AD-A276 986

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FINAL TECHNICAL REPORT

ONR GRANT: N00014-92-J-1800

R&T PROJECT: 4214064---01, DATED: 06 MAY 1992

S.O. CODE: 1121CS

AGO CODE: N68883

CAGE CODE: 42226

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APPROPRIATION: $15,259.00

TITLE: **Vertical Structure of Flow Through Tarifa Narrows**

The research undertaken in this project is part of a larger effort, in collaboration with N. Pettigrew (University of Maine, Orono), in which the PI seeks a description and a dynamical explanation of the steady, tidal, and internal wave components of the current in the Tarifa Narrows of the Strait of Gibraltar. With this effort, the PI continues his summer research program while teaching undergraduates fulltime during the academic year at Norwich University. The research funded by this ONR grant permitted the PI to work the summer of 1992 at the Ocean Process Analysis Laboratory (University of New Hampshire; W. S. Brown, director). The PI was involved primarily with the analysis of Acoustic Doppler Current Profiler (ADCP) data taken by Pettigrew and collaborators (*Proc. of Ocean '87*, 1987) during the Gibraltar Experiment (GIBEX) (Kinder & Bryden, *EOS Transact, AGU*, 1987).

The format for this final technical report will outline the principal objectives of the research and describe the progress made on each. In general, the research is not finished and continues to occupy the attention of the PI. In particular, the PI will work to finish several of the uncompleted objectives during future summers.

This document has been approved for public release and sale; its distribution is unlimited

The PI is most appreciative of the funding provided as he was able to advance on many aspects of the project.

OBJECTIVE 1: a refinement of the response method of tidal analysis for use with current records strongly contaminated by internal bores.

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The unique ADCP data set from Tarifa is profoundly characterized by the passage of a tremendous internal bore generated semidiurnally at the nearby Camarinal Sill. To avoid contamination by the bore, a modification of the response method of tidal analysis has been developed to extract the steady component of flow and the barotropic tide from the dynamically complex current; the bore is left as the residual of the analysis. This decomposition permits the steady component and the internal bore to be examined as isolated dynamical entities.

The development of this technique was finalized by the PI during the summer of 1992 while funded by this grant. A photocopy of an article which finally appeared (Hyde & Pettigrew, *JGR 98*, *#C12*, 1993) is attached.

Figure 1 (taken from Hyde & Pettigrew, 1993, page 22,755) shows an example of difference between the standard response method and the modified response method of tidal analysis. The two methods produce drastically different values for the amplitude of the tide, the phase of the tide, and the speed of the mean flow as well as the structure of the internal bore, and therefore, quite different dynamical views of the Strait.

OBJECTIVE 2: a completion of the decomposition of Tarifa ADCP time series into their steady, tidal, and internal bore components.

Approximately 25% of the Tarifa ADCP data was analyzed confidently during the summer of 1992 under the funding of this grant. In particular, the 2 periods of spring tides at the deep Tarifa instrument were finished. However, the neap tides are much more problematic and require greater dynamical insight before they can be decomposed with confidence; the same is true of the entire ADCP record in the shallows by Tarifa Harbor. Nevertheless, this analysis has provided a very interesting data set showing the vertical variation of the mean flow and the internal bore during the periods of spring tides.

Figures 2 and 3 show the results of the analysis during a period of ten semidiurnal tidal cycles of the spring tide in early April, 1986 in the deep Tarifa Narrows. Figure 2 shows a time series of the variation with depth of the velocity associated with a composite of the ten internal bores which passed the instrument during the period. Figure 3 displays the vertical variation of the north-south and east-west components of the steady flow during the period.

OBJECTIVE 3: an estimate of the transport of Atlantic water into the Mediterranean;

An estimate by Pettigrew & Hyde (*Phys Ocean Sea Strait*, NATO/ASI, 1990) of the transport of Atlantic water into the Mediterranean showed that the contribution to the transport by the internal bore is comparable to that of the steady flow.

The funding of this grant permitted a considerable improvement in the earlier estimates. Applying the modified response method to the upper layer of inflowing Atlantic water to isolate the bore, the steady flow, and the

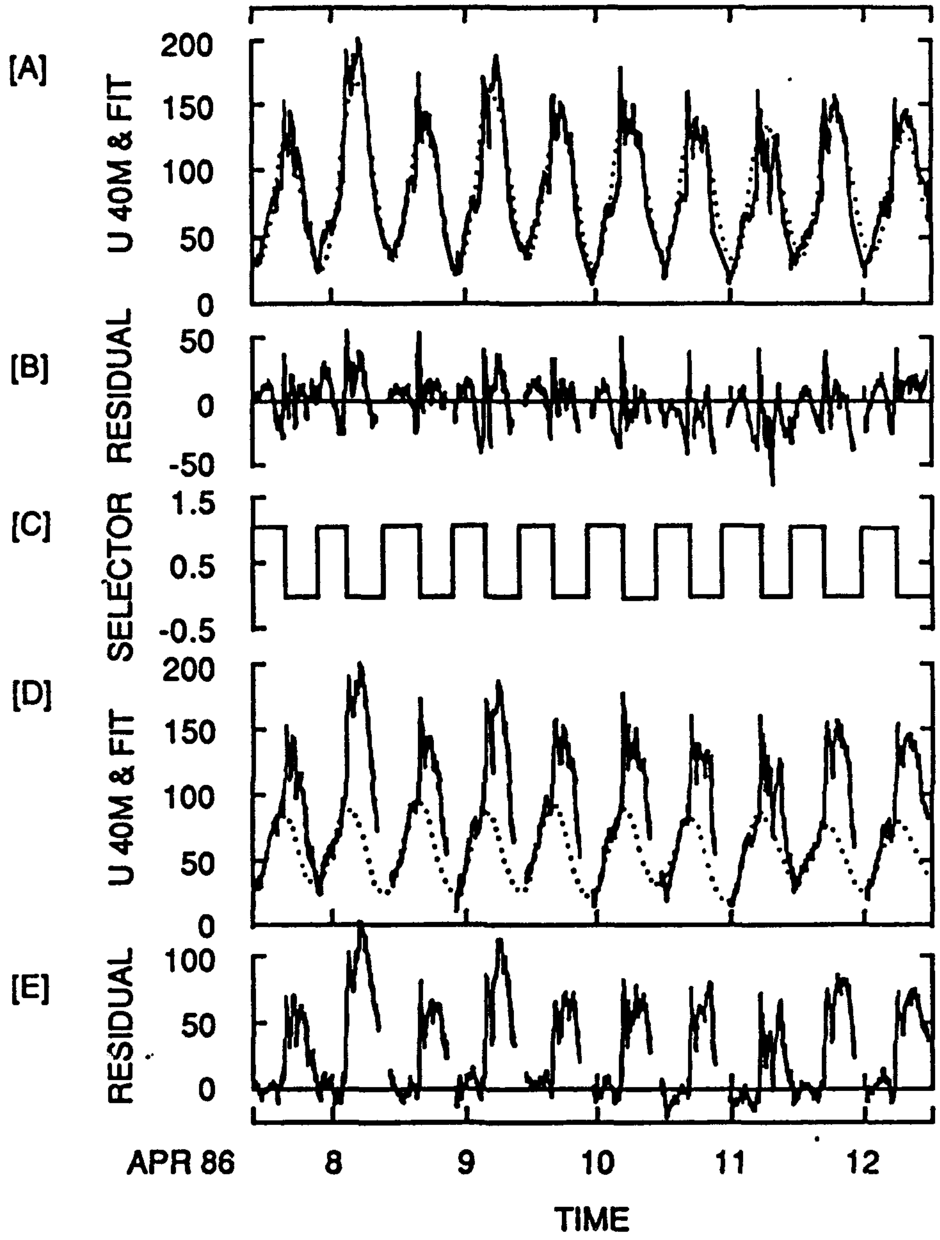

FIGURE 1 Comparison of the standard response method of tidal analysis (A & B) with the modified response method of tidal analysis (D & E). Tarifa DAPCM data at 40 meters depth.

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tide, the mass transport of Atlantic water through the Narrows was carefully decomposed. When averaged over a spring-neap cycle, it was found that the transport associated with the steady flow is 0.56 Sverdrups while that associated with the bore is 0.43 Sverdrups. The barotropic tide's contribution is negligible. Thus, the importance of the bore noted in the earlier investigation was reaffirmed. It was also found that the transport during a neap tide (1.06 Sverdrups) is slightly greater than during a spring tide (0.93 Sverdrups).

The PI has completed the computations required for this objective. A manuscript (with N. Pettigrew as lead author) now needs to be prepared.

OBJECTIVE 4: a dynamical model of the observed vertical structure of the steady component of the flow at Tarifa;

Figure 3 shows the depth dependency of the eastward (triangles) and the northward (crosses) components of the steady flow during a period of 10 spring semidiurnal tidal cycles. There is considerable shear and turning in the mean flow across the Atlantic-Mediterranean interface suggesting the existence of boundary layers straddling the interface.

At the suggestion of Pettigrew, the PI investigated the vertical structure of the mean flow using Ekman boundary layer theory. The PI fitted the data, as shown in Figure 3, with a simple Ekman model which balances pressure gradient, Coriolis, and eddy viscous forces in the interior of the fluid layers and imposes appropriate boundary conditions on the velocities and fluid stresses at the interface, at the sea surface, and at great depth. The fitting parameters are the wind stress, the pressure gradients in the two layers, the depth of the interface, and the mixing coefficient. The fit (Figure 3) is good and the values of the fitting parameters are in reasonable agreement with the physical reality of Tarifa. This success provides encouragement to continue this analysis.

Roughly 50% of the work required for this objective was accomplished by the PI during the summer of 1992 with funding from the ONR grant. There are details to be considered and there is an analysis of the data from Tarifa Harbor to be attempted before a manuscript can be prepared (with Pettigrew as lead author).

OBJECTIVE 5: a dynamical model of the temporal evolution and vertical structure of the internal bore at Tarifa.

The composite of the horizontal velocity field associated with the internal bore, shown in Figure 2, is broadly consistent with the known characteristics of a two-layer solitary wave, namely, longitudinal motion in the direction of propagation (eastward) in the thinner upper layer and opposite the direction of propagation in the thicker lower layer. However, there are also considerable, complex transverse motions. In particular, during the latter part of the passage of the wave, it appears that the transverse velocity is primarily to the left [right] of the propagation direction in the lower [upper] layer.

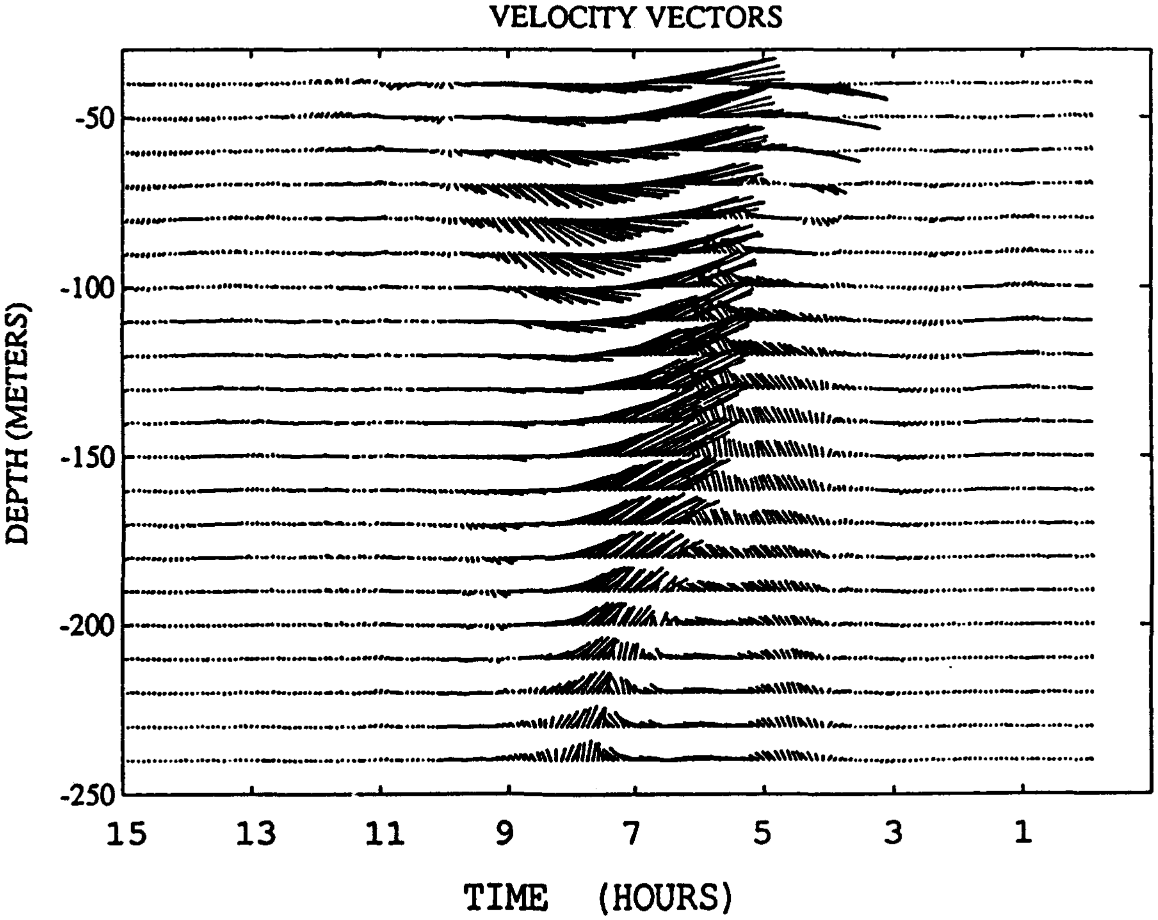

FIGURE 2 Time series composite (average of 10 semidiurnal tidal cycles) of the horizontal velocity associated with the internal bore as it passes Tarifa. Eastward current to right; northward current up. Time measured from the zero-crossing of the tidal pressure record at Tarifa. Onset of the bore at about 4 hours.

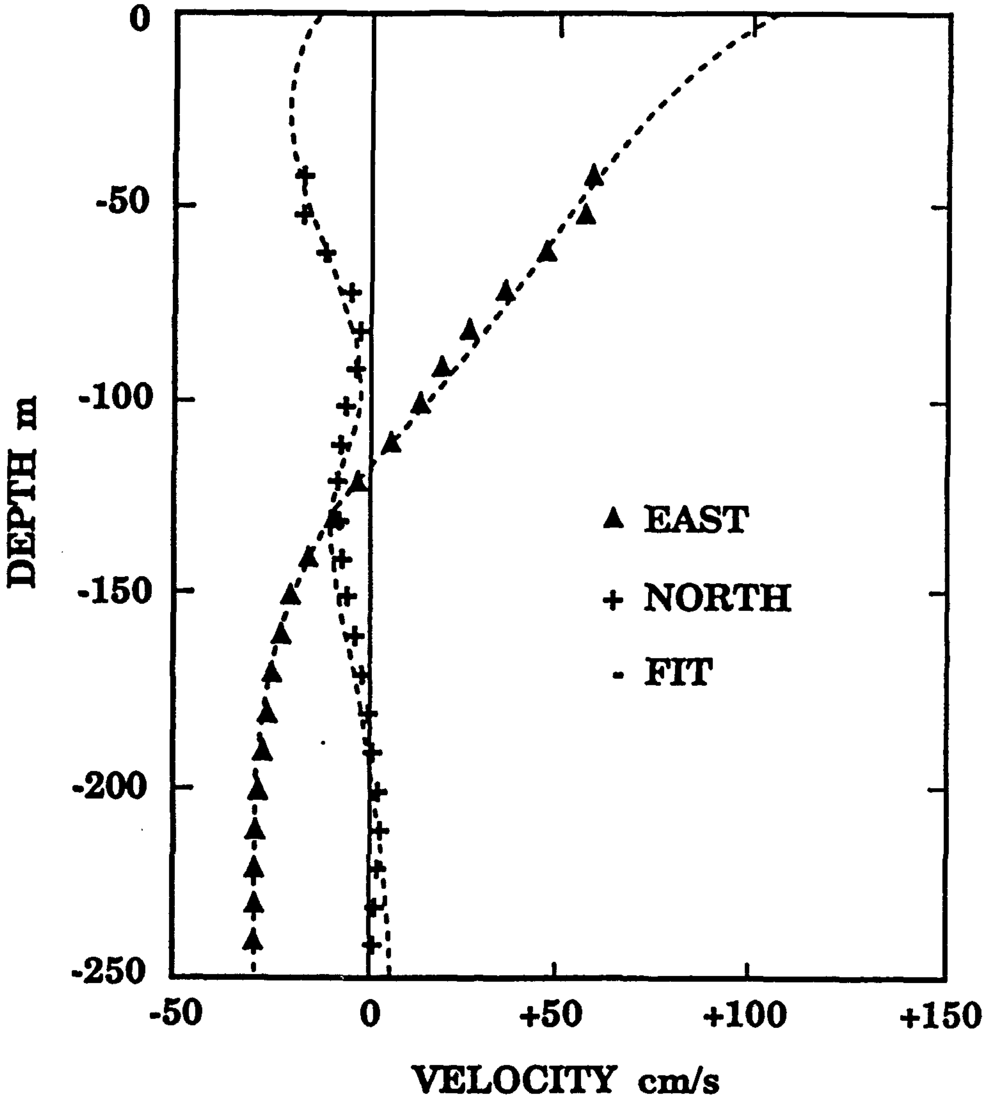

FIGURE 3 Northward and eastward components of the steady flow during a Tarifa spring tide.

An attempt to explain the dynamics of the transverse motions of the bore has not been successful so far. An asymptotic analysis of solitary waves (weakly dispersive, weakly nonlinear, long waves) influenced by rotation, which the PI submitted to JFM, was deemed inappropriate by reviewers. Instead, the modified Kadomtsev-Petviashvilli (MKP) equation (Grimshaw & Melville, *Stud Appl Math*, 1989) was suggested. However, it is not immediately obvious that the behavior in Figure 2 is explained by MKP theory. Considerably more theoretical work needs to be done in this regard. Perhaps due to its evolution, the bore exhibits several velocity fronts and a complicated depression of the interface which will frustrate any definitive explanation of the bore's dynamics.

At a minimum, the PI will attempt to publish the data showing the structure of the transverse motions of the bore.

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 98, NO. C12, PAGES 22,753–22,757, DECEMBER 15, 1993

A Modification of the Response Method of Tidal Analysis

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An easily implemented extension of the standard response method of tidal analysis is outlined. The modification improves the extraction of both the steady and the tidal components from problematic time series by calculating tidal response weights uncontaminated by missing or anomalous data. Examples of time series containing data gaps and anomalous events are analyzed to demonstrate the applicability and advantage of the proposed method.

INTRODUCTION

The analysis of tidal currents may be confounded by an inability to separate clearly the surface (barotropic) tide and the internal (baroclinic) tide. This is particularly true in regions of internal tidal generation such as the Strait of Gibraltar. Data obtained during the Gibraltar Experiment [*Kinder and Bryden*, 1987] include enormous velocity surges associated with the passage of a spectacular internal bore [*Pettigrew and Hyde*, 1990] that is approximately phase locked to the surface tide. Surge velocities often are of the order of a meter per second and may substantially exceed all of the other flow components near the pycnocline. In a fashion common to the generation of most baroclinic tides [*Rattray*, 1960; *Wunsch*, 1975], this large bore is produced as a consequence of the surface tide driving a sharply stratified water column over the Camarinal sill.

Intermittency of amplitude and phase [*Wunsch*, 1975], due presumably to temporal and spatial variations in hydrographic conditions either in the generation area or in the radiation field, often characterizes internal tide oscillations. For very long current records far from the generation region, the baroclinic component of the tide often appears as little more than noise at the barotropic tidal frequencies and may therefore be removed with a standard response analysis [*Munk and Cartwright*, 1966] of the tide. At the other extreme, in regions where local internal tidal generation is large, the currents in the tidal band may have a very complex structure that is phase locked with the surface tide. The case of the Strait of Gibraltar is a particularly dramatic example of this latter extreme. There, a very large amplitude bore severely distorts the velocity signature. (See the time series shown in Figure 1*d* which will be discussed thoroughly in section 3.) Standard tidal analyses therefore produce unreliable amplitudes and phases. Moreover, the baroclinic residual of the analysis bears little resemblance to known characteristics of the bore itself. Thus indiscriminate use of the traditional response method (or of the harmonic method) of tidal analysis may lead to fallacious views of the dynamics of the strait. A modified approach is needed.

The response method of tidal analysis was introduced [*Munk and Cartwright*, 1966] as an alternative to the harmonic method of tidal analysis [*Darwin*, 1883]. This method, developed further in the work of *Cartwright et al.* [1969] and *Zetler and Munk* [1975], is based upon the relationship between an input $T(t)$ and the output $v(t)$ of a linear system. Namely,

$$v(t) = \int_{-\infty}^{+\infty} W(t)\, T(t-\tau)\, d\tau \tag{1}$$

where $W(\tau)$ is an impulse response function. The Fourier transform of (1) results in

$$v(\omega) = H(\omega)\, T(\omega), \tag{2}$$

where H is variously referred to as the transfer function or the admittance and is obviously the Fourier transform of the coherent ratio of output v to input T.

The response method offers several advantages over the harmonic method [*Zetler et al.*, 1979; *Zetler*, 1987], including greater accuracy, clearer separation of solar gravitational and solar radiational effects, and better statistical stability. The last feature is perhaps the greatest distinction between the two methods. While a reliable estimate of tidal constituents by harmonic (or spectral) analysis requires long-term observations, the response method can analyze a relatively short record. It achieves both stability and resolution by utilizing a long-term reference series upon which high-resolution analysis can be performed and then calculating a smooth transfer function [*Munk and Cartwright*, 1966] linking the reference series to the observed time series. (See *May* [1979] for a brief but lucid discussion.) As a result, the response method, although not universally employed, has become the standard for the analysis and prediction of tidal currents.

This document presents a modification of the response method of tidal analysis which seems quite useful for isolating anomalous events (such as passages of a bore) and preventing them from contaminating tidal analyses. The procedure is an extension of and improvement upon the preliminary work of *Pettigrew and Hyde* [1990] for discriminating between the Gibraltar internal bore and its engendering tidal current. Termed "the selective response method of tidal analysis," it alters the response method formalism so that only those data that are clearly uncontaminated by anomalous events and are clearly representative of the steady flow and the tidal oscillation are included in the computation of the transfer function. While development of the selective response method was

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Paper number 93JC01441.
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motivated by the difficulties associated with the Gibraltar internal tide, the method lends itself naturally to the analysis of velocity records with missing data. Because such records are so common and because preliminary tests have been so successful, it may well be in its application to data with gaps that the modified method will find its greatest utility to oceanographers.

SELECTIVE RESPONSE METHOD OF TIDAL ANALYSIS

The selective-response method of tidal analysis is founded upon the following two modifications of the standard response method: (1) an alteration of the calculation of the response weights in which only selected data determine the optimal (least squares) fit of the tide and the mean and (2) an adjustment in the determination of the appropriate number of response weights.

The selective response method of tidal analysis applied to discretized data given at (possibly random) times t_k, $k = 1, \ldots K$, utilizes (1) in the form

$$v(t_k) = W_0 + \mathrm{Real}\,\{\Sigma_m \Sigma_n W_{mn}{}^{*} T_m(t_k - \tau_n)\} + \varepsilon(t_k) \qquad (3)$$

where the summation over m ranges from 1 to M while that over n ranges from 1 to N and where the following definitions are employed. (1) The set $\{T_m\}$ is a set of M reference series constructed from the tidal constituents obtained either from a harmonic analysis of a lengthy tidal pressure record or from tidal potential theory. For example, constituent amplitudes A_j, frequencies f_j, and phases ϕ_j form a single reference series when combined as $T_m(t_k) = \Sigma_j A_j \exp\{i(2\pi f_j t_k + \phi_j)\}$, where the sum in j ranges over the constituents from j_s to j_t, which forms the mth of the linear set of reference series. Such a set was used in this study. But equally acceptable, as need dictates, are reference sets formed by individual lines or by bilinear combination of lines. (2) The set $\{\tau_n\}$ is a set of N times (including zero) by which the reference series set $\{T_m\}$ is lagged. (3) W_0, a real number representing the mean value of the series, and $\{W_{mn}\}$, a set of MN complex numbers, are the response weights that are a measure of the response of the velocity series v to the reference series T_m. Equation (3) has $2MN+1$ weights (or, equivalently, degrees of freedom) for the fit. (4) The series $\varepsilon(t_k)$ is a time series of K elements containing the residual error between the data and the fit.

The number of reference series and the number of lags (M and N, respectively) are chosen according to the requirements of the problem. For the analysis presented in this article, all diurnal constituents are summed and all semidiurnal constituents are summed so that $M = 2$. However, M may be chosen suitably large either for a treatment of shallow-water tidal effects analogous to extended harmonic analysis [*Zetler and Cummings*, 1967] or for an application of bilinear analysis [*Munk and Cartwright*, 1966]. While the appropriate number of time lags N will be more completely addressed in the next section, note that the fits shown in this article utilize three lags corresponding to -48, 0, and +48 hours [*Cartwright et al.*, 1969].

To this point, the selective method mimics the standard method closely. With the least squares minimization of the residual ε, however, the methods diverge. For the selective method, the weights W_0 and $\{W_{mn}\}$ are computed from

$$\min \Sigma_k \{S(t_k)\,[\varepsilon(t_k)]^2\}, \qquad (4)$$

where the summation over k ranges from 1 to K and S is a real series defined for all k as

$$\begin{aligned} S(t_k) &= 1, \text{ "acceptable" datum at } t_k \\ S(t_k) &= 0, \text{ "unacceptable" datum at } t_k \end{aligned} \qquad (5)$$

which weights the least squares fit of the data so as to include ($S = 1$) or exclude ($S = 0$) a datum from the determination of the response weights. The weights W_0 and $\{W_{mn}\}$ fit only those data selected by S as acceptable and W_0 is the mean of only those data. Minimization (4) is entirely equivalent to the least squares minimization of $|\varepsilon(t_k)|$ over the $k = 1, \ldots K'$ data points which form the subset of acceptable data defined by identity (5).

Depending upon the character of the data series to be analyzed, construction of S may be quite subjective, demanding careful attention and physical intuition. One obvious instance of unacceptable data occurs with sensor malfunction (e.g., missing data or instrument glitches). A less obvious but very useful instance might be the contamination of a velocity record by anomalous events (e.g., passage of a bore) which the analyst wishes to separate from the tidal signal.

The minimization required by (4) can be performed by any standard method [*Lawson and Hanson*, 1974] although some are numerically superior to others. In particular, the normal form of the least squares equations may result in numerical instabilities that can cause problems in the response analysis [*Munk and Cartwright*, 1966].

The second modification of the standard response method of tidal analysis involves the "optimum wiggliness" criterion of *Zetler and Munk* [1975] for selection of N, the number of lag times. The criterion rests upon the variance ratios defined for each tidal band as

$$r(N) = \sigma^2(\varepsilon(N)) / \sigma^2(v), \qquad (6)$$

where $\sigma^2(\varepsilon)$ and $\sigma^2(v)$ are, respectively, the variance in the residual ε of the fit and the variance in the data v. Since ε is a fuction of the number of lags or weights N employed to fit the data, r also depends upon N.

Two mutually exclusive pieces of data of equal length, labeled A and B, are selected from the record to be analyzed. For a specified number of lags N, the weights calculated from record A are employed to fit record A and to predict record B. The residuals of the fit and of the prediction yield the variance ratio for each, namely, $r^{(AA)}(N)$ and $r^{(AB)}(N)$, respectively. This procedure is repeated using the weights from data record B to obtain the ratio $r^{(BB)}(N)$ for the fit of B and $r^{(BA)}(N)$ for the predition of A. These variance rations may be computed separately for the diurnal and semidiurnal tidal bands by summing the squares of the Fourier coefficients associated with frequencies within each separate band. The ratios obtained are $r_D{}^{(AA)}(N_D)$, $r_D{}^{(AB)}(N_D)$, $r_D{}^{(BB)}(N_D)$, and $r_D{}^{(BA)}(N_D)$ for the diurnal band and $r_S{}^{(AA)}(N_S)$, $r_S{}^{(AB)}(N_S)$, $r_S{}^{(BB)}(N_S)$, and $r_S{}^{(BA)}(N_S)$ for the semidiurnal band.

The selective method must be slightly modified from the procedure just outlined. Because some data may be excluded by the data selection series, the selective response method cannot use Fourier analysis to calculate variance ratios independently for the diurnal and semidiurnal tidal bands. Rather, the variance ratios must be computed over the entire frequency spectrum from the fundamental definition

$$r_E(N) = \langle(\varepsilon - \langle\varepsilon\rangle)^2\rangle / \langle(v - \langle v\rangle)^2\rangle, \qquad (7)$$

where a bracketed time series is simply the average over the K' acceptable data points of the series.

For both the standard and the selective methods of tidal analysis, it is expected that as N increases, $r^{(AA)}$ and $r^{(BB)}$ will decrease, the fits improving with an increasing number of degrees of freedom. Increasing the number of weights utilized allows more aperiodic noise to be included in the fit. On the other hand, ratios $r^{(AB)}$ and $r^{(BA)}$ will not necessarily decrease with increasing N because the noise of one record (which determined the weights) is not predictable in the other record. According to the optimum-wiggliness criterion, the number of diurnal and semidiurnal weights (N_D and N_S, respectively) which should be chosen are those that minimize the variance ratios of those predictions $r_D^{(AB)}$ or $r_D^{(BA)}$ and $r_S^{(AB)}$ or $r_S^{(BA)}$ respectively. Similarly, the value of N that should be chosen for the selective method is that value which minimizes $r_E^{(AB)}$ or $r_E^{(BA)}$.

The essence of the modifications to the standard-response method of tidal analysis may be summarized as follows. (1) Instead of utilizing all data points for a least squares fit, only those selected by S as acceptable contribute to the calculation of the response weights. Consequently, contamination of the fit (including the mean) by events and missing data can be avoided. (2) Because the calculation of the variance ratio selects only certain data, the choice of the number of lags or response weights for each frequency band must reflect tidal behavior across the entire frequency spectrum rather than being determined separately for each of the tidal bands.

EXAMPLES

Attention is now turned to a comparison of the standard and selective response methods of tidal analysis when each is applied to the data set that initially prompted the modifications outlined in section 2. These Doppler current records comprise three-dimensional velocity measurements with a 10-m vertical averaging interval over a depth range of 40 m to 240 m and with a 5-min time averaging interval. The observations were made in the Tarifa Narrows of the Strait of Gibraltar from March 17 through April 23, 1986.

All of the analyses presented in this section use a set of reference series derived from a long, coastal tidal-elevation station (Tarifa, Spain) record and a 6-month time series of bottom pressure. In each instance, three time lags of two summed tidal bands ($N = 3$ and $M = 2$) provide 13 ($2MN+1$) degrees of freedom for the least squares weights (including the mean).

The first example is the attempt of the standard and the selective response methods to extract the tide and mean from a very problematic time series of currents containing both data gaps and large events associated with an internal bore. Figure 1 displays a brief portion of the Doppler data for the eastward component of velocity (U) at a depth of 40 m. This record is shown in Figure 1*d*.

In this particular data record, it is difficult visually to distinguish the internal bore from the tidal oscillation. However, the signature of the bore in other data is well defined [*Pettigrew and Hyde*, 1990]. For example, at depths near the pycnocline, its velocity front is sufficiently sharp, its arrival is sufficiently phase delayed, and its duration is sufficiently brief that it is easily differentiated from the tide. Moreover, the displacement of the pycnocline by the passage of the bore aids in delineating the surge of the bore from the tidal currents. Such corroborating evidence indicates that the sudden rise in velocity from roughly 80 cm s^{-1} to over 150 cm s^{-1} during each tidal cycle is associated with the velocity front of the internal bore. At 40-m depth in the Tarifa Narrows, the arrival of the front occurs slightly after the maximum (east-ward) flow of the underlying tidal oscillation. During the pas-sage of the bore, the velocity remains elevated for almost 6 hours before falling rapidly to values of roughly 70 cm s^{-1}. Following the bore, there is usually a period of missing data. These gaps are associated with vertical excursions of the instrument (mounted on the upper flotation sphere of a subsur-face mooring) to depths from which sampling at 40 m was unattainable.

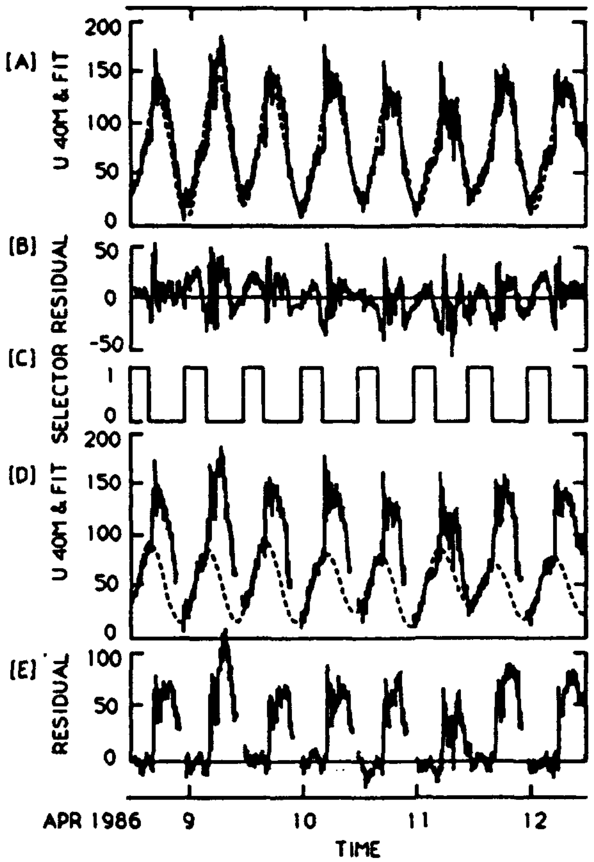

Fig. 1. Comparison of the standard and selective response methods of tidal analysis as applied to Tarifa Doppler data (eastward velocity, 40-m depth) with events and gaps. Velocity units are cm s^{-1}. (a) Velocity time series with the gaps filled by interpolation (solid line) and the mean-plus-tide fit by the standard method (dotted); (b) residual by the standard method; (c) data selection series for the selective method; (d) velocity time series (solid line) and the mean-plus-tide fit by the selective method (dotted line); (e) residual by the selective method.

The data are first fitted using the standard response method. The relatively short data gaps are filled unobtrusively using a linear interpolation. A mean and weights are calculated using the entire data record including the data associated with the internal bore. Figure 1*a* displays the fit of the mean plus tide, and Figure 1*b* shows the residual of the fit. While the fit appears to be good, it includes not just the tidal oscillation but also the motion identified with the surge of the internal bore.

The selective response method of tidal analysis provides a very different fit. If the analyst considers the data associated with both the velocity surges (passages of the bore) and the data gaps (dives of the instrument) to be unacceptable, then the selection series S is constructed as in Figure 1*c*. The fit generated by the selective response method (which uses only data points corresponding to the correct level of the instrument and to the absence of the bore) is displayed in Figure 1*d*. Figure 1*e* is the residual of this analysis.

Application of the two methods has, not too surprisingly, led to drastically different interpretations. Compare Figure 1*a* to Figure 1*d* and Figure 1*b* to Figure 1*e*. The result of the standard response method suggests a tidal amplitude of roughly 75 cm s^{-1} superimposed upon a mean flow of roughly 85 cm s^{-1} with a residual dominated by tidal harmonics having a mean of zero and an amplitude of roughly 25 cm s^{-1}. In contrast, the selective response method yields a smaller tidal oscillation (roughly 30 cm s^{-1}), a weaker steady flow (roughly 50 cm s^{-1}), and an enormous positive-definite residual (roughly 35 cm s^{-1} average contribution to the mean flow). Moreover, the fits by the two methods show a phase difference of nearly 30° with the fit of the standard response method shifted toward the advent of the bore.

Supporting evidence (for example, analyses at depths where the bore is unmistakably defined) clearly indicates that in this particular case the picture presented by the selective-response method is more indicative of the local dynamics than that presented by the standard method [see *Pettigrew and Hyde*, 1990]. The standard method fails to portray the positive definite nature of the events, erroneous tidal harmonics appear as an artifact of the analysis, and the surges of the internal bore contaminate the value of the underlying steady flow. By contrast, the selective-response method avoids these pitfalls.

The second example tests the ability of the selective response method of tidal analysis to meet the optimum-wiggliness criterion for selecting the appropriate number of weights. For the test, two mutually exclusive series of 348-hour duration each (approximately a single spring-neap tidal cycle) are selected from data of the eastward component of velocity at a depth of 240 m in Tarifa Narrows. The series are denoted A and B, respectively. At 240 m, the influence of the bore is relatively minor and there are no gaps in the data. The fits and residuals by the two methods consequently have relatively slight differences. Thus this example provides a reasonable comparison of the optimum number of weights selected by each method.

As in the previous example, the selective response method calculates a mean-plus-tide fit, rejecting data when the bore is known to be present; the data selection series therefore resembles the curve in Figure 1*c*. The standard method uses all data.

Recall that the general criterion for optimization is based upon the ratio of the variance in the residual to the variance in the data signal as defined in (6). The performance of the selective response method is perhaps most fairly assessed by comparing the variance ratio calculated by the selective method $r_E(N)$ (which encompasses the entire frequency spectrum) with the variance ratio obtained from the standard method $r_{DS}(N)$ when the Fourier components of both the diurnal and the semidiurnal bands are summed together.

For the present example, the length of the data records results in the use of 26 (complex) Fourier components for the combined diurnal (M1 ± 4.5 cpm or 0.9664 ± 0.1647 cpd) and semidiurnal (M2 ± 4.5 cpm or 1.9323 ± 0.1647 cpd) bands. The energy of the semidiurnal tide completely dominates that of the diurnal tide so that $r_{DS}(N) \equiv r_S(N)$.

Figure 2 shows that the two methods have produced equivalent results, the dependence of r_E on N being roughly that of r_{DS} on N. Specifically, the self-fit variance ratios (namely, $r^{(AA)}$ and $r^{(BB)}$) decrease with increasing N. This improvement of the fit is of course to be expected with an increasing number of degrees of freedom. However, the predicted-fit variance ratios (namely, $r^{(AB)}$ and $r^{(BA)}$) show an eventual deterioration of the fit with an increasing number of response weights. So, following the prescription of *Zetler and Munk* [1975] for this example, both methods suggest that the optimum number of weights is two for the AB ratios (that is, the prediction of cycle B from the weights of cycle A) and four for the BA ratios. A compromise of three lag times ($N = 3$) would seem reasonable for this case.

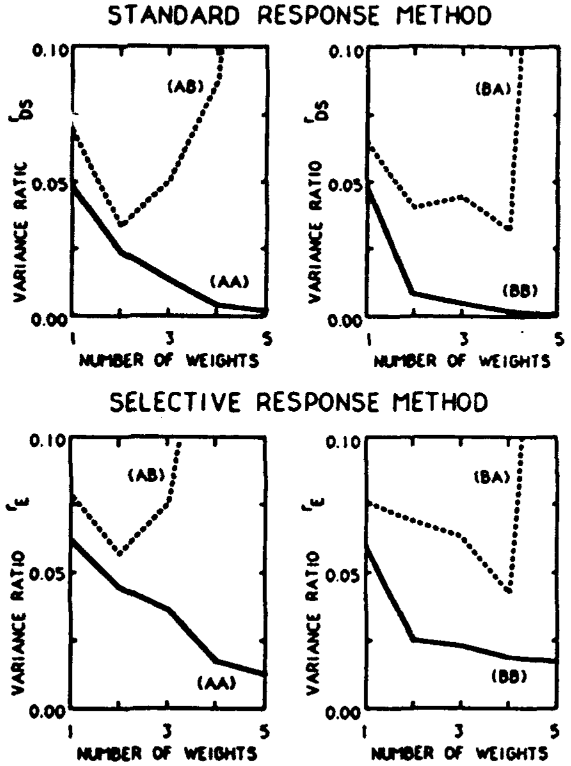

Fig. 2. Comparison of the optimum wiggliness criterion as determined by the standard and selective response methods applied to two consecutive spring-neap cycles (A and B). Ratios are of the variance in the residual to the variance in the signal (6) as a function of the number of weights. Ratios for the summed diurnal and semidiurnal frequency bands, which were obtained from cycle A weights, are $r_{DS}^{(AA)}$ (fit of A) and $r_{DS}^{(AB)}$ (prediction of B); those from cycle B weights are $r_{DS}^{(BB)}$ (fit of B) and $r_{DS}^{(BA)}$ (prediction of A). Ratios for the entire frequency spectrum, which were obtained from cycle A weights, are $r_E^{(AA)}$ (fit of A) and $r_E^{(AB)}$ (prediction of A); those from cycle B weights are $r_E^{(BB)}$ (fit of B) and $r_E^{(BA)}$ (prediction of A).

It can be concluded that use of the ratios $r_E^{(AB)}(N)$ and $r_E^{(BA)}(N)$ of the selective response method would produce results reasonably equivalent to those of the standard response method when the variance outside the diurnal and semidiurnal frequency bands is small, i.e., when $r_E(N)$ and $r_{DS}(N)$ are quite similar. Because the selective method must utilize $r_E(N)$ for this analysis, it is unable to determine a different optimal number of weights for the various separate tidal bands.

DISCUSSION

The selective-response method of tidal analysis employs a slight modification of the standard method to improve considerably the extraction of the steady and tidal components from problematic velocity time series. A data selection series (S in definition (5)) simply zero weights missing or anomalous data in the least squares determination of the response weights. Consequently, the tidal analysis remains uncontaminated by such features.

The selective response method shows promise in dealing with data gaps. Because the fit is not forced to conform to an artificial interpolation of the missing data, the globally damaging effects of interpolation are never introduced. Rather, by ignoring gaps, the method permits the response weights and the reference series to extend the fit naturally across them. Numerous test cases (not presented in this article) indicate that the selective method is capable of handling missing data not only for long periods (many tidal cycles) but also for frequent losses (more than half of the record). Quite possibly, the method proposed in this article will find its widest and most useful application to be that of bridging data gaps.

The selective response method can also be useful in isolating anomalies or events in a current record that may be phase locked to the tides. As the example of the internal bore in the Strait of Gibraltar demonstrated, the method also improves the determination of the steady and tidal flow components. Specifically, the inclusion of data associated with non-zero-averaged, phase-locked surges (such as those of an internal bore) may force the standard method to estimate poorly the steady flow and tidal amplitude, shift the phase of the tide, and leave unresolved the structure of surges. This should be of serious concern in standard tidal analyses of the Strait of Gibraltar and similar environments where the internal bore may dominate velocity time series.

Finally, it must be noted that the selective response method has its limitations and its successful usage cannot be guaranteed. The following caveats are offered. (1) The placement of the unacceptable data is a critical factor. For example, if the data gaps or anomalous events occur such that the data selection series zero weights data at the maxima and/or the minima of most of the tidal cycles, then the method will not be able to discern the correct amplitude for the fit. Test cases of this sort (not presented here) rarely proved reliable. (2) Objectives of the analysis must be clear, and definitions of "mean" and "tide" must be precise. For instance, in the analysis presented for the Strait of Gibraltar, the expressed goal is the recovery (as a residual) of the internal bore. The "mean" and the "tide" therefore have to be appropriately defined as the background upon which the surges of the internal bore are superimposed. (3) Anomalous baroclinic events may be clearly discernible at some levels of the water column and masked at others. Isolated observations within the water column may or may not reveal these features. Thus the ability to construct an effective selection time series may depend upon good fortune.

Acknowledgments. Much of the work presented in this article was completed while the authors were affiliated with the Ocean Process Analysis Laboratory (OPAL) of the Institute for the Study of Earth, Oceans and Space at the University of New Hampshire. Computations for the standard response method of tidal analysis were performed using OPAL's ocean analysis software package. The authors are particularly indebted to James Irish of the Woods Hole Oceanographic Institution for his counsel concerning theoretical and practical aspects of tidal analysis and to Bernard Zetler of the Scripps Institution of Oceanography for his guidance pertaining to the optimum-wiggliness criterion. The critiques of the anonymous referees greatly focused the article. Major support (N.R.P. and R.A.H.) was provided by contract N00014-83-K-0618 and grant N00014-89-J-1907 from the Coastal Sciences Division of the Office of Naval Research. Additional funding (R.A.H.) came from grant RII-8610679 of the Vermont-EPSCOR Program of the National Science Foundation.

REFERENCES

Cartwright, D., W.H. Munk, and B.D. Zetler, Pelagic tidal measurements--A suggested procedure for analysis, *Eos Trans. AGU, 50*, 472-477, 1969.

Darwin, G.H., Report of a committee for the harmonic analysis of tidal observations, *Br. Assoc. Rep., 1883*, 48-118, 1883.

Kinder, T.H., and H.L. Bryden, The 1985-1986 Gibraltar Experiment: Data collection and preliminary results, *Eos Trans. AGU 68*, 786-787, 793-795, 1987.

Lawson, C.L., and R.J. Hanson, *Solving Least Squares Problems*, Prentice-Hall, Englewood Cliffs, N.J., 1974.

May, P.W., Analysis and interpretation of tidal currents in the coastal boundary layer, Sc.D. dissertation, 197 pp., Mass. Inst. of Technol./Woods Hole Oceanogr. Inst., Woods Hole, Mass., 1979.

Munk, W.H., and D. Cartwright, Tidal spectroscopy and prediction, *Proc. R. Soc. London Ser. A, 259*, 533-581, 1966.

Pettigrew, N.R., and R.A. Hyde, The structure of the internal bore in the Strait of Gibraltar and its influence on the Atlantic inflow, *NATO ASI Ser. C, 318*, 493-507, 1990.

Rattray, M., On the coastal generation of internal tides, *Tellus, 12*, 54-62, 1960.

Wunsch, C., Internal tides in the ocean, *Rev. Geophys., 13*(1), 167-182, 1975.

Zetler, B.D., The evolution of modern tide analysis and prediction--some personal memories, *Int. Hydrogr. Rev., 64*(1), 123-139, 1987.

Zetler, B.D., and R.A. Cummings, A harmonic method for predicting shallow-water tides, *J. Mar. Res., 25*(1), 103-114, 1967.

Zetler, B.D., and W.H. Munk, The optimum wiggliness of tidal admittances, *J. Mar. Res., 33*, suppl., 1-13, 1975.

Zetler, B.D., D. Cartwright, and S. Berkman, Some comparisons of response and harmonic tide predictions, *Int. Hydrogr. Rev., 56*(2), 105-115, 1979.

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(Received September 20, 1991;
revised October, 19, 1992
accepted January 12, 1993.)