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#### **ABSTRACT**

We have obtained a number of results pertaining to signal detection and estimation, where the underlying random processes are imperfectly known and often possess dependency and/or nonstationarity. Our results heavily emphasize nonlocal methods, that is, methods which allow an imperfectly known distribution to vary substantially and not simply be modeled as local to a nominal. Much of this work features robustness, but we also include research involving nonparametric algorithms. Our results include the design and analysis of the classically robust saddlepoint detector for nominally Laplace noise, development of quantitative nonlocal robustness measures for signal detection, parameter estimation, and the estimation of a random variable (all with dependent data), development of a "user friendly" concept of average nonlocal robustness (a vast improvement over "worst case" or "least favorable" approaches), and an analysis of the stability of the false alarm rate of a classical "nonparametric" detector (an analysis which uses nonlocal techniques). This work underscores that traditional algorithms, while useful, are limited by their design assumptions and can offer disappointing performance when presented with realistic data which reflects imperfectly known random processes possessing dependency and/or nonstationarity. Our quantitative results not only shed light on how bad the situation can be, but how to compensate for it with improved design procedures.

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### **PUBLICATIONS SUPPORTED BY GRANT AFOSR-91-0267**

- C. Tsai and D. R. Halverson, "Nonlocal and Hybrid Robustness Measures for Signal Detection and Parameter Estimation," Proc. 1992 Conf. on Inform. Sciences and Systems, Princeton, New Jersey, March 18-20, 1992, pp. 978-983.
- W. Liu and D. R. Halverson, "Nonlocal Robustness of the Sign Detector with Nonstationary Data," Proc. 35th Midwest Symp. on Circuits and Systems, Washington D.C., August 9-12, 1992, pp. 449-452.
- M. W. Thompson, D. R. Halverson, and C. Tsai, "Robust Estimation of Signal Parameters with Nonstationary and/or Dependent Data," *IEEE Trans. Inform. Theory*, Vol. 39, No. 2, March 1993, pp. 617-623.
- 4. M. W. Thompson, D. R. Halverson, and G. L. Wise, "Robust Detection in Nominally Laplace Noise," to appear in *IEEE Trans. Commun.*.
- 5. D. R. Halverson, "Robust Estimation and Signal Detection with Dependent Nonstationary Data," to appear in *Circuits, Systems, and Signal Processing.*

## SUMMARY OF RESEARCH

The research supported by Grant AFOSR-91-0267 has focused on signal detection and estimation. It is increasingly being recognized that the grim realities of inexact statistical knowledge, together with dependency and/or nonstationarity, severely compromise the real world performance of so called "optimal" algorithms which are the product of yesterday's design procedures. In recognition of this, modern design has increasingly focused on robust and nonparametric approaches, which offer a degree of protection against the uncertain statistical knowledge. We have, in recent work, obtained a number of results pertinent to robust and nonparametric algorithms; these results are now summarized.

For example, we have employed the classical Huber-Strassen saddlepoint approach toward robustness to completely design the robust detector for nominally Laplace noise. Empirical evidence suggests that a Laplace noise model is a fairly accurate model in several important communications and signal processing contexts, and robustness is desirable since an actual noise process in these contexts would very likely deviate somewhat from being purely Laplace. Our work completely designs the saddlepoint robust detector, including threshold, and provides closed form expressions which allow for the analysis of the performance of the robust detector. We show via examples that the robustness can entail a considerable cost in detector performance; in fact, the situation can be so serious that in many cases it may be desirable to consider both robustness and performance in the design procedure. Such a dual consideration is not natural to the classical saddlepoint approach, but is readily implemented via our newly developed geometric approach. These results are delineated in #4 of the publication list.

While classical saddlepoint techniques can be useful in investigating certain questions pertaining to robustness, we believe that the exceedingly heavy reliance placed by researchers on these techniques stems more from inertia and the apparent lack of suitable alternatives, rather than a universal recognition that saddlepoint techniques are ultimately suitable in all important respects. In our recent work, a desirable alternative has been developed which approaches the concept of robustness through a perspective based on differential geometry. While this approach has found application quite naturally to robust hypothesis testing, the core of signal detection, the approach also has the advantage of possessing the potential for wider domains of application. The geometric techniques employed allow the computation of a quantitative measure of the degree of robustness of a given detector subject to a wide variation of the underlying statistical distribution function about a nominal distribution. This new approach possesses distinct advantages when compared to classical saddlepoint criteria, which are inherently nonquantitative and are relevant to only a few restricted canonical regions of admissible variation about a nominal distribution. Our approach admits an analysis based on the quantitative tradeoff of performance and robustness, thus providing the user with a much more flexible design procedure than that based on the classical techniques. This approach has provided quantitative measures of the degree of robustness for both the local case, where the results have application only for small neighborhoods of distributions near the nominal, and the more general nonlocal case. These results have been applied to compare the robustness of a Neyman-Pearson optimal detector with a detector which employs a censored version of the Neyman-Pearson optimal nonlinearity, the form of the latter arising when classical saddlepoint methodology is utilized. Our most recent work is increasingly emphasizing nonlocal methods, together with the admission of dependency and/or nonstationarity whenever possible.

For example, in some recent work we have shown how our geometric approach toward robustness can be extended to admit dependent nonstationary data for signal detection and the estimation of a random variable. Our results provide a natural quantitative measure of the robustness of a signal detector with dependent data; this measure is sensitive to essentially arbitrary perturbations in an underlying joint distribution away from the nominal. Our results show, for example, that the presence of residual dependency can result in a reduction of robustness; in particular, this reduction is approximately 50% for the linear detector. Our work therefore shows, for the first time, precisely how serious the consequences of residual dependency can be. We also show how it is possible to measure the robustness of an estimator of a random variable. Our work shows that the choice of performance fidelity criterion is crucial. For example, we find that for many popular error criteria (e.g. mean square error) any admissible estimator is completely unrobust. On the other hand, we also show that there exists a wide family of performance criteria for which some degree of robustness can be achieved. Somewhat surprisingly, in this case the best estimator has been shown by us to also often be the most robust. The conditional expectation estimator, known to be an optimal estimator under a ariety of error criteria, thus can be either completely unrobust or optimally robust, depending on the choice of error criterion. The consideration of alternative error criteria to the popular mean square error criterion is therefore highly desirable. We have also exhibited success in generalizing our original geometric approach, which is local in nature, to the nonlocal arena by employing a "worst case" perspective analogous to that used with some success by the classical methodology. Our new nonlocal techniques, however, admit the consideration of essentially arbitrary regions about the nominal distribution, and can be used to draw nonlocal conclusions for a variety of important robustness questions, including how to control detector false alarm rate as the underlying distribution varies over a region containing the nominal. These results are delineated in #5 of the publication list.

We have in addition applied our geometric methodology to robust parameter estimation. Using some of the techniques of #5 of the publication list, we have developed local robustness measures which admit dependent nonstationary data; moreover the same type of approach toward a "worst case" nonlocal generalization discussed in #5 can be undertaken. Our results show not only why censoring again can be helpful in imparting robustness, but also how limitations involving censoring can arise, including the atrophy of robustness for large numbers of samples. As with robust signal detection, our results facilitate the design of an estimator which trades off performance versus robustness subject to a cost criterion of interest of the user. These results are delineated in #3 of the publication list.

We have also more fully investigated the concept of nonlocal robustness. Classical saddlepoint approaches as well as some of our own work have employed a "least favorable" or "worst case" concept of nonlocal robustness. While such a concept can lead to useful results, it also seems reasonable that an extension of the concept to allow more than simply "near nominal" (local robustness) or "least favorable" (nonlocal robustness) would be highly desirable. In some recent past work we have shown how the concept of nonlocal robustness can be extended to admit the development of the much more general "average nonlocal robustness" measures, which include local and "worst case" measures as special cases. These "average nonlocal robustness" measures allow the user to incorporate the effect of distributions which are neither "near nominal" nor "least favorable" in the design and analysis of algorithms. Since most distributions provided by nature will very likely be neither "near nominal" nor "least favorable", an "average nonlocal robustness" approach can have great relevance to a practitioner.

In some very recent work we have extended this approach to make it much more "user friendly." While our past work blazes the trail in introducing the concept of "average nonlocal robustness" and subsequently develops appropriate robustness measures, the

methods stop short of systematically developing appropriate models for the families of admissible distributions which can be easily employed by the user. Since the choice of admissible family affects the corresponding robustness measure, this choice should not be made arbitrarily. In our more recent work we show how the theoretical approach for "average noniocal robustness" can be applied to signal detection and parameter estimation for two quite general models, one based on conditional distributions and the other based on difference equations. We show that both models can lead to the same basic input when evaluating robustness, thus allowing the user to interpret robustness results in two different, but consistent, ways. The models specialize to include all of our earlier examples, but offer the user much greater latitude in identifying appropriate families of admissible distributions than specialized examples would suggest. In addition, the technical analysis sheds philosophical light on the types of variations in an underlying distribution admitted; the variation is seen to be very rich, indeed. Moreover, we note that there are situations where disproportionate knowledge may exist regarding a distribution for various values of its argument. For example, much more may be known near the origin than on the tails, and we accordingly show how local and nonlocal robustness measures can be combined to yield a hybrid robustness measure whose function is close to local near the origin but is highly nonlocal on the tails. Such an approach allows taking advantage of increased available knowledge near the origin while admitting substantially inexact knowledge on the tails. These results are delineated in #1 of the publication list.

In addition to robustness methods, we have also investigated an alternative, the nonparametric. While we regard robustness as offering a frequently desirable middle ground between the parametric and nonparametric, there are important occasions where so little is known statistically that a nonparametric algorithm may be considered desirable. A popular "nonparametric" (or, perhaps more accurately, "distribution free") signal detector is the classical sign detector, which offers constant false alarm rate for the detection of a constant signal in i.i.d. noise with univariate distribution  $F(\cdot)$  possessing a fixed value at the origin. Historically, this detector was motivated by the admission of noise possessing an even density function, and hence  $F(0) = \frac{1}{2}$  for all admissible  $F(\cdot)$ . While the false alarm rate of the sign detector should in theory be constant, we have been told in past conversations with practitioners that this is not what often happens in the real world. Obviously, nature has seen fit to frequently provide the user with distributions that do not comply with the requirements of the admissible family. Even if one assumes a

sampling rate sufficiently slow as to obtain independent data, it would then be questionable as to whether absolute stationarity would be in force. In addition, it is questionable that F(0) would be fixed, i.e. that the underlying density, if it exists, would be even. In seeking to unravel this situation, we were of the opinion that a bridge built to the robustness arena might shed light on what was going on with this "nonparametric" detector. The issue of the lack of stability of the false alarm rate of the sign detector could then be phrased in terms of the robustness of its false alarm rate to lack of stationarity and to perturbations in F(0) away from the nominal value of  $\frac{1}{2}$ . In some very recent work, we have conducted a robustness analysis along these lines. Using our basic geometric approach, which has yielded results in other contexts, we have developed a local rebustness measure for the sign detector which admits nonstationary data. Our results show that the sign detector's false alarm rate can be highly no robust to violation of the classical assumption of stationarity with fixed known distribution value at the origin. We have also extended our local geometric methods to the nonlocal, and have shown that actual increases in the false alarm rate can be greater than 1300%, even when the noise is nominally i.i.d. with  $F(0) = \frac{1}{2}!$  We conclude the paper by illustrating how one can compensate for this lack of robustness at the design stage in order to stabilize the false alarm rate. The cost of doing so is a reduction in detection probability, but that may be considered an acceptable disadvantage since a stable false alarm rate is the supposed raison d'être for the sign detector. These results are delineated in #2 of the publication list.

We have also recently obtained a number of results which are as yet unpublished. For example, we are in the process of developing a new family of nonlocal robustness measures based on the geometric perspective combined with calculus of variations techniques. These measures have application not only within the domains of signal detection and estimation but also within other areas such as source coding. Since our methodology offers great flexibility in choosing the set of admissable distributions, we are also currently investigating appropriate models which would be consistent with what knowledge a user might have available, such as that based on, for example, density estimation techniques. In addition to research in robustness, we are also pursuing work in the nonparametric arena, and have shown that, in particular, the "nonparametric" sign detector is very sensitive to the presence of residual dependency; in addition, even the modified sign detector may not perform satisfactorily for types of dependency not admitted under its strong mixing assumption on the noise. Moreover, we have also shown that the modified sign detector may not perform well even if the noise is strong mixing with imperfectly known mixing coefficients. Since it is unlikely that a user would have available such knowledge, it appears that much work remains to be done to design satisfactory nonparametric detectors which work as intended in practical environments.

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