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THESIS

MISSILE DESIGN PC TRAP:
AN IMPROVED PC TRAP FOR TACTICAL MISSILE DESIGN

by

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The Missile Design Personal Computer Trajectory Analysis Program (Missile Design PCTRAP) is a simple and compact multi-purpose tactical missile simulation program that runs quickly on any IBM-compatible personal computer. It is an improved version of the USAF PC TRAP computer program, in that it adds guidance laws, simulates tv › extra intercept scenarios (surface-to-air and air-to-surface), and provides more simulation options, such as flight envelope generation and Monte Carlo simulations. Missile Design PC TRAP is proposed as a substitute for complex main-frame simulation models, such as TRAP for conceptual and preliminary missile design phases, trade-off studies, academic purposes, and military operational applications.

Missile Design PC TRAP can simulate the launch aircraft, the target and the missile in three-dimensions as point mass vehicles in air-to-air, surface-to-air, or air-to-surface intercept scenarios. Real time graphics display of the vehicle trajectories is available. Seven tactical missile guidance laws are derived, detailed and implemented into the Missile Design PC TRAP algorithms. The missile aerodynamic, propulsion, and physical characteristics are estimated from a small amount of input data. The program can simulate one-on-one engagements, generate launch envelopes in two planes, and perform Monte Carlo simulations with random initiation of the selected target evasive maneuvers. Its computing time is generally less than real time on a 486 33Mhz personal computer chip.

Comparison to missile flight paths generated by Missile Design PC TRAP and a more complete simulation program (TRAP) shows agreement between the simulation results. A complete description of the algorithms is offered, as well as a comprehensive user's manual.

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MISSILE DESIGN PC TRAP: AN IMPROVED PC TRAP FOR TACTICAL MISSILE DESIGN

by

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The Missile Design Personal Computer Trajectory Analysis Program (Missile Design PC TRAP) is a simple and compact multi-purpose tactical missile simulation program that runs quickly on any IBM-compatible personal computer. It is an improved version of the USAF PC TRAP computer program, in that it adds guidance laws, simulates two extra intercept scenarios (surface-to-air and air-to-surface), and provides more simulation options, such as flight envelope generation and Monte Carlo simulations. Missile Design PC TRAP is proposed as a substitute for complex main-frame simulation models, such as TRAP for conceptual and preliminary missile design phases, trade-off studies, academic purposes, and military operational applications.

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I. INTRODUCTION

A. REQUIREMENT AND PURPOSE

Of the many existing missile simulation programs, such as AASPEM, very few have all of the following:

- Simplicity of use
- Capability to run on personal computers (PC's, laptops) with minimum hardware requirements
- Capability to simulate a great variety of missile guidance and control design concepts
- Short computing times
- Compact and simple algorithms
- Simplicity in missile input data requirements
- Real time graphic display.

A digital missile simulation program that could be designed and developed to meet the above requirements would be a very helpful analytical tool for use in the conceptual or preliminary missile design phases, for trade-off studies, for academic purposes and for military operational use (such as fleet and squadron applications).

One program, the Missile Design Personal Computer Trajectory Analysis Program (Missile Design PC TRAP), was developed to specifically meet the above requirements. Missile Design PC TRAP is a simple and compact missile simulation program that runs quickly on any IBM-compatible Personal Computer (PC), which requires a small amount of missile data input, and which offers a large variety of simulation options.

B. MISSILE DESIGN PC TRAP CAPABILITIES

Missile Design PC TRAP is a three-dimensional point-mass digital missile simulation program that can simulate air-to-air, surface-to-air and air-to-surface intercept scenarios with the use of seven different guidance laws. It is capable of simulating and graphically displaying the launching aircraft, the target and the missile trajectories. Graphic displays are in real time and in color. The program can simulate one-on-one engagements, can generate launch envelopes, can perform Monte Carlo simulations with random initiation of the selected target evasive maneuvers and can be used to evaluate optimal target evasive maneuvers against a given missile. Its computing time is generally less than real time on a 80486 IBM PC.

The simple and easy input requirements to TRAP and Missile Design PC TRAP make these programs very attractive. It requires a missile data input file containing only 57 missile-related parameters describing the aerodynamic, propulsion and physical properties of the missile. For example, the missile Thrust-Time curve is approximated by only five data points. This feature allows an easy and quick missile data collection process which promotes the comparison of several missile systems to each other and the optimization of the missile design process.

With all of these desirable features, Missile Design PC TRAP provides the officer students at the Naval Postgraduate School with a missile trajectory analysis tool that is very simple to understand and use, can run on any IBM-compatible PC (including laptop computers), provides quick results and has simple input requirements. This allows the students to apply and instantaneously verify general missile theory principles to optimize their learning skills. Furthermore, since Missile Design PC TRAP is simple and very well documented in the rest of this thesis, students can easily and quickly modify its algorithms to meet their rapidly changing needs.

C. BACKGROUND

The Missile Design PC TRAP is a modified and improved version of the PC TRAP (Version 3.12) computer program which was developed by the Foreign Aerospace Science and Technology Center (FASTC) for the United States Air Force (USAF). The original PC TRAP is a condensed and abbreviated version of the main frame Trajectory Analysis Program (TRAP) used by the USAF and many other DOD organizations to conduct complete and extensive missile simulations. TRAP is available on the main frame at the Naval Postgraduate School Aeronautics and Astronautics Department and in the Warlab.

1. Trajectory Analysis Program (TRAP)

TRAP has about 30000 lines of FORTRAN-77 source code in over 300 highly modular subroutines and 91 common blocks. It can simulate, missile trajectories in three, five or six degrees of freedom, and can simulate in detail, specific guidance and control equipment such as radar, seekers and autopilots. However, TRAP is a complete simulation program which requires at least of 11 detailed input data files that describe the missile, launching aircraft, target, intercept scenario and print rate. It can even read up to 19 input data files for more detailed simulation runs. Furthermore, TRAP does not have any real time graphic display capability.

TRAP and other similar missile simulation programs, such as Flight Lab, can conduct very detailed simulations that would certainly be required in the latter stages of missile design. However, their extensive and heavy input requirements do not make them attractive analytical tools for early design, academic or military operational applications. This is why the need for a more compact and simple missile simulation program has arisen within the tactical missile community.

2. Personal Computer Trajectory Analysis Program (PCTRAP)

PC TRAP partially fulfills this arising need by providing a condensed and abbreviated version of TRAP that runs quickly on a PC. Hence, PC TRAP was developed to render a very useful missile performance evaluation tool concept (TRAP) more accessible and available to DoD personnel.

However, PC TRAP runs air-to-air combat engagements only, and uses only one coded-in guidance law (proportional navigation). Furthermore, FASTC's entire store of foreign and American air-to-air missile parameters are imbedded in the code, which makes the program **SECRET** and not user friendly for tactical missile design applications. Also, PC TRAP can only simulate one-on-one engagements and perform maximum range searches. These features considerably limit the use and availability of PC TRAP for tactical missile design and academic applications. Therefore, PC TRAP could not meet all of the above requirements stated in section A.

3. Missile Design PC TRAP

An unclassified version of TRAP 3.0 and associated documentation was released to the author by FASTC. Despite its limited applications, PC TRAP offered some very attractive features that were implemented in the algorithm of the Missile Design PC TRAP computer program. The most attractive features of PC TRAP that were kept were its limited missile data input requirement and real-time graphic display of the vehicle trajectories.

An UNCLASSIFIED version of PC TRAP was therefore extensively modified to obtain a missile computer program with the capabilities of Missile Design PC TRAP (described above), and to meet all of the above requirements in section A. The main modifications included the addition of seven guidance laws, of two additional intercept scenarios (surface-to-air and air-to-surface), and of simulation options such as launch envelope generation and Monte Carlo simulations. This thesis derives some of the fundamental theories that are implemented in the algorithms of Missile Design PC TRAP, details its algorithms, provides some sample tactical missile design study cases, and provides a user's manual for Missile Design PC TRAP.

Table 1-1 provides a brief comparative summary of the simulation capabilities of TRAP, PC TRAP, and Missile Design PC TRAP. Note that in Table 1-1, the proportional navigation guidance is abbreviated by Pro Nav.

SIMULATION CAPABILITIES	TRAP	PC TRAP	MISSILE DESIGN PC TRAP
Intercept Scenarios	Air-to-air Air-to-surface Surface-to-air	Air-to-air	Air-to-air Air-to-surface Surface-to-air
Guidance Laws	-Pro nav (rate) -Pro nav (acceleration) -Pure pursuit -Pre-programmed -Constant altitude -Constant flight path angle -Constant 'g' -Lead angle	Pro nav with constant pro nav ratio (n=4)	Homing Guidance -Pro nav -Pure pursuit -Lead angle -Augmented pro nav Command Guidance -Command pro nav -Beam rider -Command to line-of-sight (CLOS)
Input Requirements	Minimum of 11 input files (up to 19 files)	One missile input data file containing 57 data items	One missile input data file containing 57 data items
Output	Output files	-Graphic display of vehicle trajectories (real time) -Output files	-Graphics display of vehicle trajectories (real time) -Output files
Simulation Options	-Single missile flyouts -Multiple flyouts -Launch envelopes (azimuth and elevation) -Missile performance reconstruction	-Single missile flyouts -Maximum range searches -1 target	-Single missile flyouts -Maximum range searches -Launch envelopes (azimuth and elevation) -Monte Carlo simulation -Optimal target evasive maneuver evaluation -1 target
Type of Simulation	Point mass, 5 DOF, 6 DOF	Point mass	Point mass

Table 1-1. Summary of the TRAP Family Simulation Capabilities.

D. TACTICAL MISSILES - GENERAL DESCRIPTION

As defined in Ball [ref. 1], "The tactical missile is an aerospace vehicle, with varying guidance capabilities, that is self-propelled through space for the purpose of inflicting damage on a designated target." Tactical missiles may be launched from a variety of platforms including aircraft, surface ships and surface ground-bases. Typically, a search and track device located on the launching platform detects a target, assigns it to a missile system which launches one or more tactical missiles to intercept the target. Shortly after launch, the missile acquires and tracks the target to the intercept, at which point it is expected to collide with the target or guide within the missile warhead lethal radius. The measures used to quantify the performance effectiveness of a tactical missile is the miss distance, or the closest point of approach (CPA), which is the minimum distance between the missile and the target during the intercept.

A tactical missile may be employed against a variety of surface and airborne, moving and non-moving targets. The tactical missile is generally comprised of six subsystems or sections:

- Airframe
- Flight Control Section
- Guidance Section

- Fuze
- Warhead/Telemetry
- Propulsion

Figure 1-1 illustrates the location of these subsystems within the missile.

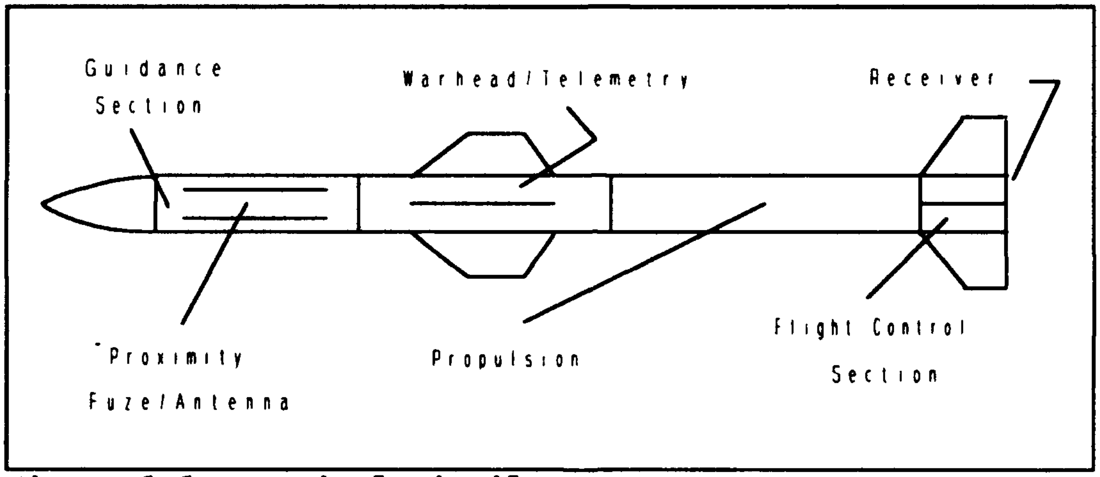


Figure 1-1. Tactical Missile Components

The airframe is the framework that carries the missile components to an intercept of the target. The guidance and fuze sections are generally located at the forward end of the airframe. A radome (for RF missile) or an IR dome (IR missile) covers the guidance section seekerhead to protect it from aerodynamic forces. The flight control section is positioned wherever the control surfaces are located. A receiver or an antenna is sometimes located at the rear end of the missile as shown at Figure 1-1.

The warhead/telemetry section is generally located behind the guidance section and in front of the missile motor. A telemetry package usually replaces the warhead section when launches are conducted in a Test & Evaluation scenario or in a training environment. Telemetry (TM) packages are used to collect and transmit missile data (circulating on electronic buses) to ground stations. The missile data are then recorded by the ground stations and used for post-flight missile performance evaluation and engineering analysis. If tactical telemetry is required, the telemetry package is located wherever the space can be found within the missile airframe.

The following discussion is a brief description of the missile functional block diagram to help the reader comprehend the distinct functions of each section. All of the missile system components must operate together to fly the missile along the correct trajectory to the target.

Figure 1-2 illustrates the functional operation of a typical tactical missile. The airframe is designed to provide the response characteristics and accelerations necessary for a successful intercept. The airframe reacts to control-surface deflections to shape the missile trajectory.

The guidance section is of particular interest in this thesis as it is the missile function responsible for implementation of guidance laws in Missile Design PC TRAP. The primary function of the guidance section is to derive

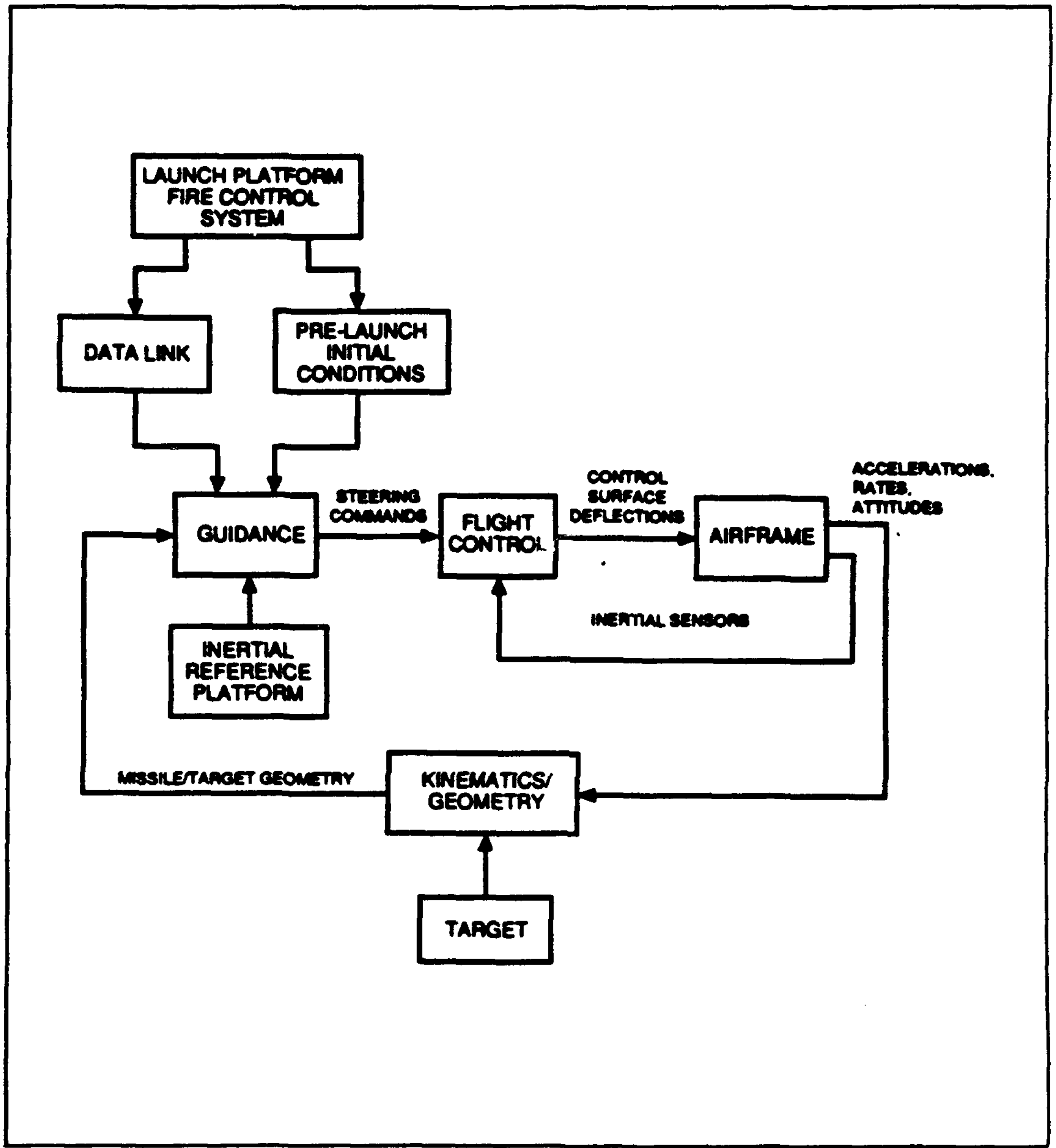


Figure 1-2. Tactical missile functional block diagram [Eichblatt]

the steering commands from the missile/target trajectory geometry. To accomplish this, the guidance section verifies that the missile is on a collision course with the target, using missile-to-target Line of Sight (LOS) angle information, by detecting whether the missile is flying too high or too low, or too much to the left or to the right with respect to the projected collision point. Then,the guidance and control system measures these LOS deviations or errors from the collision course, and transforms them into missile lateral acceleration commands. For long-range missiles that require some form of guidance prior to target acquisition and tracking by the missile seeker, the steering commands are derived from an inertial reference platform and/or launch-platform data-link transmissions during midcourse guidance. The initial conditions for a particular launch parameter are provided to the missile by the launch platform or fire-control system. Some surface-to-air missiles do not have an onboard seeker to track the target. In such instances, the steering commands are derived from an inertial reference platform and/or launch-platform data-link transmissions throughout the entire time of flight.

Once steering commands are developed by the guidance section or the launch platform guidance equipment, these missile lateral acceleration commands (a_c) are passed to the flight control section (autopilot) which uses the control surfaces to maneuver the missile quickly and efficiently to reduce the Line of Sight (LOS) deviations or errors to zero or nearly zero.

At missile/target intercept, the fuze (either proximity and/or contact) will determine when the warhead is detonated. The proximity fuze can be a small active or semi-active radar or laser system designed to detect the target within the lethal range of the warhead. The contact fuze depends on physical contact to initiate warhead detonation.

The following chapters discuss the development of the Missile Design PC TRAP computer program. Chapter II discusses the different options available to design the guidance and control sections for a tactical missile. Chapter III discusses the different guidance laws that can be used by a missile system guidance section to generate the missile lateral acceleration commands required to steer a missile towards a successful intercept of the target. All of the seven different guidance laws available in Missile Design PC TRAP are discussed and derived in two and three dimensions. Chapter IV discusses the integration of the different missile sections into a guidance loop that models the missile system. Chapter IV also provides a description of Missile Design PC TRAP and its algorithms. This description is intended to be very detailed but still straightforward enough to make this simulation program very understandable and accessible to those users interested in increasing their missile simulation knowledge, or interested in modifying the algorithm. Chapter V includes sample missile design cases that show how to optimize the use of Missile Design PC TRAP. Also included in Chapter V are results from a comparison of missile flight paths generated by Missile Design PC TRAP with the ones generated by TRAP in similar intercept scenario. Appendix A is the Missile Design PC TRAP users' manual which supplies quick and handy direction to the program user. The user's manual is an Appendix to this thesis so that it can be used as a separate document by users. Appendix B is a missile data dictionary for the missile data input file. Appendix C is a data dictionary for a very detailed output file created by **Missile Design PC TRAP**.

II. TACTICAL MISSILE GUIDANCE AND CONTROL

The tactical missile guidance section provides the navigation instructions to the missile system. The guidance section detects and tracks the target, computes the desired missile trajectory to the target and produces the electrical steering commands required to follow the desired path. The missile flight control section (or autopilot) responds to the guidance steering commands via the missile airframe and control surfaces to keep the missile on a collision course with the target. This Chapter presents an overview of missile guidance & control aspects before discussing the development of Missile Design PC TRAP in great detail.

A. PHASES OF GUIDANCE

Tactical missile guidance is generally divided into three phases: boost, midcourse and terminal. These names refer to different parts of the flight path. The transition points from one phase to the following are often used as milestones to vary some inherent guidance properties, to change the guidance law or to adopt a new type of guidance.

1. Boost Phase

The boost phase may also be called the launching or initial phase. The basic purpose of the boost phase is to accelerate the missile to supersonic speeds in the shortest time possible in order for the missile to rapidly decrease the range between itself and the target and to rapidly acquire a velocity that will give it an enormous speed advantage over the target. In other words, the booster must get the missile off to a good start or the missile will not have sufficient energy to make it to the target. This fact is especially true for surface-to-air missiles which are launched from rest, unlike aircraft-launched missiles that have the initial velocity of the launching aircraft.

The boost period lasts from the time the missile leaves the launcher until the time where the booster burns up its fuel. Some missiles use separate boosters which drop away from the missile at booster burnout. Discarding the burnt-out booster shell reduces the weight carried by the missile and enables the missile to travel farther with more maneuverability capabilities. A tactical missile can be guided or unguided during the boost phase. In cases where the missile is unguided during the boost phase, the guidance system is idle and the aerodynamic control surfaces are locked in position to guide the missile straight towards a predicted position where the missile should be at the end of the boost phase for successful target intercept.

2. Midcourse Phase

The midcourse phase of guidance is often the longest in both distance and time. This phase is the most important one in the guidance process, as it must bring the missile near the target. During this part of the flight, guidance logic changes may be required to bring the missile onto the desired collision course to ensure that it stays on this course and/or to respond quickly and adequately to target evasive maneuvers. This phase generally ends when the missile is guided to the target within the radius of the proximity fuze or when another type of guidance takes over.

3. Terminal Phase

The terminal phase is the shortest phase and is of great importance to the success of the target intercept. This last phase of guidance must have high accuracy, but more importantly a fast reaction time to counter any last second evasive maneuvers by the target. At this point in the missile flight, the missile must possess the energy required to make sharp turns that are required to overtake and score a hit on a fast-moving target.

B. GUIDANCE SYSTEM FUNCTIONS

The guidance section performs four major functions: detection, acquisition, tracking and steering.

1. Detection

Detection is the process whereby the target sensor senses a certain amount of power (in some area of the electromagnetic (EM) radiation spectrum) above that normally expected from background or internal seeker noise (the threshold valve). In some respects, the sensor unit, which is referred to as a seeker, is the most important component of the guidance section because it detects the EM power being used to guide the missile. If the sensor unit fails, there can be no missile guidance, and subsequently, no target intercept.

The kind of sensor that is used for a specific tactical missile design is determined by such factors as maximum operating range, operating conditions, band width, the kind of target information needed, the accuracy required, viewing (field-of-view) and gimbal angles, weight/size of the sensor, and the type/speed of the target.

The seeker unit can be thought of as the "eyes" of the missile. Its purpose is to detect, acquire, and track a target by sensing some unique characteristic associated with the target. This unique characteristic usually consists of the EM radiation emitted or reflected by the target in a specified band of the electromagnetic spectrum. Typical bands within the electromagnetic spectrum used in tactical missile guidance include ultraviolet, infrared, laser, visible, millimeter wave, and radar frequencies. Some missiles have seekers that can operate in more than one band at the same time or at different times (e.g., multi-mode = radar and IR detectors). All radiations may be considered as a method of transmission of energy through space.

2. Acquisition

The acquisition function is a short transition function between the target detection and target tracking functions. Acquisition is the process whereby the seeker, after experiencing one or more incidents of detection, decides (according to some pre-established criteria or algorithm) that a valid target has been detected by the guidance control system.

3. Tracking

Tracking is the process whereby the seeker continually "looks" at the target and continually specifies the angular location of the target relative to some fixed coordinate reference. This angular orientation, which is defined by an imaginary direct line between the missile and the target, is called the Line of Sight (LOS).

There are several methods available for tracking a target, depending on whether the seeker has a wide or narrow field-of-view (IFOV). The instantaneous seeker IFOV is the angular region (usually conical) about the seeker centerline, or boresight which is capable of receiving useful energy.

A seeker with a large IFOV is shown at Figure 2-1. With such a seeker, it is possible to fix the angular orientation of its centerline, which coincides with the missile mais, providing to the guidance section with an indication of the angle between the LOS (imaginary straight line from the missile to the target) and the missile centerline (LOS angle).

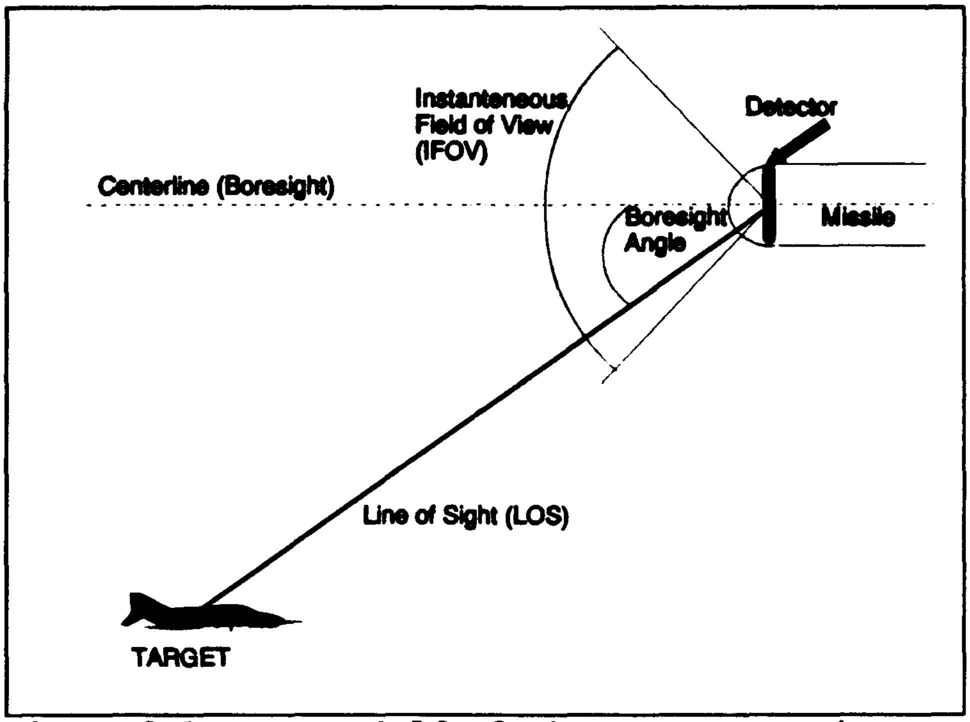


Figure 2-1. Large Field of View Representation

If a seeker has a narrow IFOV, it is usually mounted on a gimballed platform (space-stabilized platform). The seeker maintains the target within the narrow FOV by rotating the platform (as shown at Figure 2-2). If the platform is inertially stabilized, the rotation is accomplished by applying torques which are proportional to the target displacement from the IFOV center. The tracking information provided by this type of seeker is an indication of the inertial rotation rate of the line-of-sight(LOS), commonly called the LOS rate.

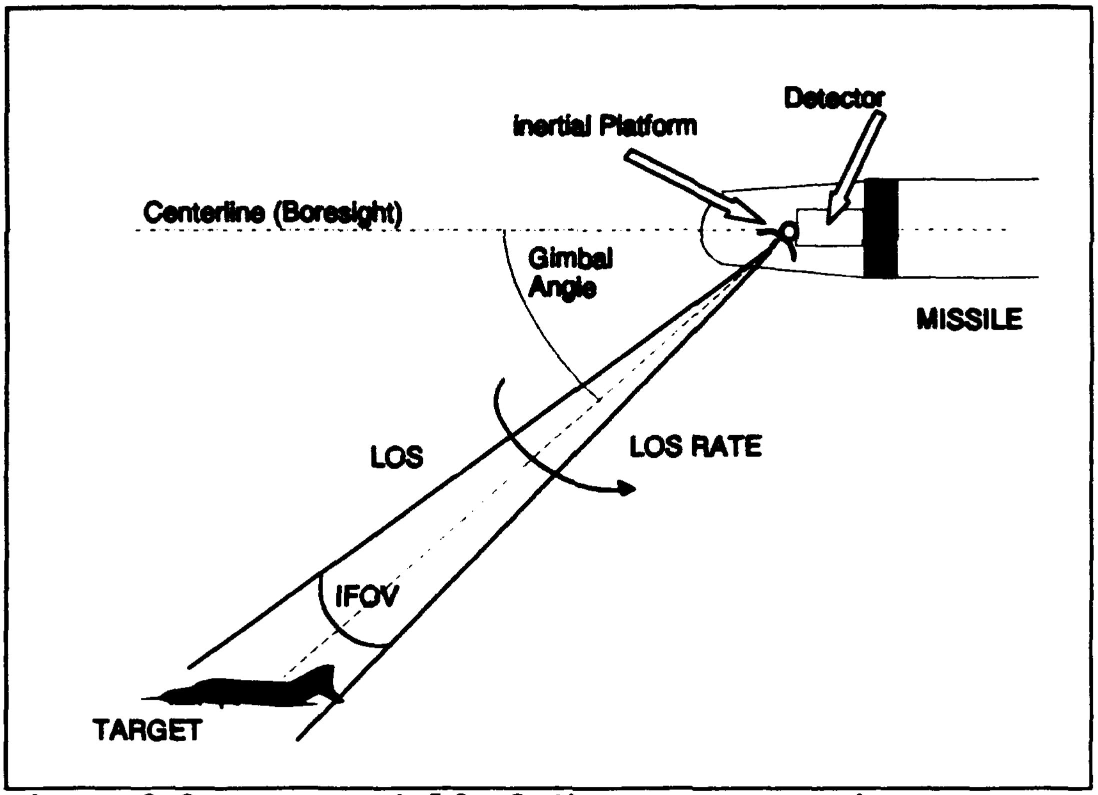


Figure 2-2. Narrow Field of View Representation

Other information which the seeker might be capable of providing to a guidance section is missile-to-target closing velocity, range and/or range rate. Radar seekers are the only ones which currently provide such information.

4. Steering

Once seeker tracking data have been obtained, they are filtered, using low pass or high pass filters, to produce a clearer "image" (less noisy) of the target flight path by extracting the pertinent target/missile kinematic variables of this specific intercept. Using these "filtered" data, the selected guidance law decides the best trajectory of the missile to the intercept with the target based upon its knowledge of the missile's capability, target capability and mission desired objectives. The guidance law ultimately produces the missile lateral acceleration commands required for a successful intercept. The fundamentals of guidance laws and their application in the Missile Design PC TRAP algorithm are described in Chapter III.

Once the missile acceleration commands are determined by the guidance law of the missile guidance section, these commands are passed to the missile control section. The control section, using pitch, yaw and roll autopilots, determines the missile fin deflection positions required to best execute the command. The fins (wings) are the missile control surfaces which are varied with the help of actuators to achieve proper aerodynamic moments and forces required to approximate the guidance acceleration and motion commands.

The functions described in the preceding two paragraphs are combined together to perform a general function called "steering". The steering function can be thought as the navigation "brain" of the missile. The more sophisticated, accurate and exact a missile steering function, the more likely the missile will be regarded as a very lethal weapon system.

C. TYPES OF GUIDANCE

Missile guidance systems may be placed in two broad categories: missile guidance using electromagnetic radiation from the target (tactical missiles) and those not using electromagnetic radiation contacts (strategic missiles).

1. Electromagnetic Radiation

This type of missile guidance includes tactical missiles, and can be further subdivided into three major categories: command guidance, homing guidance, and simultaneous use of both command and homing guidance (retransmission guidance).

a. *Command Guidance Missiles*

Command guidance missiles are those whose motion is determined by the direct EM radiation contact between friendly control points. Their guidance generally depends on the use of radio or radar links between a control point and the missile. The term command is used to describe a guidance method in which all guidance instructions, or commands, come from sources outside the missile. Therefore, command guidance missiles do not require an onboard seeker.

To receive the commands, the missile contains a "receiver" that is capable of receiving instructions from ship, ground station, or aircraft platforms. The missile rear "receiver" then converts these instructions into missile acceleration and/or motion commands which are fed to the control (autopilot) section.

(1) Command Missile System

In this type of command guidance, a tracking system that is separate from the missile is used to track both the missile and the target (i.e., the tracker is off course with the missile). Target tracking can be accomplished using radar, optical, laser or infrared systems. A typical command missile system is illustrated at Figure 2-3. The tracking system generally feeds target and missile range, closing velocity, elevation, and bearing data to a computer separated from the missile. Using the relative position and relative position rate information, the computer determines the

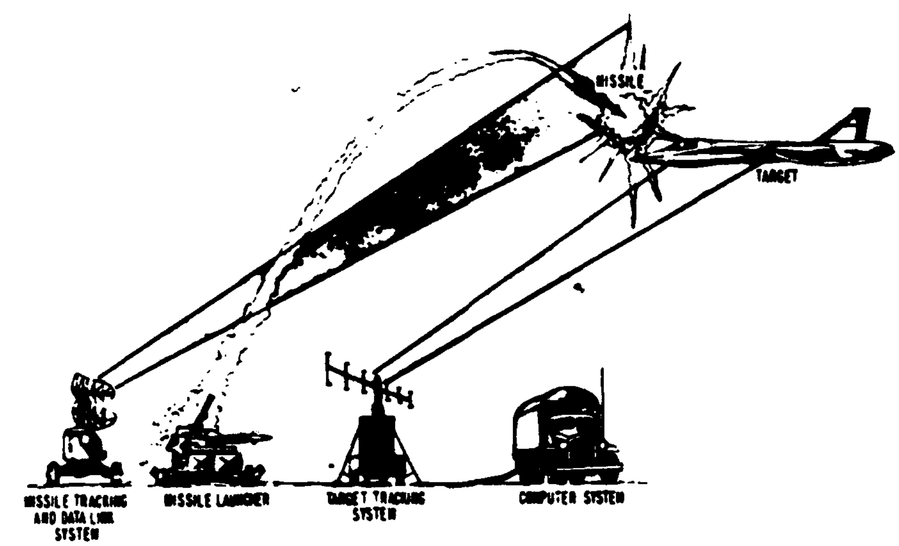


Figure 2-3. Typical Command Guided Missile System [Ball [ref.1]]

flight path the missile should follow in order to collide with the target. It then compares this computed flight path with the predicted flight path of the missile based on current tracking information and determines, using one of the different guidance laws, the correction signals required for the autopilot to move the missile control surfaces to change the current flight path to the new one.

These command signals are sent to the missile "receiver" via either the missile tracking system, or a separate command link, such as radio. It can also be sent along a wire between the launching platform and the missile.

(2) Beam-Rider System

The main difference between the beam-rider method and command guidance method described above is that the beam-rider missile guides on a tracking and guidance beam, while no command signals are passed to the missile from the launching platform. The beam-rider method is a command guidance system since the target is tracked by an EM beam transmitted by a tracking system offboard the missile. The only guidance equipment onboard the missile is a rearward-facing antenna that senses the target tracking beam. The missile guidance and control section is designed to keep the rear antenna centered in the target tracking beam. It can accomplish this by sensing the center of the beam and developing required command accelerations that will keep the rear antenna in the center of the tracking and guidance beam. The missile can thus be help of one or two tracking beams. In the one-beam tracking system illustrated the beam in Figure 2-4 (A), the beam is tracking the target directly and the missile rides this beam.

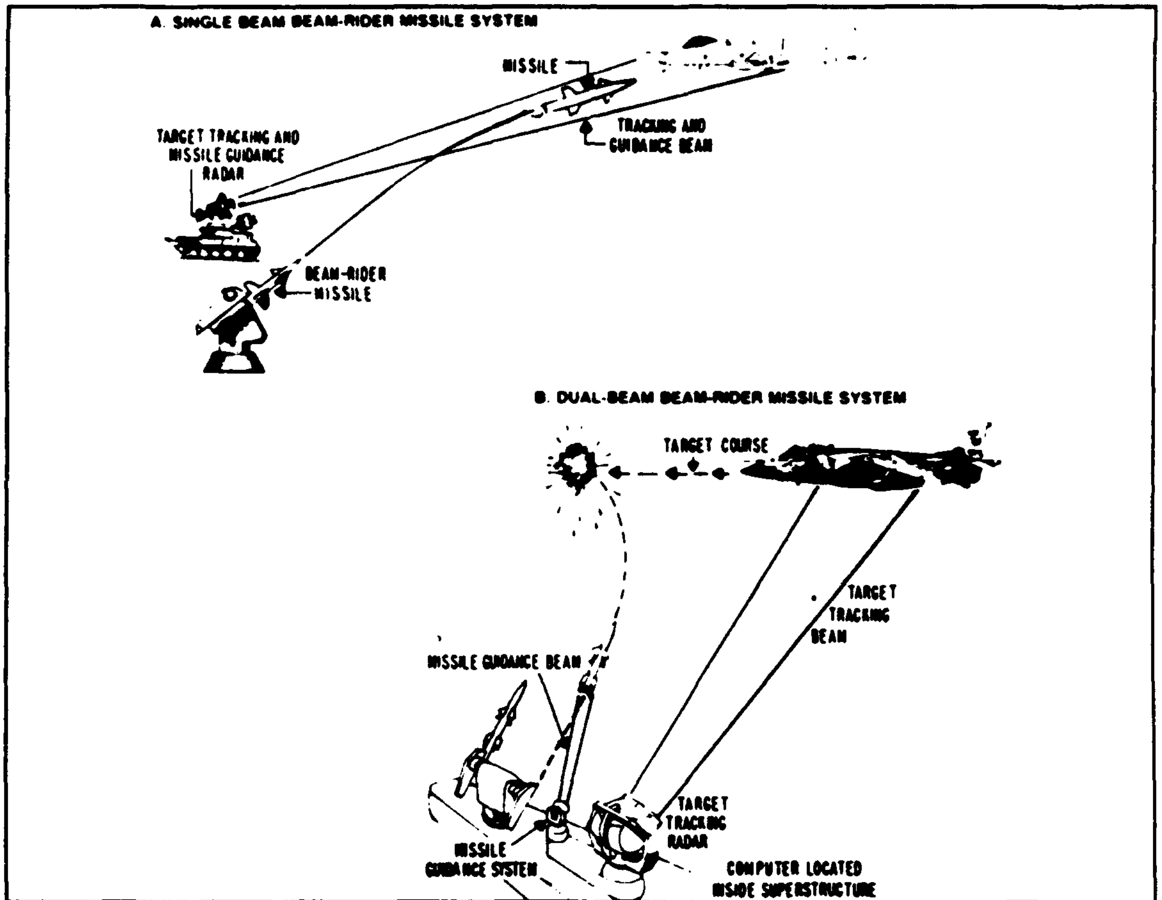


Figure 2-4. Beam-Rider Missile System [Ball [ref.1]]

The missile must always be located in a direct line between the target and source for guidance. In the case where two beams are used, one beam tracks the target directly, and a second beam is used to guide the missile as shown in Figure 2-4 (B). The second beam points at the eventual space location where the collision of the missile with the target should occur. This collision location is determined by a computer external to the missile that continuously predicts the collision point based on target/missile dynamic geometry. The collision location is continuously updated by the computer, which changes the beam pointing location accordingly. The beam-rider guidance method using two beams requires equipment that is too large and complex for aircraft use, but may be used on ship or ground-based launching platforms.

b. Homing Guidance Missiles

The expression "homing guidance" is used to describe a missile guidance system that can determine the position of the target with an onboard seeker and can formulate its own commands to guide itself to the target. An onboard homing device, usually located in the nose of the missile, detects, acquires and tracks EM radiations given off by the target. "Homing" guidance is based upon the maintenance (track) of the EM radiation contact between the missile and the target. Upon successful tracking of the target, command motion and accelerations are developed by a selected guidance law, and passed to the control section which steers the target towards an intercept point.

"Homing" guidance may be divided into three types: active homing, semi-active homing, and passive homing. They are respectively illustrated in Figures 2-5a, 2-5b, and 2-5c.

(1) Active Homing

Active homing occurs when the detection system itself is the source of the EM radiations. In a missile system using radar, for

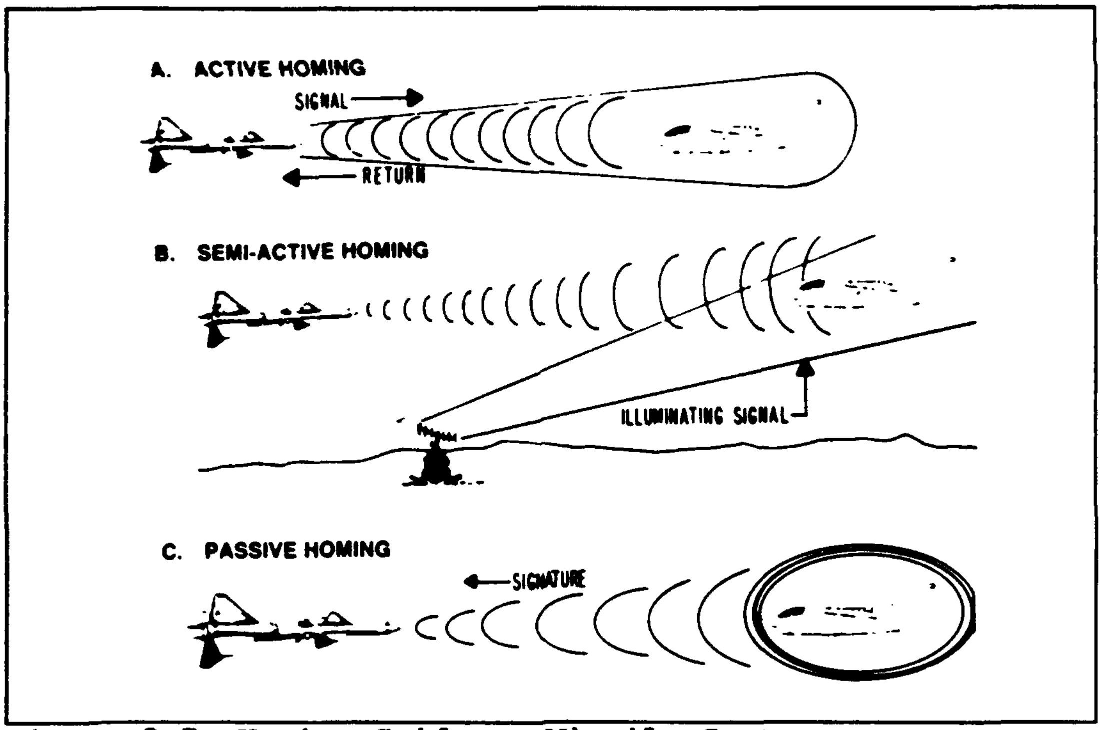


Figure 2-5. Homing Guidance Missile System

example, the transmitter and antenna located in the missile illuminate the target, and then use the radar reflections from the target for guidance. This means that once the missile is launched from the launching platform, the missile is on its own and must steer itself to the target without any further support from the launching platform.

(2) Semi-Active Homing

Semi-active homing refers to those systems where the EM radiations are actively transmitted from a source separate from the detecting agent. In a missile system using radar frequency semi-active homing, the target is illuminated by EM radiations from a transmitter not located aboard the missile. The missile has only an inboard receiver which homes on the EM radiation signal reflected off the target.

(3) Passive Homing

Passive homing refers to those systems where the target itself is the source of EM emissions or natural EM reflections. In such cases, the missile needs only to receive, detect, acquire and track the signals propagated from the target. The missile is said to be "silent" as it does not require any EM radiations transmission from friendly sources for guidance. One example is a missile using an infrared heat seeking method for guidance based on thermal radiations emitted by the target.

c. Retransmission Guidance

This type of guidance (illustrated at Figure 2-6), also known as track-via-missile (TVM), combines the advantages of command guidance with those of semi-active homing, as it uses both types of guidance simultaneously. This guidance type is used by the Patriot air defense missile system.

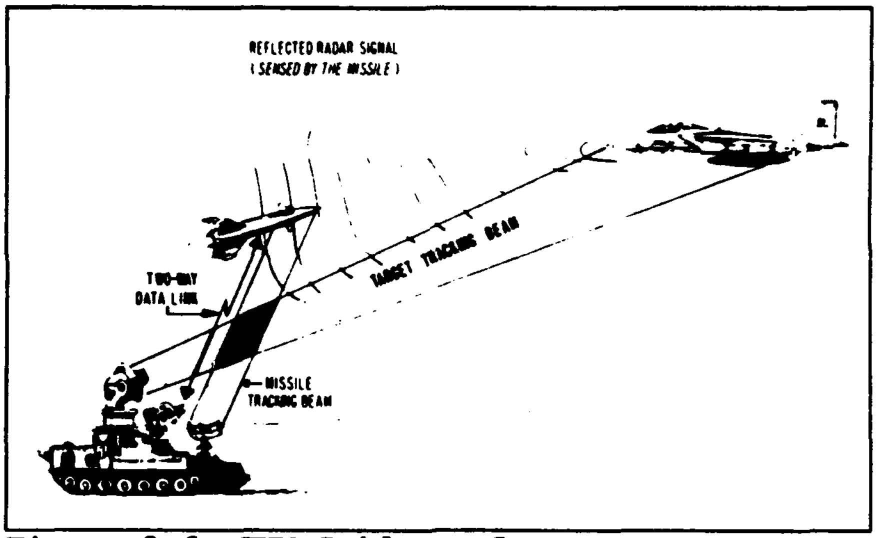


Figure 2-6. TVM Guidance System

This type of guidance is typically used in surface-to-air intercept scenarios. A multi-function radar is normally used for search and detection, and target and missile tracking systems, as in command guidance. However, in TVM systems, the radar beam tracking the target also serves as a target illuminator where reflected illumination from the target is used by an onboard missile seeker, as in semi-active homing guidance, to determine the exact position of the target with respect to the missile (LOS angle). The seeker derived data is down-linked to the ground radar for processing and generation of the required missile lateral acceleration. The appropriate acceleration commands are then sent to the missile on a data link

d. Composite Systems

Typically, no one type of guidance is best suited for all of the three phases of guidance described earlier. It is therefore the general practice in missile systems to employ more than one type of guidance, with each type operating during a given phase of missile trajectory (not simultaneously), to optimize the intercept solution. A missile guidance system using different types of guidance during one particular flight intercept is called a composite guidance system.

As an example of a missile using composite guidance, consider a missile that rides a radar beam for the entire boost and midcourse phases, and then switches to active or passive homing guidance for the duration of the terminal phase. Such a combination provides very accurate tracking and guidance during the terminal phase, and minimizes the weight and size of the onboard missile homing system equipment.

2. Non-Electromagnetic Radiation Guidance

Missile systems that do not use EM radiation contacts to guide the missile toward a target use a "self-contained guidance system" and are usually referred to as strategic missiles. These systems are most commonly applicable tc surface-to-surface scenarios. Some of the missile systems of this type use preset, terrestrial, inertial, or celestial navigation for guidance. These systems neither transmit or receive EM signals. They normally use basic principles of navigation to guide to a pre-determined target location. Missile Design PC TRAP does not model this type of missile guidance since this type of missile is not generally used for tactical missile applications.

D. MISSILE FLIGHT CONTROL SYSTEMS

A missile flight control system shall be designed to accomplish the following functions:

- Statically stabilize the missile airframe at the desired response for the planned operating conditions
- Provide maneuver control
- Generate the required missile accelerations, developed by the missile guidance system, to steer the missile to an intercept of the target

The typical missile airframe is designed to be lightly damped or slightly unstable with a relatively high natural frequency. This design, in conjunction with a stabilizing missile flight control, provides a very responsive, controlled missile that will achieve the desired lateral accelerations quickly with a minimum amount of transient response oscillations.

The missile airframe is commonly described in terms of its body axes (X_B, Y_B and Z_B) coordinate system as illustrated in Figure 2-7. As for aircraft, missile angular motion about the X_B, Y_B and Z_B axis is referred to as roll (Φ), pitch (Θ) and yaw (Ψ) respectively.

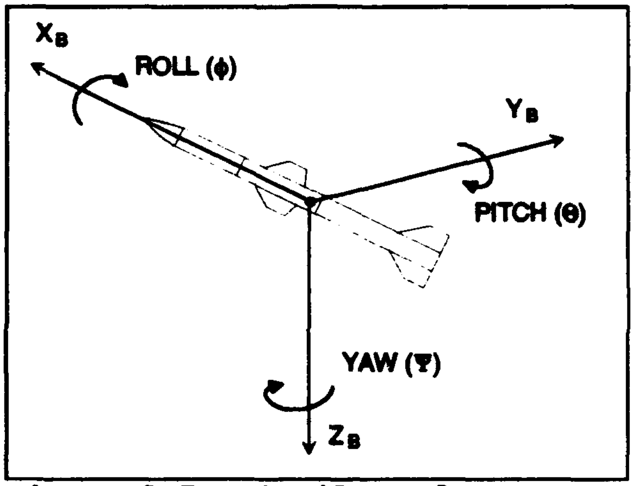


Figure 2-7. Missile Body Axes Coordinate System

Stable and controlled missile flight is achieved by controlling the airframe motions about the X_B, Y_B and Z_B axes. This is normally accomplished by using roll, pitch and yaw automatic-feedback control systems or autopilots.

Of interest in this thesis are the "aerodynamic missiles" which use aerodynamic lift to control the direction of flight. One feature of these missiles is that they are roll stabilized (i.e., there is no roll motion); thus there is no coupling between the longitudinal and lateral modes. The longitudinal mode refers to the pitch motion while the lateral mode refers to the yaw motion. This means that the missile is symmetrical about its pitch and yaw axes. This feature simplifies the design of the missile flight control system since only two types of autopilots are required: a roll autopilot that provides the missile roll stabilization, and two identical pitch and yaw autopilots which respectively control the motion of the missile about its symmetrical pitch and yaw axes.

Missile Design PC TRAP simulates skid-to-turn missiles, which are aerodynamic missiles using direct side force to turn. Unlike aircraft, skid-to-turn missiles do not bank to change their flight path direction.

Bank-to-turn missiles, which provide positive angles of attack and minimal sideslip angles, are normally propelled by turbo-jet or ramjet propulsion systems, and are normally used for long range target intercept applications. With such a configuration, bank-to-turn missiles trajectory simulations can be accomplished using a 6-degrees-of-freedom aircraft simulation model.

1. Roll Autopilots

For aerodynamic skid-to-turn missiles, the required rolling moment is achieved by differential movement of the control surfaces. Since these missiles are roll stabilized, the purpose of the roll autopilot is mainly to reduce the roll rate $d\Phi/dt$ to zero or to maintain the roll angle (Φ) to some specified reference.

ε is the error between the roll angle reference signal ($\Phi_{(ref)}$) and the missile current roll angle (Φ) as sensed by an attitude gyroscope. This error signal (ε) is then multiplied by the closed-loop gain K to give e_a, which is submitted to a compensation network to give $e_{\delta a}$, the electrical signal providing the fin deflection command to the fin control servo. $e_{\delta a}$ is then transformed into a fin deflection angle command that is transformed into the missile bank angle (Φ) that the missile must achieve to correct for the initial roll angle error (ε). Figure 2-8 is a general block diagram of a typical missile roll stabilization system. To maintain a desired roll angle, some form of an attitude reference must be used. A vertical gyroscope or a roll rate gyroscope can accomplish this task. However, in a flight control system as illustrated in Figure 2-8, use of a roll rate gyro, is not recommended as it would result in a type 0 system (Ogata [ref. 7]), which would further result in a steady-state error in roll rate in the presence of a constant disturbing rolling moment. For this reason, Figure 2-8 illustrates a vertical gyroscope which induces a feedback signal proportional to the roll angle (Φ) about the missile longitudinal axis. Still another possible method to provide an attitude reference signal is the use of an integrating gyro with its input axis along the longitudinal axis of the missile.

In the system illustrated in Figure 2-8, the control servo might be represented by a first order time lag or a second order system. The missile transfer function for δ_a input to roll angle (Φ) output is normally the transfer function of the one-degree-of-freedom rolling mode as discussed in Blakelock [ref.2]. The compensation circuit and the autopilot loop gain are

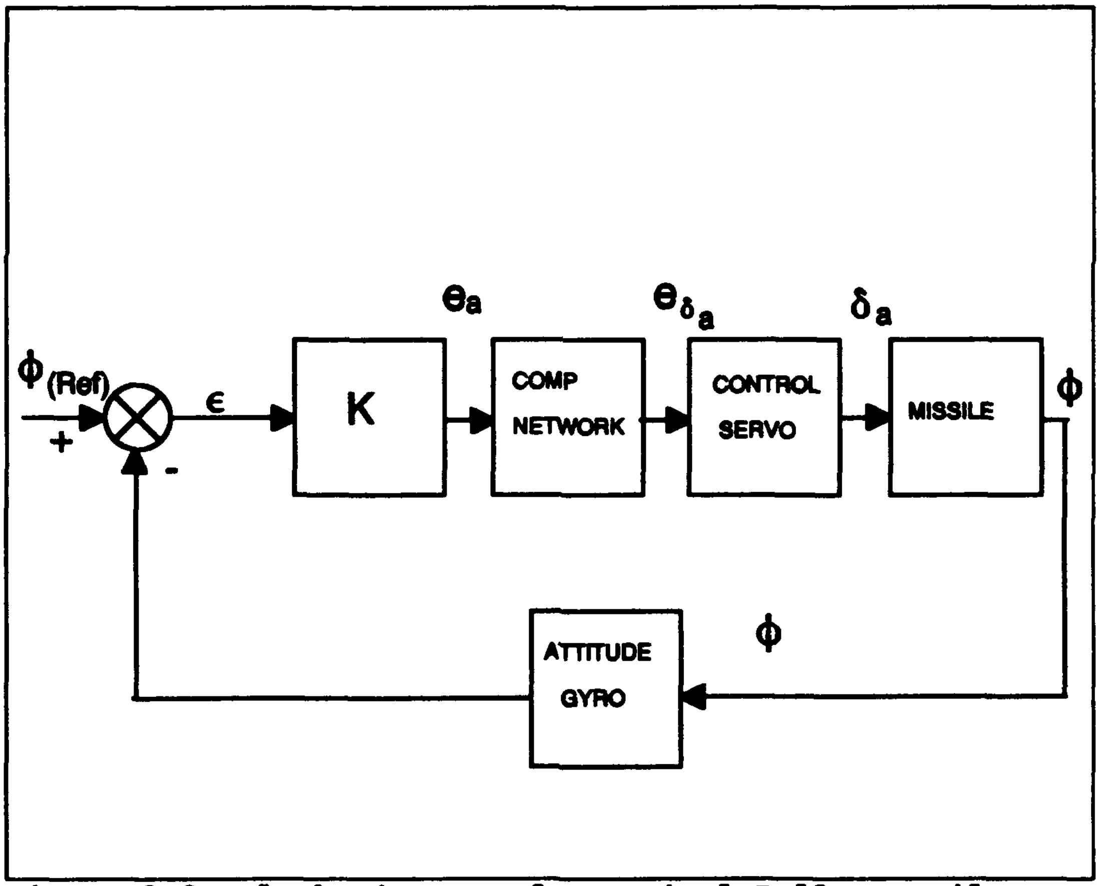


Figure 2-8. Block Diagram of a Typical Roll Autopilot

determined from the loop root locus analysis as detailed in Blakelock [ref. 2] and Garnell [ref. 4].

2. Lateral Autopilots

Control of aerodynamic missiles in the pitch and yaw planes can be accomplished either by conventional control surfaces with the canards stationary or absent, or by use of the canards with no control surfaces on the main lifting surface. As already mentioned, flight control systems for both the pitch and yaw planes are identical for aerodynamic missiles, which means that only one autopilot design is required for both planes.

A block diagram of a basic pitch/yaw lateral autopilot is shown in Figure 2-9. This lateral autopilot is composed of two inner loops that use state variable (angle of attack and pitch rate) feedback to stabilize the missile, and of an outer loop providing acceleration feedback to determine when the commanded acceleration (a_c) has been achieved. Missile state variable feedback is accomplished with the use of measurement instruments such as position gyroscopes and rate gyroscopes. Acceleration feedback is achieved with accelerometers.

As with the roll autopilot, the dynamics of the control surface servos can be described by a first order lag system or a second order system. The reference signal for the lateral autopilot is the commanded lateral acceleration (a_c), which is obtained from the missile guidance system. The acceleration command reference signal (a_c) is needed to determine when the commanded acceleration has been achieved. The achieved missile acceleration is sensed by an accelerometer and fed through gains for comparison with the commanded acceleration (a_c). The difference between the commanded and measured accelerations (a_c - a_L) will result in changes in control surface deflections until both accelerations are equal (a_c = a_L). The overall missile system response and damping is determined by the feedback gains and compensation networks based on root locus analysis Blakelock [ref. 2] and Garnell [ref. 4].

As a result of the pitch and yaw plane symmetry, longitudinal short period approximation transfer functions can be used for both pitch and yaw planes root locus analysis. Complete and detailed missile lateral autopilot root locus analyses are included in Blakelock [ref. 2].

The design of an aerodynamic missile lateral autopilot is made complicated. The fact that such missiles have large flight envelopes in which missile aerodynamic transfer function coefficients change drastically as missile velocity and altitude change. Hence, there is a need to determine a set of consistent missile physical properties and typical flight conditions to cover the entire missile flight envelope, which will be used to establish an autopilot gain schedule. To establish this schedule, a root locus analysis shall be conducted using the transfer function dynamic coefficients associated with each of these selected set of physical properties and typical flight conditions. Once again, this process is well documented in Blakelock [ref. 2].

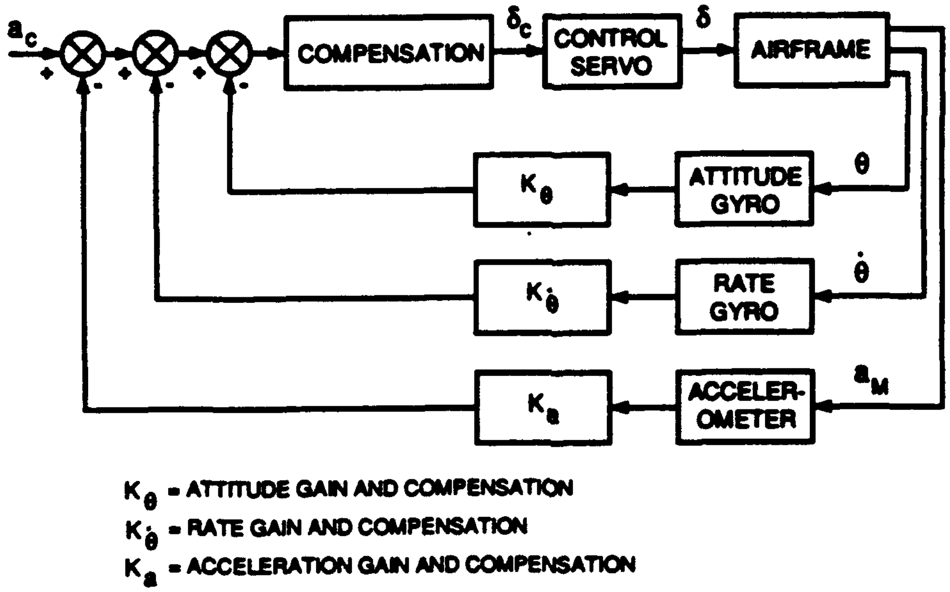


Figure 2-9. Diagram of a Basic Pitch/Yaw Lateral Autopilot [Eichblatt [ref. 3]]

III. GUIDANCE LAW FUNDAMENTALS

A. INTRODUCTION

The fundamentals of guidance laws are required at the conceptual and preliminary design stages. The selection of a specific guidance law (or combination of guidance laws) for a missile and the understanding of its kinematics is essential for evaluation and prediction of the missile guidance system performance and for missile trajectory simulation. Given a specific missile flight control system, the guidance law is the mechanism in the missile guidance system that determines the following missile performance parameters:

- Missile time of flight
- Missile acceleration requirements
- Missile maneuverability
- The missile end-game miss distance

All of these parameters are of extreme importance in missile design and missile performance evaluation and may impose major constraints on the missile design requirements. The importance of each of these parameters is generally driven by the type of target(s) that the missile is designed to defeat and by the threat and the complexity of the intercept scenario. For this reason, many different missile guidance laws are available for both homing and command guidance systems.

As in many other design problems, the guidance law design problem is a function of its complexity, as well as its cost and ease of implementation within the missile airframe. Typically, the overall performance of a guidance law in miss distance and maximum acceleration requirements improves with the complexity of the guidance law. However, implementation of complex guidance laws require a significant amount of target information which, in turn, requires complex, heavy and expensive missile guidance hardware. Subsequently, the missile designer cannot always select an optimum guidance law and is rather faced with the task of finding the proper missile guidance law that will best suit the overall design requirements.

B. GUIDANCE LAW IMPLEMENTATION IN MISSILE DESIGN PC TRAP

As mentioned earlier, there are two main types of tactical missile guidance, command guidance and homing guidance. For each type of missile guidance different guidance laws are used to generate the lateral acceleration commands required for a successful intercept of the target.

PC TRAP can only model homing guidance systems using one guidance law, proportional navigation. PC TRAP models tactical missiles in air-to-air intercept scenarios only.

Given these basic features, PC TRAP was improved (Missile Design PC TRAP) by adding the capability of simulating two more intercept scenarios, air-to-surface and surface-to-air and the capability of simulating the two main types of tactical missile guidance, command and homing guidance. Additionally, Missile Design PC TRAP can model the following seven guidance laws:

- Pure pursuit
- Lead angle (constant bearing)
- Proportional navigation
- Augmented proportional navigation
- Beam-rider
- Command to line-of-sight (CLOS)
- Command proportional navigation

The first four guidance laws are used with homing guidance systems, while the last three guidance laws are used with command guidance systems.

1. Homing Guidance

The intercept geometry for a tactical missile engagement using a homing guidance system is shown in Figure 3-1 (in two dimensions for simplicity).

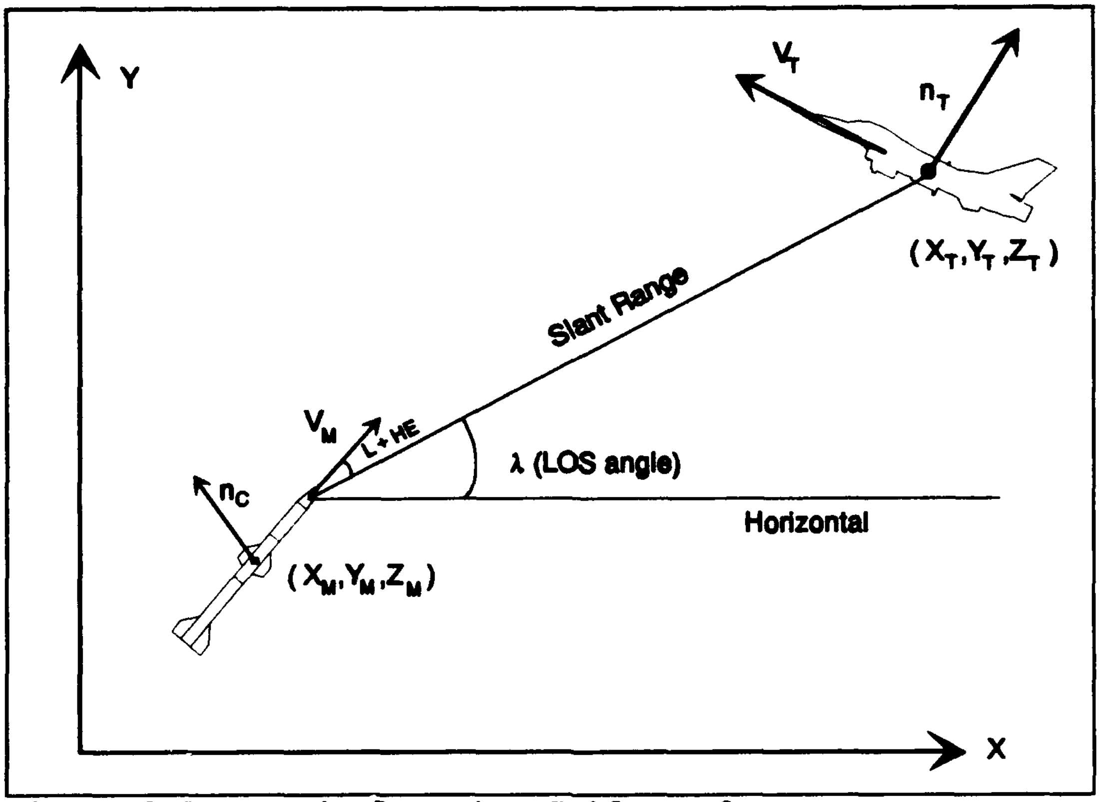


Figure 3-1. Typical Homing Guidance 2-D Intercept Geometry

Homing guidance tactical missiles have an onboard seeker or tracker that is capable of detecting, acquiring, and tracking a target using either passive, semi-active, or active homing techniques. An onboard guidance and control section steers the homing guidance tactical missiles toward a successful intercept of the target based on tracking data provided by the missile seeker according to a specific guidance law.

As shown in Figure 3-1, both missile, target velocity, and acceleration vectors are respectively shown as V_M, V_T, n_C, and n_T. The missile and target are represented as point mass vehicles located at the positions shown in Figure 3-1. The imaginary line connecting the missile to the target is the Line-of-Sight (LOS). The LOS angle (λ) is defined as the angle between the LOS and the horizontal. The angle L + HE will be defined later as the lead angle plus the initial heading error angle.

In homing tactical missiles, the missile seeker is responsible for the derivation of the time rate of change of the LOS angle ($d\lambda/dt$) which is subsequently used by the guidance law to steer the missile towards the target. Some seekers may have the capability of determining the range. The seeker data is passed to the guidance computer for implementation of the guidance law.

Missile Design PC TRAP provides a deterministic modeling of homing guidance tactical missiles. This means that at each time step during the missile simulation the target-missile intercept variables are known exactly within the program. Missile Design PC TRAP solves the position, velocity, and acceleration (state variables) of both the missile and target. From these state variables the program computes the intercept geometry at each time step and determines the LOS angle, LOS rate, and range rate when necessary. This is how the functions of a missile seeker are modeled in TRAP and Missile Design PC TRAP.

It will be seen later that the four guidance laws modeled in conjunction with homing guidance systems in Missile Design PC TRAP are very similar to one another. The intercept geometry is computed in the same way for all the homing guidance law simulations. However, it is the mathematical expressions used to generate the missile lateral accelerations (n_c) that distinguish each guidance law from the others.

2. Command Guidance

The intercept geometry for a tactical missile engagement using a command guidance system is shown in Figure 3-2 in two dimensions for simplicity.

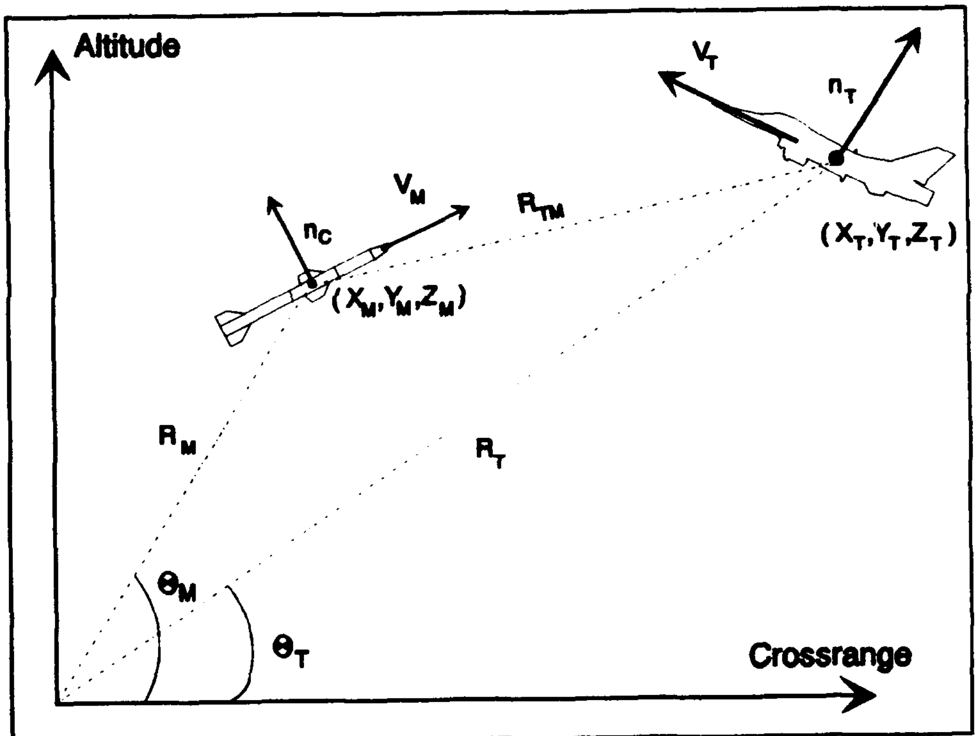


Figure 3-2. Typical Command Guidance 2-D Intercept Geometry.

Command guidance tactical misiles do not have an onboard seeker to track the target. The target is detected, acquired, and tracked by an offboard tracking system located at the origin of the fixed coordinate system shown in Figure 3-2. The missile is also tracked by this tracking system.

It can be seen in Figure 3-2 that in tactical missiles the tracking system determines the range (R_t) of the point mass target from the beam origin and the elevation angle (Θ_t) from the horizontal. Similarly, the tracking system determines the range of the missile (R_m) from the beam origin and its corresponding elevation angle (Θ_m). From these intercept parameters the intercept geometry can be established, and a guidance computer, located either onboard or offboard the missile, generates the required missile lateral acceleration to steer the command guidance tactical missile to a successful intercept of the target.

As mentioned earlier, Missile Design PC TRAP is a deterministic missile simulation model. As such, the state variables are the missile and target position, velocities, and accelerations. Hence, Missile Design PC TRAP solves the state variables at each time step and establishes the intercept geometry shown in Figure 3-2. From this intercept geometry the three command guidance laws available in Missile Design PC TRAP can be implemented as documented below.

3. Guidance Law Modeling

This section briefly explains how the different guidance laws are modeled in Missile Design PC TRAP. First, as stated above, it must be emphasized that in PC TRAP and Missile Design PC TRAP the intercept state variables are known exactly at each simulation time step. This means that both the missile and target position, velocities, and accelerations are known for each time step. These state variables are known from the solution of the linear equation of motion of the vehicle. Then, the other variables required to implement the missile guidance law can be derived from the state variables by the program.

In the following two sections the fundamentals of the guidance laws implemented in Missile Design PC TRAP are described, derived, and analyzed. All Missile Design PC TRAP guidance laws are first described and derived in two-dimensions for simplicity, as described in Zarchan [ref. 9]. Then each guidance law derivarion is expanded into three dimensions by the author for implementation in Missile Design PC TRAP. The expansion from two to three dimensions were performed by the author according to general guidelines provided in Blakelock [ref.2] and by dissecting the algorithms of PC TRAP.

C. HOMING GUIDANCE MISSILES--GUIDANCE LAWS

In a homing guidance system, the missile must be equipped with an onboard seeker which provides the target LOS (and possibly range) information required for guidance by receiving electromagnetic radiation (or energy) reflected (or emitted) by the target (i.e. radar signal or infrared radiation). The virtue of homing guidance is that measurement accuracy of target parameters is continually improving because the missile (and its seeker) are getting closer to the target as the flight progresses.

Three basic guidance laws are typically used in tactical homing guidance missiles: pursuit guidance, constant bearing (lead angle) guidance and proportional navigation guidance. Proportional navigation is the only guidance law in PC TRAP. Theoretically, all three guidance laws produce acceleration commands, perpendicular to the instantaneous missile-to-target LOS, which are proportional to the rate of change in time of the missile-to-target LOS angle.

Among these three guidance laws, the proportional navigation law is generally considered as being the "optimum" guidance law because of its great effectiveness, its lack of requirement for range to the target, and its ease of implementation. This explains why the proportional navigation guidance law is widely used in tactical missiles.

Additionally, there are some advanced guidance laws that are derived from the basic proportional navigation guidance law, and which generally improve its performance. These advanced guidance laws tend to relax the missile lateral acceleration requirements and generally yield smaller miss distances. The price paid for these more advanced guidance laws is that more target information is required for their successful implementation. The augmented proportional navigation (APN) law is an advanced guidance law that is available in Missile Design PC TRAP. The APN guidance law uses additional information, such as target maneuver information, to guide the missile to an intercept.

The kinematics of the basic proportional navigation guidance law will first be derived and then used for the derivation of all the other guidance laws discussed in this section and in the rest of this chapter.

1. Proportional Navigation

In order for a homing guided missile to use proportional navigation, the guidance system, via the seeker, must be able to measure the time rate of change of the LOS angle between the target and the missile, as well as the relative closing velocity between the target and the missile.

In practice, the seeker of tactical missiles using a radar homing system (semi-active and active) provides an effective measurement of the LOS rate and a Doppler radar provides closing velocity information. The seeker of tactical missiles using an infrared (IR) homing system (passive) measures the LOS rate, whereas the closing velocity must be estimated by the missile guidance computer. The closing velocity can be estimated with the use of accelerometers onboard the missile and an initial knowledge of the target velocity. The proportional navigation guidance law attempts to maintain an essentially constant LOS angle by generating acceleration commands that will keep the LOS rate as close to zero as possible. As illustrated at Figure 3-3, keeping a constant LOS angle (λ) between the missile and the target will ultimately cause a collision between the two vehicles.

Airborne missile systems using proportional navigation typically launch the missile with a lead angle (L) that points the missile at the predicted intercept point at time of launch. In such a case, the launching platform fire control system may estimate the target position at time of intercept, based on current target position, velocity and attitude.

Once the missile is airborne, the proportional navigation guidance law generates acceleration commands to the flight control system to maintain the missile on a collision course at constant LOS angle (λ) with

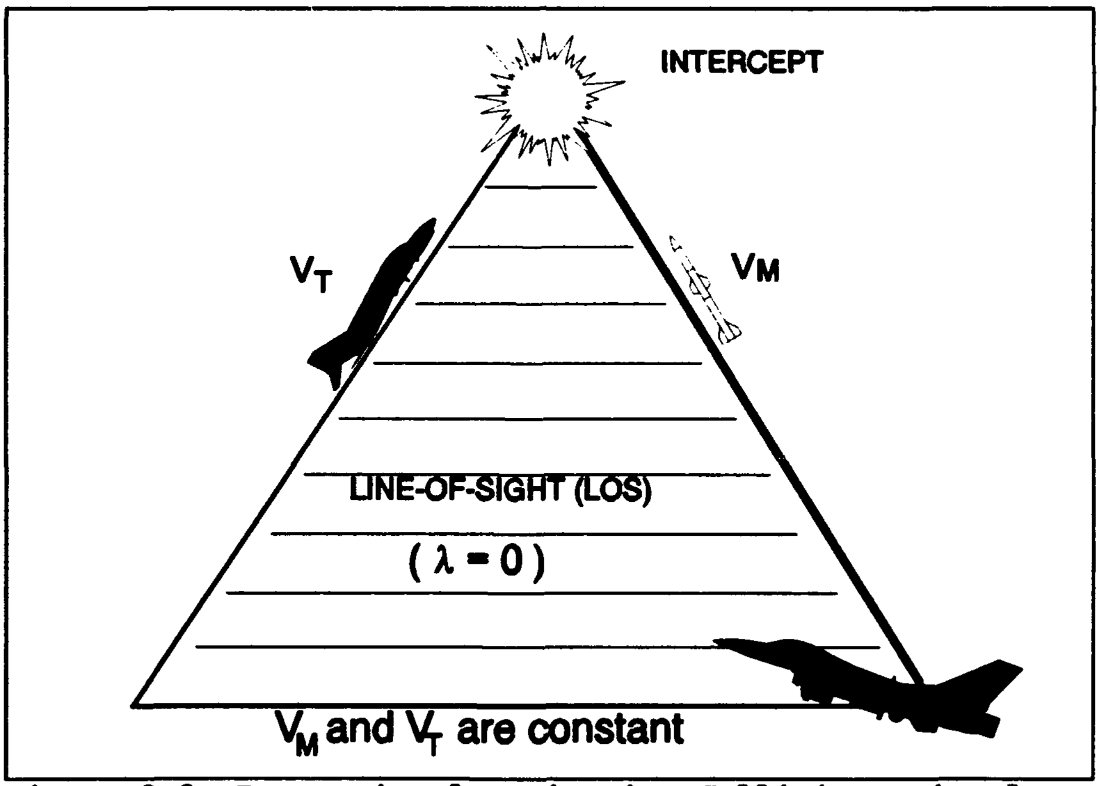


Figure 3-3. Proportional Navigation Collision Triangle

respect to the target. These acceleration commands are generated based on the following expression:

$$n_c = N\, V_c\, \frac{d\lambda}{dt}\ , \tag{1}$$

where: n_c is the missile acceleration command (m/sec²);

N is the proportional navigation constant, which is a unitless designer chosen gain generally between 2 and 6;

V_c is the missile-to-target closing velocity (m/sec); and

$d\lambda/dt$ is the time rate of change of the LOS angle, also called the LOS rate (rad/sec).

2. Two-Dimensional Intercept Geometry and Kinematics

This sub-section defines the missile-to-target intercept geometry, as defined by Zarchan [ref. 9], that shall be used to determine the parameters and the ordinary differential equations (ODE's) required to simulate a proportional navigation trajectory. In order to better understand how proportional navigation works, let us first consider a two-dimensional, point mass missile-target engagement geometry as shown at Figure 3-4. We shall use an inertial coordinate system fixed to the surface of a flat Earth model where the 1 axis is the downrange and the 2 axis can either be the altitude or the crossrange. The use of a fixed inertial coordinate system allows the integration of components of velocities and accelerations without having to include additional terms due to the Coriolis effect.

It can be seen from Figure 3-4 that the missile, with velocity magnitude V_m, is heading towards the target at an angle L + HE with respect to the line-of-sight (LOS). The angle L is the lead angle discussed earlier, and the angle HE is known as the initial heading error. This angle represents the initial (at missile launch) deviation of the missile flight path from the proportional navigation perfect collision triangle (Figure 3-3).

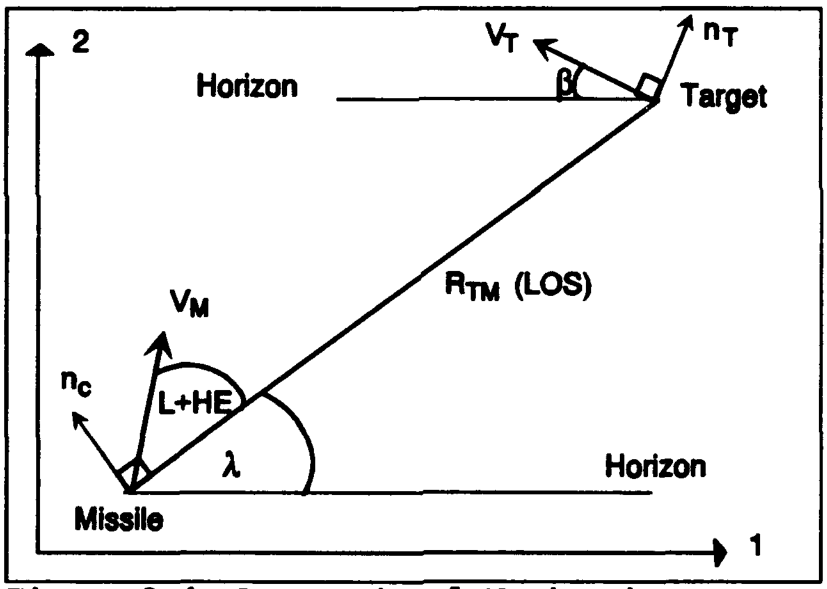


Figure 3-4. Proportional Navigation Engagement Geometry

In Figure 3-3 the imaginary line connecting the missile and the target is the Line-of-sight (LOS). The LOS makes an angle λ with respect to the fixed reference (Figures 3-3 and 3-4), and the length of the LOS represents the instantaneous missile-to-target separation, denoted $\mathbf{R}_{TM}$ (slant range). From a guidance law point of view, the goal is to make $\mathbf{R}_{TM}$ at the expected time of intercept as small as possible. The closest point of approach (CPA) between the missile and the target is the intercept miss distance.

To model the proportional navigation guidance law requires mathematical expressions for the closing velocity ($\mathbf{V}_c$) and for the LOS rate (dλ/dt). These expressions must be derived from the exactly known missile and target state variables.

The missile-to-target relative closing velocity is the time variation in slant range and is expressed as follows:

$$V_C = -\dot{R}_{TM} \; . \qquad (2)$$

At the end of an engagement, the sign of $\mathbf{V}_c$ changes indicating that the intercept has occurred. From Figure 3-4, the missile acceleration command ($\mathbf{n}_c$) are always perpendicular to the instantaneous LOS.

In PC TRAP and Missile Design PC TRAP the target velocity magnitude is constant, but the target is allowed to maneuver (the target model does not include drag for simplicity). The target acceleration (n_t) is perpendicular to the target velocity vector, and the angular velocity of the target is expressed as:

$$\frac{d\beta}{dt} = \frac{n_t}{V_t} , \qquad (3)$$

where V_t is the magnitude of the target velocity and β is the target flight path angle with respect to the horizontal, as shown in Figure 3-4.

The 1 and 2 axis components of the target velocity vector in the inertial coordinate system can be found using:

$$\begin{aligned} V_{T1} &= -V_T \cos\beta \\ V_{T2} &= V_T \sin\beta , \end{aligned} \qquad (4.a)$$

where the flight path angle of the target (β), during the intercept is obtained by integrating equation (3).

By integrating the target velocity components of equation (4.a), the target position components in the inertial coordinate system R_{T1} and R_{T2} can be found. The target velocity components can therefore defined as:

$$\begin{aligned} \dot{R}_{T1} &= V_{T1} \\ \dot{R}_{T2} &= V_{T2}. \end{aligned} \qquad (4.b)$$

Similarly, the ordinary differential equation for the missile velocity and position differential equations are given by:

$$\begin{aligned} \dot{V}_{M1} &= a_{M1} \\ \dot{V}_{M2} &= a_{M2} \\ \dot{R}_{M1} &= \dot{V}_{M1} \\ \dot{R}_{M2} &= \dot{V}_{M2} \ . \end{aligned} \qquad (5)$$

Where a_{M1} and a_{M2} are the missile acceleration components in the two-dimensional inertial coordinate system.

Knowing the missile and target positions, the LOS angle (λ) can be found as follows, using trigonometry from Figure 3-4:

$$\lambda = \arctan\frac{R_{TM2}}{R_{TM1}} \ , \qquad (6)$$

where R_{TM1} and R_{TM2} are the 1 and 2 axis components of the relative missile-to-target separations defined, respectively, as:

$$R_{TM1} = R_{T1} - R_{M1}$$
$$R_{TM2} = R_{T2} - R_{M2} \, . \tag{7}$$

Similarly, from the known missile and target velocities, the relative velocity components in the inertial coordinate system are:

$$V_{TM1} = V_{T1} - V_{M1}$$
$$V_{TM2} = V_{T2} - V_{M2} \, . \tag{8}$$

To calculate the required missile acceleration command from equation (1), we need the following expression for the LOS rate (dλ/dt):

$$\frac{d\lambda}{dt} = \frac{d}{dt}(\arctan\frac{R_{TM2}}{R_{TM1}}) = \frac{R_{TM1}\ V_{TM2} - R_{TM2}\ V_{TM1}}{R_{TM}^2}, \tag{9}$$

where:

$$R_{TM} = \sqrt{(R_{TM1}^2 + R_{TM2}^2)} \, . \tag{10}$$

$\mathbf{R_{TM}}$ is the instantaneous relative separation between the missile and the target. We also need an expression for the closing velocity, which is defined by equation (2), and is equal to

$$V_c = -\dot{R}_{TM} = -\frac{(R_{TM1}\, V_{TM1} + R_{TM2}\, V_{TM2})}{R_{TM}} \,. \qquad (11)$$

Substitution of equations (9) and (11) into equation (1), provides the missile acceleration command expression (n_c) required by the proportional navigation guidance law to steer the missile to the intercept point.

Once n_c has been found, the last state variable (missile acceleration) can be determined. As illustrated in Figure 3-4, the missile acceleration components in the inertial or fixed coordinate system shall be obtained from:

$$\begin{aligned} a_{M1} &= -\, n_c \sin\lambda \\ a_{M2} &= n_c \cos\lambda \,. \end{aligned} \qquad (12)$$

All the ordinary differential equations required to model a complete missile-to-target engagement in two dimensions with the proportional navigation guidance law have now been defined above. Using these ordinary differential equations, the missile and target state variables (position, velocity, and acceleration) can be found as well as the required missile lateral acceleration. However, the initial conditions on the ordinary differential equations are required in order to construct the two-dimensional engagement model.

a. *Initial Conditions*

In order to solve the above set of differential equations and to complete the two-dimensional engagement model, initial conditions are required by the simulation program. In TRAP and Missile Design PC TRAP, the initial position, velocity, and acceleration of the target are known based on the user's input to the programs. Similarly, the initial position of the missile is known as well as the magnitude of the missile velocity. However, as explained earlier, a homing guidance missile employing proportional navigation guidance will not usually be fired directly at the target, but may be fired in a direction to lead the target. In such a case the missile is fired with a lead angle L to point at the expected intercept point. Consequently, the initial missile velocity vector will be a function of this required lead angle.

In an ideal simulation model where both the missile and the target are flying at constant speeds, it can be seen from Figure 3-4 that, for the missile to form a perfect collision triangle (shown in Figure 3-2) with the target, the theoretical initial missile lead angle L can be found by application of the law of sines, yielding:

$$L = \arcsin\frac{V_T \sin(\beta + \lambda)}{V_M} = \arcsin\frac{V_T \sin\theta}{V_M}. \tag{13}$$

In practice, the missile is usually not launched exactly on a collision triangle, since the expected intercept point is not known precisely due to the target motion during the missile time of flight. At time of missile launch, the location of the intercept point can only be approximated by the launch platform fire control system using the above simplistic equation (13), or other more complex expressions which may require estimates of the target motion. However, using only an approximated intercept point at time of launch still provides a certain lead advantage to the missile over the target, and that advantage shall be used by the guidance law to optimize the missile trajectory. As stated earlier, any initial deviation of the missile flight path from the collision triangle is known as an initial heading error, HE. Upon missile launch, as soon as the missile seeker is enabled and allowed to acquire and track the target, the missile guidance system will determine the initial heading error HE and eliminate it as efficiently as possible.

The initial missile velocity components with respect to the 1 and 2 axis can be expressed in terms of the theoretical lead angle (L) and the actual heading error (HE) as:

$$
\begin{aligned}
V_{M1}(t=0) &= V_M \cos(L + HE + \lambda) \\
V_{M2}(t=0) &= V_M \sin(L + HE + \lambda)
\end{aligned}
\qquad (14)
$$

where V_M is the magnitude of the initial missile velocity. For air-to-air and air-to-surface scenarios, V_M is typically the speed of the launching aircraft at time of missile launch. For surface-to-air scenarios, V_M is the velocity of the missile at the exit of the launcher tube.

All the equations required to simulate a complete missile-to-target engagement in two-dimensions have been established. Extending the same ideas to the three-dimensional intercept geometry, a similar set of ordinary differential equations will be developed in the next section.

3. Three-Dimensional Intercept Geometry and Kinematics

a. *Basic Geometry*

In practice, missile-to-target intercepts occur in a three dimensional geometry as shown in Figure 3-5.

A fixed coordinate system (the earth inertial coordinate system) denoted X_E, Y_E, Z_E, is located at the point-mass missile **M** and at the point-mass target T. The three-dimensional missile-target intercept geometry can be analyzed into two different planes: the X_E-Y_E plane, which we will call the azimuth or horizontal plane, and the plane defined by the projection of the vehicle positions onto the X_E-Y_E plane with the Z_E axis, which we call the elevation or vertical plane. These two planes are shown separately in Figure 3-6. The top figure shows the elevation plane while the bottom one shows the horizontal plane.

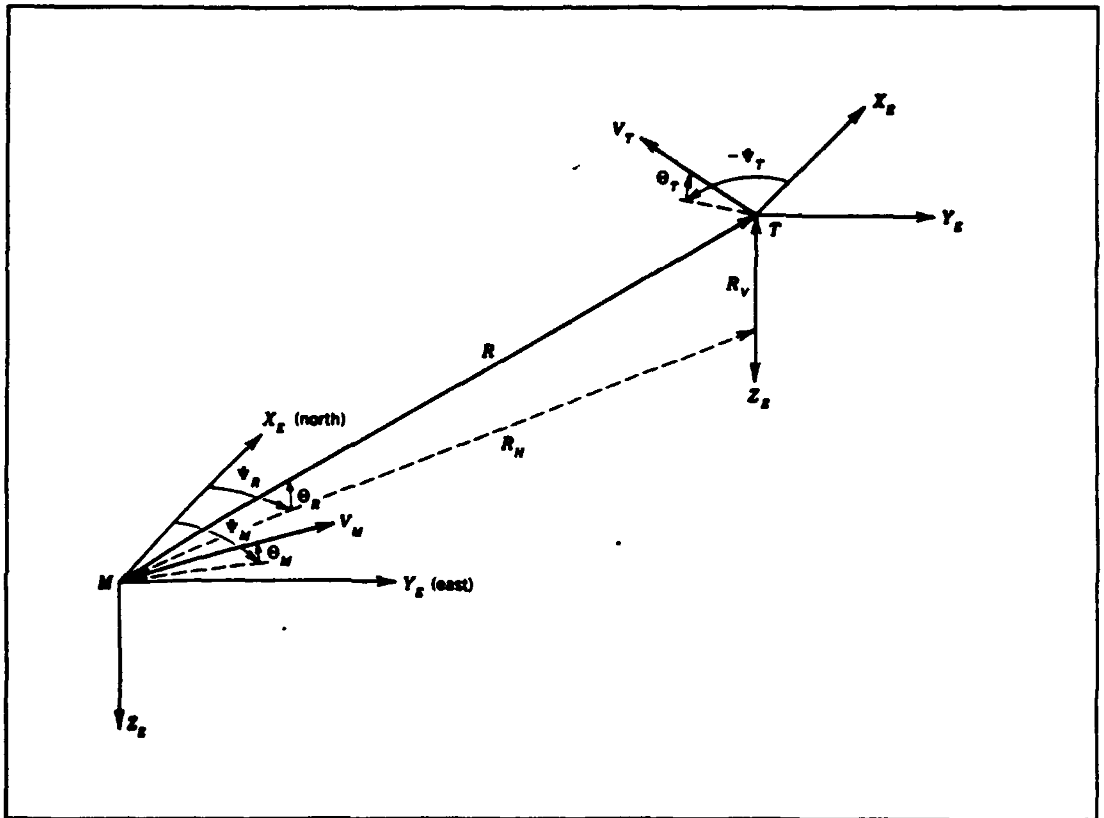


Figure 3-5. Missile-Target Intercept Geometry [Blakelock [ref. 2]]

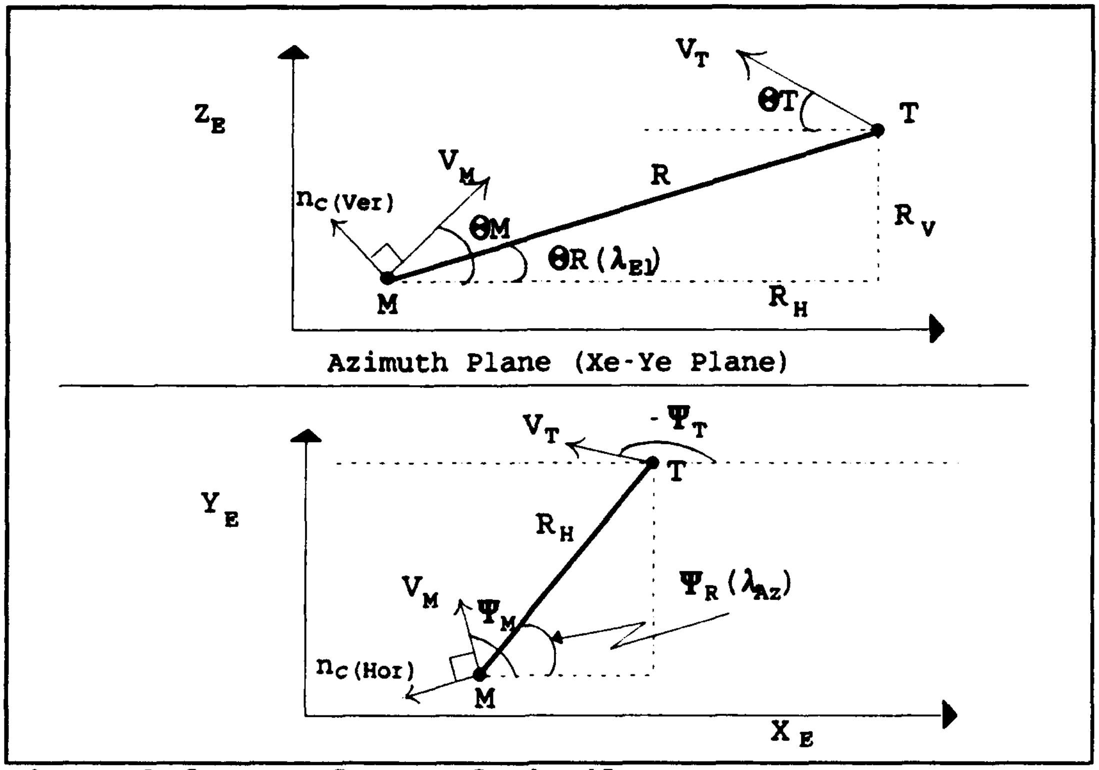


Figure 3-6. Two Planes of Missile

As shown in Figure 3-5, the missile is flying toward the target with a velocity magnitude V_M at an azimuth angle Ψ_M from the X_E axis, and at an elevation angle Θ_M from the X_E-Y_E plane. Similarly, the target has a speed V_T with an azimuth angle $-\Psi_T$, and an elevation angle Θ_T.

In Figures 3-5 and 3-6, the target and missile point mass are respectively located at coordinates **TPX**, **TPY**, **TPZ** and **MPX**, **MPY**, **MPZ** with respect to the fixed coordinate system X_E, Y_E and Z_E. From Figures 3-5 and 3-6, **R** is the total LOS vector between the missile and the target, which be decomposed into a horizontal LOS components ($\mathbf{R}_H$) in the azimuth plane, and into a vertical LOS components ($\mathbf{R}_V$) in the elevation plane. The elevation angle of the missile-to-target LOS (**R**) is Θ_R while the azimuth angle is Ψ_R.

b. Ordinary Differential Equations

The ordinary differential equations detailed for the simulation of the three dimensional proportional navigation guidance law are derived from the TRAP algorithm, which simulates the proportional navigation guidance law only. In three dimensions, a missile guidance system using the proportional navigation guidance law uses equation (1) to generate two independent acceleration lateral commands as follows:

$$n_{c(Hor)} = N \; V_{c(Hor)} \frac{d\lambda_{Az}}{dt} \quad (a)$$
$$n_{c(Ver)} = N \; V_{c(Ver)} \frac{d\lambda_{El}}{dt} \quad (b) \qquad (15)$$

where: $n_{c(Hor)}$ is the missile lateral acceleration command in the horizontal or azimuth plane (m/sec^2),

$n_{c(Ver)}$ is the missile lateral acceleration command in the vertical or elevation plane (m/sec^2),

$d\lambda_{Az}/dt$ is the LOS rate in the azimuth plane (rad/sec); and

$d\lambda_{El}/dt$ is the LOS rate in the elevation plane (rad/sec).

These two lateral acceleration commands are implemented in their respective missile plane by the missile control system as shown in Figure 3-7.

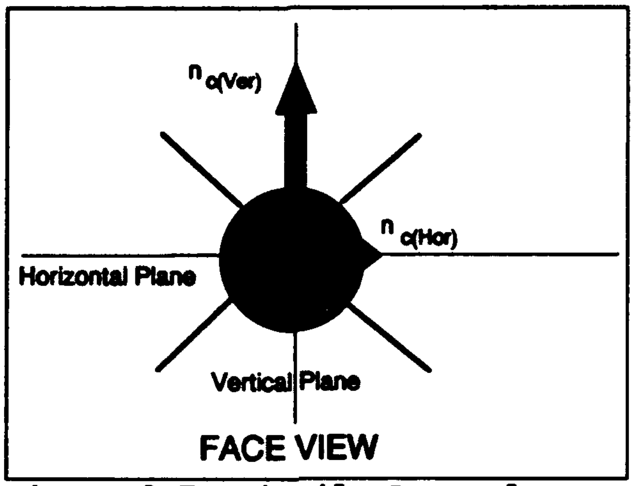


Figure 3-7. Missile Lateral Accelerations

The ordinary differential equations that must be solved to implement the equation (15) guidance law are an extension of equations (2) through (14) into the two missile planes shown in Figure 3-7. For this reason, the set of equations required to implement the homing proportional navigation guidance law in three dimensions will not be derived in full. However, for academic reasons, both set of equations required to implement the proportional navigation in the two missile planes are presented below.

(1) Horizontal Plane

The guidance law to generate the missile lateral acceleration in the horizontal plane is expressed at equation (15a). From that expression, it can be seen that expressions are needed for the LOS rate of change (dλ/dt), and for the relative closing velocity (V_C) between the target and the missile. Recall that both the missile and the target state variables (position, velocity, and acceleration) are known exactly in the simulation program.

As illustrated in Figure 3-6, the LOS angle (λ_{az}) in the horizontal plane can be found as:

$$\lambda_{az} = \Psi_R = \arctan(\frac{R_{HY_E}}{R_{HX_E}}) \ , \qquad (16)$$

where:

$$R_{HX_E} = TPX - MPX$$
$$R_{HY_E} = TPY - MPY \ . \qquad (17)$$

The rate of change of the horizontal LOS angle (λ_{az}) is given by taking the time derivative of equation (16), yielding:

$$\frac{d\lambda_{az}}{dt} = \frac{R_{HX_E} V_{HY_E} - R_{HY_E} V_{HX_E}}{R_H^2} , \qquad (18)$$

where:

$$\begin{aligned} V_{HX_E} &= V_{TX_E} - V_{MX_E} = VTX - VMX \\ V_{HY_E} &= V_{TY_E} - V_{MY_E} = VTY - VMY \\ R_H &= \sqrt{R_{HX_E}^2 + R_{HY_E}^2} . \end{aligned} \qquad (19)$$

The relative closing velocity in the horizontal plane is expressed as follows:

$$V_{C(Hor)} = -\dot{R}_H = \frac{-(R_{HX_E} V_{HX_E} + R_{HY_E} V_{HY_E})}{R_H} . \qquad (20)$$

Substitution of equations (18) and (20) into expression of equation (15a) yields the final expression for the lateral missile acceleration in the horizontal plane in feet or meter per second squared (ft/sec^2 or m/sec^2). The horizontal missile lateral acceleration is generated perpendicular to the horizontal LOS ($\mathbf{R}_H$).

(2) Vertical Plane

The guidance law for the missile lateral acceleration in the vertical plane is equation (15b). As shown in Figure 3-6, the LOS angle (λ_{El}) in the vertical plane is found as follows:

$$\lambda_{El} = \arctan(\frac{R_V}{R_H}) \tag{21}$$

where $\mathbf{R}_H$ is already defined by equation (19), and

$$\begin{aligned} R_V &= TPZ - MPZ \\ \dot{R}_V &= V_{TZ_E} - V_{MZ_E} = VTZ - VMZ \ . \end{aligned} \tag{22}$$

$$\frac{d\lambda_{El}}{dt} = \frac{R_H \, \dot{R}_V - R_V \, \dot{R}_H}{R^2} \tag{23}$$

The LOS rate of change in the vertical plane can therefore be found as:

where:

$$R = \sqrt{R_H^2 + R_V^2} \ . \qquad (24)$$

The relative closing velocity is:

$$V_{C(Ver)} = -\dot{R} = \frac{R_{HX_E}\, V_{HX_E} + R_{HY_E}\, V_{HY_E} + R_V\, \dot{R}_V}{R} \ . \qquad (25)$$

The final lateral acceleration in the vertical plane is obtained by substituting equations (23) and (25) into equation (15b).

(3) Initial Conditions

As for the two-dimensional case, initial conditions are required to solve the above ODEs. Similarly, the target initial condition (position, velocity, and acceleration) are set by the user as well as the missile initial position and velocity magnitude. As for the two dimensional case, the initial missile velocity components must be established from the initial pointing angle of the missile. In the TRAP and the Missile Design PC TRAP three dimensional models, the missile azimuth heading angle (Ψ_M) includes both the azimuth lead angle L_{Az} and the initial heading error angle HE_{Az}, while the missile elevation heading angle (Θ_M) incorporates the elevation lead angle L_{El} and the elevation heading error HE_{El}. The method to compute the lead angle in three dimensions is detailed at Chapter IV. The three-dimensional initial missile velocity components with respect to the X_E, Y_E, and Z_E axes can be expressed as follows:

$$
\begin{aligned}
V_{MX_E} &= V_M \cos(\Theta_M) \cos(\Psi_M) \\
V_{MY_E} &= V_M \cos(\Theta_M) \sin(\Psi_M) \\
V_{MZ_E} &= V_M \sin(\Theta_M).
\end{aligned} \tag{26}
$$

V_M is the missile initial airspeed along the missile longitudinal axis at launch in m/sec. For an air-launched missile, this initial speed is normally the aircraft longitudinal airspeed at launch. For surface launches, this airspeed is typically the airspeed along the missile longitudinal axis at the time where the missile leaves its launcher tube.

4. Pure Pursuit

The pure pursuit guidance law was the first tactical missile guidance technique developed and successfully implemented in homing guidance missiles, and as such, is the least complex of the homing missile guidance laws. In the pure pursuit trajectory, illustrated in two dimensions at Figure 3-8, the missiles directly toward the instantaneous location of the target at all times. Therefore, contrary to proportional navigation, the LOS between the missile and the target is maintained, by the guidance system, along the heading of the missile with respect to the target. This is shown at

Figure 3-9 which illustrates the two-dimensional geometry of a pure pursuit engagement.

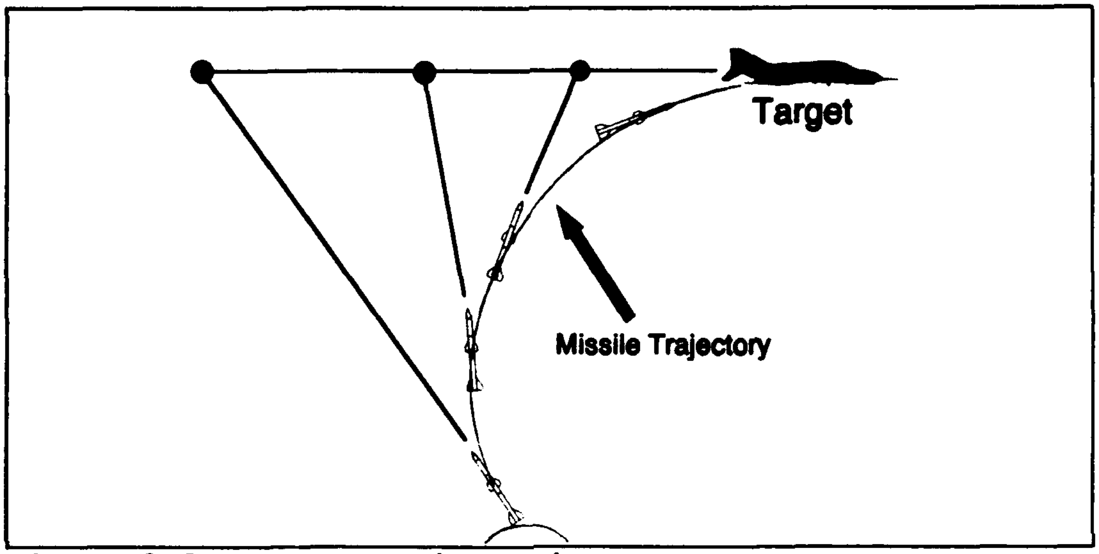


Figure 3-8. Pure Pursuit Trajectory

As the flight progresses, the missile lags behind the target and the intercept generally occurs from the rear quarter of the target. The resulting trajectory normally consumes more missile energy and time than the other homing guidance laws (proportional navigation, augmented proportional navigation, and lead angle).

The rate of turn of the missile is always equal to the LOS rate of turn ($d\lambda/dt$). Pure pursuit paths are highly curved near the end of flight, and it is possible that the missile may lack sufficient maneuverability to maintain

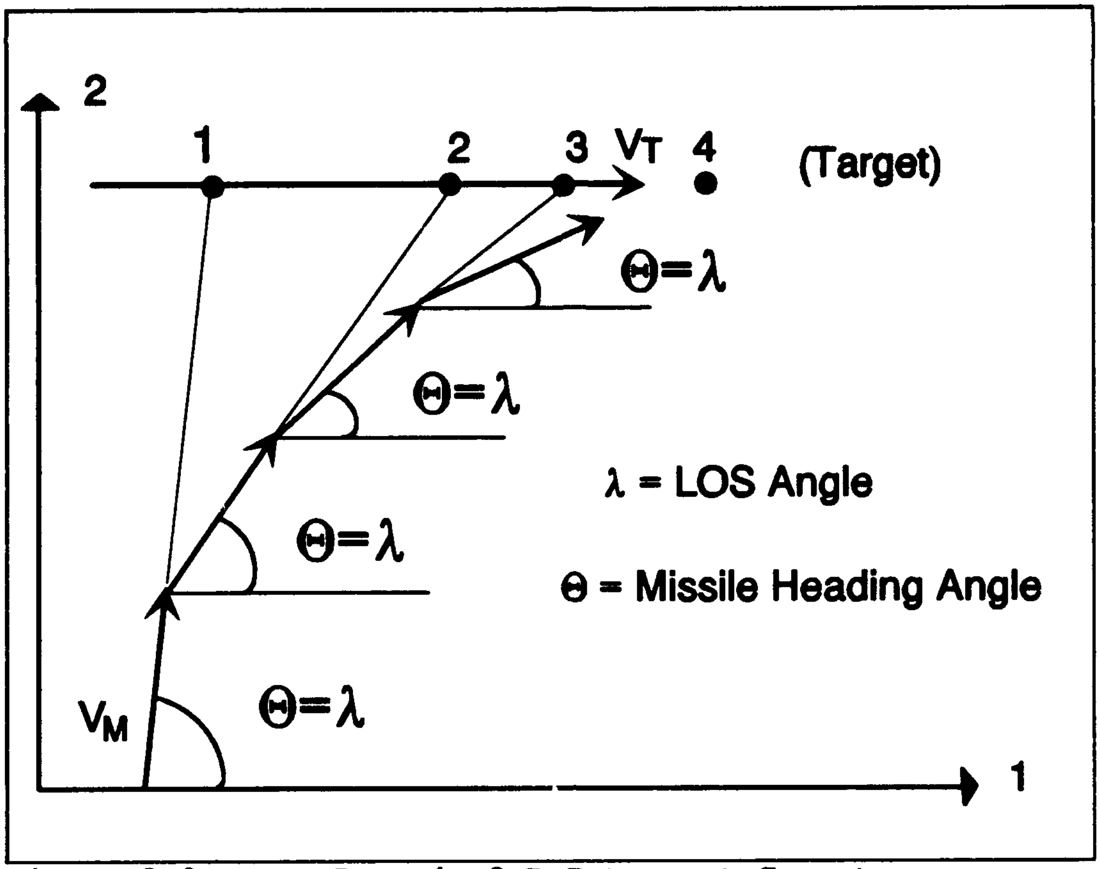


Figure 3-9. Pure Pursuit 2-D Intercept Geometry

a pure pursuit path in the terminal phase of guidance. When this is the case, the missile can be designed to continue turning at the maximum rate of which it is capable until a point is reached where a pursuit course can be resumed. The two major shortfalls of the pure pursuit guidance law are:

- The end-game maneuvers are very hard and often require more lateral acceleration than can be sustained by the missile
- The missile speed must be considerably greater than the target speed for the missile to have a real advantage over the target

The advantage of this guidance law is that it requires a minimum of target and missile information to guide towards the target. The only engagement geometry parameters that it requires to generate the acceleration commands are the target position and the missile velocity. Therefore, since the guidance signal processing is limited to looking and pointing, the guidance system avionics are relatively simple, light, low cost as well as being easy to implement.

The most common application of the pure pursuit guidance law is against slow moving and/or non maneuverable targets, or for missiles launched from a point to the rear of the target.

a. *Two-Dimensional Intercept Geometry and Kinematics*

Once the pure pursuit missile is launched, the two-dimensional intercept geometry is as shown at Figure 3-9. Unlike the proportional navigation guidance law, the pure pursuit missile guidance law does not have an initial lead angle at missile launch. Therefore, the missile is launched pointing directly at the target. The pure pursuit missile acceleration command is generated perpendicular to the missile-to-target LOS and is defined as:

$$n_c = V_M \frac{d\lambda}{dt} , \qquad (27)$$

where $d\lambda/dt$ is the LOS rate in rad/sec and V_M is the magnitude of the missile velocity in ft/sec or m/sec. A design option for this type of guidance would be to include a constant lead angle bias to accommodate faster moving targets. This is called deviated pursuit and exhibits very similar characteristics to pure pursuit except the fact that the missile has a lead advantage over the target. As for the proportional navigation guidance law, the state variables (position, velocity, and acceleration) of the missile and target are known for each simulation time step. Hence, the LOS rate is computed by Missile Design PC TRAP as detailed in equation (9) for proportional navigation.

In order to model pure pursuit trajectories in two dimensions, the model developed for the proportional navigation shall be used with the replacement of equation (1) by equation (27), and by using different initial conditions. The initial angle of the missile velocity vector with respect to the missile-to-target LOS shall simply be λ (as shown at Figure 3-9), since no lead angle is present in pure pursuit trajectories. Recall that in Missile Design PC TRAP, the initial position of both the missile and target are set by the user. However, expressions for the initial missile velocity is required.

Consequently, the initial missile velocity components can simply be expressed as:

$$\begin{aligned} V_{M1}(t=0) &= V_M \cos\lambda \\ V_{M2}(t=0) &= V_M \sin\lambda \end{aligned} \tag{28}$$

which is, in essence, equation (26) without the lead and heading error angles.

b. Three-Dimensional Intercept Geometry and Kinematics

The pure pursuit guidance law was implemented in three dimensions into the Missile Design PC TRAP by the author using the same differential equations as for the proportional navigation guidance law, as detailed above. The only difference is in the generation of the missile lateral acceleration commands, which is accomplished by using equations (15a) and (15b) with a proportional navigation constant of 1 (N = 1). This application is comparable to the expansion of the two-dimensional pure pursuit guidance law of equation (27) into both the three-dimensional elevation and azimuth planes of the missile. This is very similar to what was detailed for the three-dimensional proportional navigation model.

As for the two dimensional pure pursuit geometry case, there is no lead or heading error incorporated into the initial set up of the three-dimensional pure pursuit trajectory modelling. The three-dimensional initial missile velocity components with respect to the X_E, Y_E, and Z_E axes can be found with the use of equation (26) similarly to the proportional navigation guidance law.

5. Constant Bearing (Lead Angle)

Constant bearing guidance is also known as collision path guidance. The missile trajectory path generated by the constant bearing guidance is at the opposite extreme of the one generated by the pure pursuit guidance, while the proportional navigation path is the optimum guidance path between the latter two. The large missile accelerations obtained using the pure pursuit path may be reduced by employing a lead angle. One way to do this is to aim the missile ahead of the target at launch, so the missile traverses a straight line to a collision with a constant speed non-maneuvering target as shown at Figure 3-10. As detailed above, the lead angle principle is also used in the proportional navigation guidance law for the initial heading of the missile.

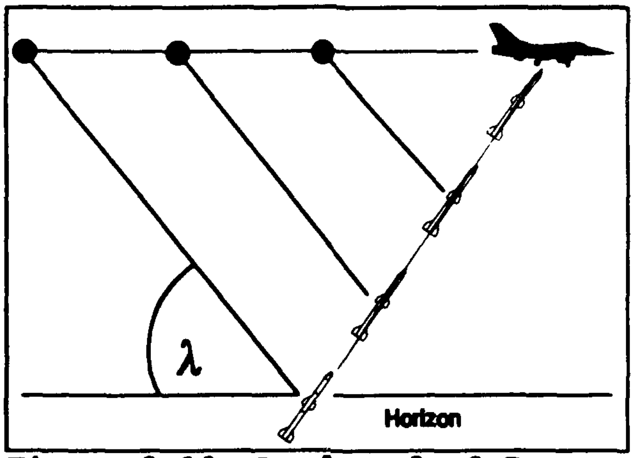


Figure 3-10. Lead Angle 2-D Intercept Scenario

When the lead angle guidance law is used in a missile, the missile converges on the target in such a manner that the LOS from the missile to the target maintains a constant direction in space. If the target maintains a constant speed and does not conduct any maneuvers, the LOS rate (dλ/dt) is zero, meaning that the missile lateral accelerations are also zero, which is a desirable quality for any guidance law. As soon as the target conducts any evasive maneuvers or if the target changes its velocity, a new intercept point is computed by the missile guidance computer, and the guidance law develops the required lateral accelerations that alters the missile flight path according to the new parameters of the intercept geometry.

a. *Two-Dimensional Intercept Geometry and Kinematics*

As for all homing guidance laws, the lead angle missile acceleration command is generated perpendicular to the missile-to-target LOS according to the following mathematical expressions:

$$n_c = N\ V_c\ \frac{d\lambda}{dt}\ , \tag{29}$$

where: n_c is the missile acceleration command (m/sec^2);

V_c is the missile-to-target closing velocity (m/sec);

dλ/dt is the LOS rate (rad/sec); and

N is the unitless proportional navigation constant set to 10 for the lead angle guidance law simulation.

Equation (29) is the same equation as for the two-dimensional proportional navigation guidance law (equation (1)) except for the value taken by the unitless proportional navigation constant (N). As mentioned earlier, for the proportional navigation guidance law, N is generally chosen to be between 2 and 6. However, for simulation of the lead angle guidance law, N is chosen to be equal to 10, which is outside the allowable range for proportional navigation. This special way of selecting N allows one to simulate the lead angle guidance law from the proportional navigation model Lindsey and Redmond [ref. 5]. This is exactly how the lead angle guidance law was implemented in Missile Design PC TRAP by the author.

b. Three-Dimensional Intercept Geometry and Kinematics

The three-dimensional intercept geometry, as computed in Missile Design PC TRAP, is exactly as detailed above for the proportional navigation guidance law. As well, the implementation of the trajectory simulation equations is the same as for the proportional navigation case, with the exception that the proportional navigation constant (N) in equation (15a) and (15b) is taken to be equal to 10 (N=10) as dictated by equations (29). For this reason, in Missile Design PC TRAP, the lead angle guidance law is simulated using the geometry and the initial conditions established for the proportional navigation guidance law, with the only difference that the commanded missile lateral accelerations are generated differently.

6. Augmented Proportional Navigation

Thus far, we have seen it has been shown that very effective guidance law and that it is relatively easy to implement. However, proportional navigation is not an optimal guidance law. As detailed in Zarchan [ref. 9], there are more advanced guidance laws that tend to relax the missile lateral acceleration requirements and that generally yield smaller miss distances. There are an infinite number of possible guidance laws. Thus, to derive the augmented proportional navigation guidance law it is necessary to state in mathematical terms, according to Zarchan's [ref.9] method [ref.], what the desired guidance law should do. It is desirable to obtain an optimal guidance law with a zero miss distance requirement and uses minimal total lateral acceleration. A mathematical way of stating the guidance problem to be solved is that it is desirable to achieve zero miss distance subject to minimizing the integral of the square of the missile lateral acceleration command, or

$$y(t_f) = 0, \; minimizing \quad \int_0^{t_f} n_c^2(t)\, dt \, . \tag{30}$$

This problem is normally solved using techniques from optimal control theory. However, it is solved in great details in Zarchan [ref. 9] using the Schwartz inequality to yield the following final optimal guidance law expression:

$$n_c = \frac{3\,(y + \dot{y}\,t_{GO} + 0.5\,n_T\,t_{GO}^2)}{t_{GO}^2} \tag{31}$$

where: y is the relative missile-to-target separation (m),

dy/dt is the relative missile-to-target separation rate (m/sec),

n_c is the missile lateral acceleration command (m/sec^2),

n_T is the target lateral acceleration (m/sec^2), and

$t_{GO} = t_f - t$ = the time-to-go before intercept (sec).

It has been shown in Zarchan [ref. 9] that equation (31) can be reduced to:

$$n_c = 3\,V_c\,\dot{\lambda} + \frac{3\,n_T}{2}\,. \tag{32}$$

It can be see that this optimal guidance law (equation 32) based on the performance criteria (cost function) established in equation (30) is simply the proportional navigation guidance law (N = 3) with an extra term to account for the maneuvering target. As stated in Zarchan [ref. 9], the proportional navigation constant (N) turns out to be 3 because it is necessary that the integral of the square of the missile acceleration be minimized. This new optimal guidance law is called augmented proportional navigation (APN).

A zero-lag APN homing loop is shown in block diagram form in Figure 3-11. The additional target maneuver term, required by the APN guidance law expressed in equation (32), appears as a feedforward term in the missile homing loop block diagram. As a result, APN generally requires less acceleration capability of the missile than proportional navigation, because APN is making use of extra information on the target instantaneous maneuver. It is therefore reasonable that this knowledge should enable the missile to maneuver in a much efficient manner.

In practice, complex guidance concepts are required to implement the APN guidance law in a tactical missile. Since the target maneuver level is not known exactly by the missile guidance computer, it must therefore be estimated from the kinematics of the intercept geometry. The optimum method to accomplish this task is with the introduction of Kalman filters in the guidance loop, which estimate missile-target relative position and velocity, as well as the target maneuver level. It is shown in Zarchan [ref. 9] that Kalman filtering combined with the APN guidance law produce substantial performance benefits and a relaxing of missile lateral acceleration requirements. However, range and time-to-go information must be available for this combination to work at its best. Time-to-go (t_{go} of TTGO) can be defined as the estimated missile flight time remaining before missile-target intercept. Typically, TTGO is estimated before missile launch by the guidance computer on the launching platform based on the target dynamics and the slant range to be traveled by the missile. If the required information is lacking, or inaccurate, the performance of this type of

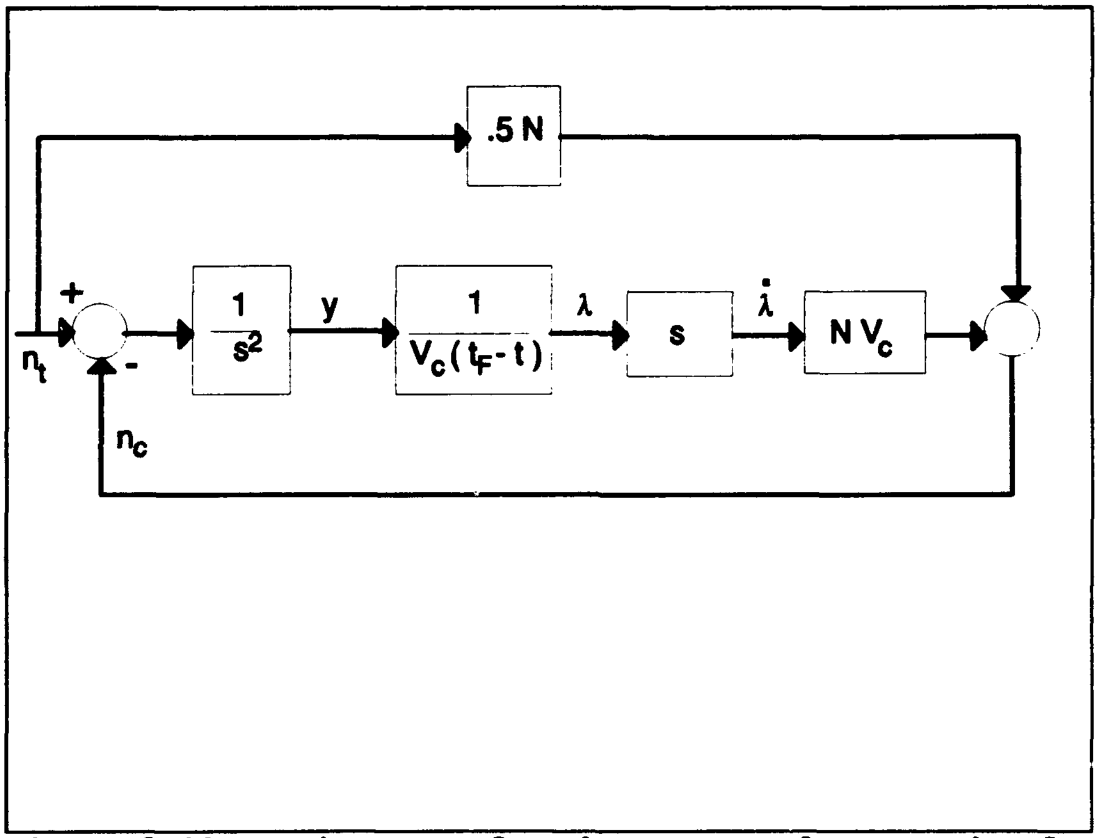


Figure 3-11. Homing Loop for the Augmented Proportional Navigation Guidance Law [Zarchan [ref. 9]]

guidance law may degrade to a point where its performance is worse than that of the basic proportional navigation guidance system.

a. *Implementation in Missile Design PC TRAP*

In the Missile Design PC TRAP, the missile and target state variables are known exactly for each simulation time step. This means that the target maneuver is known exactly for each time step during a tactical missile simulation. Hence, Missile Design PC TRAP models the APN guidance law using the exact level of the target maneuver to compute the extra term in equation (32). This means that the overall performance of the APN guidance law, as modeled in Missile Design PC TRAP, is optimistic since the exact level of target maneuver can only be estimated in practice.

The APN guidance law is modeled using the three-dimensional intercept geometry and kinematics derived above for the basic proportional navigation law. The only difference is that the two expressions of equation (15), for computing the required missile lateral acceleration in both missile planes, are replaced by the following expressions:

$$\begin{aligned} n_{c(Hor)} &= N\ V_{c(Hor)} \frac{d\lambda_{Az}}{dt} + \frac{N\ n_{T(Az)}}{2} \\ n_{c(Ver)} &= N\ V_{c(Ver)} \frac{d\lambda_{El}}{dt} + \frac{N\ n_{T(El)}}{2}\ . \end{aligned} \tag{33}$$

The proportional constant (N) does not necessarily take the value of 3 as described in equation (32). In Missile Design PC TRAP, N is determined by the user, from 2 to 6 or for the basic proportional navigation law.

Table 3-1 provides a summary of the three-dimensional mathematical expressions used in Missile Design PC TRAP to model the four homing guidance laws.

Guidance Law	Missile Plane	Mathematical Expression	Remarks
Pure Pursuit	Horizontal Vertical	$n_{c(Hor)} = V_{M(Hor)} \frac{d\lambda_{Az}}{dt}$ $n_{c(Ver)} = V_{M(Ver)} \frac{d\lambda_{El}}{dt}$	
Lead Angle	Horizontal Vertical	$n_{c(Hor)} = N\ V_{c(Hor)} \frac{d\lambda_{Az}}{dt}$ $n_{c(Ver)} = N\ V_{c(Ver)} \frac{d\lambda_{El}}{dt}$	N=10
Proportional Navigation	Horizontal Vertical	$n_{c(Hor)} = N\ V_{c(Hor)} \frac{d\lambda_{Az}}{dt}$ $n_{c(Ver)} = N\ V_{c(Ver)} \frac{d\lambda_{El}}{dt}$	$2 \leq N \leq 6$
APN	Horizontal Vertical	$n_{c(Hor)} = N\ V_{c(Hor)} \frac{d\lambda_{Az}}{dt} + \frac{N\ n_{T(Az)}}{2}$ $n_{c(Ver)} = N\ V_{c(Ver)} \frac{d\lambda_{El}}{dt} + \frac{N\ n_{T(El)}}{2}$.	$2 \leq N \leq 6$

Table 3-1. Summary of Homing Guidance Laws

D. COMMAND GUIDANCE MISSILES

Command guided missiles are missiles whose guidance instructions or commands come from sources outside the missile. A missile seeker is not present with command guidance. The general method of operation of command guidance systems is described at Chapter II.

First, this section discusses some inherent limitations of command missile guidance systems when compared to homing missile guidance systems. Then, three command guidance laws will be described, derived and analyzed in two dimensions and in three dimensions. Although Missile Design PC TRAP simulates missile trajectories in three dimensions, it is necessary to consider the more simplistic two-dimensional intercept geometry first to facilitate the development of the three-dimensional intercept geometry. This approach will be taken for the three command guidance laws discussed in this chapter.

1. Command Guidance System Limitations

One limitation of command guidance systems is that the external energy source (generally associated with the launching platform), which provides the guidance commands to the missile, must illuminate the target often enough (i.e. high data rate) to make guidance effective. This means that one energy source can only service a few targets simultaneously in a command guidance implementation.

The major limitation of a command guidance system is that, as intercept takes place further away from the location of the external energy source, measurement accuracy and hence guidance degrade as the missile approaches the intercept point. This limitation is discussed in great detail in Zarchan [ref. 9], where it is illustrated with an example where results will be presented in this section.

For his example, Zarchan [ref. 9] ran an idealistic two-dimensional simulation of a command guidance system with an input of one milliradian (mr) of noise on the measurement of the missile-to-target LOS angle. This simulation was conducted using the proportional navigation command guidance model discuss below. He subsequently simulated the same intercept scenario using the two-dimensional proportional navigation homing guidance system discussed above. Zarchan [ref. 9] then compared the noise transmission level recorded during both simulation runs by plotting the LOS rate estimates obtained from the command guidance run and the LOS rate obtained with the homing guidance run. The results are shown in Figure 3-12.

Figure 3-12 shows that the noise transmission appears to be approximately the same for both command and homing guidance for most of the flight. However, toward the end of the flight, it is obvious that there is a dramatic increase in the noise transmission of the command guidance system. This means that command guidance will generally have to contend with more noise on the LOS angle measurements than homing systems near the end of the missile flight. This excess noise may cause much larger miss distances in the case of command guidance systems.

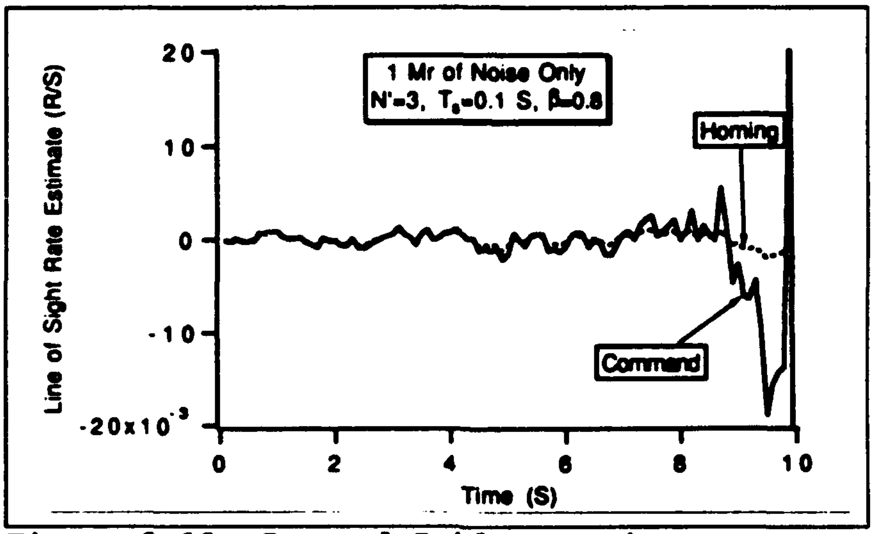


Figure 3-12. Command Guidance Noise Transmission [Zarchan [ref. 9]]

The reasons for implementing command guidance systems have more to do with cost, ease of implementation, and lack of susceptibility to countermeasures rather than performance benefits.

In this chapter, we will derive three widely used types of command guidance system: beam rider guidance, command to line-of-sight guidance (CLOS), and command proportional navigation guidance.

2. Beam Rider

The object of beam rider command guidance is to fly the missile along an electromagnetic beam (i.e. radar or laser) that is continuously pointed at the target. The beam rider uses a three-point guidance law, which means that the missile in flight is continuously located on a straight line between the beam generator and the target. Since the missile is attempting to fly along a moving beam, the missile commanded accelerations must be a function of the angular deviation of the missile from the center of the beam. If the beam is always on the target and the missile is always on the center of the beam, a successful target intercept will result. In beam rider guidance, as in pure pursuit guidance, the missile is initially fired directly at the target (i.e. along the missile-to-target LOS, which is along the beam), with no lead angle. The beam rider guidance principle was one of the first methods used in command tactical missile guidance because of its simplicity and ease of implementation.

a. Two-Dimensional Intercept Geometry and Kinematics

It can be seen from Figure 3-13 that the beam generator is located at the origin of the inertial coordinate system. The two-dimensional beam-rider intercept geometry will be defined in the inertial coordinate system shown in Figure 3-13. The 1-axis is the horizontal or crossrange and the 2-axis can either be the downrange (in the azimuth plane) or the altitude (in the vertical plane). In practice, the launching platform computer measures the important intercept variables for a beam-rider intercept scenario shown in Figure 3-13 (R_T, R_M, Θ_M, Θ_T). Using these intercept variables, the beam-rider guidance can then be derived according to Zarchan [ref. 9].

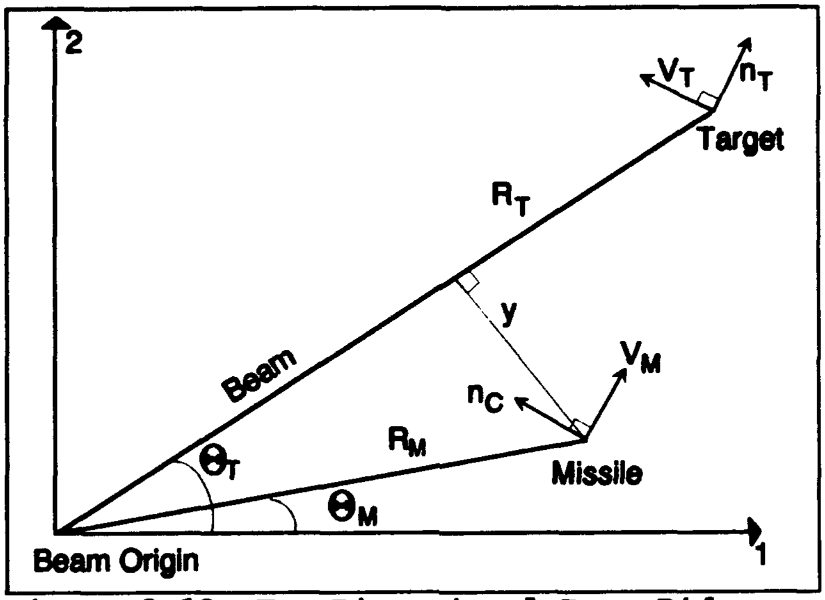


Figure 3-13. Two-Dimensional Beam Rider Intercept Geometry

From Figure 3-13, recognize that:

$$\theta_T = \arctan\frac{R_{T2}}{R_{T1}} \tag{34}$$

where R_{T1} and R_{T2} are the inertial components with respect to the 1 and 2 axis of the distance from the beam generator to the target R_{T1} and R_{T2} could also be expressed as:

$$R_{T1} = R_T \cos\Theta_T$$
$$R_{T2} = R_T \sin\Theta_T \ . \qquad (35)$$

In a similar fashion, one can express the components of the range from the beam generator to the missile (R_{M1} and R_{M2}) by first recognizing that

$$\Theta_M = \arctan\frac{R_{M2}}{R_{M1}} \qquad (36)$$

and then expressing the inertial components of the range from the beam generator to the missile as:

$$R_{M1} = R_M \cos\theta_M$$
$$R_{M2} = R_M \sin\theta_M \; . \qquad (37)$$

Equations 35 through 37 are used to determine the target and missile angles, as well as the horizontal (1 axis) and vertical (2 axis) components of the target and missile positions with respect to the energy source. The distance formula can then be used to obtain the target and missile ranges from the beam propagator as:

$$R_T = \sqrt{R_{T1}^2 + R_{T2}^2} \;\; ; \;\; R_M = \sqrt{R_{M1}^2 + R_{M2}^2} \; . \qquad (38)$$

Using geometry principles and the small angle assumption (from Figure 3-13), one can obtain a simple expression to determine the distance of the missile from the guiding beam denoted y and given by:

$$y = R_M \left(\theta_T - \theta_M\right) . \qquad (39)$$

If the missile is always on the beam, y = 0, then the missile will surely hit the target. Therefore, as in the proportional navigation homing guidance case, it is desired to minimize y, the distance of the missile from the beam, at the end of the flight. This means that one is trying to drive the miss distance to zero. The simplest possible implementation of a guidance law for a beam rider system is, therefore, to make the missile lateral acceleration commands (n_c) proportional to y. Mathematically, this translates to:

$$n_c = K\,y = K\,R_M\,(\theta_T - \theta_M) \tag{40}$$

where **K** is the beam rider guidance gain which value is typically selected to be around 10 [Zarchan [ref. 9]]. It was selected to be 10 for Missile Design PC TRAP applications.

One can see from equation (40) that the beam rider guidance command is proportional to the angular displacement off the guiding beam. The missile guidance command accelerations are generated perpendicular to the missile longitudinal axis as shown in Figure 3-13.

As detailed in Zarchan [ref. 9], beam riding guidance induces miss distances quite large in benign (no target maneuver) intercept scenarios where both homing and command proportional navigation guidance laws would yield zero miss distances. Furthermore, Zarchan [ref. 9] demonstrated that beam rider guidance requires a compensation network in the guiding loop in order to guide effectively on the target. Also, the beam rider performance, unlike that of proportional navigation, is very dependant on target speed and on the intercept geometry.

On the other hand, the beam rider guidance system is simple and can be easily implemented at relatively low cost when compared with more complex system. From a tactical point of view, beam rider guidance permits the launching of a large number of missiles into the same target control beam, since all the guidance equipment is located aboard the missile.

b. Three-Dimensional Intercept Geometry and Kinematics

The three-dimensional geometry for the beam rider guidance is shown at Figure 3-14. The three-dimensional beam rider guidance law,

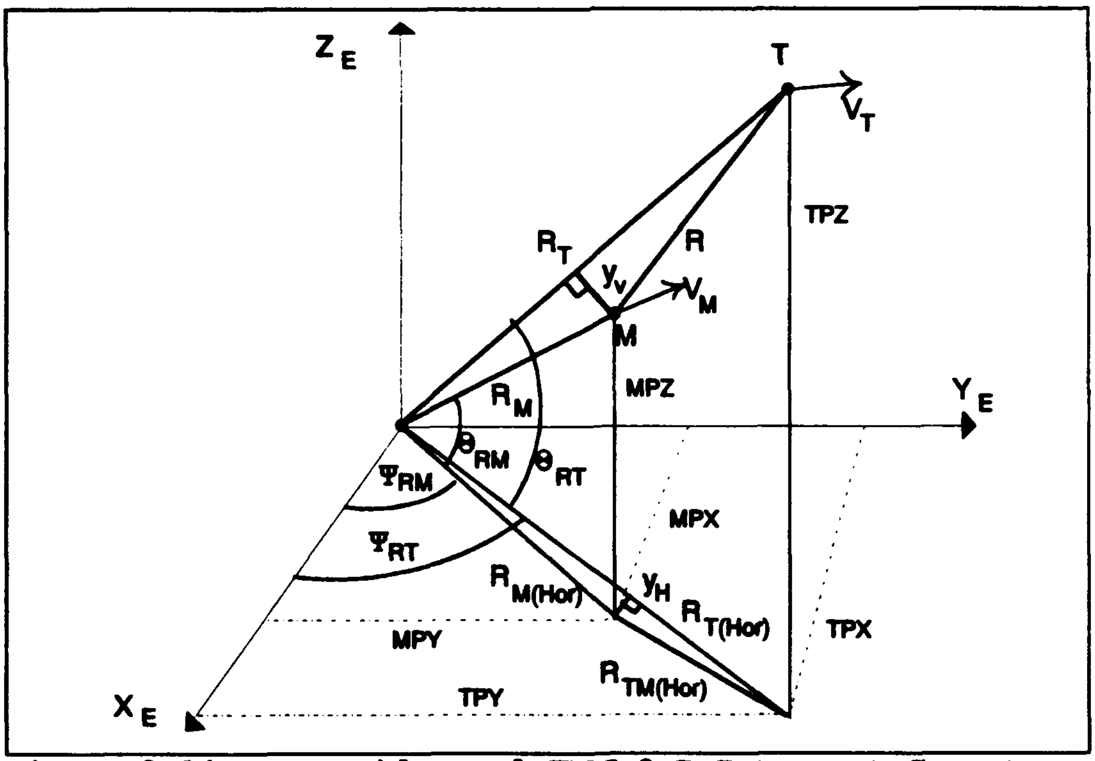


Figure 3-14. Beam Rider and CLOS 3-D Intercept Geometry

as implemented in Missile Design PC TRAP, was derived by the author as shown below.

In Figure 3-14, $\mathbf{R_M}$ and $\mathbf{R_T}$ are called the tracking lines to the missile in flight and to the target respectively. The tracking lines originate at the command guidance beam generator, which is located at the origin of the three-dimensional inertial coordinate system X_E-Y_E-Z_E. As usual, **R** is the missile-to-target LOS.

The missile position is at coordinates **MPX**, **MPY** and **MPZ**, while the target position is at coordinates **TPX**, **TPY** and **TPZ**, both with respect to the inertial coordinate system X_E-Y_E-Z_E. As usual, both the missile and target positions are known exactly for each simulation time step in Missile Design PC TRAP.

In a manner similar to the three-dimensional geometry for proportional navigation guidance missiles discussed above, the three dimensions can be defined into two distinctive planes: the horizontal or azimuth plane located on the X_E-Y_E plane, and the vertical or elevation plane, defined by the projection of the vehicle positions into the X_E-Y_E plane and the Z_E axis.

To simplify the Figure, the missile and target heading angles are not shown in Figure 3-14. However, these angles are defined above and shown in Figure 3-5 for the homing proportional navigation guidance law and remain the same for the beam-rider intercept geometry.

From Figure 3-14, one can now determine the missile and target geometry relative to the location of the beam generator. The azimuth angles of the missile (Ψ_{RM}) and of the target (Ψ_{RT}) tracking lines can be respectively found as follows:

$$\begin{aligned} \Psi_{RM} &= \arctan(\frac{MPY}{MPX}) \\ \Psi_{RT} &= \arctan(\frac{TPY}{TPX}) \ . \end{aligned} \tag{41}$$

To find similar expressions for the elevation angles of the missile (Θ_{RM})and of the target (Θ_{RT}) tracking lines, one need to determine the projections of the missile and target position onto the X_E-Y_E plane, $\mathbf{R}_{M(Hor)}$ and $\mathbf{R}_{T(Hor)}$ respectively:

$$\begin{aligned} R_{M(Hor)} &= \sqrt{MPX^2 + MPY^2} \\ R_{T(Hor)} &= \sqrt{TPX^2 + TPY^2} \ . \end{aligned} \tag{42}$$

Using equation (40), the following equations are used to obtain the elevation angles of the missile tracking line and of the target tracking line respectively:

$$\Theta_{RM} = \arctan\left(\frac{MPZ}{R_{M(Hor)}}\right)$$
$$\Theta_{RT} = \arctan\left(\frac{TPZ}{R_{T(Hor)}}\right) . \qquad (43)$$

Finally, the magnitude of the missile and target tracking lines, $\mathbf{R}_M$ and $\mathbf{R}_T$ respectively, are found as follows:

$$R_M = \sqrt{MPX^2 + MPY^2 + MPZ^2}$$
$$R_T = \sqrt{TPX^2 + TPY^2 + TPZ^2} . \qquad (45)$$

To implement the beam rider guidance law, we shall make the missile lateral acceleration command (n_C) is made proportional to the missile angular displacement off the target tracking beam (equation (40)). Therefore, the actual distance, in both the azimuth and elevation planes, of the missile to the target tracking line ($\mathbf{R}_T$) is required (Y_{Hor} and Y_{Ver}).

This distance in the vertical or elevation plane is denoted $\mathbf{y}_{Ver}$ in Figure 3-14 and is the length of the chord subtended by the arc "Θ_{RT}-Θ_{RM}" at the missile range from the beam generator ($\mathbf{R}_M$); thus for "Θ_{RT}-Θ_{RM}" in radians:

$$Y_{Ver} = R_M \left(\Theta_{RT} - \Theta_{RM}\right) . \tag{45}$$

To obtain the lateral distance in the azimuth plane of the missile from the target tracking line (Y_{Hor}), it is necessary to project the missile position into the horizontal plane as shown in Figure 3-14. This lateral distance is then the length of the chord subtended at $R_{M(Hor)}$ by the arc "Ψ_{RT}-Ψ_{RM}"; thus:

$$y_{Hor} = R_{M(Hor)} \left(\Psi_{RT} - \Psi_{RM}\right) . \tag{47}$$

As explained earlier during the proportional navigation homing guidance law discussion, the guidance system must generate a different missile lateral acceleration command for each of the two planes of a three-dimensional intercept. The two-dimensional beam rider guidance law is expressed at equation (40) and can be expanded for the three-dimensional case as follows:

$$\begin{aligned} n_{c(Hor)} &= K\, y_{(Hor)} = K\, R_{M(Hor)} \left(\Psi_{RT} - \Psi_{RM}\right) \\ n_{c(Ver)} &= K\, y_{(Ver)} = K\, R_M \left(\Theta_{RT} - \Theta_{RM}\right) . \end{aligned} \tag{48}$$

3. Command to Line-of-Sight (CLOS) Guidance

The CLOS guidance law is basically an improved version of the beam-rider guidance law. The CLOS guidance is obtained by adding a beam acceleration term to the beam-rider lateral commanded acceleration (n_c) expression (equation (40) for two-dimensions and equation (48) for three-dimensions). This addition of a beam acceleration term significantly improves the performance of the CLOS guidance law when compared with the performance of the beam-rider guidance law [Zarchan [ref. 9]].

In this section, beam acceleration terms will be developed, in both two and three dimensions, for use in Missile Design PC TRAP for implementation of the CLOS guidance law. The two-dimensional derivation of the beam acceleration term is based on Zarchan [ref. 9], while the three-dimensional derivation was done by the author.

a. Beam Acceleration - Two-Dimensional Intercept

For the development of the beam acceleration term (a_{TP}) in two dimensions, refer to the intercept geometry defined and shown in Figure 3-13. First consider the target tracking line $\mathbf{R}_T$, from the beam generator, located at the origin of the two-dimensional inertial coordinate system, to the target position. The angle Θ_T, located between $\mathbf{R}_T$ and the 1-axis in Figure 3-13, is computed using equation (34), repeated here for convenience:

$$\Theta_T = \arctan\left(\frac{R_{T2}}{R_{T1}}\right) . \tag{49}$$

Since the target tracking beam $\mathbf{R}_T$ is tracking the target at an instantaneous angle θ_T, the angular velocity and acceleration of the target tracking beam can be found by taking successive time derivatives of equations (34) and (48), yielding:

$$\dot{\theta}_T = \frac{R_{T1}\, V_{T2} - R_{T2}\, V_{T1}}{R_T^2}$$

$$\ddot{\Theta}_T = \frac{a_{T2} \cos\Theta_T - a_{T1} \sin\Theta_T - 2\, \dot{\Theta}_T\, \dot{R}_T}{R_T} \tag{50}$$

where $\mathbf{R}_T$, the range from the beam generator to the target, is defined at equation (38), and where the time derivative of $\mathbf{R}_T$ is:

$$\dot{R}_T = \frac{R_{T1}\, V_{T1} + R_{T2}\, V_{T2}}{R_T} . \tag{51}$$

Figure 3-15 shows the point-mass target acceleration geometry used to develop the beam acceleration term which we call a_{TP}. The beam acceleration term is perpendicular to the target beam R_T. As shown in Figure 3-15, the acceleration perpendicular to the beam (a_{TP}) can be expressed in terms of the inertial coordinates of target acceleration as:

$$a_{TP} = -a_{T1} \sin\Theta_T + a_{T2} \cos\Theta_T \qquad (52)$$

where a_{T1} and a_{T2} are the components of the target acceleration (n_T) with respect to the inertial coordinate system defined as:

$$\begin{aligned} a_{T1} &= n_T \cos(-\Psi_T + \frac{\Pi}{2}) \\ a_{T2} &= n_T \sin(-\Psi_T + \frac{\Pi}{2}) \, . \end{aligned} \qquad (53)$$

Equation (52) includes the first two terms of the numerator on the right-hand side of equation (50). Combining equations (50) and (52) and solving for a_{TP}, we can obtain an equivalent expression for the target tracking beam acceleration as:

$$a_{TP} = R_T \ddot{\theta}_t + 2 \dot{R}_t \dot{\theta}_t . \qquad (54)$$

In beam rider and CLOS guidance, one wants the missile to stay on the target tracking beam. Striving, therefore, to obtain:

$$\ddot{\Theta}_M = \ddot{\Theta}_T ; \qquad \dot{\Theta}_M = \dot{\Theta}_T . \qquad (55)$$

If these conditions are met (i.e., the missile stays on target tracking beam) then the missile acceleration perpendicular to the beam can be found from:

$$a_{MP} = R_M \ddot{\Theta}_M + 2 \dot{R}_M \dot{\Theta}_M , \qquad (56)$$

where the time derivative of $\mathbf{R_M}$ is given by:

$$\dot{R}_M = \frac{R_{M1}\ V_{M1} + R_{M1}\ V_{M1}}{R_M} \ . \qquad (57)$$

Substitution of equation (55) into equation (56) (assuming that $a_{TP} = a_{MP}$) yields the final two-dimensional expression for the missile tracking beam acceleration:

$$a_{MP} = R_M\ \ddot{\Theta}_T + 2\ \dot{R}_M\ \dot{\Theta}_T = a_{TP}\ , \qquad (58)$$

which is added to the beam-rider two-dimensional equation (40) to generate the two-dimensional CLOS missile lateral acceleration command as follows:

$$n_c = K\ R_M\ (\Theta_T - \Theta_M) + R_M\ \ddot{\Theta}_T + 2\ \dot{R}_M\ \dot{\Theta}_T\ . \qquad (59)$$

To summarize, adding the missile tracking beam acceleration term of equation (58) to the nominal missile lateral commanded acceleration term generated by the beam rider equation (40) yields the command to line-of-sight (CLOS) guidance law. The effect of the addition of this extra acceleration term on beam rider missile system requirements and performance is detailed in Zarchan [ref. 9].

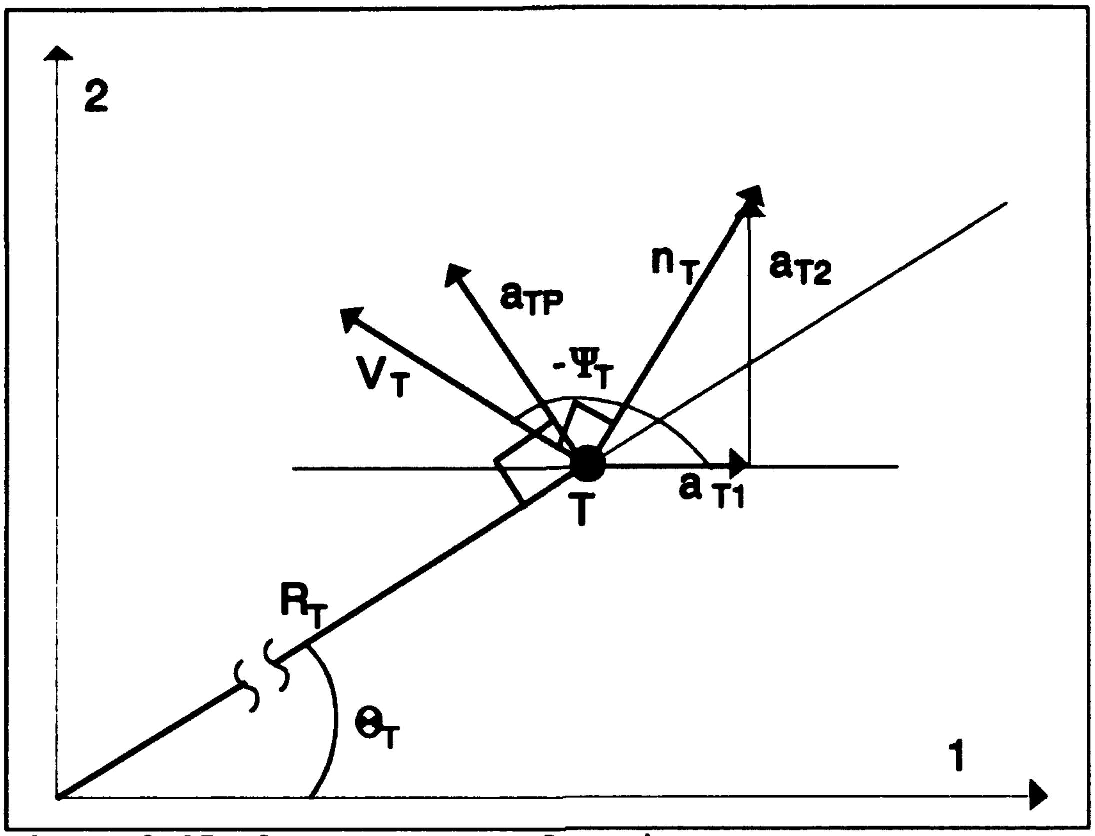


Figure 3-15. 2-D Target Acceleration

4. Beam Acceleration - Three-Dimensional Intercept

The CLOS three-dimensional intercept geometry is the same as for the beam rider one shown in Figure 3-14. To simplify the guidance law analysis, the three-dimensional kinematic equations for the CLOS guidance will not be derived in detail as this derivation was done in the preceding section for the two-dimensional CLOS guidance. The results of this preceding section will be used to develop the three-dimensional case by analogy.

It was previously seen that implementation of a three-dimensional guidance law into the Missile Design PC TRAP simulation model, two different expressions for the missile lateral acceleration commands were required: one command for each of the two missile guidance planes defined in Figure 3-7 above. By analogy with the last section, it is required to develop a missile tracking beam acceleration term (a_{TP}) for each of the two missile planes (horizontal and azimuth). The two-dimensional beam acceleration term developed above at equation (58) will be developed by the author for three-dimensional application to Missile Design PC TRAP, according to Zarchan [ref. 9].

a. Vertical (Elevation) Plane

Given that both the missile and target state variables are known in Missile Design PC TRAP, the missile tracking beam acceleration term in the missile vertical plane is the following:

$$a_{MP_{(Ver)}} = R_M \, \ddot{\Theta}_{RT} + 2 \, \dot{R}_M \, \dot{\Theta}_{RT} = a_{TP(Ver)} \, , \qquad (60)$$

where:

$$R_M = \sqrt{MPX^2 + MPY^2 + MPZ^2} \, ; \qquad (61)$$

$$\dot{R}_M = \frac{MPX\ V_{MX_E} + MPY\ V_{MY_E} + MPZ\ V_{MZ_E}}{R_M}\ ; \qquad (62)$$

$$\Theta_{RT} = \arctan\left(\frac{TPZ}{R_{T(Hor)}}\right)\ ; \qquad (63)$$

$$\dot{\Theta}_{RT} = \frac{R_{T(Hor)}\ V_{TZ_E} - TPZ\ V_{T(Hor)}}{R_T^2}\ ; \qquad (64)$$

$$\ddot{\Theta}_{RT} = \frac{a_{T2_{(Ver)}} \cos(\Theta_{RT}) - a_{T1_{(Ver)}} \sin(\Theta_{RT}) - 2\ \dot{\Theta}_{RT}\ \dot{R}_T}{R_T}\ ; \qquad (65)$$

$$R_{T(Hor)} = \sqrt{TPX^2 + TPY^2} \; ; \tag{66}$$

$$\begin{aligned} V_{T(Hor)} &= \dot{R}_{T(Hor)} = \frac{TPX\ V_{TX_E} + TPY\ V_{TY_E}}{R_{T(Hor)}} \ , \\ V_T &= \dot{R}_T = \frac{TPX\ V_{TX_E} + TPY\ V_{TY_E} + TPZ\ V_{TZ_E}}{R_T} \ . \end{aligned} \tag{67}$$

Also, by analogy from Figure 3-15:

$$\begin{aligned} a_{T1_{(Ver)}} &= -n_{T_{(Ver)}} \sin(\Theta_T) \\ a_{T2_{(Ver)}} &= n_{T_{(Ver)}} \cos(\Theta_T) \ , \end{aligned} \tag{68}$$

where Θ_T is the target heading angle in the vertical plane as shown in Figure 3-5, and $n_{T(Ver)}$ is the target total acceleration in the vertical plane.

Finally, the three-dimensional beam rider guidance law in the vertical plane is expressed as:

$$n_{C(Ver)} = K\, R_M\, (\Theta_{RT} - \Theta_{RM}) + R_M\, \ddot{\Theta}_{RT} + 2\, \dot{R}_M\, \dot{\Theta}_{RT}\ . \tag{69}$$

Equation (69) is the guidance law used in the Missile Design PC TRAP algorithm to model the CLOS guidance law in the vertical phase of the missile.

b. Horizontal (Azimuth) Plane

To determine the missile tracking beam acceleration term in the horizontal plane, consider the projection of both the target and missile positions into the horizontal plane. The acceleration term of the missile tracking beam in the horizontal plane is determined as follows:

$$a_{MP_{(Hor)}} = R_{M(Hor)}\, \ddot{\Psi}_{RT} + 2\, \dot{R}_{M(Hor)}\, \dot{\Psi}_{RT} = a_{TP(Hor)}\ , \tag{70}$$

where:

$$R_{M(Hor)} = \sqrt{MPX^2 + MPY^2}\ ; \tag{71}$$

$$\dot{R}_{M(Hor)} = \frac{MPX\ V_{MX_E} + MPY\ V_{MY_E}}{R_{M(Hor)}} ; \tag{72}$$

$$\psi_{RT} = \arctan\left(\frac{TPY}{TPX}\right) ; \tag{73}$$

$$\dot{\Psi}_{RT} = \frac{TPX\ V_{TY_E} - TPY\ V_{TX_E}}{R^2_{T(Hor)}} ; \tag{74}$$

$$\ddot{\Psi} = \frac{a_{T2_{(Hor)}} COS(\Psi_{RT}) - a_{T1_{(Hor)}} SIN(\Psi_{RT}) - 2\dot{\Psi}_{RT}\dot{R}_{T(Hor)}}{R_{T(Hor)}} \tag{75}$$

Also,

$$a_{T1_{(Hor)}} = -n_{T_{(Hor)}} \sin(\Psi_T)$$
$$a_{T2_{(Hor)}} = n_{T_{(Hor)}} \cos(\Psi_T) \,, \tag{76}$$

where Ψ_T is the target heading angle in the horizontal plane as shown in Figure 3-5, and $n_{T(Hor)}$ is the target total acceleration in the horizontal plane.

Finally, the three-dimensional beam rider guidance law in the vertical plane is expressed as:

$$n_{C(Hor)} = K\, R_{M(Hor)}\, (\Psi_{RT} - \Psi_{RM}) + a_{MP_{(Hor)}} \,. \tag{77}$$

5. Command Proportional Navigation Guidance

In Missile Design PC TRAP, command proportional navigation guidance systems are modeled with the use of a radar system as the external source and the external receiver of the electromagnetic radiation required to implement such a guidance system. It is assumed that the energy source and the receiver, which are collocated on the launching platform, track both the missile and the target.

Missile lateral acceleration commands for implementation of command proportional navigation guidance will be calculated using the expression for the proportional guidance law detailed equation (1) and repeated here for convenience (see equation (1) for definition of parameter):

$$n_c = N\, V_c \, \frac{d\lambda}{dt} \, . \tag{78}$$

a. *Two-Dimensional Geometry and Kinematics*

Figure 3-16 shows the basic two-dimensional geometry for a command proportional navigation intercept, which is similar to the geometry of the proportional navigation homing guidance developed above, except that the target is tracked from a non-moving radar system located outside of the missile. The radar system is located at the origin of the inertial coordinate system. For implementation of the proportional navigation command guidance model one needs to measure the angle and the range, with respect to the radar, of both the target and the missile (θ_T, R_T, θ_M, and R_M respectively). From Figure 3-16, missile measurements of $\mathbf{R_M}$ and θ_M and target measurements of $\mathbf{R_T}$ and θ_T are known.

In order to implement proportional navigation guidance principles (equation (78)) in the command guidance system of Figure 3-16, one needs expressions for the LOS angle rate of change ($d\lambda/dt$) and for the missile-to-target closing velocity (V_c), which were given at equations (9) and (11) respectively for homing proportional navigation guidance, and repeated

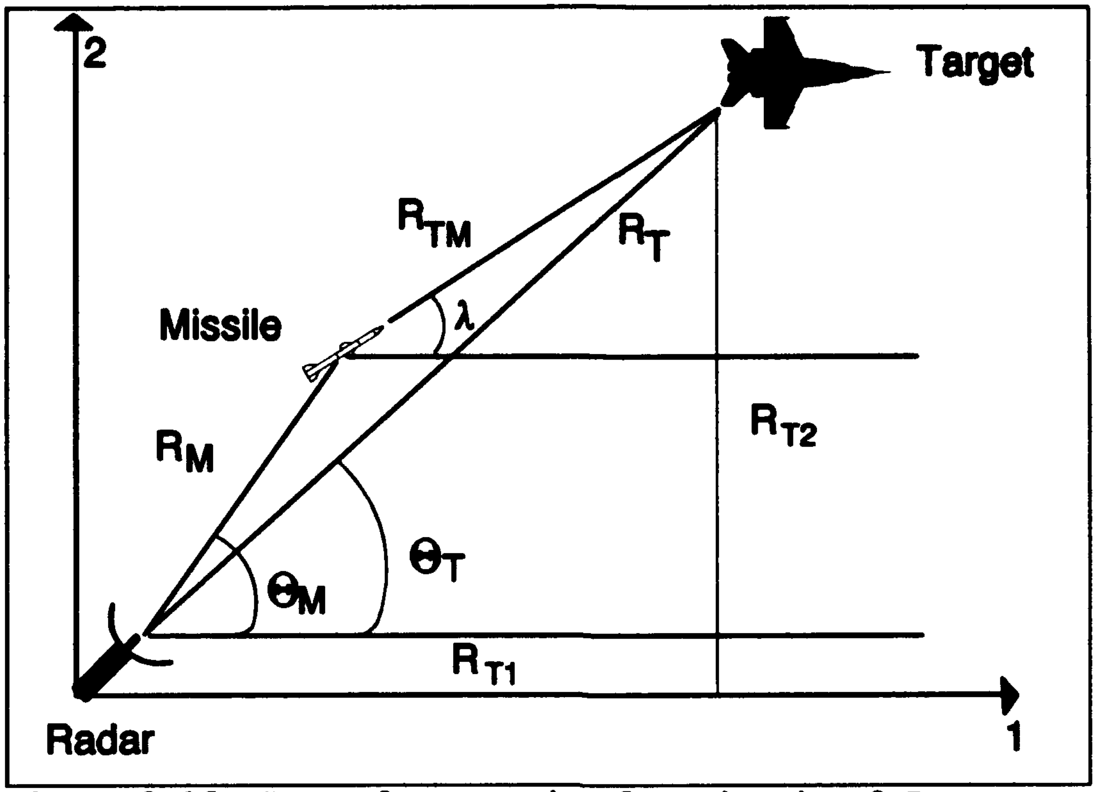


Figure 3-16. Command Proportional Navigation 2-D Intercept Geometry

here for convenience:

$$\frac{d\lambda}{dt} = \frac{R_{TM1}\, V_{TM2} - R_{TM2}\, V_{TM1}}{R_{TM}^2} \tag{79}$$

$$V_C = -\frac{(R_{TM1}\ V_{TM1} + R_{TM2}\ V_{TM2})}{R_{TM}} \ . \qquad (80)$$

However, due to the different intercept geometries between the homing and command proportional navigation, new expressions for the intercept variables included in equations (79) and (80) are required and derived below for application in the command proportional navigation guidance law.

From Figure3-16, recognize that:

$$\theta_T = \arctan\frac{R_{T2}}{R_{T1}} \qquad (81)$$

where one can express the inertial components with respect to the 1 and 2 axis of the distance from the radar to the target as:

$$\begin{aligned} R_{T1} &= R_T \cos\theta_T \\ R_{T2} &= R_T \sin\theta_T \ . \end{aligned} \qquad (82)$$

In a similar fashion, express the components of the range from the radar to the missile by first recognizing that:

$$\Theta_M = \arctan\frac{R_{M2}}{R_{M1}} \tag{83}$$

and then expressing the inertial components of the range from the radar to the missile as:

$$\begin{aligned} R_{M1} &= R_M \cos\Theta_M \\ R_{M2} &= R_M \sin\Theta_M \,. \end{aligned} \tag{84}$$

Recall from the derivation of the two-dimensional homing proportional navigation guidance law, that the relative missile-target separation and relative velocity components in the inertial coordinate system are respectively defined as:

$$\begin{aligned} R_{TM1} &= R_{T1} - R_{M1} \\ R_{TM2} &= R_{T2} - R_{M2} \,; \end{aligned} \tag{85}$$

$$V_{TM1} = V_{T1} - V_{M1}$$
$$V_{TM2} = V_{T2} - V_{M2} . \qquad (86)$$

Also recall that the instantaneous relative separation between the missile and the target is defined as:

$$R_{TM} = \sqrt{T_{TM1}^{2} + R_{TM2}^{2}} . \qquad (87)$$

The new expressions developed above in equations (85) through (87) defines the relative geometry of a two-dimensional command proportional navigation intercept. To implement this guidance law, equations (85) through (87) shall be substituted in equations (79) and (80) to obtain the complete expressions for the command proportional navigation guidance law as defined in equation (78).

6. Three-Dimensional Geometry and Kinematics

The three-dimensional intercept geometry for command proportional navigation is illustrated in Figure 3-17. The missile and target state variables are known exactly for each simulation time step. In Figure 3-17, the missile and target position are respectively V_{MX}, V_{MY}, V_{MZ} and V_{TX}, V_{TY}, V_{TZ}. As for the homing proportional navigation, the two guidance laws required to implement this guidance technique will be developed separately in their respective planes.

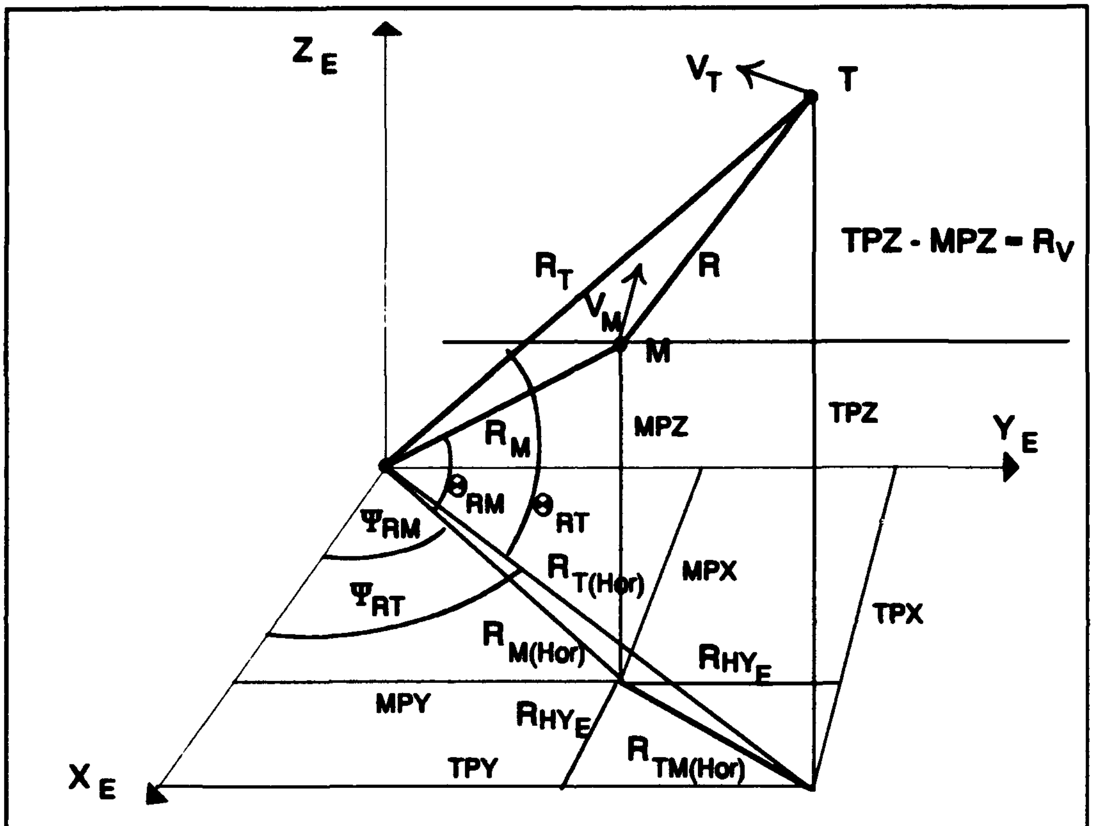


Figure 3-17. Command Proportional Navigation 3-D Intercept Geometry

a. *Horizontal Plane*

The missile lateral command for the horizontal plane is computed using the following guidance law:

$$n_{c(Hor)} = N\ V_{c(Hor)}\ \frac{d\lambda_{Az}}{dt} \tag{88}$$

where $d\lambda_{Az}/dt$, the rate of change of the horizontal LOS angle, can be found to be equal to:

$$\frac{d\lambda_{Az}}{dt} = \frac{R_{HX_E}\ V_{HY_E} - R_{HY_E}\ V_{HX_E}}{R_{TM(Hor)}{}^2}\ , \tag{89}$$

and where the relative closing velocity in the horizontal plane is expressed as follows:

$$V_{C(Hor)} = -\dot{R}_{TM(Hor)} = \frac{-(R_{HX_E}\ V_{HX_E} + R_{HY_E}\ V_{HY_E})}{R_{TM(Hor)}}\ . \tag{90}$$

Also,

$$
\begin{aligned}
R_{HX_E} &= TPX - MPX \\
R_{HY_E} &= TPY - MPY \\
V_{HX_E} &= V_{TX_E} - V_{MX_E} = VTX - VMX \\
V_{HY_E} &= V_{TY_E} - V_{MY_E} = VTY - VMY \\
R_{TM(Hor)} &= \sqrt{R_{HX_E}^2 + R_{HY_E}^2}\ .
\end{aligned}
\tag{91}
$$

The azimuth angles of the missile (Ψ_{RM}) and target (Ψ_{RT}) tracking lines can be found as per equation (41) above.

b. Vertical Plane

As for the homing proportional navigation case, the command proportional navigation guidance law to generate the missile lateral acceleration command in the vertical plane is expressed as follows:

$$
n_{c(Ver)} = N\ V_{c(Ver)}\ \frac{d\lambda_{El}}{dt} \tag{92}
$$

By analogy to the homing proportional navigation guidance law derivation, the LOS rate of change in the vertical plane can be found as follows:

$$\frac{d\lambda_{El}}{dt} = \frac{R_{TM(Hor)}\,\dot{R}_V - R_V\,\dot{R}_{TM(Hor)}}{R^2} \tag{93}$$

Similarly, the relative closing velocity is:

$$V_{C(Ver)} = -\dot{R} = \frac{R_{HX_E}\,V_{HX_E} + R_{HY_E}\,V_{HY_E} + R_V\,\dot{R}_V}{R}\,, \tag{94}$$

which is the total relative closing velocity between the missile and the target. The following expressions can be obtained from the geometry of the three-dimensional intercept:

$$\begin{aligned} R &= \sqrt{R^2_{TM(Hor)} + R^2_V} \\ R_V &= TPZ - MPZ \\ \dot{R}_V &= V_{TZ_E} - V_{MZ_E} = VTZ - VMZ\,. \end{aligned} \tag{95}$$

The elevation angles of the missile tracking line (Θ_{RM}) and of the target tracking line (Θ_{RT}) are defined in equation (43), while the magnitudes of the missile and target tracking lines, R_M and R_T respectively, are defined in equation (44).

In both homing and command proportional navigation guidance laws developed in this chapter, expressions for the relative velocity in the missile horizontal plane ($V_{c(Hor)}$) were derived and implemented in Missile Design PC TRAP. It shall be noted that the total relative velocity (V_c) could also be used instead of $V_{c(Hor)}$ for implementation of both homing and command proportional navigation guidance laws.

A summary table of the three guidance laws (three-dimensional) implemented in Missile Design PC TRAP is included in Table 3-2.

BEAM RIDER
$n_{C(Hor)} = K\ R_{M(Hor)}\ (\Psi_{RT} - \Psi_{RM})$ $n_{C(Ver)} = K\ R_M\ (\theta_{RT} - \theta_{RM})$
COMMAND TO LINE-OF-SIGHT
$n_{C(Hor)} = K\ R_{M(Hor)}\ (\Psi_{RT} - \Psi_{RM}) + R_{M(Hor)}\ d^2\Psi_{RT}/dt^2 + 2\ dR_{M(Hor)}/dt\ d\Psi_{RT}/dt$ $n_{C(Ver)} = K\ R_M\ (\theta_{RT} - \theta_{RM}) + R_M\ d^2\theta_{RT}/dt^2 + 2\ dR_M/dt\ d\theta_{RT}/dt$
COMMAND PROPORTIONAL NAVIGATION
$n_{C(Hor)} = N\ V_{C(Hor)}\ d\lambda_{Az}/dt$ $n_{C(Ver)} = N\ V_{C(Ver)}\ d\lambda_{El}/dt$

Table 2-2. Summary of Expressions for Command Missile Lateral Acceleration

IV. DESCRIPTION OF MISSILE DESIGN PC TRAP

A. INTRODUCTION

The Missile Design Personal Computer Trajectory Analysis Program (Missile Design PC TRAP) is a missile trajectory analysis program for missile preliminary design applications. It is built to assist missile designers in evaluating missile flight performances by modeling missile guidance, including the seeker, missile propulsion, missile aerodynamics and missile flight control functions. Missile Design PC TRAP is also recommended for trade-off studies, for academic purposes, such as study of the effects of target maneuvers on missile performance, and for military operational applications where real time graphic display is required and where no main frame computer is available. Missile Design PC TRAP provides point mass tactical missile simulations in three dimensions, and can be used from the early design phases to run missile flyouts to determine typical flight conditions and associated static stability and control derivatives, up to the late design phases to generate missile launch envelopes.

Missile Design PC TRAP can simulate air-to-air, surface-to-air and air-to-surface intercept scenarios. It provides the option of simulating the seven different guidance laws, given in Table 3-1 for homing guidance and in

Table 3-2 for command guidance. Two types of input data are required to run the program: a data file which contains 57 missile related data describing the missile physical, aerodynamic and propulsion characteristics, and user-friendly color-coded input menus interrogating the user on the specifics of the missile intercept scenarios. Missile Design PC TRAP can graphically portray (in color) a plotted history of the launching aircraft, the missile and the target as the simulation is being performed.

The Missile Design PC TRAP algorithms are an improved version of the PC TRAP (version 3.12) computer program developed by the Foreign Aerospace Science and Technology Center (FASTC) for the United States Air Force (USAF). The original PC TRAP is a condensed and abbreviated version of the main frame Trajectory Analysis Program (TRAP) used by the USAF to conduct complete and extensive missile simulations. As discussed earlier, TRAP is considered to be too detailed and too complex to be used as a tactical missile preliminary design tool. PC TRAP was developed by the FASTC to provide a missile simulation program that runs quickly on a PC for applications in simulators, or facilities with limited mainframe hardware capabilities (such as fleet and squadron), or for programs with real-time graphics requirements.

PC TRAP is an air-to-air missile simulation program that can simulate up to 46 real air-to-air tactical missiles from the USA, Russia, China and the Free-World. The program is therefore classified SECRET. However, the PC

TRAP version used by the author was modified by the FASTC to downgrade its security classification to UNCLASSIFIED by replacing classified missile data with data from a generic missile. The PC TRAP algorithms were used as a basis to develop the more general and versatile Missile Design PC TRAP computer program.

Missile Design PC TRAP was compiled with MICROSOFT FORTRAN Optimization Compiler Version 5.0 (1989). The program runs approximately real-time (one second of simulation for one second of flight) on 33 MHz 80486 IBM PC's with instantaneous graphic display of the simulated missile to target. The program runs much faster if the user chooses not to display the engagement graphically.

This chapter describes the Missile Design PC TRAP algorithms in great detail. The guidance loop of both homing and command guidance missiles modeled by Missile Design PC TRAP will be described, the different missile coordinate systems used during the trajectory simulations will be detailed, and a program overview will be provided. Then the input requirements to the program will be described, as well as the output options offered by Missile Design PC TRAP. The specifics of the engagement modeling will be discussed, including an extensive description of the main timing loop. Special program features, such as the launch envelop generation and the Monte Carlo simulation will be presented. Finally, the program structure will be described, including the FORTRAN source code files and the different subroutines forming Missile Design PC TRAP.

With the use of a FORTRAN compiler, Missile Design PC TRAP can easily be modified to meet user's specific simulation requirements. For this reason, the present chapter intends to be as descriptive of the program algorithms as possible.

Appendix A contains user's manual that supplies handy and quick direction to the program user. As a convention for the rest of this Chapter and Appendix A user's manual, all capital letter words written in bold font represent variables from the program source code files.

B. TACTICAL MISSILE GUIDANCE LOOPS

1. Introduction

The previous chapters introduced the general concept of tactical missile guidance and control and derived the tactical missile guidance laws modeled by Missile Design PC TRAP. The different missile systems (sections) will now be combined to illustrate how Missile Design PC TRAP models tactical missile engagements. It is convenient to represent a missile engagement model in a simplified block diagram form, shown in Figures 4-2, 4-4, and 4-5. This type of block diagram is known as a guidance loop because it is drawn as a feedback control system. Since Missile Design PC TRAP models two different types of guidance system, homing and command guidance systems, two families of guidance loops are required to represent its missile linear engagement models.

2. Homing Guidance Systems

a. *Missile-to-Target Intercept Geometry*

The important parameters common to every homing missile-target intercept scenario are shown in Figure 4-1.

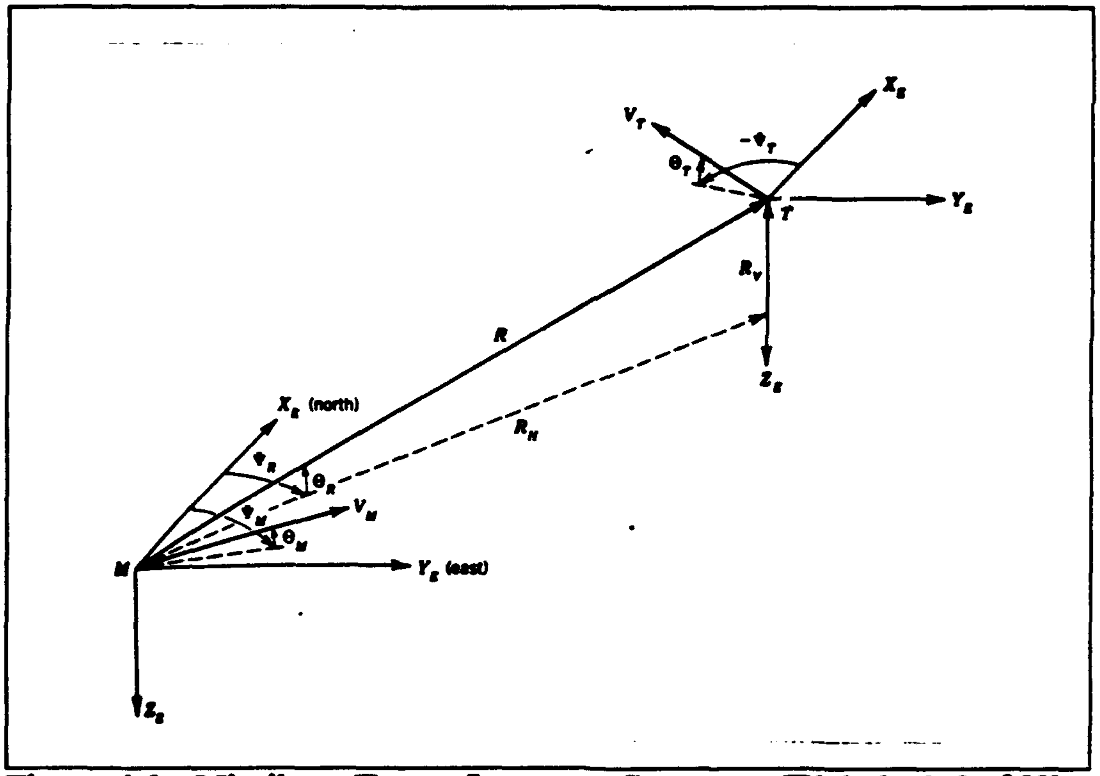


Figure 4-1. Missile-to-Target Intercept Geometry [Blakelock [ref.2]]

A moving earth reference axis system (X_E, Y_E, Z_E), the same as the fixed axis system used in Chapter III, Figure 3-5 to develop the four homing guidance laws in three dimensions, is located at the missile point mass representations M and target T. The headings of the missile Ψ_M (MPSI) and the target Ψ_T (TPSI) are measured from the X_E axis. As well, the horizontal LOS azimuth angle Ψ_R (LOSAZ) is measured from the X_E axis. The elevation angles of the missile, the target and the LOS are Θ_M (MTHETA), Θ_T (TTHETA) and Θ_R (LOSEL) respectively. In Missile Design PC TRAP, the initial target heading angle Ψ_T (TPSI) is always set to zero, meaning that the target is initially flying from south to north in the direction of to the X_E axis of the fixed coordinate system. To simplify the figure, the missile angle-of-attack (α) and (β) sideslip angles have been taken as zero.

b. Guidance Loop

The guidance loop for the homing guidance systems modeled by Missile Design PC TRAP is shown in a simplified block diagram in Figure 4-2.

Θ_{RT} is defined as the LOS direction due to target motion, while Θ_{RM} is defined as the rotation angle of the LOS due to the missile achieved lateral acceleration (n_L). In the diagram shown in Figure 4-2, Θ_{RT} acts as the reference angle from which is subtracted the LOS rotation angle due to the missile lateral movements to obtain the resulting missile-to-target LOS angle, λ. This guidance loop applies for both symmetric planes of the tactical missile. Consequently, as shown in Figure 4-1, Θ_R is the resulting LOS angle in the missile vertical plane, and Ψ_R is the resulting LOS angle in the horizontal plane.

The resultant LOS angle (λ in the general guidance loop of Figure 4-2) is measured by the onboard seeker, as it attempts to track the target. Effectively, the seeker takes the derivative of the geometric LOS

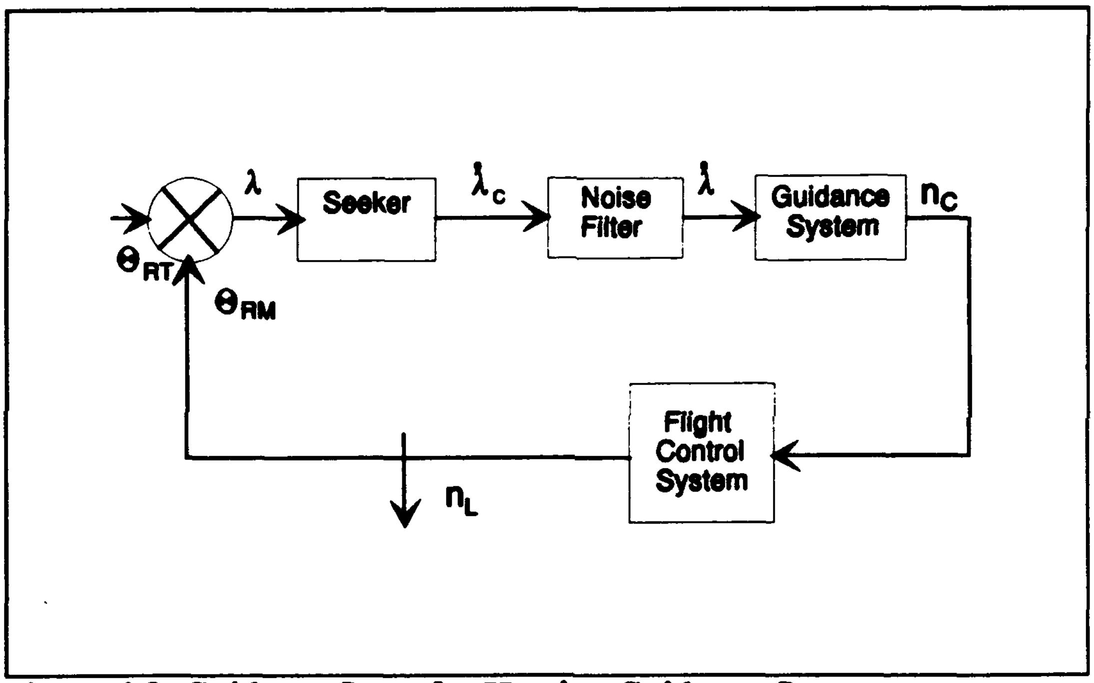


Figure 4-2. Guidance Loop for Homing Guidance Systems

angle, thus providing a measurement of the LOS rate ($d\lambda/dt$). The digital noise filter must process the noisy LOS rate measurement of the seeker and provide an estimate of the LOS rate. The output of the noise filter is the input to the guidance computer, which generates the missile lateral acceleration command (n_c) based on the pre-selected guidance law.

In tactical aerodynamic missiles, the flight control system must, by moving control surfaces, cause the missile to maneuver in such a way that the achieved lateral acceleration (n_L) matches the desired command acceleration (n_c). The missile achieved acceleration divided by the missile total velocity vector (V_M) is equal to the missile pitch or yaw rate, which, integrated yields the missile pitch or yaw angle. For the Missile Design PC TRAP three dimensional model, each missile plane has an achieved acceleration term, which, when integrated, provides the missile angles for each plane, Θ_M and Ψ_M as illustrated in Figure 4-1.

In Missile Design PC TRAP, models of the seeker and of the flight control system (autopilot) are considered to be perfect and without dynamics (perfect gain of one). As discussed later, the noise filter is modeled as a single lag system.

Generally, guidance system lags or subsystem dynamics (such as the flight control actuators) will increase the miss distance. The miss distance should always be zero in a perfect zero-lag guidance loop. As long as the lags can be represented by either linear differential or difference equations, the guidance loop will remain linear. This means that the individual miss distance caused by each of the guidance loop system lags or subsystem dynamics can be linearly added together to obtain the total engagement miss distance.

Inherent limitations of some missile system components or of the missile airframe can make the guidance loop non-linear and increase the miss distance in a non-linear fashion. For example, in Missile Design PC TRAP, the user must define the maximum lateral acceleration capability of the missile (MAXGCG), as well as the missile maximum angle-of-attack capability (MAXALP) of the missile airframe. Also, limitations on the seeker field-of-view (FOV) and on the seeker platform gimbal angles are required in the program input data file. Achievement of any of these inherent missile limitations during the engagement simulation will render the guidance loop non-linear. Once any of these limits are reached during a given simulation, the value of the variable saturates until the variable value returns below the given limit. If such limits are achieved near the end of the intercept, they will cause additional miss distance in a non-linear fashion.

In the case where the augmented proportional navigation (APN) is used as the guidance law in the guidance computer of the guidance loop illustrated at Figure 4-2, the additional target maneuver term, required by the guidance law (equation (33)), appears as a feedforward term in the guidance loop. This feedforward term provides extra information to the guidance loop, namely, knowledge of the target maneuver.

3. Command Guidance - Guidance Loop

a. *Missile-to-Target Intercept Geometry*

The guidance loops that are discussed here apply to the beam rider and CLOS guidance systems. The guidance loop for the command proportional navigation guidance law is as described in Figure 4-2, for homing guidance. To help visualizing the beam rider and CLOS intercepts, we refer to Figure 3-13, which is repeated here in Figure 4-3 for convenience. Figure 4-3 illustrates the two-dimensional beam rider and CLOS intercept geometry.

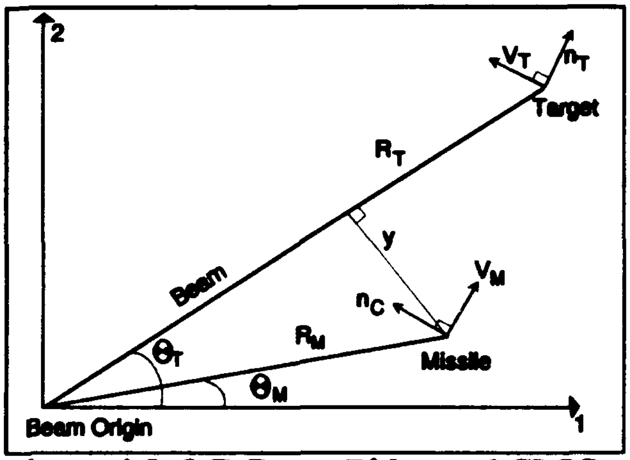


Figure 4-3. 2-D Beam Rider and CLOS Intercept Geometry

Recall that the two two-dimensional guidance laws for the beam rider and the CLOS guidance system, respectively:

$$n_c = K\, R_M\, (\Theta_T - \Theta_M)\ , \tag{96}$$

and,

$$n_c = K R_M (\theta_T - \theta_M) + R_M \ddot{\theta}_T + 2 R_M \dot{\theta}_T . \quad (97)$$

b. Beam Rider Guidance Loop

The linearized beam rider guidance loop is shown in a simplified block diagram in Figure 4-4. The Laplace transform notation

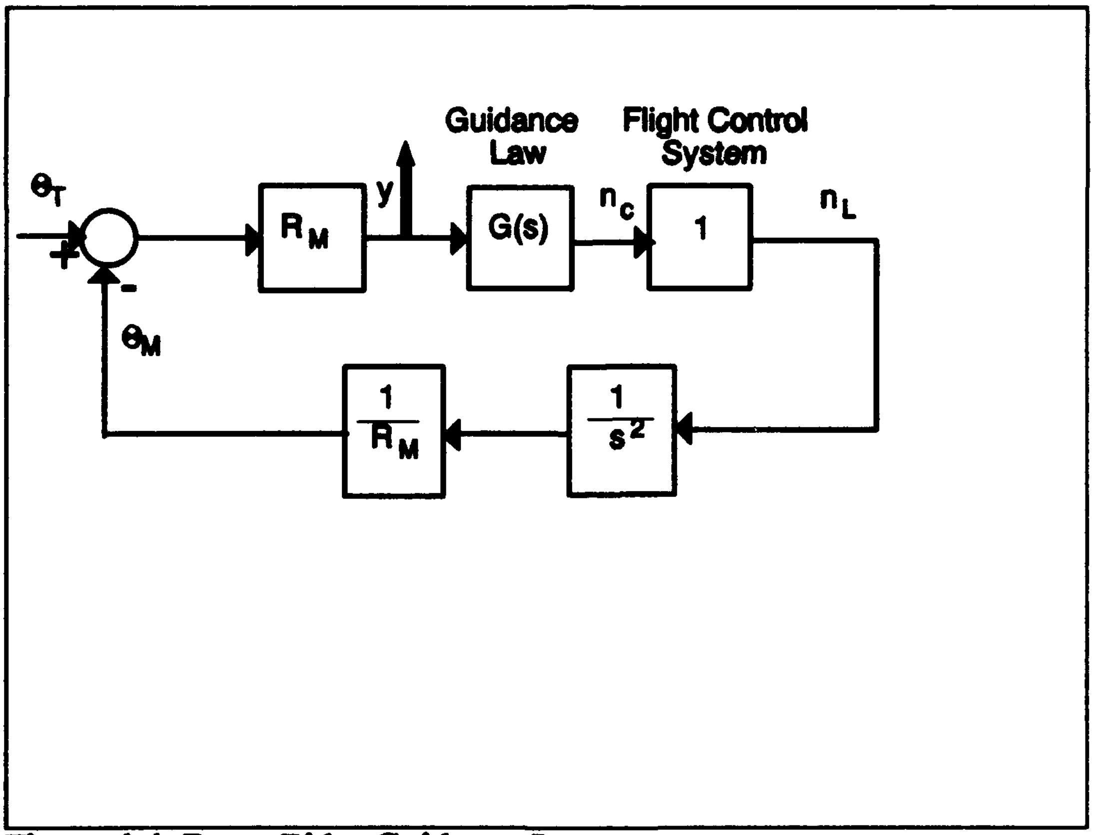


Figure 4-4. Beam Rider Guidance Loop

represent integration (s) and differentiation (1/s) of the variables in Figure 4-4.

From Figure 4-4, the reference signal to the guidance loop is the angle of the tracking line (beam) of the target, Θ_T, as illustrated in Figure 4-3. From this reference signal is subtracted the value of the missile tracking beam angle (Θ_M). As detailed in Chapter III and illustrated in Figure 4-3, the distance of the missile from the target beam, denoted **y**, is found as follows:

$$y = R_M (\Theta_T - \Theta_M) \ . \tag{98}$$

Hence, this distance can be obtained in the guidance loop as shown in Figure 4-4. This distance becomes the input to the guidance computer which generates a lateral command acceleration (n_c) proportional to the angular displacement off the target tracking beam, as defined in equation (96). As with the homing guidance system, the command acceleration is passed to the flight control system which attempts to deliver this required acceleration. The achieved acceleration (n_L) is then integrated twice and divided by R_M to give the feedback term Θ_M.

In Missile Design PC TRAP, the flight control system is modeled as being perfect and without dynamics. This means that the achieved lateral acceleration (n_L) will always equal the commanded lateral acceleration (n_c) unless the acceleration saturation limit is reached by the system, which will introduce lag mechanisms into the guidance loop.

c. *CLOS guidance Loop*

As discussed in Chapter III, adding the beam acceleration to the nominal acceleration generated by the beam rider equation (96) yielded the CLOS guidance law defined at equation (97). A CLOS guidance loop is shown in a simplified block diagram in Figure 4-5.

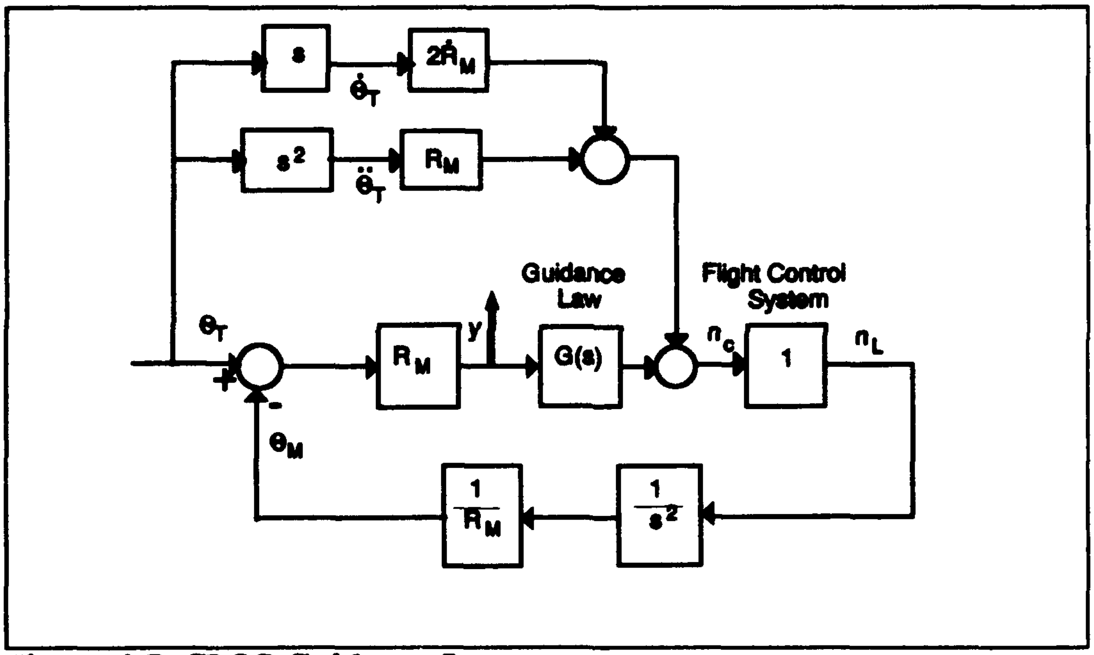


Figure 4-5. CLOS Guidance Loop

From Figure 4-5, it can be seen that the beam rider guidance loop remains unchanged and an extra feedforward path, representing the acceleration of the beam, has been added to the beam rider loop. Hence, the beam rider acceleration command (n_c) has then been modified to include the extra term shown in equation (97).

C. COORDINATE SYSTEMS

As shown in Figure 4-6, three different set of coordinates systems are alternatively used by the Missile Design PC TRAP to continuously compute the intercept geometry:

1) the stability or fixed coordinate system is the Earth (inertial) coordinate system denoted by X_S, Y_S, Z_S axis. This system originates at the surface of the Earth. It is assumed that it is a flat non-rotating Earth and that the atmosphere is at rest relative to the Earth. The flat-earth assumption can be made since the relative range of tactical

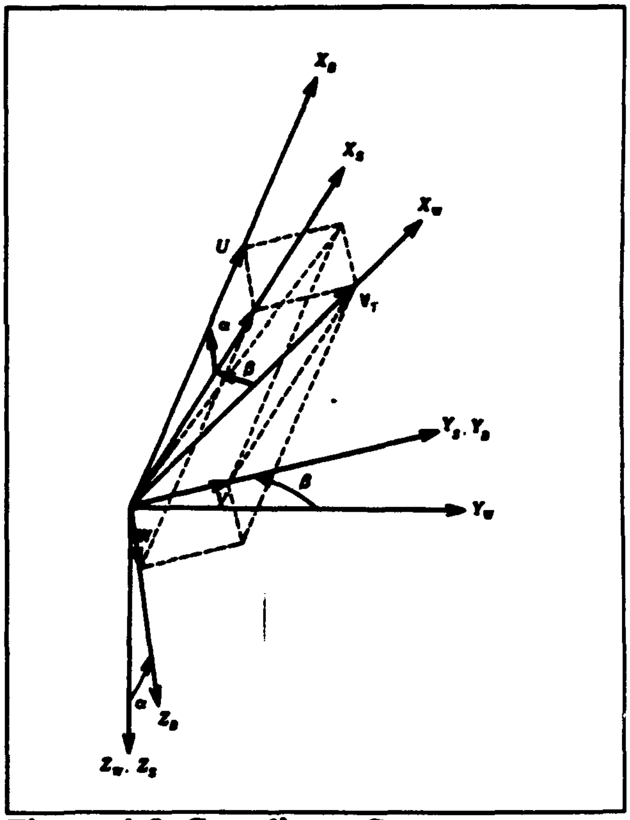


Figure 4-6. Coordinate Systems [Blakelock [ref.2]]

missiles are short enough in order not to be affected by a non-flat earth. This system is the primary reference axis system used by the program, and is primarily used to express the relative positions and velocities of the vehicles involved in the intercept;

2) the body-axis coordinate system (X_B, Y_B, Z_B) is obtained by using Euler angle transformations on the fixed coordinate system about the X_S and Z_S axes by the vehicle heading angle (Ψ) and the vehicle elevation angle (Θ) respectively. Note that since aerodynamic tactical missiles are roll stabilized, there is no rotation about the Y_S axis ($\Phi = 0$). The body-axis coordinate system is mainly used for the seeker intercept geometry resolution and to calculate the forces acting on the simulated vehicles.

3) the use of wind axes (or flight path angle axes) for the solution of the translational equations of motion rather then body axes makes lower demand on computer accuracy and time Blakelock [ref.2]. By definition, the wind axes (X_W, Y_W, Z_W) are oriented so that the X wind axis (X_W) lies along the total velocity vector V_T of the vehicle. The wind axes are then oriented with respect to the angle of attack α and the sideslip angle β as shown in Figure 4-6. The wind axes accelerations are used to calculate the equations of motion. From Figure 4-6, the body axes components of the total velocity vector (V_T) are:

$$U = V_T \cos\beta \cos\alpha$$
$$V = V_T \sin\beta \qquad (99)$$
$$W = V_T \cos\beta \sin\alpha \ .$$

D. PROGRAM OVERVIEW

This section provides the reader with a brief overview of the Missile Design PC TRAP algorithms. This overview discusses the basic steps involved in the computation of a single missile flight path as it intercepts a moving target. Each of the basic steps are discussed in greater detail in the sections below.

1. Input

Missile Design PC TRAP requires two types of data input:

1) a missile design data input file describing the propulsion, aerodynamic, guidance and physical characteristics of the missile to be modeled. The name of this data file is a program input, providing flexibility in the type and amounts of missiles that may be modeled without having to re-compile the program each time; and

2) initial missile-target intercept scenario set-up input that is interactive with the user via parameter input menus.

2. Initialization

The program computes the missile-target initial intercept geometry using the initial intercept conditions provided by the user as input data. At this point, the launching aircraft (for air-to-air and air-to-surface encounters), the missile and the target relative positions and velocities are established inside the program. This task is accomplished in the initialization part of the algorithms, just before the main timing loop.

3. Main Timing Loop

After completion of the initialization calculations, the program enters its main timing loop where it remains until termination of the intercept flight path simulation. A block diagram showing the most important computation steps included in the main timing loop is shown in Figure 4-7.

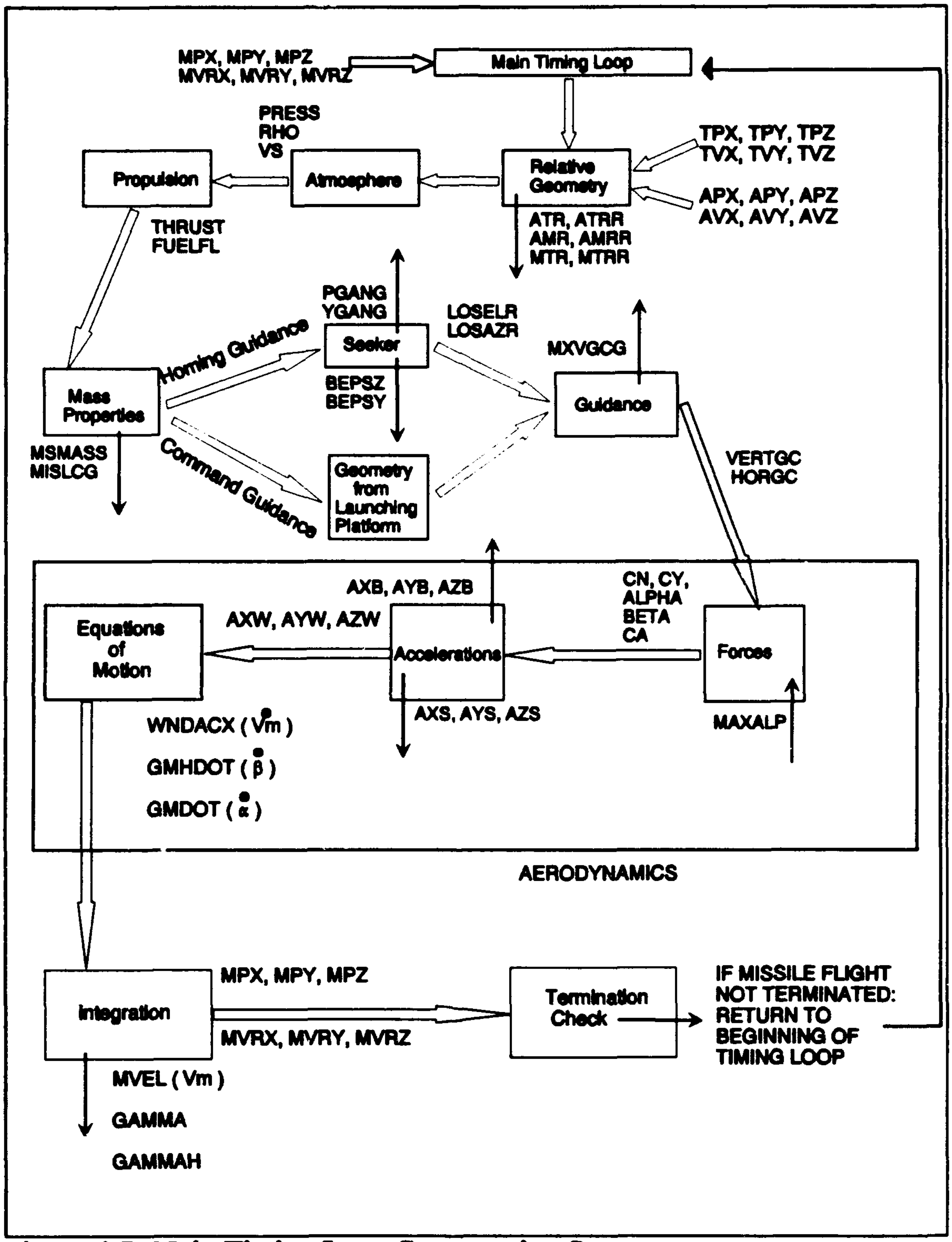


Figure 4-7. Main Timing Loop Computation Steps

The state variables that Missile Design PC TRAP solve are the missile position, velocity, and acceleration. Once the state variables are known exactly, the program can simulate the missile trajectory. The state variables for both the target and launching aircraft are the position, velocity, and acceleration of these vehicles. Target and launching aircraft simulation models are simplistic, since the drag is not computed for these vehicles by the Missile Design PC TRAP and TRAP. This allows simpler target and launching aircraft flight path generators for PC application. The first calculation in the main timing loop is involved with the intercept geometry. Missile Design PC TRAP computes relative geometry for all vehicles in the fixed coordinate system. Relative ranges and range rates between all the simulated vehicles are computed as X_S, Y_S, Z_S components as well as in magnitude. The air density (RHO), local speed of sound (VS) and static pressure (PRESS) at missile altitude are also computed.

The program then determines the vacuum thrust delivered by the solid rocket engine at the given time of flight (TIME), based on linear interpolation of the engine thrust versus time data input. The delivered thrust is computed by adjusting the current vacuum thrust to altitude. The fuel flow rate is also calculated by dividing the vacuum thrust value by the appropriate specific impulse value.

If the modeled missile has an onboard seeker, the missile seeker model then determines the rate of change of the missile-to-target LOS angles. The seeker LOS rate outputs are fed to the missile guidance section, which computes the required missile lateral acceleration commands (HORGC and VERTGC) for the missile based upon the type of homing guidance selected. For missiles without an onboard seeker, program calculations to determine the geometry variables required to compute the missile lateral acceleration command are performed according to the theory presented in Chapter III for command guidance systems.

Using the lateral acceleration commands just computed, the program determines the resultant missile angle of attack (ALPHA) and sideslip angle (BETA). Based on current missile MACH number, the program linearly interpolates through the drag coefficient data to find the correct drag coefficient value. The resultant drag coefficient is corrected for altitude and for base drag if the rocket motor is off.

From Newton's second law of motion (F=ma), a linear three-degrees of freedom (3DOF) set off is used equation of motions to compute the missile accelerations in X_B, Y_B, Z_B. This is accomplished in the missile body axes, then rotated through the angle of attack (ALPHA) to the fixed axes, then rotated through the sideslip angle (BETA) to the wind axes. It is the wind axis longitudinal (X_W) acceleration (WNDACX) that will be numerically integrated to get the updated missile velocity. Missile angle of attack and sideslip angle rates are then computed, as well as vehicle heading and flight path angles.

At this point in the missile simulation main timing loop, the program conducts a series of check to determine if any of the missile flight termination conditions are met. If none are met, the missile state variables are integrated.

The missile velocity vector (MVEL) is obtained by integration of the longitudinal wind axes acceleration (WNDACX), and the missile flight path angles are obtained by integration of the angle of attack and sideslip angle rates. Missile velocity in the body axes are then computed and rotated through the attitude angles into the fixed coordinate system. The resultant fixed coordinate axis velocities are then integrated to get the updated missile position. The time of flight (TIME) is then updated and the iteration loop begins again.

4. Output

Besides the real-time graphic display of the vehicle trajectories, Missile Design PC TRAP can produce several output files that summarize the engagement. All the output files are updated at the print rate selected by the user. Three of these output files provide the X_S, Y_S, Z_S coordinates of the launching aircraft, missile and target respectively for plot generation. Another output file contains many pertinent simulation parameters for detailed post-simulation flight analysis. Finally, the remaining output files contain limited data that could be pertinent to the missile designer. It is important to note that the output file data or formats can be changed by the user to meet her/his requirements.

E. PROGRAM INPUT

Missile Design PC TRAP is a missile trajectory analysis computer program conceived to assist missile design engineers. As such, the program is versatile, easily accessible with minimum re-compilation (FORTRAN) needs, fast, and accurate with simple intercept scenario set up. Also, the different missiles to be simulated shall be easily interchangeable.

The Missile Design PC TRAP input procedure intends to meet all of the above requirements. The input procedure is divided into two main parts: a missile data input file and an initial intercept scenario input process via parameter input menus.

1. Missile Data Input File

The TRAP and Missile Design PC TRAP data input file is a file built by the user containing specific missile related data required by the program to model a given missile. There are 57 items that need to be input into the program for successful missile modelling. The missile data input file contains specific missile data describing the propulsion, aerodynamic, guidance and physical characteristics of the missile. A dictionary listing and describing each of the required 57 items is included at Appendix B. Appendix B also contains an example data input file which format must be followed by the user to ensure that the missile data input file is successfully read and used by the program. There is also a sample missile design study case described in Chapter V. This sample case provides an example on how to build the missile data input file from the propulsion, aerodynamic and physical properties of the missile.

In PC TRAP, the missile data input files are included in the program algorithms. Any change to these input files requires recompilation of the PC TRAP source code files. This feature was modified in Missile Design PC TRAP where the name of the missile data input file is the user's choice as Missile Design PC TRAP prompts the user for the name of the missile data input file. This improved input process included in Missile Design PC TRAP provides more flexibility for the user.

Following is a general description of some of the major missile data items required by both the PC TRAP and Missle Design PC TRAP program along with suggested sources of information for these data. Data items that are not discussed below are considered to be self-explanatory as presented in the Appendix B data item dictionary.

a. *Description of Important Missile Input Data Items*

The aerodynamic tactical missiles modeled by TRAP and Missile Design PC TRAP are assumed to be symmetrical in the pitch and yaw planes, which means that aerodynamic input data applicable to one plane is automatically applicable to the other plane. **(NOTE: Each missile data item included in the input file shall be in SI units.)**

(1) Integration Time Step

DT is the integration time step. The value of the integration time step directly affects the performance of the program computations. Small values of integration time step increase the accuracy of the internal computations, and consequently provide more accurate calculations of the miss distances. However, small integration time steps increase the computation time significantly. Also, very large values of integration time step can lead to numerical integration instabilities. For these reasons, a value of 0.01 second is recommended for **DAT**. Such an integration step size offers a good compromise between computation accuracy and computation time for the boost and midcourse phases of the missile flight. With such an integration step size, the program can run approximately real time when combined with a print rate value of 0.1 second on a 80486 IBM PC. Naturally, the program runs faster with larger print rate values.

However, it was found that a 0.01 second integration step size was too large to accurately capture the miss distances during the end-game phase of missile guidance. For this reason, Missile Design PC TRAP offers the option of changing the integration step size to a smaller value (0.002 second) when the missile gets within 150 meters of the point mass target. With this change in the integration step size for the end-game phase, the miss distances can be captured to within one meter. If the integration step size is not changed for the end-game phase, the miss distances can be captured within the missile warhead lethal radius (**MDPERM**). Both TRAP and PC TRAP missile simulations models do not offer the option of changing the integration of step size during the missile terminal phase. Both TRAP and PC TRAP programs stop the simulation as soon as the missile passes within its warhead lethal radius from the target.

(2) Maximum Angle of Attack

MAXALP is the missile overall maximum angle of attack (AOA) capability. To obtain this value, a table of maximum AOA versus missile Mach number must be generated and input by the user. **MAXALP** is the greatest of the AOA values from this table. This table can be generated using the MISDATCOM computer program, which is a USAF missile stability and control computer program that provides the subsonic, transonic and supersonic missile aerodynamic coefficients and stability derivatives. MISDATCOM is available at the computer laboratory of the Department of Aeronautics and Astronautics.

(3) Coefficient of Axial Force (C_A)

The program requires power-off (motor not burning), zero lift drag values at the following Mach numbers: 0.6, 0.8 ,0.9, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.0, 4.0, 5.0 (at a Mach number of greater than 5.0), as well as a peak value and the Mach number where that occurs, and a final value of C_A and the Mach number where that occurs. If the missile original drag did not have these values, they must be extrapolated by the missile designers. These values can be obtained by running the MISDATCOM computer program at the required Mach number with zero degree of angle of attack. An example is provided in Chapter V.

(4) Lead Angle Computation

LDVFAC, **LDZFAC** and **AVGDLV** values are related to the computation of an optimum lead angle. The significance of these values is discussed below when the computation of the lead angle is detailed. If the user does not wish to use the lead angle option, the value of **AVGDLV** must then be set to zero in the program input data file. Details on how to compute the initial lead angle and on how to input the required lead angle computation values are provided below.

(5) Thrust Profile

The missile must have a thrust profile capable of being expressed with only five non-zero vacuum thrust values. Missile Design PC TRAP can model solid-rocket motors, with boost and/or boost-sustain phase. The procedure to input a thrust curve is as follows:

1) Compute the actual total impulse of the motor. Note this value.

2) Set **TIGN** and **TBO**, the starting and stopping times after launch before and after which the motor is power-off.

3) Looking at the vacuum thrust versus time curve, pinpoint five (5) non-zero points that capture as many of the highs and lows as possible. The program will linearly interpolate from zero vacuum thrust value at **TIGN**, to **VTHR1** (first vacuum thrust value) at **TTHR1** (time for first value) , to **VTHR2** to **TTHR2**, to **VTHR3** at **TTHR3**, to **VTHR4** at **TTHR4** to **VTHR5** at **TTHR5**, to zero at **TBO** (motor burn out time). By adding up the areas of the resulting triangles and trapezoids, make sure the total impulse of the estimated thrust curve equals that of the real curve.

Once again, an example demonstrating this technique is included in the sample missile design case at Chapter V.

b. Summary

The number of missile data input items that are required by TRAP and Missile Design PC TRAP to model the aerodynamic performance of a missile is small, but still large enough to provide realistic and accurate missile simulation for the preliminary design phase. Results of comparisons between the missile flight parameters estimated by Missile Design PC TRAP and the ones obtained from TRAP for similar intercept scenarios are presented in Chapter V.

The 57 missile data input items can be easily and quickly assembled into one single file. In cases where it is felt by the user that more input data are required, quick and easy modifications to the Missile Design PC TRAP algorithms can be made to accommodate those changes. This is one of the reasons why the program main loop algorithms steps are discussed in great detail below.

2. Initial Intercept Scenario Set-Up

The launching aircraft-missile-target initial intercept set-up is input via color-coded input panels or menus. When running Missile Design PC TRAP, an introductory screen will appear leading to several subsequent interactive input panels interrogating the user on the initial set-up of the intercept scenario. Since some data input requirements may differ from one type of mission to another, the sequence of input panels is different depending on the mission type (air-to-air, air-to-surface, and surface-to-air) that is to be simulated. This interactive data entry process is detailed in the Missile Design PC TRAP User's Manual included in Appendix A. Only a general description of the interactive data entry process is presented in this chapter. Note that a similar data entry process is used in PC TRAP. However, the PC TRAP data entry process is simpler due to the smaller amount of missile simulation options offered by PC TRAP when compared to Missile Design PC TRAP.

a. *Data Entry Process*

As in PC TRAP, a default capability is included in the intercept data entry process in Missile Design PC TRAP. This might help the user in his/her choice of input parameters. Also, a feature provided by this default capability is the repetition of earlier intercept parameters as default values when the program is run more than once with the same missile. This feature allows a fast data entry process when the same scenario is to be run repetitively with only slight parameter changes from run to run.

The user can elect to perform any of the following simulation options:

- One-on-one single missile flyout
- Maximum range search for a single shot
- Launch envelope generation (azimuth plane)
- Launch envelope generation (elevation plane)
- Monte Carlo simulations
- Optimal target evasive maneuver evaluation

Those options are described in great detail below and in Appendix A user's manual. Only the one-on-one single missile flyout and the maximum range search for a single shot simultation option are offered by PC TRAP. The four other simulation options were coded in Missile Design PC TRAP.

Input concerning the initial intercept scenario set-up may be entered in either SI or english units. If data is entered in SI units, all inputs are in meters (m) or in meter per second (m/sec). Inputs in English units are in nautical miles (nm) for range, feet (ft) for altitude, and knots (kts) for velocity.

Both launching aircraft and target initial conditions involve launching aircraft and target maneuvers. The target can conduct the following simplistic evasive maneuver:

- Offset from initial target heading angle. If the user chooses a 180 degree offset, this will model a turn-and-run at constant altitude
- A weave which can be described as a "zig-zag" in the horizontal plane
- A spiral

For each of these target evasive maneuver, the time of initiation of the maneuver (before target impact) is determined by the user, as well as the amount of lateral acceleration (g's) to be "pulled" by the target during the selected evasive maneuver. Target evasive maneuvers initiated at different time of flight allow the missile designer to evaluate maneuver effects on the intercept capability of the missile. This option was added to Missile Design PC TRAP.

Note that both the launching aircraft and target simulation subroutines can be replaced by a user flight path generator if desired. This allows the missile designer the flexibility of evaluating missile performance against an established external source for both the launching aircraft and target performances. To do so, the user must modify the FORTRAN source code by changing the current launching aircraft or target flight path generator subroutine(s) with the desired subroutine(s). The source code files must then be re-compiled.

One of the last inputs is the print or update interval with which the user can control both the on-screen real-time history plot print interval and the print interval within the output files. More frequent print will cause the program to run slower. The user also has the option of not having a real-time history plot printed on the screen, in which case the program runs much faster. In such cases, only the final solution is printed on the screen.

F. DETAILED ENGAGEMENT MODELING

The main module which calculates the missile flight path versus the target flight path is the subroutine **TRAP**. This subroutine requires the missile design input data and the initial intercept scenario data previously entered by the user.

1. Initialization

The initial intercept scenario input and simulation output of the program can be run in either English units or in Système Internationale (SI) units. However, the program internal calculations are performed in SI units. For this reason, if the user chooses to run the program in English units, the first step in the main module is to transfer all program inputs into SI units. The program uses meter per second squared (m/sec^2) for accelerations, meters per second (m/sec) for velocities and meter (m) for position, altitude and relative distance parameters.

The program simulates the trajectory of the launching aircraft in air-to-air and air-to-surface scenarios. In such scenarios, the missile initial conditions are those of its launching aircraft. For surface-to-air scenarios, launching aircraft initial conditions are set to zero and no launching aircraft trajectory is simulated by the program.

For surface-to-air intercept scenarios, the user must input the velocity of the missile at the launcher tube exit (MINVEL). This input value is very important as it directly influences the outcome of the simulation. If MINVEL is too small, the missile will not have the sufficient energy to overtake the gravity and drag forces acting on it, and it becomes rapidly unstable, diverging from its collision course with the target, resulting in enormous miss distances. It was found that a minimum launching velocity (MINVEL) of at least 150 m/sec (0.5 Mach) was required to get a stable missile flight path with the GENERIC missile data. The minimum launching velocity to get a stable missile flight path may vary with the given missile and with the initial intercept scenario.

a. *Geometry*

In the initialization part of the main module, the program establishes the initial intercept geometry based on the user's input. The initial geometry is determined from the initial missile-target (launching aircraft) slant range (RNGINP), the azimuth (AZ) in the horizontal plane, the target altitude (INTGPZ) and, if applicable, the launching aircraft altitude (INACPZ). The methods to determine the initial geometry vary depending on the type of scenario simulated, and on the missile guidance law used for a given simulation, as detailed in Chapter III. Refer to Figure 4-1 for the important parameters of a missile-to-target intercept scenario and to Figure 3-14 for a command missile-to-target intercept scenario.

(1) Air-to-Air Intercept Scenarios

Four guidance laws are available for air-to-air simulations: pure pursuit, proportional navigation, lead angle and augmented proportional navigation. The initial intercept geometry is similar for the four guidance laws available for air-to-air intercept geometry simulations. For air-to-air intercept engagements, the target initial position is always fixed at coordinates (0,0,**INTGPZ**) of the fixed coordinate system, which means that the X_S and Y_S axes are centered at the target, and that the Z_S-axis is centered at the negative position of the target's altitude. The initial launching aircraft position is then determined based on the initial slant range (**RNGINP**), and on the LOS azimuth (**LOSAZ**) and the LOS elevation (**LOSEL**) angles. The missile initial heading angles (**MTHETA** and **MPSI**) are initially set equal to the LOS elevation and LOS azimuth angles respectively. This means that initially, the missile velocity vector is directly pointing at the target. The introduction of a lead angle into the initial intercept geometry changes the pointing direction of the missile as discussed below. Finally, the initial missile position and total velocity are the ones of the launching aircraft (**INACPX**, **INACPY**, **INACPZ** and **INACVL** respectively).

(2) Air-to-Surface Intercept Scenarios

The guidance laws available for air-to-surface scenarios are the same as air-to-air scenarios. The initial air-to-surface intercept geometry is determined as for the case of air-to-air intercept scenarios. The launching aircraft flight path is determined by the initial geometry, which means that the launching aircraft is initially flying towards the target and consequently towards the ground. The target moves in an air-to-surface scenario, but does not maneuver.

(3) Surface-to-Air Intercept Scenarios

Three guidance laws are available for surface-to-air simulations: command proportional navigation, beam rider and CLOS. Surface-to-air initial intercept geometry is established differently by placing the origin of the fixed coordinate system at the missile launch platform. This location of the inertial coordinate axis origin allows the program to compute the required missile and target ranges from the energy source, as illustrated in Figure 3-14, for proper three-point guidance implementation. For both beam rider and CLOS guidance laws, the missile is fired directly pointing at the target, with no lead angle. However, with the command proportional navigation guidance law the missile may be launched with a lead angle, which slightly varies the missile initial conditions as discussed below.

(4) Missile Initial Flight Path Angle

For all types of missile intercept scenarios simulated by the Missile Design PC TRAP program, the initial missile angle of attack is set to zero. This means that the horizontal and vertical missile flight path angles (**GAMMAH** and **GAMMA**) are respectively equal to **MPSI** and **MTHETA**, the missile initial heading angles.

(5) Lead Angle

As discussed in earlier chapters, some missile guidance laws may require that the missile not be fired directly at the target, but rather that the missile be fired in a direction to lead the target. As detailed in Chapter III and shown at Figure 3-4, this results in a lead angle, which is the angle between the missile velocity vector and the missile-target LOS. By application of the law of sines from Figure 3-4 geometry, equation (13) can be used to compute a theoretical initial missile lead angle. However, this lead angle expression (equation (13)) is applicable to a perfect model where both the target and the missile are flying at constant airspeed. In practice, the missile speed varies greatly and equation (13) is no longer applicable.

To compute the lead angle, both the Missile Design PC TRAP and PC TRAP algorithms use an optimal lead angle computation scheme which computes a lead angle for both planes of the tactical missile. The optimum lead angle computation expressions for the missile horizontal plane and the vertical plane are respectively defined as:

$$LEADH = \arcsin\left(\frac{TVEL\ \sin(LOSAZ)}{DEN}\right)$$

$$LEADV = \arcsin\left(\frac{TVEL\ \sin(LOSEL)}{DEN}\right),\qquad (100)$$

$$DEN = LDVFAC\ MVEL + AVGDLV - LDZFAC\ APZ\ (0.001);$$

where: **TVEL** is the velocity of the target (m/sec);

LOSAZ is missile-to-target LOS angle in the azimuth plane;

LOSEL is missile-to-target LOS angle in the elevation plane;

MVEL is the velocity of the missile (m/sec);

LDVFAC is the velocity multiplier, which is a weight factor put on the missile velocity; and

LDZFAC is the altitude multiplier, which is a weight factor put on the missile altitude.

AVGDLV is the average velocity difference expression, and can be computed using the following expression from Zarchan [ref. 9] (If AVGDLV equals zero, there is no initial lead angle present):

$$AVGDLV = I_{SP}\ g\ Log(\frac{W_T}{W_G})\ ; \qquad (101)$$

where: I_{SP} is the specific impulse of the rocket motor (sec);

g is the gravitational acceleration (9.8 m/sec^2);

W_T is total missile weight at launch (Kg); and

W_G is the missile glide weight (Kg).

Once the lead angle is computed for both missile planes, it is added to the initial missile heading angles (Ψ_M and Θ_M). With no lead angle, the missile heading angles are equal to the LOS angles, and the initial missile velocity vector is pointing directly at the target.

(6) Seeker

TRAP and Missile Design PC TRAP model a narrow field of view seeker, with the seeker mounted on a gimbal platform as detailed in Chapter II. In the initialization part of the main module, important initial seeker geometry variables are defined within the program. Since the missile seeker operates in the missile body axes, the final transformation matrix from a fixed coordinate system to the missile body axes must be initialized by the program at this point. In this program, only missiles that are roll stabilized are simulated, which means that the final transformation matrix is obtained after Euler angle rotations about only two of the three missile body axes. The X_B axis is obtained after a rotation about the X axis by the angle Θ_M (**THETAM**), and the Z_B axis is obtained after rotation about the Z axis by angle Ψ_M (**PSIM**). The missile body roll axis remains the same after the Euler angle transformation. As detailed in Schmidt [ ] and Nelson [ ], the final expression for the matrix transforming the fixed coordinate system to the missile body axes is:

$$T_\Phi \; T_\Theta \; T_\Psi = \begin{pmatrix} C_\Psi C_\Theta & S_\Psi C_\Theta & -S_\Theta \\ -S_\Psi C_\Phi & C_\Psi C_\Phi & 0 \\ C_\Psi S_\Theta & S_\Psi C_\Phi & C_\Theta \end{pmatrix} \tag{102}$$

where: Ψ = **PSIM**, Θ = **THETAM** and $\Phi = 0$,

$C_\Psi = \cos(\Psi_M)$ and $S_\Psi = \sin(\Psi_M)$, and

$C_\Theta = \cos(\Theta_M)$ and $S_\Theta = \sin(\Theta_M)$.

Using the transformation of Equation (102), the relative ranges (**RNGMCX, RNGMCY, RNGMCZ**) and relative range rates (**RCRSVX, RCRSVY, RCRSVZ**) of the target from the seeker are calculated in missile body axes. Based on these relative body axes ranges, the seeker gimbal angles in pitch and yaw planes (**PGANG** and **YGANG** respectively) are determined and subsequently compared to the seeker maximum gimbal angles (**SEKGAD**). In cases where current gimbal angles exceed the maximum allowable seeker gimbal angle, the current gimbal angle value is

set to the maximum allowable value by the program. The same process is applied to the relative range rates which are compared to the maximum missile seeker LOS rate that can be physically achieved by the seeker. Nte that beam rider and CLOS guidance missiles do not have a seeker aboard the missile, which eliminates the need to model a seeker for these guidance laws for both the beam-rider and CLOS guidance laws, Missile Design PC TRAP assumes that the guidance beam is of zero width (i.e., the missile knows exactly where the beam is at each simulation time step).
a spiral.

(7) Comment About Initialization

The program initialization part is executed only once for each missile simulation conducted by Missile Design PC TRAP. At the end of this part, the launching aircraft-missile-target initial intercept geometry is all established and the program then enters its main loop that will calculate the intercept geometry starting from the above initial conditions until missile flight termination. In the initialization part, all velocities are expressed with respect to vehicle coordinates (body axis) while all positions are with respect to a fixed coordinate system. Before entering the main loop, the program transforms the body axis velocities of each simulated vehicle into fixed coordinate system velocities through an inverse Euler angle transformation, which is the inverse of the matrix detailed in equation

(102). This inverse missile Euler angle transformation from body axis components to fixed coordinate components is defined as follows:

$$(T_\Phi\ T_\Theta\ T_\Psi)^{-1} = T_\Psi^T\ T_\Theta^T\ T_\Phi^T = \begin{pmatrix} C_\Psi C_\Theta & -S_\Psi & C_\Psi S_\Theta \\ S_\Psi C_\Theta & C_\Psi & S_\Psi S_\Theta \\ -S_\Theta & 0 & C_\Theta \end{pmatrix} \quad (103)$$

where the variables are defined at equation (102).

2. Main Timing Loop

The main loop is the "heart" of the Missile Design PC TRAP program as it continuously computes the trajectory of the missile from the initial conditions, computed in the initialization part, up to the engagement termination. Most of the theory behind homing and command missile guidance systems is detailed at Chapter III. This theory is now applied by incorporating it into the Missile Design PC TRAP algorithms as shown in Figures 4-2, 4-4, and 4-5.

A block diagram detailing the major steps taken by the Missile Design PC TRAP main loop to compute the engagement geometry simulation is illustrated in Figure 4-7 above. Each main loop major steps will be discussed in details to provide the reader with an in-depth understanding of the algorithms in order to facilitate any future modification required to improve the missile design process. This discussion gives detail on the Missile Design PC TRAP modeling philosophies and capabilities as well as on the limitations of the main loop. For more precise detail, consult the source code listing for the main loop, which is included in the **UTRAP1.FOR** file.

a. Intercept Relative Geometry

The first calculation in the main timing loop is the intercept geometry. Missile Design PC TRAP computes relative geometry for all vehicles: missile-to-target geometry; launching aircraft-to-target and launching aircraft-to-missile. Relative ranges and range rates between the vehicles are computed as X_S, Y_S, Z_S components of the fixed coordinate system. The magnitudes of these relative ranges and range rates are also computed by the program at this point.

b. Atmosphere

The atmospheric equations used in the Missile Design PC TRAP models the standard atmosphere. For each vehicle operating altitude, the air density (**RHO**), the local speed of sound (**VS**) and the static pressure (**PRESS**) are computed so that the current dynamic pressure and current Mach number can be calculated.

c. Propulsion

Missile Design PC TRAP models tactical missiles propelled by solid-rocket motors, with boost and/or boost-sustain phase. As seen above, the missile thrust profile is modeled by the program using a five point vacuum thrust profile approximation. The program linearly interpolates from this approximated profile curve to determine the vacuum thrust (VTHRST) value at each simulation time step (TIME).

When the current vacuum thrust value has been determined, the fuel flow rate (FUELFL) is computed by dividing the vacuum thrust value by the appropriate specific impulse (VISP). Finally, the delivered thrust (THRUST) is calculated by adjusting the current vacuum thrust to the motor exit area (EXAREA) and to the current missile altitude.

d. Mass Properties

Missile updated mass is computed by subtracting the mass due to propellant expenditure from the current mass (MSMASS). This substraction is conducted at each time step until the value of the current missile mass reaches the user input burnout value. The location of the center-of-gravity (c.g.) (MISLCG) is shifted based on depleting propellant until it reaches its user input motor burnout location.

e. Seeker

For design missiles that have an onboard homing head or seeker, Missile Design PC TRAP models a perfect-filter seeker. In this program, it is assumed that beam rider and CLOS missiles do not have an inboard missile seeker. A general description of missile seeker functions and capabilities is presented in Chapter II.

The TRAP and Missile Design PC TRAP program model the seeker as a lagged mechanical tracking loop, continuously keeping track of the target position with respect to the missile body axes. The modeled seeker keeps track of gimbal angles and of seeker head tracking rates, and limits them to their maximum values as defined by the user in the missile input data file. The seeker also verifies that the target remains within the seeker field-of-view (IFOV). If the target moves outside the IFOV, the last tracking commands are held. The main function of the missile seeker is to calculate the rate of change of the missile-target LOS angle for implementation of the guidance law. The two seeker output values are the elevation LOS angle rate (LOSELR) and the azimuth LOS angle rate (LOSAZR) which are input to the guidance computer to generate lateral missile acceleration commands required for the missile to remain on an intercept course. The most important parameters for a narrow IFOV seeker are shown in figure 2-2, which is re-drawn in Figure 4-8 for convenience.

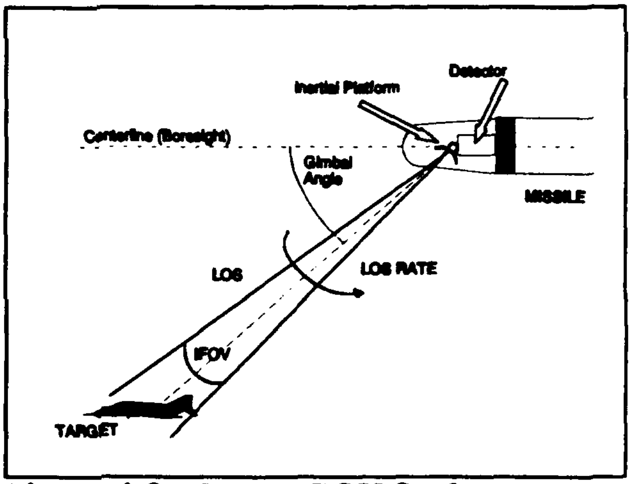


Figure 4-8. Narrow FOV Seeker

(1) Gimbal Angles

The program computes the gimbal angles by first transforming the missile-target relative ranges and range rates from X_S, Y_S, Z_S components (fixed coordinates) into missile body axes relative range and range rate components according to equation (102) (X_B, Y_B, Z_B). As shown in a two-dimensional plane in Figure 4-8, the gimbal angle is then the angle between the LOS and the missile velocity vector (the X_B axis in body axes). The program calculates the gimbal angles in both pitch and yaw missile body planes (**PGANG** and **YGANG** respectively). If any of the plane gimbal angle exceeds the maximum allowable gimbal angle defined by the user, the current gimbal angle is set equal to this maximum allowable value.

(2) LOS Rates

In practice, the seeker takes the derivative of the relative LOS angle, thus providing a measurement of the LOS rates in pitch and yaw (**LOSELR** and **LOSAZR** respectively) in the fixed coordinate system, and subsequently in the body axes system through the Euler angle transformation expressed in equation (103). In a simulation model, the LOS rates are computed based on the intercept geometry from the known position of the target and missile. These calculated LOS rates are normally noisy and must be processed through a perfect filter which will provide an improved estimate of the LOS rates. The perfect-filter seeker is modeled as a first order lag with a gain of 10 and a time constant of 0.1. This means that the current LOS rates are obtained from the calculated LOS rates using the following transfer function:

$$\frac{\dfrac{d\lambda(s)}{dt}}{\dfrac{d\lambda_C}{dt}} = \frac{10}{s + \tau} \qquad (104)$$

where: dλ/dt represent the current LOS rates passed to the guidance computer (rad/sec^2);

$d\lambda_c/dt$ represent the calculated LOS rates derived from the current relative geometry(rad/sec^2); and

τ is the filter time constant (0.1 sec).

(3) Field-of-View (IFOV)

As explained in Chapter II and shown in Figure 4-8, a gimballed seeker normally has a narrow IFOV. For adequate guidance, the seeker maintains the target within the IFOV by rotating the gimbal platform. The seeker IFOV is defined as the conical angular region about the seeker centerline which is capable of receiving useful target tracking energy.

To compute the IFOV, the program performs another coordinate transformation on the missile-target relative ranges, which are transformed from body axes to the seeker centerline axes. As shown in Figure 4-8, the seeker centerline axes (boreline) coordinate system has its X_B axis along the centerline of the missile seeker. From this new seeker centerline axis coordinate system, the seeker IFOV can be calculated (**BEPSZ** and **BEPSY** for the vertical and horizontal planes respectively) and compared with its maximum allowable value. If the target falls out of the seeker IFOV, no new tracking commands are generated by the seeker, and the last tracking commands are held.

f. Guidance

The current LOS rates in pitch and yaw are then used to generate guidance commands (**VERTGC** and **HORGC**) based on the user selected guidance law. The implementation of each available guidance law from the seeker filter output data (**LOSECR and LOSAZR**) is fully detailed in Chapter III.

As mentioned above, models of the guidance section and the flight control section are considered to be perfect and without dynamics. However, the guidance loop may be made nonlinear to account for acceleration saturation. Acceleration saturation occurs when any of the two missile lateral commanded acceleration exceeds the acceleration capability or limit fixed by the missile designer. In such a case, the program sets the lateral missile acceleration commands (VERTGC and HORGC) equal to the missile acceleration limit (MXVGCG). The current guidance lateral acceleration commands in m/sec^2 are then passed to the aerodynamics model.

g. Aerodynamics

(1) Normal and Side Forces

The TRAP and Missile Design PC TRAP aerodynamics model compute angles of attack (α in pitch plane and β in yaw plane), and the normal, side, and axial forces (F_z, F_y, and F_x respectively) in missile body axes. From the normal and side forces, the angles of attack required to pull the g's commanded (n_c) by the guidance law are estimated by the program. This is accomplished by first calculating the normal and side force coefficients, C_N and C_Y respectively, as follows:

$$n_c \, W = L = C_L \, Q \, S \; ---\rightarrow \; C_L = \frac{nW}{QS} \qquad \textbf{(105)}$$

where: n_c is the required missile lateral acceleration (m/sec^2),

W is the missile instantaneous weight (Kg),

L is the normal or side force (N),

C_L is the normal or side force coefficient,

Q is the dynamic pressure (KPa), and

S is the missile reference area (m^2).

From these values, a linear interpolation/search is performed to determine what value of angle of attack gives this force. This interpolation/search is done from the two values of C_N that are input by the user: C_N at five (5) degrees AOA (**CN5**) and C_N at 15 degrees AOA (**CN15**), which are the trimmed normal force values for the missile at these specific angles of attack. The missile is assumed to be symmetric, so that the **CN5** and **CN15** input values apply to both side and normal forces.

A maximum angle of attack (**MAXALP**) is input as the greatest of the values from the missile "maximum angle of attack vs Mach number" table. As mentionned in the PC TRAP user's manual, this may admittedly give the missile more lifting capability than realistic, but it has been found through extensive validation test by the FASTC that any large angle of attack occurs instantaneously; allowing a large angle of attack does not appear to adversely affect the results when one is only interested in a good estimate of kinematic capability.

(2) Axial Force

The computation of the axial force (drag force along the missile longitudinal axis) requires more resolution on the axial force coefficients (C_A), especially in the transonic region. As detailed above, the program requires missile power-off, zero lift axial force coefficients at 12 different Mach number, as well as a peak C_A value (**CAP**) and its corresponding Mach number (**MCAP**), and a final value (**CAF**) and its Mach number (MCAF). The peak value allows the program to correctly pinpoint that point of highest drag as the Mach number rises through the transonic regime. All of the drag coefficient values are required because it is of utmost importance to accurately represent the drag so that missile acceleration components can be estimated as accurately as possible. Errors in acceleration estimations propagate through the entire simulation computations since the acceleration components are integrated twice to get the missile position.

Based on current Mach number, the program linearly interpolates through the axial force coefficient input data to find the correct C_A, then makes correction if the missile motor is burning (i.e. when no base drag is present), and for altitude.

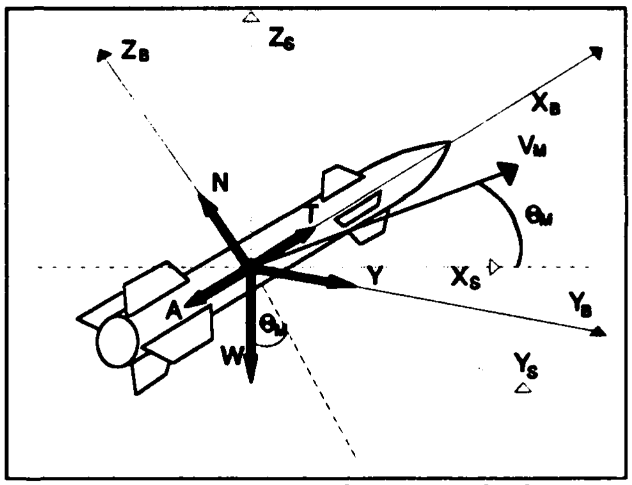


Figure 4-9. Forces acting on Missile Body

(3) Missile Acceleration Components

Once the side (Y), normal (N), and axial (A) forces acting on the missile are computed and assembled, the missile acceleration components can be computed by the program. The program computes the missile acceleration components with respect to the wind axis coordinate system. As detailed at Appendix G of Blakelock [ref.2], "all the complexities and inaccuracies resulting from the use of body axes accelerations and equations of motion are eliminated with the use of wind axes." However, to get to the wind axes acceleration components, the program first computes the missile body accelerations, which are subsequently transformed into fixed coordinate axes, and finally into the wind axis acceleration components as follows:

(4) Body axes

From Figure 4-9 and Newton's second law of motion (F=ma), the missile body acceleration components are calculated as follows:

$$A_{XB} = \frac{T}{M} - \frac{C_A\,Q\,S}{M} - g\,\sin(\Theta_M)$$
$$A_{YB} = \frac{C_Y\,Q\,S}{M} \qquad (106)$$
$$A_{ZB} = -\frac{C_N\,Q\,S}{M} + g\,\cos(\Theta_M)$$

where: A_{XB},A_{YB},A_{ZB} are the given body axes acceleration components (m/sec^2);

T is the current delivered thrust (N);

C_A, C_N and C_Y are the total axial, lift and side force coefficients;

Q is the dynamic pressure (KPa);

Θ_M is the missile heading angle in pitch (rad);

S is the missile reference area (m^2);

M is the current mass of the missile (Kg); and

g is the gravitational acceleration (9.8 m/sec^2).

(5) Fixed Coordinate System

Resolving the body axis acceleration components into fixed coordinate axes, by rotating through the missile angle of attack (α), yields:

$$\begin{aligned} A_{XS} &= A_{XB} \cos \alpha + A_{ZB} \sin \alpha \\ A_{YS} &= A_{YB} \\ A_{ZW} &= A_{ZS}. \end{aligned} \tag{107}$$

(6) Wind Axes

Finally, the wind axes missile acceleration components are obtained by rotating the fixed coordinate accelerations through the missile sideslip angle (β):

$$
\begin{aligned}
A_{XW} &= A_{XS} \cos \beta + A_{YS} \sin \beta \\
A_{YW} &= - A_{XS} \sin \beta + A_{YS} \cos \beta \\
A_{ZW} &= A_{ZS}.
\end{aligned}
\tag{108}
$$

(7) Missile Equations of Motion

In TRAP and Missile Design PC TRAP, the missile flight path is computed using a linear 3 Degree of Freedom (DOF) point mass simulation model. Ignoring the aerodynamic moments acting on the missile, the missile translational wind axis equations of motion are:

$$
\begin{aligned}
\frac{dV_M}{dt} &= \dot{V}_M = A_{XW} \\
\frac{d\beta}{dt} &= \dot{\beta} = \frac{A_{YW}}{V_M} \\
\frac{d\alpha}{dt} &= \dot{\alpha} = \frac{A_{ZW}}{V_M \cos \beta}
\end{aligned}
\tag{109}
$$

where V_M is the total missile velocity (m/sec).

The above translational missile equations of motion are numerically integrated by the program to estimate the updated missile attitude (α and β), as well as the updated missile velocity (V_M), which is, by definition, the velocity vector along the X-axis of the wind axis coordinate system. However, before integrating equation (109), the program conducts a termination check which is now detailed.

h. Integration

The last step of the main timing loop is the integration process. An Adams-Bashforth first order predictor corrector integration is used to integrate the program ordinary differential equations using the following difference equation:

$$y_{n+1} = y_n + \Delta T \, \frac{3\,\dot{y}_n - \dot{y}_{n-1}}{2} \qquad (110)$$

where: y represents the function;

y_{n+1} is the updated value of the function after integration;

y_n is the current value of the function;

dy_n/dt is the current value of the derivative of the function;

dy_{n-1}/dt is the previous value (TIME - ΔT) of the derivative of the function; and

ΔT (DT) is the integration step size (sec).

Using the above integration scheme, the wind axis longitudinal acceleration (WNDACX) is integrated to yield the updated missile velocity (MVEL). As well, the angular rates (GMDOT and GMHDOT) are integrated to get the updated flight path angles in pitch and yaw. Missile velocities in the body axes (MVBX, MVBY, MVBZ) are then computed and rotated through the missile attitude angles into the fixed coordinate system (MVRX, MVRY, MVRZ). These velocities are then integrated to get the updated missile position. From there, previous values are saved, the time counter (TIME) is updated by adding DT (the integration step size), and the iteration loop begins again.

i. Main Loop Termination

For each main timing loop iteration, the program conducts a missile flight termination check to determine if an intercept has occurred, or if the engagement geometry indicates that an intercept will never occur. This termination check is conducted by a survey of the current engagement geometry flight conditions. The following engagement geometry conditions will result in missile flight termination, in this order:

1) If missile-to-target slant range (RANGE) is within the warhead lethal radius (MDPERM). This condition is not verified by the program in the following simulation options:

- When the user does not elect to change the integration step size to a smaller value during the end-game phase
- During the Monte Carlo simulations in which the integration step size is automatically changed to a smaller value during the end-game phase of each missile flight

2) If the missile altitude (**MPZ**) has dropped below five meters (i.e., the missile is about to hit the ground). This condition is not verified for surface-to-air mission profiles.

3) If missile altitude (**MPZ**) has gone above 50 km.

4) If the missile simulation flight time (**TIME**) has exceeded the life of the missile battery (**MAXTIM**).

5) If missile velocity (**MVEL**) has dropped below a given minimum (**LOWMSV** or **LOWMSM**), and the motor has burned out (there will be no more acceleration other than gravity). For minimum velocity, if **LOWMSV** and **LOWMSM** are set to zero in the missile data input file, the program will set **LOWMSV** to launch speed for air-to-ground and air-to-air mission types.

6) If missile-to-target closing velocity (**CLOVEL**) drops below the input minimum closing velocity (**LOWCLV**).

7) If closing velocity drops below zero, indicating that the target has been passed and that an intercept has occurred.When the integration step size is allowed to be made smaller during the end-game, this condition prevails in the determination of a target intercept.

If any of these conditions are met, the program terminates and prints a summary statement of the flight to the screen. If none are met, the iteration loop begins again after integration of the missile state variables.

G. SPECIAL SIMULATION OPTIONS

In addition to the standard single missile flyouts against one target simulation options already offered in PC TRAP, Missile Design PC TRAP offers some simulation options that intend to provide a more complete performance evaluation of the missile designs. Missile Design PC TRAP can generate missile flight envelopes in both the horizontal and vertical planes. Missile Design PC TRAP can perform Monte Carlo simulation runs that can be used to evaluate the missile miss distance performance against a maneuvering target. Also, Missile Design PC TRAP can be used to evaluate optimal target evasive maneuvers.

1. Maximum Range Flight Envelopes

Missile Design PC TRAP is capable of generating maximum range flight envelopes in both the horizontal and vertical planes. This simulation was added to Missile Design PC TRAP by the author. These launch envelopes indicate the maximum range aerodynamic performance capabilities of the missile in the selected plane for a given initial intercept scenario. To generate a launch envelope, an initial missile-target intercept scenario must be set-up by the user, excluding the initial slant range and azimuth aspect angle. Instead, the user is asked to input a span of azimuth aspect angles or an altitude band within which the program will generate the launch envelope. The user is also asked to choose the resolution for this launch envelope by selecting the amount of data points at which maximum range evaluation simulation runs will be performed.

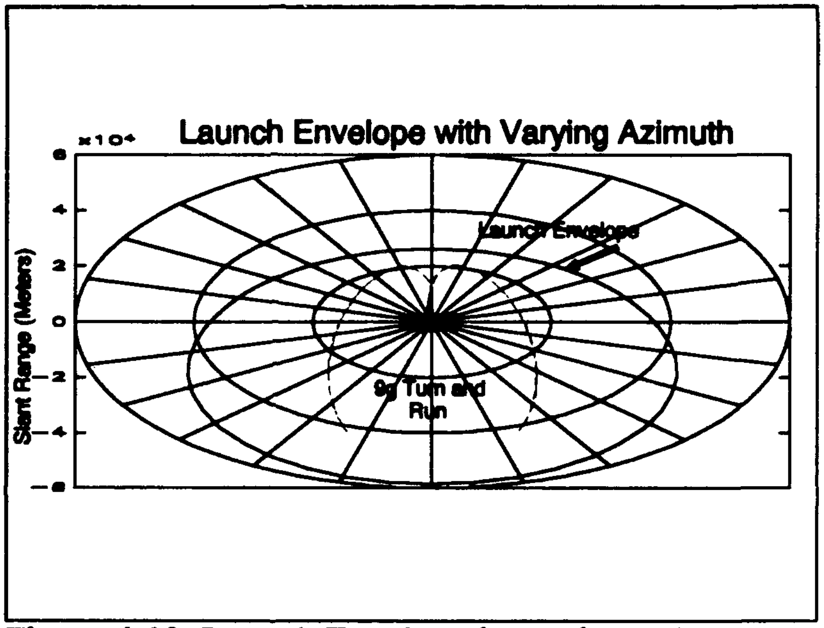


Figure 4-10. Launch Envelope in Horizontal Plane

Figure 4-10 is an example of a maximum range launch envelope with varying azimuth, generated in the horizontal plane. The azimuth aspect angle span for this launch envelope is from zero degree (the missile is initially pointing at the tail of target) up to 180 degrees (the missile is initially pointing at the nose of the target). This launch envelope was generated using 25 intervals, equally spaced within the 180 degree aspect angle span. To generate this launch envelope, the program conducted a maximum range search at each interval within the selected span of azimuth aspect angles. The maximum range is determined by performing repetitive missile shots with different initial range values until the miss distance is within the missile warhead lethal radius. The launch envelope illustrated in Figure 4-10 is shown in polar coordinates.

Similarly, Figure 4-11 provides an example of a maximum range launch envelope with varying target altitude in the elevation plane. In this case, the initial intercept set-up included the input of the azimuth aspect angle (0 degree), but no input of target altitude. Instead, an altitude band was selected to vary from 300 meters up to 10 kilometers for the generation of the launch envelope. As for the previous case, this altitude band was divided into 25 equally spaced altitude intervals. A maximum range search was then conducted by the program for each target altitude determined by the altitude band intervals.

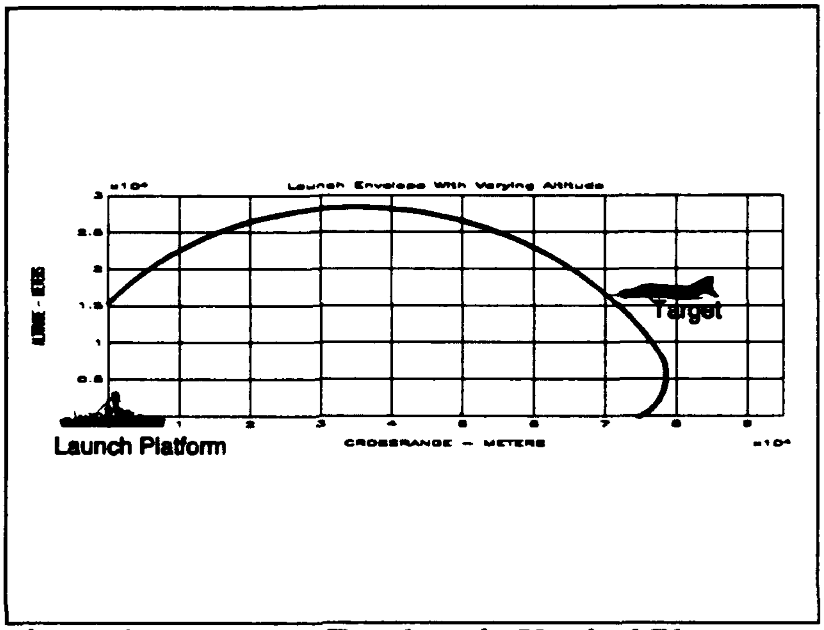


Figure 4-11. Launch Envelope in Vertical Plane

For both types of launch envelopes, the results, showing the maximum range for each intermediate value of altitude or range, are printed to an output file named **ENVEL.DAT** for plotting. Due to the large variety of intercept scenarios in which tactical missiles can be involved, it is not possible to evaluate the aerodynamic performance of a missile for each of its possible intercept scenarios. This is where missile maximum range launch envelopes become useful, as they are used to evaluate missile aerodynamic performances in a relatively large set of intercept scenarios. Then again, an infinite number of launch envelopes can be generated for each missile. This fact alone motivates the need for a missile flight analysis computer program that runs fast and that is easily accessible to DoD personnel, especially to military operational units.

2. Monte Carlo Simulations

Up to now, we have only described deterministic missile simulation techniques have been described. This means that if one runs a certain missile simulation profile several times, one will always obtain the same flight path with the same miss distance. The reason that PC TRAP and Missile Design PC TRAP are deterministic missile simulation program is due to the absence of noise and other random processes in the guidance loops. Noise analysis was not part of the scope of this thesis. However, an alternate simulation technique was implemented in Missile Design PC TRAP to provide a simulation model driven by a stochastic process.

In order to evaluate missile performance in a stochastic process, consider a target maneuver with a random starting time, that is uniformly distributed over the flight time as the stochastic variable. Several flight simulations with the target maneuver being constant from flight to flight (either plus or minus the amount of g's). However, on a given flight its initiation time is equally likely to occur anywhere during the missile flight. Such a simulation technique can be described as a Monte Carlo simulation with the target maneuver initiation time as the stochastic variable.

This technique was implemented in Missile Design PC TRAP. For each Monte Carlo simulation run, 50 missile flights are performed from the same initial intercept scenario with the same target maneuver for each flight. When the 50 missile flights of a Monte Carlo simulation run are completed, the standard deviation and mean of the 50 miss distances are computed by the program according to the formulas developed in Zarchan [ref. 9]. The results are printed to the output file named **MONTE.DAT**.

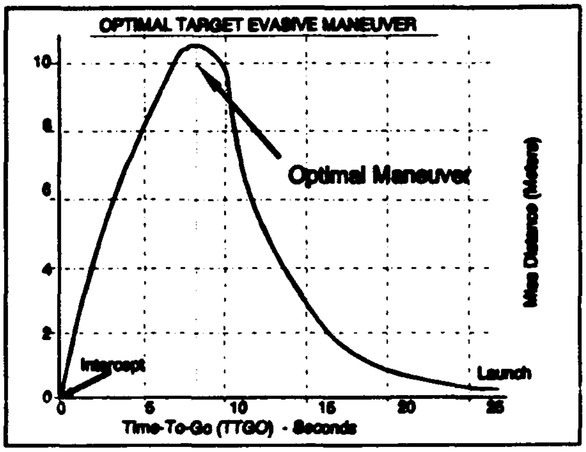
It was decided to use a sample size of 50 runs per Monte Carlo simulation, to determine the mean and standard deviation of the miss distances because 50 runs was considered to be sufficient in a trade-off between computer running time and numerical computation accuracy. It is shown in Zarchan [ref. 9] that large errors in the standard deviation estimate occur when there are less than 20 runs. The accuracy of the computation improves significantly when many samples are used in computing the standard deviation. It was found that a confidence level of 95% was obtained in the computation of the miss distance standard deviation with a sample size of 50 runs. However, the number of runs required to increase this confidence level significantly more would be very large and too costly in computing time for a small benefit in computation accuracy.

Recognizing that simulation outputs based on random inputs can vary, the Monte Carlo simulation option offered by Missile Design PC TRAP is an excellent way to evaluate missile system performance. This could be re-enforced with the fact that in practice, one never knows when a target will execute an evasive maneuver to defeat an incoming tactical missile.

3. Optimal Target Evasive Maneuver

Figure 4-12 shows miss distance results due to a step in target acceleration. The abscissa is the time-to-go (TTG) at which the acceleration was initiated. The ordinate is the miss distance corresponding to the time-to-go at which the target maneuver was initiated. Zero second t go indicates missile-target intercept, while 25 seconds t go indicates missile launch (beginning of missile flight). We can see from Figure 4-12 that a target maneuver initiated with a large or small time-to-go will cause a small miss distance. However, the miss distance curve has a maxima at approximately 7.5 seconds TTG. Therefore, the target can induce the most miss distance by initiating the given maneuver at 7.5 seconds before intercept. "From a target's point of view, an optimal maneuver is one that induces the most miss distance", as stated by Zarchan [ref. 9]. Hence, the optimal target evasive maneuver for the case illustrated in Figure 4-12 occurs at 7.5 seconds TTGO.

The concept of an optimal maneuver is useful in that it identifies the largest miss distance that the target can induced, and possibly aid in the selection of the missile guidance law and design parameters. For these reasons, a technique to produce miss distance plots similar to Figure 4-12 was implemented in Missile Design PC TRAP.



To apply this technique, the initial missile-to-target slant range is divided into 50 equally spaced intervals. Each interval corresponds to a target maneuver starting point. Then 50 missile flights are simulated with the target initiating the same maneuver at each of the above mentioned starting point. The miss distance for each flight is then recorded, as well as the corresponding target maneuver initiation time. The results are printed to the output file named **OPTIM.DAT** for plotting. In general, a curve similar to the one shown in Figure 4-12 should be obtained.

H. PROGRAM OUTPUT

Missile Design PC TRAP creates two types of output: an on-screen real-time history plot of the simulation, and output files containing pertinent data on the engagement history.

1. Real-Time Trajectory History Plot

a. Air-to-air Mission Type

For air-to-air missions, X_S and Y_S coordinates (fixed coordinate system) of the launching aircraft, the target and the missile flight paths are plotted in real simulation time on the screen. The altitude differences between the three vehicles are also plotted, with a bar graph, on the right-hand side of the X-Y time history plot. All coordinates are updated at the print interval input by the user. This feature is also present in PC TRAP.

b. Air-to-surface Mission Type

For air-to-surface missions, the altitude and the crossrange of the three vehicles are plotted in real simulation time on the screen. In Missile Design PC TRAP, the launching aircraft initial heading angle is always pointing at the target. For this reason, in air-to-surface mission types, the launching aircraft is initially flying towards the target, and subsequently towards the ground.

c. Surface-to-air Mission Type

There is no launching aircraft involved in surface-to-air missions. Consequently, only the missile and target flight paths are modeled and plotted on the screen in altitude and crossrange coordinates.

d. Generalities

For all the three mission types mentioned above, missile trajectory limitations messages may appear on the screen during the simulation. These messages simply indicates a limit conditions reached by the missile during its trajectory that may affect the result of the simulation. The consequences of the limitations on the missile trajectory is normally included with the message. During the vehicle trajectory plotting, current status of some important missile variables, such the missile Mach number, elapsed time-of-flight and range to target, are displayed in the left-hand corner of the plot.

The real-time trajectory plot is in color with a different color assigned to each vehicle. The missile flight path is shown in yellow, the target in light cyan and the launching aircraft in red. The plotting subroutine is called **PLOT** and is found in the **UTRAP.FOR** file.

The graphic display capability of TRAP and Missile Design PC TRAP makes the simulation programs very attractive. A special feature of the graphic display that was implemented in Missile Design PC TRAP is the fact that the missile trajectories generated by several guidance laws can be displayed simultaneously on the same plot for the same intercept scenario. This allows comparison of the different flight paths generated by the guidance laws.

2. Data Output Files

In the data input process, the user has the option of saving a text record of the engagement data. If the user wishes to save the engagement data output files, the following data files will be generated and saved by the program in the same directory from which is run the program:

1) **ENGAGMT.DAT** is an output file containing complete time history information on the missile flight. The data of this file provide inside detail on the intercept conditions by printing all the missile major variables during the intercept. The data is updated at every print interval. A dictionary of the output variables contained in this file is included in Appendix C.

2) **TARGET.DAT, MISSILE.DAT, SHOOTER.DAT** are output files containing the X_S, Y_S, and Z_S coordinates (fixed coordinate) of the target, missile and launching aircraft (if applicable) respectively. These data files may be used for later plotting.

3) **MACC.DAT** is an output file containing the missile time of flight, the commanded lateral accelerations in pitch and yaw, and the achieved lateral accelerations in pitch and yaw.

4) **HEADING.DAT** is an output file containing the time of flight and both the target and missile heading angle data (Θ_T, Θ_M, Ψ_T, Ψ_M).

5) **ATTITUD.DAT** is an output file containing the time of flight, the missile angle of attack (α), the missile sideslip angle (β) and the missile flight path angles (γ and γ_H) in both missile planes.

6) **VELOCIT.DAT** is an output file that contains the time of flight, the magnitude of the missile velocity, the missile-to-target closing velocity, and the magnitude of the target velocity.

Other engagement data output files can be easily generated by modification and compilation of the original program source code by the missile designers.

I. PROGRAM STRUCTURE

The Missile Design PC TRAP is a complex programs written in the FORTRAN language. Because of this complexity, it was chosen to write the program using a top-down approach that divides the numerous computations into a number of simple subroutines located in four different FORTRAN files, which diminishes the complexity of the program and increases its clearness.

The Missile Design PC TRAP computer program is executed from the FORTRAN file **MDPCTRAP.EXE**. It can be run on any IBM-compatible

The Missile Design PC TRAP computer program is executed from the FORTRAN file **MDPCTRAP.EXE**. It can be run on any IBM-compatible Personal Computer equipped with a graphics based monitor (including Laptop computers) and a math co-processor. The executable FORTRAN file results from the compilation of four different FORTRAN files containing the above described subroutines. The **UTRAP.FOR** file contains the main program driver as well as the input and graphic output subroutines of the program. The **UTRAP2.FOR** file is composed of the launching aircraft and target simulation subroutines, as well as the coordinate transformation matrices subroutines required by the program to switch from one coordinate system to another. The **UTRAP1.FOR** file contains the main module which simulates the entire missile flight path, and the **UTRAPA.FOR** file contains the subroutines required to run the Monte Carlo simulation, as well as small subroutines called directly from the main module subroutine (**UTRAP1.FOR**).

The Missile Design PC TRAP is composed of one main program driver and of the following principle subroutines:

- Subroutine **TITLE**: called by the main driver and located in the **UTRAP.FOR** file. This subroutine prints the Missile Design PC TRAP introduction panel to the screen

- Subroutine **PANEL2**: called by the main driver and located in the **UTRAP.FOR** file. This input subroutine prints the input parameter screen, prompts the user for engagement scenario parameter input, and passes inputs to main driver

- Subroutine **TRAP**: called by the main driver and located in the main module file **UTRAP1.FOR**. This subroutine is the main module of Missile Design PC TRAP which calculates the missile-target intercept scenario. This subroutine computes missile related state variables only

- Subroutine **ATMOS**: called by the subroutine **TRAP** and located in the **UTRAP2.FOR** file. This subroutine computes the atmospheric pressure, the air density and the local speed of sound at a given altitude

- Subroutine **VEHTRF**: called by the subroutine **TRAP** from the **UTRAP2.FOR** file. This subroutine uses Euler angle transformation to get fixed coordinate referenced vectors from body axes referenced ones. It is used for vehicle coordinate transformations

- Subroutine **MTMUL1**: located in the **UTRAP2.FOR** file, this subroutine is called by the main module **TRAP**. It is used to change from a fixed coordinate system into a body axes system. This subroutine is used only to calculate missile-target relative ranges and range rates from the missile seeker point of view

- Subroutine **MTMUL2**: located in the **UTRAP2.FOR** file, this subroutine is called by the main module **TRAP**. It is used for seeker related geometry calculations to change from body axes coordinates into LOS axes

- Subroutine **MTMUL3**: located in the **UTRAP2.FOR** file, this subroutine is called by the main module **TRAP**. This subroutine computes the LOS rates in the three planes of the fixed coordinate system for the seeker model

- Subroutine **TARGET**: called by **TRAP** from **UTRAP2.FOR**. Computes the target flight path based on user defined target maneuvers

- Subroutine **AC**: called by **TRAP** from **UTRAP2.FOR**. Computes the launching aircraft flight path based on user defined follow-on maneuvers

- Subroutine **DRAWGRAF**: located in **UTRAP.FOR** and called by **TRAP**. This subroutine draws on the screen the actual grid for the engagement history plot

- Subroutine **ALTSCALE**: located in **UTRAP.FOR** and called by **TRAP**. This subroutine sets up the grid size for the altitude sub-window of the engagement history plot

- Subroutine **PLOT**: located in **UTRAP.FOR** and called by **TRAP**. Plots the actual flight paths for all the vehicles involved in the simulation

- Subroutine **ALTPLT**: located in **UTRAP.FOR** and called by **TRAP**. This subroutine plots the altitude, in the altitude sub-window, for each vehicle during the engagement

- Subroutine **LEAD**: located in **UTRAPA.FOR** and called by **TRAP**. This subroutine computes the initial lead angle using the optimal lead angle computation technique (equation (97))

- Subroutine **SEEDG**: located in **UTRAPA.FOR** and called by **TRAP**. This subroutine creates two vectors containing 50 seed values each. These seed values are used to generate random numbers between 0 and 1 for the Monte Carlo simulation technique

- Subroutine **SIG**: located in **UTRAPA.FOR** and called by **TRAP**. This subroutine computes the standard deviation and mean of the miss distances recorded during the 50 missile flights required by the Monte Carlo Simulation technique

V. EXAMPLES

A. INTRODUCTION

This chapter will illustrate some possible applications of the Missile Design PC TRAP, and also demonstrate how well the simulation parameters of Missile Design PC TRAP match those of TRAP. First, a conceptual missile design will be presented from which a data input file will be built for Missile Design PC TRAP applications. Then, the performance of this conceptual missile design is evaluated using some of the simulation options offered by Missile Design PC TRAP. Finally, selected simulation parameters generated by Missile Design PC TRAP will be compared to the same parameters generated by TRAP to determine the simulation accuracy of Missile Design PC TRAP.

B. HOW TO BUILD A MISSILE INPUT DATA FILE

As mentioned in previous chapters, a missile data input file is required as an input to the Missile Design PC TRAP trajectory analysis computer program. This data input file describes the propulsion, aerodynamic, guidance and physical characteristics of the missile to be modelled. A dictionary detailing the 57 missile data input items that must be included in the missile input data file is provided in Appendix B. An example of a missile input data file is also provided in Appendix B.

This section provides an example on how to create such a missile input data file from a conceptual missile design. It is shown that some of the required missile data are the user's choice, while other data, such as aerodynamic data, must be evaluated or calculated from the conceptual design.

It should be noted that a missile input data file can also be constructed for an existing tactical missile. In such a case, the user will find the required input data information from the technical literature of this missile. However, it is important to note that missile data from existing tactical missiles are generally CLASSIFIED, which means that to simulate these missiles, Missile Design PC TRAP shall be run in a secure personal computer facility.

1. Tactical Missile Conceptual Design

The tactical missile conceptual design under consideration for this example is purely fictitious. A sketch of the missile and its important physical dimensions is shown in Figure 5-1. This missile is a surface-to-air tactical missile using command proportional navigation guidance for medium range applications.

It is propelled with a solid propellant rocket motor, equipped with both booster and sustainer phases. The missile is an aerodynamic missile using four canard fins to shape its trajectory, and four larger tail fins for stabilization. Its diameter is 21 centimeters and its length is 3.2 meters. Based on these preliminary data, a missile input data file will be built in order to model the aerodynamic performances of this conceptual tactical missile design using Missile Design PC TRAP.

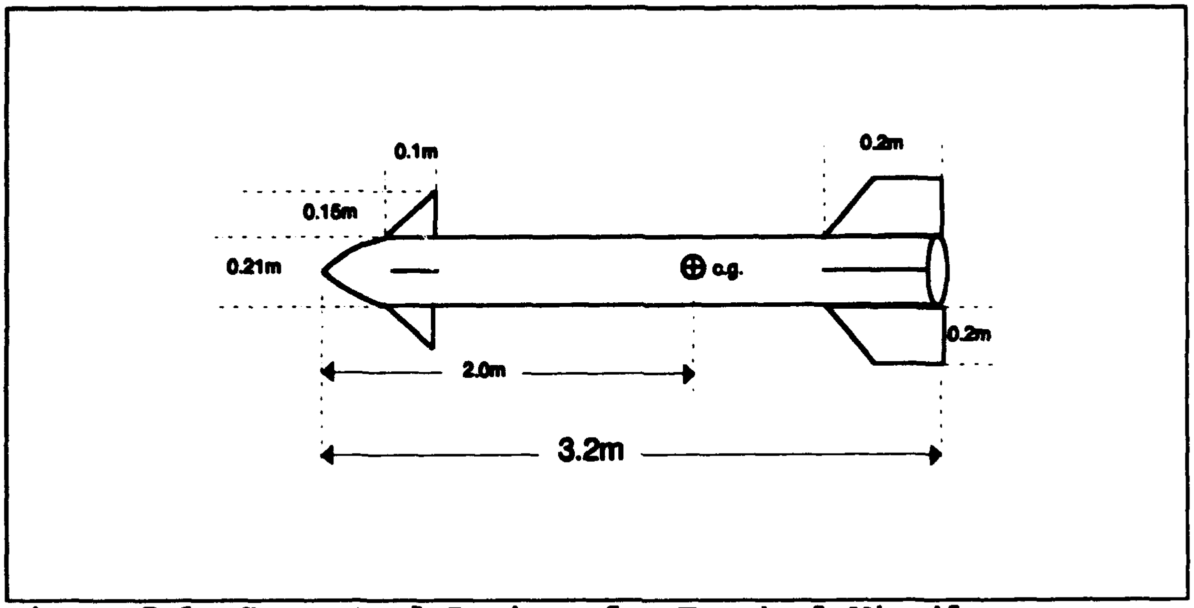


Figure 5-1. Conceptual Design of a Tactical Missile

2. Propulsion Data

Missile Design PC TRAP requires five non-zero vacuum thrust values best describing the motor thrust-time profile. Furthermore, the program requires the missile motor nozzle exit area (EXAREA), the missile motor ignition time after launch (TIGN), the time of motor burnout (TBO), the time of transition from the boost phase to the sustain phase (TISP), as well as the specific impulses for both the booster and sustainer phases (VISPS and VISPB respectively). The conceptual design vacuum thrust-time profile for the proposed missile is shown in Figure 5-2. The time of motor burn out (TBO) is 15 seconds, the missile motor nozzle exit area (EXAREA) is 0.0104 m^2, and the missile motor ignition time after launch (TIGN) is 0.0 second (right at launch). The properties of both the booster and the sustainer phases are included in Table 5-1.

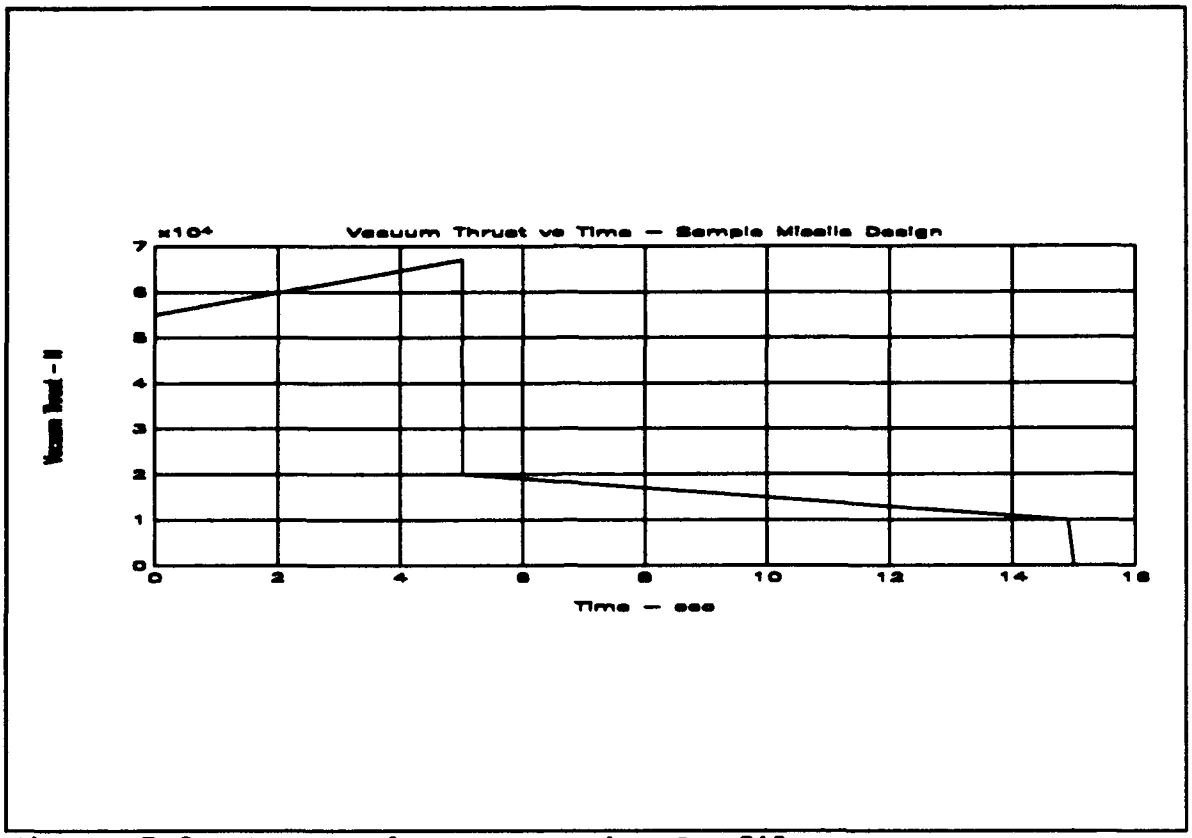


Figure 5-2. Vacuum Thrust vs. Time Profile

	Booster	Sustainer
I_{SP} (N sec/Kg)	2600	2550
Weight (Kg)	100	99
Burn Time (sec)	5	10

Table 5-1. Booster and Sustainer Characteristics

The instructions on how to select the five vacuum thrust values from the thrust-time curve are given in Chapter 4. These instructions were followed for this example as follows:

(1) Total Impulse: the equation to compute the total impulse (I_T) for a solid rocket motor is defined as follows:

$$I_T = \int_0^{t_T} F \, dt = W_P \, I_{SP} \, , \qquad (111)$$

where: W_P is the initial weight of the propellant (Kg);

F is the thrust profile (N); and

I_{SP} is the propellant specific impulse (N sec/Kg).

Using equation (111), the total impulse for the booster is 260,000 N sec, and the total impulse for the sustainer is 252,500 N sec. Hence the total impulse of the motor is 512,500 N sec.

(2) Set TIGN and TBO:

As mentioned above **TBO** is 15 seconds while **TIGN** is set to be at launch (0 second).

(3) Pinpoint Five Non-Zero Thrust Points

As detailed in the instructions given in Chapter 4, we must pinpoint five non-zero thrust values from the thrust-time curve of Figure 5-2. Those values have to be as representative of the curve behavior as possible. Then, using these five selected points, we must reconstruct the thrust profile curve and add up the areas of the resulting triangles and trapezoids under the curve to make sure that the total impulse of the estimated curve equals the total impulse of the real curve (Figure 5-2). Table 5-2 shows the five pinpointed thrust values with their corresponding time.

THRUST (N)	55000	67000	20000	20000	10000
TIME (SEC)	0.0	4.9	5	10	14.9

Table 5-2. Pinpointed Thrust Values and Corresponding Time

By computing the areas under the estimated thrust profile curve built from Table 5-2 data, a total impulse of 512,500 N sec was obtained, which is exactly the required value as computed above. Note that the total impulse is obtained by multiplying the thrust by the time in seconds, as prescribed by equation (1).

3. Missile Physical Characteristics

The physical characteristics of the missile shown in Figure 5-1 are as follows:

- Body diameter = 21 centimeters corresponding to a cross-sectional area of 0.0346 m^2 **(AREA)**

- The length of the nose is 0.8 meter and the length of the rest of the missile body is 2.4 meters

- The four canards are triangular fins with a chord of 10 centimeters and a span of 15 centimeters, with their leading edge located at 0.8 meter from the nose. Their shape is double wedge with a maximum thickness ratio of 6% located at 50% of the chord

- The four tail fins have a triangular leading edge with a rectangular expansion in the aft portion of the fins. The chord of the tail fins is 0.2 meter while their span is 0.2 meter. Their leading edges are located at 3 meters from the nose of the missile. Their shape is the same as for the canards

- The initial weight of the missile is 399 Kg **(INMSMS)** while its final weight, at motor burn out, is 200 Kg **(BOMSMS)**. The locations of the center of gravity is at 2 meters from the nose initially **(INITCG)** and at 1.6 meters from the nose at motor burn out **(BOCG)**. The center of gravity of the propellant is at *2.7 meters from the nose* **(CGPROP)**

4. Aerodynamic Data

Missile Design PC TRAP uses a minimum of aerodynamic input data to model a given missile. The program interpolates between these input data to estimate the instantaneous aerodynamic coefficients during the simulation. The missile input data file requires 12 values of axial force coefficients (C_A) at Mach numbers varying between 0.6 and 5, as well as a peak and final values and their corresponding Mach numbers. Furthermore, two values of normal force coefficients (C_N) are required: the trimmed normal force coefficients at five and fifteen degrees of angle of attack respectively (at Mach number = 1.4). If those values are not available, they can be estimated from the missile physical characteristics using the **MISDATCOM** computer program.

For the conceptual missile design which physical characteristics are detailed above and shown in Figure 5-1, **MISDATCOM** was used to compute its aerodynamic coefficients. The **MISDATCOM** input data file and associated output files for the conceptual missile design shown in Table 5-1 are included in Appendix D. The Appendix D **MISDATCOM** input file is composed of the following three cases:

- The first case is to obtain a detailed axial force coefficient versus Mach number curve, especially in the transonic region (between M=0.8 and M=1.2), to identify the peak value of axial force acting on the missile
- The second case is to obtain the axial force coefficient values at the Mach number required by Missile Design PC TRAP
- The third case provides the two required values for the trimmed normal force coefficient (C_{NTR})

The results shown in Appendix D are in a formatted output data file according to the **MISDATCOM** plot control card. The format for this type of output file is described at page 119, Example B of the **MISDATCOM** user's manual[ref. 8].

Note that the axial force coefficient at M=0.6 could not be computed by **MISDATCOM**. This value had to be extrapolated from the axial force coefficient versus Mach number curve obtained during the first case. For the third case, the values of the trimmed normal force coefficients were obtained by deflecting the four canard fins from -40 degrees to 39 degrees. It is important to note that if trimmed values cannot be obtained for the required angles of attack (5 and 15 degrees) when using **MISDATCOM**, a shift rearward of the center of gravity location might be required. Figure 5-3 shows the detailed axial force coefficient versus Mach number curve obtained from the results of the first case included in Appendix D. Note that the maximum angle of attack capability of the missile (**MAXALP**) could not be obtained from **MISDATCOM** (an engineering "guess" was required).

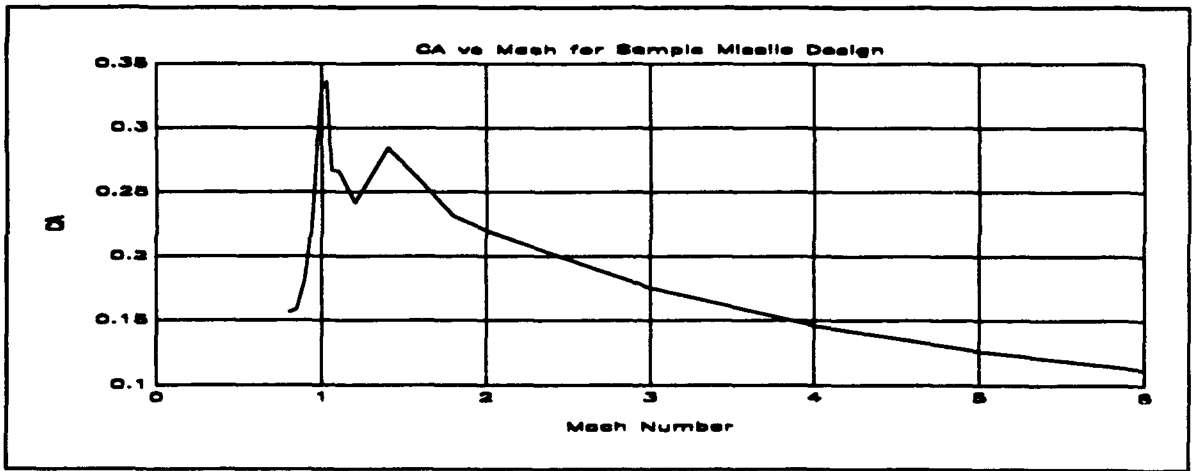


Figure 5-3. C_A vs Mach number curve

5. General Data

The rest of the input values required in the Missile Design PC TRAP input data file are general and are clearly detailed in the Appendix B input data dictionary. Most of these remaining values are normally determined by the missile design team based on desired missile handling qualities or on engineering evaluations. The complete missile data input data file, for this example, is included in Table 5-3 in a non-FORTRAN format. This data file is also included on the Missile Design PC TRAP distribution disk and is called **SAMPLE1**.

C. MISSILE PERFORMANCE EVALUATION EXAMPLES

1. SAMPLE1 Missile

The SAMPLE1 missile is the conceptual missile design proposed in the previous Section and shown in Figure 5-1. We will provide two example simulation runs to illustrate how we can evaluate the performance of this missile using Missile Design PC TRAP. We will look at a launch envelope generated for the SAMPLE1 missile and at the results of a Monte Carlo simulation run.

a. Launch Envelope with Varying Altitude

One way of verifying whether or not a conceptual missile design meets the specifications is to generate launch envelopes. SAMPLE1 is a medium range surface-to-air missile for which we will investigate its aerodynamic capability by generating a maximum range launch envelope with varying altitude.

Such a launch envelope was generated using the SAMPLE1 missile data input file shown in Table 5-3 with Missile Design PC TRAP. The resulting launch envelope is shown in Figure 5-4.

The launch envelope shown in Figure 5-4 illustrates the maximum aerodynamic range capability of the missile in the following initial intercept scenario: the initial missile velocity at launcher exit was 350 m/sec and

DT :	0.25000D-01	SAMPLE1 MISSILE DATA INPUT
AREA :	0.34600D-01	
MAXALP :	0.21500D+02	
MXVGCG :	0.36000D+02	
MAXTIM :	0.12000D+03	
CA6 :	0.15550D+00	
CA8 :	0.15650D+00	
CA9 :	0.18230D+00	
CA1 :	0.33060D+01	
CA12 :	0.24190D+01	
CA14 :	0.28430D+01	
CA16 :	0.25880D+01	
CA18 :	0.23170D+01	
CA2 :	0.22010D+00	
CA3 :	0.17600D+00	
CA4 :	0.14630D+00	
CA5 :	0.12610D+00	
CAP :	0.33610D+01	
MCAP :	0.10300D+01	
CN5 :	0.12853D+01	
CN15 :	0.43009D+01	
CAF :	0.10170D+00	
MCAF :	0.70000D+01	
RLCKON :	0.15000D+06	
SEKGAD :	0.60000D+02	
LSRLMD :	0.20000D+02	
ZFVLMD :	0.60000D+01	
TINGD :	0.40000D+00	
GBIASG :	0.10000D+01	
LOWMSV :	2.50000D+02	
LOWMSM :	0.00000D+00	
LOWCLV :	0.10000D+01	
LDVFAC :	0.10000D+01	
LDZFAC :	0.10000D+01	
AVGDLV :	0.00000D+04	equals m/sec for built-in lead angle
MDPERM :	0.17000D+02	
INMSMS :	0.39900D+03	
INITCG :	0.20000D+01	
BOMSMS :	0.20000D+03	
BOCG :	0.16000D+01	
CGPROP :	0.27000D+01	
EXAREA :	0.10400D-01	
TIGN :	0.00000D+00	
TBO :	0.15000D+02	
TISP :	0.50000D+01	
TTHR1 :	0.00000D+00	
TTHR2 :	0.49000D+01	
TTHR3 :	0.50000D+01	
TTHR4 :	0.10000D+02	
VTHR1 :	0.55000D+05	
VTHR2 :	0.67000D+05	
VTHR3 :	0.20000D+05	
VTHR4 :	0.20000D+05	
VISPB :	0.26000D+04	
VISPS :	0.25500D+04	
TTHR5 :	0.14900D+02	
VTHR5 :	0.10000D+05	

Table 5-3. SAMPLE1 Missile Input Data File

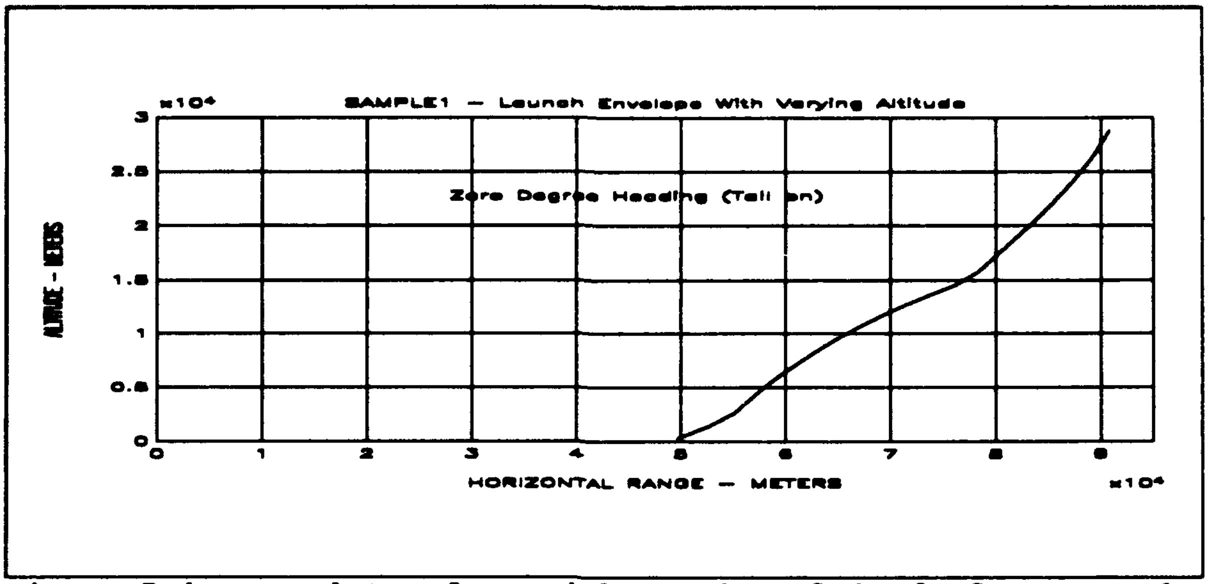


Figure 5-4. Launch Envelope with Varying Altitude for SAMPLE1 Missile

the azimuth aspect angle was zero degree (tail shot); the altitude band for the generation of the launch envelope was from 304 meters up to 30Km, and the guidance law was the proportional navigation with N=3.

The launch envelope shown in Figure 5-4 shows the maximum slant range capability of the SAMPLE1 missile for target altitudes varying from 304 meters to 30000 meters. It can be seen from Figure 5-4 that the SAMPLE1 missile has a very long range capability. We can also infer from Figure 5-4 that the maximum range capability of this surface-to-air missile increases with increasing target altitude.

b. Monte Carlo Simulation

Another simulation option to evaluate the performance of a missile is to perform a Monte Carlo simulation as described in Chapter IV. A Monte Carlo simulation run was performed with the SAMPLE1 missile data and the results are shown in Table 5-4. As a quick recall, a Monte Carlo simulation run is made of 50 missile simulations involving the same scenario with the same target maneuver. The only varying factor from simulation to simulation is the random initiation time of the selected target maneuver.

For the Monte Carlo simulation which results are shown in Table 5-4, the target maneuver was a 7g spiral. The target initial altitude was 10 Km, and the initial slant range was 15 Km at an azimuth aspect angle of 135 degrees.

The results shown in Table 5-4 could be misinterpreted. According to those results, an average miss distance of 25.09 meters was recorded during this Monte Carlo simulation run. However, 28 missile shots out of 50 had a total miss distance within the missile warhead lethal radius (17 meters) for a probability of kill equal to 56% (given a probability of fuzing equal to 100%). This probability of kill is good considering the large average miss distance shown in Table 5-4. Such a large average miss distance is caused by some missile simulations which recorded large miss distances over 200 meters and which greatly influence the statistics in a negative way. However, one must realize that those large miss distances were caused by target maneuvers occurring at a critical time during the missile flights, and that the same intercept conditions could be met in a realistic situation. Hence, one cannot ignore those data points. This is the reason why such Monte Carlo simulation results as shown in Table 5-4 shall be interpreted with great care.

MISS (X)	MISS (Y)	MISS (Z)	TOTAL MISS	-1 RIGHT	TTGO
79.79	83.55	3.68	115.59	-1	-639.6
-183.95	34.59	-182.71	261.57	-1	3.
20.25	25.89	-3.81	33.09	1	22.0
.84	.74	.07	1.12	1	7.0
-88.99	-15.37	-73.05	116.15	-1	2.5
55.19	77.15	15.04	96.05	1	1.5
.30	.34	.20	.49	-1	16.2
.07	-.02	.05	.09	1	6.4
.03	-.01	.02	.04	1	5.1
.21	.27	.19	.39	-1	15.1
-66.34	-37.09	-44.59	88.12	-1	2.4
13.18	18.70	-1.87	22.95	1	18.0
16.21	105.17	-2.72	106.45	1	1.7
81.43	90.57	2.27	121.82	1	.8
.02	-.04	.03	.06	-1	11.1
.01	-.01	.01	.02	-1	10.4
.01	-.01	.01	.02	1	2.9
-124.47	2.89	-119.07	172.27	-1	2.8
.06	-.01	.04	.07	1	6.7
9.51	11.44	-1.25	14.93	1	12.9
.01	-.02	.02	.03	-1	12.1
.19	.25	.18	.36	-1	15.9
.01	.01	.00	.01	1	6.4
69.21	76.25	13.05	103.80	1	1.4
-210.88	-40.35	-155.28	264.97	-1	3.5
-.30	-.42	.06	.52	-1	1.3
-.02	.02	.05	.06	-1	13.8
34.20	91.25	8.77	97.84	1	1.6
-14.30	-14.22	-1.21	20.20	-1	2.3
.02	-.03	.03	.05	-1	11.8
.02	-.02	.02	.04	-1	9.6
1.34	1.34	.01	1.90	-1	19.6
2.01	1.83	.12	2.72	1	8.3
.01	-.01	.01	.02	1	4.0
9.51	11.47	-1.26	14.96	1	13.1
10.20	23.14	2.74	25.43	-1	6.3
-28.40	137.15	-40.72	145.86	1	1.8
16.85	20.63	-2.63	26.77	1	19.7
-208.38	30.50	-197.45	288.69	-1	3.3
7.14	7.96	-.54	10.71	1	10.8
9.80	11.66	-1.18	15.27	1	12.7
.02	-.01	.01	.02	1	4.6
10.65	20.35	1.75	23.04	-1	6.7
.30	.34	.19	.49	-1	16.2
9.95	21.79	2.60	24.10	-1	6.4
.00	.00	.00	.00	-1	11.7
17.93	24.01	-3.61	30.18	1	21.2
6.77	7.49	-.48	10.11	1	10.5
79.36	82.30	2.81	114.36	-1	.5
.01	-.02	.02	.03	-1	10.2

	MEAN	STANDARD DEVIATION	
X AXIS	-7.27	60.33	(METERS)
Y AXIS	18.27	36.36	(METERS)
Z AXIS	-15.59	46.96	(METERS)

AVERAGE MISS DISTANCE FOR 50 RUNS: 25.09 METERS

Table 5-4 Monte Carlo Simulation Results

Similar Monte Carlo simulation results to those shown in Table 5-4 are often obtained for surface-to-air scenarios. As it will be seen below, air-to-air scenarios generally present much better Monte Carlo simulation results. Surface-to-air missiles seem to be more affected by target evasive maneuvers than air-to-air missiles.

2. GENERIC Missile

The GENERIC missile is a medium range air-to-air tactical missile which input data file to Missile Design PC TRAP is detailed in Appendix B. Two more examples will be provided with this missile. A launch envelope with varying azimuth will be shown, as well as a Monte Carlo simulation run for this air-to-air tactical missile.

a. Launch Envelope with Varying Azimuth

When one desires to evaluate the aerodynamic performance of an air-to-air missile, a launch envelope in the horizontal (or azimuth) plane may be generated. This is accomplished by setting the altitudes of both the target and the shooter. Then a span of azimuth angles for which it is desired to build the launch envelope must be established by the user. From these data, a launch envelope with varying azimuth can be generated by Missile Design PC TRAP. An example of such a launch envelope is shown in Figure 5-5. For this example, both the shooter and target were at the same altitude. The target initiated a 9g turn and run evasive maneuver at missile launch. The horizontal launch envelope shown in Figure 5-5 was built for an azimuth angle span varying from 0 to 180 degrees.

Then the resulting launch envelope was mirrored to give it the 360 degree aspect seen in Figure 5-5. Note that this launch envelope is shown in polar coordinate, which facilitates its interpretation.

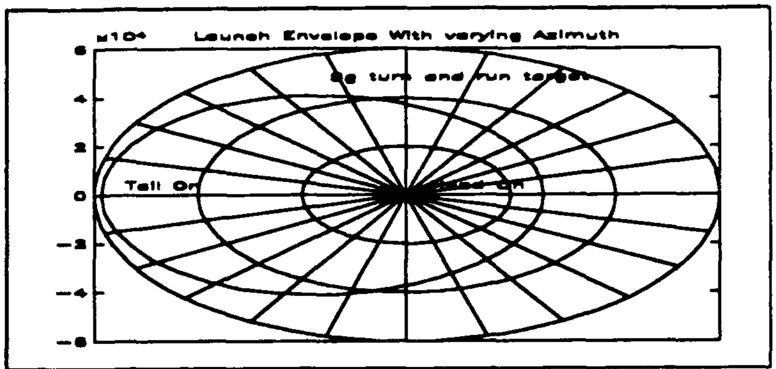


Figure 5-5. Launch Envelope with Varying Azimuth for the GENERIC Missile

b. Monte Carlo Simulation

Similarly to the above example with the SAMPLE1 missile, an example of a Monte Carlo simulation run is provided for evaluation of the GENERIC missile. The results are shown in Table 5-5 for a 9g turn and run target evasive maneuver.

As can be seen from Table 5-5, the Monte Carlo results for GENERIC are much better than the ones in Table 5-4 for SAMPLE1. We have an excellent average miss distance of 1.51 meters and very good standard deviations for both X and Y axis miss distances. Furthermore, a probability of kill of 94% was obtained as 47 simulation runs out of 50 had miss distances within the missile warhead lethal radius (13 meters). Such results demonstrate that the proportional navigation guidance law is very efficient in air-to-air intercept scenarios.

MISS (X)	MISS (Y)	MISS (Z)	TOTAL MISS	-1 RIGHT	TTGO
	(METERS)				
.17	-.20	.00	.26	-1	.3
-2.21	2.26	.00	3.16	-1	6.0
-33.52	39.49	-.05	51.79	1	4.5
-.02	-.01	.00	.03	-1	13.5
-.02	-.02	.00	.03	1	5.8
1.20	-1.34	.00	1.80	-1	24.4
1.76	-1.84	.00	2.55	1	14.5
2.02	-2.02	.00	2.85	1	14.0
2.06	-2.09	.00	2.93	1	8.3
.40	-.59	.00	.71	-1	20.1
-.02	-.01	.00	.03	-1	13.3
2.10	-2.14	.00	3.00	1	31.5
.00	-.02	.00	.02	-1	2.2
-.01	-.01	.00	.01	1	5.6
2.11	-2.11	.00	2.99	1	13.0
-.02	-.01	.00	.03	-1	13.2
1.99	-1.95	.00	2.79	1	11.2
-.07	-.02	.00	.07	-1	12.3
.99	-1.05	.00	1.44	1	6.9
.09	-.12	.00	.15	-1	15.6
.66	-.73	.00	.98	1	6.1
-.10	-.02	.00	.10	-1	11.8
-14.51	12.63	-.01	19.23	1	5.0
.14	-.43	.00	.45	-1	19.0
-1.31	1.28	.00	1.83	-1	4.1
.23	-.32	.00	.39	-1	1.2
1.68	-1.86	.00	2.51	1	17.5
-26.29	26.62	-.03	37.41	1	4.8
1.90	-1.87	.00	2.66	1	9.8
-.12	.07	.00	.14	-1	11.3
-2.20	2.22	.00	3.13	-1	5.5
1.96	-2.04	.00	2.82	1	8.1
.26	-.33	.00	.42	-1	1.1
-2.40	2.43	.00	3.42	-1	5.3
1.42	-1.53	.00	2.09	1	12.8
-2.44	2.46	.00	3.47	-1	5.6
2.42	-2.40	.00	3.40	1	11.0
1.73	-1.80	.00	2.50	1	12.3
-1.77	1.78	.00	2.51	-1	6.9
.06	-.14	.00	.15	-1	1.8
1.90	-1.92	.00	2.70	1	15.4
1.36	-1.35	.00	1.91	1	6.7
-2.45	2.46	.00	3.47	-1	5.7
.79	-1.07	.00	1.33	-1	21.6
1.65	-1.82	.00	2.45	1	35.2
-.21	.00	.00	.21	-1	11.6
.25	-.37	.00	.45	1	6.2
-.08	-.07	.00	.11	-1	13.4
2.35	-2.29	.00	3.28	1	10.1
1.67	-1.67	.00	2.36	1	8.2

	MEAN	STANDARD DEVIATION	
X AXIS	-1.05	6.50	(METERS)
Y AXIS	1.08	7.10	(METERS)
Z AXIS	.00	.01	(METERS)

Table 5-5. Monte Carlo Simulation Results

D. SIMULATION COMPARISONS BETWEEN TRAP AND MISSILE DESIGN PC TRAP

In order to evaluate the accuracy of the Missile Design PC TRAP computer program, its simulation results will be compared with the simulation results from TRAP for the same intercept scenarios. Since Missile Design PC TRAP is an abbreviated version of the main Vax TRAP computer program, the Missile Design PC TRAP simulation results shall be similar to those obtained with TRAP. The missile modeled by Missile Design PC TRAP for this investigation is a generic missile which extensive data is provided with the TRAP executable file (GENERIC). The Missile Design PC TRAP input data file was built from the detailed TRAP input data files of this generic missile.

In this section, missile trajectories generated by both programs will be compared to each other, as well as their missile lateral acceleration, thrust and velocity profiles. Also, the coefficients of normal force (C_N) and of axial force (C_A) will be compared, since TRAP uses extensive input files to estimate these coefficients while Missile Design PC TRAP uses a small amount of input values to estimate those same coefficients. Four air-to-air intercept scenarios were run on both computer programs: three involving the proportional navigation guidance law and one involving the pure pursuit guidance law. All scenarios were in a benign environment (i.e. no target maneuver). Some TRAP simulation limitations that were encountered during this investigation will first be discussed.

1. TRAP Limitations

TRAP is installed on the Vax computer system at the Department of Aeronautical and Astronautical. When TRAP was run for the purpose of this investigation, major limitations were encountered by the author.

According to the TRAP user's manual, this computer program is capable of simulating all three types of intercept scenarios: air-to-air, surface-to-air and air-to-surface. However, despite many attempts with different input parameters, the author could not simulate surface-to-air scenarios. It seems that the required input parameter combinations to simulate such a scenario could not be found. Moreover, since TRAP was developed for the USAF and that it is mainly used by USAF personnel, its user's manual discusses air-to-air scenarios only. Details on how to simulate the other two types of scenario are not provided in the user's manual. Hence, the incapability of simulating surface-to-air scenarios and the lack of instructions on how to simulate such scenarios appeared as a major limitation of the TRAP computer program.

Another limitation is the fact that the TRAP version owned by the Department of Aeronautics and Astronautics cannot model the beam rider and the command to line-of-sight (CLOS) guidance laws. It is mentioned in the description part of the TRAP user's manual that TRAP can indeed model those two guidance laws. However, it is mentioned later in the user's manual that this capability was to be added to later versions of TRAP.

Those limitations make TRAP a more complicated and less attractive simulation model. The TRAP algorithm and user's manual would need to be revised in order to render this program more user friendly and more accessible for intercept scenario simulations that are different than those from air-to-air missions. However, TRAP is a large computer program with excellent simulation capabilities for extensive and detailed air-to-air scenario simulations.

2. Scenarios Description

As mentioned above, four scenarios were simulated on bc... TRAP and Missile Design PC TRAP for result comparison purposes. The scenarios involved benign conditions since target maneuver flight paths are not the same for both programs. There would be no benefit in comparing intercept scenarios in which the targets fly different profiles from one program to the other. Hence, the scenarios that are considered for this investigation involved the same target flight paths for both simulation models. The four scenarios or profiles that are considered are each assigned a number from one to four. The intercept profiles under consideration for this investigation are detailed in Table 5-6.

As it can be seen from Table 5-6, different types of profile were selected for this investigation. There is one profile where the shooter is heading directly at the target's nose, two profiles where the shooter is heading slightly off the target's nose, and one profile where the attacker is pointing at 45 degrees off the target's tail. Additionally, the altitude difference between the shooter and target varies from profile to profile to provide different altitude bands in which the two simulation programs can be compared to each other.

3. Results

Each of the profiles described in Table 5-6 was simulated using both the missile Design PC TRAP and TRAP computer programs for a generic missile. The missile data input files to the TRAP program contain complete and detailed aerodynamic, propulsion, physical properties and guidance data. The missile data input file to Missile Design PC TRAP was built from these TRAP data files according to the requirements detailed in the Appendix B missile data input file dictionary.

Selected simulation parameters from both simulation programs were extracted for comparison purposes. These simulation parameters are plotted against each other and included in Appendix E for each profile described in Table 5-6. The selected simulation parameters that are used to compare both simulation programs are the following:

(1) missile trajectory: the missile trajectory is plotted in two planes, the horizontal (X_S and Y_S) and the vertical (X_S and Z_S) planes. This allows one to see how close both trajectories are from each other. It should be noted that for each profile flown, the trajectory of the target was exactly the same in both simulation programs.

(2) missile lateral acceleration: the missile lateral acceleration versus time of flight curves in both the azimuth and elevation planes. These are important parameters to monitor since lateral acceleration shapes the missile trajectory.

(3) delivered thrust: recall that the Missile Design PC TRAP models the rocket motor thrust profile using five points from the detailed thrust profile used in TRAP. This is the reason why it is interesting to compare the delivered thrust profiles generated by both programs.

(4) coefficient of normal force: the coefficients of normal force are estimated by both programs to determine the missile angle of attack and sideslip angle in order to achieve the required missile lateral acceleration in both missile planes. Recall that Missile Design PC TRAP uses only two trimmed coefficients of normal force to estimate the instantaneous coefficient of normal force.

(5) coefficient of axial force: this is probably the most important parameter to compare since the estimated coefficients of axial force are used to compute the drag force acting on the missile and on the longitudinal acceleration, which is integrated twice to obtain the missile position in space. This means that those coefficients must be estimated very accurately by Missile Design PC TRAP.

(6) angle of attack and sideslip angle: those angles are compared to see how accurate Missile Design PC TRAP can simulate the attitude of the missile.

(7) missile velocity profile: once again, this is a general simulation parameter that allows us to compare the performances of the missiles simulated by both programs.

Scenario	Guidance Law	Description
1	Pro Nav (N=4)	Co-altitude shot Shooter altitude 10 Km ,V_O=600 m/s Target altitude 10 Km, V_O=450 m/s Range=20.1 Km, Aspect=174.3 deg (5.7 deg off target nose)
2	Pro Nav (N=4)	Look down/shoot down shot Shooter altitude 5 Km ,V_O=400 m/s Target altitude 2 Km, V_O=350 m/s Range=9.734 Km, Aspect=180 deg (head on)
3	Pro Nav (N=4)	Look up/shoot up shot Shooter altitude 10 Km ,V_O=600 m/s Target altitude 14 Km, V_O=450 m/s Range=29.27 Km, Aspect=45 deg (45 deg off target's tail)
4	Pure Pursuit	Co-altitude shot Shooter altitude 10 Km ,V_O=600 m/s Target altitude 10 Km, V_O=450 m/s Range=20.1 Km, Aspect=174.3 deg (5.7 deg off target nose)

Table 5-6. Profile Descriptions

4. Discussion on Comparison Results

The main results of this investigation are included in Appendix E. Appendix E contains a plot comparing the results obtained from both programs for each of the seven comparison parameters detailed at the previous paragraph. Hence, there is a series of seven plots for each of the profile detailed in Table 5-6. The present paragraph discusses those results separately by profile number.

a. *Profile 1*

Profile 1 is a very simple profile were both the shooter and target are at the same altitude. Table 5-7 provides a comparative summary of the final intercept conditions obtained from both programs.

Since this profile is very simple and straightforward, the results obtained from both programs should be very similar. It is indeed the case as it can be seen from Table 5-7 where all the comparative parameters are very close to each other, except for the miss distance. The miss distance is different because Missile Design PC TRAP computes a refined miss distance, which is the real closest point of approach (CPA). On the other hand, TRAP stops the missile simulation as soon as the missile range from the target is within the missile warhead lethal radius. This means that for a warhead with a lethal radius of 13 meters, the miss distance can be as high as 13 meters while Missile Design PC TRAP attempts to capture the smallest miss distance possible. In other words, Missile Design PC TRAP should generally yield smaller miss distances. The F-pole distance is the launching aircraft-target separation at intercept.

	Time of Flight [sec]	Miss Distance [m]	F-Pole Distance [Km]	Final Missile Mach
TRAP	11.93	7.14	7.62	4.73
MD PC TRAP	11.94	1.03	7.64	4.7

Table 5-7. Profile 1 Final Intercept Conditions

From the comparative plots of Appendix E for Profile 1 (Figures E-1 to E-8), it can be seen that both TRAP and Missile Design PC TRAP missile trajectories (Figures E-1a and E-1b) are very similar, as well as their missile lateral acceleration profiles (Figure E-2). The thrust profiles (Figure E-3) are almost identical from missile launch up to about four seconds of missile time of flight (TOF). From the four to five second mark, there is a small difference in the thrust profiles. This small difference is caused by the fact that the thrust curve must be approximated by five points for Missile Design PC TRAP application, and that such an approximation may obviously cause some limitations at some point. However, when one considers that both thrust curves are still very similar despite this small limitation, the use of Missile Design PC TRAP at the conceptual design phase is very justifiable.

The comparison plots for the coefficients of normal and axial forces, Figures E-4 and E-5 respectively, demonstrate a very satisfying similarity between the values generated by both programs. In the case of the normal force coefficient (Figure E-4), both curves are almost identical from 0.5 second TOF until missile flight termination. The difference between both curves for the first 0.5 second of TOF does not seem to affect the rest of the missile flights. Similarly for the axial force coefficients, which demonstrate a small difference at the beginning of the missile TOF, but which settles nicely after the first second of flight. The overall axial force curves are not as identical as for the case of the normal force coefficients, however they are still very close to each other with the same trends.

Figure E-7 shows two sets of curves: the angle of attack and sideslip angles comparative curves. Both sets of curves show very good similarities in their portions after the first second of TOF, especially for the sideslip angle where both curves are almost identical. During the first second of TOF, we can observe transient responses which do not have the same behaviors for both program curves. These transient responses are more accentuated and acute for the TRAP simulations while they are smoother for the Missile Design PC TRAP simulations. However, these differences do not affect the rest of the simulation parameters. The last comparative plot for Profile 1 compares the missile true velocity profiles as computed by both programs. It can be seen from Figure E-7 that both velocity profiles are almost identical.

One factor that was noticed during these comparisons is the fact that the TRAP simulation involved a last second drastic maneuver to close on the target during the terminal phase of guidance. This fact can be observed from Figure E-2 which shows the missile lateral accelerations in both the horizontal and vertical planes. In the last second of missile flight, the TRAP missile pulled an extensive amount of "g's", which did not occur during the Missile Design PC TRAP simulation. This drastic maneuver can also be seen in Figures E-4 and E-6 where the coefficient of normal force, the angle of attack and the sideslip angles behavior for the TRAP simulation changes drastically in the last second of flight. Such a behavior is normally caused by the fact that for a homing missile, as the missile approaches the target, the line of sight rate information, provided by the seeker, becomes less noisy and more accurate. Hence, as the missile enters the terminal phase of guidance, the missile guidance computer must generate last second acceleration commands to compensate for the miss distance caused by the earlier noisy seeker data. This observation means that the seeker modelling must be different in the two missile simulation programs. TRAP models a more realistic seeker head while Missile Design PC TRAP models a perfect seeker head with no noise on the line of sight rate data. This fact might explain the slight difference in the total missile TOF.

b. Profile 2

Profile 2 is a look down shoot down profile where the shooting aircraft is at an altitude higher than the target. The aspect angle is head-on (180 degrees). Table 5-8 provides a comparative summary of the final intercept conditions obtained from both programs.

	Time of Flight [sec]	Miss Distance [m]	F-Pole Distance [Km]	Final Missile Mach
TRAP	8.23	10.31	3.81	3.57
MD PC TRAP	8.16	2.3	3.87	3.47

Table 5-8. Profile 2 Final Intercept Conditions

This profile is not as straightforward as Profile 1 due to the change of altitude which the missile has to go through during its flight. Such a profile was selected to observe how well Missile Design PC TRAP can correct its estimated aerodynamic parameters for altitude changes.

Table 5-8 shows a difference of 0.7 second in missile TOF, which is probably due to the change of altitude. As well, small differences in F-pole distances and final Mach numbers are observed from the terminal intercept data shown in Figure 5-5. The miss distance obtained from Missile Design PC TRAP is once again much smaller than the TRAP miss distance.

The comparative missile parameter plots are shown in Figures E-8 to E-14. Both missile trajectories are identical in the horizontal and vertical planes as shown in Figures E-8a and E-8b. The delivered thrust curve has the same behavior as the one for Profile 1, where both curves are generally very similar except for a small portion from the fourth to fifth second of missile flight.

Both normal and axial forces coefficient curves (Figures E-11 and E-12 respectively) have a transition period at the beginning of the missile flight where both simulation program values have some dissimilar trends. However, both programs generate almost identical normal and axial forces coefficients for the remaining of the time of flight.

c. *Profile 3*

Profile 3 is a look up shoot up profile where the shooting aircraft is at an altitude lower than the target. Profile 3 is a tail shot where the missile is launched at 45 degrees off the target's tail. Such a scenario requires a long missile TOF. Table 5-9 provides a comparative summary of the final intercept conditions obtained from both programs.

	Time of Flight [sec]	Miss Distance [m]	F-Pole Distance [Km]	Final Missile Mach
TRAP	40.55	4.91	21.996	2.7
MD PC TRAP	40.77	0.03	21.98	2.53

Table 5-9. Profile 3 Final Intercept Condition

This profile was tested to compare the performance of Missile Design PC TRAP with the performance of TRAP in a long time of flight scenario with some change in missile altitude. The results shown in Table 5-9 are very satisfying. Despite the long TOF, the Missile Design PC TRAP parameters are extremely close to the ones obtained from TRAP. Furthermore, the miss distance obtained with Missile Design PC TRAP (0.03 meter) is excellent and smaller than the one obtained from TRAP.

The usual comparative missile parameter plots are shown in Figures E-15 to E-21. Both missile trajectories are very close to each other as shown in Figures E-15a and E-15b. The main difference between the set of comparative plots shown in Appendix E for Profile 3 and the other two previous sets is the fact that the transition period is longer (approximately five seconds). After this transition period, all the comparative parameters generated by each simulation program are very close to each other. The normal and axial forces coefficients estimated by Missile Design PC TRAP are seem to compare extremely well with the ones generated by TRAP, as illustrated in Figures E-18 and E-19 respectively. The sideslip angle plot (Figure E-20) shows a major difference in the trend of this parameter during the transition phase. Both missiles have a sideslip angle that is opposite in direction from each other. However, since this difference occurs only during the transition phase, the remaining missile flight comparisons are not affected. In summary, Table 5-9 and Appendix E results for Profile 3 are very satisfying and demonstrate once again how well Missile Design PC TRAP represent an accurate substitute to the TRAP computer program.

d. Profile 4

Profile 4 is the same as Profile 1 except that Profile 4 uses the pure pursuit guidance law instead of the proportional navigation guidance law. Hence Profile 4 is a co-altitude shot in a front aspect at a range of 20.1 Km. Table 5-10 provides a comparative summary of the final intercept conditions obtained from both programs.

	Time of Flight [sec]	Miss Distance [m]	F-Pole Distance [Km]	Final Missile Mach
TRAP	11.90	80.03	7.65	4.68
MD PC TRAP	11.93	0.94	7.64	4.71

Table 5-10. Profile 4 Final Intercept Condition

It can immediately be seen from Table 5-10 results that there is a major difference in the miss distances for this profile. The TRAP missile has a large miss distance, so large that the missile did not even guide within its warhead lethal radius. On the other hand, the Missile Design PC TRAP guided within less than one meter from the target, which is a much superior performance than the

TRAP missile. By looking at the other comparative parameters included in Table 5-10, it can be seen that the rest of the results are similar.

The usual comparative missile parameter plots are shown in Figures E-22 to E-28. A quick glance at the missiles trajectory comparison (Figures E-22a and E-22b) shows that the trajectories generated in the horizontal plane by both simulation programs are quite different (Figure E-22a). This can be explained by the fact that both programs do not probably use the same algorithm to model the pure pursuit guidance law. The fundamentals of the pure pursuit guidance law algorithm used in Missile Design PC TRAP are detailed in Chapter III. The fundamentals for this guidance law as implemented in TRAP are not discussed in the TRAP user's manual. However, this is not a problem as such since there exist many different ways of implementing a guidance law into a missile guidance computer. The vertical trajectories of both missiles are very similar as shown in Figure E-22b.

The large miss distance achieved by the TRAP missile can be explained from Figure E-23, the missile lateral acceleration comparisons. It can be seen from Figure E-23a, the plot of the missile lateral acceleration in the horizontal plane, that both missiles pulled quite a large amount of g's in the end-game phase (last second of flight). It can also be seen from the same plot that both missiles did not achieve these accelerations in the same direction. In other words, the TRAP missile pulled last second g's in the direction opposite to where the actual target was, while the Missile Design PC TRAP pulled those g's in the right direction. This explains the large difference in miss distances. This fact can also be clearly seen in Figure E-27b, where the sideslip angles for both missiles are going in opposite directions during the last seconds of flight. As far as the other comparison parameters shown in Appendix E are concerned, they are all quite similar to each other.

e. Summary

In general, the selected simulation parameters, generated from both the TRAP and Missile Design PC TRAP trajectory analysis computer programs, that were compared for this investigation were quite similar. Minor differences were noted occasionally, but the overall performance of both missile simulation programs is very similar. This fact shall convince us that Missile Design PC TRAP represent an excellent substitute to the complex TRAP simulation program.

It was also noted that both programs seem to have the same algorithm to simulate missile trajectories using the proportional navigation guidance law since they generate very similar missile flight paths. However, this is not the case when simulating the pure pursuit guidance law, where it was seen with Profile 4 that both programs do not seem to have the same algorithms.

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

For this thesis, an improved version of PC TRAP was developed for tactical missile design applications. This improved version is called Missile Design PC TRAP. It is a simple and compact multi-purpose tactical missile simulation program that runs quickly on any IBM-compatible personal computer.

PC TRAP can simulate air-to-air missile combat engagements using only one coded-in guidance law (proportional navigation). PC TRAP can model one-on-one engagements and perform maximum range searches only. These features alone are not sufficient to use PC TRAP for tactical missile design and academic applications.

Using the algorithms of the PC TRAP computer program as a starting point, Missile Design PC TRAP was developed for tactical missile design and academic applications. The capability of simulating surface-to-air and air-to-surface intercept scenarios was added in Missile Design PC TRAP, as well as two types of missile guidance, homing and command guidance. In addition to the proportional navigation guidance law already included in PC TRAP, six other guidance laws were implemented in Missile Design PC TRAP as follows:

Homing Guidance

- Proportional navigation
- Pure pursuit
- Lead angle
- Augmented proportional navigation

Command guidance

- Command proportional navigation
- Beam rider
- Command to line-of-sight (CLOS)

The fundamentals of the seven guidance laws were described in great detail in Chapter III. The mathematical expressions to model each guidance law were derived in three dimensions by the author from the two-dimensional models described in Zarchan[ref. 9].

Four missile simulation options were implemented in Missile Design PC TRAP: maximum range launch envelope generation in the azimuth plane, maximum range launch envelope generation in the elevation plane, Monte Carlo simulation technique with the time of target evasive maneuver initiation as the stochastic variables, and optimal target evasive maneuver evaluation. These simulation options were added to the two following simulation options included in PC TRAP: single missile flyout simulation and maximum range search capability for a single missile flyout.

Both TRAP and Missile Design PC TRAP can simulate the launching aircraft and the target in three dimensions using simplistic flight path generators. Three different target maneuvers are offered in Missile Design PC TRAP: weave, offset maneuver and spiral. Only the offset target evasive maneuver is offered in PC TRAP. The time-to-go (t_{go} or TTGO) at which the target is to begin executing its evasive maneuver during a missile simulation was made an input to Missile Design PC TRAP. The launching aircraft in Missile Design PC TRAP is modeled as per the PC TRAP launching aircraft model.

The capability of capturing very small miss distances was implemented in Missile Design PC TRAP by offering the option to the user of changing the simulation integration step size (DT) for the duration of the terminal phase of the missile trajectory. It was clearly shown that with this option, Missile Design PC TRAP computes miss distances that are generally smaller than PC TRAP and TRAP.

The most attractive feature of both PC TRAP and Missile Design PC TRAP is their input procedures which require a minimum of missile aerodynamic, propulsion and physical properties data (57 missile data items as described in Appendix B). Contrary to PC TRAP in which the missile input data items are part of the program algorithms, the missile data input file containing the 57 missile-related input data items is an input to Missile Design PC TRAP. This allows the user to simulate any type of missile without having to re-compile the program for each new missile simulation.

This thesis provides a general description of a tactical missile and discusses tactical missile guidance and control functions, such as the phases of guidance and the types of guidance & control systems. The seven guidance laws implemented into Missile Design PC TRAP are detailed and derived for both homing and command guidance systems. An extensive description of the Missile Design PC TRAP algorithm is provided in order to facilitate any further modifications required to keep up with technology changes. A surface-to-air tactical missile conceptual design is offered as an example of a Missile Design PC TRAP possible application. From this conceptual design, a missile data input file to Missile Design PC TRAP is built as an example. Then a performance evaluation of this conceptual missile design is provided.

The performance of Missile Design PC TRAP was compared to the performance of the main frame TRAP simulation computer program in similar intercept scenarios (profiles). Four profiles were used for this investigation. The results showed that Missile Design PC TRAP provides overall results that are very close to the TRAP results. In fact, it was shown that Missile Design PC TRAP generally provides smaller miss distances than TRAP. Furthermore, the comparison of selected simulation parameters, (such as thrust profiles, missile trajectories, aerodynamic coefficients and missile airspeed) showed that both programs generate simulation parameters that are quite close to each other, except for a small portion at the beginning of the missile flights where a noisy transition phase was observed in each case. This fact provides a great level of confidence in the use of Missile Design PC TRAP as a compact substitute to main frame simulation models, especially to TRAP. Indeed, based on these results, Missile Design PC TRAP showed that it is an excellent substitute to more complex main frame simulation models, such as TRAP, for conceptual missile design, for trade-off studies, for academic purposes and for military operational applications. Finally, a user's manual, providing quick and handy direction on how to use the Missile Design PC TRAP computer program, is provided in Appendix A.

The following modifications to improve the already extensive capabilities of Missile Design PC TRAP are recommended for further work in the matter:

- At the moment, Missile Design PC TRAP is a deterministic simulation program, except for one simulation option (Monte Carlo) which is a target related stochastic process. More stochastic processes should be integrated, such as noise on the seeker data and on the radar signal pattern
- Improve the air-to-surface scenario to allow for scenarios such as sea skimming
- Provide on-screen graphic capability to plot user-selected simulation parameters
- Integration of an optimal guidance law using Kalman filter principles
- Obtain a version of **PC MISDATCOM** and integrate it to Missile Design PC TRAP. This way, the user would only need to input the physical properties of a tactical missile, and the Missile Design PC TRAP/PC MISDATCOM combination would compute the required aerodynamic coefficients and perform the desired simulation runs all in one input operation
- Integrate a minimum range search capability;

- **Re-write Missile Design PC TRAP in a computer language specialized in simulation**
- **Implement non-linear aerodynamics to Missile Design PC TRAP**

APPENDIX A

MISSILE DESIGN PC TRAP USER'S MANUAL

SOFTWARE USER'S MANUAL

FOR THE MISSILE DESIGN

PC TRAJECTORY ANALYSIS PROGRAM

(MISSILE DESIGN PC TRAP)

SOFTWARE USER'S MANUAL

FOR THE MISSILE DESIGN

PC TRAJECTORY ANALYSIS PROGRAM

(MISSILE DESIGN PC TRAP)

23 September 1993

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1.0 - INTRODUCTION

The Missile Design Personal Computer Trajectory Analysis Program (**Missile Design PC TRAP**) is a missile trajectory analysis program for preliminary design and academic applications. It is built to assist the missile designers in evaluating missile flight performances by modelling missile guidance, including the seeker, missile propulsion, missile aerodynamics and missile flight control functions. The **Missile Design PC TRAP** uses a linear three Degrees-of-Freedom (DOF) set of ordinary differential equations to provide point-mass missile simulations in three dimensions, and can be used from the early design phases to run missile flyouts to determine typical missile flight conditions and associated static stability and control derivatives, up to the late design phases to generate missile launch envelopes.

The **Missile Design PC TRAP** may also be used as an academic tool to study the different guidance- and geometry-related dynamics involved with the tactical missile technology. The **Missile Design PC TRAP** was built to provide a missile trajectory analysis algorithm into a form that runs quickly on a Personal Computer. Applications at the Naval Postgraduate School (NPS) include missile simulation demonstrations, missile flight analysis student projects, missile preliminary and final design performance evaluations, as well as launch envelope generation and Monte Carlo simulations. Applications outside NPS may include simulators, or facilities with limited mainframe hardware capabilities (such as fleet and squadron), or for missile simulation programs with real-time graphics requirements.

Missile Design PC TRAP can simulate air-to-air, surface-to-air and air-to-surface intercept scenarios. It has a built-in capability to simulate seven different guidance laws. The inputs to the program include a data file which contains 57 missile related data, as well as user-friendly color-coded input panels or menus interrogating the user on the initial set-up of the intercept scenarios. **Missile Design PC TRAP** can graphically portray (in color) a plotted history of the launching aircraft, the missile and the target as the simulation is being performed.

The Missile Design PC TRAP is a modified version of the PC TRAP (version 3.12) computer program developed by the Foreign Aerospace Science and Technology Center (FASTC) for the United States Air Force (USAF). The original PC TRAP is a condensed and abbreviated version of the main frame Trajectory Analysis Program (TRAP) used by the USAF to conduct complete and extensive missile simulations. This user's manual is an improved version of the PC TRAP user's manual.

Missile Design PC TRAP was compiled with MICROSOFT FORTRAN Professional Development System Version 5.1 (1991). The program runs approximately real-time (one second of simulation for one second of flight) on a 33 MHz 80486 computer chips with instantaneous graphic display of the simulated vehicles. The program runs much faster if the user chooses not to display the engagement graphically.

This manual is the user's manual that will supply handy and quick directions to the program user. This manual does not describe the algorithms, which are detailed in Chapter IV of Capt (Canadian Air Force) Daniel Gibeau's thesis. **Missile Design PC TRAP** can easily be modified to meet user's specific simulation requirements, with the use of a FORTRAN compiler. However, to help the user, it is highly recommended to read Chapter IV of Capt Gibeau's thesis before making any modifications, since Chapter IV is very descriptive of the program algorithm.

Finally, as mentioned above, **Missile Design PC TRAP** originated from PC TRAP (version 3.12) developed by FASTC for the USAF. In order to meet the missile design and academic requirements of the Naval Postgraduate School Department of Aeronautics and Astronautics for a missile trajectory analysis program, PC TRAP was extensively modified. For this reason, any question or problem concerning **Missile Design PC TRAP** shall be first directed to Professor Conrad F. Newberry at the Department of Aeronautics and Astronautics, and not directly to the FASTC or the USAF.

2.0 - MISSILE DESIGN PC TRAP USER'S MANUAL

This manual uses the following conventions:

Example of Convention	Description of Convention
COMMAND	All non-bold capital letter words indicate a command, either a Disk Operating System (DOS) command or a program application.
[ENTER]	This indicates the Carriage Return key or Enter key. This key must be pressed after each command, input parameter or program application.
' input value '	The single brackets indicate a specific input parameter or input value. **Missile Design PC TRAP** provides a blank or underline character at the cursor position where input is required.
FILENAME.EXT	This is an example of a DOS filename, no longer than eight characters for the first section of the filename, followed by a three character extension to indicate the type of file.

TIME All capital letter words written in bold font represent variables as they are found in the algorithm (source code files) of the program.

2.1 SOFTWARE AND HARDWARE REQUIREMENTS AND INSTALLATION

2.1.1 Software Requirements

The **Missile Design PC TRAP** algorithm is composed of four FORTRAN files: **UTRAP.FOR, UTRAPA.FOR, UTRAP1.FOR** and **UTRAP2.FOR**. These four files must be compiled and linked together to form the **Missile Design PC TRAP** executable file. These four FORTRAN files are included on the distribution disk. The executable file is called **MDPCTRAP.EXE**.

A FORTRAN compiler is required to re-build the **Missile Design PC TRAP** executable file after any modification is made to any of the aforementioned FORTRAN source files. It is recommended to use the Microsoft FORTRAN compiler since **Missile Design PC TRAP** uses library files for graphic display that are provided with the Microsoft FORTRAN compiler.

2.1.2 Hardware Requirements

Missile Design PC TRAP may be executed from either a floppy disk or a hard disk on any IBM-compatible computer. However, performance is significantly better from a hard disk. A math co-processor is required to run this program. **Missile Design PC TRAP** is graphics based, which means that either of the following graphic adapter is required: Color Graphics Adapter (CGA), Enhanced Graphics Adapter (EGA), the Multicolor Graphics Array (MCGA), the Variable Graphics Array (VGA), SVGA, or the equivalent AT&T graphics adapter. The program runs approximately real-time (one second of simulation for one second of flight) on a 33 MHz 80486 computer chips with instantaneous graphic display of the simulated vehicles. The program runs much faster if the user chooses not to display the engagement graphically.

2.1.3 Hard-Disk Installation

Before installing PCTRAP on a hard disk, run CHKDSK [ENTER] from DOS to determine the amount of free space available. You need at least 1.5 megabytes free for the program, the documentation, the FORTRAN files and the data files it may generate during execution. Also, **Missile Design PC TRAP** requires that your system's *CONFIG.SYS* have a "FILES=10" or greater so that multiple output files may be opened properly.

The manual installation of **Missile Design PC TRAP** on a hard disk may be accomplished as follows: At the "C:\" prompt of your hard drive (or the appropriate letter for your drive), type the following DOS command: "MD\MDPCTRAP [ENTER]". This will make a directory on your drive called C:\MDPCTRAP. Place the **Missile Design PC TRAP** distribution disk into drive "A:" and subsequently change to that drive.

Then, from the "A:\" prompt, type: COPY A:*.* C:\MDPCTRAP [ENTER]. This will copy all the files and programs from the "A:" drive onto your hard disk in the MDPCTRAP subdirectory.

Remove the original distribution disk from the "A:" drive and put it away for safe keeping. Change to the "C:\" drive and issue CD\MDPCTRAP [ENTER] (CD is a DOS command for changing directories) to enter the MDPCTRAP subdirectory. You are now ready to execute **Missile Design PC TRAP** simply by typing MDPCTRAP [ENTER] from your **Missile Design PC TRAP** subdirectory.

2.2 PROGRAM EXECUTION

Change subdirectory to MDPCTRAP on your hard drive or insert the working floppy disk into the "A:" drive and type in MDPCTRAP [ENTER]. An introductory title screen will appear as shown in

UNCLAS

NAVAL POSTGRADUATE SCHOOL
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

MISSILE DESIGN

PC TRAP

MISSILE DESIGN PCTRAP 1-vs-1 1-SHOOTER ENGAGEMENT

Version 3.5 - PC TRAP Adapted for Missile Design Purposes

Press ENTER to continue

UNCLAS

Figure A-1. Introductory Title Screen

Figure A-1.

Press [ENTER] to continue and carry on to the next menu which includes the first input panel, and which will appear as shown in Figure A-2.

```
                              UNCLAS
                MISSILE DESIGN PCTRAP 1-SHOOTER ENGAGEMENT
                MISSILE DATA AND TYPE OF ENGAGEMENT INPUT


 ENTER NAME OF INPUT FILE: _GENERIC
 (Enter EXIT to quit the program now)

  THE INPUT FILE WAS SUCCESFULLY READ


Select the type of engagement (0, 1, 2 or 3):

        1. Air-to-Air Engagement
        2. Surface-to-Air Engagement
        3. Air-to-Surface Engagement
        0. To exit now

                              UNCLAS
```

Figure A-2. Missile Data and Scenario Input Panel

The two lines prompting the user for the name of the input file will first appear by themselves. The user must enter here the name of the input file in which the 57 missile related data items are included according to the format detailed in Appendix B. A missile data file containing generic missile data is provided on the distribution disk and is called "**GENERIC**". After the user has entered the name of the input file and press [ENTER], a message will appear to indicate if the input file has been successfully read or not. If it was not successfully read, something must be wrong with the name of the file or with the format of the input file.

When the input file has been successfully read by the program, the user is then prompted to input the type of engagement or the type of scenario to be simulated by the program. As mentioned above, **Missile Design PC TRAP** can simulate missile trajectories for the three following type of scenarios:

- Air-to-Air;
- Surface-to-Air; and
- Air-to-Surface.

Different input parameters are required for each type of scenarios. This means that there is not a unique sequence of presentation of the input parameters menus. Each series of parameter menus for each type of scenario will be presented and discussed in this user's manual. Input parameters that are common to more than one type of scenarios will be discussed only once. For this reason, it is recommended to read the following Section which presents the series of input menus related to air-to-air engagements, and which explain many input parameters that are required in the other types of engagement.

2.2.1 Air-to-Air Engagements

If '1' is selected in the last input field of Figure A-2, an air-to-air engagement will be simulated by the program and the parameter menu shown at Figure A-3 will appear on the screen.

```
                         AIR-TO-AIR PARAMETERS MENU

Type of input (SELECT 1 or 2):    1. Metric (m,m,m/s)
 2                                2. English (nm,ft,kts) ?

Shooter Initial Conditions:

 Maneuvers:  ( 1, 2, or 3 )
 1           1. None
             2. Pursuit
             3. Offset          _____ G    ________ Ang
  10000.00 Altitude               .80   Velocity or Mach

                     *****  TYPE OF SIMULATION  *****

 The following options are available:
    [1] - Single shot with user defined initial range
    [2] - Single shot with maximum range search
    [3] - Launch Envelope search with varying azimuth
    [4] - Launch Envelope search with varying altitude
          (for Surface-to-air and Air-to-surface)
    [5] - Monte Carlo runs with random initiation of target maneuver
    [6] - Target Optimal Evasive Maneuver
    Type of Simulation (1,2,3,4,5): 1
                 PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-3 Air-to-Air First Parameter Menu

2.2.1.1 Units

The first panel input area is the same for all engagement types. It is for the selection of either metric (SI) or English units. Selecting '1' will require all program inputs to be in meters and cause all program outputs to be in either meters (m) or kilometers (km). Conversely, selecting '2' will treat all inputs and outputs in nautical miles (nmi) for range, feet (ft) for altitude, and knots (kts) for velocity. The program default value is '2', or English units, which can be selected by hitting [ENTER], or overridden by entering a '1' and [ENTER] if the SI units are preferred by the user. This default feature is common to each entry on each menu panel of this program. Note that if the user does not accept the default values, the new input values will be written over the default values. This means that some portions of the default values may be still seen in the input areas. However, the program only considers the new user-inputs.

2.2.1.2 Shooter Initial Conditions

For air-to-air engagements, the program requires the shooter initial conditions. The first input involves the shooter maneuver after missile launch. Some simplistic shooter maneuvers have been coded in for the user to evaluate the effects of some rudimentary shooter maneuvers on the intercept capability of the missile. Note that the launching aircraft simulation routine can be replaced by a user flight path generator. This allows the user the flexibility of evaluating missile performance with an established external source for the aircraft performance.

Altitude and velocity are the next input under shooter initial conditions. The shooter altitude is the altitude at which the missile is launched from the aircraft. The shooter velocity may be entered as either m/sec (if SI is selected), knots (if English units are selected), or Mach number. The program assumes that any velocity value input under '5' indicates that the velocity is entered in Mach number. Note that the simulated missile has a flight termination criterion that stops the missile flight when the missile speed drops down to launch speed. This action maximizes the F-Pole of the shooter aircraft.

2.2.1.3 Types of Simulation

The program can perform either one of the types of simulation shown in the type of simulation input area shown in Figure A-3. The choice made in the types of simulation input area by the user determines the next input panel that will appear on the screen.

2.2.1.3.1 User-Defined Initial Range and Azimuth

If '1' is entered, the program will perform a missile trajectory simulation with user-defined missile-to-target slant range and horizontal aspect angle. The input panel that will appear is shown at Figure A-4. The target initial conditions input area, shown at Figure A-4, will be discussed at the end of the types of simulation sub-section (paragraph 2.2.1.3) since its input parameter requirements are common to all four options shown at Figure A-3.

Range is the first input required for the input panel shown in Figure A-4. The units for the initial range input are nautical miles (for English) or meters (for SI). The range required is the actual slant range, or radar range from the point-mass launching aircraft to the point-mass target. Note that if there is an altitude difference between the target and the launching aircraft, the range input is not equal to the downrange.

```
                              PARAMETERS MENU (Contd)

        5.00 Range           45.0 _ Aspect (180=Nose)



           TARGET INITIAL CONDITIONS

     Maneuvers:  ( 1, 2, 3 or 4 ) 1
                 1. None              4. Spiral     _____ G
                 2. Weave       _____ G
                 3. Offset      _____ G  ________ Ang
       9000.00 Altitude            .90  Velocity or Mach

    Print interval(-1=No graph):    .10     (sec)

                 PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-4. Air-to-Air Parameter Menu with Range as Input

The next input field is for the initial aspect angle or heading angle between the launching aircraft and the target, in the azimuth plane. The input value must be in degree(s). In this simulation program, the target always has an initial heading angle of zero degree as set by the program, which corresponds to a straight and level flight path from left to right parallel to the X-axis on a fixed coordinate system. The launching aircraft aspect angle required for the program varies from zero degree for a tail chase initial scenario, to 180 degrees for head-on scenario. A simulation scenario requiring a 30 degrees off the tail initial scenario would therefore have an input value of 30 degrees, while a 30 degrees off the nose would be input as 150 degrees. A 90 degree initial aspect is a beam shot. Initial elevation heading or elevation aspect angles are computed by the program based on the range input and the altitude difference between the target and the launching aircraft. Shots with a horizontal aspect angle input greater than 180 degrees will not be implemented by the program.

2.2.1.3.2 Maximum Range Search

If '2' is entered, the program will perform a search to find the maximum range capability of the missile based on the input initial conditions for the given value of aspect angle. To do so, the program performs several missile flight simulations by varying the initial range until it finds the range at which the missile end-game miss distance is barely within the warhead lethal radius (**MDPERM**).

If this option is chosen by the user, the on-screen menu panel will be the same as the one shown in Figure A-4, except that the range input field will not be present on the screen. The first input field is then the launching aircraft aspect angle in the horizontal plane, which must be entered as detailed above. While performing its missile maximum range capability search, the program prints search status updates to the screen. Those updates show the current initial range being investigated, as well as the miss distance of the previous range simulation.

Sometimes, the search pattern gets hung up at one point or another. The program will stop the search by itself when the amount of iterations to obtain the maximum range becomes to high. If this occurs too much, it is recommended that the user modify the search logic to satisfy his/her requirements. Normally, it should not take more than 1 or 2 minutes to complete a maximum range search on a 486 computer chip.

2.2.1.3.3 Launch Envelope (Varying Azimuth)

If '3' is chosen, the program will perform maximum range capability searches, as detailed above, for a user-defined span of aspect angles in the azimuth plane. This is accomplished to generate a missile launch envelope that indicates missile aerodynamic performance capabilities in the horizontal plane. If this option is chosen by the user, the input menu panel shown in Figure A-5 will appear on the screen.

In this input panel, the first input field is for the minimum value (lower bound) of the azimuth aspect angle span. The second input is for the maximum value (upper bound) of the azimuth aspect angle span. These two inputs must be in degree and represent the limits for the desired span of aspect angles within which the launch envelope will be generated. The last input field is for the number of equally spaced intervals within the aspect angle span. A missile maximum range search will be conducted at each of these aspect angle interval points. For each of the searches, the maximum range and its corresponding aspect angle value are stored in a matrix within the program, which is printed in an output file called **ENVELOP.DAT** when the launch envelope search is completed. **Missile Design PC TRAP** does not display the launch envelope on the screen, which means that plot generations must be performed with an outside plotting routine.

The number of intervals determines the number of maximum range searches that will be conducted by the program. The computing

```
                         PARAMETERS MENU (Contd)

Choose an azimuth span between 0 and 180 degrees
Minimum azimuth angle:     .0
Maximum azimuth angle: 180.0
# of interval within azimuth aspect angle span
(50 Max): 25


          TARGET INITIAL CONDITIONS

Maneuvers:  ( 1, 2, 3 or 4 ) 1
            1. None              4. Spiral    _____ G
            2. Weave     _____ G
            3. Offset    _____ G   ________ Ang
  9000.00 Altitude          .90   Velocity or Mach

Print interval(-1=No graph):  -1        (sec)

              PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-5. Air-to-Air Parameter Menu for Azimuth Launch

time to generate the whole launch envelope is therefore directly proportional to the number of intervals selected by the user. The program default value is 25 intervals, which provide a very accurate launch envelope representation for a 180 degree span, and which takes approximately 35 minutes of computing time on a 486 computer chips. This is a good time when compared to the four to five hours that the main Vax TRAP computer program takes to conduct the same tasks. Note that it takes a little bit more time to generate a launch envelope in a scenario where a target maneuver is present. The maximum number of intervals is set to 50 in the program to avoid running out of memory in the process. If the user requires more than 50 intervals within a certain aspect angle span, the program can be run several times with a reduced span of aspect angle.

2.2.1.3.4 Launch Envelope (Varying Altitude)

When '4' is entered, the program will perform maximum range searches with a constant user-defined value of aspect angle at varying target altitudes. This task is performed by the program in a very similar fashion as for the latter option (paragraph 2.2.1.3.3), except that the target altitude varies from simulation to simulation.

Once this option is selected, the input panel shown in Figure A-6 appears on the screen. The aspect angle must first be entered by the user as explained above and remains constant for the entire launch envelope generation process. Then, the second and third input parameters are the minimum and maximum altitudes, respectively, which defines the altitude band for the generation of the launch envelope. The fourth field is for the number of intervals within the selected altitude band. The more complex input choice here is for the maximum altitude. The user

```
                         PARAMETERS MENU (Contd)

45.0 _ Shooting Aspect (180=Nose)

Choose an altitude span between 1 and 100 000 feet
Minimum altitude: _____   1000.00
Maximum altitude: _____25000.00
# of interval within altitude band
(50 Max): 25



          TARGET INITIAL CONDITIONS

Maneuvers:  ( 1, 2, 3 or 4 ) 1
            1. None              4. Spiral      _____ G
            2. Weave     _____ G
            3. Offset    _____ G   ________ Ang
                           .90  Velocity or Mach

Print interval(-1=No graph):  -1.00     (sec)
                  PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-6. Air-to-Air Parameter Menu for Altitude Launch Envelope

shall make sure that the missile can make it to this maximum altitude when launched from the given launching aircraft altitude. If a non-achievable maximum altitude is entered, the program will hang-up at the missile true maximum altitude as it is varying the target altitude from search to search, and the program will produce a run time error which will terminate the program at this point. Note that the second part of the input panel shown in Figure A-6, for the target initial conditions, is slightly different from the one shown in Figure A-5 because the target altitude is not a program input for case '4' of the panel shown at Figure A-6.

2.2.1.3.5 Monte Carlo Simulations

If '5' is entered from the type of simulation input area shown in Figure A-3, the program will perform a Monte Carlo simulation. The Monte Carlo simulation option provides a simulation model option driven by a stochastic process. The Monte Carlo simulation technique provides an excellent way to evaluate missile system performance.

The stochastic process is provided by a target maneuver with a random starting time (uniformly distributed over the missile flight time) as the source of error. For each Monte Carlo simulation run, 50 missile flights are performed in the same initial intercept scenario with the same target maneuver for each flight. The only different parameter from flight to flight is the time of initiation of the target maneuver, which is a parameter that greatly affects the performance of the missile. When the 50 missile flights of a Monte Carlo simulation run are completed, the standard deviation and mean of the 50 miss distances are computed by the program and provided as an output.

When the Monte Carlo type of simulation is chosen by the user, the input process is exactly the same as for the case described in Paragraph 2.2.1.3.1 for a user defined initial range and azimuth angle single shot. However, the user must ensure that a target maneuver is selected (see Paragraph 2.2.1.4) for the Monte Carlo simulation option to produce significant results.

2.2.1.3.6 Optimal Target Evasive Maneuver

If '6' is entered from the type of simulation input area shown in Figure A-3, the program will conduct an optimal target evasive maneuver evaluation. Such an evaluation provides results from 50 simulation flights in which the time of initiation of the target maneuver was equally varied from missile launched up to missile intercept. The miss distance results and their corresponding time of target maneuver initiation (time-to-go) are printed to an output file called **OPTIM.DAT**.

2.2.1.4 Target Initial Conditions

The program requires initial conditions for the target. The initial conditions input panel is as shown in Figures C-5 and C-6, and is similar for all type of simulation options discussed above.

The first input under target initial conditions involve target maneuvers. Three simplistic but realistic evasive maneuvers have been coded in for the target. This allows the user to conduct missile performance evaluation against different target evasive maneuver scenarios. The weave is a target maneuver where the target conducts a series of "S" turns in the horizontal plane with no change in altitude. The offset maneuver is a target turn away from its original horizontal plane heading angle until the target reaches a final user-defined heading angle, at which point the target resumes with a 1g turn. The spiral target maneuver is self explanatory. The spiral is the only target maneuver causing a change in target altitude.

If the user elects to use a target evasive maneuver during the missile simulation by selecting '2', '3' or '4', a supplementary input field will appear, prompting the user to enter the Time-to-Go (TTG) at which the target will initiate its maneuver. This allows the user to study the effects of different target maneuver initiation times on the missile overall performance. If '-1' is entered, the target will initiate its maneuver at missile launch. The default TTG is 10 seconds, which means that the target would initiate its maneuver 10 seconds prior to the estimated time of intercept. Note that the estimated time of intercept is based on the target flying a straight and level flight path. A target maneuver will slightly alter the estimated time of intercept, which means that the actual time-to-go may differ from the value entered by the user.

Also, if the user selects a target maneuver, the amount of "g" to be pulled by the target during this maneuver becomes a program input. The amount of g may be a positive or negative integer input ranging from -25 to 25. The sign of the input value determines the direction of the maneuver. A positive sign means that the target will turn to the left in the azimuth plane, while a negative g load will make the target turn to the right. For the weave, only positive g load values will be implemented by the program. For the offset target maneuver, the aspect angle at which the target will stop pulling the amount of "g" selected by the user is also a program input.

Once the initial target maneuver parameters are entered into the program, the next input field is the target altitude, when applicable. Enter the target altitude either in feet or in meters according to the user-defined working units. The program can handle look-up shoot-up, co-altitude and look-down shoot-down shot scenarios.

Target velocity is the next entry. As for the shooter initial velocity input, the target velocity may be entered as either m/sec (SI units), knots (English units), or Mach numbers. The program assumes that any velocity input value under '5' indicates that the velocity is entered in Mach number.

2.2.1.5 Print Rate

The last input field of the current menu panel is for history print or update interval. With this input value, the user can control the print rate. Note that more frequent print will cause the program to run slower and vice versa. Updated on the screen at the specified print interval are the plotted x and y coordinates and altitude differences, if any, of the three vehicles, as well as trajectory limitation messages. While these on-screen graphics are taking place, up to eight data output files are created at the selected print interval. A '-1' may be input if no graphics are desired, in which case only the final solution will be printed to the screen and to the output files. Non-graphics simulation run times are much smaller than graphics simulation run times. **For this reason, it is highly recommended to input '-1' in this field when generating launch envelopes.** When this input panel is completed, press [ENTER] to proceed to the next menu panel.

2.2.1.6 Homing Guidance Law Selection

The following menu panel is shown in Figure A-7. The first panel input area is for selection of the guidance law that is to be used during the missile simulation run. For air-to-air scenarios, there are four different guidance laws that can be used in this program to guide the missile towards the target. The user can select the desired guidance law by typing in its corresponding integer value from '1' to '4'. If a guidance law using proportional navigation principles is used, the program will move to an input field which will prompt the user for **N**, the proportional navigation constant, which is normally an integer between 2 and 6. Effects of the proportional navigation constant on missile performance are detailed in the literature, especially in Zarchan.

Each guidance law normally generates a different missile trajectory from the others. This is the reason why it may be interesting to compare the performances and trajectories generated by each guidance law in the same initial intercept scenario. For this reason, by entering '-1' in the **Homing Missile Guidance Law** input field, the four guidance law trajectories are simulated by the program one after the other. The four trajectories can then be immediately compared to each other on the screen via their vehicle time history plots. After the completion of each trajectory simulation, a summary of the results for this specific guidance law is presented in the upper left corner of the vehicle time history graph. Note that the user must press [ENTER] between each guidance law simulation to initiate the next simulation run. At the end of the four simulation runs, the program prints a comparative summary of the basic missile performance results achieved by each guidance law.

2.2.1.7 Integration Step Size

The next input field in the current menu panel is on the integration step size. The integration step size is fixed by the user for most of the missile flight with the input of **DELTAT** in the missile data input file. The recommended value for most of the missile flight is 0.01 second. However, if it is desired to accurately capture the magnitude of the end-game miss distance, the integration step size must be made much smaller near the end of the missile flight. On the other hand, the size of the integration step size greatly affects the simulation time. Smaller step sizes induce larger computing times.

A compromised solution was therefore integrated into the program. By entering 'Y' to the integration step size input field, the integration step size is fixed for most of the flight at value **DELTAT**, but is made smaller (at 0.0002 second) when the missile range is within 150 meters of the target. This option allows the capture of miss distances which may be as low as within one foot. However, this option takes a longer computing time towards the end of the simulation. When the integration step size is switched to the smaller value, a message appears on the screen to warn the user that the program is refining the miss

```
                        PARAMETERS MENU (Contd)

Type of Guidance Law (-2,-1,1,2,3,4,5,6,7 or 8)  3

      Homing Missile Guidance laws:

            1. Pure Pursuit
            2. Lead Angle
            3. Proportional Navigation with N= 3
            4. Augmented Proportional Navigation with N= _
            5. Optimal Guidance law
   If you want to simulate all above guidance
   laws on the same graph, enter -1
--------------------------------------------------

            DO YOU WANT A SMALLER INTEGRATION STEP SIZE
            DURING THE ENDGAME? (Y/N) [N]

--------------------------------------------------

            SAVE DATA FILES TO DISK FOR LATER REVIEW? (Y/N) [N]

            DO YOU WANT TO CHANGE ANY INPUT PARAMETERS? (Y/N) [N]
```

Figure A-7. Air-to-Air Guidance Law Input Menu

distance. It is then normal for the computing time to be longer than usual.

The default value for this input field is 'N', meaning "no change in the integration step size during the end game". To select 'N', simply press [ENTER]. When 'N' is selected, the miss distances are less accurate, and consequently the program only attempts to guide the missile within the missile warhead lethal radius (**MDPERM** in the missile input data file). When this is accomplished, the program terminates the simulation. This option should be selected when one is only interested in the general performance of the missile, or when launch envelopes generation or maximum range capability searches are desired. The use of a smaller end-game integration step during simulation runs involving the latter two options would greatly increase the computing time for no beneficial reasons.

2.2.1.8 Creation of Output Files

The user has the option of saving a text record (data files) of the simulation run if a 'Y' is entered at the "SAVE DATA FILES TO DISK FOR LATER REVIEW? [N]" prompt as shown in Figures A-5 and A-6. This prompt is the last input field for all engagement types. Complete time history information on the vehicle flights is printed to a file named **ENGAGMT.DAT** in the **MDPCTRAP** subdirectory. Due to the large amounts of simulation variables contained in this output file, a data dictionary for this output file is included in Appendix C of Capt Gibeau' thesis.

Additionally, shooter, target and missile X, Y and Z-axis coordinates are printed separately to other disk files named **SHOOTER.DAT**, **TARGET.DAT**, and **MISSILE.DAT** respectively. The vehicle coordinates are printed in feet for English units and in meters for SI units. Commanded lateral accelerations and the achieved missile lateral accelerations for both the horizontal and vertical planes are also printed to an output file called **MACC.DAT** in the following format: **TIME, CHORGC, CVERTGC, HORGC, VERTGC**. The acceleration output data are in "g's".

The **HEADING.DAT** output file contains the time of flight and both the target and missile heading angle data θ_M, θ_T, Ψ_M, Ψ_T for the two missile planes. The format for the **HEADING.DAT** output file is as follows: **TIME, TTHETA, TPSI, MPSI, MTHETA**.

The **ATTITUD.DAT** output file contains the time of flight, the missile angle of attack (α), the missile sideslip angle (β) and the missile flight path angles (γ and γ_H) in the following format: **TIME, ALPHA, BETA, GAMMA, GAMMAH**. All angle output data are in degrees.

The **VELOCIT.DAT** output file contains the time of flight, the missile Mach number, the missile velocity, the missile-to-target closing velocity and the target velocity. The format for the **VELOCIT.DAT** output file is as follows: **TIME, MXMACH, MVEL, CLOVEL, TVEL**. The velocity output data are in feet per second or meters per second according to the user's choice.

The output data included in all the Missile Design PC TRAP are detailed in the *Appendix C oc Capt Gibeau's thesis*. These output data can be extracted for later plotting.

Recall that air-to-air engagements include the simulation of three different vehicles: the launching aircraft (shooter), the target and the missile itself. The missile initial conditions are set to be the ones of the shooter initial conditions, since the missile is launched from the launching aircraft. The graphics display of air-to-air engagements shown on the screen is the azimuth plane (X and Y coordinates of all three vehicles), with an altitude scale of the three vehicles shown on the right-hand side of the graphics display window. This concludes our instructions for air-to-air engagements.

2.2.2 Surface-to-Air Engagements

The next series of menu panels starts from the beginning with the introductory panel shown at Figure A-1 above, followed by the missile data and scenario input panel of Figure A-2. If '2' is entered at the "Select the type of engagement (0, 1, 2, 3):" prompt, a surface-to-air engagement will be simulated by the program. After pressing [ENTER] to clear the menu panel shown in Figure A-2, the first surface-to-air parameters menu will appear on the screen as shown in Figure A-8 . The first input field is for the input/output choice of units, which has already been discussed above (paragraph 2.2.1.1).

2.2.2.1 Missile Initial Velocity

A surface-to-air engagement is composed of only two vehicle simulations: the target and the missile. Since the missile is launched from the ground or the sea, there is no launching aircraft. For this reason, the first surface-to-air parameters menu shown at Figure A-8 does not require shooter initial conditions. Instead, it requires the missile initial velocity at the exit of its launcher tube. This initial velocity input is very important as it greatly affects the results of the simulation. The main reason for this is the fact that a surface-to-air missile must not only overcome drag forces, but the gravity forces also. A low initial velocity may cause serious missile instabilities. This is why surface-launched missile require a big boost at the beginning of their flight. For the

```
                    SURFACE-TO-AIR PARAMETERS MENU

Type of input (SELECT 1 or 2):    1. Metric (m,m,m/s)
 2                                2. English (nm,ft,kts) ?


    Enter the Missile Velocity at
    Launcher Tube Exit: 500.00   (ft/sec)



                    *****  TYPE OF SIMULATION  *****

 The following options are available:
     [1] - Single shot with user defined initial range
     [2] - Single shot with maximum range search
     [3] - Launch Envelope search with varying azimuth
     [4] - Launch Envelope search with varying altitude
           (for Surface-to-air and Air-to-surface)
     [5] - Monte Carlo runs with random initiation of target maneuver
     [6] - Target Optimal Evasive Maneuver
     Type of Simulation (1,2,3,4,5): 1
                 PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-8. Surface-to-Air First Parameters Menu

input field prompting the missile velocity at the exit of the launcher tube, the user shall ensure that the input velocity is high enough for the missile to remain stable, and that this velocity is achievable by the missile ground-launching system. The program default value for this minimum missile speed at launch is 200 m/sec or 500 ft/sec. It was found by a trial and error method that smaller minimum velocities for the **GENERIC** missile lead to guidance problems. The units for the missile minimum velocity are ft/sec (English) or m/sec (SI).

2.2.2.2 Types of Simulation

Once the missile minimum velocity has been input and [ENTER] pressed by the user, the next input field is for the types of simulation. This option menu has been discussed in great details in paragraph **2.2.1.3** above for air-to-air engagements, and the general instructions given in that sub-section also apply for surface-to-air engagements. Hence, only special concerns related to surface-to-air engagements will be discussed here.

2.2.2.2.1 User-Defined Initial Range and Azimuth

If '1' is entered in the "types of simulation" input field, a single missile shot with user-defined azimuth aspect angle and range will be simulated by the program. After '1' has been selected, the next input menu that will appear is shown in Figure A-4.

Then, the first required input is the range, which is the missile-to-target point-mass to point-mass slant range. The units are nautical miles (English units) or meters (SI units).

The second input is for the aspect angle. Remember that you are simulating a surface-to-air engagement. The aspect angle required here is not the elevation angle which is in the vertical plane. It is rather the aspect angle in the horizontal plane, or in the plane defined by the X-Y axes, that is required by the program. The elevation angle is computed by the program based on the slant range value and the target altitude. It is important to differentiate these two different aspect angles in a three-dimensional engagement.

2.2.2.2.2 Single Shot with Maximum Range Search

This is the same option as explained above for the air-to-air engagements. Once again, the required azimuth aspect angle is in the horizontal plane.

2.2.2.2.3 Launch Envelope Search (Varying Azimuth)

This option is very well detailed for the air-to-air engagement.

2.2.2.2.4 Launch Envelope Search(Varying Altitude)

If the user intends to generate a missile launch envelope for a surface-to-air missile system, it is recommended that the user select option ('4'). This option was incorporated into the program to provide the capability to generate launch envelope specifically for surface-to-air type of engagements. This option will generate a launch envelope in the vertical plane showing the missile maximum range capability in altitude and crossrange.

As stated before for the air-to-air engagement (paragraph 2.2.1.3.4), the maximum value of the target altitude band shall be chosen very carefully to ensure that the missile can climb to this target altitude and still successfully intercept the target.

At this point, it is recommended to read the sub-section **2.2.1.3 Types of Simulation** in the above air-to-air engagement Section as this latter sub-section is much more detailed than the present one. When the type of simulation selection has been made by the user, the next input area is for the target initial conditions. Refer to paragraph **2.2.1.4 Target Initial Conditions** since the same instructions apply for surface-to-air engagements.

When the target initial conditions input area has been completed, the next input panel is for the selection of the guidance law.

2.2.2.3 Command Missile Guidance Law Selection

The next input menu is shown in Figure A-9. Three guidance laws are available for surface-to-air engagements. The guidance laws fundamentals are detailed at Chapter III of Capt Gibeau's thesis. If '6' is selected, the user will be required to input **N**, the proportional navigation constant, which may vary from 2 to 6.

```
                         PARAMETERS MENU (Contd)

Type of Guidance Law (-2,-1,1,2,3,4,5,6,7 or 8)  7

    Command Missile Guidance Laws:

          6. Command Proportional Navigation with N= _
          7. Beam Rider
          8. Command To Line Line-of-sight (CLOS)

   If you want to simulate all above guidance
   laws on the same graph, enter -2
------------------------------------------------

          DO YOU WANT A SMALLER INTEGRATION STEP SIZE
          DURING THE ENDGAME? (Y/N) [N]
------------------------------------------------

          SAVE DATA FILES TO DISK FOR LATER REVIEW? (Y/N) [N]

          DO YOU WANT TO CHANGE ANY INPUT PARAMETERS? (Y/N) [N]
```

Figure A-9. Surface-to-Air Guidance Law Input Menu

As for the air-to-air case, the three available guidance law trajectories can be simulated and plotted against each other on the screen in the same initial intercept scenario. To select this feature, the user must enter '-2' at the guidance law prompt. Once again, the [ENTER] key must be pressed between each guidance law simulation run. Once the guidance law has been chosen, the next two input fields are for the selection of the end-game integration step size and for the generation of data files. These two input fields are discussed in details at paragraphs **2.2.1.7** and **2.2.1.8** respectively. After selection of these last two input options, the program will proceed to the desired simulation run.

2.2.2.4 Trajectory Graphics Display

For surface-to-air engagements, the graphics display of the two simulated vehicles is a two-dimensional representation of their positions in the vertical plane. This means that the Y-axis of the on-screen plot is the altitude and the X-axis is the crossrange. If the user wishes to see a plot of the horizontal plane trajectory, it is recommended to generate a separate graphic display from the two vehicle time history output files **MISSILE.DAT** and **TARGET.DAT**.

2.2.3 Air-to-Surface Engagements

The next series of menu panels starts as usual with the introductory panel shown at Figure A-1, followed by the missile data and scenario input panel of Figure A-2. If '3' is entered at the "Select the type of engagement (0, 1, 2, 3):" prompt, an air-to-surface engagement will be simulated by the program. After pressing [ENTER] to clear the second menu panel shown in Figure A-2, the first air-to-surface parameters menu will appear on the screen as shown in Figure A-10.

```
                    AIR-TO-SURFACE PARAMETERS MENU

Type of input (SELECT 1 or 2):    1. Metric (m,m,m/s)
 2                                2. English (nm,ft,kts) ?

Shooter Initial Conditions:

 Maneuvers:  ( 1, 2, or 3 )
 1           1. None
             2. Pursuit
             3. Offset     _____ G    ________ Ang
  10000.00 Altitude          .80   Velocity or Mach

                 *****  TYPE OF SIMULATION  *****

 The following options are available:
   [1] - Single shot with user defined initial range
   [2] - Single shot with maximum range search
   [3] - Launch Envelope search with varying azimuth
   [4] - Launch Envelope search with varying altitude
         (for Surface-to-air and Air-to-surface)
   [5] - Monte Carlo runs with random initiation of target maneuver
   [6] - Target Optimal Evasive Maneuver
   Type of Simulation (1,2,3,4,5): 1
               PRESS RETURN TO CONTINUE PARAMETERS MENU
```

Figure A-10. Air-to-Surface First Input Menu

The program can simulate a simplistic air-to-surface model in which the missile leaves the launching aircraft and flies towards the target in a trajectory according to the guidance law selected by the user. Special surface-to-air simulation models, such as a sea skimming systems for anti-ship missiles, may be programmed in by the user. The sea skimming system is discussed in Garnell.

The air-to-surface first parameters menu shown in Figure A-10 is the same as the first air-to-air parameters menu shown in Figure A-3. However, the choice of the shooter altitude is very important here, since the target is considered to be on the ground at an altitude of one foot above sea level. This choice shall be made with the same considerations as the ones for the choice of the target altitude in surface-to-air engagements. These considerations are included in paragraph **2.2.2.2.4** above.

The rest of the input panels are similar to the ones you would see with an air-to-air engagement, which instructions are fully detailed above. Note that the guidance laws available for air-to-surface engagements are the homing missile guidance laws, since the missile is launched from an aircraft. The user must realize here again that the required aspect angle input is in the azimuth plane, and as such, is different from the aspect angle seen on the graphics display, which is the aspect angle in the elevation plane. This elevation aspect angle is computed by the program based on the input slant range and shooter altitude.

The graphics display shows the three vehicle trajectories from the elevation plane point of view. The altitude is then shown on the Y-axis, while the crossrange is shown on the X-axis. The graphics display is seen from a fixed coordinate system.

APPENDIX B

MISSILE DATA INPUT DICTIONARY

A. INTRODUCTION

In this appendix, you will find a dictionary describing the 57 missile data input items that must be included in the Missile Design PC TRAP input file. The name of this input file is a program input, which gives you the liberty of having as many missile input files as you desire. The missile data may be determined using different reference sources or computer programs, to the user's wish. The method on how to determine some of the data series, such as the propulsion thrust values which require interpolation of the missile thrust-time curve, is detailed in Chapter IV and an example is provided in Chapter V.

The second part of this appendix is an input file example which format must be followed in order for the Missile Design PC TRAP program to properly read the data. This example file contains the data of the **GENERIC** missile data provided with the program.

B. MISSILE DATA DICTIONARY

The following are the missile data input items required by Missile Design PC TRAP to model the trajectory of a user-defined missile. The items are presented in the order required in the input data file, as shown in the example file at the next Section.

DELTAT: Time step/integration rate. Value recommended is 0.01 second.

AREA: Missile cross sectional area in square meter (m^2). Make sure that you use the same reference area used to determine the aerodynamic coefficients.

MAXALP: Missile overall maximum angle of attack capability in degrees (Deg).

MXVGCG: Missile maximum g-loading in g's. This simulation program assumes a symmetric missile, which means that this maximum g loading is the same in the horizontal and vertical missile planes.

MAXTIM: Missile maximum guided flight time (time of flight) in seconds (sec). In practice, this value may correspond to the life time of the battery inboard the missile.

CA6: Missile zero lift drag coefficient at Mach=0.6, power off (no motor).

CA8: Missile zero lift drag coefficient at Mach=0.8, power off.

CA9: Missile zero lift drag coefficient at Mach=0.9, power off.

CA1: Missile zero lift drag coefficient at Mach=1.0, power off.

CA12: Missile zero lift drag coefficient at Mach=1.2, power off.

CA14: Missile zero lift drag coefficient at Mach=1.4, power off.

CA16: Missile zero lift drag coefficient at Mach=1.6, power off.

CA18: Missile zero lift drag coefficient at Mach=1.8, power off.

CA2: Missile zero lift drag coefficient at Mach=2.0, power off.

CA3: Missile zero lift drag coefficient at Mach=3.0, power off.

CA4: Missile zero lift drag coefficient at Mach=4.0, power off.

CA5: Missile zero lift drag coefficient at Mach=5.0, power off.

CAP: Missile **PEAK** zero lift drag coefficient, power off.

MCAP: Mach number at which missile maximum drag occurs.

CN5: Missile trim normal force *coefficient* (Cn_{TR}) at α=5 deg and Mach=1.4.

CN15: Missile trim normal force coefficient (Cn_{TR}) at α=15 deg and Mach=1.4.

CAF: Missile **FINAL** zero lift drag coefficient, power off

MCAF: Mach number at which missile final drag coefficient is entered.

RLCKON: Missile maximum seeker lockon capability in meters (m). This input value is also used as the starting range for maximum range searches.

SEKGAD: Maximum missile seeker **GIMBAL** angle in degrees (deg).

LSRLMD: Maximum missile seeker head tracking rate capability in degrees per second (deg/sec).

ZFVLMD: Missile seeker field of view in degrees (deg). The half angle value must be entered.

TINGD: Missile guidance delay time in second after launch (time constant after launch during which the missile does not initially guide).

GBIASG: Missile autopilot g-bias value in g's. This bias value is entered only if the user wants to use proportional navigation guidance with a g-bias in the vertical plane.

LOWMSV: Missile minimum velocity in meter per second (m/sec). Set this value to zero if the minimum velocity is the launch speed.

LOWMSM: Missile minimum Mach number. Set this value to zero if the minimum velocity is the launch speed.

LOWCLV: Minimum missile to target closing velocity (V_c) in meter per second (m/sec).

LDVFAC: Velocity multiplier in optimum lead angle computation.

LDZFAC: Altitude multiplier in optimum lead angle computation.

AVGDLV: Average ΔV (velocity difference) expression in optimum lead angle computation in meter per second (m/sec). See equation (101) in Chapter IV.

MDPERM: Missile warhead lethal radius in meters (m).

INMSMS: Missile initial mass in Kg.

INITCG: Missile initial center of gravity (c.g.) location from the nose in meter (m).

BOMSMS: Missile final (burnout) mass in Kg.

BOCG: Missile final center of gravity (c.g.) location from the nose in meter (m).

CGPROP: Missile propellant center of gravity (c.g.) location from the nose in meter (m).

EXAREA: Missile motor nozzle exit area in square meter (m^2).

TIGN: Missile motor ignition time after launch in seconds (sec).

TBO: Time of missile motor burnout in seconds after launch.

TISP: Time to transition to second specific impulse (I_{sp}) after launch in seconds (sec).

TTHR1: Time of first vacuum thrust value after launch in seconds (sec).

TTHR2: Time of second vacuum thrust value after launch in seconds (sec).

TTHR3: Time of third vacuum thrust value after launch in seconds (sec).

TTHR4: Time of fourth vacuum thrust value after launch in seconds (sec).

VTHR1: Vacuum thrust at time **TTHR1** in Newtons (N).

VTHR2: Vacuum thrust at time **TTHR2** in Newtons (N).

VTHR3: Vacuum thrust at time **TTHR3** in Newtons (N).

VTHR4: Vacuum thrust at time **TTHR4** in Newtons (N).

VISPB: Booster stage vacuum specific impulse (I_{sp}) in Newton*sec/Kg (N*sec/Kg).

VISPS: Sustainer stage vacuum specific impulse (I_{sp}) in Newton*sec/Kg (N*sec/Kg).

TTHR5: Time of fifth vacuum thrust value after launch in seconds (sec).

VTHR5: Vacuum thrust at time **TTHR5** in Newtons (N).

C. EXAMPLE OF A MISSILE INPUT DATA FILE

This example of a missile data input file shown in Table B-1 is for the **GENERIC** missile provided with the Missile Design PC TRAP program.

DT :	0.25000D-01	GENERIC MISSILE DATA INPUT	
AREA :	0.46300D-01		
MAXALP :	0.21500D+02		
MXVGCG :	0.24000D+02		
MAXTIM :	0.60000D+02		
CA6 :	0.54577D+00		
CA8 :	0.54091D+00		
CA9 :	0.66075D+00		
CA1 :	0.12875D+01		
CA12 :	0.12297D+01		
CA14 :	0.12077D+01		
CA16 :	0.10746D+01		
CA18 :	0.10046D+01		
CA2 :	0.91075D+00		
CA3 :	0.69284D+00		
CA4 :	0.60066D+00		
CA5 :	0.50848D+00		
CAP :	0.15626D+01		
MCAP :	0.10500D+01		
CN5 :	0.27298D+01		
CN15 :	0.13837D+02		
CAF :	0.42520D+00		
MCAF :	0.70000D+01		
RLCKON :	0.15000D+06		
SEKGAD :	0.60000D+02		
LSRLMD :	0.20000D+02		
ZFVLMD :	0.60000D+01		
TINGD :	0.40000D+00		
GBIASG :	0.10000D+01		
LOWMSV :	2.50000D+02		
LOWMSM :	0.00000D+00		
LOWCLV :	0.15000D+01		
LDVFAC :	0.10000D+01		
LDZFAC :	0.10000D+01		
AVGDLV :	0.00000D+04	equals 5104.8 m/sec for lead angle	
MDPERM :	0.13000D+02		
INMSMS :	0.33600D+03		
INITCG :	0.28740D+01		
BOMSMS :	0.20600D+03		
BOCG :	0.24170D+01		
CGPROP :	0.36000D+01		
EXAREA :	0.10400D-01		
TIGN :	0.00000D+00		
TBO :	0.90000D+01		
TISP :	0.90000D+01		
TTHR1 :	0.00000D+00		
TTHR2 :	0.21000D+01		
TTHR3 :	0.36000D+01		
TTHR4 :	0.42000D+01		
VTHR1 :	0.40743D+05		
VTHR2 :	0.58057D+05		
VTHR3 :	0.44778D+05		
VTHR4 :	0.22900D+05		
VISPB :	0.24516D+04		
VISPS :	0.24516D+04		
TTHR5 :	0.89000D+01	VTHR5 :	0.30000D+05

Table B-1. Example of a Missile Input Data File

APPENDIX C

ENGAGMT.DAT OUTPUT FILE DATA LIST

This appendix provides a short definition of each data item contained in the **ENGAGMT.DAT** output file. This file can be generated by Missile Design PC TRAP if desired by the user. This file is composed of the simulation variables computed inside the missile simulation for each time increment at which those variables are updated by the program.

This output file is useful when the user intends to study a specific missile scenario simulation in great details, or when the user wants to track the variation of certain missile parameters with time of flight. This output file can be modified by the user if the user requires more or different missile variables. The missile data output are printed to this file at the **print rate** indicated by the user at the beginning of the Missile Design PC TRAP. An example of the **ENGAGMT.DAT** output file is included in Figure C-1 for two time increments (the first and the last increments) plus the end game result summary. Note that the end game summary results are printed in SI units only, even if the user chooses to have the input/output data in English units.

The **ENGAGMT.DAT** missile data output items are listed below, with a short definition, in the order in which they appear in the

```
TIME   = .0000    ATR    = 5.000    AMR    = .0000    RANGE = 5.000
TPX    = .0000    TPY    = .0000    TPZ    =-.8229    TVEL  = 584.8
APX    =-3.487    APY    =-3.487    APZ    =-1.646    AVEL  = 510.5
MPX    =-3.487    MPY    =-3.487    MPZ    =-1.646    MVEL  = 510.5
ATRX   = 3.487    ATRY   = 3.487    ATRZ   = .8229    ATRR  =-102.6
AMRX   = .0000    AMRY   = .0000    AMRZ   = .0000    AMRR  = .0000
MTRX   = 3.487    MTRY   = 3.487    MTRZ   = .8229    CLOVEL= 102.6
MTVX   = 228.7    MTVY   =-356.0    MTVZ   =-84.01    RHO   = .5652E-01
MVBX   = 510.5    MVBY   = .0000    MVBZ   = .0000    VS    = 638.1
MVRX   = 356.0    MVRY   = 356.0    MVRZ   = 84.01    PRESS = 20.39
THRUST= .1773E+06 FUELFL= 36.64     MSMASS= 740.7     MISLCG= 9.429
LOSEL =-9.473     LOSAZ = 45.00     LOSELR= .4092     LOSAZR=-2.486
PGANG = .5619E-15 YGANG = .1162E-14 MTHETA=-5.298     MPSI  = 45.00
MSMACH= .8000     CN    = 2.279     CY    = .0000     CA    = .4909
ALPHA = 4.175     BETA  = .0000     GAMMA =-9.473     GAMMAH= 45.00
ATHETA=-9.473     APSI  = 45.00     TTHETA= .0000     TPSI  = .0000
WNDACX= 386.6     WNDACY= .0000     WNDACZ=-28.66     G     = 32.17
WLAMX =-.1333E-16 WLAMY = .3063     WLAMZ =-1.298     WLAMBX=-.1147E-16
WLAMBY= .2166     WLAMBZ=-1.316     DYNPRS= 9.137     QS    = 6430.

TIME   = 16.80    ATR    = 4.926    AMR    = 4.921    RANGE = .5058E-02
TPX    = 2.729    TPY    = .0000    TPZ    =-.8229    TVEL  = 584.8
APX    =-1.826    APY    =-1.826    APZ    =-1.254    AVEL  = 510.5
MPX    = 2.725    MPY    =-.3295E-02 MPZ   =-.8237    MVEL  = 1181.
ATRX   = 4.555    ATRY   = 1.826    ATRZ   = .4308    ATRR  = 72.18
AMRX   = 4.551    AMRY   = 1.823    AMRZ   = .4300    AMRR  = 712.6
MTRX   = .3756E-02 MTRY  = .3295E-02 MTRZ  = .7871E-03 CLOVEL= 680.1
MTVX   =-504.9    MTVY   =-443.1    MTVZ   =-105.9    RHO   = .6595E-01
MVBX   = 1181.    MVBY   = 14.75    MVBZ   = 12.08    VS    = 649.7
MVRX   = 1090.    MVRY   = 443.1    MVRZ   = 105.9    PRESS = 24.66
THRUST= .0000     FUELFL= .0000     MSMASS= 454.1     MISLCG= 7.930
LOSEL =-8.953     LOSAZ = 41.26     LOSELR= .1810     LOSAZR=-.7077
PGANG = .5001     YGANG =-3.691     MTHETA=-4.585     MPSI  = 21.55
MSMACH= 1.818     CN    = .2994     CY    =-.2958     CA    = .9962
ALPHA = .5483     BETA  = .5417     GAMMA =-5.134     GAMMAH= 22.09
ATHETA=-9.473     APSI  = 45.00     TTHETA= .0000     TPSI  = .0000
WNDACX=-141.2     WNDACY=-41.20     WNDACZ=-9.630     G     = 32.17
WLAMX =-.1248E-01 WLAMY = .6052E-01 WLAMZ =-.1938     WLAMBX= .1609E-02
WLAMBY= .5162E-01 WLAMBZ=-.1968     DYNPRS= 57.07     QS    = .4016E+05

   SHOT RANGE(NM)=      5.0, AZIMUTH =  45.0
 SHOOTER ALT,SPD(FT,KTS)=10000.0  510.5 TARGET ALT,SPD(FT,KTS)= 5000.0  584.8
     ** CPA WITHIN WARHEADLETHAL RADIUS**
  FLIGHT TERMINATION : CLOSING VELOCITY < MIN AND BURNOUT
  FLIGHT TIME(S)= 16.9 MISS DISTANCE(FT) =     25.81
 FPOLE(NM)=          4.926935439264470

 ***TERMINAL INTERCEPT GEOMETRY (IN SI UNITS ONLY)***:
TARGET X,Y,Z (M)  :       5069.03          .00     -1524.00
PSI, THETA (DEG)  :           .00          .00
VELOCITY (M/SEC)  :        300.83

MISSILE X,Y,Z (M):        5088.45        10.99     -1521.55
PSI, THETA (DEG)  :          1.59        16.01
VELOCITY (M/SEC)  :        596.21
MISS DISTANCE IN X,Y,Z (M):      -5.85      -5.15      -1.07
```

Table C-1. ENGAGMT.DAT Output File Example output file, from left to right then from top to bottom. The data is printed in the units selected by the user. **Note that in the following data list, the specific units of each variable output are given within square brackets "[ ]", English units first and SI units in second.** Also note that missile seeker related data does not apply to beam rider and CLOS guidance system.

TIME: The current time of missile flight simulation at which the variables where printed by the program to this file [sec].

ATR: In cases where the missile was launched by an aircraft, this variable provides the current relative range between the launching aircraft and the target. This variable may also be called the "F-Pole" in the operational world [nmi or m].

AMR: This is the relative range between the missile in flight and the launching aircraft [nmi or m].

RANGE: This is the total relative range between the missile and the target. It is also called the "slant range" [nmi or m].

TPX,TPY,TPZ: Target current position in X, Y and Z with respect to the inertial (or fixed) coordinate system. The target initial position is always (0,0,-TPZ) [nmi or m].

TVEL: Target longitudinal velocity (i.e. X-axis velocity in body axes coordinate system) [kts or m/sec].

APX,APY,APZ: Launching aircraft position in X, Y and Z with respect to the inertial (or fixed) coordinate system. If no launching aircraft (surface-to-air), the launching aircraft position is set and to (0,0,0) [nmi or m].

AVEL: Launching aircraft longitudinal velocity (i.e. X-axis velocity in body axes coordinate system) [kts or m/sec].

MPX,MPY,MPZ: Missile current position in X, Y and Z with respect to the inertial (or fixed) coordinate system [nmi or m].

AVEL: Missile wind axis velocity (i.e. X-axis velocity in wind axes coordinate system) [kts or m/sec].

ATRX: This is the X-axis component of the launching aircraft to target relative range in the fixed coordinate system [nmi or m].

ATRY,ATRZ: Y-axis and Z-axis components of the launching aircraft to target relative range in the fixed coordinate system [nmi or m]. This means that:

$$\begin{aligned} ATRX &= TPX - APX \ , \\ ATRY &= TPY - APY \ , \\ ATRZ &= TPZ - APZ \ . \end{aligned} \tag{C-1}$$

ATRR: This is the launching aircraft to target relative range rate [kts or m/sec].

AMRX,AMRY,AMRZ: X-axis, Y-axis and Z-axis components of the launching aircraft to missile relative range in the fixed coordinate system [nmi or m].

AMRR: This is the launching aircraft to missile relative range rate [kts or m/sec].

MTRX,MTRY,MTRZ: X-axis, Y-axis and Z-axis components of the target to missile relative range in the fixed coordinate system [nmi or m].

MTRR: This is the missile to target relative range rate [kts or m/sec]. This is the total range rate used in the computation of the proportional navigation guidance law [kts or m/sec].

CLOVEL: This is the missile-to-target closing velocity (V_c), which is the negative sign of **MTRR** (**CLOVEL = -MTRR**) [kts or m/sec].

MTVX,MTVY,MTVZ: X-axis, Y-axis and Z-axis components of the target to missile relative velocity in the fixed coordinate system [nmi or m]. This means the following:

$$\begin{aligned} MTVX &= TVX - MVRX \, , \\ MTVY &= TVY - MVRY \, , \\ MTVZ &= TVZ - MVRZ \, . \end{aligned} \tag{C-2}$$

RHO: This is the local air density at missile altitude [lbm/ft^3 or kg/m^3].

MVBX,MVBY,MVBZ: X-axis, Y-axis and Z-axis components of the missile total velocity in the missile body axis coordinate system [kts or m/sec].

VS: Local speed of sound at missile altitude [kts or m/sec].

THRUST: Current delivered thrust [lbf or N].

FUELFL: The current fuel flow rate for the missile solid propellant rocket engine [lbm/sec or kg/sec].

MSMASS: The instanteneous mass of the missile which varies proportional to the burnt rocket motor propellant [lbm or kg].

MISLCG: The instanteneous location of the missile center of gravity (c.g.) from the nose [ft or m].

LOSEL: The current missile-to-target Line of Sight (LOS) angle in the elevation (vertical) plane [deg].

LOSAZ: The current missile-to-target LOS angle in the azimuth (horizontal) plane [deg].

LOSELR: The current missile-to-target LOS angle rate of change in the elevation (vertical) plane [deg/sec].

LOSAZR: The current missile-to-target LOS angle rate of change in the azimuth (horizontal) plane [deg/sec].

PGANG: This is the missile-to-target looking angle from the missile seeker point of view in the vertical plane of the missile body axes coordinate system [deg].

YGANG: This is the missile-to-target looking angle from the missile seeker point of view in the horizontal plane of the missile body axes coordinate system [deg].

MTHETA: Missile heading angle in the elevation plane [deg].

MPSI: Missile heading angle in the azimuth plane [deg].

MSMACH: Missile Current Mach number based on the missile wind axis total velocity **MVEL** [kts or m/sec].

CN: This is the current coefficient of normal force (C_N) acting in the vertical plane of the missile with respect to its body axes coordinate system.

CY: This is the current coefficient of yaw force (C_Y) acting in the horizontal plane of the missile with respect to its body axes coordinate system.

CA: This is the current coefficient of axial (drag) force (C_A) acting in the longitudinal plane of the missile with respect to its body axes coordinate system.

ALPHA: This is the current value of the missile angle-of-attack [deg].

BETA: This is the current value of the missile sideslip angle [deg].

GAMMA: This is the missile flight path angle in the vertical plane [deg].

GAMMAH: This is the missile flight path angle in the horizontal plane [deg].

ATHETA: Launching aircraft heading angle in the elevation plane [deg].

APSI: Launching aircraft heading angle in the azimuth plane [deg].

TTHETA: Target heading angle in the elevation plane [deg].

TPSI: Target heading angle in the azimuth plane [deg].

WNDACX: Current missile acceleration along the X-axis of the missile wind axis coordinate system [ft/sec^2 or m/sec^2].

WNDACY: Current missile acceleration along the Y-axis of the missile wind axis coordinate system [ft/sec^2 or m/sec^2].

WNDACZ: Current missile acceleration along the Z-axis of the missile wind axis coordinate system [ft/sec^2 or m/sec^2].

DYNPRS: This is the current missile dynamic pressure (Q) [psi or Kpa].

QS: This is the current missile dynamic pressure (Q) multiplying the missile reference section (S) [lbf or N].

APPENDIX D

RESULTS FROM MISDATCOM

This Appendix contains the input and output files to the **MISDATCOM** computer program that was used to compute the aerodynamic coefficients for the example conceptual missile design of Chapter V. The input file is first included in the format used to run **MISDATCOM**. Three different input/output cases were run to obtain the missile aerodynamic coefficients required to build the input file to Missile Design PC TRAP.

The first case is to obtain a detailed curve for the missile coefficient of axial force versus flight Mach number profile. The second case was to obtain the axial force coefficients at the specific Mach numbers required for the Missile Design PC TRAP input data file as detailed in Appendix B. Finally, the third case was to compute the required trimmed coefficients of normal force at 5 and 15 degrees angle of attack.

The output file is in the PLOT format which is detailed in Example B of the **MISDATCOM** user's manual. The PLOT format was selected as it provides the required coefficients in a quick and handy output format. In the **MISDATCOM** output files that are included in this Appendix, the aerodynamic coefficient values required for the Missile Design PC TRAP are underlined in pen for each of the three cases.

```
*                              INPUT FILE TO MISDATCOM

CASEID THESIS - SURFACE TO AIR MISSILE
DIM M
HYPER
SOSE

CASEID DETAILED CA VS MACH CURVE AT SEA LEVEL
*  CASE #1 provides a detailed CA VS MACH curve, especially for the
*  transition region

$FLTCON NALPHA=2.,ALPHA=0.,2.,
ALT=0.0.,NMACH=20.,MACH=.8,.85,.9,.93,.95,.97,1.,1.03,1.06,
1.1,1.15,1.2,1.4,1.6,1.8,2.,3.,4.,5.,6.,$
$REFQ XCG=2.0,LREF=3.2,$
$AXIBOD BNOSE=0.01,TNOSE=CONICAL,LNOSE=0.8,DNOSE=0.21,LCENTR=2.4,
DCENTR=0.21,$
$FINSET2 SECTYP=NACA,SSPAN=0.0,.2,CHORD=.2,0.1,XLE=3.,3.1,
  PHIF=0.,90.,180.,270.,$
$FINSET1 SECTYP=NACA,SSPAN=0.0,.15,CHORD=.1,0.0,XLE=0.8,0.9,
  PHIF=45.,135.,225.,315.,$
NACA-1-S-1-50-6
NACA-2-S-1-50-6

PLOT
*  See Misdatcom user's manual for format of data presented
*  in the PLOT output file(FOR003.DAT)


SAVE
NEXT CASE
CASEID   REQUIRED CA VS MACH DATA
*  CASE #2 provides the CA VS Mach data points required by the
*  input file to Missile Design PC TRAP at sea level

$FLTCON NALPHA=2.,ALPHA=0.,2.,
ALT=0.0.,NMACH=13.,MACH=.8,.9,1.,1.2,1.4,1.6,1.8,2.,3.,4.,5.,6.,7.,$

PLOT

SAVE
NEXT CASE

CASEID TRIM
*
* CASE #3 is for the trimmed coefficients of normal force

$FLTCON NALPHA=3.,ALPHA=0.,5.,15.,
ALT=0.0,NMACH=1.,MACH=1.4,$
$TRIM SET=1.,PANL1=.TRUE.,PANL2=.TRUE.,PANL3=.TRUE.,
 PANL4=.TRUE.,DELMIN=-40.,DELMAX=39.,ASYM=.TRUE.$
*PRINT AERO TRIM
PLOT
NEXT CASE
```

C_A

```
UN   1  2  MBF12        CASE   1
  0.8000 18547038.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.1565    0.0000    0.0000    0.0000
  2.0000    0.5737     -0.0266     0.1553    0.0000    0.0000    0.0000

UN   2  2  MBF12        CASE   1
  0.8500 19706230.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.1591    0.0000    0.0000    0.0000
  2.0000    0.5929     -0.0274     0.1579    0.0000    0.0000    0.0000

UN   3  2  MBF12        CASE   1
  0.9000 20865420.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.1823    0.0000    0.0000    0.0000
  2.0000    0.6151     -0.0268     0.1810    0.0000    0.0000    0.0000

UN   4  2  MBF12        CASE   1
  0.9300 21560934.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.2126    0.0000    0.0000    0.0000
  2.0000    0.6330     -0.0264     0.2112    0.0000    0.0000    0.0000

UN   5  2  MBF12        CASE   1
  0.9500 22024610.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.2449    0.0000    0.0000    0.0000
  2.0000    0.6469     -0.0250     0.2434    0.0000    0.0000    0.0000

UN   6  2  MBF12        CASE   1
  0.9700 22488286.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.2819    0.0000    0.0000    0.0000
  2.0000    0.6657     -0.0210     0.2803    0.0000    0.0000    0.0000

UN   7  2  MBF12        CASE   1
  1.0000 23183800.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.3306    0.0000    0.0000    0.0000
  2.0000    0.7274      0.0058     0.3287    0.0000    0.0000    0.0000

UN   8  2  MBF12        CASE   1
  1.0300 23879312.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.3361    0.0000    0.0000    0.0000
  2.0000    0.7655     -0.0360     0.3341    0.0000    0.0000    0.0000

UN   9  2  MBF12        CASE   1
  1.0600 24574830.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.2673    0.0000    0.0000    0.0000
  2.0000    0.7653     -0.0513     0.2654    0.0000    0.0000    0.0000

UN  10  2  MBF12        CASE   1
  1.1000 25502180.      0.0000
  0.0346    3.2000      2.0000     3.2000    0.0000
  0.0000    0.0000      0.0000     0.2666    0.0000    0.0000    0.0000
  2.0000    0.7433     -0.0574     0.2648    0.0000    0.0000    0.0000
```

			C_A			
UN 11 2	MBF12	CASE 1				
1.1500	26661366.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2543	0.0000	0.0000	0.0000
2.0000	0.6683	-0.0459	0.2526	0.0000	0.0000	0.0000
UN 12 2	MBF12	CASE 1				
1.2000	27820560.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2419	0.0000	0.0000	0.0000
2.0000	0.6297	-0.0398	0.2404	0.0000	0.0000	0.0000
UN 13 2	MBF12	CASE 1				
1.4000	32457318.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2843	0.0000	0.0000	0.0000
2.0000	0.4859	-0.0174	0.2820	0.0000	0.0000	0.0000
UN 14 2	MBF12	CASE 1				
1.6000	37094076.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2588	0.0000	0.0000	0.0000
2.0000	0.4151	-0.0054	0.2581	0.0000	0.0000	0.0000
UN 15 2	MBF12	CASE 1				
1.8000	41730840.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2317	0.0000	0.0000	0.0000
2.0000	0.3686	0.0004	0.2313	0.0000	0.0000	0.0000
UN 16 2	MBF12	CASE 1				
2.0000	46367600.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2201	0.0000	0.0000	0.0000
2.0000	0.3361	0.0051	0.2199	0.0000	0.0000	0.0000
UN 17 2	MBF12	CASE 1				
3.0000	69551392.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1760	0.0000	0.0000	0.0000
2.0000	0.2422	0.0154	0.1764	0.0000	0.0000	0.0000
UN 18 2	MBF12	CASE 1				
4.0000	92735200.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1463	0.0000	0.0000	0.0000
2.0000	0.1967	0.0190	0.1469	0.0000	0.0000	0.0000
UN 19 2	MBF12	CASE 1				
5.0000	115918992.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1261	0.0000	0.0000	0.0000
2.0000	0.1733	0.0195	0.1268	0.0000	0.0000	0.0000
UN 20 2	MBF12	CASE 1				
6.0000	139102784.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1120	0.0000	0.0000	0.0000
2.0000	0.1582	0.0222	0.1128	0.0000	0.0000	0.0000

C_A

UN 21 2	MBF12	CASE 2				
0.8000	18547038.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1565	0.0000	0.0000	0.0000
2.0000	0.5737	-0.0266	0.1553	0.0000	0.0000	0.0000
UN 22 2	MBF12	CASE 2				
0.9000	20865420.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1823	0.0000	0.0000	0.0000
2.0000	0.6151	-0.0268	0.1810	0.0000	0.0000	0.0000
UN 23 2	MBF12	CASE 2				
1.0000	23183800.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.3306	0.0000	0.0000	0.0000
2.0000	0.7274	0.0058	0.3287	0.0000	0.0000	0.0000
UN 24 2	MBF12	CASE 2				
1.2000	27820560.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2419	0.0000	0.0000	0.0000
2.0000	0.6297	-0.0398	0.2404	0.0000	0.0000	0.0000
UN 25 2	MBF12	CASE 2				
1.4000	32457318.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2843	0.0000	0.0000	0.0000
2.0000	0.4859	-0.0174	0.2820	0.0000	0.0000	0.0000
UN 26 2	MBF12	CASE 2				
1.6000	37094076.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2588	0.0000	0.0000	0.0000
2.0000	0.4151	-0.0054	0.2581	0.0000	0.0000	0.0000
UN 27 2	MBF12	CASE 2				
1.8000	41730840.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2317	0.0000	0.0000	0.0000
2.0000	0.3686	0.0004	0.2313	0.0000	0.0000	0.0000
UN 28 2	MBF12	CASE 2				
2.0000	46367600.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.2201	0.0000	0.0000	0.0000
2.0000	0.3361	0.0051	0.2199	0.0000	0.0000	0.0000
UN 29 2	MBF12	CASE 2				
3.0000	69551392.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1760	0.0000	0.0000	0.0000
2.0000	0.2422	0.0154	0.1764	0.0000	0.0000	0.0000
UN 30 2	MBF12	CASE 2				
4.0000	92735200.	0.0000				
0.0346	3.2000	2.0000	3.2000	0.0000		
0.0000	0.0000	0.0000	0.1463	0.0000	0.0000	0.0000
2.0000	0.1967	0.0190	0.1469	0.0000	0.0000	0.0000

```
UN   31   2   MBF12         CASE    2       CA
     5.0000115918992.        0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000      0.0000      0.0000     0.1261     0.0000     0.0000     0.0000
     2.0000      0.1733      0.0195     0.1268     0.0000     0.0000     0.0000

UN   32   2   MBF12         CASE    2
     6.0000139102784.        0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000      0.0000      0.0000     0.1120     0.0000     0.0000     0.0000
     2.0000      0.1582      0.0222     0.1128     0.0000     0.0000     0.0000

UN   33   2   MBF12         CASE    2
     7.0000162286592.        0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000      0.0000      0.0000     0.1017     0.0000     0.0000     0.0000
     2.0000      0.1487      0.0219     0.1025     0.0000     0.0000     0.0000

UN   34   3   MTRIMMED      CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.0095      0.0004     0.3016    -0.0001     0.0002     0.0000
     5.0000      3.8375      1.2853     0.2941     0.0519     0.0454    -0.0029
    15.0000     18.3866      4.3009     0.5375     0.2913     0.1652    -0.0141
                                      CNTR
UN   35   3   MD1           CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.2805     -0.2471     1.1561    -0.2883    -0.2444     0.0161
     5.0000      1.0064     -0.4632     1.0885    -0.1801    -0.2892     0.0313
    15.0000      3.2495     -0.4630     0.8997    -0.3829    -0.2341     0.0271

UN   36   3   MD2           CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.3151     -0.2463     0.9334    -0.3283    -0.2416     0.0097
     5.0000      1.0041     -0.4205     0.8659    -0.2299    -0.2734     0.0242
    15.0000      3.3748     -0.4070     0.6780    -0.4172    -0.2300     0.0211

UN   37   3   MD3           CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.2895     -0.2069     0.6747    -0.2992    -0.2035     0.0054
     5.0000      1.0399     -0.3445     0.6146    -0.2250    -0.2267     0.0171
    15.0000      3.5516     -0.3307     0.4386    -0.3713    -0.1924     0.0148

UN   38   3   MD4           CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.2094     -0.1405     0.4449    -0.2137    -0.1390     0.0025
     5.0000      1.1100     -0.2423     0.3994    -0.1672    -0.1522     0.0103
    15.0000      3.7755     -0.2354     0.2457    -0.2616    -0.1295     0.0088

UN   39   3   MD5           CASE    3
     1.4000  32457318.       0.0000
     0.0346      3.2000      2.0000     3.2000     0.0000
     0.0000     -0.0843     -0.0545     0.3060    -0.0849    -0.0543     0.0008
     5.0000      1.2049     -0.1184     0.2804    -0.0673    -0.0575     0.0036
    15.0000      4.0095     -0.1354     0.1560    -0.0969    -0.0479     0.0031
```

APPENDIX E

RESULTS FROM COMPARISON BETWEEN TRAP AND MISSILE DESIGN PC TRAP

This Appendix contains comparative plots of selected simulation parameters generated by both TRAP and Missile Design PC TRAP in similar initial intercept scenarios (profiles). The description of the selected simulation parameters is included in Chapter V. A series of seven comparative plots is included in this Appendix for each of the four profiles used during this investigation. Before each series of plots, a profile title page is included detailing the specifics of each profile. Also located on the profile title page is a table providing the results of the miss distance components obtained for the missile simulation of the given profile.

Profile 1

Profile 1 is a co-altitude missile shot where both the target and launching aircraft are initially at the same altitude. The missile is launched from the launching aircraft at a slant range of 20.1 Km with an aspect angle of 174.3 degrees in the azimuth plane (5.7 degrees off the target's nose). The altitude of both the shooter and target is 10 Km, and their airspeed is 600 m/sec and 450 m/sec respectively. The target does not execute any evasive maneuver during the entire missile time of flight. The guidance law is proportional navigation with **N** = 4. Table E-1 provides the miss distance results obtained from both TRAP and Missile Design PC TRAP simulation flyouts.

Miss Distance Components	TRAP (Meters)	MD PC TRAP (Meters)
X_S-axis	-7.11	0.54
Y_S-axis	0.72	0.88
Z_S-axis	0.00	0.00
Total	7.14	1.03

Table E-1. Miss Distance Components Comparison - Profile 1

FIGURE E-1a: Missile Trajectory Comparison (Horizontal)

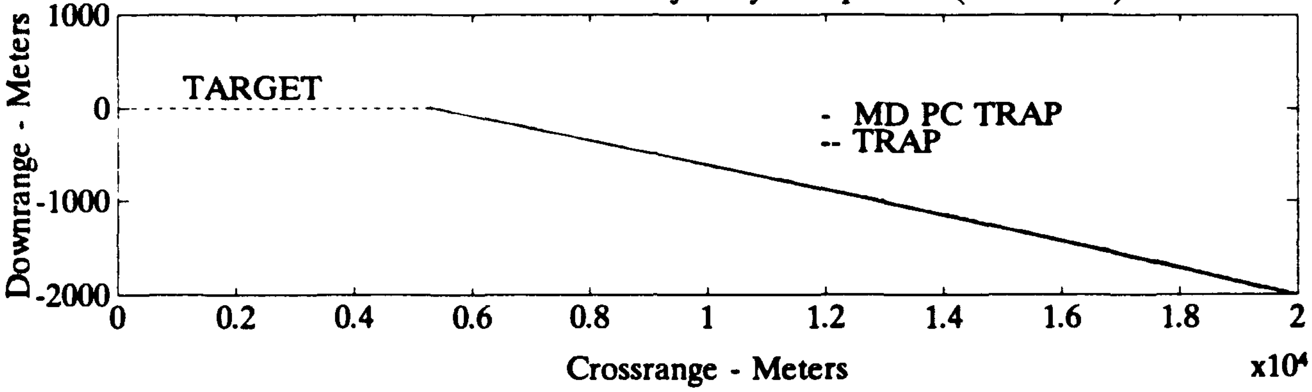


FIGURE E-1b: Missile Trajectory Comparison (Vertical)

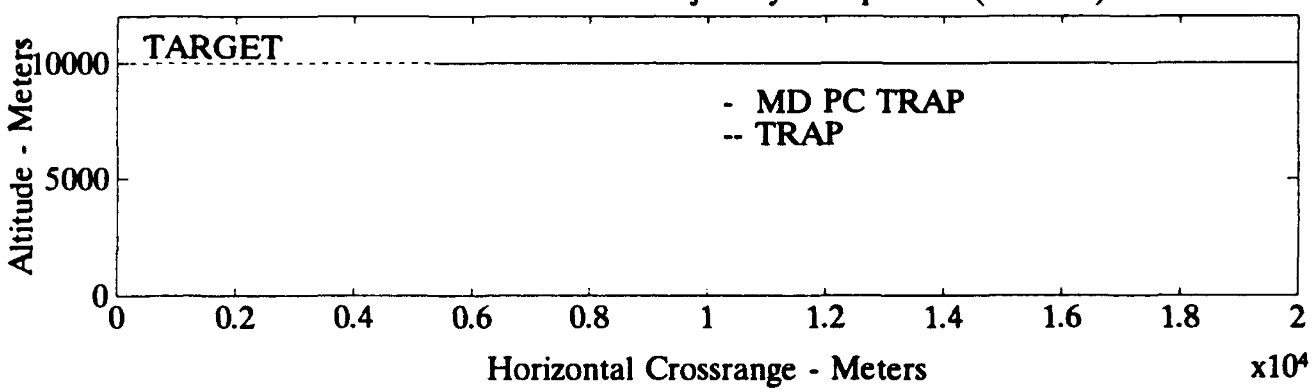


FIGURE E-2a: Missile Acceleration Comparison - Horizontal

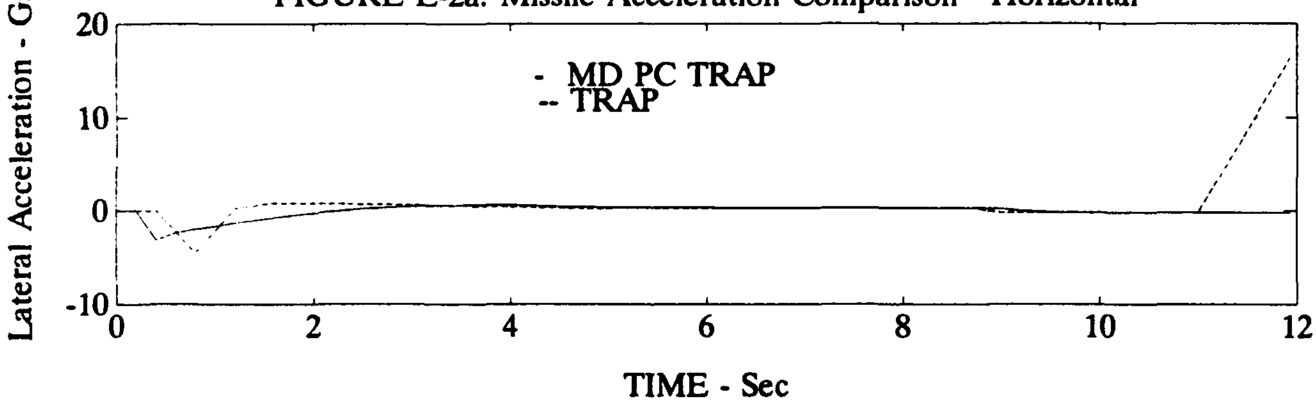


FIGURE E-2b: Missile Acceleration Comparison - Vertical

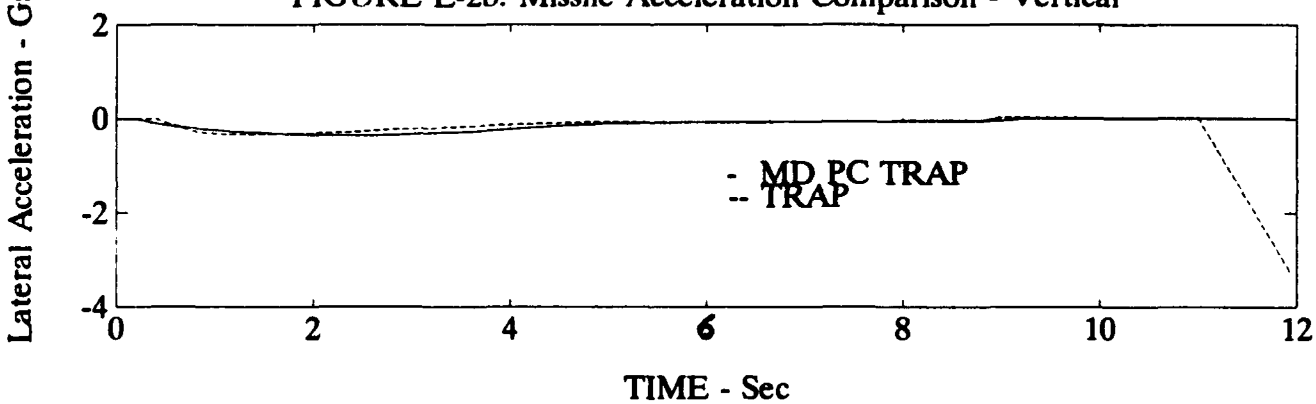


FIGURE E-3: Delivered Thrust vs Time Comparison

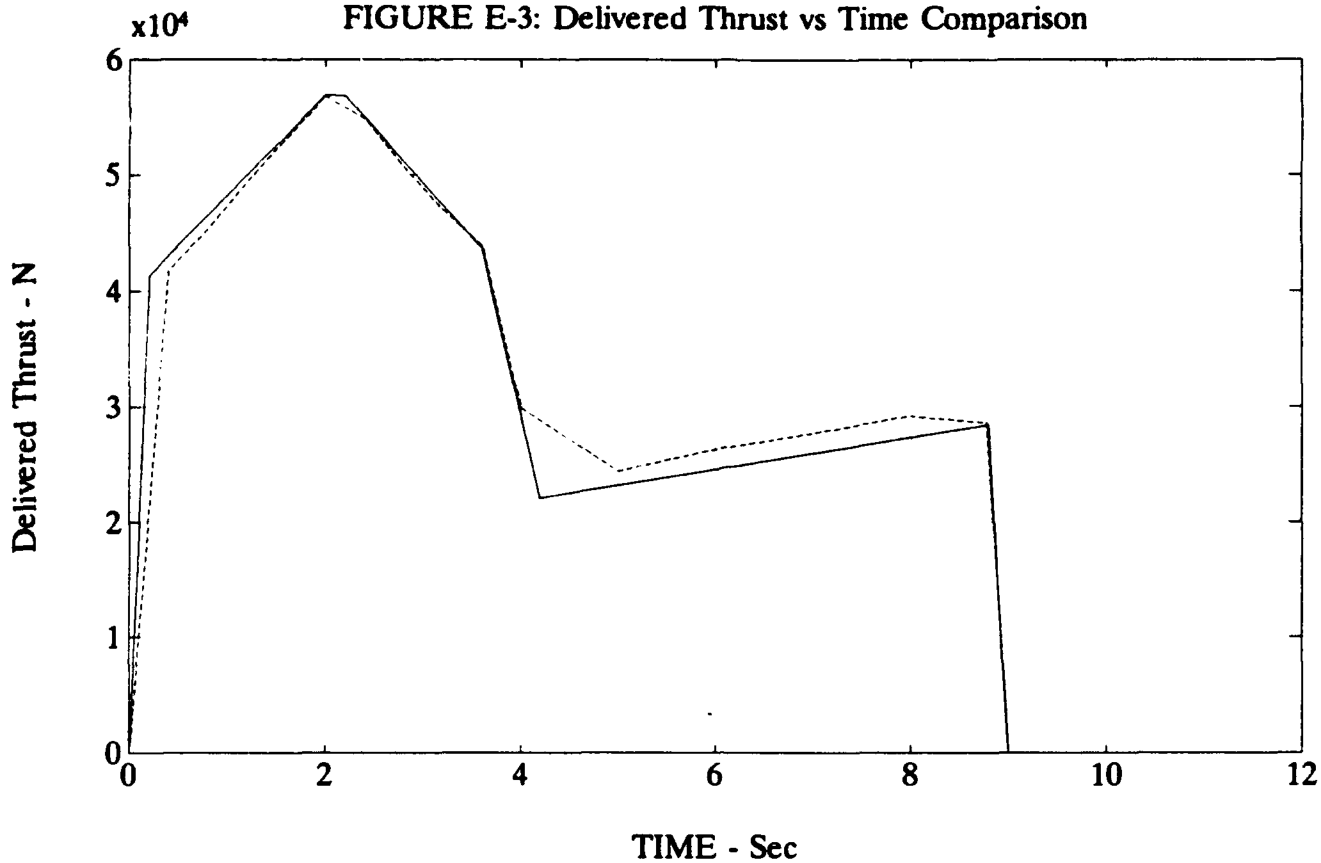


FIGURE E-4: Coefficient of Normal Force (CN) Comparison

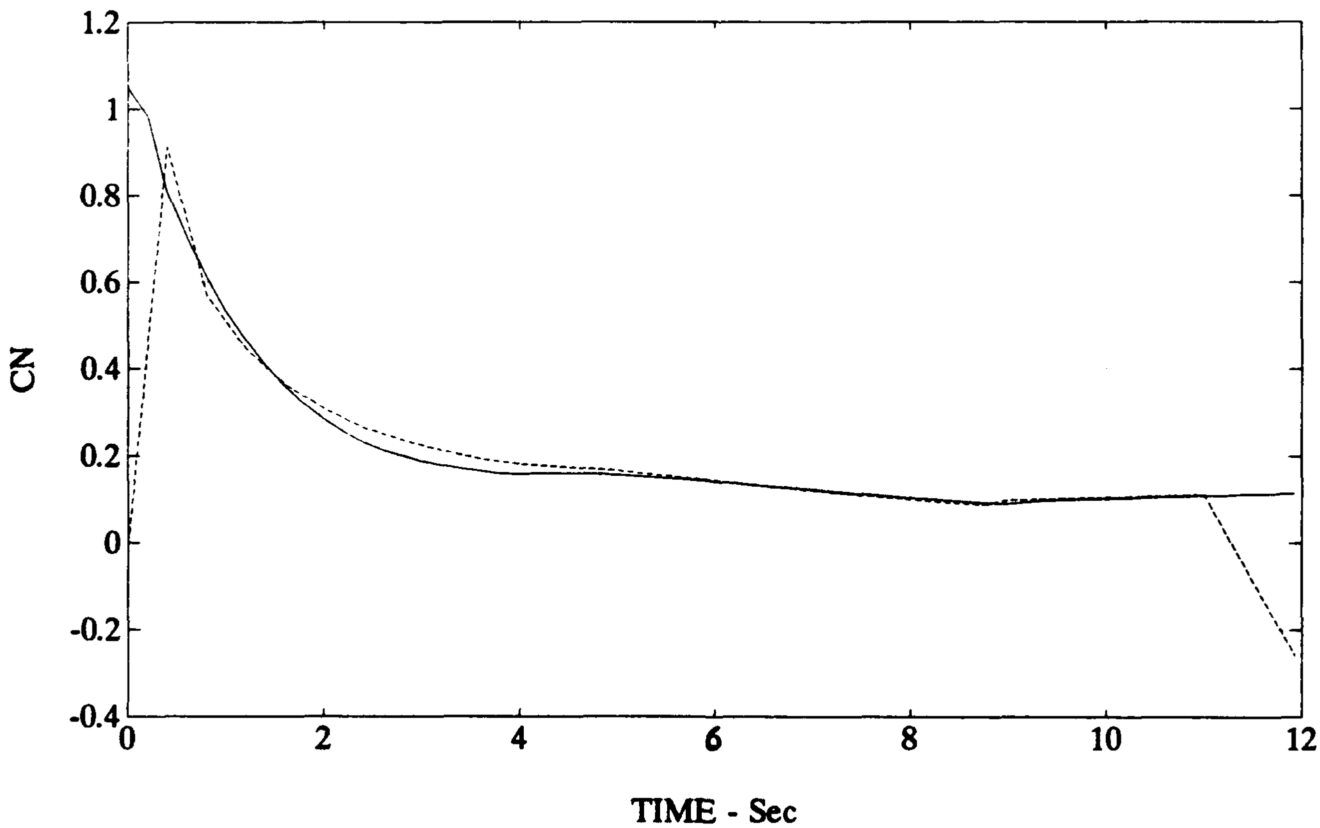


FIGURE E-5: Coefficient of Axial Force (CA) Comparison

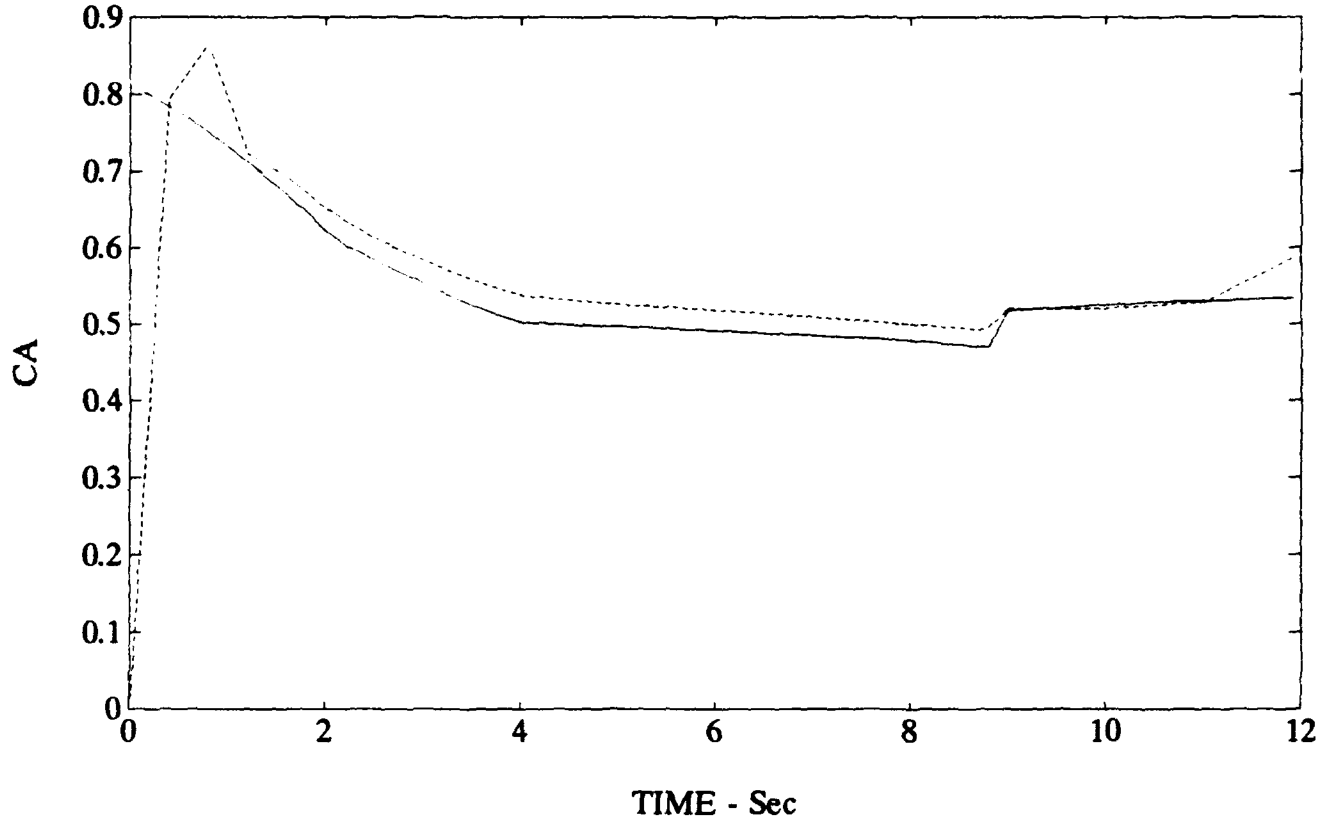


FIGURE E-6a: Missile Angle-of-Attack (ALPHA) Comparison

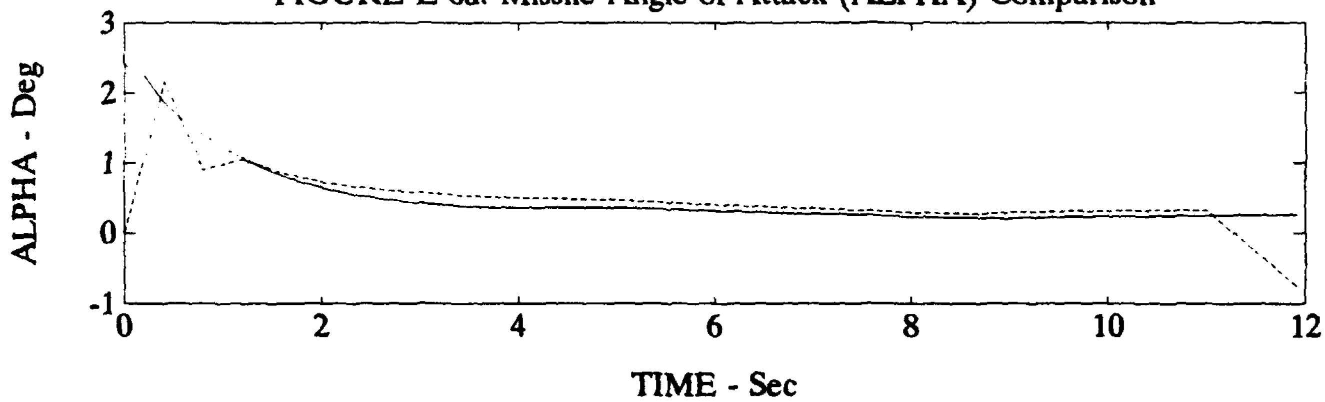


FIGURE E-6b: Missile Sideslip Angle (BETA) Comparison

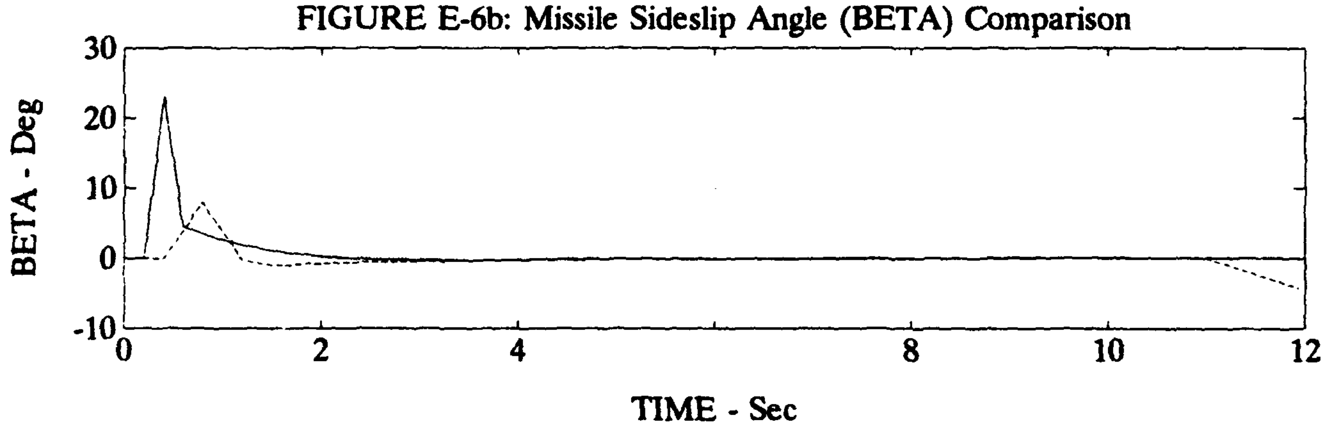
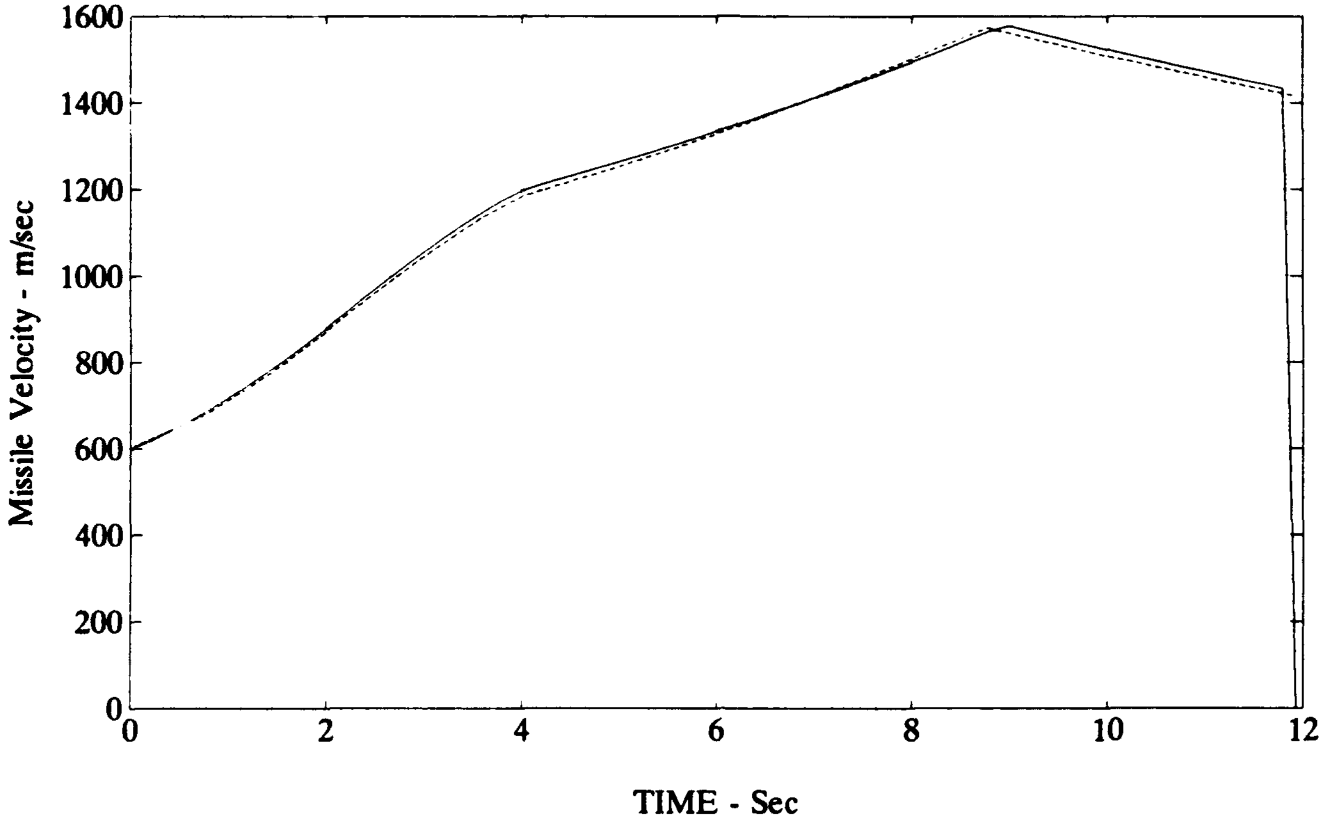


FIGURE E-7: Missile Velocity Comparison



Profile 2

Profile 2 is a look down/shoot down intercept scenario where the launching aircraft is at an altitude higher than the target. The shooter's altitude is 5 Km, while the target's altitude is 2 Km. The missile is launched from the launching aircraft at a slant range of 9.734 Km with an aspect angle of 180 degrees in the azimuth plane (head on). The airspeeds of both the shooter and target are 400 m/sec and 350 m/sec respectively. The target does not execute any evasive maneuver during the entire missile time of flight. The guidance law is proportional navigation with **N** = 4. Table E-2 provides the miss distance results obtained from both TRAP and Missile Design PC TRAP simulation flyouts.

Miss Distance Components	TRAP (Meters)	MD PC TRAP (Meters)
X_S-axis	9.78	-0.42
Y_S-axis	0.00	1.44
Z_S-axis	-3.25	-1.74
Total	10.31	2.30

Table E-2. Miss Distance components Comparison - Profile 2

FIGURE E-8a: Missile Trajectory Comparison (Horizontal)

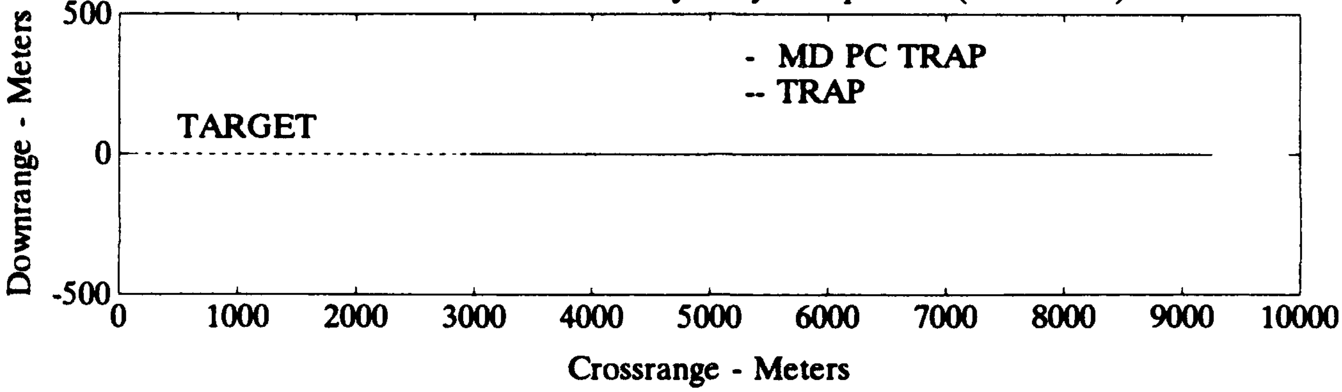


FIGURE E-8b: Missile Trajectory Comparison (Vertical)

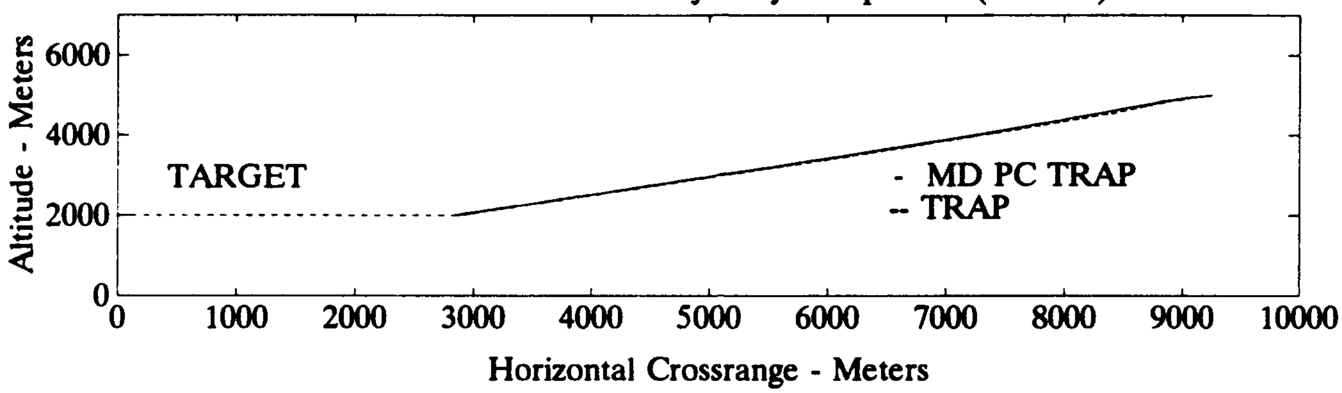


FIGURE E-9a: Missile Acceleration Comparison - Horizontal

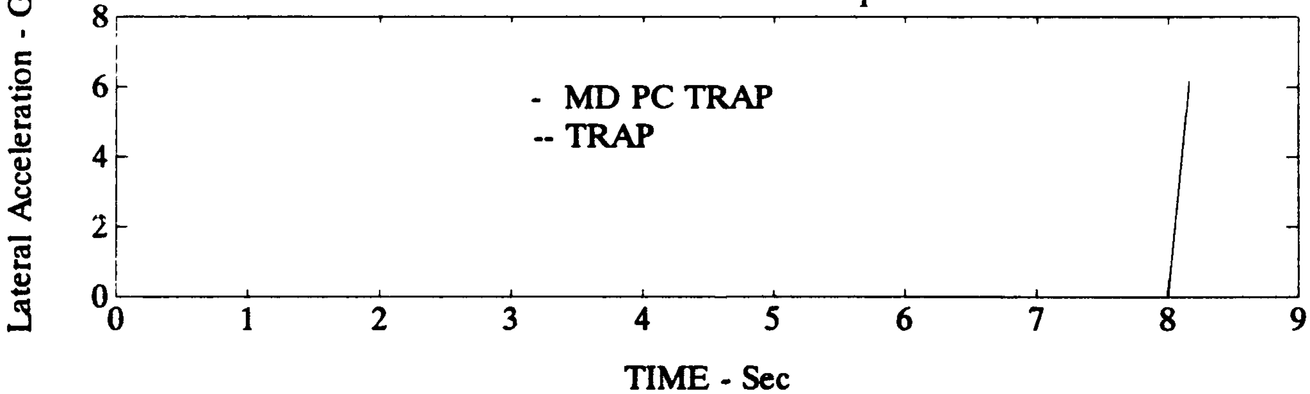


FIGURE E-9b: Missile Acceleration Comparison - Vertical

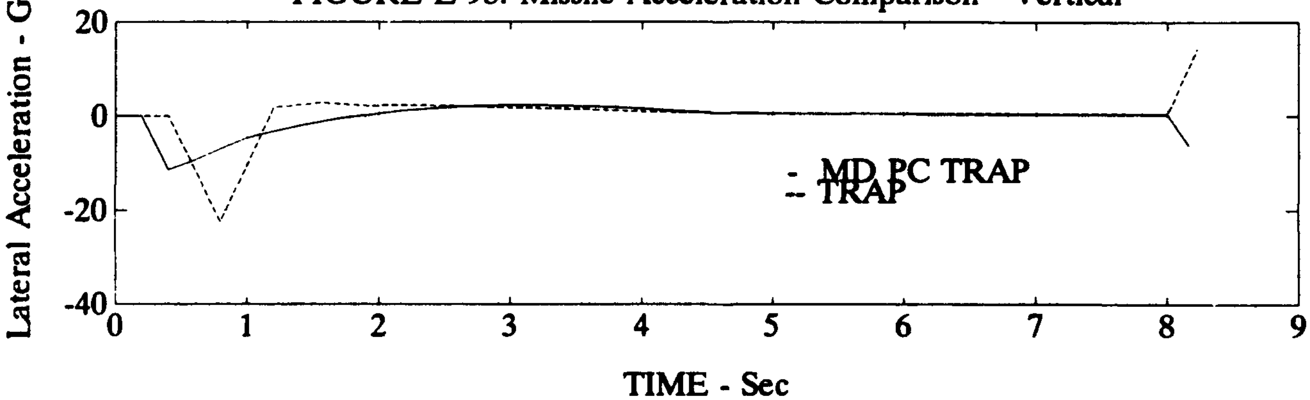


FIGURE E-10: Delivered Thrust vs Time Comparison

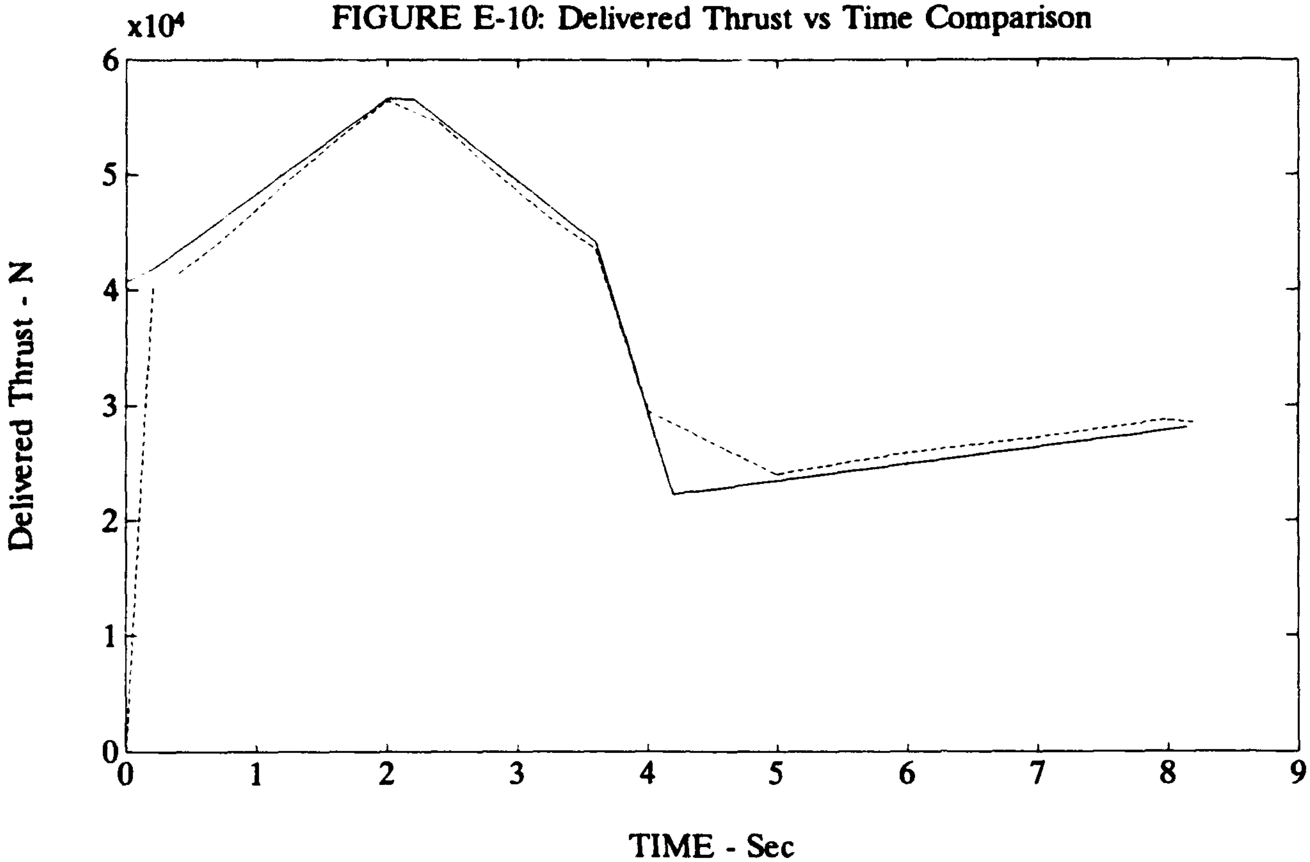


FIGURE E-11: Coefficient of Normal Force (CN) Comparison

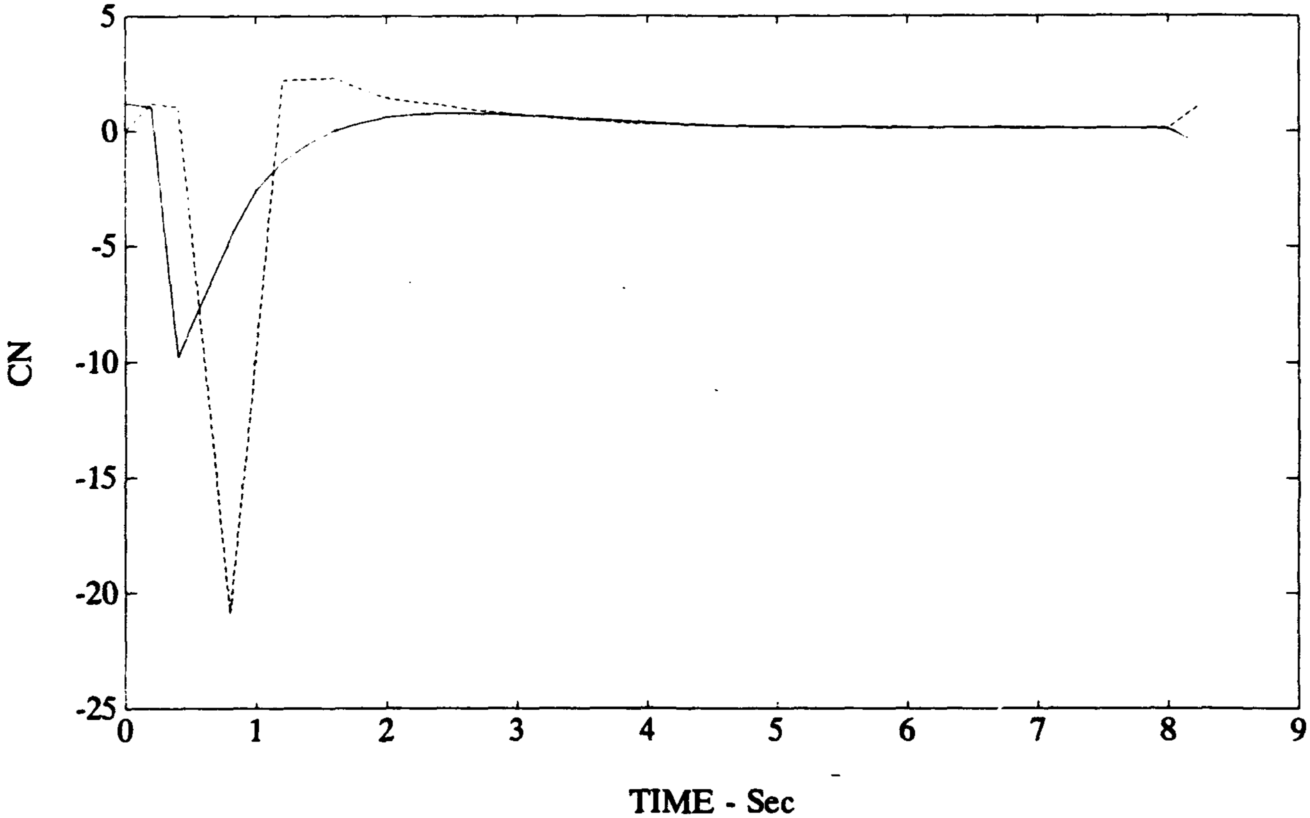


FIGURE E-12: Coefficient of Axial Force (CA) Comparison

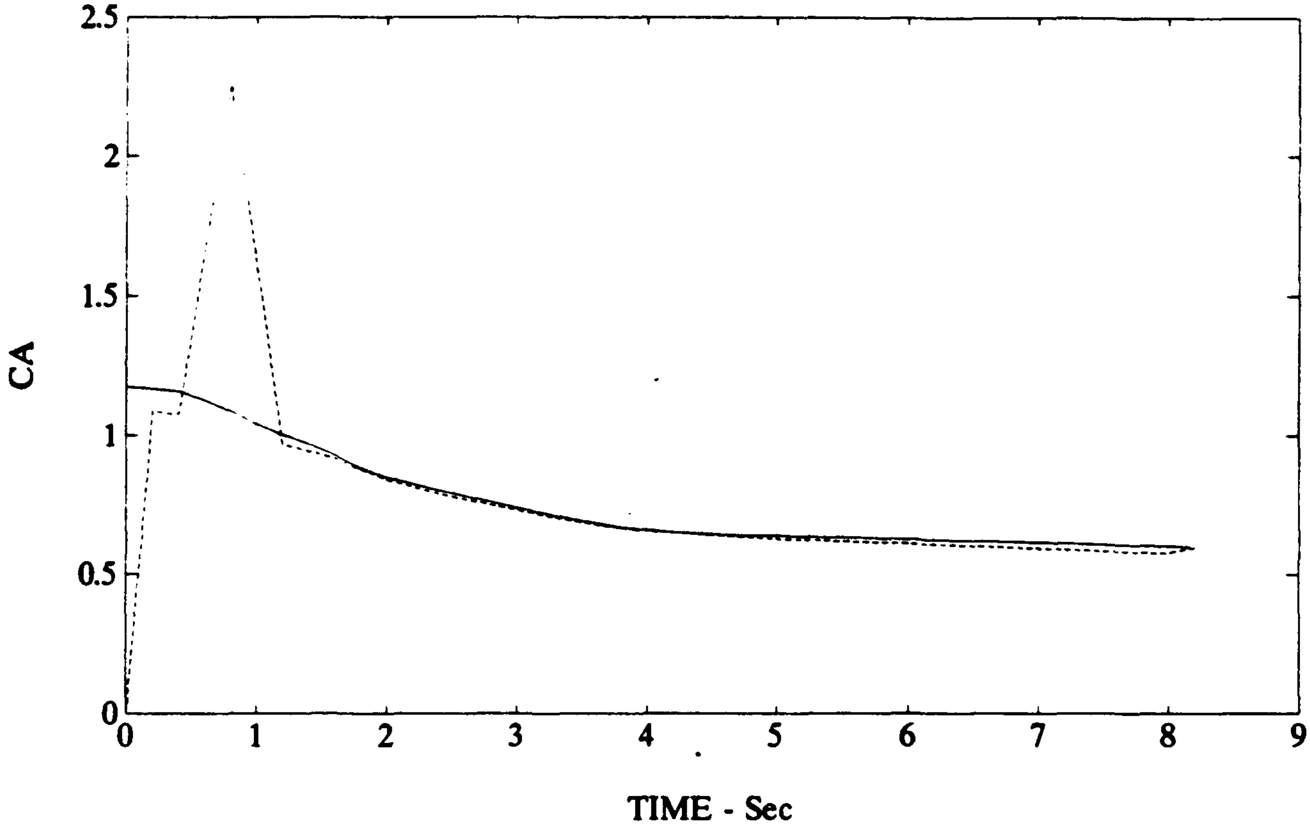


FIGURE E-13a: Missile Angle-of-Attack (ALPHA) Comparison

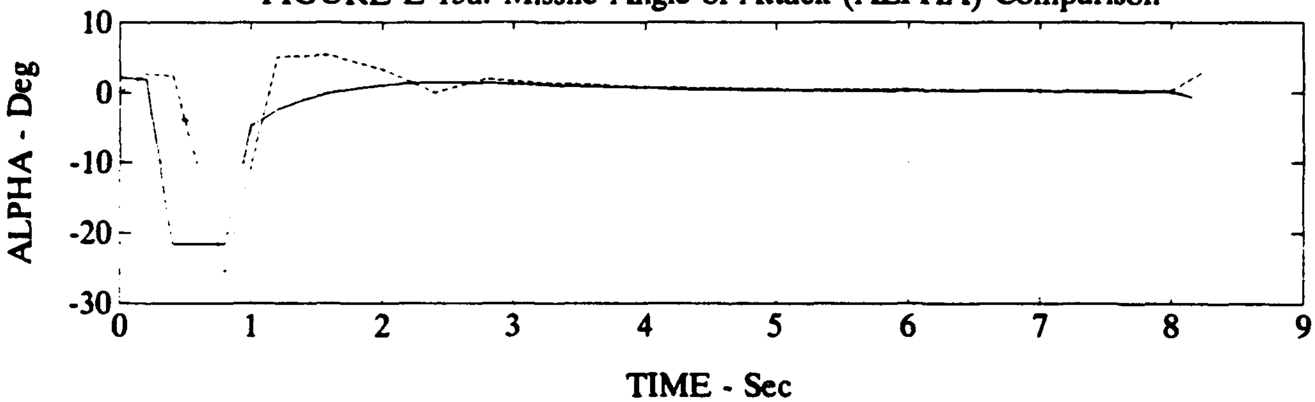


FIGURE E-13b: Missile Sideslip Angle (BETA) Comparison

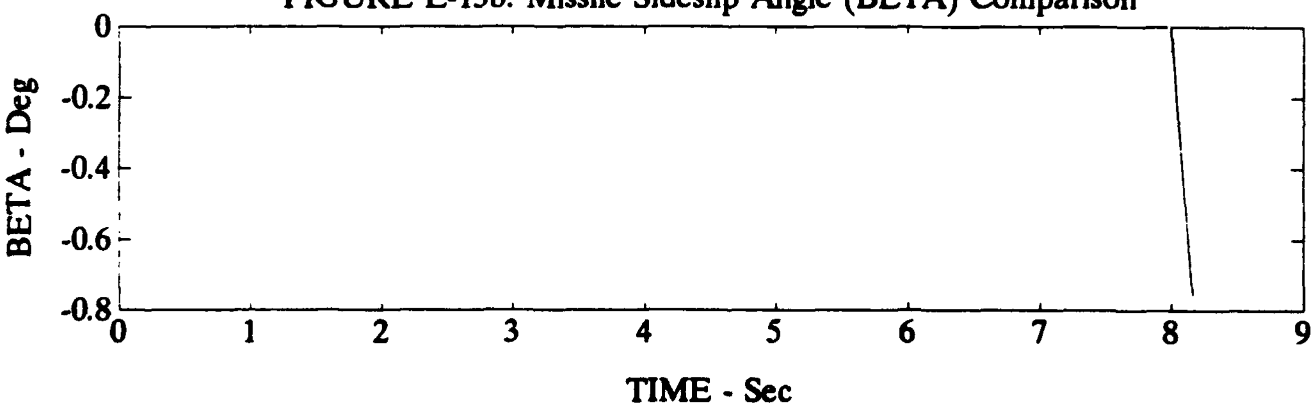
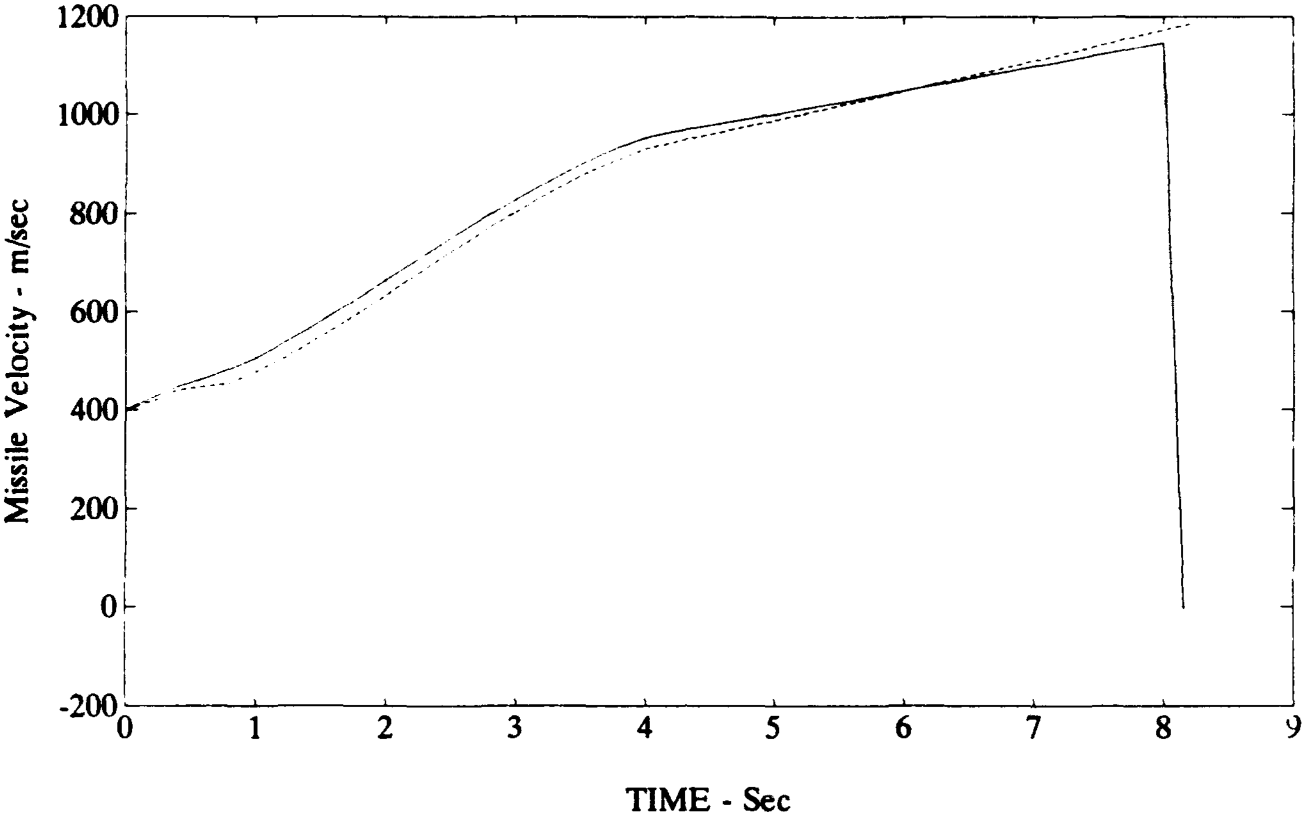


FIGURE E-14: Missile Velocity Comparison



Profile 3

Profile 3 is a look up/shoot up intercept scenario where the launching aircraft is at an altitude lower than the target. The shooter's altitude is 10 Km, while the target's altitude is 14 Km. The missile is launched from the launching aircraft at a slant range of 29.27 Km with an aspect angle of 45 degrees in the azimuth plane (45 deg off target's tail). The airspeeds of both the shooter and target are 600 m/sec and 450 m/sec respectively. The target does not execute any evasive maneuver during the entire missile time of flight. The guidance law is proportional navigation with **N** = 4. Table E-3 provides the miss distance results obtained from both TRAP and Missile Design PC TRAP simulation flyouts.

Miss Distance Components	TRAP (Meters)	MD PC TRAP (Meters)
X_S-axis	3.51	-0.01
Y_S-axis	3.37	-0.028
Z_S-axis	-0.65	0.00
Total	4.91	0.03

Table E-3. Miss Distance Components Comparison - Profile 3

FIGURE E-15a: Trajectory Comparison (Horizontal)

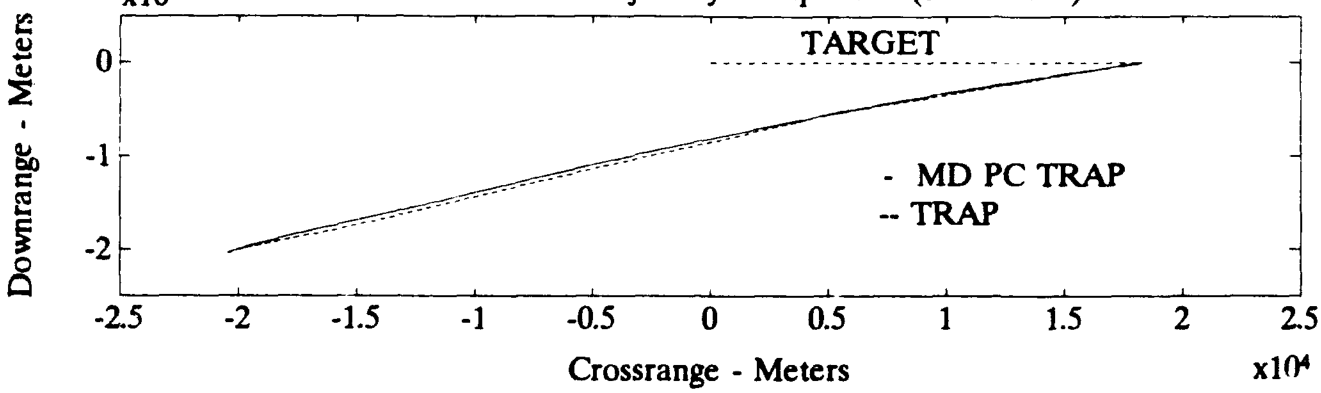


FIGURE E-15b: Trajectory Comparison (Vertical)

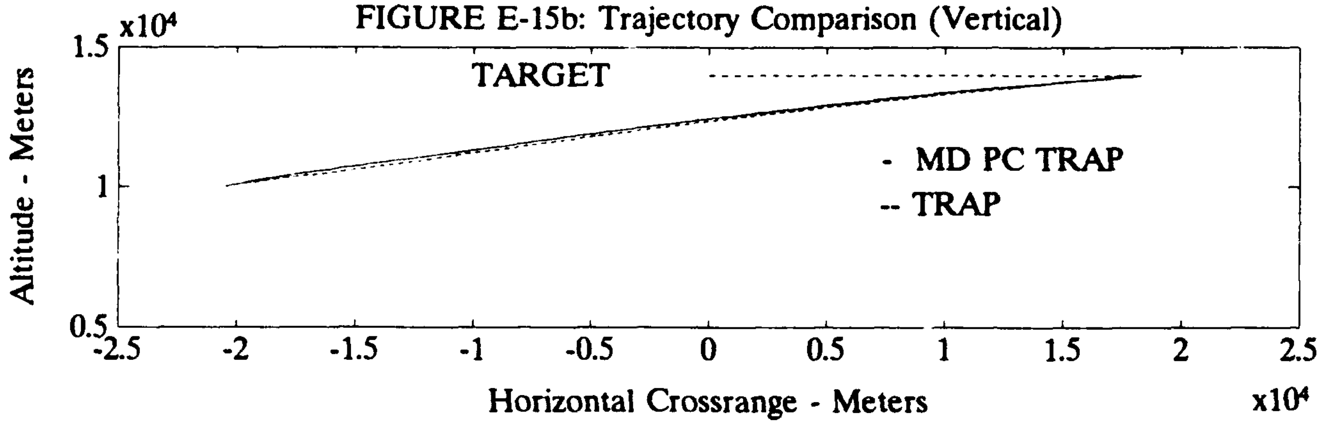


FIGURE E-16a: Missile Acceleration Comparison - Horizontal

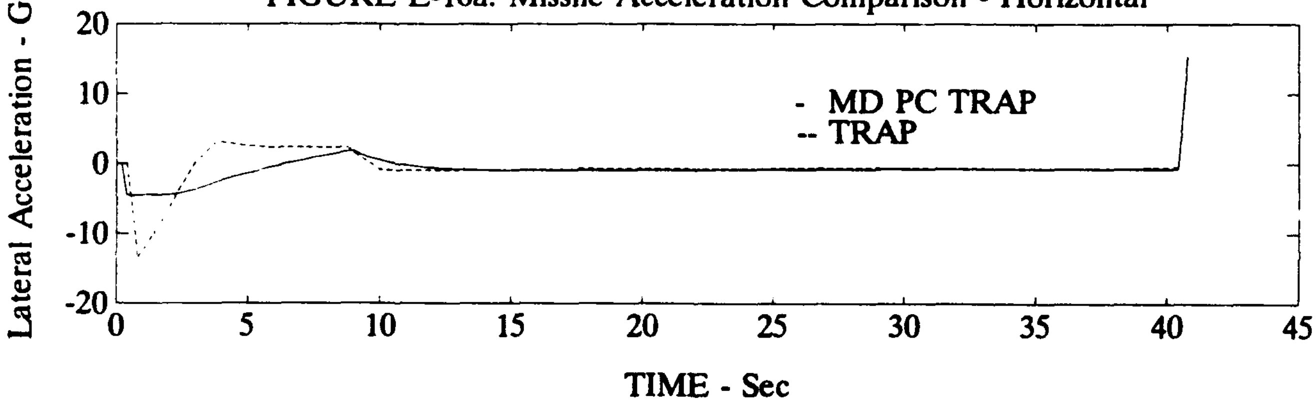


FIGURE E-16b: Missile Acceleration Comparison - Vertical

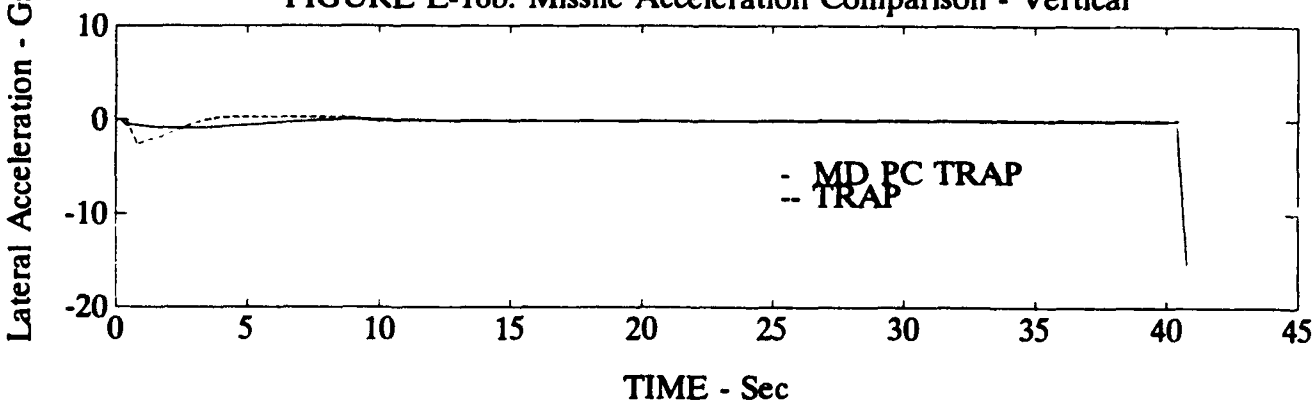


FIGURE E-17: Delivered Thrust vs Time Comparison

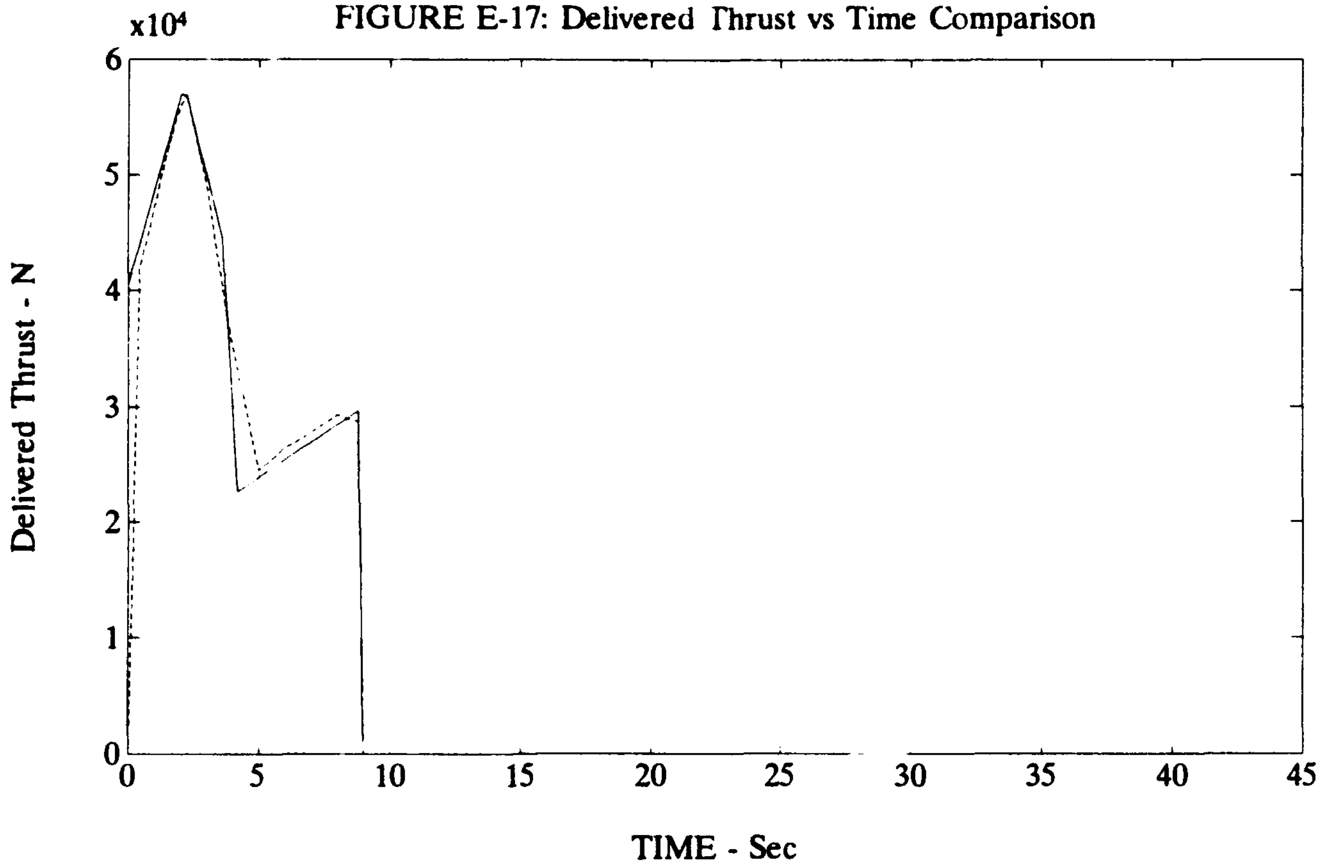


FIGURE E-18: Coefficient of Normal Force (CN) Comparison

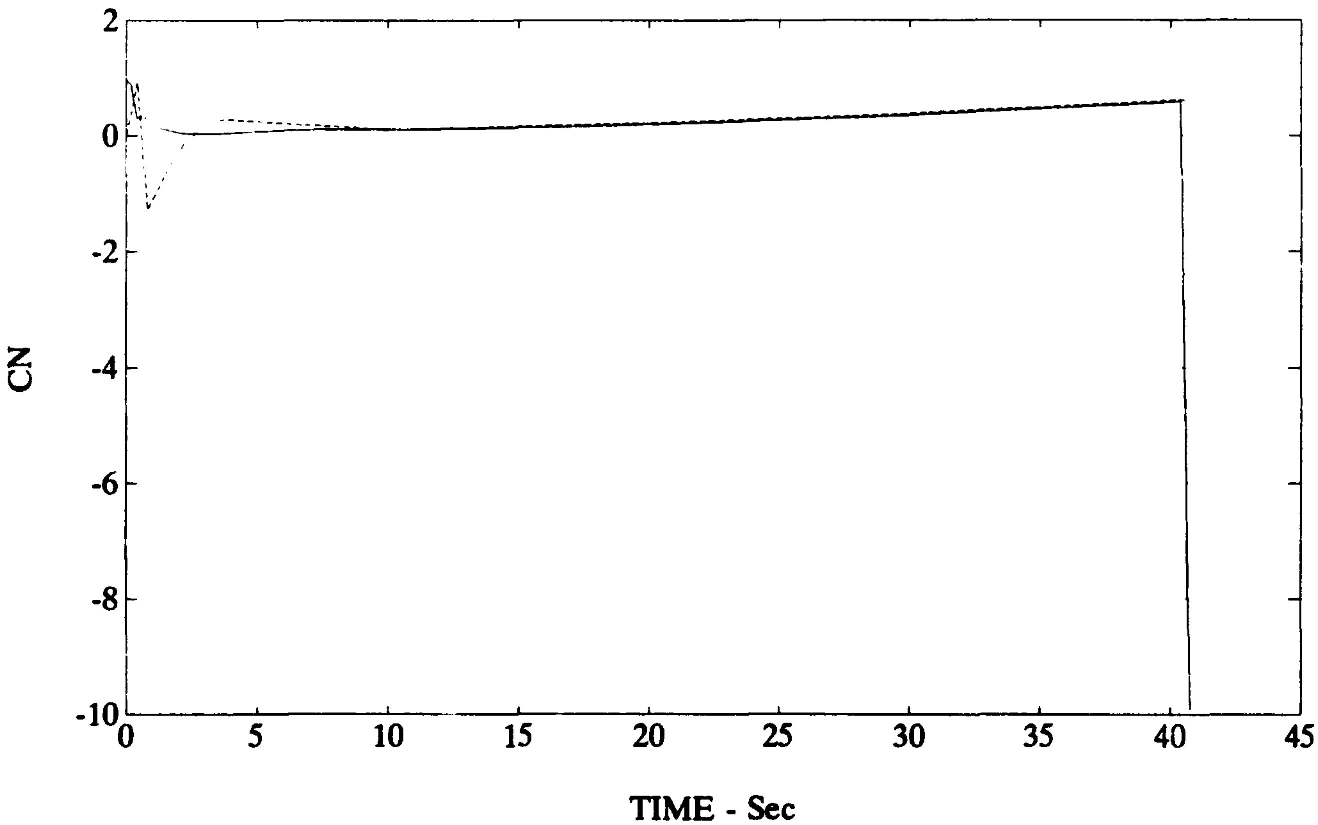


FIGURE E-19: Coefficient of Axial Force (CA) Comparison

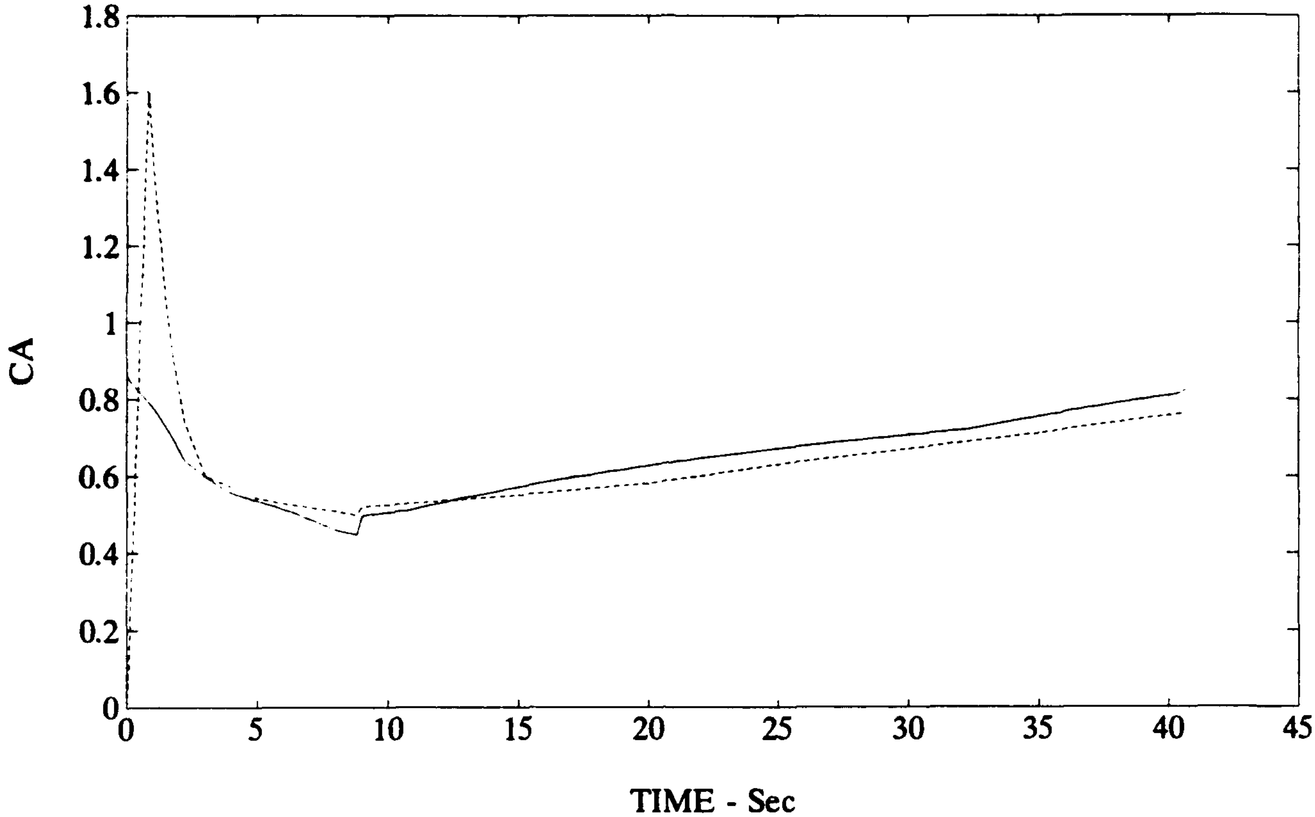


FIGURE E-20a: Missile Angle-of-Attack (ALPHA) Comparison

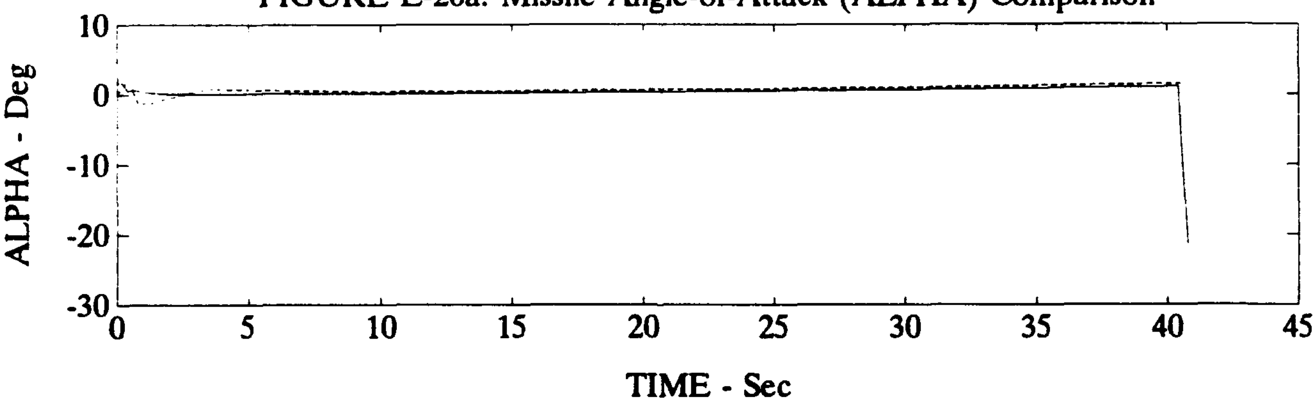


FIGURE E-20b: Missile Sideslip Angle (BETA) Comparison

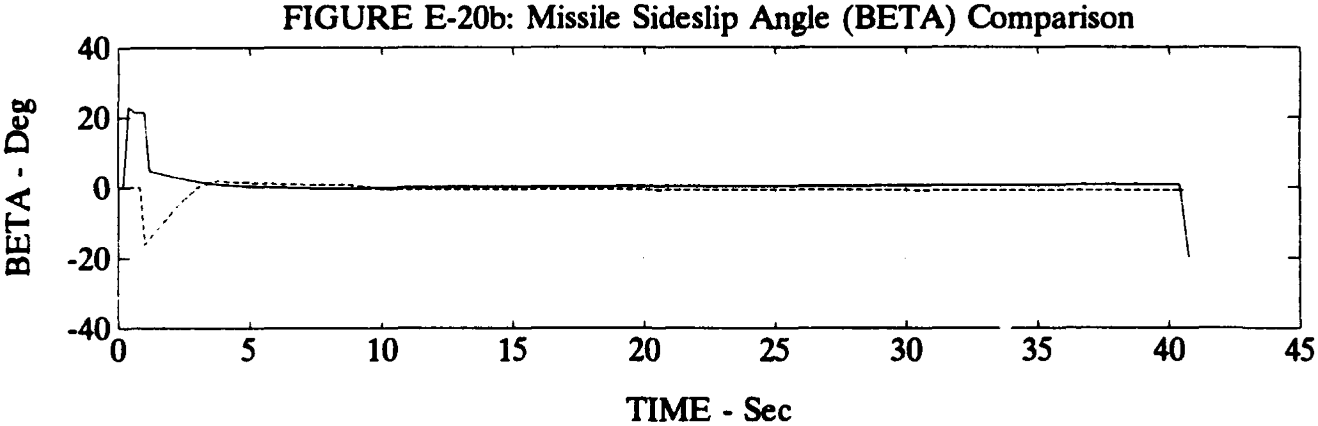
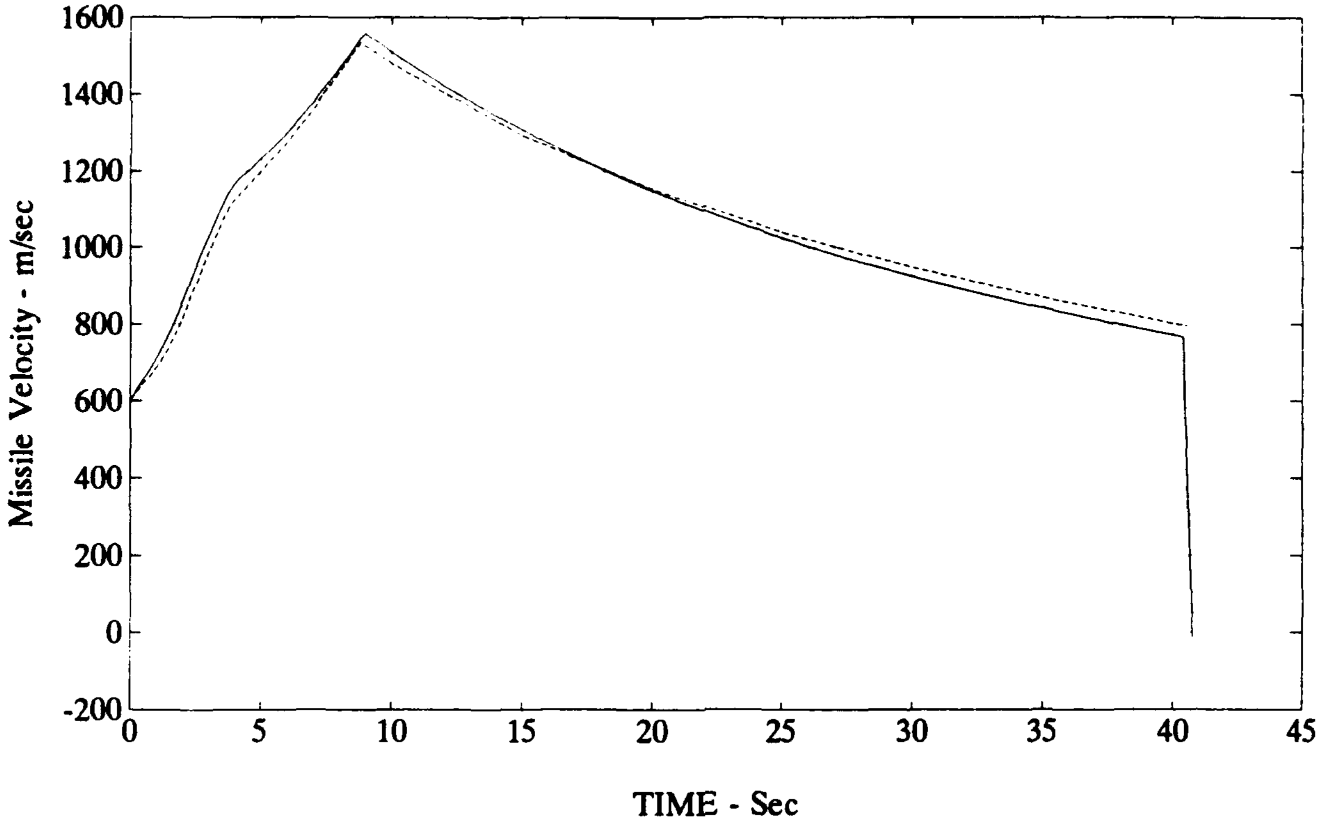


FIGURE E-21: Missile Velocity Comparison



Profile 4

Profile 4 is s a co-altitude missile shot where both the target and launching aircraft are initially at the same altitude. The missile is launched from the launching aircraft at a slant range of 20.1 Km with an aspect angle of 174.3 degrees in the azimuth plane (5.7 degrees off the target's nose). The altitude of both the shooter and target is 10 Km, and their airspeed is 600 m/sec and 450 m/sec respectively. The target does not execute any evasive maneuver during the entire missile time of flight. The guidance law is pure pursuit as detailed in Chapter III. Table E-4 provides the miss distance results obtained from both TRAP and Missile Design PC TRAP simulation flyouts.

Miss Distance Components	TRAP (Meters)	MD PC TRAP (Meters)
X_S-axis	25.04	-0.28
Y_S-axis	76.0	-0.90
Z_S-axis	0.25	0.00
Total	80.03	0.94

Table E-4. Miss Distance Components Comparison - Profile 4

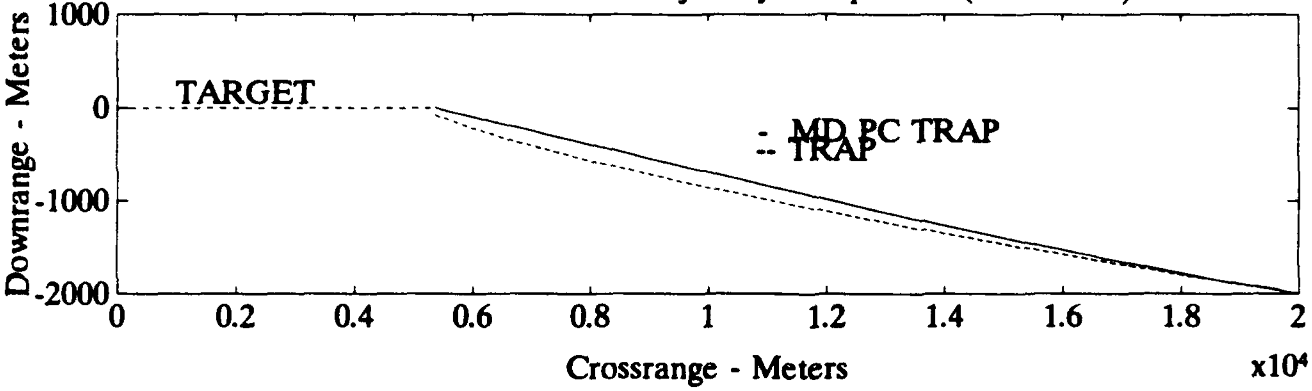


FIGURE E-22a: Missile Trajectory Comparison (Horizontal)

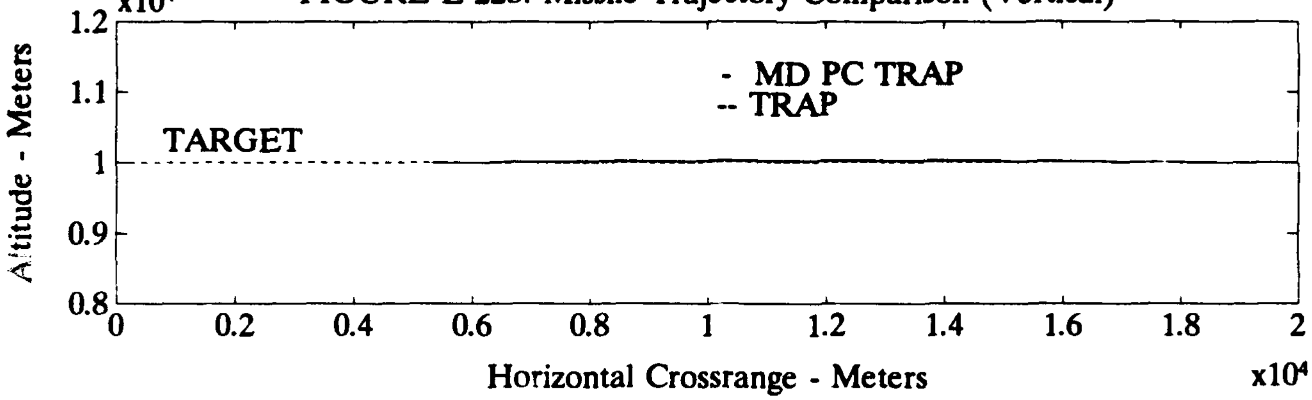


FIGURE E-22b: Missile Trajectory Comparison (Vertical)

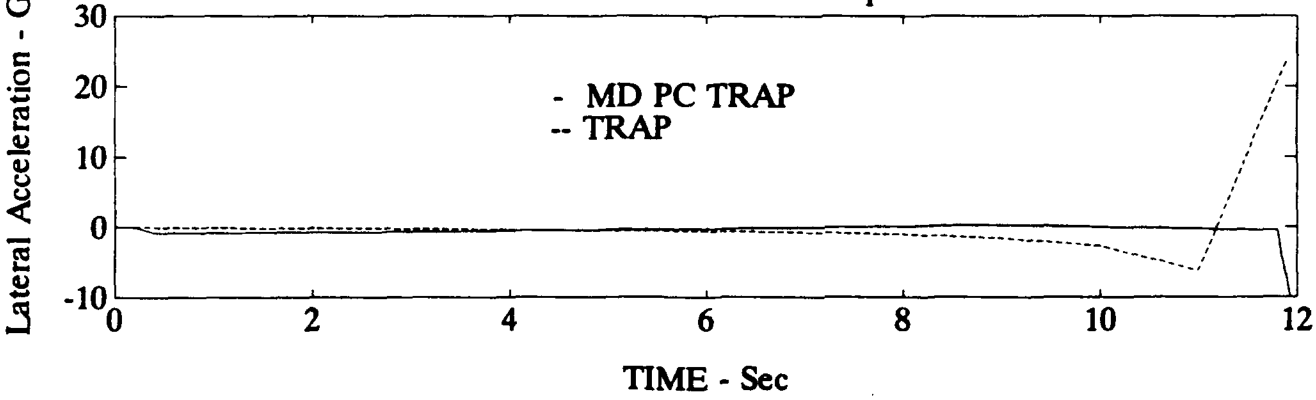


FIGURE E-23a: Missile Acceleration Comparison - Horizontal

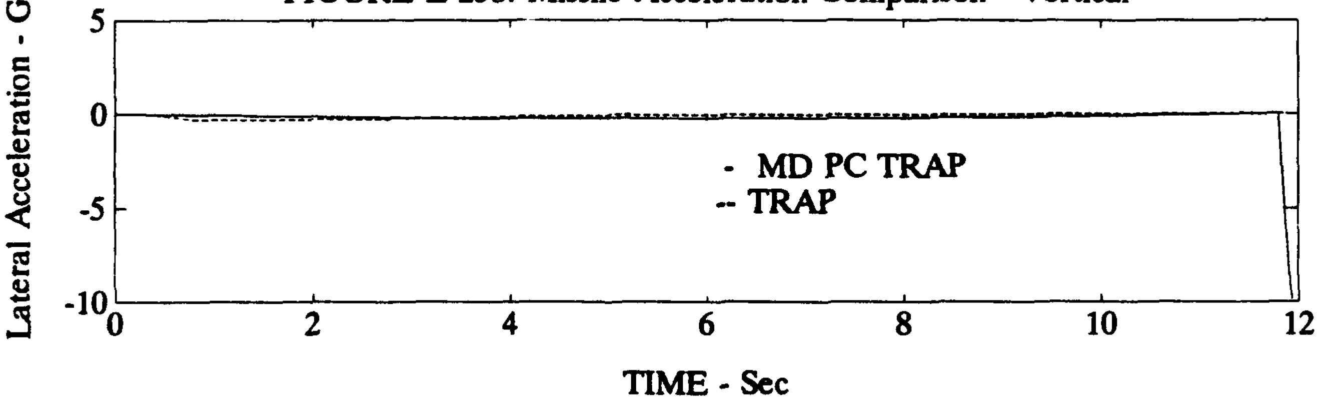


FIGURE E-23b: Missile Acceleration Comparison - Vertical

FIGURE E-24: Delivered Thrust vs Time Comparison

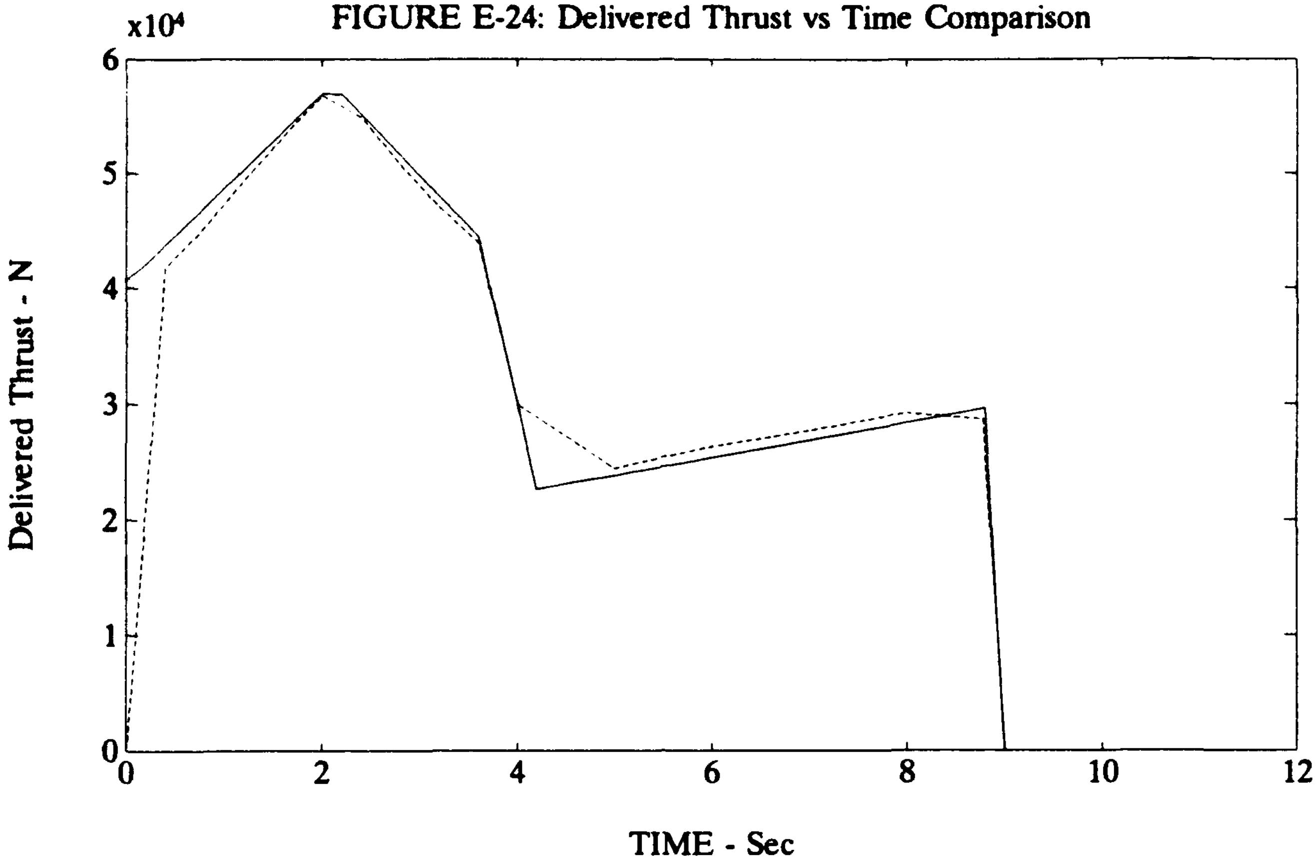


FIGURE E-25: Coefficient of Normal Force (CN) Comparison

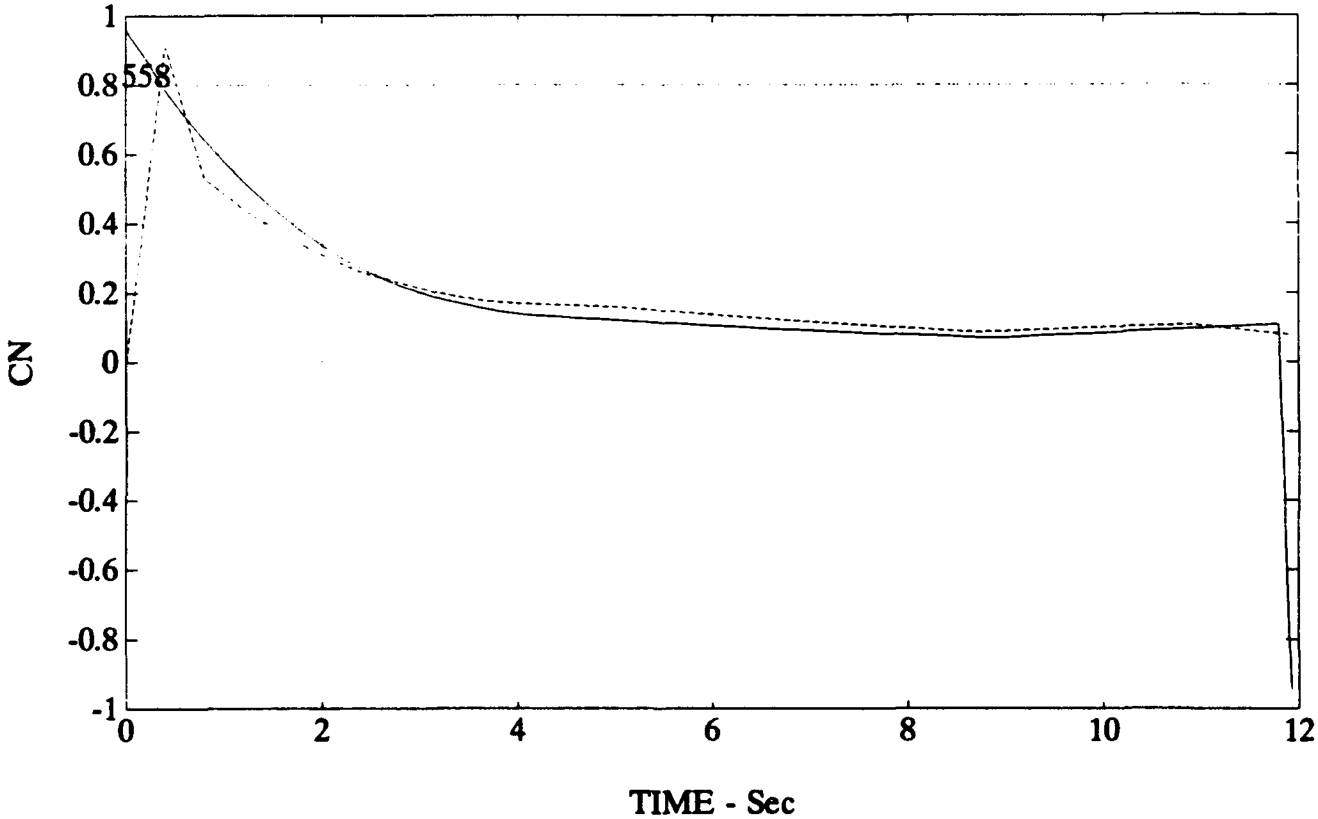


FIGURE E-26: Coefficient of Axial Force (CA) Comparison

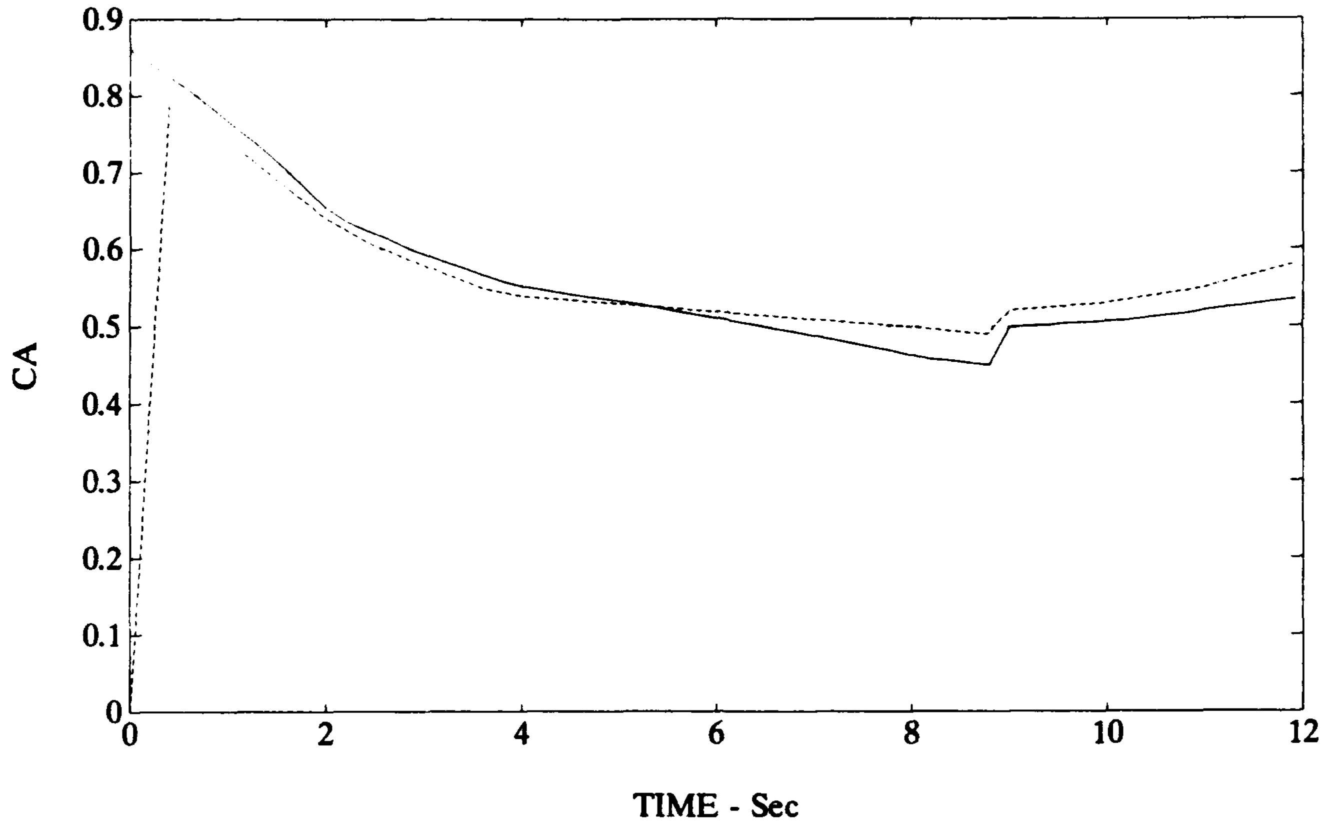


FIGURE E-27a: Missile Angle-of-Attack (ALPHA) Comparison

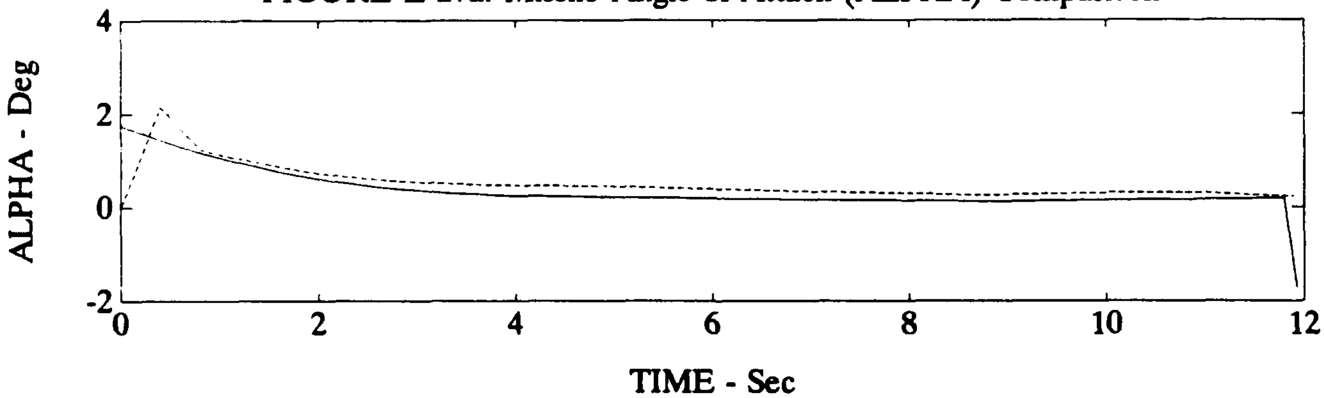


FIGURE E-27b: Missile Sideslip Angle (BETA) Comparison

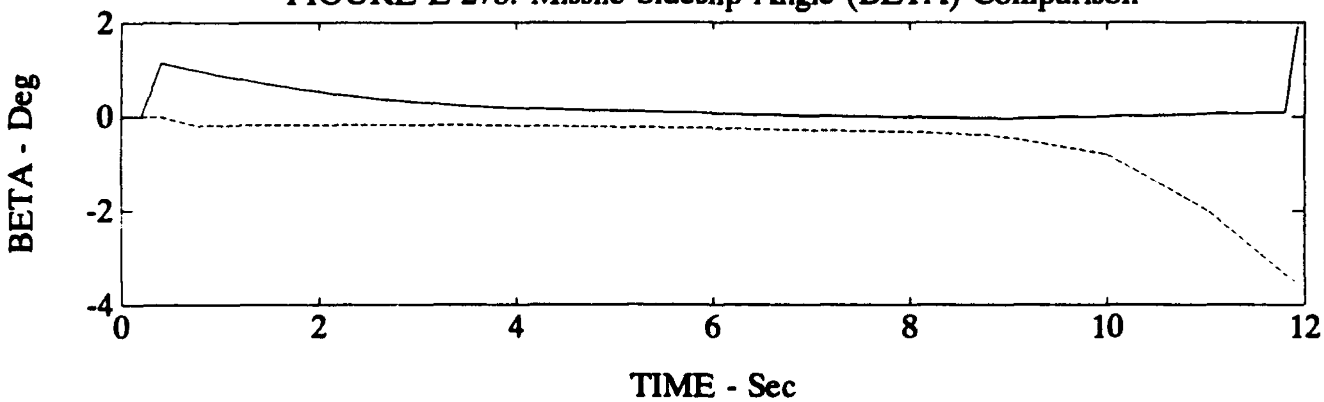
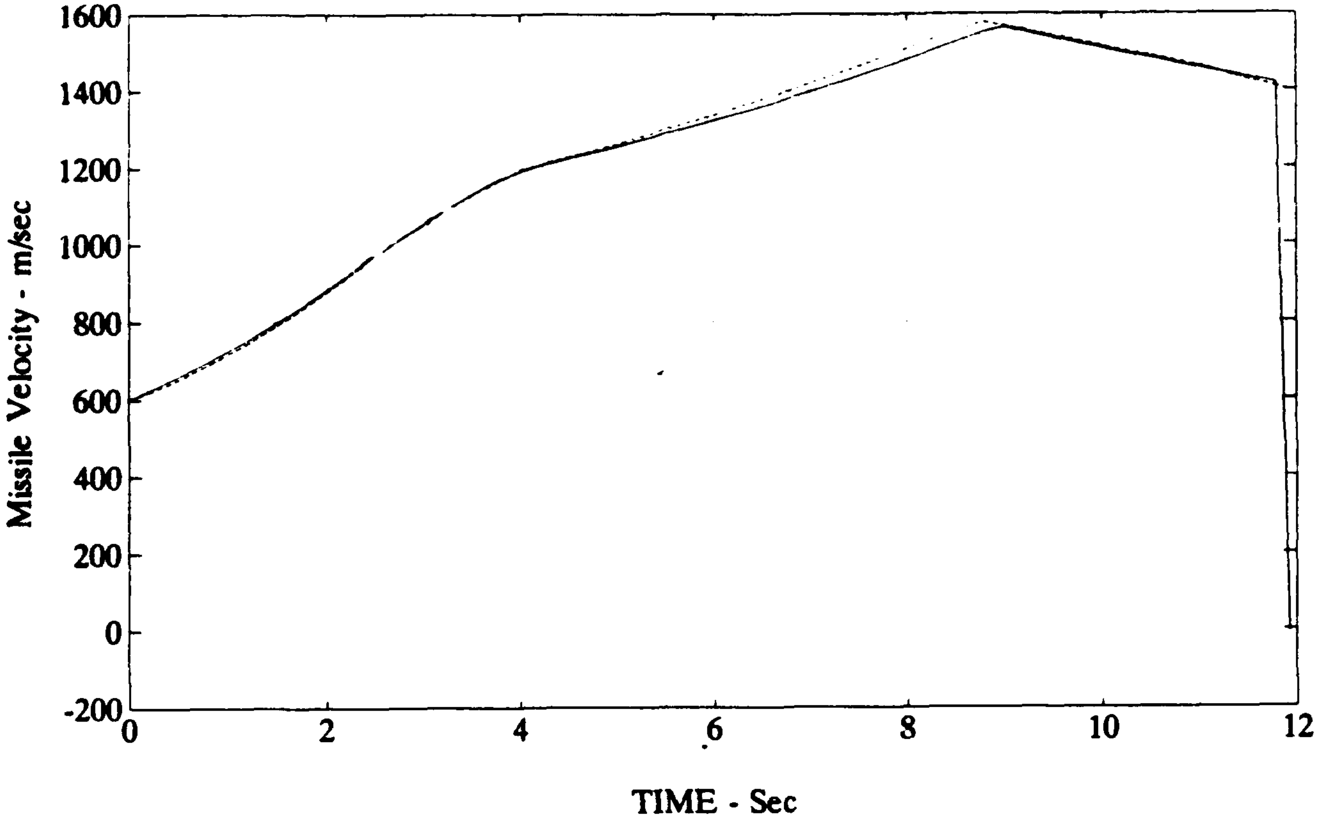


FIGURE E-28: Missile Velocity Comparison



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