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ELECTROMAGNETIC WAVE REFLECTION FROM IRREGULAR PLASMA LAYERS

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13. ABSTRACT (*Maximum 200 words*)
The general theory describing reflection of electromagnetic waves from irregular reflectors was formulated by using path integral techniques [Dashen, 1979; Flatte, 1979]. The general formulae reproduce the well known reflection coefficients for wave scattering from random rough surfaces derived by using Kirchhoff's theory or perturbation theory [Ogilvie, 1991]. The theory was used to determine the degradation of an OTH radar signal scattered from irregular Artificial Ionospheric Mirrors (AIM). The cases of density irregularities induced by fluctuations in the ambient neutral density and by fluctuations in the heater power were separately examined. Scaling laws and bounds for minimal signal loss were derived.

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SUMMARY OF RESULTS

Basic Formulae

The degradation in the performance of the Artificial Ionospheric Mirror (AIM) due to phase fluctuations induced by irregularities in the electron density of the AIM plasma were studied. The general theory was formulated on the basis of the path integral techniques [Feynmann and Hibbs, 1965; Dashen, 1979]. It was found that the presence of irregularities in the reflector reduces the signal by

$$\mathcal{L} = -4.3\left[k_o^2 < S_1^2(path) >\right] \; dB \tag{1}$$

where k_o is the free space wavenumber and S_1 is the variation in the eikonal induced by the presence of irregular structure in the reflection layer. Equation (1) when applied to specular reflection from a rough surface with random inhomogeneities having RMS height Δh, gives

$$\mathcal{L}(rough \;\; surface) = -17(k_o\Delta h)^2 \cos^2\theta \quad dB \tag{2}$$

This result is consistent with the one derived by other techniques [Beckmann and Spizzichino, 1963; Ogilvie, 1991] and reproduces the Raleigh criterion. For the AIM case and assuming a linear profile with gradient length L, the loss is given by

$$\mathcal{L}(AIM) = -4.3g \, dB \tag{3}$$

For isotropic fluctuations and reflection at an angle θ,

$$g = 2\pi(k_oL)\left(\ln\frac{1+\cos\theta}{1-\cos\theta}\right)k_o\int_o^{\infty} S_n(k)kdk \tag{4}$$

where $S_n(k)$ is the spectrum of the electron density fluctuations. For a Gaussian spectrum with correlation length ℓ_o,

$$g = \sigma_n^2 \sqrt{\frac{\pi}{2}}(k_o l_o)(k_o L) ln \, \frac{1 + \cos\theta}{1 - \cos\theta} \tag{5}$$

where σ_n the RMS relative amplitude of the fluctuations. For anisotropic fluctuations the value of ℓ_o in eq. (5) is replaced by the value of ℓ_x of the correlation length in the incidence plane in the direction perpendicular to the density gradient, i.e.

$$g_{an} = \sigma_n^2 \sqrt{\frac{\pi}{2}}(k_o l_x)(k_o L) ln \frac{1 + \cos\theta}{1 - \cos\theta} \tag{6}$$

AIM Performance

Equations (3–6) were used to compute degradation of the AIM performance to electron density fluctuations induced by irregularities in the ambient neutral density and heater control. A worst and a best case were examined. For the worst case self-absorption was neglected and the final electron density was taken as

$$n(\underline{x}) = n_o e^{\nu(\underline{x})\tau} \tag{7}$$

where $\nu(\underline{x})$ is the local ionization rate and τ the heater pulse length. For the best case it was assumed that self-absorption limits the electron density so that

$$n(\underline{x}) \sim \nu^{\alpha}(\underline{x}) \tag{8}$$

with α positive ($\alpha \approx 1 - 2$). The following results were derived.

1. Neutral density irregularities

The upper bound for the signal loss due to irregularities in the neutral density derived using eq. (7) is given

$$\mathcal{L}_N = -.1\left(\frac{\lambda}{10}\right)^2\left(\frac{L}{100m}\right)^3\left(\frac{f}{10MHz}\right)^2\left(\frac{4\times 10^3 km}{L_c}\right)$$
$$\times\ sin^2\theta\ cos^2\theta \times \left(ln\frac{1+\cos\theta}{1-\cos\theta}\right) \tag{9a}$$

In eq. (9a) $\lambda = \nu\tau(\approx 10)$, the number of growth times of the ionization, $L_c(\approx 4\times 10^3 km)$ the correlation length of the neutral density fluctuations and f the radar frequency. For the saturated case $\mathcal{L}_N$ is given again by

$$\mathcal{L}_N = -10^{-3}\alpha^2\left(\frac{L}{100m}\right)\left(\frac{f}{10MHz}\right)^2\left(\frac{4\times 10^3 km}{L_c}\right)$$
$$\times\ sin^2\theta\ cos^2\theta\ \times \left(ln\frac{1+cos\theta}{1-cos\theta}\right) \tag{9b}$$

Notice that in eq. (9b) $\alpha \approx 0(1)$ giving insignificant loss.

2. Heater induced irregularities

If the heater induces density irregularities with variance σ_n^2

$$\mathcal{L}_H = -2.1\left(\frac{\sigma_n}{.1}\right)^2\left(\frac{a_o}{10m}\right)\left(\frac{L}{100m}\right)\left(\frac{f}{10MHz}\right)^2$$
$$\times ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) \tag{10}$$

where a_o is the correlation length of the heater power density fluctuations. Equation (10) can also be cast in terms of the value σ_p of the variance of the heater power fluctuations. Using eq. (7) to connect the ionization rate to the density fluctuation that induces we find as an upper limit

$$\mathcal{L}_H = -2.1\left(\frac{\lambda}{10}\right)^2\left(\frac{\sigma_p}{10^{-2}}\right)^2\left(\frac{a_o}{10m}\right)\left(\frac{f}{10MHz}\right)^2 \times ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) \tag{11a}$$

Notice that $\sigma_p = 10^{-2}$ corresponds to —40 dB heater power density fluctuations. If we use eq. (8) which corresponds to the saturated case $\mathcal{L}_H$ will be given by

$$= -2.1\times10^{-2}\alpha^2\left(\frac{\sigma_p}{10^{-2}}\right)^2\left(\frac{a_o}{10m}\right)\left(\frac{f}{10MHz}\right)^2 \times ln\left(\frac{1+cos\theta}{1-cos\theta}\right) \tag{11b}$$

1. THEORETICAL FOUNDATIONS

1.1 General Formulation

We start with the wave equation in a medium with permittivity $\epsilon(\underline{r})$. For a monochromatic wave with frequency ω it reduces to the Helmholtz equation

$$\nabla^2\psi(\underline{r},t) + k_o^2\epsilon(\underline{r})\psi(\underline{r},t) = 0 \tag{1}$$

$$k_o^2 \equiv \frac{\omega^2}{c^2} \tag{2}$$

For short wavelength propagation, i.e. if $\psi(\underline{r})$ in the propagation direction (which we take as the x-direction) varies slowly with respect to the wavelength $1/k_o$, eq. (1) reduces to the "parabolic" wave equation

$$\frac{i}{k}\frac{\partial\hat{\psi}(\underline{r})}{\partial x} = \left[\frac{-1}{2k^2}\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right)\right] \times \hat{\psi}(\underline{r}) - \frac{1}{2}\Big(1 - \epsilon(\underline{r})\hat{\psi}(\underline{r},t)\Big) \tag{3}$$

with $\psi = \hat{\psi}(\underline{r})e^{ikx}$ and $\hat{\psi}$ a slowly varying function of x (Flatte, 1979; Dashen, 1979). If we write

$$\epsilon(\underline{r}) = \bar{\epsilon}(\underline{r}) + \delta\epsilon(\underline{r}) \tag{4}$$

i.e. an average permittivity and a fluctuating parts, eq. (3) becomes

$$\frac{i}{k_o}\frac{\partial\hat{\psi}(\underline{r})}{\partial x} = \left[\frac{-1}{2k_o^2}\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2}\right)\right]\hat{\psi}(\underline{r}) - \frac{1}{2}[1 - \bar{\epsilon}(\underline{r})]\hat{\psi}(\underline{r}) - \frac{1}{2}\delta\epsilon(\underline{r})\hat{\psi}(\underline{r}) \tag{5}$$

Equation (5) provides a clear distinction of the factors affecting the range propagation of $\hat{\psi}(\underline{r})$. The first term on the rhs describes diffraction due to the transverse curvature of $\hat{\psi}(\underline{r})$, the second term <u>refraction</u> due to the spatial variation of $\bar{\epsilon}(\underline{r})$ and the third term <u>scattering</u> due to the random variation $\delta\epsilon(\underline{r})$. If we consider propagation from a point source located at the origin of coordinates to a point $\underline{r}$ $\hat{\psi}(\underline{r})$ can be written as

$$\hat{\psi}(\underline{r}) = \frac{1}{4\pi r} u(\underline{r}) \tag{6}$$

where the factor $\frac{1}{4\pi r^2}$ accounts for energy flux conservation due to geometric spreading, while u($\underline{r}$) incorporates phase effects (including absorption) during propagation.

A formal solution of eqs. (5) and (6) can be written as a <u>path integral</u> (Feynman and Hibbs, 1965), which sums phase contributions over <u>all</u> paths [y(x), z(x)] connecting the source to the receiver. It is given by

$$\begin{aligned} u(\underline{x}) = N \int d(paths)\; exp\{ik_o S_o(path)\} \\ \times exp\{ik_o S_1(path)\} \end{aligned} \tag{7}$$

where

$$\begin{aligned} S_o(path) = \int_o^R \{\frac{1}{2}\left(\frac{\partial z(x)}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial y(x)}{\partial x}\right)^2 \\ -\frac{1}{2} - [1 - \bar{\epsilon}[x, y(x), z(x)]]\} dx \end{aligned} \tag{8}$$

$$S_1(path) = \frac{1}{2} \int_o^R \delta\epsilon[x, y(x), z(x)] dx \tag{9}$$

and N is a normalization factor adjusted to provide unity if $\delta\epsilon = 0$. We proceed next to determine the average value u($\underline{r}$). It will be given by

$$\begin{aligned} < u > = < N \int d(path)\; exp[ik_o S_o(path) \\ + ik_o S_1(path)] > \end{aligned} \tag{10}$$

We assume next that the eikonal $k_o S_1$(path) is a Gaussian variable and

(i) Interchange averages and integrations.

(ii) Use the fact that for a Gaussian variable at

$$< e^{ia} >= exp\left(-\frac{1}{2} < a^2 >\right)$$

Under these conditions eq. (10) becomes

$$\begin{aligned} < u > = N \int d(path) exp[ik_o S_o(path)] \\ \times exp\left(-\frac{1}{2}k_o^2 < S_1^2(path) >\right) \end{aligned} \tag{11}$$

where

$$< S_1^2(path) >= \frac{1}{4} < (\int_{path} [\delta\epsilon(\underline{r}(x))]^2 dx > \tag{12}$$

Equation (11) gives a physically transparent but deceptively simple looking result. The first term describes the propagation in a smooth medium, while the term

$$R = exp\left(-\frac{1}{2}k_o^2 < S_1^2(path) >\right) \tag{13}$$

decribes the modification of the propagation by the presence of fluctuations. For $R \approx 1$ we recover propagation through a smooth medium. From eq. (13) the signal reduction $\mathcal{L}$ will be

$$\mathcal{L} = -4.3\left[k_o^2 < S_1^2(path) >\right] \; dB \tag{14}$$

In practical terms the important task is the to determination of the value of $< S_1^2(path) >$ under the relevant propagation conditions. In the next section we will consider a reflection from a finite size surface. This will allow us to compare eq. (12) with standard results [Beckman and Spizzichino, 1963]. The AIM case will be examined in the subsequent sections.

1.2 Reflection by a Finite Size Surface

1.2.1 Smooth surface

We apply first the above results to the case of reflection by a smooth surface. Consider a surface with dimensions ±X and ±Y (Figure 1). The value of S_o(path) will be given [Ogilvy, 1991]

$$S_o(x_o, y_o) = Ax_o + By_o + Ch \tag{15a}$$

$$\begin{aligned} A &= sin\theta_1 - sin\theta_2\, cos\theta_3 \\ B &= -sin\theta_2\, sin\theta_3 \\ C &= -(cos\theta_1 + cos\theta_2) \end{aligned} \tag{15b}$$

and

$$< u >= N\, cos\theta_1 \int dx_o dy_o \times exp[ik_o S_o(x_o, y_o)] \times R(x_o, y_o) \tag{16}$$

For a smooth surface (i.e. R = 1) with area

$$A_M = 4XY \tag{17}$$

$$\begin{aligned} < u \overset{o}{>} &= N e^{ik_o Ch} \times cos\theta_1 \times A_M \\ &\times \frac{sink_o AX}{k_o AX} \times \frac{sink_o BY}{k_o BY} \end{aligned} \tag{18}$$

Absorbing the constant phase factor exp(1ik$_o$Ch) to the normalization N, we recover the standard formula

$$< u \overset{o}{>} = N(A_M \, cos\theta_1) \times \frac{sink_o AX}{k_o AX} \frac{sink_o BY}{k_o BY} \tag{19}$$

For specular reflection A = 0, B = 0

$$< u \underset{spec}{\overset{o}{>}} = N(A_M \, cos\theta_1) \tag{20}$$

1.2.2 Rough surface

Let us examine next specular reflection from a rough surface. From eq. (16) with A = B = 0, we find

$$< u \underset{sp}{>} = N \, cos\theta_1 \int dx_o dy_o \, R(x_o, y_o). \tag{21}$$

For a surface with random height inhomogeneities with RMS standard deviation $\triangle h$ independent of the position (x_o, y_o) we find, using eq. (13) in eq. (21),

$$< u \underset{sp}{>} = N \, cos\theta_1 \, A_M \, exp\Big(-\frac{1}{2}k_o^2 \Delta h^2 \, 4 \cos^2\theta_1\Big) \tag{22}$$

or

$$< u \underset{sp}{>} = < u \underset{sp}{\overset{o}{>}} \, exp\Big(-\frac{g_o}{2}\Big) \tag{23}$$

$$g_o \equiv 4k_o^2 \, \triangle h^2 \, cos^2\theta_1 \tag{24}$$

The loss during specular reflection will be

$$\mathcal{L}(\text{rough surface}) = -17(k_o \triangle h)^2 cos^2\theta \;\; dB \tag{25}$$

The more general formula reads

$$< u > = < u_o > \ exp\left(-\frac{g}{2}\right) \tag{26a}$$

with

$$g \equiv k_o^2 \Delta h^2 (cos\theta_1 + cos\theta_2)^2 \tag{26b}$$

For the specular case ($\theta_1 = \theta_2$), eqs. (26) reduce to eqs. (23) and (24).

1.2.3 Raleigh criterion

The Raleigh criterion is a criterion that determines the degree of roughness of a surface on the basis of a simple physical argument. Referring to Figure 2 we consider scattering in the azimuthal plane (i.e. the (x-z) plane of incidence). The difference in phase between two scattered rays is

$$\begin{aligned} \Delta\phi = k_o[(z_1 - z_2)(cos\theta_1 + cos\theta_2) + (x_2 - x_1)] \\ \times(sin\theta_1 - sin\theta_2) \end{aligned} \tag{27}$$

For specular scattering ($\theta_1 = \theta_2$) the phase difference becomes

$$\Delta\phi = 2k_o\Delta h \ cos\theta_1 \tag{28}$$

where $\Delta h = z_1 - z_2$. the interference between the two rays depends on $\Delta\phi$. For $\Delta\phi << \pi$ the waves will interfere constructively. For $\Delta\phi \sim \pi$ the waves will interfere destructively, leading to very weak contribution in the specular direction. The "Rayleigh Criterion" states that if Δh is the surface RMS deviation, the surface is smooth for $\Delta\phi < \frac{\pi}{2}$. This stated mathematically as

$$k_o\Delta h \ cos\theta_1 < \frac{\pi}{4} \tag{29}$$

From eq. (26b) we find that

$$g = 4k_o^2 \Delta h \; cos^2\theta_1 < \frac{\pi^2}{4} \tag{30}$$

which from eq. (25) corresponds to a loss of less than 10 db, a rather significant loss. A better criterion will be $k_o \Delta h \; cos\theta_1 < \frac{\pi}{8}$.

1.3. Reflection from AIM in the Presence of Fluctuations

1.3.1 Isotropic case

From eqs.(11–13) we find that the deterioration of the signal due to inhomogeneities in the AIM surface, will be given by

$$\frac{< u >}{< u >_o} = exp\left[-\frac{1}{2} k_o^2 < S_1^2(path) > \right] \tag{31}$$

where

$$< S_1^2(path) > = < \left(\int_{path} ds \; \delta\epsilon(\underline{r}(s)) \right) > \tag{32}$$

Let us first examine the case of isotropic fluctuations with scale length much shorter than the path through the cloud. In this case (Rytov et al., 1990)

$$< S_1^2(path) > = \delta\sigma_\phi^2 \tag{33}$$

The eikonal variation $\delta\sigma_\phi^2$ is given by

$$\delta\sigma_\phi^2 = \pi^2 \int_o^{\ell_o(\theta)} \frac{ds}{\bar{\epsilon}[z(s)]} \int_o^{\infty} S_\epsilon(k, z(s)) k dk \tag{34}$$

where $\ell_o(\theta)$ is the zero order path of the ray incident at an angle θ and S_ϵ(k,z) the spectrum of the fluctuations $\delta\epsilon$ at the point z. Equation (34) can be analytically evaluated when $S_\epsilon(k)$ is independent of position and the AIM layer has a linear profile with scale length L. Such a profile can be described by

$$n(z) = n_c \frac{z}{L} \tag{35}$$

where n_c is the critical density defined as

$$n_c = \frac{m\omega^2}{4\pi e^2} \tag{36}$$

In this case the zero order ray follows a parabolic trajectory given by

$$z = x\, cot\theta - \frac{1}{2}\frac{x^2}{L\, sin^2\theta} \tag{37}$$

The ray turns at a level given by

$$z_t(\theta) = L\, cos^2\theta \tag{38}$$

and exits the AIM cloud after traveling an horizontal distance $\Delta(\theta)$ given

$$\Delta(\theta) = 4L\, sin\theta\, cos\theta \tag{39}$$

Integrating eq. (34) by using the path (37) we find

$$\delta\sigma_\phi^2(\theta) = 2\pi^2 L\left(\ell n\frac{1+cos\theta}{1-cos\theta}\right)\int_o^\infty S_\epsilon(k)kdk. \tag{40}$$

For a Gaussian fluctuation spectrum

$$\psi_\epsilon(r) = \sigma_\epsilon^2 e^{-r^2/2\ell_o^2} \tag{41}$$

the integral in eq. (40) gives

$$2\pi^2\int_o^\infty S_\epsilon(k)kdk = \sigma_\epsilon^2\sqrt{\frac{\pi}{2}}\,\ell_o \tag{42}$$

so that

$$\delta\sigma_\phi^2 = \sigma_\epsilon^2 \sqrt{\frac{\pi}{2}}\, \ell_o L \left(\ell n \frac{1 + cos\theta}{1 - cos\theta} \right) \tag{43}$$

The signal deterioration will be given by

$$\mathcal{L} = -4.3\, g \;\; dB \tag{44a}$$

where

$$g = \sigma_\epsilon^2 \sqrt{\frac{\pi}{2}}\, (k_o \ell_o)(k_o L) \ell n \left(\frac{1 + cos\theta}{1 - cos\theta} \right) \tag{44b}$$

From eq. (34) it is easy to see that $\delta\sigma_\phi^2$ maximizes when $\bar{\epsilon}(z)$ becomes small, i.e. near the turning regions of the layer. It is easy to find that 50% of the contribution to $\delta\sigma_\phi^2$, comes from a fraction of the layer near the top with width ΔL given by

$$\frac{\Delta L}{z_t} = \frac{1}{2} \frac{sin\theta(1 - sin\theta)}{cos^2\theta}$$

This corresponds to about a fifth of the layer at $\theta = 45°$ and 1/6 at $\theta = 30°$. Notice that in this region the rays are almost parallel to x.

A simple but intuitive way to derive the above result is as follows. Consider phase fluctuations over a ray of length ℓ and inhomogeneities with scale ℓ_ϵ. The ray path will accommodate $N \sim \frac{\ell}{\ell_\epsilon}$ independent inhomogeneities. After a simple inhomogeneity the phase shift of the wave will be

$$\ell_\epsilon \frac{\omega}{c} \tilde{\epsilon} \approx k_o \ell_\epsilon \delta\epsilon$$

Since the random quantities corresponding to the different inhomogeneities are independent, the total mean square of the phase shift will be

$$< S_1^2 > = N < (\ell_\epsilon k_o \delta\epsilon)^2 > \approx \frac{\ell}{\ell_\epsilon} \ell_\epsilon^2 k_o^2 \sigma_\epsilon^2$$
$$\approx (k_o \ell_\epsilon)(k_o z) \sigma_\epsilon^2 \tag{45}$$

which is comparable to eq. (43).

1.3.2 Anisotropic case

For a situation with anisotropic fluctuations eq. (34) becomes

$$\delta\sigma_\phi^2 = \frac{\pi}{2} \int_o^{\ell_o(\theta)} \frac{ds}{\bar{\epsilon}[z(s)]} \int_{-\infty}^{\infty} S_\epsilon(\underline{k}_\perp; z(s)) d^2 k_\perp \tag{46}$$

where $\underline{k}_\perp$ refers to the transverse direction at every point on the path. The role of anisotropic fluctuations can be understood by considering the value of the integral

$$I = \int_{-\infty}^{\infty} S_\epsilon(\underline{k}_\perp, 0) d^2 k_\perp \tag{47}$$

for the case that the fluctuations are described by

$$\psi_\epsilon(\underline{r}) = \sigma_\epsilon^2 \, exp\left[-\frac{1}{2}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right] \tag{48}$$

In this case $\delta\epsilon(\underline{r})$ has a spectrum

$$S_\epsilon(\underline{k}) = \frac{\sigma_\epsilon^2 abc}{(2\pi)^{3/2}} exp\left[-\frac{1}{2}(a^2 k_x^2 + b^2 k_y^2 + c^2 k_z^2)\right] \tag{49}$$

Assuming propagation on the x-z plane at an angle γ to the z-axis

$$I(\gamma) = \frac{\sigma_\epsilon^2}{\sqrt{2\pi}} \frac{ac}{\sqrt{a^2 cos^2\gamma + c^2 sin^2\gamma}} \tag{50}$$

Notice that for propagation in the x-direction $\left(\gamma = \frac{\pi}{2}\right)$,

$$I\left(\frac{\pi}{2}\right) = \frac{\sigma_\epsilon^2 a}{\sqrt{2\pi}} \tag{51a}$$

and for $\gamma = 0$,

$$I(0) = \frac{\sigma_\epsilon^2 c}{\sqrt{2\pi}} \tag{51b}$$

Namely the eikonal correlation is controlled by the correlation length <u>along</u> the propagation direction. From eqs. (46–49)

$$\delta\sigma_\phi^2(\theta) = \left(\frac{\sigma_\epsilon^2}{2}\right)\sqrt{\frac{\pi}{2}}a \times \alpha$$
$$\times \int \frac{ds}{\bar{\epsilon}[z(s)]} \frac{1}{\left[sin^2\gamma(s) + \frac{a^2}{c^2}cos^2\gamma(s)\right]} \tag{52}$$

where γ(s) is the local propagation angle. For isotropic spectra a = c and we recover the result of eq. (41) with ℓ_o replaced by a. Since the integral of eq. (52) maximizes when $\bar{\epsilon}[z(s)]$ is minimum, i.e. near the turning regions, $\gamma(s) \approx \frac{\pi}{2}$ and as long as $\frac{a^2}{c^2}cos^2\gamma(s) << 1$,

$$\delta\sigma_\phi^2(\theta) = \left(\frac{\sigma_\epsilon^2}{2}\right)\left(\sqrt{\frac{\pi}{2}}\right) \int_o^{\ell_o(\theta)} \frac{ds}{\bar{\epsilon}[z(s)]} \tag{53}$$

For a linear density profile eq. (53) gives

$$\delta\sigma_\phi^2(\theta) = \sigma_\epsilon^2\left(\sqrt{\frac{\pi}{2}}\right)L\, \ell n\left(\frac{1 + cos\theta}{1 - \cos\theta}\right) \tag{54}$$

The result is similar to eq. (42) but with the isotropic scale length ℓ_o, replaced by the scale length α in the x direction. The value of g is given by

$$g(\theta) = \sigma_\epsilon^2\sqrt{\frac{\pi}{2}}(k_o a)(k_o L)\ell n\left(\frac{1 + cos\theta}{1 - cos\theta}\right) \tag{55}$$

2. SOURCES AND SPECTRA OF PERMITTIVITY FLUCTUATIONS IN AIM

For practical AIM systems there are two sources of permittivity fluctuations $\delta\epsilon(\underline{r})$, due to electron density fluctuations within the cloud. Both are associated with fluctuations in the local ionization rate. The first source is due to natural fluctuations in the ambient neutral density. The second is due to spatial variation of the incident RF power caused by scanning and heat control errors. In both cases the electron density fluctuations are induced by fluctuations in the ionization rate $\nu(\underline{x})$. In deriving the spectrum $S_n(\underline{k})$ from the spectrum of the ionization rate fluctuations $S_\nu(\underline{k})$, we use two models. An upper limit on the value of $S_n(\underline{k})$ can be found by ignoring self-absorption in the formation of the AIM cloud and assuming that

$$n(\underline{x}) = n_o e^{\nu(\underline{x})\tau} \tag{56}$$

where τ is the pulse length of heater and n_o the initial electron density. A more realistic estimate can be found by considering the saturated case (i.e. including self-absorption) in which case

$$n(\underline{x}) \sim \nu^\alpha(\underline{x}) \tag{57}$$

with $\alpha \approx 1 - 2$. The spectrum $S_n(\underline{k})$ is defined here in a normalized form, i.e.

$$S_n(\underline{k}) = \frac{1}{(2\pi)^3} \int \frac{< nn \mid \underline{x} > e^{-i\underline{k}\cdot\underline{x}}}{\overline{n}^2} \tag{58}$$

so that

$$S_n(\underline{k}) = S_\epsilon(\underline{k}). \tag{59}$$

2.1 Unsaturated Case

For the unsaturated case, we assume that the density at a point x, is related to the ionization rate by eq. (56). We, furthermore, normalize $\nu(\underline{x})$ to the pulse length τ, so that $\nu(\underline{x}) = \nu(\underline{x})\tau$. As a result

$$< n(x)n(x') >= n_o^2 < e^{\nu(x)}e^{\nu(x')} > \tag{60}$$

Writing

$$\nu(x) = \overline{\nu}(x) + \delta\nu(x) \tag{61}$$

we find

$$< n(x)n(x') >= n_o^2 e^{2\overline{\nu}(x)} < e^{\delta\nu(x)}e^{\delta\nu(x')} > \tag{62}$$

Assuming that

$$< \delta\nu(x) >= 0 \tag{63}$$

with

$$< e^{\delta\nu(x)} >= exp\left[\frac{1}{2} < \delta\nu^2(x) >\right] \tag{64}$$

we find that eq. (62) becomes

$$< n(x)n(x') >= n_o^2 e^{2\overline{\nu}(x)} e^{<\delta\nu^2>} exp[< \delta\nu(x)\delta\nu(x') >] \tag{65}$$

As a result

$$\begin{aligned} < n(x)n(x') > - < n^2(x) > &= n_o^2 e^{2\overline{\nu}(x)} exp(< \nu^2 >) \\ &\times[exp(< \delta\nu(x)\delta\nu(x') >) - 1] \end{aligned} \tag{66}$$

If we define

$$G(x-x') \equiv \frac{< n(x)n(x') > - < n^2(x) >}{< n^2(x) >} \tag{67}$$
$$= [exp(< \delta\nu(x)\delta\nu(x') >) - 1]$$

the one dimensional electron density fluctuation spectrum, will be given by

$$S_n(k) = \frac{1}{2\pi}\int dx e^{-ikx} G(x) \tag{68}$$

or

$$S_n(k) = \frac{1}{2\pi}\int dx\ e^{-ikx}\left[e^{<\delta\nu\delta\nu|x>} - 1\right] \tag{69}$$

Returning to dimensional variables and defining

$$\lambda \equiv \overline{\nu}\tau \tag{70}$$

and

$$S_\nu(x) \equiv \frac{< \delta\nu\delta\nu \mid x >}{\overline{\nu}^2} = \sigma_\nu^2 F_\nu(x) \tag{71}$$

with $F_\nu(x=0) = 1$, $F_\nu(x \to \infty) = 0$, eq. (69) becomes

$$S_n(k) = \frac{1}{2\pi}\int dx\ \ e^{-ikx}\{exp[\lambda^2\sigma_\nu^2 F_\nu(x)] - 1\} \tag{72}$$

Notice that the value of $\lambda^2\sigma_\nu^2$ is the strength parameter, since $F_\nu(x) \leq 1$. We distinguish the cases of weak, $\lambda^2\sigma_\nu^2 << 1$, and strong, $\lambda^2\sigma_\nu^2 > 1$, fluctuations.

2.1.1 Weak fluctuations

We define as weak fluctuations if $\lambda^2\sigma_\nu^2 << 1$. For example, if $\lambda \approx 10$ weak fluctuations imply $\frac{\delta\nu}{\overline{\nu}} \leq .1$. The practical applications require such as a condition. From eq. (72) we find

$$S_n(k) = \lambda^2 S_\nu(k) \tag{73}$$

Namely the spectral distribution of the density fluctuations will be similar to the ionization rate fluctuations, but their strength will increase by λ^2.

2.1.2 Strong fluctuations

In this case $\lambda^2\sigma_\nu^2 >> 1$ and

$$S_n(k) = \frac{1}{2\pi}\int dx \;\; e^{-ikx} \;\; exp(\lambda^2\sigma_\nu^2 F_\nu(x)) \tag{74}$$

In order to proceed we assume that

$$F_\nu(x) = \frac{b^2}{b^2 + x^2} \tag{75}$$

This is a Lorentzian spectrum with correlation length b. For $F_\nu(x)$ given by eq. (75) the integral in eq. (72) can be performed, using the technique of stationary phase. We find

$$S_n(k) = \sqrt{\frac{\pi}{2}\frac{1}{k}} \; exp\left(-\frac{k^2b^2}{2\sigma_\nu^2\lambda^2}\right) \tag{76}$$

The importance of this result lies in the fact that while for weak fluctuation there is essentially no spectral shift, for strong fluctuations there is a complete shift of the spectrum towards shorter wavelengths. The equivalent correlation length is smaller by $\frac{1}{\sigma_\nu\lambda}$ and can be strongly reduced by adjusting λ to large values.

2.2 Saturated Case

For the saturated case the density is connected to the ionization rate via eq. (57). In this case

$$
\begin{aligned}
&< n(x)n(x') >=< \nu^{\alpha}(x')\nu^{\alpha}(x') > \\
&=< (\overline{\nu}(x) + \delta\nu(x))^{\alpha}(\bar{\nu}(x') + \delta\nu(x'))^{\alpha} > \\
&= (\overline{\nu}(x))^{2\alpha}\left(1 + \alpha^2 \frac{< \delta\nu(x')\delta\nu(x) >}{\overline{\nu}^2}\right)
\end{aligned}
\tag{77}
$$

From eq. (77) it is straightforward to find that

$$S_n(k) = \alpha^2 S_{\nu}(k) \tag{78}$$

Namely the spectrum of the fluctuations remains the same but the amplitude is modified by the factor α^2.

3. DEGRADATION OF AIM PERFORMANCE DUE TO FLUCTUATIONS IN THE LOCAL IONIZATION RATE

3.1 Fluctuations of the Ambient Neutrals

The ionization rate is a linear function of the neutral density N. As a result, if all other factors are constant

$$S_{\nu}(k) = S_N(k) \tag{79}$$

Based on available measurements [Jursa, 1985] for the relevant altitude range of 60–80 km, $S_N(k)$ can be described by an isotropic spectrum

$$S_N(\mid \underline{k} \mid) = \frac{k_c}{k^4} \qquad k > k_c \tag{80}$$

This corresponds to an isotropic spatial autocorrelation function $< \delta N \delta N \mid \underline{r} >$ which for "small" distances has a linear decay with radius with scale length $L_c = \frac{1}{4k_c}$. The wavenumber spectrum of eq. (79) has a spectral index of p=4, i.e. decays as k^{-4}. The corresponding one dimensional wavenumber spectrum has a spectral index α = p-2 = 2. From Figure 3 and Table 1 we see that $L_c \gtrsim 4 \times 10^3 km$, giving $k_c \approx 10^{-4} km^{-1}$.

For the AIM problem the maximum size of the inhomogeneities of interest corresponds to the minimum of either the Fresnel size L_F, which is of the order of a few kms, or of $\triangle(\theta)$ defined by eq. (39). It is convenient to cast eq. (79) as

$$S_N(\underline{k}) = A\frac{k_m}{k^4} , \qquad k > k_m \tag{81}$$

where

$$A \equiv \left(\frac{k_c}{k_m}\right) \tag{82a}$$

$$k_m = max\left(\frac{1}{\triangle}, \frac{1}{L_F}\right) \tag{82b}$$

From eqs. (73), and (82)

$$S_\epsilon(k) = S_n(k) = A\lambda^2\frac{k_m}{k^4}, \quad k > k_m \tag{83}$$

For the linear profile eqs. (40) and (82) give

$$\delta\sigma_\phi^2 = 2\pi^2 L\left(ln\frac{1+cos\theta}{1-cos\theta}\right)(A\lambda^2)k_m$$

$$\times\int_{k_m}^{\infty}\frac{dk}{k^3} \tag{84}$$

$$= 2\pi^2 L\left(ln\frac{1+cos\theta}{1-cos\theta}\right)(A\lambda^2)\left(\frac{1}{2k_m}\right)$$

$$= \pi^2 L\left(ln\frac{1+cos\theta}{1-cos\theta}\right)(A\lambda^2)\frac{1}{k_m}$$

For the general AIM case $\Delta < L_F$ and $k_m = \frac{1}{\Delta}$. As a result

$$\delta\sigma_\phi^2(\theta) = \pi^2 L\lambda^2\left(ln\frac{1+cos\theta}{1-cos\theta}\right)\frac{\Delta^2(\theta)}{L_c} \tag{85}$$

From eqs. (14) and (34) the signal loss will be given by

$$\mathcal{L} = -4.3\pi\lambda^2(k_oL)(k_o\,\Delta\,(\theta))\frac{\Delta(\theta)}{L_c}$$
$$\times ln\frac{1+cos\theta}{1-cos\theta}\;dB \tag{86}$$

For a linear profile using the results of Section 1.3, eq. (86) becomes

$$\mathcal{L} = -.1\left(\frac{\lambda}{10}\right)^2\left(\frac{L}{100m}\right)^3\left(\frac{f}{10MHz}\right)^2\left(\frac{4\times 10^3 km}{L_c}\right)\times\left(ln\frac{1+cos\theta}{1-cos\theta}\right) \tag{87}$$

$$\times sin^2\theta\ cos^2\theta\ \ dB$$

Equation (87) represents an upper limit in the fluctuations, since as it can be easily verified in this case the weak fluctuation limit is always valid. For the saturated case the loss will have the same form as in (86) but with λ replaced by α. Since $\alpha \approx 0(1)$ the loss rate will be totally negligible, in this case.

3.2 Effect of Heater Induced Fluctuations

3.2.1 Loss as a function of the density fluctuation level

Fluctuations in the electron density can also arise from the finite step of the heater during painting of the AIM cloud, as well as from any other heater phase errors [Short et al., 1990]. The analysis below addresses the constraints imposed by a heater step size. The analysis relevant to this case is the anisotropic analysis discussed in Section 1.3.2. Assuming that the heater induced fluctuations have a correlation length a in the ray direction equal to the step size a_o of the ionizing beam, we find that the power loss will be given from eqs. (25) and (56) as

$$\mathcal{L} = -2.1\left[\frac{\delta n/n_o}{.1}\right]^2\left(\frac{f}{10MHz}\right)^2\left(\frac{a_o}{10m}\right)\left(\frac{L}{100m}\right)ln\left(\frac{1+cos\theta}{1-cos\theta}\right) \tag{88}$$

For a step size of 10m, f = 10 MHz and $L \approx 100m$, the round trip loss will be 4–5 db for 10% density fluctuations. To achieve the same performance with 50

MHz the fluctuation level should be reduced to 2%. As discussed by Short et al. [1990] these can be easily achieved.

3.2.2 Loss as a function of heater power fluctuations

In this case the fluctuations will be anisotropic and we should use the equations derived in Section 1.3.2 to calculate the loss. We assume that the power fluctuation spectrum is given by

$$S_p(x) = \sigma_p^2 F_p\left(\frac{x}{a}\right) \tag{89}$$

and taking $\nu \sim P$ we find

$$S_\nu(x) = \sigma_p^2 F_p\left(\frac{x}{a}\right) \tag{90}$$

For the case of weak fluctuations

$$\begin{gathered} \lambda^2\sigma_p^2 << 1 \\ S_n(x) = \lambda^2\sigma_p^2 F_p\left(\frac{x}{a}\right) \end{gathered} \tag{91}$$

For a Gaussian spectrum $F_p(\frac{x}{a})$, eq. (56) in conjunction with eq. (12b) gives

$$\begin{gathered} = -1.7\left(\frac{\lambda}{10}\right)^2\left(\frac{\sigma_p}{10^{-2}}\right)\left(\frac{a}{10m}\right)\left(\frac{L}{100m}\right)\left(\frac{f}{10MHz}\right) \\ \times ln\frac{1+cos\theta}{1+sin\theta}\ dB \end{gathered} \tag{92}$$

Notice that $\sigma_p = 10^{-2}$ corresponds to power fluctuations of 4–40 dB. According to Short et al. [1990] power fluctuations as low as —60 dB can be achieved. For the saturated case replacing λ by α we find losses by at least an order of magnitude smaller.

3.2.3 Strong fluctuations

It is interesting to examine the upper limit of loss for the case of strong fluctuations in the sense of $\lambda\sigma_p >> 1$. Such is the case if the heater power fluctuations are large (i.e. > 10 db). In this case using eq. (46) for $\delta\sigma_\phi^2$, with the spectrum given in eq. (77) we find

$$g(\theta) \approx \frac{1}{(\sigma_p\lambda)}(k_o b)(k_o L) ln\left(\frac{1+cos\theta}{1-cos\theta}\right) \tag{93}$$

Notice that the correlation length has shifted to $\frac{b}{\sigma_p\lambda}$ instead of b. The loss computed on the basis of eq. (91) is

$$\mathcal{L} = -8\left(\frac{1}{\sigma_p}\right)\left(\frac{20}{\lambda}\right)\left(\frac{b}{10m}\right)\left(\frac{L}{100}\right) ln\left(\frac{1+cos\theta}{1-cos\theta}\right) \; dB \tag{94}$$

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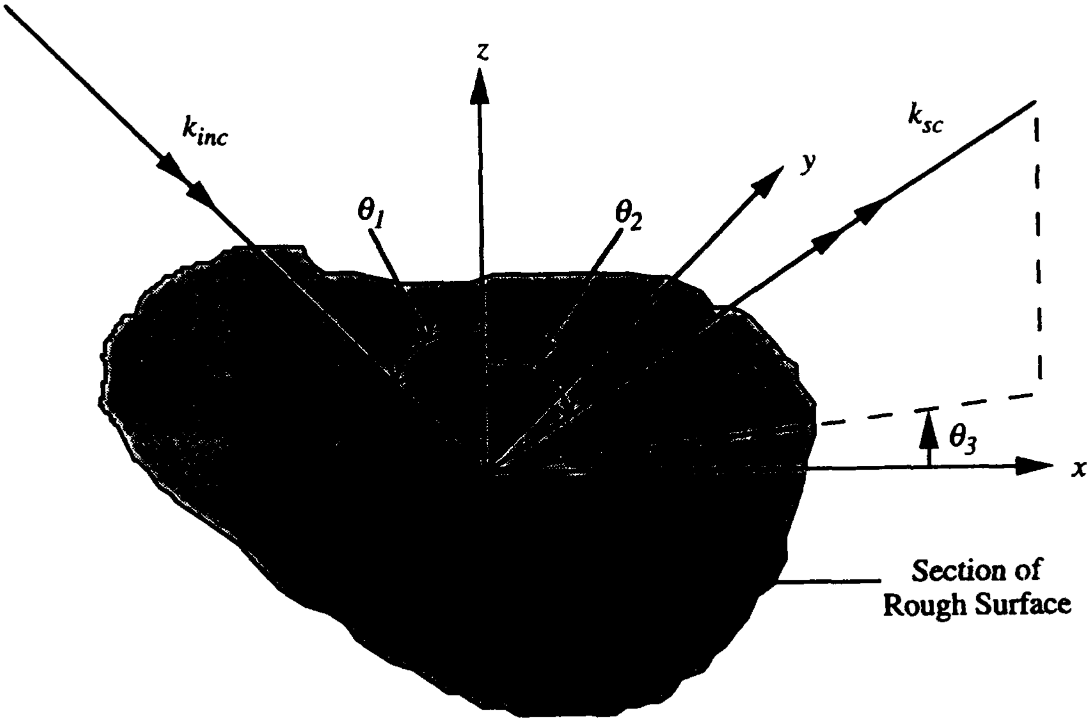

Figure 1.

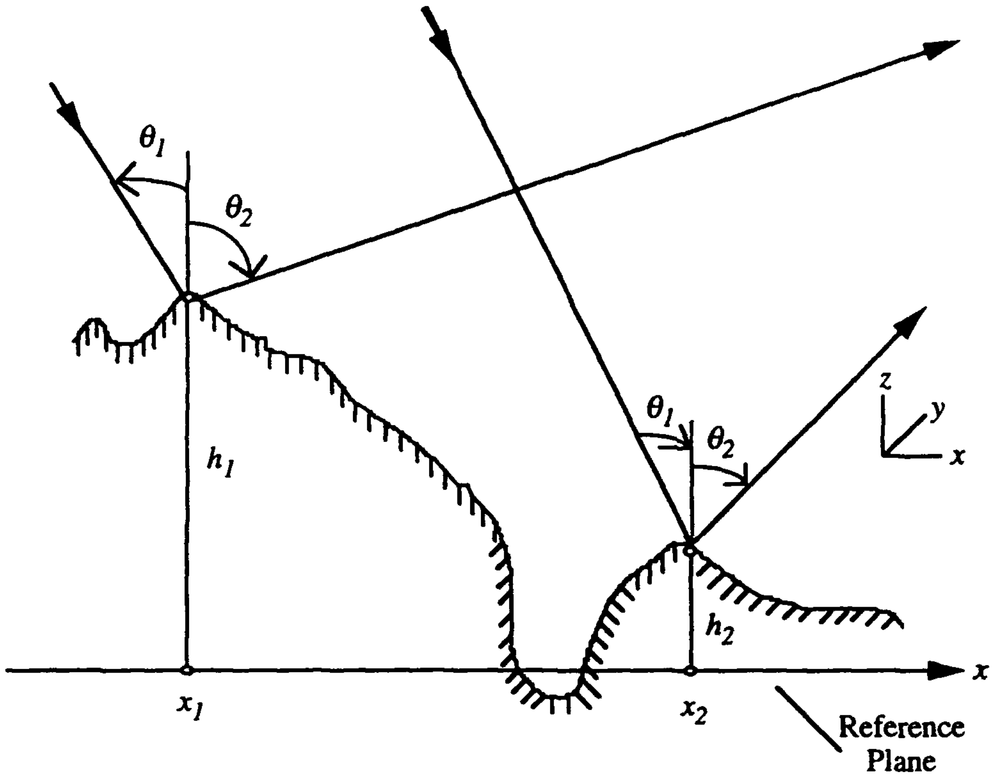

Figure 2.

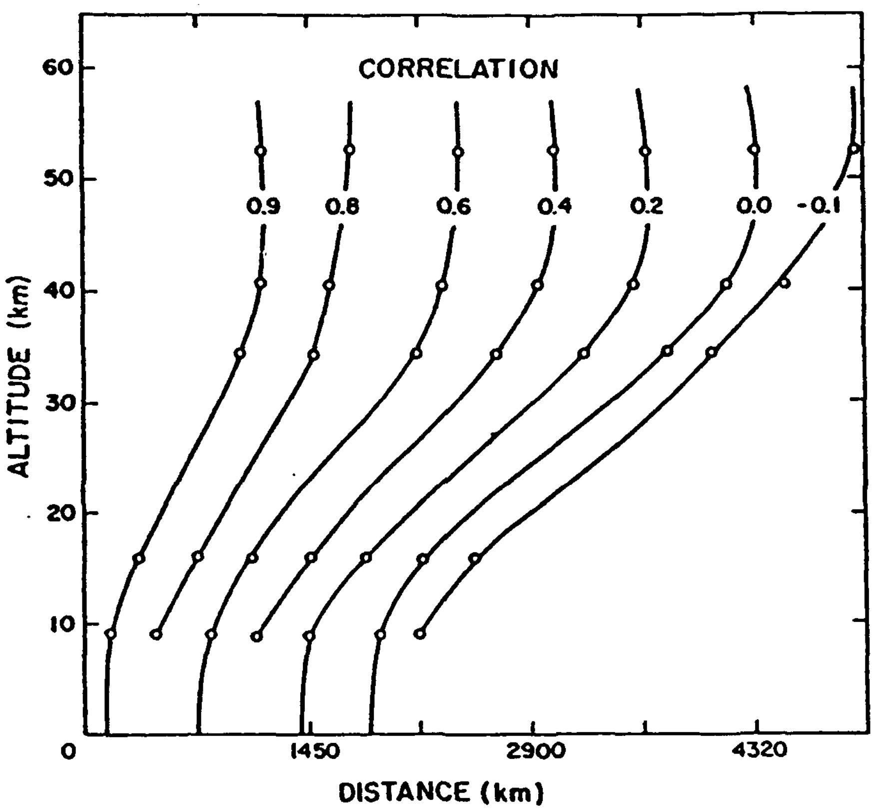
CORRELATION
0.9
0.8
0.6
0.4
0.2
0.0
-0.1
ALTITUDE (km)
60
50
40
30
20
10
0
1450
2900
4320
DISTANCE (km)

Figure 3.

Table 1. Estimated rms Differences (% of mean) Between Densities at Locations 90, 180, and 360 km Apart During the Midseason Months in the Tropics

Alt (km)	Jan			Apr			Jul			Oct		
	90	180 km	360	90	180 km	360	90	180 km	360	90	180 km	360
10	0.10	0.13	0.18	0.10	0.13	0.18	0.10	0.13	0.18	0.10	0.13	0.18
15	0.13	0.17	0.25	0.11	0.14	0.21	0.16	0.20	0.30	0.16	0.20	0.30
18	0.50	0.61	1.00	0.34	0.42	0.68	0.30	0.37	0.60	0.34	0.42	0.68
20	0.28	0.34	0.56	0.28	0.34	0.56	0.24	0.29	0.48	0.24	0.29	0.48
25	0.28	0.34	0.56	0.28	0.34	0.56	0.24	0.29	0.48	0.26	0.32	0.52
30	0.30	0.37	0.60	0.30	0.37	0.60	0.28	0.34	0.56	0.30	0.37	0.60
35	0.34	0.42	0.68	0.30	0.37	0.60	0.30	0.37	0.60	0.36	0.44	0.72
40	0.40	0.49	0.80	0.44	0.54	0.88	0.48	0.59	0.96	0.44	0.54	0.88
45	0.46	0.56	0.92	0.40	0.49	0.80	0.60	0.73	1.20	0.52	0.64	1.04
50	0.56	0.69	1.12	0.54	0.66	1.08	0.72	0.88	1.44	0.54	0.66	1.08
55	0.66	0.81	1.32	0.56	0.69	1.12	0.84	1.03	1.68	0.78	0.96	1.56
60	0.84	1.03	1.68	0.66	0.81	1.32	1.00	1.22	2.00	0.82	1.00	1.64