

### WPN - T A WEAPON TO TARGET APPLICATION PROCESS by Clyde Irvine, Jr. - Consultant, Economic Analysis ASI Systems International

WPN-T is a process that uses linear programming (LP) to create a framework for conducting Cost and Operational Effectiveness (COEA) tradeoffs between alternative weapon systems. What is a COEA? A COEA is a systems analysis approach for comparing one weapon or support system to others and selecting or recommending that system which is the "best" in terms of cost and operational effectiveness. One definition of the systems analysis approach is the following.

1. Establish the goals or objectives of the system.

2. Define the baseline or status quo system and identify alternative ways in which the system objectives can be attained.

3. Define the criteria for comparing or selecting between alternative systems.

4. Analyze and evaluate the baseline system and each alternative in terms of the selection criteria.

5. Select or recommend either the baseline system or one (or more) of the alternatives.

The above process is usually iterative especially when the initial number of alternatives is large and some sort of "neckdown" must be done to reduce the number to a manageable few.

WPN-T was developed to support a COEA of the Advanced Bomb Family (ABF). The author's approach in developing WPN-T is unique in two ways: 1) the use of linear programming to identify the conditions that establish equilibrium between the operational demand and operational supply of weapons; and, 2) the use of a basic economic analysis technique to create a framework for conducting an equal-cost and equal-effectiveness tradeoff between competing bomb families.

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The author addresses an important question that a COEA must answer: How can one do an equal-cost or equal-effectiveness tradeoff between different bomb families? There are many facets of this problem, particularly in defining weapon effectiveness in acceptable ways. Bombs come in different gross weights (500, 1000, 2000 lbs), guidance methods (unguided, laser-guided, inertial), bomb-body types (blast fragmentation, penetrators) which determine their lethality. Targets are of different types with vulnerabilities going from "soft" troops to "hard" underground command-control bunkers. There are varying numbers of each type of target, e.g., 25% soft targets, 10% hard targets, etc. Bomb delivery

methods involve low, medium and high altitude attacks. depending upon the desired circular error probability, target vulnerability, delivery threats, U.S. capabilities, the interrelationships of systems, scenario, weather, terrain, distance to target, and so on. An aircraft carrier holds the total immediate supply of bombs. It carries a limited stockpile of bombs of each bomb type and an upper limit on the number of bombs altogether. The challenge was to bring all these factors together into an analytical framework that remained comprehensive yet still comprehensible.

### THE OBJECTIVE OR GOAL OF A COEA

The objective or goal of a COEA is to provide a rational basis for comparing the existing MK80 bomb family to alternative bomb families and selecting that system which has the "best" overall cost and operational effectiveness.

### THE BASELINE SYSTEM AND ALTERNATIVES

The MK80 Series bomb family is the baseline system, i.e., the currently deployed system. The first alternative considered using other existing systems (U.S. and Allied Forces) or making improvements to the MK80, i.e., the MK80 Plus. The second alternative to the MK80 considered developing a new system altogether, i.e., the ABF.

## THE CRITERIA FOR COMPARING AND SELECTING ALTERNATIVES TO THE BASELINE

Life cycle unit cost of all-up-round bombs and guidance kits components was selected as the figure-ofmerit for cost. The number of targets-killed was selected as a measure of operational effectiveness. Criteria for selecting one bomb family over another were defined as: (1) one bomb family will kill more targets than the other for the same cost; or, (2) to kill the same number of targets one bomb family will cost less than the other. This is the equal-cost, equal-effectiveness trade-off.

# THE ANALYSIS OF ALTERNATIVES TO THE BASELINE

Part of a COEA can be formulated as an LP problem. LP is a tool of operations research. It has nothing to do with computer programming although computers are necessary to obtain solutions to all but the simplest problems. LP is a mathematical technique that selects the best "program" (course of action) from a set of feasible alternatives. LP states a problem in terms of three elements: (1) an objective function, (2) decision variations, and (3) constraints. The typical linear programming problem can be described as follows. Optimize (either maximize or minimize) some dependent variable (a linear function of independent variables) subject to a series of

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WPN-T uses linear programming techniques to address an important guestion that a COEA must answer: How can one do an equal-cost or equaleffectiveness tradeoff between different bomb families? There are many facets of this problem, particularly in defining weapon effectiveness in acceptable ways. Bombs come in different gross weights (500, 1000, 2000 lbs), guidance methods (unguided, laser-guided, inertial), bomb body types (blast fragmentation, penetrators) which determine their lethality. Targets are of different types with vulnerabilities going from "soft" troops to "hard" underground command-control bunkers. There are varying numbers of each type of target, e.g., 25% soft targets, 10% hard targets, etc. Bomb delivery methods involve low, medium, and high altitude attacks depending upon the desired circular error probability (CEP), target vulnerability, delivery threats, U.S. capabilities, the interrelationships of systems, scenario, weather, terrain, distance to target, and so on. An aircraft carrier (CVN) constitutes the total immediate supply of bombs. It carries a limited stockpile of bombs of each bomb type and an upper limit on the number of bombs altogether. The challenge was to bring all these factors together into an analytical framework that would be comprehensive, yet explainable.

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linear restrictions (constraints) involving the independent decision variables. The following paragraphs describe how the LP problem was formulated.

<u>Objective Functions</u>. An objective function defines the dependent variable (it need not be single-valued) in terms of independent variables called decision variables. There are three choices of objective functions in WPN-T:

(1) Minimize the bomb-cost to kill a given number of targets occurring according to a given percentage distribution

(2) Minimize the number of bombs necessary to kill a given number of targets occurring according to a given percentage distribution

(3) Maximize the number of targets that can be killed with a typical aircraft carrier load-out of a given bomb family.

An optimum or "best" "solution" (for any particular case) is to obtain values for any of the objective functions subject to the values of the constraints. One has to be careful about interpreting the term "best".

<u>Decision Variables.</u> The independent variables in an LP problem are usually referred to as decision variables. In WPN-T they are the following:

(1) The number of bombs of each type used on each type of target

(2) The total bomb-cost to kill each type of target

<u>Constraints.</u> The restrictions on the values that the decision variables can take are referred to as constraints. In WPN-T the constraints are:

(1) the suitability of a specific bomb configuration to a specific type of target, e.g., a laser-guided penetrator bomb would not be used against troops spread out over a large area, or blast fragmentation bombs would not be used against bunkers.

(2) the minimum number of bombs of a particular type or configuration required to kill a specific type of target

(3) percent distribution of target types

(4) the quantity of each bomb body type and related guidance kits that a specific ship can carry, including dunnage

(5) the maximum number of bombs and kits of all types that a specific ship can carry, i.e., volumetrics

(6) the permissible configurations of bomb bodies and guidance kits

(7) the investment in the stockpile, i.e., the total outlay represented by the load out

### THE ANALYTICAL FRAMEWORK AND NOTATION

The tables in Figure A illustrate the general layout of the spreadsheet used to prepare input data for the linear programming program. The table entries are the symbols used to identify the variables used in obtaining LP solutions. The "dot" notation in statistics is used to define a sum. For example, the sum of the first row of the center table at the top of Figure A is shown simply as  $(x_{1.})$ . Similarly, the sum of the first column would be (x..). The sum of every term in the matrix would be (x..). Figure A also contains the definitions for the variables. The following paragraphs define and discuss the objective functions for WPN-T.

### **OPTION I**

This option obtains the minimum total cost to kill targets subject to the constraints that the target types occur according to a specific distribution and all are to be killed. Also, the total number of bombs used cannot exceed the load out quantity. The objective function is:

minimize  $c_1 = c_1 (x_1) + c_2 (x_2) + c_3 (x_3)$ 

subject to a set of basic constraints

d1 - t1 = 0	s1 = 0, 1
d2 - t2 = 0	s2 = 0, 1
d3 - t3 = 0	s2 = 0, 1
x.1 <= a1	t <= T
x.2 <= a2	nij > 0
x.3 <= a3	$x_{ii} => 0$
p1 + p2 + p3 = 1	-)

and the additional constraint that T is a constant T\*.

Each term,  $d_i - t_i = 0$ , forces the total number of targets (T) destroyed to conform to the percentage distributions  $p_1$ ,  $p_2$ , and  $p_3$ . The terms  $x_{,j} <= a_i$  makes sure that the number of bombs used  $(x_{,j})$  do not exceed the number of bombs available  $(a_i)$ . Whether a target is susceptible (or vulnerable) to a particular bomb type is given by the term  $(s_i)$  which can be s = 0 (not vulnerable), or s=1 (vulnerable). The number of targets that are available becomes

$$t_i = (p_i)(s_i) T$$

For example, if T = 200 and p = .57 and s=1 then t = 114.

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		Avail. Trgts.   Trgts. Dstryd   Cost (\$K)   Over Kill     11   d1   c1.   d1 - t1     t2   d2   c2.   d2 - t2     t3   d3   c3.   d3 - t3     t   d   c   d3 - t3     t   biff.   (C - c)	constraints ii) > 0 for all i, j ii) => 0 for all i, j i=> 0 for all i, j -= 1 + p2 + p3 = 100% -= 1 + p2 + p3 = 100% -= 1 + p2 + p3 = 100% -= 2 + p3 = 100%-= 2 + p3 = 100% -= 2 + p3 = 100% -= 2 + p3 = 100%-= 2 + p3 = 100% -= 2 + p3 = 100% -= 2 + p3 = 100%-= 2 + p3 = 100% -= 2 + p3 = 100%-= 2 + p3 = 100% -= 2 + p3 = 10%-= 2 + p3 = 100% -= 2 + p3 = 10%-= 2 + p3 = 100% -= 2 + p3 = 10%-= 2 + p3 = 100% -= 2 + p3 = 10%-= 2 + p3 = 10% -= 2 + p3 = 10%-= 10%-= 10%-= 10%-= 10%-= 10%	3 - 13 - 0
	The Model and definitions)	W3   Total   Trgts.   Trgt.     x13   x1.   T1   P1   s1     x23   x3.   x2.   T2   P2   s2     x33   x3.   x3.   x3.   s3   s3     x.33   x.3   x.3   Totals:   P1   s1     x33   x.3   x.3   s3   s3   s3     x.33   x.3   x.3   Totals:   P   Total Targets:     u3   u   Total   Cost   C   Lowilled:	W3 Total Loadout   b3 b AUR \$k Minus   b3 b AUR \$k Minus   k Value Usage Usage   x3 x C (X - x)   x1 = x21 + x31 x21 + x31 x21 + x33   x2 = x12 + x22 + x33 x x   x1 = a1 - x.1 u1 = a1 - x.1 u2 = a2 - x.2 u3 = a3 - x.3   u1 = b1 - x.2 u3 = b1 - x.2 u3 = b1 - x.2 u3 = b3 x x	a = a1 + a2 + a3 c.1 = (c1)(x1) c.2 = (c2)(x2) c.3 = (c3)(x3) c. = c.1 + c.2 + c.3 x1 = b1 - k2 x2 = k2 x3 = b3 X = x1 + x2 + x3
Systems International	NPN-T :	III Employed Bombs W3 W1 W2 N1 W2 N1 W2 N1 W2 T1 X21 X12 T3 X31 X32 Required Use X.1 X22 Combined Use X.1 X22 Shortage Unissued: u1 u2 Available: a1 a2 Bombs-Used \$k: c.1 c.2	Initial Loadout   W1   W2     Unit Cost (\$k):   Equivalent Cost (\$k):   Equivalent Cost (\$k):   Equivalent Cost     Bomb Bodless:   bits:   bit   w2     Kits:   Kits:   bit   w2     Kits:   Equivalent AUR   x1   w2     Fquivalent AUR   x1   x2   w2     Fquivalent AUR   x1   x2   w2     Feduces W1 bomb-body stockpile   m3   w2   w2     Perfinitions   x1. = x12 + x13   x3   w2     x1. = x12 + x13   x3   x3   x3   w3     x1. = x11 + x22 + x33   x1   x1   x2   x1     x1 = (p1)(s1)T   t1 = (p1)(s1)T   t2 = (p2)(s2)T   t1 = (p1)(s1)T   t2 = (p2)(s2)T   t1 = (p1)(s1)T   t2 = (p2)(s2)T   t2 = (p1)(s1)T   t2 = (p1)(s1)T   t2 = (p1)(s1)T   t2 = (p1)(s1)T   t1 = t1 + t2 + t3   t2 = t1 + t2 + t3	d = d1 + d2 + d3 c1. = (c2)(x12) + (c3)(x13) c2. = (c1)(x21) + (c2)(x22) + (c3)(x23) c3. = (c1)(x31) + (c2)(x32) + (c3)(x33) c3. = c1. + c2. + c3. b = b1 + b3 k = k2 C = (c1)(x1) + (c2)(x2) + (c3)(x3)
ASI S		Minimum Bombs/Kii Target W1 W2 T1 Note (1) n12 T2 n21 n32 T3 n31 n32	Notes: (1) Blanks exchude (2) Any W2 usage r	

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### **OPTION II**

This option obtains the minimum number of bombs needed to kill targets subject to the constraints that all targets are to be killed according to the percentage distribution and the total cost of bombs used cannot exceed the value of the load out quantity. The objective function here is:

minimize  $x_{..} = x_{.1} + x_{.2} + x_{.3}$ 

subject to a set of basic constraints

d1 - t1 = 0	s1 = 0, 1
d2 - t2 = 0	s2 = 0, 1
d3 - t3 = 0	s2 = 0, 1
x.1 <= a1	t <= T
x.2 <= a2	nij > 0
x.3 <= a3	$x_{ij} => 0$
p1 + p2 + p3 = 1	5

and the additional constraint that T is a constant T\*

### **OPTION III**

This option obtains the maximum number of targets that can be killed subject to the constraints that all targets are to be killed according to the percentage distribution and the total number of bombs used cannot exceed the load out quantity and the total cost cannot exceed the value of the load out stockpile. The objective function is:

maximize  $d_{11} = d_{11} + d_{12} + d_{13}$ 

subject to a set of basic constraints

d1 - t1 = 0	s1 = 0, 1
d2 - t2 = 0	s2 = 0, 1
d3 - t3 = 0	s2 = 0, 1
x.1 <= a1	t <= T
x.2 <= a2	nij > 0
x.3 <= a3	$x_{ij} => 0$
p1 + p2 + p3 = 1	-

Note that T is no longer a constant. It is able to expand to a maximum value.

In Figure B, the upper left corner is a table relating Weapon Lethalities and Target Vulnerabilities. The column headings, W1, W2, W3, etc., refer to specific bomb configurations within a given bomb family. The row headings, T1, T2, T3, etc., refer to target types. The entries in the table are the minimum bombs needed of each type to kill each target type. For example, it would require 2 bombs of type W2 or 4 bombs of type W3 to kill target type T1.

The table in the middle of Figure B labeled INITIAL LOAD OUT contains logistic data. It shows, for each weapon type, the life cycle unit cost and the number of bomb bodies and kits carried. The row labeled Equivalent AUR needs some explanation. In this example 660 W1 type bombs can be dropped as "dumb" bombs. However, there are also 310 W2 guidance kits that can be used with W1 bomb bodies to create 310 W2 bombs. Hence, any use of W1 bomb bodies to make W2 smart bombs reduces the number of W1 bomb bodies available for W1 dumb bombs. If all 310 kits were used to make W2 bombs there could only be 350 W1 bombs configured. The example assumes that Bomb type W3 is never "kitted" so the initial load-out of 1100 bomb bodies equals the equivalent number of W3 bombs possible. The table at the bottomcenter of Figure B labeled Bomb Handling Tasks and Man-hours simply attaches ordnance handling time and motion data to individual weapons W1, W2, W3. The Table at the top-right contains target constraints., e.g., Target Occurrence refers to the number of each type of target expressed as a percentage of the whole. Whether a specific target is vulnerable at all is indicated in the column labeled Trgt Vulnr (1=yes).

The shaded section in Figure B contains the values of decision variables that optimize (either maximize or minimize) the objective function subject to the constraints on the decision variables. This is done for each alternative. The example illustrates the solution to the problem: maximize the number of targets that can be killed with a typical carrier load-out of a given bomb family. Each column still refers to weapon type, each row to target type. The rows below the shaded portion are explained as follows. The row labeled Required is simply the sum of the respective columns. The rows labeled Combined Use, and Shortage permit alterations to the model to automatically determine the best load-out mix. The row labeled Available starts off containing the Load out Mix, i.e., the initial number of bomb bodies and kits before any "kitting". If the best solution to the problem is to "kit" some W1 bomb bodies and make them into W2 bomb types in order to best use the available stockpile of bomb-bodies and kits, then that will be done by WPN-T, automatically. The row labeled Un-issued refers to the difference between the values in the row labeled Available (initial values) and the row labeled Required. If all Available are used (as here), Un-issued will be zero. The row totals give the quantity of all bomb types used to kill each target type.

Figure C illustrates the solution to the following problem: minimize the number of bombs needed to kill 200 targets. Notice that while only 403 bombs are used to kill 200 targets the total bomb-cost is \$7,300. The higher cost is due to the solution using more expensive (i.e., guided) bombs.

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# Weapon to Target Application (minimum bomb-usage to kill 200 targets)



Figure D illustrates the solution to the following problem: minimize the bomb-cost to kill 200 targets. Notice that more bombs (744) are used but the total bomb-cost dropped (\$3,720) because the solution used cheaper bombs (unguided).

Figure E summarizes a series of experiments for different numbers of targets (200, 300, 400 and 500.) to be killed by the same bomb family. Each line plotted in the lower part of Figure E is defined from the two points that were obtained using LP techniques: (1) the higher point on each line is the total bomb-cost for a minimum bombusage solution, and (2) the lower point is the total number of bombs used for a minimum total bomb-cost solution. This same process can be done for another bomb family and the results brought together for trade-off analyses. This is discussed below.

## A METHOD FOR OBTAINING AN EQUAL-COST, EQUAL-EFFECTIVENESS TRADE-OFF

Figure F-b shows two lines, both for the same MK80 bomb configuration. One line addresses a target-kill level of 200 targets and the other a target-kill level of 300 targets. Each line relates the cost of bombs used to the quantities of bombs used. On the horizontal axis in Figure F-b (Number of Bombs Used) we have placed a vertical line which intersects both MK80 lines. This vertical line represents a Fixed Stock of Bombs. The intersection points mark the total MK80 bomb-cost for each target-kill level for a Fixed Stock of Bombs. If we carry the Cost of Bombs Used values from E-b to E-a and plot them at the corresponding Number of Targets values on the horizontal axis we get a new line that relates the cost of bombs necessary to kill the number of targets indicated for the MK80 bomb family. Economists will recognize the technique used here as borrowed from the familiar IS-LM framework of macroeconomics. Of course the technique is not unique to macroeconomics nor restricted to those problems.

Figure G illustrates the same concept discussed in Figure F showing both the MK80 and the ABF cases. Figure G-a is the trade-off relationship between MK80 and ABF that we are looking for. That relationship is discussed below.

Figure H is simply a copy of Figure G-a so that we can discuss it without the other curves present. If we draw a horizontal line through Figure H it will intersect the ABF and MK80 lines at two points. These intersection points determine the number of targets killed at equal-cost. It is seen that, for an equal-cost the MK80 will kill more targets than the ABF. Similarly, a vertical line corresponding to a fixed stockpile of bombs intersects the MK80 and ABF lines at two points. These intersection points determine the total cost for equal-effectiveness (i.e., kill an equal number of targets) It is seen that the MK80

will cost less than the ABF to kill the same number of targets.

### **REFERENCES AND DATA REQUIREMENTS**

1. Bronson, Richard, Theory and Problems of Operations Research, Schaum's Outline Series, McGraw-Hill, 1982.

2. Eugene A. Diulio, Theory and Problems of Macroeconomic Theory, 2/ed, Schaum's Outline Series, McGraw-Hill, 1990

3. The minimum number of bombs of each type required to kill one specific target type.

4. The load-out mix of quantities of each bomb type and quantities of kits for converting them to guided weapons.

5. Life cycle unit cost (LCUC) data for each bomb component.

6. The frequency distribution of targets, i.e., the percentage of all targets of specific types and their "vulnerability" to a particular bomb type.

### **ACKNOWLEDGMENTS**

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# Weapon to Target Application (minimum bomb-cost to kill 200 targets)



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# Weapon to Target Application Summary:

s 500 Trgts	Killed							12811.5	102500
400 Trgt	Killed					10925.(	7441.(		
300 Trgts	Killed			9038.5	5581.2				
200 Trgts	Killed	7300.5	3720.5						
Bomb	Usage	403	744	719	1116	1062	1488	1405	1760



Figure E

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