of range at other values of $\gamma_0$ using the procedure described in Step 4.

Example 4:

Determine the maximum range of a disc-shape fragment assuming the following properties: $V_1 = 448$ m/s (1470 ft/sec), $M = 0.0495$ kg (0.109 lbm), $A = 0.00129$ m$^2$ (0.0139 ft$^2$), $d = 0.40$ m (1.31 ft), $t = 0.00919$ m (0.030 ft) and $\gamma_1 = 15^\circ$.

Step 1. Calculate the aspect ratio for the fragment from the assumed data. $AR = \frac{0.040}{0.00919} = 4.4$.

Step 2. Calculate the mass to area ratio for the fragment from the assumed data. $M/A = 38.4$.

Step 3. Since no graph exists for a $V_1 = 448$ m/s and an $AR = 4.4$ we must choose Figure 4-35 for $V_1 = 300$ m/s, AR = 5 to obtain an approximation for range. There is no curve in Figure 4-35 for which $M/A = 38$ but the curves are parallel and evenly spaced. Between $M/A = 72$ and $M/A = 104$ at $\gamma_1 = 40^\circ$ there is a difference in
range of 10 meters or $\frac{10}{104 - 72} = 188 \frac{m}{kN/m^2}$.

From $M/A = 104$ to $M/A = 155$ at $\theta_1 = 40^\circ$, there is a difference in range of 110 meters or $\frac{110}{155 - 104} = 2.10 \frac{m}{kN/m^2}$. This is an average of about $2 \frac{m}{kN/m^2}$.

It is required to construct a curve for $M/A = 38 \frac{kN}{m^2}$. The nearest curve to this value is $M/A = 72$ for a difference of $72 - 38 = 34 \frac{kN}{m^2}$. The value of the range for $M/A = 38$ at $40^\circ$ should be about 70 meters. Construct a curve through this point and parallel to the other curves. The range for $\theta_1 = 15^\circ$ is then read as 155 meters (509 ft).

This particular case was checked by substitution of the assumed data into the FRISB code (described in Appendix 4E). The maximum range predicted by the code was 142 meters which is about 8% lower than the curve approximation.

Several cases where the initial conditions did not match a particular graph were checked in this manner to determine the accuracy of the approximation. The error ranged from 5% to 30%. The greatest errors occurred when trying to predict the range for values of $(M/A) > 20$. These checks indicate that although the predictions from interpolation are crude, they provide a means of obtaining a rough order of magnitude for the fragment range, when gross extrapolations are required.

The FRISB code may also be used to predict the trajectory of a fragment that generates no lift as does the TRAJE code. A value of $C_L = 0$ is input into FRISB for this computation. The code can also be used to predict the trajectory of a fragment that is long and thin and spins like a helicopter rotor blade. The input involved for the use of this feature is presented in Appendix 4E along with a discussion of the analysis procedure.

### 4-5.2 Drag Fragments

Some fragments are shaped such that no one linear dimension is significantly greater than any other. The maximum range for such fragments (i.e., chunky fragments, fragments which can be approximated by a cube or sphere) can be obtained using the technique to be described.

Required input data are the initial velocity, $V_j$, of the fragment, a characteristic area $A$ of the fragment (which can be generated as shown
below), the mass of the fragment, and an initial trajectory angle or spectrum of initial trajectory angles, \( \psi \). Typical steps in the procedure for calculating the maximum range from Figures 4-37 through 4-45 are the following:

**Step 1.** Find the characteristic area of the fragment:

(a) from the equation

\[ A = V^{2/3} \]

where \( V \) is the fragment volume. \( V \) may be obtained from the mass of the fragment, \( M \), and its density, \( \rho \), from the equation

\[ V = \frac{M}{\rho} \]

(b) from the projected area of the fragment if it has three-dimensional symmetry or can be nearly approximated by a solid of three-dimensional symmetry (i.e., a sphere or a cube).

**Step 2.** Calculate the mass area ratio for the fragment, \( M/A \).

**Step 3.** Consult Figures 4-37 through 4-45 to find the figure for a \( V_i \) and \( A \) value most nearly equal that of the fragment considered. Linear interpolation can be used where necessary (see example problems).

**Step 4.** Read the maximum range from the figure for the \( M/A \) of Step 2 and the initial trajectory angles considered.

For a more precise range calculation the computer code, TRAJE, used to generate these figures may be used along with the input data for the fragment.

**Example 1. For Drag Fragments**

Assume: A nearly spherical shaped fragment that can be approximated by a sphere of radius \( R = 584 \) mm (0.192 ft) having an initial velocity of 200 m/s (656 ft/sec) and made out of steel. Find the maximum range expected for any initial trajectory angle.

**Step 1.** The characteristic area of the fragment is, by method (a)
Figure 4-37. Fragment Range for Drag Fragments,
$V_i = 300$ m/s, $A = 0.005157$ m$^2$
Figure 4-38. Fragment Range for Drag Fragments.

\[ V_0 = 300 \text{ m/s}, \quad A = 0.01073 \text{ m}^2 \]
Figure 4-39. Fragment Range for Drag Fragments.

\( V_i = 300 \text{ m/s}, A = 0.02394 \text{ m}^2 \)
Figure 4-40. Fragment Range for Drag Fragments.
\( V_i = 100 \, \text{m/s}, \ A = 0.00515 \, \text{m}^2 \)
Figure 4.41. Fragment Range for Drag Fragments.

\[ V_1 = 100 \text{ m/s}, \quad A = 0.01073 \text{ m}^2 \]
Figure 4.42. Fragment Range for Drag Fragments.

\[ V_i = 100 \text{ m/s}, \quad A = 0.02394 \text{ m}^2 \]
Figure 4.43. Fragment Range for Drag Fragments.
\[ V_i = 200 \, \text{m/s}, \quad A = 0.005157 \, \text{m}^2 \]
Figure 4-44. Fragment Range for Drag Fragments.

$V_i = 200 \text{ m/s}, \ A = 0.01073 \text{ m}^2$
Figure 4-45. Fragment Range for Dr. g. Fragments,

\[ V_i = 200 \text{ m/s}, \quad A = 0.02394 \text{ m}^2 \]
A_1 = \int_{(4/3)} R^{3.2/3} = 2.60 R^2 = 0.0087 m^2

by method (b)

A_2 = \frac{-2}{4/3} R^3 = \frac{-2}{4/3} R^3

Step 2. Calculate the mass area ratio for the fragment where

M = \frac{-V}{(4/3)} R^3

For a sphere

\rho_{steel} = 7.8 g/cm^3

desired

M = (7.8) (4/3) (-) (5.84)^3 = 6.51 kg

using A_2 from Step 1,

M/A_2 = 6.51/0.0107 = 608 kg/m^2

Step 3. The given V_i of 200 m/s and A_2 = 0.0107 m^2

corresponds to values for Figure 4-41. From this

desired

figure it can be seen that the maximum range for a

fragment having an M/A of 932 kg/m^2 is x_2 = 1800 m

and for an M/A of 280 kg/m^2 is x_1 = 900 m.

Step 4. By linear interpolation for our M/A of 608 kg/m^2

m_1 = M/A = 280 kg/m^2 \quad x_1 = 900 m

m_2 = M/A = 608 kg/m^2 \quad x_2 = ?

m_3 = M/A = 932 \quad x_3 = 1800 m

4-7
Example 3. For Drag Fragments

Assume: A chunky aluminum fragment of a mass of approximately 1.20 kg (2.64 lb) and an initial velocity of 250 m/s (820 ft/sec). Find the maximum range expected for any initial trajectory angle.

Step 1. The characteristic area of the fragment is, by method (a)

\[ A = \left( \frac{M}{\rho} \right)^{2/3} \]

from density tables

\[ \rho_{Al} = 2.7 \text{ g/cm}^3 \]

therefore

\[ A = \left( \frac{1200}{2.7} \right)^{2/3} = 58.3 \text{ cm}^2 = 5.83 \times 10^{-3} \text{ m}^2 \]

Step 2. M is given as M = 1.2 kg taking A from Step 1, thus

\[ \frac{M}{A} = \frac{1.2}{5.83 \times 10^{-3}} = 206 \text{ kg/m}^2 \]

Step 3. A conservative value for the range will be obtained by using data from Figures 4-40 and 4-43 for a fragment of characteristic area 5.16 x 10^{-3} m^2 which is slightly less than the area of the fragment considered, calculated in Step 1.

From Figure 4-40, for a fragment having \( V_i = 200 \text{ m/s} \) we may obtain a maximum range for \( M/A = 206 \text{ kg/m}^2 \) by linear interpolation from the maximum ranges, \( x \), for \( M/A \) ratios most nearly greater and less than \( M/A = 206 \text{ kg/m}^2 \):
\[ m_1 = \frac{M}{A} = 194 \text{ kg/m}^2 \quad x_1 = 700 \text{ m} \]
\[ m_2 = \frac{M}{A} = 206 \text{ kg/m}^2 \quad x_2 = \ ? \]
\[ m_3 = \frac{M}{A} = 582 \text{ kg/m}^2 \quad x_3 = 400 \text{ m} \]

\[ x_2 = (x_3 - x_1) \frac{(m_2 - m_1)}{(m_3 - m_1)} + x_1 = \frac{(206 - 194)}{(582 - 194)} (1400 - 700) \]

\[ x_2 = 700 \]
\[ x_1 = 720 \text{ m} \]

From Figure 4-40, for a fragment having a \( V_1 = 300 \text{ m/s} \) we may obtain a maximum range for \( \frac{M}{A} = 206 \text{ kg/m}^2 \) by linear interpolation as in the previous paragraph.

\[ m_1' = \frac{M}{A} = 194 \text{ kg/m}^2 \quad X_1' = 850 \text{ m} \]
\[ m_2' = \frac{M}{A} = 206 \text{ kg/m}^2 \quad X_2' = \ ? \]
\[ m_3' = \frac{M}{A} = 582 \text{ kg/m}^2 \quad X_3' = 1920 \text{ m} \]

\[ X_{1'} = (1920 - 850) \frac{(206 - 194)}{(582 - 194)} + 850 = 880 \text{ m} \]

Step 4. Interpolating for a \( V_1 = 250 \text{ m/s} \) between the values found for \( V_1 = 200 \text{ m/s} \) and \( 300 \text{ m/s} \)

\[ X_{2'} = (X_2' - X_2) \frac{(250 - 200)}{(300 - 200)} + X_2 \]

\[ = (880 - 700) \frac{1}{2} + 700 \]

\[ = 790 \text{ m} \]

A conservative value for range for this fragment is 790 m (2590 ft).
Example 3. For Drag Fragments

Assume: A titanium alloy fragment with a mass of 3.12 kg (6.84 lbm) initial velocity of 125 m/s (410 ft/sec). Find its maximum range for any initial trajectory angle.

Step 1. \[ A = \left( \frac{M}{\rho T_i} \right)^{2/3} \]

since \[ c_T = 4.46 \text{ g/cm}^3 \]

\[ A = \frac{3.12 \times 10^3}{4.46} = 79 \text{ cm}^2 = 7.0 \times 10^{-2} \text{ m}^2 \]

Step 2. \[ \frac{M}{A} = \frac{3.12}{7.9 \times 10^{-3}} = 395 \text{ kg/m}^2 \]

Step 3. For a conservative value for range use Figures 4-37 and 4-40 for an \( A = 5.15 \times 10^{-3} \text{ m}^2 \) which is less than the area of the fragment considered, from Figure 4-37.

for \( V_i = 100 \text{ m/s} \quad A = 5.15 \times 10^{-3} \text{ m}^2 \quad \frac{M}{A} = 194 \text{ kg/m}^2 \quad X_i = 420 \text{ m} \]

for \( V_i = 100 \text{ m/s} \quad A = 5.15 \times 10^{-3} \text{ m}^2 \quad \frac{M}{A} = 582 \text{ kg/m}^2 \quad X_3 = 660 \text{ m} \)

therefore, interpolating as in the previous examples for an \( \frac{M}{A} = 395 \text{ kg/m}^2 \)

for \( V_i = 100 \text{ m/s} \quad A = 5.15 \times 10^{-3} \text{ m}^2 \quad \frac{M}{A} = 395 \text{ kg/m}^2 \quad X_2 = 544 \text{ m} \)

From Figure 1-40, similarly
for $V_i = 200 \text{ m/s}$, $A = 5.15 \times 10^{-3} \text{ m}^2$, $M/A = 395 \text{ kg/m}^2$, $X_2' = 1062 \text{ m}$, interpolating for a $V_i$ of $125 \text{ m/s}$

$V_i = 125 \text{ m/s}, A = 5.15 \times 10^{-3} \text{ m}^2, M/A = 395 \text{ kg/m}^2$, $X_2'' = 673 \text{ m}$

Thus a conservative value for the range would be $673 \text{ m (2210 ft)}$.

4-5.3 For Cylindrical Propellant Tanks

Data from initial velocity measurements of fragments from Project PYRO experiments (Reference 3) were used to derive an estimate of the fragment initial velocity distribution. For medium percent yields (5 to 15%), Figures 4-46 and 4-47 present the fragment initial velocity distributions for $\text{LH}_2/\text{LH}_2$ and $\text{LH}_2/\text{RP}-1$ respectively, confined in a cylindrical missile (CBM). For medium percent yields (5 to 15%) Figures 4-48 and 4-49 show the fragment initial velocity distributions for $\text{LH}_2/\text{LH}_2$ and $\text{LH}_2/\text{RP}-1$ respectively confined by the ground surface (CBGS). In the CBGS tests, the missiles were allowed to fall back to the pad. In the CBM tests, the bulkheads between the $\text{LH}_2$ and $\text{LH}_2$ were ruptured, allowing the liquids to mix inside the missile.

Figures 4-46 through 4-49 can be used to estimate the percentage of fragments which will have an initial velocity, $V_i$, equal to or less than a particular $V_i$.

For example, if we wished to estimate the percentage of fragments which would have an initial velocity equal to or less than $1,000 \text{ m/s (3280 ft/sec)}$ for a $\text{LO}_2/\text{LH}_2$ CBM case, we would refer to Figure 4-46 and on the initial velocity axis (abcissa) at $1,000 \text{ m/s}$ go upward to the intersection with the line. Then, at the intersection point read the percentage value from the ordinate, which is 95.5%. Conversely, if we wanted to know what initial velocity 90% of the fragments would not exceed, we would enter the chart on the 90% line, go over the intersection with the curve and read downward to the initial velocity axis the value $680 \text{ m/s (2230 ft/sec)}$.

Figure 4-50 is a plot of fragment initial velocity versus yield (%) for $\text{LO}_2/\text{LH}_2$ CBM tests listed on page 86 of Reference 3. A 95th percentile has been included based on the same standard deviation as the
Figure 4-46. Initial Velocity Distribution,
CBM: LO₂/LH₂
Figure 4-47. Initial Velocity Distribution,
CBM, LO₂/(RP-1)
Figure 4-48. Initial Velocity Distribution.

CBGS, LO₂/LH₂

\( \text{ft/sec} = \text{m/s} \times 3.281 \)
Figure 4-49. Initial Velocity Distribution.
CBGS, LO$_2$/RP-1

\[ \text{INITIAL VELOCITY } U_i, \text{ meters/second} \]

\[ (\text{ft/sec} = \text{m/s} \times 3.281) \]
Figure 4-50. Initial Velocity Vs Yield, CBM, LO$_2$/LH$_2$
distribution shown in Figure 4-46. From Figure 4-50, for a yield of 50%, we would expect that 95% of the fragments would have an initial velocity of 1900 m/s (6230 ft/sec) or less.

Details on the derivation of Figure 4-46 through 4-50 are given in Appendix 4A, Sections 4A-1 and 4A-2.

4-6 Fragment Mass Distribution

4-6.1 Propellant Explosions

From the data in Reference 3, the fragment mass (weight) followed log normal distributions. That is, the logarithms of the fragment masses followed a normal or Gaussian distribution.

Figure 4-51 and 4-52 present the fragment mass (W_f) distribution for two events taken from Reference 3. These were termed Events 1 and 2, and were Saturn IV confined-by-missile (CBM) explosions. Event 1 had a percent yield of 5%, and Event 2 a yield of 1.1%.

Figure 4-53 is an average of the distribution of Events 3, 4, and 5 from Reference 3. These events were spill tests using three tanks on 120 radials with LO_2/LH_2/RP-1, and mixing on the ground (CBGS). The rationale for averaging the distribution of these events is given in Appendix 4F.

These charts can be used in the same manner as Figures 4-46 through 4-50 are used for fragment initial velocity.

4-6.2 Gas Vessel Bursts

In experiments by Pittman, (Reference 4), five tanks (two cylinders and three spheres) were ruptured by increasing pressurization until rupture. The tanks were made of the same material (Ti 6 Al 4 V alloy) with an ultimate stress (σ_u) of 1.05 GPa (150,000 psi). Pertinent data and calculated parameters for four of the tanks are given in Table 4-2, where \( \bar{W} \) is the geometric mean fragment mass, \( W(T) \) is the tank weight, \( P \) is the burst pressure, and \( E_o \) is the energy of detonation of 1 gram of TNT or 4190 J. It is interesting to note that the ratio \( \bar{W}/W(T) \) doubles while the ratio \( (P/\sigma_u) \) changes by an order of magnitude. As shown in Appendix 4F, the fragment mass follows a log normal distribution. That is, the logarithms of the fragment masses follow a normal or Gaussian distribution.
Figure 4-5: Fragment Mass Distribution from SIV CBM.

(1 lb = kg x 2.205)

LO$_2$/LH$_2$, 5% Yield
Figure 4-52. Fragment Mass Distribution from S IV CBM, LO_2/LH_2, 1.1% Yield

\( \text{lb}_m = \text{kg} \times 2.05 \)
Figure 4-53. Average of Events 3, 4 and 5. Fragment Mass Distribution, CBGS

PERCENTAGE OF FRAGMENTS WITH WEIGHT EQUAL TO OR LESS THAN $W_i$

MASS $W_i$ (grams)
Figure 4-54. Fragment Mass Distribution for Small Cylinders, $V = 0.05 \text{ m}^3$, $P = 4.2 \text{ MPa}$

$1 \text{ lb}_m = \text{ kg} \times 2.205$
### TABLE 4-2. PARAMETERS OF BURSTING TANKS

<table>
<thead>
<tr>
<th>Tank #</th>
<th>Burst Pressure (P)</th>
<th>Volume (m³)</th>
<th>Mean Frag. Mass (W)</th>
<th>W/W₀</th>
<th>P/P₀</th>
<th>PV/E₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.309</td>
<td>0.038</td>
<td>19.5</td>
<td>0.00505</td>
<td>0.0044</td>
<td>39</td>
</tr>
<tr>
<td>B</td>
<td>4.137</td>
<td>0.048</td>
<td>20.6</td>
<td>0.0044</td>
<td>0.0040</td>
<td>47</td>
</tr>
<tr>
<td>D</td>
<td>55.160</td>
<td>0.170</td>
<td>713</td>
<td>0.00917</td>
<td>0.0533</td>
<td>2,238</td>
</tr>
<tr>
<td>E</td>
<td>56.050</td>
<td>0.170</td>
<td>795</td>
<td>0.00964</td>
<td>0.0542</td>
<td>2,274</td>
</tr>
</tbody>
</table>

Tests for significant differences in means were made for Tanks A and B and for Tanks D and E. As shown in Appendix 4F no significant difference was found, so the fragment mass distribution of Tanks A and B and of Tanks D and E were averaged. These average distributions are shown in Figures 4-54 and 4-55. These figures can be used for fragment mass estimation in the same manner as Figures 4-46 through 4-50 are used for fragment initial velocity estimation.

Figure 4-56 is a plot of normalized yield (PV/E₀) versus mean fragment mass (W) for the four tanks. Two of the four points are clustered at each end of the range shown in Figure 4-56. Thus, the response in the middle of the range is still unknown. That is, the relationship of W to PV/E₀ may not be linear on a log-log-scale. Data points in the mid-ranges are needed to confirm or deny the linear relationship.

With the risks described above, one can estimate mean fragment mass for any decided ratio of PV/E₀ up to 3 kg. One could then estimate the 90th percentile of the distribution (that value of fragment mass which would equal or exceed the mass of 90% of the fragments) by using the estimate of the standard deviation (S) from the average of Tanks D and E (1.695). An example follows:

For a burst pressure (P) of 10^8 Pa (1,400 psi) and a tank volume of 0.80 m³ (2.83 ft³)

\[ PV = (0.080)(10^8) = 8 \times 10^6 \]
Figure 4-55. Fragment Mass Distribution for Titanium Alloy Spheres, $V = 0.170 \text{ m}^3$, $P = 55 \text{ MPa}$

$\left( \text{lb}_m = \text{kg} \times 2.205 \right)$
Entering the chart (Figure 4-56) on the abscissa of 1900, reading up to the intersection of the line and over the ordinate, a value of 560 g is obtained for mean fragment mass. Any Percentile (P<sub>p</sub>) value of fragment mass can then be estimated by the formula:

\[ P_p = \exp (\ln \bar{W} + Ks), \]

where \( \bar{W} \) is the mean mass, \( s \) is the estimate for the standard deviation, and \( k \) is a value from the normal distribution table. Typical values are shown in Table 4-3.

For the 90th percentile,

\[ P_{90} = \exp (\ln 560 + 1.28 \cdot 1.695) \]

\[ = \exp (8.4976) = 4.903 \text{ kg (10.79 lbm)} \]

Thus, one would expect that 90% of the fragments would have a mass equal to or less than 4.903 kg (10.79 lbm).

**TABLE 4-3. VALUES OF k OF PERCENTILES OF THE NORMAL DISTRIBUTION**

<table>
<thead>
<tr>
<th>Percentile</th>
<th>99</th>
<th>95</th>
<th>90</th>
<th>80</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>2.33</td>
<td>1.65</td>
<td>1.28</td>
<td>0.84</td>
<td>0.53</td>
</tr>
</tbody>
</table>

4-7 Probability of Fragment Arrival Versus Range

4-7.1 Propellant Explosions

From Reference 3, Figure 4-57 is a representation of fragment distance versus percent yield, with a 95 percent confidence interval. The derivation of this figure is explained in Appendix 4F.
Figure 4-56. Normalized Yield Vs Mean Fragment Mass for Bursting Tanks

\( \text{NORMALIZED YIELD } \left( \frac{PV}{E_0} \right) \)

\( ( \text{lb}_m = \text{kg} \times 2.205 ) \)
Figure 4-57. Fragment Range Vs Yield, Propellant Explosions

\[ R_{95} = 235 Y^{0.2775} \]
\[ R = 35.93 Y^{0.2775} \]

(1 ft = m \times 3.281)
Figure 4-57 can be used to estimate the mean distance of a fragment range for any given yield (percent). Also, one can obtain a 95% confidence level on maximum distance. For example, given a yield of 20%, one would expect that 95% or more of the fragments would fall within 560 meters of the explosion center.

4-7.2 Gas Vessel Bursts

In the experiment by Pittman (Reference 4), the fragments from the bursting tanks were partially contained in a circular area with a 20 foot radius with 8 ft high walls on the perimeter. Thus, the data on the fragment range was severely biased.

However, by exercising the computer programs (Appendix 4E) for fragment range as a function of fragment mass, drag, and flight angle, a distribution of fragment ranges was obtained for each of four tanks. These distributions were well fitted to members of the normal of Gaussian family.

Then, "t" tests for significant difference in mean ranges for Tanks A and B and for Tanks D and E were made. In the statistical sense, there were no significant differences in means between Tanks A and B, and between Tanks D and E.

Figures 4-58 and 4-59 present these simulated fragment range distributions for Tanks A and B and for Tanks D and E respectively. These charts can be used to estimate fragment range for similar gas vessels.

Complete details on the derivation of the simulated fragment range, goodness of fit tests, and "t" tests are given in Appendix 4F.
Figure 4.58. Fragment Range Distribution for Small Cylinders, \( v = 0.05 \text{ m/s}, P = 4.2 \text{ MPa} \)

Equal To Or Less Than \( R' \)

Percentage Of Fragments With Range
Figure 4-59. Fragment Range Distribution for Spheres,
$V = 0.170 \text{ m}^3$, $P = 55 \text{ MPa}$
LIST OF REFERENCES


APPENDIX IV. A

METHODS FOR ESTIMATING FRAGMENT INITIAL VELOCITIES

Figure 4A-1 shows the conceptual models used in analyzing bursting confinement vessels to obtain estimates of initial fragment velocities. The spherical confinement vessel case has been discussed in References 1 and 2. For this case the sphere is conceived of as fragmenting into n fragments of circular projection, and the fragments travel in a radial direction without tumbling. The energy of the confined gas is partitioned between the kinetic energy of the fragment, the energy of the gas escaping between the cracks between the fragments as they are formed, and the energy of the expansion of the internal gas. The equations of motion for the fragments are developed in Reference 2, and a computer code SPHER has been developed for the solution of these nonlinear differential equations describing the fragment motion. Solution of the differential equations is accomplished by use of Runge-Kutta integration techniques. Program SPHER appears at the end of this appendix with a description of the input-output variables.

Figure 4A-1 also shows the conceptualization of a cylinder fragmenting into n fragments. For this case, the fragments are considered to be strips which move radially from the center of the cylinder. Motion of the cylinder ends is not considered. Figure 4A-2 shows the geometric parameters of the cylinder used in the analysis. A cylinder of length L and radius R is assumed to burst into n strip fragments of width d and thickness th. A cross-section of each strip is a segment of the cross-section of the cylinder having a segment height of h and segment diameter d. In the following analysis, the projected area of each fragment is obtained from the surface area and the initial subtended angle of the fragment at the center of the cylinder, with the result being Equation (4A-11). The area of a crack about any fragment at any time is obtained by assuming the cracks only form lengthwise along the cylinder, and by obtaining an equation for the width of these cracks in terms of the initial radius of the cylinder and the radial distance r, the fragment has traveled at any time, t. The equation for fragment area is given in Equation (4A-14). One of the differential equations of motion derived on the basis of an adiabatic gas expansion and radial motion of the fragments is the same for this analysis as it was for the sphere, as are the general equations used for nondimensionalization; see Equation (4A-15). A second differential equation, Equation (4A-35), is obtained from the perfect gas law assumption, an equation for the mass flow of gas through the cracks formed between the fragments [Equation (4A-19)], and cylinder geometry considerations. Values for the nondimensional constants are also obtained through these considerations and are given in Equations (4A-36) and (4A-37).
Figure 4A-1. Conceptual Models of Bursting Confinement Vessels

a. n-Fragment sphere

b. 2-Fragment cylinder

c. n-Fragmenting cylinder
Figure 4A-2. Geometric Parameters for a Fragmenting Cylinder
This analysis does not take into account the stored energy (strain energy) in the walls of the container immediately prior to burst. The strain energy for the spherical containment vessel case can be shown to be given approximately by:

\[ SE = 2\pi R^2 h (1 - K) \frac{\sigma^2}{E} \]  

(4A-1)

where

- \( \sigma \) = the stress in the container wall
- \( SE \) = the strain energy stored in the vessel
- \( R \) = the sphere radius before burst
- \( h \) = the sphere wall thickness
- \( \nu \) = Poisson's ratio for the material
- \( E \) = the bulk modulus for the material

This equation may be written:

\[ SE = \pi R^4 P^2 \frac{(1 - \nu)}{(2 Eh)} \]  

(4A-2)

where \( P \) is the pressure of the confined gas, by substitution of the approximate relation,

\[ 2h \sigma = R P \]  

(4A-3)

The stored energy in the confined gas is

\[ GE = \frac{4\pi PR^3}{3(K - 1)} \]  

(4A-4)

Thus, the ratio, \( e \), of the strain to stored energy is

\[ e = \frac{3RP(1 - \gamma)(K - 1)}{8hE} \]  

(4A-5)

For a steel container pressurized with air having \( R/h \leq 100 \),
e = $5.25 \times 10^{-6}$ P (kg/cm²)

= $3.70 \times 10^{-7}$ P (psi)

= $5.37 \times 10^{-11}$ P (Pa)

Thus, even for this very high radius-to-thickness ratio, pressure would have to exceed $10^5$ psi to produce significant strain energy (3%) relative to the total energy available. We therefore conclude that, as a rule, strain energy can be neglected. This is emphatically not true when the vessel rupture process is similar to that of a high explosive bomb casing burst, where considerable expansion of the case occurs before rupture (up to 1.8 radii).

A computer code for the simultaneous solution of nonlinear differential equations [Equations (4A-15) and (4A-35)] (CYLIN) has been developed. This code, similar to code SPHER, uses the Runge-Kutta integration technique for the solution of the differential equations. The code, along with definitions for its input and output parameters, is given at the end of the appendix. Figures 4A-3 and 4A-4 show comparative results from the two computer codes. Figure 4A-4 shows the maximum fragment velocities predicted for spheres and cylinders of equivalent volume as a function of the number of fragments assumed. For all cases, the value for maximum fragment velocity becomes a constant when more than about 10 to 30 fragments are assumed. In general, for equal volumes and radii, the sphere fragment velocities are less than the cylinder fragment velocities. This may be because of the assumption that no energy is used in accelerating the ends of the cylinders. Thus, more energy is available for the fragments formed from the cylindrical wall. Figure 4A-3 gives comparative results for maximum fragment velocities for the two geometric cases versus mass ratio for gases of various specific heat ratios. Again, the predicted cylindrical velocities are higher than that of the spherical velocities.

Some empirical verification of the results from program SPHER is given in Appendix IV, B. Very little data are available, however, to verify this analysis. Although there are data available for fragment velocities from cylindrical confinement vessels, most of these data are from bursting artillery shells. The processes involved in the fragmentation of these shells are so dynamic that the shell may expand up to twice its diameter before cracks are formed between the fragments. Our analysis would not be applicable to this kind of event. Our analysis assumes a relatively brittle confinement vessel which does not expand significantly prior to the time that cracks are formed between the fragments. This would be the case for confinement vessels which are slowly pressurized, as opposed to the highly dynamic situation existing when a high explosive is detonated internal to the confinement vessel.
Figure 4A.3. Maximum Fragment Velocity as a Function of Mass Ratio for Spheres and Cylinders of Equivalent Volume.
Analysis for a Cylindrical Shell Fragmenting Into Strip Fragments

Variable List:

A - term in Equation (4A-16)

$A_f$ - fragment area

$a_{oo}$ - sound speed in confined gas, $\tau = 0$

$a_e$ - critical sound speed in mass flow equation

B - term in Equation (4A-16)

C - term in Equation (4A-16)

D - term in Equation (4A-16)

d - width of fragment segment

F - fragment projected area

g - nondimensionalized displacement of a fragment

h - segment height (see Figure 4A-2)

k - mass flow rate coefficient

L - cylinder length

M - mass of contained gas at any instant, $\tau$

$M_f$ - fragment mass

$M_t$ - cylindrical shell mass

n - number of fragments

P - crack perimeter about a fragment

$P_e$ - nondimensionalized pressure

$P_0$ - pressure of confined gas at any instant, $\tau$
$P_{oo}$ - initial pressure of confined gas

$R$ - initial cylinder radius

$r$ - fragment displacement at any instant, $\tau$

$s$ - segment length

$T_o$ - temperature of confined gas at any instant, $\tau$

$t_h$ - shell thickness

$V_o$ - volume of confined gas at any instant, $\tau$

$V_s$ - volume of shell material

$w$ - crack width

$x$ - nondimensionalizing constant for displacement

$\alpha$ - nondimensional constant

$\beta$ - nondimensional constant

$\theta$ - nondimensionalizing constant for time

$\theta$ - angle subtended at the center of the cylinder by a fragment $\tau = 0$

$\kappa$ - ratio of specific heats for confined gas

$\rho_o$ - confined gas density at any instant, $\tau$

$\rho^*$ - critical density of gas in mass flow equation

$\rho_s$ - density of shell material

$\tau$ - time

$\tau$ - nondimensionalized time

Prime denotes derivatives with respect to $\tau$. 
From considerations of the cylinder geometry (see Figure 4A-2):

\[ M_t = \rho s V_s = \pi \rho_s L \left[ (R + t_h)^2 - R^2 \right] \] (4A-6)

\[ A_f = \frac{2\pi RL}{n} \] (4A-7)

\[ F = L \cdot d = L \cdot 2 \left[ R^2 - (R - h)^2 \right]^{1/2} \] (4A-8)

From Figure 4A-2:

\[ A_f = L \cdot S, S = \frac{\theta}{2\pi} \cdot 2\pi R = \theta R, \theta = \cos^{-1} \left( \frac{R - h}{R} \right) \]

Therefore,

\[ A_f = L \cdot R \cdot \cos^{-1} \left( \frac{R - h}{R} \right) \] (4A-9)

From Equations (4A-6) and (4A-9),

\[ h = R \left( 1 - \cos \frac{2\pi}{n} \right) \] (4A-10)

From Equations (4A-10) and (4A-8),

\[ F = 2L \cdot R \left[ 1 - \left( \cos \frac{2\pi}{n} \right)^2 \right]^{1/2} \] (4A-11)

The crack area at any time \( t \) for fragment \( i \) is

\[ (P_w)_i = 2L \cdot w \] (4A-12)

From Figure 4A-2, it may be seen that

\[ w = \theta \left[ r(\tau) - R \right] = \frac{2\pi}{n} \left[ r(\tau) - R \right] \] (4A-13)

From Equations (4A-12) and (4A-13),

\[ (P_w)_i = \frac{4\pi L}{n} \cdot \left[ r(\tau) - R \right] = \frac{4\pi L \cdot R}{n} \left( \frac{r(\tau)}{R} - 1 \right) \] (4A-14)

4A-10
From Reference 2, Equations (4A-19), (4A-20), and (4A-21),

\[ g'' = nP_\ast \left[ 1 - \frac{(\xi')^2}{(P_\ast)^{\kappa-1/\kappa}} \right] \]

\[ X = \frac{M_t a_\infty^2}{F P_\infty} \left( \frac{2}{\kappa - 1} \right)^{1/2} \]

\[ \Theta = \frac{M_t a_\infty}{F P_\infty} \left( \frac{2}{\kappa - 1} \right)^{1/2} \]

From differentiation of the ideal gas equation,

\[ \frac{dP_\ast(\tau)}{d\tau} = \frac{1}{\rho_\ast(\tau)V_\ast(\tau)} \frac{dM(\tau)}{d\tau} + \frac{1}{T_\ast(\tau)} \frac{dT_\ast(\tau)}{d\tau} \]

\[ - \frac{1}{V_\ast(\tau)} \frac{dV_\ast(\tau)}{d\tau} \]

\[ A = B + C - D \]

Using the variable changes \( P_\ast(\tau) = P_\infty \cdot P_\ast(\xi) \), term A becomes

\[ A = \frac{1}{\Theta} \frac{P_\ast'(\xi)}{P_\ast(\xi)} \]

and using these variable changes with the ideal gas law, term C becomes

\[ C = \frac{1}{\Theta} \frac{\kappa - 1}{\kappa} \frac{P_\ast'}{P_\ast} \]

The confined gas mass flow rate through the cracks is [from Equation (4A-12), Reference 2]

\[ \frac{dM(\tau)}{d\tau} = - k\rho_\ast a_\ast Pw \]
The total crack area is

\[ P \omega = \sum (P \omega)_i = n(P \omega)_1 \]

and from Equations (4A-14) and (4A-19)

\[ = - k \rho_s a_s 4 \pi L \cdot R \left( \frac{r(\tau)}{R} - 1 \right) \]  

(4A-20)

But from the cylindrical geometry

\[ V_0(\tau) = \pi r^2(\tau) \cdot L \]  

(4A-21)

Thus, from Equations (4A-16), (4A-20) and (4A-21), term B becomes

\[ B = \frac{1}{\rho_0(\tau) V_0(\tau)} \frac{dM(\tau)}{d\tau} \]

\[ = \frac{1}{\rho_0(\tau) \cdot \pi r^2(\tau) \cdot L} \left[ - k \rho_s a_s 4 \pi L \cdot R \left( \frac{r(\tau)}{R} - 1 \right) \right] \]

\[ = - \frac{4 k \rho_s a_s R}{\rho_0(\tau)} \left[ \frac{1}{R r(\tau)} - \frac{1}{r(\tau)} \right] \]  

(4A-22)

From standard one-dimensional flow relationships

\[ \rho_s = \rho_0(\tau) \left( \frac{2}{\kappa + 1} \right)^{1/\kappa - 1} \]  

(4A-23)

\[ a_s = a_0(\tau) \left( \frac{2}{\kappa + 1} \right)^{1/2} \]

Assuming an adiabatic gas expansion

\[ a_0(\tau) = a_0^{\infty} P^{-\kappa - 1/2 \kappa} \]

and nondimensionalizing displacement and time

\[ r(\tau) = X g(\xi) \quad \tau = \theta \xi \]  

(4A-24)
we obtain from Equations (4A-22), (4A-23) and (4A-24)

\[
B = -4k\left(\frac{2}{\kappa + 1}\right)^{1/\kappa - 1}\left(\frac{2}{\kappa + 1}\right)^{1/2} a_{oo} P^{\kappa - 1/2\kappa} \\
R \left[\frac{1}{RXg(\xi)} - \frac{1}{X^2 g(\xi)^2}\right] \\

(4A-25)
\]

\[
= -4ka a_{oo} P^{\kappa - 1/2\kappa} R \left(\frac{2}{\kappa + 1}\right)^{1/\kappa - 1/2(\kappa - 1)} \\
\left[\frac{Xg(\xi)}{R} - 1\right] \\

(4A-26)
\]

From Equation (4A-16)

\[
D = \frac{1}{V_0(\tau)} \frac{dV_0(\tau)}{\tau} \\

(4A-27)
\]

Differentiating Equation (4A-21) gives

\[
\frac{dV_0(\tau)}{\tau} = \frac{d\pi r^2(\tau) - 2\pi L \cdot 2 \tau(\tau) \cdot \frac{d\tau(\tau)}{\tau}}{\tau} \\

(4A-28)
\]

Thus,

\[
D = \frac{1}{V_0(\tau)} \frac{dV_0(\tau)}{\tau} = \frac{2}{r(\tau)} \cdot \frac{d\tau(\tau)}{\tau} \\

(4A-29)
\]

Changing the variable with Equation (4A-24), term D becomes

\[
D = \frac{2}{Xg(\xi)} \frac{Xg'(\xi)}{g(\xi)} = \frac{2}{\theta} g'(\xi) \\

(4A-30)
\]

From Equations (4A-16), (4A-17), (4A-18), (4A-26), and (4A-30).

\[
\frac{1}{\theta} \frac{P_r'}{P_r} = \frac{4ka a_{oo} P^{\kappa - 1/2\kappa} R \left(\frac{2}{\kappa + 1}\right)^{1/\kappa - 1/2(\kappa - 1)} \left[\frac{Xg(\xi)}{R} - 1\right]}{X^2 g(\xi)^2} \\

(4A-13)
\]
Multiplying through Eq. (4A-32) by $\theta \kappa g^2$

$$\frac{1}{P_n^*} = \frac{4 k a_o \theta}{\theta \kappa g^2} \left( \frac{2}{\kappa + 1} \right)^{k+1/2(\kappa-1)} \frac{R^{(\kappa+1)}}{X^2} \frac{p^{\kappa-1/2 \kappa}}{p_{n_0}}$$

From Equation (4A-15),

$$\frac{\theta}{\kappa} = \frac{1}{a_o} \left( \frac{2}{\kappa - 1} \right)^{-1/2}$$

$$\frac{\theta}{\kappa^2} = \frac{F^2 P_{oo}^2}{M_t^2 a_o^4} \left( \frac{2}{\kappa - 1} \right)^{-2} \frac{M_t a_o}{FP_{oo}} \left( \frac{2}{\kappa - 1} \right)^{1/2}$$

$$= \frac{FP_{oo}^3}{M_t a_o^3} \left( \frac{2}{\kappa - 1} \right)^{-3/2}$$

(4A-34)

Assume constants $\alpha$ and $\beta$ such that Equation (4A-33) becomes

$$\frac{P_n^* g}{P_n^*} = \left[ - \alpha g + \beta \right] p_{n_0}^{\kappa-1/2 \kappa} - 2 \kappa g^2$$

(4A-35)
Then from Equations (4A-33), (4A-34), and (4A-35),

\[
\alpha = 4k\kappa a_{oo} \left( \frac{2}{\kappa + 1} \right)^{\kappa + 1/2(\kappa - 1)} \left( \frac{2}{\kappa - 1} \right)^{-1/2} \frac{1}{a_{oo}} \left( \frac{2}{\kappa - 1} \right)^{-1/2} \tag{4A-36}
\]

\[
= 4k\kappa \left( \frac{2}{\kappa + 1} \right)^{\kappa + 1/2(\kappa - 1)} \left( \frac{2}{\kappa - 1} \right)^{-1/2} \tag{4A-37}
\]

\[
\beta = 4k\kappa a_{oo} R \left( \frac{2}{\kappa + 1} \right)^{\kappa + 1/2(\kappa - 1)} \frac{FP_{oo}}{M_t a_{oo}^2} \left( \frac{2}{\kappa - 1} \right)^{-3/2}
\]

\[
= R \left( \frac{2}{\kappa - 1} \right)^{-1} \frac{FP_{oo}}{M_t a_{oo}^2} \tag{4A-38}
\]

Differential Equations (4A-15) and (4A-35) are solved simultaneously by code CYLIN using the Runge-Kutta integration technique for initial conditions:

\[ g(0) = \frac{R}{X}, \quad p_*(0) = 1.0 \quad \text{and} \quad g'(0) = 0 \]

Since \( M_t \) and \( F \) are both proportional to \( L \), \( M_t / F \) is independent of the cylinder length. Thus, length \( L \) drops out of the equations for \( \beta \), \( X \), and \( \theta \); i.e., the cylinder length does not enter into the solution for Equations (4A-15) and (4A-35).

Computer Codes CYLIN and SPHER

The following computer codes were used to solve the simultaneous nonlinear differential equations for motion of fragments emanating from bursting confinement vessels of cylindrical and spherical configuration. These codes were used to generate the data for the method of deterministic fragment initial velocity calculations given in the text of the workbook.
# Program Variable Definitions for Codes SPHER and CYLIN

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<th>Program Variable</th>
<th>Definition</th>
<th>SI</th>
<th>English</th>
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</thead>
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<td>CAPI</td>
<td>Ratio of specific heats of gas</td>
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<td>AO</td>
<td>Sound speed</td>
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<td>Initial pressure</td>
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<td>psi</td>
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<td>in.</td>
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<td>m</td>
<td>in.</td>
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<td>End length</td>
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<td>in.</td>
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<td>CT</td>
<td>Cylinder thickness</td>
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<td>in.</td>
</tr>
<tr>
<td>ET</td>
<td>End thickness</td>
<td>m</td>
<td>in.</td>
</tr>
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<td>Density</td>
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<td>lb/in³ *</td>
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<tr>
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<td>Volume</td>
<td>m³</td>
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<tr>
<td>TM</td>
<td>Total mass of confinement vessel</td>
<td>kg</td>
<td>lb **</td>
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<tr>
<td>FN</td>
<td>Number of fragments (always two)</td>
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<td>--</td>
</tr>
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<td>FK</td>
<td>Gas discharge coefficient</td>
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<td>AH</td>
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<td>XMAX</td>
<td>Maximum time of iteration</td>
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* lb, indicates English weight measurement of pounds of force. Sea level gravitation is assumed.
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<th>Program Variable</th>
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<td>B1</td>
<td>Dimensionless geometry parameter</td>
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<td>the quantity -1/κ</td>
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<td>CAP4</td>
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**Units**

**Program Variable**

**Definition**

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</table>
PROGRAM SPNER (INPUT, OUTPUT, TAPE1 = OUTPUT, TAPE2 = INPUT)

DIMENSION F(3), Y(3), N(3), P(200), T(200)

310 FORMAT (3E10.5)

312 FORMAT (E10.3)

314 FORMAT (E10.3)

315 FORMAT (3/20H GAS CHARACTERISTICS, /, 7H KAPPA=E10.3)

15H SOUND SPEED = E10.3, 7H IN/SEC, /, 10M PRESSURE = E10.3, 7H PSI,

12/ 7H VESSEL CHARACTERISTICS, /, 6H RADIUS = E10.3, 5M IN,

10H MASS = E10.3, 10H LBS, = SEC, 50, /, 10M NO. OF FRAGMENTS =

E10.3/2)

319 FORMAT (3/13M FINAL VALUES, /, 6H TIME = E10.3, 7H SEC, 7,

110M DISTANCE = E10.3, 7H INS, /, 10M VELOCITY = E10.3, 7H FT/SEC, 7,

114M ACCELERATION = E10.3, 10M IN/SEC-SEC, /, 10M PRESSURE = E10.3,

114M PSI)

320 FORMAT (52M CHARACTERISTICS OF MOTION OF FRAGMENTS (NORMALIZED),

12/ 10M T = NORM, 7X, 10G, 7X, 10H, 7X, 7X)

321 FORMAT (/, 2M PRESSURE (NORMALIZED), /, 7X, 2H T = NORM, 7X,

11M = NORM, /)

322 FORMAT (1M INITIAL CONDITIONS, /, 6M X(0) = E10.4, 7H "G(0) = E10.4,

10H G'(0) = E10.4, 7H P = NORM, E10.4, 7)

501 FORMAT (/, 1600 MUCH)

502 FORMAT (/, 1600 FORMAT (12)

500 JJ=0

READ (2,310) CAP1, AO, PO

READ (2,312) FM, TM, FK

READ (2,314) AM, XM, XMAX

READ (2,400) NENO

GO TO 551

550 PO=PO/10,

AM=0.0, E=5

551 CONTINUE

WRITE (1,315) CAP1, AO, PO, RR, TM, FN

PI = 3.1415926535

FF=2*CAP1*(RR+2) *((1/FF) - (1/FF+2))

XM=TM*(AO+2,7)*(2,0/(CAP1-1))/FF*PO

THETA=TM*(AO+(2,0/(CAP1-1)))*0.5/FF*PO

CAP2=(1.00-CAP1)/CAP1

CAP3=1.0/CAP2

CAM=(2.0*CAP1-1,0)/(2.0*CAP1-1,0)/(CAP1-1,0)

CAM2=FRAC*PI*(1,2/(CAP1+1.0))**2/(CAP1+1,0)/(CAP1+1.0)/(CAP1+1.0)/(CAP1+1.0)

11,100*(1.2/(CAP1-1.0))**0.5

B=10/(CAP1-1.0)**0.5

B=10/(CAP1-1.0)**0.5

CAP4=1.0/(CAP1-1.0)**0.5

D=(2,0/(CAP1-1.0))**0.5

11,100*(1.2/(CAP1-1.0))**0.5

CO=RR/XX

XX=0.0

Y(1)=CO

Y(2)=0.0

Y(3)=1.0

XT=0.0

YTT=0.0

WRITE (1,322) X, Y(1), Y(2), Y(3)

NA=3

F(1)=y(2)

F(2)=F4*Y(3) + ((1,00-(Y(2)**2.0)*(Y(3)**CAP2))**CAP3)

F(3)=Y(1)*Y(3) + (Y(3)**CAP4) + (A1*B1=A1*(Y(1)**2.0)) =

13.*CAP4*(Y(2)*Y(3)/Y(2))

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OF POOR QUALITY

4A-18
FF=F(2)
30 S=MDEQ (NA,Y,X,AM,0)
   50 F(1)=Y(2)
      F(2)=F(1)*Y(3)*((1.0+Y(2)+0.0)*(Y(3)+13.0)*CAP+3)
      F(3)=(Y(1)+3.0)*(Y(3)+13.0)*CAP+4)*((13.0+13.0)*CAP+3)
      IF (S<1) 40,30,40
90 CONTINUE
F2T=F(2)/F(1)
F2T=ABS(F2T)
IF (F2T<0.0) 700,700,701
700 AM=3.0+AM
701 CONTINUE
JJ=JJ+1
708 CONTINUE
YY(JJ)=Y(3)
    YYS(JJ)=Y(3)
    YY(Y(JJ)-Y(JJ+Y(2))
    YY=YY(2)
    IF (YY<0.0) 10,10,91
91 CONTINUE
GO TO 30
10 CONTINUE
T1=THETA*X
    G1=X*T(1)=x+CO
    G2=X*T(1)=X(T(1)-Y(1))
    G2=G2/12.0
    G3=(X*(THETA+2.0)+F(2)
    G4=PYY(3)
    WRITE (1,31) T1,G1,G2,G3,G4
    IF (PG=10000.) 552,552,550
552 IF (NE=1) 501,500,500
503 CONTINUE
    WRITE (1,502)
501 STOP
END

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OF POOR QUALITY
PROGRAM CYLIN (INPUT, OUTPUT, TAPE1 = OUTPUT, TAPE2 = INPUT)
DIMENSION F(2), Y(3), AA(3), RB(3), PS(200), TT(200)

310 FORMAT (3E10.5)
312 FORMAT (4E10.3)
314 FORMAT (E10.3)
315 FORMAT (5H SOUND SPEED, E13.5, 7H IN/SEC, /, 10H VELOCITY, E10.3, 4H PSI, /)
316 FORMAT (5H VELOCITY, E10.3, 5H INSEC, /, 10H PRESSURE, E10.3, 4H PSI, /)
317 FORMAT (5H VELOCITY, E10.3, 6H LB/SEC, /, 10H NO. OF FRAGMENTS, /)
318 FORMAT (3E10.5)
319 FORMAT (3E10.5, 1H TIME, E10.3, 4H SEC, /)
31A FORMAT (3E10.5, 1H DISTANCE, E10.3, 4H IN, /, 10H VELOCITY, E10.3, 4H FT/SEC, /)
31B FORMAT (3E10.5, 1H VELOCITY, E10.3, 4H IN/SEC, /, 10H PRESSURE, E10.3, /)
320 FORMAT (5H CHARACTERISTICS OF MOTION OF FRAGMENTS (NORMALIZED), /)
321 FORMAT (3E10.5, 1H TIME, NORM, X, NORM, Y, NORM, Z, /)
322 FORMAT (5H INITIAL CONDITIONS, /, 8H X(0), E10.3, 7H G(0), E10.3, /)
323 FORMAT (5H G(0), E10.3, 7H P(NORM), E10.4, 2/)  
502 FORMAT (5H, B=100 MUSH)
600 FORMAT (1I)
3000 FORMAT (E10.3)
3200 FORMAT (2, 310) CAP, AO, PO
READ (2, 310) CAP, AO, PO
READ (2, 312) FN, RR, TM, FK
READ (2, 3001) GNL
READ (2, 319) AM, XM, MAX
READ (2, 300) NEND
GO TO 551
550 FORMAT (1.2, 10)
AM = 1.0, OE = 5
561 CONTINUE
WRITE (1, 324) GNL
WRITE (1, 315) CAP, AO, PO, RR, TM, FN
PIN = 3.1415926535
FF = 2.0, GNL, RR = (1.0 - (COS(2.0 * PI / FN) + 1.0) ** 0.5)
XM = (AO + 2.0) / (2.0 * (CAP1 + 1.0)) / (FF * PO)
THETA = XM / (AO + 2.0) / (2.0 * (CAP1 + 1.0)) / (FF * PO)
CAP2 = (1.0 + (CAP1 + 1.0)) / (CAP1 + 1.0)
CAP3 = 1.0 / CAP2
CAP4 = (1.0 + (CAP1 + 1.0)) / (2.0 * CAP1)
12N = 0.0, FK = CAP4 * ((2.0 / (CAP1 + 1.0)) ** (CAP1 + 1.0) / (2.0 * (CAP1 + 1.0)))
13N = (2.0 / (CAP1 + 1.0)) ** 0.5
BI(N) = ((2.0 / (CAP1 + 1.0)) ** 0.5) * (THETA / (AO + 2.0))
CONF = 2X
X(0) = 0
Y(1) = 0
Y(2) = 0
Y(3) = 0
Y(4) = 0
Y(5) = 0
WRITE (1, 322) X, Y(1), Y(2), Y(3)
WANT
F(1) = Y(2)
F(2) = Y(4) * (1.0 + (Y(2) ** 2) * (Y(3) ** CAP2) * CAP3)

4A-20
TERM2 = CAP1 * (Y(2) * Y(1) / Y(3))
F(3) = (((Y(2) ** 2) * (Y(3) ** CAP1)) * (AL * B1 * AL * (Y(1))) * TERM2
FZ = F(2)
30 SHR KLOEG (NA, Y, F, X, AM, O)
50 F(1) = Y(2)
F(2) = NEW(3) = (((100 * (Y(2) ** 2) * (Y(3) ** CAP1)) * (AL * B1 * AL * (Y(1))) * TERM2
F(3) = (((Y(1) ** 2) * (Y(3) ** CAP1)) * (AL * B1 * AL * (Y(1))) * TERM2
IF (F = 1, 90, 30, 40
40 CONTINUE
F2 = (F2 - F(2)) / F2
F2 = ABS(F2)
IF (F2 = 10) 700, 700, 701
700 AM = 1, 5 * AM
701 CONTINUE
JJ = JJ + 1
IF (JJ = 200) 504, 503, 503
504 CONTINUE
TT = (JJ) ** 3
TT = YW(2) ** YW(2)
TT = YW(2) ** YW(2)
IF (TT = 0.01) 10, 10, 11
11 CONTINUE
GO TO 30
10 CONTINUE
T1 = THEA + X
G1 = XX ** (1) * XX * CO
G2 = XX / THEA + Y(2)
G2 = G2 / 12, 0
G3 = (YX * (THEA ** 2, 0)) * F(2)
G3 = P0 = Y(3)
WRITE (1, 314) T1, G1, G2, G3, G4
IF (PO = 12000) 552, 552, 550
552 IF (END = 1) 501, 500, 500
503 CONTINUE
WRITE (1, 502)
501 STOP
END
FUNCTION RKLOGE(N, Y, F, X, N+NT)

C 02 UCD3 RKLOGE RUNGE-KUTTA-GILL LINEAR DIFFERENTIAL EQUATION SOLVER
C 02 UCD3 RKLOGE
C MODIFIED MAY 1963 (O REMOVED FROM CALLING SEQUENCE)
C TEST OF ALGOL ALGORITHM
C DIMENSION Y(10), F(10), G(10)
C REAL X, N=INTEGER N, NT=COMMENT BEGIN INTEGER 1, J, L=REAL A
C NT=NT+1
C GO TO (1, 2, 3, 4), NT
C GO TO S(NT)
C DO 11 J=1, N
   11 Q(J)=0,
   AR, S,
   X=0 M/2,
   GO TO 13
   3 A=1.7071067812
   X=0 M/2,
   GO TO 5
   5 DO 41 I=1, N
   41 Y(I)=Y(I)+H*F(I)/6+G(I)/3,
   NT=NT+1
   RKLOGE
   GO TO 12
   12 DO 51 I=1, N
   51 Y(I)=Y(I)+H*F(L)+A*G(L)
   RKLOGE
   CONTINUE
   RETURN
END
LIST OF REFERENCES


Some experimental data for initial fragment velocities from bursting containment vessels are available to check the predictions of fragment velocities from code SPHER. For example, D. W. Boyer, et al. (Reference 1) have measured fragment velocities from bursting glass spheres of various dimensions where the contained gas was air or helium (HE). The text of the workbook, "Example 1 for Spheres," shows how the initial fragment velocity for one of these cases could be obtained from the tables. Table 4B-1 lists the data from the Boyer report and compares it with values obtained from Tables 4-1 through 4-8 of the workbook text, using the method demonstrated in the examples shown in the text. The data are also compared to values obtained by runs of code SPHER for the specific burst pressure and sphere geometries used in the experimental runs.

Results are within about 10% of one another for any of the methods used.

Table 4B-2 lists data for fragments from bursting titanium alloy spheres pumped up with nitrogen. The experimental data are from a report by Pittman (Reference 2). Pittman measured the velocities of fragments emanating from the spheres with breakwire and strobe photographic techniques. Code SPHER and the tables predict the velocities from the small diameter sphere within 10%. Velocities predicted for the larger diameter spheres are low by 15% where breakwire techniques were used to measure the fragment velocities. They agree well with the measured data where strobe photography was used to measure the fragment velocities.
### TABLE 4B-1. INITIAL FRAGMENT VELOCITIES, $V_i$, FROM BURSTING GLASS SPHERES

$(\rho = 2.6 \text{ cm}^3)$

<table>
<thead>
<tr>
<th>Sphere Characteristics</th>
<th>Pressurizing Gas Type</th>
<th>Pressure $P_a$ (Pa)</th>
<th>Initial Fragment Velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius cm</td>
<td>Wall Thickness cm</td>
<td></td>
<td>$V_i$ (Boyrer)$^c$</td>
</tr>
<tr>
<td>1.27</td>
<td>0.100</td>
<td>Air</td>
<td>$2.25 \times 10^6$</td>
</tr>
<tr>
<td>2.54</td>
<td>0.100</td>
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<td>&quot;</td>
</tr>
<tr>
<td>6.35</td>
<td>0.1</td>
<td>&quot;</td>
<td>$1.38 \times 10^6$</td>
</tr>
<tr>
<td>1.27</td>
<td>0.100</td>
<td>He</td>
<td>$2.25 \times 10^6$</td>
</tr>
<tr>
<td>2.54</td>
<td>0.100</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

$V_i$ values were obtained from Reference 1 by averaging measured values for similar cases from Table I of Reference 1 for the 1.27 and 2.54 cm radius spheres and by measurements from Figure 13 of Reference 1 for the 6.35 cm radius sphere.

### TABLE 4B-2. INITIAL FRAGMENT VELOCITIES, $V_i$, FROM BURSTING TITANIUM ALLOY SPHERES

$(\rho = 4.46 \text{ cm}^3)$

<table>
<thead>
<tr>
<th>Sphere Characteristics</th>
<th>Pressurizing Gas Type</th>
<th>Pressure $P_a$ (Pa)</th>
<th>Initial Fragment Velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius cm</td>
<td>Wall Thickness cm</td>
<td></td>
<td>$V_i$ (Pittman)$^a$</td>
</tr>
<tr>
<td>11.7</td>
<td>0.274</td>
<td>N</td>
<td>$5.51 \times 10^7$</td>
</tr>
<tr>
<td>34.3</td>
<td>0.919</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>34.3</td>
<td>0.919</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>34.3</td>
<td>0.919</td>
<td>&quot;</td>
<td>&quot;</td>
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</table>

$^a$Values taken from Reference 2.

$^1$This value was based on velocity measurements using a stroboscopic photographic technique.

$^2$These values were based on velocity measurements using breakwire measurement techniques.
LIST OF REFERENCES


APPENDIX IV.C

ESTIMATE OF INITIAL VELOCITIES OF FRAGMENTS FROM SPHERES AND CYLINDERS BURSTING INTO TWO EQUAL HALVES

The method developed by Taylor and Price\(^1\) for calculating velocities of fragments from bursting spherical gas reservoirs was modified to provide velocity calculations for fragments from both cylindrical and spherical gas vessels. These modifications were primarily geometrical in nature and will be discussed in more detail in subsequent pages. To compute the velocity of fragments from bursting spheres or cylinders which contain gas under pressure, the following assumptions were made:

1. The vessel with gas under pressure breaks into two equal halves along a plane perpendicular to the cylindrical axis, and the two container fragments are driven in opposite directions.

2. Gas within the vessel obeys ideal gas laws.

3. Originally contained gas escapes from the vessel through the opening between the fragments into a surrounding vacuum. The escaping gas travels perpendicular to the direction of motion of the fragments with local sonic velocity.

4. Energy necessary to break the vessel walls is negligible compared to the total energy of the system.

5. Drag and lift forces are ignored since the distance the fragment travels before it attains its maximum velocity and the time it takes to attain its maximum velocity are too short for drag and lift forces to have a significant effect.

A schematic depicting the essential characteristics of the Taylor and Price solution for bursting spheres is shown in Figure 4C-1. Before accelerating into an exterior vacuum, the sphere has internal volume \(V_{00}\) and contains a perfect gas of adiabatic exponent (ratio of specific heats) \(K\) and gas constant \(R\) with initial pressure \(P_{00}\) and temperature \(T_{00}\) (Figure 4C-1a). At a time \(t = 0\), rupture occurs along a perimeter \(\gamma\), and the two fragments are propelled in opposite directions due to forces applied against the area \(F\) which is perpendicular to the axis of motion of the fragments (Figure 4C-1b). The mass of the two frag-
Figure 4C-1. Parameters for Sphere Bursting into Two Halves

(a) Container mass, $M_t$

(b) Plane of rupture

(c) Negligible mass of accelerating gas
ments, $M_1$ and $M_2$, is considered large relative to the mass of the remaining gas at elevated pressure (Figure 4C-1c).

As mentioned earlier, the Taylor and Price solution for calculating velocities of halves of bursting spheres was modified to predict velocities of halves of bursting cylinders. Figure 4C-2 contains the geometric parameters associated with cylindrical vessels. The generalized fragment velocity solution and subsequent computer program allow for computation of the velocity of half of the cylinder. The vessel is assumed to break into two equal halves along a plane perpendicular to its cylindrical axis. The cylinder can have spherical segment end caps or can have flat faces. The vessel has cylindrical radius $r$, cylindrical thickness $C_t$, end cap thickness $E_t$, cylindrical length $C_f$, and end cap length $E_f$ beyond the cylindrical portion. When $C_t = C_f = 0$ and $E_f = r$, the containment vessel becomes a sphere, and the solution corresponds to that formulated by Taylor and Price. That is, a cylinder with hemispherical end caps with length-to-diameter $(L/D)$ ratio of 1.0 is a sphere.

The Taylor and Price solution, generalized to allow for cylindrical as well as spherical vessels, follows. The equations of motion and initial conditions of the two fragments are

\[
M_1 \frac{d^2 x_1}{d\tau^2} = F_P_1(\tau), \quad \text{with} \quad x_1(0) = 0, \quad \frac{dx_1(0)}{d\tau} = 0 \quad (4C-1)
\]

\[
M_2 \frac{d^2 x_2}{d\tau^2} = F_P_2(\tau), \quad \text{with} \quad x_2(0) = 0, \quad \frac{dx_2(0)}{d\tau} = 0 \quad (4C-2)
\]

where subscripts refer to each fragment and $x_1$ and $x_2$ are displacement distances and are taken along the axis of motion. To allow for cylindrical containment vessels, the cross sectional area $F$ over which the force is applied becomes

\[
F = \pi (r - C_t)^2 \quad (4C-3)
\]

The equation of state for the unaccelerated gas remaining within the confinement of the container fragments is

\[
P_0(\tau) \quad V_0(\tau) = C(\tau) \quad RT_0(\tau) \quad (4C-4)
\]
where subscript 0 denotes reservoir conditions immediately after failure, R is the gas constant, P is pressure, V is volume, T is temperature and \( C(t) \) is the mass of gas confined at high pressure as a function of time. The rate of change of the confined mass is

\[
\frac{dC(t)}{dt} = -k \pi x \rho a_a
\]  

(4C-5)

where

\[
x = x_1 + x_2
\]  

(4C-6)

and \( k \) is the coefficient of discharge of the area between the fragments and \( \rho_a \) is the gas density at critical gas velocity \( a_a \). The expression for perimeter \( \Pi \) is the same as for spheres,

\[
\Pi = 2\pi r
\]  

(4C-7)

Gas density \( \rho_a \) and \( a_a \) are standard expressions.
\[
\begin{align*}
\theta_0 &= \rho_0(\tau) \left( \frac{2}{K + 1} \right)^{1/(K-1)} \\
\xi_0 &= \xi_0(\tau) \left( \frac{2}{K + 1} \right)^{1/2}
\end{align*}
\]

where \( K \) is the adiabatic exponent (ratio of specific heats) for an ideal gas. The volume is assumed to be variable and can be described by

\[
V_0(\tau) = V_{\infty} + \Gamma X
\]

Nearly all of the gas is assumed to be accelerated with the fragments, with gas immediately adjacent to the fragments being accelerated to the velocity of the fragments. From simple-one-dimensional flow relationships,

\[
\begin{align*}
P_1(\tau) &= P_0(\tau) \left(1 - \frac{K-1}{2 [\theta_0(\tau)]^2} \frac{d\xi_1(\tau)}{d\tau} \right)^{2 K/(K-1)} \\
P_2(\tau) &= P_0(\tau) \left(1 - \frac{K-1}{2 [\theta_0(\tau)]^2} \frac{d\xi_2(\tau)}{d\tau} \right)^{2 K/(K-1)}
\end{align*}
\]

To generalize the solution, one can use the following nondimensional forms of the variables:

Dimension: \( x(\tau) = X_g(\tau) \quad \xi_1(\tau) = X_g_1(\tau) \), \( \xi_2 = X_g_2(\tau) \)

Time: \( \tau = \theta^* \)

Pressure: \( P_0(\tau) = P_{\infty} P_0(\tau) \)

From appropriate solutions and initial conditions:
\[
\begin{align*}
\frac{dx_1}{d\tau} &= \frac{X}{\theta} g_1', \\
\frac{dx_2}{d\tau} &= \frac{X}{\theta} g_2', \\
\frac{d^2 x_1}{d\tau^2} &= \frac{X}{\theta^2} g_1'', \\
\frac{d^2 x_2}{d\tau^2} &= \frac{X}{\theta^2} g_2''
\end{align*}
\]

\[
\frac{dP_\circ}{d\tau} = \frac{P_{\infty}}{\eta} P_\circ' \quad (4C-12)
\]

\[
x_1(0) = x_2(0) = \frac{dx_1(0)}{d\tau} = \frac{dx_2(0)}{d\tau} = g_1(0) = g_2(0) = g_1'(0) = g_2'(0) = 0
\]

\[P_{\circ}(0) = 1\]

where primes denote differentiation with respect to \( \tau \). The pair of characteristic values for dimension \( X \) and time \( \theta \) chosen by Taylor and Price are:

\[
X = \frac{M a}{P_{\infty}^{1/2}} \left( \frac{2}{K - 1} \right)
\]

\[
\theta = \frac{M a}{P_{\infty}^{1/2}} \left( \frac{2}{K - 1} \right)^{1/2}
\]

The final derived equations contain two dimensionless groups which define the nature of the solutions, these are

\[
\gamma = \frac{P_{\infty} V_{\infty}}{M a^{1/2} t_{\infty}}
\]

\[
\beta = k \left( \frac{2}{K + 1} \right)^{1/2} \left( \frac{2(K - 1)}{K - 1} \right) \left( \frac{2}{K - 1} \right)^{1/2} \frac{P_{\infty} V_{\infty}}{F^2}
\]
Differences between the Taylor and Price solution for spheres and our solution for cylinders, with spherical caps being a special case of cylinders, occur in the determination of area $F$ given by Equation (4C-3) and perimeter $P$ given in Equation (4C-7) where $r$ is cylindrical radius (except for the special case of a sphere where $r$ is spherical radius) instead of spherical radius. A difference also exists in the calculation of initial volume of the gas which, for the cylindrical case with spherical segment endcaps with one base, becomes

$$V_0 = \pi \left( (r - C) \right)^2 C + (E_f - E_i) \left( r - E_i \right)^2 + \frac{1}{3} (E_f - E_i)^2$$

(4C-15)

for the adiabatic case,

$$\frac{P_0 (\tau)}{P_\infty} = \frac{P \cdot \tau}{\frac{\tau}{T_0}} = \frac{T_0 (\tau)}{T_\infty} = \frac{a_0 (\tau)}{a_\infty} \frac{2K}{K - 1}$$

(4C-16)

Substitution of Equations (4C-10), (4C-12) through (4C-14), and (4C-16) into Equations (4C-1) and (4C-2) gives

$$\frac{M_1}{M_t} \frac{g_1}{1 - \frac{g_1^2}{(P_*)^K}} \frac{K}{K - 1}$$

(4C-17)

$$\frac{M_2}{M_t} \frac{g_2}{1 - \frac{g_2^2}{(P_*)^K}} \frac{K}{K - 1}$$

Differentiation of Equation (4C-4) and substitution of Equations (4C-5) through (4C-9) and (4C-11) and (4C-12) yields

$$\left( \frac{K - 1}{2} \right) \alpha + g \frac{P_*'}{P_*} = - \frac{gK}{\alpha} \frac{g(P_*)}{P_0} - Kg'$$

(4C-18)
Since fragment masses are equal \( M_1 = M_2 = M_t/2 \), the equations for the motion of the two fragments become identical, with identical initial conditions and identical solutions so that \( g_1 = g_2 = g/2 \). Equations in Equation (4C-17) then reduce to

\[
g^* = \frac{4}{K} P_\infty \left[ 1 - \frac{g^2}{4 (P_\infty) K - 1} \right] \tag{4C-19}
\]

Rearranging terms in Equation (4C-18) produces

\[
P_\infty' = \frac{3K - 1}{\alpha \frac{6K}{K - 1} \frac{g P_\infty}{P_\infty} \frac{2K}{\alpha} - Kg' P_\infty} \tag{4C-20}
\]

For initial conditions, \( g(0) = 0, g'(0) = 0, \) and \( P_\infty(0) = 1 \), nondimensional values of distance, velocity, acceleration and pressure as a function of time can be calculated by solving Equations (4C-19) and (4C-20) simultaneously using Runge Kutta method of numerical iteration. Dimensional values can then be calculated from

\[
\tau = \theta, \quad x_1(\tau) = \frac{Xg(\tau)}{2}, \quad x_1'(\tau) = \frac{X}{2\theta} g'(\tau), \tag{4C-21}
\]

\[
x_1''(\tau) = \frac{X}{2\theta^2} g''(\tau), \quad P_\infty(\tau) = P_\infty P_\infty(\tau)
\]

The computer program entitled/Frag 2/ was written in Fortran IV and exercised on a teletype tymshare terminal. The computer program is user-oriented and accepts either SI or English units and outputs either SI or English units. Rigorous English measure input is not used for length and mass measurements. Instead, inches are used instead of feet for length measurements and pounds-force (weight measure) are used instead of slugs for mass measurements in both input and output stages of the program since these units are commonly used in these types of measurements. The ratio of specific heats \( K \), speed of sound \( a_{\infty} \), initial pressure \( P_\infty \), external radius of the cylinder of sphere, and the discharge coefficient, chosen as 1.0 in all cases examined, are input parameters. The user has a choice of inputting cylinder length, end length, cylinder thickness, end thickness, and wall density; or volume, mass of the reservoir and cylinder thickness (see Figure 4C-2).
program also requires that a step size and limit be added to allow for the iteration process to begin and end. Nondimensional times are inputted for this purpose. The user has a choice of displaying nondimensional distance, velocity, acceleration and pressure as a function of nondimensional time and/or displaying dimensional distance, velocity, acceleration and pressure as a function of dimensional time. In all cases, final dimensional times, distance, velocity, acceleration and pressure are printed.

An explanation of the Runge-Kutta subroutine is contained in Table 4C-1. This is a standard computer library function which has nine arguments. After the Runge-Kutta subroutine explanation, one can find a list of the program variables, a listing of the program, and sample input and output in Table 4C-2.

**TABLE 4C-1. RUNGE-KUTTA COMPUTER LIBRARY FUNCTION**

**FILE NAME:** F16

**SUBROUTINE NAME:** RUNGE

**PURPOSE:** This subroutine employs the Fourth Order Runge-Kutta Method to solve N simultaneous first-order ordinary differential equations by calculating successive values of $Y$ according to the formula:

$$Y_{i+1} = Y_i + \frac{h}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

where $K_1 = f(x_i, Y_i)$

$$K_2 = f(x_i + \frac{h}{2}, y_i + \frac{hK_1}{2})$$

$$K_3 = f(x_i + \frac{h}{2}, y_i + \frac{hK_2}{2})$$

$$K_4 = f(x_i + h, y_i + hK3)$$
TABLE 4C-1. (cont'd)

METHOD:
The subroutine is called by the calling program five times in order to approximate successive Y(I)'s: the first time to initialize, the second time to calculate K_1(I), the third time to calculate K_2(I), the fourth time to calculate K_3(I) and the fifth time to calculate K_4(I). In addition, each time the subroutine is called, it calculates a new Y(I) and a new X(I) which are returned to the calling program where the functions (first-order differential equations) are evaluated with the new X(I) and Y(I). These values of the function are then returned to the subroutine where they are used as K_1(I), K_2(I), K_3(I), or K_4(I) and approximately accumulated to obtain Y_{i+1}(I) in the 5 calls to the subroutine.

ARGUMENTS: The subroutine RUNGE uses nine arguments: N, Y, F, X, H, M, SAVEY, PHI, K.

1. The first argument, N, represents the number of simultaneous first-order ordinary differential equations to be solved.

2. The second argument, Y, is the array name which the calling program uses to transmit the initial Y(I) values for each differential equation. Upon completion of the 5 calls to RUNGE, Y(I) will contain the new approximated values for the Y_{i+1}(I)'s.

3. The third argument, F, is the array which contains the current values of the differential equations calculated by the main program, i.e., F(J) contains the value of the Jth first-order differential equation.

4. The fourth argument, X, represents the independent variable which should be initialized in the main program before calling RUNGE. RUNGE increments X by the stepsize H.

5. The fifth argument, H, represents the step size for X.
6. The sixth argument, M, indicates which of the five passes of the subroutine is to be executed. The main program must initialize this argument as 1. RUNGE then successively increments the variable by 1 up to 5.

7. The seventh argument, SAVEY, is used within RUNGE and must be dimensioned in the calling program to be of size N.

8. The eighth argument, PHI, is also used internally by RUNGE, but must be dimensioned in the calling program to be of size N.

9. The ninth argument, K, is manipulated within RUNGE. K should be tested right after the call to RUNGE, in the calling program.

When K = 1, control should transfer to a set of code in the calling program which calculates new values for the first-order differential equations, F(I), with the current values of X and Y(I). Then RUNGE should be called again.

When K = 2, the approximation for Y(I) is completed. Values for the Y_{i+1}(I)’s are stored in Y(I) at this time, and normal flow of the calling program should resume.

LIMITATIONS AND COMMENTS:

1. The calling program must dimension SAVEY and PHI.

2. The calling program must set M = 1 before calling RUNGE.

3. The calling program must set up the N first-order differential equation values in an array F to be passed through to RUNGE when the subroutine returns with K = 1.

4. The calling program must set up separate arrays if all X and Y values for the set of differential equations are to be saved, perhaps for plotting purposes.
TABLE 4C-2. COMPUTER PROGRAM ENTITLED/FRAG/
IN FORTRAN IV

Function: This program computes the velocity of a fragment from a bursting sphere or cylinder, with or without spherical segment end caps with one base, which contains gas under pressure. It is assumed that the vessel breaks into two equal halves along a plane perpendicular to the cylindrical axis. Distance, acceleration and residual pressure as a function of time are also computed.

Input-Output Considerations: The program accepts input in either SI or English units and can print output in SI or English units, making any conversions needed internally. The program considers SI units of mass in kilograms, length in meters and time in seconds. The program considers English units of mass in pounds of force (weight measure used for convenience), length in inches and time in seconds. Input data are:

(A) Gas characteristics:
(CAP1) Adiabatic exponent (ratio of specific heats) for gas in the containment vessel.
(AO) Speed of sound in gas of vessel.
(PO) Initial pressure of gas in vessel.

(B) Vessel characteristics:
(RR) Cylinder radius
choice of
(ZN) = 1: (A) Cylinder length
(B) Length of end cap
(C) Cylinder thickness
(D) Thickness of end cap
(E) Wall density
TABLE 4C-2. (cont'd)

(ZN) = 2: (A) Volume of containment vessel
(B) Mass of reservoir
(C) Cylinder thickness

(C) Dynamic variables:
(FN) Number of fragments (always 2.)
(FK) Discharge coefficient (chosen to be 1.)
(AH) Nondimensional time increment for calculations
(XMAX) Maximum nondimensional time calculation.

(D) Input/Output format:
(ZN2) Input units
  1. = SI
  2. = English
(ZN3) Output units
  1. = SI
  2. = English
(FN1) Display nondimensional dynamic variance
  1. = Yes
  2. = No
(FN2) Display dimensional dynamic variance
  1. = Yes
  2. = No
(FN3) Make range calculation always
  2. (NO)

Variables: The definition and units of variables in this program follow.
<table>
<thead>
<tr>
<th>Program Variable</th>
<th>Variable</th>
<th>Definition</th>
<th>SI</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>FN2</td>
<td></td>
<td>if = 1., program displays normal time, distance, velocity, accelerations and pressure</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>FN3</td>
<td></td>
<td>(always 2.)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CL</td>
<td>( C_t )</td>
<td>cylinder length</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>EL</td>
<td>( E_t )</td>
<td>end length</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>CT</td>
<td>( C_t )</td>
<td>cylinder thickness</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>ET</td>
<td>( E_t )</td>
<td>end thickness</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>DEN</td>
<td></td>
<td>density</td>
<td>( \text{kg/m}^3 )</td>
<td>( \text{lb-f/in}^3 )</td>
</tr>
<tr>
<td>VOO</td>
<td></td>
<td>outside volume of vessel</td>
<td>( \text{m}^3 )</td>
<td>( \text{in}^3 )</td>
</tr>
<tr>
<td>VO</td>
<td>( V_{oo} )</td>
<td>internal volume of vessel</td>
<td>( \text{m}^3 )</td>
<td>( \text{in}^3 )</td>
</tr>
<tr>
<td>VOW</td>
<td></td>
<td>wall volume</td>
<td>( \text{m}^3 )</td>
<td>( \text{in}^3 )</td>
</tr>
<tr>
<td>TM</td>
<td>( M_Z )</td>
<td>total mass of reservoir</td>
<td>kg</td>
<td>( \text{lb-f}^0 )</td>
</tr>
<tr>
<td>PO1</td>
<td></td>
<td>unit conversion step for PO</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>RR1</td>
<td></td>
<td>unit conversion step for RR</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CL1</td>
<td></td>
<td>unit conversion step for CL</td>
<td>--</td>
<td>--</td>
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</tr>
<tr>
<td>CT1</td>
<td></td>
<td>unit conversion step for CT</td>
<td>--</td>
<td>--</td>
</tr>
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</table>

\(^3\text{lb-f indicates English weight measurement of pounds of force. Sea level gravitation is assumed.}\)

4C - 14
<table>
<thead>
<tr>
<th>Program Variable</th>
<th>Variable</th>
<th>Definition</th>
<th>SI</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>ET1</td>
<td>--</td>
<td>unit conversion step for ET</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DEN1</td>
<td>--</td>
<td>unit conversion step for DEN</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>TM1</td>
<td>--</td>
<td>unit conversion step for TM</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CAP1</td>
<td>K</td>
<td>ratio of specific heats of gas</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Ao</td>
<td>a₀₀</td>
<td>sound speed</td>
<td>m/s</td>
<td>in/sec</td>
</tr>
<tr>
<td>Po</td>
<td>P₀₀</td>
<td>initial pressure</td>
<td>Pa</td>
<td>psi</td>
</tr>
<tr>
<td>RR</td>
<td>r</td>
<td>cylinder radius</td>
<td>m</td>
<td>in</td>
</tr>
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<td>ZN1</td>
<td>--</td>
<td>if ( \text{ZI} = 1 ), input is ( C_L, E_t, C_t, E_t ), density of vessel (( A, B, C, D, E ) in program)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>ZN2</td>
<td>--</td>
<td>specifies input units 1. implies SI 2. implies English</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>ZN3</td>
<td>--</td>
<td>specifies output units 1. implies SI 2. implies English</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>FN</td>
<td>--</td>
<td>number of fragments (always 2)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>FK</td>
<td>--</td>
<td>gas discharge coefficient</td>
<td>--</td>
<td>--</td>
</tr>
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<td>ALL</td>
<td>--</td>
<td>dimensionless time interval of iteration</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>SMAX</td>
<td>--</td>
<td>maximum dimensionless time of iteration</td>
<td>--</td>
<td>--</td>
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### TABLE 4C-2, (cont'd)

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<th>Variable</th>
<th>Definition</th>
<th>SI</th>
<th>English</th>
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<tbody>
<tr>
<td>FN1</td>
<td>--</td>
<td>if $i = 1$, program displays $\theta$, $g$, $g'$, $g''$, $P_a$</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>VO1</td>
<td>--</td>
<td>unit conversion step for VO</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>PERI</td>
<td>$\pi$</td>
<td>perimeter (calculated)</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>FF</td>
<td>$F$</td>
<td>area of cross-section to which force is applied (calculated)</td>
<td>$m^2$</td>
<td>$in^2$</td>
</tr>
<tr>
<td>XX</td>
<td>$X$</td>
<td>characteristic dimension (calculated)</td>
<td>$m^2$</td>
<td>$in^2$</td>
</tr>
<tr>
<td>THETA</td>
<td>$\theta$</td>
<td>characteristic time (calculated)</td>
<td>$s$</td>
<td>sec</td>
</tr>
<tr>
<td>CAP2</td>
<td>--</td>
<td>quantity $K/(K-1)$</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CAP3</td>
<td>--</td>
<td>quantity $(3K-1)/2K$</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CAP4</td>
<td>--</td>
<td>quantity $(K+1)/(2K-1)$</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>AL</td>
<td>$\alpha$</td>
<td>dimensionless parameter</td>
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<td>--</td>
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<td>B1</td>
<td>$\beta$</td>
<td>dimensionless geometry parameter</td>
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<td>--</td>
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<td>X</td>
<td>--</td>
<td>normalized time</td>
<td>--</td>
<td>--</td>
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<tr>
<td>Y(1)</td>
<td>--</td>
<td>normalized initial fragment displacement</td>
<td>--</td>
<td>--</td>
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<tr>
<td>Y(2)</td>
<td>--</td>
<td>normalized velocity</td>
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<td>--</td>
</tr>
<tr>
<td>Y(3)</td>
<td>--</td>
<td>normalized pressure</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>F(1), F(2), F(3)</td>
<td>--</td>
<td>differential equations solved (see Equations 4C-19 and 4C-20)</td>
<td>--</td>
<td>--</td>
</tr>
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<td>Program Variable</td>
<td>Variable</td>
<td>Definition</td>
<td>SI</td>
<td>English</td>
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<td>----------</td>
<td>----------------------------------------------</td>
<td>----</td>
<td>---------</td>
</tr>
<tr>
<td>TT</td>
<td>--</td>
<td>normalized time (output)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>g</td>
<td>normalized distance (output)</td>
<td></td>
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<td>GG</td>
<td>g'</td>
<td>normalized velocity (output)</td>
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<td>GGG</td>
<td>g''</td>
<td>normalized acceleration (output)</td>
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<tr>
<td>PS</td>
<td>P_o</td>
<td>normalized pressure (output)</td>
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</tr>
<tr>
<td>T1</td>
<td>--</td>
<td>time (output)</td>
<td>s</td>
<td>sec</td>
</tr>
<tr>
<td>GL, FG1</td>
<td>--</td>
<td>distance (output)</td>
<td>m</td>
<td>in</td>
</tr>
<tr>
<td>FG2</td>
<td>--</td>
<td>velocity (output)</td>
<td>m/s</td>
<td>in/sec</td>
</tr>
<tr>
<td>FG3</td>
<td>--</td>
<td>acceleration (output)</td>
<td>m/s^2</td>
<td>in/sec^2</td>
</tr>
<tr>
<td>FG4</td>
<td>--</td>
<td>pressure (output)</td>
<td>Pa</td>
<td>psi</td>
</tr>
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TABLE 4C-2. (cont'd) ORIGINAL PAGE 18 OF POOR QUALITY

<table>
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<tr>
<th>XTRAN:</th>
<th>VER.</th>
<th>JUL 13</th>
</tr>
</thead>
</table>

<GED
APPEND/FRAG 2

DIMENSION F(3), Y(3), Z1(3), Z2(3), PS(50), TT(50), G(50), GG(50), GGG(50)

300 FORMAT (E10.3)
309 FORMAT (2/4H READ IN KAPPA, SOUND SPEED, INITIAL PRESSURE)
310 FORMAT (2/19H)
311 FORMAT (2/4H IF PARAMETERS ARE, /6X CYL. LENGTH, END LENGTH, CYL. THICK., END THICK., VALL DENS., /)
312 FORMAT (1/5H)
313 FORMAT (1/5H)
314 FORMAT (1/5H)
315 FORMAT (1/5H)
316 FORMAT (1/5H)
317 FORMAT (1/5H)
318 FORMAT (1/5H)
319 FORMAT (1/5H)
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391 FORMAT (1/5H)
392 FORMAT (1/5H)
393 FORMAT (1/5H)
394 FORMAT (1/5H)
TABLE 4C-2. (cont'd)

| 905 FORMAT | (2/24H INPUT UNITS? SI=1 ENGLISH=2) |
| 906 FORMAT | (2/24H OUTPUT UNITS? SI=1 ENGLISH=2) |
| 907 FORMAT | (2/19H SI INPUT/OUTPUT, 2/4X, 5H ITEM, 6X, |
| 9N SI UNITS, 2/6H KAPPA, 9X, E10.4, /12H SOUND SPEED, 3X, E10.4, 6H M/SEC, |
| /9H PRESSURE, 6X, E10.4, 8H PASCALS, / |
| 7N RADIUS, 8X, E10.4, 7H METERS, /12H CYL. LENGTH, 3X, E10.4, 7H METERS, / |
| 1N END LENGTH, 4X, E10.4, 7H METERS, /12H CYL. THICK, 3X, E10.4, |
| 7N METERS, /11H END THICK, 4X, E10.4, 7H METERS, /13N WALL DENSITY, |
| 2X, E10.4, 8H KG/CU M, /15H RESERVOIR HASS, |
| E10.4, 3H KG, /13H NO. OF FRAGS, 2X, E10.4) |
| 908 FORMAT | (2/24H SI INPUT/ENGLISH OUTPUT, 2/4X, 5H ITEM, 6X, 9H SI |
| 9N UNITS, 15X, 1H ENGLISH UNITS, 2/6H KAPPA, 9X, E10.4, 14X, E10.4, / |
| 12H SOUND SPEED, 3X, E10.4, 6H M/SEC, 7X, E10.4, 7H IN SEC, /9H PRESSURE, |
| 6X, E10.4, 8H PASCALS, 6X, E10.4, 4H PSI, /7H RADIUS, 8X, E10.4, 7H METERS, |
| 7N METERS, /11H END LENGTH, 4X, E10.4, 3H IN, /12H CYL. LENGTH, 3X, E10.4, 7H METERS, |
| 7X, E10.4, 3H IN, /12H CYL. THICK, 3X, E10.4, 7H METERS, 7X, E10.4, 3H IN, /12H END THICK, 3X, |
| E10.4, 7H METERS, 7X, E10.4, 3H IN |
| 928 FORMAT | (13H WALL DENSITY, 2X, E10.4, 8H KG/CU M, 6X, E10.4, |
| 10H LBF/CU IN, 1H, |
| 19H RESERVOIR HASS, E10.4, 3H KG, 11X, E10.4, 4N LBF, /13H NO. OF FRAGS, |
| 2X, E10.4, 14X, E10.4) |
| 909 FORMAT | (2/24H ENGLISH INPUT/ENGLISH OUTPUT, 2/4X, 5H ITEM, 6X, 9H SI |
| 9N UNITS, 15X, 1H ENGLISH UNITS, 2/6H KAPPA, 9X, E10.4, 14X, E10.4, / |
| 12H SOUND SPEED, 3X, E10.4, 6H M/SEC, 7X, E10.4, 7H IN SEC, /9H PRESSURE, |
| 6X, E10.4, 8H PASCALS, 6X, E10.4, 4H PSI, /7H RADIUS, 8X, E10.4, 7H IN, |
| 11X, E10.4, 7H METERS, /12H CYL. LENGTH, 3X, E10.4, 3H IN, /11X, E10.4, |
| 7N METERS, /11H END LENGTH, 4X, E10.4, 3H IN, /12H CYL. THICK, 3X, E10.4, |
| 3H IN, /12H END THICK, 3X, 3X, E10.4, 7H METERS, /12H END THICK, 3X, |
| E10.4, 3H IN, /11X, E10.4, 7H METERS) |
| 929 FORMAT | (13H WALL DENSITY, 2X, E10.4, 10H LBF/CU IN, 4X, |
| E10.4, 8H KG/CU M, 1H, |
| 19H RESERVOIR HASS, E10.4, 4N LBF, 10X, E10.4, 3H KG, /13H NO. OF FRAGS, |
| 2X, E10.4, 14X, E10.4) |
| 910 FORMAT | (2/24H ENGLISH INPUT/ENGLISH OUTPUT, 2/4X, 5H ITEM, 6X, |
| 14H ENGLISH UNITS, 2/6H KAPPA, 9Y, E10.4, /12H SOUND SPEED, 3X, E10.4, |
| 7N IN SEC, /9H PRESSURE, 6X, E10.4, 4H PSI, /7H RADIUS, 8X, E10.4, |
| 3H IN, /12H CYL. LENGTH, 3X, E10.4, 3H IN, /11K END LENGTH, 4X, E10.4, |
| 3H IN, /12H CYL. THICK, 3X, E10.4, 3H IN, /12H END THICK, 3X, E10.4 |
| 3H IN, /13H WALL DENSITY, 2X, E10.4, 10H LBF/CU IN, |
| 19H RESERVOIR HASS, E10.4, 4N LBF, /13H NO. OF FRAGS, |
| 2X, E10.4) |
| 911 FORMAT | (2/25H CHARACTERISTICS OF MOTION OF FRAGMENTS (SI UNITS), |
| 2/2X, 5H TIME, 5X, 5H DIST, 5X, 5H VEL, 4X, 7H ACCEL, 2X, 9H PRESSURE, / |
| 912 FORMAT | (2/25H CHARACTERISTICS OF MOTION OF FRAGMENTS (ENGLISH UNITS), |
| 2/2X, 5H TIME, 5X, 5H DIST, 5X, 5H VEL, 4X, 7H ACCEL, 2X, 9H PRESSURE, / |
| 913 FORMAT | (3/12H FINAL VALUES, 2/6H TIME, E10.4, 4H SEC, /10H |
| DISTANCE*, E10.4, 7H METERS, /10H VELOCITY*, E10.4, 6H M/SEC, /14H |
| ACCELERATION*, E10.4, 9H M/SQ-SEC, /10H PRESSURE*, E10.4, 5H PASCALS) |
| 914 FORMAT | (3/12H FINAL VALUES, 2/6H TIME, E10.4, 4H SEC, /10H |
| DISTANCE*, E10.4, 7H IN, /10H VELOCITY*, E10.4, 7H IN/SEC, /14H |
| ACCELERATION*, E10.4, 10H IN/SQ-SEC, /10H PRESSURE*, E10.4, 4H PSI) |

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TABLE 4C-2. (cont'd)

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DEN = DEN / 386.0886
TH = TH / 386.0886
V0 = V0 / 0.0254 ** 3.0
QE TO 106

10A A8 = A8 + 0.0254
PA = PB + 4.48222 / 0.0254 / 0.0254
RR = RR + 0.0254
CL = CL + 0.0254
EL = EL + 0.0254
CT = CT + 0.0254
ET = ET + 0.0254
DEN = DEN * (0.3086 / 9.80665) * 14.5939 / (0.0254 ** 3.0)
TH = TH * (0.3086 / 9.80665) * 14.5939
WRITE (1, 909) CAPI, CAPI, A8, A8, PB, PB, RR, RR, CL, CL, EL, EL, CT, CT, ET, ET

WRITE (1, 9091) DEN, DEN, TH, TH, FN, FN

AS = A8
PB = PB
RR = RR
CL = CL
EL = EL
CT = CT
ET = ET
DEN = DEN
TH = TH
V0 = V0 / 0.0254 ** 3.0
QE TO 106

105 WRITE (1, 910) CAPI, A8, PB, RR, CL, EL, CT, ET, DEN, TH, FN
DEN = DEN / 386.0886
TH = TH / 386.0886
QE TO 106

101 V0 * A
TH = B
CT = C
PI = 3.1415926535
IF (ZN1.EQ.1.0.AND.ZN3.EQ.1.0) GO TO 95
IF (ZN1.EQ.1.0.AND.ZN3.EQ.2.0) GO TO 94
IF (ZN1.EQ.2.0.AND.ZN3.EQ.1.0) GO TO 93
IF (ZN1.EQ.2.0.AND.ZN3.EQ.2.0) GO TO 92
95 WRITE (1, 898) CAPI, A8, PB, RR, V0, TH, CT, FN
QE TO 106
TABLE 4C-2 (cont'd)

<table>
<thead>
<tr>
<th>AO1AG</th>
<th>O.O2SA</th>
<th>P0ueSo.6O.02SA</th>
<th>.0.254,t.A4U222</th>
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<td>RRaet</td>
<td>R/0 .0254</td>
<td>V61.VI/ O.O2SA)*e3.0</td>
<td>Tl. (TM/ 14.*5939) *(9.80665/0.3046)</td>
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</table>

FloI vesve:

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<th>386 .0686</th>
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<tbody>
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<td>06</td>
<td>10.06</td>
</tr>
<tr>
<td>93</td>
<td>VRIETEL895 <a href="mailto:CAPI.AaP@.RR">CAPI.AaP@.RR</a>*11.0</td>
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</table>

CONT! (P/2.0*PfRRf Fl~.Pt(RR-CT).e 2.0 *)0.01/(CAPI1 .0)/CFT.PS) | ?XZTA'TheAGO((2.0/(CAPI1 .0))a.0.S)/(VFFPS) | CAP20CAPL/CCAPI-1.0) |

| CAPae((CAPI.I 0)/(2.00(CAPI- 1.0)) |
| AtmPG*OVI(TM*AS*Ag) | bl&IFKO (2.O/CCAPLI +1.0;) OCAPLOC (2.0/CCAPI 1.0))0*0. 5) |
| *PERI eve/ rF*Fr) | |

| X(2)0.0 | Y(2)90.0 | Y(3)u1.0 |
| WRITE (1,322) X.YCI).YC2),Y(3) |

4c-22
| TABLE 4C-2. (cont'd) |

F(1) = Y(2) 
F(2) = 4.0 * Y(3) * (1.0 - Y(2) - Y(2) / (4.0 * Y(3) * (1.0 / CAP2))) * CAP2 
F(3) = -BI * CAP1 / A1 * Y(1) * Y(3) * CAP3 - CAP1 * Y(2) * Y(3) / (CAP1 - 1.0) / 2.0 * A1 * Y(1) 
IF (FN1 - 1.0) 200, 200, 30 
200 WRITE (1, 320) 
30 CALL RUNGE (NA, X, Y, F, AH, KA, V1, V2) 
IF (KA - 1) 40, 50, 40 
50 F(1) = Y(2) 
F(2) = 4.0 * Y(3) * (1.0 - Y(2) - Y(2) / (4.0 * Y(3) * (1.0 / CAP2))) * CAP2 
F(3) = -BI * CAP1 / A1 * Y(1) * Y(3) * CAP3 - CAP1 * Y(2) * Y(3) / (CAP1 - 1.0) / 2.0 * A1 * Y(1) 
G0 TO 30 
40 IF (FN1 - 1.0) 45, 45, 201 
45 WRITE (1, 312) X, Y(1), Y(2), F(2), Y(3) 
201 CONTINUE 
JJ = JJ + 1 
TT(JJ) = X 
G(JJ) = Y(1) 
GG(JJ) = Y(2) 
GGG(JJ) = F(2) 
PS(JJ) = Y(3) 
IF (X - XMAX) 41, 10, 10 
41 CONTINUE 
G0 TO 30 
10 CONTINUE 
IF (FN2 - 1.0) 130, 130, 131 
130 IF (ZN3 - 1.0) 108, 106, 109 
108 WRITE (1, 911) 
G0 TO 110 
109 WRITE (1, 912) 
110 DB 107, 1 = JJ 
T1 = THETA * TT(1) 
G1 = (XX * G(1)) / 2.0 
G2 = (XX / (2.0 * THETA)) * GG(1) 
G3 = XX / (2.0 * THETA * THETA) * GGG(1) 
G4 = PS * P(1) 
107 WRITE (1, 312) T1, G1, G2, G3, G4 
131 FTI = THETA * X 
FG1 = (XX * Y(1)) / 2.0 
FG2 = (XX / (2.0 * THETA)) * Y(2) 
FG3 = XX / (2.0 * THETA * THETA) * F(2) 
FG4 = PS * Y(3) 
IF (ZN3 - 1.0) 111, 111, 112 
111 WRITE (1, 913) FTI, FG1, FG2, FG3, FG4 
G0 TO 113 
112 WRITE (1, 914) FTI, FG1, FG2, FG3, FG4 
113 CONTINUE 
IF (FN3 - 1.0) 204, 204, 205 
204 CONTINUE 
205 CONTINUE 
END
TABLE 4C-2. (cont'd)

READ IN KAPPA, SOUND SPEED, INITIAL PRESSURE
1.67, 6880., 500.

READ IN CYLINDER RADIUS
6.0

IF PARAMETERS ARE:
CYL. LENGTH, END LENGTH, CYL. THICK., END THICK., WALL DENS.,
ENTER 1
VOLUME, MASS OF RESERVOIR, CYL. THICK.,
ENTER 2

READ IN VALUES
0.0, 6.0, 0.012, 0.012, 0.162

INPUT UNITS? SI=1 ENGLISH=2
2

OUTPUT UNITS? SI=1 ENGLISH=2
1

READ IN NO. OF FRAGMENTS(2.), DISCHARGE COEF.
2, 1.

READ IN TIME INTERVAL, MAXIMUM TIME
0.1, 1.7

DISPLAY NONDIMENSIONAL DYNAMIC VAR.? YES=1 NO=2
1

DISPLAY DIMENSIONAL DYNAMIC VAR.? YES=1 NO=2
1

MAKE RANGE CALCULATION? YES=1 NO=2
2
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<th>ITEM</th>
<th>ENGLISH UNITS</th>
<th>SI UNITS</th>
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<td>.1670E+01</td>
<td>.1670E+01</td>
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<td>SOUND SPEED</td>
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<td>.1748E+03 M/SEC</td>
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<td>PRESSURE</td>
<td>.5000E+03 PSI</td>
<td>.3447E+07 PASCALS</td>
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<td>RADIUS</td>
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<td>.1524E+00 METERS</td>
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<td>.0000E+01 METERS</td>
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<td>.6000E+01 IN</td>
<td>.1524E+00 METERS</td>
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<td>.3048E-03 METERS</td>
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<td>WALL DENSITY</td>
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<td>.4490E+04 KG/CU M</td>
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<td>NO. OF FRAGS</td>
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INITIAL CONDITIONS

\[ x(0) = .0000E+01, w(0) = .0000E+01, g'(0) = .0000E+01, \quad P-NORM = .1000E+01 \]

**CHARACTERISTICS OF MOTION OF FRAGMENTS (NORMALIZED)**

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<th>G</th>
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TABLE 4C-2. (concl'd)

CHARACTERISTICS OF MOTION OF FRAGMENTS (SI UNITS)

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<th>VEL.</th>
<th>ACCEL.</th>
<th>PRESSURE</th>
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VALUES
TIME = 8647E-03 SEC
DISTANCE = 1486E+00 METERS
VELOCITY = 2001E+03 M/SEC
ACCELERATION = 6475E+01 M/SQ-SEC
PRESSURE = 4555E+06 PASCALS
STOP
(SWAINS)205=1
+

4C-26
The matrix of different initial conditions run on the computer is given in Table 4C-3. For these calculations, the containment vessel was chosen to be made of a titanium alloy since these alloys are often used for flight-weight containment vessels. Gases chosen were air, xenon (Xe), hydrogen (H₂), and carbon dioxide (CO₂). Table 4C-4 contains nondimensionalized input parameters and nondimensionalized final velocities. It should be noted that these nondimensional quantities are not the same as the dimensionless parameters generated in the program to calculate velocity. Rather, the nondimensional parameters in Table 4C-4 take the following form:

1. Nondimensional pressure \( \bar{P} = \frac{\text{initial pressure}}{\text{atmospheric pressure}} \)
2. Nondimensional thickness \( \bar{h} = \frac{\text{cylinder thickness}}{\text{cylinder diameter}} \)
3. Nondimensional length \( \bar{L} = \frac{\text{total length}}{\text{cylinder diameter}} \)
4. Nondimensional velocity \( \bar{V} = \frac{\text{final velocity}}{\text{sound speed of gas}} \)

For all of the cases run, the following conditions hold:

1. All vessels were assumed to be made of titanium or a titanium alloy.
2. The thickness of the containment vessel is uniform.
3. All containment vessels have hemispherical endcaps.

Figures 4C-3 through 4C-5 contain plots of \( \bar{V} \) versus \( \bar{P} \) for h/D ratios of 0.001, 0.01 and 0.1, respectively, the L/D ratio being held constant at 10.0 and as many as three curves, one for each of the gases (CO₂, air, H₂), being plotted on each figure. Figures 4-18 through 4-20 contain plots of \( \bar{V} \) versus \( \bar{P} \) for air, carbon dioxide and hydrogen gases, respectively, the L/D ratio being held constant at 10.0 and as many as three curves, one for each of the gases (0.001, 0.01, and 0.1) being plotted on a single figure. Figure 4-21 contains a plot of \( \bar{V} \) versus L/D ratio for air and a h/D ratio of 0.01. Two curves are plotted on this figure, one for each of two different initial gas pressures.

From the curves in the figures mentioned above, one can make a few conclusions.
TABLE 4C-3

INITIAL CONDITIONS

**Containment Vessel Characteristics**

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**Sonic Velocities**

- Air: 13550 ft/sec = 344.17 m/s
- Xe: 6880 ft/sec = 174.75 m/s
- $H_2$: 50000 ft/sec = 1270 m/s
- $CO_2$: 10150 ft/sec = 297.81 m/s

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<th>$P_{so}$ (psi)</th>
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Figure 4C-3. Nondimensional Velocity Vs. Nondimensional Pressure for Several Gases.
Figure 4C-4. Nondimensional Velocity Vs Nondimensional Pressure, h/D = 0.01
Figure 4C-5. Nondimensional Velocity Vs Nondimensional Pressure, h/D = 0.1
Conclusions

(1) In all cases, fragment velocity increases as initial pressure increases, but at a decreasing rate.

(2) Fragment velocities from vessels containing the heavier gases (and the lower sonic velocities) are higher than that for the lighter gases (and the higher sonic velocities) for the same initial pressure, L/D ratio and h/D ratio. The rate of change in velocity with increasing pressure, however, is greater for the lighter gases, and all the curves plotted appear to approach an asymptotic limit for a fixed h/D ratio (Figures 4C-3 through 4C-5).

(3) Fragment velocities for thin-walled vessels (low h/D ratios) are higher than those for thick-walled vessels (high h/D ratios), for the same initial pressure, L/D ratio and gas. The rate of change in velocity with increasing pressure, however, is greater for the thick-walled vessels, and all the curves plotted appear to approach an asymptotic limit for a fixed gas (Figures 4-18 through 4-20).

(4) For a fixed initial pressure, fragment velocities decrease with increasing L/D ratios. For higher initial pressures, however, the rate of decrease in the fragment velocity with increasing L/D ratios decreases (Figure 4-21).
LIST OF REFERENCES

ESTIMATION OF VELOCITIES ATTAINED BY APPURTENANCES SUBJECT TO BLAST LOADING

The situation discussed here involves the interaction of appurtenances (nearby objects) with the blast wave from propellant or pressure vessel explosions. These objects can be parts of the launch tower, storage tanks, vehicles, and objects in or attached to the upper stages of the launch vehicle itself, or they can be tools, benches, chairs, and machinery in a shop area. The types of appurtenances depend upon the location of the explosion, and, for this reason, results in this appendix are presented in such a manner that velocities of essentially any conceivable appurtenance in the blast field can be calculated.

To be able to predict velocities to which appurtenances are accelerated by explosions, one must consider the interaction of blast waves with solid objects. Figure 4D-1(1) shows schematically, in three stages, the interaction of a blast wave with an irregular object. As the wave strikes the object, a portion is reflected from the front face, and the remainder diffracts around the object. In the diffraction process, the incident wave front closes in behind the object, greatly weakened locally, and a pair of trailing vortices is formed. Rarefaction waves sweep across the front face, attenuating the initial reflected blast pressure. After passage of the front, the body is immersed in a time-varying flow field. Maximum pressure on the front face during this "drag" phase of loading is the stagnation pressure.

To predict the effect of a blast wave on an appurtenance, it is necessary to examine the net transverse pressure on the object as a function of time. This loading, somewhat idealized, is shown in Figure 4D-2. After time of arrival $T_2$, the net transverse pressure rises linearly from zero to a maximum peak reflected pressure $P_R$ in time $T_2 - T_1$. For an object with a flat face nearest the approaching blast wave, this time interval is zero. Pressure then falls linearly to drag pressure in time $T_3 - T_1$ and decays more slowly to zero in time $T_3 - T_2$.

Once the time history of net transverse pressure loading is known, the prediction of appurtenance velocity can be made. The basic assumptions are that the appurtenance behaves as a rigid body, that none of the energy in the blast wave is absorbed in breaking the appurtenance loose from its mountings or deforming it elastically or plastically, and that gravity effects can be ignored during this acceleration phase of the motion. The equation of motion of the object is then
Figure 4D-1. Interaction of Blast Wave with Irregular Object

Figure 4D-2. Time History of Net Transverse Pressure on Object During Passage of Blast Wave
\[ \dot{x}(t) = \frac{A}{M} \int_0^{(T-1)A} p(t) \, dt = \frac{A}{M} I_d \]  (4D-3)

where

\[ I_d = \text{total drag and diffraction impulse} \]

The integration in Equation (4D-3) can be performed explicitly if the pressure time history is described by suitable mathematical functions, or performed graphically or numerically if \( p(t) \) cannot be easily written in functional form. In either case, Equation (4D-3) yields the desired result—a predicted velocity for an object. The integral in Equation (4D-3) is merely the area under the curve in Figure 4D-2.

The time history of drag pressure is the modified exponential with maximum given by

\[ C_D Q = C_D \left( \frac{1}{2} \rho A u_s^2 \right) \]  (4D-4)

where
\[ C_D = \text{steady-state drag coefficient for the object} \]
\[ Q = \text{peak dynamic overpressure} \]
\[ \rho_s = \text{peak density} \]
\[ u_s = \text{peak particle velocity} \]

The characteristics of the diffraction phase of the loading can be determined easily if the peak side-on overpressure \( P_s \) or the shock velocity \( u_s \) is known, together with the shape and some characteristic dimensions of the object. The peak amplitude of the drag phase \( CDQ \) can also be determined explicitly from \( P_s \) or \( u_s \). The time history of the ensuing drag loading \( CD_q(t) \), however, is quite difficult to predict accurately for propellant blasts or blasts from gas vessel explosions.

Side-on overpressure is often expressed as a function of time by the modified Friedlander equation, (2)
\[
p(t) = P_s \left( \frac{1 - e^{-bt/T}}{T} \right)
\]

where
\[ T = \text{duration of the positive phase of the blast wave} \]

Integrating this equation gives the impulse
\[
I = \int_0^T p(t) \, dt = \frac{PT}{b} \left(1 - \frac{1 - e^{-bT}}{b} \right)
\]

The dimensionless parameter \( b \) is called the time constant, is a function of shock strength, and is reported in Chapter 6 of Reference 2. It is plotted graphically in Figure 4D-3 for a range of shock strengths, \( P \), where
\[
\bar{P} = \frac{P}{P_0}
\]

and \( p_0 \) is ambient air pressure. Ambient air pressure \( p_0 \) varies with altitude as shown in Figure 3-9. The peak reflected overpressure \( P_r \) and peak dynamic pressure \( Q \) are unique functions of \( P \) for a given ambient pressure \( p_0 \). For shocks of intermediate to weak strengths, \( \bar{P} \geq 3 \), these functions are\( (2) \)
\[
\bar{P}_r = 2\bar{P} + \frac{3\bar{P}^2}{4}
\]

and

\[
\bar{Q} = \left(\frac{5}{2}\right) \frac{\bar{P}^2}{\bar{T} + \bar{P}}
\]

where

\[
\bar{P}_r = \frac{P}{P_0}, \quad \bar{Q} = \frac{Q}{Q_0}
\]

For the time history of drag pressure, a good fit to experimental data for TNT is a slightly modified form of that employed by Glasstone, (3)

\[
q(t) = Q \left(1 - \frac{1}{T} \right)^2 e^{-bt/T}
\]

In order to estimate values for the time intervals shown in Figure 4D-2, it is necessary to obtain the shock front velocity \(U\). This is a unique function of the shock strength \(\bar{P}\) and, for \(\bar{P} \leq 3.5\), is given by (2)

\[
\bar{U}^2 = 1 + \frac{6\bar{P}}{\bar{T}}
\]

for

\[
U = \bar{U}a_o
\]

where

\(a_o\) = speed of sound in air

The manner in which \(a_o\) varies with altitude is shown in Figure 3D-2. For shock strengths \(\bar{P}\) greater than 3.5, \(\bar{P}_r\), \(\bar{Q}\), and \(\bar{U}\) for Equations (4D-8), (4D-9), and (4D-12), respectively, can be approximated from tables in Chapter 6 of Reference 2. Methods for estimating \((T_1 - t_2)\) and \((T_2 - T_1)\) are given by Norris, et al. (4) and depend on the shock front velocity given above and the geometry of the appurtenance. The first time interval can be acquired from
\( (T_1 - t_a) = \frac{x}{U} \)  \hspace{1cm} (4D-14)

where \( x \) is the distance from the front of the object to the plane facing the approaching blast wave which has the largest cross-sectional area. The latter time interval can be determined from

\( (T_2 - T_1) = \frac{4h}{U} \)  \hspace{1cm} (4D-15)

for appurtenances on the ground and

\( (T_2 - T_1) = \frac{2h}{U} \)  \hspace{1cm} (4D-16)

for appurtenances in the air, where \( h \) is the minimum transverse dimension of the largest mean presented area. Time interval \( (T_2 - t_2) \) is equivalent to \( T \) and can be acquired by rearranging Equation (4D-6) giving

\[
T = \frac{1b}{P \cdot 1 - \left(1 - e^{\frac{-b}{b}}\right)} \hspace{1cm} (4D-17)
\]

The integral in Equation (4D-3) is just the area under the curve in Figure 4D-2. Using the law of similar triangles, time interval \( (T_1 - T_2) \) (see Figure 4D-2) can be obtained from

\[
\frac{(T_2 - T_1)}{(T_l - T_1)} = \frac{C_D q(T_2)}{P_r} \hspace{1cm} (4D-18)
\]

Solving Equation (4D-18) for \( T_l \),

\[
T_2 = \frac{C_D q(T_2)}{P_r} T_1 \]

\[
T_l = \frac{C_D q(T_2)}{P_r} \hspace{1cm} (4D-19)
\]

The total area under the curve in Figure 4D-2 is then

\[ \text{Area} = \int_{T_1}^{T_2} \]
\[ I_d = \frac{\text{Area}_{\Delta t_j P_{r_{j-1}}}}{2} - \int_{T_2}^{T_t} C_D q(T_2)^T dT \]

where \( \Delta t_j P_{r_{j-1}} \) represents the triangle with vertices \( t_0, P_{r_{j-1}}, \) and \( \Delta t_j [C_D q(T_2)]^T_{t_0} \) represents the triangle with vertices \( t_0, [C_D q(T_2)]^T_{t_0}, \) and \( T_1. \) The integral in Equation (4D-20) can be solved by substituting Equation (4D-11) for \( q(T) \). Making similar substitutions and solving the integral, Equation (4D-20) becomes

\[ I_d = \frac{P_{r_{j-1}}(T_t - T_2)}{2} \]

After substituting appropriate values for the variables and evaluating the middle portion of Equation (4D-21) over the time interval \( T_2 \) to \( T_t \), total drag and diffraction impulse \( I_d \) can be determined. Substituting total impulse \( I_d \), the mean presented area \( A \) and total mass \( M \) of the object into Equation (4D-3), the maximum velocity of the appurtenance can be found.

Surely, this is a needlessly long procedure to follow every time an estimate of appurtenance velocity is required. In order to simplify the velocity calculation, Equation (4D-3) can be put into nondimensional form. This was done by substituting known environmental variables into Equations (4D-4) and (4D-21) and rearranging terms. After performing all of the substitutions, a tremendously long equation evolves, which reduces to the simple functional form

\[ \frac{M}{P_0 A (K_i + X)} = \frac{P_s}{P_0} \left( \frac{C_D I_a}{D_s} \right) \]

where

4D.8
$M = \text{mass of object}$

$V = \text{velocity of object}$

$a_0 = \text{velocity of sound in air}$

$p_0 = \text{atmospheric pressure}$

$A = \text{mean presented area of object}$

$K = \text{constant (4 if appurtenance is on the ground and 2 if appurtenance is in air)}$

$H = \text{minimum transverse dimension at location of largest presented area of object}$

$X = \text{distance from the front of object to location of largest cross-sectional area}$

$P_s = \text{peak incident overpressure}$

$C_D = \text{drag coefficient}$

$I_s = \text{peak incident specific impulse}$

The manner in which $p_0$ and $a_0$ vary with altitude is shown in Figures 3-D and 3D-2, respectively. Representative values for drag coefficient $C_D$ can be found in Figure 4D-4. Equation (4D-22) states that nondimensional appurtenance velocity is a function of nondimensional pressure and nondimensional impulse. A computer program was written to determine combinations of nondimensional pressures and impulses required to produce various nondimensional velocities. A graphical representation of the nondimensional pressure $P_s$ and nondimensional impulse $I_s$ combinations producing various values of nondimensional velocity $V$ is shown in Figure 4D-5. Objects of various sizes and shapes, as shown in the key for symbols, were used as input into the program to test the nondimensional scaling law. As expected, all calculated points were near the appropriate nondimensional velocity curves. Thus, Figure 4D-5 can be used to calculate the velocity of any type of appurtenance. Care should be taken when interpolating between curves and extending curves. Estimates made by extending the curves to lower nondimensional impulses are especially hazardous. An example for calculating appurtenance velocity follows.
<table>
<thead>
<tr>
<th>SHAPE</th>
<th>SKETCH</th>
<th>$C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right Circular Cylinder (long rod), side-on</td>
<td>Flow</td>
<td>1.20</td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
<td>0.47</td>
</tr>
<tr>
<td>Rod, end-on</td>
<td>Flow</td>
<td>0.82</td>
</tr>
<tr>
<td>Disc, face-on</td>
<td>Flow</td>
<td>1.17</td>
</tr>
<tr>
<td>Cube, face-on</td>
<td>Flow</td>
<td>1.05</td>
</tr>
<tr>
<td>Cube, edge-on</td>
<td>Flow</td>
<td>0.90</td>
</tr>
<tr>
<td>Long Rectangular Member, face-on</td>
<td>Flow</td>
<td>2.05</td>
</tr>
<tr>
<td>Long Rectangular Member, edge-on</td>
<td>Flow</td>
<td>1.55</td>
</tr>
<tr>
<td>Narrow Strip, face-on</td>
<td>Flow</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Figure 4D-4. Drag Coefficients, $C_D$, for Various Shapes
Figure 4D-5. Nondimensional Appurtenance Velocity $\bar{V}$ as a Function of Nondimensional Pressure $P_a$ and Nondimensional Impulse $I_g$. 
Example

Suppose a square cement block is subjected to an explosion. At the block it was determined that

\[ P_a = 1.0 \times 10^5 \text{ Pa (14.5 psi)} \]
\[ I_s = 1.9 \times 10^4 \text{ Pa-sec (2.756 psi-sec)} \]

The explosion occurs at sea level. Therefore,

\[ P_o = 1.0135 \times 10^5 \text{ Pa} \]
\[ a_o = 340 \text{ m/s (1115 ft/sec)} \]

The block has characteristics [see Equation (4D-22)]

\[ X = 0 \text{ m} \]
\[ H = 2.5 \text{ m (8.2 ft)} \]
\[ C_D = 1.05 \]
\[ A = 25 \text{ m}^2 (269 \text{ ft}^2) \]
\[ M = 2.8 \times 10^5 \text{ kg (6.17 \times 10^5 \text{ lb})} \]

Since the appurtenance is on the ground,

\[ K = 4 \]

Nondimensional pressure \( \bar{P} \) is then

\[ \frac{P_s}{P_o} = \frac{1.0 \times 10^5 \text{ Pa}}{1.0135 \times 10^5 \text{ Pa}} \]
\[ \bar{P}_s = 0.99 \]

Nondimensional impulse \( \bar{I}_s \) is

\[ \bar{I}_s = \frac{C_D I_s a_o}{P_s (KH + X)} = \frac{(1.05)(1.9 \times 10^4)(340)}{(1.0 \times 10^5)(4)(2.5) + (0)} \]

4D-12
I = 6.8

Locating the point \((P_s, I_s)\) on Figure 4D-5, one can find that it lies very near the 5.0 nondimensional velocity \(V\) curve. Choosing \(V\) equal to 5.0, the appurtenance velocity can be calculated as follows:

\[
\frac{MV_a}{p_o A(KH + X)} = 5.0
\]

\[
V = \frac{(5.0)p_o A(KH + X)}{Ma_0}
\]

\[
V = \frac{(5.0)(1.0 \times 10^5 \text{ Pa})(25 \text{ m}^2)(4)(2.5 \text{ m}) + (0)}{(2.8 \times 10^5 \text{ kg})(340 \text{ m/s})}
\]

\[
V = 1.33 \text{ m/s (4.36 ft/sec)}
\]

The listing of variables and list of the computer program follow.

COMPUTER PROGRAM ENTITLED /NDAPVE/ IN FORTRAN IV

Function:

This program computes nondimensional appurtenance velocities for various nondimensional pressures and nondimensional impulses.

Input data are:

(A) Nondimensional values for calculations

(PB, BB) Ordered pairs of nondimensional peak incident overpressure \(P\) and dimensionless time constant \(b\).

(PBB, UB, QB, PRB) Ordered quadruplets of nondimensional values of incident overpressure \(P\), shock front velocity \(U\), dynamic pressure \(Q\), and reflected pressure \(P_r\).
(NDV) Nondimensional velocity values.

(E) Blast wave characteristics

[P(I)] Peak incident overpressure

[PI(I)] Specific impulse

(C) Ambient conditions

[PO(I)] Atmospheric pressure $p_0$

[AO(I)] Speed of sound $a_o$

(D) Appurtenance characteristics

(X) Distance $X$ from front of object to location of largest cross-sectional area.

(H) Minimum transverse distance $H$ of the mean presented area of the appurtenance.

(CD) Drag coefficient $C_D$

(A) Mean presented area $A$

(TM) Mass $M$ of appurtenance

Variables:

The definition and units of the variables in this program are given in the following table.
<table>
<thead>
<tr>
<th>Program Variable</th>
<th>Variable</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>PR</td>
<td>P</td>
<td>nondimensional pressure for calculating b</td>
<td>...</td>
</tr>
<tr>
<td>BB</td>
<td>h</td>
<td>dimensionless time constant h</td>
<td>...</td>
</tr>
<tr>
<td>PRR</td>
<td>P</td>
<td>nondimensional peak incident pressure for calculating G, Q, P_r</td>
<td>...</td>
</tr>
<tr>
<td>CB</td>
<td>U</td>
<td>nondimensional shock front velocity</td>
<td>...</td>
</tr>
<tr>
<td>QD</td>
<td>Q</td>
<td>nondimensional peak dynamic pressure</td>
<td>...</td>
</tr>
<tr>
<td>PRR</td>
<td>P_r</td>
<td>nondimensional peak reflected pressure</td>
<td>...</td>
</tr>
<tr>
<td>NDV</td>
<td>V</td>
<td>nondimensional velocity for internal calculations</td>
<td>...</td>
</tr>
<tr>
<td>PDI</td>
<td>P_s</td>
<td>peak incident overpressure values</td>
<td>Pa</td>
</tr>
<tr>
<td>PDI</td>
<td>L_s</td>
<td>incident specific impulse values</td>
<td>Pa.s</td>
</tr>
<tr>
<td>POH</td>
<td>p_0</td>
<td>atmospheric pressure values</td>
<td>Pa</td>
</tr>
<tr>
<td>AOH</td>
<td>a_s</td>
<td>speed of sound values</td>
<td>m/s</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>distance from front of object to location of largest mean presented area</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>H</td>
<td>minimum transverse distance of mean presented area of appurtenance</td>
<td>m</td>
</tr>
<tr>
<td>CB</td>
<td>C_d</td>
<td>drag coefficient</td>
<td>...</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>mean presented area</td>
<td>m^2</td>
</tr>
<tr>
<td>IM</td>
<td>M</td>
<td>mass of appurtenance</td>
<td>kg</td>
</tr>
<tr>
<td>G</td>
<td>--</td>
<td>number of p_0, a_s input combinations</td>
<td>...</td>
</tr>
<tr>
<td>V</td>
<td>--</td>
<td>number of PDI input values</td>
<td>...</td>
</tr>
<tr>
<td>R</td>
<td>--</td>
<td>number of POH input values</td>
<td>...</td>
</tr>
<tr>
<td>S</td>
<td>--</td>
<td>number of NDV input values</td>
<td>...</td>
</tr>
<tr>
<td>PRR</td>
<td>P</td>
<td>nondimensional peak incident pressure for a particular iteration</td>
<td>...</td>
</tr>
<tr>
<td>T</td>
<td>T, t</td>
<td>total duration of blast wave</td>
<td>s</td>
</tr>
<tr>
<td>PRR</td>
<td>P_r</td>
<td>nondimensional peak reflected pressure for a particular iteration</td>
<td>...</td>
</tr>
</tbody>
</table>
## TABLE 4D-1. (CONT'D)

<table>
<thead>
<tr>
<th>Program Variable</th>
<th>Variable</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>QBAR</td>
<td>Q</td>
<td>nondimensional dynamic pressure for a particular iteration</td>
<td></td>
</tr>
<tr>
<td>UBAR</td>
<td>U</td>
<td>nondimensional shock front velocity for a particular iteration</td>
<td></td>
</tr>
<tr>
<td>PR</td>
<td>P_r</td>
<td>peak reflected pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>Q</td>
<td>Q</td>
<td>peak dynamic pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>U</td>
<td>U</td>
<td>shock front velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>T_1</td>
<td>T_1</td>
<td>time corresponding to occurrence of peak reflected pressure</td>
<td></td>
</tr>
<tr>
<td>T_2</td>
<td>T_2</td>
<td>time corresponding to second interaction of distracted and drag phase of loading curves</td>
<td></td>
</tr>
<tr>
<td>Q_1:3</td>
<td>q(T_2)</td>
<td>dynamic pressure at time T_2</td>
<td>Pa</td>
</tr>
<tr>
<td>CD1_2</td>
<td></td>
<td>intermediate calculation for T_1</td>
<td></td>
</tr>
<tr>
<td>L_1</td>
<td>L</td>
<td>time at which distracted phase of loading is over</td>
<td></td>
</tr>
<tr>
<td>APR1</td>
<td></td>
<td>intermediate calculation for p_1</td>
<td></td>
</tr>
<tr>
<td>AC1_3</td>
<td></td>
<td>intermediate calculation for p_3</td>
<td></td>
</tr>
<tr>
<td>P_1</td>
<td>P</td>
<td>intermediate calculation for p_1</td>
<td></td>
</tr>
<tr>
<td>P_2_3</td>
<td></td>
<td>intermediate calculation for p_3</td>
<td></td>
</tr>
<tr>
<td>T_2_2</td>
<td></td>
<td>intermediate calculation for p_3</td>
<td></td>
</tr>
<tr>
<td>X_1_2</td>
<td></td>
<td>intermediate calculation for p_1</td>
<td></td>
</tr>
<tr>
<td>X_2_2</td>
<td></td>
<td>intermediate calculation for p_1</td>
<td></td>
</tr>
<tr>
<td>X_2_2</td>
<td></td>
<td>intermediate calculation for p_1</td>
<td></td>
</tr>
<tr>
<td>X_2_2</td>
<td></td>
<td>calculated appearance vectors</td>
<td></td>
</tr>
<tr>
<td>XSTD_10_k</td>
<td>V</td>
<td>calculated nondimensional velocity V</td>
<td></td>
</tr>
<tr>
<td>HSTD</td>
<td></td>
<td>1 means calculated V is bar from input V</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 means calculated V is near input V</td>
<td></td>
</tr>
</tbody>
</table>
### Program Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>final calculated ( \bar{V} )</td>
<td>...</td>
</tr>
<tr>
<td>V</td>
<td>final calculated ( V )</td>
<td>...</td>
</tr>
<tr>
<td>P</td>
<td>peak incident pressure for a particular final calculated ( V ) and input values</td>
<td>Pa</td>
</tr>
<tr>
<td>I</td>
<td>specific impulse for particular final calculated ( V ) and input values</td>
<td>Pa-s</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>atmospheric pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>( c_a )</td>
<td>speed of sound</td>
<td>m/s</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>nondimensional pressure for particular calculated ( V ) and input parameters</td>
<td>...</td>
</tr>
<tr>
<td>( \bar{T} )</td>
<td>nondimensional impulse for particular calculated ( V ) and input parameters</td>
<td>...</td>
</tr>
<tr>
<td>PPARXO</td>
<td>subroutines variable used to calculate ( b )</td>
<td>...</td>
</tr>
<tr>
<td>PPARX</td>
<td>subroutines variable used to calculate ( b )</td>
<td>...</td>
</tr>
<tr>
<td>PFOX</td>
<td>subroutines variable used to calculate ( E, Q, \bar{P} )</td>
<td>...</td>
</tr>
<tr>
<td>PX</td>
<td>subroutines variable used to calculate ( E, Q, \bar{P} )</td>
<td>...</td>
</tr>
</tbody>
</table>

**Indicators on front of calculator:**

- \( NNDV = XXLI = \pm 1.0 \) indicates \( P = 0 \)
- \( NNDV = XXLI = \pm 2.0 \) indicates \( I_1 \) or \( I_2 \) for \( I_3 \)
- \( \pm 3.0 \) indicates input \( V \) is greater than maximum calculated \( \bar{V} \)
- \( \pm 4.0 \) indicates input \( V \) is less than minimum calculated \( \bar{V} \)

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PROGRAM UNLVL 47/V 45 OPT=1

CBAR=1.0/2.0*PRAR#2.0/(7.0*PHAR)
UBAR=SCRT(1.0/6.0*PHAR)/7.0
GO TO 30

305 CALL PAMUQP(PHAR, UBAR, ORAR, PRBRR, PHB, UB, QB, PB)
IF (UBAR, 0.0, 0.0) GO TO 30

307 PR=PHAR=PR(J)
GR=GR=PO(J)
UBAR=UB(J)

11=I/U
IF (G, 0.0, 0.0) GO TO 30B
T2=4.0*H/6+T1
GO TO 30A

309 T2=2.0*H/6+T1

30A CONTINUE
IF (t2, 0.0, t2) GO TO 32
C1=U1*2.0+2.0*EXP(*H2*2/2)
C2=C1*2.0*PI
C3=C1/2.0
C4=C1*2.0/2.0
C5=C1/2.0
EXP=C1*2.0*H/2

11M=I=1.0/2.0*H
IF (A, 0.0, 1.0) GO TO 10
IF (F1, 0.0, 2.0) GO TO 13
IF (NDV(3), GT, NDV(K)) GO TO 10
FIN=O

14 P(K)=P(K)
P(K)=P(K+1)+P(K+1)/2.0
GO TO 12

13 IF (NDV(3), GT, NDV(K)) GO TO 19
P(K+1)=P(K)
P(K)=P(K+1)+P(K+1)/2.0
GO TO 12

10 CONTINUE
101=1
NDV(K)=3.0
GO TO 16
17 NDV(3)=9.0
GO TO 16

30 NDV(K)=T(K)=1.0
GO TO 16

32 NDV(K)=T(K)=2.0
16 NDV(M, J, L)=NDV(K)

X(K,J, L)=T(K)
PP(M, J, L)=P(K)
PP(M, J, L)=P(J)
ADD(M, J, L)=ADD(J)
ADD(M, J, L)=ADD(J)
ADD(M, J, L)=ADD(J)
Y(M, J, L)=P(K)/PO(J)
PRINT OUT RESULTS
PPI=IMPULSE
PPI=PRESSURE
VX=VELOCITY OF APPURTEANCE
NNX=VX/VX2=1.0 INDICATES PP > GT
NNX=VX/VX2=2.0 INDICATES PP = GT
NNX=VX/VX2=3.0 INDICATES PP < GT
NNX=VX/VX2=4.0 INDICATES PP < GT

Y=NONDIMENSIONAL PRESSURE
Z=NONDIMENSIONAL IMPULSE

IF(G,E0,?) GO TO 400
PRINT 44X,K,M,C,D,A,TM
GO TO 401
400 PRINT 500X,K,M,C,D,A,TM
401 PRINT 501
DO 20 K=1,5
DO 20 J=1,5
20 PRINT 572X,K,M,J,L,A00(M,J,L),PPJ(M,J,L),VX3(M,J,L),

GO TO 300
300 CONTINUE
190 FORMAT(2F15.5)
191 FORMAT(2F15.5)
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193 FORMAT(2F15.5)
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367 Format(2F15.5)
368 Format(2F15.5)
369 Format(2F15.5)
SUBROUTINE PARAMB(E,PBAR,PB,BB)
DIMENSION PR(30), PB(30)
REAL PHAM
IF (PHAM LE PB(E1)) GO TO 20
B=0.0
GO TO 11
20 GO TO 10 IF=1,21
IF (PHAM LT PB(I)) GO TO 9,
K=1
PBAR=PBARS(PR(K)-PB(I))
PBAR=PBAR+PR(I)
IF (PR(K)=PHAM) GO TO 10
B=(BU(K)-BR(I))/(PH(K)-PH(I))+(PBAR-PB(I)) *BB(I)
GO TO 11
9 B=BB(I)
GO TO 11
10 CONTINUE
11 RETURN
END
SUBROUTINE PARUDP(PRAN, UBAR, QBAR, PARR, PHR, UB, OR, PBX)
DIMENSION PBX(5), UB(5), OR(5), PHR(5)
REAL PARR, UBAR, QBAR, PBX
IF (PHR .LE. PBX(5)) GO TO 20
UBAR = UBAR * PARR = 0.0, 0
GO TO 11
20 DO 10 10, 16, 5
IF (PRAN, EQ, PHR(1)) GO TO 9
K = 1
PBX = ARS(PB1(K) = PB2(1))
PRAN = PB2(PRAN = PB1(1))
IF (PBX .LT. PB2) GO TO 10
UBAR = UBAR(UB) / (PB2) * PB2(1) + (PHR = PRAN(1)) * UB(1)
QBAR = QBAR(UB) / (PB2) * PB2(1) + (PHR = PRAN(1)) * OR(1)
PBX = PBX(UB) / (PB2) * PB2(1) + (PHR = PRAN(1)) * PBX(1)
GO TO 11
9 UBAR = UB(1)
QBAR = QBAR(1)
PARR = PARR(1)
GO TO 11
10 CONTINUE
11 RETURN
END


APPENDIX IV, E

ANALYSES FOR FRAGMENT TRAJECTORIES

Analysis for Obtaining Fragment Range and Terminal Velocities for Disc-Shaped Fragments

<table>
<thead>
<tr>
<th>List of Variables</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>planform area - m^2</td>
</tr>
<tr>
<td>AP</td>
<td>projected area - m^2</td>
</tr>
<tr>
<td>AR</td>
<td>disc aspect ratio (diameter/thickness)</td>
</tr>
<tr>
<td>C</td>
<td>chord or width of rotor blade - m</td>
</tr>
<tr>
<td>C_D</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>C_L</td>
<td>lift coefficient</td>
</tr>
<tr>
<td>D_P</td>
<td>profile drag - N</td>
</tr>
<tr>
<td>I</td>
<td>mass moment of inertia of rotor</td>
</tr>
<tr>
<td>L</td>
<td>lift - N</td>
</tr>
<tr>
<td>M</td>
<td>mass - kg</td>
</tr>
<tr>
<td>Q</td>
<td>torque - N-m</td>
</tr>
<tr>
<td>R</td>
<td>radius of rotor</td>
</tr>
<tr>
<td>T</td>
<td>thrust - N</td>
</tr>
<tr>
<td>V_c</td>
<td>rotor blade velocity along rotational axis</td>
</tr>
<tr>
<td>V_i</td>
<td>initial velocity of fragment</td>
</tr>
<tr>
<td>V_t</td>
<td>tip velocity of rotor</td>
</tr>
<tr>
<td>V_in</td>
<td>induced velocity from thrust</td>
</tr>
<tr>
<td>X</td>
<td>range - m</td>
</tr>
</tbody>
</table>
The range of disc-shaped flying fragments from an explosion was determined from the fragment accelerations due to lift and drag forces.

The forces acting on the particle are as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>altitude - m</td>
</tr>
<tr>
<td>X</td>
<td>horizontal velocity</td>
</tr>
<tr>
<td>Y</td>
<td>vertical velocity</td>
</tr>
<tr>
<td>X</td>
<td>horizontal acceleration</td>
</tr>
<tr>
<td>Y</td>
<td>vertical acceleration</td>
</tr>
<tr>
<td>a</td>
<td>airfoil curve slope</td>
</tr>
<tr>
<td>b</td>
<td>number of rotor blades</td>
</tr>
<tr>
<td>d</td>
<td>disc diameter</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>r</td>
<td>disc radius</td>
</tr>
<tr>
<td>t</td>
<td>disc thickness</td>
</tr>
<tr>
<td>Δt</td>
<td>time increment</td>
</tr>
<tr>
<td>α</td>
<td>trajectory angle - rad</td>
</tr>
<tr>
<td>α_i</td>
<td>initial trajectory angle - rad</td>
</tr>
<tr>
<td>ρ</td>
<td>density of air - kg/m³</td>
</tr>
<tr>
<td>θ</td>
<td>angle of attack of disc - rad</td>
</tr>
<tr>
<td>ω</td>
<td>angular velocity - rad/sec</td>
</tr>
</tbody>
</table>
The acceleration in the Y direction is:

\[ \ddot{Y} = -g - \frac{AC_D \rho (\dot{X}^2 + \dot{Y}^2)}{2M} \sin \alpha + \frac{AC_L \rho (\dot{X}^2 + \dot{Y}^2)}{M} \cos \alpha \]  

(4E-1)

and for the X direction

\[ \ddot{X} = -\frac{AC_D \rho (\dot{X}^2 + \dot{Y}^2)}{2M} \cos \alpha - \frac{AC_L \rho (\dot{X}^2 + \dot{Y}^2)}{M} \sin \alpha \]  

(4E-2)

where

\[ A = \text{area of fragment} \]
\[ C_D = \text{drag coefficient} \]
\[ C_L = \text{lift coefficient} \]

At \( t = 0 \)

\[ \dot{X} = V_i \cos \alpha_i \]  

(4E-3)

\[ \dot{Y} = V_i \sin \alpha_i \]  

(4E-4)

where

\[ V_i = \text{initial velocity} \]
\[ \alpha_i = \text{initial trajectory angle} \]

It is assumed that the fragment is spinning about its Y axis. This motion gives the required stability for flight and allows the fragment to maintain a constant angle with respect to the relative wind.
A complete program (FRISB) was written to determine the particle trajectory from these equations. The second order differential equations were solved simultaneously using a fourth order Runge-Kutta method. These solutions gave the velocities, which were then numerically integrated to yield the distances.

The lift coefficient based on the planform area was determined from

\[ C_L = 1.82 \tan(\theta) \quad \text{(see Reference 1)} \quad (4E-5) \]

which is valid for a "thin" circular wing. It was assumed that the particle retained a constant angle of attack throughout the flight, which resulted in a constant lift coefficient. The maximum lift coefficient would be for an approximately a 10° angle of attack. Larger angles would result in stall or total loss of lift. Using \( \theta = 10^\circ \) in Equation (4E-5), \( C_L = 0.32 \).

The drag coefficient was chosen from Hoerner \(^2\) for a fragment with a rectangular cross section

For a "thin" disc or \( d/t \geq 3 \), the drag coefficient based on the projected area is constant at 0.85.

As can be seen by examining Equations (4E-1) through (4E-4), the trajectory of a fragment is governed by a number of dependent and independent variables. The dependent variables are displacements \( X \) and \( Y \); the independent variable list includes \( \theta_1, V_1, M, A, \) and \( AP \). To construct a useful system of graphs, a spectrum of values for the independent variables was input to FRISB, which gave the flight trajectories.

The input data were selected by assuming a disc aspect ratio

\[ AR = \frac{d}{t} \quad (4E-6) \]

and values of fragment mass and density. The mass is expressed as

\[ M = \pi r^2 t \rho = \frac{2 \pi r^3 \rho}{AR} \]
and

\[ r = \left( \frac{M \cdot AR}{2 \pi \rho} \right)^{1/3} \]  

(4E-8)

Now the planform or lift area is given as

\[ A = \pi \left( \frac{M \cdot AR}{2 \pi \rho} \right)^{2/3} \]

(4E-9)

The projected or drag area is

\[ AP = dt \]

(4E-10)

where \( d \) is calculated from Equation (4E-8), and \( t \) is determined from Equation (4E-6) for a given value of \( AR \). The independent variables used in various computer runs are shown in Table 4-1 of the text. The results of these runs are plotted in Figures 4-27 through 4-36 of the text. The graph shows the maximum range of the fragment versus the initial trajectory angle. For a given velocity and aspect ratio, a family of curves is shown for various values of \( M/A \). For several cases the lift force would act to pull the fragment up into a completely vertical flight. When this occurred, it was assumed that the fragment became unstable and fell straight to the ground. This phenomenon generally occurred for relatively high initial velocities and/or trajectory angles. A line has been drawn on Figures 4-28 through 4-35 to depict this occurrence. All points to the right of the line represent a "normal" flight, and those to the left are for the fragments that attained a vertical flight.

The value 300 m/s (938 ft/s) was used as a maximum initial velocity for the test cases. This velocity is very close to Mach 1 for STP conditions, and velocities above this would result in the need for a more complex aerodynamic analysis. The FRISB code would most likely predict larger ranges that would actually occur at higher velocities due to the increased drag at supersonic speeds. Therefore, for the purpose of estimation, the results from FRISB could be used for higher velocities.

Some fragments may not be disc or spherical shaped, but long and thin as a helicopter blade. The disc analysis is not valid for such a fragment because of the differing mechanisms of flight. The disc depends upon the forward motion to generate lift, where a whirling blade would generate most of its lift from the thrust due to the whirling motion, much the same as a helicopter rotor blade. Due to the possible need for such an analysis, the FRISB code was adapted with a subprogram to compute the trajectory of a fragment that flies, such as a helicopter rotor blade. The fragment geometry is as follows:
The induced velocity is obtained from (see Reference 3):

\[
V_{in} = V_t \left[ -\left( \frac{abc}{16\pi R} + \frac{V_c}{2V_t} \right) 
+ \sqrt{\left( \frac{abc}{16\pi R} + \frac{V_c}{2V_t} \right)^2 + \frac{abc\alpha}{8\pi R} - \frac{abcV_c}{8\pi RV_t}} \right]
\]  

(4E-11)

The thrust is determined by

\[
T = \frac{1}{2} C_L V_t^2 \rho \frac{bcR}{g}
\]  

(4E-12)

The acceleration in the \( y \) direction due to the thrust is

\[
Y = \sqrt{\left( \frac{T_R}{M} \cos \alpha - g \right)^2 + \left( \frac{T_R}{M} \sin \alpha \right)^2}^{1/2}
\]  

(4E-13)

The change in vertical velocity is

\[
4E-\]

\[
V_t = \text{velocity at which blade moves along vertical axis}
\]

\[
V_{in} = \text{velocity induced by thrust of blade}
\]

\[
\alpha = \text{pitch or trajectory angle}
\]

\[
V_R = \text{tip speed} = Ru
\]
\( \Delta V_c = \dot{V} \Delta t \) \hfill (4E-14)

The torque produced by the rotating blade is

\[
Q = T \left( \frac{V_c + V_{in}}{V_t} \right) + D_p \cdot R
\]

\( D_p \) is the profile drag, which is given by:

\[
D_p = \frac{1}{2} \rho C_D \frac{V^2 C_k}{g}
\] \hfill (4E-16)

The change in angular rotation is

\[
- \Delta \omega = \frac{Q \Delta t}{I}
\] \hfill (4E-17)

where \( I \) is the mass moment of inertia of the blade.

A fragment flying in this fashion would eventually lose its angular velocity due to drag and thus lose its lift. This procedure differs from that for a disc in that it assumes that the lift force comes from the rotation of the fragment and not the forward motion. The drag due to the forward motion of the fragment blade is considered when the computation is returned from the subroutine to the main program.

Input Data

1st data card
NN - number of test runs included in data check

2nd data card
N - number of differential equations to be solved by Runge-Kutta = 2
X - initial time = 0.0
H - time increment (0.1 sec is usually sufficient)
*AP - projected area and drag area - m^2
OPER = 1 for disc - 2 for rotor blade

*If rotor subprogram is used, this is read in as anything because it will not be used.
| 3rd data card | **RO** - density of air - \( \text{kg/m}^3 \)  |  
|              | **A** - lift area (planform) - \( \text{m}^2 \)  |  
|              | **CL** - lift coefficient  |  
|              | **CD** - drag coefficient  |  
|              | **AMASS** - segment mass - \( \text{kg} \)  |  
|              | **ALPHAO** - initial trajectory angle - \( \text{rad} \)  |  
|              | **VO** - initial fragment velocity - \( \text{m/s} \)  |  
|              | **G** - acceleration of gravity - \( \text{m/s}^2 \)  |  

| 4th data card | **EMLGA** - angular velocity - \( \text{rad/s} \)  |  
| (only if using rotor analysis) | **R** - radius of blade - \( \text{m} \)  |  
|              | **ASLIF** - lift curve slope of rotor airfoil - \( \text{2} \times \)  |  
|              | **C** - chord or width of rotor  |  
|              | **N** - number of rotor blades - 2  |  
|              | **EMI** - mass moment of inertia - \( \text{N}\cdot\text{s}^2\cdot\text{m} \)  |
PROGRAM FR1SB (INPUT,OUTPUT,TAPES ROUTPUT)

THIS PROGRAM SOLVES THE SECOND ORDER DIFFERENTIAL EQUATIONS THAT
GOVERN THE TRAJECTORY OF AN EXPLOSION FRAGMENT. THE SECOND ORDER
EQUATIONS ARE WRITTEN AS FIRST ORDER AND SOLVED BY RUNGE KUTTA
TECHNIQUES. THE SOLUTIONS ARE THEN INTEGRATED TO OBTAIN THE FRAGMENT
TRAJECTORY.

THE INPUT VARIABLES ARE: X-INITIAL TIME, N-NUMBER OF EQUATIONS,
R=AIR DENSITY, A=FRAGMENT PLANFORM AREA, CL-LIFT COEFFICIENT, CD-DRAG
COEFFICIENT, AMASS-PARTICLE MASS, ALPHAO-INITIAL TRAJECTORY ANGLE,
V=INITIAL VELOCITY, T=TIME OF ATTACH, AP-PROJECT
AREA, OPER-SIGNALS USE OF ROTOR SUMROUTINE, OPEN=1-DISC, OPER=2-ROTOR

DIMENSION Y(2,F(2),AA(2),BB(2),DSUMY(1000),DSUMX(1000),DXDOT(2).
INTEGER OPER
READ 25,NN
ON 99,90,NN
READ 99,90,NN,AMASS,ALPHA0,V0
IF (OPER=2) GO TO 314
READ 1000,EMEGA,H,ASLP,C,B,EMI
CONTINUE
PRINT 106
PRINT 112,CL,CD
PRINT 116
PRINT 117,AMASS
PRINT 118
PRINT 293,A
PRINT 114,ALPHA0
PRINT 115,VO
PRINT 116
PRINT 111,H
PRINT 110
PRINT 14
CONTINUE
LX=0
I=0
VC = 0.
ALPHA=ALPHA0.
Y(0)=0.
SUM=0.
SUM=X.
ICNT = 0
Y(0)=0.
Y(2)=0.
CONTINUE
ICNT = ICNT + 1
YDOT = Y(1) + Y(0)*SIN(ALPHA0)
IF (OPER=2) YDOT = YDOT + VC*COS(ALPHA0)
XDOT = Y(2) + VO*COS(ALPHA0)
IF (OPEN, E2, 2) XDOT = XDOT + VC * SIN(ALPHA)
DXDOT(1) = XDOT
DALLPHA = ATAN(YDOT/XDOT)
ALPHA = DALLPHA

IF (ALPHA, GT, 1.5) GO TO 398
IF (ALPHA, LT, -1.4) GO TO 398
IF (OPEN, E1, 2) GO TO 320
CALL ROUTER* (X, Y, RIG, CL, CD, ALPHAD, ASL, C, B, EMI, RO, VC, DVC, H, G, AMASS, A)
310 CONTINUE

320 CONTINUE
A(1) = Y(1)
H(1) = Y(2)
BETA = SQRT(PAP + CD*AMASS)
CIM = SQRT(PAP + CL*AMASS)
IF (PPE, E4, 2) GAM = 0.0

330 CONTINUE
YSPXS = Y(1)**2 + Y(2)**2 + 2*Y(1)*Y(2)*VC*COO(ALPHAD) + 2*Y(2)*VC*COO(ALPHAD)
IF (ALPHAD, EQ, 0.0) GO TO 22

340 CONTINUE
A3(1) = Y(1)
B3(2) = Y(2)
YDOTO = YDOT = YDOT + VC*COO(ALPHAD)

350 CONTINUE
S = SQRT(DT + DTF + DTV)
SUM = SUM + DYS
SUMX = SUMX + DISX
LL = LL + 1
LL = 0

360 CONTINUE
I = I + 1
SUMY(I) = SUMY
SUMX(I) = SUMX
SUM = SUM + YDOT + XDOT + ALPHAD

370 CONTINUE
IF (SUMY, LT, SUM) GO TO 993
IF (SUMX, LT, SUM) GO TO 994

380 CONTINUE
I = ALPHAD + (SUMX / SUMY)
SUMX = SUMX / I
SUMY = SUMY / I
CALL EXTRPL (X, Y, Z, XMIN, XMAX, YMIN, YMAX, SMIN)
PRINT 3000
3001 FORMAT(1H1)
CALL PLOT1 (2, 2)
CALL PLOT2 (XMAX, 0., YMAX, 0.)
CALL PLOT3 (1, 0, DSUMX, DSUMY, 1)

4E-10
CALL PLOTH(0.0,1M.)

999 CONTINUE
25 FORMAT(I10)
100 FORMAT(8F10.6)
106 FORMAT(11H1)

110 FORMAT(8X,TIME=,12X,DISPLACEMENT,6X,DISPLACEMENT,9X,VELOCITY,11X,VELOCITY,ICF,ALPHA)

111 FORMAT(10X,TIME INCREMENT = ,F5.2,SEC)

112 FORMAT(10X,LIFT COEFFICIENT = ,F10.5,DRAG COEFFICIENT = ,F10.5)

113 FORMAT(10X,INITIAL MASS = ,F10.6,KG)

114 FORMAT(10X,INITIAL TRAJECTORY ANGLE = ,F10.5,RADIANS)

115 FORMAT(10X,INITIAL VELOCITY = ,F10.3,SEC)

116 FORMAT(14H1)

263 FORMAT(10X,FRAGMENT AREA = ,F10.4,MMETER2)

1000 FORMAT(6F10.5)

99 FORMAT(I10,3F10.5)

STOP

. END

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE PLOT(NV, NH)
COMMON/GRAPH/POINT(91, 101)

C********** INPUT **********

NV = NO. OF VERTICAL GRID LINES (MUST EQ. 11, 63, OR 2)
NH = NO. OF HORIZONTAL GRID LINES (MUST EQ. 5, 3, OR 2)

C THIS IS A FORTRAN PROGRAM WHICH BUILDS A PLOT GRID FOR A PRINTER-PRINT
C THE FOLLOWING PROGRAMS HAVE TO BE USED IN CONJUNCTION WITH THIS PROGRAM
C A SUBROUTINE PLOT
C B SUBROUTINE PLOT
C C SUBROUTINE PLOT

C********** BLANK OUT THE PLOT AREA
VC = NV-1
HC = NH-1
C = 1

C BLANK OUT THE PLOT AREA
DO 1 I = 1, NV
DO 1 J = 1, NH
1 POINT(I, J) = BL

C DETERMINE TYPE OF GRID TO BE USED
JS = 100/(NV = 1)
IB = 100/(NH = 1)

C BUILD THE HORIZONTAL GRID LINES
DO 2 I = 1, NV
DO 2 J = 1, NH
2 POINT(I, J) = HC

C BUILD THE VERTICAL GRID LINES, AND PUT CHARACTER AT INTERSECTING
C POINTS
DO 3 J = 1, NV
DO 3 I = 1, NH
3 IF(POINT(I, J) .EQ. MC) GO TO 9
POINT(I, J) = VC
GO TO 7
9 POINT(I, J) = C
CONTINUE

C PUT CHARACTER AT EACH TENTH SPOT OF THE PERIMETER OF THE GRID
DO 5 J = 1, NV
DO 5 I = 1, NH
5 POINT(I, J) = C
CONTINUE

RETURN
END
FUNCTION RKLDOG(N,Y,X,H,N,NT)
52 D2 UCSD RKLDOG RUNGE-KUTTA-GILL LINEAR DIFFERENTIAL EQUATION SOLVER
52 D2 UCSD RKLDOG
52 MODIFIED MAY 1963 (Q REMOVED FROM CALLING SEQUENCE)
52 TEST OF ALCOL ALGORITHM
52 DIMENSION Y(10),Fx(10),Q(10)
52 REAL X,N=INTEGER,N,NT=COMMENT-BEGIN INTEGER I,J,L,REAL A
52 NT=NT+1
11 GO TO (1,2,3,4),NT
12 GO TO 3(NT)
11 DO 11 J=1,N
12 Q(J)=Y(J)
13 A=Y(1)/H/2.
14 GO TO 5.
2 A=2*K24321881.
15 GO TO 5.
3 A=1.7071067812.
16 X=X+H/2.
17 GO TO 5.
4 DO 4 J=1,N
19 NT=NT+1
41 RKLDOG=2.
20 GO TO 6.
5 GO 51 L=1,N
21 Y(L)=Y(L)+M(L)(L)+M(L)(L)
22 Q(L)=2.*A*(M(L)(L)+M(L)(L))
23 RKLDOG=1
51 CONTINUE
6 RETURN
END
SUBROUTINE ROTOR(EMEG, R, CD, ALPHAD, ASLP, D, B, EME, RD, VC, DVC, M, G, A)
Z2 = VC
T = EMEG*R
VA = VT/2.
DD = ASLP*B*C/(2*(1.1917*R))
V1 = VT*(1.1917*R) - VC/(2*(1.1917*R)) + (DD/16. + VC/(2*(1.1917*R))**2. + (DD*ALPH
1/16. - (DD*VC)/(B*,VT)**2)
T = .5*CL*V(A**2) + R**9*C + R/G
AC = (((*G/A*ASS)*COS(ALPHA) = G) + ((T*G/A*ASS) * SIN(ALPHA))**2
1)**.5
DP = .5*R*D*CD*V**2. + D32/G
O = (T*V + V)*/V4 + DP = R
OMEGA = G*V/EME
EMEG = EMEG = DMEGA
VC = ACC =
DVC = VC = Z2
PRINT 191, EMEG, VC, T, D
/4, 4)
DE=
3
SUBROUTINE PLOT(XMAX,XMIN,YMAX,YMIN)
COMMON/GRAPH/1/X(100),Y(100),XF,YF,YL,XPTS

******* INPUT ********

XMAX = XL = THE MAX. SCALE VALUE FOR THE X-AXIS
XMIN = XF = THE MIN. SCALE VALUE FOR THE X-AXIS
YMAX = YL = THE MAX. SCALE VALUE FOR THE Y-AXIS
YMIN = YF = THE MIN. SCALE VALUE FOR THE Y-AXIS

C. THIS IS A FORTRAN PROGRAM WHICH SETS THE PLOT_SCALE.

***************

XL = XMAX
YL = YMAX
XF = XMIN
YF = YMIN
RETURN
END
SUBROUTINE PLOT3(CHAR, XARAY, YARAY, N)
DIMENSION XARAY(200), YARAY(200)
COMMON/GRAPH1/X(100), Y(100), XF, XL, YF, YL, NPTS
COMMON/GRAPH2/POINT(101, 101), CM
C
C ******** INPUT ********
C
C CHAR = PLOT CHARACTER TO BE USED
C XARAY = VALUES OF X TO PLOT
C YARAY = VALUES OF Y TO PLOT
C N = NO. OF POINTS (OR DIMENSION OF XARAY AND YARAY) TO PLOT
C IF XARAY(1) .GT. 100 THE ARRAY WILL BE SCALCED TO .LE. 100
C
C THIS IS A FORTRAN PROGRAM WHICH SCALES ARRAYS TO BE PLOTTED
C
C **********************
C
CH = CHAR
ISTEP = N/100 + 1
DO 1 I=1, N, ISTEP
J = J + 1
X(J) = XARAY(I)
Y(J) = YARAY(I)
NPTS = J
CALL PLOT
RETURN
END
SUBROUTINE PLOT%(NCY, NCX, LABEL)
COMMON/GRAPH1/X(100),Y(100),XF,XF,YF,YL:N
COMMON/GRAPH2/POINT(41,101),CM
COMMON/GRAPH3/POINT(11),T(S).
DIMENSION LABEL(26).

C. ********** INPUT **********
C.
NCX = SWITCH INDICATING Y-AXIS CAPTION
C. = 0, NO CAPTION
C. = 1, UP TO 12 RCD CHARACTERS IN TITLE
NCY = SWITCH INDICATING X-AXIS CAPTION
C. = 0, NO CAPTION
C. = 1, UP TO 72 RCD CHARACTERS TO BE PRINTED AS A CAPTION
ABOVE THE PLOT, AND UP TO 72 RCD CHARACTERS TO BE PRINTED
ON A LINE BELOW THE PLOT
C.
THIS IS A FORTRAN PROGRAM WHICH PRINTS THE SELTS, CAPTIONS AND SCALES
C. OF THE DESIRED GRAPHS
C.

C. ********** INTERNAL PROCEDURES **********
C.
IF(NCY .EQ. 0) GO TO 1
PRINT 101, (LABEL(IX),IX=1,1)
101 FORMAT(1H1,/2X,4H***,12A1,IX,3H***,/,)
1 DO 10 I=1,15
10 IF(MOD(I,10) .EQ. 1) GO TO 91
PRINT 110, (POINT(I,J),J=1,101)
110 FORMAT(2X,10I1)
GO TO 90
90 IF(NCX .EQ. 0) GO TO 1
PRINT 120, (LABEL(IY),IY=1,2), (POINT(IY,J),J=1,101)
120 FORMAT(2X,2X,10I1)
GO TO 3
3 PRINT 110, (POINT(I,J),J=1,101)
3 DO 55 I=17,91
55 IF(MOD(I,10) .EQ. 1) GO TO 55
PRINT 110, (POINT(I,J),J=1,101)
GO TO 55
55 INDEX = 6 - (I - 11)/10
PRINT 120, (INDEX), (POINT(I,J),J=1,101)
GO TO 50
50 CONTINUE
PRINT 130, (S(I),I=1,11,2)
130 FORMAT(1H0,8X,E15.3,S(6X,E15.3))
IF(NCY .EQ. 0) GO TO 9
RETURN
END
SUBROUTINE EXTVAL(X,N,XMAX,XMIN)
DIMENSION X(N)

C FIND MAXIMUM AND MINIMUM VALUES IN THE ARRAY X WHICH HAS N ENTRIES
XMAX=X(1)
XMIN=X(1)
DO 200 I=2,N
   IF (XMAX.GT.X(I)) GO TO 100
   XMAX=X(I)
100 IF (XMIN.LT.X(I)) GO TO 200
   XMIN=X(I)
200 CONTINUE
XMAX=10**LOG10(XMAX)
XMIN=10**LOG10(XMIN)
RETURN
END
Computer Code TRAJE for Obtaining the Range and Terminal Velocities of Drag Fragments

The following computer code, TRAJE, was used to generate figures for fragment range versus initial trajectory angle for fragments which experience no lift forces. It can be used to determine the terminal velocity of fragments at their maximum range. The program is based on a perturbation technique described in Reference 4. The following table lists the program variables used in TRAJE that were not previously defined in FRISB.

<table>
<thead>
<tr>
<th>Program Variable</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM</td>
<td>input fragment mass</td>
<td>kg</td>
</tr>
<tr>
<td>AF</td>
<td>input fragment characteristic area</td>
<td>m²</td>
</tr>
<tr>
<td>CD</td>
<td>input fragment drag coefficient</td>
<td></td>
</tr>
<tr>
<td>NCD</td>
<td>input branch constant (for NCD &gt; 1 the velocity dependent drag coefficient is introduced)</td>
<td></td>
</tr>
<tr>
<td>DT</td>
<td>input time increment</td>
<td>s</td>
</tr>
<tr>
<td>TM</td>
<td>input maximum time limit</td>
<td>s</td>
</tr>
<tr>
<td>V₀</td>
<td>initial fragment velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>AL₀</td>
<td>initial trajectory angle</td>
<td>rad</td>
</tr>
<tr>
<td>BBT</td>
<td>drag constant</td>
<td>N/kg</td>
</tr>
<tr>
<td>TT</td>
<td>test time</td>
<td>s</td>
</tr>
<tr>
<td>XQ</td>
<td>fragment horizontal displacement</td>
<td>m</td>
</tr>
<tr>
<td>Y</td>
<td>fragment vertical displacement</td>
<td>m</td>
</tr>
<tr>
<td>VX</td>
<td>fragment instantaneous horizontal velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>VY</td>
<td>fragment instantaneous vertical velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>ALP</td>
<td>instantaneous trajectory angle</td>
<td>rad</td>
</tr>
<tr>
<td>V</td>
<td>total fragment velocity</td>
<td>m/s</td>
</tr>
</tbody>
</table>
MITE (1,10) T, X, Y, V, V, ALP
GO TO 100
100 MITE (1,10) T, X, Y, V, V, ALP
C0M1NUE
IF (T < 100), 100, 100
CALC, "&", INPUT PARAMETER
GO TO 100
IF (T = 100), 100, 100
IF (T < 100) 100, 100
GO TO 100
IF (T < 100) 100, 100
GO TO 100
IF (T < 100) 100, 100
GO TO 100
IF (T < 100) 100, 100
GO TO 100
NET (T, ALPHA, DATA) / GRAPH (P2, P3)
CALC, CH0O, FOR WH, T
CALL D1L (DTSALPHA, T), DARY
C0M1NUE
CONTINUE
CONTINUE
CONTINUE
END
SUBROUTINE DYL (DT, ALP, VI, DX, DY, VX, VY, DA)
REAL LNP, 
COMMON/VAR/NET, AF, EM
I=1
FORMAT (10,25)
S
RED &V

DEFINITION OF LOG FUNC.
IF (U, 1, U**2) 201, 201, 201
201 LNP = (1.0*(U**2)) (((1.0/3.0)*U)**2) - (U/2.0)*U
GO TO 202
202 S = (1.0*U)
END
REFERENCES


APPENDIX IV. F

STATISTICAL FITTING TO FRAGMENT DATA

4F-1 Derivation of Figures 4-46 through 4-49

From the initial fragment velocity data on grouped tests by propellant and configuration given on page 102 of Reference 1, the following estimated means and standard deviations for the log-normal (to the base e) distributions are shown in Table 4F-1.

<table>
<thead>
<tr>
<th></th>
<th>CEM</th>
<th>CEM</th>
<th>CBGS</th>
<th>CBGS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LO₂/LH₂</td>
<td>LO₂/RP-1</td>
<td>LO₂/LH₂</td>
<td>LO₂/RP-1</td>
</tr>
<tr>
<td>Estimated Mean</td>
<td>5.2759</td>
<td>5.5249</td>
<td>4.0410</td>
<td>4.7739</td>
</tr>
<tr>
<td>Estimated Std. Dev.</td>
<td>0.9875</td>
<td>0.5249</td>
<td>0.7715</td>
<td>0.6387</td>
</tr>
</tbody>
</table>

These distributions are plotted on Figures 4-46 through 4-49, respectively. The goodness of fit statistics (W) are given in Reference 1.

4F-2 Derivation of Figure 4-50

The data from page 86 of Reference 1 for fragment initial velocity measurements were used to determine the regression line (least squares fit) shown of initial velocity, \( U \), in Figure 4-50. The 95th percentile \( U_{95} \) estimate was constructed by taking the estimate for the standard deviation \( \sigma \) from the CEM \( LO₂ / LH₂ \) group in Section 4F-1 above, establishing a point 1.65 \( \sigma \) above the point \( U = 73.96 \), \( Y = 1\% \) by the formula:

\[
U_{95} = e^{\ln 73.96 + 1.65 (0.9875)}
\]

\[
= 377.24 \text{ meters} (1237.7 \text{ ft})
\]

To determine the geometric mean \( u \) meters/second, raise \( e \) to the power shown.
A line was then extended from the point \( U = 377.24, \ Y = 1 \), parallel to the regression line for \( U \) on \( Y \) (\( U = 73.96, Y = 0.4296 \)).

The values for nine tests from Reference 1 are presented in Table 4F-2.

**TABLE 4F-2. MEAN AND MAXIMUM FRAGMENT INITIAL VELOCITY FOR \( \text{LO}_2/\text{LH}_2 \) CDM**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Yield (%)</th>
<th>Mean Velocity m/s</th>
<th>Maximum Velocity Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>053</td>
<td>4</td>
<td>110.3</td>
<td>240</td>
</tr>
<tr>
<td>001</td>
<td>29</td>
<td>457.2</td>
<td>1100</td>
</tr>
<tr>
<td>118</td>
<td>20</td>
<td>215.4</td>
<td>340</td>
</tr>
<tr>
<td>199</td>
<td>8</td>
<td>201.2</td>
<td>455</td>
</tr>
<tr>
<td>200</td>
<td>17</td>
<td>268.2</td>
<td>504</td>
</tr>
<tr>
<td>210</td>
<td>7</td>
<td>198.1</td>
<td>462</td>
</tr>
<tr>
<td>212</td>
<td>27</td>
<td>240.8</td>
<td>383</td>
</tr>
<tr>
<td>213</td>
<td>35</td>
<td>301.8</td>
<td>459</td>
</tr>
<tr>
<td>265</td>
<td>10</td>
<td>210.3</td>
<td>441</td>
</tr>
</tbody>
</table>

Figure 4F-1 shows the regression line, the estimated 95th percentile line, and the mean and maximum observed velocity points.

**4F-3 Rationale for Averaging Fragment Mass Distribution for Events 3, 4, and 5**

The estimated means and standard deviations (log-normal to the base e) for the fragment mass for the five events from Reference 1 are shown in Table 4F-3.

The estimated means and standard deviations of events 3, 4, and 5 were fairly close to each other, and the events were of the same type. Therefore, it seemed reasonable to apply a "t" test (see Reference 8) for significant difference in means. This test was applied to events 3 and 4 since the difference in estimated means was the greatest for any pair of estimated means from events 3, 4, and 5.
Figure 4F-1. Initial Velocity Vs Yield, CBM, LO₂/LH₂
TABLE 4F-3. ESTIMATED MEAN AND STANDARD DEVIATION FOR FRAGMENT MASS DISTRIBUTION (LOG NORMAL TO BASE e) FOR FIVE EVENTS

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Fragment Mass Distribution</th>
<th>Mean</th>
<th>Standard Dev.</th>
<th>Percent Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>7.7226</td>
<td>0.8020</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>9.3940</td>
<td>1.142</td>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>9.7761</td>
<td>1.178</td>
<td>23.0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>10.1488</td>
<td>1.0367</td>
<td>24.4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>10.0522</td>
<td>0.8838</td>
<td>62.6</td>
</tr>
</tbody>
</table>

The "t" test is applied using the following steps:

1. The pooled estimate for the standard deviation (Sp) is calculated by:
   \[ Sp = \sqrt{\frac{S_1^2 + S_2^2}{2}} \]

2. The "t" statistic is calculated by
   \[ t = \frac{(\bar{W}_1 - \bar{W}_2)}{\left(S_p\right) / \sqrt{n}} \]

3. The "t" statistic is then compared to a value in the t distribution table, \( t_\alpha;2(n-1) \), where \( \alpha \) is the probability of type 1 error or risk of accepting the hypothesis that there is no significant difference in means where there is, and \( 2(n-1) \) is a parameter in the t distribution (degree of freedom) used to find the tabled value.

4. If the calculated value of \( t \) (from step 2) is between \( \pm t_\alpha;2(n-1) \) (the table value), we accept the hypothesis.

\* To determine geometric mean weight, raise e to power shown in table.
that there is no significant difference in the means. If not, we reject the hypothesis, and conclude there is a significant difference in the means.

Choosing an $\alpha$ of 0.20, the table value of $t$ for 16 degrees of freedom is $\pm 1.337$.

The formula for calculating "t" can be further simplified to:

$$t = \frac{\sqrt{n} (\bar{W}_1 - \bar{W}_2)}{S_1^2 + S_2^2}$$ (4F-1)

Using the above formula, the calculated value of $t$ is:

$$t = \frac{3 (10.1488 - 9.7761)}{1.0367^2 + 1.1787^2} = 0.712$$

Since the calculated value of $t$ of 0.712 lies between the table value $\pm 1.337$, we accept the hypothesis that there is no significant difference in means and can use a single distribution for the fragment mass distribution for events 3, 4, and 5. Averaging the means of events 3, 4, and 5 yielded an average value of 9.9924, and for the standard deviation an average value of 1.0331. These values were used to construct Figure 4-53.

4F-4 Fragment Mass Distributions For Gas Vessel Bursts

The fragment mass data from each of the tanks were sorted in ascending order; the values for the mass for the 10th to the 90th percentiles in 10% steps were identified. Table 4F-4 is a listing of these values.

Figures 4F-2 through 4F-5 are plots of the percentile points on log normal probability paper for tanks A, B, D and E, respectively.

Table 4F-5 is a listing of the estimated means and standard deviations for the log normal (to the base e) distributions.

A "W" statistic (see Reference 9) for goodness of fit was calculated for each of the distributions. The approximate probability of obtaining the calculated test statistic, given that the chosen distribution is correct, was then determined. The results are shown in Table 4F-6.
TABLE 4F-4. PERCENTILES FOR PLOTTING FRAGMENT MASSES OF TANKS A, B, D, AND E

<table>
<thead>
<tr>
<th>Percent</th>
<th>Tank A</th>
<th>Tank B</th>
<th>Tank D</th>
<th>Tank E</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.2</td>
<td>1.1</td>
<td>85</td>
<td>61</td>
</tr>
<tr>
<td>20</td>
<td>3.7</td>
<td>4.6</td>
<td>199</td>
<td>199</td>
</tr>
<tr>
<td>30</td>
<td>5.2</td>
<td>6.6</td>
<td>454</td>
<td>454</td>
</tr>
<tr>
<td>40</td>
<td>11.0</td>
<td>24.0</td>
<td>624</td>
<td>738</td>
</tr>
<tr>
<td>50</td>
<td>15.0</td>
<td>31.0</td>
<td>1731</td>
<td>1277</td>
</tr>
<tr>
<td>60</td>
<td>42.0</td>
<td>38.0</td>
<td>2015</td>
<td>1617</td>
</tr>
<tr>
<td>70</td>
<td>53.0</td>
<td>63.0</td>
<td>2156</td>
<td>1873</td>
</tr>
<tr>
<td>80</td>
<td>96.0</td>
<td>92.0</td>
<td>2270</td>
<td>2270</td>
</tr>
<tr>
<td>90</td>
<td>145.0</td>
<td>125.0</td>
<td>2639</td>
<td>3036</td>
</tr>
</tbody>
</table>

TABLE 4F-5. LISTING OF ESTIMATED MEANS AND STANDARD DEVIATIONS FOR LOG-NORMAL DISTRIBUTION (TO THE BASE e) FOR GAS VESSELS

<table>
<thead>
<tr>
<th>Tank No.</th>
<th>Estimated Mean</th>
<th>Estimated Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.9730</td>
<td>1.4821</td>
</tr>
<tr>
<td>B</td>
<td>3.0327</td>
<td>1.6829</td>
</tr>
<tr>
<td>D</td>
<td>6.5698</td>
<td>1.8080</td>
</tr>
<tr>
<td>E</td>
<td>6.6782</td>
<td>1.5815</td>
</tr>
</tbody>
</table>
Figure 4F-2. Fragment Mass Distribution, Tank A

(1b_m = kg \times 2.205)
Figure 4F-3. Fragment Mass Distribution, Tank B

\( \text{(lb}_m = \text{kg} \times 2.205) \)
Figure 4F-4. Fragment Mass Distribution, Tank D

\(1 \text{ lb}_m = \text{kg} \times 2.205\)
Figure 43-5. Fragment Mass Distribution, Tank E.
TABLE 4F-6. SUMMARY OF "W" TEST ON NORMALITY FOR
FRAGMENT MASS DISTRIBUTION FOR
TANKS A, B, D, AND E

<table>
<thead>
<tr>
<th>Tank No.</th>
<th>&quot;W&quot;</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.948</td>
<td>.69</td>
</tr>
<tr>
<td>B</td>
<td>.929</td>
<td>.43</td>
</tr>
<tr>
<td>D</td>
<td>.851</td>
<td>.08</td>
</tr>
<tr>
<td>E</td>
<td>.956</td>
<td>.34</td>
</tr>
</tbody>
</table>

As it is customary to consider values exceeding 2 to 10% as adequate grounds for not rejecting the hypothesis that the data belong to the chosen distribution, the fits for tanks A, B, and E are more than adequate, and for D somewhat questionable.

Except for Tank D, the other values compare favorably with those obtained for the log-normal fragment distribution for Events 1, 2, 3, 4 and 5 for the propellants (Reference 1).

4F-5 Rationale For Averaging Fragment Mass Distributions For
Tanks A and B and Tanks D and E

A "t" test for significant difference in means was made for each of the pairs of tanks, with the following results:

For tanks A and B, the calculated value of "t" was 0.314 versus the table value of ± 1.337.

For tanks D and E, the calculated value of "t" was 1.354 versus the table value of ± 1.337.

Thus, no significant difference in means for either pair was found, and a single distribution for each pair was derived by averaging the respective pair of means and standard deviations. Table 4F-7 presents the results, and the distributions are charted in Figures 4-54 and 4-55.
TABLE 4F-7. ESTIMATED MEAN, AND STANDARD DEVIATION (TO THE BASE 10) FOR TANKS A AND B, AND D AND E

<table>
<thead>
<tr>
<th>Tanks</th>
<th>Estimated Mean</th>
<th>Estimated Standard Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>2.7984</td>
<td>1.583</td>
</tr>
<tr>
<td>D and E</td>
<td>6.6240</td>
<td>1.695</td>
</tr>
</tbody>
</table>

4F-6 Derivation of Figure 4F-5, Fragment Distance Versus Percent Yield For Propellant Explosions

Table 4F-8 is taken from Reference 1, with some minor corrections. For each of the five events, a 95% upper confidence limit was put on the estimated mean (M) and a 90% upper confidence limit was established for the estimated standard deviation (S), using methods outlined in Reference 8.

The confidence limit on the mean was calculated using the following formula:

\[ CL = M \pm \frac{S}{\sqrt{n}} \cdot t(n; 95) \]

where \( n \) is the number of fragments and \( t(n; 95) \) is the value of the t distribution with \( n \) degrees of freedom at the 95th percentile.

The confidence interval for the standard deviation was calculated using the following formula:

\[ CL = \left[ \frac{\chi^2 \left( \sum X_i^2 \right) - \left( \sum X_i \right)^2}{n} } {\chi^2 (n - 1); 90} \right]^{-1/2} \]

where \( X_i \) is the distance of the ith fragment, \( n \) is the number of fragments, and \( \chi^2 (n - 1); 90 \) is the value of a chi square distribution with \( n - 1 \) degrees of freedom at the 90th percentile.

Then, using the new upper confidence level values of M and S, the \( R_{95} \) in which 95% of the fragments should fall was calculated as follows:
### TABLE 4F-8. CONFIDENCE LIMITS ON MEAN, STANDARD DEVIATION, AND DISTANCE CONTAINING 95% OF FRAGMENTS

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Sample Size</th>
<th>Yield</th>
<th>TNT</th>
<th>Normal Distribution</th>
<th>Distance (m)</th>
<th>Max Dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>%</td>
<td></td>
<td></td>
<td>R</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>34</td>
<td>5</td>
<td>1.55 x 10(^2)</td>
<td>136.3</td>
<td>58.13</td>
<td>3.20</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>1.1</td>
<td>1.08 x 10(^3)</td>
<td>102.7</td>
<td>61.87</td>
<td>10.05</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
<td>23</td>
<td>4.03 x 10(^2)</td>
<td>253.0</td>
<td>160.77</td>
<td>16.55</td>
</tr>
<tr>
<td>4</td>
<td>86</td>
<td>24.4</td>
<td>4.28 x 10(^2)</td>
<td>225.6</td>
<td>104.24</td>
<td>11.23</td>
</tr>
<tr>
<td>5</td>
<td>81</td>
<td>62.6</td>
<td>1.007 x 10(^2)</td>
<td>300.5</td>
<td>174.96</td>
<td>31.44</td>
</tr>
</tbody>
</table>
The interval from the mean (M) to $R_{95}$ is indicated for each event on Figure 4F-6 by a bar.

A line was then drawn parallel to the regression line, and just touching the longest bar. Thus, the distances read from this line could be expected to encompass at least 95% of the fragments resulting from a given yield.

**4F-7 Derivation of Simulated Fragment Range Distribution for Gas Vessel Bursts**

For each tank, the fragments were divided into classes by fragment area. The computer program FRISB was then exercised taking into account fragment shape (from drag) for initial angles of 15°, 30°, 45°, 60°, and 75° to determine a range for each angle for each class of fragment. Table 4F-9 presents the results of the range simulation.

In the table the fragments have been divided into groups or classes with the average planform area $\overline{AL}$ and the average mass $M$ of each class listed. The area data were obtained by measurement from the photographs of Reference 2. Mass data were given in the reference for each fragment, $\overline{AD}$ is the average drag area for each class of fragment; this area is calculated on the basis of the thickness of the fragments and a characteristic width dimension equal to the square root of the fragment planform area. The maximum range of the classes of fragments for various values of initial trajectory angle ($\alpha = 5^\circ$, $\alpha = 3^\circ - 15^\circ$) was calculated using code FRISB assuming the average characteristics for the fragments in each class, and that the initial velocity was the maximum initial velocity for the fragments measured for each tank.

The fragment ranges were ordered, and the percentiles were determined and are shown in Table 4F-10. Then the percentiles were plotted on normal probability paper and are shown in Figures 4F-7 through 4F-10. The estimates for the mean and standard deviation for each distribution were then calculated and are shown in Table 4F-11.

A "W" goodness of fit statistic was calculated for each distribution and the probability of obtaining the calculated test statistic value, given that the chosen distribution is correct was then determined. The results are shown in Table 4F-12.
AT LEAST 95% OF FRAGMENTS FOR YIELD \( Y \), ARE EXPECTED TO FALL WITHIN THIS DISTANCE

- \( R_{95} = 235 \gamma^{0.2775} \)
- \( R = 95.93 \gamma^{0.2775} \)

Indicates interval from mean distance (m) to 95th percentile of range (R_{95})

Indicates mean distance and event number

\( (\pi = m \times 3.281) \)

Figure 4F-6. Mean Distance vs Yield with Estimated Range Containing 95% of the Fragments (R_{95})
### TABLE 4E-9. COMPUTER SIMULATED FRAGMENT RANGE FOR GAS VESSEL BURSTS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Mass of Detonant (kg)</th>
<th>Mass of Detonant (lbm)</th>
<th>Vessel Pressure (kPa)</th>
<th>Vessel Pressure (MPa)</th>
<th>Range (m)</th>
<th>Range (ft)</th>
</tr>
</thead>
</table>

- (continued)
### TABLE 4F-10. PERCENTILES FOR SIMULATED RANGES FOR GAS VESSEL BURSTS

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Tank A</th>
<th>Tank B</th>
<th>Tank D</th>
<th>Tank E</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.566</td>
<td>0.532</td>
<td>6.4</td>
<td>6.9</td>
</tr>
<tr>
<td>20</td>
<td>0.806</td>
<td>0.747</td>
<td>11.9</td>
<td>20.8</td>
</tr>
<tr>
<td>30</td>
<td>2.330</td>
<td>1.98</td>
<td>23.5</td>
<td>25.4</td>
</tr>
<tr>
<td>40</td>
<td>2.947</td>
<td>2.95</td>
<td>28.7</td>
<td>45.4</td>
</tr>
<tr>
<td>50</td>
<td>5.107</td>
<td>4.33</td>
<td>43.8</td>
<td>56.0</td>
</tr>
<tr>
<td>60</td>
<td>6.473</td>
<td>6.38</td>
<td>63.8</td>
<td>77.4</td>
</tr>
<tr>
<td>70</td>
<td>8.679</td>
<td>7.39</td>
<td>66.5</td>
<td>96.3</td>
</tr>
<tr>
<td>80</td>
<td>11.120</td>
<td>10.25</td>
<td>108.4</td>
<td>114.6</td>
</tr>
<tr>
<td>90</td>
<td>12.969</td>
<td>11.00</td>
<td>131.6</td>
<td>143.3</td>
</tr>
</tbody>
</table>

### TABLE 4F-11. ESTIMATES FOR MEAN AND STANDARD DEVIATION FOR SIMULATED RANGE DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Tank No.</th>
<th>Estimate for Mean (M)</th>
<th>Estimate for Standard Deviation (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.67</td>
<td>6.72</td>
</tr>
<tr>
<td>B</td>
<td>5.06</td>
<td>3.92</td>
</tr>
<tr>
<td>D</td>
<td>53.05</td>
<td>44.05</td>
</tr>
<tr>
<td>E</td>
<td>65.1</td>
<td>46.20</td>
</tr>
</tbody>
</table>
Figure 4F-7. Simulated Range Distribution for Tank A

(1 ft = m x 0.3048)
Figure 4F-7. Simulated Range for Tank D

RANGE R, meters

(\text{m} = \text{m} \times 1.281)

PERCENTAGE

0 10 20 30 40 50 60 70 80 90 100

140
Figure 4F-10. Simulated Range for Tank E
TABLE 4F-12. SUMMARY OF "W" TEST ON NORMALITY FOR SIMULATED FRAGMENT RANGE DISTRIBUTION FOR GAS VESSEL BURSTS

<table>
<thead>
<tr>
<th>Tank No.</th>
<th>&quot;W&quot;</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.929</td>
<td>.46</td>
</tr>
<tr>
<td>B</td>
<td>.921</td>
<td>.37</td>
</tr>
<tr>
<td>D</td>
<td>.933</td>
<td>.47</td>
</tr>
<tr>
<td>E</td>
<td>.956</td>
<td>.74</td>
</tr>
</tbody>
</table>

As shown from the probability column of Table 4F-12, the normal distributions are fairly good fits to the data.

4F-8 Rationale For Combining Simulated Range Distribution For Tanks A and B and for Tanks D and E

A "t" test for significant differences in means was made for each of the pairs of tanks, with the following results.

For tanks A and B, the calculated value of "t" was 0.235 and for tanks D and E was 0.558 versus the table value of ± 1.337. Thus, no significant difference in means for either pair was found, and a single distribution for the simulated range for each pair was derived by averaging the respective pair of means and standard deviations. Table 4F-13 presents the results, and the distributions are charted in Figures 4-58 and 4-59.

TABLE 4F-13. ESTIMATED MEAN AND STANDARD DEVIATION FOR SIMULATED RANGE DISTRIBUTION FOR TANKS A AND B, AND TANKS D AND E

<table>
<thead>
<tr>
<th>Tanks</th>
<th>Estimated Mean</th>
<th>Estimated Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>5.4</td>
<td>5.5</td>
</tr>
<tr>
<td>D and E</td>
<td>59.2</td>
<td>45.1</td>
</tr>
</tbody>
</table>
LIST OF REFERENCES


CHAPTER V

EFFECTS OF FRAGMENTS

5-1 Damage Estimates to Structures and Facilities

5-1.1 General

One can see from Chapter IV that there are wide variations in characteristics of fragments generated during accidental explosions of the types covered in this handbook. Large sections of pressure vessels, and appurtenances accelerated by the explosions, can be quite massive (over 500 kg or 1000 lb), while some fragments thrown to large distances are quite light (less than 1 g or 0.002 lb). Impact velocities can range from a few meters per second to several hundred meters per second. Also, in Chapter IV, methods are given for estimating the probability that fragments of given mass and impact velocity will strike a structure located a specified distance from an accident. An important characteristic of fragments from these accidents is that they are of low velocity, and large average mass, compared to fragments from munitions which are intended to cause damage by penetration or perforation.

Conventional structures which can be damaged by fragments include frame or masonry residences, light to heavy industrial buildings, office building, public buildings, mobile homes, cars, and others too numerous to name. Damage can be superficial, such as denting of metal panels or breakage of panes of glass. But, massive fragments can cause more extensive damage such as perforation of wooden roofs, severe crushing of mobile homes or cars, etc. Most of the fragments will be nonpenetrating and will cause damage by imparting impulsive loads during impact. Methods similar to those used to establish threshold damage levels under blast loadings can also be used to establish thresholds for impact damage by fragments, i.e., lower limits for superficial damage. The methods will be somewhat simpler because the impacts will almost certainly be of short enough duration to be purely impulsive for almost any "target" structure or structural component. Impact conditions with large fragments which can be certain to cause significant structural damage can probably also be established by equating kinetic energy in the fragment to energy absorption capability for typical roof panels, roof supporting beams, etc.

Launch facilities for liquid-propellant rockets present some special "targets" to fragments from accidental explosions which can be especially susceptible to perforation damage. These are thin-walled tanks for storage or transport of energetic propellant liquids such as
LH₂, RP-1, or LO₂. Fragment impact conditions which would cause perforation of such tanks can be estimated from ballistic-perforation formulas for munitions fragments and from hailstone impact data. These formulas will be presented and discussed.

5.1-2 Impact of Fragments on Thin Metal Targets

The following methods can be used for fragment impact on metal sheets or plates. The details of the formulation of these methods are presented in Appendix 5A.

The $V_{50}$ limit velocity is defined as the velocity at which a projectile will have a 50% chance of penetrating a given target. Knowing the properties of the projectile (fragment) and the target, $V_{50}$ can be obtained from Figure 5-1.

In this figure, $a$ is the radius of the fragment (assuming a spherical shape), $h$ is the thickness of the target, $\rho_f$ is the density of the fragment (or projectile), $\rho_t$ is the density of the target material, and $\tau_t$ is the yield stress of the target material.

The solid line in Figure 5-1 gives the relationship between limit velocity and target thickness. As the graph shows, there is uncertainty in this relation. For hard fragments which are less likely to deform, a lower nondimensional limit velocity (more conservative) should be chosen. For softer fragments, a higher limit velocity can be used. At this time, it is not known whether this relationship holds for values of $h/a$ greater than about 2.2.

This method is good for the impact of a fragment with its velocity normal to the target surface. For oblique impacts, the normal component of the velocity should be used. According to one report, (1) for oblique impacts, the penetration velocity is minimum at an angle of 30° from the normal direction. The difference between the penetration velocities at 0° and at 30° may be as great as 20%. Therefore, if oblique impact is expected, the penetration velocity obtained by use of Figure 5-1 should be multiplied by 0.8.

For fragment velocities less than $V_{50}$, the permanent deflection at the impact point on the target can be determined. Figure 5-2 is a graph of nondimensional deflection versus nondimensional velocity.

For given fragment properties, a given target, and a given normal component of fragment velocity, $h$ can be obtained. Of course, for very low fragment velocities, there is no permanent deflection.
This method was developed for impacts not very close to the edge of a sheet or plate. For fragment impact near the edge of an unsupported or simply-supported sheet or plate, the deflection may be twice the deflection that would be otherwise expected.

This analysis has been formulated for spherical fragments. To apply this to fragments of other shapes, let

\[ a = \left( \frac{m}{4\pi \rho} \right)^{1/3} \]

where \( m \) is the mass of the fragment. More research must be done to determine other effects of fragment shape.

Table 5-1 is a list of the important properties (density and yield stress) of a few selected fragment and target materials.

### TABLE 5-1. MATERIAL PROPERTIES
(References 2, 3)

<table>
<thead>
<tr>
<th>density ( \rho )</th>
<th>yield stress ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{kg}{m^3} )</td>
<td>( \frac{lb/ft^3}{Pa} )</td>
</tr>
<tr>
<td>Steel</td>
<td></td>
</tr>
<tr>
<td>1015</td>
<td>7850</td>
</tr>
<tr>
<td>1018</td>
<td>1015</td>
</tr>
<tr>
<td>1020 (large grained)</td>
<td>3.66 ( \times 10^8 )</td>
</tr>
<tr>
<td>1020 (sheet)</td>
<td>4.42 ( \times 10^8 )</td>
</tr>
<tr>
<td>3.11 ( \times 10^8 )</td>
<td>45,000</td>
</tr>
<tr>
<td>Aluminum Alloys (sheet)</td>
<td>2770</td>
</tr>
<tr>
<td>2024-O</td>
<td>8.85 ( \times 10^7 )</td>
</tr>
<tr>
<td>2024-T3</td>
<td>3.66 ( \times 10^8 )</td>
</tr>
<tr>
<td>2024-T4</td>
<td>3.66 ( \times 10^8 )</td>
</tr>
<tr>
<td>Titanium Alloy</td>
<td>4520</td>
</tr>
<tr>
<td>6Al4V</td>
<td>1.11 ( \times 10^9 )</td>
</tr>
</tbody>
</table>
Example 1:

Given fragment radius $a = 0.020 \text{ m} (0.066 \text{ ft})$ and density $\rho = 7000 \text{ kg/m}^3 (440 \text{ lbm/ft}^3)$, target density $\rho_t = 7000 \text{ kg/m}^3 (440 \text{ lbm/ft}^3)$ and yield stress $\sigma_y = 4.4 \times 10^8 \text{ Pa} (6.38 \times 10^4 \text{ psi})$ and fragment velocity $V = 200 \text{ m/s (660 ft/sec)}$, find the minimum target thickness $h$ to prevent penetration. Solution: Nondimensional velocity is computed, and $h/a$ is read from Figure 5-1:

$$
\frac{c_p V_{50}}{\sqrt{c_t \sigma_y}} = \frac{(7000 \text{ kg/m}^3)(200 \text{ m/s})}{\left[(4.4 \times 10^8 \text{ N/m}^2)(7000 \text{ kg/m}^3)\right]^{1/2}} = 0.798
$$

Then from Figure 5-1, $h/a = 0.202$. For the given $a$, $h = 0.0040 \text{ m} (0.013 \text{ ft})$.

Example 2:

Given fragment radius $a = 0.010 \text{ m} (0.033 \text{ ft})$ and density $\rho = 7000 \text{ kg/m}^3 (440 \text{ lbm/ft}^3)$, target density $\rho_t = 7000 \text{ kg/m}^3 (440 \text{ lbm/ft}^3)$, yield stress $\sigma_y = 4.4 \times 10^8 \text{ Pa} (6.38 \times 10^4 \text{ psi})$ and thickness $h = 0.0010 \text{ m} (0.0033 \text{ ft})$, find $V_{50}$. Solution: $h/a$ is computed, and the nondimensional velocity is read from Figure 5-1:

$$
h/a = \frac{0.0010 \text{ m}}{0.010 \text{ m}} = 0.10
$$

Then from Figure 5-1, $\frac{c_p V_{50}}{\sqrt{c_t \sigma_y}} = 0.43$. For the given properties, $V_{50} = 110 \text{ m/s (360 ft/s)}$.

Example 3:

Given the same fragment and target properties as in Example 2, and a fragment velocity less than the limit velocity $V = 52.0 \text{ m/s (171 ft/s)}$, find the deflection at the impact point on the target:

$$
\frac{c_p V}{\sqrt{c_t \sigma_y}} = \frac{(7000 \text{ kg/m}^3)(52.0 \text{ m/s})}{\left[(4.4 \times 10^8 \text{ N/m}^2)(7000 \text{ kg/m}^3)\right]^{1/2}} = 0.207
$$
From Figure 5-2, \( \frac{Ah}{a^2} = 0.068 \). Then for example, \( a = 0.0068 \) m (0.022 ft).

5-1.3 Impact of Fragments on Roofing Materials

Nearly any impact of a fragment upon the roof of a building will cause at least some superficial damage. Damage which only affects the appearance but which does not interfere with the performance of the roofing will not be discussed here. Serious damage includes cracking and complete penetration.

Because of the many kinds of roofing and the scarcity of data of fragment impact upon roofing materials, the following discussion will be kept as general as possible, presenting only the lower limits of damage for groupings of roofing materials, with the understanding that these are not known very accurately.

The analysis for the impact upon metal targets leads one to believe that the important projectile property is momentum. Until more information is obtained, it must be assumed that momentum is also important in impact upon roofing materials. (The following discussion is based upon data in Reference 4 in which synthetic hailstones were projected at roofing materials targets. The velocities in the tests correspond to the terminal fall velocities of hailstones of the particular sizes used).

The roofing materials can be separated into three classes: asphalt shingles, built-up roofs (alternate layers of bitumen and reinforcing membranes, often topped with pebbles or crushed stone), and miscellaneous materials (asbestos cement shingles, slate, cedar shingles, clay tile, and sheet metal). Lower limits of fragment momentum for serious damage to common roofing materials are given in Table 5-2.

For oblique impact, the component of the velocity normal to the surface of the roof should be used in the calculation of momentum.

Aged shingles may sustain serious damage at a lower fragment momentum than that which is given in the table. Also, the tests were conducted at room temperature. The limiting momentum would be greater for shingles at a higher temperature, and less for shingles at a lower temperature.
<table>
<thead>
<tr>
<th>Roofing Material</th>
<th>Minimum Fragment Momentum (kg/m/s)</th>
<th>lb (ft/sec)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shingles</td>
<td>0.710</td>
<td>5.13</td>
<td>crack shingle</td>
</tr>
<tr>
<td></td>
<td>6.12</td>
<td>44.1</td>
<td>damage deck</td>
</tr>
<tr>
<td>Built-up roof</td>
<td>&lt;0.710</td>
<td>&gt;5.13</td>
<td>crack tar flood coat</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>14.5</td>
<td>crack surface of conventional built-up roof without top layer of stones</td>
</tr>
<tr>
<td></td>
<td>&gt;4.43</td>
<td>&gt;31.9</td>
<td>with a 14 kg/m² top layer of slag, there was no damage up to 4.43 kg m/s, which was the maximum momentum of the test</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.003 m (1/8&quot;) asbestos</td>
<td>0.710</td>
<td>5.13</td>
<td></td>
</tr>
<tr>
<td>0.006 m (1/4&quot;) asbestos cement shingles</td>
<td>1.27</td>
<td>9.16</td>
<td></td>
</tr>
<tr>
<td>0.006 m (1/4&quot;) green slate</td>
<td>1.27</td>
<td>9.16</td>
<td></td>
</tr>
<tr>
<td>0.006 m (1/4&quot;) grey slate</td>
<td>0.710</td>
<td>5.13</td>
<td></td>
</tr>
<tr>
<td>0.013 m (1/2&quot;) cedar shingles</td>
<td>0.710</td>
<td>5.13</td>
<td></td>
</tr>
<tr>
<td>0.019 m (3/4&quot;) red clay tile</td>
<td>1.27</td>
<td>9.16</td>
<td></td>
</tr>
<tr>
<td>Standing seam terne metal</td>
<td>4.43</td>
<td>31.9</td>
<td>plywood deck cracked</td>
</tr>
</tbody>
</table>
Example 4:

A fragment with a mass of 0.25 kg (0.55 lbm) and a velocity of 20.0 m/s (65.6 ft/s) strikes a roof of asphalt shingles in a direction normal to the surface of the roof. What kind of damage can be expected?

Solution: Calculate the momentum of the fragment, and compare it to the values in Table 5-2.

\[
\text{Momentum } = mv = (9.25 \text{ kg}) (20.0 \text{ m/s}) = 5.0 \text{ kg m/s}
\]

According to Table 5-2, this momentum will crack the shingle, but it will not damage the deck.

5-2 Damage Estimates to People from Secondary Fragments

5-2.1 Penetrating Fragments

Fragments can be divided into two categories, penetrating and nonpenetrating. Due to a limited amount of available data, penetrating fragments will refer to fragments weighing up to 0.015 kg (0.033 lbm) and area-to-mass ratio \(A/M\) up to 0.09 m\(^2\)/kg where \(A\) is the cross-sectional area of a fragment along its trajectory and \(M\) is the mass of the fragment. Nonpenetrating fragments will refer to fragments weighing 4.54 kg (10 lbm) or more. Only a summary of the methods for determining fragment damage to people will be presented here. The development of these methods is given in Appendix 5B for the convenience of the interested reader.

To determine whether a fragment can cause severe body penetration damage, it is necessary to determine its striking velocity \(V\) in (m/s) and \(A/M\) ratio; these parameters can, in general, be determined from other portions of this handbook with the exception of parameters from glass window fragments which follow. The ballistic limit velocity \(V_{50}\) (in m/s), which is the velocity at which half of the missiles incident on the body are expected to perforate the skin with enough residual velocity to cause severe damage is (5.6)

\[
V_{50} = 1247.1 \left( \frac{A}{M} \right) + 22.03 \text{ m/s} \tag{5-1}
\]

for

\[
A/M \leq 0.09 \text{ m}^2/\text{kg}, \quad M \leq 0.015 \text{ kg}
\]

where

- \(A\) is the cross-sectional area of the fragment along its trajectory in m\(^2\)
- \(M\) is the mass of the fragment in kg
$V_{50}$ is the ballistic limit velocity in m/s

If $V \geq V_{50}$, then one can expect some serious wounds from body penetration.

To determine if a glass fragment from a window pane broken by the blast wave can cause severe body penetration damage, it is necessary to calculate an effective peak overpressure $P_e$

where

$$P_e = \frac{P}{P_o} \quad (5-2)$$

for windows oriented side-on or back-on to the approaching blast wave where $P_o$ is atmospheric pressure in Pa, see Figure 5-3.

or

$$P_e = \frac{P}{P_o} \quad (5-3)$$

for windows oriented face-on to the approaching blast wave.

For $P_s \leq 3.5$ (7)

$$\frac{P}{P_o} = \frac{1}{2} \frac{2 + (v + 1) \frac{V}{V}}{(v - 1) \frac{V}{V} + 2} \quad (5-4)$$

where $v = 1.4$

For $P_s > 3.5$, $P_e$ can be acquired directly from Figure 5-4. The ratio $A/M$ should be chosen as the smaller of

$$\frac{A}{M} = \frac{1}{1} \quad (5-5)$$

or
Figure 5.3: Atmospheric Pressure vs. Altitude Above Sea Level

\( p(h) = p_0 \times 1.45 \times 10^{-4} \times h \)

\( h = m \times 3.261 \)
Figure 5-4. Scaled Reflected Overpressure Vs Scaled Side-on Overpressure
\[
\frac{A'}{A^*} = \frac{1}{\sqrt{A^*}}
\]

(5-6)

where

- \( t \) is the thickness of the window pane in meters
- \( \rho \) is the density of the glass which is approximately 2471 kg/m³ (from Reference 8)

\( A' \) is the geometric mean frontal area of the glass fragment in m² expressed by

\[
A' = 6.4516 \times 10^{-4} \cdot \left(2.4 - \sqrt{12.5 + (5.8566 \times 10^{-5} P_e)^2}\right)
\]

(5-7)

for \( P_e \) in the range 0 Pa to 96.5 kPa. (From References 8 and 9).

Striking velocity \( v \) (in m/s) is

\[
v = \left[(0.2539 + (1.896 \times 10^{-4}) (t - 7.62 \times 10^{-4}) - 0.928\right]

\times 3.3443 P_e^{0.547}
\]

(5-8)

for \( P_e \) in range 690 Pa to 689 KPa and \( t \geq 7.62 \times 10^{-4} \) m.

If \( v \geq V_{50} \), then one can expect some serious wounds from body penetration.

5.2.2 Nonpenetrating Fragments

Criteria for body damage from nonpenetrating objects are contained in Table 5-3. It should be noted that damage is dependent on fragment mass and velocity only. The table also only contains one fragment mass value. One can logically assume that larger masses propelled at the same velocities shown in the table will produce more damage than the 4.54 kg (10 lb) mass presented in the table.
TABLE 5-3. TENTATIVE CRITERIA FOR INDIRECT BLAST EFFECTS FROM NONPENETRATING FRAGMENTS
(References 10, 11, 12)

<table>
<thead>
<tr>
<th>Mass</th>
<th>Event</th>
<th>Extent of Damage</th>
<th>Impact Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.54 kg (10 lb)</td>
<td>Cerebral Concussion</td>
<td>Mostly &quot;safe&quot;</td>
<td>3.05 m/s (10 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Threshold</td>
<td>1.57 m/s (15 ft/sec)</td>
</tr>
<tr>
<td></td>
<td>Skull Fracture</td>
<td>Mostly &quot;safe&quot;</td>
<td>3.05 m/s (10 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Threshold</td>
<td>4.57 m/s (15 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Near 100%</td>
<td>7.01 m/s (23 ft/sec)</td>
</tr>
</tbody>
</table>

5-2.3 Example Calculations for Determining Damage Estimates to People from Secondary Fragments

Example 1:

Environmental Conditions:

\[
P_b = 3.3 \times 10^4 \text{ Pa (0.53 psi)}
\]

\[
P_o = 1.16 \times 10^3 \text{ Pa s (0.0163 psi/sec)}
\]

\[
P_0 = 1.0135 \times 10^5 \text{ Pa (14.2 psi)}
\]

Penetrating fragment - Mass M = 0.015 kg (0.033 lbm)

\[
\text{Velocity } V = 131 \text{ ft/sec}
\]

Cross sectional area A along trajectory of fragment = 0.0016 m² (0.0172 ft²)

Windows in a nearby building are 3.175 m (10 ft) thick and are struck by the approaching blast wave.

Nonpenetrating fragment - Mass M = 3.00 kg (6.6 lbm)

\[
\text{Velocity } V = 20 \text{ m/s (70 ft/sec)}
\]
From Equation (5-3)

\[ P_e = \overline{P_r} p_0 = (0.864) (101,350) = 87,567 \text{ Pa} \]

(b) \[
\left( \frac{\Delta}{M} \right)_1 = \frac{1}{t_0} = \frac{1}{(3.175 \times 10^{-3} \text{ m})(2.471 \times 10^{3} \text{ kg/m}^3)} = 0.1275 \text{ m}^2/\text{kg} \]
From Equation (5-7):

\[
A' = (6.4516 \times 10^{-4})
\]

Thus \(2.4 = \ln(12.5 + (5.8566 \times 10^{-5}) (8.7567 \times 10^{4})^{2} \cdot \frac{1}{2}
\]

\[
A' = 0.1402 \text{ m}^2
\]

Therefore,

\[
\left(\frac{A}{M}\right)^{-1} = \frac{1}{(2471) \sqrt{0.1402}} = 0.00108 \text{ m}^2/\text{kg}
\]

The smaller value for \(A/M\) (0.00108 m²/kg) is the better choice.

(c) \(V_{50}\) for \(A/M = 0.00108 \text{ m}^2/\text{kg}\) can be calculated from Equation (5-1):

\[
V_{50} = (1247.1) (0.00103) + 22.03
\]

\[
V_{50} = 23.4 \text{ m/s (76.8 ft/sec)}
\]

(d) From Equation (5-8), the striking velocity \(V\) is

\[
V = (10.2539) + (1.896 \times 10^{-4}) (3.175 \times 10^{-3} - 7.62 \times 10^{-4} - 0.928 (0.3443) (8.7567 \times 10^{-4} 0.547
\]

\[
V = 53.0 \text{ m/s (174 ft/sec)}
\]

(e) Since \(V > V_{50}\), one can expect some severe body penetration damage.
Nonpenetrating fragments:

The mass (4.00 kg) and velocity (2 m/s) of the nonpenetrating fragment are less than the mass and velocity required for the mostly "safe" damage condition shown in Table 5-3. Thus, one would not expect deaths to result from nonpenetrating fragments.
LIST OF REFERENCES


APPENDIX V.A

EFFECTS OF FRAGMENTS ON STRUCTURES

The structures that are considered here are metal plates and sheets. There does not appear to be any effect of the curvature of the target; therefore, it is reasonable to use data for flat targets and apply them to any general shape that may be of interest.

The methods described in Section 5-1 are based upon an examination of data of fragment and hailstone impact upon metal sheets and plates. In these studies, synthetic hailstones (ice spheres) were fired at target sheets of aluminum alloys, and various shapes of fragments were fired at steel targets. A model analysis was performed, using the methods described in Reference 4. The parameters of interest are listed in Table 5A-1.

TABLE 5A-1. LIST OF PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>radius of fragment (assuming spherical shape)</td>
</tr>
<tr>
<td>h</td>
<td>thickness of target</td>
</tr>
<tr>
<td>v</td>
<td>velocity of fragment</td>
</tr>
<tr>
<td>δ</td>
<td>permanent deflection of target at point of impact</td>
</tr>
<tr>
<td>ρ</td>
<td>density of fragment (projectile)</td>
</tr>
<tr>
<td>ρ</td>
<td>density of target</td>
</tr>
<tr>
<td>σ</td>
<td>yield stress of target material</td>
</tr>
</tbody>
</table>

This analysis is concerned with plastic deformation, which makes the parameter σ more important than the modulus of elasticity of the target material. Also, the fragment is assumed to be a rigid body, which makes the strength of the fragment an unnecessary parameter. The model analysis and a study of the data resulted in the nondimensional terms (Table 5A-2).
TABLE 5A-2. NONDIMENSIONAL TERMS

\[
\begin{align*}
&\frac{\rho \cdot V}{\sqrt{\sigma_t \cdot t}} \quad \text{dimensionless projectile velocity} \\
&(\frac{\delta h}{a^2}) \quad \text{dimensionless target deflection} \\
&(\frac{h}{a}) \quad \text{dimensionless target thickness}
\end{align*}
\]

When \( \frac{\delta h}{a^2} \) is plotted versus \( \frac{\rho \cdot V}{\sqrt{\sigma_t \cdot t}} \), the data follow a straight line with some scatter in the data points (see Figure 5-2 in text). The line intersects the horizontal axis at a positive value of velocity. This is expected because there is a finite fragment velocity below which no permanent target deflection occurs.

Letting the velocity in the dimensionless projectile velocity term be the limit velocity, the locus of \( \frac{\rho \cdot V}{\sqrt{\sigma_t \cdot t}} \) versus \( \frac{h}{a} \) is linear with the data points lying within about 15% of the values on the line (see Figure 5-1 in the text). The hailstone impact data fall in the region above the line (higher limit velocity), and the steel fragment data fall on and below the line (lower limit velocity). This indicates a possible effect of fragment strength. For this reason it may be desirable to be more conservative with steel fragments, choosing a lower limit velocity, and less conservative with aluminum fragments, choosing a higher limit velocity.
LIST OF REFERENCES


APPENDIX V.B

DAMAGE ESTIMATES TO PEOPLE FROM SECONDARY FRAGMENTS

5B-1 Penetrating Fragments

Undoubtedly a great deal of research has been conducted to produce classified wound ballistics equations for the military. Although through unclassified equations of this type do not exist, some publicly available body penetration data have been accumulated in recent times and some relatively simple analyses have been performed. Methods for predicting body damage from fragments using fragment parameters available in this document are presented in this appendix. More reliable damage criteria will undoubtedly be produced as the state-of-the-art improves.

Sperrazza and Kokinakis(1,2) concerned themselves with a ballistic limit velocity $V_{50}$ for animal targets. The $V_{50}$ velocity is the striking velocity at which one expects half the impacting missiles to perforate an object. They found that this velocity depended on the area to mass ratio, that is

$$V_{50} \propto \frac{A}{M}$$

(5B-1)

where $A$ is cross-sectional area of the projectile along the trajectory, and $M$ is the mass of the projectile. They fired steel cubes, spheres and cylinders of various masses up to 0.015 kg (0.033 lbm) into 3 mm (0.118 in.) thick isolated skin (human and goat) to establish a ballistic limit. One of their assumptions was that, if the projectile penetrates the skin, its residual velocity would be sufficient enough to cause severe damage. This cautious assumption is appropriate for establishing a certain margin of safety in the calculation. Their conclusions were that, in the range of their data for steel cubes, spheres and cylinders, $V_{50}$ depended linearly on projectile A/M ratio. Specifically,

$$V_{50} = 1747.1 \frac{\text{kg}}{\text{m} \cdot \text{s}} \left( \frac{A}{M} \right) + 22.03 \text{ m/s}$$

(5B-2)

where $A/M$ is in $\text{m}^2/\text{kg}$, and $V_{50}$ is in m/s.

"Equation (5B-2) has been adjusted for SI units."
Kokinakis later fired plastic sabots end-on into 20\% gelatin that was 1 cm thick. The sabots were fired end-on since this represents the "worst case," and 20\% gelatin was used because this ballistically simulates isolated human skin. The linear relation of $V_{50}$ versus $A/M$ formulated by Sperrazza and Kokinakis (1) is plotted in Figure 5B-1. The average values for these experiments are located on this graph. Circles on the figure represent the initial experiments using steel cubes, spheres and cylinders weighing up to 0.015 kg (0.033 lbm), and each average value represents as many as 30 data points. The line drawn on the graph is a least squares fit to these average values. Upward pointed triangles represent the average values for the subsequent experiments with end-on plastic sabots. These average values also lie near the line drawn for the prior study, thus adding a degree of confidence in the analysis.

Unfortunately, other authors have not presented their penetration data in the same form as Sperrazza and Kokinakis. Glasstone (3) expressed the probability of glass fragments penetrating the abdominal cavity in terms of the mass of the glass fragments. To compare Glasstone's conclusions with that of Sperrazza and Kokinakis, it is necessary to make a few assumptions. The first assumption is that the glass fragment velocity for 50\% probability of penetration of the abdominal cavity is biologically equivalent to the ballistic limit velocity $V_{50}$ for penetrating isolated human skin. This assumption is true provided that, after the glass fragment penetrates the skin, it does not encounter too much resistance before it perforates the abdominal cavity. Glasstone only specifies the mass of the glass required for penetration and does not give its cross-sectional area, thickness or density. For the purpose of comparing the conclusions of Glasstone with those of Sperrazza and Kokinakis, it was assumed that glass fragments are propelled edge-on, which is probably the worst case, and that they are square with thicknesses of 3.175 mm (1/8 in.) to 6.35 mm (1/4 in.). It was also assumed that the glass fragments have an average density of 2471 kg/m$^3$. (4) With these assumptions, it is not difficult to calculate $A/M$.

If the glass fragment has a thickness $t$, and edge length $y$, then for volume

$$V = y^2 t$$

(5B-3)

where

$V$ = volume of the fragment

$y$ = edge length

$t$ = thickness

Thus, the mass $m$ of the fragment is
Figure 5B-1. Ballistic Limit ($V_{50}$) Vs. Fragment Area/Mass for Isolated Human and Goat Skin
where \( \rho \) is the density of the glass. Rearranging Equation (5B-4) gives the edge length,

\[
y = \sqrt[3]{\frac{M}{c \rho t}}
\]  

(5B-5)

The area-to-mass ratio \( A/M \), assuming edge-on impact, is

\[
\frac{A}{M} = \frac{1}{4} \frac{y^2}{M}
\]  

(5B-6)

or from Equations (5B-4), (5B-5) and (5B-6),

\[
\frac{A}{M} = \sqrt[3]{\frac{1}{c \rho t}}
\]  

(5B-7)

Glasstone's criteria for 50% probability of glass fragments penetrating the abdominal cavity are shown in Table 5B-1. This table also contains the estimates for \( A/M \) for glass thicknesses of 3.175 mm (1/8 in.) and 0.35 mm (1/4 in.). The velocity values and calculated values for \( A/M \) which fall in the range of values used by Sperrazza and Kokinakis are plotted as squares in Figure 5B-1. The dashed lines indicate a range of \( A/M \) values for thickness values from 2.175 mm (1/8 in.) to 6.35 mm (1/4 in.). Even with the crude assumptions mentioned above, the calculated points fall very near the line drawn on Figure 5B-1.

White [5] also related skin penetration velocity to the masses impacting fragments. He concluded that slight skin laceration occurred when spherical bullets with mass 0.0087 kg (0.019 lbm) were propelled into the body at 57.0 m/s (100 ft/sec). Assuming that the density \( \rho \) of steel is 7.925 kg/m^3, the \( A/M \) ratio can be calculated from

\[
\frac{A}{M} = \frac{r^2}{M}
\]  

(5B-8)

where \( r \) is the radius of the spherical penetrator or

\[
\frac{A}{M} = \left( \frac{3M}{4 \pi \rho r^2} \right)^{2/3}
\]  

(5B-9)
TABLE SB-1. 50 PERCENT PROBABILITY OF CLASS A/M FRAGMENTS PENETRATING ABDOMINAL CAVITY(3)

<table>
<thead>
<tr>
<th>Mass of Glass Fragment</th>
<th>Impact Velocity (m/s) (ft/sec)</th>
<th>A/M [3.175 mm (1/8 in.) thick] m²/kg</th>
<th>A/M [6.35 mm (1/4 in.) thick] m²/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>125 (410)</td>
<td>0.1136</td>
<td>0.1603</td>
</tr>
<tr>
<td>0.0005</td>
<td>84 (275)</td>
<td>0.0507</td>
<td>0.0717</td>
</tr>
<tr>
<td>0.001</td>
<td>75 (245)</td>
<td>0.0358</td>
<td>0.0507</td>
</tr>
<tr>
<td>0.01</td>
<td>55 (180)</td>
<td>0.0113</td>
<td>0.0160</td>
</tr>
</tbody>
</table>

*Table SB-1 has been adjusted for SI units.

Using Equation (SB-9) and the mass and density mentioned above, A/M becomes 0.0148 m²/kg. The velocity value given above (57.0 m/s) and the calculated value for A/M are plotted on Figure SB-1 as a downward pointed triangle. This point appears to be a little higher than expected, especially since only slight skin laceration is expected at these velocities instead of 50% penetration.

Custard, et al.,(6) like Glasstone, specify velocity as a function of mass only for 50% penetration. Making the assumptions that the thickness of the glass can vary from 3.175 mm (1/8 in.) to 6.35 mm (1/4 in.), that the fragments travel edge-on and are square, and that the density of glass is 2471 kg/m³, A/M was calculated from Equation (SB-7). The results are plotted on Figure SB-1 as diamonds and agree fairly well with the conclusions of Sperrazza and Kokinakis. Thus, for values of A/M up to 0.06 m²/kg and values of M up to 0.016 kg (0.033 lbm), the functional relationship expressed in Equation (SB-7) and drawn as a solid line in Figure SB-1 is an adequate representation of 50% probability of skin penetration by a projectile that can result in serious wounds.

Estimates of velocities, presented areas and masses (up to 0.016 kg (0.033 lbm)) of fragments from a propellant or gas explosion can be acquired from other portions of this document and compared with Figure SB-1 to determine if penetration is likely. No estimate, however, of the velocity, mass and area of window glass fragments has been made elsewhere in this report. Since wounding from flying glass is a major concern, a method for determining these parameters will be included here. Fletcher, Richmond and Jones(4, 7) conducted blast experiments to obtain information on glass fragments from breaking window panes. From their statistical analysis of the data, they were able to establish functional relationships among several variables. To be able to be used in conjunction with the work of Sperrazza.
and Koltsarić, it is necessary to obtain the velocity and area to mass ratio of the glass fragments from the work of Fletcher et al. After converting their equations to SI units, the geometric mean frontal area \( A' \) of fragments becomes

\[
A' = 0.15 \cdot 10^{-4} \left( \frac{2.4 \cdot 12.5 \cdot 600.0 \cdot 10^{-5} P_e}{P_e} \right)^2
\]  

(5B-10)

for \( P_e \) in range 6 to 60.6 kPa, where \( P_e \) is effective peak overpressure in Pascals.

The effective peak overpressure \( P_e \) is equivalent to incident peak overpressure \( P_0 \) for windows oriented side-on or back-on to the approaching blast wave and reflected peak overpressure \( P_r \) for windows oriented face-on to the approaching blast wave. The handbook contains methods for determining \( P_e \). For \( P_e \leq 3.5 \) \( (P_e = P_0/P_0 \) where \( P_0 \) is atmospheric pressure), \( P_e = P_0/P_0 \) can be determined from

\[
\frac{P_e}{P_0} = \frac{(x+1)}{x-1}\frac{P_0}{P_0} + 2
\]  

(5B-11)

where \( x \neq 1.4 \).

Atmospheric pressure \( P_0 \) can be acquired from Figure 5.3 at sea level and various altitudes above sea level. For \( P_e > 3.5 \), \( P_e \) can be acquired from Figure 5.4. Effective peak pressure \( P_e \) is the

\[
P_e = P_0
\]  

(5B-12)

for windows oriented side-on or back-on to the approaching blast wave, or

\[
P_e = P_r
\]  

(5B-13)

for windows oriented face-on to the approaching blast wave.

In cases where all fragments are intact, then mass \( M \) can be determined from

\[
M = \frac{P_e}{V_{fm}}
\]  

(5B-14)
If all glass fragments travel flat side forward, then
\[ A = A' \]  
(5B-15)

If all glass fragments travel edge forward, then
\[ A = \frac{t}{A'} \]  
(5B-16)

Thus, for these two cases, the ratio \( A/M \) is
\[ \frac{A}{M} = \frac{1}{t} \]  
(5B-17)
or
\[ \frac{A}{M} = \frac{1}{t / A'} \]  
(5B-18)

Whichever gives the lower value for \( A/M \) should be chosen for safety reasons. The geometric mean velocity can be acquired from Fletcher, et al. (4.7)

After converting their equations to SI units,
\[ (0.2539) + (1.896 \times 10^{-4}) (t - 7.62 \times 10^{-4}) \]
\[ = 0.3443 \left( \frac{P_e}{0.547} \right) \]  
(5B-19)

for \( P_e \) in the range 690 Pa to 689 kPa and \( t \geq 7.62 \times 10^{-4} \) m.

A summary of the methods for determining the combinations of parameters which may produce serious penetration damage from fragments less than 0.015 kg (0.033 lbm) is given here for convenience. To determine whether a fragment can cause severe body penetration damage, it is necessary to determine its striking velocity \( V \) (in m/s) and \( A/M \) ratio. Here, \( A \) is the cross-sectional area of the projectile along its trajectory (in m²) and \( M \) is the mass of the projectile (in kg). These parameters can, in general, be determined from other portions of this handbook. The ballistic limit velocity \( V_{50} \) (in m/s) is then
\[ V_{50} = 1247.1 \frac{A}{M} \]  
(11.70)

for \( A/M \leq 0.001 \) m²/kg, where \( A \) is the cross-sectional area of the fragment along its trajectory (in m²), \( M \) is the mass of the fragment in kg, and
\[ V_50 \text{ is the ballistic limit velocity in m/s. If } V \geq V_{50}, \text{ then one can expect some serious wounds from body penetration.} \]

To determine whether a glass fragment from a window pane broken by the blast wave can cause severe body penetration damage, it is necessary to calculate an effective peak overpressure \( P_e \) where

\[ P_e = \frac{P_5 p_0}{(5B-21)} \]

for windows oriented side-on or back-on to the approaching blast wave

(\( p_0 \) is atmospheric pressure in Pa; see Figure 5B-2), or

\[ P_e = \frac{P_r p_0}{(5B-22)} \]

for windows oriented face-on to the approaching blast wave.

For \( \frac{P_e}{p_0} \leq 3.5 \),

\[ \frac{P_r}{p_0} = 2 \frac{P_5}{p_0} + \frac{(y + 1) P_5^2}{y (y - 1) (\rho / \rho_0)^{2/3} + 2} \]  

where \( y = 1.4 \).

For \( \frac{P_e}{p_0} > 3.5 \), \( P_r \) can be acquired directly from Figure 5B-3. The ratio \( \frac{A}{A'} \) should be chosen as the smaller of

\[ \frac{2}{M} = \frac{1}{A'} \]  

(5B-24)

or

\[ \frac{A}{A'} = \frac{1}{2} \frac{1}{A'} \]  

(5B-29)

where \( t \) is the thickness of the window pane in meters, \( \rho \) is the density of the glass which is approximately 2571 kg/m\(^3\), \( (4) \) and \( A' \) is the geometric cross-sectional area of the glass fragment in m\(^2\) expressed by

\[ A' = 0.4 (1 - 5) \left( \frac{4}{5} \right)^{1/4} \]  

(5B-26)

for \( P_e \) in the range 0 Pa to 76.5 kPa.
Striking velocity $V$ (in m/s) is

$$V = (0.2539) + (1.896 	imes 10^{-4}) (t - 7.62 	imes 10^{-4}) x 0.547 P_e$$

for $P_e$ in the range 690 Pa to 689 kPa, and $t \geq 7.62 	imes 10^{-4}$ m. If $V \geq V_50$, then one can expect some serious wounds from body penetration.

5B-2 Nonpenetrating Fragments

Criteria for body damage from nonpenetrating fragments are rather limited. Table 5B-2 contains tentative damage criteria for indirect blast effects involving nonpenetrating objects. It should be noted that the table applies to a fragment of only one mass. One can logically assume that larger masses propelled at the same velocities shown in the table will produce more damage than the 5.54 kg (10 lb) mass presented in the table.

**TABLE 5B-2. TENTATIVE CRITERIA FOR INDIRECT BLAST EFFECTS FROM NONPENETRATING FRAGMENTS(9-11)**

<table>
<thead>
<tr>
<th>Mass</th>
<th>Event</th>
<th>Extent of Damage</th>
<th>Impact Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.54 kg (10 lb)</td>
<td>Cerebral Concussion</td>
<td>Mostly &quot;safe&quot;</td>
<td>3.05 m/s (10 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Threshold</td>
<td>4.57 m/s (15 ft/sec)</td>
</tr>
<tr>
<td></td>
<td>Skull Fracture</td>
<td>Mostly &quot;safe&quot;</td>
<td>3.05 m/s (10 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Threshold</td>
<td>4.57 m/s (15 ft/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Near 100%</td>
<td>7.01 m/s (23 ft/sec)</td>
</tr>
</tbody>
</table>
LIST OF REFERENCES


CHAPTER VI

RISK ASSESSMENT AND INTEGRATED EFFECTS

6-1 Risk Assessment

A systematic, effective approach is required to identify the type, magnitude, and probability of occurrence of undesired events in the operation of any given system. Certain well-known related techniques have been used in industry (chemical, aerospace, nuclear and defense) to accomplish this result. These systems analyses are alternately referred to as "Hazard Analysis," "Safety Analysis," "Risk Assessment" and "Reliability Analysis."

Three basic related systematic methods are employed, either singly or in combination, to accomplish these types of analyses. These methods are:

(1) Event Tree - Starts with an event that initiates a possible accident and develops the possible consequences of the event by considering the response of engineered safety systems that would be called upon as a result of the initial event.

(2) Fault Tree - Provides a method for determining the probabilities needed for the event trees. Fault trees employ a logic almost the reverse of event trees in that they start with an undesired event and identify the ways it may have been caused.

(3) FMECA - The Failure Mode Effects and Criticality Analysis is a systematic procedure for identifying each failure mode of the system and for evaluating the consequences. The FMECA starts with the components of the system, works up through the subsystem to the system level, and identifies the effects of each failure mode on the system operation.

An example of a simple event tree is shown in Figure 6-1 and a simple fault tree in Figure 6-2 for a chemical spill in a storage tank subsystem. Given an outcome from the event tree, a spill magnitude is defined. For each spill magnitude, a hazard or risk assessment can be made. From the fault tree, the probabilities of failure are determined to assign to each branch of the event tree. Then, the probability (or expected frequency) of occurrence...
EVENT

Pipe Break
Vessel Rupture
Transfer Overflow
Transfer Leak

SAFETY SYSTEM # 1
SAFETY SYSTEM Backup # 2
SAFETY SYSTEM Backup # 3

P1

P2

P3

P4

\( P_A \) Very Large Spill

\( P_B \) Large Spill

\( P_C \) Moderate Spill

\( P_D \) Small or No Spill

MAGNITUDE

\( 1 - P2 \)

\( 1 - P3 \)

\( 1 - P4 \)

Works

Works

Works

P = PROBABILITY OF FAILURE
Figure 6-2. Typical Fault-Tree Diagram: Combining of Failure Probabilities
is determined by the proper algebraic combination of the branch probabilities leading to the outcome.

Examples of these types of analyses can be found in references 1, 2, 3, 4, and 5.

For gas vessel or propellant systems, the figures and methods described in Chapters I through V can be used to assess the possible effects of any given undesired event based on the characteristics of fragments (mass initial velocity, and range) or blast phenomena.

The workbook does not give methods for estimating probabilities of catastrophic events occurring, nor does it employ the methods of risk assessment just discussed. It does, however, allow prediction of severity of certain classes of accidental explosion, as well as effects, under the assumption that an explosion does occur.

6-2 Prediction of Relative Blast and Fragment Effects

In Chapters II through IV, we give a number of examples of specific calculations for use of graphs, tables or simple equations each of which provides an estimate of some aspect of explosion hazards or their effects. But here we give a series of more complete examples. Each example represents a possible accident which can occur at aerospace launch or test facilities. Our assessment, as is true for all such estimates which can be made from this workbook, starts with the assumption that an accident has occurred and does not consider the probability of occurrence. Five different accident "scenarios" are presented, and estimates of blast and fragmentation effects are made for each scenario. The quantities of propellant, volumes and types of compressed gases, masses of vehicle structure, etc., are often approximate, but they are realistic values. The problems are intended to illustrate the way in which the data in Chapters II through IV can be used to estimate relative blast and fragment characteristics, for some "typical" accidents.

6-2.1 Scenario #1. Fall-back of Space Shuttle during launch.

The scenario for this accident is a failure of thrust just after lift-off of the space shuttle. This multi-stage vehicle is a large, winged orbiter and landing vehicle similar in configuration to a large aircraft, which is boosted by the solid propellant rocket engines and a liquid-fueled engine. At lift-off, all three engines are firing and delivering thrust. The vehicle is assumed to rise at most a few meters, and then to fall back onto the launch pad with sufficient impact velocity to rupture the liquid rocket tankage. The propellants mix on the ground surface and reach an ignition source after some time delay.
Data for types and amounts of propellant in the Space Shuttle at lift-off, flow rates, and estimates of structural weight are given in Table 6-1. Because this accident occurs immediately after lift-off, all propellant weights are (conservatively) assumed to be the same as at lift-off. Assuming fall-back under gravity from a height of 10 meters (33 ft), the impact velocity is $U = 14.0 \text{ m/s (45.9 ft/sec)}$.

The propellants in the solid propellant boosters are assumed not to be explosive. This assumption seems well founded, based on the extensive testing of detonability of solid propellants in Project SOPHY. Furthermore, the quantities of liquid propellants listed for the first four subsystems in Table 6-1 are relatively small, and the type of accident postulated would be unlikely to rupture tanks containing these propellants. So, we assume for this example problem that only the external tank ruptures and spills its propellants in a CBGS type of accident. For estimating fragmentation effects, we also use only the structural mass of the damaged external tankage.

A number of other hazards are obviously associated with this type of accident, but some are not calculable from methods given in this workbook. In particular, the trajectories and impact effects of the propulsive solid propellant boosters, which are burning and thrusting when fall-back occurs, cannot be estimated. Presumably, the orbiter with its human payload can escape by igniting escape rockets in the event of fall-back. If we can estimate its flight location when explosion occurs, some predictions of blast effects can possibly be made. But, we have too little data in hand at present to make this estimate.

6-2.1.1 Yield

The yield is calculated by the methods in Chapter 1. According to Table 1-2, Equation 1-2 and Figure 1-7 are used to determine a value for the yield, which is compared with a value obtained from Figure 1-1. The smaller value is used:

Equation 1-2: $Y = 10^\% + \frac{4.43\%}{\text{m/s}} U (U < 4.4\text{ m/s})$

In Figure 1-7, choose $t_{\text{ignition}}$ for maximum $Y$: $t_{\text{ignition}} = 0.6\text{ sec.}$
<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Propellant</th>
<th>Quantity (kg)</th>
<th>Flow Rate (kg/s)</th>
<th>Flow Rate (lbm/s)</th>
<th>Structural Mass (kg)</th>
<th>Structural Mass (lbm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aux. Power Units (APU)</td>
<td>Hydrozene</td>
<td>281</td>
<td>0.04</td>
<td>0.09</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Orbital Maneuvering (OMS)</td>
<td>MMH</td>
<td>424.0</td>
<td>3.29</td>
<td>7.24</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>N2O4</td>
<td>7,010</td>
<td>5.42</td>
<td>11.9</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Reaction Control (RCU)</td>
<td>MMH</td>
<td>1,270</td>
<td>0.523</td>
<td>1.15</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>N2O4</td>
<td>2,050</td>
<td>0.841</td>
<td>1.85</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Power</td>
<td>LI2</td>
<td>83.6</td>
<td>944</td>
<td>0.000221</td>
<td>0.000486</td>
<td>--</td>
</tr>
<tr>
<td>Reactant</td>
<td>Max. mission</td>
<td>592</td>
<td>1100</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Storage System</td>
<td>LO2</td>
<td>710</td>
<td>0.00176</td>
<td>0.00387</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Thruster</td>
<td>30 day</td>
<td>4,260</td>
<td>9370</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>External Tank</td>
<td>LI2</td>
<td>163,000</td>
<td>909</td>
<td>2,000</td>
<td>178,000</td>
<td>392,000</td>
</tr>
<tr>
<td></td>
<td>LO2</td>
<td>6,018,000</td>
<td>5,450</td>
<td>12,000</td>
<td>178,000</td>
<td>392,000</td>
</tr>
<tr>
<td>Environmental Control System</td>
<td>NH3</td>
<td>46.4</td>
<td>102</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1,986,000</td>
<td>4,020</td>
<td>8840</td>
<td>34,500</td>
<td>76,000</td>
</tr>
</tbody>
</table>
Then
\[
\frac{Y \cdot 100}{Y_{100}} = 62, \quad Y = \frac{62 \cdot Y_{100}}{100} = 44.6\%.
\]

From Figure 1-1 using a total propellant and oxidizer mass of 711000 kg, \( y = 0.06 \). The multiplier is 370\%. \( Y = (0.06)(370\%) = 22.2\% \). Choose the lower value of \( Y \) which is 22.2\%.

6.2.1.2 Overpressure and Specific Impulse

The overpressure and specific impulse are calculated by the methods in Chapter II. The effective mass is calculated from Equation 2-9:
\[
W = W_{t}X_{100}Y
\]
Table 2-1 gives the procedure for finding the overpressure and specific impulse as functions of distance:
\[
W = W_{t}X_{100}Y = (711000 \text{ kg}) \frac{22.2\%}{100} = 153000 \text{ kg} (348,000 \text{ lb})
\]

According to Table 2-1, the overpressure is read from Figure 2-16, and the specific impulse is read from Figure 2-17.

The following (Table 6-2) is a list of overpressures \( P_{r} \) and specific impulses \( I_{e} \) from Figures 2-16 and 2-17. The solid lines in these figures were used. The other terms in Table 6-2 will be explained later.

6.2.1.3 Effect of Blast Waves on Structures

The methods described in Section 3-1 are used to determine the effects of blast waves on structures.

Glass Breakage

Equation 3-1 relates overpressure for glass breakage to the properties of the glass pane.

\[
\left[ \frac{P_{r}X}{Y^{2}} \right] = \frac{1.0 + 3.08\left( \frac{X}{Y} \right)^{2} + \left( \frac{Y}{X} \right)^{4}}{8.0 \cdot 0.79 \cdot 0.11 \left( \frac{X}{Y} \right)^{2} - 0.79 \left( \frac{Y}{X} \right)^{4}}^{1/2}
\] (3-1)
### Table 6-2. Overpressures and Impulses

<table>
<thead>
<tr>
<th>R (m)</th>
<th>$\frac{R}{W^{1/3}}$</th>
<th>$P_s$ (Pa)</th>
<th>$\overline{P}_s$ (Pa·s)</th>
<th>$I_s$ (Pa·s/kg $^{1/3}$)</th>
<th>$I_s$ (Pa·s/kg $^{1/2}$)</th>
<th>$I_s$ (Pa·s/kg $^{1/3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.4</td>
<td>0.6</td>
<td>$1.400 \times 10^6$</td>
<td>13.8</td>
<td>$2.09 \times 10^4$</td>
<td>386</td>
<td>1.22 $\times 10^4$</td>
</tr>
<tr>
<td>54.1</td>
<td>1.0</td>
<td>$5.60 \times 10^5$</td>
<td>5.53</td>
<td>$1.29 \times 10^4$</td>
<td>238</td>
<td>7.54 $\times 10^3$</td>
</tr>
<tr>
<td>108</td>
<td>2.0</td>
<td>$1.52 \times 10^5$</td>
<td>1.50</td>
<td>$7.03 \times 10^3$</td>
<td>139</td>
<td>4.11 $\times 10^3$</td>
</tr>
<tr>
<td>216</td>
<td>4.0</td>
<td>$4.71 \times 10^4$</td>
<td>0.465</td>
<td>$3.82 \times 10^3$</td>
<td>70.6</td>
<td>2.23 $\times 10^3$</td>
</tr>
<tr>
<td>541</td>
<td>10.0</td>
<td>$1.31 \times 10^4$</td>
<td>0.129</td>
<td>$1.71 \times 10^3$</td>
<td>31.6</td>
<td>1.00 $\times 10^3$</td>
</tr>
<tr>
<td>1080</td>
<td>20.0</td>
<td>$5.22 \times 10^3$</td>
<td>0.0515</td>
<td>$9.24 \times 10^2$</td>
<td>17.1</td>
<td>5.40 $\times 10^2$</td>
</tr>
</tbody>
</table>

(ft = m X 3.281)

( psi = Pa X 1.450 X 10^{-4} )

( psi·sec = Pa·s X 1.450 X 10^{-4} )
for a blast wave striking a window head-on.

\[ P_s = 2P_s \]

for a blast wave traveling parallel to the surface of the glass.

It can be shown that \( P_s \) increases as \( \frac{X}{r} \) increases. For the lowest \( P_s \) to break any kind of glass that is likely to be found, choose \( X = 0.20 \) and \( X = 0.300 \) m (0.98 ft). Then, \( P_s = 2420 \) Pa (0.351 psi) for glass breakage with the glass surface perpendicular to the direction of travel of the blast wave. \( P_s = 4840 \) Pa (0.702 psi) for glass breakage with the direction of propagation of the blast wave parallel to the surface of the window.

Looking at the data in Table 6-2, it can be seen that, for no breakage with windows facing the source of the blast wave, the distance must be much greater than 1080 m (3500 ft). (By extrapolating curve 1, the \( i \) vs \( P_s \) curve in the next figure, Figure 6-3, it appears that the distance for an overpressure of 2420 Pa is near 2000 m (6000 ft).) With the glass surface parallel to the direction of travel of the blast wave, the distance for no glass breakage must be about 1100 m (3600 ft).

Building Damage

The degree of building damage as a function of distance from the source of the blast wave can be observed in Figure 6-3. Figure 6-3 is a copy of Figure 3-2, with the (impulse, overpressure) points for selected distances \( R \). The \((i, P_s)\) paths for four of the scenarios to be discussed in this chapter are included in this figure. The paths cross the limit of damage lines. By finding the values of \( R \) at which they cross one knows the minimum distance at which a given degree of safety exists. One can interpolate between two adjacent values of \( R \) by assuming a logarithmic scale for \( R \) on Figure 6-3. Alternatively, one can determine the overpressure or impulse at the point of the intersection of the \((i, P_s)\) path and the damage limit curve, and then go back to the figure in the text where the overpressure or impulse was obtained and find the corresponding distance. The resulting building damage for this scenario (No. 1) is presented in Table 6-3.

Overturning of Object:

One can use the methods of Section 3-1.3 to determine the minimum distance from this explosion at which a Saturn V rocket may be placed, for which it will not be overturned by the blast wave. Assume that the fueled Saturn V is standing vertically without restraints. Assume standard atmospheric temperature and pressure. Let \( h = 110 \) m (360 ft), \( h_{cg} = 30 \) m (100 ft), \( b = 10.1 \) m (33 ft), \( H = 10.1 \) m (33 ft), \( h_{bl} = h/2 = 55 \) m (180 ft), \( m = 2.8 \times 10^7 \)
Figure 5.3. Pressure Vs Impulse Diagram for Building Damage with Blast Curves for Scenarios 1 Through 4
TABLE 6-3. BUILDING DAMAGE

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of minor structural damage</td>
<td>1200</td>
<td>3900</td>
</tr>
<tr>
<td>threshold of major structural damage (some load-bearing members fail)</td>
<td>470</td>
<td>1500</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>270</td>
<td>890</td>
</tr>
</tbody>
</table>

kg ($6.2 \times 10^6\text{ lb}_m$), and $C_p = 1.2$. Then, $A = 1100\text{ m}^2\text{ (12000 ft}^2)$, $h/b = 10.9$, and $h_f^2/h = 0.473$. The procedure is to find the minimum impulse $i_c$ for overturning. Setting this equal to the applied impulse $i$ allows one to find the maximum distance for which the vehicle will overturn. For all larger distances, the vehicle will not overturn.

$h/b$ is too large for Figure 3-4, so use Equation 3-4.

$$\frac{i_c h_{bf}}{mg^1/2b^3/2} = \sqrt[3]{\left[\frac{2}{3} + \frac{h^2}{b^2} + \frac{2h^2}{b^2} \left(\frac{h_f^2}{h^2}\right)\right]} \left[\sqrt[4]{\left(\frac{h^2}{b^2}\right)\left(\frac{h_f^2}{h^2}\right)} \left(\frac{b}{h}\right)\left(\frac{h_f}{h}\right)^3\right]$$

= 1.23. Then $i_c = 5810\text{ Pa} \cdot \text{s} (8427\text{ psi} \cdot \text{sec})$

Let $i_c = 5810\text{ Pa} \cdot \text{s}$. Then $i_c / p H = 1.77 (0.000257\text{ psi} \cdot \text{sec})$

This is the minimum nondimensional applied impulse for overturning which is shown on Figure 6-4, a copy of Figure 3-3. The points $\left(\text{a } C_p, i_c / p H, P\right)$ were plotted on Figure 6-4 for several values of $R$. The place where this curve (curve #4) crosses the line of minimum applied impulse for overturning gives the value of $R$ for the "threshold" of overturning. For Scenario 1, this distance is 220 m (720 ft). At distances greater than 220 m (720 ft), a Saturn V will not be overturned by a blast wave from this explosion.

6-2.1.4 Effect of Blast Waves on Humans

The methods described in Section 3-2 are used to determine the effects of blast waves on humans.
Lung Damage

Figure 3-10 in Section 3-2 is used to determine the extent of damage to the lungs of humans by blast waves. Figure 6-5 is a copy of Figure 3-10 with the \( P_s \), \( \frac{1/2}{1/3} \), and \( \frac{m}{p} \) points for selected values of \( R \). \( p_0 = 1.013 \times 10^5 \text{ Pa} \) (14.7 psi). The value of \( m \) is chosen as 5.00 kg (11 lb). The same procedure as the one that was used for building damage is also used here to determine the values of \( R \) for various degrees of lung damage. The results are presented in Table 6-4.

**TABLE 6-4. LUNG DAMAGE**

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of lung damage</td>
<td>170</td>
<td>560</td>
</tr>
<tr>
<td>99%</td>
<td>100</td>
<td>330</td>
</tr>
<tr>
<td>90%</td>
<td>100</td>
<td>330</td>
</tr>
<tr>
<td>50%</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>10%</td>
<td>70</td>
<td>230</td>
</tr>
<tr>
<td>1%</td>
<td>70</td>
<td>230</td>
</tr>
</tbody>
</table>

Ear Damage

Figure 3-11 is used to determine the extent of damage to the ears of humans by blast waves. Figure 6-6 is a copy of Figure 3-11 in Section 3-2 with the \( (P_t, I) \) points for selected values of \( R \). Table 6-5 shows the results. Note that a value of \( R \) cannot be determined for TTS because the curve for Scenario #1 never reaches the TTS damage limit curve, and it is inadvisable to try to extrapolate this curve.
Figure 6.5. Survival Curves for Lung Damage with Curves for Scenarios 1 Through 4
TABLE 6-5. EAR DAMAGE

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>temporary threshold shift (TTS)</td>
<td>over 2000</td>
<td>over 6000</td>
</tr>
<tr>
<td>(20% - no loss)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>threshold of eardrum rupture</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% eardrum rupture</td>
<td>270</td>
<td>890</td>
</tr>
<tr>
<td>100%</td>
<td>130</td>
<td>430</td>
</tr>
</tbody>
</table>

Skull Fracture

Figure 3-12 is used to determine the chance of Skull Fracture due to bodily translation and impact caused by blast waves. Figure 6-7 is a copy of Figure 3-12 with the \( \frac{L_s}{m^{1/3}} \) points for selected values of \( R \).

The body mass, \( m \), is chosen as 5.00 kg (11 lb). The results are presented in Table 6-6.

TABLE 6-6. SKULL FRACTURE

<table>
<thead>
<tr>
<th>Chance of Skull Fracture</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td>350</td>
<td>1100</td>
</tr>
<tr>
<td>threshold</td>
<td>310</td>
<td>1000</td>
</tr>
<tr>
<td>50%</td>
<td>270</td>
<td>890</td>
</tr>
<tr>
<td>near 100%</td>
<td>240</td>
<td>790</td>
</tr>
</tbody>
</table>

Body Translation and Impact

Figure 3-16 is used to determine the probability of mortality from whole body translation and impact upon a hard surface caused by blast waves. Figure 6-8 is a copy of that figure with the \( \frac{L_s}{m^{1/3}} \) points for selected values of \( R \). The body mass \( m \) is chosen as 5.00 kg. Table 6-7 shows the results. Note that no value of distance is given for "near 100% mortality" because it is undesirable to extrapolate the curves in Figure 6-8.
Figure 6.7. Skull Fracture, 0 m Altitude, with Curves for Scenarios 1, 2, and 3
TABLE 6-7. MORTALITY FROM BODY IMPACT

<table>
<thead>
<tr>
<th>Degree of Mortality</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td>340</td>
<td>1120</td>
</tr>
<tr>
<td>threshold of lethality</td>
<td>240</td>
<td>790</td>
</tr>
<tr>
<td>50% mortality</td>
<td>160</td>
<td>520</td>
</tr>
</tbody>
</table>

6-2.1.5 Fragment Characteristics

The initial fragment velocity distribution is obtained from Figure 4-48. 95% of the fragments have an initial velocity less than or equal to 500 m/s (1600 ft/sec).

The fragment mass distribution is obtained from Figure 4-53. 95% of the fragments have a mass less than or equal to 120 kg (264 lb).

The fragment range is obtained from Figure 4-57. 95% of the fragments will strike the ground at a distance less than or equal to 580 m (1900 ft).

6-2.1.6 Appurtenances

As an example of an appurtenance, a cement block identical to that described in Example 1 of Section 4-4 will be used. Suppose that it is located at a distance of 108 m (350 ft) from the source of the blast wave, and one must know its velocity after being picked up by the blast wave.

At \( R = 108 \text{ m} \), \( P = 1.52 \times 10^5 \text{ Pa} \), \( P_s = 1.50 \), and \( I_s = 7.03 \times 10^3 \text{ Pa} \cdot \text{s} \).

The nondimensional impulse is

\[
\overline{I} = \frac{C D^3 a}{P_s (KH + X)} = \frac{1.05 \times 10^5 \times 10^{-3} \times 10^3}{1.52 \times 10^5 \left[ 4(2.5) - 0 \right]} = 1.65 \text{.}
\]

Locating the \((\overline{P}_s, \overline{I})\) point on Figure 4-20, one finds that \( \overline{V} = 3.5 \). Then,

\[
\overline{V} = \frac{\sqrt{P_s A (KH + X)}}{Ma_o} = \frac{(3.5)(1.03 \times 10^5)(6.25)}{(2.8 \times 10^5)(340)} \left[ 4(2.5) - 0 \right] = 0.24 \text{ m/s (0.76 ft/sec).}
\]
This cement block would be thrown at a very low velocity.

1.2 Effect of Fragments on Humans

The effect on humans of flying glass from windows broken by blast waves will be considered. In Example 1 of Section 5-2, the 50% penetration limit velocity \( V_{50} \) for typical glass was found to be 23.4 m/s (76.8 ft/sec). Equation 5-8 relates striking velocity \( V \) to overpressure \( P_e \), which may be \( P_e^g \) or \( P_e^r \), depending upon the orientation of the glass with the blast wave.

\[
V = 0.2336 + (1.206 \times 10^{-4})(t - 7.62 \times 10^{-4}) - 0.928 \quad \text{m/s (270 ft/sec)}
\]

Setting \( V = V_{50} \) one can solve for the minimum \( P_e \) for human injury from flying glass. Letting \( t = 3.175 \times 10^{-3} \text{ m} \) \( P_e = 1.96 \times 10^4 \text{ Pa} \) (284 psi).

For windows where the direction of propagation of the blast wave is parallel to the glass surface, \( P_e = P_r \). Then \( P_r = 1.26 \times 10^4 \text{ Pa} \). From Figure 2-16, \( R = 380 \text{ m} \) (1200 ft). At any distance less than 380 m, there is likelihood of human injury by flying glass from windows with surfaces parallel to the direction of propagation of the blast wave.

For windows where the direction of propagation of the blast wave is normal to the glass surface, \( P_e = P_g \). Solving 5-4 for \( P_g \) and manipulating gives \( P_g = 3.45 \times 10^3 \text{ Pa} \) (507 psi). From Figure 2-16, \( R = 540 \text{ m} \) (1760 ft). Tables 5-8 and 5-9 summarize the damage and injury that can be expected.

2-2.2 Scenario 2. Explosion of Space Shuttle Propellant Tanks in Early Stages of Flight

In this "scenario", the Space Shuttle has lifted off from the pad and started its ascent. A failure occurs in the tankage for the main liquid propellant rocket engine, causing the \( \text{LH}_2 \) and \( \text{LO}_2 \) to mix and ignite within the tanks (CBM). The explosion occurs at about 100 m (328 ft) altitude, where ground reflections do not affect the blast wave properties. This altitude is not great enough, however, to affect blast wave characteristics other than by lack of ground reflection. The initial quantities of propellants at lift-off are given in Table 5-1. Assuming 30 seconds of propellant consumption, remaining weights are:

- **External Tank** (11, 750 kg (167, 000 lb) in)
  - \( \text{LH}_2 \): 344 kg (757, 000 lb) in
  - \( \text{LO}_2 \): 344 kg (757, 000 lb) in
- **Solid Propellants** (885, 000 kg (1, 930, 000 lb) in)
### Table 6-8 Scenario 1: Damage to Structures

<table>
<thead>
<tr>
<th>Damage to Structures</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of overturning Saturn V</td>
<td>220</td>
<td>720</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>270</td>
<td>886</td>
</tr>
<tr>
<td>threshold of major structural damage</td>
<td>350</td>
<td>1148</td>
</tr>
<tr>
<td>% of fragments will strike the ground within this distance</td>
<td>580</td>
<td>1897</td>
</tr>
<tr>
<td>threshold of side-on glass breakage</td>
<td>1100</td>
<td>3607</td>
</tr>
<tr>
<td>threshold of minor structural damage</td>
<td>1200</td>
<td>3937</td>
</tr>
<tr>
<td>threshold of face-on glass breakage</td>
<td>2600</td>
<td>8530</td>
</tr>
</tbody>
</table>

### Table 6-9 Scenario 1: Human Injury

<table>
<thead>
<tr>
<th>Human Injury</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% - 10% survival (lung damage)</td>
<td>70</td>
<td>230</td>
</tr>
<tr>
<td>50% survival (lung damage)</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>90 - 99% survival (lung damage)</td>
<td>100</td>
<td>328</td>
</tr>
<tr>
<td>50% ear drum rupture</td>
<td>130</td>
<td>426</td>
</tr>
<tr>
<td>50% mortality from body impact</td>
<td>160</td>
<td>525</td>
</tr>
<tr>
<td>threshold of lung damage</td>
<td>170</td>
<td>558</td>
</tr>
<tr>
<td>near 100% chance of skull fracture</td>
<td>240</td>
<td>787</td>
</tr>
<tr>
<td>threshold of lethality from body impact</td>
<td>240</td>
<td>787</td>
</tr>
</tbody>
</table>
### Table 6-9 Scenario 1: Human Injury (Concluded)

<table>
<thead>
<tr>
<th>Human Injury</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of eardrum rupture, 50% chance of skull fracture</td>
<td>270</td>
<td>889</td>
</tr>
<tr>
<td>threshold of skull fracture</td>
<td>310</td>
<td>1000</td>
</tr>
<tr>
<td>mostly safe from body impact injury</td>
<td>340</td>
<td>1100</td>
</tr>
<tr>
<td>mostly safe from skull fracture</td>
<td>350</td>
<td>1100</td>
</tr>
<tr>
<td>threshold of injury from flying glass (side-on)</td>
<td>380</td>
<td>1200</td>
</tr>
<tr>
<td>threshold of injury from flying glass (face-on)</td>
<td>640</td>
<td>2100</td>
</tr>
<tr>
<td>temporary threshold shift (hearing)</td>
<td>over 2000</td>
<td>over 6000</td>
</tr>
</tbody>
</table>
In this type of accident, we can estimate explosive yield and blast effects, as well as initial velocities of fragments, making the same restrictive assumptions as for Scenario I. But, fragment trajectories and impact conditions cannot be estimated, because all of the graphs in Chapter IV relating to these properties of fragments are based on explosions which occur on the ground surface. The methods for computing fragment trajectories are applicable, but the appropriate computer programs have not been exercised for initial conditions of significant altitudes above the ground. Because we cannot estimate impact conditions, we, of course, cannot use the methods of Chapter V for predicting damaging effects of fragments.

6-2.2.1 Yield

The yield is calculated by the methods in Chapter I. According to Table 1-2, values for the yield are obtained from Figures 1-1 and 1-6, and the lower value is used:

From Figure 1-1, for a fuel and oxidizer mass of $5.20 \times 10^5$ kg, $y = 0.06$. The multiplier is 370%. $Y = (0.06)(370\%) = 22.2\%$

In Figure 1-6, choose ignition for maximum $Y$. Then $Y = 58\%$. Using the lower value, $Y = 22.2\%$.

The overpressure and impulse curves for propellant explosions were obtained from data of explosions occurring on the ground. An argument similar to that used in Section A-2-b can be used here. The ground explosion can be approximated by the vessel in Figure A2-4b, where half of the energy is released above the ground. To apply the analysis to an explosion in free air, the energy (or yield) must be halved. Then $Y = 11.1\%$.

6-2.2.2 Overpressure and specific impulse

The overpressure and specific impulse are calculated by the methods in Chapter II. The effective mass is calculated from Equation 2-9:

$W = W_e \times \frac{y}{100}$. Table 2-1 gives the procedure for finding the overpressure and specific impulse as functions of distance:

$W = W_e \times \frac{y}{100} = (5.20 \times 10^5$ kg $\times \frac{11.1\%}{100\%}) = 57,500 \text{ kg} = 126,000 \text{ lb}$

According to Table 2-1, the overpressure is read from Figure 2-14, and the specific impulse is read from Figure 2-15.

The following (Table 6-10) is a list of overpressures ($P_o$) and specific impulses ($I$) from Figure 2-14 and 2-15. The solid lines in these figures were used.
### Table 6.10: Overpressures and Impulses

<table>
<thead>
<tr>
<th>( \chi ) (% s)</th>
<th>( P_0 ) (Pa)</th>
<th>( P_1 ) (Pa)</th>
<th>( \Delta P ) (Pa)</th>
<th>( \Delta p ) (Pa/s/km(^{1/3}))</th>
<th>( I_0 ) (m/s(^2))</th>
<th>( I_f ) (m/s(^2))</th>
<th>( \frac{I_f}{P_0} ) (m/s(^2)/Pa(^{1/3}))</th>
<th>( \frac{I_f}{P_0} ) (m/s(^2)/Pa(^{1/3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.4</td>
<td>3.40 × 10^4</td>
<td>7.48 × 10^4</td>
<td>4.08 × 10^4</td>
<td>1.54 × 10^4</td>
<td>1.54 × 10^4</td>
<td>1.24 × 10^4</td>
<td>0.28 × 10^4</td>
<td>0.28 × 10^4</td>
</tr>
<tr>
<td>21.2</td>
<td>0.009</td>
<td>1.01 × 10^6</td>
<td>11.2</td>
<td>7.70 × 10^4</td>
<td>1.01 × 10^4</td>
<td>9.02 × 10^4</td>
<td>5.28 × 10^4</td>
<td>5.28 × 10^4</td>
</tr>
<tr>
<td>16.4</td>
<td>2.000</td>
<td>3.99 × 10^6</td>
<td>23.7</td>
<td>4.23 × 10^4</td>
<td>2.11 × 10^4</td>
<td>2.47 × 10^4</td>
<td>7.77 × 10^4</td>
<td>7.77 × 10^4</td>
</tr>
<tr>
<td>7.21</td>
<td>1.25 × 10^6</td>
<td>3.11 × 10^6</td>
<td>1.15 × 10^4</td>
<td>4.27 × 10^4</td>
<td>1.05 × 10^4</td>
<td>4.17 × 10^4</td>
<td>5.21 × 10^4</td>
<td>5.21 × 10^4</td>
</tr>
<tr>
<td>16.4</td>
<td>3.60 × 10^6</td>
<td>6.29 × 10^6</td>
<td>2.44 × 10^6</td>
<td>4.00 × 10^4</td>
<td>1.00 × 10^4</td>
<td>1.26 × 10^4</td>
<td>0.872</td>
<td>0.872</td>
</tr>
<tr>
<td>16.4</td>
<td>0.004</td>
<td>1.03 × 10^6</td>
<td>1.38</td>
<td>1.20 × 10^4</td>
<td>1.38 × 10^4</td>
<td>1.59 × 10^4</td>
<td>5.000</td>
<td>5.000</td>
</tr>
<tr>
<td>2.00</td>
<td>4.20 × 10^5</td>
<td>0.0406</td>
<td>8.41 × 10^5</td>
<td>1.16 × 10^2</td>
<td>1.16 × 10^2</td>
<td>7.49 × 10^2</td>
<td>2.48</td>
<td>2.48</td>
</tr>
</tbody>
</table>

\( \chi \) in m \times 3.281

\( \text{psi} \cdot \text{sec} = \text{Pa} \times 1.450 \times 10^{-4} \)

\( \text{psi} = \text{Pa} \times 1.450 \times 10^{-4} \)
2.2.3 Effect of Blast Waves on Structures

The methods described in Section 3.1 are used to determine the effects of blast waves on structures.

Glass Breakage

In Scenario #1, it was found that the minimum overpressure to break a typical pane of glass facing the explosion is 2420 Pa. For a pane of glass parallel to the path of the blast wave, the minimum overpressure required to break it is 4840 Pa. We are concerned with damage on the ground. The blast wave will be reflected by the ground. Therefore, the reflected overpressure is important here. Replace $P_1$ by $P$ and calculate the new $P$.

From Equation 2-3, $P = 2420$ Pa (0.35$g$ psi), $r_1P = 1210$ Pa (0.186 psi). For $P = 4840$ Pa (0.702 psi), $r_2P = 2420$ Pa (0.35$g$ psi). As in this particular calculation, $P_r$ and $r_1$ will be used in place of $P_1$ and $r_1$ throughout this scenario. From Figure 2-14, one would expect to be able to obtain the distance at which these overpressures would be observed for Scenario #2. For $P = 2420$ Pa, or $P = 1210$ Pa a value of $R/W^{1/3}$ cannot be read from Figure 2-14, but it can be seen that the distance $R$ must be over 1000 m (3000 ft). Therefore, glass breakage will occur on the ground below the explosion.

Building Damage

The $(i, P)$ points for selected values of $R$ for Scenario #2 are included in Figure 6-3. Using the same techniques as in Scenario #1, but using the reflected overpressures and impulses, Table 6-11 was assembled. (The reflected impulses are calculated by Equation 2-4). For this explosion at an altitude of 1000 m, there might be building damage on the ground (possibly minor structural damage).

**TABLE 6-11. BUILDING DAMAGE**

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of minor structural damage</td>
<td>over 800</td>
<td>over 2600</td>
</tr>
<tr>
<td>threshold of major structural damage</td>
<td>450</td>
<td>1500</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>300</td>
<td>980</td>
</tr>
</tbody>
</table>
### Effect of Blast Waves on Humans

The methods described in Section 3-2 are used to determine the effects of blast waves on humans.

**Lung Damage**

\[ \frac{1}{P_s} \frac{1}{2} \frac{1}{3} \] points for selected values of \( R \) for Scenario 2 are included in Figure 6-5. Table 6-12 was obtained from this figure. There will be no lung damage to humans from this explosion.

#### Table 6-12. Lung Damage

<table>
<thead>
<tr>
<th>Chance of Survival</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of lung damage</td>
<td>170</td>
<td>560</td>
</tr>
<tr>
<td>99.9%</td>
<td>100</td>
<td>330</td>
</tr>
<tr>
<td>99.7%</td>
<td>100</td>
<td>330</td>
</tr>
<tr>
<td>99%</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>90%</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>10%</td>
<td>90</td>
<td>300</td>
</tr>
<tr>
<td>0%</td>
<td>90</td>
<td>300</td>
</tr>
</tbody>
</table>

**Ear Damage**

The \( (P_s, l) \) points for selected values of \( R \) for Scenario 2 are included in Figure 8-9. Table 6-13 was assembled using this figure. The data could not be extrapolated to find the distance for "Temporary Threshold Shift."
TABLE 6-13. EAR DAMAGE

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of eardrum rupture</td>
<td>340</td>
<td>1100</td>
</tr>
<tr>
<td>50% eardrum rupture</td>
<td>150</td>
<td>490</td>
</tr>
</tbody>
</table>

Skull Fracture

\[
\frac{1}{s}
\]

The \( \{P_s, 1/3\} \) points for selected values of \( R \) for Scenario -2 are included in Figure 6-7. Table 6-14 is the result.

TABLE 6-14. SKULL FRACTURE

<table>
<thead>
<tr>
<th>Chance of Skull Fracture</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td>390</td>
<td>1290</td>
</tr>
<tr>
<td>threshold</td>
<td>360</td>
<td>1200</td>
</tr>
<tr>
<td>50%</td>
<td>340</td>
<td>1100</td>
</tr>
<tr>
<td>near 100%</td>
<td>320</td>
<td>1000</td>
</tr>
</tbody>
</table>

Body Translation and impact

\[
\frac{1}{s}
\]

The \( \{P_s, 1/3\} \) points for selected values of \( R \) for Scenario =2 are included in Figure 6-8. In the result, Table 6-15, note that no value of \( R \) is given for "near 100% mortality" because extrapolation is not recommended.
TABLE 6-15. MORTALITY FROM BODY IMPACT

<table>
<thead>
<tr>
<th>Degree of Mortality</th>
<th>(m) Distance</th>
<th>(ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>threshold of lethality</td>
<td>370</td>
<td>1200</td>
</tr>
<tr>
<td>50% mortality</td>
<td>300</td>
<td>1000</td>
</tr>
</tbody>
</table>

There would be injury to people on the ground due to the blast wave.

6-2.2.5 Fragment Characteristics

The initial fragment velocity distribution is obtained from Figure 4-50; 95% of the fragments have an initial velocity less than or equal to 1000 m/s (3300 ft/s).

The fragment mass distribution cannot be determined with the methods that are presently available. Figures 4-51 and 4-52 show the mass distribution for CBM explosions with yields of 5% and 1.1%. There is obviously a strong dependence of mass distribution upon yield, and it is not valid to attempt to extrapolate the information in these figures to a yield of 11.1%.

6-2.2.6 Effect of Fragments on Humans

The effect on humans of flying glass from windows broken by blast waves will be considered. In Scenario 1, it was shown that for injury from flying glass, \( P = 9.45 \times 10^3 \text{ Pa} \) (137 psi) with the blast wave striking the pane of glass head-on, and \( P = 1.96 \times 10^4 \text{ Pa} \) (2.84 psi) with the blast wave propagating in a direction parallel to the surface of the glass.

As for other types of damage in this Scenario, the reflected over-pressure must be used in place of \( P \). Then, \( P = 9.45 \times 10^3 \text{ Pa} \) for normal blast impact and \( P = 1.96 \times 10^4 \text{ Pa} \) for blast impact parallel to the glass surface. The corresponding values of \( P \) are then \( 4.04 \times 10^3 \text{ Pa} \) (60.0 psi) and \( 9.45 \times 10^3 \text{ Pa} \) (137 psi). From Figure 2-14 of Chapter II, at a distance \( R \) less than 150 m (492 ft), there is likelihood of human injury by flying glass from windows normal to the direction of travel of the blast wave. The corresponding distance for windows parallel to the direction of travel is 422 m (1400 ft).

There will be no injury by flying glass to humans on the ground.
6-2.3 Scenario #3. Fall Back of the Titan Centaur Vehicle During Launch

The scenario for this accident is nearly identical to the first, except that the launch vehicle is different. The Titan Centaur is a three-stage, liquid-fueled rocket vehicle with a mass fraction of 0.85. The first two stages are fueled with the hypergolic propellant combination N₂O₄ and Aerozine 50, and the third stage with LO₂ and LH₂. The missile is assumed to rise a short distance when loss of thrust in the first stage allows it to fall back on the launch pad. The impact is assumed to be sufficiently violent that at least the first stage ruptures and spills its propellant on the launch pad. Impact velocity is 14.0 m/s (45.9 ft/s). Table 6-16 gives propellant masses and estimates of structural masses for each stage, based on the mass fraction of 0.85.

Several possible assumptions regarding severity of the fall-back accident can be made. We will make two such assumptions, and predict effects for both. These assumptions are:

(1) Only the first stage ruptures and explodes, and

(2) Both first and second stages rupture and explode

For the first assumption, the structure of the first stage is assumed to be the only source of fragments, while for the second assumption, the structures of both first and second stages are fragment sources.

**TABLE 6-16. PROPELLANTS AND STRUCTURES AT LIFT-OFF OF TITAN CENTAUR ROCKET**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Propellants</th>
<th>Propellant Masses (kg)</th>
<th>Propellant Masses (lb)</th>
<th>Structure Masses (kg)</th>
<th>Structure Masses (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titan First</td>
<td>N₂O₄</td>
<td>75 900</td>
<td>167,000</td>
<td>20 600</td>
<td>45,300</td>
</tr>
<tr>
<td></td>
<td>Aerozine 50</td>
<td>40 400</td>
<td>88,900</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Titan Second</td>
<td>N₂O₄</td>
<td>10 500</td>
<td>22,900</td>
<td>5 360</td>
<td>11,800</td>
</tr>
<tr>
<td></td>
<td>Aerozine 50</td>
<td>10 900</td>
<td>24,000</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Centaur (Third)</td>
<td>LO₂</td>
<td>11 400</td>
<td>25,100</td>
<td>2 410</td>
<td>5300</td>
</tr>
<tr>
<td></td>
<td>LH₂</td>
<td>2 270</td>
<td>4990</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
6-2.3.1 Yield

The yield is calculated by the methods in Chapter 1. According to Table 1-2 values for the yield are obtained from Figure 1-1 and Table 1-1, and the lower value is used:

From Figure 1-1, for a fuel and oxidizer mass of 116,000 kg (255,000 lbm) (only the first stage), or 146,000 kg (321,000 lbm) (first and second stages), \( y = 0.05 \). The multiplier is 240%. \( Y = (0.06)(240\%) = 14.4\% \). From Table 1-1, \( Y = 1.5\% \). Using the lower value, \( Y = 1.5\% \).

6-2.3.2 Overpressure and Specific Impulse

The overpressure and specific impulse are calculated by the methods in Chapter II. The effective mass is calculated from Equation 2-9.

\[ W = W \times \frac{Y}{100} \]

Table 2-1 gives the procedure for finding the overpressure and specific impulse as functions of distance:

\[ W_1 = W \times \frac{Y}{100} = (1.16 \times 10^5 \text{ kg}) \times \frac{1.5\%}{100\%} = 1740 \text{ kg (3820 lbm)} \]

(only the first stage)

\[ W_{1+2} = (1.46 \times 10^5 \text{ kg}) \times \frac{1.5\%}{100\%} = 2190 \text{ kg (4820 lbm)} \]

(first and second stages)

According to Table 2-1, the overpressure is read from Figure 2-7, and specific impulse is read from Figure 2-8.

The following (Table 6-17) is a list of overpressures \( (P_s) \) and specific impulses \( (I_s) \) from Figures 2-7 and 2-8. The solid lines in the figures were used.

6-2.3.3 Effect of Blast Waves on Structures

The methods described in Section 3-1 are used to determine the effects of blast waves on structures.

Glass Breakage

In Scenario #1, it was found that the minimum overpressure to break a typical pane of glass facing the explosion is 2420 Pa (0.351 psi). For a pane of glass parallel to the path of the blast wave, the minimum overpressure required to break it is 4840 Pa (0.702 psi). From Figure 2-7 one would expect to be able to obtain the distance at which these overpressures would be observed for Scenario #3. For \( P_s = 2420 \text{ Pa or 4840 Pa} \), a value of 6.30

6-30
### Table 6.12. Overpressures and Impulses

<table>
<thead>
<tr>
<th>$\rho$ (g/cm³)</th>
<th>$p_0$ (Pa)</th>
<th>$p_1$ (Pa)</th>
<th>$t_0$ (s)</th>
<th>$t_1$ (s)</th>
<th>$p_{1/2}$ (Pa)</th>
<th>$p_{1/2} / S$ (Pa/cm²)</th>
<th>$R$ (m)</th>
<th>$\Delta p$ (Pa)</th>
<th>$\Delta t$ (s)</th>
<th>$\Delta p / \Delta t$ (Pa/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24</td>
<td>2.445</td>
<td>3.41 x 10³</td>
<td>0.16</td>
<td>0.02</td>
<td>4.78</td>
<td>4.78</td>
<td>5.02</td>
<td>2100</td>
<td>1600</td>
<td>350</td>
</tr>
<tr>
<td>0.40</td>
<td>2.445</td>
<td>3.41 x 10³</td>
<td>0.16</td>
<td>0.02</td>
<td>4.78</td>
<td>4.78</td>
<td>5.02</td>
<td>2100</td>
<td>1600</td>
<td>350</td>
</tr>
<tr>
<td>0.60</td>
<td>2.445</td>
<td>3.41 x 10³</td>
<td>0.16</td>
<td>0.02</td>
<td>4.78</td>
<td>4.78</td>
<td>5.02</td>
<td>2100</td>
<td>1600</td>
<td>350</td>
</tr>
<tr>
<td>0.80</td>
<td>2.445</td>
<td>3.41 x 10³</td>
<td>0.16</td>
<td>0.02</td>
<td>4.78</td>
<td>4.78</td>
<td>5.02</td>
<td>2100</td>
<td>1600</td>
<td>350</td>
</tr>
<tr>
<td>1.00</td>
<td>2.445</td>
<td>3.41 x 10³</td>
<td>0.16</td>
<td>0.02</td>
<td>4.78</td>
<td>4.78</td>
<td>5.02</td>
<td>2100</td>
<td>1600</td>
<td>350</td>
</tr>
</tbody>
</table>

(R = m x 1.281)

($p_{\text{atm}} = Pa \times 1.44 x 10^4$)

($p_{\text{atm}} = Pa \times 1.44 x 10^4$)
R/W cannot be read from Figure 2-7, but it can be seen that R/W must be over 60, corresponding to $R > 720$ m (2400 ft) for only one stage exploding and $R > 780$ m (2500 ft) for both the first and second stages exploding.

Building Damage

The $(i, P, \gamma)$ points for selected values of $R$ for Scenario 3 are included in Figure 5. Using the same technique as in Scenario 1, Table 6-18 was assembled.

**TABLE 6-18. BUILDING DAMAGE**

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Stage</td>
<td>1st and 2nd Stages</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(m)</td>
<td>(ft)</td>
<td>(m)</td>
</tr>
<tr>
<td>threshold of minor structural damage</td>
<td>380</td>
<td>1200</td>
<td>416</td>
</tr>
<tr>
<td>threshold of major structural damage</td>
<td>140</td>
<td>460</td>
<td>160</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>95</td>
<td>310</td>
<td>100</td>
</tr>
</tbody>
</table>

Overturning of Objects - Two Stages Exploding

The $(a, C, \alpha, \phi, \psi, \theta)$ points for selected values of $R$ are included in Figure 6-4. For $R < 95$ m (310 ft), a Saturn V rocket can be expected to overturn as a result of this explosion.

6.2.3.4 Effect of Blast Waves on Humans

The methods described in Section 3-2 are used to determine the effects of blast waves on humans.

Line Damage

The $(\bar{P}, -\frac{1}{2}, \frac{1}{3})$ points for selected values of $R$ for Scenario 3 are included in Figure 6-5. Table 6-19 was obtained from this figure.
TABLE 6-19. LUNG DAMAGE

<table>
<thead>
<tr>
<th>Chance of Survival</th>
<th>Distance</th>
<th>1st Stage</th>
<th>1st and 2nd Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m)</td>
<td>(ft)</td>
<td>(m)</td>
</tr>
<tr>
<td>threshold of lung damage</td>
<td>120</td>
<td>390</td>
<td>130</td>
</tr>
<tr>
<td>99%</td>
<td>53</td>
<td>170</td>
<td>55</td>
</tr>
<tr>
<td>90%</td>
<td>46</td>
<td>150</td>
<td>52</td>
</tr>
<tr>
<td>50%</td>
<td>39</td>
<td>130</td>
<td>45</td>
</tr>
<tr>
<td>10%</td>
<td>35</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>1%</td>
<td>32</td>
<td>100</td>
<td>36</td>
</tr>
</tbody>
</table>

Ear Damage

The \((P_{0.5}, L_0)\) points for selected values of \(R\) for Scenario 3 are included in Figure 6-6. Table 6-20 was assembled using this figure. The data could not be extrapolated to find the distance for "Temporary Threshold Shift".

TABLE 6-20. EAR DAMAGE

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance</th>
<th>1st Stage</th>
<th>1st and 2nd Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of eardrum rupture</td>
<td>260</td>
<td>850</td>
<td>280</td>
</tr>
<tr>
<td>50% eardrum rupture</td>
<td>210</td>
<td>700</td>
<td>120</td>
</tr>
</tbody>
</table>

Skull Fracture

The \((P_{0.5}, L_0)\) points for selected values of \(R\) for Scenario 3 are included in Figure 6-7. Table 6-21 is the result. The curves in Figure 6-6 should not be extrapolated to find the damage limits beyond "mostly safe."
TABLE 6-21. SKULL FRACTURE

<table>
<thead>
<tr>
<th>Chance of Skull Fracture</th>
<th>distances</th>
<th>1st Stage (m)</th>
<th>1st and 2nd Stages (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td></td>
<td>110</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>360</td>
<td>390</td>
</tr>
</tbody>
</table>

Body Translation and Impact

The \( P_s \) points for selected values of \( R \) for Scenario 43 are included in Figure 6-8. In the result, Table 6-22, note that a value of \( R \) is given only for "mostly safe" because extrapolation is not recommended.

TABLE 6-22. MORTALITY FROM BODY IMPACT

<table>
<thead>
<tr>
<th>Degree of Mortality</th>
<th>distances</th>
<th>1st Stage (m)</th>
<th>1st and 2nd Stages (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mostly safe</td>
<td></td>
<td>110</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>360</td>
<td>390</td>
</tr>
</tbody>
</table>

6-2.3.5 Fragment Characteristics

The fragment mass and initial velocity distributions cannot be determined by the methods in Chapter IV because the figures there are not to be used for hypergolic propellants.

The fragment range is obtained from Figure 4-57. 95% of the fragments will strike the ground at a distance less than or equal to 270 m (889 ft).

6-2.3.6 Appurtenances

As an example of an appurtenance, a cement block will be used as in Scenario 11. The stand-off distance is 52 m (170 ft), where \( P_s = 1.2 \times 10^6 \) Pa, \( P = 2.4 \) and \( \rho = 1140 \) Pa - s. Then \( t = 0.208 \), locating the impulse point on Figure 4-26, one finds that \( V = 5.8 \). Then \( V = 0.76 \) m/s (1.7 ft/s). Thus, this cement block would be thrown at a low velocity.
The effect on humans of flying glass from windows broken by blast waves will be considered. In Scenario #1, it was shown that for injury from flying glass, $P_s = 0.45 \times 10^3$ Pa with the blast wave striking the pane of glass head-on, and $P_s = 1.96 \times 10^4$ Pa with the blast wave propagating in a direction parallel to the surface of the glass. From Figure 2-7, at a distance $R/W^{1/3}$ less than 38.0, there is likelihood of human injury by flying glass from windows parallel to the direction of travel of the blast wave. The corresponding distance $R/W^{1/3}$ for windows normal to the direction of travel cannot be read from the graph. The maximum distances from the explosion for human injury from flying glass are shown in Table 6-23.

The damage and injury to be expected in Scenario #3 are summarized in Tables 6-24 and 6-25.

**TABLE 6-23. MINIMUM SAFE DISTANCE FOR INJURY BY FLYING GLASS**

<table>
<thead>
<tr>
<th>Distance</th>
<th>1st Stage (m)</th>
<th>1st and 2nd Stages (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>460</td>
<td>490</td>
</tr>
</tbody>
</table>

**TABLE 6-24. SCENARIO 3: DAMAGE TO STRUCTURES**

<table>
<thead>
<tr>
<th>Damage to Structures</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn V overturning</td>
<td>95</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>310</td>
</tr>
<tr>
<td>threshold major structural damage</td>
<td>140</td>
</tr>
<tr>
<td>$95%$ of fragments will strike the ground within this distance</td>
<td>270</td>
</tr>
<tr>
<td>threshold minor structural damage</td>
<td>380</td>
</tr>
<tr>
<td>threshold of side wall glass breakage</td>
<td>$&gt; 720$</td>
</tr>
</tbody>
</table>

Threshold of side wall glass breakage
### TABLE 6-25. SCENARIO 3: HUMAN INJURY

<table>
<thead>
<tr>
<th>Human Injury</th>
<th>Distance 1st Stage (m)</th>
<th>Distance 1st and 2nd Stages (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% survival (lung damage)</td>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>10% survival (lung damage)</td>
<td>35</td>
<td>110</td>
</tr>
<tr>
<td>50% survival (lung damage)</td>
<td>39</td>
<td>130</td>
</tr>
<tr>
<td>90% survival (lung damage)</td>
<td>46</td>
<td>150</td>
</tr>
<tr>
<td>99% survival (lung damage)</td>
<td>53</td>
<td>170</td>
</tr>
<tr>
<td>50% eardrum rupture, mostly safe from skull fracture and body impact</td>
<td>110</td>
<td>360</td>
</tr>
<tr>
<td>threshold of lung damage</td>
<td>120</td>
<td>390</td>
</tr>
<tr>
<td>threshold of eardrum rupture</td>
<td>260</td>
<td>850</td>
</tr>
<tr>
<td>threshold injury from flying glass (side-on)</td>
<td>460</td>
<td>1500</td>
</tr>
</tbody>
</table>

---

### 6-2.4 Scenario #4. Pressure Burst of a Centaur Pressurant Tank During Pneumatic Strength Testing in a Shop Area

One of the spherical pressurant tanks for the Centaur launch vehicle is assumed to fail catastrophically while it is being proof tested in a shop area. The vessel is spherical, made of Ti-6 Al-4V alloy, has a volume of 0.1206 m³ (4.26 ft³), and fails at the design pressure of 20.7 MPa (3000 psi), and at room temperature of 25 °C (77 °F). The vessel is being pressurized with helium, with a ratio of 1.07. The radius of the vessel is 0.507 m (1.66 ft), and the wall thickness is 4.60 mm (0.181 ft). The density of the titanium alloy is 4.50 Mg/m³ (279 lb/ft³), making the mass of the vessel 24.4 kg (53.5 lbm).
6-2.4.1 Overpressure and Specific Impulse

Determine the starting point and locate it on Figure 2-18. Use the nearest curves to find the overpressure versus distance behavior. Use Figure 2-21 to determine the specific impulse versus distance behavior:

\[ p_1 = 2.07 \times 10^7 \text{ Pa (g)} = 2.08 \times 10^7 \text{ Pa (absolute)} \]
\[ p_a = 1.013 \times 10^5 \text{ Pa. Then } \frac{p_1}{p_a} = 205.3 \]
\[ T_1 = T_a = 208^\circ \text{K. Then } \frac{T_1}{T_a} = 1.00. \]

For \( v_1 = 1.667 \), from Figure 2-21, \( \bar{p}_s \) so = 4.8.

\[
\bar{R}_o = \left[ \frac{\frac{4\pi}{3} \left( \frac{p_1}{p_a} - 1 \right)}{v_1 - 1} \right]^{1/3} = \left[ \frac{\frac{4\pi}{3} \left( 2.08 \times 10^7 \text{ Pa} - 1 \right)}{1.013 \times 10^5 \text{ Pa} - 1.667 - 1} \right]^{1/3} = 0.0920
\]

In Figure 2-16 the point \( (\bar{R}, \bar{P}) \) lies nearest the 4th curve from the bottom. This gives \( \bar{P}_s \) versus \( \bar{R}_o \) and

\[
\bar{R} = \bar{R}_o \left[ \frac{\frac{4\pi}{3} \left( \frac{p_1}{p_a} - 1 \right)}{v_1 - 1} \right]^{1/3}
\]

\[
= \bar{R}(0.307 \text{ m}) \left[ \frac{\frac{4\pi}{3} \left( 2.08 \times 10^7 \text{ Pa} - 1 \right)}{1.013 \times 10^5 \text{ Pa} - 1.667 - 1} \right] = \bar{R}(3.336) \text{ (r in m)}
\]

\[
\bar{p}_s = \frac{p_s - p_a}{p_a}
\]
Then \( p_s - p_a = p_a \bar{p}_s = (1.013 \times 10^5 \text{ Pa}) \bar{p}_s \) in Pa.
Overpressures and specific impulses versus distance are listed in Table 6-26 and graphed in Figures 6-9 and 6-10.

6.2.4.2 Effect of Blast Waves on Structures

The methods described in Section 3-1 are used to determine the effects of blast waves on structures.

Glass Breakage

In Scenario 1, it was found that the minimum overpressure to break a typical pane of glass facing the explosion is 2420 Pa. For a pane of glass parallel to the path of the blast wave, the minimum overpressure required to break it is 4840 Pa. From Figure 6-9 one can obtain the distance at which these overpressures would be observed for Scenario 4. For \( P_s = 2420 \text{ Pa} \), it can be seen that the distance \( R \) must be 28 m (92 ft). For \( P_s = 4840 \text{ Pa} \), the distance \( R \) is 10 m (32 ft).

Building Damage

The \( (i, P_s) \) points for selected values of \( R \) for Scenario 4 are included in Figure 6-3. Using the same techniques as in Scenario 1, Table 6-27 was assembled. It is unlikely that any building damage will result.
### Table D-26. Overpressures and Specific Impulses for a Bursting Pressurant Tank

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \rho )</th>
<th>( p' )</th>
<th>( \frac{1}{T} )</th>
<th>( \frac{1}{P_o \cdot s} )</th>
<th>( \frac{1}{P_o \cdot s^1/3} )</th>
<th>( \frac{1}{P_o^{1/2} \cdot s^1/3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>0.007</td>
<td>4.5</td>
<td>4.4 x 10^5</td>
<td>1.1</td>
<td>1100</td>
<td>60.0</td>
</tr>
<tr>
<td>0.10</td>
<td>0.33</td>
<td>4.4</td>
<td>3.9 x 10^6</td>
<td>1.3</td>
<td>280</td>
<td>6.0</td>
</tr>
<tr>
<td>0.20</td>
<td>0.67</td>
<td>2.7</td>
<td>2.7 x 10^5</td>
<td>2.7 x 10^{-1}</td>
<td>76</td>
<td>66</td>
</tr>
<tr>
<td>0.30</td>
<td>1.0</td>
<td>1.9</td>
<td>1.9 x 10^5</td>
<td>1.9 x 10^{-1}</td>
<td>150</td>
<td>60</td>
</tr>
<tr>
<td>0.50</td>
<td>1.7</td>
<td>1.0</td>
<td>1.0 x 10^5</td>
<td>3.0 x 10^{-2}</td>
<td>94</td>
<td>65</td>
</tr>
<tr>
<td>0.90</td>
<td>3.3</td>
<td>0.49</td>
<td>6.6 x 10^4</td>
<td>5.4 x 10^{-2}</td>
<td>55</td>
<td>32</td>
</tr>
<tr>
<td>1.3</td>
<td>1.7</td>
<td>0.16</td>
<td>4.2 x 10^4</td>
<td>2.6 x 10^{-2}</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
<td>0.01</td>
<td>1.6 x 10^3</td>
<td>5.4 x 10^{-3}</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>44</td>
<td>150</td>
<td>0.014</td>
<td>1.4 x 10^3</td>
<td>4.1 x 10^{-3}</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>100</td>
<td>330</td>
<td>1.4 x 10^{-3}</td>
<td>1.4 x 10^{-2}</td>
<td>4.7 x 10^{-4}</td>
<td>0.48</td>
<td>0.28</td>
</tr>
<tr>
<td>1000</td>
<td>330</td>
<td>1.4 x 10^{-4}</td>
<td>1.1 x 10^{-1}</td>
<td>4.6 x 10^{-6}</td>
<td>0.047</td>
<td>0.027</td>
</tr>
</tbody>
</table>

\( \rho = \text{m} \times 3.281 \)
\( \text{psi} = \text{Pa} \times 1.450 \times 10^{-4} \)
\( \text{psi}-\text{sec} = \text{Pa} \cdot \text{s} \times 1.450 \times 10^{-4} \)
Figure 6-3. Overpressure Vs Distance for Pressurant Tank Burst
Figure 6-10. Specific Impulse Vs Distance for Pressurant Tank Burst
TABLE 6-27. BUILDING DAMAGE

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of minor structural damage</td>
<td>1.3</td>
<td>4.3</td>
</tr>
<tr>
<td>threshold of major structural damage</td>
<td>0.6</td>
<td>2</td>
</tr>
<tr>
<td>threshold of partial demolition</td>
<td>0.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

6-2.4.3 Effect of Blast Waves on Humans

The methods described in Section 3-2 are used to determine the effects of blast waves on humans.

Lung Damage

\[ \frac{1}{s} \]

The \((P_r, \frac{1}{2}, \frac{1}{3})\) points for selected values of \(R\) for Scenario #1 are included in Figure 6-5. Table 6-28 was obtained from this figure. The "1% survivability" limit cannot be determined, but it certainly lies within the gas vessel, and therefore is of no interest.

TABLE 6-28. LUNG DAMAGE

<table>
<thead>
<tr>
<th>Chance of Survival</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of lung damage</td>
<td>1.2</td>
<td>3.0</td>
</tr>
<tr>
<td>99%</td>
<td>0.54</td>
<td>1.7</td>
</tr>
<tr>
<td>90%</td>
<td>0.48</td>
<td>1.6</td>
</tr>
<tr>
<td>50%</td>
<td>0.41</td>
<td>1.4</td>
</tr>
<tr>
<td>10%</td>
<td>0.35</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Ear Damage

The \((P_r, \frac{1}{5})\) points for selected values of \(R\) for Scenario #4 are
included in Figure 6-6. Table 6-29 was assembled using this figure.
Although it is inadvisable to extrapolate the curves in Figure 6-6, one can estimate values of R for "threshold of eardrum rupture" and "50% eardrum rupture".

**Table 6-29. EAR DAMAGE**

<table>
<thead>
<tr>
<th>Degree of Damage</th>
<th>Distance (m)</th>
<th>Distance (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold of eardrum rupture</td>
<td>4 (approx.)</td>
<td>13 (approx.)</td>
</tr>
<tr>
<td>50% eardrum rupture</td>
<td>1.6 (approx.)</td>
<td>5 (approx.)</td>
</tr>
<tr>
<td>TTS</td>
<td>147</td>
<td>480</td>
</tr>
</tbody>
</table>

**Skull Fracture**

The \( \frac{1}{3} \) points for selected values of R for Scenario #4 could not be included in Figure 6-7. Because extrapolation is not recommended, little information on safe stand-off distances (with respect to skull fracture) can be obtained. It appears that distances greater than about 1 m (3 ft) will be safe for humans for this type of injury.

**Body Translation and Impact**

The \( \frac{1}{3} \) points for selected values of R for Scenario #4 could not be included in Figure 6-8. As above, it seems that a safe distance for humans, considering whole body translation and impact, is about 1 m (3 ft).

**6-2.4.4 Appurtenances**

This vessel burst occurs in a laboratory. A reasonable object for an appurtenance might be a hammer. This is grossly idealized as two vertical cylinders, one on top of the other. The top cylinder, the head, has a radius of 0.01905 m (0.75 in), a length of 0.1016 m (4.0 in), and a mass of 0.918 kg (2.0 lb). The lower cylinder (the handle) has a radius of 0.01905 m (0.75 in), a length of 0.254 m (10 in), and a mass of 0.679 kg (1.5 lb). The total mass is 1.60 kg (3.537 lb), and the cross-sectional area facing the blast wave A is 0.0135 m² (0.145 ft²). \( C_D = 1.20 \), \( K = 4 \).
Then, \( X = 0 \), and \( H = 0.01905 \) m (0.06250 ft). Assume that the hammer is located 0.067 m (2.19 ft) from the center of the vessel. At this distance, \( P_s = 2.74 \times 10^5 \) Pa (39.7 psi), \( P_g = 2.7 \), and \( I_s = 275 \) Pa·s (0.0399 psi·s).

For Figure 4-26, \( \bar{T} = \frac{C_1 A}{P_s (K + X)} \)

\[
= \frac{(1.20)(275)(331)}{(2.74 \times 10^5)(4(0.01905) + 0.7)} = 5.23.
\]

From that graph, \( \bar{v} = 20 \). Then \( \bar{v} = \frac{\sqrt{P_s A(K + X)}}{\sqrt{P_s}} = \frac{(20)(1.013 \times 10^5)(0.0125)(4(0.01905) + 0.7)}{(0.679)(331)} = 9 \) m/s (30 ft/s). A hammer can be thrown at a high enough velocity to be dangerous.

6-2.4.5 Effect of Fragments on Humans

The effect on humans of flying glass from windows broken by blast waves will be considered. In Scenario #1, it was shown that for injury from flying glass, \( P_s = 9.45 \times 10^3 \) Pa with the blast wave striking the pane of glass head-on, and \( P_g = 1.96 \times 10^4 \) Pa with the blast wave propagating in a direction parallel to the surface of the glass. From Figure 6-9, at a distance \( R \) less than 9.8 m (32 ft), there is likelihood of human injury by flying glass from windows normal to the direction of travel of the blast wave. The corresponding distance for windows parallel to the direction of travel is 5.7 m (19 ft).

A summary of damage and injury for Scenario 4 is given in Table 6-30.

6-2.4.6 Fragment Barrier

A metal sheet or plate can be used as a barrier to stop fragments from a bursting pressurant tank. It was found that the initial fragment velocity is 211 m/s. Assume that the barrier is close enough to the vessel so that the fragment velocity does not decrease significantly before the fragment strikes the barrier.

For a barrier, assume a steel sheet or plate \( \rho = 7850 \) kg/m$^3$, \( \sigma = 3.11 \times 10^5 \) N/m$^2$. The barrier may surround the vessel, but effects of curvature and oblique impact will be ignored.
TABLE 6-30. SCENARIO 4: DAMAGE TO STRUCTURES AND INJURY

<table>
<thead>
<tr>
<th>Damage and Injury</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% survival (lung damage)</td>
<td>0.35</td>
</tr>
<tr>
<td>50% survival (lung damage)</td>
<td>0.41</td>
</tr>
<tr>
<td>90% survival (lung damage)</td>
<td>0.48</td>
</tr>
<tr>
<td>99% survival (lung damage)</td>
<td>0.54</td>
</tr>
<tr>
<td>mostly safe from skull fracture and mortality</td>
<td>~1</td>
</tr>
<tr>
<td>due to body impact</td>
<td></td>
</tr>
<tr>
<td>threshold of lung damage</td>
<td>1.2</td>
</tr>
<tr>
<td>50% eardrum rupture</td>
<td>~1.6</td>
</tr>
<tr>
<td>threshold of eardrum rupture</td>
<td>~4</td>
</tr>
<tr>
<td>threshold of injury from flying glass (face-on)</td>
<td>9.8</td>
</tr>
<tr>
<td>threshold of injury from flying glass (side-on)</td>
<td>5.9</td>
</tr>
<tr>
<td>threshold of side-on glass breakage</td>
<td>16</td>
</tr>
<tr>
<td>threshold face-on glass breakage</td>
<td>28</td>
</tr>
<tr>
<td>temporary threshold shift</td>
<td>147</td>
</tr>
</tbody>
</table>
The fragment mass is determined by choosing the 90th percentile of distribution of fragment mass (the mass which would equal or exceed the mass of 90% of the fragments). From Figure 4-55, the fragment mass is then 6.6 kg (15 lb.). Assume that the fragment is a sphere of radius \( a \).

\[
a = \left( \frac{\frac{m}{4 \pi \rho}}{\frac{4}{3}} \right)^{1/3} \left( \frac{6.6 \text{ kg}}{4520 \text{ kg/m}^3} \right)^{1/3} = 0.0704 \text{ m (0.23 ft)}
\]

The nondimensional limit velocity is

\[
\frac{a \cdot V_{50}}{\sqrt{\frac{1}{(3.11 \times 10^8 \frac{N}{m^2})(7850 \frac{kg}{m^3})}}} = 0.900
\]

From Figure 5-1, \( h/a \) to stop the fragment is 0.23. Then \( h = 0.23 \cdot a = (0.23)(0.0704) = 0.016 \text{ m} = 0.052 \text{ ft} = 0.63 \text{ in}.\)

A steel plate of 0.016 m (0.63 in) thickness would stop 50% of the fragments with the given initial velocity. Before such a barrier is used, the blast loading should also be determined.

6-2.5 Scenario 55. Rotor Burst of an Aircraft Gas Turbine Running on a Test Stand

This problem is quite different from the previous four. It does not involve accidental explosions at all and is included to demonstrate the utility of some of the methods given in the workbook to safety problems other than the primary intended ones.

The problem is as follows: Predict the maximum range and terminal velocity for a fan blade fragment of a gas turbine engine assuming that the blade failure occurs at its connection to the rotor disk.

**For Case I.** Assume the spinning blade leaves through the engine inlet with a deflection that tips it into the horizontal plane within an initial trajectory greater than 33°, with no loss of kinetic energy. Although the twist in blade may not contribute to the assumption of lift forces, assume that the blade is a spinning body or deformed into a boomerang-like shape. Identify the angle-of-attack and trajectory angle for maximum fragment range.

**For Case II - The fan blade is deflected and deforms by curling into a cylinder or sphere-like shape upon leaving the inlet to the gas turbine with...**
a loss of 10% of the translational kinetic energy. In this case, predict the
maximum possible range and terminal velocity using drag coefficients for
a cylinder or sphere.

Fragment characteristics are as follows:

Geometrical shape (see Fig. 6-11)
Rotational kinetic energy = 17,400 J
Translational kinetic energy = 200 J
Mass = 0.5 kg
Fan blade tip speed = 373 m/s
Fan speed = 123.1 RPS
Fan jet velocity = 255 m/s
Location of fan blade center of
gravity (CG) from CL of engine = 0.342 m
Location of fan blade CG from
failure = 0.116 m

These types of jet engine rotor failures have occasionally occurred during
takeoff and climb-out. Some have also occurred on static jet engine test
stands. Documented evidence shows that trajectories have been as much as
57° forward, relative to the plane of failure for fan rotors. Containment
barriers cannot be provided at the inlet of engines on static test stands
without upsetting inlet performance data. Thus, it becomes important to
identify hazards to people and risk of damage to facilities for such failures.

Case I was run under two sets of assumptions. Since FRISB was
written for "disc" fragments, it was necessary to assume that the blade
curled into a disc shape. The planform area of the blade fragment was set
equal to the surface area of a representational disc. The planform thickness
of the blade as viewed from the front was set equal to the thickness of a
representational disc. The optimal lift coefficient for a circular wing
(CL = 0.32) was taken, assuming gyro stability for the circular disc and assuming
a constant angle of attack. A drag coefficient of 0.85 was obtained from
Tables relating drag coefficient to the thickness-to-diameter ratio of a plate-
shaped fragment. The area over which pressure drag acted was taken as the
thickness times the diameter of the representational disc. The initial velocity
of the disc was assumed to be the initial translational velocity of the fragment.
The rotational velocity of the fragment only contributes to the gyro stability
of the disc. Results were obtained from the spectrum of initial trajectory
angles from 30 to 70° in five-degree intervals. The results given in Table
6-31 are for the maximum range obtained.

As a check on the results obtained for Case I under the assumptions
of the disc geometry, the trajectory characteristics of the fragment were
determined under a different set of assumptions relating to lift. For these
"thrust" assumptions, the blade was assumed to spin about an axis normal to its planform area, producing a thrust like a helicopter rotor blade, and thus lift. All other sources of lift were ignored. The fragment was approximated from drawings to have the shape of a NACA 4412 airfoil. The rotational kinetic energy contributing to the thrust is assumed to dissipate as a result of pressure drag forces as a function of the average linear velocity of the blade in rotation. The rotation of the blade contributes only to lift. The initial velocity of the blade is the initial translational velocity as in the disc case. The drag resulting from the translational velocity of the blade acts to dissipate the translational kinetic energy.

For Case II in which no lift is considered, the fragment was assumed to deform into a cylindrical shape, and a pressure drag coefficient of 0.85 was used (which is approximately correct for cylindrical fragments traveling at Mach numbers less than 1/2). Trajectory differential equations were solved both by the use of Runge-Kutta techniques and the time interval perturbation technique as of Zaker. Results were the same. Predictably, for the no lift case the maximum range is less than for the case where lift is assumed. The terminal velocity is also slightly less. See Table 6-31.

<table>
<thead>
<tr>
<th>TABLE 6-31</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
</tr>
<tr>
<td>deg</td>
</tr>
<tr>
<td>Case I</td>
</tr>
<tr>
<td>Disc</td>
</tr>
<tr>
<td>Thrust</td>
</tr>
<tr>
<td>Case II</td>
</tr>
<tr>
<td>No lift</td>
</tr>
</tbody>
</table>

Table 6-31 and Figure 6-11 show the results of predictions obtained for both cases for the optimum initial trajectory angle $\alpha_i$ and under various assumptions to be described. The maximum range in meters $R_m$ is given, as well as the terminal velocity $V_t$ in meters per second, and the terminal ballistic angle of attack in degrees $\theta$. From the table it may be seen that the maximum range expected under any of these sets of assumptions is 112 meters (367 ft) and the terminal velocity is only slightly less than the initial translational velocity of 30 meters per second (98 ft/sec). To put the damage potential of this fragment in perspective, a spherical fragment having the same mass with this terminal velocity could penetrate a 0.6-mm-thick (0.0236 in) steel plate or a 1.6-mm-thick (0.0629 in) aluminum plate.
Figure 6-11. Maximum Range as a Function of Initial Trajectory Angle, Cases I and II, Turbine Rotorblade Example.
LIST OF REFERENCES


CHAPTER VII
DISCUSSION OF RESULTS

Chapters I through VI in this workbook give a number of simplified prediction methods which hopefully can be used by a typical safety engineer to estimate damaging effects for certain classes of accidental explosions in aerospace launch facilities and flight vehicles. All prediction methods employ relatively simple graphs or equations, and require at most the use of a desk calculator or slide rule to make estimates. Example calculations are given in each chapter to illustrate use of the graphs.

Generally, the first step in predicting the effects of an accident is to estimate the total explosive yield or energy. For accidents involving the liquid propellants commonly used in rocket vehicles, a number of curves are presented in Chapter I for prediction of explosive yields given the propellant mixture and type of postulated accident. These graphs are based primarily on scaled results from Project PYRO tests, with upper limits established by other related work. This chapter also contains scaled graphs for estimating explosive energy release for gas vessel bursts, given the vessel characteristics and size, type of gas, and initial conditions. These graphs are based primarily on computer code solutions rather than experimental data.

Once explosive yields or energies are estimated from Chapter I, they can be used as inputs to graphs in Chapter II to obtain predictions of blast wave characteristics over a range of distances from the explosive sources. The primary blast wave properties included in Chapter II are side-on peak overpressure \( P_s \) and side-on impulse \( I_s \), although we include discussions of other properties such as reflected peak overpressure \( P_r \) and time histories of drag pressure \( q \). Again, blast data for liquid propellants are based primarily on experiment, while data for bursting gas vessels are estimated from computer code solutions for such waves.

Chapter III is devoted to prediction of damaging effects of blast waves, and is designed to give such predictions once blast pressures and impulses are known. The chapter therefore contains a number of scaled \( P-I \) (pressure-impulse) curves which are associated with various levels of damage or injury to a variety of structures, structural elements, and people. From different graphs, one can estimate minor damage through complete collapse of residences, threshold for glass breakage, incipient damage to beams or plates, and incipient toppling or overturning of a vehicle. Probability of injury or mortality of people can be predicted.
from other scaled graphs. These give estimates of threshold of ear
damage, various probabilities of mortality as functions of size of an
individual as well as blast wave properties, tertiary blast damage from
individuals being tumbled by a blast wave, and probability of injury from
flying glass from windows destroyed by blast waves. The prediction
methods for blast damage to structures are based on bomb damage studies
from World War II and a variety of other analytic and experimental
sources. Extensive work by Lovelace Foundation over a number of years
is the primary basis for the estimates of injury and mortality to people.

Knowledge or estimate of some of the general characteristics of
vessels involved in accidental explosions, and explosive yield estimates
from Chapter I, will allow one to estimate characteristics of fragments
generated by those explosions. The first set of graphs allows estimation
of fragment initial velocities. These, and fragment mass and shape
distributions, allow use of other graphs to predict fragment terminal
velocities and impact conditions. Finally, several curves give probabi-
li ty of fragment arrival versus range. Most of the predictions from this
chapter are statistical functions because of the inherent statistical nature
of fragmentation. The graphs for fragment initial velocities are based on
exercise of computer program developed under this contract and a pre-
vious one, supported by limited experimental data. Graphs for mass and
shape distributions of fragments are, on the other hand, based entirely
on fits of statistical functions to experimental "missile maps".

Once fragment impact conditions are known, one should then be
able to estimate effects of such impacts on structures, facilities and
people. Chapter V provides data for making some such predictions.
These include incipient damage to light structures or panels, and blunt
object impact injury to people. Some data on thresholds of penetration
of light panels are also given. This part of the workbook is, because of
lack of sufficient experimental data or analyses, and in some instances
the classified nature of fragment impact effects on humans, less complete
than other chapters. Some of the graphs for impact damage are based
on tests and analysis of what one might think to be an unrelated field,
 i.e., hail damage to aircraft and ground structures.

The sixth chapter includes a brief discussion of the more complete
field of risk assessment, of which damage estimation is a part. We
point out that a more complete analysis is needed if one is to estimate
the probability of an accident occurring, which type of accident is more
probable than another, etc. Most of Chapter VI is devoted to a series of
illustrations of use of the methods of Chapters I through V in predicting
both blast and fragmentation effects for five accident "scenarios". The
five postulated accidents include several which could occur during launch
or flight of multi-stage rocket vehicles, or during test of components in
a test bay or laboratory. Hopefully, study of these examples should lead
the reader through the workbook methods for reasonably complete
estimates of effects of certain accidents, as opposed to calculation of
separate parts of the effects given in previous chapters.

Throughout the workbook, the "tolerances", or possible errors,
involved in various calculations are indicated. These are included in
various ways, such as error bands about groups of measured data, a
range of probabilities of mortality or fragment impact, etc. A wide
error band indicates either large inherent spread of data or considerable
uncertainty in the method of estimation because it has not been validated
by experiment. Factors of safety are not included as such, although use
of upper bounds to loading parameters such as blast pressure and impulse
or impact velocities of fragments will assure conservation.
CHAPTER VIII
CONCLUSIONS

Some conclusions can be drawn regarding the applications of this workbook, and limitations to its use. These are discussed in this brief chapter.

It is possible using methods given in the workbook to make reasonable estimates of blast wave characteristics over a wide range of distances from the source of the accident. These characteristics can then be used to predict damage to structures, for a number of types of structures and damage modes, and can also be used to predict various levels of injury to and probability of mortality of humans. Confidence in blast damage and injury prediction is good, given knowledge of the blast wave properties, because of extensive past testing and analysis. The blast damage prediction methods are cast in a format which allow their use for other types of explosions.

Prediction methods are given for estimating initial velocities, ranges, masses, and impact conditions for fragments generated by propellant explosions and gas vessel bursts. The methods for predicting initial velocities are reasonably well founded on theoretical analyses and experimental data, and apply over a wide range of simulated burst conditions. Methods for predicting fragment ranges and impact conditions have a good theoretical basis, and can be used for other predictions involving flight through the air of high-velocity objects. Such predictions can be made for objects launched over a very wide range of initial Mach numbers. Methods for predicting fragment mass and shape distributions are entirely based on statistical fits to quite limited data, and therefore involve considerable uncertainty, as well as being impossible to accurately extrapolate.

Some predictions can be made of fragment impact effects on structures and structural elements from graphs and equations given in the workbook. These effects are much less well-known than are blast effects, so only limited predictions are possible. Some effects of fragment impact on humans can also be predicted, but these predictions are limited by security restrictions on wounding potential of fragments. Throughout the workbook, limitations such as this are noted when they are known.

This workbook is hopefully presented in a manner which allows easy use by typical safety engineers. For readers who are interested in the detail behind the relatively simple equations or graphs used to make predictions, a number of detailed appendices are included in appropriate
We believe that the workbook is the first to provide safety engineers with relatively simple estimates of blast and fragment hazards for accidental explosions in liquid-propellant fueled flight vehicles.

As noted before, some parts of the workbook have wider potential application than explosive hazard prediction for liquid-fueled rockets. The sections on fragment trajectory prediction, or the associated computer programs, can be used to predict ranges and impact conditions for many types of fragments or objects thrown into the air. The sections on blast effects apply for blast loads from any source. The methods for estimating fragment impact damage, though limited, are independent of the sources of these fragments or impacting objects.
CHAPTER IX
RECOMMENDATIONS

This workbook will allow prediction of blast and fragmentation effects for a wide variety of explosive accidents which could occur at aerospace launch and test facilities. It is based on a rather exhaustive review of existing test and accident data; analyses of blast and shock wave physics and effects of such waves on objects and humans; and analyses of fragment velocities, trajectories, impact conditions, and effects of impacts. A number of supporting studies have been made in generating relatively simple application formulas or graphs - these are reported in appropriate appendices.

The bases for the prediction methods given in this workbook range from a firm foundation of extensive testing and analysis, through analyses supported by limited testing or accident reporting, to some predictions which are quite speculative because of little or no corroborating evidence. Predictions in the latter case could often be improved by additional research. Also, some of the methods which have been developed here have potentially wide application to problems in hazards prediction other than explosive effects for flight-weight aerospace vehicles. We therefore give in this short chapter a list of recommendations for areas in which we feel there is a need for additional testing, analysis, or correlation of prediction methods.

Some of these areas are:

1. Definition of fragmentation characteristics for bursting gas storage bottles. Existing data consist of only five tests for two bottle geometries, one material, and one gas. There are no reliable "missile maps" for such bursts. Curves for fragment range presented in the workbook are based on computer-generated predictions, which should be validated by test.

2. Definition of blast wave characteristics for burst of cylindrical gas storage vessels, either analytically or experimentally. Present methods are limited to essentially spherically symmetric cases.

The analysis in this workbook is based upon data generated by a computer program. It has been confirmed to some extent by experimental data, but more experiments are needed. More work must be done to determine specific
impulse versus distance for gas vessel bursts. Then, a good nomograph should be developed.

The effects of changing the vessel shape from spherical to cylindrical should be investigated, both theoretically and experimentally.

In this analysis of the blast wave, the energy required to burst a pressure vessel and accelerate the fragments was neglected. Analytical and experimental studies would allow this effect to be included in the calculation of the blast wave parameters.

This workbook deals with very particular types of fragments and targets. The analysis should be extended to include shapes of fragments other than spherical and targets other than metal sheets and plates.

Targets that might be studied include wood, concrete, brick and glass. Also, it may not be desirable to treat the side of a fuel tank as an unsupported metal sheet or plate. There is probably some effect of the liquid, and one is concerned with the possibility of a fragment igniting a fuel tank, in addition to only puncturing it.

Oblique impacts by fragments deserve more study, and the effects of the strength of fragments is presently unknown. A model-scale experimental program is recommended to fill this void.

We recommend extension of the present work to accidental explosions in thick-walled storage vessels typical of ground transport and storage vessels. The current work is directed toward explosions of flight-weight hardware. Blast and fragmentation characteristics can be drastically different for heavier vessels, and consequently, so can the effects on structures, facilities, vehicles and humans. Included in this work should be exercise of the SPHER and CYLIN codes for typical initial conditions for massive vessels.

The analyses used to develop scaled curves for fragment range and impact conditions, for both lifting and drag-type fragments, have potentially wider application than generation of some of the scaled curves of Chapter IV. We
recommend that those programs be used to develop more
non-r Hoe graphs which include other parameters such as
initial altitude of an explosion, flight velocity as a function
of this altitude, and additional mass to area ratios typical
of ground storage vessels. We also recommend that the
programs be used for other types of accidents involving
high-velocity missiles or fragments such as the pieces
from a turbine rotor burst postulated in the fifth applica-
tion problem in Chapter VI.

(6) The existing codes for predicting initial fragment velocities
are limited to one space dimension and time (1-D codes).
We recommend the development of a limited 2-D code for
cylinder fragmentation, based on a combination of the
assumptions inherent in the FRAG-2 and CYLIN codes used
in this workbook. Such a code should more accurately
predict initial fragment velocities for real vessels, but
would certainly be more complex and more expensive to
run than either of the existing codes.

(7) As a part of studies of accidental explosions of thick-walled
vessels, we recommend a literature search for tests or
accident cases, where data on projection of large parts of
tanks could be compared with predictions from FRAG-2,
which assumes that a tank separates into two pieces which
are propelled by the exhausting fluids. Rail tank car
accident reports, and burst tests of tank cars conducted at
White Sands Missile Range, would perhaps be appropriate
sources of such data.

(8) We recommend using the programs for prediction of frag-
ment ballistics to generate tables for range and terminal
impact conditions, such as was done in a limited sense
for fragments from bursting gas spheres in Chapter IV.
These predictions would supplement, or could conceivably
supplant, missile map data for this class of explosion.

The specific recommendations listed before can be supplemented
by a last general recommendation. This workbook contains, we believe,
the most accurate assessments which can be made based on the current
state of the art. In almost all areas covered, either ongoing or future
studies may well alter the prediction methods, or the results of applying
the prediction methods. The workbook is so organized that alterations or
modifications can be made to individual chapters, without a complete
revision of the entire book. It is strongly recommended that revision be
considered on some regular schedule, say at two-year intervals.
LIST OF SYMBOLS

A, B, C, D - terms in equations for bursting vessel motion

A - presented target area
- area of an object presented to the blast front
- planform or top surface area of fragment
- cross-sectional area of a fragment along its trajectory
- mean presented area of object

\( AD \) - average drag area

\( AL \) - average planform area

\( A_i \) - fragment area

\( A^1 \) - geometric mean frontal area of glass fragment

a - radius of a fragment
- airfoil curve slope

\( a_0 \) - critical sound speed in mass flow equation

\( a_\infty \) - ambient sound velocity

\( a_{00} \) - sound speed in confined gas, \( \tau = 0 \)

b - vehicle track width or depth of target base
- number of rotor blades

\( C_D, C_d \) - drag coefficient

\( C_l \) - lift coefficient

\( C_t \) - cylinder length

\( C_t \) - cylinder thickness

\( G(\tau) \) - mass of gas confined at high pressure as a function of time
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_p$</td>
<td>profile drag</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter of fragment</td>
</tr>
<tr>
<td>$d_1$</td>
<td>width of fragment segment</td>
</tr>
<tr>
<td>$E$</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>$E_b$</td>
<td>effective blast energy</td>
</tr>
<tr>
<td>$E$</td>
<td>bulk modulus</td>
</tr>
<tr>
<td>$E_t$</td>
<td>end cap length</td>
</tr>
<tr>
<td>$E_t$</td>
<td>end cap thickness</td>
</tr>
<tr>
<td>$e$</td>
<td>stored energy ratio</td>
</tr>
<tr>
<td>$F$</td>
<td>fragment projected area</td>
</tr>
<tr>
<td>$g$</td>
<td>nondimensionalized displacement of a fragment</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>$H$</td>
<td>minimum transverse dimension at location of largest presented area of object</td>
</tr>
<tr>
<td>$h$</td>
<td>thickness of a target, glass, or plate</td>
</tr>
<tr>
<td>$h$</td>
<td>sphere wall thickness</td>
</tr>
<tr>
<td>$h$</td>
<td>segment height</td>
</tr>
<tr>
<td>$I$</td>
<td>mass moment of inertia of rotor</td>
</tr>
<tr>
<td>$I$</td>
<td>nondimensional specific impulse</td>
</tr>
<tr>
<td>$I_d$</td>
<td>total drag and diffraction impulse</td>
</tr>
<tr>
<td>$I_s$</td>
<td>incident specific impulse</td>
</tr>
<tr>
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</tr>
<tr>
<td>$i$</td>
<td>side-on impulse</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS (Cont'd)

- \( i \) - threshold impulse
- \( K \) - adiabatic exponent (ratio of specific heats)
- \( \alpha \) - constant (4 if appurtenance is on the ground and 2 if appurtenance is in air)
- \( \mu \) - value from the normal distribution table
- \( k \) - mass flow rate coefficient
- \( L \) - cylinder length
- \( L/D \) - length-to-diameter ratio
- \( M \) - mass of fragment; of contained gas
- \( \bar{M} \) - average mass
- \( M_f \) - fragment mass
- \( M_t \) - cylindrical shell mass
- \( m \) - total mass of target; of fragment
- \( n \) - number of fragments
- \( P \) - pressure of confined gas
- \( P_c \) - crack perimeter about a fragment
- \( \bar{P} \) - nondimensional pressure
- \( P_e \) - peak overpressure
- \( P_0 \) - pressure of confined gas at any instant
- \( P_{oo} \) - initial pressure of confined gas
- \( P_r \) - threshold applied maximum reflected pressure
- \( \rho \) - reflected peak overpressure
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
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<tr>
<td>( P_r )</td>
<td>peak reflected pressure</td>
</tr>
<tr>
<td>( P_s )</td>
<td>peak side-on overpressure</td>
</tr>
<tr>
<td>( P_o )</td>
<td>nondimensional pressure</td>
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<td>( P_0 )</td>
<td>ambient air pressure</td>
</tr>
<tr>
<td>( p(t) )</td>
<td>net transverse pressure</td>
</tr>
<tr>
<td>( Q )</td>
<td>peak dynamic pressure</td>
</tr>
<tr>
<td>( Q_o )</td>
<td>peak dynamic overpressure</td>
</tr>
<tr>
<td>( R )</td>
<td>gas constant</td>
</tr>
<tr>
<td>( S_{\text{radius}} )</td>
<td>sphere radius before burst</td>
</tr>
<tr>
<td>( R )</td>
<td>radius of rotor</td>
</tr>
<tr>
<td>( r )</td>
<td>initial cylinder radius</td>
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<tr>
<td>( r )</td>
<td>distance from center of a vessel</td>
</tr>
<tr>
<td>( S_e )</td>
<td>cylinder radius</td>
</tr>
<tr>
<td>( S_e )</td>
<td>disc radius</td>
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<tr>
<td>( s )</td>
<td>fragment displacement at any instant</td>
</tr>
<tr>
<td>( S_{\text{E}} )</td>
<td>strain energy stored in vessel</td>
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<tr>
<td>( s )</td>
<td>estimate for the standard deviation</td>
</tr>
<tr>
<td>( s )</td>
<td>segment length</td>
</tr>
<tr>
<td>( T )</td>
<td>duration of the positive phase of the blast wave</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>temperature of confined gas at any instant</td>
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LIST OF SYMBOLS (Cont'd)

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<thead>
<tr>
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<tbody>
<tr>
<td>$T_{oo}$</td>
<td>temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>thickness of fragment; disc; window pane</td>
</tr>
<tr>
<td>$t_a$</td>
<td>time of arrival</td>
</tr>
<tr>
<td>$t_h$</td>
<td>shell thickness</td>
</tr>
<tr>
<td>$U$</td>
<td>shock velocity</td>
</tr>
<tr>
<td>$U_{p}$</td>
<td>peak particle velocity</td>
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<tr>
<td>$V$</td>
<td>velocity of fragment</td>
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<td>volume</td>
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<tr>
<td>$\bar{V}$</td>
<td>initial velocity ratio</td>
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<tr>
<td>$V_c$</td>
<td>rotor blade velocity along rotational axis</td>
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<td>$V_i$</td>
<td>volume of the vessel before it bursts</td>
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<td>$V_{in}$</td>
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<td>volume of confined gas at any instant</td>
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<tr>
<td>$V_s$</td>
<td>volume of shell material</td>
</tr>
<tr>
<td>$V_t$</td>
<td>tip velocity of rotor</td>
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<tr>
<td>$V_{50}$</td>
<td>ballistic limit velocity</td>
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<tr>
<td>$W$</td>
<td>effective mass of propellant</td>
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<tr>
<td>$W_m$</td>
<td>mean mass</td>
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<tr>
<td>$W_T$</td>
<td>total mass of propellant and oxidizer</td>
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<tr>
<td>$w$</td>
<td>crack width</td>
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**LIST OF SYMBOLS (Cont'd)**

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<thead>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>displacement of an object</td>
</tr>
<tr>
<td></td>
<td>distance from the front of object to location of largest cross-sectional area</td>
</tr>
<tr>
<td></td>
<td>the distance from the front of the object to the plane facing the approaching blast wave</td>
</tr>
<tr>
<td>X</td>
<td>horizontal velocity</td>
</tr>
<tr>
<td>X</td>
<td>horizontal acceleration</td>
</tr>
<tr>
<td>x</td>
<td>short halfspan</td>
</tr>
<tr>
<td></td>
<td>nondimensionalizing constant for displacement</td>
</tr>
<tr>
<td>Y</td>
<td>terminal blast yield</td>
</tr>
<tr>
<td>Y</td>
<td>vertical velocity</td>
</tr>
<tr>
<td>Y</td>
<td>vertical acceleration</td>
</tr>
<tr>
<td>y</td>
<td>edge length</td>
</tr>
<tr>
<td></td>
<td>long halfspan</td>
</tr>
<tr>
<td>α</td>
<td>nondimensional constant</td>
</tr>
<tr>
<td>α_i</td>
<td>initial trajectory angle</td>
</tr>
<tr>
<td>β</td>
<td>nondimensional constant</td>
</tr>
<tr>
<td>δ</td>
<td>permanent deflection of target at point of impact</td>
</tr>
<tr>
<td>δ_t</td>
<td>time increment</td>
</tr>
<tr>
<td>θ</td>
<td>nondimensionalizing constant for time</td>
</tr>
<tr>
<td>φ</td>
<td>angle of attack of disc</td>
</tr>
<tr>
<td></td>
<td>angle subtended at the center of the cylinder by a fragment ( \theta )</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS (Cont'd)

\( \kappa \) - ratio of specific heats for confined gas
\( \gamma \) - ratio of specific heats for confined gas
\( \nu \) - Poisson's ratio for the material
\( t_0 \) - nondimensionalized time
\( \rho \) - density of glass, air
\( c_0 \) - confined gas density at any instant
\( \rho_p \) - density of a fragment (or projectile)
\( \rho_f \) - density of fragment
\( \rho_s \) - density of shell material
\( \rho_s^* \) - peak density in shock wave
\( \rho_t \) - density of target
\( \rho_c \) - critical density of gas in mass flow equation
\( \sigma_t \) - yield stress of target material
\( \sigma_y \) - yield stress of glass; plate
\( \sigma \) - stress in container wall
\( t \) - time
\( s_i \) - impulse shape factors
\( q_p \) - pressure shape factors
\( \omega \) - angular velocity
CONVERSION FACTORS

The following table provides multiplying factors for converting numbers and miscellaneous units to corresponding new numbers and SI units.

The first two digits of each numerical entry represent a power of 10. An asterisk follows each number which expresses an exact definition. For example, the entry "1 -02 2.54" expresses the fact that 1 inch = 2.54 x 10\(^{-2}\) meter, exactly, by definition. Most of the definitions are extracted from National Bureau of Standards documents. Numbers not followed by an asterisk are only approximate representations of definitions, or are the results of physical measurements.

<table>
<thead>
<tr>
<th>To convert from</th>
<th>to</th>
<th>multiply by</th>
</tr>
</thead>
<tbody>
<tr>
<td>atmosphere</td>
<td>newton/meter(^2)</td>
<td>+05 1.013 25(^*)</td>
</tr>
<tr>
<td>bai</td>
<td>newton/meter(^2)</td>
<td>+05 1.00(^*)</td>
</tr>
<tr>
<td>British thermal unit (mean)</td>
<td>joule</td>
<td>+03 1.055 67</td>
</tr>
<tr>
<td>calorie (mean)</td>
<td>joule</td>
<td>+00 4.190 02</td>
</tr>
<tr>
<td>dyne</td>
<td>newton</td>
<td>-05 1.00(^*)</td>
</tr>
<tr>
<td>erg</td>
<td>joule</td>
<td>-07 1.00(^*)</td>
</tr>
<tr>
<td>Fahrenheit (temperature)</td>
<td>Celsius</td>
<td>(t_c = (5/9)(t_F - 32))</td>
</tr>
<tr>
<td>foot</td>
<td>meter</td>
<td>-01 3.048(^*)</td>
</tr>
<tr>
<td>inch</td>
<td>meter</td>
<td>- 02 2.54(^*)</td>
</tr>
<tr>
<td>lb(_f) (pound force, avoirdupois)</td>
<td>newton</td>
<td>+00 4.448 221 651 250 5(^*)</td>
</tr>
<tr>
<td>lb(_m) (pound mass, avoirdupois)</td>
<td>kilogram</td>
<td>-01 4.535 923 7(^*)</td>
</tr>
<tr>
<td>pascal</td>
<td>newton/meter(^2)</td>
<td>+00 1.00(^*)</td>
</tr>
<tr>
<td>pound force (lb(_f) avoirdupois)</td>
<td>newton</td>
<td>+00 4.448 221 615 200 5(^*)</td>
</tr>
<tr>
<td>pound mass (lb(_m) avoirdupois)</td>
<td>kilogram</td>
<td>-01 4.535 923 7(^*)</td>
</tr>
<tr>
<td>poundal</td>
<td>newton</td>
<td>-01 1.382 549 543 76(^*)</td>
</tr>
<tr>
<td>slug</td>
<td>kilogram</td>
<td>+01 1.459 390 29(^*)</td>
</tr>
<tr>
<td>To convert from</td>
<td>to</td>
<td>multiply by</td>
</tr>
<tr>
<td>------------------------------</td>
<td>------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>foot/second²</td>
<td>meter/second²</td>
<td>-01 3.048</td>
</tr>
<tr>
<td>inch/second²</td>
<td>meter/second²</td>
<td>-02 2.547</td>
</tr>
<tr>
<td>gram/centimeter³</td>
<td>gram/centimeter³</td>
<td>+03 1.000</td>
</tr>
<tr>
<td>lbm/inch³</td>
<td>kilogram/meter³</td>
<td>+04 2.767 900 5</td>
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<tr>
<td>lbm/foot³</td>
<td>kilogram/meter³</td>
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</tr>
<tr>
<td>slug/foot³</td>
<td>kilogram/meter³</td>
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<tr>
<td>lbf/foot²</td>
<td>newton/meter²</td>
<td>+01 4.788 025 8</td>
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<tr>
<td>lbf/inch² (psi)</td>
<td>newton/meter²</td>
<td>+03 6.894 757 2</td>
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<tr>
<td>foot/second</td>
<td>meter/second</td>
<td>-01 3.048</td>
</tr>
<tr>
<td>inch/second</td>
<td>meter/second</td>
<td>-02 2.547</td>
</tr>
<tr>
<td>foot³</td>
<td>meter³</td>
<td>-02 2.581</td>
</tr>
<tr>
<td>inch³</td>
<td>meter³</td>
<td>-05 1.638 706 4</td>
</tr>
</tbody>
</table>
GLOSSARY OF TERMS

Air-embolic insult - air bubbles circulating in the blood which can contribute to the collapse of the heart.

Angle of attack - angle between fragment horizontal axis and the relative wind vector.

Applied impulse - actual impulsive loading applied to a "target".

Appurtenance - a piece of equipment or an object located near a source of an explosion, which can be accelerated by the blast wave from the explosion.

Blast yield - energy release in an explosion inferred from measurements of the characteristics of blast waves generated by the explosion.

Blunt trauma - injury caused by a nonpenetrating object.

Burst pressure - the pressure at which a gas storage vessel bursts or fails.

CBGS - Confined by Ground Surface. This abbreviation designates a liquid propellant explosion occurring on the ground after spill and mixing.

CBM - Confined by Missile. This abbreviation designates an explosion within the tankage of a liquid propellant vessel or rocket.

Critical threshold impulse - blast wave impulse which determines the impulse asymptote for an isodamage contour.

Drag coefficient - ratio of drag force to dynamic force exerted by wind pressure on a reference area.

Edema - abnormal accumulation of fluid in connective tissues causing local swelling.

Energy of detonation - the energy in an explosion which drives a blast wave.

Explosive yield - energy released in an explosion, often expressed as a percent or fraction of energy which would be released by the same mass of a standard high explosive such as TNT.

FMECA - abbreviation for Failure Mode Effects and Criticality Analysis. A systematic procedure for identifying failure modes of a system and for evaluating consequences of failures.

Fall-back - an accident in which a launch vehicle settles or falls back to earth in initial stages of launch.

Fault tree - a method employed in risk assessment for determining probabilities for event trees (see event tree).

Fibrotic foot - fine scars of the lungs.

Free-field impulse - see side-on impulse.

Free-field pressure - see side-on overpressure.

HVI - High Velocity impact. This abbreviation designates a liquid propellant explosion occurring after a vehicle with unburned propellant impacts the earth at relatively high velocity.

Ignition time - time after beginning of an accident involving liquid propellants at which initiation of an explosion occurs.

Induced velocity - velocity along rotor axis, induced by thrust generated by the whirling blade.

Initial trajectory angle - angle of fragment's horizontal axis relative to the ground surface at the beginning of the flight.

Isodamage line - Loci of combinations of overpressure and impulse which produce the same level of blast damage to a given "target".

Lift coefficient - ratio of lift force to dynamic force exerted by wind pressure on a reference area.

Limit velocity (V50) - impact velocity, for a fragment or missile striking a target, at which 50% perforations occur.

Major structural damage - damage to a residence involving partial or total collapse of roof, partial demolition of one or two external walls, or severe damage to load-bearing partitions requiring replacement.
Minor structural damage - damage to a residence involving window breakage and wall and support cracking.

Overpressure - pressure in a blast wave above atmospheric pressure.

Partial demolition - damage to a residence in which 50% to 75% of the external brick work is destroyed, or the building is rendered so unsafe that it must be demolished.

Planform area - the area viewed by looking down on the fragment - top surface area.

Plate aspect ratio - ratio of length to width for a rectangular plate.

Projected area - area of fragment viewed perpendicular to the top surface area.

Pulmonary hemorrhage - internal bleeding occurring in the lungs.

Reflected impulse - integral of reflected pressure-time history.

Relative wind vector - vector along which the fragment flies.

Side-on impulse - integral of time history of side-on overpressure.

Side-on overpressure - blast wave overpressure in an undisturbed blast wave.

Standoff distance - distance from center of an explosion.

Temporary threshold shift - the case where 90 percent of those exposed to a blast wave advancing at normal angle of incidence to the earth are not likely to suffer an excessive degree of hearing loss.

Terminal yield - blast yield from measurements made far enough from an explosion that the waves are similar to those generated by a specified mass of TNT.

Threshold bending impulse - blast wave impulse which produces an incipient bending failure.

Threshold membrane impulse - blast wave impulse which produces an incipient stretching failure.
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