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Instantaneous Variance Estimation by Spectral Collapse

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Abstract

Mean-square consistent estimators for both the two-dimensional spectrum and the instantaneous variance of non-stationary white noise are obtained from a single time series. Applications to real sonar data indicate that these estimators can be adapted to both detect and recover transients.

1 Introduction

To motivate this paper, a brief outline of the background leading to our results on transient recovery is given. In 1991, Herd and Gerr [12] presented a collection of graphical methods to display the presence of periodic correlation in non-stationary time series. If a time series is periodically correlated, then the method of diagonal smoothing is a consistent estimator of the two-dimensional spectrum under suitable mixing conditions [9]. Once an estimate of the spectrum is obtained, the associated spectral coherence, the coherent and incoherent collapse can graphically display the periodic correlation.

The authors of this paper applied these graphical methods to the STRAP dataset of the DFDT project [1]. This dataset contains 16,384 array snapshots from a nine-sensor random array recording in a real underwater acoustic environment. An area of spectral coherence was observed to exceed a false-alarm threshold of $P_0 = 0.001$. Both the coherent and incoherent collapse of the spectral coherence display obvious periodic patterns. An FFT was then applied to the coherent collapse and a collection (typically three) of very clear pulses were obtained. On examination of this pulse train and the original time series, R. Madan made the conjecture that this method had recovered transients from the time series. This paper uses the model of non-stationary white noise (NSWN) [8] to provide a theoretical explanation for Madan's observation.

2 Non-Stationary White Noise

Non-stationary white noise processes have been in the electrical engineering literature for a number of years. Standard manipulations typically operate using a generalized function formalism. For example, Papoulis [15] delineates non-stationary white noise as those random process $\{x(t)\}$ with zero mean and covariance function of the form

$$R_{xx}(t_1,t_2) = E[x(t_1)\overline{x(t_2)}] = \sigma_x^2(t_1)\delta(t_1-t_2). \quad (1)$$

Here $\delta(t)$ denotes the Dirac delta function and the overline denotes the complex conjugate. The function $\sigma_x^2(t)$ will be called the instantaneous variance. Thus, R_{xx} is a generalized function supported on the main diagonal of the $t_1 \times t_2$ plane. Since $\{x(t)\}$ is non-stationary, it is of interest to determine its two-dimensional spectrum $S_{xx}(f_1,f_2)$ defined as the Fourier transform of the covariance function [13]:

$$\begin{aligned} S_{xx}(f_1,f_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i2\pi(f_1t_1-f_2t_2)}R_{xx}(t_1,t_2)dt_1dt_2 \\ &= \widehat{\sigma_x^2}(f_1-f_2). \end{aligned} \quad (2)$$

Here $\widehat{\sigma_x^2}(f)$ denotes the one-dimensional Fourier transform of the instantaneous variance. Thus, the spectrum of $\{x(t)\}$ is constant along lines of constant difference frequency in the $f_1 \times f_2$ plane. We note this observation was previously obtained by Ogura [14] for time series (discrete time). This diagonal structure implies that $S_{xx}(f_1,f_2)$ should be amenable to estimation via the diagonal smoothing technique used in periodically correlated processes [12] and forms the basis for the estimators used in this paper. However, there are technical issues with continuous-time white noise processes which can be overcome via a form of Brownian motion.

3 Technical Issues

Discrete-time NSWN $\{x_n\}$, their associated Fourier representation, and the accompanying development of the harmonizable theory has been undergoing a rigorous exploration by Houdré [6], [7]. In particular, his non-stationary white noise model provides an excellent example to distinguish between L and R harmonizable random processes [8].

However, continuous-time NSWN $\{x(t)\}$ requires additional technical considerations. For example, the middle term of equation (1) is merely a formalism [3, page 273]. Two approaches to a NSWN random process can be made. First, there is the Gel'fand-Vilenkin [3] theory of generalized random processes. This theory provides the justification for the formal but non-trivial manipulations found in engineering texts. Second, there is the integration theory for non-homogeneous Brownian motion as discussed in Doob [2]. This theory permits non-stationary white noise to be approached as the infinitesimal of a non-homogeneous Brownian motion. Both approaches have their merits and both can be made consistent by introducing the following ordinary random process $\{B(t)\}$.

Definition 3.1 A random process $\{B(t)\}$ will be called an L^1 Brownian motion provided it has zero mean and there is a positive function $\sigma_x^2 \in L^1(\mathbf{R})$ which represents the covariance function as

$$R_{BB}(t_1, t_2) = E[B(t_1)\overline{B(t_2)}] = \int_{-\infty}^{\min(t_1,t_2)} \sigma_x^2(t)\, dt.$$

Standard statistical arguments show that $\{B(t)\}$ admits a version which is real-valued, Gaussian, separable, jointly measurable, and has continuous sample paths. These properties permit the interpretation of $\{B(t)\}$ as a generalized random process with derivative $x(t) = B'(t)$ satisfying the formalism

$$\int_a^b g(t)x(t)dt = \int_a^b g(t)dB(t).$$

From this, it is straight-forward to obtain

$$R_{B'B'}(t_1, t_2) = \frac{\partial^2 R_{BB}}{\partial t_1 \partial t_2}(t_1, t_2) = \sigma_x^2(t_1)\delta(t_1 - t_2).$$

Thus, a meaning can be attached to equation (1), the associated covariance function R_{xx}, and the spectral estimators.

4 Spectral Estimators

Theorem 4.1 shows diagonal smoothing estimates the 2D spectrum of NSWN provided the usual tradeoffs regarding the growths of the time and frequency windows are made to force consistency of the estimator.

Theorem 4.1 Suppose $\{x(t)\}$ is a non-stationary white noise random process with instantaneous variance $\sigma_x^2(t)$ obtained as the generalized derivative of an L^1 Brownian motion. Assume

(NWN-1) $\{x(t)\}$ is real-valued

(NWN-2) $\{x(t)\}$ is Gaussian

(NWN-3) $\sigma_x^2 \in L^1(\mathbf{R})$ and $\widehat{\sigma_x^2} \in L^1(\mathbf{R})$.

Let the finite Fourier transform of the sample be denoted

$$\hat{x}(b; f) = \int_{-b}^{b} e^{-i2\pi f t} x(t)dt.$$

Consider smoothing the "raw" 2D spectrum along a diagonal line segment through (f_1, f_2):

$$S_{xx}(b, F; f_1, f_2) = \frac{1}{2F}\int_{-F}^{F} \hat{x}(b; f_1 + f)\overline{\hat{x}(b; f_2 + f)}\, df.$$

Then for each (f_1, f_2) in the frequency plane, there holds:

$$\lim_{\substack{b\to\infty \\ F\to\infty}} S_{xx}(b, F; f_1, f_2) = \widehat{\sigma_x^2}(f_1 - f_2)$$

in quadratic mean, provided $b/F \to 0$.

Theorem 4.2 shows the spectral collapse used by Hurd and Gerr [12] can be adapted to estimate the instantaneous variance. What makes this result of interest is the appearance of the Wigner time-frequency distribution and suggests a connection between diagonal smoothing and time-frequency estimators.

Theorem 4.2 Suppose $\{x(t)\}$ is a non-stationary white noise random process with instantaneous variance $\sigma_x^2(t)$ obtained as the generalized derivative of an L^1 Brownian motion. Assume (NWN-1), (NWN-2), and (NWN-3) hold and make the additional assumption:

(NWN-4) There is an $\alpha > 1/2$ such that $\sigma_x^2(t) = \mathcal{O}[|t|^{-\alpha}]$ as $t \to \infty$.

Define the Wigner time-frequency distribution by taking the inverse Fourier transform of the spectral estimate along lines of constant sum frequency:

$$W_{xx}(b, F, U; t, f) = \int_{-U}^{U} e^{+i2\pi u t} S_{xx}(b, F; f + u/2, f - u/2)\, du.$$

Coherent collapse over frequency gives:

$$\sigma_x^2(b, F, U, V; t) = \frac{1}{2V}\int_{-V}^{V} W_{xx}(b, F, U; t, f)\, df.$$

Then for each $t \in \mathbf{R}$, there holds:

$$\lim_{\substack{b\to\infty \\ F\to\infty \\ U\to\infty \\ V\to\infty}} \sigma_x^2(b, F, U, V; t) = \sigma_x^2(t)$$

in quadratic mean, provided $U/b^{2\alpha-1} \to 0$ and $U^2 b/F \to 0$.

5 Applications

While Theorem 4.1 estimates the Fourier transform $\widehat{\sigma_x^2}(f_1 - f_2)$ of the instantaneous variance of a NSWN process $\{x(t)\}$, some comments are required to make the connection to the estimators associated with spectral coherence as used by Hurd and Gerr [12].

First, Hurd and Gerr [12] work with time series $\{x_n\}$ rather than with continuous-time random processes $\{x(t)\}$. Therefore, the spectral estimator of Theorem 4.1 will be implemented using a discretization of $S_{xx}(b, F; f_1, f_2)$. The notation will change accordingly. Specifically, the finite Fourier transform will be implemented by applying an FFT to the available stretch of data:

$$\hat{x}(N; m) = \sum_{n=0}^{N-1} e^{-i2\pi m n/N} x_n.$$

We are tacitly using the number of time samples N to replace the continuous time interval $[-b, b]$.

Second, diagonal smoothing is performed by averaging over a diagonal of frequency bins instead of an interval. Henceforth, we will use L frequency bins to correspond to the diagonal smoothing interval $[-F, F]$ and estimate the 2D spectrum as

$$S_{xx}(N, L; m_1, m_2) = \frac{1}{L}\sum_{l=0}^{L} \hat{x}(N; m_1 + l)\overline{\hat{x}(N; m_2 + l)}.$$

Third, the false-alarm threshold of Goodman [4] is used as follows: Define the spectral coherence function

$$C_{xx}(N, L; m_1, m_2) = \frac{S_{xx}(N, L; m_1, m_2)}{\sqrt{S_{xx}(N, L; m_1, m_1) S_{xx}(N, L; m_2, m_2)}},$$

provided the denominator does not vanish. Assume the components of the FFT $\hat{x}(N; m)$ are complex Gaussian with zero mean and locally constant variance. That is, for each frequency bin there holds $E[|\hat{x}(N; m)|^2] = E[|\hat{x}(N, m + l)|^2]$ for $l = 0, \ldots, L$. Then

$$\text{Prob}(|C_{xx}(N, L; m_1, m_2)|^2 > \gamma_0^2) = (1 - \gamma_0^2)^{L-1}.$$

Thus, a false-alarm threshold γ_0 for probability P_0 is given by $(1 - \gamma_0)^{L-1} = P_0$. In the plots of spectral coherence, only those features exceeding the false-alarm threshold determined by L will be displayed.

Fourth, Hurd and Gerr [12] also calculate the collapsed spectral coherence which they define as the mean value of the estimated spectral coherence along a diagonal of constant difference frequency:

$$c_{xx}(N, L; m) = \frac{1}{N - m}\sum_{m'=0}^{N-m} C_{xx}(N, L; m + m', m').$$

From the preceding remarks, it is reasonable to conjecture that the collapsed coherence will also be a reasonable estimator for $\widehat{\sigma_x^2}$. In light of Theorem 4.2, we also conjecture that the inverse FFT of $c_{xx}(N, L; m)$ is also a reasonable estimator for $|\sigma_x^2(0)|^{-2}\sigma_x^2(t_n)$. While Hurd and Gerr [12] collapse over the entire frequency plane, we will restrict the collapse to those regions of the frequency plane which display a high level of coherence.

Example 5.1 Amplitude-Modulated Noise: In this example, a total of $N = 4,096$ time samples simulating a NSWN signal $\{s_n\}$ in additive noise were collected. The time series $\{x_n\}$ has the form

$$x_n = s_n + z_n = \sum_{p=1}^{3} 1_{[0,.2]}(t_n - \tau_p)u_n + z_n.$$

The τ_p's are pulse arrival times uniformly selected from the observation interval. The time samples t_n correspond to a sample rate of 300 Hertz to match the STRAP dataset. The forcing term $\{u(t)\}$ is IID uniform $[0, 1]$ noise. The noise term $\{z_n\}$ is IID, Gaussian with zero mean, a variance σ_z^2, and is independent from the signal. Thus, $\{x_n\}$ is NSWN with instantaneous variance

$$\text{Var}[x_n] = \frac{1}{12}\sum_{p=1}^{3} 1_{[0,.2]}(t - t_p) + \frac{1}{5}.$$

Figure 1 displays both the pulse train and its noise-corrupted version where the sample variance of $\{z(t)\}$ is 0.2101. Figure 2 displays a contour plot of the estimated spectral coherence obtained by 16-bin smoothing and thresholding with the probability of false alarm $P_0 = 0.05$. Note the emergence of a square patch of coherence in the region $\mathcal{R} = [0, 5] \times [0, 5]$. A coherent collapse was performed over $\mathcal{R}$, and a zero-padded inverse FFT was applied. The results are displayed in figure 3 which plots the original signal and an estimate of the instantaneous variance (scaled by $|\sigma_x^2(0)|^{-2}$) recovered from the noise-corrupted signal.

Example 5.2 STRAP DATASET: This dataset contains 16,384 array snapshots obtained from a 9-sensor random array recording in a real underwater acoustic environment. For this paper, only the time series obtained from sensor #1 will be discussed as similar results are obtained from the remaining sensors. While space does not permit the display of the time series in this paper, it will be presented in the talk. There are hints of transients of similar shape near 2, 28, and 53 seconds. The spectral coherence was estimated over the region $[4, 13] \times [4, 13]$ Hertz in the (f_1, f_2) frequency plane using $L = 32$ bin frequency smoothing. Figure 4 displays the resulting spectral coherence using $P_0 = 0.001$ to set the threshold. When the coherent collapse of the spectral coherence was computed over the square, periodic patterns were observed in the magnitude of the collapse. An FFT was then performed and the results are shown in figure 5. There are three clear spikes which seem to be associated with the previously mentioned transients.

6 Concluding Remarks

Theorems 4.1 and 4.2 demonstrate that nonstationary white noise provides one model in which transient recovery via the Fourier transform of the collapsed spectrum makes sense. However, there remain a number of topics regarding the random process, the estimators, and applications which are open for exploration.

Regarding the random process, both the Gaussian assumption and the L^1 bound on the instantaneous variance were made to obtain tractable covariance bounds for Theorems 4.1 and 4.2. Since these kinds of bounds are also obtained using various mixing conditions [11], it seems reasonable to conjecture that both the Gaussian assumption and the L^1 bound may be relaxed and Theorem 4.1 will still be valid. Moreover, it also seems reasonable to conjecture Theorem 4.1 will still be valid if the white-noise process $\{x(t)\}$ were replaced by process $\{y(t)\}$ which had a covariance R_{yy} with the same general "shape" as the white-noise covariance R_{xx}. One example $\{y(t)\}$ could be obtained by passing $\{x(t)\}$ through a linear filter $y = h * x$.

Regarding the estimators, it seems worthwhile to explore other means to smooth the raw spectrum, especially when working with limited time series. In this regard, the work of Thomson [16] seems especially promising as a means to include the length of the time series in the estimation scheme. The appearance of the Wigner distribution in Theorem 4.2 is of interest in that the work on cross-term suppression in the time-frequency distributions should carry over to give better smoothing windows for the two-dimensional spectrum. In this regard, we point out the work of Wilbur and McDonald [18] as having made the connection between the Wigner distribution and cyclostationary processes.

Regarding the applications, it is worthwhile to point out that the use of the false-alarm threshold is promising as a means of transient detection. However, comparisons need to be made with this method against other methods of transient detection and estimation, such as a short-time variance estimator or time-frequency distributions to also detect transients. In this regard, the work of Hearon and Amin [5] should be pointed out as establishing a statistical criteria for kernel selection for a time-frequency distribution.

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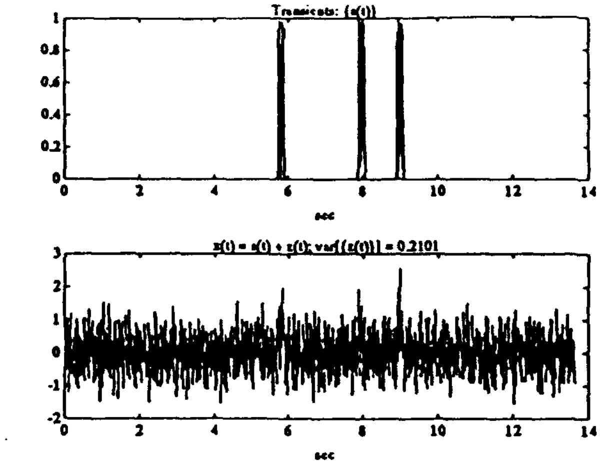

Figure 1: **Upper Plot:** Random pulse train. **Lower Plot:** Pulse train in $N(0, \sigma_z^2)$ noise.

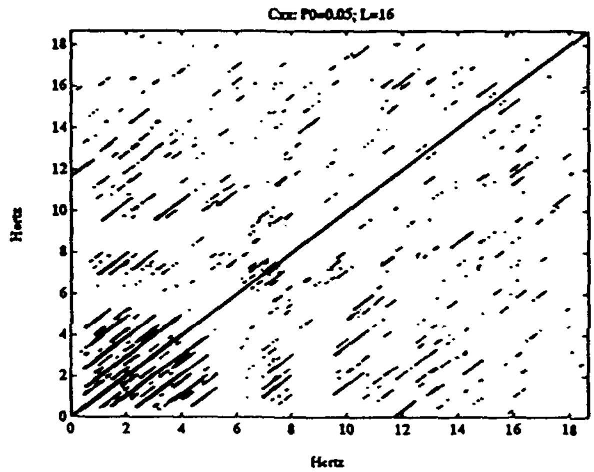

Figure 2: Estimated spectral coherence: $L = 16$ bin diagonal smoothing; $P_0 = 0.05$.

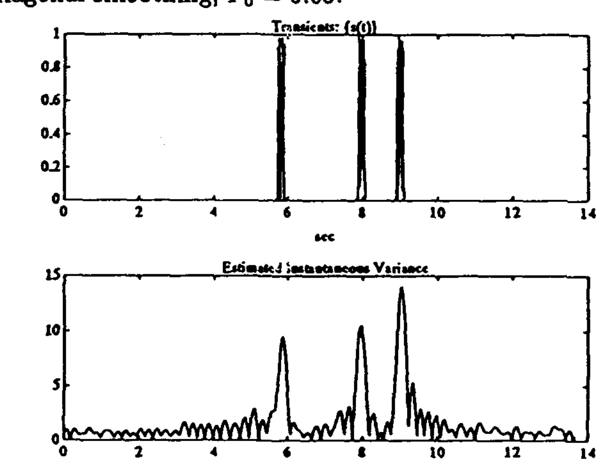

Figure 3: **Upper Plot:** Random pulse train. **Lower Plot:** Instantaneous variance (scaled by $|\sigma_x^2(0)|^{-2}$) obtained from the inverse FFT of the collapsed spectral coherence over the region $\mathcal{R} = [0, 5] \times [0, 5]$.

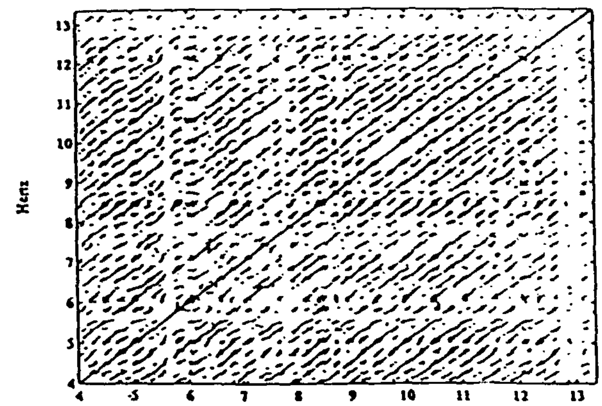

Figure 4: Estimated spectral coherence of the time series of sensor # 1 with 32-bin diagonal smoothing and $P_0 = 0.001$.

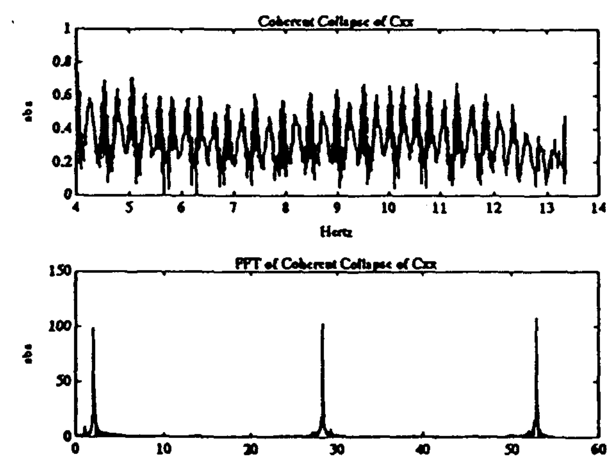

Figure 5: **Upper Plot:** Magnitude of coherent collapse of the spectral coherence. **Lower Plot:** Magnitude of the inverse FFT (zero-padded to 16,384 points) of the coherent collapse.