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## SUBSPACE ESTIMATION WITHOUT EIGENVECTORS FOR ADAPTIVE BEAMFORMING

by

Mylène Toulgoat, Christoph Gierull and Ross M. Turner

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## SUBSPACE ESTIMATION WITHOUT EIGENVECTORS FOR ADAPTIVE BEAMFORMING

by

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## ABSTRACT

This report describes a new fast projection technique for adaptive beamforming. It is based on a technique proposed by Yeh where Gram-Schmidt orthogonalization is applied to  $N$  rows of the covariance matrix to obtain the signal subspace. In Yeh's technique,  $N$  is equal to  $L$ , the number of jammers. This technique, applied to bearing estimation, requires a priori knowledge of the number of sources. Even though it gives good results when the number of sources is much smaller than the number of array elements, the performance of Yeh's technique degrades severely as the number of jammers increases. We improve the technique by taking a larger number of rows in the orthogonalization: we add a thresholding procedure to the technique in order to optimize  $N$ . We compared this new technique, called Subspace Estimation without Eigenvectors (SEWE), with other fast projection techniques. SEWE was shown to give superior performance in terms of the Signal-to-Noise-plus-Jammer Ratio achievable with a given computational load, except when a very high SNJR is required.

## RÉSUMÉ

Ce rapport décrit une nouvelle technique de projection rapide pour la formation autoadaptative des faisceaux. Cette technique est basée sur une méthode proposée par Yeh où une orthogonalisation de Gram-Schmidt est appliquée à  $N$  colonnes de la matrice de covariance pour obtenir le sous-espace du signal. Dans la technique de Yeh, le nombre de colonnes,  $N$ , est égal au nombre de brouilleurs,  $L$ . Cette technique, utilisée pour l'estimation des directions des sources, requiert une connaissance a priori du nombre de sources. Même si elle donne de bons résultats lorsque le nombre de brouilleurs est très faible par rapport au nombre d'éléments, la performance de la technique de Yeh se dégrade rapidement lorsque le nombre de brouilleurs augmente. Nous améliorons la méthode en lui ajoutant un seuil ce qui permet d'optimiser la valeur de  $N$  en augmentant le nombre colonnes utilisées pour faire l'orthogonalisation. Cette nouvelle technique est comparée avec d'autres techniques de projection. On démontre qu'elle donne une meilleure performance en terme du Rapport Signal-sur-Bruit-plus-Brouillage (RSBB) qu'on peut atteindre avec une charge de calcul donnée, sauf dans le cas de RSBB très élevé.

## EXECUTIVE SUMMARY

Jamming of radar systems via the antenna sidelobes or the antenna mainbeam can seriously degrade radar performance. Thus jamming can provide a crucial advantage to enemy forces if radar electronic counter-counter-measures (ECCM) are not effective.

Adaptive antenna nulling provides an effective ECCM but at the cost of increased equipment complexity and capability. A major cost factor is the need for a very high speed, real-time, computational capability. The computational capability required is so high it limits the application of this effective ECCM.

This report describes and evaluates a computationally efficient algorithm that reduces the requirements for high speed computation by 10 times in some cases.

The new method, called Subspace Estimation Without Eigenvectors (SEWE), uses the concept of signal and noise subspace. Here the noise subspace is the jamming space. The algorithm calculates a projection matrix which represents a subspace orthogonal to the jammer subspace. The projection matrix is applied to a weight vector that steers the array in a particular direction. When the resulting adapted weight vector is applied to superimposed signal and jamming, the jamming signals are greatly reduced while the loss of signal is small. This occurs because the adapted weight vector is approximately orthogonal to the jamming signals.

We compared the SEWE technique with other fast projection techniques. SEWE was shown to give superior performance in terms of the achievable Signal-to-Noise-plus-Jammer Ratio (SNJR) for a given computational load, except for the case where very high SNJR is required.

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# SUBSPACE ESTIMATION WITHOUT EIGENVECTORS FOR ADAPTIVE BEAMFORMING

## 1.0 Introduction

Array signal processing techniques based on eigenvector projection are highly effective for both interference suppression and for estimation of signal angle-of-arrival. Projection methods employ the concept of signal and noise subspaces. In the case of jamming signals, the noise space represents the jamming space. Jamming is cancelled by projecting the array data vectors into a space orthogonal to the noise space, ie the signal space. In the case of angle-of-arrival estimation, the array data vectors are projected into a space orthogonal to the signal space. These techniques estimate the noise and signal subspace with an error that decreases when an increasing number of sample vectors is used in the calculation of the covariance matrix. Averaging is exploited by incorporating many data vectors into an estimated covariance matrix which approaches the true covariance matrix asymptotically as the observation period increases.

Bühning [1,2] proposed an eigenvector-based projection technique for adaptive beamforming; the eigenvectors corresponding to the strongest eigenvalues of the covariance matrix are used to calculate a projection matrix orthogonal to the jammer subspace. This projection matrix is applied to the array steering vector which is projected into a subspace orthogonal to the interference. The resulting adapted steering weight vector is, ideally, orthogonal to the jamming signals. Variations of the eigenvector-based adaptive nulling algorithms have been studied by Gabriel [3]. He showed that the sidelobes of low sidelobe antennas were less perturbed when eigenvector-based projection techniques were used as an alternative to the Sample-Matrix-Inversion (SMI) technique of Reed et al [4]. Because the signal or interference subspace is estimated with a finite sample size, the cancellation of the jamming is imperfect.

Projection techniques based on eigenanalysis are computationally demanding particularly for arrays with a large number of elements. This limits the applicability of these techniques where high-speed real-time processing is required. Fast projection techniques based on Data Vector Orthogonalization (DVO) [5,6,7,8] avoid calculation-intensive eigenanalysis, but at a cost in performance. The loss in performance comes from the inability of these techniques to incorporate averaging to better estimate the signal or interference subspace. In contrast, eigen-based methods become progressively more accurate as the number of data vectors incorporated in the covariance matrix increases. In the most advanced version of DVO, Nickel [8] introduces a double-threshold procedure that improves performance; an internal threshold rejects data vectors that do not add sufficient information about the signal or interference subspace, while an external threshold keeps the procedure going until a performance criterion is satisfied. Toulgoat and Turner [9,10,11] have developed a technique called Data Vector Selection and Orthogonalization (DVSO) which provides a better estimate of the interference subspace than does DVO, albeit at the price of a higher computational load. The required load is, however, still much less than that of the eigenvector techniques. In addition, Toulgoat and Turner [9,10,11] have used DVSO with averaging procedures to develop two algorithms that provide a better trade off in performance



versus computational load. The first technique, called Data Vector Selection, Orthogonalization and Weight AVERaging (DVSO-WAVER), averages repeated computations of the adapted weight vectors to give a final adapted weight vector for nulling the jamming. The second, called DVSO-COVAR, uses a data selection process for the choice of data vectors used in the formation of the covariance matrix. Gram-Schmidt (GS) orthogonalization is then applied to the columns of the covariance matrix to obtain a good estimate of the signal subspace. These two methods provide essentially the same Signal-to-Noise-plus-Jammer Ratio (SNJR) as the SMI method but require far fewer computations.

The data-vector selection technique used in DVSO-COVAR is similar to a technique that Reilly and Law [12] used for direction finding. However, [12] applies the selection process to the permutation and selection of columns of a covariance matrix. In contrast, in DVSO-COVAR, the data selection is applied prior to the calculation of the covariance matrix, in order to choose the data vectors to be used in the formation of the covariance matrix. A procedure equivalent to that of [12] is then applied for selecting a subset of the columns vectors of the covariance matrix.

Yeh [13,14] proposed another technique based on Gram-Schmidt orthogonalization. This technique was first applied to bearing estimation; the projection matrix is computed by using any  $L$  rows of the covariance matrix where  $L$  is the number of jammers. It is computationally more efficient than projection techniques using eigenvectors especially for the case where the number of jammers is much smaller than the number of array elements. In this report, the Yeh technique is applied to adaptive beamforming and shown to give good results when the number of jammers is much smaller than the number of array elements. We propose an improved version of the Yeh technique incorporating a threshold procedure, called Subspace Estimation Without Eigenvectors (SEWE). This new algorithm is compared with eigenvector decomposition, DVSO and DVSO-COVAR techniques.

## 2.0 Signal and jammer characteristics

Radar systems usually transmit pulsed signals that are narrowband with respect to the carrier frequency. Since the targets of interest are in the far field, the received signals are plane waves. When no reflection is received from a target, the data vector,  $\mathbf{x}(t_n)$ , is defined by:

$$\mathbf{x}(t_n) = \sum_{i=1}^L j_i(t_n) \mathbf{a}(\theta_i) + \mathbf{n}(t_n) = \mathbf{A} \mathbf{j}_n + \mathbf{n}_n$$

where  $L$  is the number of jammers,  $\mathbf{j}_n = [j_1(t_n), \dots, j_K(t_n)]$  is the jammer vector with  $j_i(t_n)$  being complex gaussian amplitude of the  $i^{\text{th}}$  jammer at time  $t_n$ ,  $\mathbf{n}_n$  is the receiver noise vector with mutually independent components. Its power is  $E\{\mathbf{n}_n^2\} = K\sigma^2$  where  $K$  is the number of array elements. The sampling rate is selected such that the quantities  $j_i(t_n)$  and  $j_i(t_{n+1})$  are statistically independent. The matrix  $\mathbf{A}$  represents the directions of the jammers with  $\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]$ . The quantity  $\mathbf{a}(\theta_i)$  is a deterministic vector representing the direction of arrival of the  $i^{\text{th}}$  jammer. Without any loss of generality, we assume a uniform linear array with equally spaced elements.

In that case,  $a(\theta_i)$  is defined as

$$a(\theta_i)^T = [1, \exp(j \frac{2\pi d}{\lambda} \sin(\theta_i)), \dots, \exp(j \frac{2\pi d(K-1)}{\lambda} \sin(\theta_i))] \quad (1)$$

where  $d$  is the inter-element spacing,  $\theta_i$  is the direction of arrival of the  $i^{\text{th}}$  jammer,  $\lambda$  is the jammer wavelength.

When projection techniques are applied to radar systems for adaptive nulling of jammers, it is important that sample vectors be taken in the absence of signal and clutter. This is a standard radar problem; one solution is to sample at time intervals corresponding to ranges where it is known that there are no targets or clutter returns.

### 3.0 Performance criterion

The ultimate measure of performance of an adaptive array is the Signal-to-Noise-plus-Jammer Ratio (SNJR). We attempt to maximize this quantity by using an adapted weight vector,  $w_s$ , that discriminates against jamming while causing a minimal loss of useful signal. The adapted weight vector,  $w_s$ , is calculated from a steering vector,  $w_r$ , which specifies the direction for which the SNJR is to be maximized:

$$w_s = \alpha T w_r$$

where  $T$  is a transformation matrix and  $\alpha$  an arbitrary constant. The optimum solution for maximisation of the SNJR is given by  $T = \Phi^{-1}$  where  $\Phi$  is the true covariance matrix. For an infinite Jammer-to-Noise Ratio (JNR) in each channel the transformation can be written as

$$T = I - A(A^H A)^{-1} A^H$$

where  $A$  is the matrix of the directions of the jammers given by (1). The transformation matrix  $T$ , projects the steering vector into a subspace orthogonal to the subspace which is spanned by the directional vectors of the jammers (jammer subspace). Instead of using the direction vectors of the jammers to build up the projection matrix,  $T$ , one can use any  $L$  basis vectors which span this subspace. The projection matrix,  $T$ , is written as:

$$T = I - VV^H \quad (2)$$

where  $V = [v_1, \dots, v_L]$  is the matrix of the  $L$  basis vectors. When sensor noise is present (especially for low JNR), we may need more than  $L$  basis vectors to better define the jammer subspace.

With this adapted weight vector the SNJR is given by:

$$SNJR(\theta) = \frac{E\{w_a^H a(\theta)P\}}{E\{w_a^H \Phi w_a\}} \frac{\sigma^2}{K} = \frac{a(\theta)^H T a(\theta)P}{a(\theta)^H T \Phi T a(\theta)} \frac{\sigma^2}{K}$$

where  $a(\theta)$  is the steering vector with  $\theta$  being the steering direction. The quantity  $\sigma^2/K$  is the normalization factor.

We calculate a two-dimensional SNJR to evaluate the performance in all directions. It is defined as follows:

$$SNJR_{2D} = \sum_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} SNJR(\theta)$$

An area of one beamwidth around each jammer is excluded since the SNJR degrades severely at those locations. The beamwidth, BW, is defined as follows:

$$BW = 2 \frac{\lambda}{K}$$

where  $L$  is the length of aperture;  $L = K \lambda/2$ .

### 3.1 Test case

The array has 64 elements with half-wavelength spacing between each element. Performance is evaluated for three different scenarios: scenario #1 with  $L=3$  jammers located at  $\theta_1=-33.4^\circ$ ,  $\theta_2=-12.7^\circ$  and  $\theta_3=-8.6^\circ$ ; scenario #2 with  $L=5$  jammers located at  $\theta_1=-41.3^\circ$ ,  $\theta_2=-24.2^\circ$ ,  $\theta_3=-9.2^\circ$ ,  $\theta_4=-6.3^\circ$  and  $\theta_5=-3.4^\circ$ ; scenario #3 with  $L=10$  jammers located at  $\theta_1=-45.2^\circ$ ,  $\theta_2=-41.3^\circ$ ,  $\theta_3=-37.6^\circ$ ,  $\theta_4=-24.2^\circ$ ,  $\theta_5=-18.1^\circ$ ,  $\theta_6=-15.1^\circ$ ,  $\theta_7=-12.1^\circ$ ,  $\theta_8=-9.2^\circ$ ,  $\theta_9=-6.3^\circ$  and  $\theta_{10}=-3.4^\circ$ . We use two different values for the JNR: 26.9 dB and 40 dB. The noise power,  $\sigma^2$ , is set at 1.

## 4.0 Yeh-Projection technique

### 4.1 Description of the technique

The Yeh-projection technique [13,14] calculates the projection matrix using any  $L$  columns of the covariance matrix, where  $L$  is the number of jammers. The sample covariance matrix is defined as follows:

$$R = \frac{1}{M} \sum_{i=1}^M x_i x_i^H = [x_1, \dots, x_K] \quad (3)$$

where  $M$  is the number of samples used in the formation of the covariance matrix and  $r_j$  is the  $j^{\text{th}}$  column of the covariance matrix. The jammer subspace,  $T$ , is calculated by an orthonormalization of the  $L$  columns of the covariance matrix,  $R$ , that have been estimated. The projection matrix is then calculated with equation (2).

Yeh [13,14] used this projection matrix to evaluate a spectral estimator and then determine the positions of the sources. It gives poor results when the Signal-to-Noise Ratio is low [13]. Moreover, the direction finding application requires the technique to be able to discriminate between two sources which are very close together. In contrast, adaptive beamforming has only to cancel jamming signals which are very strong with respect to the receiver noise power; furthermore, there is no need for two different nulls if the signals are very close together. For those reasons, the projection technique of Yeh is well suited for jammer nulling. The projection matrix is used to calculate an adapted weight vector,  $w_a$ , from a steering weight vector,  $w_s$ ,

$$w_a = Tw_s$$

where the steering weight vector,  $w_s$ , is given by

$$w_s = [1, \exp(2j\frac{\pi}{\lambda} d \sin(\theta_s)), \dots, \exp(2j\frac{\pi}{\lambda} (K-1) d \sin(\theta_s))]$$

Besides its good performance in terms of SNJR, Yeh's technique has a small computational load compared with other adaptive beamforming algorithms such as Bühring technique, SMI or the DVSO-COVAR technique. The computational load for Yeh's technique is given in terms of the number of complex multiplications (c.m.) required to find the adapted weight vector:

$$MKL + L^2K$$

where  $M$  is the number of samples used in the formation of the covariance matrix,  $K$  is the number of array elements and  $L$  is the number of jammers. Note that only  $L$  columns of the covariance matrix have to be calculated, which reduces the number of complex multiplications to  $MKL$  (c.m.) compared with  $M(K+1)K/2$  (c.m.) for the calculation of the lower triangle of the covariance matrix. If  $L \ll K$ , this greatly reduces the computational load incurred in the calculation of the covariance matrix.

The next section presents simulation results for the Yeh-projection technique as applied to adaptive beamforming.

## 4.2 Simulation results-Yeh's technique

In this section, Yeh's technique is compared with conventional Fourier beamforming with Taylor antenna tapering to control the sidelobes. The comparison is carried out by scanning the array over the entire range of angles,  $\theta$ , i.e. for  $-1 \leq \sin(\theta) \leq 1$ . As the scan angle passes the jamming signal there is a very large decrease in SNJR; however the region of decreased SNJR is, in general, very much narrower for the Yeh technique.

Figures 1, 2, 3 show the normalized SNJR versus  $\sin(\theta)$  for scenarios 1 ( $L=3$ ), 2 ( $L=5$ ) and 3 ( $L=10$ ) for both the Yeh technique and Fourier beamforming with Taylor weights. For all of the scenarios, the JNR is 26.9 dB. For the small ratio  $L/K$  in scenario 1, the SNJR curve for the Yeh technique approaches the optimum value of 0 dB except for areas around the jammer. The SNJR for Fourier beamforming with Taylor weighting is severely degraded everywhere. As the number of jammers increases or as they get closer together, the performance of the Yeh-projection degrades progressively as seen in Figure 3. Here the number of jammers has increased to ten with a cluster of six closely spaced. In Figure 3, the SNJR for the Yeh technique is 8 dB below SNJR maximum.

The performance of the Yeh technique can be improved by using a larger number of columns in the orthonormalization. To evaluate the number of columns required, the normalized  $\text{SNJR}_{2D}$  is plotted versus the number of columns,  $N$ , used in the orthonormalization. Figure 4 gives  $\text{SNJR}_{2D}$  versus  $N$  for scenarios 1 ( $L=3$ ), 2 ( $L=5$ ) and 3 ( $L=10$ ) respectively with  $\text{JNR}=26.9$  dB. In the first scenario where  $L=3$ , the optimum value for  $N$ ,  $N_{\text{opt}}$ , equals the number of jammers,  $L$ . For  $L=5$ , the optimum value for  $N$  stands at  $N_{\text{opt}}=8$ , while the value  $N_{\text{opt}}=19$  gives the maximum  $\text{SNJR}_{2D}$  when  $L=10$ . We observe, in general, that  $\text{SNJR}_{2D}$  is very poor for  $N < L$ , and that the  $\text{SNJR}_{2D}$  decreases slowly as a function of  $N$  for  $N > N_{\text{opt}}$ . Figure 5 gives the same results for  $\text{JNR}=40$  dB. In this case,  $N_{\text{opt}}$  tends to be slightly lower as expected.

Figures 6, 7 and 8 display SNJR versus  $\sin(\theta)$  curves for  $N=L$  and  $N=N_{\text{opt}}$  for the same scenarios as above with the  $\text{JNR}=26.9$  dB. As expected, using  $N=N_{\text{opt}}$  instead of  $N=L$  greatly improves the performance of the Yeh technique for scenarios 2 and 3 while leaving the performance unchanged for scenario 1. Yeh's technique can then be modified by using  $N_{\text{opt}}$  columns instead of  $L$  for the Gram-Schmidt orthogonalization. Since the quantity  $N_{\text{opt}}/L$  increases with the number of jammers, it is not possible to find a single value for  $N_{\text{opt}}$  convenient for every scenario. In order to determine  $N_{\text{opt}}$  for each scenario, we have devised a thresholding procedure for Yeh's technique. This procedure allows the number of sample vectors,  $N$ , to increase beyond  $L$  to approximately,  $N_{\text{opt}}$ , eliminating the requirement for a priori knowledge of the number of jammers and their disposition and power. In the next section we present a new version of Yeh's technique, called Subspace Estimation Without Eigenvectors (SEWE).

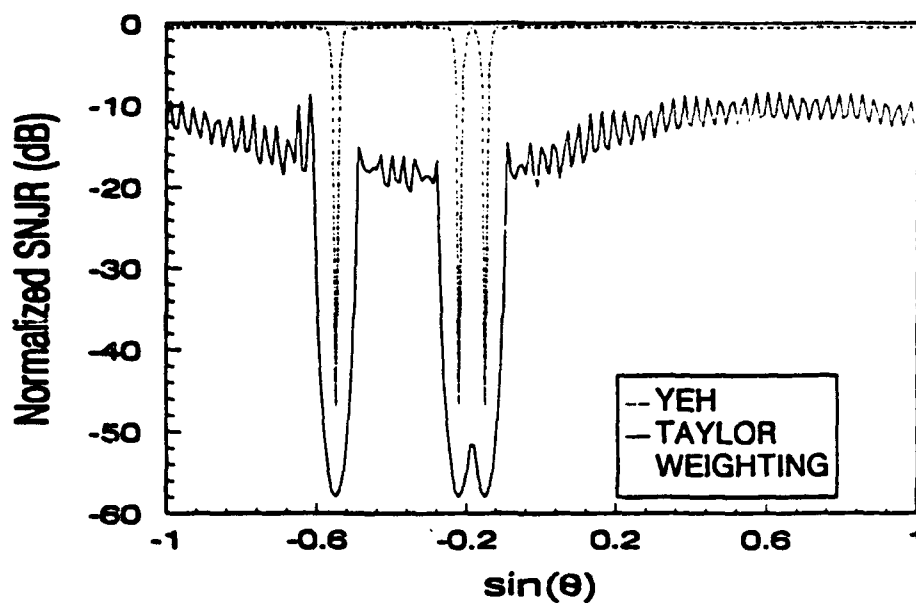


Figure 1 Normalized SNJR versus  $\sin(\theta)$  for  $L=3$ ,  $K=64$  and  $JNR=26.9$  dB.

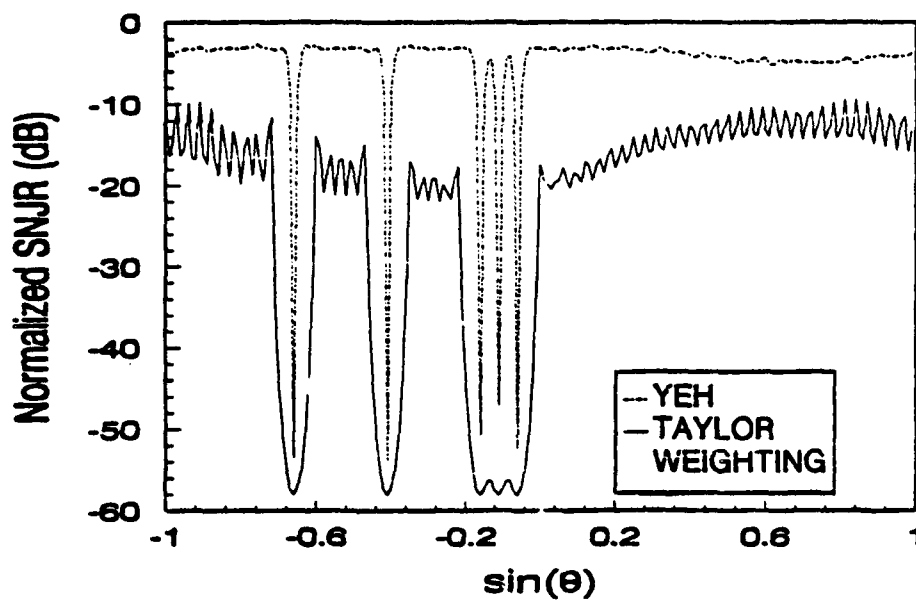


Fig. 2 Normalized SNJR versus  $\sin(\theta)$  for  $L=5$ ,  $K=64$  and  $JNR=26.9$  dB.

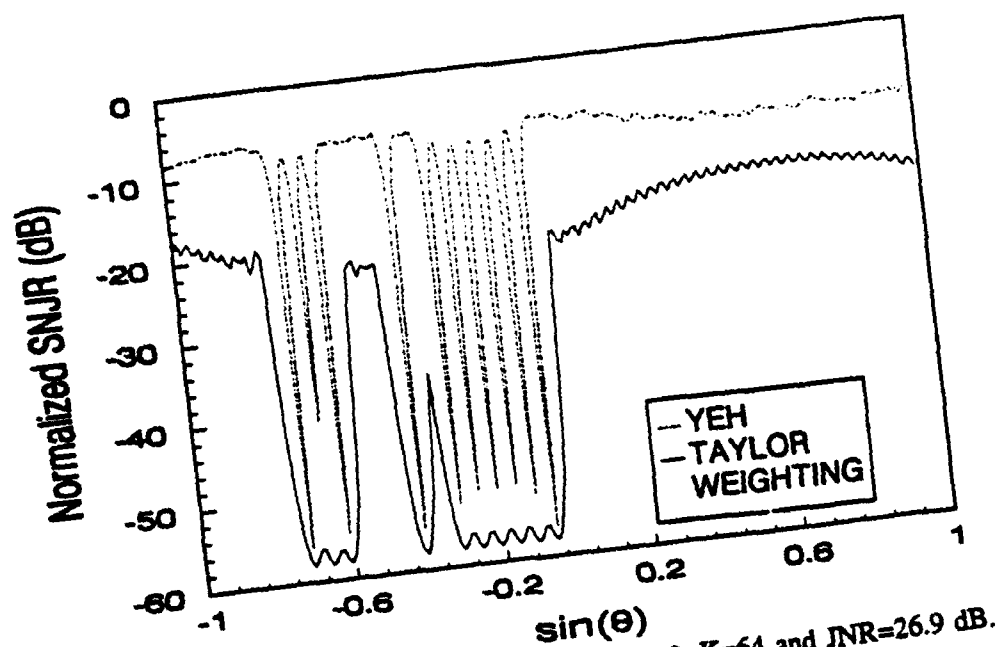


Figure 3 Normalized SNJR versus  $\sin(\theta)$  for  $L=10$ ,  $K=64$  and  $JNR=26.9$  dB.

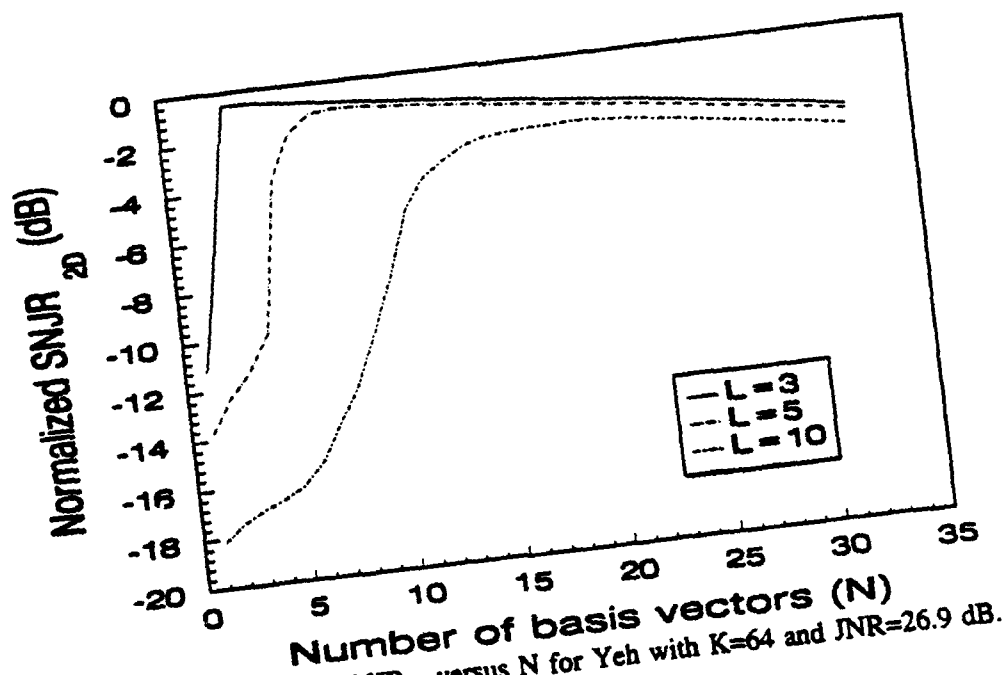


Figure 4 Normalized SNJR<sub>20</sub> versus N for Yeh with  $K=64$  and  $JNR=26.9$  dB.

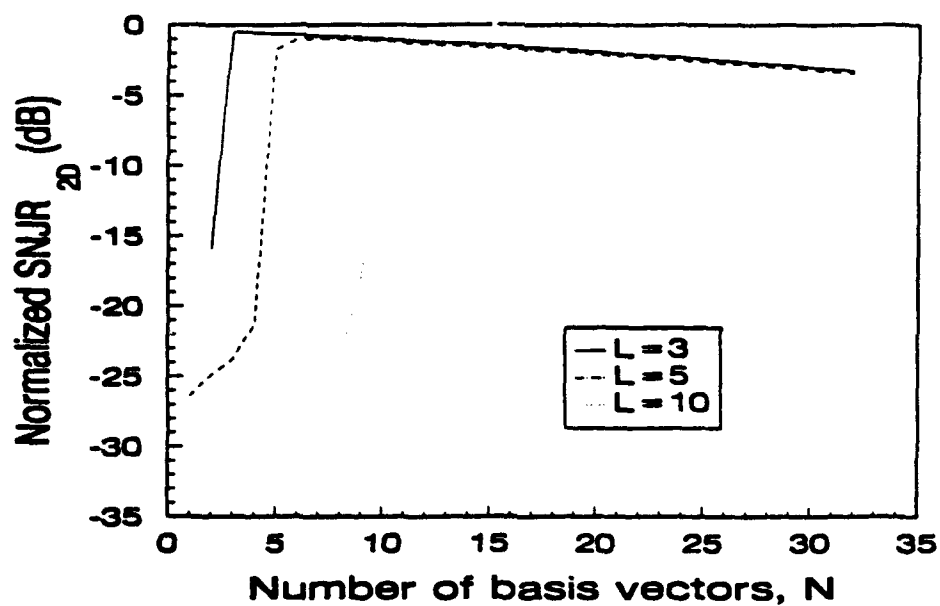


Figure 5 Normalized SNJR<sub>2D</sub> versus N for Yeh with K=64 and JNR=40 dB.

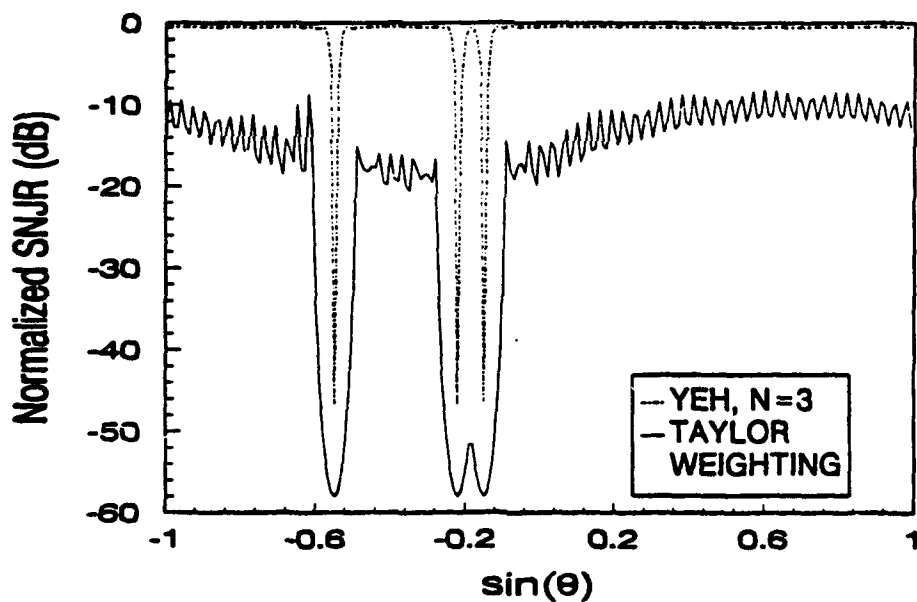


Figure 6 Normalized SNJR versus  $\sin(\theta)$  for L=3, K=64 and JNR=26.9 dB.



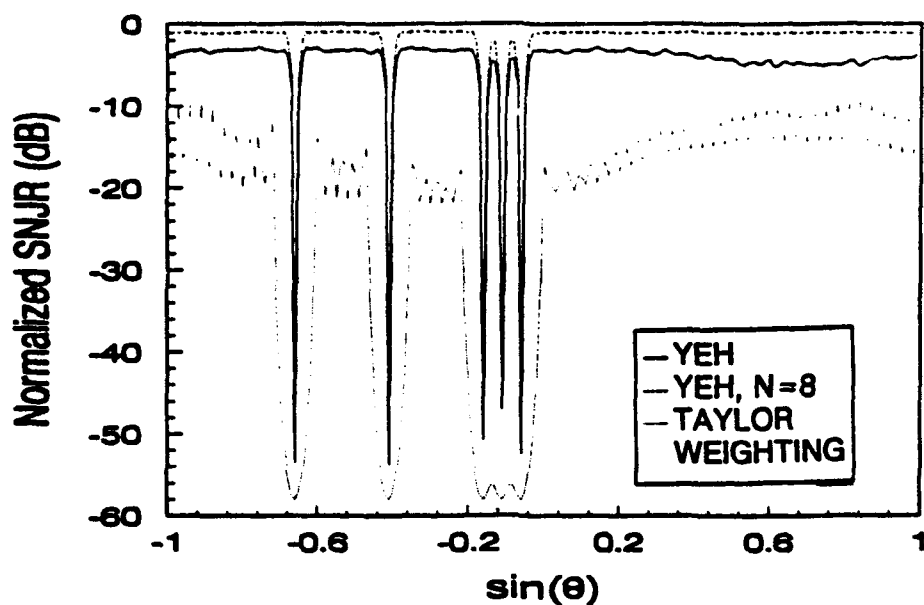


Figure 7 Normalized SNJR versus  $\sin(\theta)$  for  $L=5$ ,  $K=64$  and  $JNR=26.9$  dB

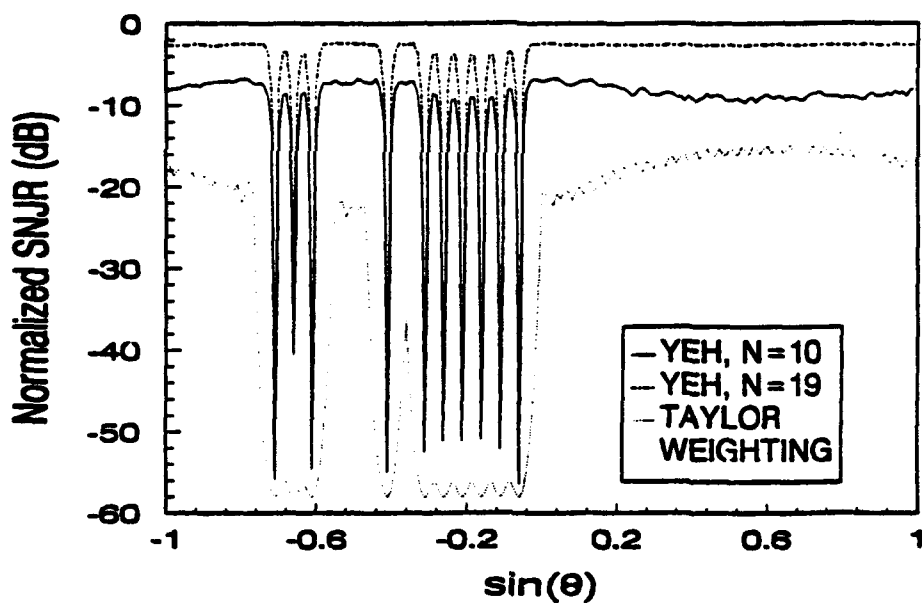


Figure 8 Normalized SNJR versus  $\sin(\theta)$  for  $L=10$ ,  $K=64$  and  $JNR=26.9$  dB.

### 5.0 Subspace Estimation Without Eigenvectors (SEWE)

The SEWE technique trades off a small increase in the computational load for an improved SNJR. It is shown that the addition of a thresholding procedure to the orthonormalization procedure considerably improves the performance of Yeh's technique at the cost of a moderate increase in computational load. The thresholding procedure has two advantages: firstly, a priori knowledge of the number of jammers is not required; secondly, the procedure leads to selection of values for  $N$  close to  $N_{opt}$ .

We investigate two different statistics. The first statistic is defined as the power of the residual basis vector after orthogonalization,  $T_1$  [5,6]:

$$T_{1,m} = |u_m|^2 \quad (4)$$

where  $u_m$  is the residual basis vector after orthogonalization at the  $m^{\text{th}}$  step. This is compared to a threshold,  $\Delta_1$ , which is proportional to the noise power. The procedure terminates when the power of the residual basis vector is approximately equal to the noise power. The second statistic  $T_{2,m}$  is based on the volume of the generalized parallelepiped defined by the vertices of the vectors,  $[r_1, \dots, r_m]$  where  $r_i$  is the  $i^{\text{th}}$  column vector of the covariance matrix [8]. Noting that  $u_i$  is the  $i^{\text{th}}$  orthogonal basis vector generated from the column vectors,  $r_1, r_2$  up to  $r_i$ , the volume of the parallelepiped is given by:

$$\prod_{i=1}^m |u_i|$$

It is also noted that  $|r_i|^2 \geq |u_i|^2$ . A test statistic based on the square of the volume is given by

$$T_{2,m} = \prod_{i=1}^m \frac{|u_i|^2}{|r_i|^2} \quad (5)$$

We observe that  $T_{2,m} \leq 1$ , with  $T_{2,m}=1$  when the  $r_i$ 's and  $u_i$ 's are colinear. Secondly, because the  $u_i$ 's are generated from the  $r_i$ 's by the Gram-Schmidt process,  $T_{2,m}$  is independent of any overall scale factors or gain factors i.e. multiplying all of the data vectors by a constant will not change  $T_{2,m}$ . In general as  $m$  increases, a point will be reached where the jamming space is well represented by the  $N=m$  vectors,  $u_1$  to  $u_N$ .  $T_{2,m}$  decreases rapidly as  $m$  increases beyond  $N$ . The optimum value for  $N$ ,  $N_{opt}$ , is selected when the quantity,  $T_2^{1/2m}$ , which is the length of the edge of a cube of volume equal to  $(T_2)^{1/2}$  falls below a threshold,  $\Delta_2$ . Since the statistic  $T_2$  is always smaller than 1,  $\Delta_2$  is chosen accordingly.

The details of the new algorithm follow. As in Yeh's technique, the covariance matrix is calculated one column at a time:

$$r_{lm} = \sum_{i=1}^N x_{li} x_{im}^H \quad (6)$$

where  $r_{lm}$  is the  $l^{\text{th}}$  element of the  $m^{\text{th}}$  column of the covariance matrix.

The algorithm is presented here with a statistic  $T_m$  and a threshold  $\Delta$ , which are equal to either  $T_{1,m}$  and  $\Delta_1$ , or  $T_{2,m}$  and  $\Delta_2$  depending on the statistics investigated.

The technique is implemented as follows:

1) Set  $m=1$  and calculate  $r_{lm}$  for  $l=1$  to  $K$ .

2)  $u_m = r_m = [r_{1m} \ r_{2m} \ \dots \ r_{Km}]^T$

3) Calculate a statistic  $T_m$  according to equation (4) or (5)

4)

If  $T_m > \Delta$

$$v_m = u_m / |u_m|$$

else

$v_m = 0$ ; the procedure terminates indicating no jammers are present.

5) Set  $m=m+1$

6) Calculate  $r_{lm}$  for  $l=1$  to  $K$  according to (6).

7)

$$u_m = r_m - \sum_{i=1}^{m-1} (v_i^H r_m) v_i$$

8) Calculate the statistics  $T_m$  according to (4) or (5).

9)

If  $T_m > \Delta$

$$v_m = u_m / |u_m|$$

else

$v_m = 0$ ; the procedure terminates with  $N$  basis vectors where  $N=m-1$ .

Calculate  $w_i$  as,

$$w_s = w_s - \sum_{i=1}^N (v_i^H w_s) v_i$$

The choice of a value for  $\Delta$  depends on the choice of the statistics. If  $T_m = T_{1,m}$ ,  $\Delta$  is chosen proportional to noise power. If  $T_m = T_{2,m}$ ,  $\Delta_2$  is related to a length. The addition of a threshold in the procedure increases the computational load. The new computational load is given by:

$$C = N[ (M - .75)K + N(K - \frac{M}{2}) + \frac{M}{2} + (K + .5) ]$$

where  $N$  depends on the threshold value. As the threshold  $\Delta$  decreases, the computational load increases.

### 5.1 Simulation results - SEWE technique

We first determine through simulation the optimum values for  $\Delta_1$  and  $\Delta_2$ . We also analyze the effect of the number of jammers and of the Jammer-to-Noise Ratio, JNR, on the selection of the threshold. Figure 9 gives normalized  $SNJR_{2D}$  versus  $\Delta_1$  for scenarios 1, 2 and 3 with  $JNR=26.9$  dB. All three scenarios have a maximum  $SNJR_{2D}$  for a value of  $\Delta_{1,opt}$  between .65 mW and 3.2 mW. As  $\Delta_1$  gets larger than  $\Delta_{1,opt}$ , the  $SNJR_{2D}$  decreases very quickly for scenarios 2 and 3. This region corresponds to a small number of basis vectors (Same area as  $N < L$  in section 4.2). On the other hand, as  $\Delta_1$  gets smaller than  $\Delta_{1,opt}$ , the  $SNJR_{2D}$  gently rolls off (Case where  $N > N_{opt}$  in section 4.2). Setting the threshold's value to 1.3 mW gives performance near optimum  $SNJR_{2D}$  ( $\pm 0.1$  dB).

$SNJR_{2D}$  versus  $\Delta_1$  is shown in Figure 10 for the same scenarios as above with  $JNR=40$  dB. In this case, a value of 1.3 mW for  $\Delta_1$  gives the near maximum value ( $\pm 0.15$  dB) for the three scenarios. The value of the threshold is only slightly affected by the number of jammers and the JNR. However, the threshold depends on the number of array elements,  $K$ , and is also proportional to the squared power of the noise power,  $p_n$ .

We then investigate the second statistic  $T_2$  and the threshold,  $\Delta_2$ . Figures 11 and 12 give  $SNJR_{2D}$  versus  $\Delta_2$  for scenarios 1, 2 and 3 with  $JNR=26.9$  and 40 dB respectively. We observe basically the same behavior as for  $\Delta_1$ . A value of 0.005 for  $\Delta_2$  gives near maximum  $SNJR_{2D}$  ( $\pm 0.15$  dB) for SEWE technique for the three scenarios with the two different JNR's. This threshold is independent of the noise power, a very practical advantage.

Figures 13, 14 and 15 give  $SNJR_{2D}$  versus  $\sin(\theta)$  for scenarios 1, 2 and 3 respectively when using  $T_2$ . Results for conventional beamforming and the Yeh-projection technique are also shown on the same graph. It is clear from these results that adding a thresholding procedure to Yeh's technique improves the performance. The next section compares the SEWE technique with DVSO-COVAR and an eigenvector-based projection technique.

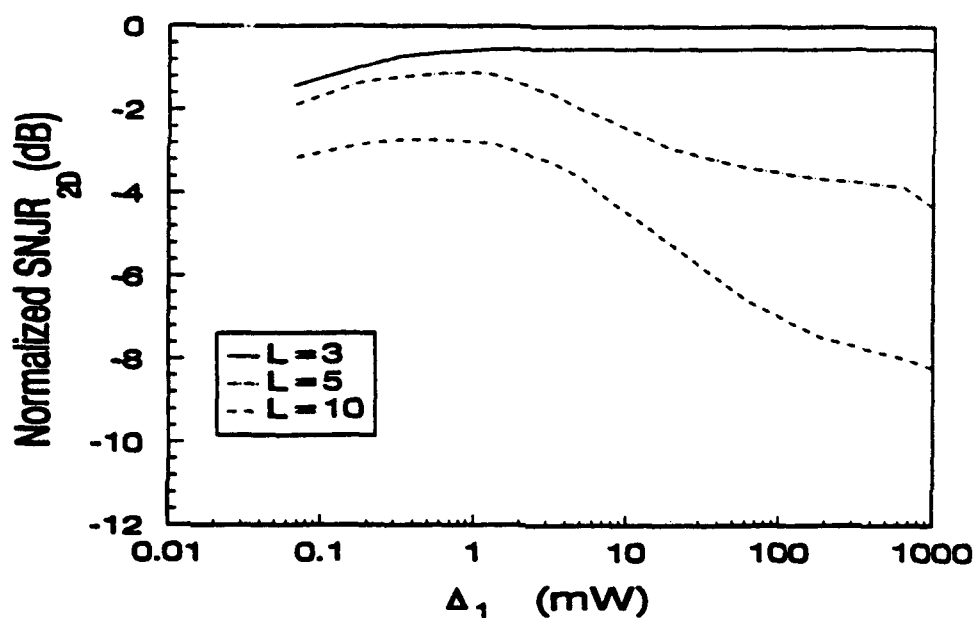


Figure 9 Normalized  $\text{SNJR}_{2D}$  versus  $\Delta_1$  for SEWE with  $K=64$  and  $\text{JNR}=26.9$  dB

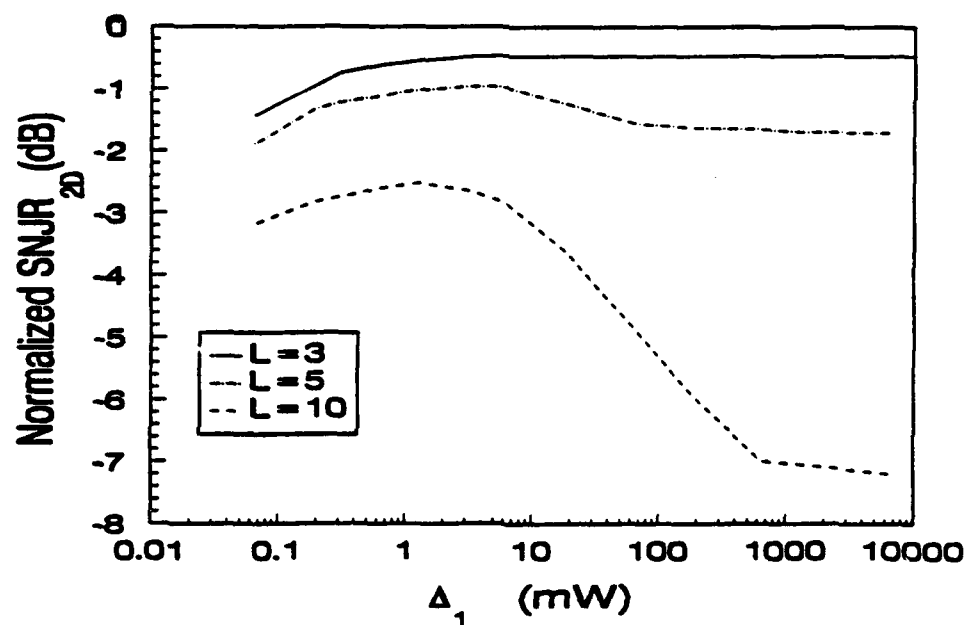


Figure 10 Normalized  $\text{SNJR}_{2D}$  versus  $\Delta_1$  for SEWE with  $K=64$  and  $\text{JNR}=40$  dB.

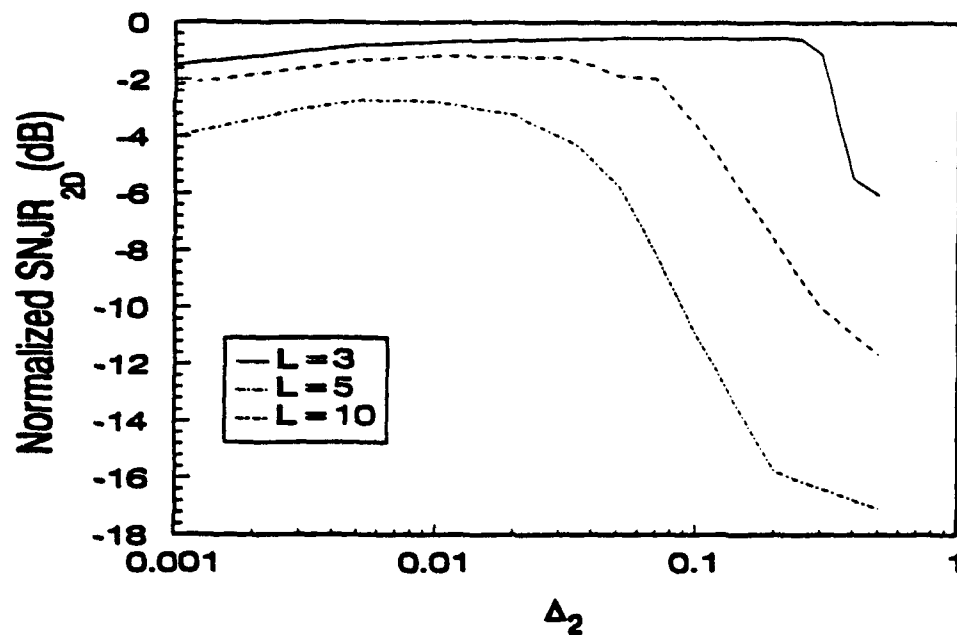


Figure 11 Normalized SNJR<sub>20</sub> versus  $\Delta_2$  for SEWE with K=64 and JNR=26.9 dB.

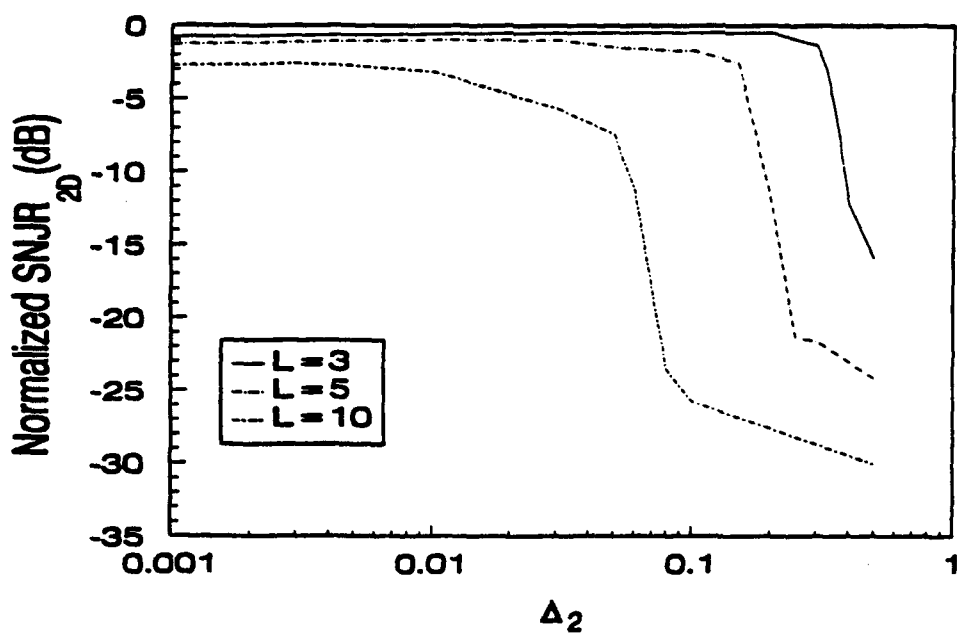


Figure 12 Normalized SNJR<sub>20</sub> versus  $\Delta_2$  for SEWE with K=64 and JNR=40 dB.

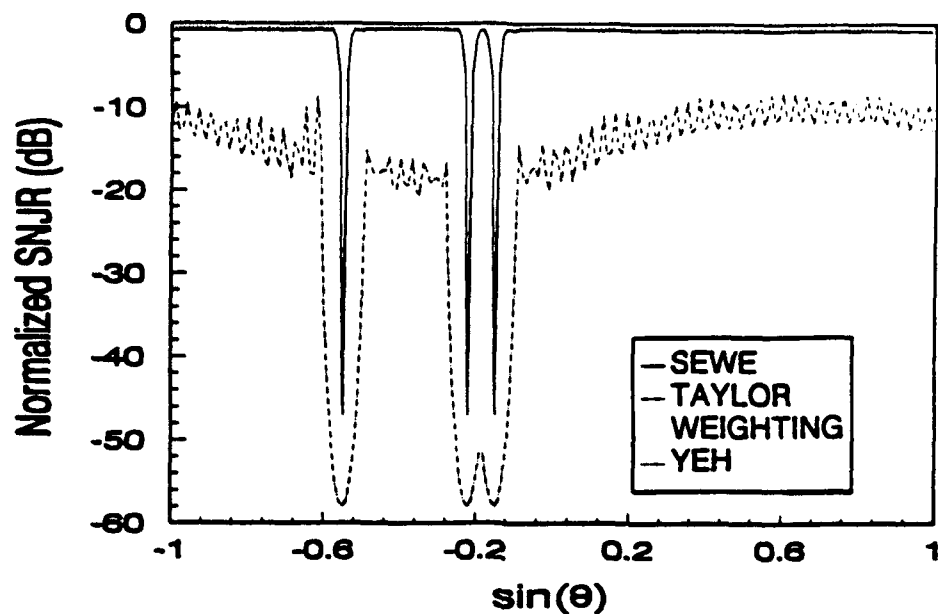


Figure 13 Normalized SNJR versus  $\sin(\theta)$  for SEWE with  $K=64$  and  $JNR=40$  dB.

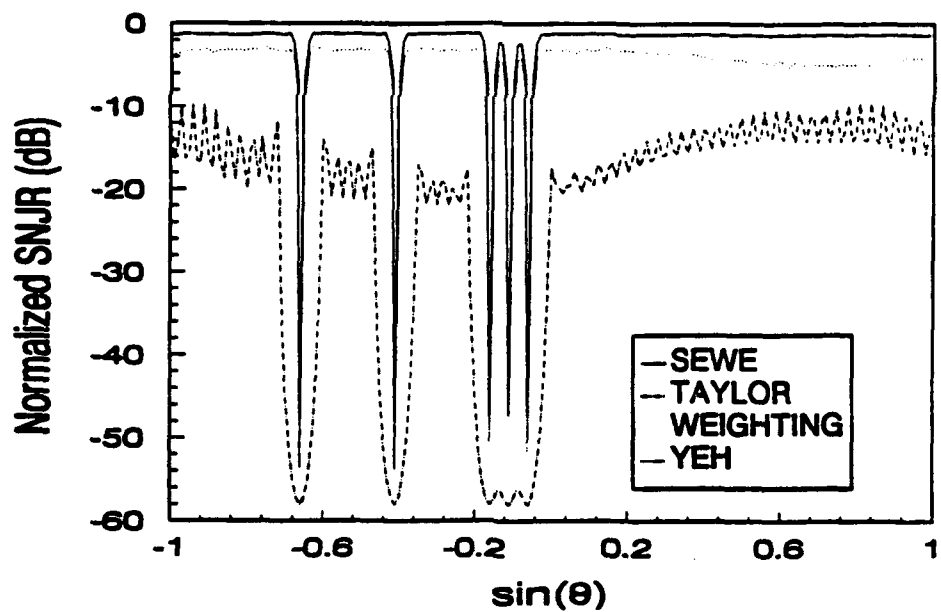


Figure 14 Normalized SNJR versus  $\sin(\theta)$  for SEWE with  $L=5$ ,  $K=64$  and  $JNR=26.9$  dB

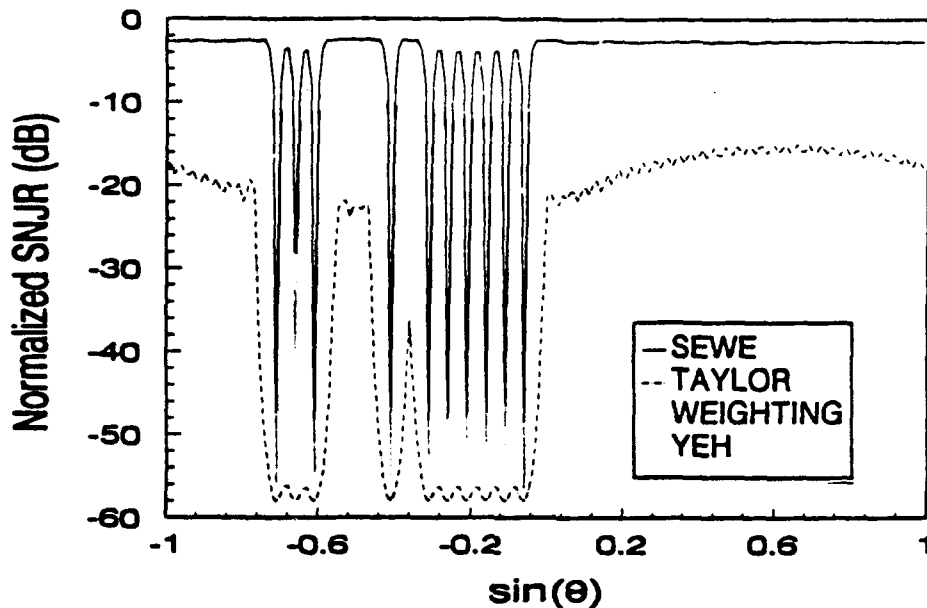


Figure 15 Normalized SNJR versus  $\sin(\theta)$  for SEWE with  $L=10$ ,  $K=64$  and  $JNR=26.9$  dB

## 6.0 Comparison with other projection techniques

The SEWE technique is compared with three projection techniques: DVSO, DVSO-COVAR and an eigen-decomposition based projection method. A brief description is given of the projection techniques. This is followed by simulation results illustrating the comparative performance of the various techniques.

### 6.1.1 DVSO technique

The basis of the Data Vector Selection and Orthogonalization (DVSO) [9, 10, 11] technique is the selection of the best  $M$  vectors from a larger group of  $N$  vectors which are sampled when the transmitting signal is absent and stored before the process begins. The  $M$  vectors are selected on the basis that they yield the highest SNJR as compared to any other set of  $M$  vectors selected from the set of  $N$ . The selection process, the Gram-Schmidt orthogonalization and the computation of the adapted weight,  $w_s$ , are combined in a single procedure. The computational load is given by [16]:

$$C_{DVSO} = K(0.5 + 0.5N + 2M + MN + M^2) + 0.5MN$$



### 6.1.2 DVSO-COVAR technique

DVSO-COVAR [11] employs the DVSO method in calculating the covariance matrix and processing its columns to obtain an adapted weight vector. The major feature of DVSO-COVAR is its selection process: the best data vectors are chosen to form the covariance matrix. We are thus able to obtain a good estimate of the covariance matrix using fewer, but higher quality, data vectors than in the conventional approach. The DVSO method is applied in order to find the  $M$  best data vectors out of a set of  $N$  vectors,  $\{\mathbf{x}_i\}$ . This process is repeated  $Q$  times to give

$$M_T = \sum_{i=1}^Q M_i$$

data vectors which are used to calculate the covariance matrix  $R$ :

$$R = \frac{1}{M_T} \sum_{i=1}^{M_T} \mathbf{x}_i \mathbf{x}_i^H$$

The final step applies the DVSO method to the columns of the covariance matrix to find an adapted weight vector.

The number of computations required by the DVSO-COVAR technique is data-dependent; the mean number of calculations is estimated by Monte-Carlo simulations. For a single trial evaluation of the projection matrix, the number of multiplications is [16]:

$$\sum_{i=1}^Q C_b(M_i, N, K) + M_T(K+1)K/2 + C_b(H, N, K)$$

where

$$C_b = K(0.5 + 0.5N + 2M + MN + M^2) + 0.5MN$$

### 6.1.3 Eigenvector-based projection technique

This technique is based on an eigendecomposition of the covariance matrix. The covariance matrix is calculated as in Yeh's technique with equation (3). This Hermitian matrix is then decomposed in terms of its eigenvectors as

$$R = \sum_{i=1}^K \lambda_i \mathbf{v}_i \mathbf{v}_i^H$$

where  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_K$  is the set of ordered eigenvalues and  $\{v_1, \dots, v_K\}$  are the corresponding eigenvectors.

The Akaike Criterion (AIC) [15] is used to select the signal space eigenvectors. The number of these vectors is the value  $p$  which minimizes the function

$$AIC(p) = (K-p) \ln \frac{\frac{1}{K-p} \sum_{i=p+1}^K \lambda_i}{\prod_{i=p+1}^K \lambda_i^{\frac{1}{K-p}}} + p(2K-p),$$

The computational load of the eigendecomposition technique is given in terms of the number of complex multiplications as [16]:

$$C = C_{cm} + C_{ev}$$

The first term,  $C_{cm}$ , is the number of multiplications required to calculate the covariance matrix:

$$C_{cm} = \frac{M(K)(K+1)}{2},$$

where  $M$  is the number of data vectors used in the formation of the covariance matrix and  $K$  is the number of array elements. The second term,  $C_{ev}$ , is associated with the calculation of the eigenvalues and associated eigenvectors. The total number of multiplications for the evaluation of the eigenvalues and some associated eigenvectors is

$$C_{ev} = C_{tr} + C_{QL} + C_{pow},$$

where  $C_{tr}$ , the number of operations required for the tridiagonalization, is given by

$$C_{tr} = \frac{2}{3}K^3 + \frac{3}{4}K^2 + \frac{25}{12}K - 7,$$

The number of computations for the QL algorithm is

$$C_{QL} = 6K^2 - 3K,$$

The number of operations required for the power method,  $C_{pow}$ , is

$$C_{pow} = 6K^2n_{ev} + 12Kn_{ev}^2 + 6f_s(n_{ev}^3) + 8n_{ev}^3,$$

where  $n_{ev}$  is the number of eigenvectors to be calculated and  $f_1(n_{ev}^3)$  is the number of computations required for the full eigen solution of a  $n_{ev} \times n_{ev}$  matrix.

## 6.2 Simulation results - projection techniques

This section compares SEWE technique with DVSO, DVSO-COVAR and an eigen decomposition technique in terms of SNJR versus computational load. We performed simulations for the three scenarios discussed previously with JNR=26.9 dB. In DVSO-COVAR, the parameter N was set to approximately  $(1.5 L)$ , while Q was varied from 1 to 20. Figures 16, 17 and 18 give  $SNJR_{20}$  versus the number of complex multiplications for DVSO, DVSO-COVAR, SEWE and an eigenvector-based projection technique for scenarios 1 ( $L=3$ ), 2 ( $L=5$ ) and 3 ( $L=10$ ) respectively. When  $L \ll K$  as in scenario 1, the SEWE technique achieves near optimum performance (only .53 dB below maximum value while DVSO-COVAR is 0.2 dB below) with 20 times fewer computations than DVSO-COVAR. For larger L (scenario 3), the  $SNJR_{20}$  was 2.72 dB below the optimum in SEWE method as opposed to .34 dB for DVSO-COVAR. But SEWE required 10 times fewer computations than DVSO-COVAR. For all the three scenarios, eigen decomposition-based method achieves the optimum  $SNJR_{20}$  but at the cost of a huge computational load. DVSO gives sub-optimum performance but with far fewer computations.

These results show that SEWE technique is certainly the best choice for cases where  $L \ll K$  (scenarios 1 and 2 for example). It is also a good choice for a larger number of jammers if one is willing to trade-off some performance for a large reduction in the number of computations.

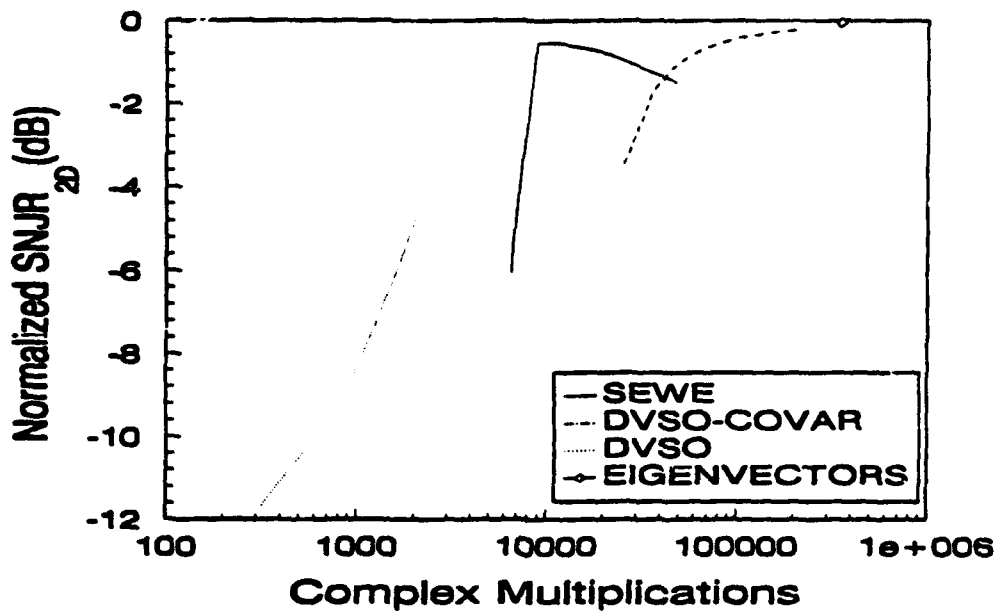


Figure 16 Normalized SNR<sub>2D</sub> versus Complex Multiplications with L=3, K=64 and JNR=26.9 dB

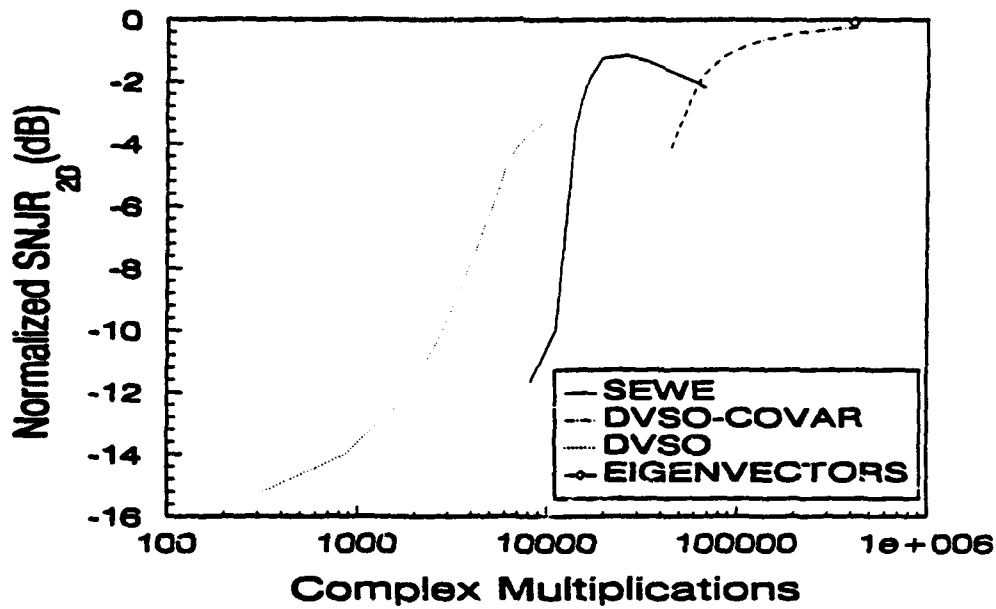


Figure 17 Normalized SNR<sub>2D</sub> versus Complex Multiplication for L=5, K=64 and JNR=26.9 dB.

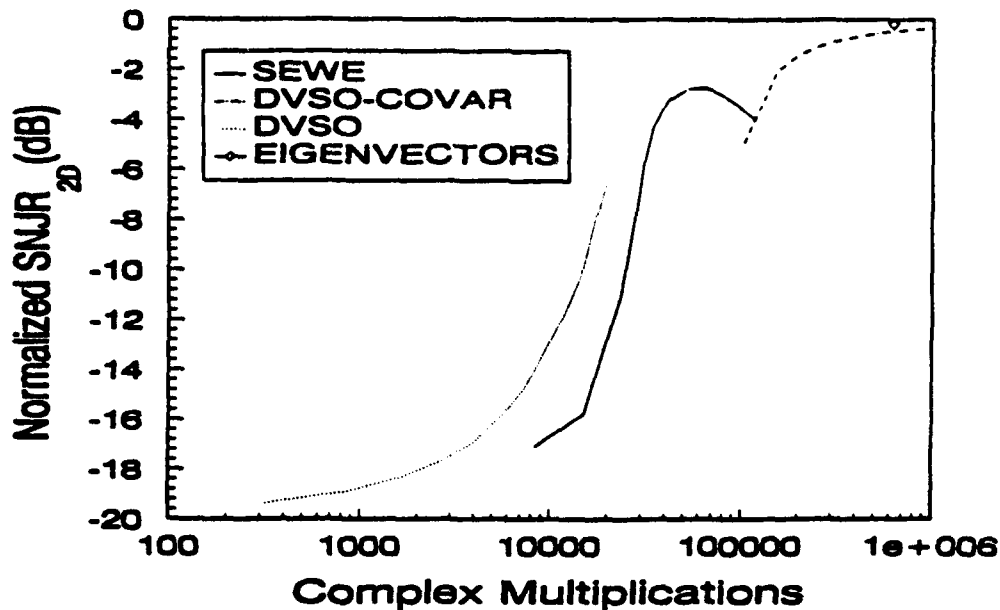


Figure 18 Normalized  $SNJR_{20}$  versus Complex Multiplication for  $L=10$ ,  $K=64$  and  $JNR=26.9$  dB.

## 7.0 Conclusions

This report has shown that the Yeh projection technique significantly improves the performance of adaptive beamforming provided a priori information is available on the number of jammers. An improvement to the Yeh technique has been developed and presented; this improved technique, called Subspace Estimation without Eigenvectors (SEWE), does not require a priori knowledge of the jammer scenario. The SEWE technique has been compared with other fast projection methods and also with an eigenvector decomposition technique. The SEWE technique demonstrated superior performance in terms of Signal-to-Noise-plus-Jammer Ratio (SNJR) versus computational load, except when a very high SNJR is required; in this latter case, DVSO-COVAR and eigenvector projection perform somewhat better. It should be noted however, that the best performance for SEWE for a 64 element array with 10 jammers was a loss of only 2 dB with respect to the optimum. This loss was considerably less for smaller number of jammers, e.g. 3 and 5 jammers where the loss was 0.2 dB and 1.2 dB respectively.

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## 9.0 References

1. Bühring, W., "Adaptive Orthogonal Projection for Rapid Converging Interference Suppression", *Electron. Lett.*, 1978, 14, pp. 515-516.
2. Bühring, W., "Adaptive Antenna with Rapid Convergence", *IEE Conf. on Antennas and Prop.*, IEE Conf. Publ. 169, 1978, pp.51-54.
3. Gabriel, W.F., "Using Spectral Estimation Techniques in Adaptive Processing Antenna Systems", *IEEE Trans. AP-34*, No.3, March, 1986, p.29.
4. Reed, L.S., Mallett, J.D., Brennan, L.E., "Rapid Convergence Rate in Adaptive Arrays", *IEEE Trans. Vol. AES-10*, No.6, November 1974, pp.853-864.
5. Hung, E.K.L., Turner, R.M., "A Fast Beamforming Algorithm for Large Arrays", *IEEE Trans. Vol. AES-19*, No-4, July 1983, pp.598-607.
6. Hung, E.K.L., Turner, R.M., Herring, R.W., "Fast and Effective Adaptive Beamforming and Spectral line Estimation by Direct Orthogonalization of the Data Vectors", *IEEE Asilomar Conference on Circuits, Systems and Computers*, Oct. 31 - Nov.2, 1983, pp.276-281.
7. Nickel, U., "Angular superresolution by antenna array processing", *International conf. on radar*, Paris, April 24-28, 1989, pp. 48-58.
8. Nickel, U., "Some Properties of Fast Projection Methods of the Hung-Turner Type", *Signal Processing III: Theories and Applications*, I.T. Young et al (eds.), *Signal Processing III: Theories and Application*, North Holland, 1986, pp.1165-1168.
9. Toulgoat, M., Turner, R.M., "Sample Vectors Selection Applied to the Estimation of the Signal Subspace for Adaptive Nulling and Estimation of Direction of Arrival", *DREO Signal Processing Workshop*, Defence Research Establishment Ottawa, May 6-8, 1991.

10. Toulgoat, M., Turner, R.M., "The Application of Sample Vector selection to Adaptive Nulling: Performance Comparison for Techniques Combining Gram-Schmidt Orthogonalization with Averaging", Canadian Conference on Electrical and Computer Engineering, Quebec, Sep. 25-27, 1991.
11. Toulgoat, M. and Turner, R.M., "Data Selection for Fast Projection Techniques Applied to Adaptive Nulling: A Comparative Study of Performance (U)", DREO Technical Report # 1100, Dec.1991.
12. Reilly, J.P. and Law, M.K., "A Fast High Performance Array Processing Technique for Angle-of-Arrival Estimation and Detection of the Number of Incident Signals", Canadian J. Elect. & Comp. Eng., Vol. 14, pp.38-45, 1989.
13. Yeh, C.-C., "Projection Approach to Bearing Estimation", IEEE Trans., vol. ASSP-34, pp.1347-1349, 1986.
14. Yeh, C.-C., "Simple computation of projection matrix for bearing estimations", IEE Proc., Vol. 134, Pt.F. No-2, April 1987.
15. Wax, M. and T. Kailath, "Detection of Signals by Information Theoretic Criteria, IEEE Trans. Acoust. Speech Signal Process., vol. ASSP-33, pp. 387-392, April 1985.
16. Toulgoat, M. and Turner, R.M., "Data Selection for Fast Projection Techniques: A Comparative Study of Direction Finding Performance (U)", DREO Technical Report # 1135, Dec.1992.

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