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SCHEDULING PEACETIME ROTATION OF PAKISTAN ARMY UNITS

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Scheduling Peacetime Rotation of Pakistan Army Units

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Abstract

Since Pakistan has varying climates and terrains, the Pakistan Army rotates its units between peacetime locations so that no unit endures inequitable hardship or enjoys unfair advantage. Army policy specifies strict constraints on unit rotations, such as the length of a unit's stay in any location, the number of units moving at any time, and the allowable replacements for any moving unit. Scheduling rotations manually in accordance with these rules, as is currently practiced, is extremely difficult and time-consuming. This paper presents an integer programming model that finds feasible, minimum-cost schedules for the Pakistan Army's desired planning horizons. The model also ensures that the units are positioned at the end of the planning horizon so that feasible schedules exist for future planners. The model is implemented with commercially available optimization software. Schedules are obtained for realistic test problems in less than an hour on a personal computer.

The Pakistan Army peacetime rotation problem, as described in the abstract, was chosen by Colonel Baig as a research topic for a masters thesis, Baig [1992], directed by his co-authors. He received guidance on problem definition from Brigadier General Haroon Bashir Khan, who was then serving as the Army Attaché at the Pakistan Embassy in Washington. The model presented in this paper captures all aspects of the peacetime rotation problem as described by the general.

The peacetime rotation problem is a unique scheduling/timetabling problem. It shares the diversity and large number of constraints typically found in the construction of sport schedules. (*E.g.*, see Andrue and Corominas [1989] for the 1992 Summer Olympic Games, Shell [1985] for the National Basketball Association, Cain [1977] and Shell [1985] for Major League Baseball, and Ferland and Fleurent [1991] for the National Hockey League). However, the peacetime rotation problem has relatively few events compared to the hundreds or thousands of games that make up a typical sports schedule. An exact procedure is therefore undertaken in this paper instead of the heuristic procedures used successfully for large scale timetabling problems Carter [1986]. The following sections present: 1) policies defining the problem, 2) the integer programming formulation, 3) model refinements for increased tractability, 4) computational experience, and 5) conclusions.

1 Pakistan Army Peacetime Rotation Policies

The Pakistan Army classifies military locations into Peace Areas (PAs), Semi-Hard Areas (SHAs) and Hard Areas (HAs). This classification accounts for Pakistan's diverse terrain, ranging from desert to lofty mountains, and temperatures, ranging from below freezing to above 40 degrees Celsius. The geographic classifications also account for available facilities at the locations and the proximity to major metropolitan areas. To ensure that personnel serve equally in all three areas, unit personnel are rotated between locations during peacetime on a regular basis.

Military units in the Pakistan Army are classified in three ways: according to their *operational* role (strike or defensive), according to their *functional* role (Armor, Artillery, Engineers, Infantry, Signals, Supply, etc), and according to whether their equipment is supplied by Eastern or Western bloc nations. (See Cohen [1984] for the roles of the East and West in equipping the Pakistan Army.) Separating the units into distinct categories based on these three attributes simplifies subsequent mathematical analysis and is useful for elucidating rotation policy.

The General Headquarters of the Pakistan Army has established rotation policies, whose salient features are as follows.

1. A unit can move only if replaced by a unit of the same category, *i.e.*, having the same operational and functional roles, and operating the same type of equipment. This results in a natural division of the overall problem into separate rotation problems for each category. The largest category, currently, has 87 units.

2. If a unit at location A moves to location B, then a unit of the same category at location B must move to location A in the same year. This policy, referred to as *mutual replacement*, simplifies transfer of operational and administrative responsibilities at both locations.

3. Equipment does not move, only personnel do.

4. No more than one unit can move from the same brigade in the same year. A brigade is composed of three units of the same category.

5. Some units (such as Engineers, Signals, Reconnaissance and Support Battalions) do not belong to a brigade. They fall under direct control of a Division. There can be no more than one of these units moving from the same location in the same year.

6. Each unit's tenure requirement varies by location classification as follows:

   - Peace Area (PA) 5-7 years,
   - Semi-Hard Area (SHA) 2-4 years,
   - Hard Area (HA) 1-3 years.

7. An individual unit must rotate according to the cycle of locations: PA → SHA → PA → HA → PA, as shown in Figure 1.

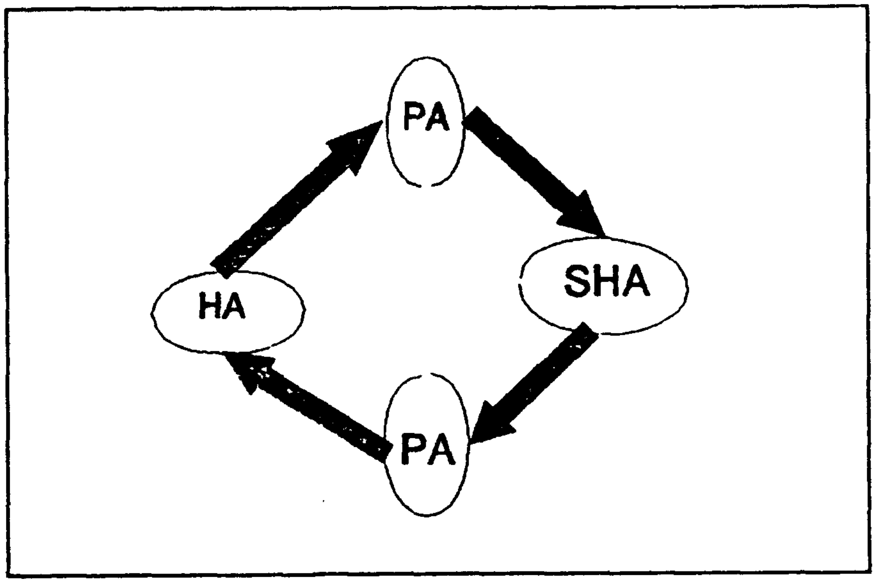

Figure 1: *Each unit must rotate through Peace Areas, Semi-Hard Areas and Hard Areas in the indicated order. Alternate Peace Areas in the cycle must be different. The length of stay in each location must fall within a prescribed range.*

8. There is no restriction placed on which SHA or HA locations a unit visits on its cycle. However, a unit must not return to its previous PA location.

Currently, peacetime rotation schedules are developed manually by planners at the General Headquarters of the Pakistan Army. A five-year schedule is developed on a yearly basis. Due to the large problem size and complex policy structure, the current method suffers from the following drawbacks:

- The units may be positioned at the end of the planning horizon in a way that precludes future schedules from satisfying the policy constraints.
- The schedule requires hundreds of man-hours to develop.
- It is difficult to evaluate proposed policy changes.
- The schedule may not be developed impartially.

- The schedule may incur excess cost by transferring units more often or over greater distances than necessary.

These limitations motivated the development of an integer programming model to assist with rotation scheduling.

2 Peacetime Rotation Model

Our integer programming model for the Pakistan Army's *peacetime rotation* problem minimizes the total cost associated with all scheduled moves, while ensuring that all policy constraints are satisfied. The model is valid for time horizons up to 15 years, which satisfies the Pakistan Army's planning requirement for five-year rotation schedules.

- Indices:

i, i'	units,
l, l'	locations,
t, t'	years.

- Given Data:

$initl_i$	the initial location for unit i,
min_l	minimum stay allowed at location l,
max_l	maximum stay allowed at location l,
$mcost_{ill'}$	movement cost for unit i from l to l',
$stay_{il}$	number of years unit i has been at location l at the start of the first time period,
$\mathcal{P}$	set of peace area locations,
$\mathcal{H}$	set of hard area locations,

S set of semi-hard area locations.

- Derived Sets: The following sets are used to enforce rotation policies. Their derivation is discussed in section 3.

$U_{ll't}$ set of all units eligible to move from location l to l' in year t,

F_{ilt} set of all possible locations from which unit i could have moved if it arrives at location l in year t,

T_{ilt} set of all possible locations to which unit i can move if situated at location l in year t.

- Decision Variables:

$x_{ill't}$ 1 if unit i moves from location l to location l' in year t, and 0 otherwise.

- Formulation:

$$\text{minimize} \sum_i \sum_l \sum_{l'} \sum_t mcost_{ill'} x_{ill't}$$

$$\sum_{t=min_l - stay_{il}}^{max_l - stay_{il}} \sum_{l' \in T_{ilt}} x_{ill't} = 1 \quad \forall il \ \ s.t. \ \ l = initl_i \tag{1}$$

$$\sum_{i \in U_{l'lt}} x_{il'lt} = \sum_{i \in U_{ll't}} x_{ill't} \quad \forall ll't \tag{2}$$

$$\sum_{l'} \sum_{i \in U_{ll't}} x_{ill't} \le 1 \quad \forall lt \tag{3}$$

$$\sum_{t'=1}^{t} \sum_{l' \in F_{ilt'}} x_{il'lt'} \le \sum_{t'=1+min_l}^{t+max_l} \sum_{l' \in T_{ilt'}} x_{ill't'} \quad \forall ilt \tag{4}$$

$$\sum_{l' \in T_{ilt}} x_{ill't} \le \sum_{t'=t-max_l}^{t-min_l} \sum_{l' \in F_{ilt'}} x_{il'lt'} \quad \forall ilt \tag{5}$$

$$\sum_{l\in\mathcal{P}}\sum_{l'\in\mathcal{H}}\sum_{t} x_{ill't} \leq 1 \quad \forall i \tag{6}$$

$$\sum_{l\in\mathcal{P}}\sum_{l'\in\mathcal{S}}\sum_{t} x_{ill't} \leq 1 \quad \forall i \tag{7}$$

$$\sum_{l\in\mathcal{H}}\sum_{l'\in\mathcal{P}}\sum_{t} x_{ill't} \leq 1 \quad \forall i \tag{8}$$

$$\sum_{l\in\mathcal{S}}\sum_{l'\in\mathcal{P}}\sum_{t} x_{ill't} \leq 1 \quad \forall i \tag{9}$$

$$\sum_{l'\in\mathcal{H}}\sum_{t} x_{il'lt} + \sum_{l'\in\mathcal{S}}\sum_{t} x_{il'lt} \leq 1 \quad \forall i, l \in \mathcal{P} \tag{10}$$

$$\sum_{i}\sum_{l\in\mathcal{H}}\sum_{l'\in\mathcal{P}}\sum_{t} x_{ill't} = \sum_{i}\sum_{l\in\mathcal{S}}\sum_{l'\in\mathcal{P}}\sum_{t} x_{ill't} \tag{11}$$

Constraint (1) ensures each unit's first move is completed within the minimum and maximum tenure requirements at its initial location. Constraint (2) enforces the mutual replacement policy. Constraint (3) allows no more than one unit to move from the same location in the same year. If two or more brigades have the same geographical location, this location is broken into separate brigade locations. Thus, constraint (3) also enforces the restriction that no more than one unit can move from the same brigade.

Constraints (4) and (5) control the tenure requirements for all moves taking place after the first move. Constraint (4) restricts the number of times a unit enters location l up to year t to be less than or equal to the number of times it leaves the location up to year $t + max_l$. These constraints ensure that a unit leaves a location in year t only if it arrived at that location in an appropriate earlier year. (They can alternatively be formulated with noncumulative inequalities, but the cumulative form provides better computational performance.) Constraint (5) requires a unit leaving location l in year t to have arrived there $t - max_l$ to $t - min_l$ years earlier. Constraints (4) and

(5) cannot prevent a unit from moving away from the same location more than once in the last few years. Constraints (6) - (9) eliminate this problem by allowing each unit at most one move between different areas during the planning horizon (a valid restriction for planning horizons of 15 years or less).

Constraint (10) prevents a unit from moving to the same PA more than once. If the planning horizon is less than seven years, this constraint is unnecessary.

Constraint (11) states the total number of moves from HAs to PAs should be equal to the total number of moves from SHAs to PAs. This constraint helps position units appropriately at the end of the planning horizon, as discussed further in section 3.

3 Model Refinement

Even though we can treat the Pakistan Army's rotation scheduling problem with separate models for each operational/functional area, the independent models can still be quite large. The largest operational/functional area has 87 units spread over 30 locations. Straightforward application of the preceding model for these units over an eight-year horizon would require over 600,000 binary variables. Fortunately, characteristics of the policies can be exploited to identify many impossible unit movements and eliminate the corresponding variables. The transition eligibility parameter $OK_{ill't}$ is defined for this purpose. It has value 1 if and only if unit i is eligible to move from location l to l' in year t. The idea is to make this parameter zero as often as possible without sacrificing model fidelity or optimality. The sets $U_{ll't}$, T_{ilt}, F_{ilt} defined in section 2 are readily generated from this parameter. The logic for deriving $OK_{ill't}$ is as follows.

1. Rotation policies and tenure limits severely restrict the set of allowable moves. For example, a unit that has moved from a HA and has been at a PA for three years can not possibly move for two more years and

then only to a SHA. Subsequent moves are similarly restricted. These situations are detailed below in four exhaustive cases. Let OPA_i denote the current or most recent PA location for unit i in the initial year, $Stay = \min\{stay_{il}, min_l\}$ for the unit's initial location l, and let "\" denote set exclusion.

a) If unit i is currently located at a PA and previously served in a SHA,

$$OK_{ill't} = \begin{cases} 1 \ \ if \ \ l \in \mathcal{P}, & l' \in \mathcal{H}, & 6 \leq t + Stay \leq 8, \\ 1 \ \ if \ \ l \in \mathcal{H}, & l' \in \{\mathcal{P} \setminus OPA_i\}, & 7 \leq t + Stay \leq 11, \\ 1 \ \ if \ \ l \in \{\mathcal{P} \setminus OPA_i\}, & l' \in \mathcal{S}, & 12 \leq t + Stay \leq 18, \\ 0 \ \ otherwise. \end{cases}$$

b) If unit i is currently located at a PA and previously served in a HA,

$$OK_{ill't} = \begin{cases} 1 \ \ if \ \ l \in \mathcal{P}, & l' \in \mathcal{S}, & 6 \leq t + Stay \leq 8, \\ 1 \ \ if \ \ l \in \mathcal{S}, & l' \in \{\mathcal{P} \setminus OPA_i\}, & 8 \leq t + Stay \leq 12, \\ 1 \ \ if \ \ l \in \{\mathcal{P} \setminus OPA_i\}, & l' \in \mathcal{H}, & 13 \leq t + Stay \leq 19, \\ 0 \ \ otherwise. \end{cases}$$

c) If unit i is currently located at a SHA,

$$OK_{ill't} = \begin{cases} 1 \ \ if \ \ l \in \mathcal{S}, & l' \in \{\mathcal{P} \setminus OPA_i\}, & 3 \leq t + Stay \leq 5, \\ 1 \ \ if \ \ l \in \{\mathcal{P} \setminus OPA_i\}, & l' \in \mathcal{H}, & 8 \leq t + Stay \leq 12, \\ 1 \ \ if \ \ l \in \mathcal{H}, & l' \in \mathcal{P}, & 9 \leq t + Stay \leq 15, \\ 0 \ \ otherwise. \end{cases}$$

d) If unit i is currently located at a HA.

$$OK_{ill't} = \begin{cases} 1 \; if \; l \in \mathcal{H}, & l' \in \{\mathcal{P} \setminus OPA_i\}, \quad 2 \le t + Stay \le 4, \\ 1 \; if \; l \in \{\mathcal{P} \setminus OPA_i\}, & l' \in \mathcal{S}, \quad 7 \le t + Stay \le 11, \\ 1 \; if \; l \in \mathcal{S}, & l' \in \mathcal{P}, \quad 9 \le t + Stay \le 15. \\ 0 \; otherwise. \end{cases}$$

For a detailed example, suppose unit i is currently located at a PA and its old location is a HA (case b). This unit can move to a SHA after completing 5 to 7 years of stay at the PA, so it is eligible to move only when $t + Stay$ is 6, 7 or 8 years. The unit's next move, to any PA except OPA_i must take place 2 to 4 years later, *i.e.*, in one of the years 8 through 12.

2. The mutual rotation policy also helps eliminate many variables. Consider a unit at location l that is eligible to move to l' in year t after executing step 1. That move can be scheduled only if another unit is eligible to move from location l' to location l in the same year. Therefore, any $OK_{ill't}$ that was 1 after step 1 is changed to 0 unless:

$$\sum_{i' \neq i} OK_{i'l'lt} \ge 1.$$

3. We can extend the idea of step 2 to subsequent moves. For unit i to be eligible for a move to location l' in year t, there must be another unit i' eligible to replace unit i at location l' between $t + min_{l'}$ and $t + max_{l'}$. Therefore, any $OK_{ill't}$ that remains 1 after the first two steps is changed to 0 unless:

$$\sum_{i' \neq i} \sum_{l} \sum_{t'=t+min_{l'}}^{t+max_{l'}} OK_{i'l'lt'} \ge 1.$$

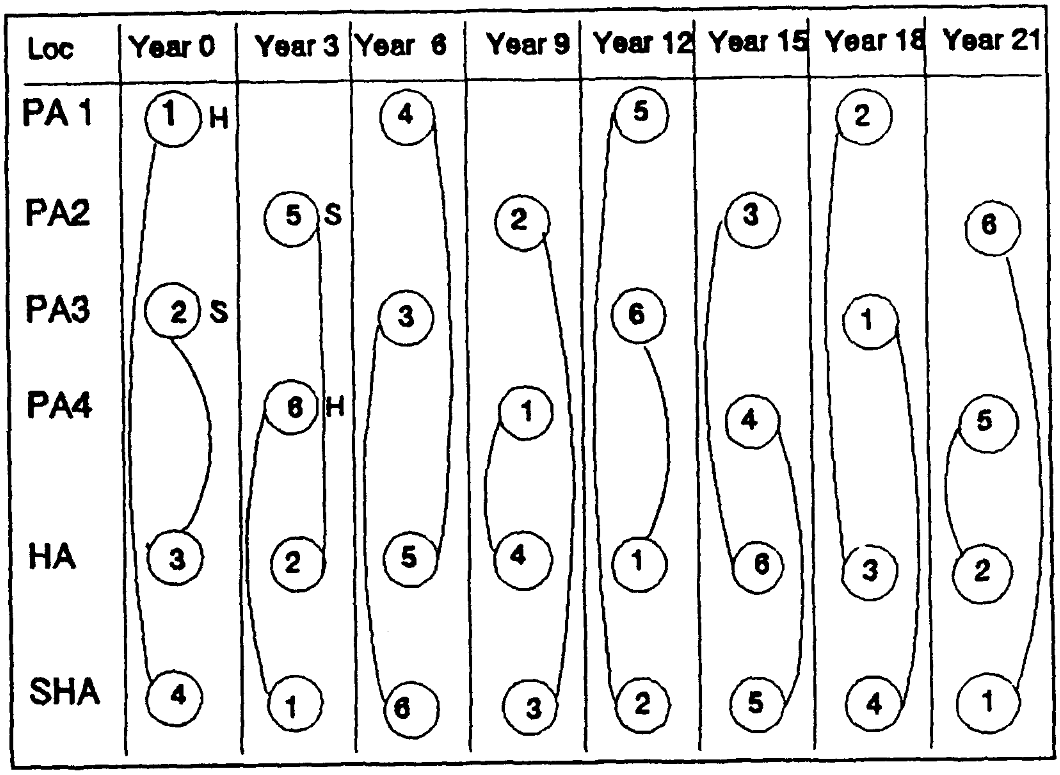

Figure 2: *A feasible six-unit rotation pattern that can be extended indefinitely. Nodes in the figure represent units undergoing moves in the indicated year. Edges represent mutual replacement.*

3.1 Conditions for Future Feasibility

The Pakistan Army needs to ensure that units are positioned at the end of the planning horizon so that feasible schedules exist for future planners. Sufficient conditions are developed for this purpose. These conditions are explained with the help of Figure 2.

Figure 2 shows an indefinitely repeatable rotation schedule that satisfies all restrictions of the rotation policy outlined in section 1. This figure contains 4 PAs, 1 HA, and 1 SHA locations where each location has one unit. The circles contain unit identifiers and the arcs between the circles indicate units exchanging locations. For the units initially located at PAs (1, 2, 5, and 6), their last area is shown with H (for HA) or S (for SHA).

It is possible to add units in multiples of 6 (4 PA units, 1 HA unit, 1 SHA unit) up to the maximum of three at each location. Each 6 additional units (1 at each existing location) could be feasibly added to the above schedule by allowing moves in similar 3 year increments starting at year 1 for the first 6 units and at year 2 for the second 6. New locations with units following the same pattern could also be added.

The positioning of units in Figure 2 obeys the following conditions at all times.

Condition 1 The total number of units in PAs is twice the number of units in HAs and SHAs.

Condition 2 The number of units in HAs and SHAs is equal.

Condition 3 Half the units in PAs previously served in SHAs and half previously served in HAs.

The three conditions above are not necessary to guarantee the existence of a feasible solution in the future. However, as proven in the appendix, if the conditions are satisfied for the six years preceding the current horizon and are enforced throughout the horizon, then future feasibility is guaranteed. Unfortunately, some operational/functional areas have not always conformed to the conditions. Therefore, the test problems of section 4 are solved with constraint (11) instead of the more restrictive form of the constraint

$$\sum_i \sum_{l \in \mathcal{H}} \sum_{l' \in \mathcal{P}} x_{ill't} = \sum_i \sum_{l \in \mathcal{S}} \sum_{l' \in \mathcal{P}} x_{ill't} \quad \forall t \tag{12}$$

which would guarantee feasible rotations in the future with appropriate starting conditions. Though not guaranteed, constraint (11) empirically yielded rotation schedules with ending conditions that allowed future feasible rotations.

4 Computational Experience

The Pakistan Army peacetime rotation model is implemented in the General Algebraic Modeling System, GAMS, [1992] and solved using XA [1987] and OSL [1991]. All computational results are obtained using a 486/33 personal computer with 16 megabytes of RAM. The goal of our work was to develop an implementable scheduling model. We believe our choice of commercially available software and a personal computer represents the best chance of implementation for the following reasons:

1. The Pakistan Army can implement the model for a reasonable cost.
2. The software is stable, well documented, and the user can benefit from future software improvements.
3. Software maintenance cost is low.
4. The software is portable to new platforms and operating systems.
5. Algebraic modeling languages such as GAMS allow easy modification and addition of constraints.

We develop eight test problems as described in Table 1. Problem names are vague and data is hypothetical for security reasons, but problem sizes are representative of actual Pakistan Army situations. The computing time required for GAMS to generate test problems is given in Table 1. A large portion of the GAMS time was taken deriving the parameter $OK_{ll't}$.

All the test problems satisfy Condition 3 of Section 3 for the initial time period. All but INFANTRY 1 also satisfy Conditions 1 and 2. The test problems are all feasible. Experimentation with other starting conditions often resulted in infeasibility.

The test problems of Table 1 were solved for integer solutions with a 10% optimality tolerance (*i.e.*, termination occurs when the first solution guaranteed within 10% of optimal is obtained), using both the XA and OSL

Problem Type	Problem Size	GAMS Generation Time (seconds)
INFANTRY 1		
6 years	2,562 constraints	530
87 units	12,241 binary variables	
30 locations	74,571 nonzeros	
INFANTRY 2		
6 years	1,599 constraints	236
72 units	5,746 binary variables	
24 locations	35,037 nonzeros	
ARTILLERY		
6 years	1,088 constraints	154
54 units	3,140 binary variables	
21 locations	18,530 nonzeros	
7 years	1,563 constraints	200
54 units	5,263 binary variables	
21 locations	31,967 nonzeros	
8 years	2,135 constraints	258
54 units	7,828 binary variables	
21 locations	49,393 nonzeros	
ENGINEERS		
6 years	913 constraints	89
36 units	2,193 binary variables	
19 locations	14,829 nonzeros	
7 years	1,253 constraints	116
36 units	3,421 binary variables	
19 locations	23,379 nonzeros	
8 years	1,684 constraints	147
36 units	4,949 binary variables	
19 locations	35,430 nonzeros	

Table 1: *Test problem description and model generation time on a 486/33 personal computer.*

Problem Type Horizon (Year)	Solution Time seconds (solver)	Iterations
INFANTRY 1		
$t = 6$	3,575 (XA)	37,919 (XA)
	2,338 (OSL)	7,062 (OSL)
INFANTRY 2		
$t = 6$	1,568 (XA)	21,647 (XA)
	452 (OSL)	3,178 (OSL)
ARTILLERY		
$t = 6$	95 (XA)	2,519 (XA)
	173 (OSL)	2,196 (OSL)
$t = 7$	679 (XA)	14,969 (XA)
	211 (OSL)	1,998 (OSL)
$t = 8$	∞ (XA)	∞ (XA)
	455 (OSL)	3,642 (OSL)
ENGINEERS		
$t = 6$	165 (XA)	4,735 (XA)
	105 (OSL)	1,394 (OSL)
$t = 7$	2,855 (XA)	54,769 (XA)
	344 (OSL)	3,381 (OSL)
$t = 8$	8,950 (XA)	105,918 (XA)
	1,627 (OSL)	11,212 (OSL)

Table 2: *Test Problem Solution Times*

solvers. All tuning parameters were left at their default values. Table 2 demonstrates these results and highlights OSL's superior performance in all but one case. XA was unable to solve the linear programming relaxation of the 8-year Artillery problem apparently due to cycling.

For completeness of the computational testing, all problems were also solved to optimality (*i.e.*, with the optimality tolerance set to zero). The integrality gap (difference between the optimal linear and integer programming objective function values) is zero in five of the eight problems tested and less than 3% in the other cases. For the five examples with no integrality gap, solving with zero tolerance took about the same time as solving with a 10% tolerance; but the other three examples took significantly longer. Since the integrality gap was so small, the added computational time for the zero tolerance yielded no improvement in solution quality. Though this behavior cannot be guaranteed, we recommend using the 10% optimality tolerance (the GAMS default) for future instances of this problem.

5 Conclusion

Computational experience with the model demonstrates that optimal unit rotations can be developed for 6-8 year horizons on a personal computer in less than an hour, using the GAMS modeling language and the OSL solver. The Pakistan Army requires five-year rotation schedules for planning purposes, so the 6-8 year schedules we have obtained are more than adequate in scope. The computation times are considered acceptable and represent a significant improvement over the hundreds of man-hours currently used to solve the problem.

This work offers some general lessons for practical application of optimization modeling.

- Use general purpose software when possible. Compared to special-purpose algorithms, the costs of development and long-term maintenance are much lower. Also, general-purpose solvers can much more readily adapt to changes in the problem.

- Using an algebraic modeling language to generate the model facilitates rapid assessment of computational tractability and allows several solvers to be tested competitively with minimal effort. In our experience, integer programming applications are too varied for any one solver to always outperform the others. (E.g., though XA lost to OSL in this case, it has been our preferred solver in other applications.)

- When computational effort prior to optimization is devoted to the elimination of unnecessary variables, this refinement can often make large-scale instances of real-world problems tractable.

- Mathematical analysis leading to additional constraints beyond those specified by the original problem statement, such as the constraints on the ending conditions, can lead to better solutions; and, in the case of integer programming, carefully chosen extra constraints often make the model easier to solve.

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Appendix

Theorem If Conditions 1, 2 and 3 of Section 3 are satisfied for six years, they are sufficient to guarantee a future feasible rotation schedule.
PROOF:

Condition 3 stipulates that half the units in PAs must have previously served in HAs and half in SHAs. This status is maintained on a yearly basis provided that whenever a unit moves from a HA to a PA, another unit moves from a SHA to a PA. The units at HAs and SHAs can therefore be separated into pairings which satisfy one of the following three cases.

- CASE 1. A unit has been at a HA for 1 year and another unit has been at a SHA for 1 year.
- CASE 2. A unit has been at a HA for 2 years and another unit has been at a SHA for 2 years.
- CASE 3. A unit has been at a HA for 3 years and another unit has been at SHA for 3 years.

It is shown for each case that it is possible for the units at the HA and SHA to feasibly rotate in the same year with a unit currently at a PA location. Any six units satisfying Conditions 1, 2 and 3 for the last 6 years and feasibly rotating can be used. Without loss of generality, the six units are numbered according to Figure 2 (i.e., unit **3** is at HA). For clarity, the unit identifiers are bold faced. Also, as in Figure 2, year 0 is considered the first year available to change a unit's location.
CASE 1

Unit **3** has stayed 1 year at a HA. This implies unit **3** replaced unit **6**, 1 year ago and unit **6** replaced unit **1** either 2, 3 or 4 years ago.

Unit **4** has stayed 1 year at a SHA. This implies unit **4** replaced unit **5**, 1 year ago and unit **5** replaced unit **2** either 3, 4 or 5 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit **6** (**5**) replacing unit **1** (**2**) either 3 or 4 years ago.

These conditions provide the following rotation eligibilities:

Unit **4** is eligible to move in years 1, 2 or 3 and Unit **1** is eligible to replace unit **4** in years 2, 3 or 4 if unit **1** was replaced 3 years ago, or in years 1, 2 or 3 if unit **1** was replaced 4 years ago.

Unit **3** is eligible to move in years 0, 1 or 2 and Unit **2** is eligible to replace unit **3** in years 2, 3 or 4 if unit **2** was replaced 3 years ago, or in years 1, 2 or 3 if unit **2** was replaced 4 years ago.

Therefore, it is feasible for both unit **3** and unit **4** to rotate in year 2.

CASE 2

Unit **3** has stayed 2 years at a HA. This implies unit **3** replaced unit **6**, 2 years ago and unit **6** replaced unit **1** either 3, 4 or 5 years ago.

Unit **4** has stayed 2 years at a SHA. This implies unit **4** replaced unit **5**, 2 years ago and unit **5** replaced unit **2** either 4, 5 or 6 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit **6** (**5**) replacing unit **1** (**2**) either 4 or 5 years ago.

These conditions provide the following rotation eligibilities:

Unit **4** is eligible to move in years 0, 1 or 2 and Unit **1** is eligible to replace unit **4** in years 1, 2 or 3 if unit **1** was replaced 4 years ago, or in years 0, 1 or 2 if unit **1** was replaced 5 years ago.

Unit **3** is eligible to move in years -1, 0 or 1 and Unit **2** is eligible to replace unit **3** in years 1, 2 or 3 if unit **2** was replaced 4 years ago, or in years 0, 1 or 2 if unit **2** was replaced 5 years ago.

Therefore, it is feasible for both unit **3** and unit **4** to rotate in year 1.

CASE 3

Unit **3** has stayed 3 years at a HA. This implies unit **3** replaced unit **6**, 3 years ago and unit **6** replaced unit **1** either 4, 5 or 6 years ago.

Unit **4** has stayed 3 year at a SHA. This implies unit **4** replaced unit **5**, 3 years ago and unit **5** replaced unit **2** either 5, 6 or 7 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit **6** (**5**) replacing unit **1** (**2**) either 5 or 6 years ago.

These conditions provide the following rotation eligibilities:

Unit **4** is eligible to move in years -1, 0 or 1 and Unit **1** is eligible to replace unit **4** in years 0, 1 or 2 if unit **1** was replaced 5 years ago, or in years -1, 0 or 1 if unit **1** was replaced 6 years ago.

Unit **3** is eligible to move in years -2,-1 or 0 and Unit **2** is eligible to replace unit **3** in years 0, 1 or 2 if unit **2** was replaced 5 years ago, or in years -1, 0 or 1 if unit **2** was replaced 6 years ago.

Therefore, it is feasible for both unit **3** and unit **4** to rotate in year 0. □

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