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SOLUTION PROCEDURES FOR LARGE-SCAALE COMBINATORIAL OPTIMIZATION

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13. ABSTRACT (Maximum 200 words)

Results of research performed under this grant have shown that problems having thousands, and sometimes millions, of variables can be solved using present-day technology based on mathematical results that utilize the structure underlying the problem and that incorporate related advances of the mathematical theory into a general approach called "branch-and-cut". The term "branch-and-cut" and the ideas encompassing it, are the direct result of this research effort. Now the two leading commercial codes for solving integer programming problems. OSL (an code marketing by IBM) and CpleX (a code marketed by the CpleX corporation) both incorporate cutting plane ideas and use the term "branch-and-cut" in their marketing literature.

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Summary of Research Results under AFOSR Grant F49620-90-C-0022:

Many problems of practical significance fall into the class of *combinatorial optimization problems*. These include the optimal design of VLSI circuits, i.e. the laying out of circuits to minimize the area dedicated to wires, the dispatching and routing of vehicles, the scheduling of crews to flights, the optimal selection of a portfolio of projects or investments as well as many other problems in business, economics and engineering. It has been presumed until recently that problems having more than a few hundred decision variables could not be solved to optimality, and so heuristics were usually employed producing, in general, *suboptimal* solutions, i.e. feasible solutions without any guarantee as to their "closeness" to a best possible problem solution.

Results of research performed under this grant have shown that problems having thousands, and sometimes millions, of variables can be solved using present-day technology based on mathematical results that utilize the structure underlying the problem and that incorporate related advances of the mathematical theory into a general approach called "branch-and-cut". The term "branch-and-cut" and the ideas encompassing it, are the direct result of this research effort. Now the two leading commercial codes for solving integer programming problems, OSL (an code marketing by IBM) and CpleX (a code marketed by the CpleX corporation) both incorporate cutting plane ideas and use the term "branch-and-cut" in their marketing literature. There are over 100 citations to this term in the literature.

For general zero-one problems, Hoffman and Padberg have solved real-world capital budgeting problems having up to 6000 integer variables, as well as problems arising from the design of automobiles having up to 2756 variables. No software package to date is capable of performing better on these set of test problems, than those results we report in the *ORSA Journal on Computing* paper which was published in 1991. There has been renewed interest recently in the capital budgeting research due to the fact that many major US Corporations are using reengineering techniques to restructure their businesses and capital budgeting models provide a tool to measure the effectiveness of various projects/processes.

Since the work cited in the *ORSA Journal on Computing*, Padberg and Rinaldi have solved traveling salesman problems, using essentially the same algorithmic design and solved problems having over 2000 cities. This work was published in *SIAM Review* also in 1991. In 1993, a problem having 3000 variables was solved by Applegate, Bixby, Chvatal and Cook using a parallel branch-and-cut approach modelled after the research of Cannon and Hoffman.

The past three years, Hoffman and Padberg have concentrated on the solution to crew-scheduling problems provided by the airline industry. The results of this research are reported in *Management Science*. In this paper, the largest crew-scheduling problems ever solved are reported. This paper has received over 250 requests for reprints and over 50 researchers have requested the data sets. Only one of the problems which we have labelled "hard" within this test set has been solved by another researcher. That problem, labelled NW04 in our data set, was solved by Jonathan Epstein on a CM5 machine using 125 processors. With that many processors, his branch and bound code, fine-tuned to the CM5 architecture, took over one hour to solve. Our branch-and-cut approach takes about 20 minutes on *one* processor, again proving the viability and superiority of the branch-and-cut approach.

The exact optimization of such large-scale problems – considered impossible only a few years ago – are the result of an overall algorithm or procedure called *branch-and-cut*. The major components of this algorithm consist of automatic reformulation procedures, heuristics which provide "good" feasible integer solutions, and cutting plane procedures which tighten the linear programming relaxation to the combinatorial problem under consideration – all of which is embedded into a tree-search framework. Whenever possible, the procedure permanently fixes variables (by reduced cost implications and logical implications) and does comparable conditional fixing throughout the search-tree. These four components are combined in such a way as to guarantee optimality of the solution obtained at the end of the calculation. However, the algorithm may also be stopped *early* to produce suboptimal solutions along with a bound on the remaining error. The cutting planes generated by the algorithm are facets of the convex hull of feasible integer solutions or good polyhedral approximations thereof and as such they are the "tightest cuts" possible. Lifting procedures assure that the cuts generated are valid throughout the search tree. In all instances, because of the power of the cutting-plane procedures, the search trees are extraordinarily short and the total execution times for the algorithm are within reasonable limits.

In addition to the continuation of work on branch-and-cut solution procedures, Karla Hoffman working with Carl Harris and Leslie-Ann Yarrow has used combinatorial optimization techniques to problems related to statistical experimental design. For creating latin-hypercube designs, statistical literature had routinely recommended using randomly generated latin-hypercube designs even though it was known that such designs have bias. This research has proved that optimal designs that match exactly the correlation of the underlying proplutaion not only exist, but can be computed in reasonable times.

Dr. Hoffman is working with her student Peter Ball, and with Roy Marsten on parallel implementations of scheduling problems in the airline industry. The approach taken integrates the column-generation phase with the solution approach, thereby providing new approaches that allow the *global* solution to the *integer* crew-scheduling problem to be found. The approach is a column-generation approach based on dual-price information and path following procedures. Such procedures requires that only a very small fraction of the variables of the problem need to be explicitly generated.

Manfred Padberg, working with his PhD students, D. Alevras and G. Kramer has produced three software packages (for public domain distribution) called DODEAL, ENTSP, ENZERO. DODEAL is an implementation of the double desription method, and ENTSP and ENZERO are enumeration procedures for the traveling salesman problem and the general zero-one problem.

Dr. Padberg's research with D. Alevras considers optimization problems over the unit hypercube, and optimization of order preserving assignments with contiguous assignments. D. Alevras is will be defending his dissertation during the Spring, 1994 semester.

Dr. Padberg's research with G. Cramer examines the polyhedral structure of the asymmetric traveling salesman problem, and develops heuristic solutions for that problem. Dr. Padberg's research with his student M. Rigjal examines scheduling problems with linear assignment and quadratic interaction costs. Mr. Rigjal is expected to defend his dissertation in May, 1994.

Publications under AFOSR Grant F49620-90-C-0022

Papers published in refereed journals:

Cannon, T.L. and Hoffman, K.L. 'Large-scale linear programming on distributed workstations", *Annals of Operations Research* Vol 22 (1990) 181-217.

Domich,P.D., Hoffman, K.L. Jackson, R.H.F. and McLane, M. "A microcomputer based solver for the facility location problem" *Management Science*. Volume 37 (1991) 960-979.

Hoffman,K.L. and Padberg, M. "Solving large-scale set-partitioning problems arising in the airline industry" *Management Science* 39 (1993) 657-682.

Hoffman, K.L. and Padberg, M. "Techniques for improving the LP- representation of zero-one linear programming problems" *ORSA Journal on Computing*, Volume 3 (1991) 121-134.

Padberg, M. and Rinaldi, G. "A Branch-and-Cut Approach to a Travelling Salesman Problem with Side Constraints" *Management Science*,Vol 35, (1989)1393-1412

Padberg, M. and Rinaldi, G. "An efficient algorithm for the minimum capacity cut problem" *Mathematical Programming Series A*, Vol 47, (1990)pp19-36.

Padberg, M. and Rinaldi,G "A branch-and-cut algorithm for the resolution of large-scale symmetric traveling saleman problems" *SIAM Review* Vol 33, (1991)60-100.

Padberg, M. and Rinaldi, G. "Facet identification for the symmetric traveling saleman polytope" *Mathematical Programming* Vol 47 (1990) 219-257.

Padberg, M. "The Boolean Quadric Polytope: Some Characteristics, Facets and Relatives" *Mathematical Programming, Series B*,Vol 45 (1989) 139-172.

Padberg, M. and Alevras, D. "Order Preserving Assignments" to appear in *Naval Research Logistics Quarterly* 41 (1994) 1-27.

Padberg, M. and Sung, T-Y. "An analytical derivation of max-flow mincut", *Computational Mathematics and Applications.*

Padberg, M. and Sung, T-Y. "An analytical comparison of travelling salesman problems", *Mathematical Programming* 52 (1991) 315-358.

Published books:

Padberg, M. (1993) *Linear Programming: Lecture Notes*, New York University.

Papers submitted to refereed journals (and not yet published):

Alevras, D., Padberg, M. and M.P. Rijal "The convex hull of a linear congruence constraint in zero-one variables" submitted to *Zeitschrift für Operations Research.*
Alevras, D. and Padberg, M. "Order preserving assignments with contiguity" submitted to *Discrete Mathematics.*
Grötschel, M. and Padberg, M. "Ulysses 2000: In Search of Optimal Solutions to Hard Combinatorial Problems" accepted for publication in *Scientific American*
Harris,C, Hoffman,K.L., and Yarrow,L.A. " Obtaining optimal latin hypercube sampling plans", submitted to *OR Spektrum.*
Harris,C., Hoffman, K.L. and Yarrow, L.A. "Using Lagrangian Relaxation to solve latin hypercube sampling plans", submitted to Annals of Operations Research.
Padberg, M. "Lehman's forbidden minor characterization of ideal 0-1 matrices", submitted to *Annals of Discrete Mathematics.*
Rushmeier, R., Hoffman, K. and Padberg, M. "Recent advances in exact optimzation of airline scheduling problems", submitted to *Operations Research.*

Honors/Awards/Prizes:

Dr. Karla L. Hoffman was awarded the Distinguished Faculty Award by the President and Board of Visitors of George Mason University, on April 30, 1989. Each school within the university honors one faculty member per year with this award.
Dr. Karla L. Hoffman was elected Treasurer of the Operations Research Society of America for a three year term beginnning May, 1993.
Dr. Karla Hoffman continues her role as Associate Editor of the journals *Mathematical Programming, Series B* and Computational Optimization and Applications.

In 1990, Dr. Manfred Padberg received an Alexander-von-Humbolt Research Award for senior U.S. scientists. He was also invited to work at the Ecole Polytechnique during his sabbatical leave from New York University (academic year 1989-1990). During his stay in Paris, Dr. Padberg twice taught a course in combinatorial optimization in the graduate program run jointly by Ecole Polytechnique and the University of Paris (Sorbonne). He also accepted a position as Visiting Distinguished Research Professor at George Mason University, and resides at George Mason University during the summer terms.

Manfred Padberg gave a plenary talk on Integer Programming techniques at the ORSA/TIMS meeting in San Fransisco, Fall, 1992.

Manfred Padberg was appointed to the editorial board of *Computational Mathematics and its Applications* in 1992.

Primary data samples:

All data sets related to the reports cited above are available from the authors by contacting the principal investigators. Approximately ten researchers have requested this data to date and all have received the data in machine-readable form. The data sets are also being collected by Robert Bixby of Rice University to be distributed in machine readable form.

This research has produced not results for which patents were applied for or issued.