MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

A CORRELATION ALGORITHM: THE CONCEPTUAL FRAMEWORK

L.G. TAFF
Group 91

TECHNICAL REPORT 789

2 FEBRUARY 1988

LEXINGTON MASSACHUSETTS

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the Department of the Air Force under Contract F19628-85-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Hugh L. Southall, Lt. Col., USAF
Chief, ESD Lincoln Laboratory Project Office

ABSTRACT

The lack of a utilitarian solution to the frame-to-frame correlation problem poses insurmountable difficulties for the successful passive observation of a collection of co-moving, nearly co-located objects. This is exactly the task faced in a scenario with respect to intercontinental ballistic missiles of the multiple re-entry vehicle delivery type. This report presents the conceptual framework for a potentially viable correlation algorithm. As well, the formal mathematical explication of the technique is included and applied to a specific illustrative example.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

A CORRELATION ALGORITHM: THE CONCEPTUAL FRAMEWORK

I. INTRODUCTION

This report proposes a method of solving the frame-to-frame correlation problem. The technique that is elucidated below is an amalgam of three disparate elements. One is a solution for the position and apparent motion of the entire complex of objects. Second is a mechanism for discriminating between the objects of interest and any other signals that might engender confusion. The final aspect provides a basis for identifying a particular detection in one frame with an above-threshold signal in another frame; that is, it is an algorithm for effecting the correlation *per se*. I expect that the synergism of these components will impart a power to this method not yet demonstrated by its competitors.

What do I mean by the frame-to-frame correlation problem? Imagine an optical telescope equipped with a light sensing device (e.g., a camera). Suppose that this combination is capable of exposure onto an appropriate recording medium (be it photographic film, videotape, charge coupled device, or so forth). The record of one such exposure I call a "frame." (Whether integration of the signal or averaging of the signal has occurred in order to form the "frame" is irrelevant to the larger point addressed in this report. Obviously this cavalier attitude must give way when one considers the details of the signal processing. Nevertheless, I shall continue to describe the problem, and my proposed solution to it, at the highest possible level.)

The telescope/camera/recording system is programmed to acquire a sequence of frames numbered by the time-ordering index $n = 1,2,...,N$. What has been imaged and recorded? In one scenario it is a target complex composed of the warheads, decoys, debris, and so on, of a multiple independent re-entry vehicle-type intercontinental ballistic missile. As these objects will shine by reflected sunlight, and are in a tightly bound, highly eccentric geocentric orbit, the sequence of frames clearly shows motion. Thus, the relative locations of the parts of the target complex will change from one frame to another. This fact is the genesis of the frame-to-frame correlation problem.

The pure correlation problem is confounded by many factors. I shall consider four obvious contributors toward masking the identity of each constituent of the target complex. One is that the celestial sphere is densely covered with stars, nebulosities, and galaxies. Therefore, each frame contains detectable signals from the fixed background as well as from the objects of interest. A second complication is that the optical system has a minimum finite resolution owing to the existence of diffraction. Hence, it is possible that the angular separation of two targets, or of a target and a star, is less than the system's resolution. This crowding problem means that what were separate signals on one frame may be merged into a single one on the next frame, and then separate again on a succeeding frame. Thus, the number of detections per frame will not be constant. A third ingredient contributing to the entanglement of real and false signals is the sources of noise in the system. Whether they be cosmic ray collisions, random electronic fluctuations in the circuitry, or from some other cause, the net effect is to produce a spurious signal that cannot, by itself, be distinguished from a true one. The fourth component I shall mention is associated with variations in the intrinsic brightnesses of the targets themselves. These can result from the rotation of a nonuniformly reflecting body or just the ordinary statistical fluctuations in the arrival of photons from an object near the system's limiting magnitude. In the former instance there will be a periodic fading in and out of the signal, whereas in the latter case the appearance of an above-threshold presence will be a temporally random event.

There are other factors which could be invoked to prove the veracity of the claim that the recording process is not a clean one. However, providing an exhaustive catalog of these is not my intention. The list just enumerated is appropriate to all wavelengths of the electromagnetic spectrum. In addition, it has counterparts with respect to active modes of sensing. The essential point is that no observing process will be pristine nor "noise-free" in the larger sense. This fact further complicates the solution of the frame-to-frame correlation problem.

Section II contains a nonmathematical description of the three facets of the algorithm. As the exposition progresses it will become increasingly clear to the reader that they interact in a subtle fashion. Accordingly, the algorithm is inherently iterative in concept. Following the textual account, a brief, formal, abstract quantitative sketch is presented. While the details of implementation await constructive criticism, a fully worked out (simple) numerical example is also included.

The concepts described herein have wider applicability. One obvious one is to the Optical Aircraft Measurements Program. The developers of this technology are looking at a similar phenomenon, in a passive mode, but in the infrared portion of the spectrum. However, in order to simplify the exposition of (presumably) novel ideas, I shall only consider the topic of frame-to-frame correlation.

II. THE CONCEPTUAL FRAMEWORK

A group of objects is contemporaneously co-moving. They occupy a limited spatial extent. Moreover, the patterns of their geometrical and kinematical dispersions, as defined by (say) the moment of inertia tenor and the velocity co-variance matrix, are characteristic of this type of complex. The fact that the distribution function $f(\mathbf{r},v,I,t)$ has a particular form shall be exploited when we discriminate between authentic signals emanating from the members of the target complex and other sources of detectable events. The distribution function depends on spatial location $\mathbf{r}$, velocity $\mathbf{v}$, intensity I, and the time t. While performing this separation we shall deduce, as a by-product, the centroid of the complex in the four-dimensional product space of location and velocity on the instrumental focal plane. When projected back onto the celestial sphere, this localization can serve as an estimator for the position and angular velocity of the center of mass of the threat cloud.

An observer, with a telescope, camera, and recording medium (the "instrument"), is some distance away from the assemblage. This topocentric distance is large enough so that the bunching of the components is apparent. On the other hand, the observer is near enough so that the fact that the conglomeration is composed of many individual parts is manifest. The observer causes the recording of a succession of frames $n = 1,2,...,N$ at the ordered times $t_1,t_2,...,t_N$ ($t_n > t_m$ if $n > m$). On image number n there are N_n detections whose locations can be ascertained.

The act of recording is presumed to be short compared to the temporal spacing between frames. Furthermore, the total temporal duration necessary to acquire the N frames is a small fraction of the orbital period of the complex's center of mass. Hence, it is an excellent approximation to neglect the dynamical aspects of the motion in favor of a purely kinematical description. Thus, a linear polynomial in the time should prove to be sufficient to represent the projected trajectory of an actual signal. (If curvature can be ascertained, then the inclusion of quadratic terms would be necessary. The degree of the polynomial, indeed the functional form of the empirical description, is a detail.)

The reader must appreciate the fact that dispensing with the (well-known) dynamics is not an essential, or even necessary, approximation for the algorithm's success (or power). Were it possible to meaningfully deduce the dynamical state from the quality and quantity of data gathered by the observer, then it would be beneficial to the performance of the algorithm to incorporate it. The reason should be clear; every systematically incorrect assumption built into the mathematical implementation must result in a less accurate representation of the observed phenomena than would a more realistic model. Therefore, it is desirable to include the full dynamics, if it could be done so, in a self-contained, physically significant fashion. As there is no known procedure to do so with the required accuracy on the time scales available, a purely empirical kinematical model that will precisely (and to all intents rigorously) describe the motion is the preferred option.

Because the target complex is in rapid motion, the signals from one element of it will not remain static on the observer's collection of frames. By forming the ratio of the difference between two locations and the corresponding times of exposure, an approximation to the projected angular velocity may be obtained. Indeed, the spatial compactness of the distribution function f(**r**,v,I,t) coupled with the smallness of the $(t_N - t_1)$ to (orbital period) ratio insures that all components of the threat cloud will have the same apparent angular velocity no matter which pair of frames is utilized. The geometrical center of the complex is found by averaging the positions of the true signals. (This is easily quantified. An arc second measurement precision combined with a 10-s time spacing between frames will yield an angular speed precision of approximately $0''.1$/s. If the observer is about 2000 km away, then this implies a tangential speed resolution of a meter per second and a tangential spatial resolution of 10 m.)

At this point in the depiction of the constituents of the algorithm, two have been discussed — namely, how the centroids of the true signals will be determined and how the genuine signals will be separated from the insubstantial ones. The last aspect instructs us how to perform the first two tasks; for if we do not already know which detections are interesting and to which other ones (on the other frames) they correlate, then we will not have a related pair of positions to difference. Thus, no estimate for the projected angular velocity would be forthcoming. Now it should be clear why this entire process must be an iterative one.

A trial assignment of matched $\mathcal{N}$-tuples $[\mathcal{N} = \max_n (N_n)]$ yields a set of velocities and locations. The distributions of these variables can be compared to projection of f(**r**,v,I,t). Eliminating those signals that do not conform to f in some statistically well-defined sense yields a new set of matched $\mathcal{N}$-tuples. These are utilized to compute positions and angular velocities which are, in turn, compared to f ... Note that a model for the six-dimensional location and velocity distribution function of the false signals is necessary too.

The third facet of the algorithm is a statistical measure of the relative probability of a correlation hypothesis. A correlation hypothesis is just one possible matched $\mathcal{N}$-tuple. (More explicitly it is the statement that the third object in frame one is also the eighteenth object in frame two, does not appear in frame three, is the eleventh object in frame four, and so on over all frames and all objects therein.) The necessary relative probabilities can be computed from the probabilities of false alarms, detections, and the conditional probability of target reality given its detection. These probabilities are relative because all (unimportant) normalizations have been discarded for simplicity of presentation. However, I shall consistently renormalize each of them so that all correlation hypotheses may be intercompared on the same scale. The most probable hypothesis is chosen to compute the positions and angular velocities that are to be compared with f(**r**,v,I,t).

Let me restate the iteration process: By some mechanism, say picking out the brightest object in each frame and declaring it to be one and the same parcel of the target complex, an initial value for the position and angular velocity centroids is found. This starting point, the constraints imposed by f(**r**,v,I,t) and the distribution function for the bogus signals, perhaps some other *a priori* information, and the limitation of linear motion (or an alternative kinematical hypothesis) allow every detection in every frame to be assigned to the true or false target bins.

Once this is accomplished, the true detections are matched by a correlation hypothesis. After due consideration of all such possible hypotheses, via the objective statistical measure just described, the best one is found. Utilizing this particular assignment of detections we can produce a new estimate for the position and angular velocity centroids. With these refined values we again compare the geometrical patterns of the detections and their angular velocity spread to the theoretical distribution functions, thereby differentiating between true and false signals in a more accurate manner. Then, with a revised register of real signals, we construct all possible matchings over the N frames, picking the most probable one. This is used to ...

III. THE CENTROIDS

As the mass of each member of the target complex is unknown, we cannot compute the position of the center of mass or its angular velocity. We could calculate a luminosity-weighted average position and angular velocity. If this were coupled with a mass-luminosity relationship, then we could estimate the center-of-mass values. Further discussion of this topic would take me too far afield. So, with the understanding that massive re-entry vehicles will not be differentiated from balloon-type decoys, we proceed by treating each detection equally.

Similarly, the exact nature of the computation of the location and velocity centroid will not be discussed herein. Actually, the former is trivial, but the latter does pose the possibility of constructing a sophisticated statistical estimator. Suppose that for a single object we have its Cartesian coordinates on each frame $\{(x_n,y_n)\}$, n = 1,2,...,N. Then at the crudest level we could use

$$\dot{x} = \frac{x_n - x_{n-1}}{t_n - t_{n-1}} \quad , \quad \dot{y} = \frac{y_n - y_{n-1}}{t_n - t_{n-1}}$$

for any value of $n \epsilon [2,N]$ to approximate the projection of the angular velocity. Taking the average of these N - 1 assessments would be one step better. However, if the motion is truly linear, within the inherent statistical uncertainties of the measurement process, then the forms

$$\dot{x} = \frac{x_n - x_m}{t_n - t_m} \quad , \quad \dot{y} = \frac{y_n - y_m}{t_n - t_m}$$

are valid for all n,m in-between 1 and N so long as n is not equal to m. Utilizing an average of all independent approximations of this type, properly weighted, is better yet.

Similarly, should a more complex kinematical model be chosen, then an appropriately sophisticated statistical estimation procedure for the parameters of the model can be created. No matter how it is accomplished, the final values for $\dot{x}$ and $\dot{y}$, when coupled with the plate scale and the coordinate transformation from the focal plane back to the sky, fix the angular velocity centroid.

IV. THE DISTRIBUTION FUNCTIONS

As mentioned earlier, I need two distribution functions. One, $f(\mathbf{r},\mathbf{v},I,t)$, models the three-dimensional location and velocity distributions of the target complex. We need to project these onto the instrument's focal plane. Naturally, the direction cosines of this projection are unknown. Hence, only generalities about the space and velocity dependences of f will really count and these can be adequately represented by the lowest-order moments of f. If it should happen that the measurement errors dominate the intrinsic speed dispersion, for example, then a Gaussian model is appropriate for the marginal velocity distributions.

The other distribution function represents the noise and background source location-velocity behavior. We can expect this to be dependent on place on the celestial sphere but not on the time. It may be necessary to include an admixture of distributions — one for the natural objects and one for the noise sources. Let me symbolize this total distribution function by F.

The purpose to which these distributions are to be put has already been stated. The distribution functions to which the data will be compared involve model parameters **q** and **Q** as well as a relative weight factor. The free parameter vectors **q** and **Q** involve the aforementioned projection factors, properties of the low-order moments of f and F, and so on. The relative weight factor is the ratio of the number of true to false signals. Multiplying by the total number of detections fixes the absolute number of threat cloud members.

The fashion in which all this is used is to form the likelihood function for a favored correlation hypothesis. The likelihood is maximized by solving for **q** and **Q**. However, the formal aspects of the analysis are independent of these details. For instance, let $g(x,y,\dot{x},\dot{y},I,t;\mathbf{q})$ and $G(x,y,\dot{x},\dot{y},I,t;\mathbf{Q})$ represent the projections of f and F onto the product space of location and velocity associated with the instrumental focal plane. Furthermore, if there is a total of N_t targets and N_b background/noise sources, then the probability of a detection being a real target is given by

$$P = \frac{N_t g}{N_t g + N_b G} \quad .$$

When it is desirable to do so, I shall separate $N_b G$ into $N_b'\gamma + N_b''\Gamma$ where $\gamma(\Gamma)$ is the projected distribution function of the natural (noise) sources and N_b' (N_b'') is the total number of them. Note that N_b'' especially will vary from frame to frame.

V. CORRELATION HYPOTHESES

Time is quantized in practice because frames are acquired at specific instants. Therefore, I shall change notation and write $g_n(x,y,\dot{x},\dot{y},I;\mathbf{q})$ for $g(x,y,\dot{x},\dot{y},I,t_n;\mathbf{q})$, and so on.

The target space T is the five-dimensional product space of location (x,y), velocity $(\dot{x},\dot{y})$, and intensity (I) on the frames. Each frame n = 1,2,...,N has a target probability density $g_n(x,y,\dot{x},\dot{y},I;\mathbf{q})$ defined over this space. One reason each frame has its own probability distribution of targets is that the target complex moves. Thus, if the telescope/camera/recording system is not tracking the target cloud, then g_n will be non-zero where g_m was not and vice versa $(n \neq m)$.

The probability of a real target within dx/2 of x, dy/2 of y, having velocity components within $d\dot{x}/2$ of $\dot{x}$ and $d\dot{y}/2$ of $\dot{y}$, and having an intensity (i.e., brightness) within dI/2 of I is

$$g_n(x,y,\dot{x},\dot{y},I;\mathbf{q})dxdyd\dot{x}d\dot{y}dI \quad .$$

For any subset S of T the integral $(dxdyd\dot{x}d\dot{y}dI \equiv dV)$

$$\int_S g_n dV$$

is the fraction of real targets on frame n within S. If S is equal to T, then the integral is equal to unity.

A simple model for g_n is the following: g_n is non-zero over a rectangular area of dimensions a × b which moves north and east at constant speed v making an angle θ with respect to the east-west line. The center of the rectangle is at (X_n,Y_n) on frame n. Finally, the intensity dependent part is a uniform distribution (all components of the target complex are equally bright).

The number of targets N_t is some (unknown) positive integer. Because of crowding and photon statistics, the number of detectable targets will vary from frame to frame.

The probability density of a false target is similarly defined and the symbol $G_n(x,y,\dot{x},\dot{y},I;\mathbf{Q})$ is used to denote it. On the average there are N_b background sources. G and N_b may be a function of n because the telescope/camera was moved between frames, because of the statistics of noise, because of photon statistics in the case of natural background sources (e.g., stars), because the lengths of the exposures for frames may be different (so that more and fainter stars appear on a longer exposure), and so on. For simplicity I assume that the telescope/camera does not move and that all exposures reach the same limiting magnitude. Therefore, N_b will, on the average, be the same for all frames.

The integral

$$\int_S G_n(x,y,\dot{x},\dot{y},I;\mathbf{Q})dV$$

is the fraction of false targets on frame n within S. If S is equal to T, then the integral is equal to unity. On some occasions it will be useful to decompose G_n into its natural (γ_n) and noise (Γ_n) parts. I then write for N'_b stars per frame and N''_b noise sources per frame (on the average)

$$G_n = \frac{N_b' \gamma_n + N_b'' \Gamma_n}{N_b} \quad .$$

A simple model for Γ_n is one independent of velocity, uniformly distributed over the whole area of a frame, with a Gaussian intensity distribution. Similarly, a simple *a priori* model for the stellar distribution γ_n is that it is independent of n (fixed pointing and sensitivity), independent of velocity, with a Poisson distribution over the area of the frame, and with a Gaussian intensity distribution. Moreover, we could fix N_b' to be equal to the average number of stellar sources at this galactic latitude (to the instrument's limiting magnitude).

For each real target on frame n there is a certain probability of detection. Symbolize this by $D_n(x,y,\dot{x},\dot{y},I;\mathbf{q})$. D_n could vary over the frame because of emulsion problems on a photographic plate, dead pixels on a charge coupled device, and so forth. D_n might be a function of velocity because of blurring caused by motion. Note that false targets do not need a detection probability as the definition of G presumes their detection (otherwise they would not cause any problems). In particular,

$$g_n \; D_n \; dV$$

is the probability of detection of a real target on frame n in the five-dimensional product space volume element dV.

Finally, even if a real target is successfully detected we may not measure its aspects precisely (or accurately). Thus, to be complete, we need to assign a conditional probability that a particular real target, detected at $(x,y,\dot{x},\dot{y},I)$ really has a location at (x,y), velocity $(\dot{x},\dot{y})$, and intensity I. This will be symbolized by $P_n(x,y,\dot{x},\dot{y},I;\mathbf{q})dV$. Because measurement error is not the real issue in frame-to-frame correlation, this probability density will be given short shrift below.

A *two-frame* correlation hypothesis is an assignment of a subset of the N_n detections on frame n to a subset of the N_m detections on frame m. The cardinal number $\mathcal{N}$ of this subset has $\min(N_n,N_m)$ as an upper bound. The correlation hypothesis is true if and only if all real target detections on frame n are matched with all real target detections on frame m *and* they represent the same targets in each of the $\mathcal{N}$ instances.

Number the detections which we shall try to correlate on frame n by the index η; on frame m the corresponding index is μ. They both run from unity to $\mathcal{N}$. The probability that detection η on frame n is real and correlated with real detection μ on frame m is

$$\int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta \int g_m(\mu;\mathbf{q})D_m(\mu;\mathbf{q})P_m(\mu)dV_\mu$$

where I have used the shorthand

$$g_n(\mu;\mathbf{q}) \equiv g_n(x_\mu,y_\mu,\dot{x}_\mu,\dot{y}_\mu,I_\mu;\mathbf{q}) \quad .$$

Thus, the total probability of pairs of detected true targets being matched (from the same real targets) is

$$\prod_{\substack{\eta,\mu=1\\ \text{matched}}}^{\mathcal{N}} \int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta \int g_m(\mu;\mathbf{q})D_m(\mu;\mathbf{q})P_m(\mu)dV_\mu \quad .$$

Not all detections on frame n are matched. They will not be matched either because they represent a false target on frame n (from the natural background or noise) or because their counterparts on frame m were not detected. The probability of these two events is

$$\int G_n(\eta;\mathbf{Q})dV_\eta + \int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta \int g_m(\mu;\mathbf{q})\,[1 - D_m(\mu;\mathbf{q})]dV_\mu \quad .$$

The probability of a correlation hypothesis includes the product of this quantity over all no matches. There is an exactly analogous expression with n and m interchanged (and η with μ as appropriate). Therefore, the *total relative probability* of a two-frame correlation hypothesis is given by

$$\prod_{\substack{\eta,\mu=1\\ \text{matched}}}^{\mathcal{N}} \left[\int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta \int g_m(\mu;\mathbf{q})D_m(\mu;\mathbf{q})P_m(\mu)dV_\mu\right] \times$$

$$\prod_{\substack{\eta,\mu=1\\ \text{not matched}}}^{\mathcal{N}} \left\{\int G_n(\eta;\mathbf{Q})dV_\eta + \int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta \int g_m(\mu;\mathbf{q})\,[1 - D_m(\mu;\mathbf{q})]dV_\mu\right\} \times$$

$$\prod_{\substack{\eta,\mu=1\\ \text{not matched}}}^{\mathcal{N}} \left\{\int G_m(\mu;\mathbf{Q})dV_\mu + \int g_m(\mu;\mathbf{q})D_m(\mu;\mathbf{q})P_m(\mu)dV_\mu \int g_n(\eta;\mathbf{q})\,[1 - D_n(\eta;\mathbf{q})]dV_\eta\right\}$$

$$= \prod_{\substack{\eta,\mu=1\\ \text{matched}}}^{\mathcal{N}} [p_n(\eta)p_m(\mu)] \times$$

$$\prod_{\substack{\eta,\mu=1\\ \text{not matched}}}^{\mathcal{N}} [A_n(\eta) + p_n(\eta)p'_m(\mu)] \times$$

$$\prod_{\substack{\eta,\mu=1\\ \text{not matched}}}^{\mathcal{N}} [A_m(\mu) + p_m(\mu)p'_n(\eta)] \quad .$$

Because this is a relative probability, I can renormalize by dividing through by the factors missing from the partial products. This leads to using

$$\prod_{\substack{\eta,\mu=1 \\ \text{matched}}}^{\mathcal{N}} \frac{p_n(\eta)p_m(\mu)}{[A_n(\eta) + p_n(\eta)p'_m(\mu)]\,[A_m(\mu) + p_m(\mu)p'_n(\eta)]}$$

for the probability of a two-frame correlation hypothesis.

Now consider what a *three-frame* correlation hypothesis would involve. For frames ℓ,m,n, there are N_ℓ, N_m, N_n detections. The maximum number $\mathcal{N}$ of possible correlates is limited by $\min(N_\ell, N_m, N_n)$. The probability that detection λ on frame ℓ is real and correlated with real detections μ on frame m and η on frame n is

$$\int g_\ell(\lambda;\mathbf{q})D_\ell(\lambda;\mathbf{q})P_\ell(\lambda)dV_\lambda \int g_m(\mu;\mathbf{q})D_m(\mu;\mathbf{q})P_m(\mu)dV_\mu \times$$
$$\int g_n(\eta;\mathbf{q})D_n(\eta;\mathbf{q})P_n(\eta)dV_\eta = p_\ell(\lambda)p_m(\mu)p_n(\eta) \quad .$$

Not all detections on frame ℓ are matched on both frames m and n. A detection will not be matched either because it represents a false target on frame ℓ or because one or both of its counterparts on frames m or n was not detected. These events have a probability of

$$A_\ell(\lambda) + p_\ell(\lambda)\,[p'_m(\mu)p_n(\eta) + p_m(\mu)p'_n(\eta) - p'_m(\mu)p'_n(\eta)] \quad .$$

Incorporating all factors, the relative probability of a three-frame correlation hypothesis is

$$\prod_{\substack{\lambda,\mu,\eta=1 \\ \text{matched}}}^{\mathcal{N}} p_\ell(\lambda)p_m(\mu)p_n(\eta) \times$$

$$\prod_{\substack{\lambda,\mu,\eta=1 \\ \text{not matched}}}^{\mathcal{N}} \left\{A_\ell(\lambda) + p_\ell(\lambda)\,[p'_m(\mu)p_n(\eta) + p_m(\mu)p'_n(\eta) - p'_m(\mu)p'_n(\eta)]\right\} \times$$

$$\prod_{\substack{\lambda,\mu,\eta=1 \\ \text{not matched}}}^{\mathcal{N}} \left\{A_m(\mu) + p_m(\mu)\,[p'_\ell(\lambda)p_n(\eta) + p_\ell(\lambda)p'_n(\eta) - p'_\ell(\lambda)p'_n(\eta)]\right\} \times$$

$$\prod_{\substack{\lambda,\mu,\eta=1 \\ \text{not matched}}}^{\mathcal{N}} \left\{A_n(\eta) + p_n(\eta)\,[p'_\ell(\lambda)p_m(\mu) + p_\ell(\lambda)p'_m(\mu) - p'_\ell(\lambda)p'_m(\mu)]\right\} \quad .$$

After renormalization, we will deal with

$$\prod_{\substack{\lambda,\mu,\eta=1 \\ \text{matched}}}^{\mathcal{N}} \frac{p_\ell(\lambda)p_m(\mu)p_n(\eta)}{(\text{all ways to fail matching})_{\lambda\mu\eta}}$$

where (all ways to fail matching)$_{\lambda\mu\eta}$ is equal to the product of the last three factors in the previous expression. The procedure for further generalization should now be clear.

VI. A NUMERICAL EXAMPLE

I shall briefly illustrate the algorithm on a simple example. First, to make the problem tractable, I shall ignore intensity information. Thus, a given pixel is either on or off. I also have a small frame area and only three frames.

Figure 1 shows the locations of all background sources in my 12 × 15 frame ($N_b' = 45$). There are five locations which represent threshold sources. These five pixels are sometimes on. Figure 2 has a 1, 2, or 3 (in Souvenir type font) in the corresponding pixel to indicate on which frames these stars were detected. Also contained in Figure 2 are the locations and frame numbers for which the noise pixels are on (in Universe type font, $N_b'' = 5$). Figure 3(a) shows all background pixels which are on and all lit target location pixels from all three frames (i.e., their union). The target complex moves from the lower left to the upper right. The target complex itself is shown in Figure 3(b) ($N_t = 6$). Figures 4, 5, and 6 show the succession of individual frames without the target complex (a) and with it (b). Finally, Figure 7 shows those 40 pixels always lit, and therefore presumably part of the fixed background, while Figure 8 shows the complement of all pixels at least once lit (= 105). Those pixels never lit can be disregarded. Note that disregarding never-lit pixels and always-lit pixels is permissible because of the neglect of intensity information. There are 35 suspect pixels left (over all three frames). These represent threshold stars, noise pixels, and the three locations of the target complex.

The initial model for the target complex is a rectangle of size a × b moving at constant speed v making an angle θ with respect to the x-axis (θ positive counterclockwise). Initial estimates for these parameters are a = 2, b = 4 (g = 1/8 on these 8 pixels, zero elsewhere), v = 6.5 pixels/interframe duration, $\theta = 47°$. These estimates were derived utilizing a ruler and protractor after an impartial examination of Figures 4, 5, and 6. Remember that $N_t = 6$ [over a 2 × 4 array, see Figure 3(b)], v = 6 pixels/interframe duration, and $\theta = 51°$.

The *a priori* model for the natural background sources is a stellar density of 50 stars at or above threshold with 7 ($= \sqrt{50}$) being at threshold. Thus, $N_b' = 46.5$ (= 50 − 7/2). Therefore, we take γ to be uniform over the frame and likewise for Γ. The average number of noise sources per frame is to be $3 = N_b''$ (so $N_b = 49.5$). [Since there are 40 pixels always lit, our initial estimate for N_b' might have been $40 + \sqrt{40}/2 = 43$. This leaves 12 pixels on the first frame and 9 on the last two to be noise pixels plus target complex. As we have already assumed that the target complex occupies 8 pixels, there must be 2 noise pixels per frame. Note that the initial model ignores crowding. Without intensity information, there is no good method of deciding the overlap issue.] Thus,

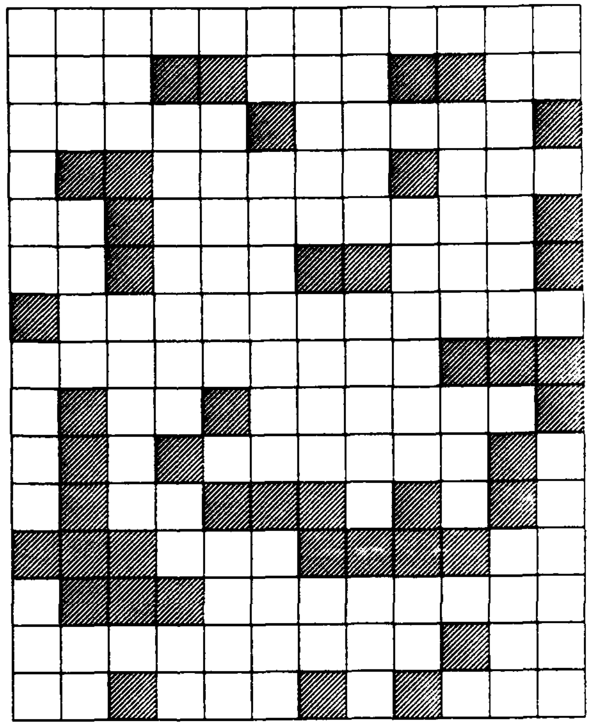
$$G = 0.275/49.5 \quad .$$

The next step is to use the formula on page 9 to evaluate the probability of a detection being a real target. The values for g change from frame to frame because the target complex has moved. We handle this by using the assumed centroids, obtained by looking at Figures 4, 5, and 6 as discussed above. As only 35 pixels are in question, only their P values have been calculated.

Because of the form of g (= 1/8 over the 2 × 4 pixels centered on the assumed centroid, 0 elsewhere), the P values are either 0.784 or 0 depending on whether or not they fall onto the presumed 2 × 4 array, properly centered and properly oriented.

After completing this step, the three target complex locations are shown in Figure 9. All but 2 noise pixels have been eliminated from further consideration and only one threshold star has not been removed. The structure of the target complex is so simple, when combined with the relatively dense presence of stars and with the absence of intensity formation, that further iteration leads to no refinement of Figure 9 as a result of any reasonable correlation hypothesis.

The new estimate for v is 6.8 pixels/interframe duration and $\theta = 46°$. The crowding of stars in association with the simple intensity model makes it impossible to distinguish between a pixel containing a star (above threshold) and a star plus a component of the target complex. Therefore, a refinement of the results beyond this iteration does not seem possible.



87696-1

Figure 1. Background sources on standard frame.

							1				3
				1							3
			1								
									2		
						2,3					
			2		3						
									1,3		
	2										
										3	
			1	1							
		1,3					1				
		3	1		2	2					

87696-2

Figure 2. Frame numbers of noise pixels on (Universe type font) and threshold stars on (Souvenir type font).

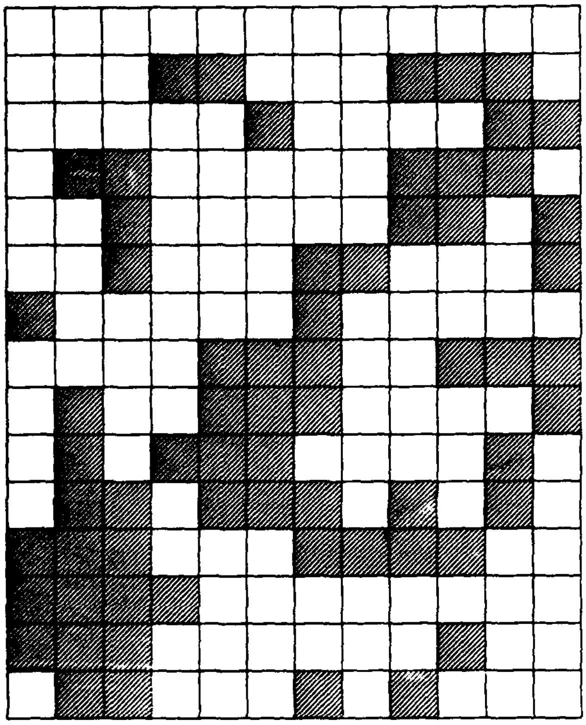


Figure 3(a). Background sources plus all three target complex locations.

87696-3

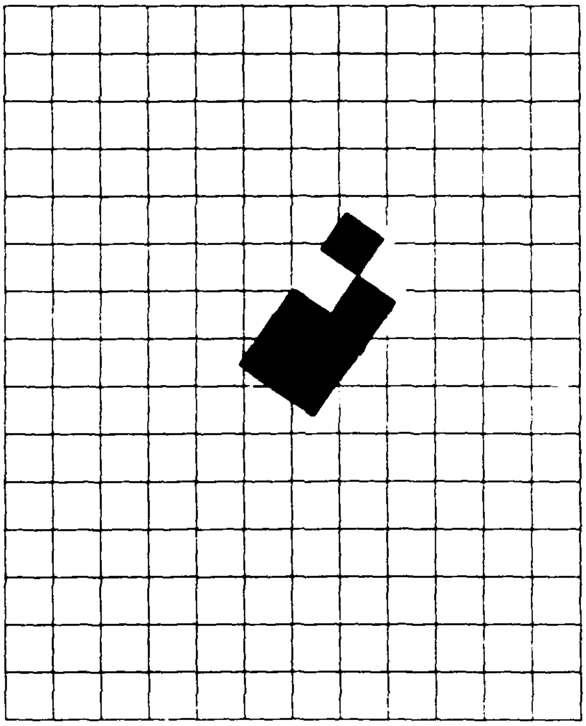


Figure 3(b). Actual target complex configuration.

87696-4

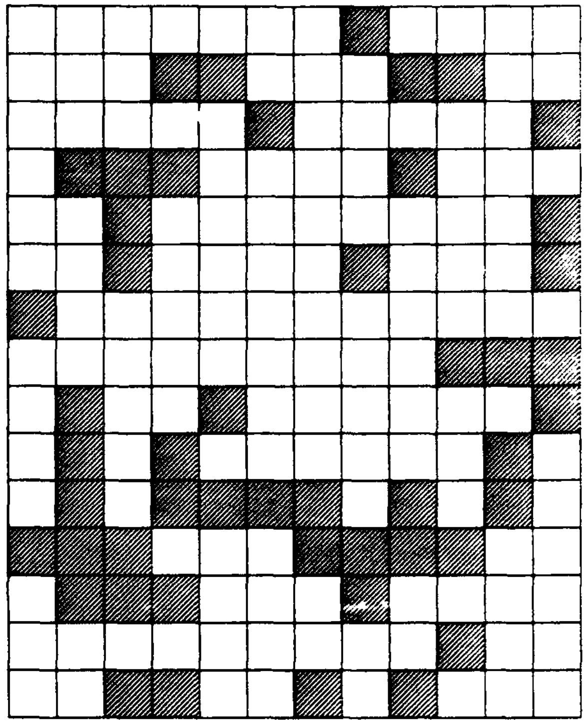


Figure 4(a). Frame 1: background plus noise.

87696-5

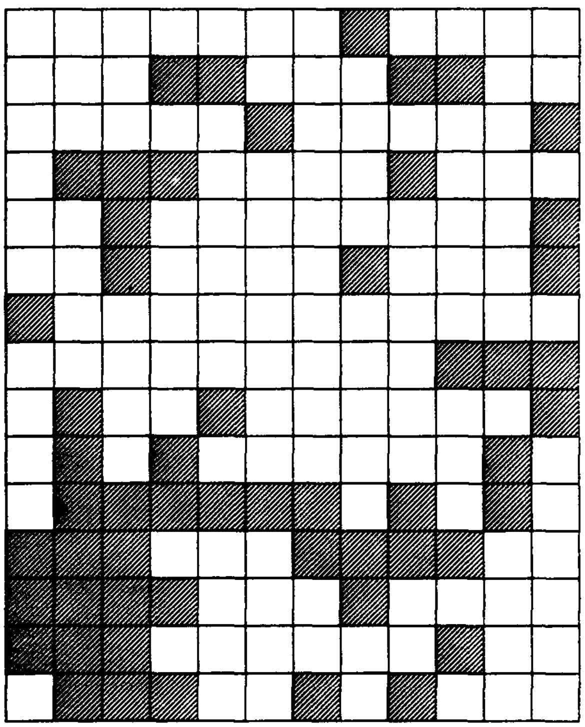


Figure 4(b). Frame 1: background plus noise plus target complex.

87696-6

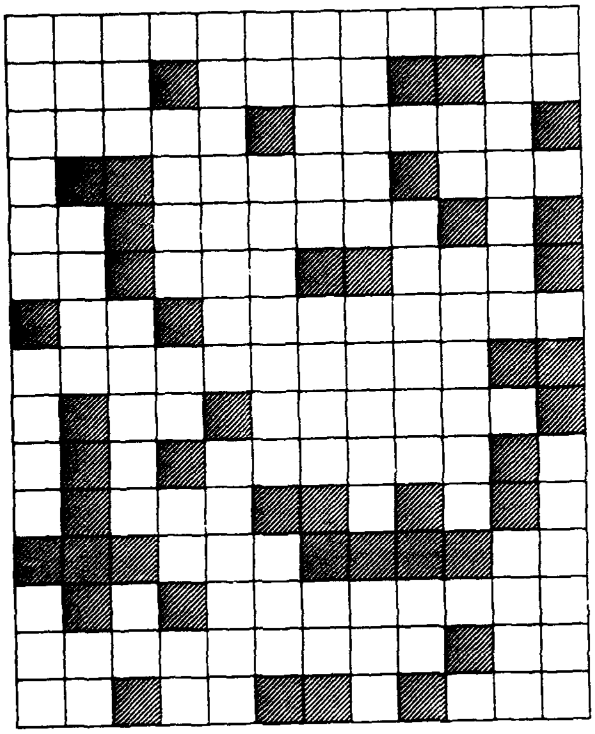


Figure 5(a). Frame 2: background plus noise.

87696-7

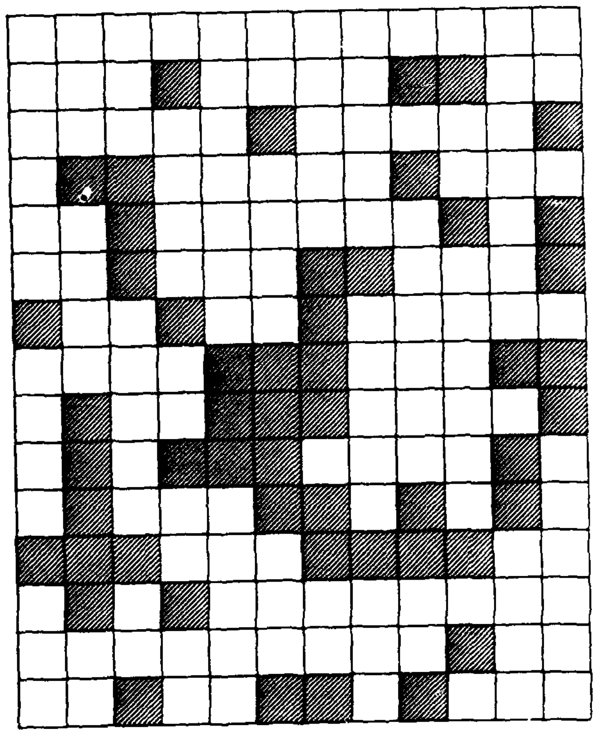


Figure 5(b). Frame 2: background plus noise plus target complex.

87696-8

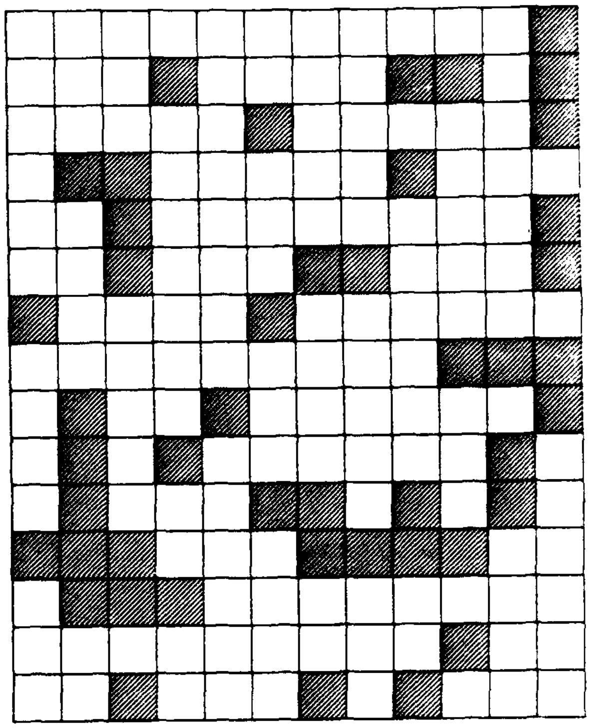


Figure 6(a). Frame 3: background plus noise.

87696-9

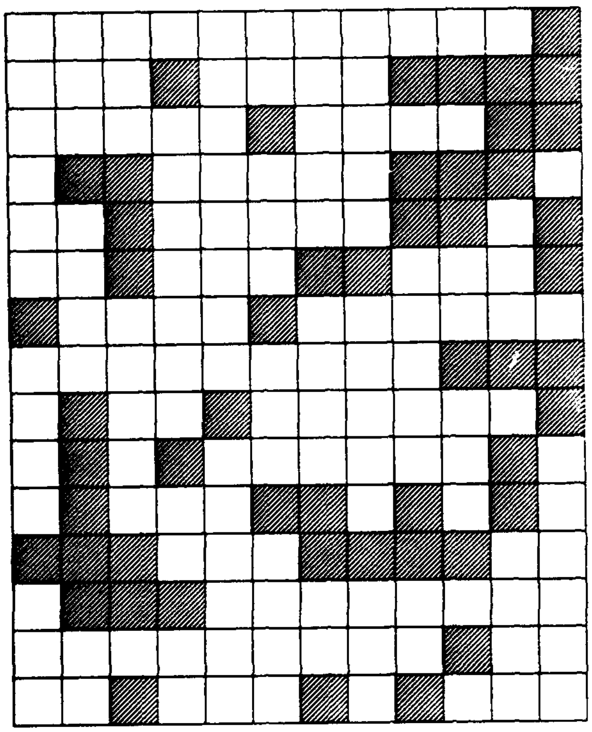


Figure 6(b). Frame 3: background plus noise plus target complex.

87696-10

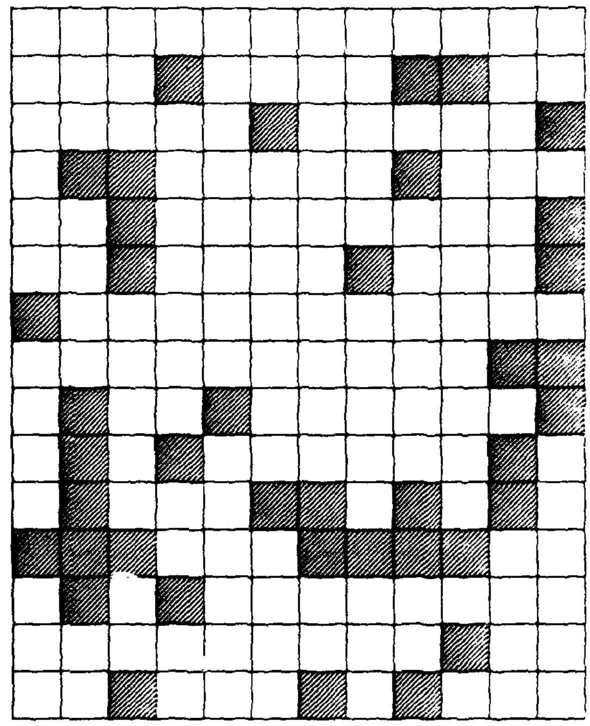


Figure 7. Pixels lit on all frames.

87696-11

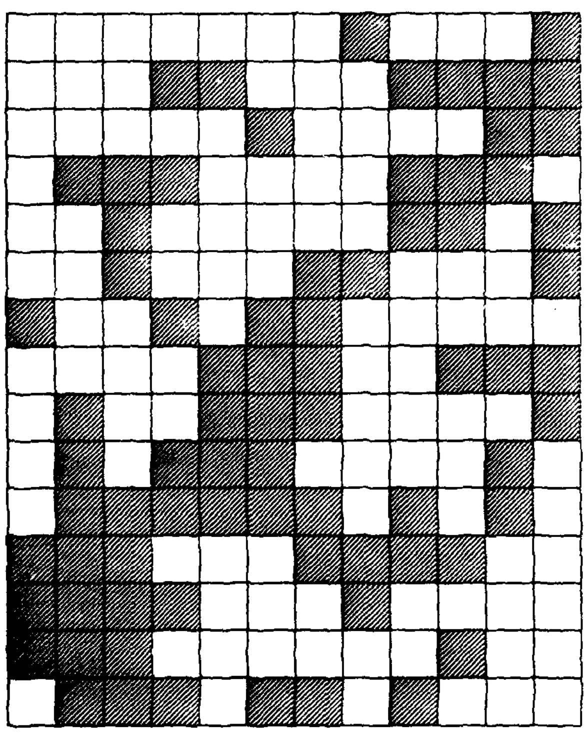
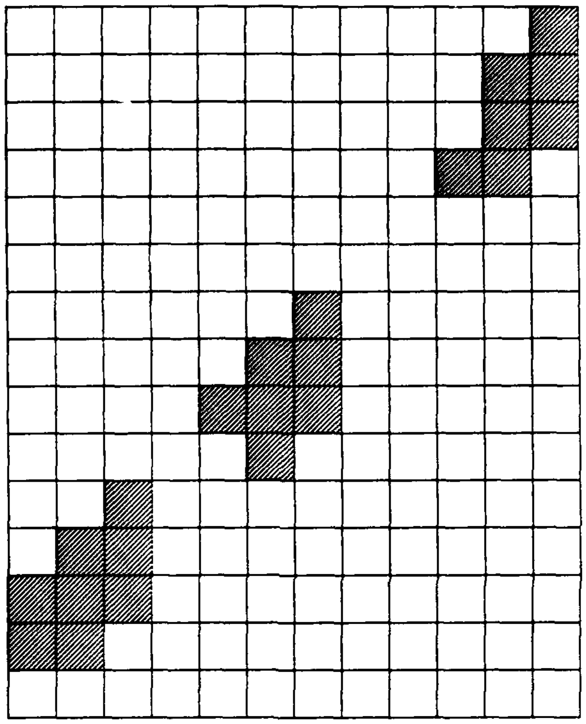


Figure 8. Complement of at least once-lit pixels.

87696-12



88289-1

Figure 9. Presumed target complex after one iteration.

VII. SUMMARY

I have described a method of resolving the frame-to-frame correlation problem. The unique property of this algorithm is its simultaneous reliance on a tripartite structure intimately intermixed in execution. This three-pronged approach seeks to concomitantly derive the position and angular velocity of the target complex, to separate true signals from false ones, and to make the best possible association of a set of detections on one frame with a set of detections on another frame. Whether or not this conceptual framework will prove to be efficacious in practice awaits a full computational test.

ACKNOWLEDGMENTS

Two people materially aided the development of my ideas. One is Dr. W.R. Stromquist of D.H. Wagner Associates. To him I owe the origin of the statistical basis for evaluating different correlation hypotheses. The second individual is Ms. D.M. Jonuskis. To her I owe thanks for a complete search of the relevant astronomical literature. This turned up the essence of the discrimination concept utilizing the distribution functions. In addition, whatever clarity this presentation possesses is to her credit; her questions considerably sharpened my own thinking on this subject.

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report 789	5. MONITORING ORGANIZATION REPORT NUMBER(S) ESD-TR-87-096
6a. NAME OF PERFORMING ORGANIZATION Lincoln Laboratory, MIT 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Electronic Systems Division
6c. ADDRESS (City, State, and Zip Code) P.O. Box 73 Lexington, MA 02173-0073	7b. ADDRESS (City, State, and Zip Code) Hanscom AFB, MA 01731
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Air Force Systems Command, USAF 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F19628-85-C-0002
8c. ADDRESS (City, State, and Zip Code) Andrews Air Force Base Washington, DC 20334	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: 12424F PROJECT NO.: 77 TASK NO.: WORK UNIT ACCESSION NO.:

11. TITLE (Include Security Classification)

A Correlation Algorithm: The Conceptual Framework

12. PERSONAL AUTHOR(S)

Laurence G. Taff

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Report	FROM ______ TO ______	1988, February, 2	40

16. SUPPLEMENTARY NOTATION

None

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	correlation; frame-to-frame correlation; SDI; optical discrimination

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The lack of a utilitarian solution to the frame-to-frame correlation problem poses insurmountable difficulties for the successful passive observation of a collection of co-moving, nearly co-located objects. This is exactly the task faced in a scenario with respect to intercontinental ballistic missiles of the multiple re-entry vehicle delivery type. This report presents the conceptual framework for a potentially viable correlation algorithm. As well, the formal mathematical explication of the technique is included and applied to a specific illustrative example.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Lt. Col. Hugh L. Southall, USAF	22b. TELEPHONE (Include Area Code) (617) 981-2330 22c. OFFICE SYMBOL ESD/TML

DD FORM 1473, 84 MAR

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