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SOME CONCEPTS FOR TARGET TRAJECTORY PREDICTIONS

BY G. W. GROVES W. D. BLAIR J. E. GRAY
SYSTEMS RESEARCH AND TECHNOLOGY DEPARTMENT

MARCH 1994

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NAVAL SURFACE WARFARE CENTER
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FOREWORD

This report describes the results of an initial investigation of new methods for predicting the future trajectory of maneuvering targets. With the advent of highly maneuverable threats, target trajectory prediction is critical for the success of ship self-defense systems. This report discusses many of the concepts associated with trajectory prediction with respect to the dynamic constraints of the target and its self-imposed limitations. The prediction methods have applications in command and decision systems and weapons control systems, such as the PHALANX Close-In Weapons System, NATO Sea Sparrow, etc.

Appreciation is given to Ernest Ohlmeyer of the Aeromechanics Branch, Naval Surface Warfare Center Dahlgren Division (NSWCDD), for his helpful suggestion to use proportional navigation techniques for goal-based trajection prediction for maneuvering targets.

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ABSTRACT

The classical problem of weapons control involves predicting the future position of a maneuvering target. Methods of target trajectory prediction for ship self-defense against a maneuvering antiship cruise missile (ASCM) are studied. Several prediction schemes are described and studied in the context of a typical single ASCM engagement. Methods of utilizing the estimated threat acceleration and/or a "goal" hypothesis are considered. "Terminal Parameters" (TP) for effectively characterizing the threat of a target to the ship are introduced, along with some Measures of Effectiveness (MOE) for comparing the different prediction methods. These TPs and MOEs are illustrated through a simulated engagement involving a hypothesized threat track. An enhancement of defensive capability achieved by utilizing a goal hypothesis is demonstrated.

CONTENTS

ILLUSTRATIONS

TABLES

CHAPTER 1

INTRODUCTION

One of the most important and difficult problems facing the surface Navy is self-defense against fast, maneuvering antiship cruise missiles (ASCMs), which may skim the sea surface and have a small radar cross section. Among the many tasks that need to be completed to successfully address this problem is that of improving techniques for threat-trajectory prediction. The ability of future ASCMs to carry out sharp and deceptive maneuvers interjects much uncertainty into the trajectory-prediction process.

This report describes some simple algorithms for predicting the future trajectory of an incoming ASCM. The algorithms are separated into two general classes. One uses only the threat state estimates (position, velocity, and perhaps acceleration) at the instant of firing and is called "State-Based." The other class assumes additional knowledge of the threat's destination or goal and is called "Goal-Based." The latter method is more pertinent to self-defense than to situations involving a task force where alternative threat destinations are feasible.

The threat speed for each of the new predictors is assumed constant throughout the duration of each predicted trajectory. Several effects are thereby neglected: energy conversion between potential and kinetic caused by rising or descending, the usual frictional loss of energy, and the possibility of the threat varying its thrust or drag coefficient to confuse the defenders. While these effects can be important in some circumstances, it is worthwhile to gain understanding of the simpler constant-speed case before introducing complications. For a threat with limited capability of controlling its longitudinal acceleration, and whose trajectory is not very steep, these simplifications are appropriate. In the present state of ASCM technology, rapid speed variation is not feasible, especially in the endgame portion of the flight.

State-Based predictors include the well-known constant-velocity (CV) and constant-acceleration (CA) predictors. The new state-based predictors include Constant Turning Rate (CTR), Exponentially Decreasing Turning Rate (EDTR), and Helical. The CTR predictor is intended as a replacement of CA predictor, which does not maintain constant threat speed. The EDTR predictor transitions from CTR to CV, and thus avoids prolonged continuation of a turning maneuver while maintaining constant speed. The Helical predictor, which produces a constant-curvature and constant-torsion trajectory, is proposed for cases in which there is reliable threat jerk information. The trajectory resulting from constant lateral and longitudinal acceleration (CLLA) is discussed. In this trajectory the acceleration vector, loosely speaking, is constant when referred to the velocity vector, which is quite different from the CA predictor, in which the acceleration vector is constant in an inertial frame.

The Goal-Based predictors include the Minimax (MM), the Earliest-Arrival (EA), and several Proportional-Navigation (PN) predictors. The MM predictor minimizes the maximum lateral-acceleration magnitude required to reach the ship. The EA predictor assumes that the threat will get to the ship as soon as possible while maintaining constant speed. The PN predictors steer the threat toward the ship through PN formulas that specify an appropriate acceleration at any instant (i.e., ignoring the initial-acceleration information). The Twisted PN (TPN) predictor is proposed for cases in which it is desirable to make use of the initial estimated threat acceleration. The TPN predictor produces a trajectory with nonvanishing torsion.

This report is organized as follows. The analysis uses a simple vector formalism as introduced in Chapter 2. Chapter 3 introduces the traditional and new state-based prediction methods. The goal-based prediction methods are described in Chapter 4. Chapter 5 introduces the concept of Terminal Parameters (TPs), which translate the initial threat state estimates into values that facilitate the command and control process (e.g., whether to fire, when to fire, how to predict the threat trajectory). Measures of Effectiveness (MOEs) are introduced in Chapter 6, along with a comparison of the methods through a test scenario. Chapter 7 considers modifications needed when the predicted trajectory would otherwise take the threat through the sea surface. A general summary of the work and concluding remarks are given in Chapter 8.

CHAPTER 2

PROBLEM FORMULATION

PLANAR TRAJECTORIES

Let the threat position, velocity, and acceleration at time $t=0$ be denoted by $\mathbf{x}_0, \dot{\mathbf{x}}_0, \ddot{\mathbf{x}}_0$, respectively. Let $\mathbf{x}(t)$ be the predicted threat position based on these values. In a large number of cases, it is desirable to assume that the threat will continue to fly in the plane osculating to the true trajectory at the time $t=0$. The formula

$$\mathbf{x}(t) = \mathbf{x}_0 + \alpha(t)\dot{\mathbf{x}}_0 + \beta(t)\ddot{\mathbf{x}}_0 \tag{2-1}$$

ensures that the threat remain on this plane. In this case, the trajectory is completely determined by the two scalar functions $\alpha(t)$ *and* $\beta(t)$. Note that these functions and their derivatives are chosen to have the initial values

$$\begin{array}{ll} \alpha(0)=0 & \beta(0)=0 \\ \dot{\alpha}(0)=1 & \dot{\beta}(0)=0 \\ \ddot{\alpha}(0)=0 & \ddot{\beta}(0)=1 \end{array} \tag{2-2}$$

For constant-speed motion, the constraint relation

$$|\dot{\mathbf{x}}_0|^2(1-\dot{\alpha}^2) = |\ddot{\mathbf{x}}_0|^2\dot{\beta}^2 \tag{2-3}$$

is required by (2-1).

CONSTRAINT ON THE THREAT ACCELERATION

For the prediction schemes discussed in this report, the threat speed is assumed to remain constant throughout the prediction time. This requires that the acceleration always remain perpendicular to the velocity. If the initial conditions or state estimates obey this constraint, the prediction formulas satisfy the constraint for all time. When the initial values are obtained from sensor information by means of some track filtering scheme, this constraint will not generally be satisfied. In this case, one of the two vectors can be altered. Since the estimated velocity value is more reliable than the acceleration estimate, the acceleration is altered when necessary; assuming that the estimated acceleration is of some value, it should be altered as little as possible. The following criteria were considered to determine a modified acceleration, $\ddot{\mathbf{x}}_{0A}$, to replace the initial estimated acceleration.

The modified acceleration should be orthogonal to the estimated velocity. The modified acceleration should lie in the plane of the velocity and the estimated acceleration. The magnitude of the modified acceleration will be set equal to the component of the estimated acceleration that is perpendicular to the velocity and given by

$$\ddot{\mathbf{x}}_{0A} = \ddot{\mathbf{x}}_0 - \frac{\dot{\mathbf{x}}_0 \cdot \ddot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|^2}\dot{\mathbf{x}}_0 \qquad (2\text{-}4)$$

where $\dot{\mathbf{x}}_0$ is the estimated velocity vector and $\ddot{\mathbf{x}}_0$ is the estimated acceleration vector as provided by a track-filtering algorithm such as a Kalman filter.

CHAPTER 3

STATE-BASED PREDICTORS

The trajectory predictors can conveniently be divided into two classes: state-based and goal-based. The state-based predictors depend only on the estimated threat state (position, velocity, and perhaps acceleration), and are discussed in this chapter, while the goal-based predictors are discussed in Chapter 4.

CV TRAJECTORY

The CV trajectory is given by (2-1), where

$$\begin{aligned} \alpha(t) &= t \\ \beta(t) &= 0 \end{aligned} \tag{3-1}$$

The CV threat continues to fly in a straight line at constant speed. The constancy of the speed is demonstrated by the fact that the expressions (3-1) satisfy the condition (2-3). The estimated initial threat acceleration is not used in the CV predictor.

CA TRAJECTORY

The CA trajectory is given by (2-1), where

$$\begin{aligned} \alpha(t) &= t \\ \beta(t) &= t^2/2 \end{aligned} \tag{3-2}$$

Here, the acceleration vector is constant in magnitude and direction if referred to an inertial frame. Since the expressions (3-2) do not satisfy the condition (2-3) for constant speed, the speed generally varies.

CTR TRAJECTORY

In order to achieve a trajectory that is qualitatively similar to CA, but which obeys the constant-speed constraint, the CTR predictor is used. The trajectory is given by (2-1) with

$$\begin{aligned} \alpha(t) &= \omega_0^{-1}\sin(\omega_0 t) \\ \beta(t) &= \omega_0^{-2}[1-\cos(\omega_0 t)] \end{aligned} \tag{3-3}$$

where ω_0 is the constant turning rate given by

$$\omega_0 = \frac{|\ddot{\mathbf{x}}_0|}{|\dot{\mathbf{x}}_0|} \tag{3-4}$$

The trajectory is of course a circular arc. Since the expressions (3-3) satisfy the condition (2-3), the speed remains constant over the time of prediction.

EDTR TRAJECTORY

It is unreasonable to expect that a nonzero turning rate will persist indefinitely, because the threat would be flying in circles. The EDTR trajectory allows for the turning rate to decay to zero. The resulting trajectory begins similar to a circular arc and ends up as a CV trajectory. This EDTR trajectory should not be confused with the linear exponentially decaying acceleration predictor, which results in a variable-speed trajectory.

The constraint (2-3) is satisfied by setting

$$\dot{\alpha} = \cos[\mu(t)], \qquad \dot{\beta} = \omega_0^{-1}\sin[\mu(t)] \tag{3-5}$$

where $\mu(t)$ is an undetermined function. No essential additional assumption has been made by this substitution. Satisfying the initial conditions on α and β of (2-2) requires that

$$\mu(0) = 0, \qquad \dot{\mu}(0) = \omega_0 \tag{3-6}$$

The first of these relations is obtained by setting t equal to zero in (3-5) and applying the initial conditions (2-2). The second is obtained by setting t equal to zero in the derivative of (3-5) and applying (2-2). The form of the lateral acceleration needs to be specified. Differentiating the position vector (2-1) twice and taking its magnitude gives

$$|\ddot{\mathbf{x}}| = \dot{\mu}\,|\dot{\mathbf{x}}_0| \tag{3-7}$$

where $\dot{\alpha}(t)$ and $\dot{\beta}(t)$ are eliminated by using (3-5). For the exponentially decreasing turning rate, we take an exponentially-decreasing lateral acceleration with time constant τ and set the above expression equal to

$$|\ddot{\mathbf{x}}| = |\ddot{\mathbf{x}}_0|\,e^{-t/\tau} \tag{3-8}$$

This requires that

$$\dot{\mu} = \omega_0 e^{-t/\tau} \tag{3-9}$$

This differential equation, along with the initial values, determines μ to be

$$\mu(t) = \omega_0 \tau\,(1 - e^{-t/\tau}) \tag{3-10}$$

The functions α and β can now be obtained by integrating their previously determined derivatives given by

$$\alpha(t) = \int_0^t \cos[\omega_0\tau(1-e^{-\acute{t}/\tau})]\,d\acute{t} \tag{3-11}$$

$$\beta(t) = \omega_0^{-1}\int_0^t \sin[\omega_0\tau(1-e^{-\acute{t}/\tau})]\,d\acute{t} \tag{3-12}$$

Making the substitution

$$\lambda = \omega_0\tau e^{-\acute{t}/\tau} \tag{3-13}$$

in (3-11) and (3-12) gives

$$\alpha(t) = -\tau \int_{\omega_0\tau}^{\omega_0\tau e^{-t/\tau}} \cos(\omega_0\tau-\lambda)\frac{d\lambda}{\lambda} \tag{3-14}$$

and

$$\beta(t) = -\frac{\tau}{\omega_0} \int_{\omega_0\tau}^{\omega_0\tau e^{-t/\tau}} \sin(\omega_0\tau-\lambda)\frac{d\lambda}{\lambda} \tag{3-15}$$

The sine and cosine integral functions can be used to express the solution of these integrals. In the notation of Abramowitz and Stegun[1] they are expressed as

$$Si(z) = \int_0^z \frac{\sin t}{t}dt = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)(2n+1)!} \tag{3-16}$$

$$\begin{aligned} Ci(z) &= \gamma + \ln z + \int_0^z \frac{\cos t - 1}{t}dt \\ &= \gamma + \ln z + \sum_{n=1}^{\infty} \frac{(-1)^n z^{2n}}{2n(2n)!} \end{aligned} \tag{3-17}$$

where $\gamma = 0.57722$ is Euler's constant. The sums in the expressions for $Si(z)$ and $Ci(z)$ converge slowly for large argument, for which Abramowitz and Stegun provide good approximations in the form of rational algebraic expressions. The result can be expressed in the form

$$\begin{aligned} \alpha(t) = -\tau\{&\cos(\omega_0\tau)[Ci(\omega_0\tau e^{-t/\tau}) - Ci(\omega_0\tau)] \\ &+ \sin(\omega_0\tau)[Si(\omega_0\tau e^{-t/\tau}) - Si(\omega_0\tau)]\} \end{aligned} \tag{3-18}$$

$$\beta(t) = \tau\omega_0^{-1}\{-\sin(\omega_0\tau)\,[Ci(\omega_0\tau e^{-t/\tau}) - Ci(\omega_0\tau)] + \cos(\omega_0\tau)\,[Si(\omega_0\tau e^{-t/\tau}) - Si(\omega_0\tau)]\} \tag{3-19}$$

Approximations of (3-18) and (3-19) for small and for large values of t are readily obtained. For $t \gg \tau$ (3-18) and (3-19) approach the asymptotic values

$$\alpha(t) \to -\tau\{\cos(\omega_0\tau)\,[\gamma + \ln(\omega_0\tau) - t\tau^{-1} - Ci(\omega_0\tau)] + \sin(\omega_0\tau)\,[\omega_0\tau e^{-t/\tau} - Si(\omega_0\tau)]\} + O(e^{-2t/\tau}) \tag{3-20}$$

and

$$\beta(t) \to -\tau\omega_0^{-1}\{\sin(\omega_0\tau)\,[\gamma + \ln(\omega_0\tau) - t\tau^{-1} - Ci(\omega_0\tau)] - \cos(\omega_0\tau)\,[\omega_0\tau e^{-t/\tau} - Si(\omega_0\tau)]\} + O(e^{-2t/\tau}) \tag{3-21}$$

where $O(e^{-2t/\tau})$ means "order of" and indicates a quantity whose ratio to $e^{-2t/\tau}$ remains bounded as $t \to \infty$. For $t \ll \tau$ the same expressions approach the asymptotic values

$$\alpha(t) = t - \frac{\omega_0^2}{6}t^3 + O(t^4)\,, \qquad \beta(t) \to \frac{t^2}{2} - \frac{t^3}{6\tau} + O(t^4) \tag{3-22}$$

where here $O(t^4)$ indicates a quantity whose ratio to t^4 remains bounded as $t \to 0$. The time derivatives of the asymptotic expressions (3-20) and (3-21) for large t indicate that the velocity vector finally approaches

$$\dot{\mathbf{x}}_f = \dot{\mathbf{x}}(\infty) = \dot{\mathbf{x}}_0\cos(\omega_0\tau) + \omega_0^{-1}\ddot{\mathbf{x}}_0\sin(\omega_0\tau) \tag{3-23}$$

The angle between the initial and final velocities is determined by the relation

$$\frac{\dot{\mathbf{x}}_0 \cdot \dot{\mathbf{x}}_f}{|\dot{\mathbf{x}}_0|^2} = \cos(\omega_0\tau) \tag{3-24}$$

which shows that the total change in heading (amount of turning) is equal to $\omega_0\tau$.

It is seen that for positive τ (decreasing turning rate), the velocity vector eventually approaches the final constant value given by (3-23). Thereafter, the trajectory is a straight line, which can be represented by

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{h} + \dot{\mathbf{x}}_f t \tag{3-25}$$

which defines the constant displacement vector $\mathbf{h}$. This vector is evaluated by discarding the exponentially decreasing terms in (3-20) and (3-21) and comparing (2-1) with (3-25). The result is

$$
\begin{aligned}
\boldsymbol{h} = \; & \tau\{-\cos(\omega_0\tau)\,[\gamma + \ln(\omega_0\tau) - Ci(\omega_0\tau)] \\
& \qquad + \sin(\omega_0\tau)\, Si(\omega_0\tau)\}\dot{\boldsymbol{x}}_0 \\
& + \frac{\tau}{\omega_0}\{-\sin(\omega_0\tau)\,[\gamma + \ln(\omega_0\tau) - Ci(\omega_0\tau)] \\
& \qquad - \cos(\omega_0\tau)\, Si(\omega_0\tau)\}\ddot{\boldsymbol{x}}_0
\end{aligned}
\tag{3-26}
$$

The length of this vector is

$$
|\boldsymbol{h}| = \tau|\dot{\boldsymbol{x}}_0|\{[\gamma + \ln(\omega_0\tau) - Ci(\omega_0\tau)]^2 + Si^2(\omega_0\tau)\}^{1/2} \tag{3-27}
$$

Figure 3-1 shows both the actual trajectory (solid curve) given by (2-1) and a hypothetical trajectory (dashed line) given by (3-25) representing the motion of a hypothetical missile that travels during its entire flight with the constant velocity equal to the real missile's final velocity. This hypothetical missile began its flight from the initial position at the beginning of the ray shown on Figure 3-1 at the same time the real missile was at its initial position.

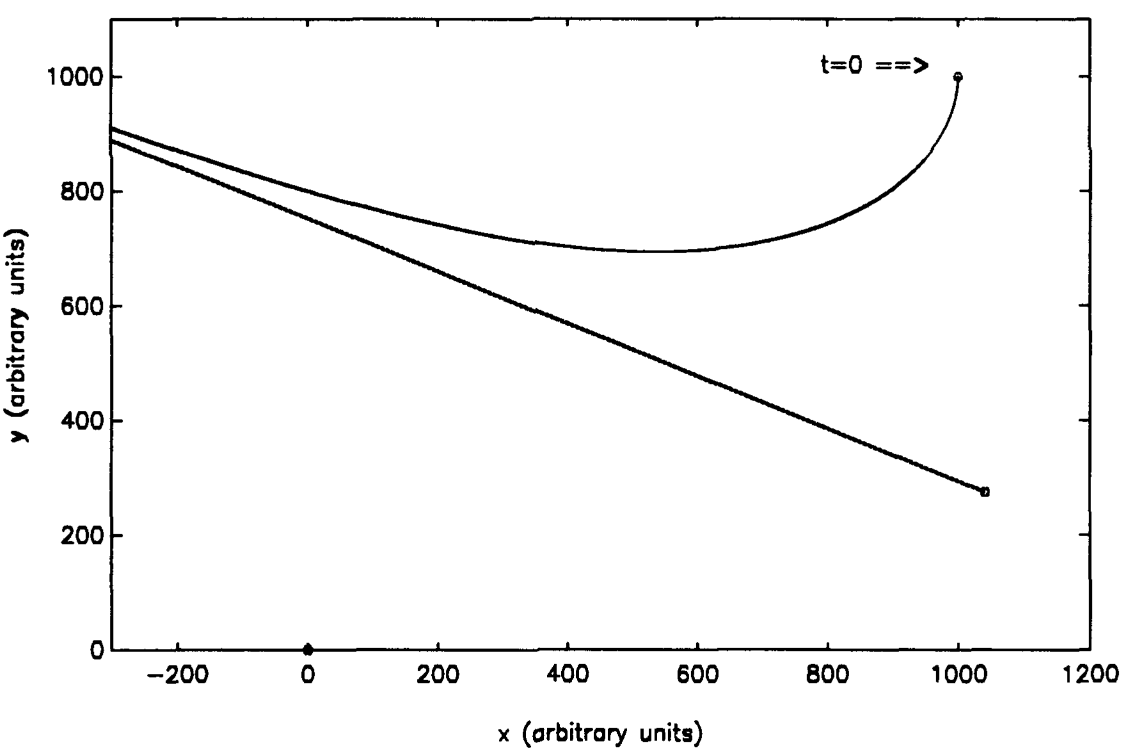

FIGURE 3-1. TYPICAL EDTR TRAJECTORY (SOLID) AND ASYMPTOTE (DASHED) AS RAY DEPARTING FROM A POINT THAT IS SYNCHRONOUS WITH INITIAL THREAT POSITION

Projected along its linear trajectory, the hypothetical missile begins its flight a distance of

$$\frac{\boldsymbol{h}\cdot\dot{\boldsymbol{x}}_f}{|\dot{\boldsymbol{x}}_0|} = -\tau|\dot{\boldsymbol{x}}_0|[\gamma + \ln(\omega_0\tau) - Ci(\omega_0\tau)] \qquad \textbf{(3-28)}$$

behind the initial position of the real missile (but eventually catches up and asymptotically coincides in position with the real missile). The "offset" of the hypothetical trajectory; i.e., the minimum distance of the hypothetical missile from the initial position of the real missile, is given by

$$\delta = \{|\boldsymbol{h}|^2 - [\boldsymbol{h}\cdot\dot{\boldsymbol{x}}_f/|\dot{\boldsymbol{x}}_0|]^2\}^{1/2} = \tau|\dot{\boldsymbol{x}}_0|Si(\omega_0\tau) \qquad \textbf{(3-29)}$$

An appropriate value of the die-off time constant, τ, must be selected. If this value greatly exceeds the prediction time, the EDTR will not perceptibly differ from CTR. It may be easiest to base the value of τ on the expected total change of heading, $\omega_0\tau$.

HELICAL TRAJECTORY

The helix, being a curve having constant curvature and constant torsion, is the simplest reasonable nonplanar trajectory. It requires knowledge of the threat jerk, *in addition to the* position, velocity and acceleration, from the estimated state vector. Although the jerk estimate may be rather poor, it is constrained to improve prediction in a manner to be described below. It should be noted that prediction models based on planar motion tacitly assume that the jerk lies in the velocity-acceleration plane. This assumption may be worse in some instances than using jerk information from the estimated state vector.

The helix is described by a simple, well-known formula that expresses the position vector as a function of time. The constants of this representation are expressed as the initial position, $\boldsymbol{x}_0$, velocity, $\dot{\boldsymbol{x}}_0$, acceleration, $\ddot{\boldsymbol{x}}_0$, and jerk, $\dddot{\boldsymbol{x}}_0$. The helical trajectory is represented by

$$\begin{aligned}\boldsymbol{x}(t) = \; & \boldsymbol{x}_0 + (\dot{\boldsymbol{x}}_0 + \omega_0^{-2}\dddot{\boldsymbol{x}}_0)\,t \\ & + \omega_0^{-2}[1-\cos(\omega_0 t)]\ddot{\boldsymbol{x}}_0 - \omega_0^{-3}\dddot{\boldsymbol{x}}_0\sin(\omega_0 t)\end{aligned} \qquad \textbf{(3-30)}$$

where

$$\omega_0 = \frac{|\dddot{\boldsymbol{x}}_0|}{|\ddot{\boldsymbol{x}}_0|} \qquad \textbf{(3-31)}$$

Since for constant speed the acceleration is always perpendicular to the velocity,

$$\dot{\mathbf{x}}\cdot\ddot{\mathbf{x}} = 0 \tag{3-32}$$

Taking the derivative of (3-32), it is seen that

$$\dot{\mathbf{x}}\cdot\dddot{\mathbf{x}} + |\ddot{\mathbf{x}}|^2 = 0 \tag{3-33}$$

which indicates that the angle between the jerk and the velocity lies between $\pi/2$ and $3\pi/2$. Taking derivatives of (3-30) for direct substitution into (3-32), using (3-31) and (3-33), it is seen that

$$\dot{\mathbf{x}}\cdot\ddot{\mathbf{x}} = \omega_0^{-2}\ddot{\mathbf{x}}_0\cdot\dddot{\mathbf{x}}_0[\cos(\omega_0 t) - \cos(2\omega_0 t)] \equiv 0 \tag{3-34}$$

This shows that the initial jerk must be perpendicular to the initial acceleration in order that the expression (3-34) vanish for all time. In a similar manner it is found that

$$\ddot{\mathbf{x}}\cdot\dddot{\mathbf{x}} = \ddot{\mathbf{x}}_0\cdot\dddot{\mathbf{x}}_0\cos(2\omega_0 t) \tag{3-35}$$

This shows that the jerk is always perpendicular to the acceleration, since initially this is so. Since the left side of (3-35) is (half) the derivative of the acceleration dotted onto itself, the acceleration magnitude remains constant. Also, since ω_0 remains constant, (3-31) requires that the jerk magnitude remain constant.

Note that the helical trajectory reduces to the familiar constant turning rate (in a circular trajectory) if

$$\dddot{\mathbf{x}}_0 = -\omega_0^2\dot{\mathbf{x}}_0 \tag{3-36}$$

This suggests that it may be convenient to represent the jerk as a deviation from that required to sustain circular motion. Accordingly, the vector $\mathbf{p}$ is defined as this deviation and we have

$$\mathbf{p}(t) = \dddot{\mathbf{x}}(t) + \omega_0^2\dot{\mathbf{x}}(t) \tag{3-37}$$

Thus, (3-36) and (3-37) show that $\mathbf{p}(t) \equiv 0$ in the special case where the helical motion degenerates to pure circular motion. The right side of (3-37) can be evaluated by taking time derivatives of (3-30) to show that the vector, $\mathbf{p}$, remains constant. It is also seen that $\mathbf{p}$ is parallel to the nonfluctuating part of $\ddot{\mathbf{x}}$. Thus, $\mathbf{p}$ is parallel to the axis of the helix. Accordingly, we write

$$\mathbf{p}(t) = \mathbf{p}_0 \tag{3-38}$$

The position vector can then be written as

$$\begin{aligned}\mathbf{x}(t) = {} & \mathbf{x}_0 + \omega_0^{-2}\mathbf{p}_0 t + (\omega_0^{-1}\dot{\mathbf{x}}_0 - \omega_0^{-3}\mathbf{p}_0)\sin(\omega_0 t) \\ & + \omega_0^{-2}[1 - \cos(\omega_0 t)]\ddot{\mathbf{x}}_0\end{aligned} \tag{3-39}$$

Also,

$$\dot{\mathbf{x}}\cdot\mathbf{p}_0 = \omega_0^{-2}|\mathbf{p}_0|^2 \tag{3-40}$$

follows from (3-37), (3-38), and (3-39), showing that the angle between the velocity vector and $\mathbf{p}_0$ is constant.

As noted, the initial estimated values of the threat's state vector must satisfy certain constraints. As in the case of the threat acceleration vector, the jerk must also be made to satisfy some constraints. If no meaningful estimate of the jerk is available then there is no reason to use the helical prediction. Accordingly, it is assumed here that reasonable values of the threat position, velocity, acceleration, and jerk are available. As the estimated acceleration value is more reliable than the jerk, the acceleration estimate will be adjusted first by replacing its estimated value by $\ddot{\mathbf{x}}_{0A}$ defined in (2-4). To find a suitable replacement, $\dddot{\mathbf{x}}_{0A}$, for the estimated jerk vector, the following criteria are considered.

(a) The resulting jerk vector should be perpendicular to the acceleration; i.e., $\ddot{\mathbf{x}}_{0A}\cdot\dddot{\mathbf{x}}_{0A} = 0$.

(b) The relation $\dot{\mathbf{x}}_0\cdot\dddot{\mathbf{x}}_{0A} + |\ddot{\mathbf{x}}_{0A}|^2 = 0$, as required by (3-33).

(c) The $\dddot{\mathbf{x}}_{0A}$ should be equal to the component of $\dddot{\mathbf{x}}_0$ which is perpendicular to $\ddot{\mathbf{x}}_{0A}$.

The first two criteria are essential. It turns out that in general the third cannot be satisfied exactly. It may be desirable to satisfy criterion (c) as well as possible, in a least-square-error sense.

Let us represent the vector, $\dddot{\mathbf{x}}_{0A}$, which is to replace the estimated jerk, by

$$\dddot{\mathbf{x}}_{0A} = \mu\dot{\mathbf{x}}_0 + \nu\dot{\mathbf{x}}_0\times\ddot{\mathbf{x}}_{0A} \tag{3-41}$$

which satisfies criterion (a) for all values of μ *and* ν, which are scalar quantities to be determined. Criterion (b) is satisfied by requiring

$$\mu = -\frac{|\ddot{\mathbf{x}}_{0A}|^2}{|\dot{\mathbf{x}}_0|^2} \tag{3-42}$$

The value of ν is still to be determined. It is noted that so far no use at all has been made of the estimated jerk vector. Criterion (c) is relaxed to minimize (with respect to ν) the discrepancy between $\dddot{\mathbf{x}}_{0A}$ and the component $\mathbf{V}$ of $\dddot{\mathbf{x}}_0$ which is perpendicular to $\ddot{\mathbf{x}}_{0A}$. This component is equal to

$$\mathbf{V} = \dddot{\mathbf{x}}_0 - \frac{\dddot{\mathbf{x}}_0\cdot\ddot{\mathbf{x}}_{0A}}{|\ddot{\mathbf{x}}_{0A}|^2}\ddot{\mathbf{x}}_{0A} \tag{3-43}$$

Thus, "equal" is replaced by "close as possible" in a least-square-error sense whereby the quantity $|\dddot{\mathbf{x}}_{0A} - \mathbf{Y}|$ is minimized with respect to ν. This determines ν to have the value

$$\nu = \frac{(\dot{\mathbf{x}}_0 \times \dddot{\mathbf{x}}_{0A}) \cdot \dddot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|^2 |\dddot{\mathbf{x}}_{0A}|^2} \tag{3-44}$$

and the modified jerk $\dddot{\mathbf{x}}_{0A}$ is now determined by (3-41).

CONSTANT LATERAL AND LONGITUDINAL ACCELERATIONS

Let a_T be the magnitude of the constant longitudinal or thrusting acceleration, and let a_L be the magnitude of the constant lateral acceleration. Only the magnitudes are constant. The vector accelerations will be denoted by $\mathbf{a}_T$ and $\mathbf{a}_L$, respectively, and are not constant. Gravity is neglected, and the trajectory is assumed to lie in a fixed plane, the "flight plane". Note that if the flight plane is horizontal, this assumption is not needed. Let this plane be determined by a normal unit vector, $\mathbf{n}$. Let $\mathbf{x}(t)$ be the position at time t. According to these definitions,

$$a_T = \frac{\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}}}{|\dot{\mathbf{x}}|} \tag{3-45}$$

and

$$\ddot{\mathbf{x}} = \mathbf{a}_T + \mathbf{a}_L, \qquad \textit{where} \qquad \mathbf{a}_T \cdot \mathbf{a}_L = 0 \tag{3-46}$$

The longitudinal acceleration vector is

$$\mathbf{a}_T = \frac{a_T \dot{\mathbf{x}}}{|\dot{\mathbf{x}}|} = \frac{(\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}})\, \dot{\mathbf{x}}}{|\dot{\mathbf{x}}|^2} \tag{3-47}$$

Let the lateral acceleration vector be represented by

$$\mathbf{a}_L = \mu \mathbf{n} \times \dot{\mathbf{x}} \tag{3-48}$$

where μ is a scalar to be determined. This form ensures that the lateral acceleration will lie in the flight plane and be perpendicular to the velocity vector. Dotting this expression onto itself gives

$$|\mathbf{a}_L|^2 = a_L^2 = \mu^2 [\,|\dot{\mathbf{x}}|^2 - (\mathbf{n} \cdot \dot{\mathbf{x}})^2] \tag{3-49}$$

which allows μ to be expressed in terms of other quantities. The last term within the brackets on the right is zero, because the velocity vector lies in the flight plane. Substituting this value of μ back into the expression for the lateral acceleration (3-48) gives

$$\mathbf{a}_L = \frac{(\mathbf{n} \times \dot{\mathbf{x}})\, a_L}{|\dot{\mathbf{x}}|} \tag{3-50}$$

The total acceleration is the sum of the longitudinal and lateral accelerations, in accordance with (3-46), and we have

$$\ddot{\mathbf{r}} = \frac{a_L(\mathbf{n}\times\dot{\mathbf{r}}) + a_T\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} \tag{3-51}$$

Let a cartesian coordinate system on the flight plane be established with unit vectors $\mathbf{i}$ and $\mathbf{j}$ mutually perpendicular and perpendicular to $\mathbf{n}$. Let u and v be the velocity components in this system, according to

$$\dot{\mathbf{r}} = u\mathbf{i} + v\mathbf{j} \tag{3-52}$$

In accordance with (3-51), the acceleration components are given by

$$\dot{u} = \frac{-a_L v + a_T u}{\sqrt{u^2 + v^2}} \tag{3-53}$$

and

$$\dot{v} = \frac{a_L u + a_T v}{\sqrt{u^2 + v^2}} \tag{3-54}$$

Equations (3-53) and (3-54) comprise a system of first order differential equations in the velocity components. To find a solution, it is convenient to rewrite them in terms of the speed, w, and direction θ. Accordingly,

$$\begin{aligned} u &= w\cos\theta \\ v &= w\sin\theta \end{aligned} \tag{3-55}$$

Substituting these expressions into (3-53) and (3-54), we obtain

$$\begin{aligned} \dot{w}\cos\theta - w\dot{\theta}\sin\theta &= -a_L\sin\theta + a_T\cos\theta \\ \dot{w}\sin\theta + w\dot{\theta}\cos\theta &= a_L\cos\theta + a_T\sin\theta \end{aligned} \tag{3-56}$$

Multiplying these equations by $\cos\theta$ and $\sin\theta$, respectively, and adding, we obtain

$$\dot{w} = a_T \tag{3-57}$$

Similarly, multiplying the two equations of (3-56) successively by $-\sin\theta$ and $\cos\theta$ and adding, we obtain

$$w\dot{\theta} = a_L \tag{3-58}$$

Equations (3-57) and (3-58) show that the simple effects of the two acceleration components are more easily seen in terms of the speed and direction than on the rectilinear velocity components. The integrals of (3-57) and (3-58) are

$$w = w_0 + a_T t \tag{3-59}$$

and

$$\theta = \theta_0 + \frac{a_L}{a_T} \ln\left(1 + \frac{a_T t}{w_0}\right) \tag{3-60}$$

Thus, the turning rate varies in a simple way. The time rate of turning is given by

$$\dot{\theta} = \frac{a_L}{w} = \frac{a_L}{w_0 + a_T t} \tag{3-61}$$

while the turning rate per traversed arc length is

$$\frac{\partial\theta}{\partial s} = \frac{\partial\theta/\partial t}{\partial s/\partial t} = \frac{\dot{\theta}}{w} = \frac{a_L}{(w_0 + a_T t)^2} \tag{3-62}$$

The reciprocal of this quantity is the radius of curvature, which is seen to vary quadratically with t.

The solutions (3-59) and (3-60) are then substituted back into the expressions for the rectilinear velocity components to give

$$u = \nu\left[u_0 \cos\left(\frac{a_L \ln \nu}{a_T}\right) - v_0 \sin\left(\frac{a_L \ln \nu}{a_T}\right)\right] \tag{3-63}$$

and

$$v = \nu\left[v_0 \cos\left(\frac{a_L \ln \nu}{a_T}\right) + u_0 \sin\left(\frac{a_L \ln \nu}{a_T}\right)\right] \tag{3-64}$$

where u_0 and v_0 are the velocity components at the initial time $t = 0$, and ν is written for the expression,

$$\nu = 1 + \frac{a_T t}{\sqrt{u_0^2 + v_0^2}} \tag{3-65}$$

Let the corresponding cartesian coordinates be denoted by x and y. The equations

$$\begin{aligned} \dot{x} &= u \\ \dot{y} &= v \end{aligned} \tag{3-66}$$

can be integrated upon substitution of (3-63) and (3-64) for the velocity components. It is required to find expressions for the integrals

$$I = \int_0^t (1 + At') \cos[B \ln(1 + At')]\, dt' \tag{3-67}$$

and

$$J = \int_0^t (1+At')\sin[B\ln(1+At')]\,dt' \tag{3-68}$$

where

$$A = \frac{a_T}{\sqrt{u_0^2+v_0^2}}, \qquad B = \frac{a_L}{a_T} \tag{3-69}$$

A simple substitution,

$$\lambda = \frac{a_L}{a_T}\ln(1+At') \tag{3-70}$$

allows straightforward evaluation of the integrals. The results are

$$I = \frac{\sqrt{u_0^2+v_0^2}}{4a_T^2+a_L^2}\left\{\nu^2\left[2a_T\cos\left(\frac{a_L\ln\nu}{a_T}\right)+a_L\sin\left(\frac{a_L\ln\nu}{a_T}\right)\right]-2a_T\right\} \tag{3-71}$$

and

$$J = \frac{\sqrt{u_0^2+v_0^2}}{4a_T^2+a_L^2}\left\{\nu^2\left[2a_T\sin\left(\frac{a_L\ln\nu}{a_T}\right)-a_L\cos\left(\frac{a_L\ln\nu}{a_T}\right)\right]+a_L\right\} \tag{3-72}$$

The coordinates can then be expressed as

$$x(t) = \int u\,dt = u_0 I - v_0 J + x_0 \tag{3-73}$$

and

$$y(t) = \int v\,dt = v_0 I + u_0 J + y_0 \tag{3-74}$$

It is convenient to refer the cartesian components to a system that is determined by the initial $\dot{\mathbf{x}}_0$ and $\ddot{\mathbf{x}}_0$. Let the unit vectors $\mathbf{i}$ and $\mathbf{j}$ be defined by

$$\mathbf{i} = \frac{\dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|}, \qquad \mathbf{j} = \frac{\mathbf{n}\times\dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|} \tag{3-75}$$

where $\mathbf{n}$ is the unit vector perpendicular to both $\dot{\mathbf{x}}_0$ and $\ddot{\mathbf{x}}_0$, given by

$$\mathbf{n} = \frac{\dot{\mathbf{x}}_0\times\ddot{\mathbf{x}}_0}{\sqrt{|\dot{\mathbf{x}}_0|^2|\ddot{\mathbf{x}}_0|^2-(\dot{\mathbf{x}}_0\cdot\ddot{\mathbf{x}}_0)^2}} \tag{3-76}$$

The acceleration components a_T and a_L are also obtained from the initial velocity and acceleration by taking

$$a_T = \frac{\ddot{\mathbf{x}}_0 \cdot \dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|} \tag{3-77}$$

and

$$a_L = \left| \ddot{\mathbf{x}}_0 - \frac{a_T \dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|} \right| \tag{3-78}$$

The above formulas for a_T and a_L simply perpetuate the values they apparently had at time t_0.

The prediction algorithm can be summarized as follows. For a given initial state, compute a_T and a_L from (3-77) and (3-78), the unit vector $\mathbf{n}$ from (3-76), and the unit cartesian vectors $\mathbf{i}$ and $\mathbf{j}$ from (3-75). Then, for any desired time t the position vector is given by $\mathbf{x}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, where $x(t)$ and $y(t)$ are given by (3-73) and (3-74) with I and J given by (3-71) and (3-72).

CHAPTER 4

GOAL-ORIENTED APPROACH TO PREDICTION FOR SELF-DEFENSE

The goal-based approach predicts future threat behavior on the assumption that the threat is targeted on the ship. If this assumption is not valid, the ship is safe although some defensive rounds may be wasted. Clearly, for self-defense, failure to make this assumption could be risky.

Since not all threat states represent a danger to the ship, the first section of this chapter, "Domain of Threat Capability," attempts to exclude goal-based prediction from those targets whose state vector indicates that the ship is an unlikely target. The next section, "Constraint on Initial Acceleration," considers the issue of constraining the acceleration estimate so that the target with a maximum acceleration limit will intercept the ship. Because the threat acceleration can exhibit quick changes, it is safer to assume a value that will drive the threat toward the ship. The remainder of the chapter discusses several particular models for prediction, each of which embodies the goal-oriented feature through different criteria.

DOMAIN OF THREAT CAPABILITY

It is reasonable to assume that a threat in a given initial state would be capable of reaching any ship within a certain region and unable to reach a ship outside this region. Inverting this concept, for a given ship position, there are some target states that would be capable of hitting the ship, while all other states would not. For a given initial target state and ship position, it is necessary to determine first if the threat poses any danger to the ship. Excluding from consideration those states considered not dangerous implies some limitations on threat maneuverability. One such limitation might be to assume that it cannot overfly the ship and return to hit it. The following restrictions attempt to formalize such limitations in a simple way. The threat is assumed unable to reach any point in the halfspace behind itself. It is also assumed that the theat is unable to reach any point within the "exclusion" torus, which is centered on the threat's initial position and has its axis of symmetry coincident with the velocity vector. The radius b_1 of the torus generating circle is equal to the radius of curvature defined by the hypothesized maximum lateral acceleration, a_1. That is,

$$b_1 = \frac{|\dot{\mathbf{x}}_0|^2}{a_1} \tag{4-1}$$

Furthermore, the distance from the axis of symmetry to the center of the generating circle is also b_1. This is the limiting case of a torus on the verge of intersecting itself. All of the space not in the threat's rear halfspace nor in its exclusion torus is considered to comprise its domain of capability. The sign of $\mathbf{x}_0 \cdot \dot{\mathbf{x}}_0$ determines if the ship is in the excluded halfplane. To determine whether or not the ship lies within the exclusion torus, the minimax acceleration a_m, to be described below, is computed, and if $a_m > a_1$, the ship is within this torus.

CONSTRAINT ON INITIAL ACCELERATION

For the goal-oriented hypothesis, the initial threat acceleration is constrained so that the trajectory plane includes the origin. This is quite different from the acceleration constraint discussed in Chapter 2. Let the adjusted acceleration vector, to be substituted for the estimated state acceleration, be denoted by $\ddot{\mathbf{x}}_{0B}$. This new acceleration vector satisfies the following criteria:

1. The adjusted acceleration is perpendicular to the velocity. If $\ddot{\mathbf{x}}_{0B}$ represents the adjusted acceleration, then

$$\ddot{\mathbf{x}}_{0B} \cdot \dot{\mathbf{x}}_0 = 0 \tag{4-2}$$

2. The adjusted acceleration lies in the plane of the threat position and velocity. This condition is applied by setting

$$\ddot{\mathbf{x}}_{0B} = \mu \mathbf{x}_0 + \nu \dot{\mathbf{x}}_0 \tag{4-3}$$

where μ and ν are scalar constants to be determined.

3. The magnitude of the adjusted acceleration is equal to the magnitude of the estimated acceleration component that is perpendicular to the velocity. That is,

$$|\ddot{\mathbf{x}}_{0B}| = \left|\ddot{\mathbf{x}}_0 - |\dot{\mathbf{x}}_0|^{-2} (\ddot{\mathbf{x}}_0 \cdot \dot{\mathbf{x}}_0)\, \dot{\mathbf{x}}_0\right| \tag{4-4}$$

The vector within the outer bars on the right side is the component of the estimated acceleration perpendicular to the velocity.

These three conditions are sufficient to determine the values of μ and ν except for sign ambiguity which is resolved by requiring that the adjusted acceleration be directed toward the origin, or $\ddot{\mathbf{x}}_{0B} \cdot \mathbf{x}_0 \leq 0$. Dotting $\dot{\mathbf{x}}_0$ to both sides of (4-3) and applying (4-2) allows one of the two constants to be expressed in terms of the other. For example, we can set

$$\nu = -\mu \frac{(\mathbf{x}_0 \cdot \dot{\mathbf{x}}_0)}{|\dot{\mathbf{x}}_0|^2} \tag{4-5}$$

and express $\ddot{\mathbf{x}}_{0B}$ by

$$\ddot{\mathbf{x}}_{0B} = \mu\left[\mathbf{x}_0 - \frac{(\mathbf{x}_0\cdot\dot{\mathbf{x}}_0)}{|\dot{\mathbf{x}}_0|^2}\dot{\mathbf{x}}_0\right] \tag{4-6}$$

Squaring (4-4) gives

$$|\ddot{\mathbf{x}}_{0B}|^2 = |\ddot{\mathbf{x}}_0|^2 - |\dot{\mathbf{x}}_0|^{-2}(\dot{\mathbf{x}}_0\cdot\ddot{\mathbf{x}}_0)^2 \tag{4-7}$$

Equating this with the square of (4-6) gives

$$\mu = \pm\left[\frac{|\dot{\mathbf{x}}_0|^2|\ddot{\mathbf{x}}_0|^2 - (\dot{\mathbf{x}}_0\cdot\ddot{\mathbf{x}}_0)^2}{|\mathbf{x}_0|^2|\dot{\mathbf{x}}_0|^2 - (\mathbf{x}_0\cdot\dot{\mathbf{x}}_0)^2}\right]^{1/2} \tag{4-8}$$

Substituting this value into (4-6), with the appropriate sign of μ, gives

$$\ddot{\mathbf{x}}_{0B} = \left[\frac{|\dot{\mathbf{x}}_0|^2|\ddot{\mathbf{x}}_0|^2 - (\dot{\mathbf{x}}_0\cdot\ddot{\mathbf{x}}_0)^2}{|\mathbf{x}_0|^2|\dot{\mathbf{x}}_0|^2 - (\mathbf{x}_0\cdot\dot{\mathbf{x}}_0)^2}\right]^{\frac{1}{2}}\left[\frac{\mathbf{x}_0\cdot\dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|^2}\dot{\mathbf{x}}_0 - \mathbf{x}_0\right] \tag{4-9}$$

The magnitude of this modified acceleration is consistent with (4.7) and never exceeds $|\ddot{\mathbf{x}}_0|$. The sign of (4-9) was chosen so that the resulting trajectory will be concave toward the origin.

MM PREDICTOR

"Minimax" indicates the predicted threat track that minimizes the maximum lateral acceleration needed to reach the origin after passing through a terminal CV segment of specified length, s. It is not surprising that minimax requires that the initial trajectory segment be a circular arc. The entire track lies in the $(\mathbf{x}_0, \dot{\mathbf{x}}_0)$ plane (i.e., the plane of the estimated initial threat position and velocity). If the initial distance $|\mathbf{x}_0|$ is less than s, then in general, no minimax track is possible.

It is convenient to define the position of the center of curvature of the circular-arc segment at time t_0 by the expression

$$\mathbf{x}_c = \mu\mathbf{x}_0 - (\mu-1)\frac{\mathbf{x}_0\cdot\dot{\mathbf{x}}_0}{|\dot{\mathbf{x}}_0|^2}\dot{\mathbf{x}}_0 \tag{4-10}$$

At this time the radius vector is

$$\boldsymbol{x}_c - \boldsymbol{x}_0 = (\mu - 1)\left[\boldsymbol{x}_0 - \frac{\boldsymbol{x}_0 \cdot \dot{\boldsymbol{x}}_0}{|\dot{\boldsymbol{x}}_0|^2}\dot{\boldsymbol{x}}_0\right] \tag{4-11}$$

which lies in the $(\boldsymbol{x}_0, \dot{\boldsymbol{x}}_0)$ plane and is seen to be perpendicular to $\dot{\boldsymbol{x}}_0$. Here, μ is a constant scalar to be determined. The condition

$$|\boldsymbol{x}_c|^2 = s^2 + |\boldsymbol{x}_c - \boldsymbol{x}_0|^2 \tag{4-12}$$

is a statement of the Pythagorean theorem in the right triangle defined by the origin, the point where the circular arc transitions to the CV segment, and the position of $\boldsymbol{x}_c$. Substituting (4-10) into (4-12) gives a linear equation in μ (because the quadratic terms cancel) from which the value

$$\mu = \frac{|\dot{\boldsymbol{x}}_0|^2(s^2 + |\boldsymbol{x}_0|^2) - 2(\boldsymbol{x}_0 \cdot \dot{\boldsymbol{x}}_0)^2}{2[|\boldsymbol{x}_0|^2|\dot{\boldsymbol{x}}_0|^2 - (\boldsymbol{x}_0 \cdot \dot{\boldsymbol{x}}_0)^2]} \tag{4-13}$$

is obtained. To determine interception parameters (PIP, time of flight, etc.), it is convenient to first calculate θ, the arc length traversed during the circular flight segment. If α is the angle at $\boldsymbol{x}_c$ subtended by the CV flight segment, then $\alpha + \theta$ is the angle at the threat between its velocity vector and the origin. The CV segment and $\boldsymbol{x}_c$ form the two legs of a right triangle, from which it is seen that

$$\cos\alpha = \frac{|\boldsymbol{x}_0 - \boldsymbol{x}_c|}{|\boldsymbol{x}_c|} \tag{4-14}$$

There is also the relation

$$\cos(\alpha + \theta) = -\frac{\boldsymbol{x}_c \cdot (\boldsymbol{x}_0 - \boldsymbol{x}_c)}{|\boldsymbol{x}_c||\boldsymbol{x}_0 - \boldsymbol{x}_c|} \tag{4-15}$$

Together (4-14) and (4-15) serve to determine θ. The turning rate within the circular arc, ω, is given by (3-4). The time spent in the circular-arc segment is θ/ω, which thus occurs during the interval

$$t_0 \le t \le t_0 + \frac{\theta}{\omega} \tag{4-16}$$

EA PREDICTOR

It is assumed that the threat will immediately achieve a lateral acceleration in the $(\boldsymbol{x}_0, \dot{\boldsymbol{x}}_0)$ plane, with maximum allowable magnitude being the "limiting acceleration" a_l, which must be specified, and directed to approach the ship. The predicted track will follow the resulting circular arc until the velocity vector points toward the origin, whereupon a terminal CV track is taken. The formalism of the MM predictor, discussed above, is applicable except that a different μ is needed. Equating the two alternative expressions for the radius of curvature gives

$$|\boldsymbol{x}_c - \boldsymbol{x}_0| = \frac{|\dot{\boldsymbol{x}}_0|^2}{a_1} \tag{4-17}$$

Then substituting $\boldsymbol{x}_c$ from (4-10) gives

$$\mu = 1 - [|\boldsymbol{x}_0|^2 |\dot{\boldsymbol{x}}_0|^2 - (\boldsymbol{x}_0 \cdot \dot{\boldsymbol{x}}_0)^2]^{-1/2} |\dot{\boldsymbol{x}}_0|^3 a_1^{-1} \tag{4-18}$$

instead of (4-13). The simple track just described gives the EA predictor, provided

$$|\boldsymbol{x}_c - \boldsymbol{x}_0| \le |\boldsymbol{x}_c| \tag{4-19}$$

which implies that

$$\sin^2\epsilon \le \frac{1 + \mu - \mu^2}{2(1-\mu)} \tag{4-20}$$

where ϵ is the angle between $\boldsymbol{x}_0$ and $\dot{\boldsymbol{x}}_0$. This relation is obtained by eliminating $\boldsymbol{x}_0 \cdot \dot{\boldsymbol{x}}_0$ in terms of ϵ in (4-18), substituting (4-10) and (4-17) into the inequality (4-19), and eliminating $|\dot{\boldsymbol{x}}_0|$ between the two resulting relations. An interception is still possible if this condition is violated, but this requires a more complicated track, and for the moment will not be considered further.

PN METHODS

Proportional Navigation (PN) is often used to guide a pursuing object (a missile) to intercept a pursued object (the target). The required missile acceleration is expressed as a function of the missile and target positions and velocities. Let $\boldsymbol{x}_m$ and $\boldsymbol{x}_t$ denote the missile and target positions in the usual PN formulation, with the missile-to-target relative position vector being $\boldsymbol{x} = \boldsymbol{x}_t - \boldsymbol{x}_m$. Here the target position is replaced by the ship position at the origin and the missile becomes the threat. Thus, $\boldsymbol{x}_t$ is replaced by $\boldsymbol{0}$, and $\boldsymbol{x}_m$ is replaced by $-\boldsymbol{x}$.

One particular PN law, designated here as Standard PN (SPN), has been the subject of a number of studies. It considers the rotation rate of the Line of Sight (LOS) and specifies an acceleration that will asymptotically reduce this rotation rate to zero. A parameter n, called the PN gain, determines the speed at which the LOS rotation rate is nulled. The acceleration is taken perpendicular to the LOS, and not necessarily perpendicular to the velocity. Accordingly, the speed in general is not constant. This is referred to as "True PN," in contrast with "Pure

PN," which conserves missile speed.[2] The formula for the control acceleration for pure PN is given by

$$\ddot{\mathbf{x}}_{SPN} = \frac{n}{|\mathbf{x}|^4}\mathbf{x}\cdot\dot{\mathbf{x}}[(\mathbf{x}\cdot\dot{\mathbf{x}})\mathbf{x} - |\mathbf{x}|^2\dot{\mathbf{x}}] \tag{4-21}$$

A Pure PN can be obtained from True PN by taking as the required acceleration just the c component of SPN which is perpendicular to the velocity. Here, MPN represents this modification of SPN. The formula for the control acceleration for MPN is given by

$$\ddot{\mathbf{x}}_{MPN} = n(\mathbf{x}\cdot\dot{\mathbf{x}})^2|\mathbf{x}|^{-4}|\dot{\mathbf{x}}|^{-2}[|\dot{\mathbf{x}}|^2\mathbf{x} - (\mathbf{x}\cdot\dot{\mathbf{x}})\dot{\mathbf{x}}] \tag{4-22}$$

Adler[3] proposed a convenient three-dimensional (Pure) PN law, which here is designated as APN and in our notation is written

$$\ddot{\mathbf{x}}_{APN} = n|\mathbf{x}|^{-2}[|\dot{\mathbf{x}}|^2\mathbf{x} - (\mathbf{x}\cdot\dot{\mathbf{x}})\dot{\mathbf{x}}] \tag{4-23}$$

where n is again a numerical gain, which must be specified. Adler showed that a value of n greater than two is required for practical intercept trajectories. The value $n=4$ gives reasonable trajectories for both MPN and APN. As both PN formulas determine a required acceleration in the plane of the threat velocity and position, the resulting trajectory lies in a plane that includes the origin.

The PN lateral acceleration magnitude monotonically decreases if the pursued object is stationary, as the ship in the situation under discussion. Therefore, if the threat is initially capable of achieving the required PN acceleration, it is reasonable to assume that the required acceleration remains achievable. When the acceleration is initially unachievable, two obvious strategies suggest themselves. One is for the threat to fly a CTR segment at maximum allowed lateral acceleration a_l until the PN acceleration becomes achievable. Another is to adjust n downward. In the latter case, an appropriate value of n is limited by

$$n \le \frac{|\mathbf{x}|^2}{|\dot{\mathbf{x}}|}[|\mathbf{x}|^2|\dot{\mathbf{x}}|^2 - (\mathbf{x}\cdot\dot{\mathbf{x}})^2]^{-1/2}a_l \tag{4-24}$$

for Adler PN.

As indicated in Table 4-1, some closed-form solutions to PN trajectories and range-time histories have been obtained, in the special case of a stationary target.

TWISTED PN (TPN) PREDICTOR

The TPN trajectory is not necessarily confined to a single plane. If, at the moment of

TABLE 4-1. EXISTENCE OF PN SOLUTION

	True or Pure PN	Trajectory Solution	Range vs Time
SPN	True	a	a
MPN	Pure	d	b
APN	Pure	d	c
TPN	Pure	b	b

Notes:

(a) Cochran et al[4] give solutions for n=3 and n=4 for general CV target motion.
(b) No known solutions are available.
(c) Groves and Gray[5] give solutions for n=3 and n=4
(d) Groves and Gray[5] give solutions for general n.

observation, the threat is moving in the plane of its estimated velocity and acceleration, it cannot simultaneously remain on this plane and also intercept the ship if the ship does not lie on the plane. The TPN predictor gradually rotates the osculating plane of threat motion until it contains the target ship.

The common PN predictors determine a required acceleration $\ddot{\mathbf{x}}_{PN}$ based on current values of the estimated threat position and velocity. They assume that the acceleration can be abruptly changed at will. The resulting trajectory, of course, is torsionless (planar). Here, a slight generalization of the usual PN procedure is used. It will be assumed that the jerk, but not the acceleration, can be assigned at will.

Any suitable PN trajectory that is targeted on the origin can be used. However, instead of using $\ddot{\mathbf{x}}_{PN}$ immediately, the current acceleration is to be 'moved' gradually to the desired value $\ddot{\mathbf{x}}_{PN}$. One of the simplest ways of achieving such a gradual change in the acceleration is to apply a jerk that is proportional to the difference between the current and the desired acceleration. The applied jerk must satisfy the basic constraint

$$\dot{\mathbf{x}}\cdot\dddot{\mathbf{x}} + |\ddot{\mathbf{x}}|^2 = 0 \qquad (4\text{-}25)$$

(obtained by differentiating $|\dot{\mathbf{x}}|^2 = |\dot{\mathbf{x}}_0|^2$ twice). A convenient prediction formula is obtained by taking

$$\dddot{\mathbf{x}} = \tau^{-1}(\ddot{\mathbf{x}}_{PN} - \ddot{\mathbf{x}}) - \frac{\dot{\mathbf{x}}|\ddot{\mathbf{x}}|^2}{|\dot{\mathbf{x}}|^2} \tag{4-26}$$

This expression satisfies the basic constraint (4-25) and tends to null the acceleration discrepancy as time elapses. The parameter τ is a time constant, which can be assigned somewhat arbitrarily, but controls the rate at which the acceleration reaches the desired level. When the acceleration becomes essentially equal to that required by PN the jerk becomes

$$\dddot{\mathbf{x}} = -\frac{\dot{\mathbf{x}}|\ddot{\mathbf{x}}|^2}{|\dot{\mathbf{x}}|^2} \tag{4-27}$$

and is directed in the opposite direction of the velocity. The motion will henceforth be planar. Momentarily the threat is in a circular-arc trajectory, as can be seen by the following argument. A solution of (4-27) for which both $|\ddot{\mathbf{x}}|$ and $|\dot{\mathbf{x}}|$ are constants is

$$\ddot{\mathbf{x}} = \frac{|\ddot{\mathbf{x}}|^2}{|\dot{\mathbf{x}}|^2}(\mathbf{x}_c - \mathbf{x}) \tag{4-28}$$

where $\mathbf{x}_c$ is a constant. It is seen that (4-28) represents an acceleration directed always toward the point $\mathbf{x}_c$, and has magnitude consistent with circular motion, and that $|\ddot{\mathbf{x}}|$ and $|\dot{\mathbf{x}}|$ are constant as required. Of course, this motion is hypothetical because the resulting acceleration would continually stray from, and need to be brought back toward that required by PN.

The jerk formula (4-26) contains two parameters $(n \text{ and } \tau)$ that need to be specified. If the time constant τ is chosen too large, the resulting predicted trajectory will miss the origin. It seems reasonable to choose τ to be some fraction of the remaining time to intercept. As this time continually decreases, it is worthwhile to use a time-varying τ. One convenient possibility is to take

$$\tau(t) = \frac{|\mathbf{x}|}{m|\dot{\mathbf{x}}|} \tag{4-29}$$

where m is a selectable nondimensional parameter that determines the rate at which the required acceleration approaches the value given by the PN formula. This ensures that the trajectory pass through the origin. The TPN formula (4-26) then becomes

$$\dddot{\mathbf{x}} = -\frac{mn|\dot{\mathbf{x}}|^3}{|\mathbf{x}|^3}\mathbf{x} - \left[\frac{mn|\dot{\mathbf{x}}|\mathbf{x}\cdot\dot{\mathbf{x}}}{|\mathbf{x}|^3} - \frac{|\ddot{\mathbf{x}}|^2}{|\dot{\mathbf{x}}|^2}\right]\dot{\mathbf{x}} - \frac{m|\dot{\mathbf{x}}|}{|\mathbf{x}|}\ddot{\mathbf{x}} \tag{4-30}$$

for APN, while for MPN, simple obvious modifications need to be made. However, the two parameters, n and m, must be specified.

CHAPTER 5

TERMINAL PARAMETERS

Decisions and hypotheses are to be based on the latest available values of $\mathbf{x}_0, \dot{\mathbf{x}}_0, \ddot{\mathbf{x}}_0$. However, it is convenient to translate these into other parameters more closely related to the decision-making process. These subsequent parameters are used to facilitate conceptualizing the possible future threat trajectories. It is emphasized that these Terminal Parameters (TPs) depend only on the threat state.

Some of the TPs depend on an assumed lateral acceleration magnitude a_l that the threat is incapable of exceeding. This value may be based on the identity of the threat, or may be hypothesized. The larger the value of a_l, the more difficult the prediction problem becomes.

The TPs are based on the supposition that the initial guess of the threat acceleration $\ddot{\mathbf{x}}_0$ is so poorly determined that the hypothesis $\ddot{\mathbf{x}}_0 = \mathbf{0}$ is as good as any other. The proposed TPs defined in the following list will be considered.

a_m	-- The MM acceleration
t_m	-- The MM arrival time
t_{EA}	-- The earliest arrival time
γ_{acq}	-- Look angle at acquisition
t_{LA}	-- The latest arrival time

The MM acceleration is that required to achieve a MM trajectory, described in Chapter 4. Note that its value depends on the parameter s, the length of the final CV flight segment. The t_m is the total flight time in the CTR and the CV segments. The t_{EA} is the total flight time assuming that the threat will fly the EA trajectory, as described in Chapter 4.

The definition of the look angle at acquisition, γ_{acq}, requires a hypothetical defensive missile or bullet to be launched against the threat. "Acquisition" then means "of the threat by the defensive missile," and the "look angle" is between the LOS from defensive missile to threat and the velocity of the defensive missile. A small look angle is of course advantageous to the defensive side. In the examples presented here, the defensive missile is a bullet assumed to fly a CV trajectory. This greatly simplifies the computations. However, the same concepts exist for defensive missiles flying other types of trajectories. While the criterion for acquisition here is artificially simple, it could have been made to depend on a large number of environmental factors and technical details of the defensive missile.

The latest arrival (LA) time is useful in some contexts, but its definition is not as circumscribed as the EA time. For this reason, there was no discussion of an LA trajectory in Chapter 4. The LA time is based on the threat continuing a reasonable trajectory that delays as long as reasonably practical before turning toward the ship. This vague criterion is implemented in a definite way in order to permit illustrative values to be computed.

For a given current threat state, $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$, downrange and crossrange distances (a, b) to the ship from the threat are defined by

$$\begin{aligned} a &= |\boldsymbol{x}|\cos\theta = -\boldsymbol{x}\cdot\dot{\boldsymbol{x}}/|\dot{\boldsymbol{x}}| \\ b &= |\boldsymbol{x}|\sin\theta = |\boldsymbol{x}\times\dot{\boldsymbol{x}}|/|\dot{\boldsymbol{x}}| \end{aligned} \tag{5-1}$$

where θ is the current look angle of the ship as seen by the threat (i.e., the angle between the vectors $-\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$). Recall that a_1 represents the maximum achievable lateral acceleration. Then $b_1 = |\dot{\boldsymbol{x}}|^2/a_1$ is the minimum achievable turn radius. The domain of capability (described in Chapter 4) is the region

$$a > 0 \quad and \quad a^2 + (b - b_1)^2 > b_1^2 \tag{5-2}$$

in the $(\boldsymbol{x}, \dot{\boldsymbol{x}})$ plane. If the above condition is not satisfied, it is assumed that it is not possible to hit the ship. If the condition <u>is</u> satisfied, various trajectories to the ship are possible. In particular, there is a unique CTR trajectory to the ship with an achievable turning rate.

The LA trajectories are defined as follows. If

$$a > b_1 \quad and \quad b > b_1 \tag{5-3}$$

the trajectory length is taken to be the larger of two quantities. The first is the time taken to travel the distance $a + b$ minus the amount cut off by rounding the corner with a circular arc of radius b_1. The second is the time taken to fly a CTR trajectory to the ship. There will be definite regions of the (a, b) plane where one of these alternatives is to be taken as the LA time, but these regions have not been delineated. If

$$b < b_1 \tag{5-4}$$

the LA trajectory will consist of a CV continuation until the latest opportunity to turn, with maximum achievable lateral acceleration, to be able to hit the ship. If

$$a < b_1 \quad and \quad b > b_1 \tag{5-5}$$

the LA trajectory is the CTR trajectory which will hit the ship.

CHAPTER 6

COMPARISON OF THE METHODS

The prediction methods discussed in previous chapters are compared in Table 6-1. Each column gives the characteristics of one of the prediction methods, while each row represents a particular feature for all the predictors.

MEASURES OF EFFECTIVENESS (MOE)

The MOEs are intended to quantify the ability of a defensive missile or bullet to kill the threat. The following parameters measure the effectiveness of the prediction scheme in a given scenario:

Look Angle $\gamma(t, t_1)$
Heading Error $\phi(t, t_1)$
CPA Distance $\delta(t_1)$

where t_1 is the launch time of the defensive missile, and t is time during the engagement. It is noted that these MOEs are related to a particular defensive missile to be fired at the threat and will depend on the type of trajectory assumed by the missile. Thus, they are defined only in the context of such a defensive-missile system. In the examples discussed below, the defensive missile was a bullet flying a CV trajectory. The Look Angle and the Heading Error vary along the missile trajectory, and thus depend on t, whereas the missile-to-threat distance at the closest point of approach (CPA Distance) depends only on information available at launch time; i.e., on the threat state for any given prediction method.

The Look Angle is the angle between the LOS from missile to threat and the missile velocity. (In a more detailed study, the Look Angle would be defined in terms of the body axis instead of the missile velocity.) The Look Angle is an important consideration in the case of a missile with terminal homing. If the Look Angle is too large at acquisition time, acquisition will be impossible because of seeker limitations.

The Heading Error is defined here as the angle between the velocity of the defensive missile and the direction from the current missile position to the Perfect-Prediction PIP (PPPIP). While

TABLE 6-1. COMPARISON OF THE PREDICTION METHODS

PREDICTION METHOD	CA	CTR	EDTR	HELIX	MM	EA	SPN	MPN	APN	TPN
CONSTANT SPEED		*	*	*	*	*		*	*	*
GOAL ORIENTED					*	*	*	*	*	*
USES $\ddot{\mathbf{r}}_0$	*	*	*	*						*
USES $\dddot{\mathbf{r}}_0$				*						
PARAMETERS			τ		S	a_1	N	N	N	N,M
ASSUMED TRAJECTORY	PARABOLA	CIRCLE	SPIRAL	HELIX	CIRCLE + LINE	CIRCLE + LINE				
HOW TO COMPUTE	CLOSED FORM	CLOSED FORM	CLOSED FORM	CLOSED FORM	CLOSED FORM	CLOSED FORM	CLOSED FORM	RUNGE KUTTA	CLOSED FORM	RUNGE KUTTA
PLANAR	*	*	*		*	*	*	*	*	
TRAJECTORY PLANE	$\dot{\mathbf{r}}_0, \ddot{\mathbf{r}}_0$	$\dot{\mathbf{r}}_0, \ddot{\mathbf{r}}_0$	$\dot{\mathbf{r}}_0, \ddot{\mathbf{r}}_0$	N/A	$\mathbf{r}_0, \dot{\mathbf{r}}_0$	$\mathbf{r}_0, \dot{\mathbf{r}}_0$	$\mathbf{r}_0, \dot{\mathbf{r}}_0$	$\mathbf{r}_0, \dot{\mathbf{r}}_0$	$\mathbf{r}_0, \dot{\mathbf{r}}_0$	N/A

the PIP indicates the projected intercept point based on the predicted threat trajectory, the PPPIP is based on the true threat trajectory. It is the threat's true position at the time when the missile, having been launched at time t_1, could just arrive provided it were shot in the appropriate direction (which is not necessarily the direction it was shot) as shown in Figure 6-1. The heading error is approximately the amount by which the missile must turn, during the time remaining until intercept, to obtain zero miss. The word "approximately" is used here because the PPPIP is defined in terms of where the intercept would have occurred had the missile been fired in the appropriate direction. Even if the intercept can be achieved, it is at a slightly different point than PPPIP, because the total arc length from launch point to PPPIP for the actual shot (assuming that it is capable of reaching the PPPIP) is different from the arc length based on the hypothetical well-aimed shot.

The Closest-Point of Approach (CPA) distance is the shortest distance between the defensive missile and the threat that will occur (based on the actual missile and threat trajectories).

TEST SCENARIO

A threat trajectory is considered that lies in a single plane and consists of several CV segments joined by circular arcs. The threat speed is a constant Mach 2 throughout, and the lateral acceleration in all turns is 6g, as shown in Figure 6-2. The various track prediction schemes are illustrated by showing the predicted tracks determined at various points along the actual threat trajectory. Each of these predicted tracks branch from the threat track at a certain launch time. Each predicted track ends at the PIP determined for a hypothetical flight of a constant-velocity bullet shot from the origin at launch time. Defense with a higher-speed bullet results in PIPs lying closer to the point at which each predicted track branches from the actual threat track.

Figures 6-3 through 6-8 show the predicted CV, MM, EA and TPN trajectories for various parameter choices. Note that the scale varies somewhat between figures. It is seen that without the goal-based hypothesis, the predicted tracks can diverge sharply from the true threat trajectory, as the CV tracks of Figure 6-3. Figure 6-9 shows the PIPs for the CV predictor and corresponding actual synchronous threat positions, joined by lines. Figure 6-10 shows a comparison of the PIPs for the APN, CV, and MM Predictors, which again illustrates the smaller divergence of the goal-based PIPs from the true threat trajectory. This is not surprising in view of the fact that this particular threat track is goal-based.

Figure 6-11 shows the variation of the MM acceleration as function of launch time for two values of s. The sharp rise in the curve for $s = 5\,km$, near the end of the trajectory, occurs at the point where the threat is no longer able to reach the ship. Figures 6-13 and 6-14 show a comparison of the variation of the various terminal time parameters for a pair of values for the limiting acceleration and s. It is seen that the MM times are not very different in the two cases,

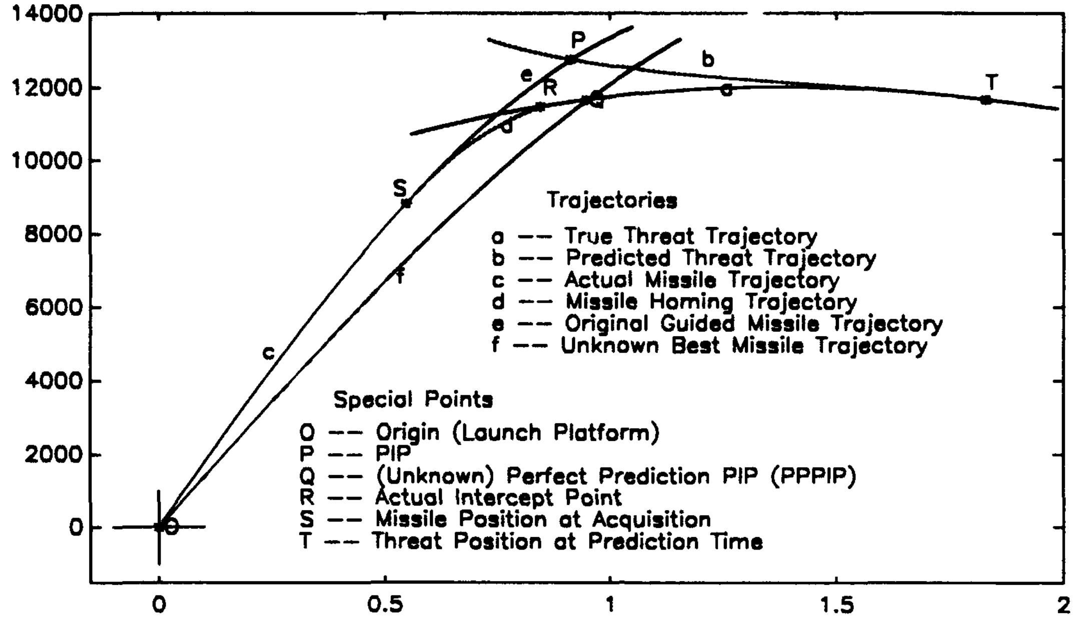

FIGURE 6-1. TRUE AND HYPOTHETICAL THREAT AND MISSILE TRACKS

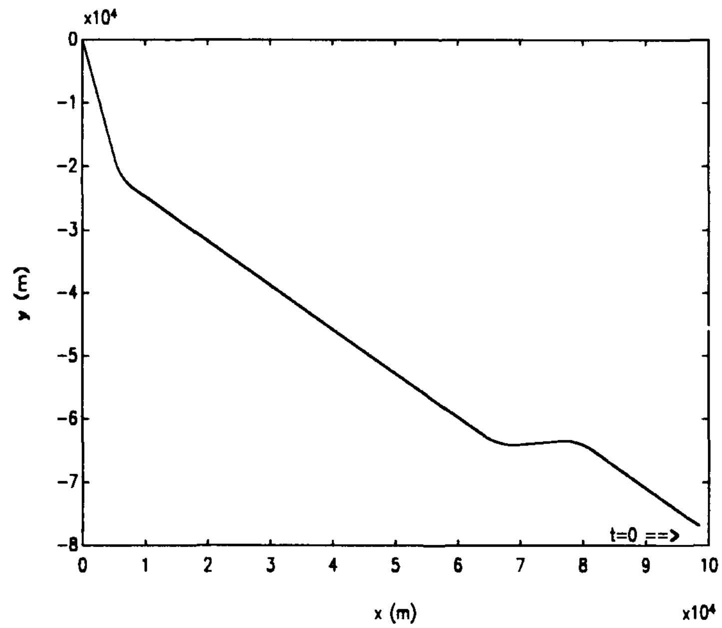

FIGURE 6-2. HYPOTHETICAL THREAT TRAJECTORY FOR THE STUDIES

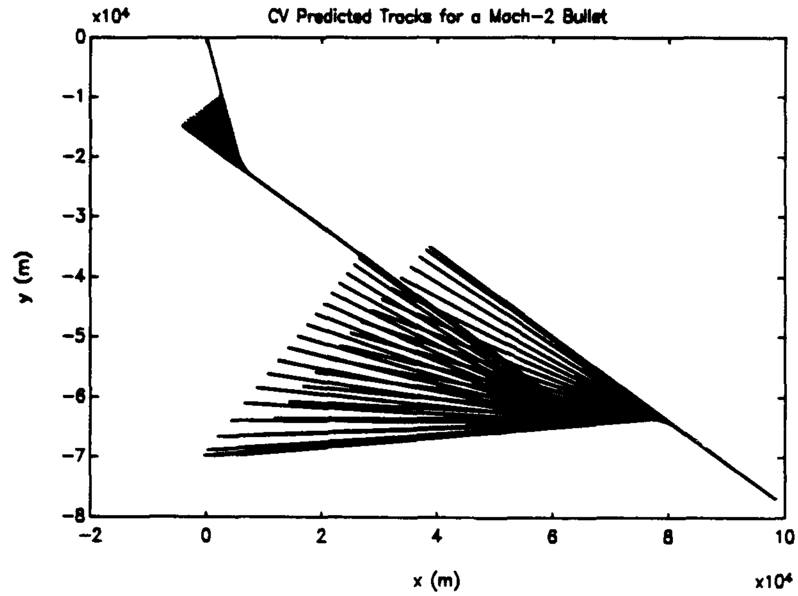

FIGURE 6-3. CV TRACKS FOR A MACH-2 BULLET

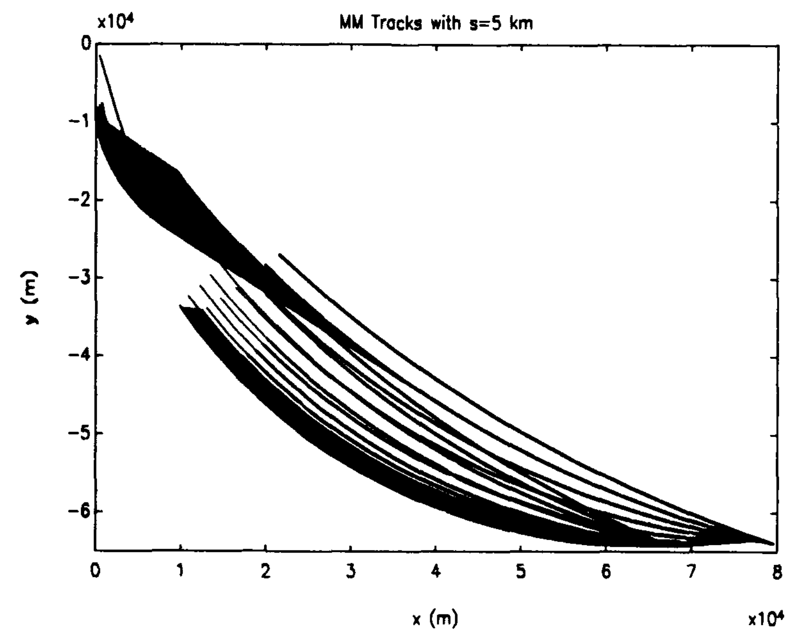

FIGURE 6-4. MM TRACKS WITH s=5 km, MACH-1 BULLET

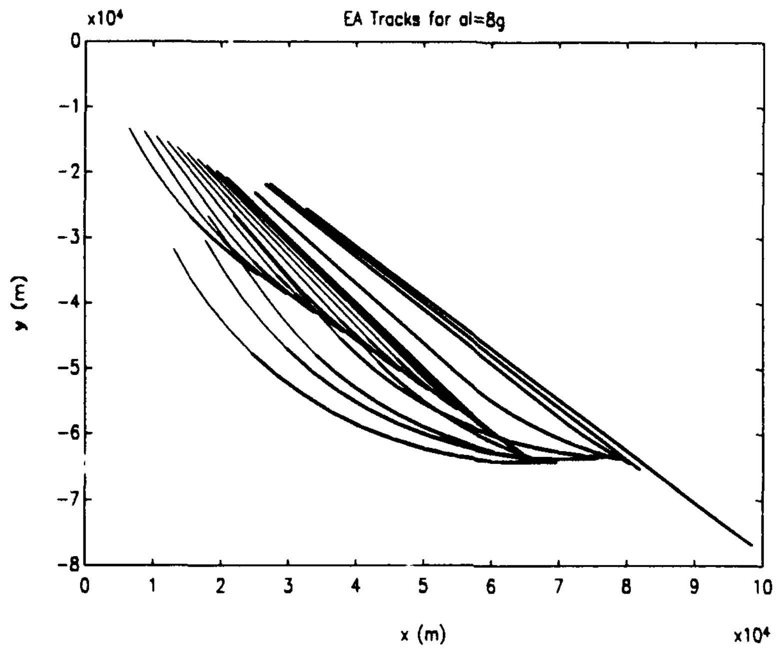

FIGURE 6-5. EA TRACKS WITH $a_1 = 8g$, MACH-1 BULLET

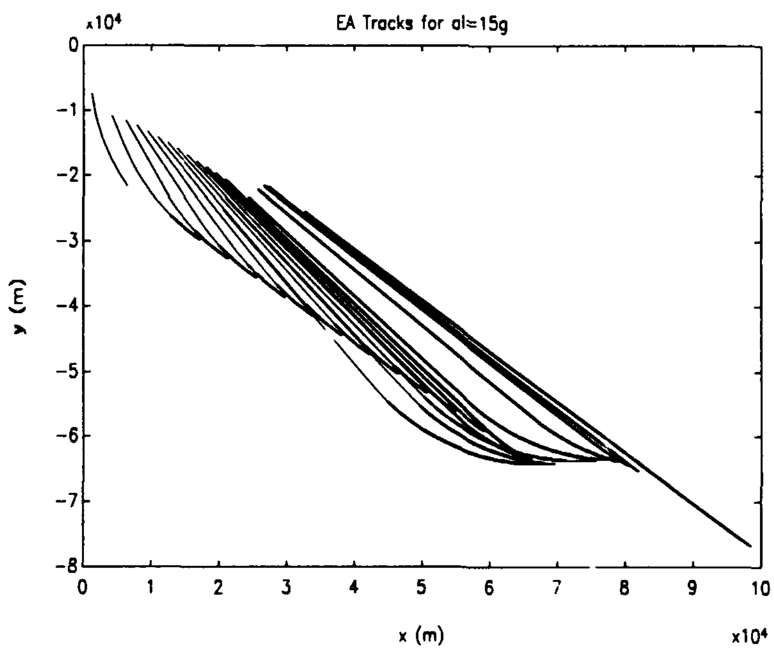

FIGURE 6-6. EA TRACKS WITH $a_1 = 15g$, MACH-1 BULLET

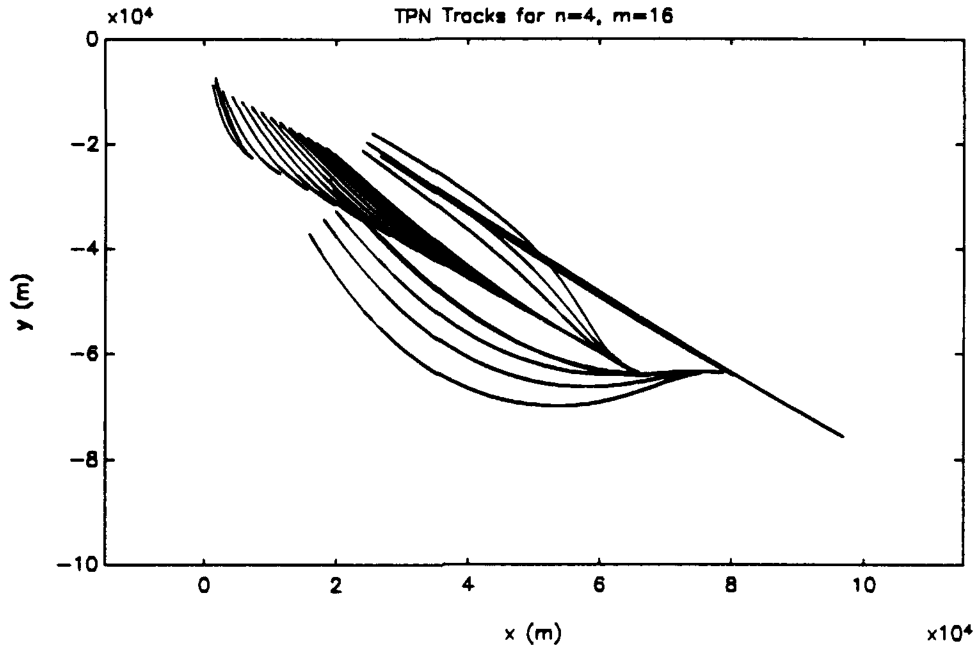

FIGURE 6-7. TPN TRACKS WITH n=4, m=12, MACH-2 BULLET

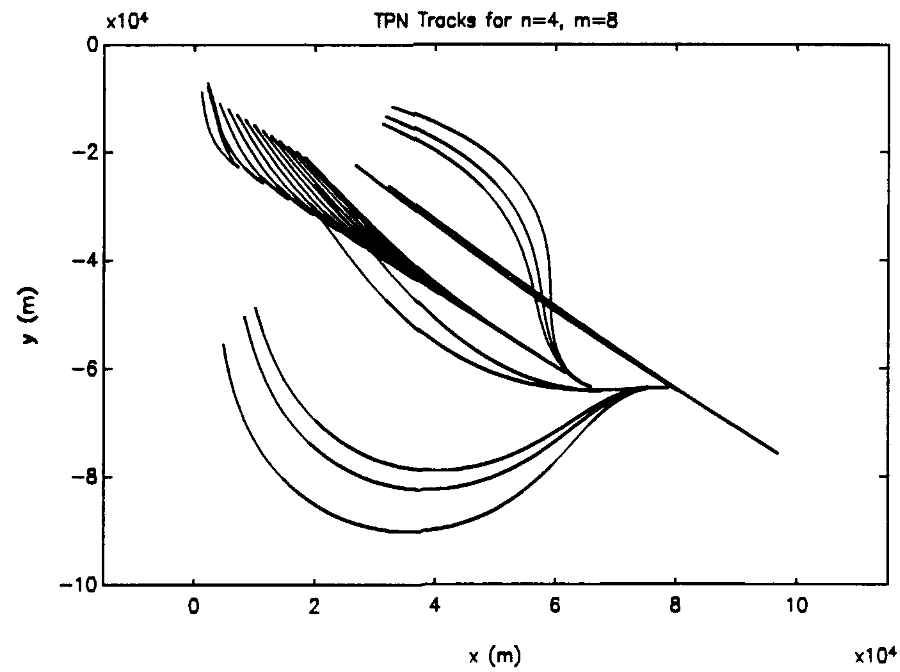

FIGURE 6-8. TPN TRACKS WITH n=4, m=8, MACH-2 BULLET

this difference being illustrated in Figure 6-13.

Figures 6-15 to 6-20 show the variation in look angle and heading error for some of the predictors. It is seen that at time when the threat is carrying out even a modest maneuver these MOEs assume very disadvantageous (large) values. Figures 6-21 and 6-22 compare the various predictors according to their variation in look angle. Figures 6-23 and 6-24 compare the same predictors' time profile of heading error. Figures 6-25 and 6-26 compare various predictors' time profile of miss distance. Figure 6-27 illustrates the superiority of the (goal-based) PN predictor over the state-based predictors, as is seen by the lower magnitudes of both the look angle and heading error as the threat nears its goal. The PIP tracks for two of the PN predictors are shown in Figure 6-28.

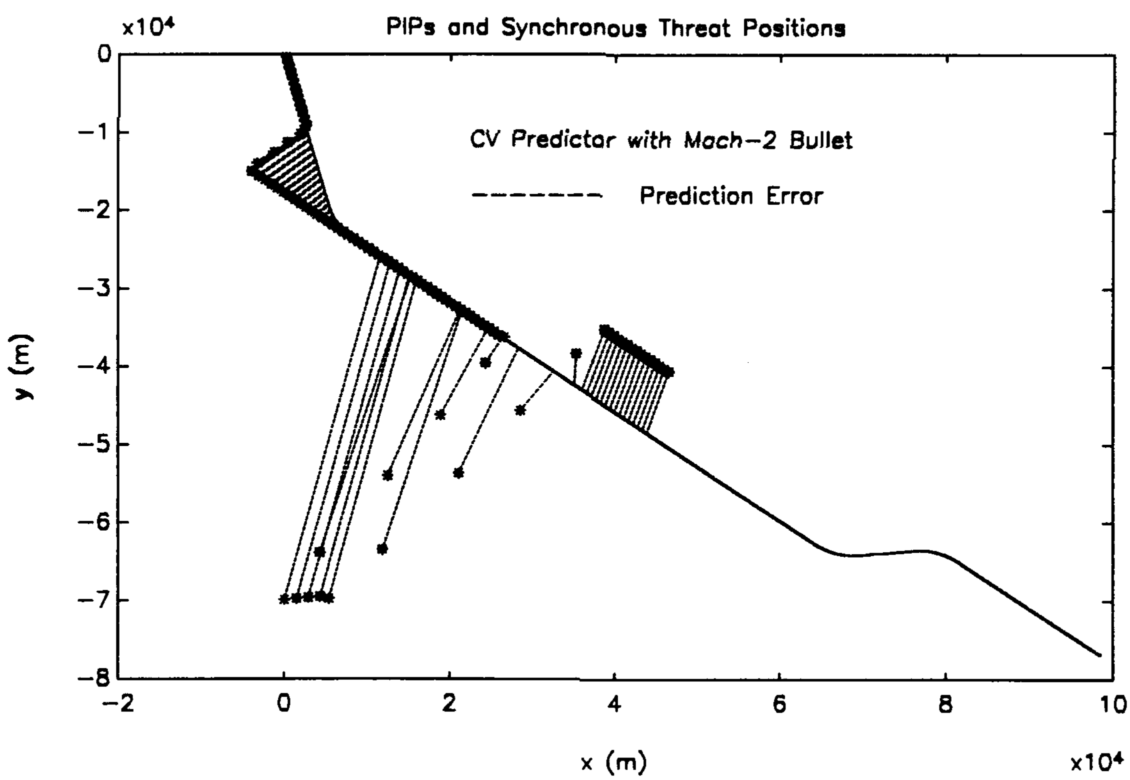

FIGURE 6-9. PIPs AND SYNCHRONOUS THREAT POSITIONS FOR CV PREDICTOR

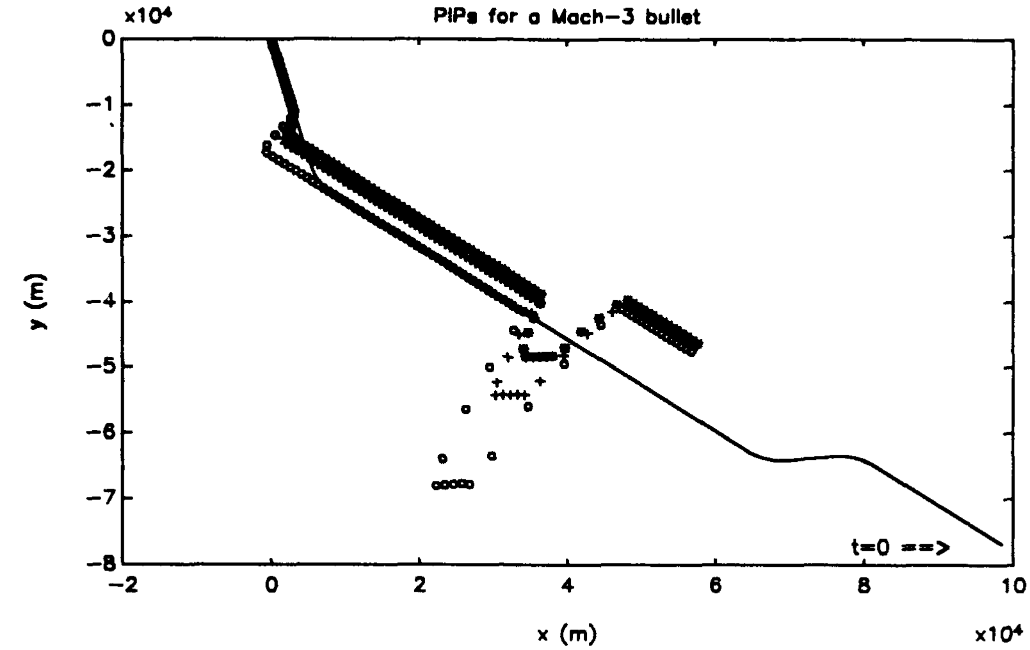

FIGURE 6-10. COMPARATIVE PIP POSITIONS (o = CV, + = MM, * = APN)

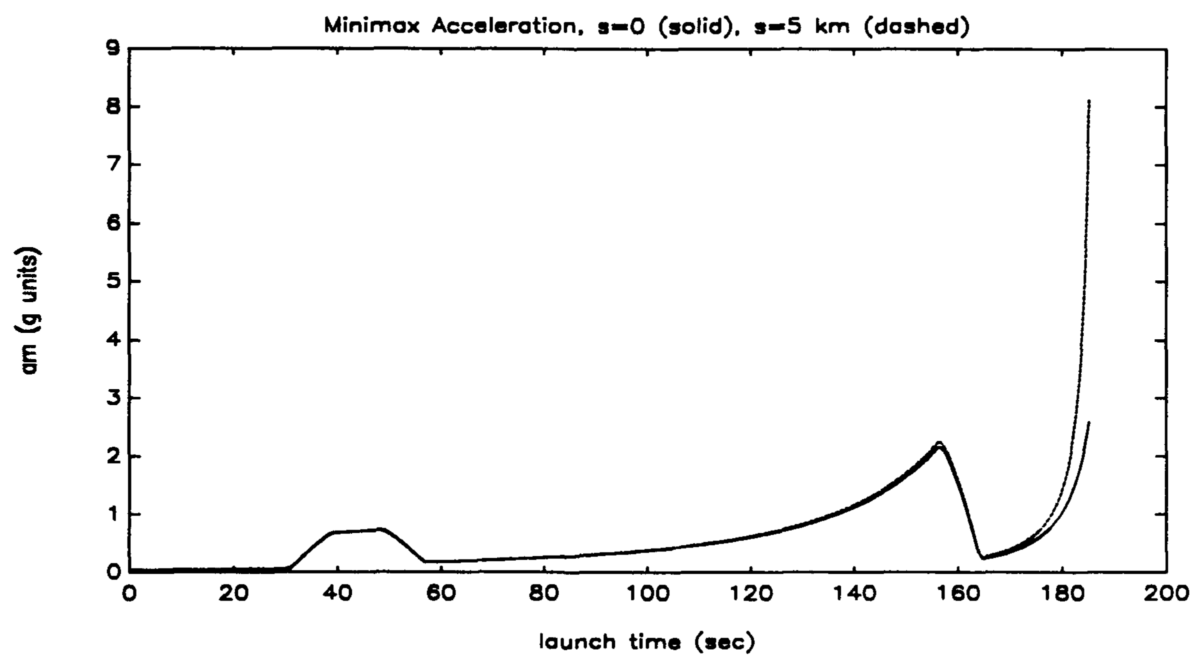

FIGURE 6-11. MM ACCELERATION FOR TWO VALUES OF s

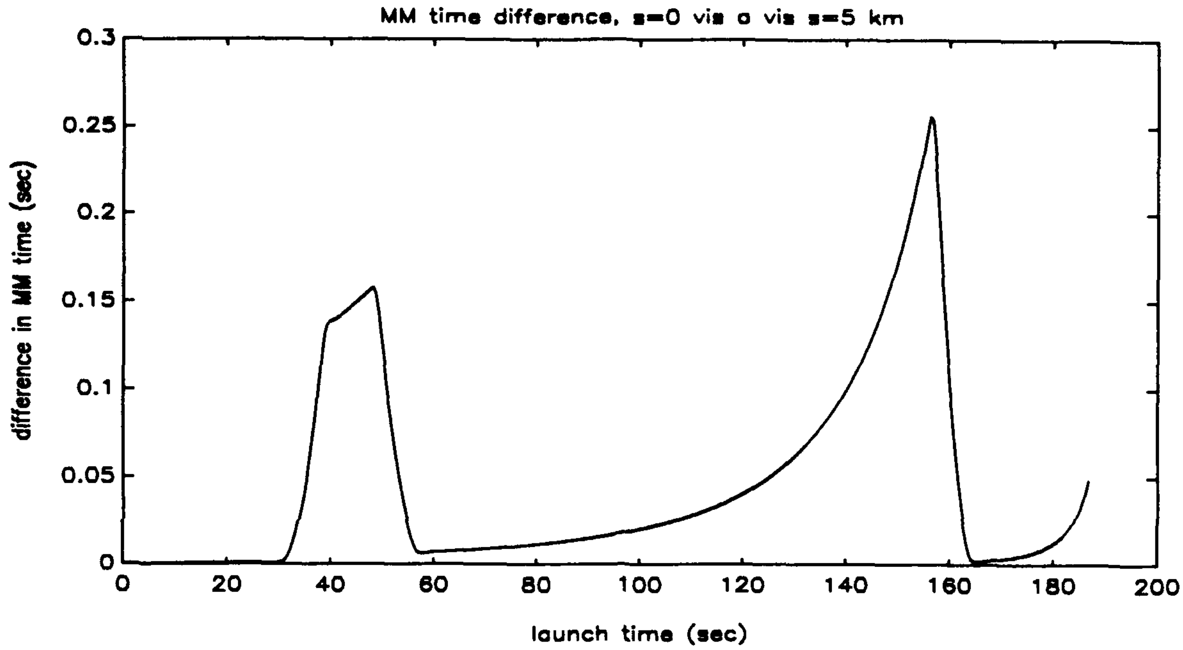

FIGURE 6-12. TIME DIFFERENCE BETWEEN THE MM ARRIVAL TIMES FOR TWO VALUES OF s. (THE ACTUAL MM TIMES ARE SHOWN IN FIGURES 6-13 AND 6-14)

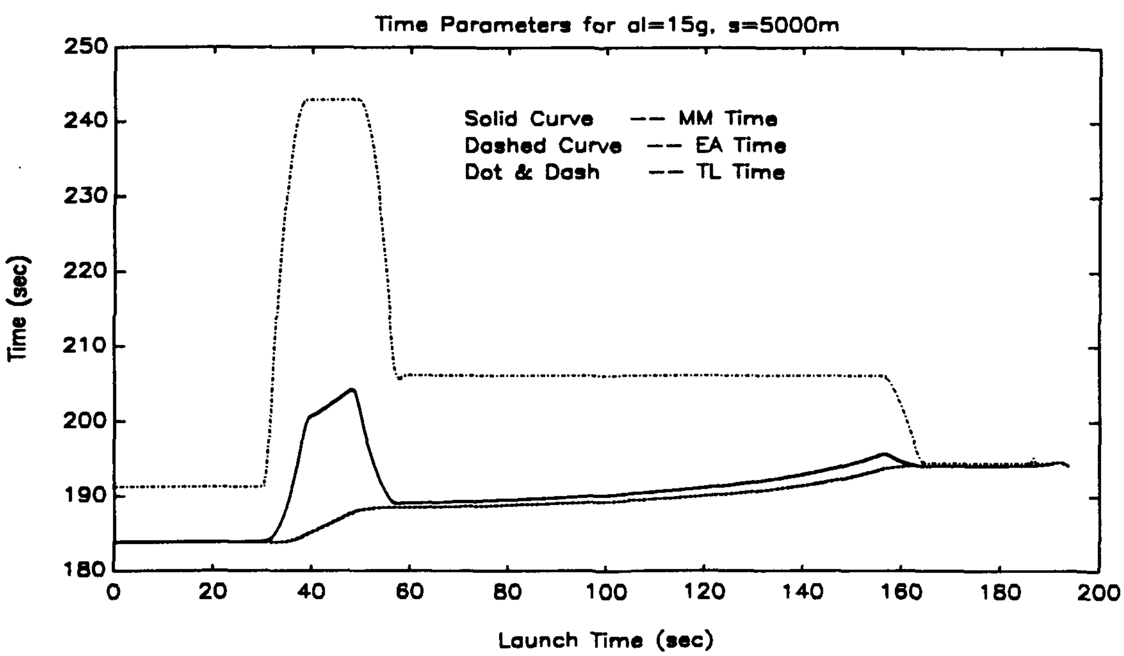

FIGURE 6-13 VARIATION OF TIME PARAMETERS WITH LAUNCH TIME

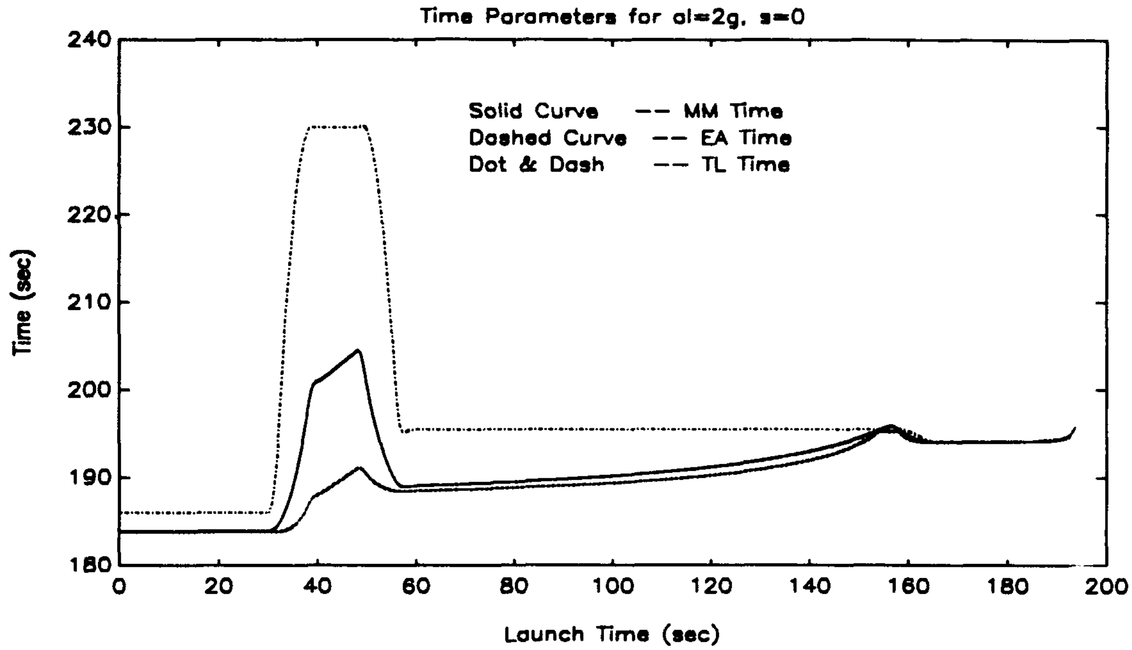

FIGURE 6-14. VARIATION OF TIME PARAMETERS WITH LAUNCH TIME

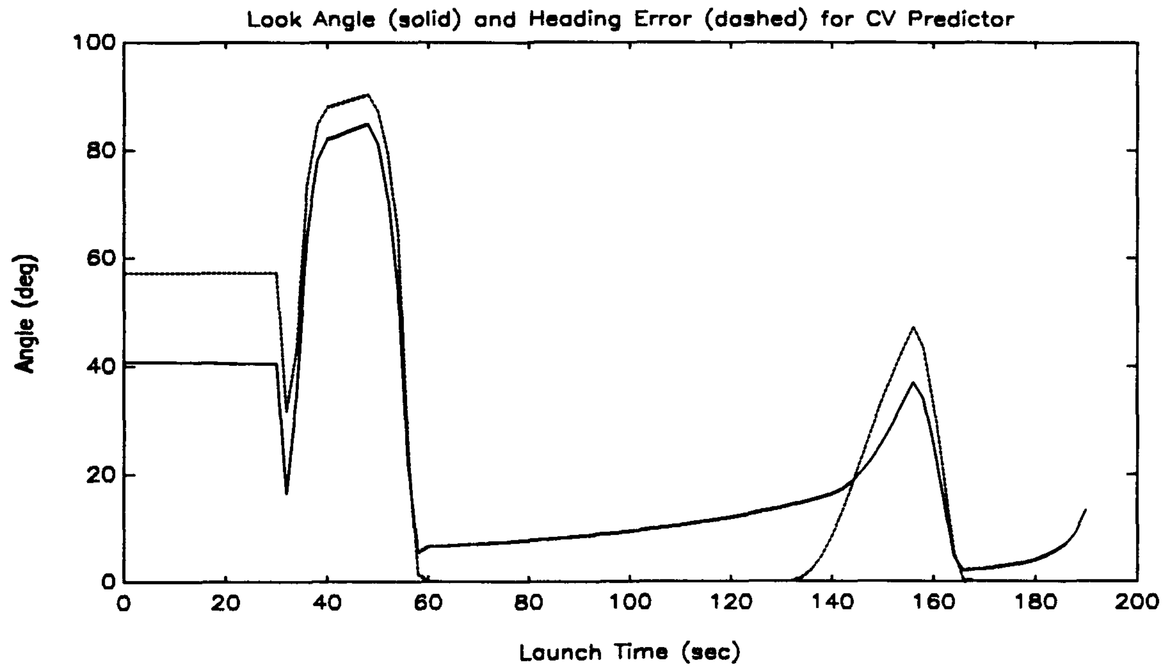

FIGURE 6-15. LOOK ANGLE AND HEADING ERROR FOR CV PREDICTOR

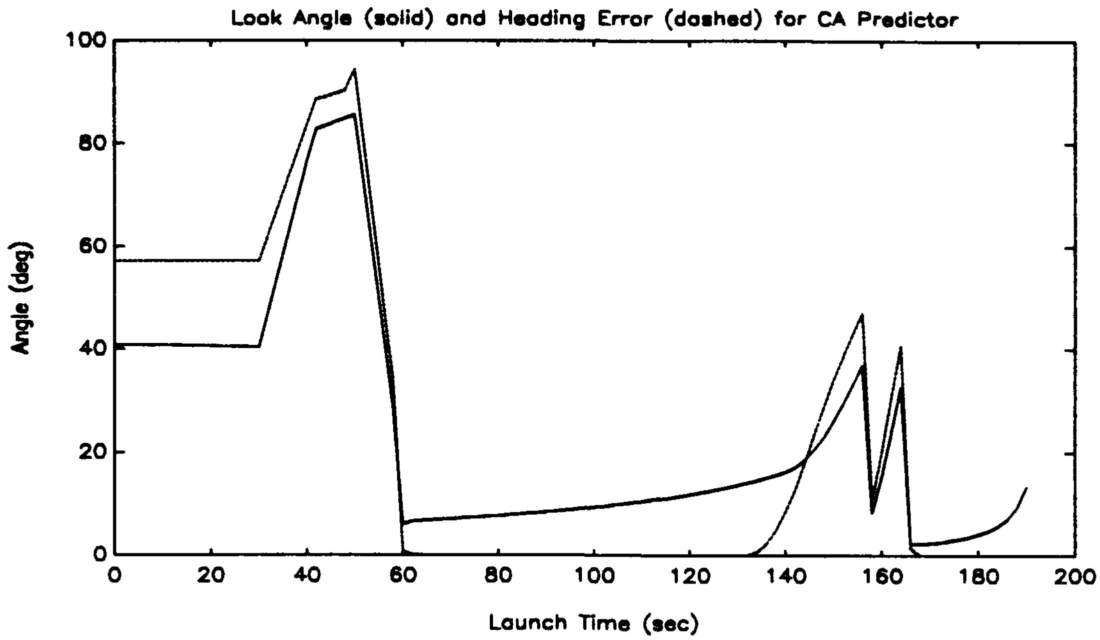

FIGURE 6-16. LOOK ANGLE AND HEADING ERROR FOR CA PREDICTOR

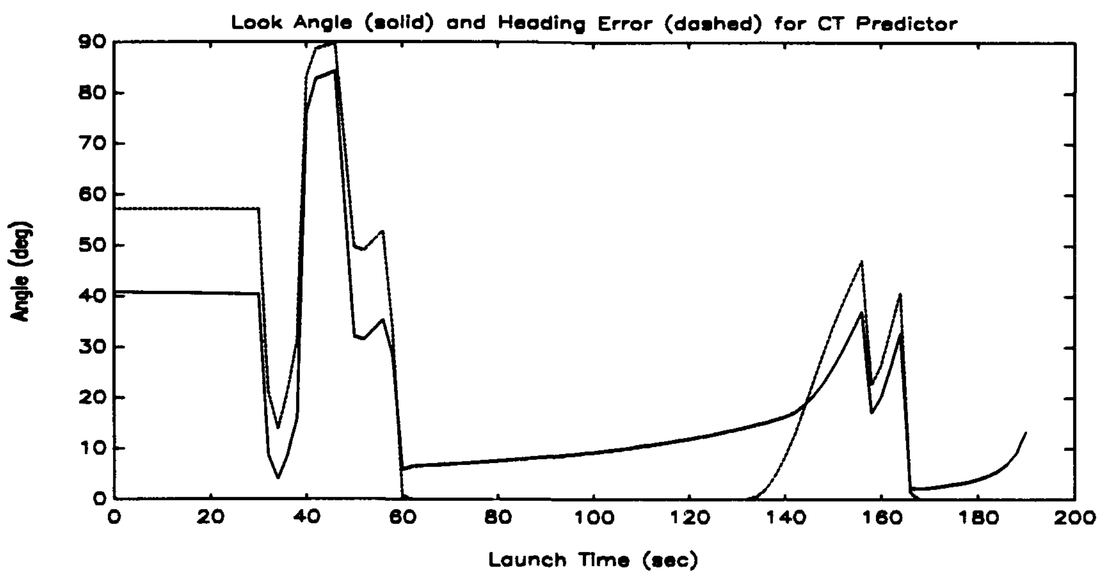

FIGURE 6-17. LOOK ANGLE AND HEADING ERROR FOR CTR PREDICTOR

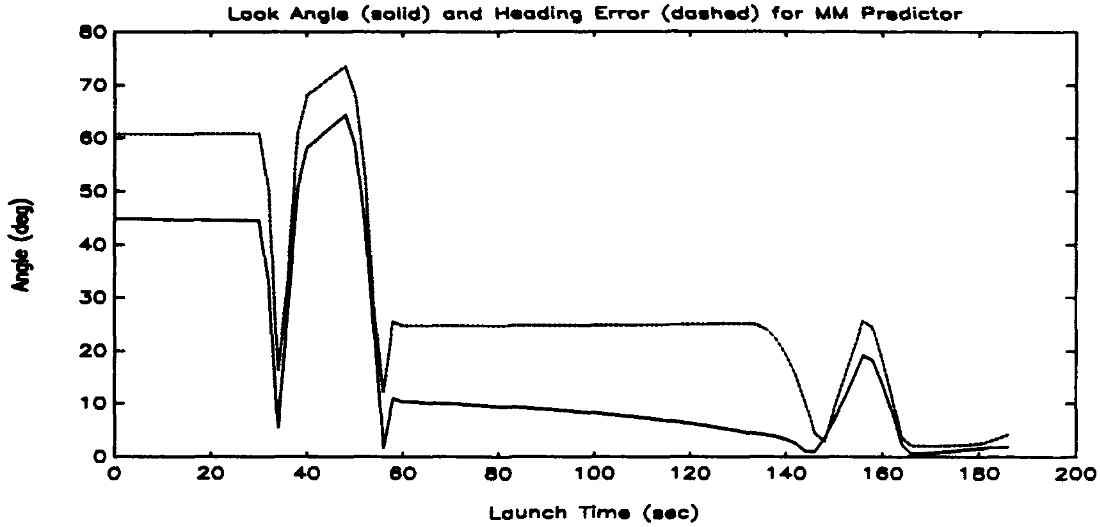

FIGURE 6-18. LOOK ANGLE AND HEADING ERROR FOR MM PREDICTOR

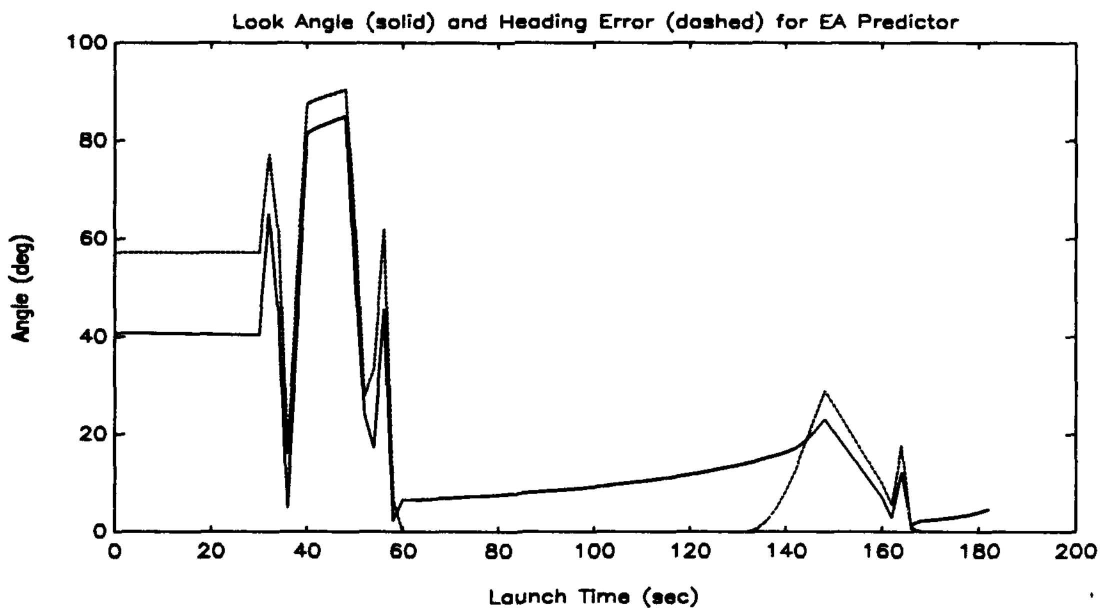

FIGURE 6-19. LOOK ANGLE AND HEADING ERROR FOR EA PREDICTOR

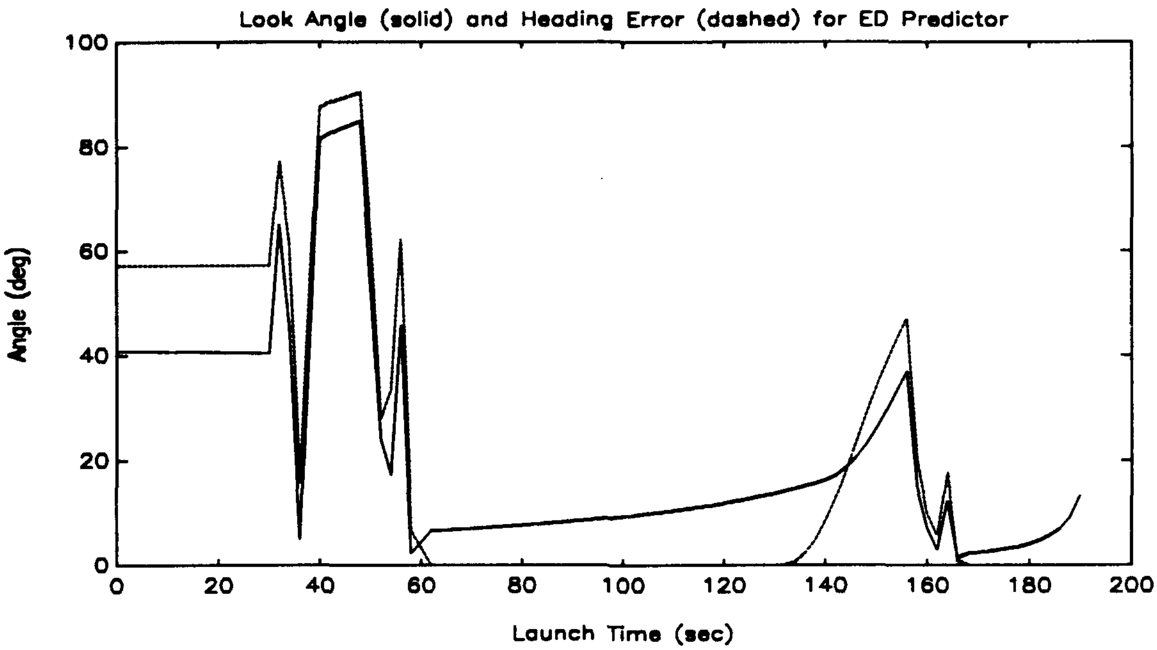

FIGURE 6-20. LOOK ANGLE AND HEADING ERROR FOR EDTR PREDICTOR

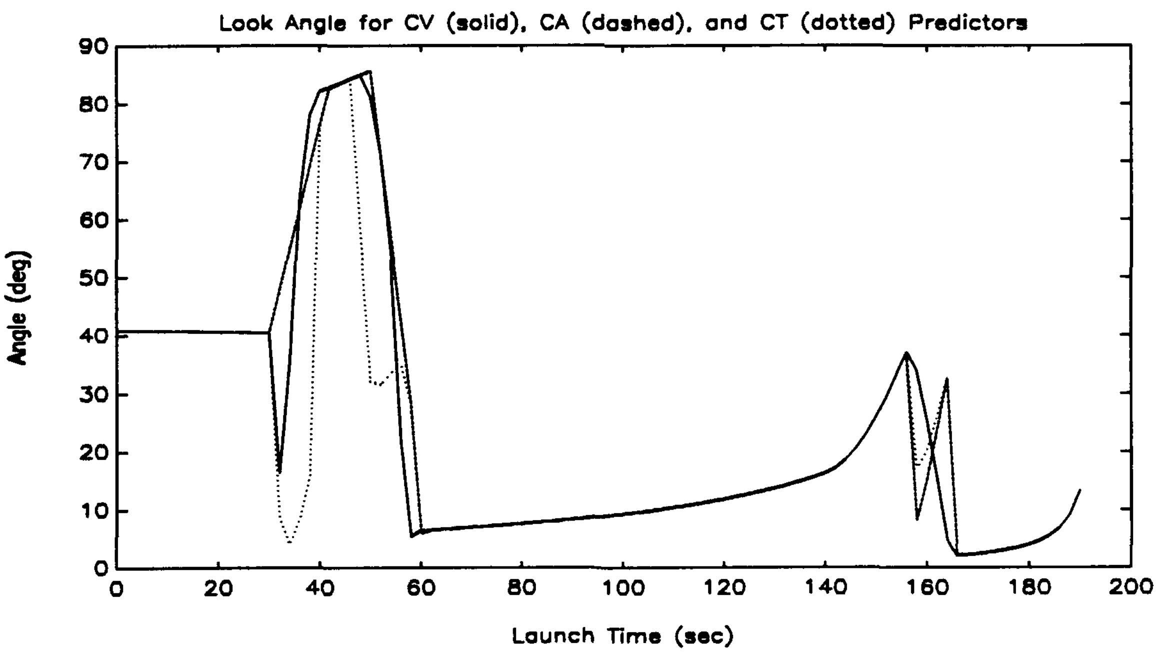

FIGURE 6-21. LOOK ANGLE FOR CV, CA, AND CTR PREDICTORS

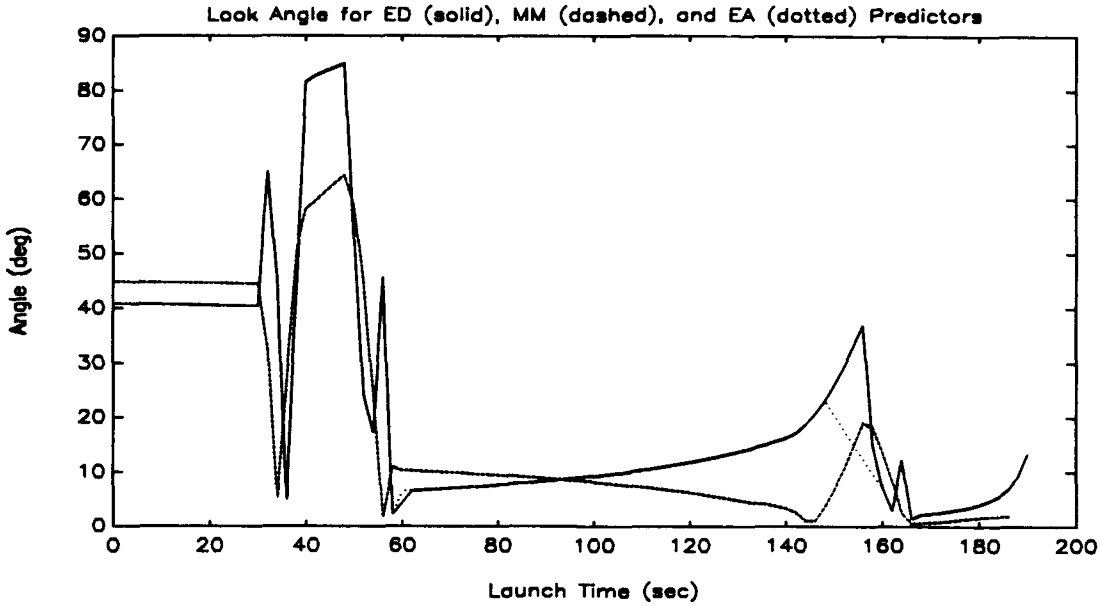

FIGURE 6-22. LOOK ANGLE FOR EDTR, MM, AND EA PREDICTORS

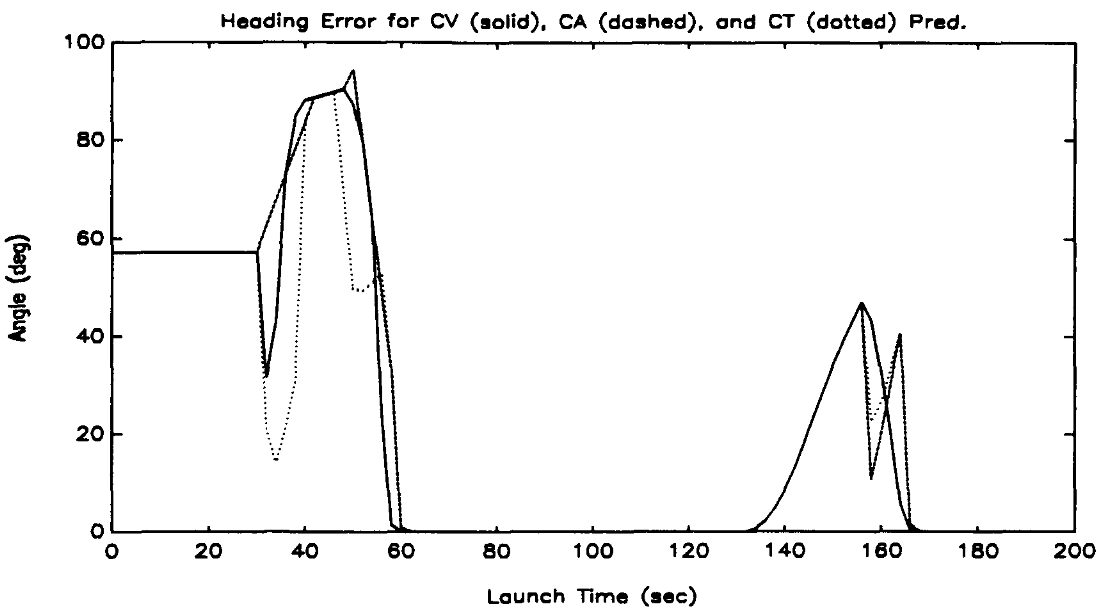

FIGURE 6-23. HEADING ERROR FOR EDTR, MM, AND EA PREDICTORS

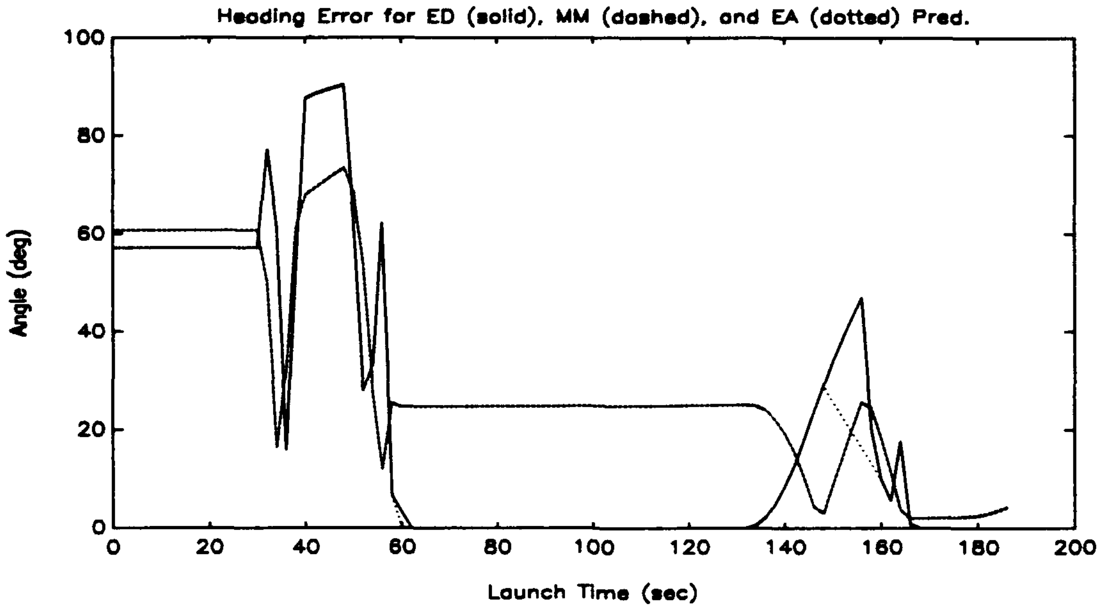

FIGURE 6-24. HEADING ERROR FOR CV, CA, AND CTR PREDICTORS

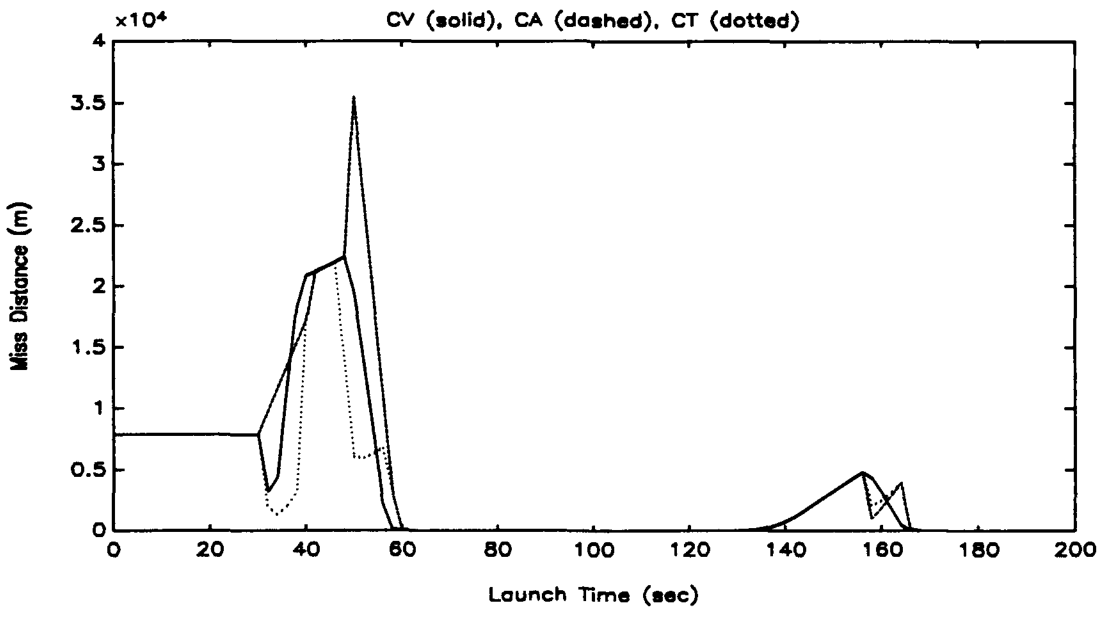

FIGURE 6-25. MISS DISTANCE FOR CV, CA, AND CTR PREDICTORS

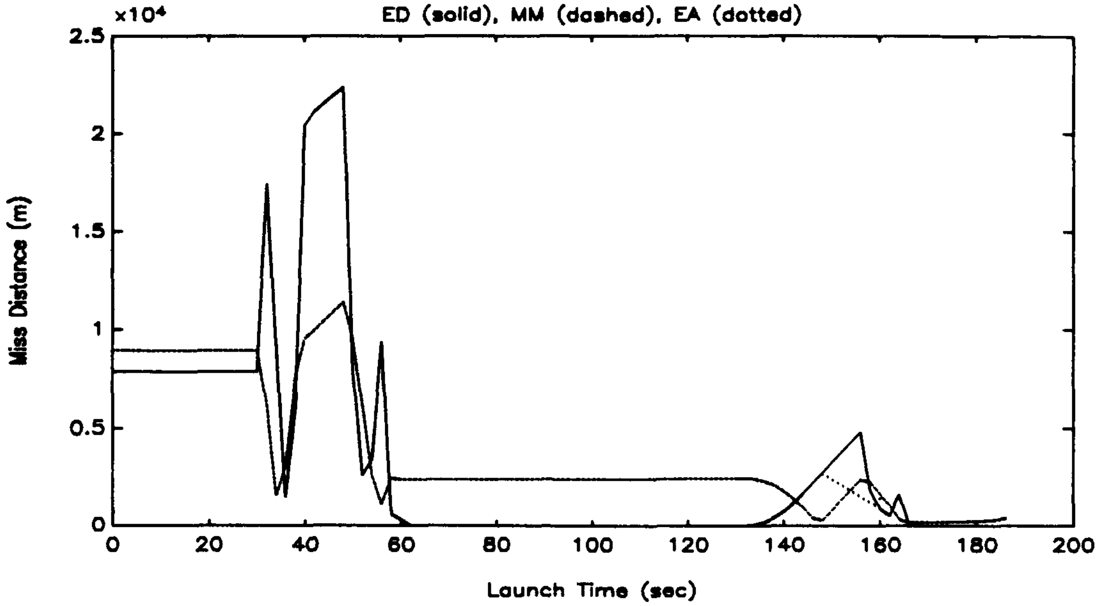

FIGURE 6-26. MISS DISTANCE FOR EDTR, MM, AND EA PREDICTORS

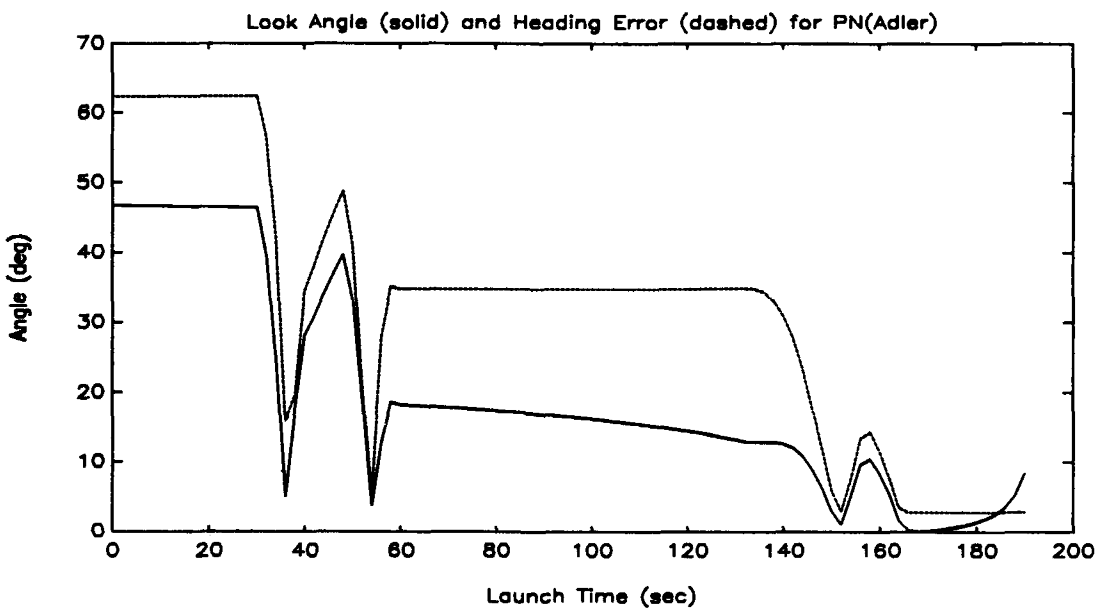

FIGURE 6-27. LOOK ANGLE AND HEADING ERROR FOR PN PREDICTOR

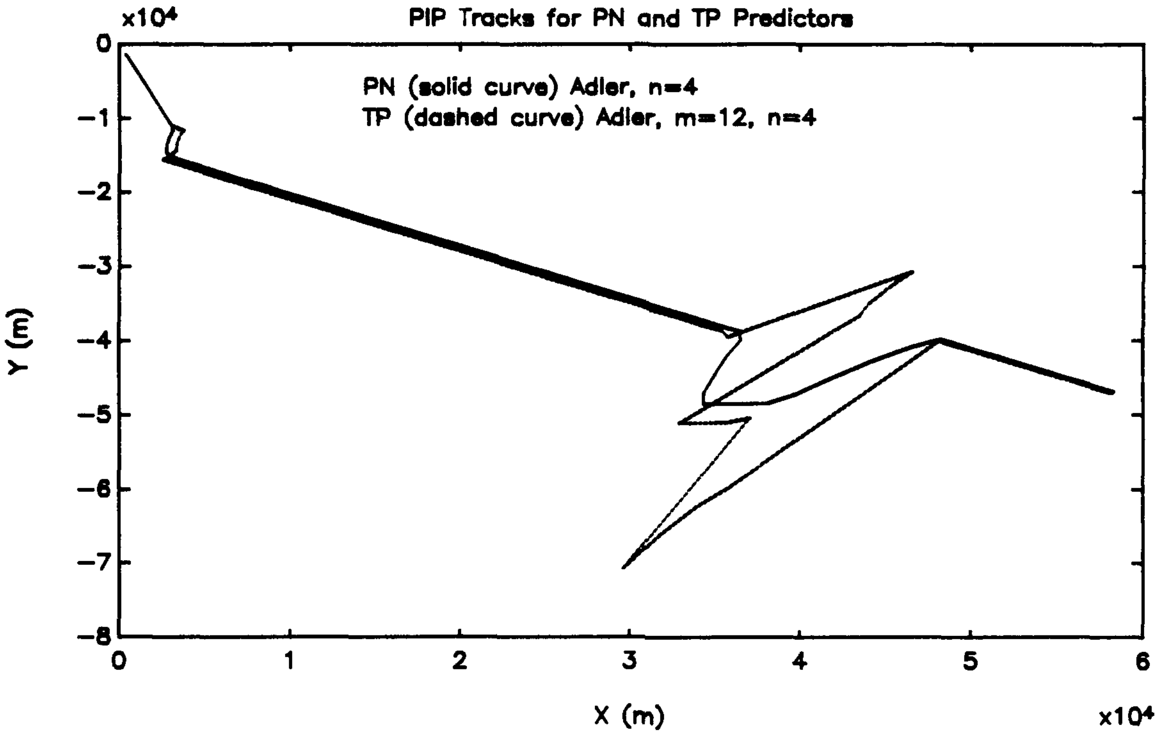

FIGURE 6-28. PIP TRACKS FOR PN AND TPN PREDICTORS

CHAPTER 7

SEA-SURFACE CONSTRAINED THREAT TRAJECTORY PREDICTION

The previous discussion has assumed that the predicted threat trajectories do not intersect the sea surface. This is not always the case. Diving threats, in particular, will tend to generate predicted trajectories intersecting the sea surface. Modifications are proposed here so that the predicted trajectories will lie entirely above the sea surface. Similar modifications are needed to redefine some of the Terminal Parameters so that they be based on "dry" trajectories. Only the planar trajectories are studied in this chapter, and for these the sea is to be represented by the line where the sea surface plane intersects the flight plane. This will be referred to as the "sea line."

Let $\boldsymbol{n}$ be a unit vector in the flight plane, perpendicular to the sea line and directed so that $\boldsymbol{n}\cdot\boldsymbol{x} > 0$. Because the threat will never cross the sea line, $\boldsymbol{n}$ remains constant and can be determined from initial conditions.

MM TRAJECTORY

The constraint is adjusted if the end of the circular-arc segment is below the sea line. This occurs if

$$\left(\boldsymbol{x}_0 + \frac{\sin\theta}{\omega}\dot{\boldsymbol{x}}_0 + \frac{1-\cos\theta}{\omega^2}\ddot{\boldsymbol{x}}_0\right)\cdot\boldsymbol{n} < 0 \tag{7-1}$$

With the definitions of Chapter 4, the vector quantity within the parentheses is clearly the threat position at the transition point between the CTR and CV segments. If the situation (7-1) arises, ideally, a trajectory avoiding the sea line but reaching the origin with minimal lateral-acceleration magnitude should be found. The following simplification is hypothesized to provide such a trajectory, but no demonstration of this has been found. The idealized trajectory consists of one circular arc in the flight plane with CV segments before and after it. The trajectory having these properties, plus the minimum possible lateral-acceleration magnitude, is the one that is determined by the following scheme. Define the "dive point,"

$$\boldsymbol{x}_d = \boldsymbol{x} - \frac{\boldsymbol{n}\cdot\boldsymbol{x}}{\boldsymbol{n}\cdot\dot{\boldsymbol{x}}}\dot{\boldsymbol{x}} \tag{7-2}$$

where the threat would impact the sea line if a CV trajectory were taken. Then, at a distance

$$a = \min[|\mathbf{x}_d| - s, |\mathbf{x} - \mathbf{x}_d|] \tag{7-3}$$

before reaching the dive point; that is, when the threat is at the position

$$\mathbf{x}_1 = \mathbf{x}_d - \frac{a\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|} \tag{7-4}$$

a circular-arc segment is begun. The total arc length θ in this segment is given by

$$\cos\theta' = -\frac{\dot{\mathbf{x}}\cdot\mathbf{x}_d}{|\dot{\mathbf{x}}||\mathbf{x}_d|} \tag{7-5}$$

where the radius of curvature is

$$b = \frac{a}{\tan(\theta'/2)} \tag{7-6}$$

If b turns out to be too small (requiring an acceleration magnitude that is too large), no MM trajectory is possible. The center of curvature is at the point

$$\mathbf{x}_c = \mathbf{x}_d + \frac{\sqrt{a^2+b^2}\,(\mathbf{x}_1 + \mathbf{x}_2 - 2\mathbf{x}_d)}{|\mathbf{x}_1 + \mathbf{x}_2 - 2\mathbf{x}_d|} \tag{7-7}$$

where

$$\mathbf{x}_2 = \frac{|\mathbf{x}_d| - a}{|\mathbf{x}_d|}\mathbf{x}_d \tag{7-8}$$

is the point on the sea line where the circular-arc segment ends and the final CV sea-skimming run over the distance s is begun. The initial acceleration in the circular arc is given by

$$\ddot{\mathbf{x}} = \frac{|\dot{\mathbf{x}}|^2}{b}\frac{\mathbf{x}_c - \mathbf{x}_1}{|\mathbf{x}_c - \mathbf{x}_1|} \tag{7-9}$$

Figures 7-1 though 7-5, show on an arbitrary scale some typical MM tracks with a run-in distance of $s = 2000\,m$ and sea constraint. In Figure 7-1, the condition (7-1) does not arise, and the trajectory is not affected by the presence of the sea. In Figures 7-2 through 7-5, there is a final CV segment, at least as long as s, along the sea line, with successively greater lateral-acceleration magnitudes required in the CTR segment.

Since no demonstration that the foregoing trajectory does indeed minimize the maximum lateral-acceleration magnitude has been given, there remains a possibility that a double-arc trajectory, first turning away from and then toward the sea line, would satisfy the constraints and result in a lower maximum lateral-acceleration magnitude. The geometry is a bit messy, and this possibility has not been investigated.

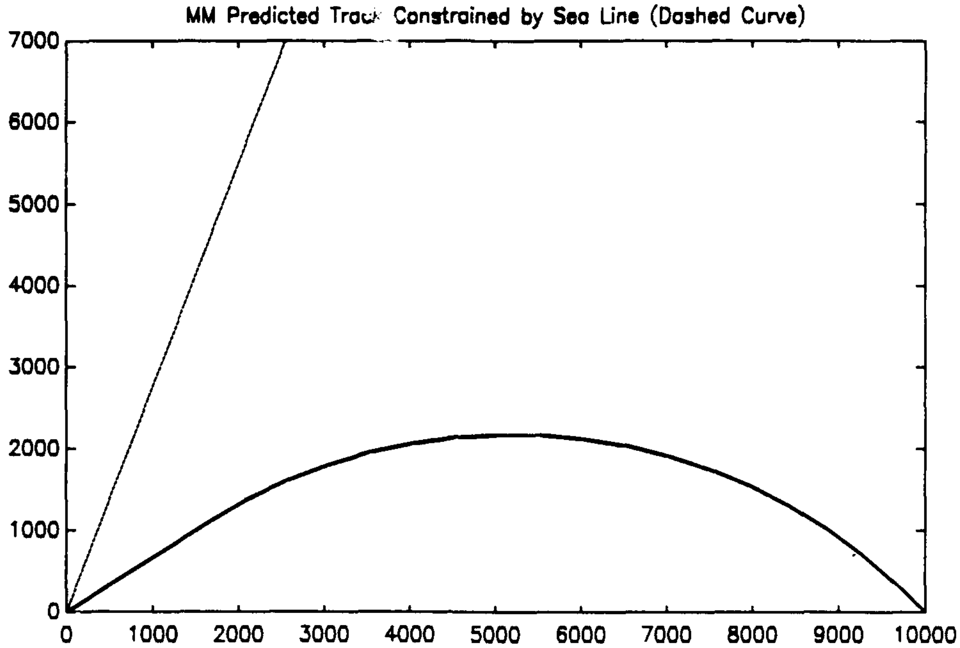

FIGURE 7-1. EXAMPLE IN $(x, \dot{x})$ PLANE WHERE SEA INDUCES NO MODIFICATION OF TRAJECTORY

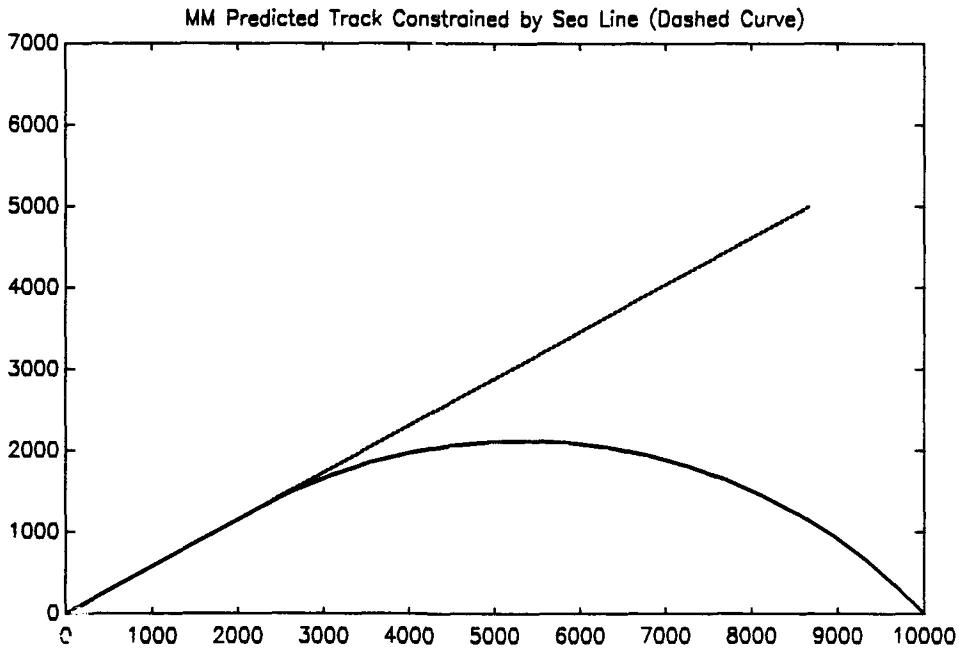

FIGURE 7-2. EXAMPLE OF TRAJECTORY IN $(x, \dot{x})$ PLANE

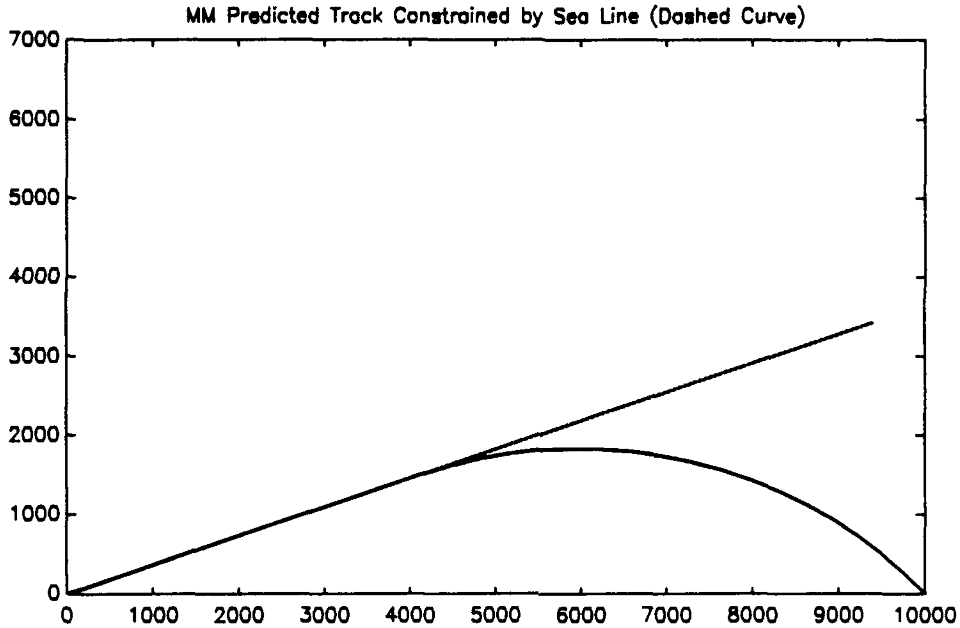

FIGURE 7-3. EXAMPLE OF TRAJECTORY IN $(x, \dot{x})$ PLANE

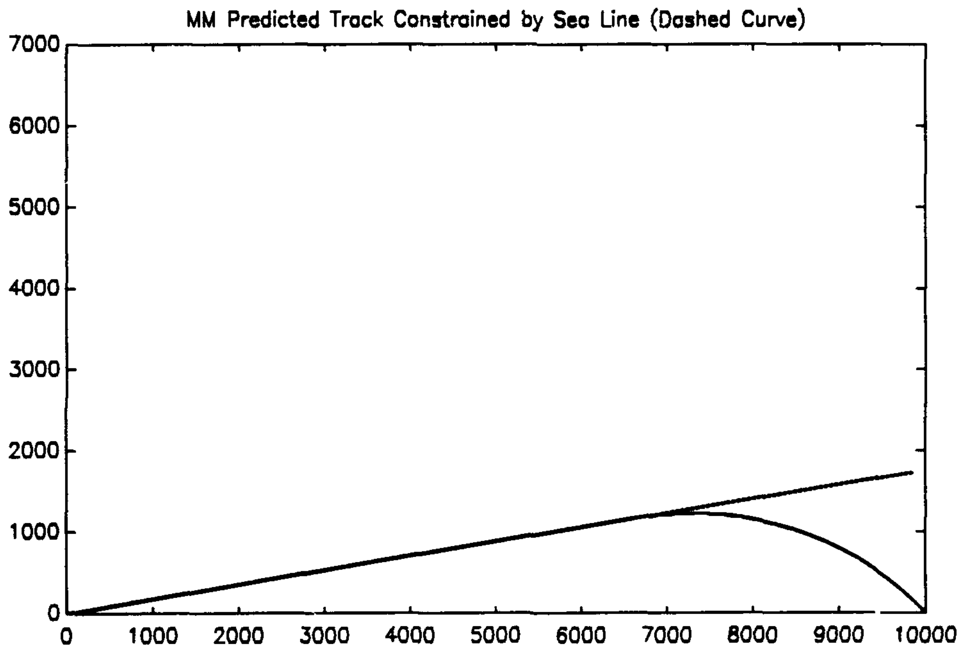

FIGURE 7-4. EXAMPLE OF TRAJECTORY IN $(x, \dot{x})$ PLANE

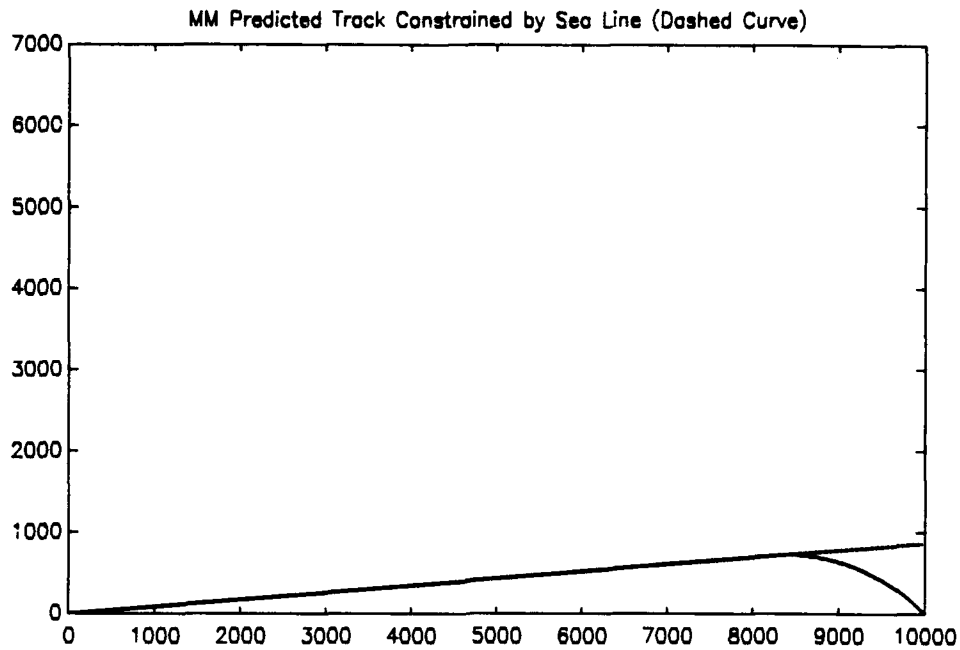

FIGURE 7-5. EXAMPLE OF TRAJECTORY IN $(\boldsymbol{x}, \dot{\boldsymbol{x}})$ PLANE

EA TRAJECTORY

The effect of the sea line on the EA predictor is quite simple. If the unconstrained EA trajectory crosses the line, there is simply no constrained EA trajectory at all. This is because the limiting lateral-acceleration magnitude, a_1, is specified. A sharper turn to avoid the sea line is impossible. A simple check is made, as in the MM case discussed in the previous section, but with the computations based on the μ computed as in Chapter 4.

PN TRAJECTORY

The predicted trajectories can be kept away from the sea line by an additional term in the required acceleration. Let $\boldsymbol{h}$ be a vector whose magnitude is the distance between the threat and the sea line and in the direction of $\boldsymbol{n}$. This is equal to

$$\boldsymbol{h} = (\boldsymbol{n} \cdot \boldsymbol{x})\boldsymbol{n} \tag{7-10}$$

If the threat were to fly CV, it would hit the sea line in the time

$$t_s = -\frac{|\mathbf{h}|}{\mathbf{n}\cdot\dot{\mathbf{x}}} \tag{7-11}$$

It is desirable to introduce an incremental threat acceleration only if the threat would otherwise cross the sea line. Let this incremental acceleration be denoted by $\ddot{\mathbf{x}}_s$. This will be taken perpendicular to the threat velocity, and with magnitude that increases inversely with distance from the sea line, and directly with velocity component toward the sea line. These requirements are met by the expression

$$\begin{aligned} |\ddot{\mathbf{x}}_s| &= \frac{\dot{\mathbf{x}}\cdot\mathbf{n}}{t_s} \qquad && if \quad t_s > 0 \\ |\ddot{\mathbf{x}}_s| &= 0 && if \quad t_s \le 0 \end{aligned} \tag{7-12}$$

The incremental threat acceleration must have the form

$$\ddot{\mathbf{x}}_s = \mu\left[\mathbf{x} - \frac{\mathbf{x}\cdot\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|^2}\dot{\mathbf{x}}\right] \tag{7-13}$$

in order to lie in the $(\mathbf{x},\dot{\mathbf{x}})$ plane and be perpendicular to the threat velocity. The scalar quantity μ is determined in order that the incremental acceleration have the appropriate magnitude (7-12). Accordingly,

$$\mu = \frac{(\mathbf{n}\cdot\dot{\mathbf{x}})^2}{(\mathbf{n}\cdot\mathbf{x})}\left[|\mathbf{x}|^2 \frac{(\mathbf{x}\cdot\dot{\mathbf{x}})^2}{|\dot{\mathbf{x}}|^2}\right]^{-\frac{1}{2}} \tag{7-14}$$

However, improvement is still needed. The above-described incremental acceleration can be appreciable even in cases where the threat would not cross the sea line, especially in cases where the threat velocity is directed nearly normal to the sea line. (It does not matter if the sea line is crossed at the point occupied by the ship.) Therefore, it is advantageous to multiply $\ddot{\mathbf{x}}_s$ by a factor that is relatively small for normal incidence (of the threat velocity toward the sea line) and increases to unity for glancing incidence. There is considerable arbitrariness in specifying such a factor, but here it has been found convenient to use $\sin^8\delta$, where δ is the angle between the threat velocity and $\mathbf{n}$. This angle is given by the relation

$$\cos\delta = \frac{\mathbf{n}\cdot\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|} \tag{7-15}$$

Thus,

$$\sin^8\delta = \left[1 - \frac{(\mathbf{n}\cdot\dot{\mathbf{x}})^2}{|\dot{\mathbf{x}}|^2}\right]^4 \tag{7-16}$$

This factor reduces unwanted modification of the PN law in cases where no modification is needed. In addition, there is no a priori justification for applying the incremental acceleration, as just defined, with unit gain. Let ν be an arbitrary gain, to be adjusted to achieve the desired effect. The incremental acceleration is then represented by

$$\ddot{\mathbf{x}}_S = -\nu \frac{(\mathbf{n}\cdot\dot{\mathbf{x}})^2}{\mathbf{n}\cdot\mathbf{x}}\left[1-\frac{(\mathbf{n}\cdot\mathbf{x})^2}{r^2}\right]^4\left[r^2-\frac{(\mathbf{x}\cdot\dot{\mathbf{x}})^2}{|\dot{\mathbf{x}}|^2}\right]^{-\frac{1}{2}}\left[\mathbf{x}-\frac{\mathbf{x}\cdot\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|^2}\dot{\mathbf{x}}\right] \qquad (7\text{-}17)$$

This expression may appear a bit complicated, but it serves the purpose and is easy to compute. Figures 7-6 to 7-9 show the resulting APN and MPN trajectories for various values of ν. The value $\nu = 0$ results in the unmodified PN law and in these cases results in a trajectory that crosses the sea line. Small positive values (on (0,.3)) cause numerical difficulties near the sea line and are not shown. The results indicate that the value $\nu = .5$ is appropriate. Figures 7-6 and 7-7 show two examples of Adler PN, and Figures 7-8 and 7-9 show two examples of MPN.

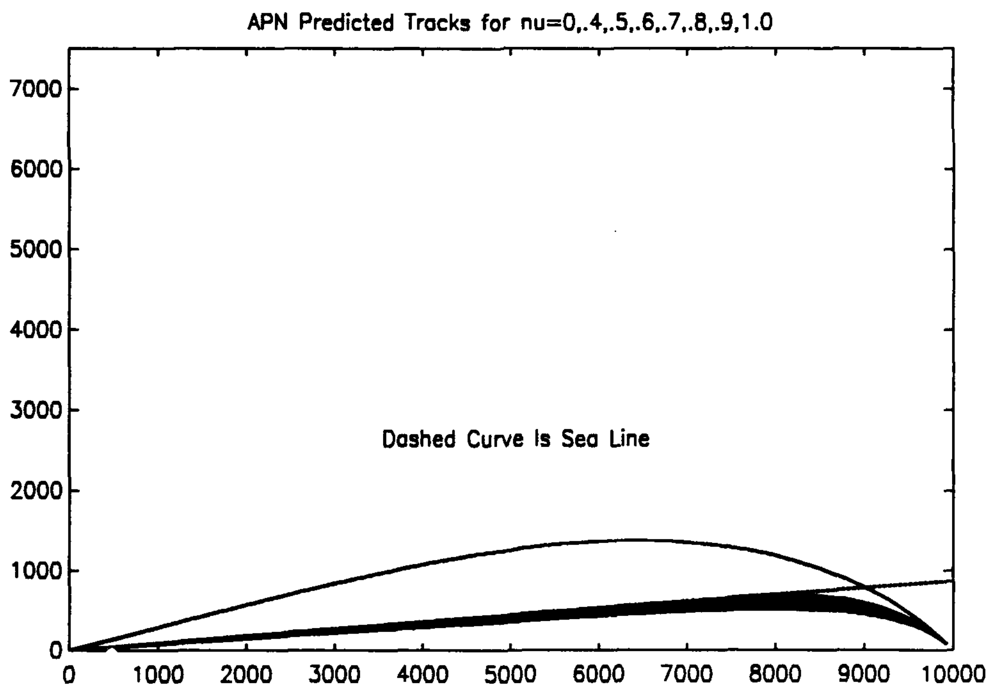

FIGURE 7-6. EXAMPLE OF TRAJECTORY IN $(\mathbf{x}, \dot{\mathbf{x}})$ PLANE (ARBITRARY SCALE)

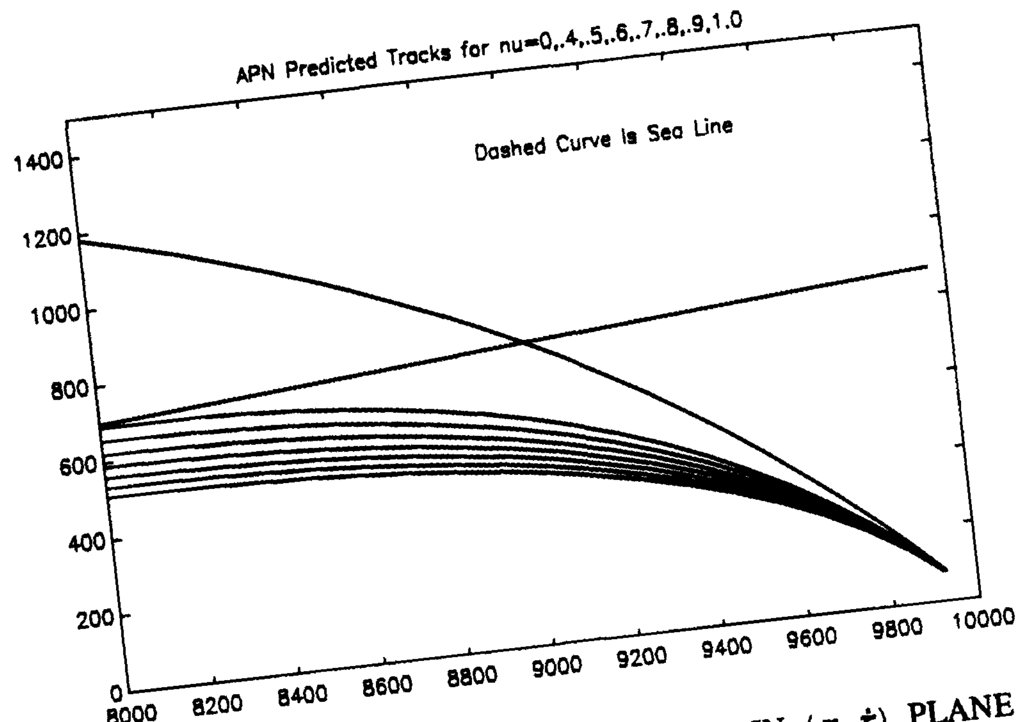

FIGURE 7-7. EXAMPLE OF TRAJECTORY IN $(x, \dot{x})$ PLANE

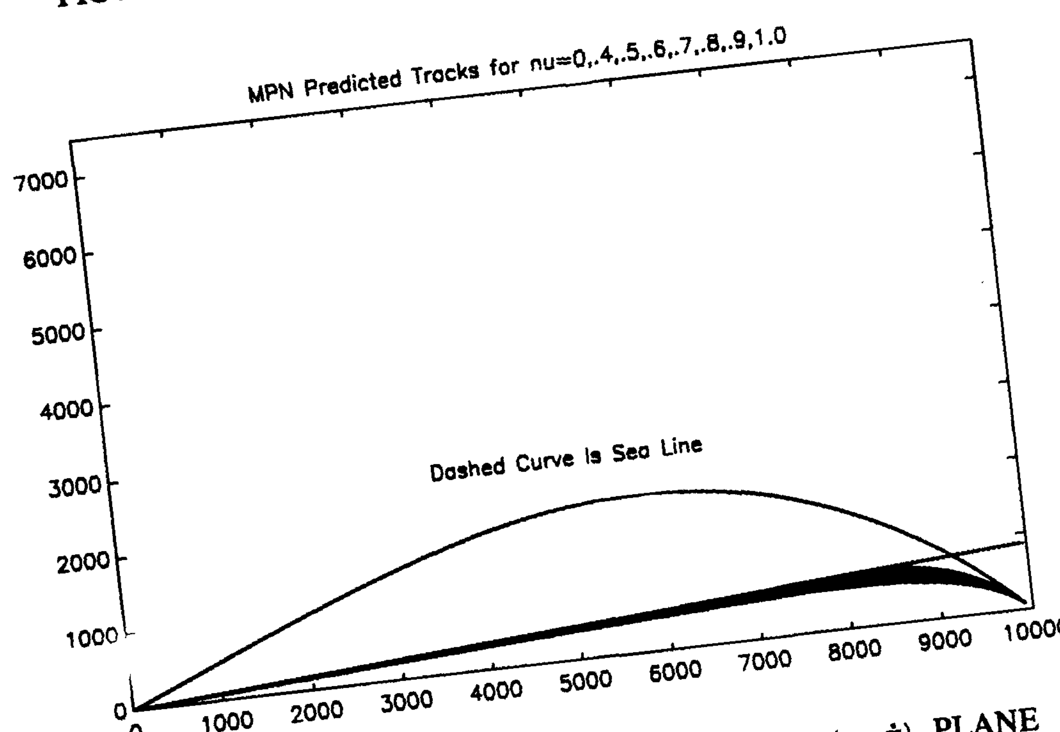

FIGURE 7-8. EXAMPLE OF TRAJECTORY IN $(x, \dot{x})$ PLANE

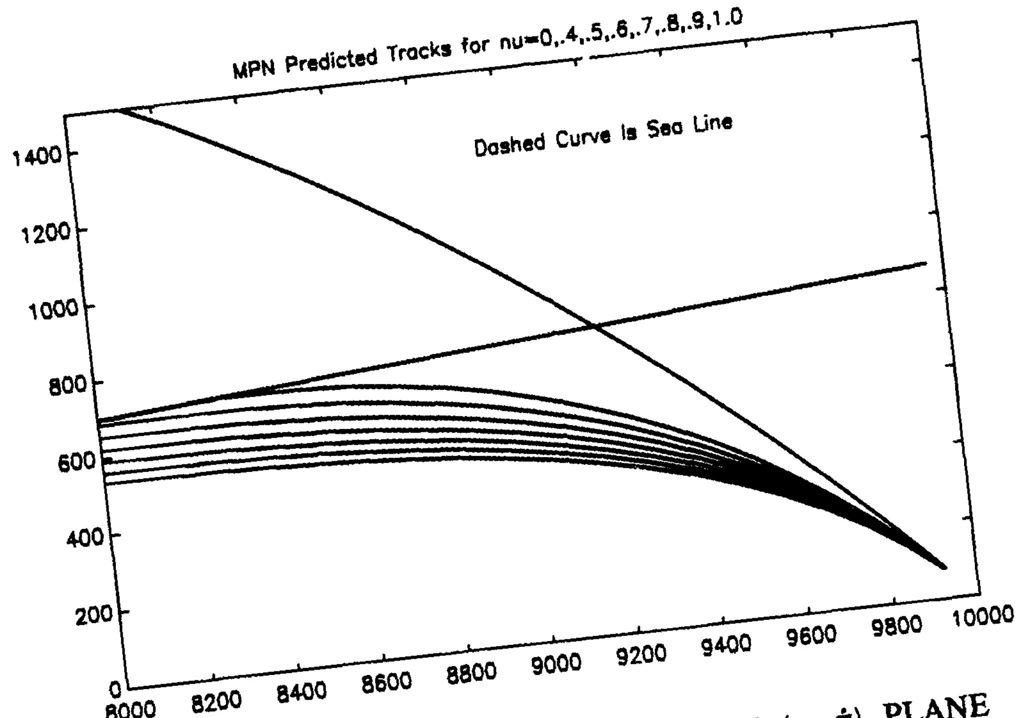

FIGURE 7-9. EAMPLE OF TRAJECTORY IN $\langle x, \dot{x} \rangle$ PLANE

CHAPTER 8

CONCLUSIONS

The state-based predictors are easily deceived by maneuvering threats into producing PIPs that lie far from the actual threat track whose destination is the ship. These state-based predictors are distinguished by the degree of confidence placed on the estimated threat acceleration and jerk. With minimal confidence in the estimates of acceleration and jerk, one can do no better than a CV linear extrapolation of the initial threat position. Maximal confidence in these estimates leads one to the Helical predictor. Between these extremes are the CTR and EDTR predictors, which ignore the initial jerk, with the former applying the initial lateral acceleration throughout the prediction while the latter has the acceleration decay exponentially to zero.

The goal-based predictors were found to be useful when the destination of the threat is somewhat unambiguous. All ignore the initial estimated threat acceleration except the TPN, which may be used when this value is deemed reliable and important. The large prediction errors typically generated by the state-based predictors are considerably reduced. However, the state-based predictors are usually more accurate for short times.

Most of the predictors used the constant-speed constraint. While this constraint may not be realized by all threats, taking speed variation into account requires threat models that are considerably more complicated. The only exceptions were the CA and the CLLA predictors, in which the speed can vary. The CA predictor is not recommended except for short-term predictions. The CLLA predictor may be useful if the variation in speed can be estimated. Methods for adjusting the initial threat state estimates to satisfy the constant-speed constraint were outlined.

A number of Terminal Parameters were defined in Chapter 5 for the purpose of facilitating decision making by providing the user with more immediately-usable information. For example, the parameter t_{EA}, the earliest arrival time, might be monitored for each threat to ensure that the target is engaged in a timely manner. The other Terminal Parameters similarly represent the voluminous essential information into a more concisely manageable form.

The concepts discussed in this report need to be developed further. The next advance in methodologies for dealing with maneuvering threats may involve specifics of the different threat types, as knowledge of their projected capabilities becomes known. Future prediction schemes may use target ID when available, or characterize its behavior in an adaptive (on line) algorithm.

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