INTERNAL RATE OF RETURN IN DEFENSE ANALYSIS

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COMBAT SCARS
INTERNAL RATE OF RETURN IN DEFENSE ANALYSIS

Properly used NPV and IRR complement each other very well. Further, IRR can be quite useful when used alone. For one thing, IRR can be used to compare a proposed alternative cost stream with a baseline using different base years. Unlike NPV calculations, using the same study period, or having the same size project is not necessary for IRR to be meaningful. For example, one could compare a larger U.S. Air Force aircraft life cycle cost with a smaller Royal Air Force fleet costs over different periods using historical data without having to make the base year, exchange rate, or other adjustments necessary to have a meaningful NPV comparison. IRR also provides an indication of risk differentials, whereas, NPV provides little risk insight.
The paper addresses the criticisms of IRR and demonstrates practical, reliable defense applications for the IRR methodology in comparative defense analyses.

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INTERNAL RATE OF RETURN IN DEFENSE ANALYSIS

Introduction

Discounted Cash Flow (DCF) includes the present value (PV) (or net present value (NPV)) and the internal rate of return (IRR) methods of analyzing cash flows. DCF provides insight into financial management not possible using other techniques. The NPV of the time-phased costs over the economic life of an investment project is the best single-number measure of its life-cycle cost.

Internal rate of return (IRR) is rarely used in defense analysis. A minor reason is that some IRR calculation requires cash inflow or revenue as well as outflow Since defense generates no revenue, there is no IRR for a single cost stream. However, a strength of IRR is in comparing project cost streams directly, a critical aspect of defense Functional Economic Analysis (FEA). IRR in this case is based on the differential between, say a baseline and alternative cost streams with investments. The technique is explained below under mutually exclusive projects and demonstrated in the appendix.

The major reason for IRR not being used in defense analysis centers around the extensive criticism of IRR found in corporate finance and financial management textbooks. These criticisms of IRR are the focus this paper.[1] These criticisms overstate the minor difficulties associated with IRR and understate the coexistent difficulties with NPV. This has resulted in IRR not being exploited to its potential. The aim of this paper is to put the criticisms of IRR into perspective and put the two DCF measures into balance.

Net Present Value

NPV is well accepted for sound reasons, but it has limitations. For one thing, to solve for NPV, one must first calculate the "opportunity cost of capital," also called the "discount rate." This rate is used in the discounting equation to calculate NPV. While generally a given in theoretical discussions, figuring out the cost of capital can be a difficult and time-consuming process. This is especially so in large, complex organizations. To get around this difficulty, the Department of Defense (DoD) has historically prescribed a 10 percent rate but not restricted analysts from using other rates. Office of Management and Budget (OMB) policy is being revised to reflect the lower market interest rates. Said another way, the discount rate has changed over time. We should expect it to continue to change.

A second difficulty with using NPV alone is that risk is assumed to be equal among competing projects. Risk is seldom equal in practice. Portfolio diversification is an acknowledged risk reduction technique. Similarly several smaller projects should have less risk than one large one, all other things being equal. NPV favors larger projects whereas smaller ones inherently have less risk. Risk of program disapproval or cancellation is inherent in defense analyses, thus the emphasis on "affordability," e.g., lower cost.

A final difficulty with using NPV is that it is largely limited to comparing projects *within* an organization, presumably with the same opportunity cost of capital (discount rate). Using different rates can change NPV rankings. NPV is therefore not very useful for comparisons *between* organizations--especially those of different size. For example, Anglo-American NPV defense industry comparisons can be meaningless while IRR comparisons can be illuminating. Even with these minor difficulties, NPV is the best absolute measure of value of an outflow-inflow stream. IRR is the best relative measure. Both have difficulties, but IRR is strongest where NPV is weakest.

Internal Rate of Return

The second discounted cash flow measure, IRR, has traditionally been defined as the [*sic*, any] discount rate at which NPV is equal zero. NPV has been applauded and IRR criticized for decades. While the focus of the criticism has been on using IRR in capital budgeting decisions, the unfavorable coverage has spilled over into other areas. Analysis of the reasons given for supposed IRR inferiority is the focus of this paper.

IRR is used extensively despite the textbook criticism. It is often favored by business people. For one thing, IRR is very good for screening projects. NPV is highly sensitive to the discount rate, while IRR bypasses the problem of deciding the "correct" one. Because IRR is a rate or ratio, not an absolute amount, it is more useful for comparing unlike investments, say stocks and bonds. It also is more useful for making comparisons between different periods, between different sized firms and in making international comparisons. The intention here is not to argue for either point of view, but instead to put the issue into balance. The aim is to show:

- NPV and IRR have essentially equivalent utility;
- they are *complementary* ways of looking at a problem or opportunity;
- IRR and NPV together gives a better analysis than either alone;
- if properly viewed, NPV and IRR give identical signals, including capital budgeting decisions;
- IRR is particularly useful for comparing different sized projects, where it receives some of its greatest criticism; and
- IRR is useful alone.

We will critically examine the professed reasons for the superiority of NPV over IRR in capital budgeting.[2]

Criticism Number One

IRR is the same even if the cash flows are reversed or inverted. For example, the IRR is 25 percent for both of the following:

Project	Year 0	Year 1	NPV@10%	IRR	DCF File
A	-$1,000.00	+$1,250.00	+$136.36	25%	S05A.DC!
B	+$1,000.00	-$1,250.00	-$136.36	25%	S05B.DC!

Table 1. Positive versus Negative Cash Flows

This criticism, simply stated, is that IRR does not keep track of the sign. This is misdirected. If you borrow money at 10 percent, you will pay the interest, not receive it. Interest rate tables use positive amounts and rates. Interest rate calculation routines use positive principal.[3] We are accustomed to keeping track of borrower or lender outside the actual calculations. Why should IRR be treated differently from its interest rate counterpart? Why would we even analyze a project that will lose money or a defense project with higher costs and lower benefits?[4] Project B should be eliminated in preliminary screening, but the reason is not always as obvious as it is in Table 1. The criticism is typically presented as it is in Table 2, but without the total.

Project	Year 0	Year 1	Year 2	Year 3	Total	NPV @ 10%	IRR	DCF File
C	+$1,000.00	-$3,600.00	+$4,300.00	-$1,760.00	-$60.00	-$41.32	60%	S05C.DC!

Table 2. "Conflicting" IRR/NPV Signals

Although the IRR is 60 percent, NPV is negative at a all discount rates including zero. (Note that undiscounted net cash flow is a *negative* $60.00). This is simply a variation of negative versus positive cash flows discussed above. Look at the project from the perspective of making money not losing it. Then IRR and NPV give identical decisions.[5]

It is tempting to correct this supposed problem by simply redefining IRR. Internal rate of return is, at a minimum, misleading. The Project C "IRR" is a rate of payment, not a rate of return. If "IRR" were simply redefined, much of its criticism would go away. Interest as something we either pay or receive. We should treat IRR the same.[6]

This is a good place to deviate briefly and examine the relationship between IRR and NPV: there is no simple, intuitive, or linear relationship between the two. You may wish to try the following yourself using S05C.DC! or S05D.DC!. See Table 3.

Discount Rate	20.00%	40.00%	60.00%	80.00%	100.00%	DCF File
NPV (net positive)	+$32.41	+$18.75	$0.00	-$25.38	-$55.00	S05C.DC!
	├── normal ──┤			├── noninterest ──┤		
NPV (net negative)	-$32.41	-$18.75	$0.00	+$25.38	+$55.00	S05D.DC!
	├── inverted ──┤			├── nonsense ──┤		

Table 3. NPV/IRR Relationship

Note that tripling the discount rate from 20 to 60 percent, only changed NPV by $32.31--hardly worth analyzing. Yet, if NPV were in millions of dollars, we would pay a lot closer attention, though IRRs would be unchanged. Large changes in the discount rate may change NPV significantly or very little, depending on the size of the cash flows. IRR, on the other hand, is dependent more so on the structure of the cash flows. NPV is a better *absolute* measure; IRR is a better *relative* measure. *The two measures complement each other.*

With net-positive cash flows, NPV decreases from maximum at a zero percent discount rate and converges on zero as it increases. This is normal. But once past zero NPV, where IRR is determined, NPV is negative at all discount rates. This latter area is of no interest.

Finally, with net-negative cash flows, NPV also converges on zero NPV with an increasing discount rate. After zero, NPV increases with an increasing discount rate. This implies that although we were losing money at all discount rates below 60 percent, the project became profitable at 80 percent. Worse still, the higher the discount rate, the more attractive it becomes. This suggests we can turn around an unfavorable project by increasing our opportunity cost of capital. This is nonsense, but incorporated then ignored in the negative-positive criticism.

Criticism Number Two

IRR can supposedly give a different decision from NPV on mutually exclusive projects. See Table 4.

Project	Year 0	Year 1	NPV @ 0%	NPV @10%	IRR	DCF File
E	-$1,000.00	+$2,000.00	+$1,000.00	+$818.18	100.0%	S05E.DC!
F	-$10,000.00	+$15,000.00	+$5,000.00	+$3,636.36	50.0%	S05F.DC!

Table 4. Mutually Exclusive Projects

The criticism is that although the IRR of Project E is greater, investing in Project F will make you $3,636.36 better off and is therefore preferred. Of course this is correct! We would clearly expect that an investment that is 10 times larger would create the larger NPV, but it is only 4.44 times larger. We will get a larger NPV by investing in two bonds at 5 percent yield than by investing in one at 6 percent. But this does mean we should always look for the lowest yield and buy more of them. We should expect the same in capital budgeting. Being able to invest in only one of the two projects is called a "constrained-financing" assumption in capital budgeting parlance--constrained to $10,000.00 in this example. Risk is also assumed equal for mutually exclusive projects. Such assumptions are somewhat artificial and raise several practical and theoretical questions:

- Who would not prefer to invest in ten projects like Project E (or Project E ten times, NPV = $8,818.18) instead of Project F?
- Who would not prefer that people developing proposals look for projects with the highest possible rate of return also?
- Should the higher rate of return receive favorable consideration for being of lower absolute market risk (i.e., less money is at risk)?
- Should we eliminate Project E simply because it is the smaller of the two?
- Can we invest in both now?
- Can we invest in one now, one later?

A focus on NPV to the exclusion of IRR would build in a bias for large projects over smaller, perhaps more cost-effective ones. What Criticism Number Two instead confirms is that having both NPV and IRR gives a better picture of the problem or opportunity than either alone.

One way to overcome the supposed mixed signals is to normalize the larger project to the smaller one (or vice versa). See Table 5. Project F is 10 times larger than Project E, so divide Project F cash flows by 10. Project F is normalized (or scaled) to Project E.

Another way to overcome the supposed mixed signals is to evaluate the difference (or "deltas") between the two. This is done in S05G.DC!, Table 5, and Figure 1.

Project	Year 0	Year 1	NPV @ 0%	NPV @10%	IRR	DCF File
E	-$1,000.00	+$2,000.00	$1,000.00	+$818.18	100.0%	S05E.DC!
F	-$10,000.00	+$15,000.00	$5,000.00	+$3,636.36	50.0%	S05F.DC!
F (normalized)	-$1,000.00	+$1,500.00	$500.00	+$363.64	50.0%	S05F.DC!
G (differential)	-$9,000.00	+$13,000.00	$4,000.00	+$2,818.18	44.4%	S05G.DC!
H (realistic)	-$9,000.00	+$11,800.00	$2,800.00	+$1,727.27	31.1%	S05H.DC!

Table 5. Normalized, Differential and Realistic Comparisons

The technique of evaluating differences (also called incremental flows) bypasses the problem with different size projects. It is also the most practical way to analyze the difference between alternatives with cash outflows only--a FEA for example. Figure 1 illustrates NPV at all relevant discount rates (those between zero in the IRR). See the appendix for an example of applying differential cash flows in a defense analysis.

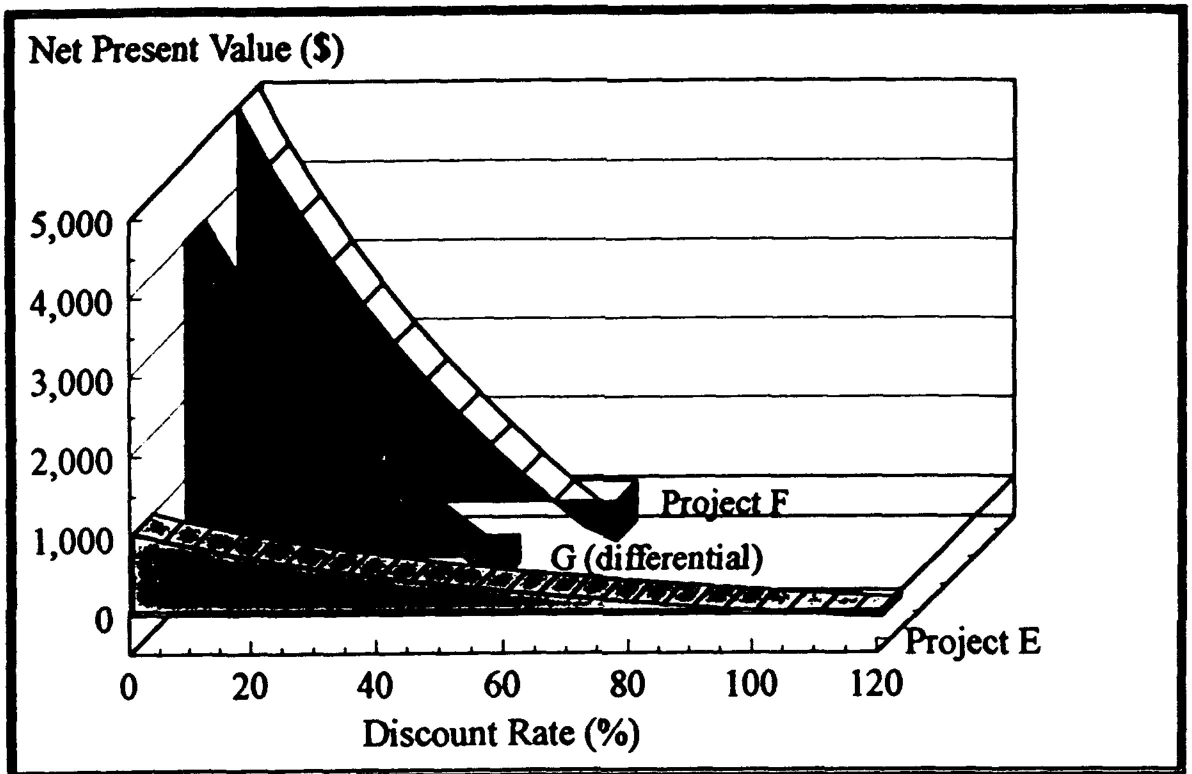

Figure 1. Mutually Exclusive Projects

NPV and IRR can be used together when evaluating different sized projects. If there is an apparent conflict, simply understand what is causing it and present the information differently if necessary. Of course, more realistically comparable investments, such as Projects F and H, IRR and NPV give the same answer regardless. Still, both NPV and IRR give a clearer picture than either alone. It is not a case of either-or; we can have both.

Although it will not always compensate for the supposed mixed NPV/IRR signals when comparing projects with very large differences, the following technique can close the gap. Different sized projects are more directly comparable if financing is included in the analysis. The traditional approach is to represent the cost of capital used to finance the project with a single discount rate, excluding the cash flows used to finance the project. Instead, financing (or opportunity cost if using internal funds) cash flows are usually only implied. In the following example, cash flows from financing the project explicitly included in the mutually exclusive projects:

Project	Year 0	Year 1	NPV @ 10%	IRR	DCF File
E	-$1,000.00	+$2,000.00	+$818.18	100.0%	S05E.DC!
Financing	+$1,000.00	-$1,100.00	$0.00	10.0%	
Difference	-$2,000.00	+$3,100.00	$818.18	55.0%	S05I.DC!
F	-$10,000.00	+$15,000.00	+$3,636.36	50.0%	S05F.DC!
Financing	+$10,000.00	-$11,000.00	$0.00	10.0%	
Difference	$20,000.00	$26,000.00	$3,636.36	30.0%	S05J.DC!

Table 6. Financing Included

Assume a 10 percent loan to finance the project. While NPV remains the same for both projects, IRRs declines to 55 percent and 30 percent respectively. The difference between the project life cycle financing costs and project revenues (a variant of the differential cash flow technique) can give a better picture. An advantage to using this approach would be that the impact the project itself might have on financing costs can be incorporated. For example, very large projects might require external financing at higher rates than current ones.[7] The opportunity cost of capital is typically assumed to be constant and equivalent for both large and small projects.

Criticism Number Three

More than one IRR is possible with multiple sign changes. Additional IRRs can occur if the signs of the cash flows change more than once. Not uncommonly, this criticism is combined with Criticism Number One (negative versus positive flows) and presented as the inverted project K. This unnecessarily complicates the picture. We will use project K for demonstration, with positive cash flows, a positive NPV, and IRRs of 25 and 400 percent. (With the DCF program, it makes no difference which one you use.) See Table 7 and Figure 2.

Project	Year 0	Year 1	Year 2	NPV @ 10%	"IRRs"	DCF File
K	+$400.00	-$2,500.00	+$2,500.00	+$193.39	25% & 400%	S05K.DC!
Inverted	-$400.00	+$2,500.00	-$2,500.00	-$193.39	25% & 400%	S05K.DC!

Table 7. Sign Changes

Which IRR is correct? The answer is that both are mathematically correct if you use the criterion: *an* [*sic*, any] IRR at which NPV is zero. According to "Descartes' rule of signs" there can be as many different IRRs as there are changes in the sign of the flows ("- + -" is two sign changes). With more sign changes, there could be more IRRs. Given there are two IRRs, which if either is meaningful?

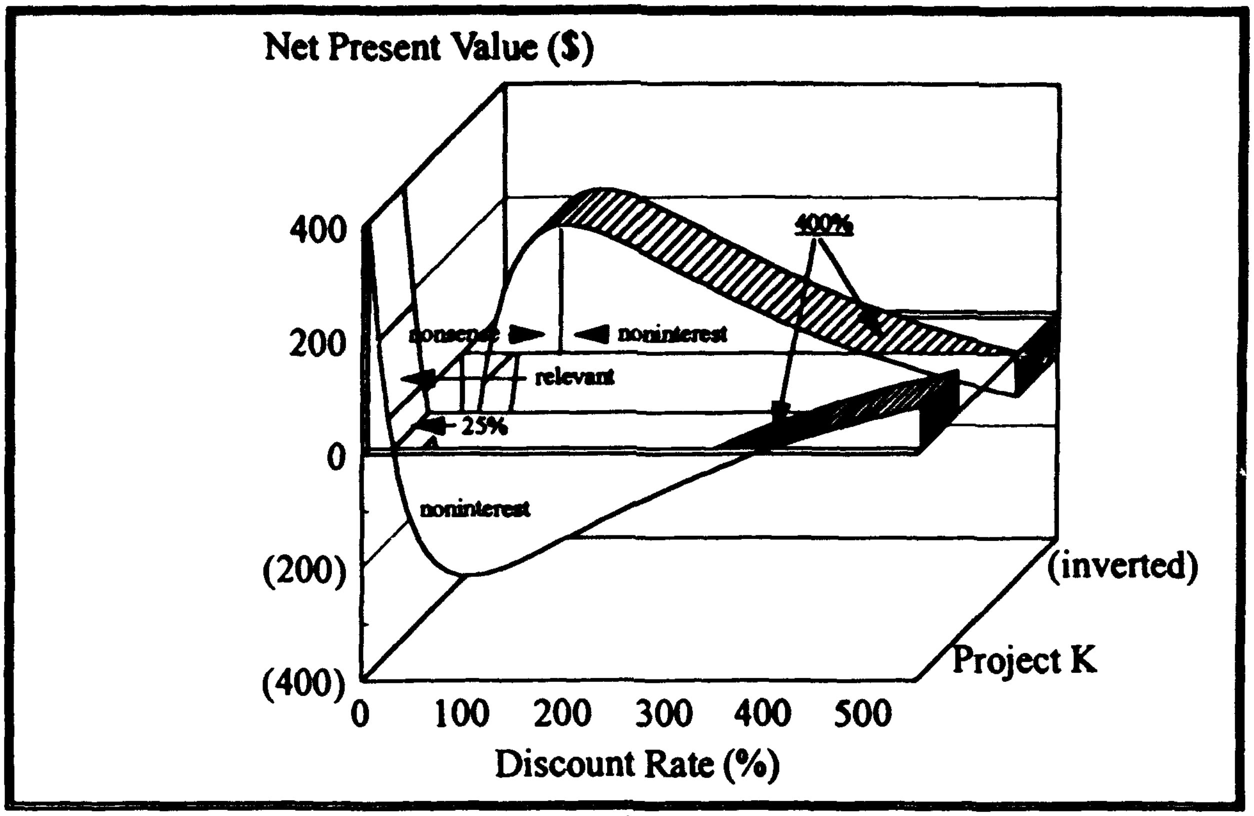

Figure 2. Conflicting Rates of Return

If we accept that both IRRs are mathematically correct, one solution is simply to take the smaller, more conservative 25 percent.

A second way is to use the IRR closest to the net return on outflows. Ignoring when they occurred, outflows are $2,500.00 and inflows $2,900.00. The $400.00 net is the NPV at a zero percent discount rate. It is also 16 percent of outflows. This return on outflows does not consider the time value of money; therefore, it should be close to the relevant IRR. The closest IRR is 25 percent. On the other hand, 400 percent is 25 times larger than the return on outflows. While "mathematically correct," 400 percent is not meaningful.

A third way is to use the IRR that is consistent with NPV converging on zero as the discount rate increases. In other words, look for the same pattern we find with an annuity or bond yield. We cannot arrive at 400 percent without going through a nonsense zone where NPV increases with increasing discount rates. While mathematically correct, given the widely-accepted definition, this second IRR is not meaningful. The rate calculated before entering a nonsense zone is 25 percent.

Sign Change Demonstration

If you're still not convinced, see Table 8. This might be the type of analysis a credit union manager would perform on a member account, but it is simplified to make the calculations obvious.

Month	Borrowed (Payback)	Unpaid Balance	Payments at Selected IRRs					
			12% (1X)	120% (10X)	240% (20X)	480% (40X)	840% (70X)	960% (80X)
0	-$5,000							
1		-$5,000	-$50	-$500	-$1,000	-$2,000	-$3,500	-$4,000
2		-$5,000	-$50	-$500	-$1,000	-$2,000	-$3,500	-$4,000
3	+$20,000	-$5,000	-$50	-$500	-$1,000	-$2,000	-$3,500	-$4,000
4		$15,000	$150	$1,500	$3,000	$6,000	$10,500	$12,000
5		$15,000	$150	$1,500	$3,000	$6,000	$10,500	$12,000
6		$15,000	$150	$1,500	$3,000	$6,000	$10,500	$12,000
7	+$1,000	$15,000	$150	$1,500	$3,000	$6,000	$10,500	$12,000
8		$16,000	$160	$1,600	$3,200	$6,400	$11,200	$12,800
9	-$3,000	$16,000	$160	$1,600	$3,200	$6,400	$11,200	$12,800
10		$13,000	$130	$1,300	$2,600	$5,200	$9,100	$10,400
11		$13,000	$130	$1,300	$2,600	$5,200	$9,100	$10,400
12	-$13,000	$13,000	$130	$1,300	$2,600	$5,200	$9,100	$10,400
Total	-0.00-	$116,000	$1,160	$11,600	$23,200	$46,400	$81,200	$92,800

TABLE 8. Sign Change Demonstration

Assume interest either received or paid on your credit union account is "1 percent per month on the balance," i.e., $10.00 per month per $1,000.00 balance or 12 percent per year. Assume that you make the interest payment accept cash payment monthly, i.e., principal changes are a separate transaction. IRR varies directly with the rates implicit in the interest payments you enter, but never exceeds the highest rate implied.

Figure 3 illustrates 240 percent implied cash flows and a 240 percent IRR.[8] This is what we would expect, since IRR is a composite of individual rates which determine the cash flows. The Net Cash Flow column has two sign changes, and Descartes' rule says that there can be an additional IRR. Sure enough, there is another one at 697.14 percent. But this second IRR doesn't behave the way the first one does. See Figure 4.

Discounted Cash Flow

Prepared by	R. Martin
Date prepared	06-15-1994
Description	[illegible] - 240% Implied Rate
Study period	12 months
Discount rate	697.14 percent
Discount method	End-of-month

Project Month	Borrowed (-)	Payments (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
0	-$5,000.00	$0.00	$5,000.00	1.0000	$5,000.00
1	$0.00	-$1,000.00	-$1,000.00	0.6325	-$632.53
2	$0.00	-$1,000.00	-$1,000.00	0.4001	-$400.10
3	$20,000.00	-$1,000.00	-$21,000.00	0.2531	-$5,314.53
4	$0.00	$3,000.00	$3,000.00	0.1601	$480.23
5	$0.00	$3,000.00	$3,000.00	0.1013	$303.76
6	$0.00	$3,000.00	$3,000.00	0.0640	$192.14
7	$1,000.00	$3,000.00	$2,000.00	0.0405	$81.02
8	$0.00	$3,200.00	$3,200.00	0.0256	$82.00
9	-$3,000.00	$3,200.00	$6,200.00	0.0162	$100.49
10	$0.00	$2,600.00	$2,600.00	0.0103	$26.66
11	$0.00	$2,600.00	$2,600.00	0.0065	$16.86
12	-$13,000.00	$2,600.00	$15,600.00	0.0041	$63.99
Total	$0.00	$23,200.00	$23,200.00	2.7143	-$0.01

Uniform monthly inflow without terminal value	$0.00
Internal rate of return (IRR)(sign +/-: 3)	240.00 percent
IRR—discount rate differential (outflow)	457.14 percent

Figure 3. Sign Change Demonstration (240% Implied IRR)

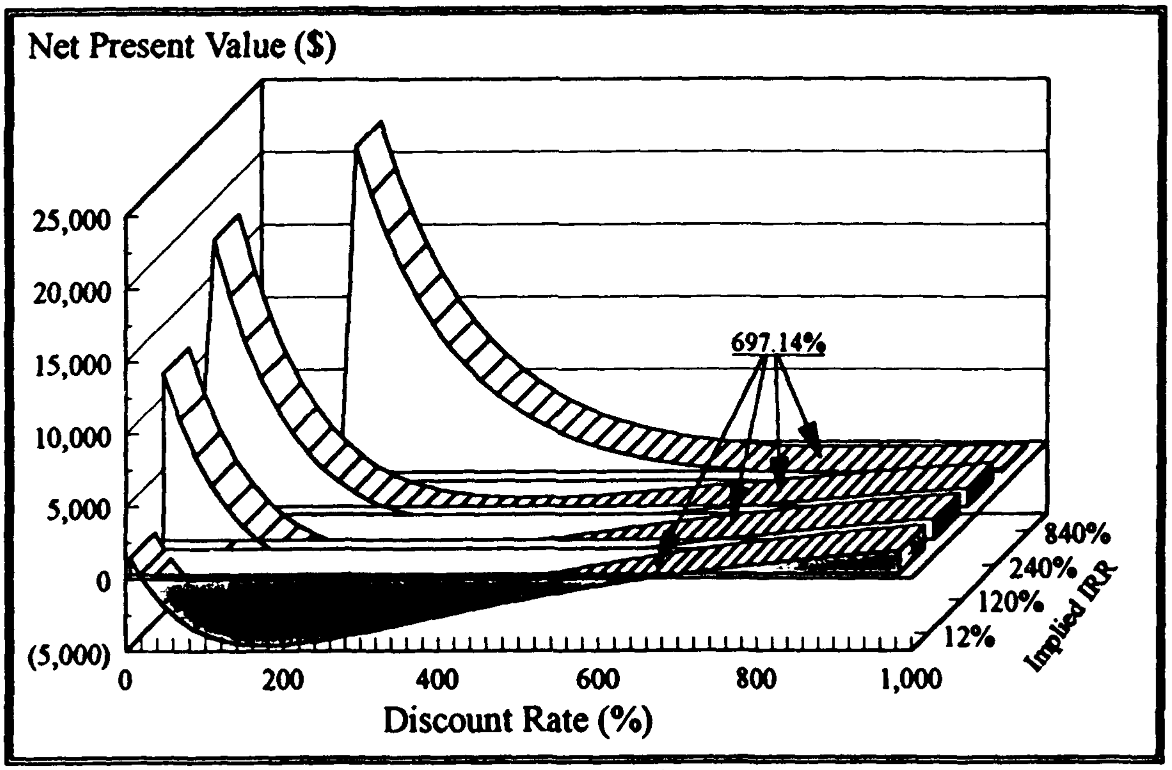

Figure 4. Sign Change Demonstration

While the first IRRs are responsive to changes in interest payments, the additional IRR is not. All give a NPV of zero at 697.14 percent.[9] This second rate fails on reasonableness, but it serves a purpose: 697.14 percent provides a useful upper bound for meaningful IRR calculations.[10] IRR can be calculated and bounded into a relevant range for practical use.

IRR Algorithms

Typical IRR algorithms calculate any "mathematically correct" IRR. In doing so, developers have accepted the sign change criticisms and agreed that all IRRs are the same. These algorithms can be found in, for example, spreadsheet financial functions, in function calls in high-level programming languages, and in financial programs. These algorithms may require you to provide a "seed," i.e., enter a rate. The seed is used to do a half-interval search to calculate an IRR. If you enter a seed close to 25 percent, it would more likely than not be the IRR returned. Conversely, if you provide a seed close to a second or subsequent rate, it should be the one returned. But what is "close" when you don't know what the choices are? Negative rates are also possible with many algorithms, even with positive cash flows, resulting in a nonsense IRR. The result is that you are uncertain what the program will calculate if the cash flows contain multiple sign changes. Financial analysis programs typically warn you of the prospects, but pretty much leave it up to you to figure out what to do about it. For these and other reasons, spreadsheets are prone to provide "ERR" as the solution to IRR calculations or to lock up. When either happens, the IRR function has likely had trouble as explained in this paper or perhaps exceeded precision limits.[11]

Criticism Number Four

Under some circumstances IRR is incalculable. This is perhaps the most serious criticism of IRR. But, as IRR critics overlook, it is not much of a problem in practice and it applies to NPV as well. The only difference is that it is not as obvious with NPV as it is with IRR. IRR is incalculable in at least four circumstances.. First, IRR cannot be calculated if cash flows are all-positive. See Project L below:

Project	Year 1		Year 2		NPV @10%	IRR	DCF File
L	+$300.00		+$700.00		$851.24	None	S05L.DC!
Project	Half 1	Half 2	Half 3	Half 4	NPV @10%	IRR	DCF File
M (L/2)	-$1,400.00	+$1,700.00	+$400.00	+$300.00	$800.87	100%	S05M.DC!

Table 9. All-Positive Annual Flows

Cash flows are $300.00 and $700.00 in years 0 and 1 respectively. There is no return, internal or otherwise, because there is no investment. A project with all-positive cash flows is a gift. It implies output without input--rare in legitimate enterprise.

The appearance of an all-positive cash flow situation can occur though, if we lump the actual cash flows from shorter periods into longer ones. Automated report generation routines could give flows similar to those in S05L.DC!. Using more frequent intervals could correct the apparent problem. For example, +$300.00 in year 1 of Project L could have been the aggregate of quarterly or semiannual flows of which one or more were negative. If this were the case, simply using data that more closely corresponds to the period over which cash flows occur would result in a solution. In this example, semiannual flows make IRR calculation possible, and does a better job of calculating NPV as well.

A second circumstance in which IRR is incalculable is when cash flows are all-negative. This, like the all-positive case, needs no further consideration: it's a giveaway. It follows the same reasoning as the all-positive case.

A third circumstance in which a positive IRR is incalculable is when net cash flows are zero--say +$1,000.00 in year 1 and -$1,000.00 in year 2[12]. Intuitively we would conclude that this was a favorable circumstance in which we were given an interest-free loan for one year. There may be an opportunity cost of capital implied in such an unlikely transaction; however, it is incalculable without entering the actual cash flows. At 10 percent, such an exchange would be worth $100.00 to the recipient, presuming the $1,000.00 were invested. If so, we should include the cash flow from the investment in the calculation giving net cash flows of $100.00, not zero. We can handle this situation by including project financing.

The fourth circumstance under which a positive IRR is incalculable is more subtle. It occurs with certain other combinations of cash flows not described above. With a positive cash flow of $0.01, the DCF program accepts the data and calculates the IRR normally. The IRR, 0.0010 percent is in rounding. If we increase the revenue in year 1 by 1¢ ($0.01), the program reports "IRR incalculable--cash flows are zero." If we decrease year 1 revenue by 1¢ (-$0.01) to $999.99, NPV is negative at any positive discount rate and the DCF program reports "IRR incalculable--NPV diverging from zero."[13]

Project	Year 0	Year 1	NPV@ 0%	IRR	DCF File	IRR Message
N1	-$1,000.00	+$1,000.01	+$0.01	0.00%	S05N.DC!	None
N2	-$1,000.00	+$1,000.00	$0.00	None	S05N.DC!	Cash flows zero
N3	-$1,000.00	+$999.99	-$0.01	None	S05N.DC!	NPV diverging
N4	-$1,000.00	<$1,000.00	varies	None	S05N.DC!	NPV diverging

Table 10. IRR Incalculable Messages

For any inflow in period 1 less than less than the outflow in period zero, a positive IRR is incalculable. This is an implausible cash flow arrangement in that we will get back less than we put into it. We should therefore reject project N3 and N4 on cash flow grounds without discounting. Nevertheless, we will pursue the criticism as it has traditionally been presented. The following cash flows also trigger "IRR incalculable--NPV diverging from zero" in the DCF program:

Project	Year 0	Year 1	Year 2	NPV @ 10%	IRR	DCF File
P	+$1,000.00	-$3,000.00	+$2,500.00	+$338.85	None	S05P.DC!

Table 11. Complex Cash Flows

There is something implausible about Project P flows as well. We should be suspicious of a project that yields +$1,000.00 in year zero. We could, for example, invest +$1,000.00 from year 0 at opportunity cost of capital and generate additional positive cash flows in subsequent years.[14] If there were no penalties for canceling the project we would do so in year 1 and initiate it again, giving us another all-positive cash flow situation. Still, we concede Project P as possible and that IRR is incalculable.[15]

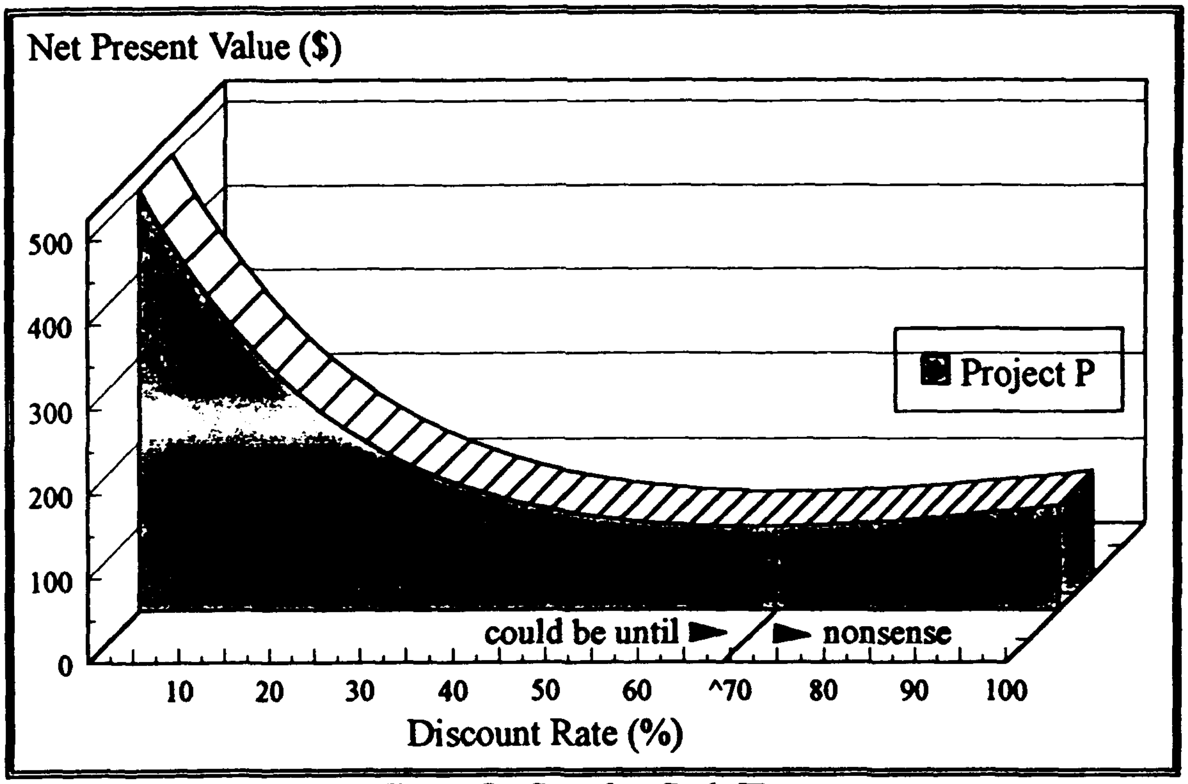

Figure 5. Complex Cash Flows

While this is an IRR limitation, but NPV is suspect as well because it is positive at all discount rates. Said another way, the discount rate is irrelevant. Or worse, past 66.7 percent we could increase NPV by simply increasing our opportunity cost of capital--increase value by increasing cost. This is absurd. Recommendation: If you receive a " . . .diverging from zero" message, look carefully at your input data. If you are certain your data are okay, try a couple of different discount rates above and below your cost of capital to figure out if you are in the nonsense area of the DCF plot.[16]

Summarizing, a positive IRR is incalculable with all-positive, all-negative, net-zero, or complex cash flows. What are the implications of this? None as far as IRR is concerned, and NPV may be suspect as well. This is especially true with the "diverging from zero" message associated with complex cash flows. Simultaneously, IRR gives no decision, e.g., it is incalculable. Under these circumstances, you should look carefully at the structure of the cash flows. If the cash flows are okay, at least an incalculable IRR has highlighted a potentially troublesome NPV.

Criticism Number Five

The final criticism centers on the "reinvestment assumption." Critics correctly point out that IRR implicitly assumes that positive cash flows generated are reinvested at the IRR, not the discount rate. (Negative cash flows would be disinvested as well, but generally aren't mentioned, presumably to simplify the demonstration.) They argue that the discount rate, not the IRR, represents the opportunity cost of capital. This in turn makes IRR *inappropriate* for appraising or ranking projects. Not uncommonly, this criticism is mixed in with one or more of the other four major criticisms addressed above.[17]

To correct for contradictions resulting from using implicit assumptions, simply make them explicit. For example, if you want to know what the IRR is with cash flows reinvested at the discount rate, run the DCF program twice. In the second run, enter the actual cash flows generated from reinvested funds for all projects. If the funds can be used to retire debt, include the interest avoided as a positive cash flow to the project. Of course, it is essential to treat all projects the same in this regard if comparisons are to be meaningful. While this technique would have been cumbersome several decades ago when some of the more popular textbooks were written, modern electronic spreadsheets or the DCF program make it straightforward. Usually, however, such adjustments are unnecessary. Commonly, reinvestment at the IRR is an appropriate assumption. It is the one made in a bond yield problem, which is, after all, an investment decision.

Other Criticisms

This concludes our discussion of the major criticisms of IRR as they apply to capital budgeting. The criticisms cited above are not the only ones you will find, but most of the remaining ones are either trifling or they relate to the extent to which IRR fails to conform to (admittedly useful) guidelines for a capital budgeting decision. For example, it is useful to be able to add the estimated NPV from several projects to arrive at a total NPV for all of them. This is sometimes called the "*value additivity principle*." We cannot add IRRs for a meaningful number, but we can average them. We can calculate an IRR for multiple projects by simply combining the cash flows. We should *expect* IRRs from these combined cash flows to be different and not usable alone if the projects are different size.[18]

Conclusion

It has, I believe, been shown that, practically speaking and properly viewed, IRR yields the same decision as does NPV except under some extreme circumstances that present few limitations in practice. No attempt has been made to suggest that IRR is superior to NPV. They are best used together because:

- Both NPV and IRR can yield nonsense answers in extreme circumstances, but if both are considered together these circumstances are highlighted and therefore understood.

- NPV and IRR give consistent answers if handled and viewed properly. The DCF program helps insure they are.

- Business people favor relative measures such as IRR over absolute ones such as NPV. Even if it were superior, NPV has limited utility if not used.

- IRR and NPV together give an indication of risk as well as return.

- IRR is not affected by large cash flows. This makes returns comparable across different investments, industries, or even across international boundaries.

- IRR is useful alone in virtually all time-value-of-money problems.

NOTES

1. See *Managerial Finance*, Ninth Edition by J. Fred Weston and Thomas E. Copeland (Forth Worth: The Dryden Press), 1992, pp 309-320. The first edition was published in 1962. "*Summarizing the comparison between the NPV and IRR criteria*, we see that the IRR has many difficulties that invalidate it as a generally acceptable capital budgeting rule (p. 352, emphasis in original).

2. I wrote an IBM personal-computer based Discounted Cash Flow program to the specifications implicit in this paper. The files referred to in the paper are demonstration and sample files Sample DCF files S05A.DC! through S05P.DC! provide the relevant calculations for the tables and figures in this paper. The other 49 of the 51 groups of files calculate annuities through yields. IRR is the same rate solved for in virtually all time-value-of-money calculations. I will be replacing the Economic Analysis sample files with the CIM FEA demonstration in the appendix. I would be pleased to demonstrate the program any time at the workshop outside the presentation.

3. Loan payment routines could handle negative numbers the same way the DCF program does. However, the loan payment routines I evaluated used positive principal only. One allowed a negative interest rate.

4. Even with this aside, how do you get a positive cash flow in the first period (year) of a project? Some have suggested investment tax credit. While that might be possible, I would hate to enter an Internal Revenue Service audit with nothing spent and $1,000.00 claimed in tax credits. Even a 100 percent tax credit on the amount spent would be zero net cash flow, not $1,000.00. Still, a positive cash flow in period zero can be rationalized outside capital budgeting theory--a retainer for services to be rendered for example. We therefore concede some plausibility for Project C and evaluate the criticism as it has traditionally been stated.

5. Using the DCF program, enter cash flows as they occur, but look for negative cash flows and "(outflow)" just before IRR.

6. Either internal rate of either payment or outflow would be okay. Using the DCF program, total net cash flow, total discounted cash flow (NPV), and uniform periodic revenue with and without terminal value all show the direction of the flows (+/-). Also, "(outflow)" just before IRR indicates negative cash flows. IRR calculations cease and the reason printed if the program detects a noninterest or nonsense zone. NPV, on the other hand, is determined by the data entered. The DCF program deals only with positive rates from the point of view of either lender or borrower.

7. Increases in debt-equity ratios suggest higher risk to lenders, who in turn increase rates. This in turn would change the cost of capital.

8. The rate at which NPV is zero is between 697.14 and 697.15 percent. This is rounding error in the DCF program.

9. NPV is 0.058 percent of undiscounted net cash flow and IRR 763 percent at the maximum overshoot ($47.47 / $81,200). It is also in the nonsense zone. In electronics where oscillations are studied extensively, such near-solutions are called "lost in the noise." While the implied rate of 840 percent is the responsive IRR, this is an extreme case for both NPV and IRR. The 697.14 percent rate is 58 times a nominal 12 percent interest rate and perhaps 25 times what could be considered a good return on investment. Rates of return become meaningless at such high multiples. One way to insure only a insure only a responsive IRR is to limit its size, say to 100 percent. Some IRR algorithms do this. The view taken here is that it is unnecessary to limit IRR to less than 500 or 1,000 percent.

10. If the implied rate is close to 697.14 percent, such as 720 percent, the DCF program could settle on either of the two rates. Otherwise the DCF program will cease calculation and present the 697.14 percent IRR, e.g., it will not enter the nonsense zone. Multiple sign changes are flagged with a "(Sign +/-: *n*)," where *n* is the number of times signs changed.

11. Some of the IRR equations can include large or small numbers that exceed precision limits, especially floating-point limits, in programming languages. For example, a spreadsheet algorithm can appear to lock up (or actually lock up) when calculating extremely high IRRs. Since very large percentages become meaningless in a practical sense, placing a limit of, say 500 or 1,000 percent provides a satisfactory answer without lockup. I placed limits in these extremes when developing the DCF program. The DCF program calculates the first (smallest), *positive* IRR possible, but operating only in the relevant range. If cash flows are net-negative, "(outflow)" is printed before the IRR in the DCF output report. It calculates the first IRR consistent with NPV converging on zero with an increasing discount rate. It ceases calculation and notifies you if it enters a noninterest/nonsense zone. The program also cautions you with "(Sign +/-:n)" with multiple sign changes.

12. The DCF program treats zero and negative IRRs as incalculable, e.g., no *positive* rate exists. See Criticism Number One for an explanation of why. If you calculate the IRRs on S005N2.DC! and S00N3.DC! cash flows you will likely receive 0.00 and a -0.0010 percent IRRs respectively.

13. Some IRR algorithms calculate a zero IRR for N2 and a negative IRR for N3 and N4. Negative IRRs are logically inconsistent with positive discount rates. The reason IRR is incalculable is that any positive discount rate drives NPV more negative--away from zero. It is immediately in the nonsense zone. If we invert the flows and try again, the same thing happens. This implies negative interest from either borrower or lender point of view. DCF is unnecessary; the project should be rejected.

14. An IRR solution may still be possible if NPV turns again at subsequently higher discount rates and again converges on zero. However, this subsequent rate, if it exists, is questionable. The DCF program will not attempt to calculate IRRs in the nonsense zone.

15. IRR may be calculable with complex cash flows using other assumptions. For example, if you use either continuous or midyear discounting with S05P.DC!, IRR is 30.74 percent. In doing so you are accepting the default assumption of not discounting year zero. This is equivalent to accepting that the year zero flow occurs at the beginning of year one.

16. IRR calculations enter the nonsense zone at a $100.00 NPV and a 66.7 percent discount rate. The point at which NPV begins to increase with an increasing discount rate is not a meaningful number for analysis and is therefore not presented in the DCF output. I left a development feature in the program so you can find out what it was. Immediately after receiving " . . .diverging from zero" in a DCF output, save the file. Change the name first if you prefer. Then turn the preview option "On," list the files and select the one you just saved. The approximate discount rate at which NPV began diverging from zero will be displayed on the top line of the preview screen, off to the right, followed by the NPV at which it occurred. This will tell you where the nonsense zone begins.

17. Combining Criticism One (negative versus positive flows) and Two (mutually exclusive projects), then concluding that IRR is at fault is a favorite. But projects that will lose money do not compare well with those that will make money. IRR has nothing to do with

it. Nor should smaller projects with less money at risk be expected to generate equivalent NPVs. Combining two or more of them into one demonstration should not change the expectation.

18. Although not often formally criticized, high rates deserve mention. It puts IRR in what the British call the "too difficult" category (it's not really worth the trouble). Thousands, or millions of percent IRR are possible, although not very realistic or meaningful. A data entry error would be the more likely reason for an IRR in the hundreds of percent at acceptable risk. With bad data, typical IRR algorithms appear to "lock up" when attempting to calculate very large IRRs. Other algorithms, in spreadsheets for example, typically display "ERR" and leave it up to the user to determine what causes it. The DCF program attempts only to calculate an IRR up to 1,000 percent using annual discounting or proportionally greater using smaller periods. S05Q.DC! demonstrates a very high rate of return.

INTERNAL RATE OF RETURN IN DEFENSE ANALYSIS

Appendix

The purpose of this appendix is to illustrate internal rate of return (IRR) use in a defense application. The cost data uses are taken from the *Corporate Information Management (CIM) Functional Economic Analysis (FEA) Guidebook*, Version 1.1, 15 January 1993.

Figure 1 summarizes estimated and recurring and investment costs from the *Guidebook*. Net Present Value (NPV) analysis, using a spreadsheet NPV function[1] and the DCF program[2] gives the NPV at a 7 percent discount rate indicated in the bottom row. The differences (deltas) in columns C, E, and G are calculated by subtracting the sum of recurring and investment costs from recurring costs.[3]

Figure 2 provides the IRRs calculated in spreadsheet and by the DCF program. No IRR is calculable for the baseline. The spreadsheet IRR function gives "ERR." The DCF program gives "IRR incalculable--cash flows are all negative" or "...all positive" if cash flows are inverted.

The spreadsheet IRR function also gives "ERR" for the delta between baseline recurring cost and total Alternative A cost.[4] The DCF program gives the IRRs indicated using three different equations.

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[1] Lotus 1-2-3, Version 4.0 for Windows.

[2] Development version 1.1N, October 1992, Copyright © 1992, all rights reserved.

[3] Some IRR algorithms give unusual answers (or warn you that you may get unusual answers) if you enter positive cash flows in the early years followed by negative cash flows in the later years. The safest thing to do, if you are potentially using one of these algorithms, is calculate the difference as shown in Figure 1. With the DCF program if makes no difference.

[4] Lotus 1-2-3 provided no reason for not being able to calculate the IRR nor could I find out using on-screen help or the documentation. It might be an upper limit placed on IRRs, say 100 or 200 percent. It could also be overflow in the calculations. "ERR" is common in the IRR function of all spreadsheets I've used.

Figure 3 is a screen capture of the DCF program calculating the Alternative A Differential. The spreadsheet IRR function and the DCF program gave essentially the same answer using end-of-year discounting on the Alternative B delta. Both gave the same end-of-year NPV at a 7 percent discount rate of $449.0 for six years (Figure 3, Total Discounted Cash Flow, Figure 1 Column E, last row).

Figure , calculates DCF using middle-of-year (M-O-Y) discounting.

Figure 5 recalculates NPV by reentering the IRR as the discount rate, confirming the IRR.

Figure 6 is IRR confirmation using continuous discounting.

Figure 7 is the data entry screen.

Analysis

Assuming equal benefits, Alternative A is recommended:

- NPVs and IRRs confirm a significantly larger cost avoidance (savings).
- IRRs (119.84 through 231.53 percent) are significantly larger than for either the As-Is (no IRR, all negative) or Alternative B (24.66 through 27.97 percent).
- Risk is lowest for Alternative A.[5]

Figure 8 extends the study period to 20 years, carrying period 6 (FY99) cash flows out to the other 14 years. Please note that NPV increases significantly as expected, but IRR changes very little.

[5] For a rough comparison, U.S. industry actual rates of return are around 16 percent. A target rate of return for new, moderate risk investment might be around 50 percent. Estimated IRR for alternative B is only a little above U.S. actual and below what industry might use. Alternative A is well above it.

CIM FEA Costs

Period FY	B/L Recurring (A)	B/L Total (B)	Delta (C) (A-B)	Alt A (D)	A Delta (E) (A-D)	Alt B (F)	B Delta (G) (A-F)
1. FY94	$2,923.6	$2,942.7	($19.1)	$2,942.7	($19.1)	$2,942.7	($19.1)
2. FY95	$2,870.4	$2,891.6	($21.2)	$2,846.9	$23.5	$2,884.8	($14.4)
3. FY96	$2,821.1	$2,844.5	($23.4)	$2,751.1	$70.0	$2,826.9	($5.8)
4. FY97	$2,783.4	$2,797.5	($14.1)	$2,655.2	$128.2	$2,768.9	$14.5
5. FY98	$2,744.1	$2,750.4	($6.3)	$2,559.4	$184.7	$2,711.0	$33.1
6. FY99	$2,703.3	$2,703.3	$0.0	$2,463.6	$239.7	$2,653.1	$50.2
Total	$16,845.9	$16,930.0	($84.1)	$16,218.9	$627.0	$16,787.4	$58.5
E-O-Y NPV @ 7%	$13,423.6	$13,494.3	($70.7)	$12,974.6	$449.0	$13,390.6	$32.9

Data source: *CIM Functional EA Guidebook*, 15 Jun 93, pp. C-9 through C-11

Figure 1

IRR Calculations

Method (equation)	Algorithm	B/L Delta	A Delta	B Delta
End-of-year	Lotus 1-2-3	ERR	ERR	27.96%
End-of-year	DCF Program	...all negative	231.25%	27.97%
Middle-of-year	DCF Program	...all negative	231.53%	27.97%
Continuous	DCF Program	...all negative	119.84%	24.66%

Data source: *CIM Functional EA Guidebook*, 15 Jun 93, pp. C-9 through C-11

Figure 2

Alternative A Differential

Discounted Cash Flow

Prepared by	R. Martin
Date prepared	09-10-1993
Description	CIM FEA Differential
Study period	6 years
Discount rate	7.00 percent
Discount method	End-of-year

Project Year	B/L Recurring (-)	Alternative A (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
1	$2,923.60	$2,942.70	$19.10	0.9346	$17.85
2	$2,870.40	$2,846.90	-$23.50	0.8734	-$20.53
3	$2,821.10	$2,751.10	-$70.00	0.8163	-$57.14
4	$2,783.40	$2,655.20	-$128.20	0.7629	-$97.80
5	$2,744.10	$2,559.40	-$184.70	0.7130	-$131.69
6	$2,703.30	$2,463.60	-$239.70	0.6663	-$159.72
Total	$16,845.90	$16,218.90	-$627.00	4.7665	-$449.03

Uniform annual inflow without terminal value	-$94.20
Internal rate of return (IRR)(outflow)	231.25 percent
IRR—discount rate differential	224.25 percent

Data source: *CIM Functional EA Guidebook*, 15 Jan 93, pp. C-9 through C-11

Figure 3

Middle-of-Year Discounting

Discounted Cash Flow

Prepared by	R. Martin
Date prepared	09-10-1993
Description	CIM FEA Differential
Study period	6 years
Discount rate	7.00 percent
Discount method	Middle-of-year

Project Year	B/L Recurring (-)	Alternative A (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
1	$2,923.60	$2,942.70	$19.10	0.9673	$18.48
2	$2,870.40	$2,846.90	-$23.50	0.9040	-$21.24
3	$2,821.10	$2,751.10	-$70.00	0.8449	-$59.14
4	$2,783.40	$2,655.20	-$128.20	0.7896	-$101.23
5	$2,744.10	$2,559.40	-$184.70	0.7379	-$136.30
6	$2,703.30	$2,463.60	-$239.70	0.6897	-$165.31
Total	$16,845.90	$16,218.90	-$627.00	4.9334	-$464.74

Uniform annual inflow without terminal value	-$94.20
Internal rate of return (IRR)(outflow)	231.53 percent
IRR—discount rate differential	224.53 percent

Data source: *CIM Functional EA Guidebook*, 15 Jan 93, pp. C-9 through C-11

Figure 4

IRR Confirmation

Discounted Cash Flow

Prepared by	R. Martin
Date prepared	09-10-1993
Description	CIM FEA Differential
Study period	6 years
Discount rate	231.53 percent
Discount method	Middle-of-year

Project Year	B/L Recurring (-)	Alternative A (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
1	$2,923.60	$2,942.70	$19.10	0.6508	$12.43
2	$2,870.40	$2,846.90	-$23.50	0.1963	-$4.61
3	$2,821.10	$2,751.10	-$70.00	0.0592	-$4.14
4	$2,783.40	$2,655.20	-$128.20	0.0179	-$2.29
5	$2,744.10	$2,559.40	-$184.70	0.0054	-$1.00
6	$2,703.30	$2,463.60	-$239.70	0.0016	-$0.39
Total	$16,845.90	$16,218.90	-$627.00	0.9312	$0.00

Uniform annual inflow without terminal value	$0.00
Internal rate of return (IRR)(outflow)	231.53 percent
IRR—discount rate differential (outflow)	0.00 percent

Data source: *CIM Functional EA Guidebook*, 15 Jan 93, pp. C-9 through C-11

Figure 5

Continuous Confirmation

Discounted Cash Flow

Prepared by	R. Martin
Date prepared	09-10-1993
Description	CIM FEA Differential
Study period	6 years
Discount rate	119.84 percent
Discount method	Continuous

Project Year	B/L Recurring (-)	Alternative A (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
1	$2,923.60	$2,942.70	$19.10	0.5827	$11.13
2	$2,870.40	$2,846.90	-$23.50	0.1758	-$4.13
3	$2,821.10	$2,751.10	-$70.00	0.0530	-$3.71
4	$2,783.40	$2,655.20	-$128.20	0.0160	-$2.05
5	$2,744.10	$2,559.40	-$184.70	0.0048	-$0.89
6	$2,703.30	$2,463.60	-$239.70	0.0015	-$0.35
Total	$16,845.90	$16,218.90	-$627.00	0.8338	$0.00

Uniform annual inflow without terminal value	$0.00
Internal rate of return (IRR)(outflow)	119.84 percent
IRR—discount rate differential	0.00 percent

Data source: *CIM Functional EA Guidebook*, 15 Jan 93, pp. C-9 through C-11

Figure 6

Study Period Increased

Discounted Cash Flow

D i s c o u n t e d   C a s h   F l o w

Prepared by	R. Martin
Date prepared	09-10-1993
Description	CIM FEA Differential
Study period	20 years
Discount rate	7.00 percent
Discount method	End-of-year

Project Year	B/L Recurring (-)	Alternative A (+)	Net Cash Flow	Discount Factor	Discounted Cash Flow
1	$2,923.60	$2,942.70	$19.10	0.9346	$17.85
2	$2,870.40	$2,846.90	-$23.50	0.8734	-$20.53
3	$2,821.10	$2,751.10	-$70.00	0.8163	-$57.14
4	$2,783.40	$2,655.20	-$128.20	0.7629	-$97.80
5	$2,744.10	$2,559.40	-$184.70	0.7130	-$131.69
6	$2,703.30	$2,463.60	-$239.70	0.6663	-$159.72
7	$2,703.30	$2,463.60	-$239.70	0.6227	-$149.27
8	$2,703.30	$2,463.60	-$239.70	0.5820	-$139.51
9	$2,703.30	$2,463.60	-$239.70	0.5439	-$130.38
10	$2,703.30	$2,463.60	-$239.70	0.5083	-$121.85
11	$2,703.30	$2,463.60	-$239.70	0.4751	-$113.88
12	$2,703.30	$2,463.60	-$239.70	0.4440	-$106.43
13	$2,703.30	$2,463.60	-$239.70	0.4150	-$99.47
14	$2,703.30	$2,463.60	-$239.70	0.3878	-$92.96
15	$2,703.30	$2,463.60	-$239.70	0.3624	-$86.88
16	$2,703.30	$2,463.60	-$239.70	0.3387	-$81.19
17	$2,703.30	$2,463.60	-$239.70	0.3166	-$75.88
18	$2,703.30	$2,463.60	-$239.70	0.2959	-$70.92
19	$2,703.30	$2,463.60	-$239.70	0.2765	-$66.28
20	$2,703.30	$2,463.60	-$239.70	0.2584	-$61.94
Total	$54,692.10	$50,709.30	-$3,982.80	10.5940	-$1,845.87

Uniform annual inflow without terminal value	-$174.24
Internal rate of return (IRR)(outflow)	233.44 percent
IRR—discount rate differential	226.44 percent

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Figure 8