NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

PREDICTING ANTENNA PARAMETERS FROM ANTENNA PHYSICAL DIMENSIONS

by

Steven E. Lundholm

December, 1993

Thesis Advisor: R. Clark Robertson

Best
Available
Copy

REPORT DOCUMENTATION PAGE	Form Approved OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December, 1993	Master's Thesis

4. TITLE AND SUBTITLE

PREDICTING ANTENNA PARAMETERS FROM ANTENNA PHYSICAL DIMENSIONS

5. FUNDING NUMBERS

6. AUTHOR(S)

Lundholm, Steven E.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or U.S. Government

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report details the development and provides the documentation for custom computer software applications that evaluate antenna parameters. The applications are written in Mathcad 3.1 for the following antenna types: linear, planar, and circular arrays; folded dipole; caged dipole; parabolic reflectors with helical and spiral feeds. Inputs to the applications are limited to the antenna's physical dimensions. In some cases, ground parameters are required.

The chapters are structured to provide the user with the necessary information needed to use and interpret the software for each antenna type. The software applications are provided as appendices and give examples of each antenna type.

Outputs of the applications provide various numerical and performance predictions, as well as far-field radiation patterns. The results computed are consistent with predictions provided in applicable publications, as well as those calculated by numerical antenna analysis programs such as ELNEC.

14. SUBJECT TERMS

Radiation Pattern, Radiated Power, Directivity, Gain, Polarization, Efficiency, Effective Height/Area, Bandwidth, Wavelength, Effective Isotropic Radiated Power, Input Impedance, Reflection Coefficient

15. NUMBER OF PAGES 216

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

Approved for public release; distribution is unlimited

PREDICTING ANTENNA PARAMETERS FROM ANTENNA PHYSICAL DIMENSIONS

by

Steven E. Lundholm
Lieutenant, United States Navy
B.S., University of Minnesota, 1983

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1993

Author: ______________________
Steven E. Lundholm

Approved by: ______________________
R. Clark Robertson, Thesis Advisor

____
Richard W. Adler, Second Reader

____
Michael A. Morgan, Chairman,
Department of Electrical and
Computer Engineering

ABSTRACT

This report details the development and provides the documentation for custom computer software applications that evaluate antenna parameters. The applications are written in Mathcad 3.1 for the following antenna types: linear, planar, and circular arrays; folded dipole; caged dipole; parabolic reflectors with helical and spiral feeds. Inputs to the applications are limited to the antenna's physical dimensions. In some cases, ground parameters are required.

The chapters are structured to provide the user with the necessary information needed to use and interpret the software for each antenna type. The software applications are provided as appendices and give examples of each antenna type.

Outputs of the applications provide various numerical and performance predictions, as well as far-field radiation patterns. The results computed are consistent with predictions provided in applicable publications, as well as those calculated by numerical antenna analysis programs such as ELNEC.

TABLE OF CONTENTS

I. BACKGROUND AND PURPOSE

This thesis details the development of user friendly mathematical applications capable of computing the radiation pattern and other pertinent antenna parameters of an antenna or antenna system based on available information, primarily dimensional information obtained from photographs. The applications developed are compatible with any IBM personnel computer using MS-DOS version 3.2 or higher and a math coprocessor. The applications are written in Mathcad version 3.1 engineering software and can be run using either Mathcad 3.1 or Mathcad version 4.0. These applications are intended for use by the Naval Maritime Intelligence Center (NAVMARINTCEN).

As already mentioned, user inputs are limited to antenna dimensions based on physical measurements primarily obtained from photographs, although other source data can be used. The applications are flexible to the extent that other parameters may be estimated and used as inputs to increase the accuracy of the computations to gain better insight into the antenna's performance. The Mathcad applications provide various performance predictions as well as a graphical representation of the antenna's far-field radiation pattern. The necessary background information needed to interpret the application's formulas and displays are provided in the corresponding thesis chapters.

Dietrich [Ref. 1] and Gerry [Ref. 2] have completed the first and second reports of this project, respectively. This thesis is the last in a series of three reports intended to fulfill the NAVMARINTCEN statement of work.

II. INTRODUCTION

The increasingly sophisticated design by foreign countries of antennas warrants the development of improved analytical methods to provide timely and accurate technical assessment of these antennas. Through the process of reverse engineering, foreign communication, navigation, Identification, Friend or Foe (IFF), and radar antennas are evaluated to determine their capabilities, limitations, and vulnerabilities. Having limited information available concerning the parameters of an antenna, usually restricted to photographs that provide only the type and physical dimensions of the antenna, current methods of antenna evaluation are slow and tedious.

With the availability of high-speed personal computers matched with sophisticated off-the-shelf engineering software, the assessment of antennas using only photographic information may now be obtained rapidly. The objective of this report is to provide NAVMARINTCEN with user friendly Mathcad software to achieve this type of performance in their evaluation of antenna systems.

As with any computer analysis of antennas, the accuracy of the results depend upon the sophistication of software used, the complexity of calculations, the reliability of input data obtained from photographs, and the accuracy of any required input data that must be estimated. This report and the Mathcad applications are written with existing engineering equations for the type of antenna analyzed and inputs are limited to physical dimensions. Additional information such as ground effects and characteristic impedance may be included to increase the accuracy of the analysis. To this end, this report should provide an initial understanding as to the capabilities of the antenna system in question.

The chapters of this report document the equations and assumptions used for each specific antenna type and is written as a comprehensive reference for the software. Copies of each application are included as appendices to provide the user with a printed illustration of the software.

III. THE UNIFORM LINEAR ARRAY ANTENNA

Arrays are highly directional antenna system with narrow, steerable beams and low side lobes used for long distance communication and radar systems. The linear array is one geometrical configuration that consists of radiating elements lying along a straight line. As a result of the dependance of inter-element spacing (d) to the wavelength (λ), linear arrays are inherently narrowband antenna systems.

In this Mathcad application uniformly spaced, equally excited, isotropic point sources aligned on the z-axis are used as the radiating elements. In using isotropic point sources, polarization of the array is not calculated and an assumption is made that the antenna input resistance (R_{in}) is equal to the radiation resistance (R_r). This assumption results in calculations for gain (G) that are idealized values. The following inputs are needed to implement the software:

N = number of isotropic radiating elements

d = inter-element spacing between adjacent elements

θ_o = direction of main lobe *

f = frequency of interest *

I_o = antenna feed current *

Z_o = characteristic feed impedance *

The first two inputs are obtained from visual data such as photographs, and the * indicates input parameters that are either known or are estimated. The linear array geometry is illustrated in Figure 3.1.

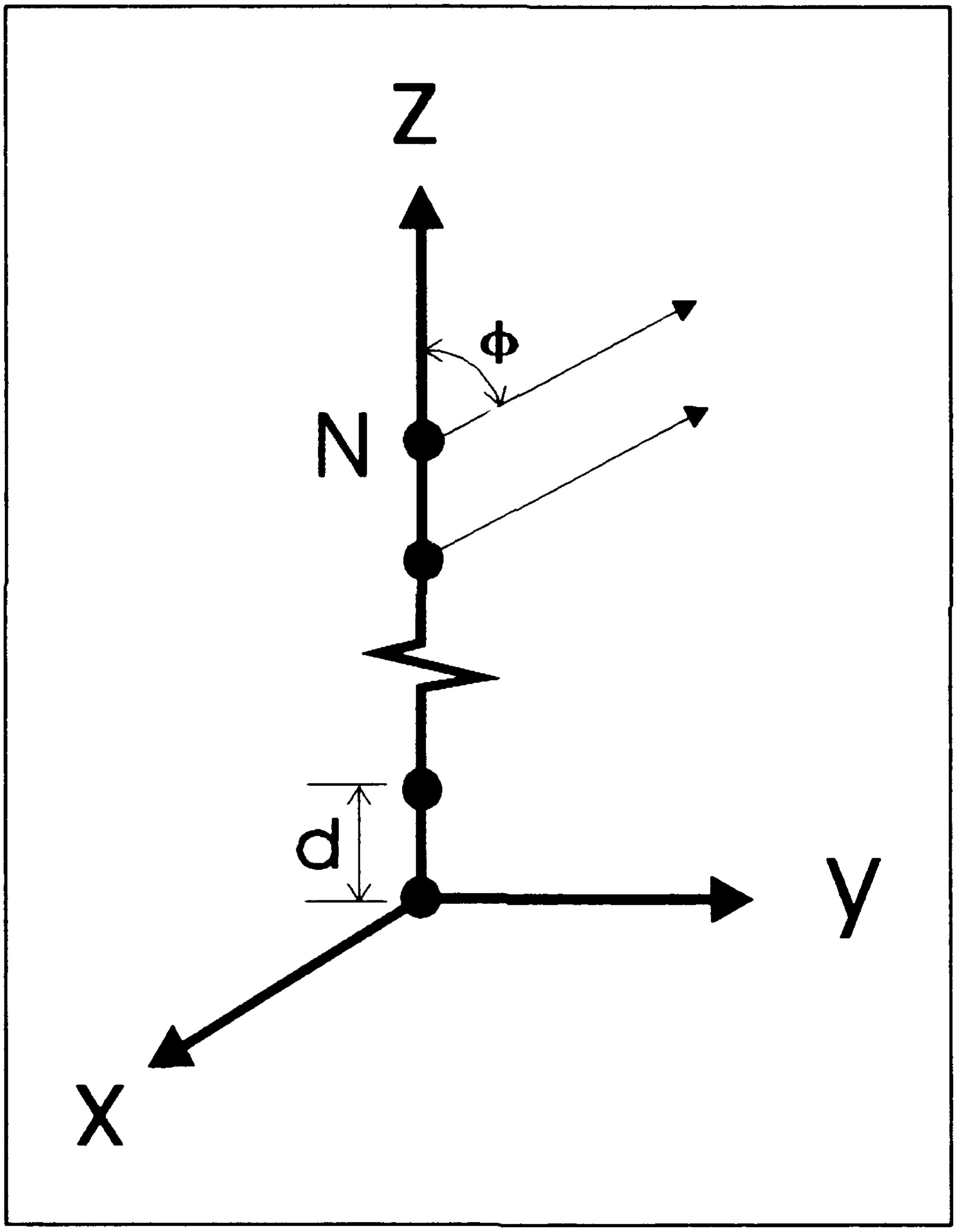


FIGURE 3.1 Geometry of N-element linear array of isotropic point sources

The total electric field of a linear array in the far-field, neglecting mutual coupling between adjacent elements, is determined by the vector addition of the fields of individual radiating elements. It can be shown that the total field is equivalent to the product of the field of a single element, selected at a reference point, and a factor which is referred to as the array factor (AF):

$$\boldsymbol{E}(total) = [\boldsymbol{E}(element)]\ x\ [AF] \qquad (3.1)$$

In (3.1), **E** is the vector electric field intensity, while AF is a scalar quantity. This result is referred to as pattern multiplication for arrays of identical elements. The array factor is a function of the geometry of the array and the element excitation amplitude and phase. For uniform excitation, by varying the inter-element spacing (d) and/or the phase (β) between the elements, the characteristics of the array factor, and consequently the total electric field, of the array can be controlled. [Ref. 3: pp. 204-207]

This Mathcad application is written for an N-element linear -ray with equally spaced, uniformly excited isotropic radiating elements. In addition, element phase is assumed to vary linearly along the length of the array. The radiation pattern of a specific element type is neglected in (3.1) since in normal usage it will have little effect when the array consists of a large number of elements. However, the user of this software should be able to use information regarding the type of element and the orientation of the elements to narrow the range of possible θ_o's. The normalized array factor (AF_n) is [Ref. 3: pp. 212-214]:

$$AF_n(\theta) = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi(\theta)\right)}{\sin\left(\frac{1}{2}\psi(\theta)\right)}\right] \quad (dimensionless) \qquad (3.2)$$

where

$$\psi(\theta) = kd\cos(\theta) + \beta \quad (radians) \qquad (3.3)$$

$$\beta = -kd\cos(\theta_o) \quad (radians) \qquad (3.4)$$

In (3.4), θ_o is the direction of the maximum array factor value (i.e., for a broadside array $\theta_o = \pi/2$, for an end-fire array with the main beam directed at 180°, $\theta_o = \pi$, and for a phased array with the main beam directed to ±60°, $\theta_o = \pi/3$), β is the phase excitation difference between the elements, k is the wavenumber ($2\pi/\lambda$), and d is the inter-element spacing.

With the linear array aligned along the z-axis, the array factor is a function of θ but not of ϕ. The visible region of the array in terms of θ, $\psi(\theta)$, and the inter-element spacing (d/λ) is:

$$0 < \theta < \pi \quad (radians) \qquad (3.5)$$

$$\beta - kd < \psi(\theta) < \beta + kd \quad (radians) \tag{3.6}$$

d/λ determines how much of the array factor appears in the visible region as defined by (3.5) and (3.6). Although d/λ provides guidance in determining the frequency to select for this Mathcad application, it should be noted that most arrays are designed with the inter-element spacing less than one wavelength, usually close to a half-wavelength. Thus, linear arrays are characteristic narrowband antenna systems. [Ref. 4: p. 123]

An observation point is considered to be in the far-field if all of the following are satisfied [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \quad (meters) \tag{3.7}$$

$$r \geq 5(Nd) \quad (meters) \tag{3.8}$$

$$r \geq \frac{2(Nd)^2}{\lambda} \quad (meters) \tag{3.9}$$

The array length, Nd, in (3.8) and (3.9) is the maximum dimension of the antenna (the array length is assumed to include a distance of d/2 beyond each end element [Ref. 5: p.55]). The minimum distance to the far-field is found by selecting the largest value of (3.7), (3.8), and (3.9).

Radiation intensity ($U(\theta)$) is related to the power radiated in a given direction and is independent of the distance to the observation point [Ref. 3: p. 27; Ref. 4: p. 33]. For linear arrays $U(\theta)$ is given by [Ref. 3: pp. 229-233]:

$$U(\theta) = (AF_n)^2 \quad (W/solid\ angle) \tag{3.10}$$

Radiated power (P_{rad}) is the total power radiated by the antenna and is obtained by integrating $U(\theta)$ over a surface surrounding the antenna [Ref. 4: p. 33]. Since the geometry of the linear array is aligned on the z-axis and the array factor is independent of ϕ, P_{rad} is thus [Ref. 3: pp. 229-233]:

$$P_{rad} = 2\pi \int_0^{\pi} U(\theta) \sin(\theta)\, d\theta \quad (W) \tag{3.11}$$

Directivity (D_o) is the maximum value of directive gain where directive gain is the ratio of $U(\theta)$ in a specific direction to the average radiation intensity [Ref. 4: pp. 34-36]. Hence, D_o is given by [Ref. 3: pp. 229-233]:

$$D_o = \frac{4\pi U_{max}}{P_{rad}} \quad (dimensionless) \tag{3.12}$$

In (3.12), U_{max} equals unity and occurs at θ_o for the uniform linear array.

Effective isotropic radiated power (EIRP) is a commonly used communications term that is defined as the product of antenna gain and total power accepted by the antenna from the transmitter. EIRP is determined as [Ref. 4: p. 62]:

$$EIRP = P_{rad} D_o \quad (W) \tag{3.13}$$

Assuming that the power delivered to the array consisting of N-isotropic point sources is equivalent to the radiated power, we can estimate the radiation resistance (R_r) of the antenna as [Ref. 3: p. 55]:

$$R_r = \frac{2\,(P_{rad})}{|I_o|^2} \quad (\Omega) \tag{3.14}$$

Using the same logic, we obtain the input resistance (R_{in}) of the array made up of isotropic point sources as equivalent to the radiation resistance:

$$R_{in} = R_r \quad (\Omega) \tag{3.15}$$

The exact gain (G) of the array cannot be calculated with the data assumed, but an idealized value can be computed based on the assumption made to determine R_{in} and the assumptions made to calculate total antenna efficiency (ε_t). The total antenna efficiency is given by [Ref. 3: pp. 44-45]:

$$\varepsilon_t = \varepsilon_r \varepsilon_{cd} = 1 - |\Gamma|^2 \quad (dimensionless) \tag{3.16}$$

where

$$\varepsilon_{cd} = 1 \quad (dimensionless) \tag{3.17}$$

$$\varepsilon_r = 1 - |\Gamma|^2 \quad (dimensionless) \tag{3.18}$$

$$\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o} \quad (dimensionless) \tag{3.19}$$

In (3.17), ε_{cd} is the conduction and dielectric efficiency which is unity for an ideal lossless antenna. In (3.19), Z_o is the characteristic impedance of the transmission line and Γ is voltage reflection coefficient. Therefore, the total antenna efficiency is equal to the mismatch efficiency of (3.18) and the ideal gain for a linear array is [Ref. 3: pp. 43-44]:

$$G = \varepsilon_t D_o \quad (dimensionless) \tag{3.20}$$

$$G(dB) = 10\log_{10}(\varepsilon_t D_o) \quad (dB) \tag{3.21}$$

As a result of constructing the linear array with isotropic point sources, the polarization loss factor (PLF) and antenna polarization are not calculated. Therefore, the maximum effective aperture (A_{em}) of the array when used as a receiver is calculated assuming negligible polarization mismatches. The A_{em} is then [Ref. 3: p 63]:

$$A_{em} = \frac{G(\lambda)^2}{4\pi} PLF \quad (m^2) \tag{3.22}$$

where

$$PLF = 1 \quad (dimensionless) \tag{3.23}$$

The assumptions used in finding R_r and A_{em} result in an idealized value in calculating the effective height (h_{em}). The effective height is [Ref. 6: p. 42]:]

$$h_{em} = 2\sqrt{\frac{R_r A_{em}}{\eta_o}} \quad (meters) \tag{3.24}$$

where η_0 is the characteristic impedance of free space (120π).

Two displays are produced in this Mathcad application. The first display is the rectangular representation of the magnitude of the array factor from $-\pi$ to π. The second display is the polar representation of the magnitude of the array factor.

The Hansen-Woodyard end-fire array was not addressed in this application nor were arrays of uniform spacing but nonuniform amplitude (i.e., Dolph-Tschebyscheff array). In addition, nonuniformly spaced, uniformly excited arrays were not addressed by this application.

The results obtained with this application are identical to those found in [Ref. 3: pp. 216-240] with respect to array factor patterns and directivity. Table 3.1 is a comparison of data from [Ref. 3] for a 10-element equally excited, uniformly spaced, end-fire linear array while Figure 3.2 is a comparison of the array factor pattern of the same array. Of note, the array factor patterns from this application and the patterns from [Ref. 3] are identical as expected.

Table 3.1 End-fire Array Data Comparison

ANTENNA DIMENSIONS	REFERENCE DIRECTIVITY (D_o)	CALCULATED DIRECTIVITY (D_o)
$N = 10$ $d = \lambda/4$ m $\theta_o = 0°$ $\beta = -kd$	10.0 dB	10.003 dB

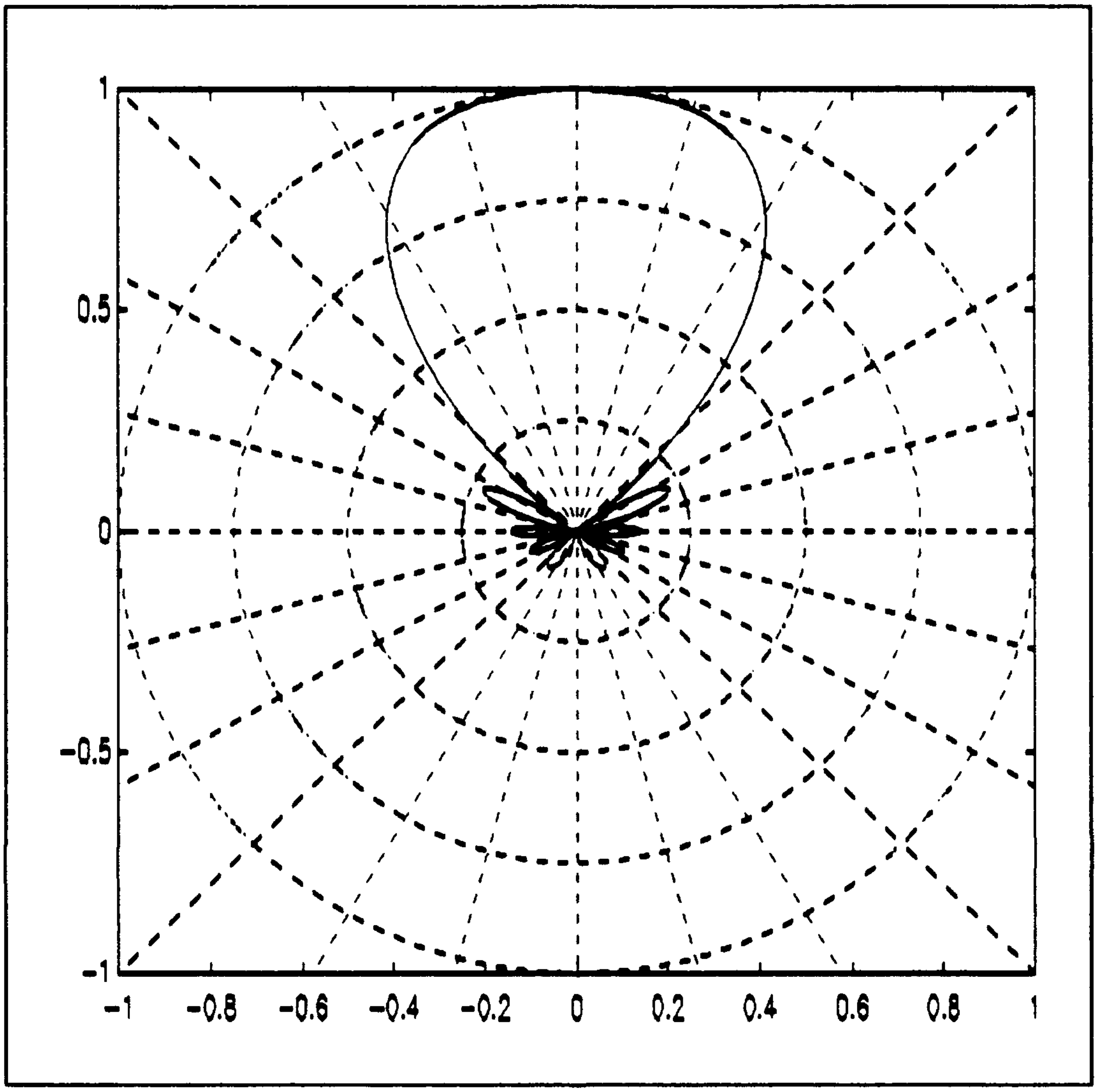


FIGURE 3.2 Array factor pattern of 10-element equally excited, uniformly spaced end-fire linear array with d = $\lambda/4$ m, λ_o = 0°, and β = -kd

Table 3.2 is a comparison of data from [Ref. 3] for a 10-element equally excited, uniformly spaced, broadside linear array, and Figure 3.3 is a comparison of the array factor pattern of the same array. The array factor patterns from this application and the patterns from [Ref. 3] are identical.

Table 3.2 Broadside Array Data Comparison

ANTENNA DIMENSIONS	REFERENCE DIRECTIVITY (D_o)	CALCULATED DIRECTIVITY (D_o)
$N = 10$ $d = \lambda/4$ m $\theta_o = \pi/2$ $\beta = 0$	6.99 dB	7.14 dB

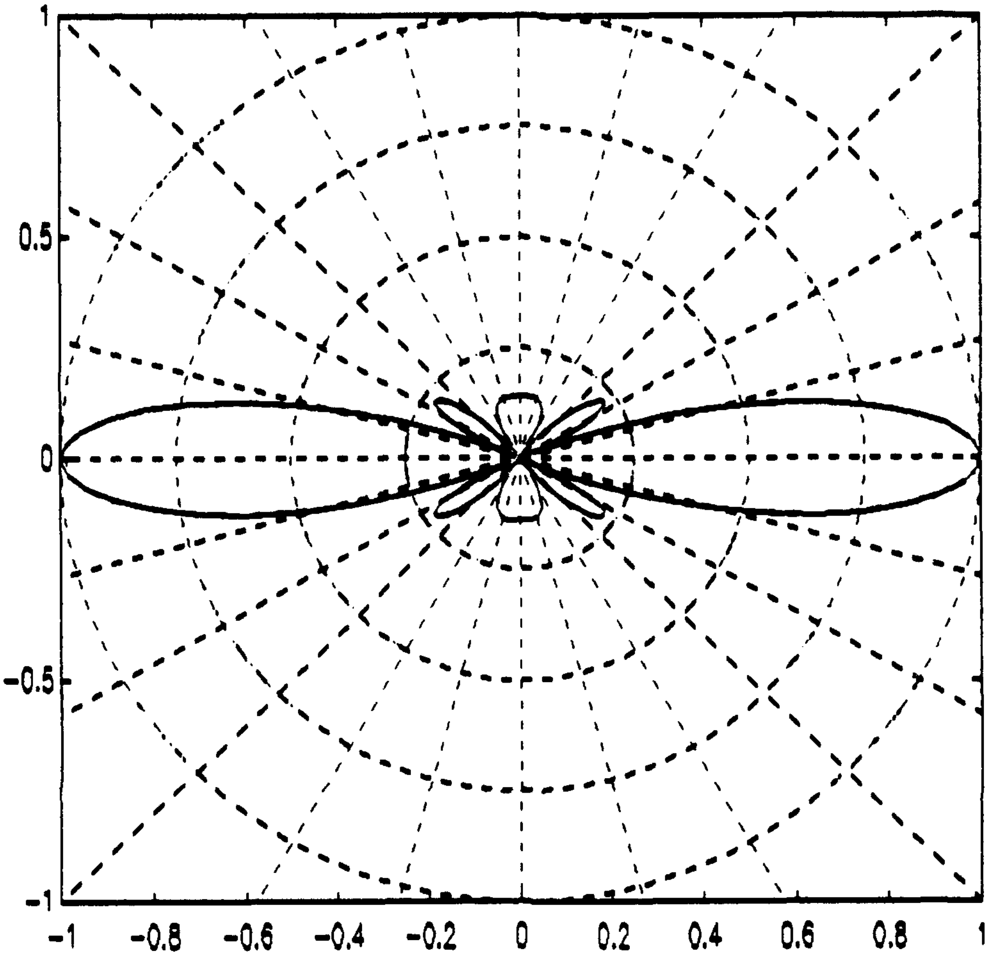


FIGURE 3.3 Array factor pattern of 10-element equally excited, uniformly spaced broadside linear array with d = $\lambda/4$ m, $\theta_o = \pi/2$, and $\beta = 0$

IV. THE PLANAR ARRAY ANTENNA

Planar arrays, frequently used long distance communications and radar systems, exhibit characteristics analogous to those of linear arrays but have additional capability to control and shape the radiation pattern. Planar arrays are more versatile than linear arrays, providing symmetrical patterns with lower side lobes, and can be used to scan the main beam of the antenna array toward any point in space [Ref. 3: p. 261]. As with linear arrays, planar arrays are narrowband antenna systems due to the dependance of the inter-element spacing (d_x and d_y) on the wavelength (λ). The following inputs are required analyze a planar array:

M = number of isotropic radiating elements in the x-direction

N = number of isotropic radiating elements in the y-direction

d_x = inter-element spacing between adjacent elements in the x-direction

d_y = inter-element spacing between adjacent elements in the y-direction

f = frequency of interest *

θ_o = direction of main lobe at $\theta = \theta_o$ *

ϕ_o = direction of main lobe at $\phi = \phi_o$ *

I_o = antenna feed current *

Z_o = characteristic feed impedance *

The first four inputs are data obtained primarily from visual data such as photographs, and the * indicates input parameters that are either known or estimated. The planar array geometry is illustrated in Figure 4.1.

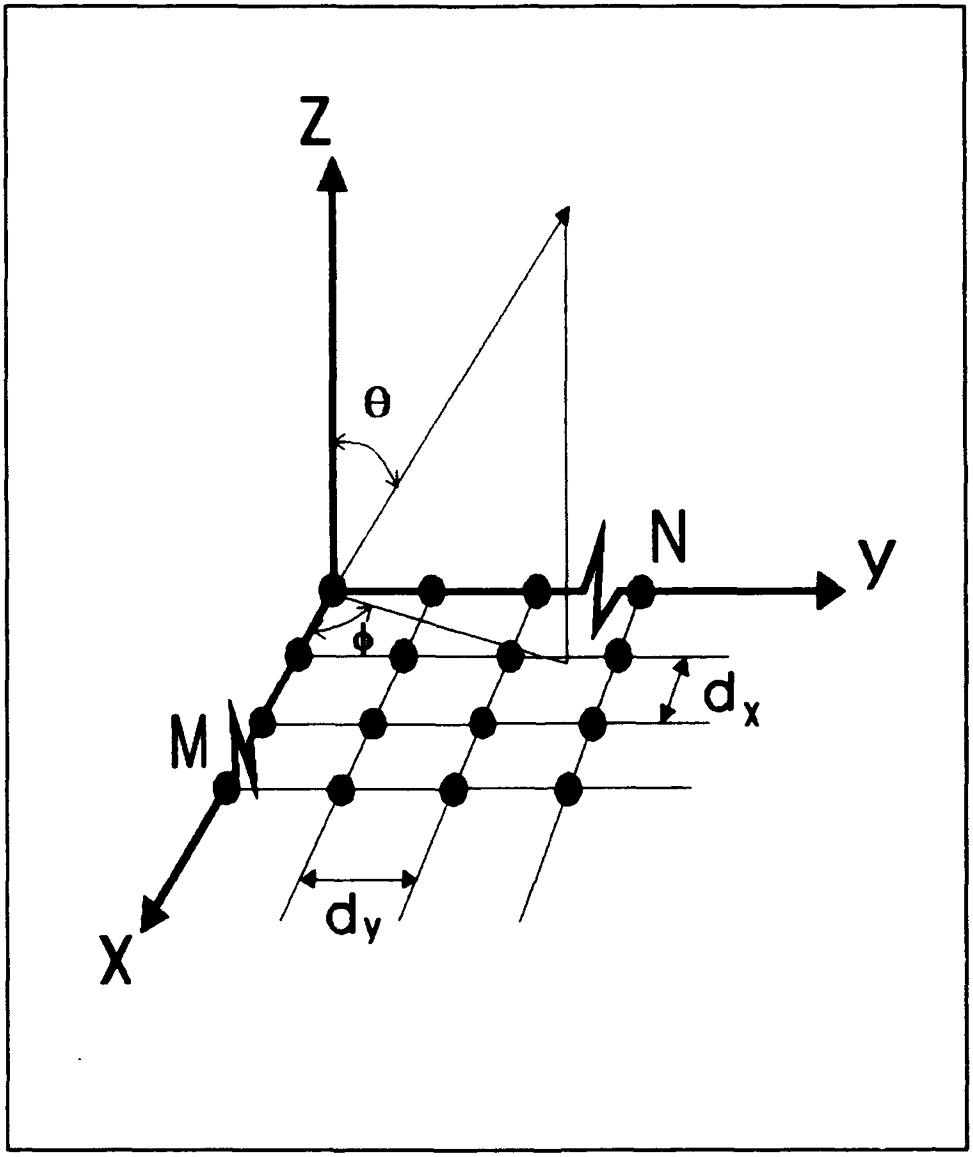


FIGURE 4.1 Geometry of M x N element planar array of isotropic point sources

This Mathcad application is written for an M x N planar array with uniformly spaced, equally excited, identical radiating elements positioned on the x-y plane. This application may also be used to approximate the array factor and field patterns for arrays that have elliptical or circular geometries by assuming an ellipse is equivalent to a rectangle or a circle is equivalent to a square, respectively [Ref. 6: p.187]. Since isotropic point sources are assumed as array elements, the polarization of the array is not calculated, and an assumption is made that the antenna input resistance (R_{in}) is equivalent to the radiation resistance (R_r). This assumption results in values for gain (G) that are idealized. As with the linear array, the radiation pattern of a specific element type is neglected in the analysis of the planar array since in normal usage it will have little effect for a planar array with a large number of elements. However, as with the linear array, the user of this software should be able to use information regarding the type of element and the orientation of the elements to narrow the range of the possible θ_o,s and ϕ_o's.

The normalized array factor (AF_n) is calculated for far-field observations neglecting mutual coupling between adjacent elements with equal amplitude excitation and is [Ref. 3: pp. 260-263]:

$$AF_n(\theta,\phi) = \left[\frac{\sin\left(\frac{M}{2}\psi_x(\theta,\phi)\right)}{M\sin\left(\frac{\psi_x(\theta,\phi)}{2}\right)}\right]\left[\frac{\sin\left(\frac{N}{2}\psi_y(\theta,\phi)\right)}{N\sin\left(\frac{\psi_y(\theta,\phi)}{2}\right)}\right] \quad (4.1)$$

where

$$\psi_x(\theta,\phi) = kd_x\sin(\theta)\cos(\phi) + \beta_x \quad (radians) \quad (4.2)$$

$$\psi_y(\theta,\phi) = kd_y\sin(\theta)\sin(\phi) + \beta_y \quad (radians) \quad (4.3)$$

$$\beta_x = -kd_x\sin(\theta_o)\cos(\phi_o) \quad (radians) \quad (4.4)$$

$$\beta_y = -kd_y\sin(\theta_o)\sin(\phi_o) \quad (radians) \quad (4.5)$$

Equation (4.1) is the normalized array factor that is a function of both θ and ϕ where (4.2) and (4.3) are the array factor phase shifts in the x and y directions, respectively. (4.4) and (4.5) are the progressive phase shift between adjacent elements in the x and y directions, respectively, with the main beam directed along $\theta = \theta_o$ and $\phi = \phi_o$. In (4.2) through (4.5), k is the wavenumber ($2\pi/\lambda$).

When the inter-element spacing between the elements (d_x and d_y) equals or is greater than the wavelength (λ), multiple maxima of equal magnitude are formed. To avoid creating multiple maxima, the inter-element spacing must be less than a wavelength at the frequency of operation [Ref. 3: p. 263]. With the dependance of inter-element spacing on the wavelength, planar arrays are characteristic narrowband antenna systems.

An observation point is considered to be in the far-field if all of the following are satisfied [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \quad (meters) \tag{4.6}$$

$$r \geq 5\sqrt{(Md_x)^2 + (Nd_y)^2} \quad (meters) \tag{4.7}$$

$$r \geq \frac{2((Md_x)^2 + (Nd_y)^2)}{\lambda} \quad (meters) \tag{4.8}$$

The length of an array is assumed to include a distance d/2 beyond each end element [Ref. 5: p. 55]. Therefore, the maximum dimension of the planar array is:

$$\sqrt{(Md_x)^2 + (Nd_y)^2} \quad (meters) \tag{4.9}$$

The minimum distance to the far-field is found selecting the largest value of (4.6), (4.7), and (4.8).

Directivity (D_0) for planar arrays is determined approximately by [Ref. 7: p. 78]:

$$D_o = 4\pi \frac{AF_n AF_n^*|_{max}}{\int_0^{2\pi}\int_0^{\pi/2}\left(AF_n AF_n^*\right)\sin(\theta)\, d\theta\, d\phi} \quad (W) \tag{4.10}$$

The maximum value of the numerator in (4.10) is unity at $\theta = \theta_0$ and $\phi = \phi_0$ since the array factor is normalized. The denominator is the radiated power (P_{rad}) of the antenna where

P_{rad} is obtained by integrating over all angles around the antenna above the x-y plane ($0 \le \theta \le \pi/2$) [Ref. 3: p. 28; Ref. 7: p. 78]. This assumes the array is always pointed toward one side or the other of the x-y plane.

Effective isotropic radiated power (EIRP) for planar arrays is defined as the product of antenna gain and the total power radiated. EIRP is determined as [Ref. 4: p. 62]:

$$EIRP = P_{rad} D_o \quad (W) \tag{4.11}$$

Assuming that the power delivered to planar arrays constructed of M x N isotropic radiating elements is equivalent to the radiated power, we estimate the radiation resistance (R_r) of the array as [Ref. 3: p. 55]:

$$R_r = \frac{2\,(P_{rad})}{|I_o|^2} \quad (\Omega) \tag{4.12}$$

Since the radiating elements are assumed to be ideal point sources, the input resistance of a planar array (R_{in}) is assumed to be equivalent to the radiation resistance:

$$R_{in} = R_r \quad (\Omega) \tag{4.13}$$

The gain (G) of a planar array cannot be calculated given the information assumed to be available, but an idealized value can be derived. Another factor affecting the gain is the assumption made when determining the total antenna efficiency (ε_t). The total antenna efficiency for an antenna is [Ref. 3: pp. 44-45]:

$$\varepsilon_t = \varepsilon_r \varepsilon_{cd} = 1 - |\Gamma|^2 \quad (dimensionless) \tag{4.14}$$

where

$$\varepsilon_{cd} = 1 \quad (dimensionless) \tag{4.15}$$

$$\varepsilon_r = 1 - |\Gamma|^2 \quad (dimensionless) \tag{4.16}$$

$$\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o} \quad (dimensionless) \tag{4.17}$$

In (4.14), ε_{cd} is the conduction and dielectric efficiency, which is unity for an ideal lossless antenna. In (4.17), Z_o is the characteristic impedance of the transmission line and Γ is the voltage reflection coefficient. Therefore, the total antenna efficiency is equal to the mismatch efficiency of (4.15) when ideal lossless antenna is assumed. The ideal gain of a planar array in dimensionless and decibel quantities is then [Ref. 3: pp. 43-44]:

$$G = \varepsilon_t D_o \quad (dimensionless) \tag{4.18}$$

$$G(dB) = 10\log_{10}(\varepsilon_t D_o) \quad (dB) \tag{4.19}$$

The maximum effective aperture (A_{em}) for a planar array is derived by assuming polarization mismatches are negligible (this application is constructed using isotropic radiating elements for which polarization loss factor (PLF) and antenna polarization are not calculated) [Ref. 3: p. 63]:

$$A_{em} = \frac{G(\lambda)^2}{4\pi} PLF \qquad (m^2) \tag{4.20}$$

$$PLF = 1 \qquad (dimensionless) \tag{4.21}$$

The effective height (h_{em}) of the planar array is now estimated by [Ref. 6: p. 42]:

$$h_{em} = 2\sqrt{\frac{R_r A_{em}}{\eta_o}} \qquad (m) \tag{4.22}$$

In (4.22), η_o is the characteristic impedance of free space (120π).

Four two-dimensional array pattern displays are produced in this Mathcad application: the first is in the x-z plane; the second is in the y-z plane; the third display generated is a plane perpendicular to the x-y plane when $\phi = \pi/4$; the last display is a plane perpendicular to the x-y plane when $\phi = \phi_o$. The last display is redundant when ϕ is either 0, $\pi/4$, or $\pi/2$.

Results obtained with this application are the same as those found in [Ref. 3: pp. 260-274]. Directivity was calculated using the approximate equation (4.10). An alternative approximation for directivity is given by either [Ref. 3: p. 272-273]:

$$D_o = \pi \cos(\theta_o) D_x D_y \quad (dimensionless) \qquad (4.23)$$

or

$$D_o = \frac{\pi^2}{\Omega_A(rads^2)} \quad (dimensionless) \qquad (4.24)$$

In (4.23), D_x and D_y are the directivities of broadside linear arrays of length and number of elements Md_x, M, and Nd_y, N in the x and y directions, respectively. In (4.24), Ω_A is the beam solid angle.

Table 4.1 is a comparison of directivity computed for a 10x10 planar array. The reference directivity is derived using (4.24) while the calculated directivity is from (4.10).

Table 4.1 Planar Array Data Comparison

ANTENNA DIMENSIONS	REFERENCE DIRECTIVITY (D_o)	CALCULATED DIRECTIVITY (D_o)
$M = N = 10$ $d_x = d_y = \lambda/2$ m $\theta_o = 30°$ $\phi_o = 45°$ $\beta_x = \beta_y = -\pi/(2\sqrt{2})$	23.67 dB	24.07 dB

Figure 4.2 and Figure 4.3 are comparisons of array factor patterns for a 5x5 planar array with patterns from [Ref. 3]

and the calculated patterns. Figure 4.2 is a representation of the array factor patterns in the x-z and y-z planes, and the scale of this figure has been increased in order to increase the clarity of the pattern. Figure 4.3 is the array pattern in the plane at $\phi = 45°$, where the maximum beam is directed at $\phi_o = 45°$. The array patterns produced by this application are identical to the patterns given in [Ref. 3].

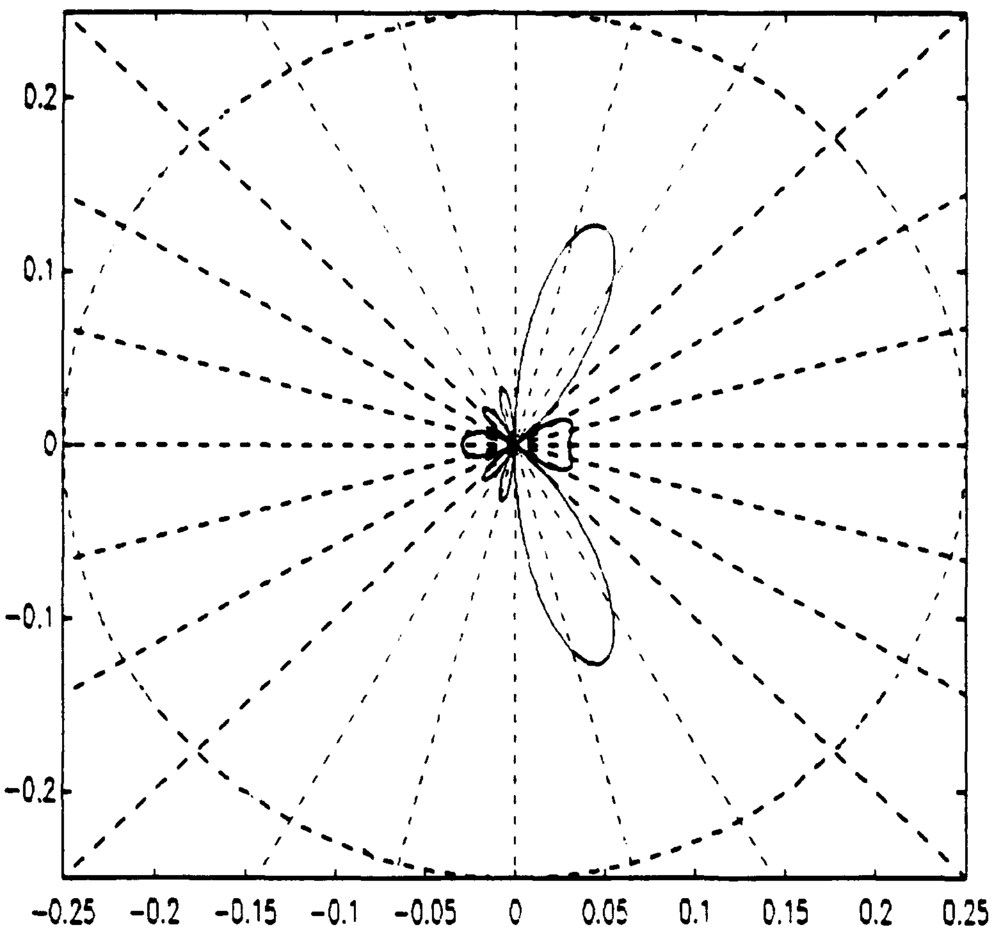


FIGURE 4.2 Array factor pattern of a 5x5 element, equally excited, planar array with $d_x = d_y = \lambda/2$ m, $\theta_o = 30°, \phi_o = 45°$, in the x-z and y-z planes

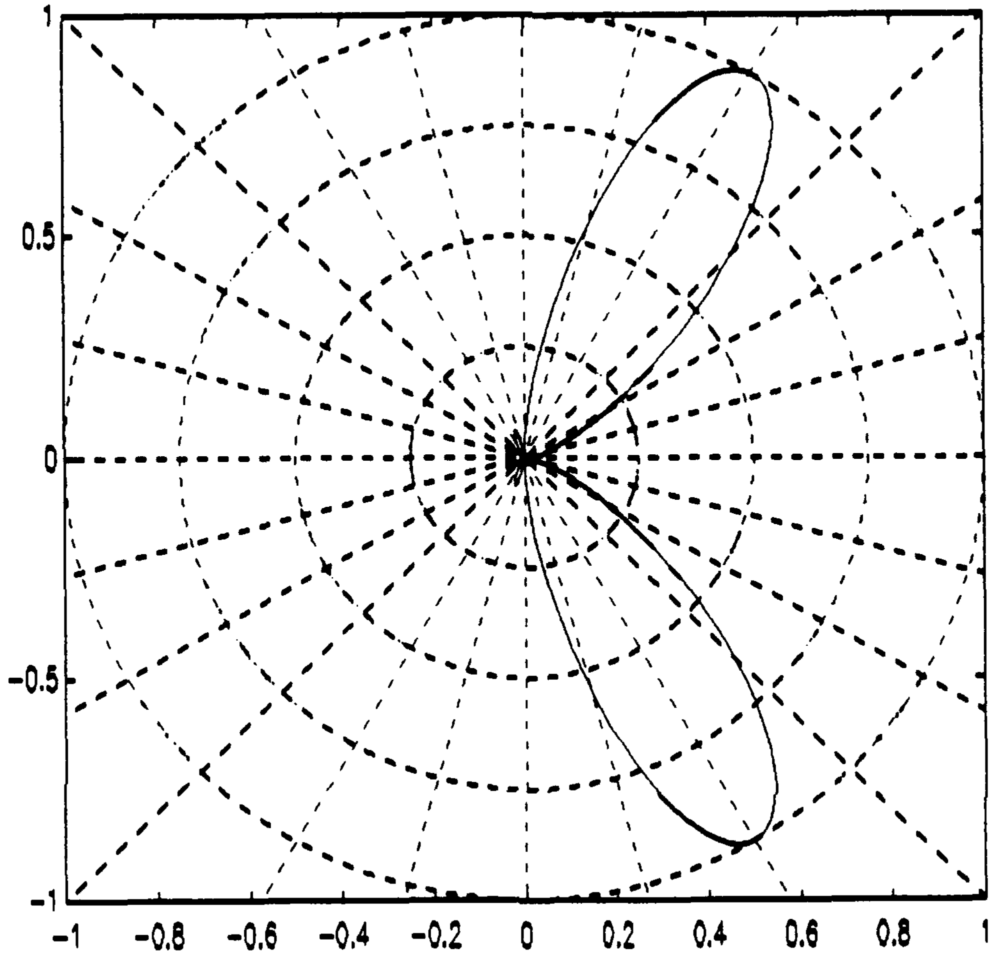


FIGURE 4.3 Array factor pattern of a 5x5 element, equally excited, planar array with $d_x = d_y = \lambda/2$ m, $\theta_o = 30°$, $\phi_o = 45°$, in the plane $\phi = 45°$

V. THE CIRCULAR ARRAY ANTENNA

Circular arrays, or ring arrays, are antennas in which the radiating elements are placed on a circle with no elements positioned inside the circle. Circular antenna arrays with elements positioned inside the circle are planar arrays, and these antennas are addressed in Chapter IV. Circular array antenna are used for radio direction finding, air and space navigation, underground propagation, radar, and sonar systems [Ref. 3: p. 274]. As with linear and planar arrays, circular arrays with uniformly spaced, equally excited elements are narrowband antenna systems due to the dependance of the number of elements (N) and the circumference of the circle to the wavelength (λ) [Ref. 3: p. 277].

The following inputs are required to analyze a circular array:

N = number of isotropic radiating elements

a = radius of the circle

f = frequency of interest *

θ_o = direction of main lobe at $\theta = \theta_o$ *

ϕ_o = direction of main lobe at $\phi = \phi_o$ *

I_o = antenna feed current *

Z_o = characteristic feed impedance *

The first two inputs are data primarily obtained from visual data such as photographs, and the * indicates input parameters that are either known or estimated. The planar array geometry is illustrated in Figure 5.1.

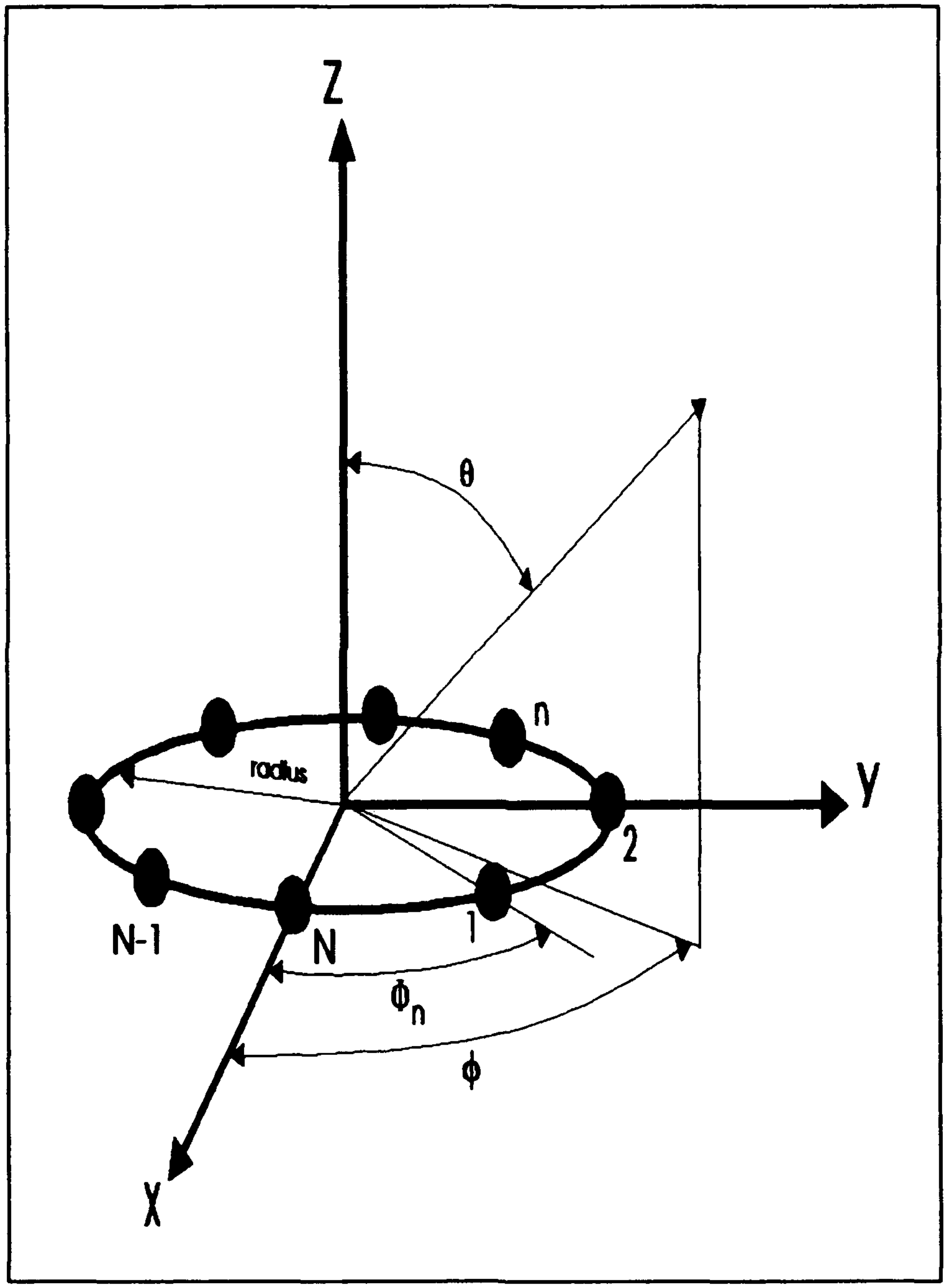


FIGURE 5.1 Geometry of N-element circular array of isotropic point sources

This Mathcad application is written for N-element circular arrays with uniformly spaced, equally excited, isotropic radiating elements on the x-y plane. The polarization of the array is not calculated since isotropic point sources are used, and the antenna input resistance (R_{in}) is assumed to be equivalent to the radiation resistance (R_r). This assumption results in the computation of ideal gain (G). As with the linear and planar arrays, the radiation pattern of a specific element type is neglected in the analysis of the circular array since in normal usage it will have little effect for a circular array with a large number of elements. However, as with both the linear and planar arrays, the user of this application should be able to use information regarding the type of element and the orientation of the elements to narrow the range of possible θ_o's and ϕ_o's.

The normalized array factor (AF) is calculated for far-field observations neglecting mutual coupling between adjacent elements with equal amplitude excitation and is [Ref. 3: pp. 274-278; Ref. 4: 350-354]:

$$AF(\theta,\phi) = \frac{1}{N}\sum_{n=1}^{N} e^{j[ka\sin\theta\cos(\phi-\Phi_n) + \alpha_n]} \quad (dimensionless) \quad (5.1)$$

where

$$\Phi_n = 2\pi\left(\frac{n}{N}\right), \; n = 1, 2, \ldots N \quad (radians) \quad (5.2)$$

$$\alpha_n = -ka\sin\theta_o\cos(\phi_o-\Phi_n) \quad (radians) \quad (5.3)$$

Equation (5.1) is the normalized array factor. Equation (5.2) is the angular position of the nth element on the x-y plane. Equation (5.3) is the phase excitation of the nth element with the position of the main beam directed at $\theta = \theta_o$ and $\phi = \phi_o$, and k is the wavenumber ($2\pi/\lambda$) and a is the radius of the array.

An observation point is considered to be in the far-field if all of the following are satisfied [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \quad (meters) \tag{5.4}$$

$$r \geq 5(2a) \quad (meters) \tag{5.5}$$

$$r \geq \frac{2(2a)^2}{\lambda} \quad (meters) \tag{5.6}$$

The minimum distance to the far-field is found by taking the maximum of (5.4), (5.4), and (5.6), where the diameter of the circle is taken to be the maximum dimension of the antenna.

To determine the directivity (D_o) for a circular array, the approximate equations of radiation intensity ($U(\theta,\phi)$) for arrays and radiated power (P_{rad}) are used. Radiation intensity is [Ref. 3: pp. 229-233]:

$$U(\theta,\phi) = (AF)^2 \quad (W/solid\ angle) \tag{5.7}$$

The radiated power is [Ref. 3: pp. 28]:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta,\phi)\sin(\theta)\,d\theta\,d\phi \quad (W) \tag{5.8}$$

In (5.8), P_{rad} is computed by integrating over all angles above the x-y plane ($0 \le \theta \le \pi/2$) [Ref. 7: p. 78]. This assumes that the array has a pattern maxima above the x-y plane with no radiation below the x-y plane. The directivity for the circular array is [Ref. 3: pp. 229-233]:

$$D_o = \frac{4\pi U_{max}}{P_{rad}} \quad (dimensionless) \tag{5.9}$$

In (5.9), U_{max} is unity and occurs at $\theta = \theta_o$ and $\phi = \phi_o$ since the array factor is normalized.

Effective isotropic radiated power (EIRP) is determined by [Ref. 4: p. 62]:

$$EIRP = P_{rad} D_o \quad (W) \tag{5.10}$$

Assuming that the power delivered to a circular array of N isotropic, uniformly spaced radiating elements is equivalent to the radiated power, we can estimate then the radiation resistance (R_r) of the array as [Ref. 3: p. 55]:

$$R_r = \frac{2\,(P_{rad})}{|I_o|^2} \quad (\Omega) \tag{5.11}$$

Since the radiating elements are assumed to be ideal point sources, the input resistance of a circular array (R_{in}) is estimated to be equivalent to the radiation resistance:

$$R_{in} = R_r \quad (\Omega) \tag{5.12}$$

Since the input resistance is estimated, the gain (G) of a circular array cannot be calculated, but an ideal value can be derived. As with linear and planar arrays, addressed in Chapter III and Chapter IV, respectively, the total antenna efficiency (ε_t) is [Ref. 3: pp. 44-45]:

$$\varepsilon_t = \varepsilon_r \varepsilon_{cd} = 1 - |\Gamma|^2 \quad (dimensionless) \tag{5.13}$$

where

$$\varepsilon_{cd} = 1 \quad (dimensionless) \tag{5.14}$$

$$\varepsilon_r = 1 - |\Gamma|^2 \quad (dimensionless) \tag{5.15}$$

$$\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o} \quad (dimensionless) \tag{5.16}$$

In (5.14), ε_{cd} is the conduction and dielectric efficiency, which is unity for an ideal lossless antenna. In (5.16), Z_o is the characteristic impedance of a transmission line, and Γ is the voltage reflection coefficient. Therefore, the total antenna efficiency is equal to the mismatch efficiency of (5.15) when an ideal lossless antenna is assumed, and the gain of a circular array is [Ref. 3: pp. 43-44]:

$$G = \varepsilon_t D_o \quad (dimensionless) \tag{5.17}$$

$$G(dB) = 10\log_{10}(\varepsilon_t D_o) \quad (dB) \tag{5.18}$$

The maximum effective aperture (A_{em}) of a circular array is derived assuming polarization mismatches are negligible (this Mathcad application is constructed using isotropic radiating elements for which polarization loss factor (PLF) and antenna polarization are not calculated) [Ref. 3: p. 63]:

$$A_{em} = \frac{G(\lambda)^2}{4\pi} PLF \qquad (m^2) \tag{5.19}$$

$$PLF = 1 \qquad (dimensionless) \tag{5.20}$$

The effective height (h_{em}) of a circular array is estimated by [Ref. 6: p. 42]:

$$h_{em} = 2\sqrt{\frac{R_r A_{em}}{\eta_o}} \qquad (m) \tag{5.21}$$

In (5.21), η_o is the characteristic impedance of free space (120π).

Four two-dimensional array pattern displays are produced in this Mathcad application: the first is the x-z plane; the second is the y-z plane; the third is a plane perpendicular to the x-y plane when $\phi = \pi/4$; the last is a plane perpendicular to the x-y plane when $\phi = \phi_o$. The last display is redundant when ϕ is either 0, $\pi/4$, or $\pi/2$.

The directivity of a uniform circular array approaches the number of elements (N) as the radius (a) of the array becomes very large [Ref. 3: p. 277]. Table 5.1 provides the directivity of a uniform circular array as a function of the radius of the array.

Table 5.1 Circular Array Directivity Data

RADIUS (m)	NUMBER OF ELEMENTS	DIRECTIVITY (D_o)
1.59	10	13.70 dB
10	10	12.83 dB
20	10	12.89 dB
50	10	11.14 dB
100	10	10.33 dB

Figures 5.2 and 5.3 provide comparisons of the array radiation patterns computed for a 10-element, uniformly spaced, equally excited circular array with a radius of 1.59 meters with those in [Ref. 3]. Figure 5.2 is the radiation pattern in the x-z plane, and the radiation pattern in the y-z plane is shown in Figure 5.3. For these patterns, $\theta_o = \phi_o = 0°$. The array radiation patterns generated by this application are identical to the patterns given in [Ref. 3].

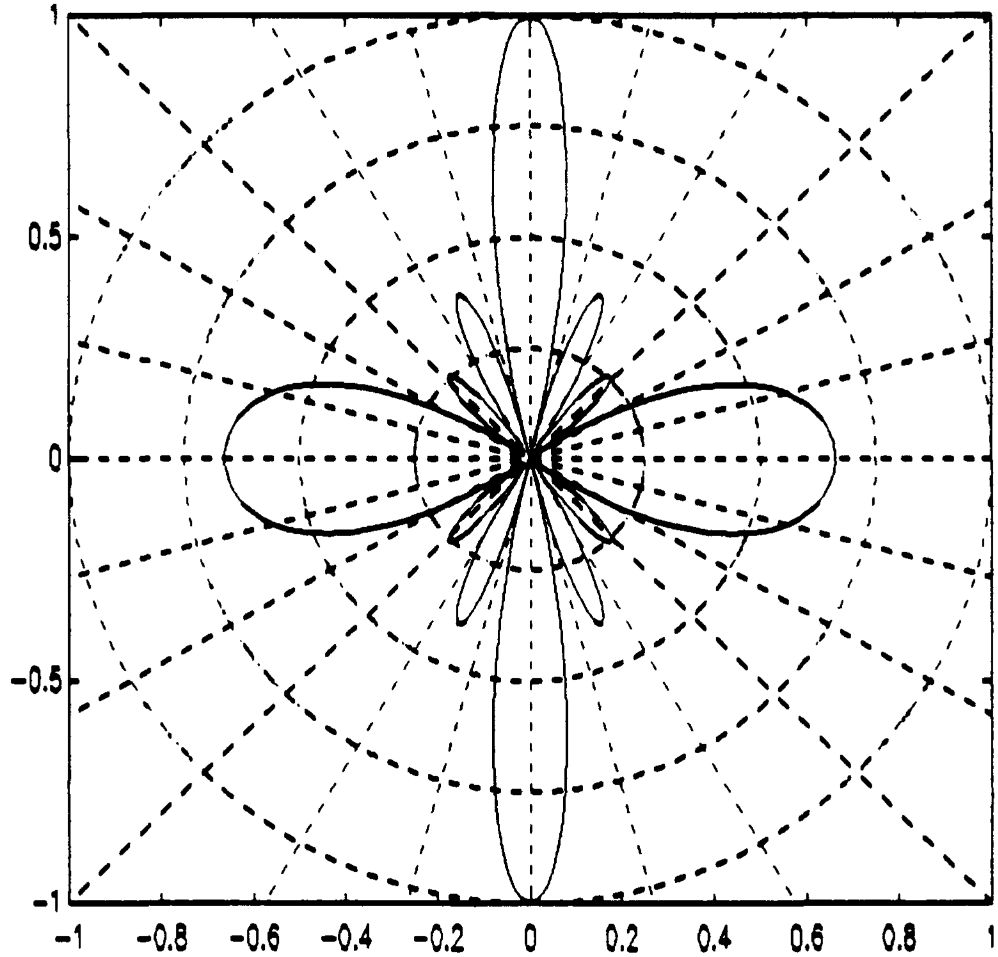


FIGURE 5.2 Array factor pattern of a 10-element, uniformly spaced, equally excited circular array with a = 1.59 m, and $\theta_o = \phi_o = 0°$, in the x-z plane

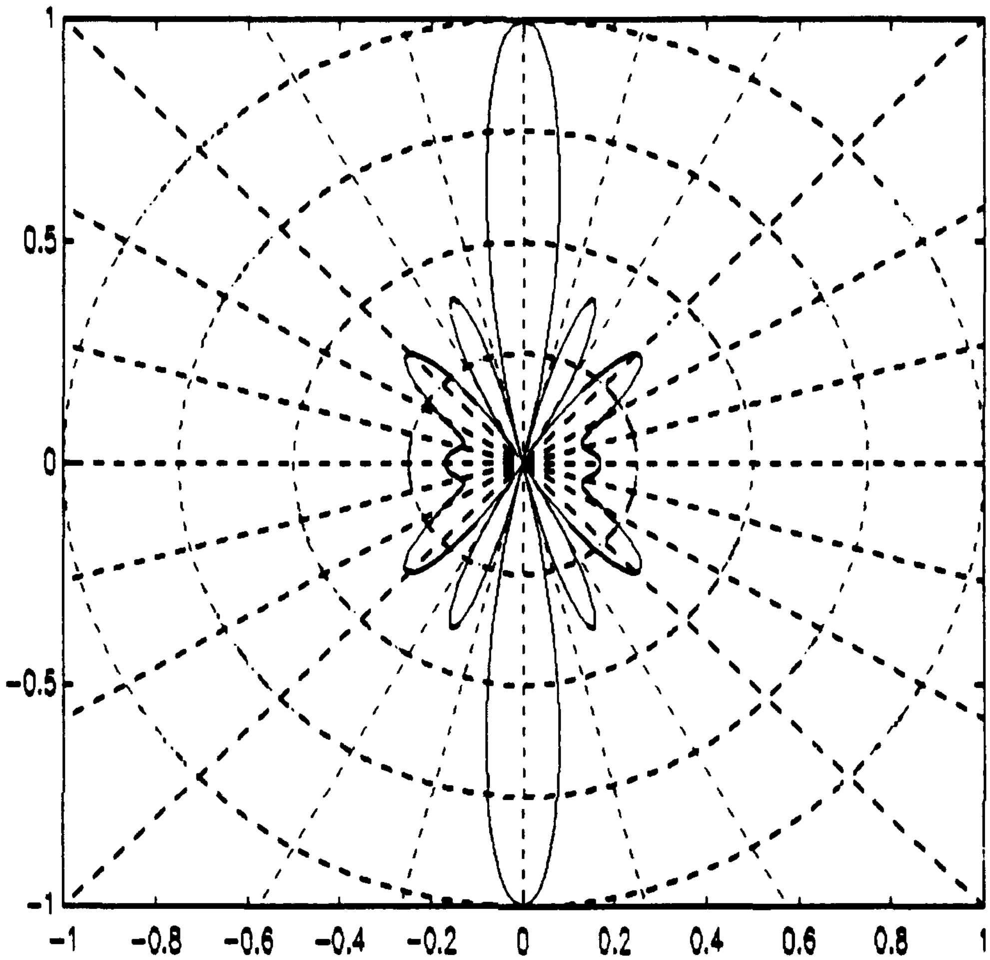


FIGURE 5.3 Array factor pattern of a 10-element, uniform spaced, equally excited circular array with a = 1.59 m, and $\theta_o = \phi_o = 0°$, in the y-z plane

VI. THE FOLDED DIPOLE ANTENNA

The folded dipole is a center-fed wire antenna that performs in a manner similar to that of a linear, center-fed thin-wire antenna with the added advantage of a more easily matched input impedance [Ref. 8: p. 7-36]. The folded dipole antenna consists of two parallel dipoles connected at the ends to form a narrow wire loop. The folded dipole antenna is illustrated in Figure 6.1.

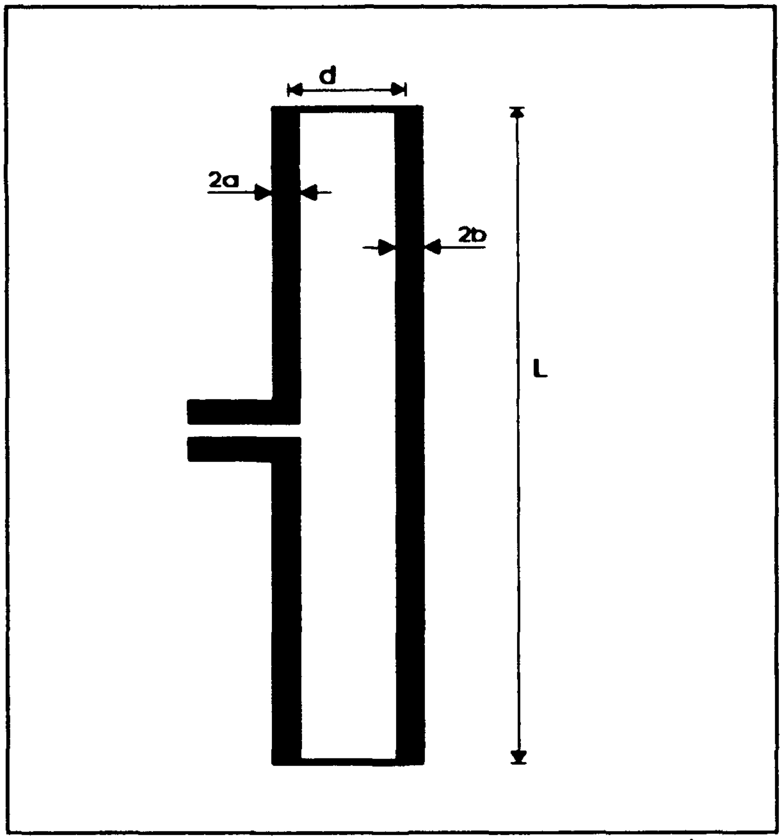


FIGURE 6.1 Geometry of folded dipole antenna

The folded dipole can be assumed to behave as a thin-wire dipole when the following are satisfied: the length of the antenna is much greater than the diameter of the feed conductor ($L \gg 2a$); the distance between the conductors is greater than the radius of the feed conductor ($d > a$); $(2\pi d/\lambda)^2 \ll 1$; all conductors are assumed to be perfect [Ref. 9: p. 172];

This Mathcad application calculates the parameters of a folded dipole by assuming the antenna behaves as a thin-wire dipole with an equivalent radius (a_e). The current maximum is normalized to one amp. The equivalent radius is [Ref. 8: p. 7-38]:

$$a_e \approx \exp\left[\frac{a^2 \ln(a) + b^2 \ln(b) + 2ab \ln(d)}{(a+b)^2}\right] \quad (meters) \qquad (6.1)$$

This application provides the characteristics of the antenna both in free space and positioned horizontally over the earth. Although the folded dipole positioned vertically over the earth occurs frequently in VHF/UHF communications, it is not addressed in this research.

The following inputs are required to analysis a folded dipole:

a = radius of feed conductor

b = radius of second conductor

d = spacing between conductors

L = length of antenna

h = height of antenna above ground plane

f = frequency of interest *

ε_{cd} = conduction/dielectric efficiency of conductors *

ε_r = relative dielectric constant of ground plane *

σ = conductivity of ground plane *

r_{ff} = distance for far-field parameter calculations

σ_w = incoming wave electric field unit vector for antenna in free space *

σ_a = unit polarization vector for antenna in free space*

σh_w = incoming wave electric field unit vector for horizontally positioned antenna *

θh_p = antenna polarization direction for horizontally positioned antenna *

The first five inputs are physical dimensions obtained through photographs, the * indicates input parameters that are either known or estimated.

A. FOLDED DIPOLE IN FREE SPACE

Assuming the folded dipole behaves as a thin-wire dipole with $a_e \ll d \ll L$, we have the far-field electric field intensity of the antenna aligned with the z-axis as [Ref. 3: p. 120]:

$$E(\theta) = j\eta_o \frac{I_o e^{-jkr_{ff}}}{2\pi r_{ff}} \left[\frac{\cos\left(\frac{kL}{2}\cos(\theta)\right) - \cos\left(\frac{kL}{2}\right)}{\sin(\theta)} \right] \quad (V/m) \qquad (6.2)$$

In (6.2), η_o is the intrinsic impedance of free space, equal to 120π, and $k = 2\pi/\lambda$ is the wavenumber. The normalized electric field is found by dividing (6.2) by the maximum value of the electric field which occurs at $\theta = \pi/2$ for L less than about 1.25λ. Clearly, the radiation pattern of the folded dipole is identical to that of a conventional dipole of length L.

An observation point is considered to be in the far-field if all of the following are satisfied [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \quad (meters) \tag{6.3}$$

$$r \geq 5L \quad (meters) \tag{6.4}$$

$$r \geq \frac{2L^2}{\lambda} \quad (meters) \tag{6.5}$$

The minimum distance to the far-field is found by comparing the values of (6.3), (6.4), and (6.5) and selecting the maximum value. The distance between the conductors is not taken into account in determining the minimum distance to the far-field since it is assumed that the folded dipole has an over all effective radius (a_e).

Directivity (D_o) for the folded dipole is found by determining the radiation intensity ($U(\theta)$) and radiated power (P_{rad}). Since the radiation pattern is symmetric in the x-y plane, $U(\theta)$ and P_{rad} are functions of θ only. Radiation intensity is [Ref. 3: p. 28]:

$$U(\theta) = \frac{I_{ff}^2}{2\eta_o}|E(\theta)|^2 \quad (W/\,solid\ angle) \tag{6.6}$$

The radiated power is [Ref. 3: pp. 28]:

$$P_{rad} = 2\pi\int_0^{\pi} U(\theta)\sin(\theta)\,d\theta \quad (W) \tag{6.7}$$

Directivity for a folded dipole is [Ref. 3: pp. 29]:

$$D_o = \frac{4\pi U_{max}}{P_{rad}} \quad (dimensionless) \tag{6.8}$$

where $U_{max} = U(\pi/2)$.

Effective isotropic radiated power (EIRP) is determined by [Ref. 4: p. 62]:

$$EIRP = P_{rad}D_o \quad (W) \tag{6.9}$$

The folded dipole is unique in its capability to act as a step up impedance transformer when the antenna is resonant. The antenna effectively operates as an unbalanced transmission line, and it can be analyzed by decomposing the current into two distinct modes: a transmission line mode [Figure 6.2] and an antenna mode [Figure 6.3]. [Ref. 3: p. 341; Ref. 4: pp. 205-207]

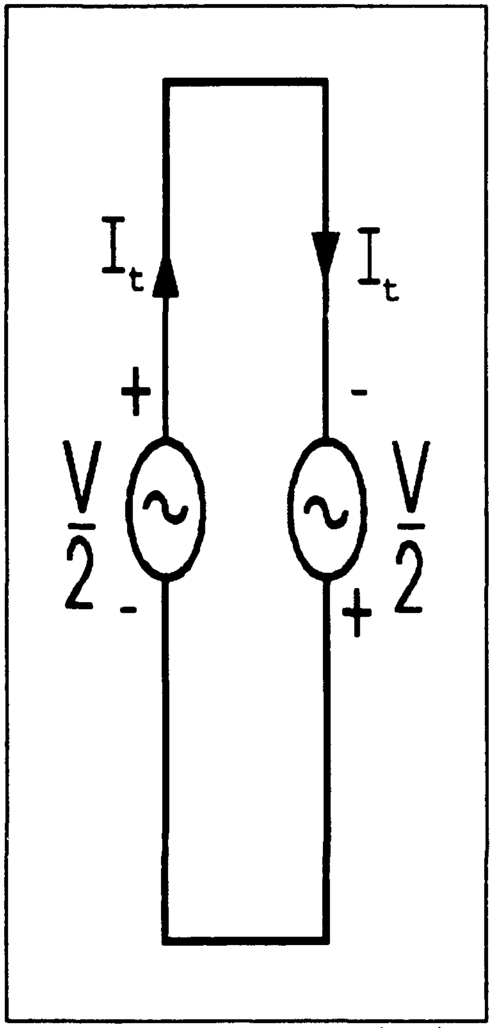


FIGURE 6.2 Transmission line mode

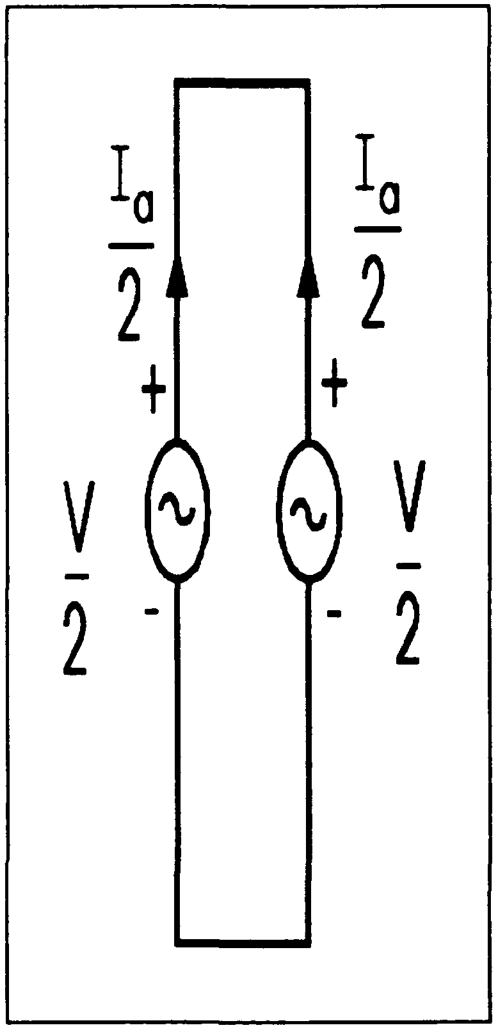


FIGURE 6.3 Antenna mode

The input impedance (Z_{in}) at the feed for the folded dipole is given in (6.10), and the radiation resistance (R_{in}) for the antenna is the real component of (Z_{in})) [Ref. 3: p.342]:

$$Z_{in} = \frac{4 Z_t Z_d}{2 Z_d + Z_t} \quad (\Omega) \qquad (6.10)$$

In (6.10), Z_t is the input impedance of the transmission line mode and is obtained from the impedance transfer equation

$$Z_t = jZ_o \tan\left(\frac{kL}{2}\right) \quad (\Omega) \tag{6.11}$$

where Z_o in (6.11) is the characteristic impedance of a two-wire transmission line and is given in (6.12) [Ref. 3: pp. 341-342; Ref. 8: p. 7-39]:

$$Z_o = \frac{\eta_o}{\pi}\cosh^{-1}\left(\frac{d}{2\sqrt{ab}}\right) \quad (\Omega) \tag{6.12}$$

The input impedance of a linear dipole of length L and equivalent radius a_e is calculated by [Ref. 3: pp. 124, 127, 294, and 342]:

$$Z_d = R_d + jX_d \quad (\Omega) \tag{6.13}$$

$$R_d = \frac{R_r}{\sin^2\left(\frac{kL}{2}\right)} \quad (\Omega) \tag{6.14}$$

$$X_d = \frac{X_r}{\sin^2\left(\frac{kL}{2}\right)} \quad (\Omega) \tag{6.15}$$

$$\begin{aligned} R_r = {} & \frac{\eta_o}{2\pi}[\gamma + \ln(kL) - C_i(kL)] \\ & + \frac{\eta_o}{2\pi}\left[\frac{1}{2}\sin(kL)\,[S_i(2kL) - 2S_i(kL)]\right] \\ & + \frac{\eta_o}{2\pi}\left[\frac{1}{2}\cos(kL)\left(\gamma + \ln\left(\frac{kL}{2}\right) + C_i(2kL) - 2C_i(kL)\right)\right] \end{aligned} \tag{6.16}$$

$$X_r = \frac{\eta_o}{4\pi}[2S_i(kL)+\cos(kL)[2S_i(kL)-S_i(2kL)]]$$

$$-\frac{\eta_o}{4\pi}\left[\sin(kL)\left(2C_i(kL)-C_i(2kL)-C_i\left(\frac{2ka_e^2}{L}\right)\right)\right] \quad (6.17)$$

In (6.16) and (6.17), $C_i(x)$ and $S_i(x)$ are the cosine and sine integrals, respectively, [Ref. 3: pp. 124 and 743-746]:

$$C_i(x) = \int_{\infty}^{x}\frac{\cos(\tau)}{\tau}d\tau = \gamma+\ln(x)-\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^{2n}}{2n(2n)!} \quad (6.18)$$

$$S_i(x) = \int_0^x\frac{\sin(\tau)}{\tau}d\tau = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^{(2n-1)}}{(2n-1)(2n-1)!} \quad (6.19)$$

and $\gamma = 0.57721$ (Euler's constant).

Mathcad computes summations more quickly than integrals. In this application, the summation forms of (6.18) and (6.19) are used with the index n ranging from 1 to 50 to ensure proper convergence. The maximum value of the argument of either $C_i(x)$ or $S_i(x)$ is $x = 4\pi L/\lambda$, and for $L = \lambda$ the succeeding terms in the summation rapidly decrease for $n > 20$.

The gain (G) of a folded dipole is obtained as the product of the antenna reflection efficiency (ε_t) and the directivity of (6.8) where the antenna reflection efficiency is [Ref. 3: pp. 43-45]:

$$\varepsilon_t = \varepsilon_r\varepsilon_{cd} = \varepsilon_{cd}(1-|\Gamma|^2) \quad (dimensionless) \quad (6.20)$$

where

$$\varepsilon_r = 1 - |\Gamma|^2 \quad (dimensionless) \tag{6.21}$$

$$\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o} \quad (dimensionless) \tag{6.22}$$

In (6.21) ε_r is the mismatch efficiency and ε_{cd} is the conduction and dielectric efficiency in (6.22). In (6.22), Z_o is the characteristic impedance of the transmission line given in (6.12). Therefore, the gain of the antenna is:

$$G = \varepsilon_t D_o \quad (dimensionless) \tag{6.23}$$

$$G(dB) = 10\log_{10}(\varepsilon_t D_o) \quad (dB) \tag{6.24}$$

The folded dipole in free space aligned on the z-axis is horizontally polarized with respect to the x-y plane. Therefore, given the direction of the incoming wave's unit polarization vector (σ_w), the polarization loss factor (PLF) is [Ref. 3: pp. 48-53]:

$$PLF = |\vec{\sigma}_w \cdot \vec{\sigma}_a^*|^2 \quad (dimensionless) \tag{6.25}$$

The maximum effective aperture (A_{em}) of a folded dipole is [Ref. 3: p.63]:

$$A_{em} = \frac{G(\lambda)^2}{4\pi} PLF \quad (m^2) \tag{6.26}$$

The effective height (h_{em}) of the folded dipole is [Ref. 6: p. 42]:

$$h_{em} = 2\sqrt{\frac{R_r A_{em}}{\eta_o}} \qquad (m) \qquad (6.27)$$

The bandwidth of the folded dipole antenna is dependant on the length of the wires and has better bandwidth characteristics than a conventional dipole of the same length. It can be assumed that the bandwidth of the folded dipole is essentially the same as that of a conventional dipole of an radius a_e ($a < a_e < d/2$) [Ref. 3: p. 346]. The bandwidth is increased when the distance between the conductors (d) is increased and/or when the conductor radii a and b are increased [Ref. 10: p. 190].

Tables 6.1 and 6.2 are comparisons of measured and calculated input impedance data for the folded dipole as a function of its electrical length. The radii of the conductors (a, b) are the same (0.0005 m) while the spacing between the conductors (d) is held constant at 0.00625 m and λ = 1 m [Ref. 8: pp. 7-40 - 7-41].

TABLE 6.1 Folded Dipole Antenna Data Comparison

ANTENNA LENGTH (L)	MEASURED $Re(Z_{in})$ Ω	CALCULATED $Re(Z_{in})$ Ω
λ/π	2000	6533 **
$2\lambda/\pi$	2400	2592
$3\lambda/\pi$	0.0	0.1
$4\lambda/\pi$	300	324
$5\lambda/\pi$	1700	1685
$6\lambda/\pi$	0.5	1.0
$7\lambda/\pi$	100	100

TABLE 6.2 Folded Dipole Antenna Data Comparison

ANTENNA LENGTH (L)	MEASURED $Im(Z_{in})$ Ω	CALCULATED $Im(Z_{in})$ Ω
λ/π	5000	5437
$2\lambda/\pi$	-650	-1204 **
$3\lambda/\pi$	-90	-85
$4\lambda/\pi$	1000	1061
$5\lambda/\pi$	700	793
$6\lambda/\pi$	-190	-174
$7\lambda/\pi$	600	633

To further test the validity of the Mathcad application, a numerical antenna analysis program was also used. The program used was ELNEC, a powerful, easy-to-use program for modeling and analyzing a wide variety of antenna types including ground effects and parasitic structures. The fundamental computation portion of ELNEC is the same as MININEC (Version 3), that was developed by the Naval Ocean Systems Center [Ref. 11: pp. 5-7].

Table 6.3 is a comparison of the gain for a folded dipole in free space with different dimensions [Ref. 8: p. 3-29]. For each case, the radii of the conductors (0.0005 m) and the distance between the conductors (12.5a m) are the same. As can be seen, the comparison between the gains obtained from various sources is very good.

TABLE 6.3 Folded Dipole Antenna Data Comparison

ANTENNA DIMENSION	MEASURED Gain (G)	ELNEC Gain (G)	Mathcad Gain (G)
$L = \lambda/4$ m $d = L/13$ m	1.71 dB	1.70 dB	1.64 dB
$L = \lambda/2$ m $d = L/25.5$ m	2.14 dB	2.20 dB	2.15 dB

The computed parameters in Tables 6.1, 6.2, and 6.3 are essentially the same as those measured or computed with ELNEC with exception of the ones marked with **. These discrepancies result in using the transmission line model and equivalent radius to analysis the folded dipole in this application.

As previously mentioned, the folded dipole is a center-fed wire antenna that performs in a manner similar to a linear, center-fed, thin-wire antenna. Therefore, the radiation pattern of a folded dipole is also similar to that of a thin-wire antenna. The folded dipole radiation pattern in free space is illustrated in Figure 6.4.

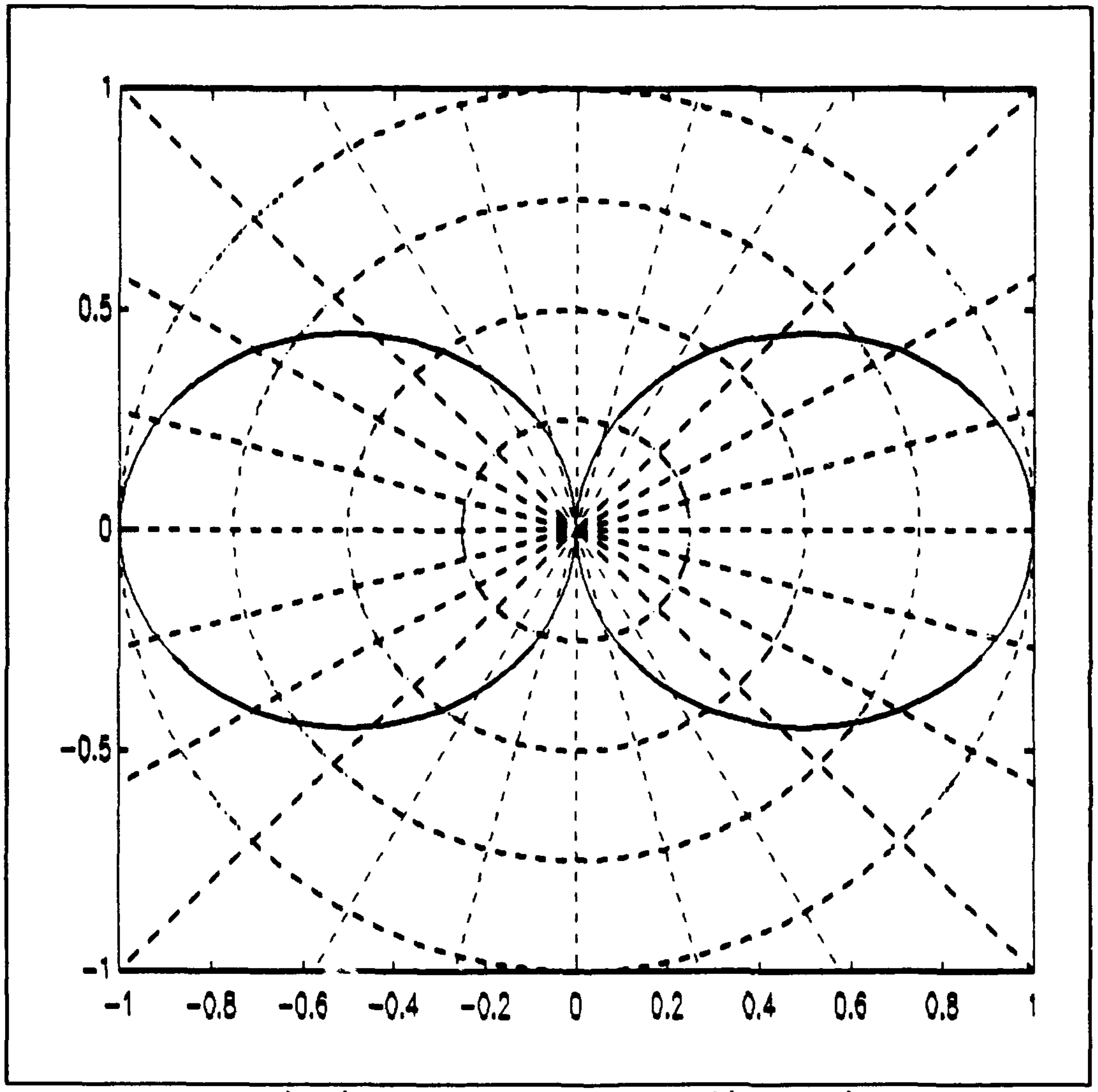


FIGURE 6.4 Radiation Pattern folded dipole in free space with $L = \lambda/2$ m, $a = b = 0.0005$ m, and $a_e = 0.004$ m

Figure 6.5 is a comparison of the radiation pattern of a folded dipole in free space obtained using ELNEC and the Mathcad application. The patterns are not identical, but are similar and oriented in the same direction as expected. The difference can be attributed to the fact that ELNEC takes into account mutual coupling between the conductors while the

Mathcad application models the folded dipole as a thin-wire, center-fed dipole.

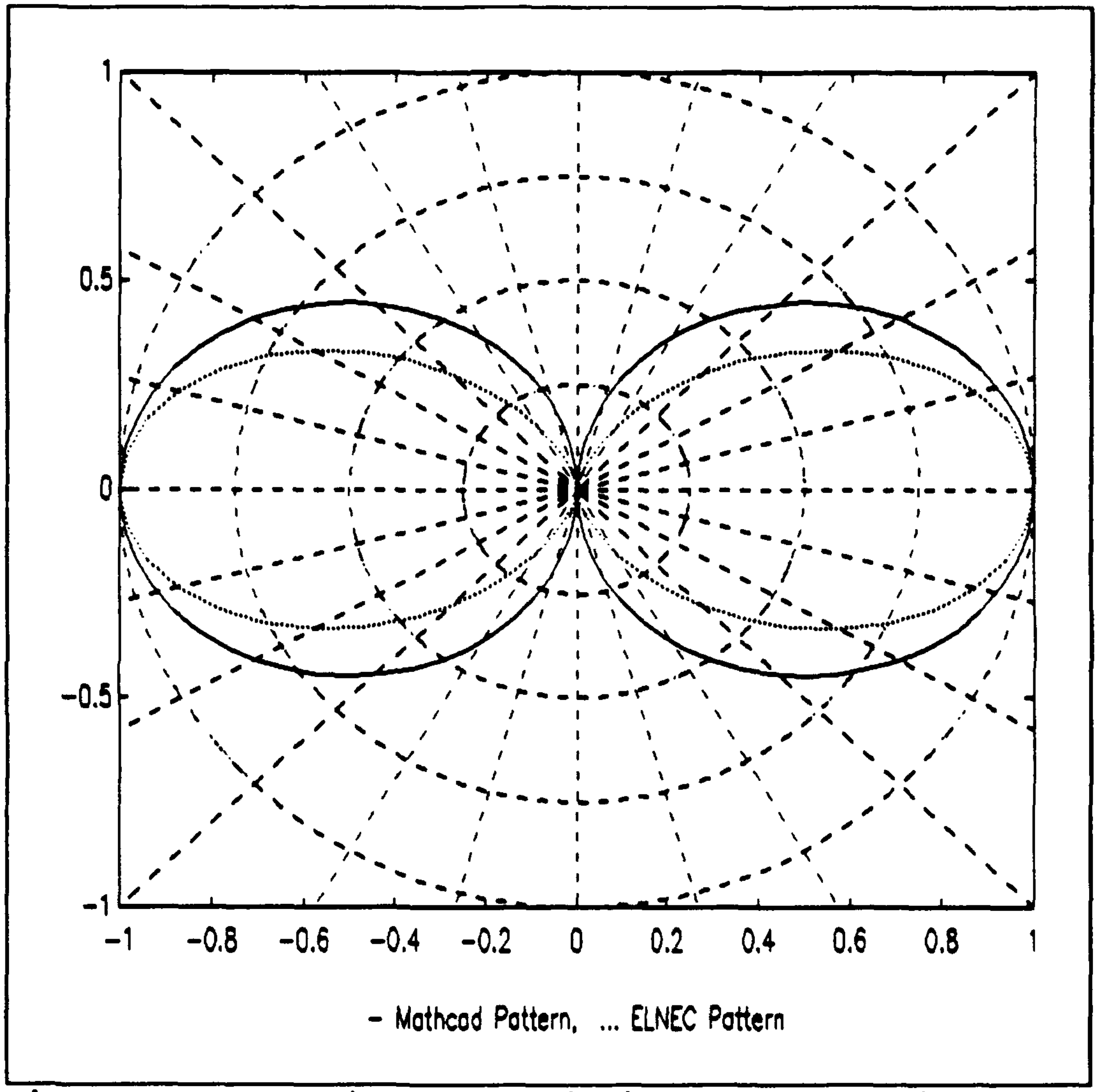


Figure 6.5 Comparison of radiation patterns using ELNEC and Mathcad of a Folded dipole in free space with $L = \lambda/2$ m, $a = b = 0.0005$ m, and $a_e = 0.004$ m

B. FOLDED DIPOLE POSITIONED HORIZONTALLY OVER THE EARTH

Through the use of image theory, the far-field electric field intensity of a folded dipole oriented horizontally above a flat earth can be obtained. The θ component of the electric field of a horizontal dipole (parallel to the y-axis) in free space is [Ref. 3: pp. 143-144]:

$$E\theta(\theta,\phi) = j\eta_o \frac{I_o e^{-jkr_{ff}}}{2\pi r_{ff}} \left[\frac{\cos\left(\frac{kL}{2}\sin(\theta)\sin(\phi)\right) - \cos\left(\frac{kL}{2}\right)}{\sqrt{1-\sin^2(\theta)\sin^2(\phi)}} \right] \quad (V/m) \qquad (6.28)$$

From image theory, the θ component of the electric field over ground is [Ref. 4: pp. 229-235]:

$$E\theta_{gp}(\theta) = E\theta(\theta,\phi)\left(e^{jkh\cos(\theta)} - \Gamma_v e^{-jkh\cos(\theta)}\right) \quad (V/m) \qquad (6.29)$$

In (6.28) and (6.29), $E_\theta(\theta,\phi)$ is valid only above the ground ($0 \le \theta \le \pi/2$) [Ref. 3: pp. 135-142]. In (6.29), h is the distance from the earth to the antenna and the vertical reflection coefficient (Γ_v) of the ground is [Ref. 4: pp. 231-233]:

$$\Gamma_v = \frac{\varepsilon'_r\cos(\theta) - \sqrt{\varepsilon'_r - \sin^2(\theta)}}{\varepsilon'_r\cos(\theta) + \sqrt{\varepsilon'_r - \sin^2(\theta)}} \quad (dimensionless) \qquad (6.30)$$

where

$$\varepsilon'_r = \varepsilon_r - j\frac{\sigma}{2\pi f\varepsilon_o} \quad (dimensionless) \qquad (6.31)$$

In (6.30) and (6.31), (ε'_r) is the relative complex effective dielectric constant of the ground, (ε_r) is the relative dielectric constant of the ground, and (σ) is the conductivity of the ground. For a perfect ground plane, $\sigma \to \infty$ and $\Gamma_v = 1$.

The Φ component of the electric field of a horizontal dipole (parallel to the y-axis) in free space is [Ref. 3: pp. 135-142]:

$$E\phi(\theta,\phi) = j\eta_o \frac{I_o e^{-jkr_{ff}}}{2\pi r_{ff}} \left[\frac{\cos\left(\frac{kL}{2}\sin(\theta)\sin(\phi)\right) - \cos\left(\frac{kL}{2}\right)}{\sqrt{1-\sin^2(\theta)\sin^2(\phi)}} \right] \quad (V/m) \qquad (6.32)$$

Now from image theory, the Φ component of the electric field over earth is [Ref. 4: pp. 229-235]:

$$E\phi_{vp}(\theta) = E\phi(\theta,\phi)\left(e^{jkh\cos(\theta)} + \Gamma_h e^{-jkh\cos(\theta)}\right) \quad (V/m) \qquad (6.33)$$

In (6.33), (Γ_h) is the horizontal reflection coefficient of the ground and is [Ref. 4: pp. 231-232]:

$$\Gamma_h = \frac{\cos(\theta) - \sqrt{\varepsilon'_r - \sin^2(\theta)}}{\cos(\theta) + \sqrt{\varepsilon'_r - \sin^2(\theta)}} \quad (dimensionless) \qquad (6.34)$$

When $\sigma \to \infty$, $\Gamma_h = -1$.

The radiation intensity ($U_h(\theta,\phi)$) of a folded dipole over a earth must take into account both the θ and Φ components of the electric field. Therefore, the radiation intensity is [Ref. 3: p. 28]:

$$U_h(\theta) = \frac{r_{ff}^2}{2\eta_o}\left[|E\theta_{gp}(\theta)|^2 + |E\phi_{gp}(\theta)|^2\right] \quad (W/\,solid\ angle) \qquad (6.35)$$

The radiated power (Ph_{rad}) follows (6.7) except that the field is present only above the earth. Thus,

$$Ph_{rad} = 2\pi\int_0^{\frac{\pi}{2}} U_h(\theta)\sin(\theta)\,d\theta \quad (W) \qquad (6.36)$$

Directivity (Dh_o) and effective isotropic radiated power (EIRPh) for a horizontal folded dipole over the earth are identical to (6.8) and (6.9), respectively, with the results of (6.36) inserted.

Input resistance (Rh_{in}) for the folded dipole over the earth plane is found by (where I_o in (6.32) is normalized to one amp) [Ref. 3: p. 124]:

$$Rh_{in} = \frac{Rh_r}{\sin^2\left(\frac{kL}{2}\right)} \quad (\Omega) \qquad (6.37)$$

where the radiation resistance (Rh_r) is,

$$Rh_r = \frac{2\,Ph_{rad}}{|I_o^2|} \quad (\Omega) \qquad (6.38)$$

The voltage reflection coefficient (ΓH), gain (Gh), maximum effective aperture (Ah_{em}), and maximum effective height (hh_{em}) for a horizontal folded dipole over the earth are determined using the equations for those parameters for a folded dipole in free space, as listed above.

The Polarization Loss Factor (PLFh) for the antenna over the earth can be determined when the direction of the main beam of the polarized antenna (θh_p) is estimated. The antenna polarization vector (σh_a) is then calculated, and estimating the unit polarization vector of the incoming wave (σh_w), the PLF is found from (6.25).

Table 6.4 is a comparison of measured [Ref. 8: p. 3-29], ELNEC, and Mathcad software gains for a horizontal folded dipole over a reflecting sheet. The radii of the conductors (0.0067 m) are the same. As can be seen, the Mathcad software gain is similar to the measured and ELNEC gains.

TABLE 6.4 Folded Dipole Antenna Data Comparison

ANTENNA DIMENSION	MEASURED Gain (G)	ELNEC Gain (G)	Mathcad Gain (G)
$L = \lambda/2$ m $d = L/25.5$ m $h = \lambda/8$ m	7.14 dB	6.98 dB	7.19 dB

No input impedance data has been found to compare a folded dipole orie: ì horizontally over the earth with that computed by this Mathcad application.

Figures 6.6 and 6.7 are the H-plane and E-plane radiation patterns of a folded dipole oriented horizontally over the earth, respectively. Since the folded dipole is similar to a thin-wire center-fed dipole of length L, the illustrations below are similar to those of a dipole of length L.

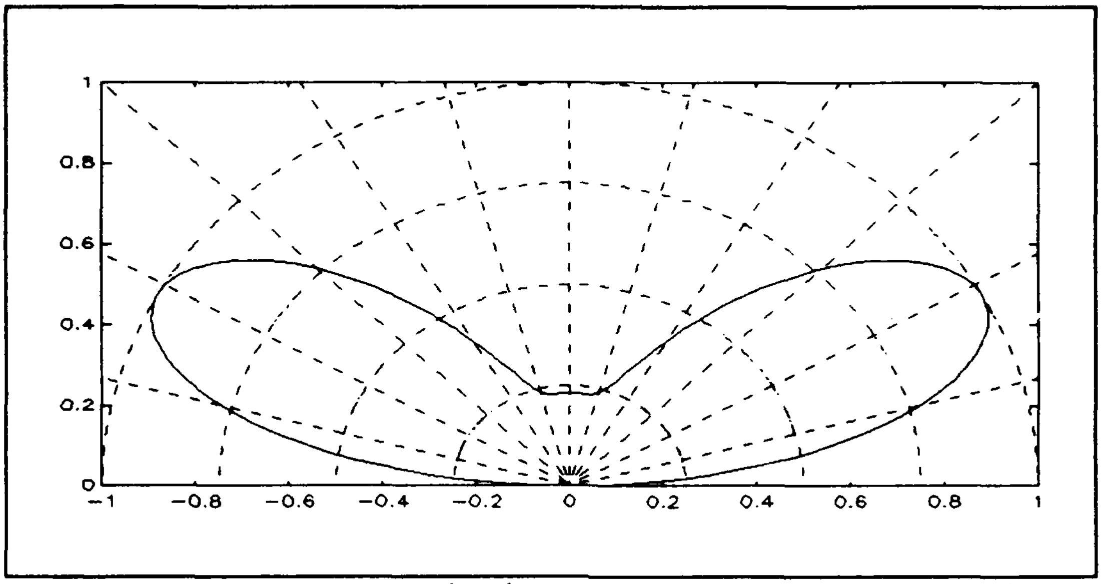


FIGURE 6.6 H-plane radiation pattern of a folded dipole with $L = \lambda/2$ m, $a = b = 0.0005$ m, $d = 0.00625$ m, $h = 0.5$ m, $\sigma = 0.01$ S/m, $\varepsilon_r = 15$

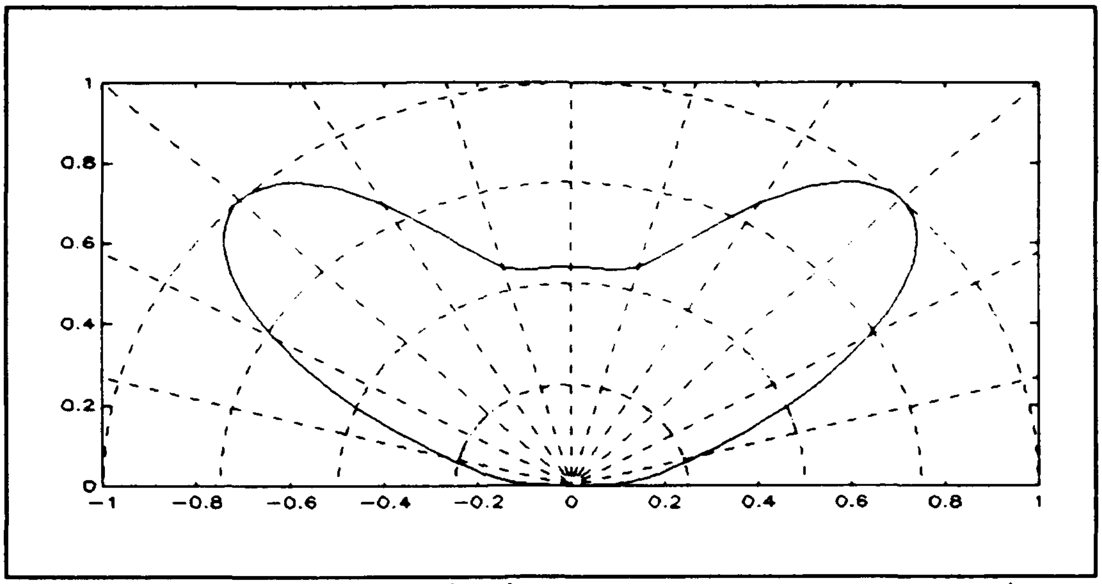


FIGURE 6.7 E-plane radiation pattern of a folded dipole with L = $\lambda/2$ m, a = b = 0.0005 m, d = 0.00625 m, h = 0.5 m, σ = 0.01 S/m, ε_r = 15

Figures 6.8 and 6.9 are comparisons of the H-plane and E-plane radiation patterns between those obtained with ELNEC and Mathcad software of a folded dipole oriented horizontally over the earth, respectively. As expected, the patterns are similar. As previosly mentioned, the difference can be attributed to the fact that ELNEC takes into account mutual coupling between the conductors while the Mathcad application is modeled after a thin-wire, center-fed dipole.

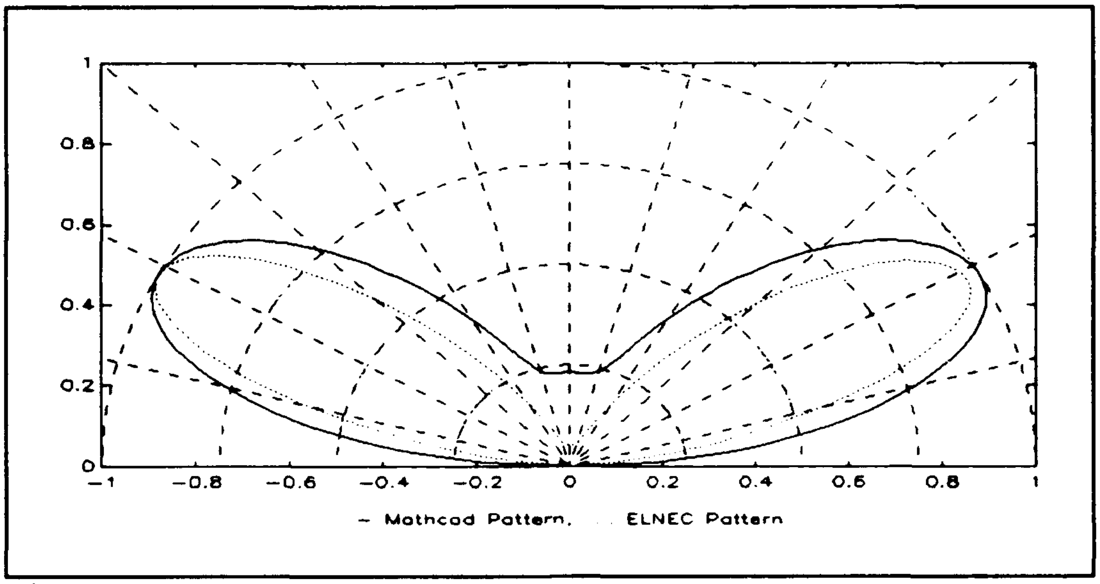


Figure 6.8 Comparison between ELNEC and Mathcad of H-plane radiation patterns of a folded dipole with L = $\lambda/2$ m, a = b = 0.0005 m, d = 0.00625 m, h = 0.5 m, σ = 0.01 S/m, ε_r = 15

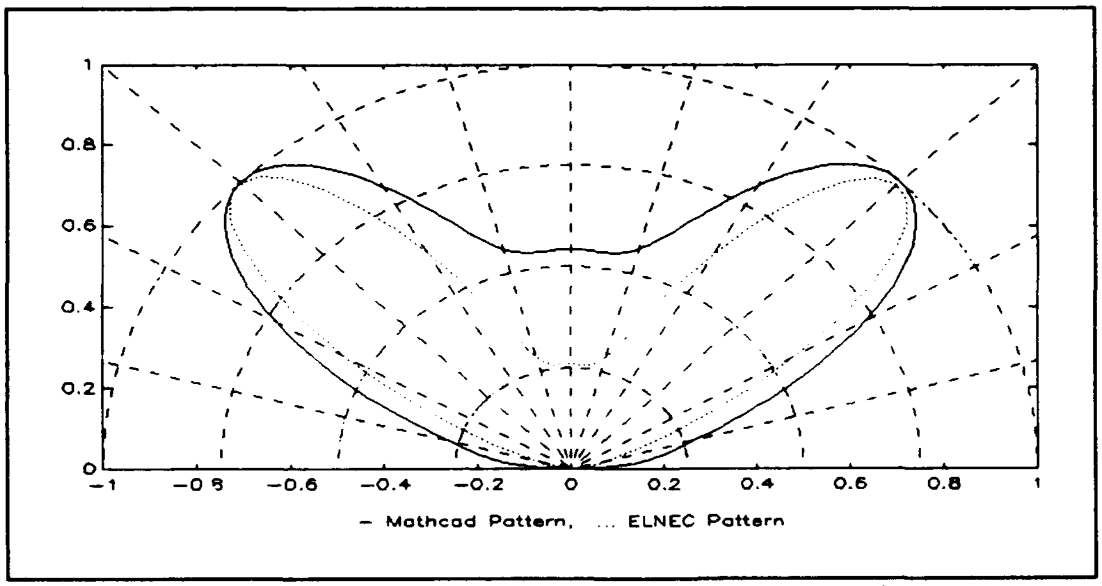


Figure 6.9 Comparison between ELNEC and Mathcad of E-plane radiation patterns of a folded dipole with L = $\lambda/2$ m, a = b = 0.0005 m, d = 0.00625 m, h = 0.5 m, σ = 0.01 S/m, ε_r = 15

VII. THE CAGED DIPOLE ANTENNA

The caged dipole antenna is used in communications for its broadband antenna characteristics. Its performance is similar to that of a thick cylindrical dipole. Since thick cylindrical dipoles are usually too clumsy to use except at very short wavelengths, thick dipoles are replaced with a caged dipole which consists of a number of thin wire dipoles arranged to form a circular cross section. The antenna is usually supported by a central conductor and support members that provide rigidity but contribute little to the electric properties of the caged dipole. The caged dipole (without the central conductor and the support members) is illustrated in Figure 7.1 [Ref. 12: pp. 172-174].

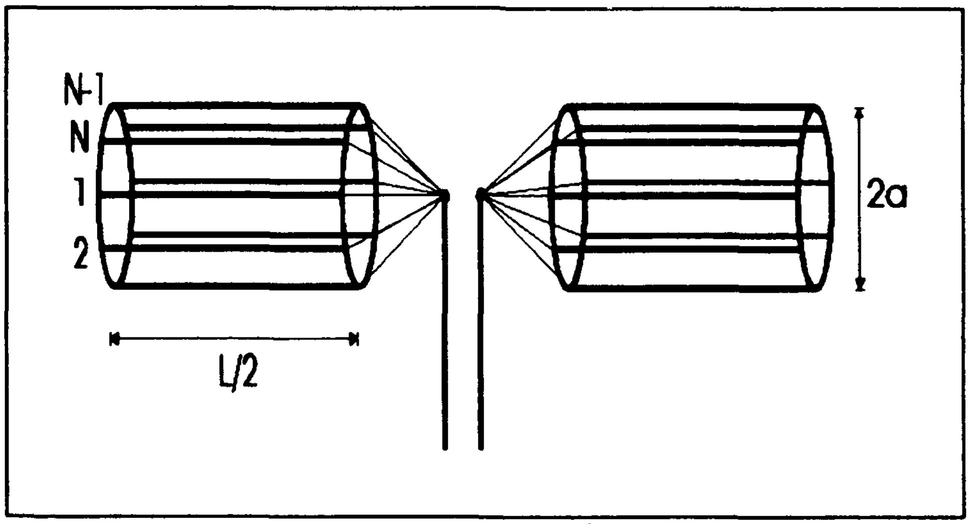


FIGURE 7.1 Geometry of caged dipole antenna

This Mathcad application models the caged dipole antenna as a circular array of center-fed, equally excited, uniformly spaced, thin-wire dipoles. This approach is taken to account for the number of conductors that are present in the caged dipole. As with the arrays previously considered, mutual coupling between the dipoles is neglected. In this manner, this application simulates the characteristics of a thick cylindrical dipole. The thin-wire dipoles are modeled after finite length dipoles in free space that have their length much greater than their diameter (L > wire diameter), and the dipoles are assumed to be perfect conductors. Since the central conductor and the support wires have little effect on the electrical properties of a caged dipole, mutual coupling from these components are not taken into account in this application. As seen in Figure 7.1, the inner ends of a caged dipole, the feed points, maybe coned in order to reduce base capacity [Ref. 12: pp. 172-174]. This effect is also not addressed.

This application provides the characteristics of the antenna in free space as well as and oriented both vertically and horizontally over the earth.

A. CAGED DIPOLE IN FREE SPACE

The following inputs are required to analyze a caged dipole in free space:

N = number of conductors

a = radius of antenna

L = length of antenna

f = frequency of interest *

I_o = antenna feed current *

Z_o = input impedance *

ε_{cd} = conduction/dielectric efficiency of conductors *

r_{ff} = distance for far-field parameter calculations *

σ_w = incoming wave electric field unit vector for antenna *

σ_a = unit polarization vector for antenna *

The first three inputs are physical dimensions obtained primarily from photographs, and the * indicates input parameters that are either known or estimated.

Assuming the caged dipole behaves as a circular array with equally excited (I_n), uniformly spaced elements in the x-y plane with a radius of (a), we have the array factor ($AF(\theta,\phi)$) [Ref. 3: pp. 274-278]:

$$AF(\theta,\phi) = \sum_{n=1}^{N} I_n e^{j[ka\sin\theta\cos(\phi-\Phi_n)]} \quad (dimensionless) \quad (7.1)$$

where the angular position in the nth element in the x-y plane is:

$$\Phi_n = 2\pi\left(\frac{n}{N}\right),\ n = 1, 2, \ldots N \quad (radians) \quad (7.2)$$

Equation (7.1) is derived from:

$$AF(\theta,\phi) = \sum_{n=1}^{N} I_n e^{j[ka\sin\theta\cos(\phi-\Phi_n)+\alpha_n]} \quad (dimensionless) \qquad (7.3)$$

where the phase excitation of the nth element is:

$$\alpha_n = 0 = -ka\sin\theta_o\cos(\phi_o-\Phi_n) \quad (radians) \qquad (7.4)$$

In (7.3), and the phase excitation of the nth element (α_n) is zero as the result of desiring the main beam of the array to be directed perpendicular to the length of the antenna ($\theta_o = \pi/2$). In (7.1), k is the wavenumber ($2\pi/\lambda$).

Assuming each conductor of a caged dipole behaves as a thin-wire dipole, we have the far-field electric field intensity of a single conductor aligned with the z-axis as [Ref. 3: p. 120]:

$$E(\theta) = j\eta_o \frac{I_n e^{-jkr_{ff}}}{2\pi r_{ff}}\left[\frac{\cos\left(\frac{kL}{2}\cos(\theta)\right) - \cos\left(\frac{kL}{2}\right)}{\sin(\theta)}\right] \quad (V/m) \qquad (7.5)$$

In (7.5), $\eta_o = 120\pi$ and is the intrinsic impedance of free space.

As previously discussed in Chapter III, the total electric field of an array constructed with identical elements is the product of the field of a single element and the array factor as a consequence of the principal of pattern multiplication [Ref. 3: p. 207]. Therefore, the total electric field of a caged dipole in free space is approximated as:

$$E_t(\theta,\phi) = E(\theta)\,AF(\theta,\phi) \qquad (V/m) \tag{7.6}$$

An observation point is considered to be in the far-field if all of the following are satisfied [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \qquad (meters) \tag{7.7}$$

$$r \geq 5\sqrt{L^2 + (2a)^2} \qquad (meters) \tag{7.8}$$

$$r \geq \frac{2(L^2 + (2a)^2)}{\lambda} \qquad (meters) \tag{7.9}$$

The minimum distance to the far-field is found by taking the maximum of (7.7) through (7.9). In (7.8) and (7.9), the maximum dimension of the antenna is the diagonal length of the array.

Directivity (D_o) for a caged dipole is found by determining the radiation intensity ($U(\theta,\phi)$) and radiated power (P_{rad}). Radiation intensity is [Ref. 3: p. 28]:

$$U(\theta,\phi) = \frac{I_{ff}^2}{2\eta_o}|E_t(\theta,\phi)|^2 \qquad (W/\,solid\ angle) \tag{7.10}$$

The radiated power is [Ref. 3: pp. 28]:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin(\theta)\,d\theta\,d\phi \qquad (W) \tag{7.11}$$

Directivity for a caged dipole is [Ref. 3: pp. 29]:

$$D_o = \frac{4\pi U_{max}}{P_{rad}} \quad (dimensionless) \tag{7.12}$$

where $U_{max} = U(\pi/2,0)$.

Effective isotropic radiated power (EIRP) is determined by [Ref. 4: p. 62]:

$$EIRP = P_{rad} D_o \quad (W) \tag{7.13}$$

Input resistance (R_{in}) for a caged dipole in free space is found from [Ref. 3: p. 124]:

$$R_{in} = \frac{R_r}{\sin^2\left(\frac{kL}{2}\right)} \quad (\Omega) \tag{7.14}$$

where the radiation resistance (R_r) is:

$$R_r = \frac{2P_{rad}}{|I_n|^2} \quad (\Omega) \tag{7.15}$$

The gain (G) of a caged dipole is obtained as the product of the antenna reflection efficiency (ε_t) and the directivity, where the antenna reflection efficiency is ([Ref. 3: pp. 43-45]:

$$\varepsilon_t = \varepsilon_r \varepsilon_{cd} = \varepsilon_{cd}(1-|\Gamma|^2) \quad (dimensionless) \tag{7.16}$$

where

$$\varepsilon_r = 1-|\Gamma|^2 \quad (dimensionless) \tag{7.17}$$

$$\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o} \quad (dimensionless) \tag{7.18}$$

The parameter ε_r is the mismatch efficiency, ε_{cd} is the conduction and dielectric efficiency which is unity for a lossless conductor, and Z_o is the characteristic impedance of a transmission line. Therefore, the gain of the antenna is:

$$G = \varepsilon_t D_o \quad (dimensionless) \tag{7.19}$$

$$G(dB) = 10\log_{10}(\varepsilon_t D_o) \quad (dB) \tag{7.20}$$

A caged dipole in free space aligned parallel to the z-axis is horizontally polarized with respect to the x-y plane. Therefore, given the direction of the incoming wave's unit polarization vector (σ_w), the polarization loss factor (PLF) is [Ref. 3: pp. 48-53]:

$$PLF = |\vec{\sigma}_w \cdot \vec{\sigma}_a^*|^2 \quad (dimensionless) \tag{7.21}$$

The maximum effective aperture (A_{em}) of a caged dipole is [Ref. 3: p.63]:

$$A_{em} = \frac{G(\lambda)^2}{4\pi} PLF \quad (m^2) \tag{7.22}$$

The effective height (h_{em}) of the caged dipole is [Ref. 6: p. 42]:

$$h_{em} = 2\sqrt{\frac{R_r A_{em}}{\eta_o}} \quad (m) \tag{7.23}$$

The bandwidth of a thick cylindrical dipole antenna is dependent on the ratio of the length of the antenna to the diameter of the antenna (L/2a). When L/2a ≈ 5000 the acceptable bandwidth of a thick cylindrical dipole is about 3% of the center frequency, and when L/2a ≈ 260 the bandwidth is about 30%. Hence, if we assume a linear relationship between bandwidth and L/2a, and since the caged dipole's performance is similar to that of a thick cylindrical dipole, then the bandwidth of a caged dipole is [Ref. 3: p. 333]:

$$BW = f_{high} - f_{low} \quad (Hz) \tag{7.24}$$

where the upper and lower frequencies are:

$$f_{high} = f_c + F\left(\frac{L}{2a}\right) f_c \quad (Hz) \tag{7.25}$$

$$f_{low} = f_c - F\left(\frac{L}{2a}\right) f_c \quad (Hz) \tag{7.26}$$

In (7.25) and (7.25), f_c is the center frequency and F(L/2a) is the linear relationship function between antenna length and diameter. This relationship is obtained by solving $y = mx + f_c$ for m using $x_1 = 5000$, $y_1 = 3\%$ and $x_2 = 260$, $y_2 = 30\%$. The relationship becomes:

$$F\left(\frac{L}{2a}\right) = -5.696 \cdot 10^{-5} \left(\frac{L}{2a}\right) + 0.3148 \quad (dimensionless) \tag{7.27}$$

Table 7.1 is a comparison of the gain of a thick cylindrical dipole with that of a caged dipole (N = 8) in free space computed both by ELNEC and Mathcad software [Ref. 8: p. 2-38]. As previously mentioned in Chapter VII, ELNEC is an easy-to-use numerical analysis program for modeling and analyzing different antenna types [Ref. 11: pp. 5-7]. As can be seen, the comparison between the measured, ELNEC, and Mathcad application values are similar.

TABLE 7.1 Caged Dipole Antenna Data Comparison

ANTENNA DIMENSIONS	MEASURED Gain (G)	ELNEC Gain (G)	Mathcad Gain (G)
$L = \lambda/2$ m $a = 0.025$ m $N = 8$	2.14 dB	1.87 dB	1.88 dB

Figure 7.2 is the radiation pattern of a caged dipole (N = 8) in free space. It is essentially equivalent to that of a center-fed, thick cylindrical dipole having the same conductor length and diameter [Ref. 8: p. 2-38]:

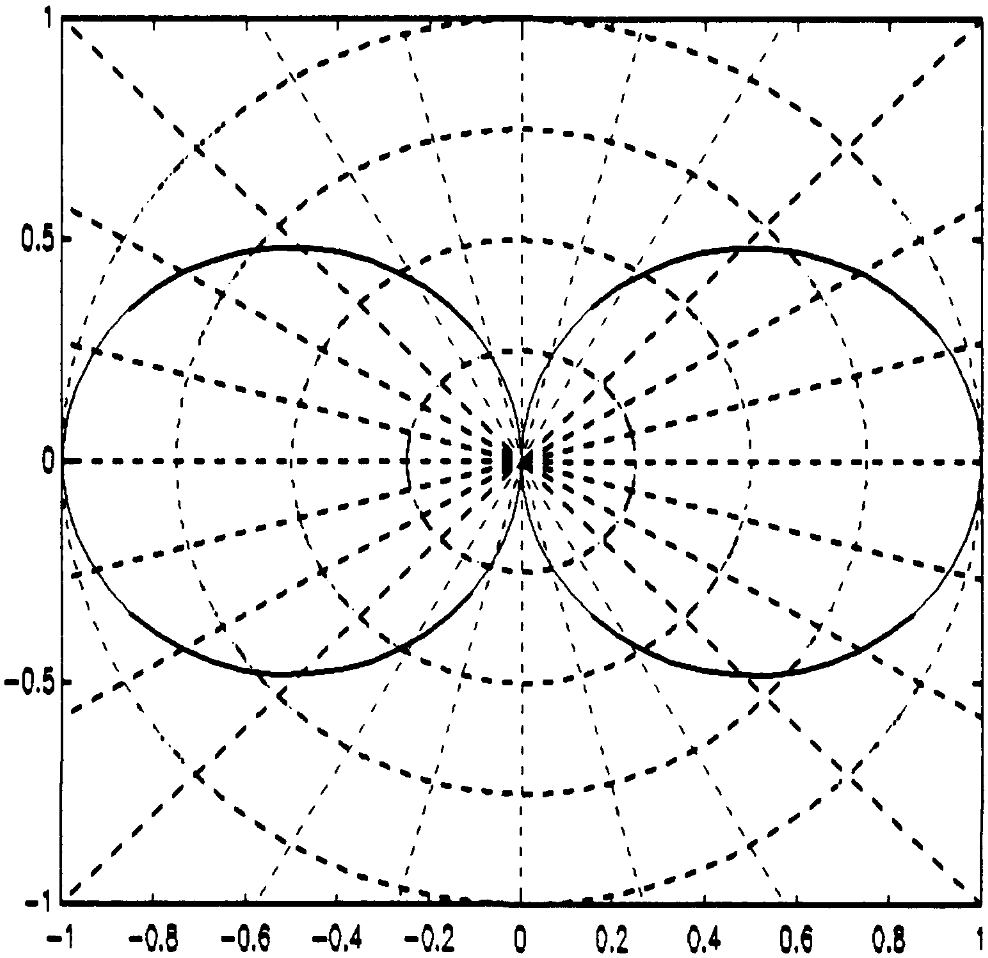


FIGURE 7.2 Radiation pattern of a caged dipole in free space with L = λ/2 m, N = 8, and a = 0.125 m

Figure 7.3 is a comparison of the radiation pattern of a caged dipole (N = 8) obtained using ELNEC and Mathcad. The patterns are not identical, but are similar and oriented in the same direction as expected. The difference can be attributed to the fact that ELNEC takes into account mutual coupling between elements while the Mathcad application ignores these effects.

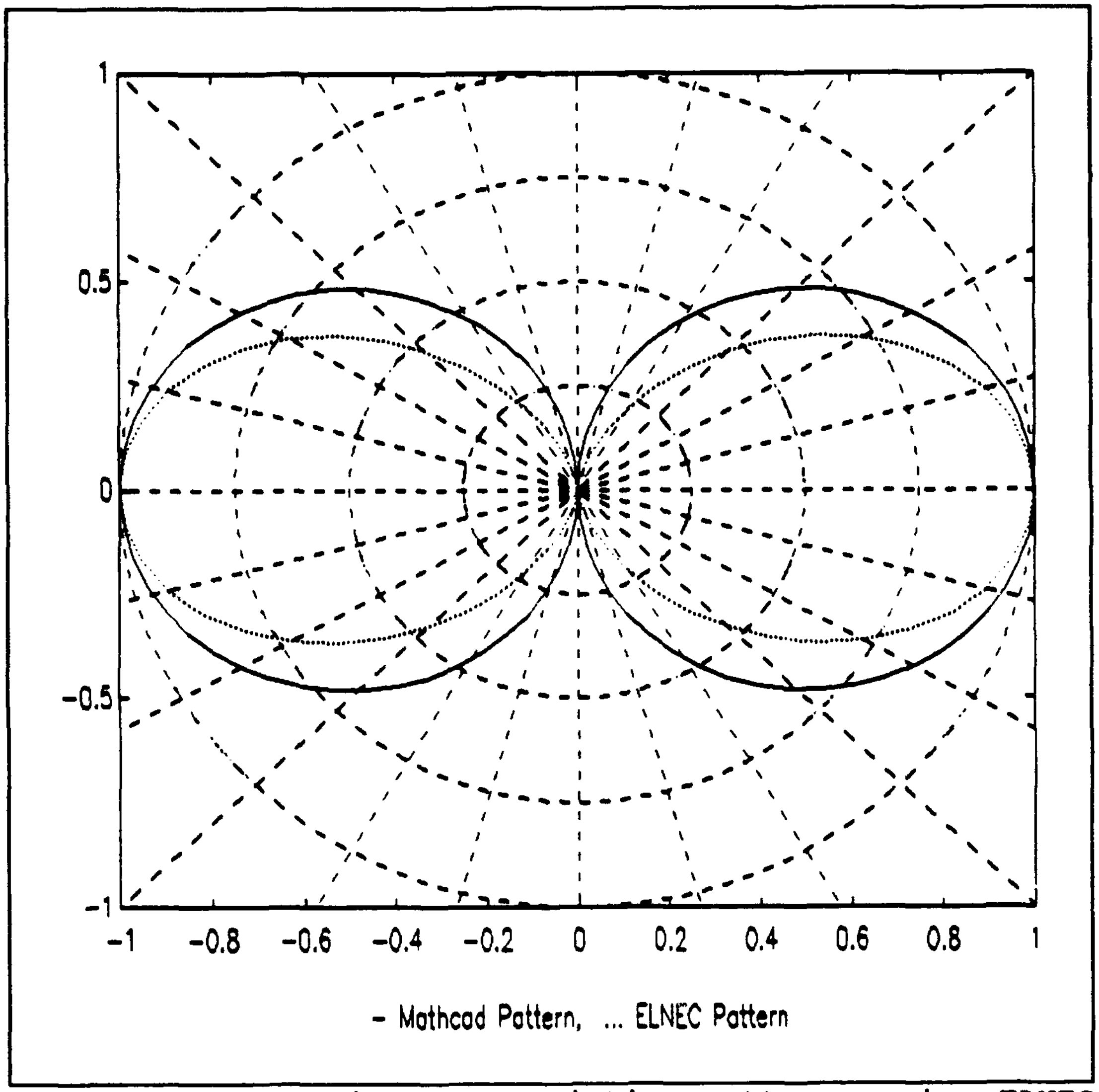


FIGURE 7.3 Comparison of radiation patterns using ELNEC and Mathcad of a caged dipole in free space with $L = \lambda/2$ m, $N = 8$, and $a = 0.125$ m

B. CAGED DIPOLE ORIENTED VERTICALLY OVER EARTH

The following known or estimated inputs are additional data required to analyze a caged dipole oriented vertically over the earth. The height of the antenna above ground is a physical dimension obtained through photographs:

h = antenna height above ground

ε_r = relative dielectric constant of earth

σ = conductivity of earth

σv_w = incoming wave electric field unit vector for vertical antenna

θv_p = antenna polarization direction for vertically orientated antenna

As with the caged dipole in free space, the analysis of a caged dipole positioned vertically above the ground does not take into account the central conductor and support members since they produce negligible effects on the antenna's electrical properties.

From image theory and pattern multiplication for arrays, the total far-field electric field intensity of a caged dipole oriented vertically above a flat earth is [Ref. 4: pp. 229-235]:

$$Ev_t(\theta,\phi) = Ev(\theta)\,AFv(\theta,\phi)\left(e^{jkh\cos(\theta)} + \Gamma_v e^{-jkh\cos(\theta)}\right) \quad (V/m) \qquad (7.28)$$

In (7.28), Ev(θ) is the far-field electric field intensity of a finite length dipole in free space which is given by (7.5), and $AFv(\theta,\phi)$ is the array factor for a circular array which is (7.1). The total electric field intensity is valid only above the ground ($0 \le \theta \le \pi/2$) [Ref. 3: pp. 135-142]. The distance from the earth to the center of the antenna is (h), and the vertical reflection coefficient (Γ_v) of the ground is [Ref. 4: pp. 231- 233]:

$$\Gamma_v(\theta) = \frac{\varepsilon'_r\cos(\theta) - \sqrt{\varepsilon'_r - \sin^2(\theta)}}{\varepsilon'_r\cos(\theta) + \sqrt{\varepsilon'_r - \sin^2(\theta)}} \quad (dimensionless) \qquad (7.29)$$

where

$$\varepsilon'_r = \varepsilon_r - j\frac{\sigma}{2\pi f\varepsilon_o} \qquad (dimensionless) \qquad (7.30)$$

In (7.29) and (7.30), (ε'_r) is the relative complex effective dielectric constant of the ground, (ε_r) is the relative dielectric constant of the ground, (σ) is the conductivity of the ground, and ε_o is the permittivity of free space ($1/(36\pi) \times 10^{-9}$). For a perfect ground plane, $\sigma \to \infty$ and $\Gamma_v = 1$.

The radiation intensity ($U_v(\theta,\phi)$) of a caged dipole over earth is [Ref. 3: p. 28]:

$$U_v(\theta,\phi) = \frac{r_{ff}^2}{2\eta_o}\left[|Ev_t(\theta,\phi)|^2\right] \qquad (W/solid\ angle) \qquad (7.31)$$

The radiated power (Pv_{rad}) is present only above the earth. Thus, [Ref. 3: p. 28]:

$$Pv_{rad} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} Uv(\theta,\phi)\sin(\theta)\,d\theta\,d\phi \quad (W) \qquad (7.32)$$

Directivity (Dv_o) and effective isotropic radiated power (EIRPv) for a vertically oriented caged dipole over the earth are given by (7.12) and (7.13), respectively, with the results of (7.32) inserted. Uv_{max} in (7.12) is simply the maximum value of (7.31) which, in Mathcad, is found by constructing (7.31) as a vector and then finding the maximum value of the vector.

Input resistance (Rv_{in}), voltage reflection coefficient (ΓV), gain (Gh), maximum effective aperture (Av_{em}), and maximum effective height (hv_{em}) for a vertical caged dipole over the earth are determined using the equations for those parameters for a caged dipole in free space.

The Polarization Loss Factor (PLFv) for the antenna over the earth can be determined when the direction of the main beam of the polarized antenna (θv_p) is estimated. The antenna polarization vector (σv_a) is then calculated and, estimating the unit polarization vector of the incoming wave (σv_w), PLF is found from (7.21).

Table 7.2 is a comparison of the gain of a vertical caged dipole over a flat earth obtained with both ELNEC and the Mathcad application. As can be seen, the gains are similar. As previously mentioned, the difference in the gains can be attributed to the fact that ELNEC takes into account mutual coupling between the elements as well as ground effects. The Mathcad application ignores mutual coupling between elements and approximates ground effects.

TABLE 7.2 Caged Dipole Antenna Gain Comparison

ANTENNA DIMENSIONS	ELNEC Gain (G)	Mathcad Gain (G)
$L = \lambda/2$ m $a = 0.125$ m $N = 8$ $h = 0.25$ m $\sigma = 0.01$ S/m $\varepsilon_r = 15$	4.25 dB	4.50 dB

Since the caged dipole behaves in a manner similar to that of a thin dipole except for bandwidth characteristics, the radiation pattern of a vertically oriented caged dipole over ground will resemble that of a thin vertical dipole over ground. Figure 7.4 is a plot of the radiation pattern of a caged dipole over earth. There is a very close resemblance to the radiation pattern of a thin vertical dipole over ground [Ref. 4: p. 234]. Figure 7.5 is a comparison of radiation patterns computed by ELNEC and Mathcad software. As expected, the patterns are similar.

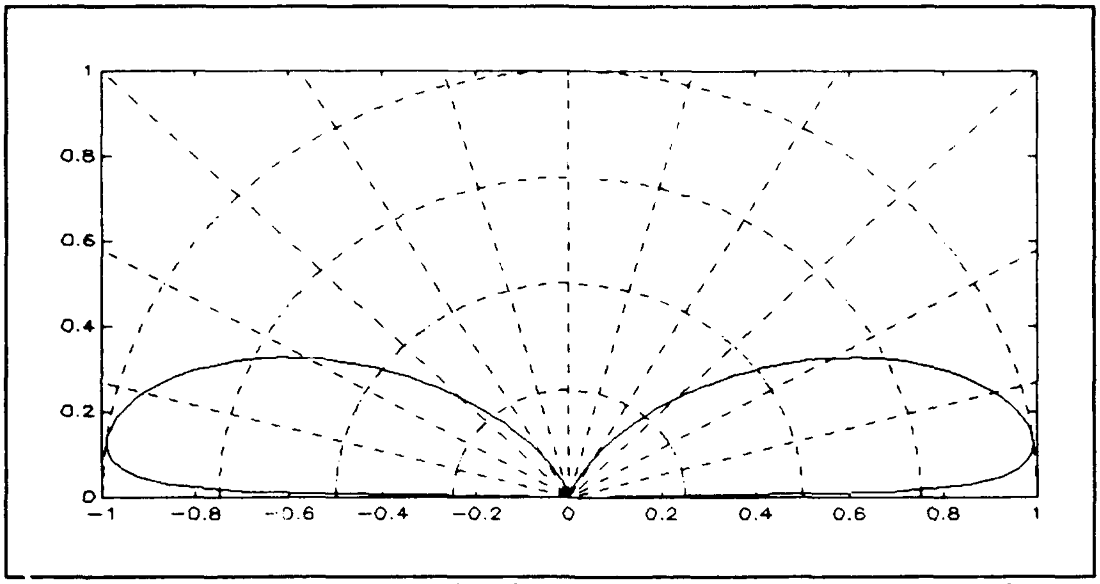


FIGURE 7.4 E-plane radiation pattern of a caged dipole with L = $\lambda/2$ m, N = 8, a = 0.125 m, h = 0.25 m, σ = 0.01 S/m, and ε_r = 15

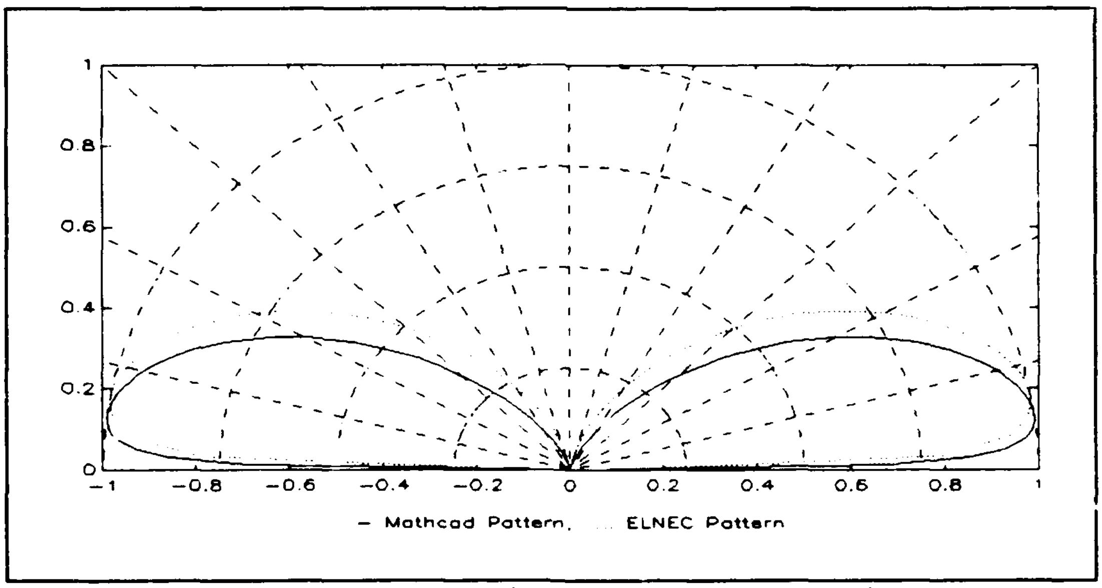


FIGURE 7.5 E-plane radiation pattern comparison between ELNEC and Mathcad of a caged dipole with L = $\lambda/2$ m, N = 8, a = 0.125 m, h = 0.25 m, σ = 0.01 S/m, and ε_r = 15

C. CAGED DIPOLE ORIENTED HORIZONTALLY OVER EARTH

The following known or estimated inputs are additional data required to analyze a caged dipole oriented horizontally over the earth:

h = antenna height above ground

ε_r = relative dielectric constant of earth

σ = conductivity of earth

σh_w = incoming wave electric field unit vector for vertical antenna

θh_p = antenna polarization direction for vertically orientated antenna

As with the evaluations of a caged dipole both in free space and oriented vertically over the earth, the analysis of a caged dipole positioned horizontally above the ground does not take into account the central conductor and support members since they produce negligible effects on the antenna's electrical properties.

Through the use of image theory, the far-field electric field intensity of a caged dipole oriented horizontally above a flat earth can be obtained. The θ component of the electric field of a horizontal dipole (parallel to the y-axis) in free space is [Ref. 3: pp. 143-144]:

$$E\theta h(\theta,\phi) = j\eta_o \frac{I_n e^{-jkr_{ff}}}{2\pi r_{ff}} \left[\frac{\cos\left(\frac{kL}{2}\sin(\theta)\sin(\phi)\right) - \cos\left(\frac{kL}{2}\right)}{\sqrt{1-\sin^2(\theta)\sin^2(\phi)}} \right] \quad (V/m) \qquad (7.33)$$

The θ component of the total electric field over ground is [Ref. 4: pp. 229-235]:

$$E\theta h_t(\theta,\phi) = E\theta h(\theta,\phi)\, AFh(\theta,\phi) \left(e^{jkh\cos(\theta)} - \Gamma_v e^{-jkh\cos(\theta)}\right) \quad (V/m) \qquad (7.34)$$

The array factor ($AFh(\theta,\phi)$) in (7.34) for a horizontal array is determined by [Ref. 3: pp. 274 and 776]:

$$AFh(\theta,\phi) = \sum_{n=1}^{N} I_n e^{jka\sin\theta\sin\phi\sin\Phi_n} \quad (dimensionless) \qquad (7.35)$$

In (7.33) through (7.35), the electric field ($E\theta h(\theta,\phi)$), the total electric field ($E\theta h_t(\theta,\phi)$), and array factor ($AFh(\theta,\phi)$) are valid only above the ground ($0 \le \theta \le \pi/2$) [Ref. 3: pp. 135-142]. In (7.34), (h) is the distance from the earth to the center of the antenna, and the vertical reflection coefficient (Γ_v) of the ground is given by (7.29).

The Φ component of the electric field of a horizontal dipole (parallel to the y-axis) in free space is [Ref. 3: pp. 135-142]:

$$E\phi h(\theta,\phi) = j\eta_o \frac{I_n e^{-jkr_{ff}}}{2\pi r_{ff}} \left[\frac{\cos\left(\frac{kL}{2}\sin(\theta)\sin(\phi)\right) - \cos\left(\frac{kL}{2}\right)}{\sqrt{1-\sin^2(\theta)\sin^2(\phi)}} \right] \quad (V/m) \qquad (7.36)$$

From image theory, the Φ component of the total electric field over earth is [Ref. 4: pp. 229-235]:

$$E\phi h_t(\theta,\phi) = E\phi h(\theta,\phi)\, AFh(\theta,\phi)\left(e^{jkh\cos(\theta)} + \Gamma_h e^{-jkh\cos(\theta)}\right) \quad (V/m) \qquad (7.37)$$

In (7.37), ($AFh(\theta,\phi)$) is given by (7.35), and (Γ_h) is the horizontal reflection coefficient of the ground and is [Ref. 4: pp. 231-232]:

$$\Gamma_h = \frac{\cos(\theta) - \sqrt{\varepsilon'_r - \sin^2(\theta)}}{\cos(\theta) + \sqrt{\varepsilon'_r - \sin^2(\theta)}} \quad (dimensionless) \qquad (7.38)$$

When $\sigma \rightarrow \infty$, $\Gamma_h = -1$.

The radiation intensity ($U_h(\theta,\phi)$) of a caged dipole over a earth must take into account both the θ and Φ components of the electric field. Therefore, the radiation intensity is [Ref. 3: p. 28]:

$$Uh(\theta,\phi) = \frac{r_{ff}^2}{2\eta_o}\left[|E\theta h_t(\theta,\phi)|^2 + |E\phi h_t(\theta,\phi)|^2\right] \quad (W/solid\ angle) \qquad (7.39)$$

The radiated power (Ph_{rad}) is present only above the earth. Thus,

$$Ph_{rad} = \int_0^{2\pi}\int_0^{\pi/2} Uh(\theta,\phi)\sin(\theta)\, d\theta\, d\phi \quad (W) \qquad (7.40)$$

The remaining parameters (i.e., Dh_o, EIRPh, Rh_{in}, etc.) for a horizontal caged dipole over the earth are calculated using the equations for these parameters listed previously.

Table 7.3 is a comparison of the gain of a horizontal caged dipole over a flat earth obtained with ELNEC and the

Mathcad application. As can be seen, the gains are similar. As previously mentioned, the difference in gains can be attributed to the fact that ELNEC takes into account mutual coupling between the elements as well as ground effects. The Mathcad application ignores mutual coupling between elements and approximates ground effects.

TABLE 7.3 Caged Dipole Antenna Gain Comparison

ANTENNA DIMENSIONS	ELNEC Gain (G)	Mathcad Gain (G)
$L = \lambda/2$ m $a = 0.025$ m $N = 8$ $h = 1.0$ m $\sigma = 0.01$ S/m $\varepsilon_r = 15$	4.99 dB	4.73 dB

As previously mentioned with regard to the vertically oriented caged dipole over the earth, the caged dipole behaves in a manner similar to that of a thin cylindrical dipole. Therefore, the radiation patterns of a horizontally oriented caged dipole are similar to those of horizontal thin cylindrical dipole which in turn resembles that of a thin dipole. Figures 7.6 and 7.7 illustrate the H-plane and E-plane patterns, respectively, of a caged dipole over the earth [Ref. 4: pp. 231-237]. The radiation patterns of the horizontally orientated caged dipole over a flat earth are very similar to those of a horizontal thin dipole over earth. Figures 7.8 and 7.9 compare the radiation patterns between those computed with ELNEC and those computed by Mathcad software. As expected, the patterns are similar.

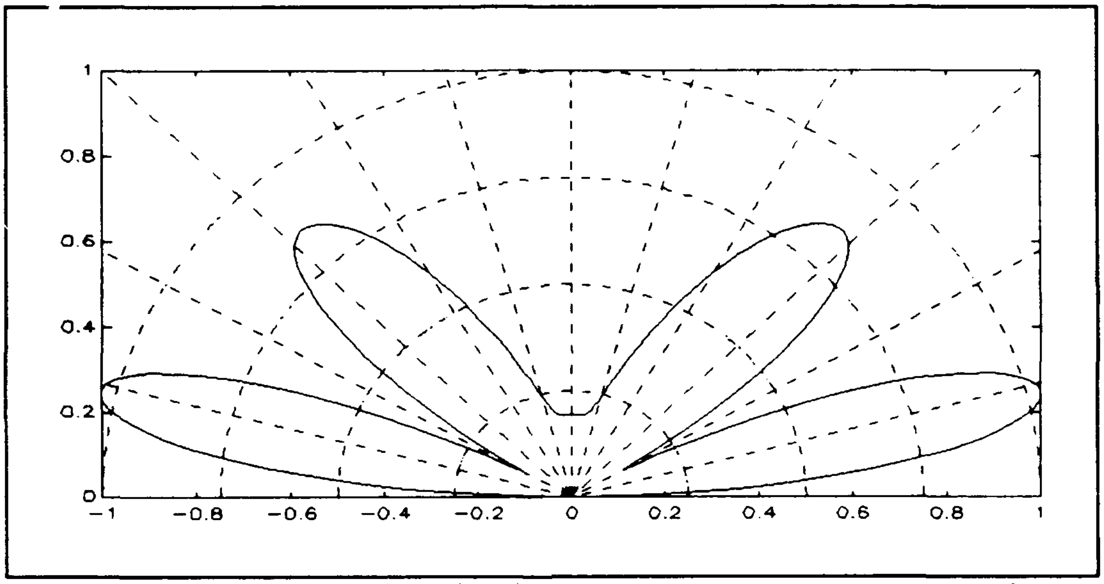


FIGURE 7.6 H-plane radiation pattern of a caged dipole with L = $\lambda/2$ m, N = 8, a = 0.125 m, h = 1.0 m, σ = 0.01 S/m, and ε_r = 15

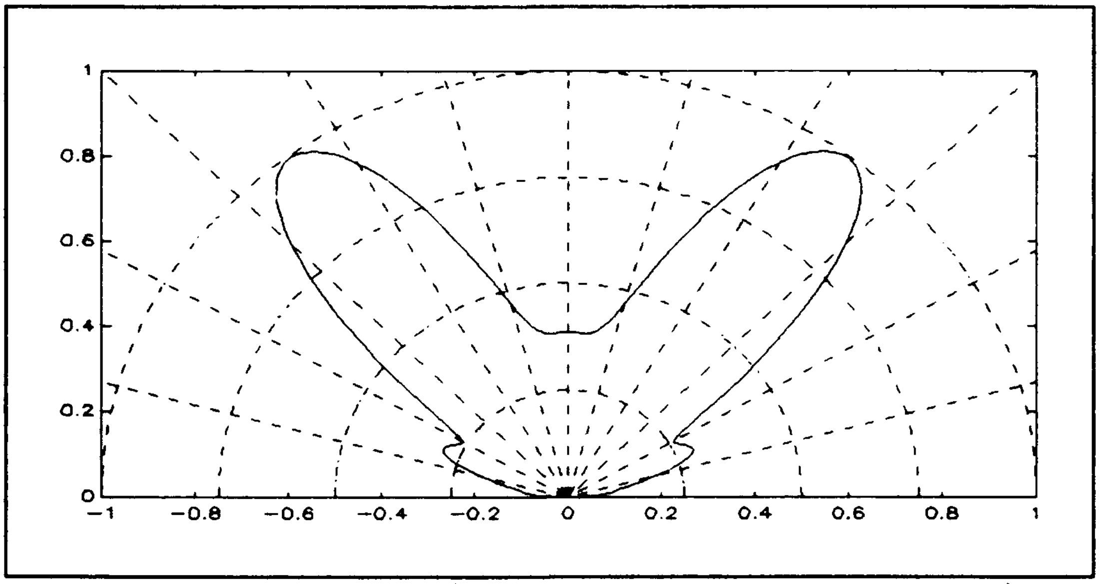


FIGURE 7.7 E-plane radiation pattern of a caged dipole with L = $\lambda/2$ m, N = 8, a = 0.125 m, h = 1.0 m, σ = 0.01 S/m, and ε_r = 15

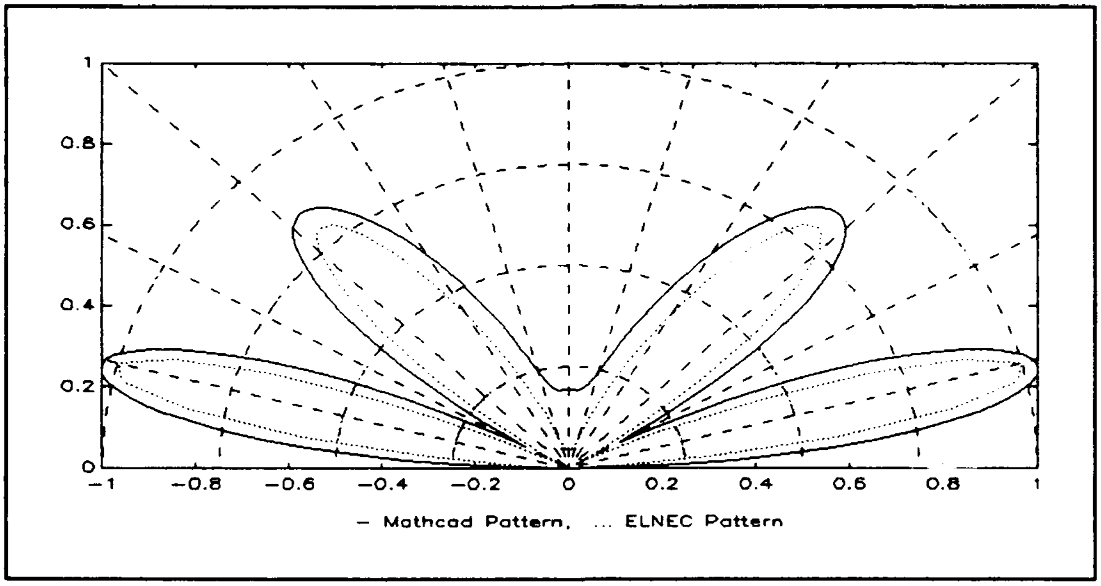


FIGURE 7.8 Comparison between ELNEC and Mathcad of H-plane radiation patterns of a caged dipole with L = $\lambda/2$ m, N = 8, a = 0.125 m, h = 1.0 m, σ = 0.01 S/m, and ε_r = 15

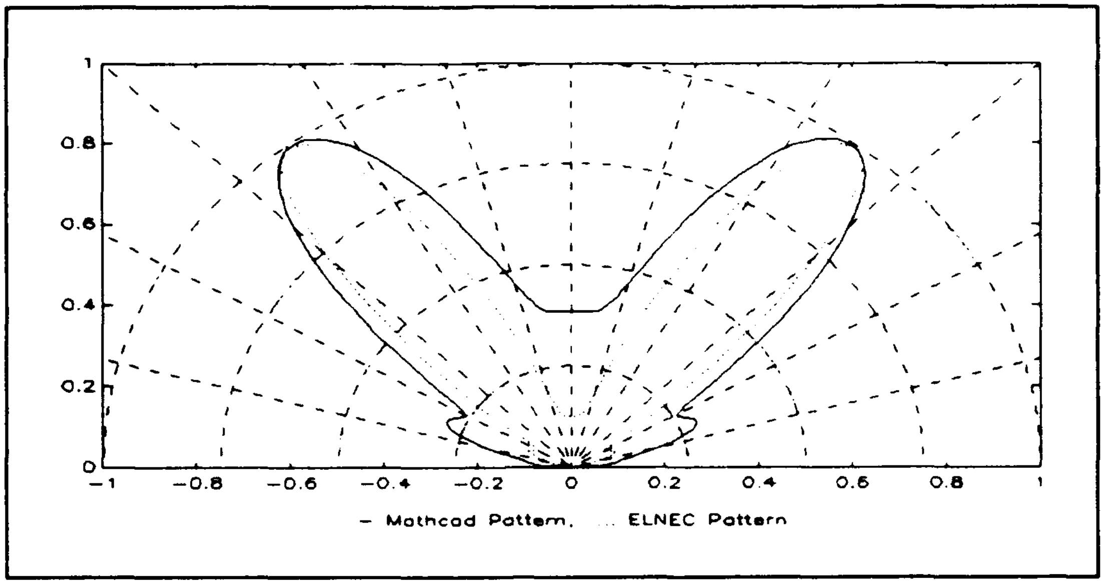


FIGURE 7.9 Comparison between ELNEC and Mathcad of E-plane radiation patterns of a caged dipole with $L = \lambda/2$ m, $N = 8$, $a = 0.125$ m, $h = 1.0$ m, $\sigma = 0.01$ S/m, and $\varepsilon_r = 15$

VIII. PARABOLIC REFLECTORS

Many communications and radar operations require very large values of antenna gain (G) that are difficult to achieve with a single device. One of the most popular ways to construct a very high gain antenna is to reflect the electromagnetic energy of a relatively small antenna off a metallic parabolic dish in the far-field of the feed device [Ref. 13: p. 12-2]. If the source antenna is located at the focus of the parabola and if the radius (a) of the parabolic dish periphery is large with respect to wavelength (λ), then the reflector may be approximated as a uniform phase aperture antenna. Gain in excess of 30 dB is common for this type of structure [Ref. 4: pp. 423-424]. The geometry of a parabolic reflector is illustrated in Figure 8.1, where the parameters include the focal distance (f), the distance (ρ) from the origin to any point (P) on the reflector surface, and the distance (r') from the origin to the projection of point (P) onto the $z = 0$ plane. The primed angles in Figure 8.1 correspond to the parameters associated with the feed horn. The Mathcad application user should note that the focal distance should exceed the minimum distance to far-field for the feed antenna.

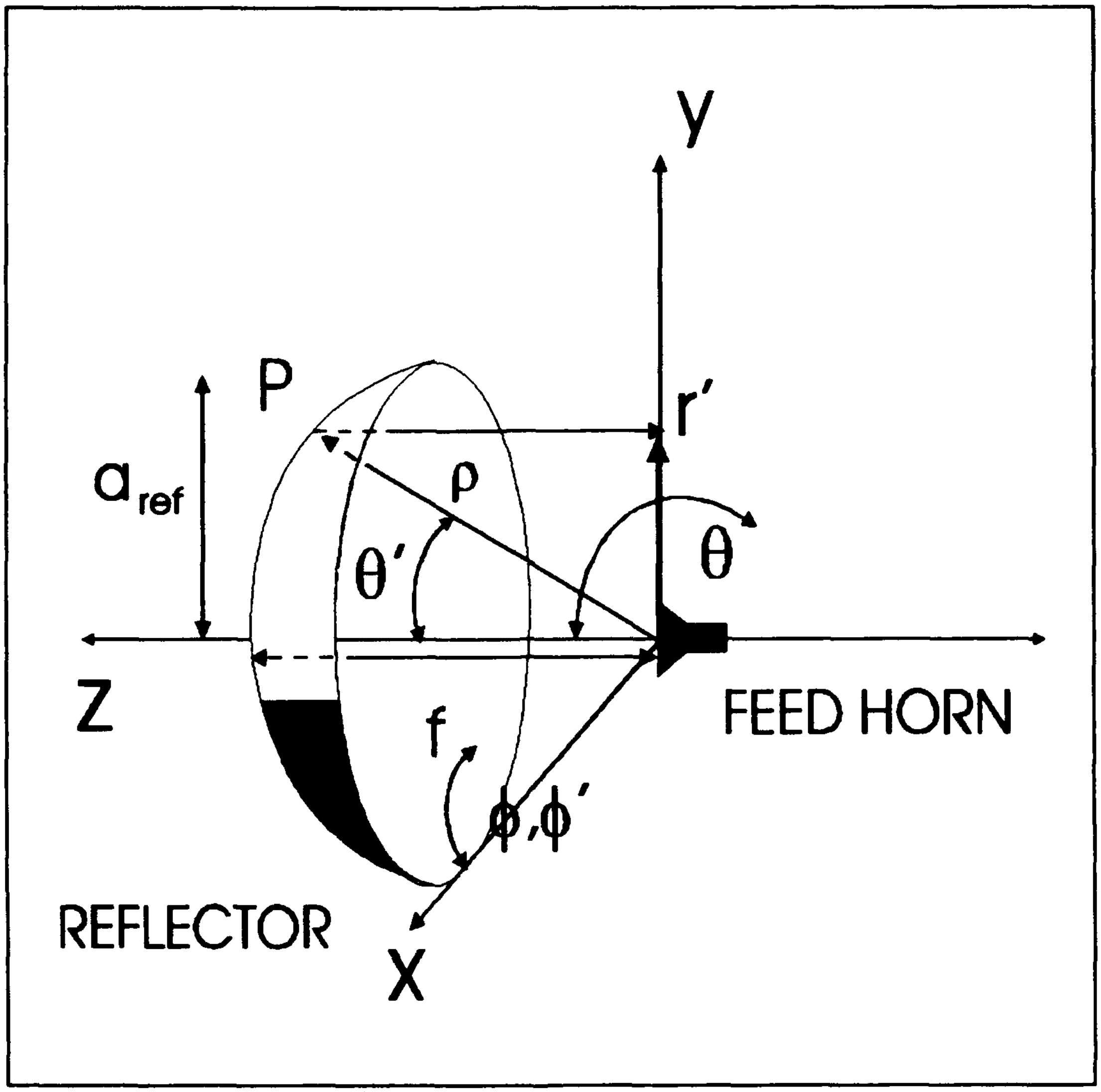


FIGURE 8.1 Parabolic Reflector Geometry

The Mathcad parabolic reflector applications are unique in that the software is not written as a stand alone package. Rather, the reflector software is run as an addendum to the programs that already exist for various antennas [Ref. 2: pp. 4-15, 67-114]. Four types of feeds are analyzed: helical, planar equiangular spiral, pyramidal horn, and conical horn.

The following additional inputs are required to analyze parabolic reflectors:

foc = reflector focal length

a_{ref} = radius of the mouth of the reflector

E_o = electric field scale factor *

ε_b = blockage efficiency *

ε_{sp} = spar efficiency *

r_{ff2} = secondary far field observation distance *

x,y,z = polarization loss factor coordinates *

σ_w = incoming wave unit polarization vector *

t1, t2 = radiated power increments for ϕ and θ

i1 = secondary field increments

N,M = summation increments

The first two inputs are physical dimensions obtained primarily through photographs, inputs with the * are parameters that are either known or estimated, and the last inputs are only used to affect the processing time of the application.

In order to calculate the far-field patterns of a parabolic reflector, the electric field at the aperture of the device must be known. Since the electric field patterns of interest are in the far-field, it is assumed that the circular aperture created by the reflector is located at the origin and is parallel to the z = 0 plane. The Cartesian components of the aperture electric field are expressed as [Ref. 14: pp. 121-123; Ref. 15: pp. 29-31]:

$$E_{ax}(r',\phi') = [-E_{\theta f}(\theta'(r'),\phi')\cos(\phi') + E_{\phi f}(\theta'(r'),\phi')\sin(\phi')]e^{jk(\rho(r')-2foc)} \quad (V/m) \tag{8.1}$$

$$E_{ay}(r',\phi') = [E_{\theta f}(\theta'(r'),\phi')\sin(\phi') + E_{\phi f}(\theta'(r'),\phi')\cos(\phi')]e^{jk(\rho(r')-2foc)} \quad (V/m) \tag{8.2}$$

In (8.1) and (8.2), ($E_{ax}(r',\theta')$) and ($E_{ay}(r',\theta')$) are the aperture electric fields and k is the wavenumber ($2\pi/\lambda$).

The relationships between (θ'), (ρ), and (r') needed to evaluate (8.1) and (8.2) are expressed as [Ref. 4: p. 426]:

$$\theta' = 2\tan^{-1}\left(\frac{r'}{2foc}\right) \quad (radians) \tag{8.3}$$

$$\rho(r') = \frac{4(foc)^2+(r')^2}{4foc} \quad (m) \tag{8.4}$$

The electric field from the entire parabolic assembly is commonly referred to as the secondary pattern. Referring to Figure 8.1, the electric field in the -z half-plane is considered the secondary pattern. The secondary field of a parabolic reflector is calculated using the electric field integral equation solution and computing the component vector potentials ($P_x(\theta,\phi)$), ($P_y(\theta,\phi)$) as follows [Ref. 4: p. 426]:

$$P_{\bar{x}}(\theta,\phi) = \int_0^{2\pi}\int_0^{a_{ref}} E_{ax}(r',\phi')\left[e^{jkr'\sin(\theta)(\cos(\phi')\cos(\phi)+\sin(\phi')\sin(\phi))}\right] * r'dr'd\phi' \quad (V/m) \tag{8.5}$$

$$P_{\bar{y}}(\theta,\phi) = \int_0^{2\pi}\int_0^{a_{ref}} E_{ay}(r',\phi')\left[e^{jkr'\sin(\theta)(\cos(\phi')\cos(\phi)+\sin(\phi')\sin(\phi))}\right] \tag{8.6}$$

$$*\,r'\,dr'\,d\phi' \qquad (V/m)$$

Double integrals in Mathcad take significant.y longer to converge to an answer than do single integrals [Ref. 15: p. 200]. To reduce the processing time of the applications, (8.5) and (8.6) were each separated into two single integrals. Therefore, for the applications, (8.5) and (8.6) are each computed as:

$$P(\theta,\phi) = \int_0^{2\pi} A(\theta,\phi,\phi')\,d\phi' \qquad (V/m) \tag{8.7}$$

where

$$A(\theta,\phi,\phi') = \int_0^{a_{ref}} E_a(r',\phi')\,e^{jkr'\sin(\theta)[\cos(\phi')\cos(\phi)+\sin(\phi')\sin(\phi)]}\,r'\,dr' \tag{8.8}$$

Any contribution from feed system back lobes is ignored, and the component vector potentials calculated in (8.7) and (8.8) are applied to the subsequent formulas to compute the secondary field [Ref. 4: p. 383]:

$$E_\theta(\theta,\phi) = \frac{jke^{-jkr_{ff2}}}{2\pi r_{ff2}}[P_x(\theta,\phi)\cos(\phi) + P_y(\theta,\phi)\sin(\phi)] \qquad (V/m) \tag{8.9}$$

$$E_\phi(\theta,\phi) = \frac{jke^{-jkrff2}}{2\pi r_{ff2}}[-P_x(\theta,\phi)\sin(\phi) + P_y(\theta,\phi)\cos(\phi)]\cos(\theta) \tag{8.10}$$

The radiation intensity ($U(\theta,\phi)$), radiated power (P_{rad}), and directivity (D_o) of the secondary field pattern are expressed using the intrinsic impedance of free space ($\eta_o = 120\pi$) as follows [Ref. 3: pp. 27-30]:

$$U(\theta,\phi) = \frac{(r_{ff2})^2}{2\eta_o}\left[|E_\theta(\theta,\phi)|^2 + |E_\phi(\theta,\phi)|^2\right] \quad (W/solid\ angle) \qquad (8.11)$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U(\theta,\phi)\sin(\theta)\,d\theta\,d\phi \quad (W) \qquad (8.12)$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} \quad (dimensionless) \qquad (8.13)$$

In (8.13), (U_{max}) is the maximum value of radiation intensity, obtained by creating (U_{max}) as a matrix and finding the maximum value of the matrix.

The computer processing time required to evaluate (8.12) is prohibitively long. Thus, the parabolic reflector Mathcad applications use the following numerical approximation to evaluate radiated power [Ref. 3: pp. 37-42]:

$$P_{rad} = \left(\frac{2\pi}{N}\right)\left(\frac{\pi}{2M}\right)\sum_{B=1}^{N}\left(\sum_{A=1}^{M} U(\theta_A,\phi_B)\sin(\theta_A)\right) \quad (W) \qquad (8.14)$$

In (8.14), (N and M) are user selected integers. This approximation for P_{rad} reduces the processing time of the applications substantially. As N and M increase, the results obtained with (8.14) converge to that obtained with (8.12).

When N = M = 3, the processing time for only P_{rad} of the helical feed reflector option is approximately four hours on a 33 MHz, 386 machine, and when N and M is increased to 5, the time to calculate P_{rad} for the same application is ten hours. For the applications, we chose N = M = 10 to evaluate P_{rad} with reasonable accuracy. With this choice, it takes approximately one day for the application to run, but this is much less than the thirty-six hours it takes if (8.12) is used.

Parabolic reflectors are members of the aperture antenna family. The gain of an aperture antenna is the product of its aperture efficiency (ε_{ap}) and directivity. Therefore, the gain of a parabolic reflector is written as [Ref. 6: p. 575]:

$$G = \varepsilon_{ap} D_o \quad (dimensionless) \qquad (8.15)$$

Aperture efficiency is the product of several separate terms, including spillover efficiency (ε_s), taper efficiency (ε_t), phase (or random surface error) efficiency (ε_p), polarization efficiency (ε_x), blockage efficiency (ε_b), spar efficiency (ε_{sp}), and ohmic efficiency (ε_{ohmic}).

Spillover efficiency (ε_s) is the fraction of power radiated by the feed system that is intercepted and reflected by the parabolic dish. Spillover efficiency is the most important efficiency term for any feed system and is defined as [Ref. 6: pp. 583]:

$$\varepsilon_s = \frac{\int_0^{2\pi}\int_0^{\theta_o} U_f(\theta') \sin(\theta')\, d\theta' d\phi'}{\int_0^{2\pi}\int_0^{\pi} U_f(\theta') \sin(\theta')\, d\theta' d\phi'} \quad (dimensionless) \qquad (8.16)$$

In (8.16), ($U_f(\theta')$) is the radiation intensity function of the feed antenna and (θ_o) is the value of (8.3) when (r') equals the radius of the mouth of the reflector (a_{ref}).

Taper efficiency (ε_t) accounts for the lack of amplitude uniformity of the feed pattern on the surface of the reflector. Taper efficiency, which is a dimensionless quantity, is expressed as [Ref. 3: pp. 626-627]:

$$\varepsilon_t = 2\cot^2\left(\frac{\theta_o}{2}\right) \frac{\left|\int_0^{2\pi}\int_0^{\theta_o} \sqrt{\frac{4\pi U_f(\theta')}{P_{rad}}} \tan\left(\frac{\theta'}{2}\right) d\theta' d\phi'\right|^2}{\int_0^{2\pi}\int_0^{\theta_o} \frac{4\pi U_f(\theta'}{P_{rad}} \sin(\theta')\, d\theta' d\phi'} \qquad (8.17)$$

Phase efficiency (ε_p) is a function of the power loss that occurs if the field at the mouth of the dish is not in phase at every point in the aperture. Phase efficiency is approximately given by [Ref. 4: pp. 434-435]:

$$\varepsilon_p \approx e^{\frac{-4\pi 6\cdot 10^{-5} a_{ref}}{\lambda}} \quad (dimensionless) \qquad (8.18)$$

The power that is lost due to cross-polarized electric fields in the aperture of the reflector determines polarization efficiency (ε_x). Polarization efficiency is difficult to precisely calculate, but can be reasonably approximated as 0.98 [Ref, 4: p. 435].

Blockage (ε_b) and spar (ε_{sp}) efficiencies are determined by the presence of feed structures, support struts, and possible signal processing equipment in front of the reflector's aperture. Precise computation of blockage and spar efficiencies requires an extensive method of moments simulation and is not attempted by the Mathcad parabolic reflector applications. However, if the radius (a_f) of the blocking structure at the focal point and the number of spars (N) that are ($\lambda/2$) thick is known, blockage efficiency and spar efficiency can be estimated using Tables 10.1 and 10.2, respectively [Ref. 4: p.436].

TABLE 8.1 Blockage Efficiency

a_f/a_{ref}	0.05	0.10	0.20
ε_b	0.99	0.96	0.84

TABLE 8.2 Spar Efficiency

	$a_{ref} = 5/\lambda$	$a_{ref} = 50/\lambda$	$a_{ref} = 100/\lambda$
N = 3	0.95	0.99	1.00
N = 4	0.94	0.99	1.00

Ohmic efficiency (ε_{ohmic}) for a reflector is usually very high but difficult to accurately calculate. The Mathcad parabolic reflector applications assume ohmic efficiency is 0.98.

With all reflector efficiency terms known or estimated, the aperture efficiency (ε_{ap}), effective aperture (A_e), and isotropic radiated power (EIRP) are given by [Ref. 3: pp. 623-630]:

$$\varepsilon_{ap} = \varepsilon_s \varepsilon_t \varepsilon_p \varepsilon_x \varepsilon_b \varepsilon_{sp} \varepsilon_{ohmic} \quad (dimensionless) \tag{8.19}$$

$$A_e = \varepsilon_{ap} \pi (a_{ref})^2 \quad (m^2) \tag{8.20}$$

$$EIRP = P_{rad} D_o \quad (W) \tag{8.21}$$

The magnitude of the feed current (I_o) for the source antenna cannot be estimated based on measured geometry. However, if the feed current is normalized to 1 amp, radiation resistance (R_r) for the entire reflector structure is:

$$R_r = \frac{2 P_{rad}}{|I_o|^2} \quad (\Omega) \tag{8.22}$$

In (8.22), (P_{rad}) is the radiated power from the aperture of the reflector, not of the feed antenna.

The Mathcad parabolic reflector applications use the radiation resistance computed for the parabolic reflector antenna system to determine effective height (h_{em}) by [Ref. 6: p. 42]:

$$h_e = 2\sqrt{\frac{R_I A_e}{\eta_o}} \quad (m) \qquad (8.23)$$

The bandwidth of a parabolic reflector is assumed to be the same as its feed antenna.

The polarization loss factor (PLF) of the parabolic reflector for a user selected point in the secondary far-field and a specific wave unit polarization vector (σ_w) is found using the electric field components of (8.9) and (8.10) as follows [Ref. 3: 51-53]:

$$PLF = |\vec{\sigma}_w \cdot \vec{\sigma}_a^*|^2 \quad (dimensionless) \qquad (8.24)$$

where the unit polarization vector of the antenna is:

$$\vec{\sigma}_a(x,y,z) = \frac{\vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z}{\sqrt{|E(x,y,z)|}} \quad (dimensionless) \qquad (8.25)$$

and the Cartesian components of the electric field are [Ref. 3: p. 776]:

$$E_x = E_\theta \cos(\theta)\cos(\phi) - E_\phi \sin(\phi) \quad (V/m) \qquad (8.26)$$

$$E_y = E_\theta \cos(\theta)\sin(\phi) + E_\phi \cos(\phi) \quad (V/m) \qquad (8.27)$$

$$E_z = -E_\theta \sin(\theta) \quad (V/m) \qquad (8.28)$$

All of the secondary field parameters calculated by the Mathcad parabolic reflector applications assume that the observation point is in the far-field. Thus, all of the subsequent formulas must hold for the application calculations to be valid [Ref. 4: pp. 24-25]:

$$r \geq 1.6\lambda \qquad (m) \tag{8.29}$$

$$r \geq 10\,a_{ref} \qquad (m) \tag{8.30}$$

$$r \geq \frac{8(a_{ref})^2}{\lambda} \qquad (dimensionless) \tag{8.31}$$

Table 8.3 is a comparison of reflector directivity computed by the Mathcad applications to predictions made for an identical antenna with unity aperture efficiency [Ref. 3: p. 634]. As can be seen, the comparisons vary by approximately 8 dB. This can be attributed to the fact that P_{rad} was approximated using (8.14) instead of (8.12), and also to the fact that the numerical tolerance variable (TOL in Mathcad's math menu) for the application was increased to 0.5 verses 0.001 in order to reduce processing time. This increase in TOL reduces both the accuracy of (8.14) and the processing time of the application.

TABLE 8.3 Parabolic Reflector Data Comparison

ANTENNA PARAMETER	PREDICTED DATA	CALCULATED DATA
DIRECTIVITY	42 dB	50.2 dB

Figure 8.2 is a comparison of measured data fed by an optimum horn with that calculated by the Mathcad application fed by an equiangular spiral for a parabolic reflector (a_{ref} = 0.320 m, f = 0.147 m) operating at 35 GHz [Ref. 14: p. 126]. As can be seen, the comparison is good. The Mathcad pattern of Figure 8.2 is not symmetric which can be attributed to the number of increments used to increase the speed of processing.

The reflectors fed by pyramidal and conical horn antennas are not included in this report. The applications have been developed and are included on the diskette containing the software. The complexity of the software results in prohibitively long processing time for the two applications (i.e., the pyramidal horn fed reflector application was still calculating P_{rad2} after seven days when run on a 50 MHz, 486 personal computer using Mathcad 4.0).

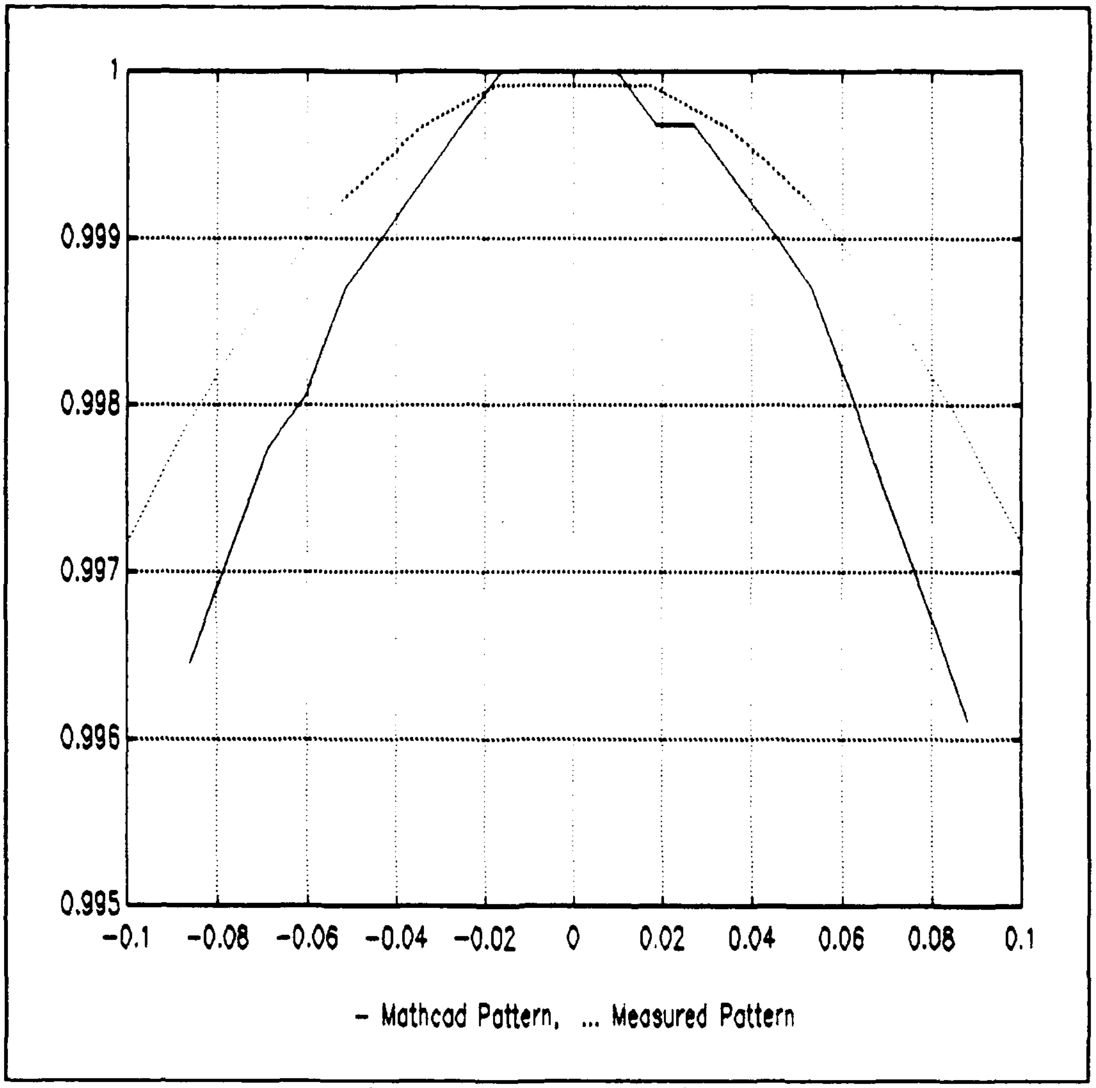


FIGURE 8.2 Comparison of Parabolic Reflector E-Plane Electric Field Patterns

IX. REMARKS AND CONCLUSIONS

This thesis and the accompanying software applications are intended to provide the Naval Maritime Intelligence Center (NAVMARINTCEN) with software for the analysis of various antenna types where information regarding the antenna is restricted primarily to the antenna type and the physical dimensions of the antenna. The goal of this research was to provide user friendly software that an engineer familiar with basic antenna types can easily use and interpret. Although the software was developed for that purpose, the complexity of formulas that apply to many of the antenna types of necessity reduced the simplicity of the applications.

The software applications developed are written in Mathcad 3.1 and are compatible with any IBM personal computer that supports Mathcad 3.1 or Mathcad 4.0 for Windows. Many of the applications require extensive processing time. In order to reduce processing time, which results from the complexity of various integrals, approximate numerical techniques were implemented to evaluate the integrals and the numerical tolerance of the more complicated applications was reduced. Even with the processing time reduced, some programs require up to five days to complete when run with Mathcad 4.0 on a 50 MHz, 486 personal computer. Applications that did not exceed a day to complete were run on a 33 MHz, 386 personal computer with Mathcad 4.0. All applications were run using Mathcad 4.0 since Mathcad 4.0 is approximately twice as fast as Mathcad 3.1. As previously mentioned, the applications were written in Mathcad 3.1 due to sponsor requirements. Mathcad 3.1 is compatible with Mathcad 4.0, but Mathcad 4.0 is not compatible with Mathcad 3.1.

For this thesis, a number of the antennas could not be adequately researched through the use of texts. In some cases, such as the caged dipole, very little information on the antenna could be found in either texts or in professional journals. For the caged dipole, the antenna is modeled as a circular array of center-fed, equally excited, uniformly spaced, thin-wire dipoles to take into account the number of conductors that are present in the caged dipole. This model provides good results when compared to predicted (the caged dipole's performance is similar to that of a thick cylindrical dipole) and ELNEC results.

It was found that Mathcad is not the ideal tool to analyze antennas due to its long processing time. An additional drawback of Mathcad is that when a function is in the beginning of an application and the same function is placed later in the application, Mathcad does not store the values of the function, but re-evaluates the function. This re-evaluation increases processing time significantly, especially when the function itself takes a very long time (hours vice minutes) to reach a numerical solution.

Nevertheless, the Mathcad applications developed in this research allow the user to analyze several antenna types (i.e., arrays with isotropic point sources, parabolic reflectors with the various feed antennas) that could not easily be evaluated using some antenna numerical analysis programs such as NEC, ELNEC, or MININEC. For the other antenna types researched, the antenna numerical analysis programs mentioned would be easier to setup, run, and provide parameters closer to those of a real antenna. The disadvantage of this approach is that it requires an ability to evaluate antennas using a program such as ELNEC, while the Mathcad applications dc not require any advanced programming knowledge on the part of the user.

LINEAR ARRAY
MATHCAD SOFTWARE ARRAY_LN.MCD

This linear array application is based on N isotropic point sources uniformly spaced and aligned on the z-axis. Mutual interference between adjacent sources are ignored when calculating antenna parameters. Since isotropic point sources are used as the radiating elements, polarization of the antenna can not be determined. The gain can only be idealized when it is assumed that the antenna input resistance is equivalent to the radiation resistance. Bandwith calculations are not carried out as a result of the narrowband characteristics of arrays, and in large part to the dependance inter-element spacing (d) has with the wavelength (λ). The following limits should be adhered to when analyzing Broadside, End-Fire, and Phased (scanning) antennas:

$0 \le \theta_0 \le \pi$ (Scanning Antennas)

$\theta_0 = \pi/2$ (Broadside Antennas)

$\theta_0 = 0$ or π (Ordinary End-Fire Antennas)

(Note: Mathcad equations can not use symbolic subscripts. Therefore, parameters do not contain subscripts (i.e., AF_n will be written AFn).)

The linear array antenna Mathcad application will compute the following parameters (Items with * indicate parameters that can be computed but their results are idealized as the result of using isotropic point sources as the radiating elements):

λ = Wavelength
r_{min} = Minimum Distance to the Far-Field
β = Phase Difference Between Adjacent Elements
ψ = Array Factor Phase Shift
AF_n = Normalized Array Factor
$U(\theta)$ = Radiation Intensity
P_{rad} = Radiated Power
D_o = Directivity
EIRP = Effective Isotropic Radiated Power
R_r = Radiation Resistance
R_{in} = Antenna Input Resistance *
Γ = Voltage Reflection Coefficient *
ε_t = Antenna Efficiency *
G = Gain *
A_{em} = Maximum Effective Aperture *
h_{em} = Maximum Effective Height *

The following known or estimated data must be entered:

N = Number of Isotropic Radiating Elements
f = Frequency of Interest
d = Distance Between Adjacent Elements
θ_o = Direction of Main Lobe
I_o = Antenna Feed Current
Z_o = Characteristic Feed Impedence

Enter input data here

$N := 10$	(elements)	$d := 0.25$	(meters)
$f := 3 \cdot 10^8$	(Hz)	$\theta o := 0$	(radians)
$Io := 1.0$	(A)	$Zo := 75.0$	(Ω)

Define constants and calculate wavelength:

$c := 2.9979 \cdot 10^8$	(meters/sec)	$\eta o := 120 \cdot \pi$	(Ω)
$\lambda := \frac{c}{f}$	(meters/cycle)	$k := \frac{2 \cdot \pi}{\lambda}$	(m^{-1})
$\lambda = 0.999$	(meters/cycle)	$\alpha d := 1$	(dimensionless)

Calculate linear array parameters:

Define angular offset θ:

$$\theta := \left(0 - 10^{-4}\right), \left(\frac{2\cdot\pi}{180}\right) .. \left(2\cdot\pi - 10^{-4}\right)$$ (radians)

Minimum Distance to Far-Field r_{min}:

$$r_0 := 1.6\cdot\lambda$$ (meters)

$$r_1 := 5\cdot N\cdot d$$ (meters)

$$r_2 := \frac{2\cdot(N\cdot d)^2}{\lambda}$$ (meters)

$$rmin := \max(r)$$ (meters)

$$rmin = 12.509$$ (meters)

Array Factor AF_n:

$$\beta := -k\cdot d\cdot\cos(\theta o)$$ (radians)

$$\psi(\theta) := k\cdot d\cdot\cos(\theta) + \beta$$ (radians)

$$AFn(\theta) := \frac{1}{N}\cdot\left[\frac{\sin\left(\frac{N}{2}\cdot\psi(\theta)\right)}{\sin\left(\frac{1}{2}\cdot\psi(\theta)\right)}\right]$$ (dimensionless)

Radiation Intensity U:

$$U(\theta) := (AFn(\theta))^2$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := 2 \cdot \pi \cdot \int_0^\pi U(\theta) \cdot \sin(\theta)\, d\theta$$ (W)

$$Prad = 1.256$$ (W)

Directivity D_o:

$$Umax := 1$$ (W/solid angle)

$$Do := \frac{4 \cdot \pi \cdot Umax}{Prad}$$ (dimensionless)

$$Do = 10.007$$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$$EIRP := Prad \cdot Do$$ (W)

$$EIRP = 12.566$$ (W)

Radiation Resistance R_r:

$$Rr := \frac{2 \cdot Prad}{(|Io|)^2} \qquad (\Omega)$$

$$Rr = 2.512 \qquad (\Omega)$$

Input Resistance R_{in}:

$$Rin := Rr \qquad (\Omega)$$

$$Rin = 2.512 \qquad (\Omega)$$

Voltage Relection Coeffecient Γ:

$$\Gamma := \frac{Rin - Zo}{Rin + Zo} \qquad \text{(dimensionless)}$$

$$\Gamma = -0.935 \qquad \text{(dimensionless)}$$

Reflection Efficiency ε_t:

$$\varepsilon t := \varepsilon cd \cdot \left[1 - (|\Gamma|)^2\right] \qquad \text{(dimensionless)}$$

$$\varepsilon t = 0.125 \qquad \text{(dimensionless)}$$

Gain G:

$G := \epsilon t \cdot Do$ (dimensionless) $GdB := 10 \cdot \log(G)$ (dB)

$G = 1.255$ (dimensionless) $GdB = 0.986$ (dB)

Maximum Effective Aperture A_{em}:

$$Aem := \frac{G \cdot \lambda^2}{4 \cdot \pi}$$ (m^2)

$Aem = 0.1$ (m^2)

Maximum Effective Height h_{em}:

$$hem := 2 \cdot \sqrt{\frac{Rr \cdot Aem}{\eta o}}$$ (m)

$hem = 0.052$ (m)

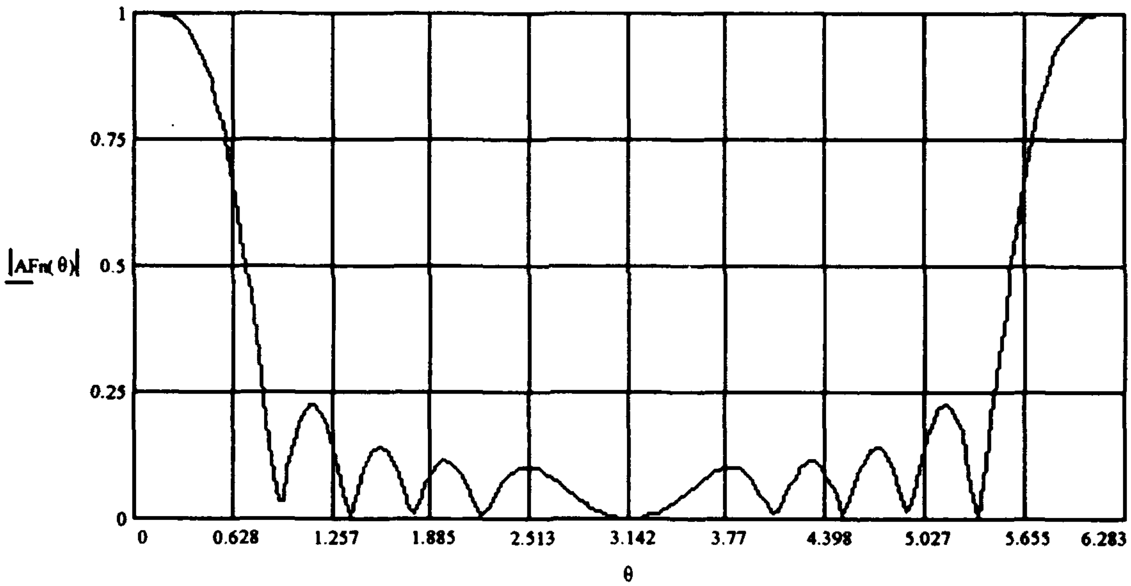
NORMALIZED ARRAY FACTOR RECTANGULAR PLOT

Number of radiating elements:

N = 10

Direction of Main Lobe (radians):

$\theta o = 0$



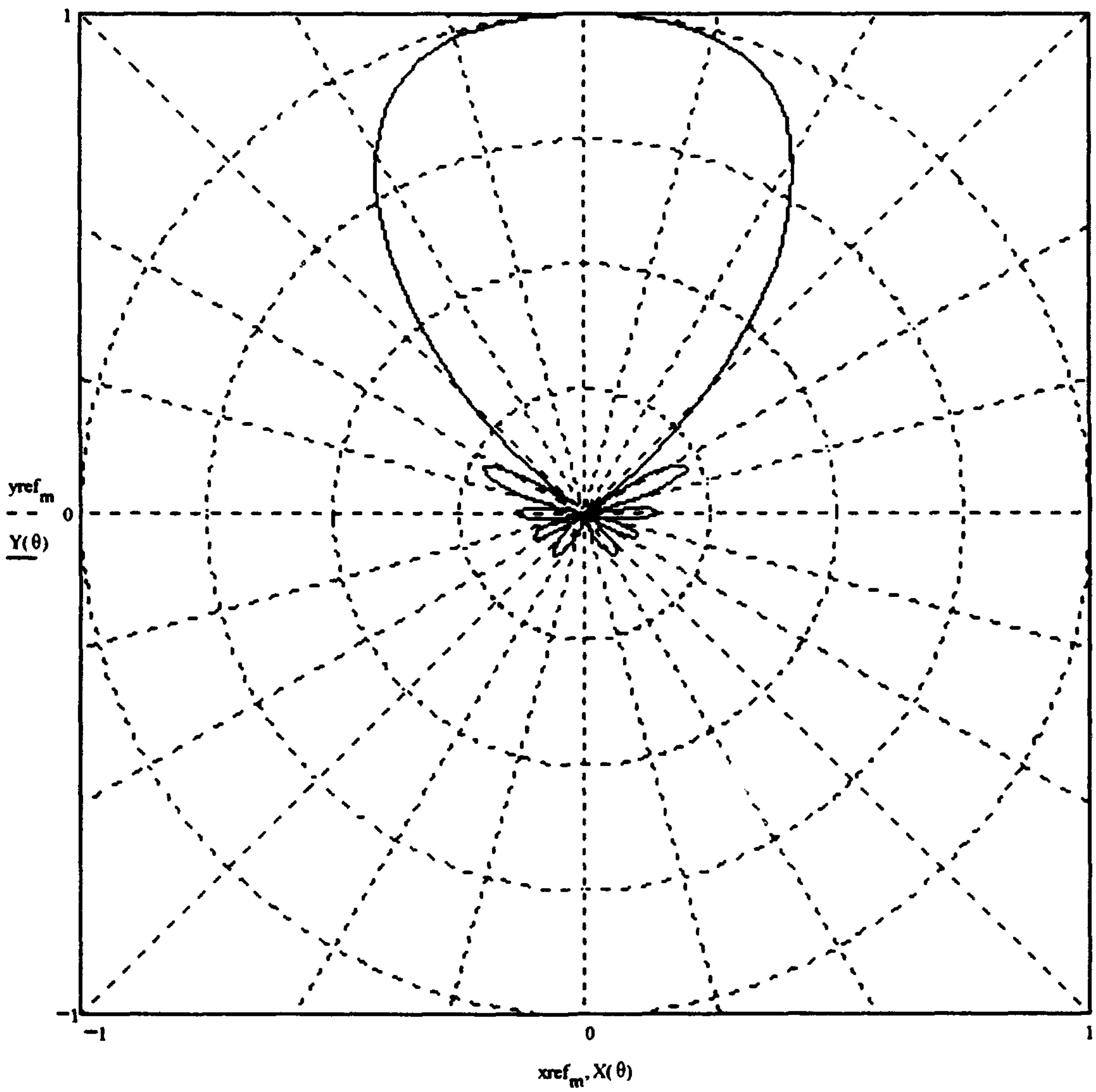
NORMALIZED ARRAY FACTOR POLAR PLOT

Number of Elements: $N = 10$

Direction of Main Beam (radians): $\theta o = 0$

$X(\theta) := |AFn(\theta)| \cdot \cos\left(\theta + \frac{\pi}{2}\right)$ $Y(\theta) := |AFn(\theta)| \cdot \sin\left(\theta + \frac{\pi}{2}\right)$

θ = 0°



θ = 180°

PLANAR ARRAY

MATHCAD SOFTWARE ARRAY_PL.MCD

This planar array application is based on M and N isotropic point sources uniformly spaced and aligned on the x-axis and y-axis respectively. Mutual interference between adjacent elements are neglected when calculating antenna parameters. Since isotropic point sources are used as the radiating elements, polarization of the antenna can not be determined. The gain can only be idealized when it is assumed that the antenna input resistance is equivalent to the radiation resistance and the total antenna efficiency is calculated for a ideal lossless antenna.

(Note: Mathcad equations can not use symbolic subscripts. Therefore, parameters do not contain subscripts (i.e., AF_n will be written AFn).)

The planar array antenna Mathcad application will compute the following parameters (Items with * indicate parameters that can be computed but their results are idealized as the result of using isotropic point sources as the radiating elements):

λ = Wavelength

L_x = Antenna Dimension in x-direction

L_y = Antenna Dimension in y-direction

r_{min} = Minimum Distance to the Far-Field

β_x = Progressive Phase Shift Between Adjacent Elements in x-direction

βy = Progressive Phase Shift Between Adjacent Elements in y-direction

ψ_x = Array Factor Phase Shift in x-direction

ψ_y = Array Factor Phase Shift in x-direction

$AF_n(\theta,\phi)$ = Normalized Array Factor

$U(\theta,\phi)$ = Radiation Intensity

P_{rad} = Radiated Power

D_o = Directivity

EIRP = Effective Isotropic Radiated Power

R_r = Radiation Resistance

R_{in} = Antenna Input Resistance *

Γ = Voltage Reflection Coefficient *

ε_t = Antenna Efficiency *

G = Gain *

A_{em} = Maximum Effective Aperture *

h_{em} = Maximum Effective Height *

The following known or estimated data must be entered:

M = Number of Isotropic Radiating Elements in x-direction
N = Number of Isotropic Radiating Elements in y-direction
f = Frequency of Interest
d_x = Distance Between Adjacent Elements in x-direction
d_y = Distance Between Adjacent Elements in y-direction
θ_o = Direction of Main Beam at $\theta = \theta o$
ϕ_o = Direction of Main Beam at $\phi = \phi o$
I_o = Antenna Feed Current
Z_o = Characteristic Feed Impedence
j = Number of Increments in Degrees for Far-Field Radiation Pattern

---

Enter input data here

$M := 5$	(elements)	$dx := 0.5$	(meters)
$N := 5$	(elements)	$dy := 0.5$	(meters)
$f := 3 \cdot 10^8$	(Hz)	$\theta o := \frac{\pi}{6}$	(radians)
$Io := 1.0$	(A)	$\phi o := \frac{\pi}{4}$	(radians)
$Zo := 75.0$	(Ω)	$j := 180$	(degrees)

Define constants and calculate wavelength:

$c := 2.9979 \cdot 10^8$ (meters/sec) $\eta o := 120\,\pi$ (Ω)

$\lambda := \frac{c}{f}$ (meters/cycle) $k := \frac{2\,\pi}{\lambda}$ (m^{-1})

$\lambda = 0.999$ (meters/cycle) $\alpha d := 1$ (dimensionless)

Calculate linear array parameters:

Define angular offset θ and ϕ:

$$\theta := \left(0 - 10^{-4}\right), \left(-10^{-4} + \frac{2\cdot\pi}{j}\right) .. \left(2\cdot\pi - 10^{-4}\right)$$ (radians)

$$\phi := \left(0 - 10^{-4}\right), \left(-10^{-4} + \frac{2\cdot\pi}{j}\right) .. \left(2\cdot\pi - 10^{-4}\right)$$ (radians)

Minimum Distance to Far-Field r_{min}:

$r_0 := 1.6 \cdot \lambda$ (meters)

$r_1 := 5 \cdot \sqrt{(M \cdot dx)^2 + (N \cdot dy)^2}$ (meters)

$r_3 := \frac{2 \cdot \left[\sqrt{(M \cdot dx)^2 + (N \cdot dy)^2}\right]^2}{\lambda}$ (meters)

$rmin := \max(r)$ (meters) $rmin = 25.018$ (meters)

Array Factor AF_n:

$$\beta x := -k \cdot dx \cdot \sin(\theta o) \cdot \cos(\phi o)$$ (radians)

$$\beta y := -k \cdot dy \cdot \sin(\theta o) \cdot \sin(\phi o)$$ (radians)

$$\psi x(\theta,\phi) := k \cdot dx \cdot \sin(\theta) \cdot \cos(\phi) + \beta x$$ (radians)

$$\psi y(\theta,\phi) := k \cdot dy \cdot \sin(\theta) \cdot \sin(\phi) + \beta y$$ (radians)

$$AFn(\theta,\phi) := \left[\frac{\sin\left(\frac{M}{2}\cdot\psi x(\theta,\phi)\right)}{M\cdot\sin\left(\frac{\psi x(\theta,\phi)}{2}\right)}\right]\cdot\left[\frac{\sin\left(\frac{N}{2}\cdot\psi y(\theta,\phi)\right)}{N\cdot\sin\left(\frac{\psi y(\theta,\phi)}{2}\right)}\right]$$ (dimensionless)

Radiation Intensity U:

$$U(\theta,\phi) := AFn(\theta,\phi)\cdot\overline{AFn(\theta,\phi)}$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := \int_0^{2\cdot\pi}\int_0^{\frac{\pi}{2}} U(\theta,\phi)\cdot\sin(\theta)\, d\theta\, d\phi$$ (W)

$$Prad = 0.206$$ (W)

Directivity D_o:

$Umax := 1$ (W/solid angle)

$Do := \frac{4 \cdot \pi \cdot Umax}{Prad}$ (dimensionless)

$Do = 61.09$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$EIRP := Prad \cdot Do$ (W)

$EIRP = 12.566$ (W)

Radiation Resistance R_r:

$Rr := \frac{2 \cdot Prad}{(|Io|)^2}$ (Ω)

$Rr = 0.411$ (Ω)

Input Resistance R_{in}:

$Rin := Rr$ (Ω)

$Rin = 0.411$ (Ω)

Voltage Relection Coeffecient Γ:

$$\Gamma := \frac{Rin - Zo}{Rin + Zo}$$ (dimensionless)

$\Gamma = -0.989$ (dimensionless)

Reflection Efficiency ε_t:

$$\varepsilon t := \varepsilon cd \cdot \left[1 - (|\Gamma|)^2\right]$$ (dimensionless)

$\varepsilon t = 0.022$ (dimensionless)

Gain G:

$G := \varepsilon t \cdot Do$ (dimensionless) $GdB := 10 \cdot \log(G)$ (dB)

$G = 1.326$ (dimensionless) $GdB = 1.225$ (dB)

Maximum Effective Aperture A_{em}:

$$Aem := \frac{G \cdot \lambda^2}{4 \cdot \pi}$$ (m^2) $Aem = 0.105$ (m^2)

Maximum Effective Height h_{em}:

$$hem := 2 \cdot \sqrt{\frac{Rr \cdot Aem}{\eta o}}$$ (m) $hem = 0.021$ (m)

TWO-DIMENSIONAL ANTENNA PATTERN: (x-z) plane

Number of Elements along x-axis: $M = 5$

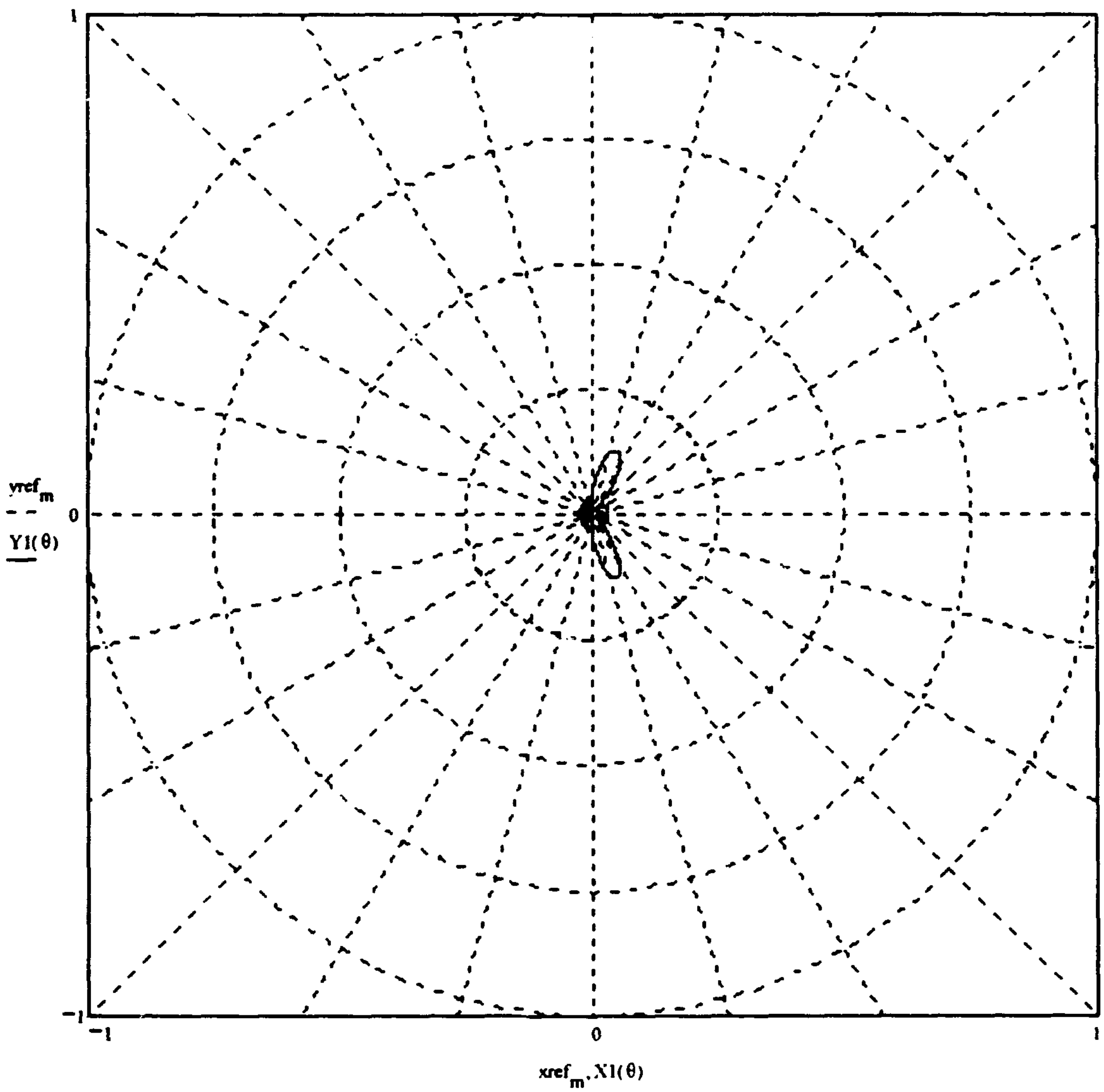
Number of Elements along y-axis: $N = 5$

Direction of Main Beam: $\theta o = 0.524$ (radians)

$\phi o = 0.785$ (radians)

$$X1(\theta) := |AFn(\theta,0)| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y1(\theta) := |AFn(\theta,0)| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0^{\circ}$



TWO-DIMENSIONAL ANTENNA PATTERN: (y-z) plane

Number of Elements along x-axis: $M = 5$

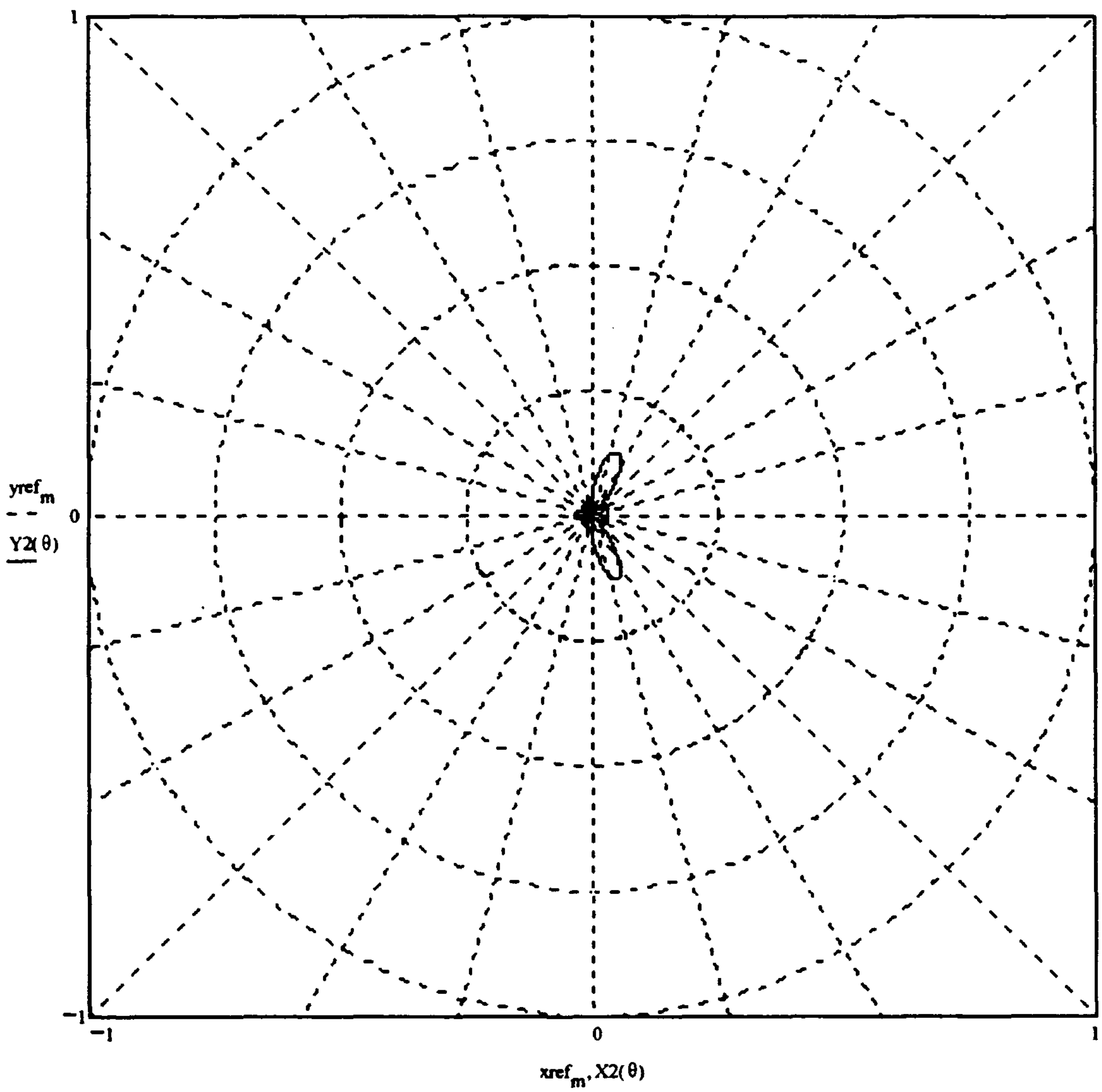
Number of Elements along y-axis: $N = 5$

Direction of Main Beam: $\theta o = 0.524$ (radians)

$\phi o = 0.785$ (radians)

$$X2(\theta) = \left| AFn\left(\theta, \frac{\pi}{2}\right) \right| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y2(\theta) = \left| AFn\left(\theta, \frac{\pi}{2}\right) \right| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0°$



TWO-DIMENSIONAL ANTENNA PATTERN: $\phi = \pi/4$

Number of Elements along x-axis: $M = 5$

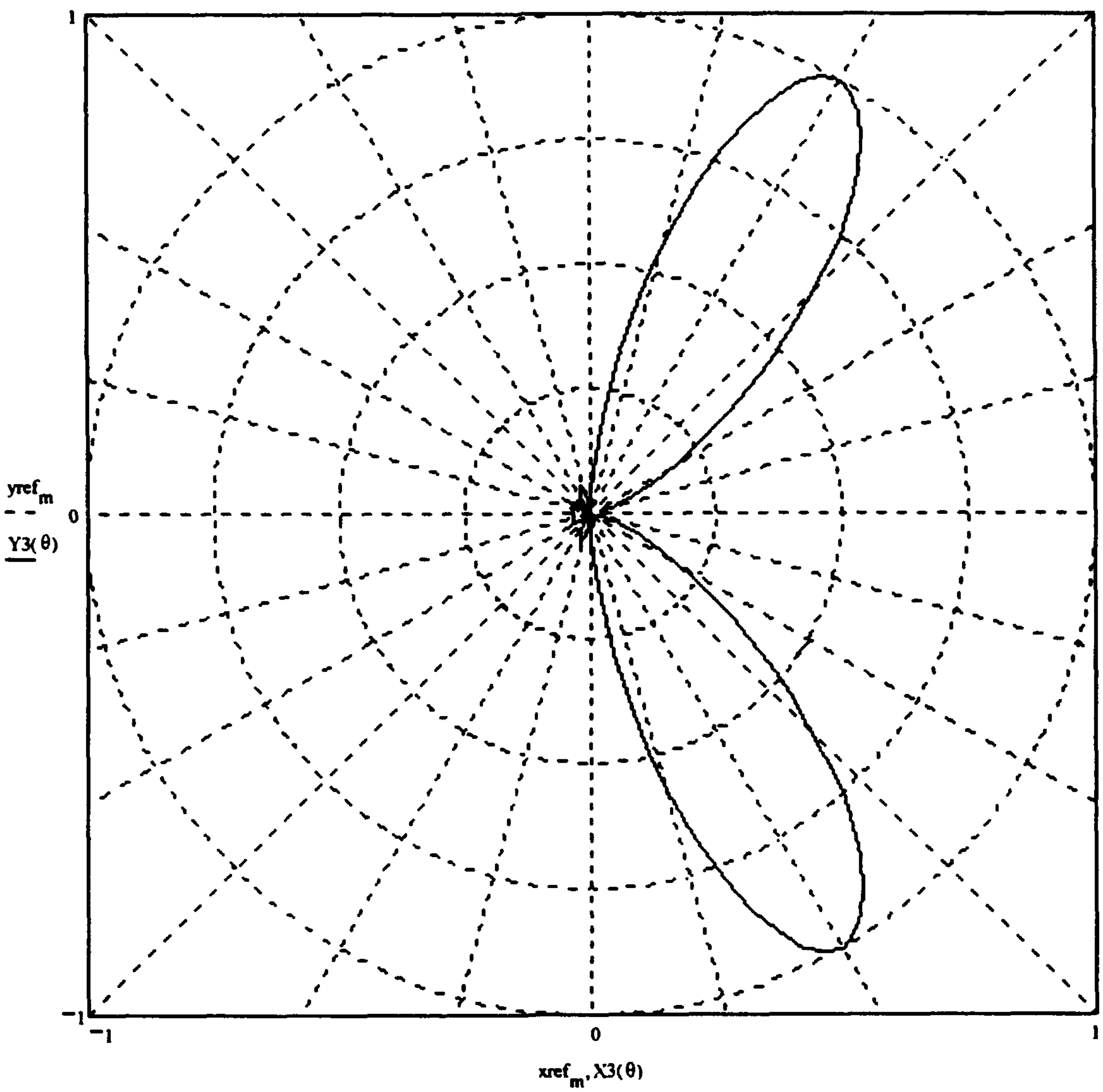
Number of Elements along y-axis: $N = 5$

Direction of Main Beam: $\theta o = 0.524$ (radians)

$\phi o = 0.785$ (radians)

$$X3(\theta) := \left|AFn\left(\theta, \frac{\pi}{4}\right)\right| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y3(\theta) := \left|AFn\left(\theta, \frac{\pi}{4}\right)\right| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0°$



TWO-DIMENSIONAL ANTENNA PATTERN: $\phi = \phi_o$

Number of elements along x-axis: $M = 5$

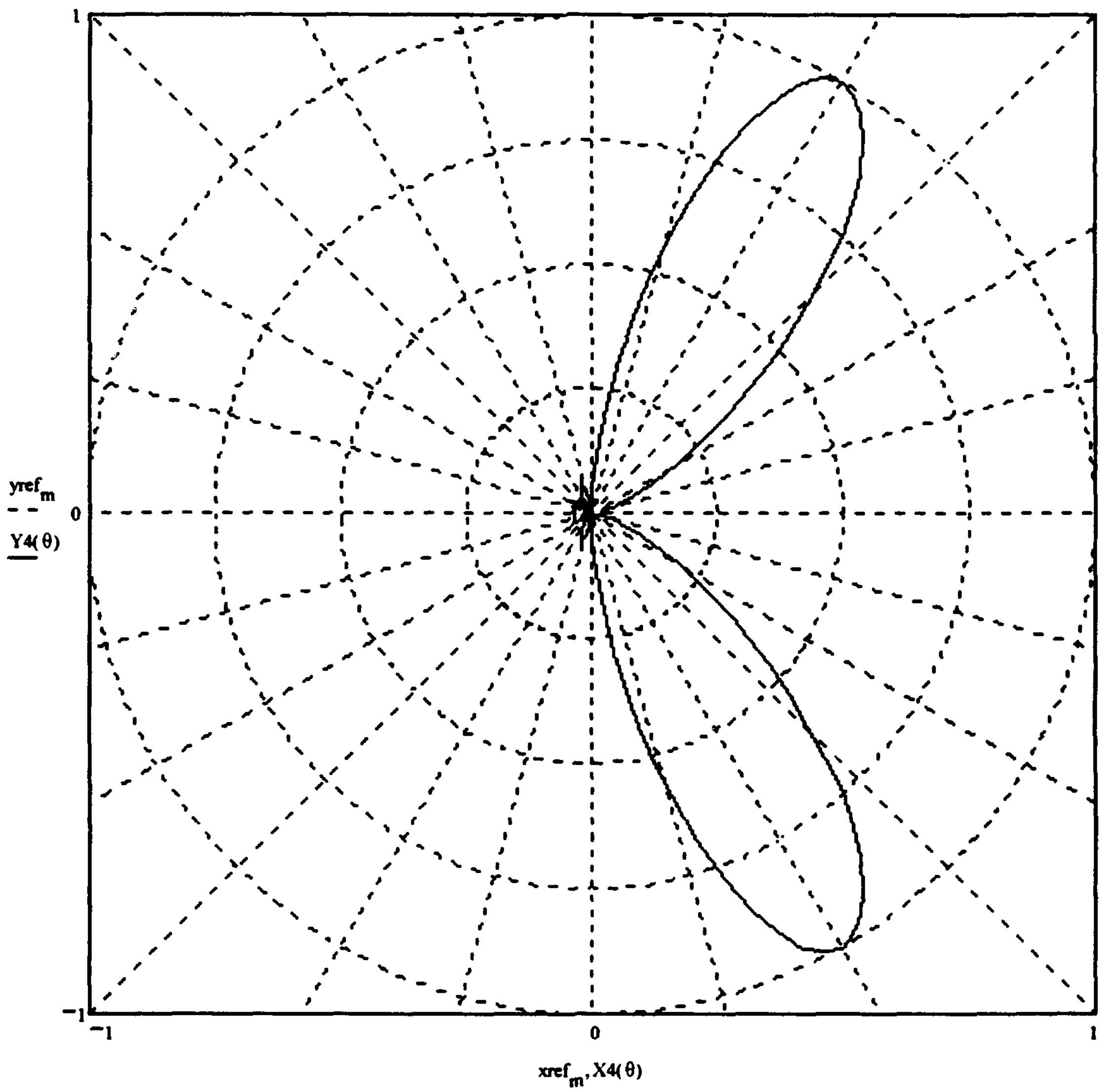
Number of Elements along y-axis: $N = 5$

Direction of Main Beam: $\theta o = 0.524$ (radians)

$\phi o = 0.785$ (radians)

$$X4(\theta) := \left|AFn(\theta, \phi o)\right| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y4(\theta) := \left|AFn(\theta, \phi o)\right| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0^{\circ}$



CIRCULAR ARRAY
MATHCAD SOFTWARE ARRAY_CI.MCD

This circular array application is based on N isotropic point sources uniformly spaced on the x-y plane along a circular ring of constant radius. Mutual interference between adjacent elements are neglected when calculating antenna parameters. Since isotropic point sources are used as the radiating elements, polarization of the antenna can not be determined. The gain can only be idealized when it is assumed that the antenna input resistance is equivalent to the radiation resistance and the total antenna efficiency is calculated for a ideal lossless antenna.

(Note: Mathcad equations can not use symbolic subscripts. Therefore, parameters do not contain subscripts (i.e., AF_n will be written AFn).)

The circular array antenna Mathcad application will compute thefollowing parameters (Items with * indicate parameters that can becomputed but their results are idealized as the result of usingisotropic point sources as the radiating elements):

λ = Wavelength

r_{min} = Minimum Distance to the Far-Field

α_n = Phase Excitation of the nth Element

$AF(\theta,\phi)$ = Normalized Array Factor

$U(\theta,\phi)$ = Radiation Intensity

P_{rad} = Radiated Power

D_o = Directivity

EIRP = Effective Isotropic Radiated Power

R_r = Radiation Resistance

R_{in} = Antenna Input Resistance *

Γ = Voltage Reflection Coefficient *

ε_t = Antenna Efficiency *

G = Gain *

A_{em} = Maximum Effective Aperture *

h_{em} = Maximum Effective Height *

The following known or estimated data must be entered:

N = Number of Isotropic Radiating Elements
f = Frequency of Interest
a = Radius of circle
θ_o = Direction of Main Beam at $\theta = \theta o$
ϕ_o = Direction of Main Beam at $\phi = \phi o$
I_o = Antenna Feed Current
Z_o = Characteristic Feed Impedence
j = Number of Increments in Degrees for Far-Field Radiation Pattern

---

Enter input data here

$N := 10$	(elements)	$a := \frac{10}{2 \cdot \pi}$	(meters)
$f := 3 \cdot 10^8$	(Hz)	$\theta o := 0$	(radians)
$Io := 1.0$	(A)	$\phi o := 0$	(radians)
$Zo := 75.0$	(Ω)	$j := 360$	(degrees)

Define constants and calculate wavelength:

$c := 2.9979 \cdot 10^8$	(meters/sec)	$\eta o := 120 \cdot \pi$	(Ω)
$\lambda := \frac{c}{f}$	(meters/cycle)	$k := \frac{2 \cdot \pi}{\lambda}$	(m^{-1})

Define constants and calculate wavelength:

$\lambda = 0.999$ (meters/cycle) $\quad$ $\text{ocd} := 1$ (dimensionless)

Calculate linear array parameters:

Define angular offset θ and ϕ:

$$\theta := \left(0 - 10^{-4}\right), \left(-10^{-4} + \frac{2\cdot\pi}{j}\right) .. \left(2\cdot\pi - 10^{-4}\right)$$ (radians)

$$\phi := \left(0 - 10^{-4}\right), \left(-10^{-4} + \frac{2\cdot\pi}{j}\right) .. \left(2\cdot\pi - 10^{-4}\right)$$ (radians)

Minimum Distance to Far-Field r_{min}:

$r_0 := 1.6\cdot\lambda$ (meters) $\quad$ $r_1 := 5\cdot(2\cdot a)$ (meters)

$$r_2 := \frac{2\cdot(2\cdot a)^2}{\lambda}$$ (meters)

$\text{rmin} := \max(r)$ (meters) $\quad$ $\text{rmin} = 20.278$ (meters)

Array Factor AF:

$n := 1 .. N$ (elements) $\quad$ $\Phi_n := 2\cdot\pi\cdot\frac{n}{N}$ (radians)

$$\alpha_n := -k\cdot a\cdot\sin(\theta o)\cdot\cos\left(\phi o - \Phi_n\right)$$ (radians)

Array Factor AF:

$$AF(\theta,\phi) := \frac{1}{N} \cdot \left[\sum_{n} e^{j \cdot \left(k \cdot a \cdot \sin(\theta) \cdot \cos\left(\phi - \Phi_{n}\right) + \alpha_{n} \right)} \right]$$ (dimensionless)

Radiation Intensity U:

$$U(\theta,\phi) := AF(\theta,\phi) \cdot \overline{AF(\theta,\phi)}$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := \int_{0}^{2 \cdot \pi} \int_{0}^{\frac{\pi}{2}} U(\theta,\phi) \cdot \sin(\theta) \, d\theta \, d\phi$$ (W)

$$Prad = 0.536$$ (W)

Directivity D_{o}:

$$Umax := 1$$ (W/solid angle)

$$Do := \frac{4 \cdot \pi \cdot Umax}{Prad}$$ (dimensionless)

$$Do = 23.427$$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$$EIRP := Prad \cdot Do$$ (W)

$$EIRP = 12.566$$ (W)

Radiation Resistance R_r:

$$Rr := \frac{2 \cdot Prad}{(|Io|)^2} \qquad (\Omega)$$

$$Rr = 1.073 \qquad (\Omega)$$

Input Resistance R_{in}:

$$Rin := Rr \qquad (\Omega)$$

$$Rin = 1.073 \qquad (\Omega)$$

Voltage Relection Coeffecient Γ:

$$\Gamma := \frac{Rin - Zo}{Rin + Zo} \qquad \text{(dimensionless)}$$

$$\Gamma = -0.972 \qquad \text{(dimensionless)}$$

Reflection Efficiency ε_t:

$$\varepsilon t := \varepsilon cd \cdot \left[1 - (|\Gamma|)^2\right] \qquad \text{(dimensionless)}$$

$$\varepsilon t = 0.056 \qquad \text{(dimensionless)}$$

Gain G:

$G := \alpha \cdot Do$ (dimensionless) $GdB := 10 \cdot \log(G)$ (dB)

$G = 1.303$ (dimensionless) $GdB = 1.149$ (dB)

Maximum Effective Aperture A_{em}:

$Aem := \frac{G \cdot \lambda^2}{4 \cdot \pi}$ (m^2) $Aem = 0.104$ (m^2)

Maximum Effective Height h_{em}:

$hem := 2 \cdot \sqrt{\frac{Rr \cdot Aem}{\eta o}}$ (m) $hem = 0.034$ (m)

TWO-DIMENSIONAL ANTENNA PATTERN: (x-z) plane

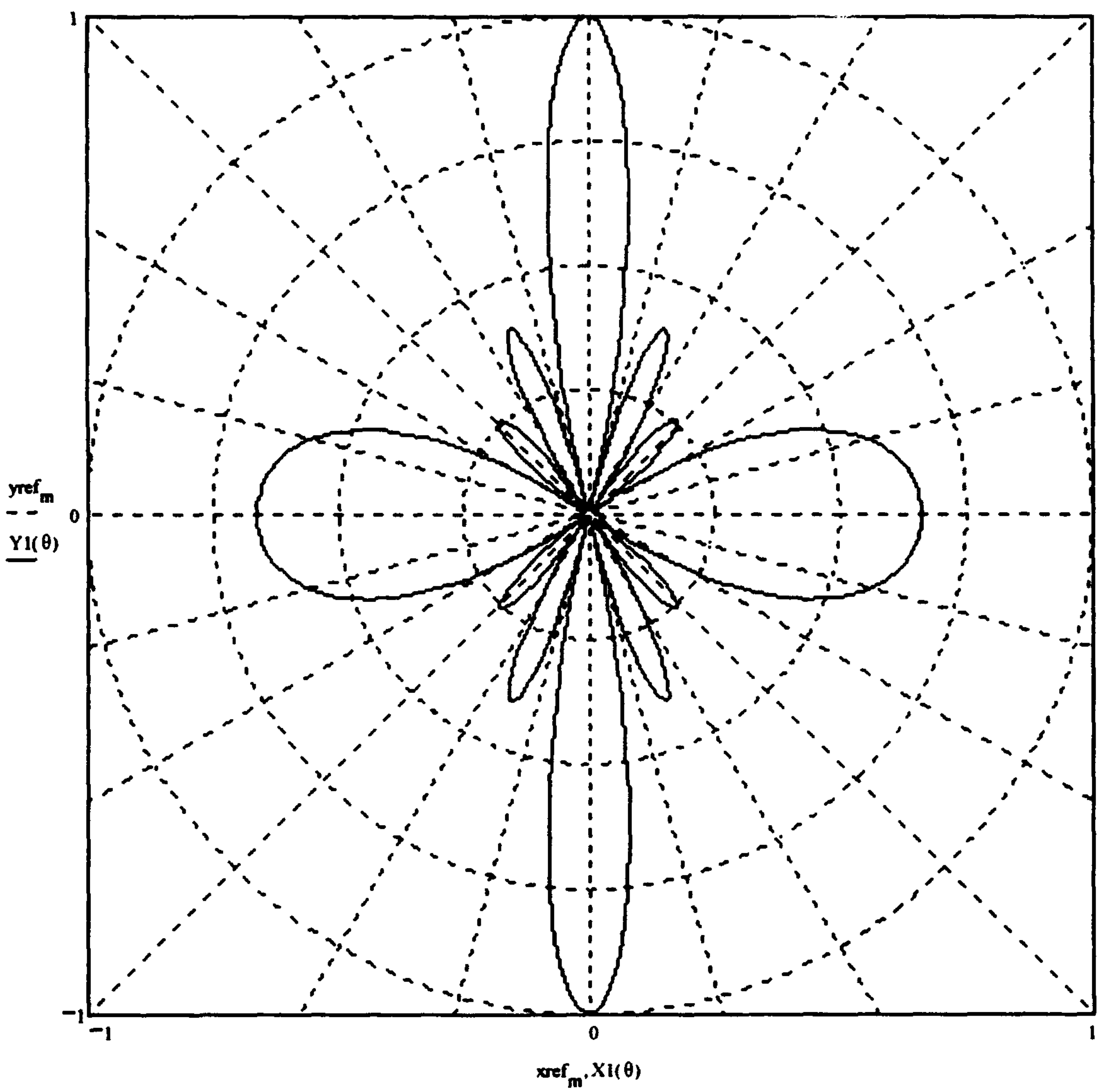
Number of Elements in x-y plane: $N = 10$

Direction of Main Beam: $\theta o = 0$ (radians)

$\phi o = 0$ (radians)

$$X1(\theta) := |AF(\theta,0)| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y1(\theta) := |AF(\theta,0)| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0^{\circ}$



TWO-DIMENSIONAL ANTENNA PATTERN: (y-z) plane

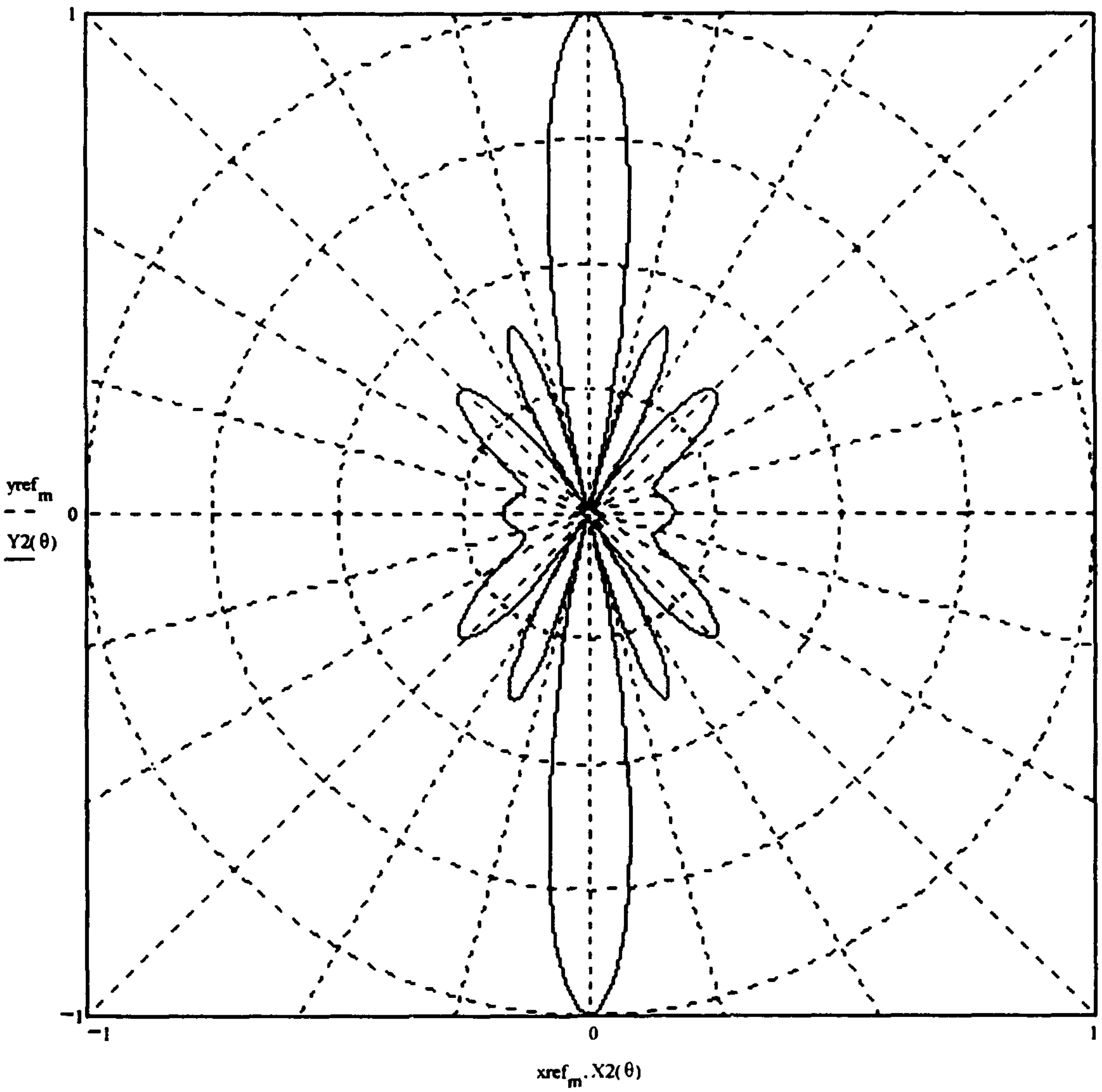
Number of Elements in x-y plane: $N = 10$

Direction of Main Beam: $\theta o = 0$ (radians)

$\phi o = 0$ (radians)

$$X2(\theta) = \left|AF\left(\theta,\frac{\pi}{2}\right)\right| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y2(\theta) = \left|AF\left(\theta,\frac{\pi}{2}\right)\right| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0°$



TWO-DIMENSIONAL ANTENNA PATTERN: $\phi = \pi/4$

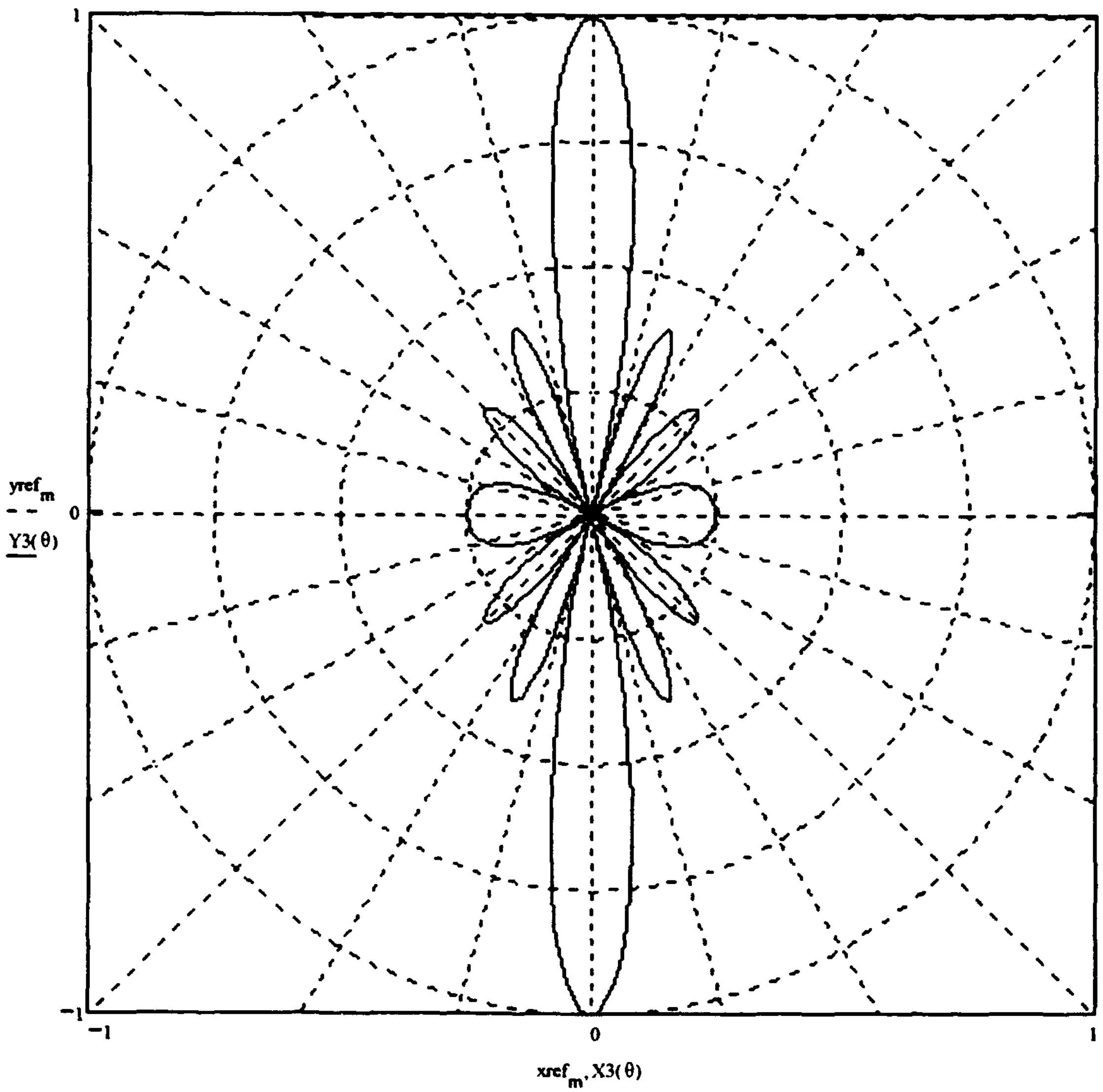
Number of Elements in x-y plane: $N = 10$

Direction of Main Beam: $\theta o = 0$ (radians)

$\phi o = 0$ (radians)

$$X3(\theta) := \left|AF\left(\theta, \frac{\pi}{4}\right)\right| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y3(\theta) := \left|AF\left(\theta, \frac{\pi}{4}\right)\right| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0°$



TWO-DIMENSIONAL ANTENNA PATTERN: $\phi = \phi_o$

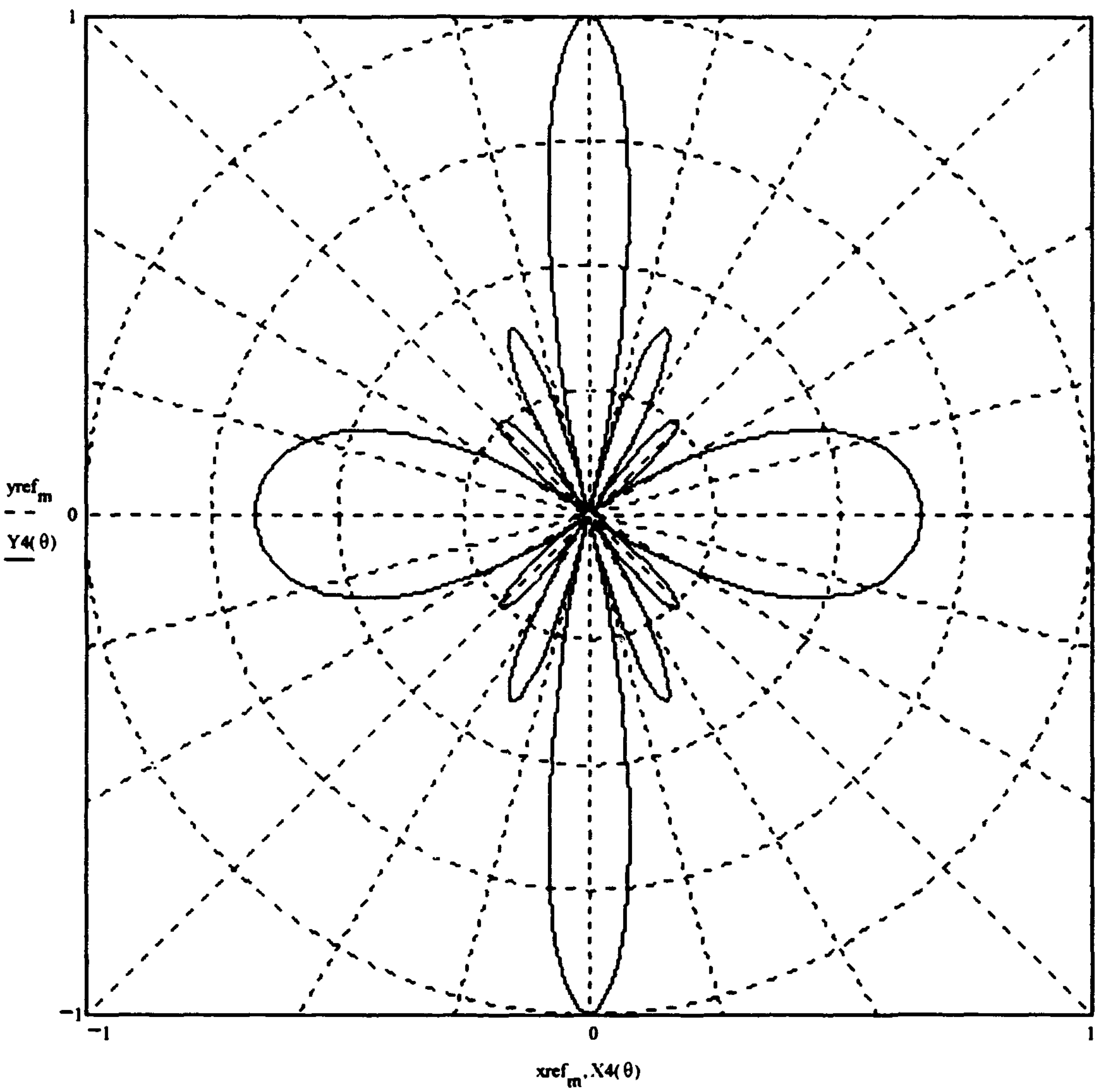
Number of Elements in x-y plane: $N = 10$

Direction of Main Beam: $\theta o = 0$ (radians)

$\phi o = 0$ (radians)

$$X4(\theta) := |AF(\theta, \phi o)| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y4(\theta) := |AF(\theta, \phi o)| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

$\theta = 0^\circ$



FOLDED DIPOLE
MATHCAD SOFTWARE FOLDED.MCD

To provide impedance matching to coaxial input cables and maintain good directional pattern characteristics, a folded dipole is used instead of a single dipole element. The geometry of a folded dipole is such that the spacing between dipole elements (d) is much less than the wavelength (λ) and the length of an element (L).

This Mathcad application calculates the parameters of a folded dipole aligned vertically on the z-axis and a folded dipole positioned horizontally above the earth. For the calculations, the antenna is assumed to behave as a cylindrical dipole with equivalent radius (a_e).

(Note: Mathcad equations can not use symbolic subscripts. Therefore, parameters do not contain subscripts (i.e., R_{in} will be written Rin).)

The folded dipole antenna Mathcad application will compute the following parameters (* indicates antenna parameters that are calculated for a folded dipole in free space and positioned horizontally over the earth):

λ = Wavelength
a_e = Equivalent Radius
R_r = Radiation Resistance
R_d = Input Resistance of Linear Dipole
X_r = Reactance of Linear Dipole
X_d = Input Reactance of Linear Dipole
Z_d = Input Impedance of Linear Dipole
Z_o = Characteristic Impedance of Two-wire Transmission Line
Z_t = Characteristic Impedance Transform of Folded Dipole
Z_{in} = Characteristic Impedance of Folded Dipole
Z_o = Maximum Current of Folded Dipole
r_{min} = Minimum Distance to the Far-Field
$E(\theta)$ = Electric Field *
$E_n(\theta)$ = Normalized Electric Field *
$U(\theta)$ = Radiation Intensity *
U_{max} = Maximum Radiation Intensity *
P_{rad} = Radiated Power *
D_o = Directivity *
EIRP = Effective Isotropic Radiated Power *
R_{in} = Antenna Input Resistance *
Γ = Voltage Reflection Coefficient *

ε_t = Antenna Efficiency *

σ_a = Antenna Polarization Vector *

PLF = Polarization Loss Factor *

G = Gain *

A_{em} = Maximum Effective Aperture *

h_{em} = Maximum Effective Height *

ε_{rp} = Relative Complex Effective Dielectric Constant of Ground

Γ_v = Vertical Reflection Coefficient of Ground Plane

Γ_h = Horizontal Reflection Coefficient of Ground Plane

The following known or estimated data must be entered:

f = Frequency of Interest

a = Radius of Conductor

b = Radius of Conductor

d = Distance Between Conductors

L = Length of Antenna

h = Height of Antenna Above Earth

ε_{cd} = Conduction/Dielectric Efficiency

ε_r = Relative Dielectic Constant of Ground

σ = Conductivity of Ground

r_{ff} = Far Field Distance

i = Number of Increments in Degrees for Far Field Radiation Pattern

σ_w = Incoming Wave Electric Field Unit Vector For Free Space Antenna

σ_{wh} = INcoming Wave Electric Field Unit Vector For Horizontal Antenna

θ_{ph} = Antenna Polarization Direction For Horizontal Antenna

---

Enter input data here

$\sigma w := \begin{pmatrix}1\\0\\0\end{pmatrix}$	(dimensionless)	$\sigma wh := \begin{pmatrix}0\\0\\1\end{pmatrix}$	(dimensionless)
$f := 3 \cdot 10^8$	(Hz)	$\theta ph := 0$	(radians)
$a := 0.0005$	(meters)	$rff := 1 \cdot 10^5$	(meters)
$b := a$	(meters)	$i := 90$	(radians)
$d := 12.5 \cdot a$	(meters)	$\varepsilon cd := 1$	(dimensionless)
$L := 0.5$	(meters)	$\sigma := 25$	(siemens/meter)
$h := .5$	(meters)	$\varepsilon r := 15$	(dimensionless)

Define constants:

$\sigma a := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (dimensionless)

$c := 2.9979 \cdot 10^{8}$ (meters/sec) $\eta o := 120 \cdot \pi$ (Ω)

$\lambda := \frac{c}{f}$ (meters/cycle) $k := \frac{2 \cdot \pi}{\lambda}$ (m^{-1})

$\lambda = 0.999$ (meters/cycle) $\gamma := 0.57721$ (dimensionless)

$\varepsilon o := \frac{1}{36 \cdot \pi} \cdot 10^{-9}$ (F/m) $z := 0 .. 2 \cdot i$ (radians)

Define angular offset θ:

$\theta := 0, \frac{2 \cdot \pi}{i} .. 2 \cdot \pi$ (radians)

Define cosine [Ci(x)] and sine [Si(x)] integrals:

$n := 1 .. 50$

$\mathrm{Ci}(x) := 0.57721 + \ln(x) - \sum_{n} \frac{(-1)^{n-1} \cdot x^{2 \cdot n}}{(2 \cdot n) \cdot (2 \cdot n)!}$ $\mathrm{Si}(x) := \sum_{n} \frac{(-1)^{n-1} \cdot x^{(2 \cdot n - 1)}}{(2 \cdot n - 1) \cdot (2 \cdot n - 1)!}$

FOLDED DIPOLE ALIGNED ON THE Z-AXIS IN FREE SPACE:

Equivalent radius a_e:

$$ae = \exp\left[\frac{a^2\cdot\ln(a) + b^2\cdot\ln(b) + 2\cdot a\cdot b\cdot\ln(d)}{(a+b)^2}\right]$$ (meters)

$$ae = 0.002$$ (meters)

Electric field $E(\theta)$:

$$E(\theta) = j\cdot\eta o\cdot\frac{e^{-j\cdot k\cdot rff}}{2\cdot\pi\cdot rff}\cdot\left(\frac{\cos\left(\frac{k\cdot L}{2}\cdot\cos(\theta)\right) - \cos\left(\frac{k\cdot L}{2}\right)}{\sin(\theta)}\right)$$ (V/m)

$$En(\theta) := \frac{E(\theta)}{E\left(\frac{\pi}{2}\right)}$$ (V/m)

Minimum Distance to Far-Field r_{min}:

$r_0 := 1.6\cdot\lambda$ (meters) $r_1 := 5\cdot L$ (meters)

$$r_2 := \frac{2\cdot(L)^2}{\lambda}$$ (meters)

$rmin := \max(r)$ (meters)

$rmin = 2.5$ (meters)

Radiation Intensity $U(\theta)$:

$$U(\theta) := \frac{rff^2}{2 \cdot \eta o} \cdot (|E(\theta)|)^2$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := 2 \cdot \pi \cdot \int_0^{\pi} U(\theta) \cdot \sin(\theta)\, d\theta$$ (W)

$$Prad = 36.64$$ (W)

Directivity D_o:

$$Umax := U\left(\frac{\pi}{2}\right)$$ (W/solid angle)

$$Do := \frac{4 \cdot \pi \cdot Umax}{Prad}$$ (dimensionless)

$$Do = 1.641$$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$$EIRP := Prad \cdot Do$$ (W)

$$EIRP = 60.132$$ (W)

Radiation Resistance R_r:

$$Rr := \frac{\eta o}{2 \cdot \pi} \cdot \left[\gamma + \ln(k \cdot L) - Ci(k \cdot L) + \frac{1}{2} \cdot \sin(k \cdot L) \cdot (Si(2 \cdot k \cdot L) - 2 \cdot Si(k \cdot L)) \ldots \atop + \frac{1}{2} \cdot \cos(k \cdot L) \cdot \left(\gamma + \ln\left(\frac{k \cdot L}{2}\right) + Ci(2 \cdot k \cdot L) - 2 \cdot Ci(k \cdot L) \right) \right] \quad (\Omega)$$

$$Rr = 73.281 \quad (\Omega)$$

Input Radiation Resistance of linear dipole R_d:

$$Rd := \frac{Rr}{\left(\sin\left(\frac{k \cdot L}{2}\right)\right)^2} \quad (\Omega)$$

$$Rd = 73.281 \quad (\Omega)$$

Reactance of linear dipole X_r:

$$Xr := \frac{\eta o}{4 \cdot \pi} \cdot \left[2 \cdot Si(k \cdot L) + \cos(k \cdot L) \cdot (2 \cdot Si(k \cdot L) - Si(2 \cdot k \cdot L)) \ldots \atop + \left[-1 \cdot \sin(k \cdot L) \cdot \left(2 \cdot Ci(k \cdot L) - Ci(2 \cdot k \cdot L) - Ci\left(\frac{2 \cdot k \cdot ae^2}{L}\right) \right) \right] \right] \quad (\Omega)$$

$$Xr = 43.142 \quad (\Omega)$$

Input reactance of linear dipole X_d:

$$Xd := \frac{Xr}{\left(\sin\left(\frac{k \cdot L}{2}\right)\right)^2} \quad (\Omega)$$

$$Xd = 43.142 \quad (\Omega)$$

Input impedance of linear dipole Z_d:

$$Zd := Rd + j \cdot Xd \qquad (\Omega)$$

$$Zd = 73.281 + 43.142i \qquad (\Omega)$$

Characteristic impedance of two-wire transmission line Z_o:

$$Zo := \frac{\eta o}{\pi} \cdot \operatorname{acosh}\left(\frac{d}{2 \cdot \sqrt{a \cdot b}}\right) \qquad (\Omega)$$

$$Zo = 302.312 \qquad (\Omega)$$

Characteristic impedance transform for folded dipole Z_t:

$$Zt := j \cdot Zo \cdot \tan\left(\frac{k \cdot L}{2}\right) \qquad (\Omega)$$

$$Zt = -2.747 \cdot 10^{5} i \qquad (\Omega)$$

Characteristic impedance of folded dipole Z_{in}:

$$Zin := \frac{4 \cdot Zt \cdot Zd}{2 \cdot Zd + Zt} \qquad (\Omega)$$

$$Zin = 293.307 + 172.465i \qquad (\Omega)$$

Input Resistance R_{in}:

$Rin := Re(Zin)$ (Ω) $\qquad$ $Rin = 293.307$ (Ω)

Voltage Relection Coeffecient Γ:

$\Gamma := \frac{Rin - Zo}{Rin + Zo}$ (dimensionless) $\Gamma = -0.015$ (dimensionless)

Reflection Efficiency ε_t:

$\varepsilon t := \varepsilon cd \cdot \left[1 - (|\Gamma|)^2\right]$ (dimensionless) $\varepsilon t = 1$ (dimensionless)

Gain G:

$G := \varepsilon t \cdot Do$ (dimensionless) $GdB := 10 \cdot \log(G)$ (dB)

$G = 1.641$ (dimensionless) $GdB = 2.15$ (dB)

Polarization loss factor PLF:

$PLF := \left(\left|\sigma w \cdot \overline{\sigma a}\right|\right)^2$ (dimensionless) $PLF = 1$ (dimensionless)

Maximum Effective Aperture A_{em}:

$Aem := \frac{G \cdot \lambda^2}{4 \cdot \pi} \cdot PLF$ (m^2) $Aem = 0.13$ (m^2)

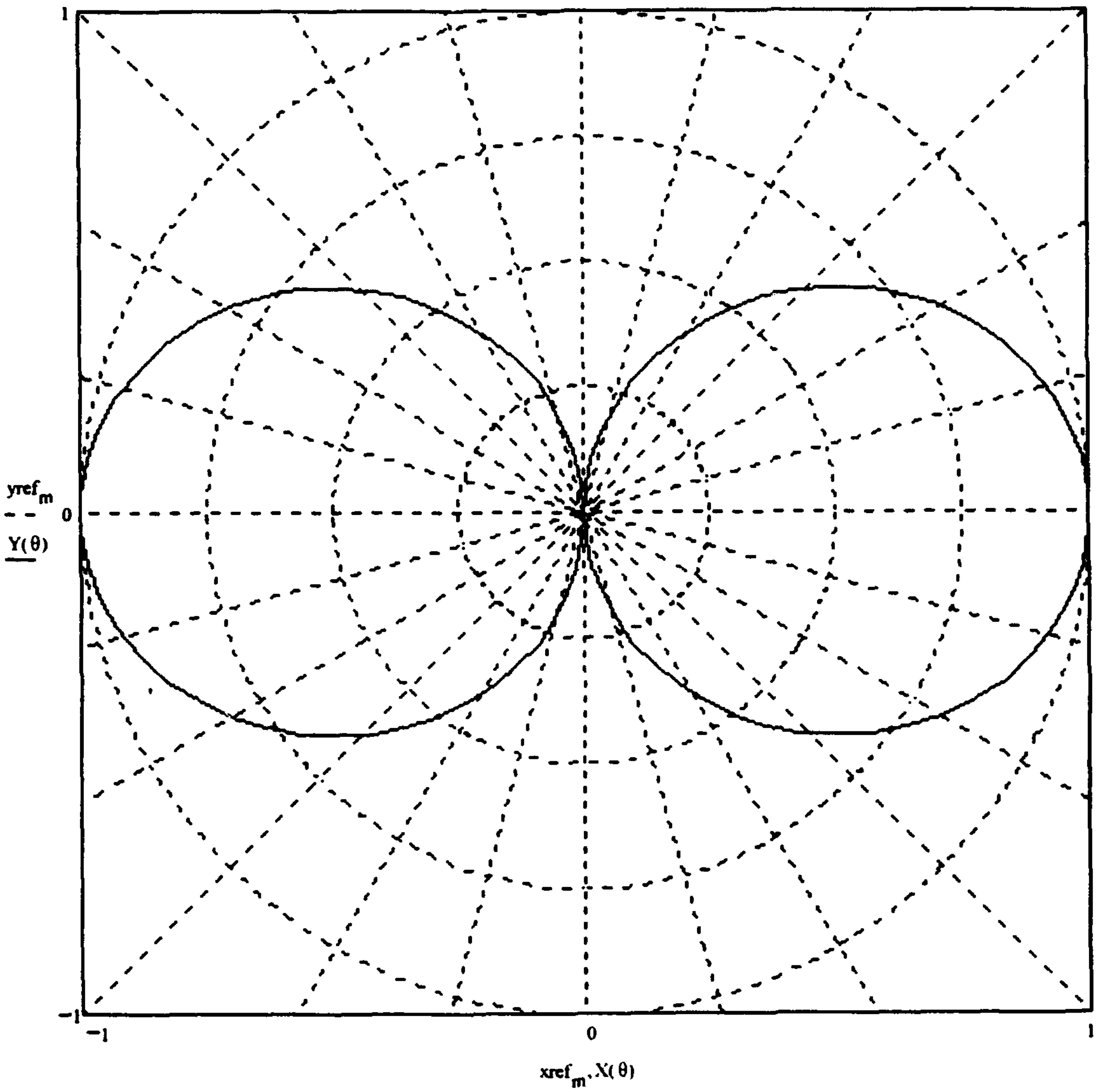
Maximum Effective Height h_{em}:

$hem := 2 \cdot \sqrt{\frac{Rr \cdot Aem}{\eta o}}$ (m) $hem = 0.318$ (m)

Far Field radiation pattern of vertical folded dipole in free space.

$$X(\theta) := |En(\theta)| \cdot \cos\left(\theta + \frac{\pi}{2}\right) \qquad Y(\theta) := |En(\theta)| \cdot \sin\left(\theta + \frac{\pi}{2}\right)$$

θ = 0°



θ = 180°

HORIZONTALLY POSITIONED FOLDED DIPOLE OVER THE EARTH:

Define angular offsets θ1 and θ2:

$$\theta 1 := -\frac{\pi}{2}, -\frac{\pi}{2}+\frac{\pi}{2\cdot i} .. \frac{\pi}{2}$$ (radians)

$$\theta 2_z := -\frac{\pi}{2}+\frac{\pi}{2\, i}\cdot z$$ (radians)

Relative complex effective dielectric constant ε_{rp}:

$$\varepsilon rp := \varepsilon r - j\cdot\frac{\sigma}{2\cdot\pi\cdot f\cdot\varepsilon o}$$ (dimensionless)

ϕ component of E-field Eϕ:

$$E\phi(\theta 1) := j\cdot\eta o\cdot\frac{e^{-j\cdot k\cdot rff}}{2\cdot\pi\cdot rff}\cdot\left[\frac{\cos\left(\frac{k\cdot L}{2}\cdot\sin(\theta 1)\cdot\sin(0)\right)-\cos\left(\frac{k\cdot L}{2}\right)}{\sqrt{1-\sin(\theta 1)^2\cdot\sin(0)^2}}\right]$$ (V/m)

$$E\phi 1_z := j\cdot\eta o\cdot\frac{e^{-j\cdot k\cdot rff}}{2\cdot\pi\cdot rff}\cdot\left[\frac{\cos\left(\frac{k\cdot L}{2}\cdot\sin\left(\theta 2_z\right)\cdot\sin(0)\right)-\cos\left(\frac{k\cdot L}{2}\right)}{\sqrt{1-\sin\left(\theta 2_z\right)^2\cdot\sin(0)^2}}\right]$$ (V/m)

θ component of E-field Eθ:

$$E\theta(\theta 1) := j\cdot\eta o\cdot\frac{e^{-j\cdot k\cdot rff}}{2\cdot\pi\cdot rff}\cdot\left[\frac{\cos\left(\frac{k\cdot L}{2}\cdot\sin(\theta 1)\cdot\sin\left(\frac{\pi}{2}\right)\right)-\cos\left(\frac{k\cdot L}{2}\right)}{\sqrt{1-\sin(\theta 1)^2\cdot\sin\left(\frac{\pi}{2}\right)^2}}\right]$$ (V/m)

$$E\theta 1_z := j\cdot\eta o\cdot\frac{e^{-j\cdot k\cdot rff}}{2\cdot\pi\cdot rff}\cdot\left[\frac{\cos\left(\frac{k\cdot L}{2}\cdot\sin\left(\theta 2_z\right)\cdot\sin\left(\frac{\pi}{2}\right)\right)-\cos\left(\frac{k\cdot L}{2}\right)}{\sqrt{1-\sin\left(\theta 2_z\right)^2\cdot\sin\left(\frac{\pi}{2}\right)^2}}\right]$$ (V/m)

Vertical plane wave reflection coefficient Γ_v:

$$\Gamma v(\theta 1) := \frac{\varepsilon rp \cdot \cos(\theta 1) - \sqrt{\varepsilon rp - \sin(\theta 1)^2}}{\varepsilon rp \cdot \cos(\theta 1) + \sqrt{\varepsilon rp - \sin(\theta 1)^2}}$$ (dimensionless)

$$\Gamma v1_z := \frac{\varepsilon rp \cdot \cos\left(\theta 2_z\right) - \sqrt{\varepsilon rp - \sin\left(\theta 2_z\right)^2}}{\varepsilon rp \cdot \cos\left(\theta 2_z\right) + \sqrt{\varepsilon rp - \sin\left(\theta 2_z\right)^2}}$$ (dimensionless)

Horizontal plane wave reflection coefficient Γ_h:

$$\Gamma h(\theta 1) := \frac{\cos(\theta 1) - \sqrt{\varepsilon rp - \sin(\theta 1)^2}}{\cos(\theta 1) + \sqrt{\varepsilon rp - \sin(\theta 1)^2}}$$ (dimensionless)

$$\Gamma h1_z := \frac{\cos\left(\theta 2_z\right) - \sqrt{\varepsilon rp - \sin\left(\theta 2_z\right)^2}}{\cos\left(\theta 2_z\right) + \sqrt{\varepsilon rp - \sin\left(\theta 2_z\right)^2}}$$ (dimensionless)

E-field (ϕ component) with ground plane reflection $E\phi_{gp}$:

$$E\phi gp(\theta 1) := E\phi(\theta 1) \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 1)} + \Gamma h(\theta 1) \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 1)}\right)$$ (V/m)

$$E\phi gp1_z := \left|\left[E\phi 1_z \cdot \left(e^{j \cdot k \cdot h \cdot \cos\left(\theta 2_z\right)} + \Gamma h1_z \cdot e^{-j \cdot k \cdot h \cdot \cos\left(\theta 2_z\right)}\right)\right]\right|$$ (V/m)

$$E\phi n(\theta 1) := \frac{|E\phi gp(\theta 1)|}{\max(E\phi gp1)}$$ (V/m)

E-field (θ component) with ground plane reflection $E\theta_{gp}$:

$$E\theta gp(\theta 1) := E\theta(\theta 1)\cdot\left(e^{j\cdot k\cdot h\cdot\cos(\theta 1)} - \Gamma v(\theta 1)\cdot e^{-j\cdot k\cdot h\cdot\cos(\theta 1)}\right)$$ (V/m)

$$E\theta gp1_z := \left|\overrightarrow{\left[E\theta 1_z\cdot\left(e^{j\cdot k\cdot h\cdot\cos(\theta 2_z)} - \Gamma v1_z\cdot e^{-j\cdot k\cdot h\cdot\cos(\theta 2_z)}\right)\right]}\right|$$ (V/m)

$$E\theta n(\theta 1) := \frac{|E\theta gp(\theta 1)|}{\max(E\theta gp1)}$$ (V/m)

Radiation Intensity Uh(θ1):

$$Uh(\theta 1) := \frac{rff^2}{2\cdot\eta o}\cdot\left[(|E\phi gp(\theta 1)|)^2 + (|E\theta gp(\theta 1)|)^2\right]$$ (W/solid angle)

Radiated Power Ph_{rad}:

$$Phrad := 2\cdot\pi\cdot\int_0^{\frac{\pi}{2}} Uh(\theta 1)\cdot\sin(\theta 1)\,d\theta 1$$ (W)

$$Phrad = 70.415$$ (W)

Directivity Dh_o:

$$Ugp\_max_z := \frac{rff^2}{2\cdot\eta o}\cdot\left[\left(\left|\overrightarrow{E\phi gp1_z}\right|\right)^2 + \left(\left|\overrightarrow{E\theta gp1_z}\right|\right)^2\right]$$ (W/solid angle)

$$Uhmax := \max(Ugp\_max)$$ (W/solid angle)

Directivity Dh_o:

$$Dho = \frac{4 \cdot \pi \cdot Uhmax}{Phrad}$$ (dimensionless)

Dho = 3.938 (dimensionless)

Effective Isotropic Radiated Power EIRPh:

EIRPh := Phrad·Dho (W)

EIRPh = 277.3 (W)

Input Resistance Rh_{in}:

$$Rhin := \frac{2 \cdot Phrad}{\sin\left(\frac{k \cdot L}{2}\right)^2}$$ (Ω)

Rhin = 140.83 (Ω)

Voltage Relection Coeffecient ΓH:

$$\Gamma H := \frac{Rhin - Zo}{Rhin + Zo}$$ (dimensionless)

ΓH = −0.364 (dimensionless)

Reflection Efficiency ϵh_t:

$\epsilon ht := \epsilon cd \cdot \left[1 - (|\Gamma H|)^2\right]$ (dimensionless) $\epsilon ht = 0.867$ (dimensionless)

Gain Gh:

$Gh := \varepsilon ht \cdot Dho$ (dimensionless) $GhdB := 10 \cdot \log(Gh)$ (dB)

$Gh = 3.415$ (dimensionless) $GhdB = 5.334$ (dB)

Polarization loss factor PLFv:

$Ehmax := |E\theta n(\theta ph)|$ (V/m) $Lobes := \mathrm{floor}\left(\frac{2 \cdot h}{\lambda} + 1\right)$

$$\sigma ah := \begin{bmatrix} 0 \\ 0 \\ \frac{Ehmax}{Lobes} \end{bmatrix}$$ (dimensionless)

$$PLFh := \left(\left|\sigma wh \cdot \overline{\sigma ah}\right|\right)^2$$ (dimensionless)

$PLFh = 7.736 \cdot 10^{-4}$ (dimensionless)

Maximum Effective Aperture Av_{em}:

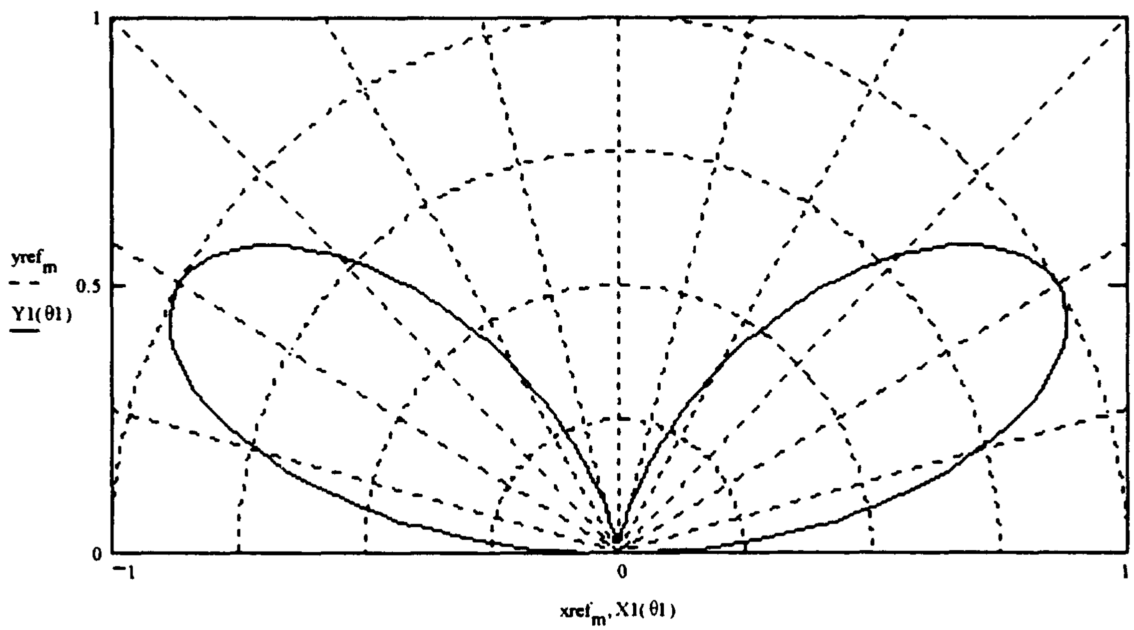
$Ahem := \frac{Gh \cdot \lambda^2}{4 \cdot \pi} \cdot PLFh$ (m^2) $Ahem = 2.099 \cdot 10^{-4}$ (m^2)

Maximum Effective Height hv_{em}:

$hhem := 2 \cdot \sqrt{\frac{Rhin \cdot Ahem}{\eta o}}$ (m) $hhem = 0.018$ (m)

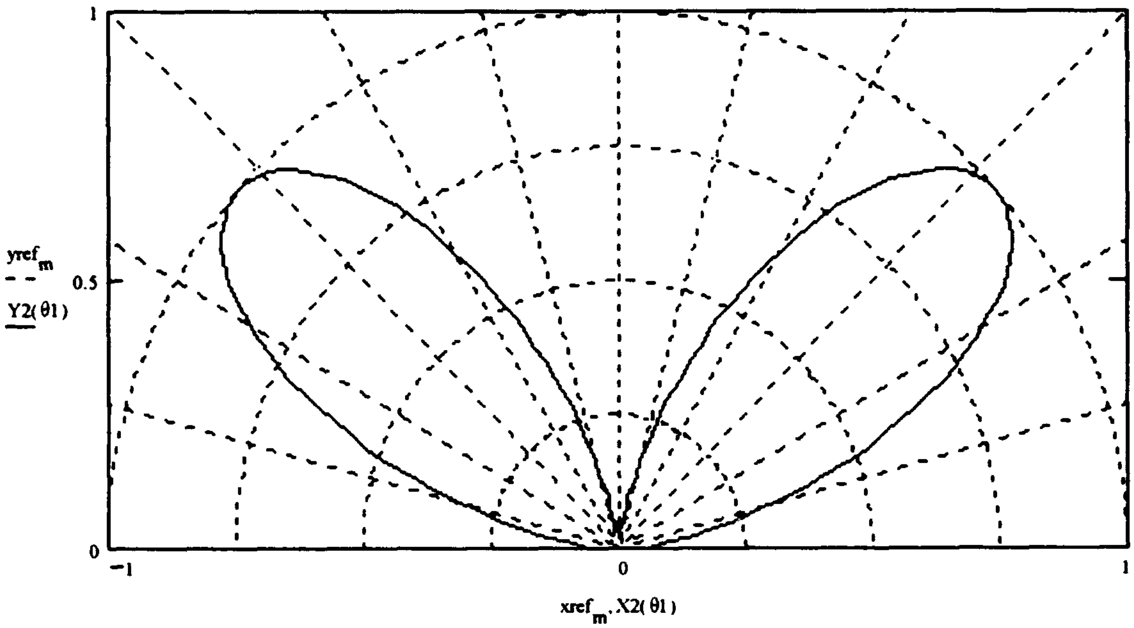
H-Plane Radiation Pattern (y-z plane) of Horizontal Folded Dipole Over The Earth:

$$Y1(\theta 1) = |E\phi n(\theta 1)| \cdot \sin\left(\theta 1 + \frac{\pi}{2}\right) \qquad X1(\theta 1) = |E\phi n(\theta 1)| \cdot \cos\left(\theta 1 + \frac{\pi}{2}\right)$$



E-Plane Radiation Pattern (x-z plane) of Horizontal Folded Dipole Over The Earth:

$$Y2(\theta 1) := |E\theta n(\theta 1)| \cdot \sin\left(\theta 1 + \frac{\pi}{2}\right) \qquad X2(\theta 1) := |E\theta n(\theta 1)| \cdot \cos\left(\theta 1 + \frac{\pi}{2}\right)$$



CAGED DIPOLE ANTENNA
MATHCAD SOFTWARE CAGED_DI.MCD

This caged dipole application is modeled after a circular array with the radiating elements as dipoles. The dipoles are modeled after conductors with infinitely small radii and antenna length greater than $\lambda/4$. Mutual interference between adjacent elements are neglected when calculating antenna parameters.

(Note: Mathcad equations can not use symbolic subscripts. Therefore, parameters do not contain subscripts (i.e., AF_n will be written AFn).)

The caged dipole antenna Mathcad application will compute the following parameters:

VERTICAL CAGED DIPOLE IN FREE SPACE:

λ = Wavelength
r_{min} = Minimum Distance to the Far-Field
$AF(\theta,\phi)$ = Normalized Array Factor
$E(\theta,\phi)$ = E-Field of Finite Length Dipole
$E_t(\theta,\phi)$ = Total E-Field
$E_{tn}(\theta,\phi)$ = Total Normalized E-Field
$U(\theta,\phi)$ = Radiation Intensity
P_{rad} = Radiated Power
D_o = Directivity
EIRP = Effective Isotropic Radiated Power
R_r = Radiation Resistance
R_{in} = Input Resistance
Γ = Voltage Reflection Coefficient
ε_t = Antenna Effeciency
G = Gain
PLF = Polarization Loss Factor
A_{em} = Maximum Effective Aperture
h_{em} = Maximum Effective Height
flow = Low Frequency
fhigh = High Frequency
BW = Bandwidth

VERTICAL CAGED DIPOLE OVER EARTH:

ε_{rp} = Relative Complex Effective Dielectric Constant
$AFv(\theta1,\phi)$ = Normalized Array Factor

$Ev(\theta 1,\phi)$ = E-Field of Finite Length Dipole

$\Gamma_v(\theta 1)$ = Vertical Plane Wave Reflection Coefficient

$Ev_t(\theta 1,\phi)$ = Total E-Field

$Ev_{tn}(\theta 1,\phi)$ = Total Normalized E-Field

$Uv(\theta 1,\phi)$ = Radiation Intensity

Pv_{rad} = Radiated Power

Dv_o = Directivity

EIRPv = Effective Isotropic Radiated Power

Rv_r = Radiation Resistance

Rv_{in} = Input Resistance

ΓV = Voltage Reflection Coefficient

ϵv_t = Antenna Efficiency

Gv = Gain

PLFv = Polarization Loss Factor

Av_{em} = Maximum Effective Aperture

hv_{em} = Maximum Effective Height

HORIZONTAL CAGED DIPOLE OVER EARTH:

$AFh(\theta 1,\phi 1)$ = Normalized Array Factor

$\Gamma_h(\theta 1)$ = Horizontal Plane Wave Reflection Coefficient

$E\theta h(\theta 1)$ = θ Component of E-Field

$E\theta h_t(\theta 1,\phi 1)$ = θ Component of Total E-Field

$E\theta h_{tn}(\theta 1,\phi 1)$ = θ Component of Total Normalized E-Field

$E\phi h(\theta 1)$ = ϕ Component of E-Field

$E\phi h_t(\theta 1,\phi 1)$ = ϕ Component of Total E-Field

$E\phi h_{tn}(\theta 1,\phi 1)$ = ϕ Component of Total Normalized E-Field

$Uh(\theta 1,\phi 1)$ = Radiation Intensity

Ph_{rad} = Radiated Power

Dh_o = Directivity

EIRPh = Effective Isotropic Radiated Power

Rh_r = Radiation Resistance

Rh_{in} = Input Resistance

ΓH = Voltage Reflection Coefficient

ϵh_t = Antenna Efficiency

Gh = Gain

PLFh = Polarization Loss Factor

Ah_{em} = Maximum Effective Aperture

hh_{em} = Maximum Effective Height

The following known or estimated data must be entered:

N = Number of Dipoles
a = Radius of Antenna
L = Length of Antenna
h = Antenna Height Above Ground
f = Frequency of Interest
I_o = Antenna Feed Current
Z_o = Input Impedance
ε_r = Relative Dielectric Constant of Ground
σ = Conductivity of Ground
σ_w = Incoming Wave E-Field Unit Vector for Antenna (Free Space)
σ_a = Antenna Unit Polarization Vector (Free Space)
σv_w = Incoming Wave E-Field Unit Vector for Vertical Antenna
θv_p = Direction of Incoming Unit Vector (Vertical Antenna)
σh_w = Incoming Wave E-Field Unit Vector for Horizontal Antenna
θh_p = Direction of Incoming Unit Vector (Horizontal Antenna)
j = Number of Increments in Degrees for Far Field Radiation Pattern

---

Enter input data here:

$\sigma w := \begin{pmatrix}1\\0\\0\end{pmatrix}$ (dimensionless) $\sigma a := \begin{pmatrix}1\\0\\0\end{pmatrix}$ (dimensionless)

$\sigma vw := \begin{pmatrix}1\\0\\0\end{pmatrix}$ (dimensionless) $\theta vp := \frac{80 \cdot \pi}{180}$ (dimensionless)

$\sigma hw := \begin{pmatrix}0\\0\\1\end{pmatrix}$ (dimensionless) $\theta hp := 0$ (dimensionless)

$N := 8$ (elements) $Io := 1.0$ (A)

$a := \frac{1}{8}$ (meters) $Zo := 75.0$ (Ω)

$L := 0.5$ (meters) $j := 90$ (increments)

$h := 0.25$ (meters) $\sigma := 10^2$ (siemens/m)

$f := 3 \cdot 10^8$ (Hz) $\epsilon r := 15$ (dimensionless)

Define constants and calculate wavelength:

$c := 2.9979 \cdot 10^8$ (meters/sec) $\eta o := 120 \cdot \pi$ (Ω)

$\lambda := \frac{c}{f}$ (meters/cycle) $k := \frac{2 \cdot \pi}{\lambda}$ (m^{-1})

$In := \frac{Io}{N}$ (A) $rff := 10^5$ (meters)

$\epsilon o := \frac{1}{36 \cdot \pi} \cdot 10^{-9}$ (farads/m) $\epsilon cd := 1$ (dimensionless)

$\lambda = 0.999$ (meters/cycle) $z := 0 .. 2 \cdot j$ (increments)

Calculate caged dipole parameters parameters:

Define angular offset θ:

$\theta := 0, \frac{2 \cdot \pi}{j} .. 2 \cdot \pi$ (radians) $\phi := 0, \frac{2 \cdot \pi}{j} .. 2 \cdot \pi$ (radians)

Minimum Distance to Far-Field r_{min}:

$r_0 := 1.6 \cdot \lambda$ (meters)

$r_1 := 5 \cdot \sqrt{L^2 + (2 \cdot a)^2}$ (meters)

$r_4 := \frac{2 \cdot \left[L^2 + (2 \cdot a)^2 \right]}{\lambda}$ (meters)

$rmin := max(r)$ (meters) $rmin = 2.795$ (meters)

VERTICAL CAGED DIPOLE IN FREE SPACE:

Array Factor AF(θ):

$n := 1 .. N$ (elements) $\Phi_n := 2 \cdot \pi \cdot \frac{n}{N}$ (radians)

$$AF(\theta,\phi) := \sum_n In \cdot e^{j \cdot k \cdot a \cdot \sin(\theta) \cdot \cos\left(\phi - \Phi_n\right)}$$ (dimensionless)

E-Field of finite dipole E(θ):

$$E(\theta) := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left(\frac{\cos\left(\frac{k \cdot L}{2} \cdot \cos(\theta)\right) - \cos\left(\frac{k \cdot L}{2}\right)}{\sin(\theta)} \right)$$ (V/m)

Total E-Field $E_t(\theta)$ & Normalized E-Field $E_{tn}(\theta)$:

$$Et(\theta,\phi) := E(\theta)\cdot AF(\theta,\phi)$$ (V/m)

$$Etn(\theta,\phi) := \frac{Et(\theta,\phi)}{E\left(\frac{\pi}{2}\right)\cdot AF\left(\frac{\pi}{2},0\right)}$$ (V/m)

Radiation Intensity $U(\theta)$:

$$U(\theta,\phi) := \frac{rff^2}{2\cdot\eta o}\cdot\left(\left|Et(\theta,\phi)\right|\right)^2$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := \int_0^{2\cdot\pi}\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\,d\theta\,d\phi$$ (W)

$$Prad = 0.441$$ (W)

Directivity D_o:

$$Umax := U\left(\frac{\pi}{2},0\right)$$ (W/solid angle) $$Do := \frac{4\cdot\pi\cdot Umax}{Prad}$$ (dimensionless)

$$Do = 1.543$$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$$EIRP := Prad\cdot Do$$ (W) $$EIRP = 0.681$$ (W)

Radiation Resistance R_r:

$Rr := \frac{2 \cdot Prad}{(|In|)^2}$ (Ω) $Rr = 56.487$ (Ω)

Input Resistance R_{in}:

$Rin := \frac{Rr}{\sin\left(\frac{k \cdot L}{2}\right)^2}$ (Ω) $Rin = 56.487$ (Ω)

Voltage Relection Coeffecient Γ:

$\Gamma := \frac{Rin - Zo}{Rin + Zo}$ (dimensionless) $\Gamma = -0.141$ (dimensionless)

Reflection Efficiency ε_t:

$\varepsilon t := \varepsilon cd \cdot \left[1 - (|\Gamma|)^2\right]$ (dimensionless) $\varepsilon t = 0.98$ (dimensionless)

Gain G:

$G := \varepsilon t \cdot Do$ (dimensionless) $GdB := 10 \cdot \log(G)$ (dB)

$G = 1.513$ (dimensionless) $GdB = 1.798$ (dB)

Polarization Loss Factor PLF:

$PLF := \left(\left|\sigma w \cdot \overline{\sigma a}\right|\right)^2$ (dimensionless) $PLF = 1$ (dimensionless)

Maximum Effective Aperture A_{em}:

$Aem := \frac{G \cdot \lambda^2}{4 \cdot \pi} \cdot PLF$ (m^2) $Aem = 0.12$ (m^2)

Maximum Effective Height h_{em}:

$hem := 2 \cdot \sqrt{\frac{Rr \cdot Aem}{\eta o}}$ (m) $hem = 0.268$ (m)

Define Frequency Relationship F(x):

$F(x) := -5.696 \cdot 10^{-5} \cdot x + 0.31481$ (dimensionless)

Frequency Range f_{low} and f_{high}:

$flow := f - F\left(\frac{L}{2 \cdot a}\right) \cdot f$ (Hz) $flow = 2.056 \cdot 10^8$ (Hz)

$fhigh := f + F\left(\frac{L}{2 \cdot a}\right) \cdot f$ (Hz) $fhigh = 3.944 \cdot 10^8$ (Hz)

Bandwidth BW:

$BW := fhigh - flow$ (Hz) $BW = 1.888 \cdot 10^8$ (Hz)

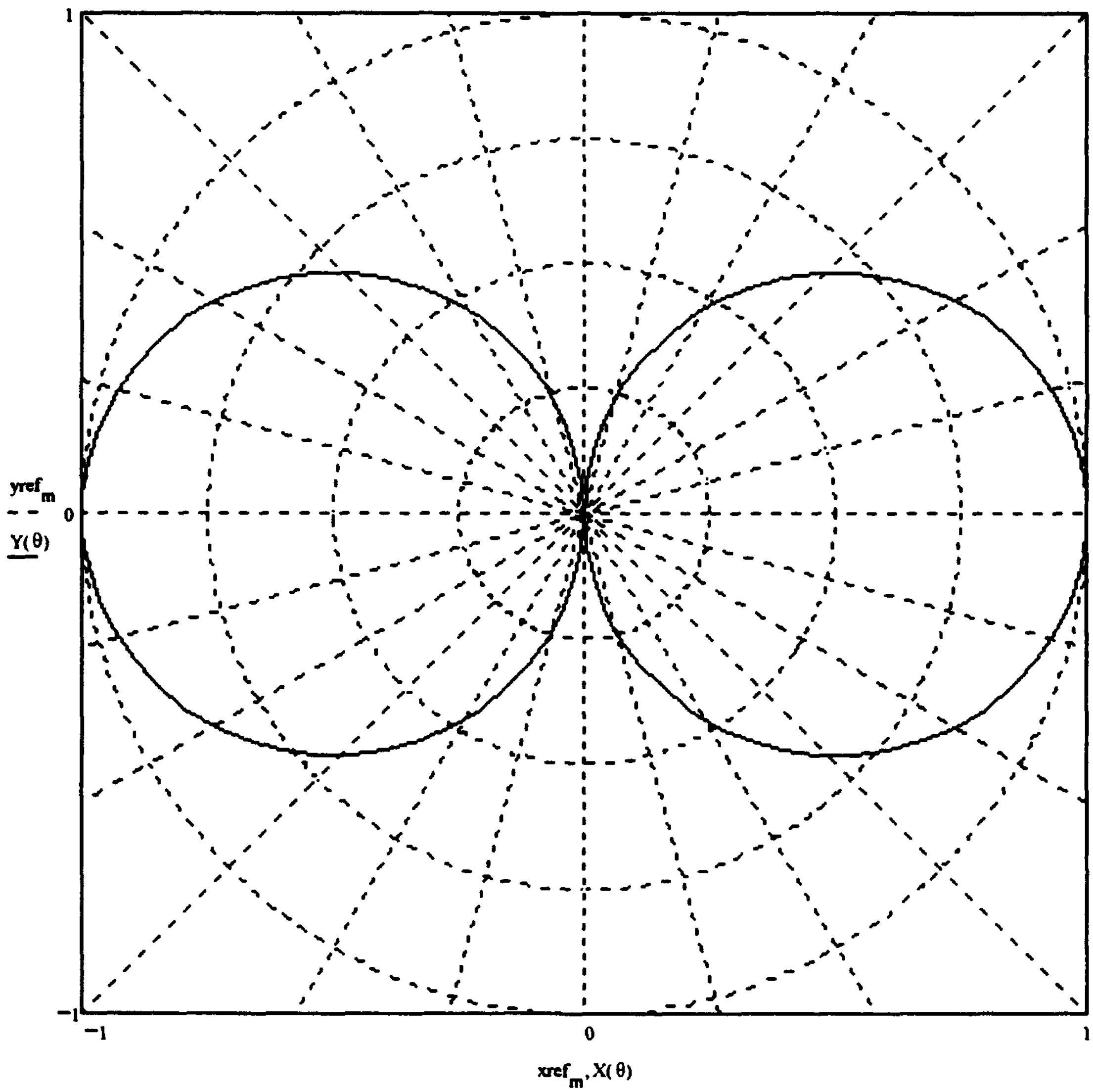
E-FIELD PATTERN: This far-field radiation pattern is for the vertical caged dipole in free space:

Number of Dipole Conductors: $N = 8$

Radius of Antenna: $a = 0.125$ (meters)

$$X(\theta) := |Etn(\theta,\phi)| \cdot \cos\left(\theta - \frac{\pi}{2}\right) \qquad Y(\theta) := |Etn(\theta,\phi)| \cdot \sin\left(\theta - \frac{\pi}{2}\right)$$

θ = 0°



θ = 180°

VERTICAL CAGED DIPOLE OVER EARTH:

Define angular offset θ1 and θ2:

$$\theta 1 := -\frac{\pi}{2}, -\frac{\pi}{2} + \frac{\pi}{2 \cdot j} .. \frac{\pi}{2}$$ (radians)

$$\phi 2_z := \frac{\pi}{j} \cdot z$$ (radians)

$$\theta 2_z := -\frac{\pi}{2} + \frac{\pi}{2 \cdot j} \cdot z$$ (radians)

Relative complex effective dielectric constant ε_{rp}:

$$\varepsilon rp := \varepsilon r - j \cdot \frac{\sigma}{2 \cdot \pi \cdot f \cdot \varepsilon o}$$ (dimensionless)

Array Factor AFv(θ):

$$AFv(\theta 1, \phi) := \sum_n In \cdot e^{j \cdot k \cdot a \cdot \sin(\theta 1) \cdot \cos(\phi - \Phi_n)}$$ (dimensionless)

$$AFvgp_z := \sum_n In \cdot e^{j \cdot k \cdot a \cdot \sin(\theta 2_z) \cdot \cos(\phi 2_z - \Phi_n)}$$ (dimensionless)

E-Field of finite dipole Ev(θ1):

$$Ev(\theta 1) := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left(\frac{\cos\left(\frac{k \cdot L}{2} \cdot \cos(\theta 1)\right) - \cos\left(\frac{k \cdot L}{2}\right)}{\sin(\theta 1)} \right)$$ (V/m)

$$Evgp_z := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left[\frac{\cos\left(\frac{k \cdot L}{2} \cdot \cos(\theta 2_z)\right) - \cos\left(\frac{k \cdot L}{2}\right)}{\sin(\theta 2_z)} \right]$$ (V/m)

Vertical plane wave reflection coefficient $\Gamma_V(\theta 1)$:

$$\Gamma v(\theta 1) := \frac{\varepsilon rp \cdot \cos(\theta 1) - \sqrt{\varepsilon rp - \sin(\theta 1)^2}}{\varepsilon rp \cdot \cos(\theta 1) + \sqrt{\varepsilon rp - \sin(\theta 1)^2}}$$ (dimensionless)

$$\Gamma v1_z := \frac{\varepsilon rp \cdot \cos(\theta 2_z) - \sqrt{\varepsilon rp - \sin(\theta 2_z)^2}}{\varepsilon rp \cdot \cos(\theta 2_z) + \sqrt{\varepsilon rp - \sin(\theta 2_z)^2}}$$ (dimensionless)

<u>Total E-Field $Ev_t(\theta 1)$ & Normalized E-Field $Ev_{tn}(\theta 1)$:</u>

$$Evt(\theta 1, \phi) := Ev(\theta 1) \cdot AFv(\theta 1, \phi) \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 1)} + \Gamma v(\theta 1) \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 1)}\right)$$ (V/m)

$$Evtgp_z := \left|\left[Evgp_z \cdot AFvgp_z \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 2_z)} + \Gamma v1_z \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 2_z)}\right)\right]\right|$$ (V/m)

$$Evtn(\theta 1, \phi) := \frac{|Evt(\theta 1, \phi)|}{\max(Evtgp)}$$ (V/m)

<u>Radiation Intensity $Uv(\theta 1)$:</u>

$$Uv(\theta 1, \phi) := \frac{rff^2}{2 \cdot \eta o} \cdot \left(|Evt(\theta 1, \phi)|\right)^2$$ (W/solid angle)

$$Uv1_z := \frac{rff^2}{2 \cdot \eta o} \cdot \left(\left|\overrightarrow{Evtgp}\right|\right)^2$$ (W/solid angle)

<u>Radiated Power Pv_{rad}:</u>

$$Pvrad := \int_0^{2 \cdot \pi} \int_0^{\frac{\pi}{2}} Uv(\theta 1, \phi) \cdot \sin(\theta 1) \, d\theta 1 \, d\phi$$ (W)

$$Pvrad = 0.483$$ (W)

<u>Directivity Dv_o:</u>

$$Uvmax := Uv(\theta vp, 0)$$ (W/solid angle)

Directivity Dv_o:

$$Dvo := \frac{4 \cdot \pi \cdot Uvmax}{Pvrad}$$ (dimensionless)

$Dvo = 4.415$ (dimensionless)

Effective Isotropic Radiated Power EIRPv:

$EIRPv := Pvrad \cdot Dvo$ (W) $EIRPv = 2.133$ (W)

Radiation Resistance Rv_r:

$$Rvr := \frac{2 \cdot Pvrad}{(|In|)^2}$$ (Ω) $Rvr = 61.857$ (Ω)

Input Resistance Rv_{in}:

$$Rvin := \frac{Rvr}{\sin\left(\frac{k \cdot L}{2}\right)^2}$$ (Ω) $Rvin = 61.857$ (Ω)

Voltage Relection Coeffecient ΓV:

$$\Gamma V := \frac{Rvin - Zo}{Rvin + Zo}$$ (dimensionless) $\Gamma V = -0.096$ (dimensionless)

Reflection Efficiency εv_t:

$\varepsilon vt := \varepsilon cd \cdot \left[1 - (|\Gamma V|)^2\right]$ (dimensionless) $\varepsilon vt = 0.991$ (dimensionless)

Gain Gv:

$Gv := \varepsilon vt \cdot Dvo$ (dimensionless) $GvdB := 10 \cdot \log(Gv)$ (dB)

$Gv = 4.374$ (dimensionless) $GvdB = 6.409$ (dB)

Polarization Loss Factor PLFv:

$Evmax := Evtn(\vartheta vp, 0)$ (V/m) $lobesv := \mathrm{floor}\left(2 \cdot \frac{h}{\lambda} + 1\right)$

$$\sigma va := \begin{bmatrix} \frac{Evmax}{lobesv} \\ 0 \\ 0 \end{bmatrix}$$ (dimensionless)

$PLFv := \left(\left|\sigma w \cdot \overline{\sigma va}\right|\right)^2$ (dimensionless) $PLFv = 0.986$ (dimensionless)

Maximum Effective Aperture A_{em}:

$Avem := \frac{Gv \cdot \lambda^2}{4 \cdot \pi} \cdot PLFv$ (m^2) $Avem = 0.343$ (m^2)

Maximum Effective Height hv_{em}:

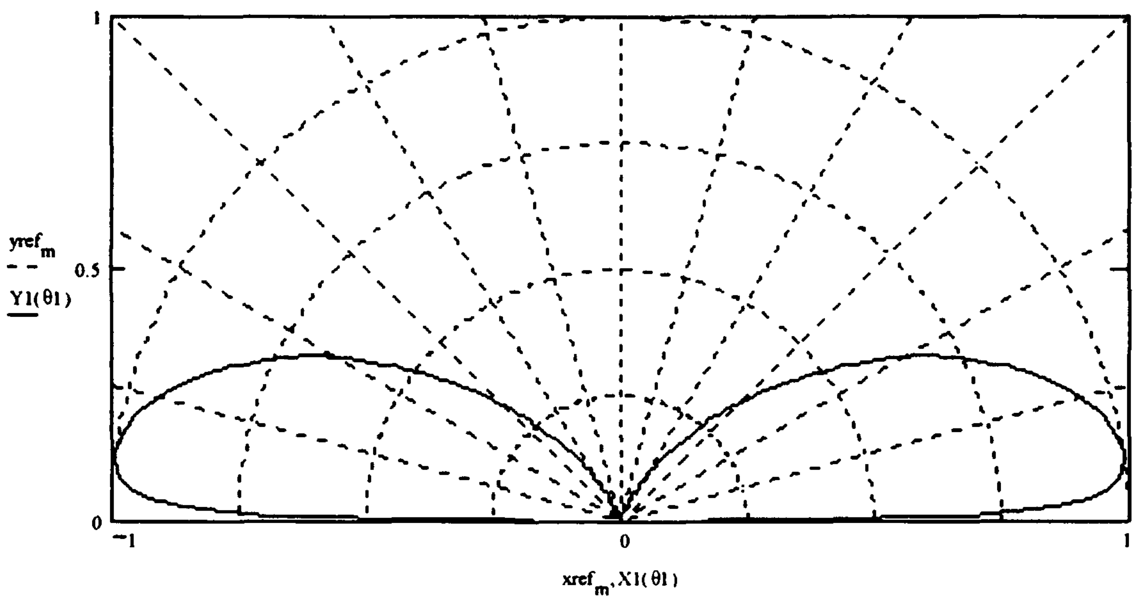
$hvem := 2 \cdot \sqrt{\frac{Rvr \cdot Avem}{\eta o}}$ (m) $hvem = 0.474$ (m)

E-FIELD PATTERN: This far-field radiation pattern is for the vertical caged dipole over earth:

Number of Dipole Conductors:	$N = 8$	(elements)
Radius of Antenna:	$a = 0.125$	(meters)
Height of Antenna Above Earth:	$h = 0.25$	(meters)

$$X1(\theta 1) := \left|Evtn(\theta 1, \phi)\right| \cdot \cos\left(\theta 1 + \frac{\pi}{2}\right) \qquad Y1(\theta 1) := \left|Evtn(\theta 1, \phi)\right| \cdot \sin\left(\theta 1 + \frac{\pi}{2}\right)$$

$\theta = 0°$



HORIZONTAL CAGED DIPOLE OVER EARTH:

Define angular offset $\phi 1$ and $\phi 2$:

$$\phi 1 := 0, \frac{\pi}{j} .. 2 \cdot \pi$$ (radians)

Array Factor $AFh(\theta 1, \phi 1)$:

$n := 1 .. N$ (elements) $\qquad \Theta_n := 2 \cdot \pi \cdot \frac{n}{N}$ (radians)

$$AFh(\theta 1, \phi 1) := \sum_n In \cdot e^{j \cdot k \cdot a \cdot \left(\sin(\phi 1) \cdot \sin(\theta 1) \cdot \sin\left(\Theta_n\right) + \cos(\theta 1) \cdot \cos\left(\Theta_n\right)\right)}$$ (dimensionless)

$$AFh1_z := \sum_n In \cdot e^{j \cdot k \cdot a \cdot \left(\sin\left(\phi 2_z\right) \cdot \sin\left(\theta 2_z\right) \cdot \sin\left(\Theta_n\right) + \cos\left(\theta 2_z\right) \cdot \cos\left(\Theta_n\right)\right)}$$ (dimensionless)

θ component of E-Field of finite dipole $E\theta h(\theta 1)$:

$$E\theta h(\theta 1) := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left[\frac{\cos\left(\frac{k \cdot L}{2} \cdot \sin(\theta 1)\right) - \cos\left(\frac{k \cdot L}{2}\right)}{\sqrt{1 - \sin(\theta 1)^2}}\right]$$ (V/m)

$$E\theta h1_z := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left[\frac{\cos\left(\frac{k \cdot L}{2} \cdot \sin\left(\theta 2_z\right)\right) - \cos\left(\frac{k \cdot L}{2}\right)}{\sqrt{1 - \sin\left(\theta 2_z\right)^2}}\right]$$ (V/m)

Total E-Field $E\theta h_t(\theta 1, \phi 1)$:

$$E\theta ht(\theta 1, \phi 1) := E\theta h(\theta 1) \cdot AFh(\theta 1, \phi 1) \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 1)} - \Gamma v(\theta 1) \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 1)}\right)$$ (V/m)

$$E\theta ht1_z := \left|\overline{\left[E\theta h1_z \cdot AFh1_z \cdot \left(e^{j \cdot k \cdot h \cdot \cos\left(\theta 2_z\right)} - \Gamma v1_z \cdot e^{-j \cdot k \cdot h \cdot \cos\left(\theta 2_z\right)}\right)\right]}\right|$$ (V/m)

Normalized E-Field $E\theta h_{tn}(\theta 1, \phi 1)$:

$$E\theta htn(\theta 1, \phi 1) := \frac{|E\theta ht(\theta 1, \phi 1)|}{\max(E\theta ht1)}$$ (V/m)

Horizontal plane wave reflection coefficient $\Gamma_h(\theta 1)$:

$$\Gamma h(\theta 1) := \frac{\cos(\theta 1) - \sqrt{\varepsilon rp - \sin(\theta 1)^2}}{\cos(\theta 1) + \sqrt{\varepsilon rp - \sin(\theta 1)^2}}$$ (dimensionless)

$$\Gamma h1_z := \frac{\cos(\theta 2_z) - \sqrt{\varepsilon rp - \sin(\theta 2_z)^2}}{\cos(\theta 2_z) + \sqrt{\varepsilon rp - \sin(\theta 2_z)^2}}$$ (dimensionless)

ϕ component of E-Field of finite dipole $E\phi h(\theta 1)$:

$$E\phi h(\theta 1) = j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left(1 - \cos\left(\frac{k \cdot L}{2}\right)\right)$$ (V/m)

$$E\phi h1_z := j \cdot \eta o \cdot \frac{In \cdot e^{-j \cdot k \cdot rff}}{2 \cdot \pi \cdot rff} \cdot \left(1 - \cos\left(\frac{k \cdot L}{2}\right)\right)$$ (V/m)

Total E-Field $E\phi h_t(\theta 1, \phi 1)$ & Normalized E-Field $E\phi h_{tn}(\theta 1, \phi 1)$:

$$E\phi ht(\theta 1, \phi 1) := E\phi h(\theta 1) \cdot AFh(\theta 1, \phi 1) \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 1)} + \Gamma h(\theta 1) \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 1)}\right)$$ (V/m)

$$E\phi ht1_z := \left|\left[E\phi h1_z \cdot AFh1_z \cdot \left(e^{j \cdot k \cdot h \cdot \cos(\theta 2_z)} + \Gamma h1_z \cdot e^{-j \cdot k \cdot h \cdot \cos(\theta 2_z)}\right)\right]\right|$$ (V/m)

$$E\phi htn(\theta 1, \phi 1) := \frac{|E\phi ht(\theta 1, \phi 1)|}{\max(E\phi ht1)}$$ (V/m)

Radiation Intensity Uh(θ1,ϕ1):

$$Uh(\theta 1,\phi 1) := \frac{rff^2}{2\cdot\eta o}\cdot\left[\left(\left|E\theta ht(\theta 1,\phi 1)\right|\right)^2+\left(\left|E\phi ht(\theta 1,\phi 1)\right|\right)^2\right]$$ (W/solid angle)

$$Uh1_z := \frac{rff^2}{2\cdot\eta o}\left[\left(\left|\overrightarrow{E\theta ht1}\right|\right)^2+\left(\left|\overrightarrow{E\phi ht1}\right|\right)^2\right]$$ (W/solid angle)

Radiated Power Pv_{rad}:

$$Phrad := \int_0^{2\cdot\pi}\int_0^{\frac{\pi}{2}} Uh(\theta 1,\phi 1)\cdot\sin(\theta 1)\, d\theta 1\, d\phi 1$$ (W)

$Phrad = 1.059$ (W)

Directivity Dh_o:

$Uhmax := Uh(\theta hp, 0)$ (W/solid angle)

$$Dho := \frac{4\cdot\pi\cdot Uhmax}{Phrad}$$ (dimensionless)

$Dho = 5.051$ (dimensionless)

Effective Isotropic Radiated Power EIRPh:

$EIRPh := Pvrad\cdot Dvo$ (W)

$EIRPh = 2.133$ (W)

Radiation Resistance Rv_r:

$$Rhr := \frac{2\cdot Phrad}{\left(\left|In\right|\right)^2}$$ (Ω)

$Rhr = 135.58$ (Ω)

Input Resistance Rh_{in}:

$$Rhin := \frac{Rhr}{\sin\left(\frac{k \cdot L}{2}\right)^2} \quad (\Omega) \qquad Rhin = 135.58 \quad (\Omega)$$

Voltage Relection Coeffecient ΓH:

$$\Gamma H := \frac{Rhin - Zo}{Rhin + Zo} \quad \text{(dimensionless)} \qquad \Gamma H = 0.288 \quad \text{(dimensionless)}$$

Reflection Efficiency εv_t:

$$\varepsilon ht := \varepsilon cd \cdot \left[1 - \left(\left| \Gamma H \right| \right)^2 \right] \quad \text{(dimensionless)} \qquad \varepsilon ht = 0.917 \quad \text{(dimensionless)}$$

Gain Gh:

$$Gh := \varepsilon ht \cdot Dho \quad \text{(dimensionless)} \qquad GhdB := 10 \cdot \log(Gh) \quad \text{(dB)}$$

$$Gh = 4.633 \quad \text{(dimensionless)} \qquad GhdB = 6.659 \quad \text{(dB)}$$

Polarization Loss Factor PLFh:

$$Ehmax := E\phi htn(\theta hp, 0) \quad \text{(V/m)} \qquad lobesh := \text{floor}\left(2 \cdot \frac{h}{\lambda} + 1\right)$$

$$\sigma ha := \begin{bmatrix} 0 \\ 0 \\ \frac{Ehmax}{lobesh} \end{bmatrix} \quad \text{(dimensionless)}$$

$$PLFh := \left(\left| \sigma hw \cdot \overline{\sigma ha} \right| \right)^2 \quad \text{(dimensionless)} \qquad PLFh = \quad \text{(dimensionless)}$$

Maximum Effective Aperture Ah_{em}:

$$Ahem := \frac{Gh \cdot \lambda^2}{4 \cdot \pi} \cdot PLFh \quad (m^2)$$

$$Ahem = 35.536 \quad (m^2)$$

Maximum Effective Height hh_{em}:

$$hhem := 2 \cdot \sqrt{\frac{Rhr \cdot Ahem}{\eta o}} \quad (m)$$

$$hhem = 7.15 \quad (m)$$

E-PLANE RADIATION PATTERN: This far-field radiation pattern in the (x-z) plane is for the horizontal caged dipole over earth:

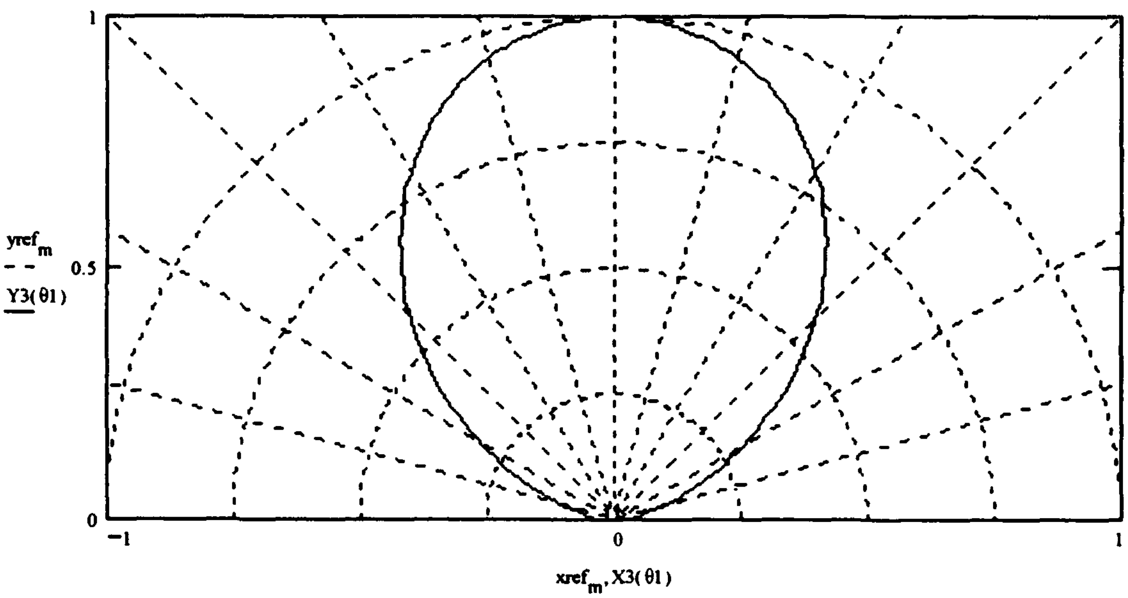
Number of Dipole Conductors: $N = 8$ (elements)

Radius of Antenna: $a = 0.125$ (meters)

Height of Antenna Above Earth: $h = 0.25$ (meters)

$$X3(\theta 1) := \left|E\theta htn\left(\theta 1, \frac{\pi}{2}\right)\right| \cdot \cos\left(\theta 1 + \frac{\pi}{2}\right) \qquad Y3(\theta 1) := \left|E\theta htn\left(\theta 1, \frac{\pi}{2}\right)\right| \cdot \sin\left(\theta 1 + \frac{\pi}{2}\right)$$

$\theta = 0°$

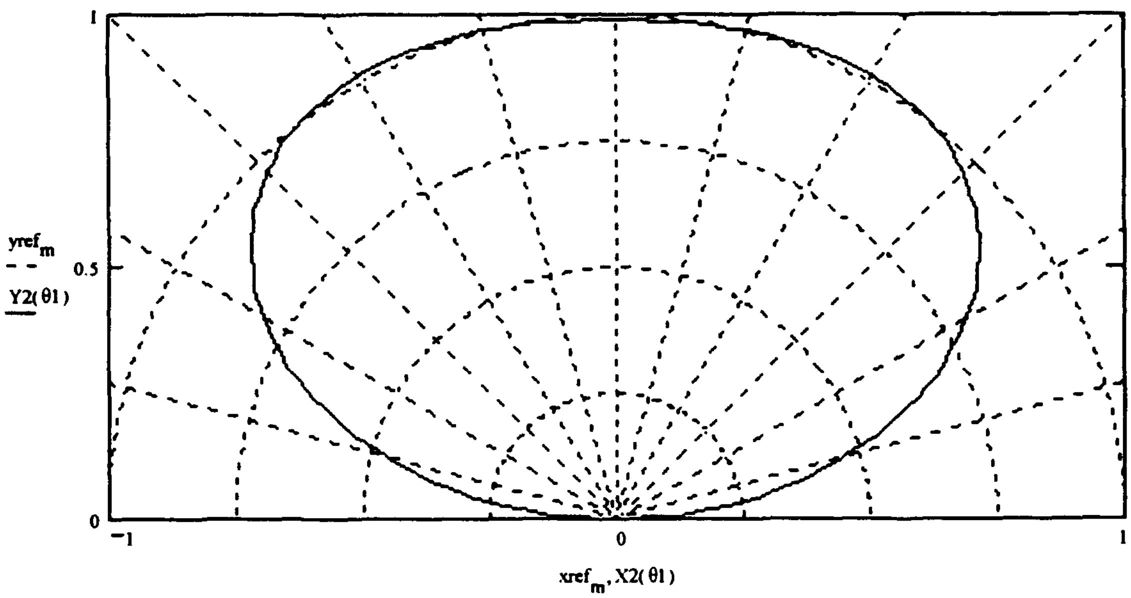


H-PLANE RADIATION PATTERN: This far-field radiation pattern in the (y-z) plane is for the horizontal caged dipole over earth:

Number of Dipole Conductors:	$N = 8$	(elements)
Radius of Antenna:	$a = 0.125$	(meters)
Height of Antenna Above Earth:	$h = 0.25$	(meters)

$$X2(\theta 1) := |E\phi htn(\theta 1, 0)| \cdot \cos\left(\theta 1 + \frac{\pi}{2}\right) \qquad Y2(\theta 1) := |E\phi htn(\theta 1, 0)| \cdot \sin\left(\theta 1 + \frac{\pi}{2}\right)$$

$\theta = 0^\circ$



THE HELICAL ANTENNA (REFLECTOR OPTION)
MATHCAD SOFTWARE-DISH_HEL.MCD

When built to the proper specifications, the helical antenna possesses many qualities which make it suitable for a wide variety of communications applications. If the following conditions are satisfied the helix will exhibit a highly directional axial main lobe, low side lobe level, negligible mutual interference with adjacent antennas, low voltage standing wave ratio (VSWR), and resistive input impedance over a wide frequency band:

$$.8 \le C_\lambda \le 1.15$$
$$n > 3$$
$$12 \le \alpha \le 14$$

(Note: λ in a subscript indicates the dimension is in wavelengths. Mathcad equations can not use symbolic subscripts. Therefore, the symbol λ will immediately follow the parameter in equations (i.e., Cλ) in lieu of subscripts.)

The helical antenna Mathcad application will compute the following parameters (Items with * indicate parameters that are calculated for both axial and peripheral feed geometries):

C = Circumference of Helix
λ = Wavelength
α = Pitch Angle
Do = Directivity
p = Relative Phase Velocity
$E_{\theta,\phi}$ = Electric Field Components
Ψ = Array Factor Phase Shift
U = Radiation Intensity
Prad = Radiated Power
R = Antenna Input Resistance*
Γ = Voltage Reflection Coefficient*
ε_r = Reflection Efficiency*
h_{em} = Maximum Effective Height*
G = Gain*
EIRP = Effective Isotropic Radiated Power*
A_{em} = Maximum Effective Aperture*
AR = Axial Ratio
PLF = Polarization Loss Factor
BW = Bandwidth
f_{high} = Upper Frequency Limit
f_{low} = Lower Frequency Limit
Acceptable Conductor Diameter
$E_{x,y,z}$ = Electric Field Cartesian Coordinates

θ_p, ϕ_p = Unit Polarization Vector Coordinate Angles

σ_a = Antenna Unit Polarization Vector

r_{min} = Mininum Distance to the Far-Field

The following data must be input based on known or estimated data:

D = Diameter of Helix (Center to Center)
S = Spacing Between Turns (Center to Center)
L = Length Along Conductor of One Turn
n = Number of Turns
d = Diameter of Helical Conductor
f = Frequency of Interest
m = Desired Mode
i = Number of Increments in Degrees for Far Field Radiation Pattern
I_o = Antenna Feed Current
Z_o = Characteristic Feed Impedance
σ_w = Incoming Wave Electric Field Unit Vector

Enter input data here:

$$\sigma w := \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{j}{\sqrt{2}} \\ 0 \end{bmatrix}$$ (dimensionless)

$D := \frac{0.1}{\pi}$ (meters) $L := 0.097$ (meters)

$n := 10$ (turns) $f := 3 \cdot 10^{10}$ (Hz)

$m := 1$ (dimensionless) $Z_0 := 150$ (Ω)

$S := 0.023$ (meters)

$d := 0.005$ (meters)

$x := 1$ (meters)

$y := 1$ (meters)

$\iota := 360$ (degrees)

$Io := 1$ (A)

$z := 1000$ (meters)

Calculate helical geometric parameters and define constants:

$c := 2.9979 \cdot 10^{8}$ (meters/sec)

$\eta_0 := 120 \cdot \pi$ (Ω)

$C := \pi \cdot D$ (meters)

$\lambda := \frac{c}{f}$ (meters/cycle)

$C = 0.1$ (meters)

$\lambda = 9.993 \cdot 10^{-3}$ (meters/cycle)

$C\lambda := \frac{C}{\lambda}$ (dimensionless)

$\alpha := \mathrm{atan}\left(\frac{S}{C}\right)$ (radians)

$C\lambda = 10.007$ (dimensionless)

$\alpha = 0.22607$ (radians)

$L\lambda := \frac{L}{\lambda}$ (dimensionless)

$S\lambda := \frac{S}{\lambda}$ (dimensionless)

$L\lambda = 9.70679$ (dimensionless)

$S\lambda = 2.30161$ (dimensionless)

$\alpha d := \frac{180}{\pi} \cdot \alpha$ (degrees)

$k := 2 \cdot \frac{\pi}{\lambda}$ (m^{-1})

$\alpha d = 12.95276$ (degrees)

Calculate helical antenna parameters:

Define angular offset θ from helical axis:

$$\theta := 0, \frac{2\cdot\pi}{i} .. 2\cdot\pi$$ (radians)

$$\phi := 0, \left(\frac{\pi}{i}\right) .. 2\cdot\pi$$ (radians)

Minimum Distance to the Far-Field r_{min}:

$$r_0 = 1.6\cdot\lambda$$ (meters)

$$r_1 := 5\cdot n\cdot S$$ (meters)

$$r_2 = \frac{2\cdot(n\cdot S)^2}{\lambda}$$ (meters)

$$rmin := \max(r)$$ (meters)

$$rmin = 10.58741$$ (meters)

Relative Phase Velocity p:

$$p := \frac{L\lambda}{S\lambda + m + \left(\frac{1}{2\cdot n}\right)}$$ (dimensionless)

$$p = 2.89616$$ (dimensionless)

Array Factor Phase Shift ψ:

$$\psi(\theta) := 2\cdot\pi\cdot\left(S\lambda\cdot\cos(\theta) - \frac{L\lambda}{p}\right)$$ (radians)

Electric Field Components E_θ, E_ϕ:

$$E(\theta) := \left| \frac{\sin\left(\frac{\pi}{2 \cdot n}\right) \cdot \sin\left(\frac{n \cdot \psi(\theta)}{2}\right)}{\sin\left(\left(\frac{\psi(\theta)}{2}\right)\right)} \cdot \cos(\theta) \right|$$ (V/m)

$$E\theta(\theta) := E(\theta)$$ (V/m)

$$E\phi(\theta) := j \cdot E(\theta)$$ (V/m)

Radiation Intensity $U(\theta)$:

$$U(\theta) := \frac{1}{\eta_0} \cdot (|E(\theta)|)^2$$ (W/solid angle)

Radiated Power P_{rad}:

$$Prad := \int_0^{2 \cdot \pi} \int_0^{\pi} U(\theta) \cdot \sin(\theta)\, d\theta\, d\phi$$ (W)

$$Prad = 1.67351 \cdot 10^{-3}$$ (W)

Directivity D_o:

$$Do := 12 \cdot C\lambda^2 \cdot n \cdot S\lambda$$ (dimensionless)

$$Do2 := \frac{4 \cdot \pi \cdot U(0)}{Prad}$$ (dimensionless)

$$Do = 2.7658 \cdot 10^4$$ (dimensionless)

$$Do2 = 19.91822$$ (dimensionless)

Axial Ratio AR:

$AR := \left| L\lambda \cdot \left(\sin(\alpha) - \frac{1}{p} \right) \right|$ (dimensionless)

$AR = 1.17586$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$EIRP := Prad \cdot Do$ (W) $EIRP2 := Prad \cdot Do2$ (W)

$EIRP = 46.286$ (W) $EIRP2 = 0.03333$ (W)

Polarization Loss Factor PLF:

$\theta p := \operatorname{atan}\left(\frac{\sqrt{x^2 + y^2}}{z} \right)$ (radians) $\phi p := \operatorname{atan}\left(\frac{y}{x} \right)$ (radians)

$\theta p = 1.41421 \cdot 10^{-3}$ (radians) $\phi p = 0.7854$ (radians)

$Ex := E\theta(\theta p) \cdot \cos(\theta p) \cdot \cos(\phi p) - E\phi(\theta p) \cdot \sin(\phi p)$ (V/m)

$Ey := E\theta(\theta p) \cdot \cos(\theta p) \cdot \sin(\phi p) + E\phi(\theta p) \cdot \cos(\phi p)$ (V/m)

$Ez := E\theta(\theta p) \cdot \sin(\theta p) \cdot -1$ (V/m)

$$\sigma a := \frac{1}{\sqrt{(|Ex|)^2 + (|Ey|)^2 + (|Ez|)^2}} \cdot \begin{pmatrix} Ex \\ Ey \\ Ez \end{pmatrix}$$ (dimensionless)

$$\sigma a = \begin{bmatrix} 0.5 - 0.5j \\ 0.5 + 0.5j \\ -9.99999 \cdot 10^{-4} \end{bmatrix}$$ (dimensionless)

Polarization Loss Factor PLF:

$PLF := \left(\left| \sigma w \cdot \overline{\sigma a} \right| \right)^2$ (dimensionless) $PLF = 1$ (dimensionless)

Radiation Resistance R_r:

$Rr := 2 \cdot \frac{Prad}{(|Io|)^2}$ (Ω) $Rr = 3.34702 \cdot 10^{-3}$ (Ω)

Dual Parameters

Axial Feed / **Peripheral Feed**

Input Resistance R:

$Ra := 140 \cdot \sqrt{C\lambda}$ (Ω) $Rp := \frac{150}{\sqrt{C\lambda}}$ (Ω)

$Ra = 4.42874 \cdot 10^2$ (Ω) $Rp = 47.41756$ (Ω)

Voltage Reflection Coeffecient Γ:

$\Gamma a := \frac{Ra - Z_0}{Ra + Z_0}$ (dimensionless) $\Gamma p := \frac{Rp - Z_0}{Rp + Z_0}$ (dimensionless)

$\Gamma a = 0.49399$ (dimensionless) $\Gamma p = -0.51962$ (dimensionless)

Reflection Efficiency ε_r:

$\varepsilon ra := 1 - (|\Gamma a|)^2$ (dimensionless) $\varepsilon rp := 1 - (|\Gamma p|)^2$ (dimensionless)

$\varepsilon ra = 0.75597$ (dimensionless) $\varepsilon rp = 0.72999$ (dimensionless)

<u>Gain G:</u>

$Ga := \varepsilon ra \cdot Do$ (dimensionless)

$Gadb := 10 \cdot \log(\varepsilon ra \cdot Do)$ (dB)

$Ga = 2.09088 \cdot 10^{4}$ (dimensionless)

$Gadb = 43.20328$ (dB)

$Ga2 := \varepsilon ra \cdot Do2$ (dimensionless)

$Gadb2 := 10 \cdot \log(\varepsilon ra \cdot Do2)$ (dB)

$Ga2 = 15.05765$ (dimensionless)

$Gadb2 = 11.77757$ (dB)

$Gp := \varepsilon rp \cdot Do$ (dimensionless)

$Gpdb := 10 \cdot \log(\varepsilon rp \cdot Do)$ (dB)

$Gp = 2.01902 \cdot 10^{4}$ (dimensionless)

$Gpdb = 43.0514$ (dB)

$Gp2 := \varepsilon rp \cdot Do2$ (dimensionless)

$Gpdb2 := 10 \cdot \log(\varepsilon rp \cdot Do2)$ (dB)

$Gp2 = 14.54017$ (dimensionless)

$Gpdb2 = 11.62569$ (dB)

<u>Maximum Effective Aperture A_{em}:</u>

$Aema := \frac{\varepsilon ra \cdot \lambda^{2} \cdot Do}{4 \cdot \pi} \cdot PLF$ (m^2)

$Aema = 0.16615$ (m^2)

$Aema2 := \frac{\varepsilon ra \cdot \lambda^{2} \cdot Do2}{4 \cdot \pi} \cdot PLF$ (m^2)

$Aema2 = 1.19657 \cdot 10^{-4}$ (m^2)

$Aemp := \frac{\varepsilon rp \cdot \lambda^{2} \cdot Do}{4 \cdot \pi} \cdot PLF$ (m^2)

$Aemp = 0.16044$ (m^2)

$Aemp2 := \frac{\varepsilon rp \cdot \lambda^{2} \cdot Do2}{4 \cdot \pi} \cdot PLF$ (m^2)

$Aemp2 = 1.15545 \cdot 10^{-4}$ (m^2)

Maximum Effective Height h_{em}:

$hema := 2 \cdot \sqrt{Rr \cdot \frac{Aema}{\eta_0}}$ (m)

$hemp := 2 \cdot \sqrt{Rr \cdot \frac{Aemp}{\eta_0}}$ (m)

$hema = 2.42912 \cdot 10^{-3}$ (m)

$hemp = 2.38701 \cdot 10^{-3}$ (m)

$hema2 := 2 \cdot \sqrt{Rr \cdot \frac{Aema2}{\eta_0}}$ (m)

$hemp2 := 2 \cdot \sqrt{Rr \cdot \frac{Aemp2}{\eta_0}}$ (m)

$hema2 = 6.51873 \cdot 10^{-5} \; 2$ (m)

$hemp2 = 6.40574 \cdot 10^{-5}$ (m)

Bandwidth:

$fhigh := \frac{1.15 \cdot c}{C}$ (Hz)

$flow := \frac{.8 \cdot c}{C}$ (Hz)

$fhigh = 3.44759 \cdot 10^{9}$ (Hz)

$flow = 2.39832 \cdot 10^{9}$ (Hz)

$BW := fhigh - flow$ (Hz)

$BW = 1.04927 \cdot 10^{9}$ (Hz)

Acceptable Conductor Diameter:

$dmin := .005 \cdot \lambda$ (m)

$dmax := .05 \cdot \lambda$ (m)

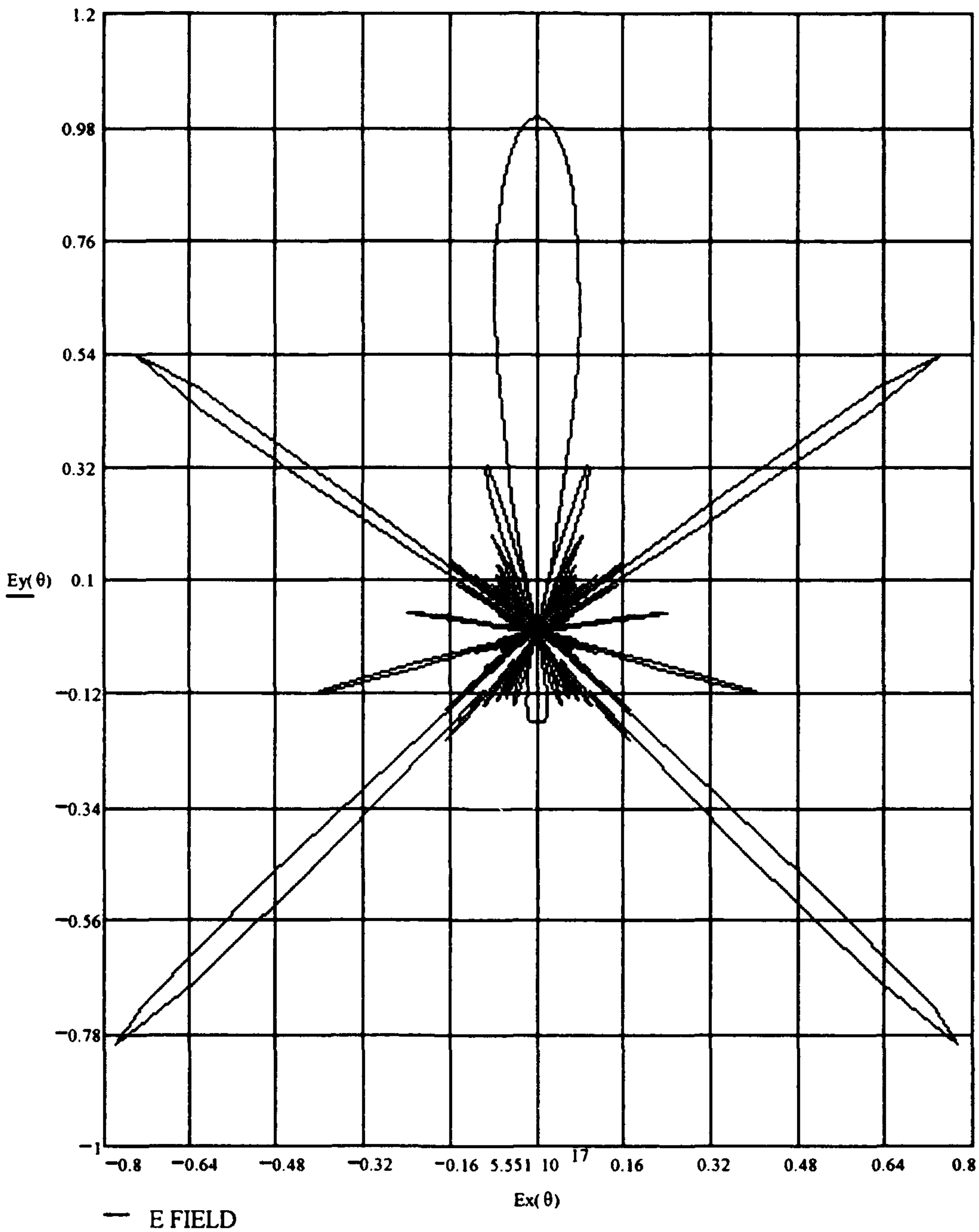
$dmin = 4.9965 \cdot 10^{-5}$ (m)

$dmax = 4.9965 \cdot 10^{-4}$ (m)

HELICAL ANTENNA FAR-FIELD RADIATION PATTERN

For the purpose of this far-field radiation pattern, the helical antenna axis is equivalent to the Ex = 0 grid line. The pattern is essentially symetric when rotated about the antenna's axis.

$$Ex(\theta) := E\left(\theta - \frac{\pi}{2}\right)\cdot\cos(\theta) \qquad Ey(\theta) := E\left(\theta - \frac{\pi}{2}\right)\cdot\sin(\theta)$$



PARABOLIC REFLECTOR ANALYSIS WITH HELICAL ANTENNA FEED

The helical antenna Mathcad application (Reflector Option) will compute the following additional parameters (Note: a 2 or 22 will follow most secondary field parameters in this application, and inorder to speed up computation time corresponding vectors are created with subscript A & B:

$P_{x,y}$ = E-Field Integral and Summation Equation Vector Components

$E_{ax,ay}$ = Aperture E-Field Components

θ',ρ = E-Field Integral and Summation Equation Parameters

E_θ = Secondary Field Theta E-Field Component

E_ϕ = Secondary Field Phi E-Field Component

U = Secondary Field Radiation Intensity

P_{rad} = Secondary Field Radiated Power

D_o = Secondary Field Directivity

G = Secondary Field Gain

ε_{ap} = Aperture Efficiency

ε_s = Spillover Efficiency

ε_t = Taper Efficiency

ε_p = Phase Efficiency

ε_x = Polarization Efficiency

ε_{ohmic} = Ohmic Efficiency

A_e = Reflector Effective Area

h_e = Reflector Effective Height

EIRP = Reflector Effective Radiated Power

$E_{x,y,z}$ = Secondary Field Cartesian Components

σ_a = Reflector Unit Polarization Vector

PLF = Reflector Polarization Loss Factor

r_{min} = Minimum Distance to Secondary Far-Field

θ_o = Reflector Periphery Coaltitude

θ_p,ϕ_p = Theta and Phi for Polarization Loss

The following additional data must be inputted based on known or estimated data:

E_o = Electric Field Scale Factor

a_{ref} = Radius of the Mouth of the Reflector

foc = Reflector Focal Length

ε_b = Blockage Efficiency

ε_{sp} = Spar Efficiency

t1,t2 = Radiated Power Increments for Phi and Theta

il = Secondary Field Increments
N,M = Summation Increments
r_{ff2} = Secondary Field Observation Distance
x,y,z = Polarization Loss Factor Coordinates
σ_w = Incoming Wave Unit Polarization Vector

Enter additional input data here:

foc := 5	(meters)	aref := 5	(meters)
	(Note: (f) should be greater than minimum distance to the far-field for the feed antenna)		
		εb := .96	(dimensionless)
εsp := .95	(dimensionless)	t1 := 10	(increments)
t2 := 10	(increments)	i1 := 10	(increments)
N := 10	(increments)	M := 10	(increments)
rff2 := 10^3	(meters)	x := 1000	(m)
$\sigma w2 := \begin{bmatrix} \frac{j}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$	(dimensionless)	y := 1000	(m)
		z := 1000	(m)

Establish Integral and Summation Increments:

$\phi 1 := 0, \frac{2\cdot\pi}{t1} .. 2\cdot\pi$ (radians)

$A := 1 .. N$ (increments)

$\phi 2 := 0, \frac{2\cdot\pi}{i1} .. 2\cdot\pi$ (radians)

$B := 1 .. M$ (increments)

$\theta 2 := -\left(\frac{\pi}{36}\right) - 10^{-6}, \frac{\pi}{i1\cdot 18} - \frac{\pi}{36} - 10^{-6} .. \frac{\pi}{36} - 10^{-6}$

$\phi 11_B := 2\cdot\pi\cdot\frac{B}{M}$ (radians)

$rp := 0, \frac{aref}{t2} .. aref$ (m)

$\phi 22_B := 2\cdot\pi\cdot\frac{B}{M}$ (radians)

$rp1_A := aref\cdot\frac{A}{N}$ (m)

$\theta 22_A := \left(\frac{\pi}{18}\right)\cdot\frac{A}{N} - \frac{\pi}{36}$ (radians)

$\theta 22_0 := -\frac{\pi}{36}$ (radians)

Calculate reflector geometric parameters:

$\theta o := 2\cdot\operatorname{atan}\left(\frac{aref}{2\cdot foc}\right)$ (radians)

$r_0 := 1.6\cdot\lambda$ (m)

$\theta o = 0.9273$ (radians)

$r_1 := 10\cdot aref$ (m)

$\rho(rp) := \frac{4\cdot foc^2 + rp^2}{4\cdot foc}$ (m)

$r_2 := \frac{8\cdot aref^2}{\lambda}$ (m)

$\theta p(rp) := 2\cdot\operatorname{atan}\left(\frac{rp}{2\cdot foc}\right)$ (radians)

$rmin := \max(r)$ (m)

$\rho 1_A := \frac{4\cdot foc^2 + \left(rp1_A\right)^2}{4\cdot foc}$ (m)

$rmin = 2.0014\cdot 10^4$ (m)

$\theta p1_A := 2\cdot\operatorname{atan}\left(\frac{rp1_A}{2\cdot foc}\right)$ (radians)

Aperture Electric Fields E_{ax}, E_{ay}:

$$Eax(rp,\phi 1) := (E\theta(\theta p(rp))\cdot -\cos(\phi 1) + E\phi(\theta p(rp))\cdot \sin(\phi 1))\cdot e^{j\cdot k\cdot(\rho(rp) - foc\cdot 2)} \qquad (V/m)$$

$$Eay(rp,\phi 1) := (E\theta(\theta p(rp))\cdot \sin(\phi 1) + E\phi(\theta p(rp))\cdot \cos(\phi 1))\cdot e^{j\cdot k\cdot(\rho(rp) - foc\cdot 2)} \qquad (V/m)$$

$$Eax1_{A,B} := \left[E\theta\left(\theta p1_A\right)\cdot\left(-\cos\left(\phi 11_B\right)\right) + E\phi\left(\theta p1_A\right)\cdot\sin\left(\phi 11_B\right)\right]\cdot e^{j\cdot k\cdot\left(\rho 1_A - foc\cdot 2\right)} \qquad (V/m)$$

$$Eay1_{A,B} := \left(E\theta\left(\theta p1_A\right)\cdot\sin\left(\phi 11_B\right) + E\phi\left(\theta p1_A\right)\cdot\cos\left(\phi 11_B\right)\right)\cdot e^{j\cdot k\cdot\left(\rho 1_A - foc\cdot 2\right)} \qquad (V/m)$$

Electric Field Integral Equation Vector Components $P_{x,y}$:

$$Ax(\theta 2,\phi 2,\phi 1) := \int_0^{aref} Eax(rp,\phi 1)\cdot e^{j\cdot k\cdot rp\cdot\sin(\theta 2)\cdot(\cos(\phi 1)\cdot\cos(\phi 2) + \sin(\phi 1)\cdot\sin(\phi 2))}\cdot rp\, drp \qquad (V/m)$$

$$Px(\theta 2,\phi 2) := \int_0^{2\cdot\pi} Ax(\theta 2,\phi 2,\phi 1)\, d\phi 1 \qquad (V/m)$$

$$Ay(\theta 2,\phi 2,\phi 1) := \int_0^{aref} Eay(rp,\phi 1)\cdot e^{j\cdot k\cdot rp\cdot\sin(\theta 2)\cdot(\cos(\phi 1)\cdot\cos(\phi 2) + \sin(\phi 1)\cdot\sin(\phi 2))}\cdot rp\, drp \qquad (V/m)$$

$$Py(\theta 2,\phi 2) := \int_0^{2\cdot\pi} Ay(\theta 2,\phi 2,\phi 1)\, d\phi 1 \qquad (V/m)$$

$$Ax1_{A,B} := \int_0^{aref} Eax1_{A,B}\cdot e^{j\cdot k\cdot rp1\cdot\sin\left(\theta 22_A\right)\cdot\left(\cos\left(\phi 11_B\right)\cdot\cos\left(\phi 22_B\right) + \sin\left(\phi 11_B\right)\cdot\sin\left(\phi 22_B\right)\right)}\cdot rp1\, drp1 \qquad (V/m)$$

$$Px1_{A,B} := \int_0^{2\cdot\pi} Ax1_{A,B}\, d\phi 11 \qquad (V/m)$$

$$Ay1_{A,B} := \int_0^{aref} Eay1_{A,B}\cdot e^{j\cdot k\cdot rp1\cdot\sin\left(\theta 22_A\right)\cdot\left(\cos\left(\phi 11_B\right)\cdot\cos\left(\phi 22_B\right) + \sin\left(\phi 11_B\right)\cdot\sin\left(\phi 22_B\right)\right)}\cdot rp1\, drp1 \qquad (V/m)$$

$$Py1_{A,B} := \int_0^{2\cdot\pi} Ay1_{A,B}\, d\phi 11 \qquad (V/m)$$

Secondary Field Electric Field Components E_θ, E_ϕ:

$$E\theta2(\theta2,\phi2) := \frac{j\cdot k\cdot e^{-j\cdot k\cdot rff2}}{2\cdot\pi\cdot rff2}\cdot(Px(\theta2,\phi2)\cdot\cos(\phi2) + Py(\theta2,\phi2)\cdot\sin(\phi2))$$ (V/m)

$$E\phi2(\theta2,\phi2) := \frac{j\cdot k\cdot e^{-j\cdot k\cdot rff2}}{2\cdot\pi\cdot rff2}\cdot(Px(\theta2,\phi2)\cdot -\sin(\phi2) + Py(\theta2,\phi2)\cdot\cos(\phi2))\cdot\cos(\theta2)$$ (V/m)

$$E\theta3_{A,B} := \frac{j\cdot k\cdot e^{-j\cdot k\cdot rff2}}{2\cdot\pi\cdot rff2}\cdot\left(Px1_{A,B}\cdot\cos\left(\phi22_B\right) + Py1_{A,B}\cdot\sin\left(\phi22_B\right)\right)$$ (V/m)

$$E\phi3_{A,B} := \frac{j\cdot k\cdot e^{-j\cdot k\cdot rff2}}{2\cdot\pi\cdot rff2}\cdot\left[Px1_{A,B}\cdot\left(-\sin\left(\phi22_B\right)\right) + Py1_{A,B}\cdot\cos\left(\phi22_B\right)\right]\cdot\cos\left(\theta22_A\right)$$ (V/m)

Secondary Field Radiation Intensity U:

$$U2(\theta2,\phi2) := \frac{rff2^2}{2\cdot\eta_0}\cdot\left[(|E\theta2(\theta2,\phi2)|)^2 + (|E\phi2(\theta2,\phi2)|)^2\right]$$ (W/solid ang)

$$U3_{A,B} := \frac{rff2^2}{2\cdot\eta_0}\cdot\left[\left(\left|E\theta3_{A,B}\right|\right)^2 + \left(\left|E\phi3_{A,B}\right|\right)^2\right]$$ (W/solid ang)

Secondary Field Radiated Power P_{rad}:

$$Prad2 := \left(\frac{2\cdot\pi}{N}\right)\cdot\left(\frac{\pi}{2\cdot M}\right)\cdot\sum_B\left(\sum_A U2\left(\theta22_A,\phi22_B\right)\cdot\sin\left(\theta22_A\right)\right)$$ (W)

$$Prad2 = 0.19318$$ (W)

Secondary Field Directivity D_o:

$$Do22 := \frac{4\cdot\pi\cdot\max(U3)}{Prad2}$$ (dimensionless)

$$Do22 = 1.04762\cdot 10^5$$ (dimensionless)

Spillover Efficiency ε_s:

$$\varepsilon s := \frac{\int_0^{\theta o} U(\theta p)\cdot \sin(\theta p)\, d\theta p}{\int_0^{\pi} U(\theta p)\cdot \sin(\theta p)\, d\theta p}$$ (dimensionless)

$\varepsilon s = 0.19013$ (dimensionless)

Taper Efficiency ε_t:

$$\varepsilon t := \frac{\left[\left|\int_0^{\theta o} \sqrt{\frac{U(\theta p)\cdot 4\cdot \pi}{Prad}}\cdot \tan\left(\frac{\theta p}{2}\right) d\theta p\right|\right]^2 \cdot 2\cdot \cot\left(\left(\frac{\theta o}{2}\right)\right)^2}{\int_0^{\theta o} \frac{U(\theta p)\cdot 4\cdot \pi}{Prad}\cdot \sin(\theta p)\, d\theta p}$$ (dimensionless)

$\varepsilon t = 0.50837$ (dimensionless)

Phase Efficiency ε_p:

$$\varepsilon p := e^{\frac{-4\cdot \pi\cdot 6\cdot 10^{-5}\cdot aref}{\lambda}}$$ (dimensionless)

$\varepsilon p = 0.68574$ (dimensionless)

Aperture Efficiency ε_{ap}:

$\varepsilon x := .98$ (dimensionless) $\varepsilon ohmic := .98$ (dimensionless)

$\varepsilon ap := \varepsilon s\cdot \varepsilon t\cdot \varepsilon p\cdot \varepsilon x\cdot \varepsilon b\cdot \varepsilon sp\cdot \varepsilon ohmic$ (dimensionless) $\varepsilon ap = 0.05805$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$EIRP22 := Prad2 \cdot Do22$ (W)

$EIRP22 = 2.02379 \cdot 10^{4}$ (W)

Gain G:

$G22 := \varepsilon ap \cdot Do22$ (dimensionless) $G22dB := 10 \cdot \log(G22)$ (dimensionless)

$G22 = 6.08192 \cdot 10^{3}$ (dB) $G22dB = 37.84041$ (dB)

Effective Aperture A_e:

$Ae22 := \varepsilon ap \cdot \pi \cdot aref^{2}$ (m^2)

$Ae22 = 4.55961$ (m^2)

Radiation Resistance R_r:

$$Rr22 = 2 \cdot \frac{Prad2}{(|Io|)^{2}}$$ (Ω)

$Rr22 = 0.38636$ (Ω)

Effective Height h_e:

$$he22 := 2 \cdot \sqrt{Rr22 \cdot \frac{Ae22}{\eta_0}}$$ (m)

$he22 = 0.13672$ (m)

Polarization Loss Factor PLF:

$\theta p2 := \mathrm{atan}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$ (radians) $\phi p2 := \mathrm{atan}\left(\frac{y}{x}\right)$ (radians)

$\theta p2 = 0.95532$ (radians) $\phi p2 = 0.7854$ (radians)

$E\theta 4 := E\theta 2(\theta p2, \phi p2)$ (V/m) $E\phi 4 := E\phi 2(\theta p2, \phi p2)$ (V/m)

$Ex2 := E\theta 4 \cdot \cos(\theta p2) \cdot \cos(\phi p2) - E\phi 4 \cdot \sin(\phi p2)$ (V/m)

$Ey2 := E\theta 4 \cdot \cos(\theta p2) \cdot \sin(\phi p2) - E\phi 4 \cdot \cos(\phi p2)$ (V/m)

$Ez2 := -E\theta 4 \cdot \sin(\theta p2)$ (V/m)

$\sigma a2 := \frac{1}{\sqrt{(|Ex2|)^2 + (|Ey2|)^2 + (|Ez2|)^2}} \cdot \begin{pmatrix} Ex2 \\ Ey2 \\ Ez2 \end{pmatrix}$ (dimensionless)

$\sigma a2 = \begin{pmatrix} 0.12034 + 0.48533j \\ 0.12034 + 0.48533j \\ -0.60556 - 0.36501j \end{pmatrix}$ (dimensionless)

$PLF2 := \left(\left|\sigma w2 \cdot \overline{\sigma a2}\right|\right)^2$ (dimensionless)

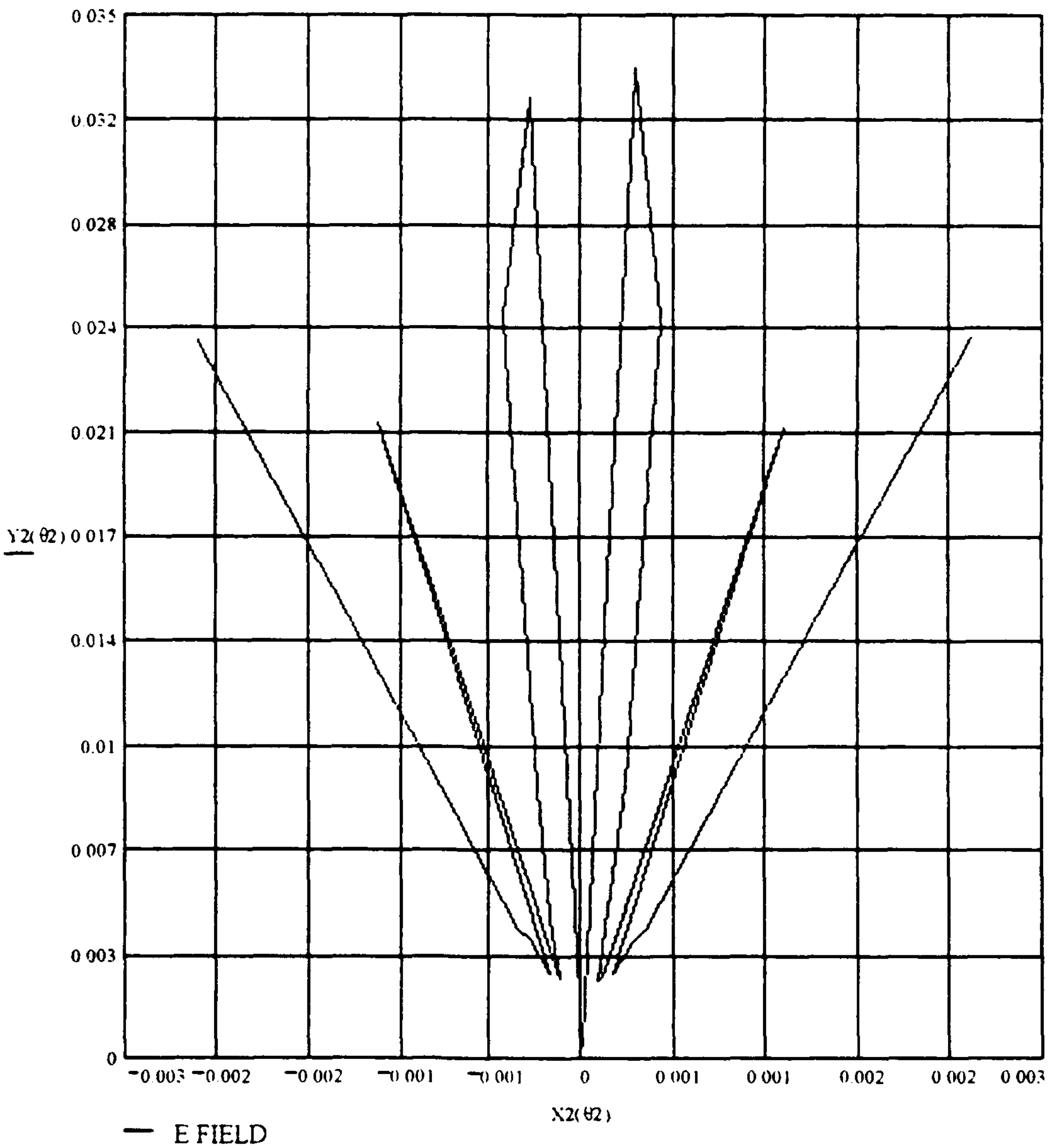
$PLF2 = 0.25003$ (dimensionless)

PARABOLIC REFLECTOR WITH HELICAL FEED FAR-FIELD ELEVATION PATTERNS

E-FIELD RADIATION PATTERN:

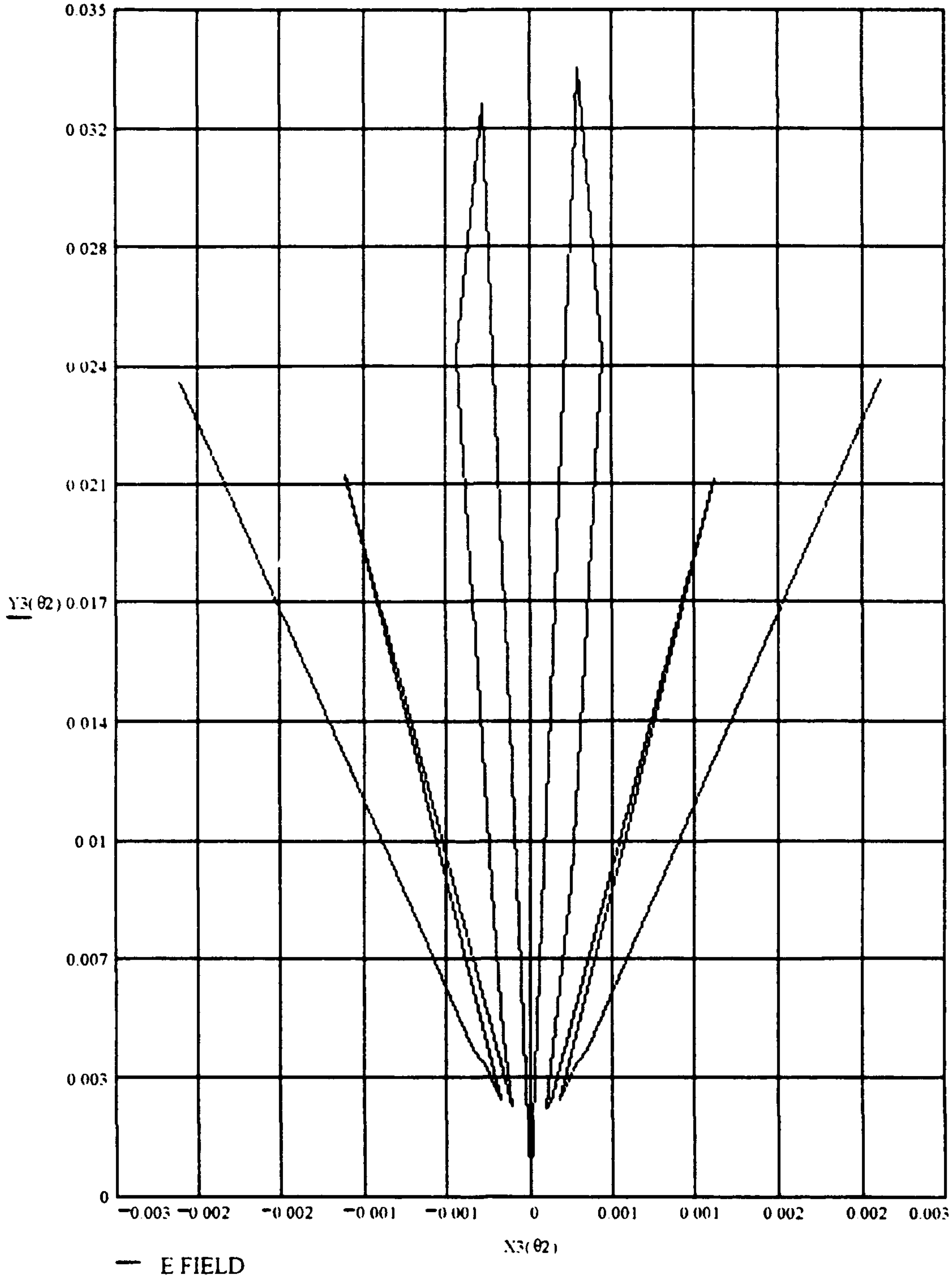
$i1 := 20$ $\theta2 := -\left(\frac{\pi}{36}\right) - 10^{-6}, \frac{\pi}{i1\ 18} - \frac{\pi}{36} - 10^{-6} .. \frac{\pi}{36} - 10^{-6}$

$X2(\theta2) := \left|E\theta2\left(\theta2, \frac{\pi}{2}\right)\right| \cos\left(\theta2 + \frac{\pi}{2}\right)$ $Y2(\theta2) := \left|E\theta2\left(\theta2, \frac{\pi}{2}\right)\right| \sin\left(\theta2 + \frac{\pi}{2}\right)$



H-FIELD RADIATION PATTERN:

$$X3(\theta 2) := |E\phi 2(\theta 2, 0)| \cdot \cos\left(\theta 2 + \frac{\pi}{2}\right) \qquad Y3(\theta 2) := |E\phi 2(\theta 2, 0)| \cdot \sin\left(\theta 2 + \frac{\pi}{2}\right)$$



THE SPIRAL ANTENNA (REFLECTOR OPTION)
MATHCAD SOFTWARE-DISH_SPI.MCD

Spiral antennas are a family of two or three dimensional devices that possess frequency independant parameters over a wide bandwidth. Spiral antennas are commonly used for direction finding, satellite tracking and missile guidance.

The planar spiral may be of the Archemedean, log-spiral, or equiangular type. All three radiate two main, circularly polarized lobes perpendicular to the plane of the antenna. Additional gain for planar spirals may be achieved by placing a metal cavity on the side of the antenna with the unwanted lobe. The cavity may be empty or be filled with electromagnetic energy absorbing material. These applications principly examine the equiangular planar spiral and do not account for cavity backed effects.

(Note: Mathcad equations cannot use symbolic subscripts. Therefore, symbols like λ will immediately follow the parameter in equations.)

The spiral antenna Mathcad applications will compute the following parameters for equiangular planar spirals:

k = Wavenumber

λ = Wavelength

D_o = Directivity

$E\phi$ = Electric Field Component

U = Radiation Intensity

U_{max} = Maximum Radiation Intensity

P_{rad} = Radiated Power

G = Gain

EIRP = Effective Isotropic Radiated Power

A_{em} = Maximum Effective Aperture

BW = Bandwidth

r_{min} = Minimum Distance to Far-Field

R_r = Radiation Resistance

h_{em} = Maximum Effective Height

f_{high} = Upper Operating Frequency

f_{low} = Lower Operating Frequency

r_n = Any point on the n^{th} edge of a spiral

ε_{ex} = Equiangular Planar Spiral Expansion Ratio

Zi = Planar Spiral Input Impedance

Γ = Voltage Reflection Coefficient

ε_{rv} = Reflection Efficiency

PLF = Polarization Loss Factor

λ_{high} = Upper Operating Wavelength

λ_{low} = Lower Operating Wavelength

A = Planar Spiral Electric Field Amplitude

The following data must be input based on known or estimated data:

M = Mode

N = Number of Spiral Arms

f = Frequency of Interest

i = Number of Increments for Far Field Radiation Patterns

r_{ff} = Distance of Far-Field Calculations

I_o = Input Current at Antenna Terminals

r_o = Spiral Feed Point

a = Flare Rate

δ_{n+1} = Angular Arm Width of n^{th} Spiral Arm

ϕ_r = Azimuth to Compute Expansion Ratio

R = Overall Radius

E_o = Source Strength Constant for Planar Spirals

σ_w = Wave Unit Polarization Vector

Z_o = Characteristic Impedance of Feed Assembly

σ_a = Equiangular Planar Spiral Unit Polarization Vector

THE PLANAR SPIRAL ANTENNAS

Enter input data here:

$N := 2$	(arms)	$f := 3.5 \cdot 10^{10}$	(Hz)
$M := 1$	(mode)	$Io := 1$	(amps)
(Note: M_{max} is N-1)			
$i := 30$	(increments)	$rff := 1 \cdot 10^{3}$	(meters)
$a := .221$	(dimensionless)	$ro := .001$	(m)
$R := .01$	(m)	$Eo := 10^{3}$	(V/m)

$$\delta := \begin{bmatrix} 0 \\ \frac{\pi}{2} \\ \pi \\ \frac{3\pi}{2} \end{bmatrix}$$ (radians)

$$\sigma w := \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$ (dimensionless)

$Zo := 100$ (Ω)

Calculate planar spiral antenna geometric parameters and define constants:

$c := 2.9979 \cdot 10^{8}$ (meters / sec)

$\eta_0 := 120 \cdot \pi$ (Ω)

$\lambda := \frac{c}{f}$ (meters / cycle)

$\varepsilon_0 := \frac{1}{36 \cdot \pi} \cdot 10^{-9}$ (Farads / m)

$\lambda = 8.56543 \cdot 10^{-3}$ (meters / cycle)

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$ (H / m)

$$\sigma a := \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$ (dimensionless)

Calculate planar spiral antenna parameters :

Define angular offset θ from y-z axis:

$\theta := -\frac{\pi}{2} - 10^{-6}, \frac{\pi}{1} - \frac{\pi}{2} - 10^{-6} .. \frac{\pi}{2} - 10^{-6}$ (radians)

$\phi := 0, \frac{2\pi}{1} .. 2 \cdot \pi$ (radians)

Distance to Far-Field r_{min}:

$rr_0 := 1.6 \cdot \lambda$ (m) $rr_1 := 10 \cdot R$ (m)

$rr_2 := \dfrac{8 \cdot R^2}{\lambda}$ (m)

$rmin := \max(rr)$ (m)

$rmin = 0.1$ (m)

Wavenumber k:

$k := \dfrac{2\pi}{\lambda}$ (m^{-1})

$k = 7.33552 \cdot 10^2$ (m^{-1})

Radial Distance to n^{th} Spiral Edge r:

$r(n, \phi r) := ro \cdot e^{a(\phi r - \delta_{n-1})}$ (m)

$r(1, 2\pi) = 4.00917 \cdot 10^{-3}$ (m)

Expansion Ratio ε_{ex}:

$\varepsilon ex(n, \phi r) := \dfrac{r(n, \phi r + 2\pi)}{r(n, \phi r)}$ (dimensionless)

$\varepsilon ex(1, 2\pi) = 4.00917$ (dimensionless)

Bandwidth BW:

Equiangular Spiral

$\lambda high := 4 \cdot ro$ (m)

$fhigh := \dfrac{c}{\lambda high}$ (Hz)

Log-Periodic Spiral

$\lambda high1 := 20 \cdot ro$ (m)

$fhigh1 := \dfrac{c}{\lambda high1}$ (Hz)

$fhigh = 7.49475 \cdot 10^{10}$ (Hz) $fhigh1 = 7.49475 \cdot 10^{10}$ (Hz)

$\lambda low := 4 \cdot R$ (m)

$flow := \frac{c}{\lambda low}$ (Hz)

$flow = 7.49475 \cdot 10^{9}$ (Hz)

$BW := fhigh - flow$ (Hz) $BW1 := fhigh1 - flow$ (Hz)

$BW = 6.74528 \cdot 10^{10}$ (Hz) $BW1 = 7.49475 \cdot 10^{9}$ (Hz)

Electric Field $E(\underline{\theta,\phi})$ and Electric Field Amplitude $A(\underline{\theta})$:

$w := 0 .. i$ (increments)

$$E(\theta) := \frac{Eo \cdot k^3 \cdot \cos(\theta) \cdot (1 + j \cdot a \cdot \cos(\theta))^{-1 - j \cdot \frac{M}{a}} \cdot \tan\left(\frac{\theta}{2}\right)^M \cdot e^{j \cdot \left[M \cdot \left(\theta + \frac{\pi}{2}\right) - k \cdot rff\right]}}{\sin(\theta)^2 \cdot rff}$$ (V/m)

$$A(\theta) := \frac{\cos(\theta) \cdot \tan\left(\frac{\theta}{2}\right)^M \cdot e^{\left(\frac{M}{a}\right) \cdot \operatorname{atan}(a \cdot \cos(\theta))}}{\sin(\theta) \cdot \sqrt{1 + a^2 \cdot \cos(\theta)^2}}$$ (V/m)

$$\psi(\theta) := \frac{M}{2 \cdot a} \cdot \ln\left(1 + a^2 \cdot \cos(\theta)^2\right) + \operatorname{atan}(a \cdot \cos(\theta))$$ (dimensionless)

$$E\phi(\theta) := A(\theta) \cdot e^{-j \cdot \psi(\theta)} \cdot \frac{e^{j \cdot \left[M \cdot \left(\theta + \frac{\pi}{2}\right) - k \cdot rff\right]}}{rff}$$ (V/m)

$$A1_w := \frac{\cos\left(-\frac{\pi}{2}+\pi\cdot\frac{w}{i}\right)\cdot\tan\left(\frac{-\frac{\pi}{2}+\pi\cdot\frac{w}{i}}{2}\right)^M\cdot e^{\left(\frac{M}{a}\right)\cdot\operatorname{atan}\left(a\cdot\cos\left(-\frac{\pi}{2}+\pi\cdot\frac{w}{i}\right)\right)}}{\sin\left(-\frac{\pi}{2}+\pi\cdot\frac{w}{i}\right)\cdot\sqrt{1+a^2\cdot\cos\left(-\frac{\pi}{2}+\pi\cdot\frac{w}{i}\right)^2}}$$ (V/m)

Radiation Intensity U(θ):

$$U(\theta) := \frac{1}{2\cdot\eta_0}\cdot\left(A(\theta)^2\right)$$ (W / solid angle)

$$U1_w := \frac{1}{2\cdot\eta_0}\cdot\left[\left(A1_w\right)^2\right]$$ (W / solid angle)

$$Umax := \max(U1)$$ (W / solid angle)

$$Umax = 2.2285\cdot 10^{-3}$$ (W / solid angle)

Radiated Power P_{rad}:

$$Prad := 4\cdot\pi\cdot\int_0^{\frac{\pi}{2}} U(\theta)\cdot\sin(\theta)\,d\theta$$ (W)

$$Prad = 7.94528\cdot 10^{-3}$$ (W)

Directivity D_o:

$$D_0 := \frac{4\cdot\pi\cdot Umax}{Prad}$$ (dimensionless)

$$D_0 = 3.52462$$ (dimensionless)

Radiation Resistance Rr:

$$Rr := \frac{2\ Prad}{(|Io|)^2} \quad (\Omega)$$

$$Rr = 0.01589 \quad (\Omega)$$

Input Impedance Zi:

$$Zi := \frac{N\ 30\ \pi}{\sin\left(\pi \frac{M}{N}\right)} \quad (\Omega)$$

$$Zi = 1.88496 \cdot 10^2 \quad (\Omega)$$

Voltage Reflection Coefficient Γ:

$$\Gamma := \frac{Zi - Zo}{Zi + Zo} \quad \text{(dimensionless)}$$

$$\Gamma = 0.30675 \quad \text{(dimensionless)}$$

Reflecton Efficiency ε_{rv}:

$$\varepsilon rv := 1 - (|\Gamma|)^2 \quad \text{(dimensionless)}$$

$$\varepsilon rv = 0.90591 \quad \text{(dimensionless)}$$

Gain G:

$$G := \varepsilon rv\ D_0 \quad \text{(dimensionless)}$$

$$GdB := 10 \log(G) \quad (dB)$$

$$G = 3.19298 \quad \text{(dimensionless)}$$

$$GdB = 5.04196 \quad (dB)$$

Effective Isotropic Radiated Power (EIRP):

$$EIRP := \frac{Prad\ G}{\varepsilon rv} \quad (W)$$

$$EIRP = 0.028 \quad (W)$$

Polarization Loss Factor (PLF):

$$PLF := \left(|\rho w \cdot \overline{\rho a}|\right)^2 \quad \text{(dimensionless)}$$

$$PLF = 1 \quad \text{(dimensionless)}$$

Maximum Effective Aperture (A_{em}):

$$Aem := \frac{\lambda^2 \, D_0 \, ecd \, PLF}{4 \, \pi} \qquad (m^2)$$

$$Aem = 1.86416 \cdot 10^{-5} \qquad (m^2)$$

Maximum Effective Height (h_{em}):

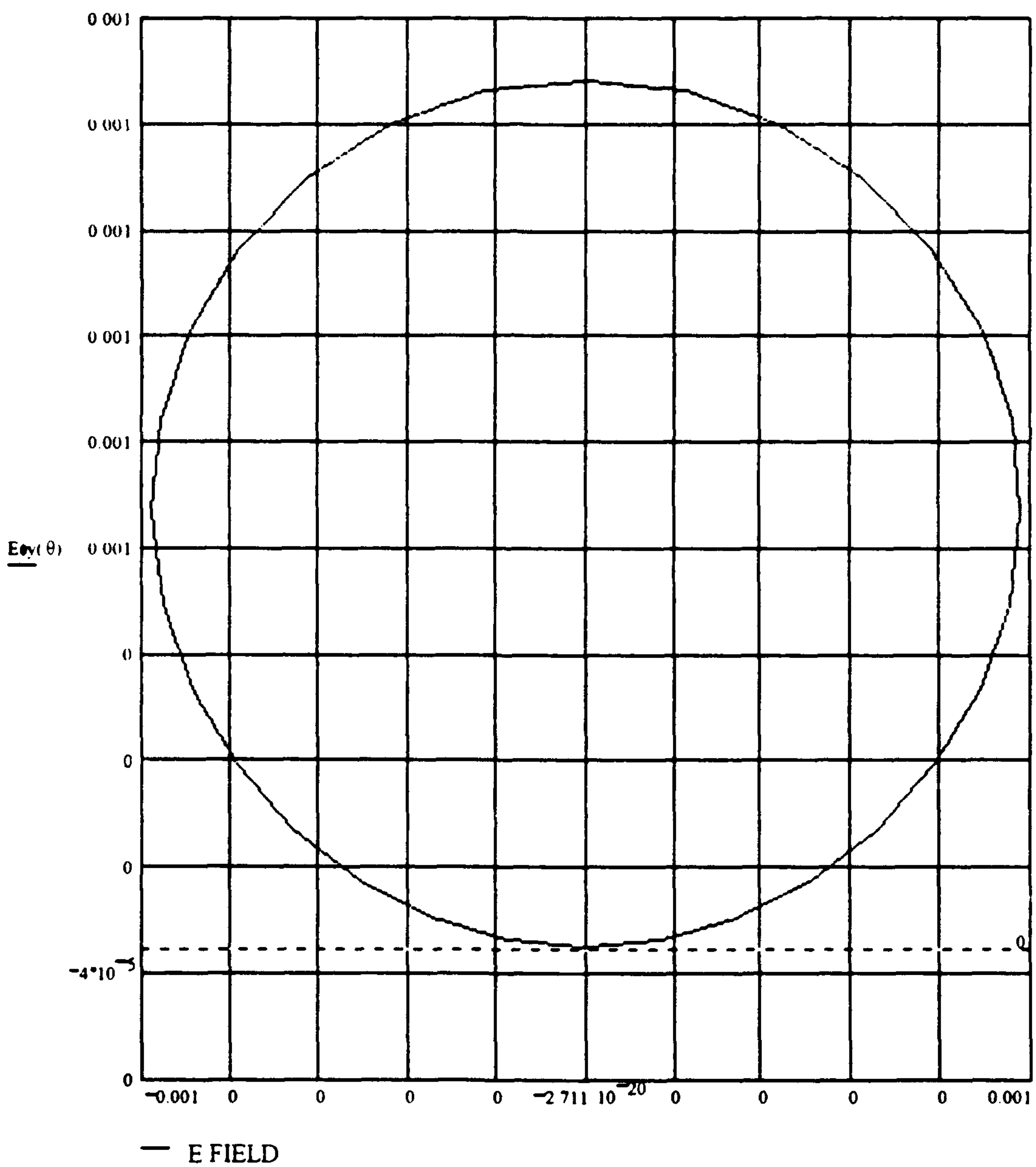
$$hem := \sqrt{\frac{Rr \, Aem}{\eta_0}} \cdot 2 \qquad (m)$$

$$hem = 5.6063 \cdot 10^{-5} \qquad (m)$$

THE EQUIANGULAR PLANAR SPIRAL ANTENNA FAR-FIELD ELEVATION PATTERN

For the purpose of this far-field radiation pattern, the spiral antenna lies parallel to the Ey = 0 grid line and is centered at the origin. The magnitude of the electric field pattern is rotationally symmetric with respect to the Ex=0 grid line. The equiangular planar spiral antenna possesses a mirror image radiator pattern in the -y half plane.

$$E\phi x(\theta) := |E\phi(\theta)| \cos\left(\theta + \frac{\pi}{2}\right) \qquad E\phi y(\theta) := |E\phi(\theta)| \sin\left(\theta + \frac{\pi}{2}\right)$$



PARABOLIC REFLECTOR ANALYSIS WITH SPIRAL ANTENNA FEED

The spiral antenna Mathcad application (Reflector Option) will compute the following additional parameters (Note: a 2 or 22 will follow most secondary field parameters in this application, and inorder to speed up computation time corresponding vectors are created with subscript A & B:

$P_{x,y}$ = E-Field Integral and Summation Equation Vector Components

$E_{ax,ay}$ = Aperture E-Field Components

θ',ρ = E-Field Integral and Summation Equation Parameters

E_{θ} = Secondary Field Theta E-Field Component

E_{ϕ} = Secondary Field Phi E-Field Component

U = Secondary Field Radiation Intensity

P_{rad} = Secondary Field Radiated Power

D_c = Secondary Field Directivity

G = Secondary Field Gain

ε_{ap} = Aperture Efficiency

ε_s = Spillover Efficiency

ε_t = Taper Efficiency

ε_p = Phase Efficiency

ε_x = Polarization Efficiency

ε_{ohmic} = Ohmic Efficiency

A_e = Reflector Effective Area

h_e = Reflector Effective Height

EIRP = Reflector Effective Radiated Power

$E_{x,y,z}$ = Secondary Field Cartesian Components

σ_a = Reflector Unit Polarization Vector

PLF = Reflector Polarization Loss Factor

r_{min} = Minimum Distance to Secondary Far-Field

θ_o = Reflector Periphery Coaltitude

θ_p,ϕ_p = Theta and Phi for Polarization Loss

The following additional data must be inputted based on known or estimated data:

a_{ret} = Radius of the Mouth of the Reflector

foc = Reflector Focal Length

ε_b = Blockage Efficiency

ε_{sp} = Spar Efficiency

t1,t2 = Radiated Power Increments for Phi and Theta

$i1$ = Secondary Field Increments
N, M = Summation Increments
r_{ff2} = Secondary Field Observation Distance
x, y, z = Polarization Loss Factor Coordinates
σ_w = Incoming Wave Unit Polarization Vector

<u>Enter additional input data here:</u>

$aref := 0.32$ (meters)

$foc := 0.147$	(meters)	$\epsilon b := 96$	(dimensionless)
$\epsilon sp := 95$	(dimensionless)	$t1 := 10$	(increments)
$t2 := 10$	(increments)	$i1 := 10$	(increments)
$N := 10$	(increments)	$M := 10$	(increments)
$rff2 := 10^3$	(meters)	$x := 1000$	(m)
		$y := 1000$	(m)
$\sigma w2 := \begin{bmatrix} \frac{j}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$	(dimensionless)	$z := 1000$	(m)

Establish Integral and Summation Increments:

$\phi 1 := 0, \frac{2 \cdot \pi}{t1} .. 2 \cdot \pi$ (radians)

$\phi 2 := 10^{-3}, \frac{2 \cdot \pi}{i1} + 10^{-3} .. 2 \cdot \pi + 10^{-3}$ (radians)

$\theta 2 := -\left(\frac{\pi}{36}\right) - 10^{-3}, \frac{\pi}{i1 \cdot 18} - \frac{\pi}{36} - 10^{-3} .. \frac{\pi}{36} - 10^{-3}$

$rp := 0, \frac{aref}{t2} .. aref$ (m)

$rp1_A := aref \cdot \frac{A}{N}$ (m)

$A := 1 .. N$ (increments)

$B := 1 .. M$ (increments)

$\phi 11_B := 2 \cdot \pi \cdot \frac{B}{M}$ (radians)

$\phi 22_B := 2 \cdot \pi \cdot \frac{B}{M}$ (radians)

$\theta 22_A := \left(\frac{\pi}{18}\right) \cdot \frac{A}{N} - \frac{\pi}{36}$ (radians)

$\theta 22_0 := -\frac{\pi}{36}$ (radians)

Calculate reflector geometric parameters:

$\theta o := 2 \cdot \mathrm{atan}\left(\frac{aref}{2 \cdot foc}\right)$ (radians)

$\theta o = 1.65544$ (radians)

$\rho(rp) := \frac{4 \cdot foc^2 + rp^2}{4 \cdot foc}$ (m)

$\theta p(rp) := 2 \cdot \mathrm{atan}\left(\frac{rp}{2 \cdot foc}\right)$ (radians)

$\rho 1_A := \frac{4 \cdot foc^2 + \left(rp1_A\right)^2}{4 \cdot foc}$ (m)

$\theta p1_A := 2 \cdot \mathrm{atan}\left(\frac{rp1_A}{2 \cdot foc}\right)$ (radians)

$r_0 := 1.6 \cdot \lambda$ (m)

$r_1 := 10 \cdot aref$ (m)

$r_2 := \frac{8 \cdot aref^2}{\lambda}$ (m)

$rmin := \max(r)$ (m)

$rmin = 95.64028$ (m)

Aperture Electric Fields E_{ax}, E_{ay}:

$$Eax(rp,\phi 1) := E\phi(\theta p(rp)) \cdot \sin(\phi 1) \cdot e^{j \cdot k \cdot (\rho(rp) - foc2)}$$ (V/m)

$$Eay(rp,\phi 1) := E\phi(\theta p(rp)) \cdot \cos(\phi 1) \cdot e^{j \cdot k \cdot (\rho(rp) - foc2)}$$ (V/m)

$$Eax1_{A,B} := E\phi\left(\theta p1_A\right) \cdot \sin\left(\phi 11_B\right) \cdot e^{j \cdot k \cdot \left(\rho 1_A - foc2\right)}$$ (V/m)

$$Eay1_{A,B} := E\phi\left(\theta p1_A\right) \cdot \cos\left(\phi 11_B\right) \cdot e^{j \cdot k \cdot \left(\rho 1_A - foc2\right)}$$ (V/m)

Electric Field Integral Equation Vector Components $P_{x,y}$:

$$Ax(\theta 2,\phi 2,\phi 1) := \int_0^{aref} Eax(rp,\phi 1) \cdot e^{j \cdot k \cdot rp \cdot \sin(\theta 2) \cdot (\cos(\phi 1) \cdot \cos(\phi 2) + \sin(\phi 1) \cdot \sin(\phi 2))} \cdot rp \, drp$$ (V/m)

$$Px(\theta 2,\phi 2) := \int_0^{2\pi} Ax(\theta 2,\phi 2,\phi 1) \, d\phi 1$$ (V/m)

$$Ay(\theta 2,\phi 2,\phi 1) := \int_0^{aref} Eay(rp,\phi 1) \cdot e^{j \cdot k \cdot rp \cdot \sin(\theta 2) \cdot (\cos(\phi 1) \cdot \cos(\phi 2) + \sin(\phi 1) \cdot \sin(\phi 2))} \cdot rp \, drp$$ (V/m)

$$Py(\theta 2,\phi 2) := \int_0^{2 \cdot \pi} Ay(\theta 2,\phi 2,\phi 1) \, d\phi 1$$ (V/m)

$$Ax1_{A,B} := \int_0^{aref} Eax1_{A,B} \cdot e^{j \cdot k \cdot rp1 \cdot \sin\left(\theta 22_A\right) \cdot \left(\cos\left(\phi 11_B\right) \cdot \cos\left(\phi 22_B\right) + \sin\left(\phi 11_B\right) \cdot \sin\left(\phi 22_B\right)\right)} \cdot rp1 \, drp1$$ (V/m)

$$Px1_{A,B} := \int_0^{2 \cdot \pi} Ax1_{A,B} \, d\phi 11$$ (V/m)

$$Ay1_{A,B} := \int_0^{aref} Eay1_{A,B} \cdot e^{j \cdot k \cdot rp1 \cdot \sin\left(\theta 22_A\right) \cdot \left(\cos\left(\phi 11_B\right) \cdot \cos\left(\phi 22_B\right) + \sin\left(\phi 11_B\right) \cdot \sin\left(\phi 22_B\right)\right)} \cdot rp1 \, drp1$$ (V/m)

$$Py1_{A,B} := \int_0^{2 \cdot \pi} Ay1_{A,B} \, d\phi 11$$ (V/m)

Secondary Field Electric Field Components E_θ, E_ϕ:

$$E\theta2(\theta2,\phi2) := \frac{j \cdot k \cdot e^{-j \cdot k \cdot rff2}}{2 \cdot \pi \cdot rff2} \cdot (Px(\theta2,\phi2) \cdot \cos(\phi2) + Py(\theta2,\phi2) \cdot \sin(\phi2))$$ (V/m)

$$E\phi2(\theta2,\phi2) := \frac{j \cdot k \cdot e^{-j \cdot k \cdot rff2}}{2 \cdot \pi \cdot rff2} \cdot (Px(\theta2,\phi2) \cdot -\sin(\phi2) + Py(\theta2,\phi2) \cdot \cos(\phi2)) \cdot \cos(\theta2)$$ (V/m)

$$E\theta3_{A,B} := \frac{j \cdot k \cdot e^{-j \cdot k \cdot rff2}}{2 \cdot \pi \cdot rff2} \cdot \left(Px1_{A,B} \cdot \cos\left(\phi22_B\right) + Py1_{A,B} \cdot \sin\left(\phi22_B\right)\right)$$ (V/m)

$$E\phi3_{A,B} := \frac{j \cdot k \cdot e^{-j \cdot k \cdot rff2}}{2 \cdot \pi \cdot rff2} \cdot \left[Px1_{A,B} \cdot \left(-\sin\left(\phi22_B\right)\right) + Py1_{A,B} \cdot \cos\left(\phi22_B\right)\right] \cdot \cos\left(\theta22_A\right)$$ (V/m)

Secondary Field Radiation Intensity U:

$$U2(\theta2,\phi2) := \frac{rff2^2}{2 \cdot \eta_0} \left[\left(\left| E\theta2(\theta2,\phi2) \right| \right)^2 + \left(\left| E\phi2(\theta2,\phi2) \right| \right)^2 \right]$$ (W/solid ang)

$$U3_{A,B} := \frac{rff2^2}{2 \cdot \eta_0} \left[\left(\left| E\theta3_{A,B} \right| \right)^2 + \left(\left| E\phi3_{A,B} \right| \right)^2 \right]$$ (W/solid ang)

Secondary Field Radiated Power P_{rad}:

$$Prad2 := \left(\frac{2 \cdot \pi}{N}\right) \cdot \left(\frac{\pi}{2 \cdot M}\right) \cdot \sum_B \left(\sum_A U2\left(\theta22_A, \phi22_B\right) \cdot \sin\left(\theta22_A\right) \right)$$ (W)

$$Prad2 = 1.84163 \cdot 10^{-11}$$ (W)

Secondary Field Directivity D_o:

$$Do22 := \frac{4 \cdot \pi \cdot \max(U3)}{Prad2}$$ (dimensionless)

$$Do22 = 4.59318 \cdot 10^5$$ (dimensionless)

Spillover Efficiency ε_s:

$$\varepsilon s := \frac{\int_{10^{-6}}^{\theta o} U(\theta p)\cdot \sin(\theta p)\, d\theta p}{\int_{10^{-6}}^{\frac{\pi}{2}-10^{-6}} U(\theta p)\cdot \sin(\theta p)\, d\theta p}$$ (dimensionless)

$\varepsilon s = 1.00042$ (dimensionless)

Taper Efficiency ε_t:

$$\varepsilon t := \frac{\left[\left|\int_0^{\theta o} \sqrt{\frac{U(\theta p)\cdot 4\cdot \pi}{Prad}}\cdot \tan\left(\frac{\theta p}{2}\right) d\theta p\right|\right]^2 \cdot 2\cdot \cot\left(\left(\frac{\theta o}{2}\right)\right)^2}{\int_0^{\theta o} \frac{U(\theta p)\cdot 4\cdot \pi}{Prad}\cdot \sin(\theta p)\, d\theta p}$$ (dimensionless)

$\varepsilon t = 0.45135$ (dimensionless)

Phase Efficiency ε_p:

$$\varepsilon p := e^{\frac{-4\cdot\pi\cdot 6\cdot 10^{-5}\cdot aref}{\lambda}}$$ (dimensionless)

$\varepsilon p = 0.97222$ (dimensionless)

Aperture Efficiency ε_{ap}:

$\varepsilon x := .98$ (dimensionless) $\varepsilon ohmic := .98$ (dimensionless)

$\varepsilon ap := \varepsilon s\cdot \varepsilon t\cdot \varepsilon p\cdot \varepsilon x\cdot \varepsilon b\cdot \varepsilon sp\cdot \varepsilon ohmic$ (dimensionless) $\varepsilon ap = 0.38452$ (dimensionless)

Effective Isotropic Radiated Power EIRP:

$$EIRP22 := Prad2 \cdot Do22$$ (W)

$$EIRP22 = 8.45893 \cdot 10^{-6}$$ (W)

Gain G:

$$G22 := ecap \cdot Do22$$ (dimensionless)

$$G22dB := 10 \cdot \log(G22)$$ (dimensionless)

$$G22 = 1.76615 \cdot 10^{5}$$ (dB)

$$G22dB = 52.47028$$ (dB)

Effective Aperture A_e:

$$Ae22 := ecap \cdot \pi \cdot aref^2$$ (m^2)

$$Ae22 = 0.1237$$ (m^2)

Radiation Resistance R_r:

$$Rr22 := 2 \cdot \frac{Prad2}{(|Io|)^2}$$ (Ω)

$$Rr22 = 3.68326 \cdot 10^{-11}$$ (Ω)

Effective Height h_e:

$$he22 := 2 \cdot \sqrt{Rr22 \cdot \frac{Ae22}{\eta_0}}$$ (m)

$$he22 = 2.19868 \cdot 10^{-7}$$ (m)

Polarization Loss Factor PLF:

$\theta p2 := \operatorname{atan}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$ (radians) $\phi p2 := \operatorname{atan}\left(\frac{y}{x}\right)$ (radians)

$\theta p2 = 0.95532$ (radians) $\phi p2 = 0.7854$ (radians)

$E\theta 4 := E\theta 2(\theta p2, \phi p2)$ (V/m) $E\phi 4 := E\phi 2(\theta p2, \phi p2)$ (V/m)

$Ex2 := E\theta 4 \cdot \cos(\theta p2) \cdot \cos(\phi p2) - E\phi 4 \cdot \sin(\phi p2)$ (V/m)

$Ey2 := E\theta 4 \cdot \cos(\theta p2) \cdot \sin(\phi p2) - E\phi 4 \cdot \cos(\phi p2)$ (V/m)

$Ez2 := -E\theta 4 \cdot \sin(\theta p2)$ (V/m)

$$\sigma a2 := \frac{1}{\sqrt{(|Ex2|)^2 + (|Ey2|)^2 + (|Ez2|)^2}} \cdot \begin{pmatrix} Ex2 \\ Ey2 \\ Ez2 \end{pmatrix}$$ (dimensionless)

$$\sigma a2 = \begin{pmatrix} -0.40672 + 0.03531j \\ -0.40672 + 0.03531j \\ 0.81344 - 0.07063j \end{pmatrix}$$ (dimensionless)

$$PLF2 := \left(\left|\sigma w2 \cdot \overline{\sigma a2}\right|\right)^2$$ (dimensionless)

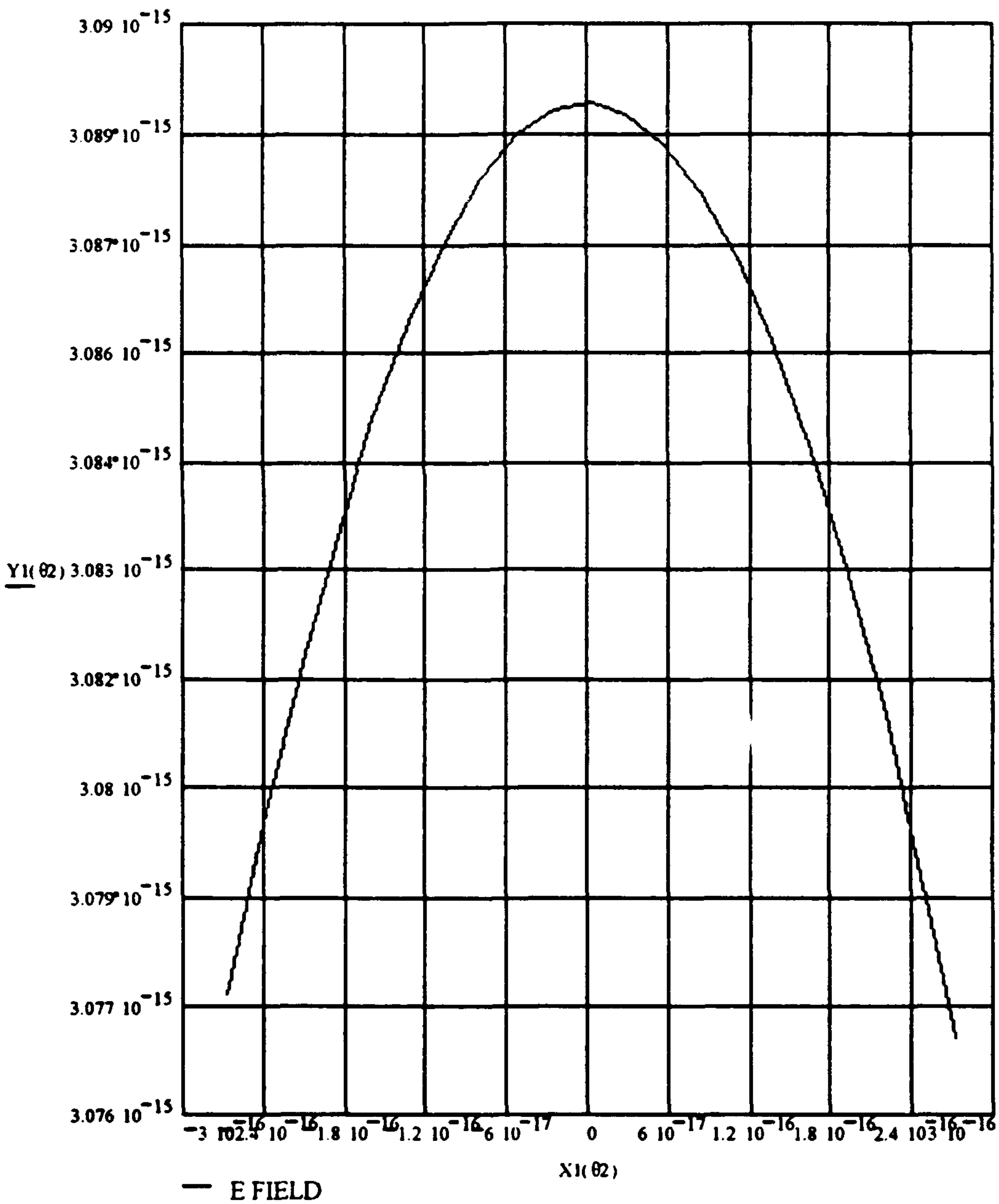
$PLF2 = 0.16667$ (dimensionless)

PARABOLIC REFLECTOR WITH SPIRAL FEED ANTENNA

E-PANE RADIATION PATTERN:

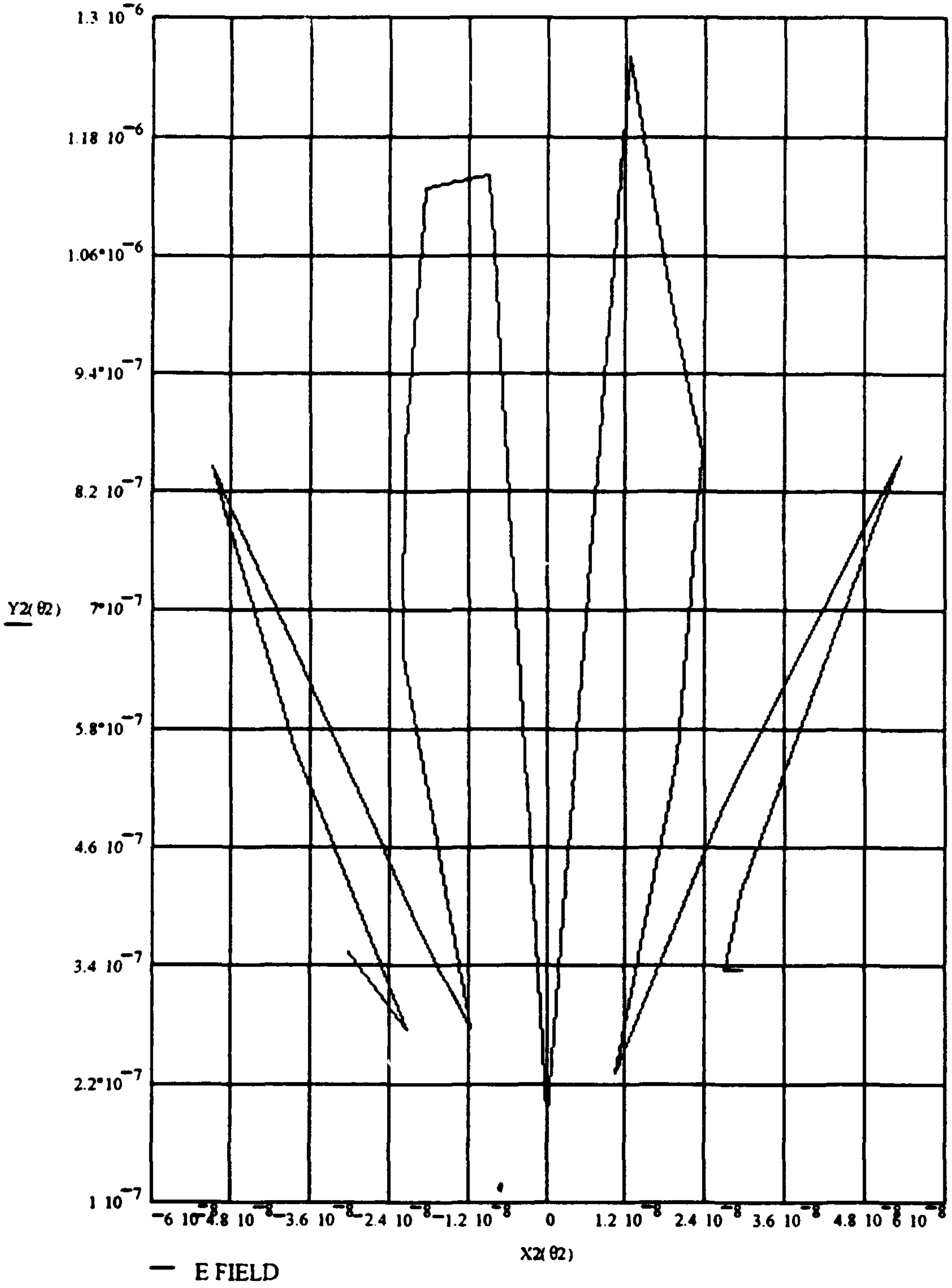
$i1 := 20$ $\theta 2 := -\left(\frac{\pi}{36}\right) - 10^{-3}, \frac{\pi}{i1 \cdot 18} - \frac{\pi}{36} - 10^{-3} .. \frac{\pi}{36} - 10^{-3}$

$X1(\theta 2) := \left|E\theta 2\left(\theta 2, \frac{\pi}{2}\right)\right| \cdot \cos\left(\theta 2 + \frac{\pi}{2}\right)$ $Y1(\theta 2) := \left|E\theta 2\left(\theta 2, \frac{\pi}{2}\right)\right| \cdot \sin\left(\theta 2 + \frac{\pi}{2}\right)$



H-PLANE RADIATION PATTERN:

$$X2(\theta 2) := \left|E\phi 2(\theta 2, 0)\right| \cos\left(\theta 2 + \frac{\pi}{2}\right) \qquad Y2(\theta 2) := \left|E\phi 2(\theta 2, 0)\right| \sin\left(\theta 2 + \frac{\pi}{2}\right)$$



REFERENCES

1. Dietrich, D. S., *Predicting Radiation Characteristics From Antenna Physical Dimensions*, Master's Thesis, Naval Postgraduate School, Monterey, CA, December 1992.

2. Gerry, D. D., *Mathcad Computer Applications Predicting Antenna Parameters From Antenna Physical Dimensions And Ground Characteristics*, Master's Thesis, Naval Postgraduate School, Monterey, CA, June 1993.

3. Balanis, C. A., *Antenna Theory Analysis and Design*, John Wiley & Sons, Inc., 1982.

4. Stutzman and Thiele, *Antenna Theory and Design*, John Wiley & Sons, Inc., 1981.

5. Elliot, R. S., *Beamwidth and Directivity of Large Scanning Arrays*, First of Two Parts, The Microwave Journal, Vol. VI, December 1963.

6. Kraus, John D., *Antennas*, McGraw-Hill Book Company, 1988.

7. Elliot, R. S., *Beamwidth and Directivity of Large Scanning Arrays*, Last of Two Parts, The Microwave Journal, Vol. VII, January 1964.

8. Lo and Lee, *Antenna Handbook Theory, Applications, and Design*, Van Nostrand Reinhold Company Inc., 1988.

9. Harrison and King, *Folded Dipoles and Loops*, IRE Transactions on Antennas and Propagation, Vol. AP-9, March 1961.

10. German and Brooks, *The Effects of the Physical Parameters on the Bandwidth of a Folded Dipole*, IRE Transactions on Antennas and Propagation, Vol. AP-6, April 1958.

11. Lewallen, R. W., *Reference Manual, ELNEC ver. 2.25*, 1992.

12. Smith, R. A., *Aerials for Metre and Decimetre Wave-Lengths*, Cambridge at the University Press, 1949.

13. Jasik, Henry, *Antenna Engineering Handbook*, McGraw-Hill Book Company, 1961.

14. Jones, E. M. T., *Paraboloid Reflector and Hyperboloid Lens Antennas*, IRE Transactions on Antennas and Prop-gation, Vol. AP-2, July 1954.

15. Rusch and Potter, *Analysis of Reflector Antennas*, Academic Press, 1970.

16. Mathcad 3.0, *Mathcad 3.0 User's Guide*, Mathsoft, Inc., 1991.

INITIAL DISTRIBUTION LIST

1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 52 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Chairman, Code EC Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5002	1
4.	Professor R. Clark Robertson, Code EC/Rc Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5002	2
5.	Professor Richard W. Adler, Code EC/Ab Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5002	1
6.	LT Steven E. Lundholm 1140 3rd St. N.W. New Brighton, MN 55112	1
7.	NAVMARINTCEN ATTN: Mr. Ron Ullom DI433 4301 Suitland Washington D.C. 20395-5020	4
8.	Professor M.A. Morgan, Code EC/Mw Department of Electrical and Computer Engineering Naval Postgraduate School Monterey, CA 93943-5002	1