EFFECTS OF GROUND REFLECTIONS ON FREQUENCY MODULATED SIGNALS

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It is well known that ground reflections cause ambiguities in radar detection and tracking. A more serious problem is the interference that occurs when the phase difference between the direct signal and the ground reflected signal is small. This has been well studied in the context of the usual pulsed signals. There is little that can be done to entirely eliminate this problem. In this report the effect on frequency modulated signals of reflections from surfaces with different statistics is investigated. The ground reflection studied in this report is the coherent reflection. Four different kinds of rough surfaces (normal, exponential, Rayleigh and skew-normal) are considered. In each case an explicit expression for the received signal is obtained. Although the normal distribution is the most commonly used model, in applications the surfaces often have asymmetric characteristics. The three other distributions considered in this report are asymmetric. It is found that the received signals for normal and exponential surfaces have quite different amplitude characteristics. It is found that the Gaussian model may be a reasonable approximation for such asymmetric surfaces only when the Rayleigh roughness parameter is small.

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Preface

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Effects Of Ground Reflection On Frequency Modulated Signals

1. INTRODUCTION

It is well-known that radar detection is often adversely affected by ground reflections [Evans, 1966; Smith and Mrstik, 1979][1,2]. Although several techniques have been devised to alleviate this problem there are still situations where the problem can be acute. Particularly in low-angle radar tracking, the problem is sometimes irremediable [Barton, 1974][3]. The fact that the ground surface is almost always irregular introduces further complications and demands careful study [Papa et al, 1983][4]. It must be noted here that all studies thus far on the effects of ground reflections have been restricted to conventional pulsed radars. However, chirp radars use frequency modulated signals and the effects of ground reflections on such signals may be quite different from those on ordinary pulsed signals. This report concentrates on the effect of ground reflections on frequency modulated signals.

2. DESCRIPTION OF THE PROBLEM

The geometry of the problem is shown in Figure 1. The ground is represented as a perfectly conducting random surface whose mean

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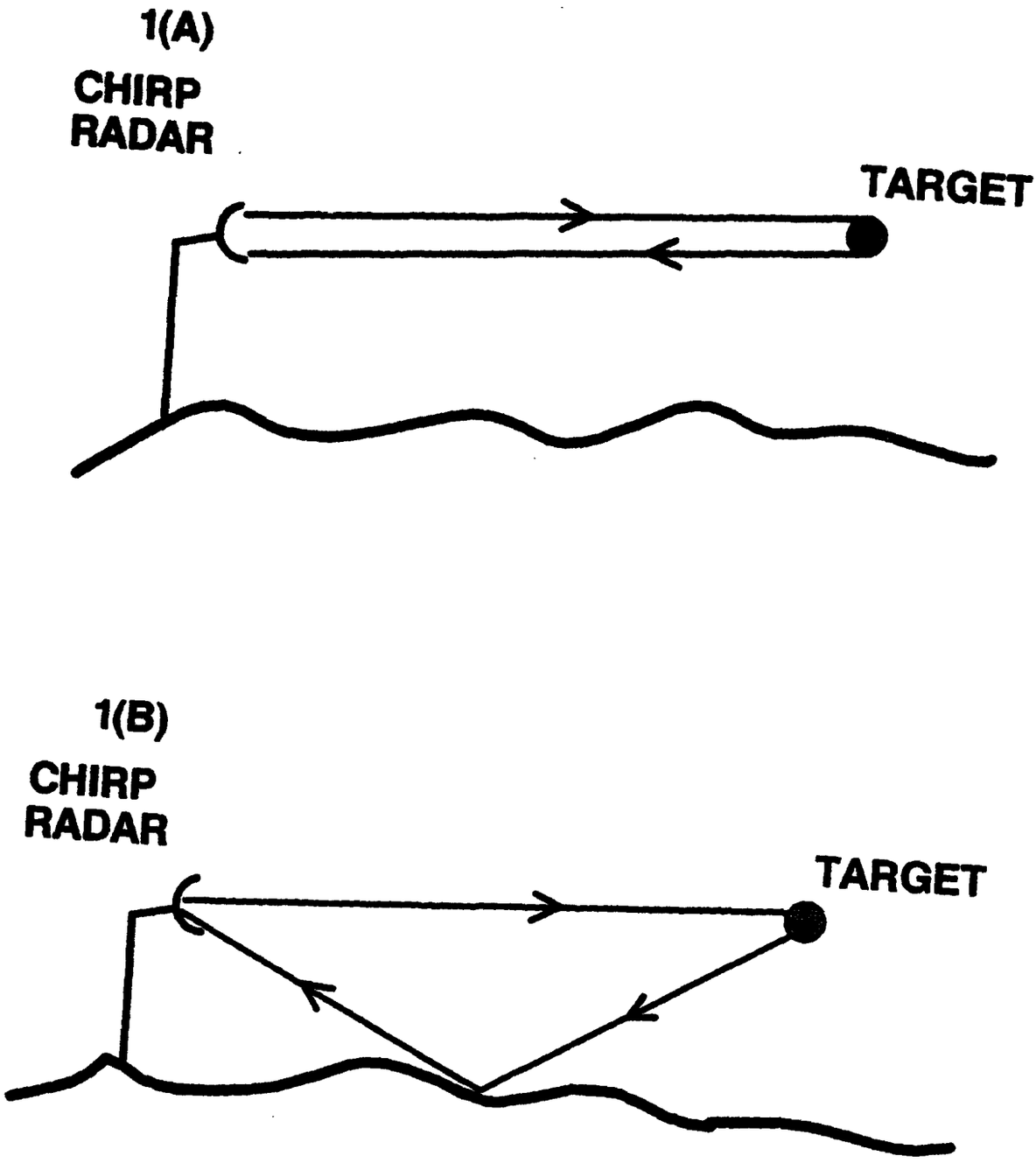

FIGURE.1 SCATTERING GEOMETRY

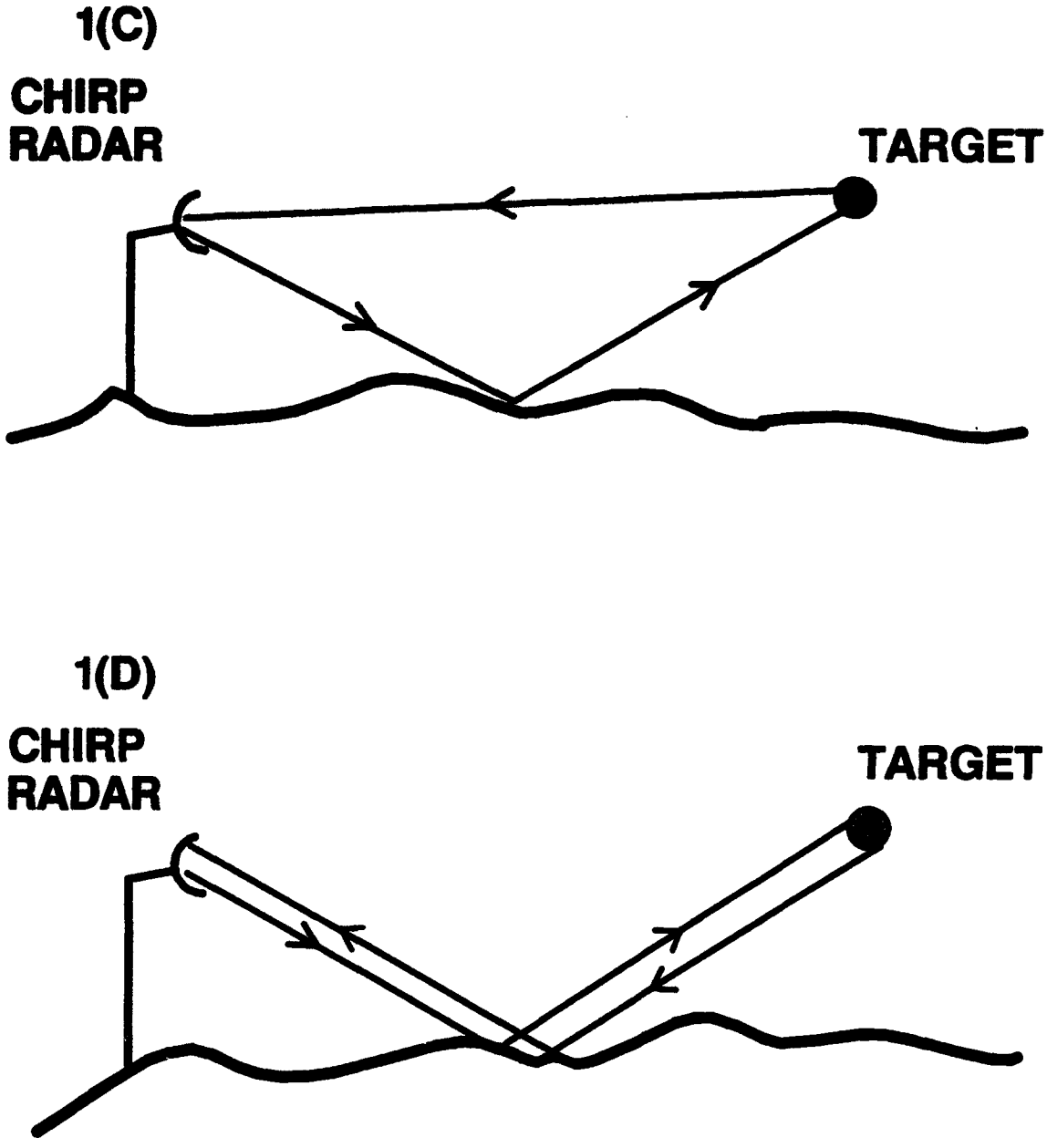

FIGURE.1 SCATTERING GEOMETRY

coincides with the xy-plane. Moreover the surface is assumed to be isotropic and smoothly varying in slopes so that the Kirchhoff method may be used for analysis.

The main quantity of interest to us in this report is the average power of the signal received by the chirp radar. There are four contributions to this quantity as shown in Figure 1. The target is assumed to have unit reflectivity. Both the target and the radar antenna are assumed to be isotropic. The ground reflection that we refer to in this report is the specular reflection or the coherent reflection.

Let $f_1(t)$, $f_2(t)$, $f_3(t)$ and $f_4(t)$ represent the received signals corresponding to the four cases shown schematically in Figure 1. The angle between the coherent ray and the z-axis is denoted as θ while the reflection coefficient associated with this ray is denoted as R. With these notations we have the relation

$$f_2(t) = R\, f_1(t+D) \tag{1}$$

where D is the delay caused by the path difference between the direct signal (Figure 1a) and the ground reflected signal (Figure 1b). Also it is clear that

$$f_3(t) = f_2(t) \tag{2}$$

For the same reason

$$f_4(t) = R^2\, f_1(t+2D) \tag{3}$$

Thus the received signal f(t) is related to $f_1(t)$ as

$$f(t) = f_1(t) + 2R\, f_1(t+D) + R^2\, f_1(t+2D) \tag{4}$$

Hereinafter we shall use capital letters to denote Fourier transforms of corresponding signals. For example

$$F_1(\omega) = \int_{-\infty}^{\infty} dt\ f_1(t)\ \exp(i\omega t) \tag{5}$$

is the Fourier transform of the signal $f_1(t)$. Fourier transformation of Eq.(4) leads to

$$F(\omega) = \left[\ 1 + 2Re^{-i\omega D} + R^2 e^{-i2\omega D}\ \right] F_1(\omega) \tag{6}$$

The signal transmitted by the chirp radar $f_0(t)$ is a linear frequency-modulated pulse defined as:

$$f_0(t) = \begin{cases} \exp\left[- i(\omega_c t + .5\ mt^2)\right] & \text{for } |t| \le T_o \\ 0 & \text{for } |t| > T_o \end{cases} \tag{7}$$

where ω_c is the angular carrier frequency, T_o is half the pulse width, and m is the parameter determining the extent of frequency modulation.

On reception the signal passes through a matched filter whose transfer function $H(\omega)$ is given as

$$H(\omega) = \exp\left[- \frac{i}{2m} (\omega_c - \omega)^2 \right] \tag{8}$$

This results in an effective pulse compression [Cook, 1960; Cohen, 1987][5,6]. Denoting the signal after this filtering as g(t) and its Fourier transform as $G(\omega)$,

$$G(\omega) = H(\omega)\ F(\omega) \tag{9}$$

On substituting Eq.(6) into Eq.(9)

$$G(\omega) = G_0(\omega) + G_1(\omega)\ e^{-i\omega D} + G_2(\omega)\ e^{-i2\omega D} \tag{10}$$

where

$$G_0(\omega) = H(\omega)\ F_1(\omega) \tag{11a}$$

$$G_1(\omega) = 2\ H(\omega)\ R\ F_1(\omega) \tag{11b}$$

$$G_2(\omega) = R^2\ H(\omega)\ F_1(\omega) \tag{11c}$$

Inverse Fourier transformation of Eq.(10) leads to

$$g(t) = g_0(t) + g_1(t+D) + g_2(t+2D) \tag{12}$$

Notice that $g_0(t)$ is the direct signal while $g_1(t+D)$ and $g_2(t+2D)$ are the ground reflected signals with delays D and 2D respectively. Quite often in practice these delays are very small and are much less than the pulse width of the signal. This inevitably results in an interference pattern that can be detrimental to radar detection and tracking. Clearly this problem is most acute when $g_1(t)$ or $g_2(t)$ is of the same magnitude as $g_0(t)$. For this reason, a significant part of this report will be devoted to the study of $g_1(t)$ and $g_2(t)$ for various kinds of rough surfaces.

In Section 3 we study the direct signal (Figure 1a). As mentioned above there are two ground reflected signals :- $g_1(t)$ and $g_2(t)$. One ground reflection is involved in $g_1(t)$ whereas $g_2(t)$ undergoes two such reflections (Figure 1d). These reflections are indeed coherent reflections and they invariably depend on the statistical

characteristics of the surface under consideration. In fact the coherent reflection coefficient R is given as $\chi(v_z)$ [Beckmann and Spizzichino, 1963][7] where χ is the characteristic function and $v_z = 2\omega\sqrt{\mu_0}\sqrt{\varepsilon_0}\cos\theta$. We consider several statistical distributions and in each case study the characteristics of ground reflected signals. The characteristic functions of the four statistical distributions considered in this report are given in Appendix A. The corresponding coherent reflection coefficients are given in Appendix B. Section 4 is devoted to the analysis of $g_1(t)$. For each statistical distribution, explicit expressions are derived for $g_1(t)$. In Section 5 similar expressions for $g_2(t)$ are obtained for normal and exponential distributions. The results thus obtained are studied in comparison to one another in Section 6. The conclusions are presented in Section 7.

3. DIRECT SIGNAL

Since we have assumed that the target has unit reflectivity, the direct signal $f_1(t)$ is identical to the transmitted signal $f_0(t)$ apart from the time delay, which depends on the range. For the analysis in this report, this time delay is irrelevant. We therefore let $f_1(t) = f_2(t)$. Thus

$$F_1(\omega) = \int_{-T_o}^{T_o} dt \exp\left[-i(\omega_c t + 0.5mt^2)\right] e^{i\omega t} \qquad (13)$$

It follows from Eq.(11a) that

$$g_0(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, H(\omega)\, F_1(\omega)\, e^{-i\omega t}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \exp\left[-\frac{i}{2m} (\omega_c - \omega)^2 \right] e^{-i\omega t}$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left[-i(\omega_c \tau + 0.5\, m\tau^2) \right] e^{i\omega\tau}$$

$$= \frac{1}{2\pi} \exp\left[-\frac{i}{2\pi} \omega_c^2 \right] \int_{-T_o}^{T_o} d\tau \exp\left[-i(\omega_c \tau + 0.5\, m\tau^2) \right] A_1 \qquad (14)$$

where

$$A_1 = \int_{-\infty}^{\infty} d\omega \exp\left[-\frac{i}{2m} (\omega^2 - 2\omega\omega_c) - i\omega(t - \tau) \right]$$

$$= (2\pi m)^{.5}\, e^{-i\pi/4} \exp\left[\frac{i}{2m} (\omega_c - mt + m\tau)^2 \right] \qquad (15)$$

On substituting Eq.(15) into Eq.(14) and evaluating the integral we obtain

$$g_0(t) = \left(\frac{m}{2\pi} \right)^{.5} \exp\left[i\left(.5mt^2 - \omega_c t - \pi/4 \right) \right] 2\, T_o\, \mathrm{sinc}\,[mT_o t] \qquad (16)$$

Thus the direct signal has the anticipated sharp peak at $t = 0$ and a 4 dB pulse width of $\frac{\pi}{mT_o}$. Noting that the pulse width of the transmitted signal is $2T_o$ we see that the ratio of pulse compression achieved is $2mT_o^2/\pi$.

4. GROUND REFLECTED SIGNAL I

Taking the inverse Fourier transform of Eq.(11b) we get

$$g_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; 2R \; H(\omega) \; F_1(\omega) \; e^{-i\omega t} \tag{17}$$

We note that the reflection coefficient R for a normal distribution is nondispersive since its statistical characteristics are symmetrical. However, in applications, one notices that many surfaces do not possess this property. To study such situations we consider as examples three other distributions that have asymmetric characteristics. The reflection coefficients R associated with these statistical distributions are given in Appendix B. Below we analyze each case for the ground reflected signal I.

4.1 Normal Distribution

Substituting Eqs.(8), (13) and (B3) into Eq.(17) we get

$$g_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; 2 \; \exp\left[-.5a^2\omega^2 - \frac{i}{2m} \left(\omega_c - \omega \right)^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \; \exp\left(- i\omega_c\tau - .5im\tau^2 + i\omega\tau - i\omega t \right) \tag{18a}$$

$$= \frac{1}{\pi} \; \exp\left[- \frac{i}{2m} \omega_c^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \; \exp\left(- i\omega_c\tau - .5im\tau^2 \right) A_2 \tag{18b}$$

where we have used Eq.(B3) which gives $R = \exp\left(-0.5\alpha^2\omega^2\right)$ for the reflection coefficient.

$\alpha = 2\left(\mu_o\varepsilon_o\right)^{0.5}\sigma\cos\theta$.

$$A_2 = -\int_{-\infty}^{\infty} d\omega \exp\left[-\left(\frac{\alpha^2}{2} - \frac{1}{2m} \right) \omega^2 - i\left(t - \tau - \omega_c/m \right) \right]$$

$$-\left(\pi/p_0 \right)^{.5} \exp\left[-\frac{1}{4p_0} \left(\omega_c/m + \tau - t \right)^2 \right] \qquad (19)$$

and where

$$p_0 = .5\alpha^2 + \frac{i}{2m} \qquad (20)$$

Substituting Eq.(19) back into Eq.(18) and simplifying, we obtain

$$g_1(t) = \left(\pi p_0 \right)^{-.5} \exp\left[-\frac{i}{2m}\omega_c^2 - \frac{1}{4p_0} \left(t - \omega_c/m \right)^2 \right] A_3 \qquad (21)$$

where

$$A_3 = -\int_{T_o}^{T_o} d\tau \exp\left(-a_1\tau^2 - b_1\tau \right) \qquad (22)$$

$$a_1 = \frac{im}{2} + \frac{1}{4p_0} \qquad (23a)$$

and

$$b_1 = \omega_c \left(i + \frac{1}{2p_0 m} \right) - \frac{t}{2p_1} \qquad (23b)$$

The integral in Eq.(22) may readily be expressed in terms of error functions as follows

$$A_3 = 0.5 \left(\pi/a_1 \right)^{0.5} \exp\left[\frac{b_1^2}{4a_1} \right]$$

$$\cdot \left\{ \operatorname{erf}\left[\sqrt{a_1} \left(T_o + \frac{b_1}{2a_1} \right) \right] - \operatorname{erf}\left[\sqrt{a_1} \left(- T_o + \frac{b_1}{2a_1} \right) \right] \right\} \qquad (24)$$

Thus the explicit solution for the ground reflected signal I for a normal distribution of the ground surface is contained in Eqs.(21), (23) and (24).

4.2 Exponential Distribution

From Eqs.(8), (13), (B5) and (17) we get

$$g_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; 2 \frac{e^{- i\alpha\omega}}{1 - i\alpha\omega} \exp\left[- \frac{i}{2m} \left(\omega_c - \omega \right)^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left(- i\omega_c \tau - .5im\tau^2 - i\omega\tau - i\omega t \right)$$

$$= \frac{1}{\pi} \exp\left[- \frac{i}{2m} \omega_c^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left(- i\omega_c \tau - .5im\tau^2 \right) A_4 \qquad (25)$$

where

$$A_4 = \int_{-\infty}^{\infty} d\omega \, \frac{1}{1-i\alpha\omega} \exp\left(- ia\omega^2 + ib\omega \right) \tag{26}$$

where

$$a = \frac{1}{2m} \tag{27a}$$

$$b = \omega_c/m + \tau - t - \alpha \tag{27b}$$

Integration of Eq.(26) results in

$$A_4 = - \exp\left[i\frac{b^2}{4a} \right] \frac{\pi}{\alpha} \exp\left(- 2a\beta_1^2 \right) \operatorname{erfc}\left(i\sqrt{(2a)}\beta_1 \right) \tag{28}$$

where

$$\beta_1 = 0.5 \left\{ \left[\frac{b}{2a} - \frac{1}{\alpha} \right] + i \left[\frac{b}{2a} + \frac{1}{\alpha} \right] \right\} \tag{29}$$

Substituting Eq.(28) in Eq.(25)

$$g_1(t) = - \frac{1}{\alpha} \exp\left[- \frac{i}{2m} \omega_c^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left(- i\omega_c \tau - .5im\tau^2 \right)$$

$$\exp\left[i\frac{b^2}{4a} - 2a\beta_1^2 \right] \operatorname{erfc}\left(i\sqrt{(2a)}\, \beta_1 \right) \tag{30}$$

Use of Eq.(27b) leads to the simplified form:

$$g_1(t) = - \frac{1}{\alpha} \exp\left[- \frac{i}{2m} \omega_c^2 + i\frac{\Omega_1}{4a} \right] A_5 \tag{31}$$

where

$$\Omega_1 = \omega_c/m - t - \alpha \tag{32}$$

$$A_5 = -\int_{T_o}^{T_o} d\tau \exp\left(-im\tau(t+\alpha)\right) \cdot \exp\left[-2a\beta_1^2\right] \operatorname{erfc}\left(i\sqrt{(2a)}\beta_1\right) \tag{33}$$

It does not appear possible to express A_5 in terms of known functions. But we note the following. Since $\omega_c - mt - m\alpha \gg m\tau$ in our domain of interest, namely, $T_o \le t$, if $\tau \le T_o$ it is clear that

$\exp\left[-2a\beta_1^2\right] \operatorname{erfc}\left(i\sqrt{(2a)}\beta_1\right)$ is a slowly varying function of τ. We therefore approximate it by its mean value

$$\exp\left[-2a\beta_2^2\right] \operatorname{erfc}\left(i\sqrt{(2a)}\beta_2\right)$$

where

$$\beta_2 = 0.5\left\{\left[\frac{\Omega_1}{2a} - \frac{1}{\alpha}\right] + i\left[\frac{\Omega_1}{2a} + \frac{1}{\alpha}\right]\right\} \tag{34}$$

When this approximation is used, Eq.(33) becomes

$$A_5 = \exp\left[-2a\beta_2^2\right] \operatorname{erfc}\left(i\sqrt{(2a)}\beta_2\right) A_6 \tag{35}$$

where

$$A_6 = -\int_{T_o}^{T_o} d\tau \exp\left(-im\tau(t+\alpha)\right)$$

$$= 2T_o \operatorname{sinc}\left(mT_o(t+\alpha)\right) \tag{36}$$

Thus from Eqs.(31), (35), and (36)

$$g_1(t) = -\frac{1}{\alpha} \exp\left[-\frac{i}{2m}\omega_c^2 + i\frac{\Omega_1^2}{4a}\right]$$

$$\cdot \exp\left[-2a\beta_2^2\right] \operatorname{erfc}\left(i\sqrt{(2a)}\beta_2\right)$$

$$\cdot 2T_o \operatorname{sinc}\left[mT_o(t+\alpha)\right] \tag{37}$$

4.3 Rayleigh Distribution

Using Eqs.(8), (13), and (B6) in Eq.(17) we get

$$g_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, 2 \exp\left[-i\sqrt{\pi\tilde{\alpha}}\omega - \frac{i}{2m}\left(\omega_c - \omega\right)^2\right]$$

$$\cdot \left\{1 + i\sqrt{\pi\tilde{\alpha}}\omega \operatorname{erfc}\left(-i\tilde{\alpha}\omega\right) \exp\left(-\tilde{\alpha}^2\omega^2\right)\right\}$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left[-i\omega_c\tau - .5im\tau^2 + i\omega\tau - i\omega t\right] \tag{38}$$

$$= \frac{1}{\pi} \exp\left[-\frac{i}{2m}\omega_c^2\right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left[-i\omega_c\tau - .5im\tau^2\right]\left[I_1 + I_2 + I_3\right] \tag{39}$$

where

$$I_1 = \int_{-\infty}^{\infty} d\omega \exp\left[i\omega(\tau - t) - \frac{i}{2m}\left(\omega^2 - 2\omega\omega_c\right) - i\sqrt{\pi\tilde{\alpha}}\omega\right] \tag{40a}$$

$$I_2 = \int_{-\infty}^{\infty} d\omega \exp\left[i\omega(\tau - t) - \frac{i}{2m}\left(\omega^2 - 2\omega\omega_c\right)\right]$$

$$\cdot i\sqrt{\pi\tilde{\alpha}}\omega \exp\left[-i\sqrt{\pi\tilde{\alpha}}\omega - \tilde{\alpha}^2\omega^2\right] \tag{40b}$$

$$I_3 = -\int_{-\infty}^{\infty} d\omega \exp\left[i\omega(\tau - t) - \frac{i}{2m}\left(\omega^2 - 2\omega\omega_c\right)\right]$$

$$\cdot\ i\sqrt{\pi}\tilde{\alpha}\omega \quad \exp\left(- i\sqrt{\pi}\tilde{\alpha}\omega - \tilde{\alpha}^2\omega^2\right) \operatorname{erf}(- i\tilde{\alpha}\omega) \tag{40c}$$

I_1 and I_2 may be readily evaluated in terms of elementary functions as follows:

$$I_1 = \left(2\pi m\right)^{.5} \exp\left[- i\pi/4 + i\frac{m}{2}\left(\omega_c/m + \tau - t - \sqrt{\pi}\tilde{\alpha} \right)^2 \right] \tag{41}$$

$$I_2 = - i\tilde{\alpha}\pi \frac{b_2}{2a_2^{1.5}} \exp\left[\frac{b_2^2}{4a_2} \right] \tag{42}$$

where

$$a_2 = \tilde{\alpha}^2 + \frac{i}{2m} \tag{43a}$$

$$b_2 = - i\left(\omega_c/m + \tau - t - \sqrt{\pi}\tilde{\alpha} \right) \tag{43b}$$

It turns out that I_3 can be expressed in terms of a confluent hypergeometric function.

$$I_3 = - \frac{2\tilde{\alpha}^2}{a_2^{1.5}} \Phi_1 \left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{\tilde{\alpha}^2}{a_2}, \frac{b_2^2}{4a_2} \right] \tag{44}$$

where

$$\Phi_1(a,b;c,c';z,\zeta) = \sum_{k=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{(a)_{k+\ell}\,(b)_k}{(c)_k\,(c')_\ell} \frac{z^k\,\zeta^\ell}{k!\;\ell!} \tag{45}$$

$$(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)} \tag{46}$$

With these results we can write $g_1(t)$ as

$$g_1(t) = \frac{1}{\pi} \exp\left[-\frac{i}{2m}\omega_c^2\right] [J_1 + J_2 + J_3] \tag{47}$$

where

$$J_\ell = -\int_{T_o}^{T_o} d\tau \exp\left(-i\omega_c\tau - .5im\tau^2\right) I_\ell \ , \ \ell = 1,2,3 \tag{48}$$

J_1 is readily evaluated as

$$J_1 = (2\pi m)^{.5}\, 2T_o \exp\left[-i\pi/4 + i\frac{m}{2}\Omega_2^2\right] \operatorname{sinc}\left[\left(t + \sqrt{\pi\tilde{\alpha}}\right)mT_o\right] \tag{49}$$

$$\Omega_2 = \omega_c - mt - m\sqrt{\pi\tilde{\alpha}} \tag{50}$$

After some rearrangements J_2 may be written as

$$J_2 = -\tilde{\alpha}\pi\left(\frac{1}{m}\right)\frac{1}{2a_2^{1.5}} \exp\left[\frac{\Omega_2^2}{4a_2m^2}\right] B \tag{51}$$

where

$$B = -\int_{T_o}^{T_o} d\tau\ (m\tau + \Omega_2) \exp\left(-a_3\tau^2 - b_3\tau\right) \tag{52}$$

and where

$$a_3 = \frac{1}{4a_0} + \frac{im}{2} \tag{53a}$$

$$b_3 = \frac{\Omega_2}{2a_0 m} + i\omega_c \tag{53b}$$

$$a_0 = \tilde{\alpha}^2 + \frac{i}{2m} \tag{53c}$$

On evaluating the integral in Eq.(52).

$$B = \exp\left[\frac{b_3^2}{4a_3} \right] (j_1 + j_2) \tag{54}$$

where

$$j_1 = \frac{m}{2a_3} \left\{ \exp\left[- a_3 \left(- T_o + \frac{b_3}{2a_3} \right)^{.5} \right] \right.$$

$$\left. - \exp\left[- a_3 \left(T_o + \frac{b_3}{2a_3} \right)^{.5} \right] \right\} \tag{55a}$$

$$j_2 = \left[\Omega_2 - \frac{mb}{2a_2} \right] \frac{1}{2} \left(\frac{\pi}{a_3} \right)^{.5}$$

$$\left[\operatorname{erf}\left\{ \sqrt{a_3} \left[T_o + \frac{b_3}{2a_3} \right] \right\} - \operatorname{erf}\left\{ \sqrt{a_3} \left[- T_o + \frac{b_3}{2a_3} \right] \right\} \right] \tag{55b}$$

It is apparent that the integral in J_3 is too complicated for analytic evaluation. But since $\omega_c/m - t - \sqrt{\pi\tilde{\alpha}} \gg \tau$ in $-T_o \le \tau, t \le T_o$ we note that Ψ_1 is a slowly varying function of τ in the domain of the integrand. This enables us to approximate

$\Psi_1 \left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{\tilde{\alpha}^2}{a_2}, \frac{b_2^2}{4a_2} \right]$ by its mean value

$\Psi_1 \left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{\tilde{\alpha}^2}{a_2}, -\frac{\Omega_2^2}{4a_2} \right]$.

Thus

$$J_3 = - \frac{\sqrt{\pi}\tilde{\alpha}^2}{a_2^{1.5}} \Psi_1 \left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{\tilde{\alpha}^2}{a_2}, -\frac{\Omega_2^2}{4a_2} \right] A_7 \tag{56}$$

where

$$A_7 = \int_{-T_o}^{T_o} d\tau \exp\left(-i\omega_c\tau - .5im\tau^2\right)$$

$$= \exp\left[\, i\,\frac{\omega_c^2}{2m}\,\right]\frac{1}{2}\,(\pi/m)^{.5}\,(1-i)$$

$$\left\{ \operatorname{erf}\left[\, .5\sqrt{m}\,(1+i)\left(\, T_o+\omega_c/m\,\right)\,\right]\right.$$

$$\left. - \operatorname{erf}\left[\, .5\sqrt{m}\,(1+i)\left(-\, T_o+\omega_c/m\,\right)\,\right]\right\} \qquad (57)$$

Equation (47), together with Eqs.(49), (51), (54), (55), (56), and (51) provide the explicit solution for the $g_1(t)$.

4.4 Skew-Normal Distribution

Substituting Eqs.(8), (13), and (B8) into Eq.(17) we have

$$g_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\; 2\exp\left[\, -\frac{i}{2m}\,\left(\,\omega_c-\omega\right)^2\,\right]$$

$$\cdot\,\frac{1}{2\sigma}\exp\left[\, i\,\left(\,\frac{2}{\pi}\,\right)^{.5}\,(\alpha_1-\alpha_2)\,\omega\,\right]$$

$$\cdot\left\{\sigma_1\exp\left(\,-.5\alpha_1^{\,2}\,\omega^2\,\right)\operatorname{erfc}\left[\,-\frac{i}{\sqrt{2}}\,\alpha_1\omega\,\right]\right.$$

$$\left. +\,\sigma_2\exp\left(\,-.5\alpha_2^{\,2}\,\omega^2\,\right)\operatorname{erfc}\left[\,-\frac{i}{\sqrt{2}}\,\alpha_2\omega\,\right]\right\}$$

$$\cdot\int_{-T_o}^{T_o} d\tau\exp\left(\,-i\omega_c\tau - .5im\tau^2 + i\omega\tau - i\omega t\,\right) \qquad (58)$$

Let

$$g_1(t) = s_1(t) + s_2(t) \tag{59}$$

where

$$s_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; 2 \exp\left[-\frac{i}{2m} \left(\omega_c - \omega \right)^2 \right]$$

$$\cdot \frac{1}{2\sigma} \exp\left[i \left(\frac{2}{\pi} \right)^{.5} (\alpha_1 - \alpha_2) \, \omega \right]$$

$$\cdot \sigma_1 \exp\left(-.5\alpha_1^2 \, \omega^2 \right) \operatorname{erfc}\left[-\frac{i}{\sqrt{2}} \alpha_1 \omega \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left(-i\omega_c \tau - .5im\tau^2 + i\omega\tau - i\omega t \right) \tag{60a}$$

$$s_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; 2 \exp\left[-\frac{i}{2m} \left(\omega_c - \omega \right)^2 \right]$$

$$\cdot \frac{1}{2\sigma} \exp\left[i \left(\frac{2}{\pi} \right)^{.5} (\alpha_1 - \alpha_2) \, \omega \right]$$

$$\cdot \sigma_2 \exp\left(-.5\alpha_2^2 \, \omega^2 \right) \operatorname{erfc}\left[\frac{i}{\sqrt{2}} \alpha_2 \omega \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left(-i\omega_c \tau - .5im\tau^2 - i\omega\tau - i\omega t \right) \tag{60b}$$

Let us first consider $s_1(t)$. The RHS of Eq.(60a) may be rearranged as follows.

$$s_1(t) = \frac{\sigma_1}{2\pi\sigma} \int_{-T_o}^{T_o} d\tau \exp\left(- i\omega_c \tau - .5im\tau^2 \right) A_8 \tag{61}$$

where

$$A_8 = \int_{-\infty}^{\infty} d\omega \exp\left[- \frac{i}{2m} \left(\omega_c - \omega \right)^2 + i\omega(\tau - t) \right]$$

$$\cdot \exp\left[-.5\alpha_1^2 \omega^2 + i \left(\frac{2}{\pi} \right)^{.5} (\alpha_1 - \alpha_2)\, \omega \right] \operatorname{erfc}\left[- \frac{i}{\sqrt{2}} \alpha_1 \omega \right]$$

$$= \exp\left[- \frac{i}{2m} \omega_c^2 \right] \int_{-\infty}^{\infty} d\omega \exp\left(- p_1\omega^2 - q_1\omega \right) \operatorname{erfc}\left(\gamma_1 \omega \right) \tag{62}$$

and where

$$p_1 = .5\left(\alpha_1^2 + i/m \right) \tag{63a}$$

$$q_1 = - i \left[\frac{\omega_c}{m} + \left(\frac{2}{m} \right)^{.5} (\alpha_1 - \alpha_2) + \tau - t \right] \tag{63b}$$

and

$$\gamma_1 = - i\alpha_1/\sqrt{2} \tag{63c}$$

The integral in Eq.(62) may be represented in terms of the hypergeometric function mentioned earlier.

$$A_8 = \exp\left[-\frac{i}{2m}\omega_c^2 \right] \left\{ \left(\frac{\pi}{p_1} \right)^{.5} \exp\left[\frac{q_1^2}{4p_1} \right] \right.$$

$$\left. + \frac{\gamma_1 q_1}{p_1^{1.5}} \Psi_1\left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{\gamma_1^2}{p_1}, \frac{q_1^2}{4p_1} \right] \right\} \tag{64}$$

Substituting Eq.(64) into Eq.(61) and rearranging we get

$$s_1(t) = .5(\pi p_1)^{-.5} \frac{\sigma_1}{\sigma} \exp\left[-\frac{i}{2m}\omega_c^2 - \frac{\Omega^2}{4p_1} \right] A_3(\alpha_1, \lambda_1)$$

$$+ \frac{\sigma_1}{2\pi\sigma} \exp\left[-\frac{i}{2m}\omega_c^2 \right] A_9 \tag{65}$$

where

$$\Omega = \frac{\omega_c}{m} + \left(\frac{2}{m} \right)^{.5}(\alpha_1 - \alpha_2) - t \tag{66}$$

$$\alpha_1 = .25/p_1 + i.5m \tag{67a}$$

$$\lambda_1 = .5\Omega/p_1 + i\omega_c \tag{67b}$$

and

$$A_9 = -\int_{T_o}^{T_o} d\tau \exp\left(-i\omega_c\tau - .5im\tau^2 \right)$$

$$\frac{\gamma_1 q_1}{p_1^{1.5}} \Psi_1\left[\frac{3}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{\gamma_1^2}{p_1}, \frac{q_1^2}{4p_1} \right] \tag{68}$$

Note that A_3 is the same integral as in Eq.(22) - Only the variables are now different. Here the result may immediately be obtained from Eq.(24).

In the integral in Eq.(68) we may employ the same approximation as before, namely, identify the slowly varying part of the integrand by its mean value. To be more specific,

$$q\,\Psi_1\left[\frac{3}{2}\,,\,\frac{1}{2}\,;\,\frac{3}{2}\,,\,\frac{3}{2}\,;\,-\frac{\gamma_1^2}{p_1}\,,\,\frac{q_1^2}{4p_1}\right]$$

$$\simeq -\,i\,\Omega\,\Psi_1\left[\frac{3}{2}\,,\,\frac{1}{2}\,;\,\frac{3}{2}\,,\,\frac{3}{2}\,;\,-\frac{\gamma_1^2}{p_1}\,,-\frac{\Omega^2}{4p_1}\right] \qquad (69)$$

Thus

$$A_9 = -\,i\,\Omega\,\frac{\gamma_1}{p_1^{1.5}}\,\Psi_1\left[\frac{3}{2}\,,\,\frac{1}{2}\,;\,\frac{3}{2}\,,\,\frac{3}{2}\,;\,-\frac{\gamma_1^2}{p_1}\,,-\frac{\Omega^2}{4p_1}\right] A_7 \qquad (70)$$

Equation (65) along with Eqs.(70) and (57) provides the solution for $s_1(t)$.

We note from Eq.(60) that the integrand in $s_2(t)$ is very similar in structure to that in $s_1(t)$. Thus the procedure for evaluating $s_2(t)$ is identical to that of $s_1(t)$. We therefore omit the steps and simply present the results.

$$s_2(t) = .25\left(a_2p_2\right)^{-.5}\,\frac{\sigma_2}{\sigma}\,\exp\left[-\frac{i}{2m}\,\omega_c^2-\frac{\Omega^2}{4p_2}+\frac{b_2}{4a_2}\right]$$

$$\left[\operatorname{erf}\left\{\sqrt{a_2}\left[T_o+\frac{b_2}{2a_2}\right]\right\}-\operatorname{erf}\left\{\sqrt{a_2}\left[-T_o+\frac{b_2}{2a_2}\right]\right\}\right]$$

$$-\,.25\,\Omega\,(\pi m)^{-.5}\,\frac{\sigma_2}{\sigma}\,\frac{\gamma_2}{p_2^{1.5}}(1+i)\,\Psi_1\left[\frac{3}{2}\,,\,\frac{1}{2}\,;\,\frac{3}{2}\,,\,\frac{3}{2}\,;-\frac{\gamma_2^2}{p_2}\,,\,\frac{\Omega^2}{4p_2}\right]$$

$$\left\{\operatorname{erf}\left[.5\sqrt{m}\,(1+i)\left(T_o+\omega_c/m\right)\right]\right.$$

$$\left.-\operatorname{erf}\left[.5\sqrt{m}\,(1+i)\left(-T_o+\omega_c/m\right)\right]\right\} \qquad (71)$$

where

$$\alpha_2 = .25/p_2 + i.5m \tag{72a}$$

$$\beta_2 = .5\Omega/p_2 + i\omega_c \tag{72b}$$

$$p_2 = .5\left(\alpha_2^2 + i/m\right) \tag{72c}$$

$$\gamma_2 = i\alpha_2/\sqrt{2} \tag{72d}$$

This completes our derivation for $g_1(t)$.

5. GROUND REFLECTED SIGNAL-II

On taking the inverse Fourier transform of Eq.(11c)

$$g_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, R^2\, H(\omega)\, F_1(\omega)\, e^{-i\omega t} \tag{73}$$

In this section we evaluate Eq.(73) for two cases - normal distribution and exponential distribution.

5.1 Normal Distribution

Substituting Eqs.(8), (13) and (B3) into Eq.(73) we have

$$g_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \exp\left[-\frac{i}{2m} (\omega_c^2 - \omega) - \alpha^2\omega^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left\{ -i\omega_c\tau - .5im\tau^2 + i\omega\tau \right\} \qquad (74)$$

On comparing Eq.(74) with Eq.(18a) we can immediately write down the result as

$$g_2(t) = .5\, g_1(t, 2\alpha^2) \qquad (75)$$

5.2 Exponential Distribution

From Eqs.(8), (13), (B5) and (73) we have

$$g_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \left\{ \frac{e^{-i\tilde{\alpha}\omega}}{1 - i\tilde{\alpha}\omega} \right\}^{.5} \exp\left[-\frac{i}{2m} (\omega_c - \omega)^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left\{ -i\omega_c\tau - .5im\tau^2 + i\omega(\tau - t)^2 \right\}$$

$$= \frac{1}{2\pi} \exp\left[-\frac{i}{2m} \omega_c^2 \right]$$

$$\cdot \int_{-T_o}^{T_o} d\tau \exp\left\{ -i\omega_c\tau - .5im\tau^2 \right\} B_1 \qquad (76)$$

where

$$B_1 = -\int_{-\infty}^{\infty} d\omega \left\{ \frac{e^{-i\tilde{\alpha}\omega}}{1-i\tilde{\alpha}\omega} \right\}^2 \exp\left[-\frac{i}{2m} (\omega_c - 2\omega\omega_c)^2 + i\omega(\tau - t) \right]$$

$$= \frac{2\pi}{\tilde{\alpha}^2} \exp\left\{ .25i\pi + .25i\tilde{b}^2/a \right\}$$

$$\cdot \left[\left(\frac{a}{\pi} \right)^{.5} - ia\beta_2 \exp\left(-a\beta_2^2\right) \operatorname{erfc}\left(i\beta_2\sqrt{a}\right) \right] \tag{77}$$

where

$$\tilde{b} = b - \alpha \tag{78a}$$

$$\beta_2 = e^{i\pi/4} \left[\frac{\tilde{b}}{2a} + \frac{i}{\alpha} \right] \tag{78b}$$

Substituting Eq.(77) into Eq.(76) and simplifying we get

$$g_2(t) = \frac{1}{\tilde{\alpha}^2} \exp\left\{ .25i\pi - \frac{i}{2m}\omega_c^2 + .5i\Omega_3 m \right\} \left[\left(\frac{a}{\pi} \right)^{.5} \vartheta_1 + \vartheta_2 \right] \tag{79}$$

where

$$\vartheta_1 = -\int_{-T_o}^{T_o} d\tau \exp\left\{ -im\tau(t+2\alpha) \right\} \tag{80a}$$

$$\vartheta_2 = -ia \int_{-T_o}^{T_o} d\tau \exp\left\{ -im\tau(t+2\alpha) \right\}$$

$$\cdot \beta_2 \exp\left(-a\beta_2^2\right) \operatorname{erfc}\left(i\beta_2\sqrt{a}\right) \tag{80b}$$

$$\Omega_3 = \Omega_1 - \alpha \tag{80c}$$

Let us first consider the integral in Eq.(80b). Notice that $\operatorname{erfc}\left(i\beta_2\sqrt{a}\right)$ is a slowly varying function of τ and hence may be approximated by its mean value $\bar{\beta}_2 \exp(-a\bar{\beta}_2^2)\operatorname{erfc}(i\bar{\beta}_2\sqrt{a})$. Here

$$\bar{\beta}_2 = e^{i\pi/4}\left[\frac{\Omega_3}{2a} + \frac{i}{\bar{\alpha}}\right] \tag{81}$$

Thus

$$\mathcal{I}_2 = -\,ia\bar{\beta}_2 \exp(-a\bar{\beta}_2^2)\operatorname{erfc}(i\bar{\beta}_2\sqrt{a})\,\mathcal{I}_1 \tag{82}$$

$\mathcal{I}_1$ is readily evaluated as

$$\mathcal{I}_1 = 2T_o \operatorname{sinc}\left(m(t+2\bar{\alpha})T_o\right) \tag{83}$$

Putting all these together

$$g_2(t) = \frac{1}{\bar{\alpha}^2}\exp\left\{.25i\pi - \frac{i}{2m}\omega_c^2 + .5i\Omega_3 m\right\}$$

$$\left[\left(\frac{a}{\pi}\right)^{.5} - ia\bar{\beta}_2 \exp(-a\bar{\beta}_2^2)\operatorname{erfc}(i\bar{\beta}_2\sqrt{a})\right]\mathcal{I}_1 \tag{84}$$

6. DISCUSSION

In Sections 4 and 5 we have calculated the ground reflected signal of the chirp radar generated by four different statistical distributions. In this section we analyze these results and offer some physical interpretations. We first look at GRS-I.

6.1 Ground Reflected Signal - I

For normal distribution $g_1(t)$ is given by Eqs.(21) and (24). Although it is elementary to compute this expression to obtain a numerical value, the present form does not offer a physically transparent picture. In other words, the characteristics of the results are not immediately evident from the expressions in Eqs.(21) and (24).

One way to gain insight is to examine the local behavior. Recall that the signal has a peak at $t = 0$. Moreover our primary interest is in the study of the relative amplitude of the ground reflected signal with respect to the direct signal. We therefore look at the case when $t = 0$. We further simplify our task by restricting our attention to the region where $\alpha^2\omega_c^2 \ll 1$. Thus to a first order in $\alpha^2\omega_c^2$

$$g_1(0) = 2\left(\frac{2m}{\pi}\right)^{.5} T_o\,[1-.5\alpha^2\omega_c^2] \tag{85}$$

Note that $\alpha\omega_c$ is the Rayleigh parameter. Hence $\alpha^2\omega_c^2 \ll 1$ refers to the small roughness region. When $\alpha\omega_c = 0$, that is, when the surface is perfectly smooth, Eq.(85) reduces to

$$|g_1(0)| = 2T_o\left(2m/\pi\right)^{.5} \tag{86}$$

Also from Eq.(16) we see that

$$|g_0(0)| = T_o\left(2m/\pi\right)^{.5} \tag{87}$$

This implies that

$$|g_1(0)| = 2\,|g_0(0)| \tag{88}$$

From Appendix B it is clear that when $\alpha\omega_c = 0$, $R = 1$, hence it follows that

$$g_1(t) = 2g_0(t) \tag{89}$$

Thus it is seen that Eq.(85) is in agreement with Eq.(89). Also, Eq.(85) suggests that $|g_1(0)|$ should decrease with increasing $\alpha\omega_c$. This is in agreement with the physical fact that the coherent return should decrease with an increase in surface roughness.

We turn our attention now to the exponential case. As before we look at the special case when $\alpha^2\omega_c^2 \ll 1$ and $t = 0$. It turns out that to a first order in $\alpha^2\omega_c^2$, $|g_1(0)|_{exp}$ has the same expression as Eq.(85). This means that to a first order in $\alpha^2\omega_c^2$, coherent returns from normally distributed and exponentially distributed surfaces are identical.

It is apparent that the results obtained for surfaces with Rayleigh and skew-normal statistics have fairly complicated structures. Even from a computational point of view the resulting expressions in the above cases are unmanageable. It turns out that for parameters appropriate for our problem the series representation of the hypergeometric function is not suitable for actual computations. Unless further study of these special functions are undertaken the value of these results may not be fully appreciated.

6.2 Ground Reflected Signal - II

Consider the results obtained for normally distributed surfaces. When $t = 0$ and $\alpha^2\omega_c^2 \ll 1$ we have

$$|g_2(0)| = T_o \left[2m/\pi \right]^{.5} \left\{ 1 - 2\,\alpha^2\omega_c^2 \right\} \qquad (90)$$

Notice that when $\alpha\omega_c = 0$, Eq.(90) leads to

$$|g_2(0)| = |g_0(0)| \qquad (91)$$

which agrees with the physical fact that $g_2(t) = g_0(t)$ when $\alpha\omega_c = 0$. Apart from having all similar characteristics as the corresponding $|g_1(0)|$ we note from Eq.(90) that $|g_2(0)|$ decreases much faster with $\alpha\omega_c$ than $|g_1(0)|$.

6.3 Numerical Results

Figure 2 is a plot of the computed results for GRS-I. More specifically it is a plot of $|\,g_1(t)/g_0(t)|$ versus t. The parameters chosen are to an extent arbitrary but at the same time pertinent to our problem. The solid lines and the dotted lines correspond respectively to normal and exponential statistics. The main signal centered around t=0 is the compressed pulse. The rest are sidelobes. When $\alpha = 0.01$ (Figure 2a) the normal and the exponential results coincide and $|\,g_1(t)/g_0(t)|_{max} = 2$. Since $\alpha = 0.01$ corresponds to $\alpha\omega_c = 0.015$ both these results are in agreement with our predictions. When $\alpha = 0.07$ or $\alpha\omega_c = 0.104$ (Figure 2b) we still see that the normal and exponential results coincide. But notice now that the amplitude of the coherent signal is reduced. For further increase in α we note that $|\,g_1(t)/g_0(t)|_{normal}$ decreases faster than $|\,g_1(t)/g_0(t)|_{exponential}$. The point to note is that in some situations these results can be quite different from one another.

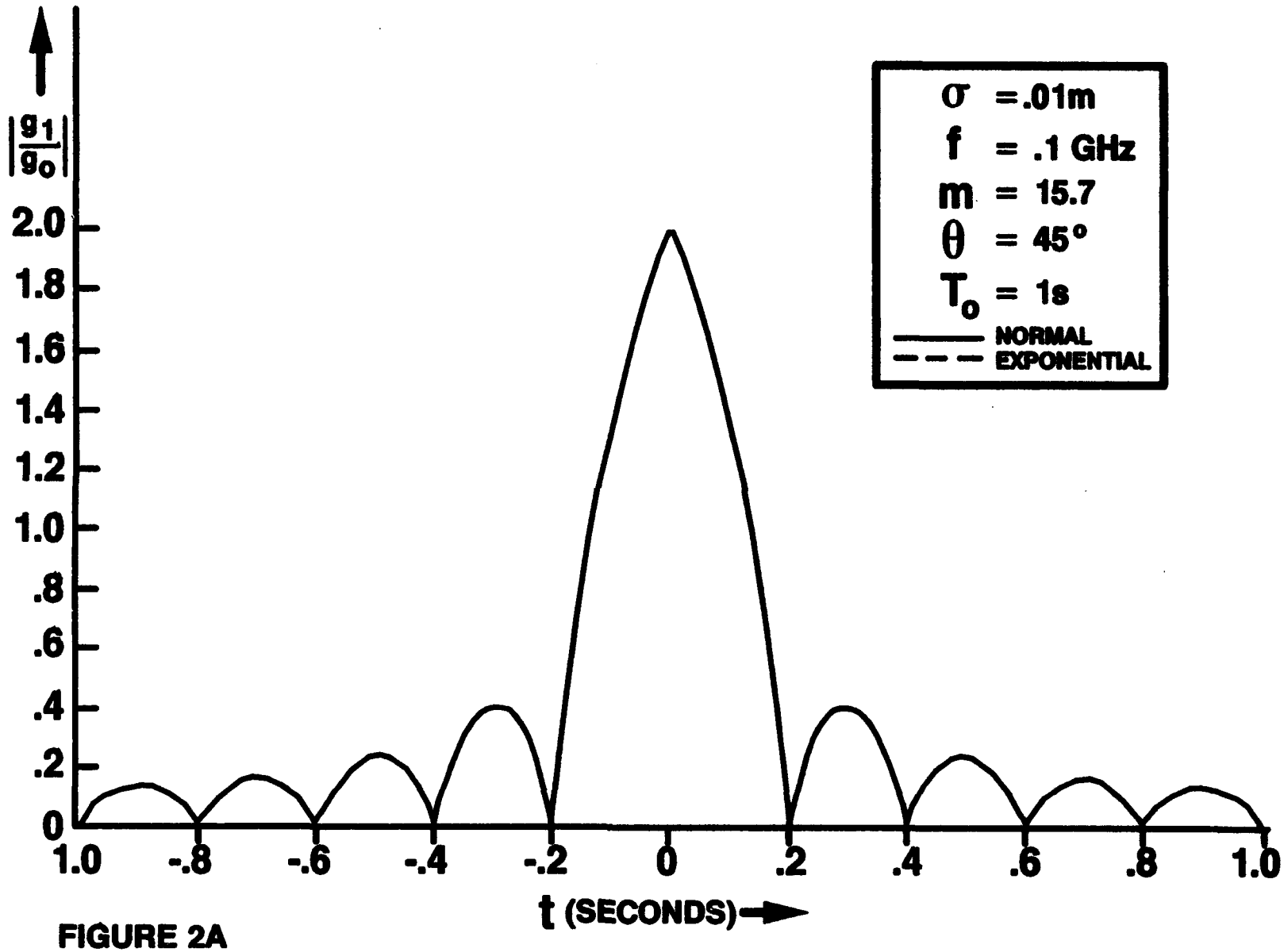
σ =.01m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
$\left|\frac{g_1}{g_o}\right|$
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 2A

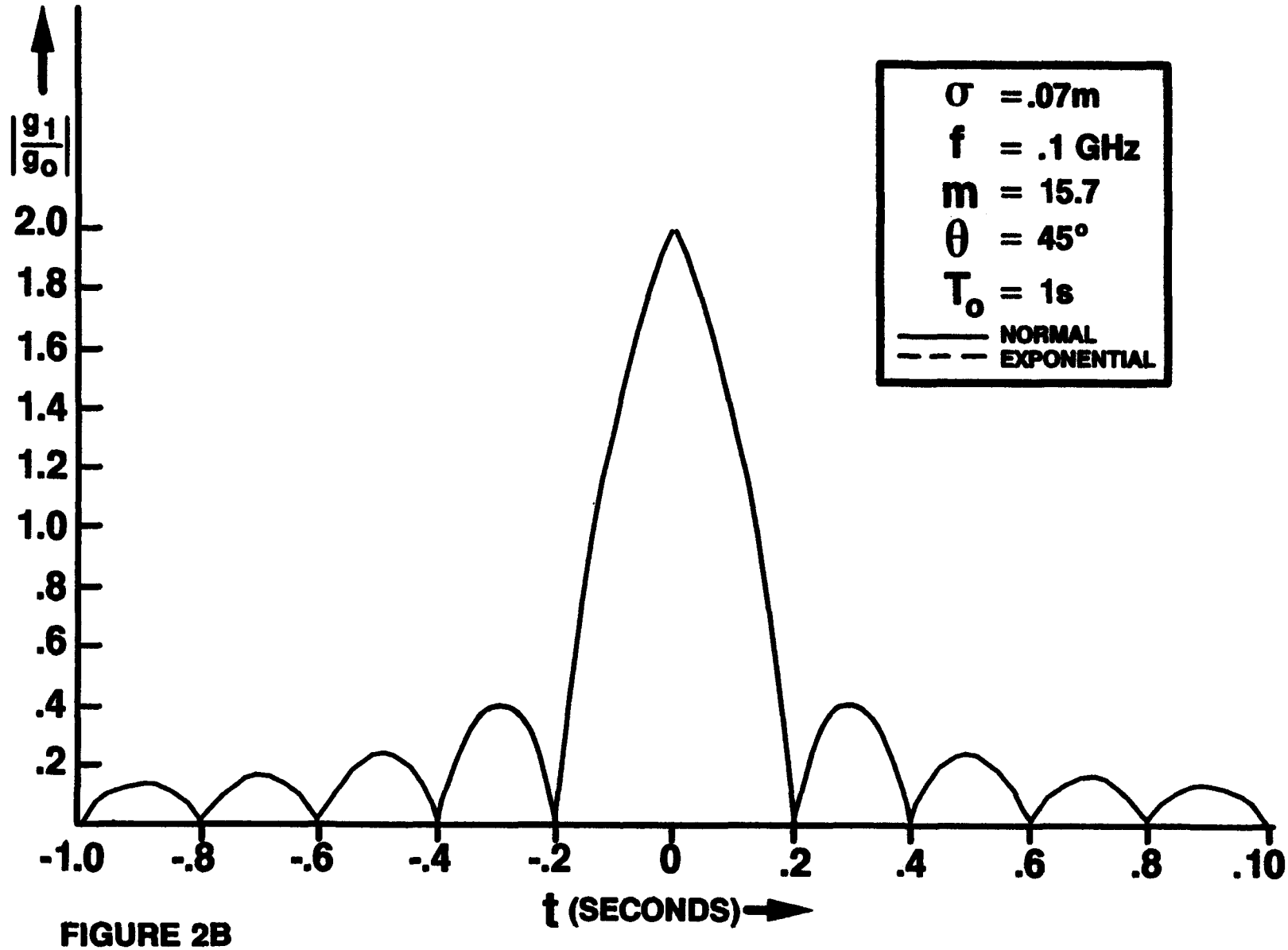
$\left|\frac{g_1}{g_o}\right|$
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
.10
t (SECONDS)
σ =.07m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL

FIGURE 2B

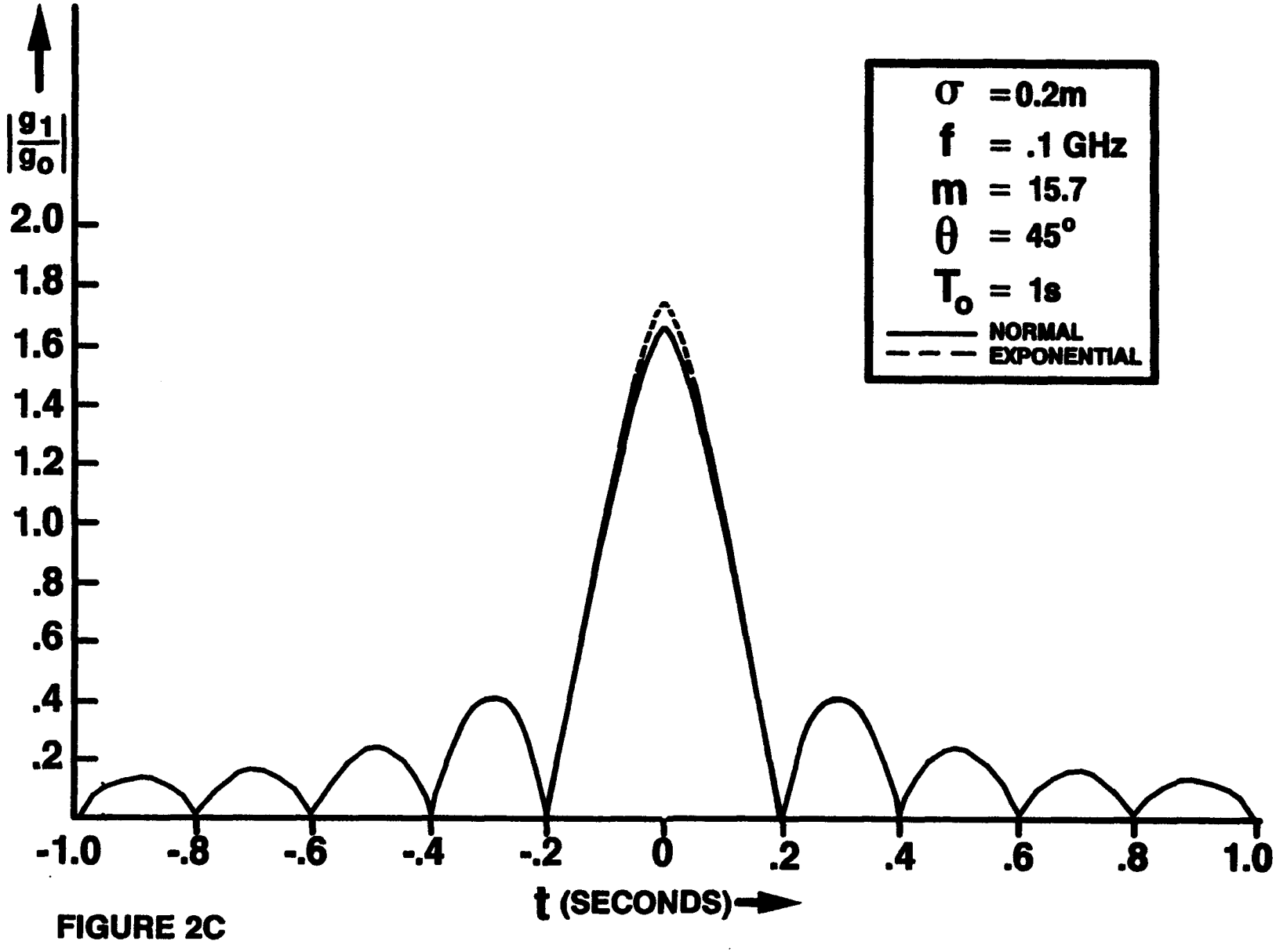
σ =0.2m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
$\left|\frac{g_1}{g_o}\right|$
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 2C

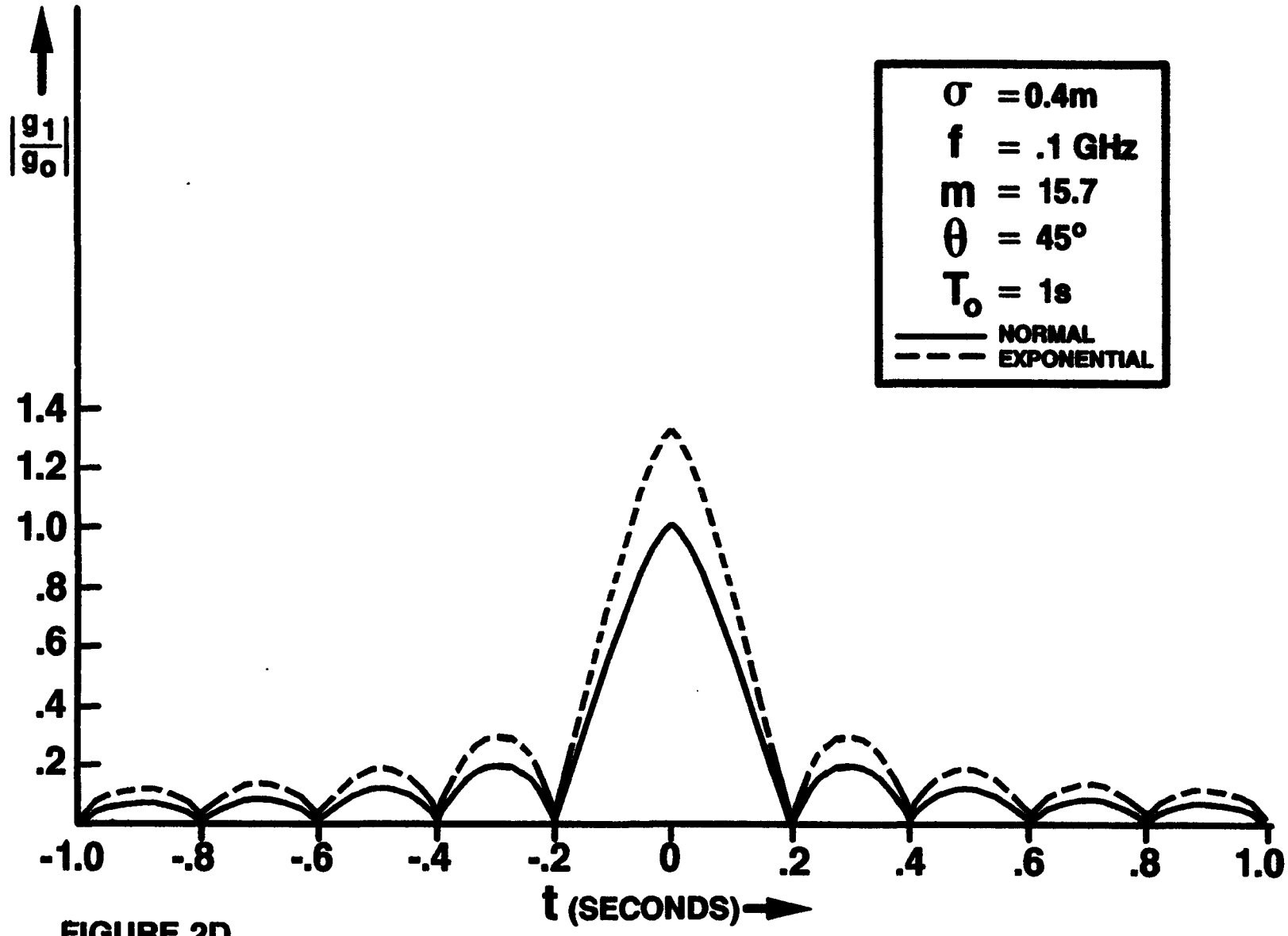
σ =0.4m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
|g1/g0|
1.4
1.2
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 2D

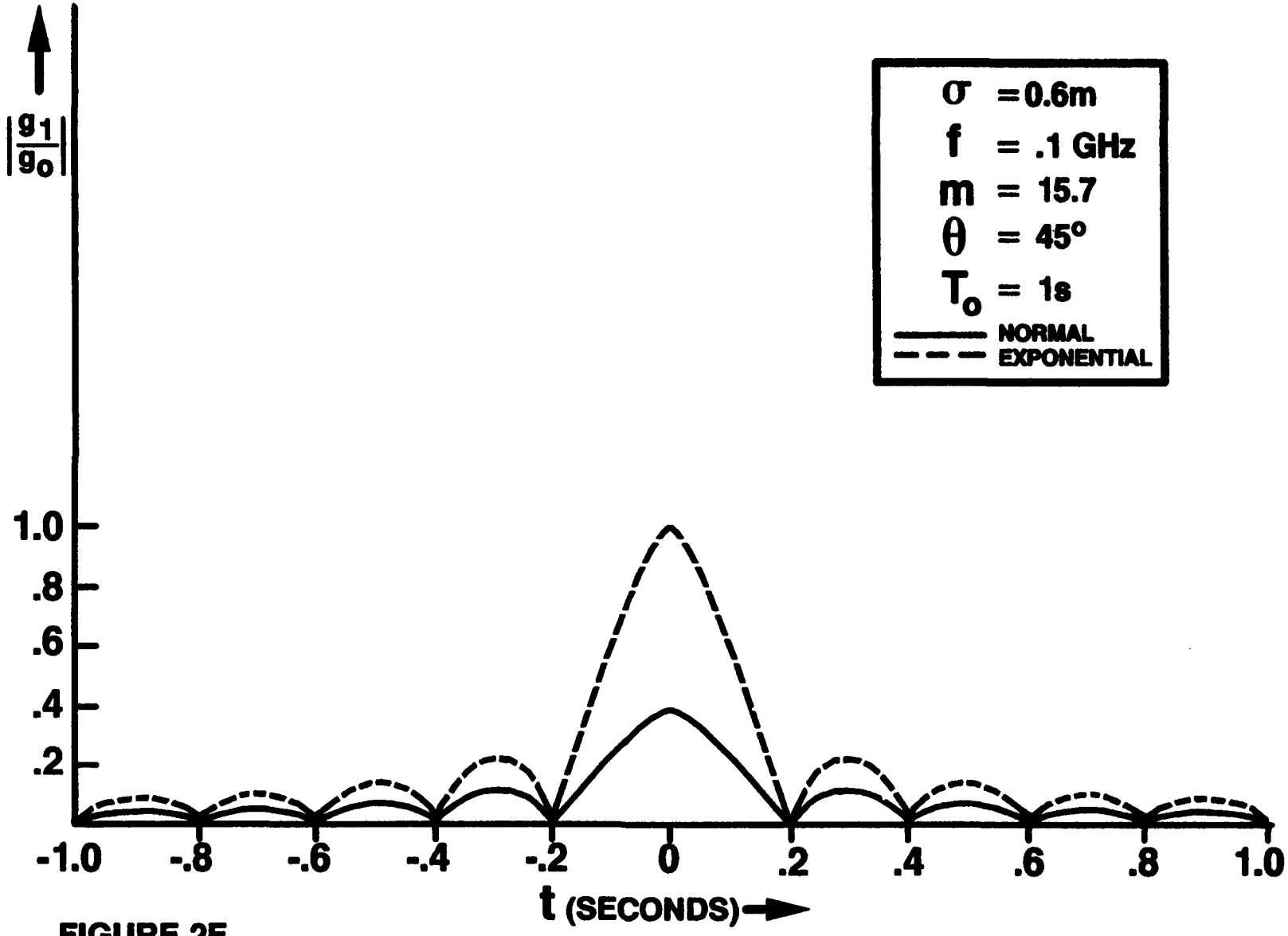
σ = 0.6m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
$\left|\frac{g_1}{g_o}\right|$
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 2E

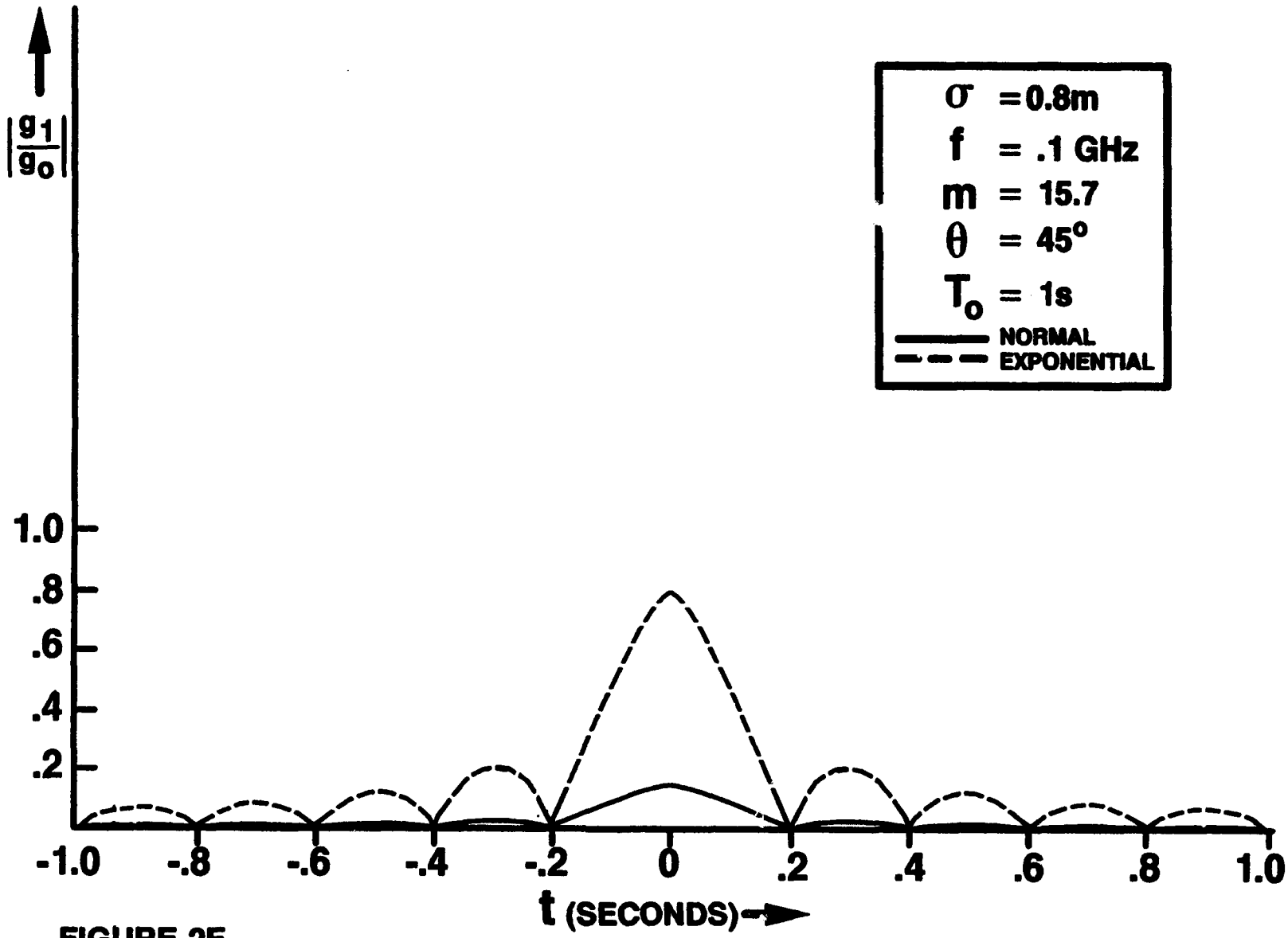
$\left|\frac{g_1}{g_o}\right|$
σ =0.8m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
1.0
.8
.6
.4
.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 2F

In Figure 3 we have plotted GRS-II. Recalling the fact that for $\alpha\omega_c = 0$, $g_2(t) = 2g_1(t)$ we have appropriately chosen the scale so that the results for $g_2(t)$ may be easily compared with those of $g_1(t)$. It is evident that $g_2(t)$ has all the characteristics of $g_1(t)$. But notice that the rate of decrease of $|g_2(t)|$ with $\alpha\omega_c$ is much higher than that of $|g_1(t)|$ - a fact clearly in agreement with our predictions.

In applications, one often models the ground as a normally distributed surface. This is primarily because such a model facilitates easy analysis. But there is growing experimental evidence that contradicts this assumption. For example, terrain and a large class of sea surfaces do not have the symmetric roughness geometry of a normally distributed rough surface. To be more precise, the valleys are often more shallow and the hills more steep.

The exponential distribution is a good candidate for an asymmetric rough surface. The results obtained in this report bear out that the assumption of normal statistics can be meaningful only when $\alpha\omega_c$ is small. Otherwise this assumption can lead to significant errors.

The exponential distribution is a rather extreme case of asymmetric distribution. A model more suitable for application is the Rayleigh distribution. For terrain, perhaps the most appropriate distribution to use is the skew-normal. Further study of the results pertaining to these models is left for future work.

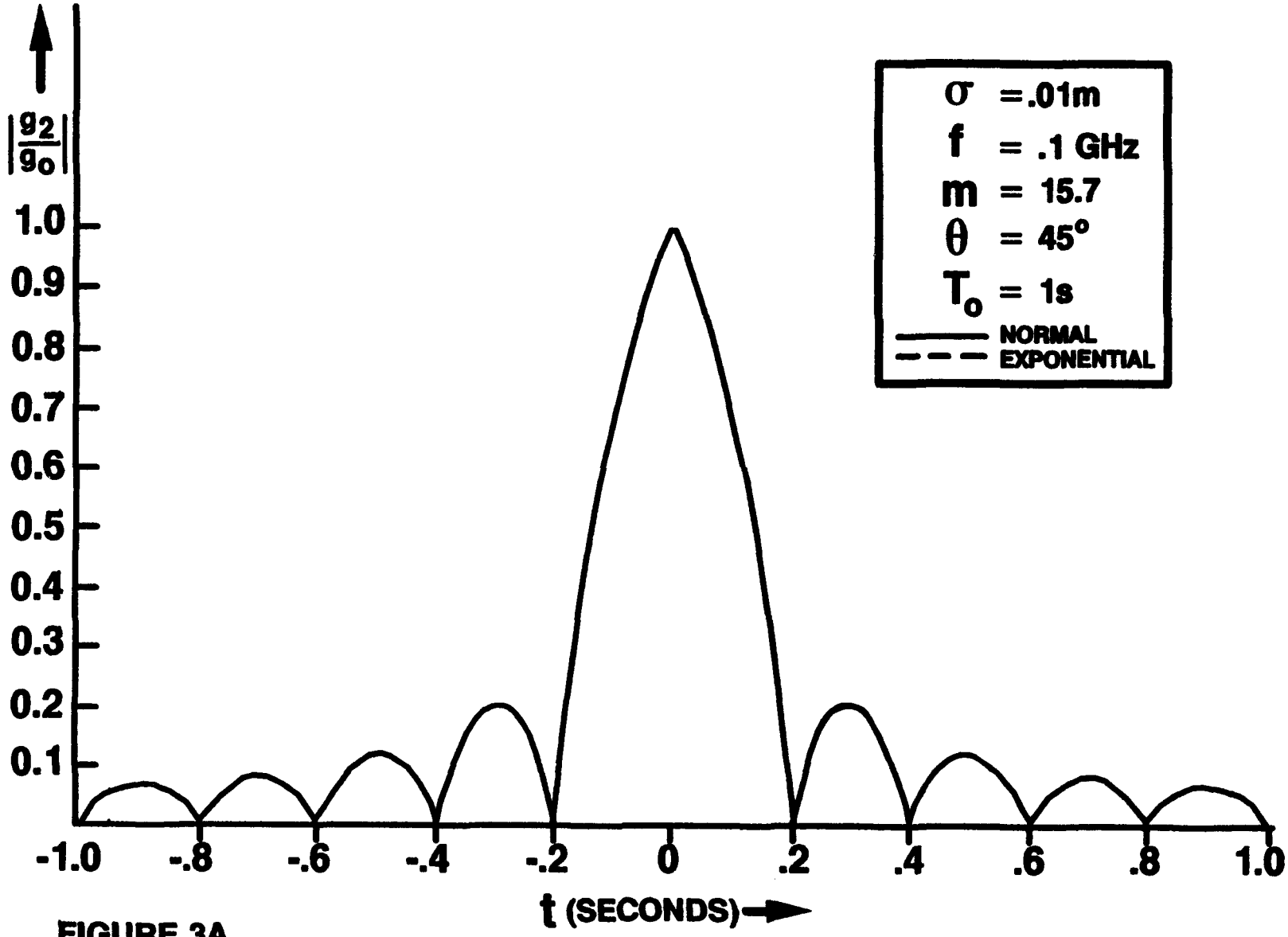
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)
σ =.01m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL

FIGURE 3A

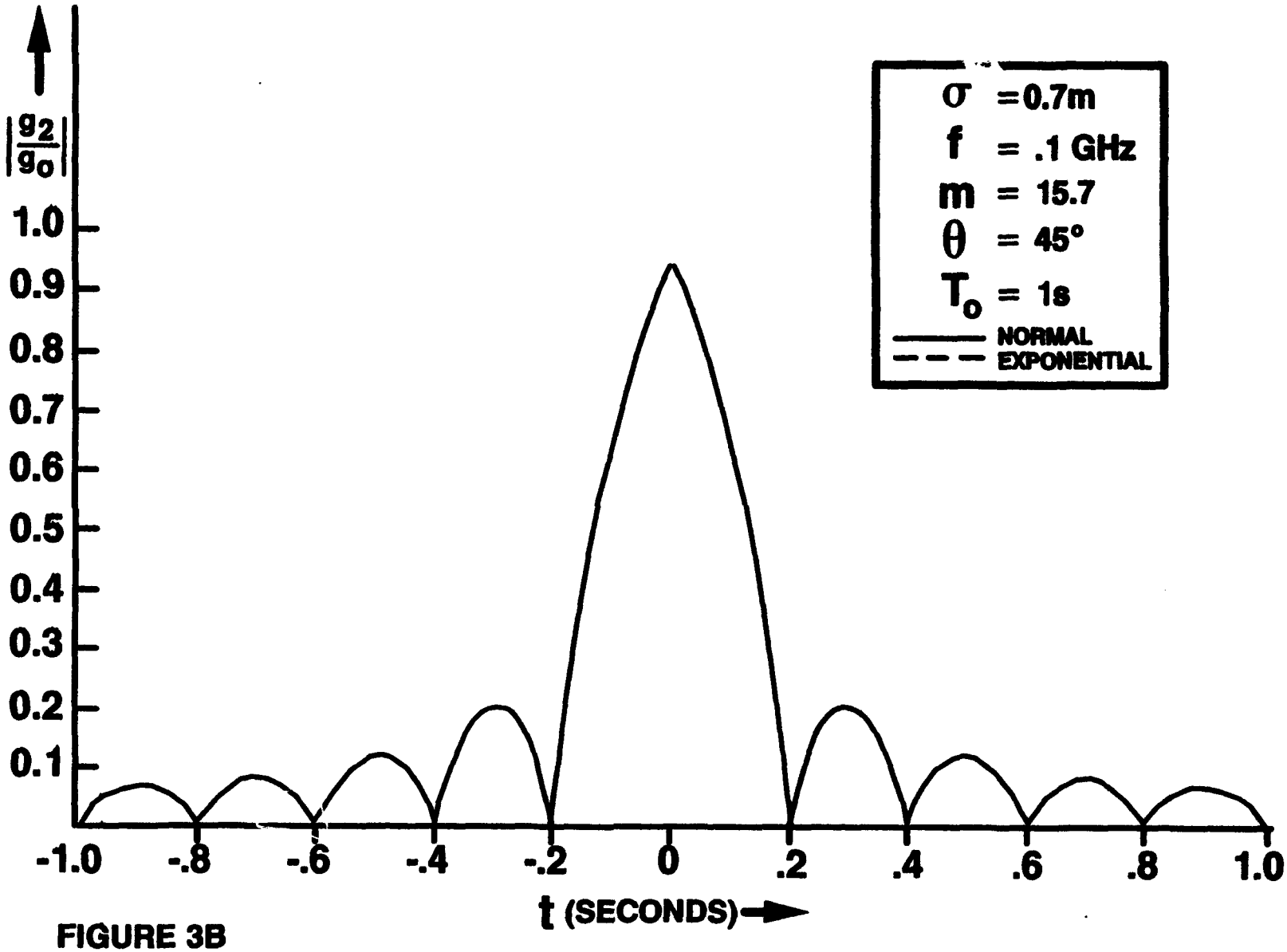
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)
σ =0.7m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL

FIGURE 3B

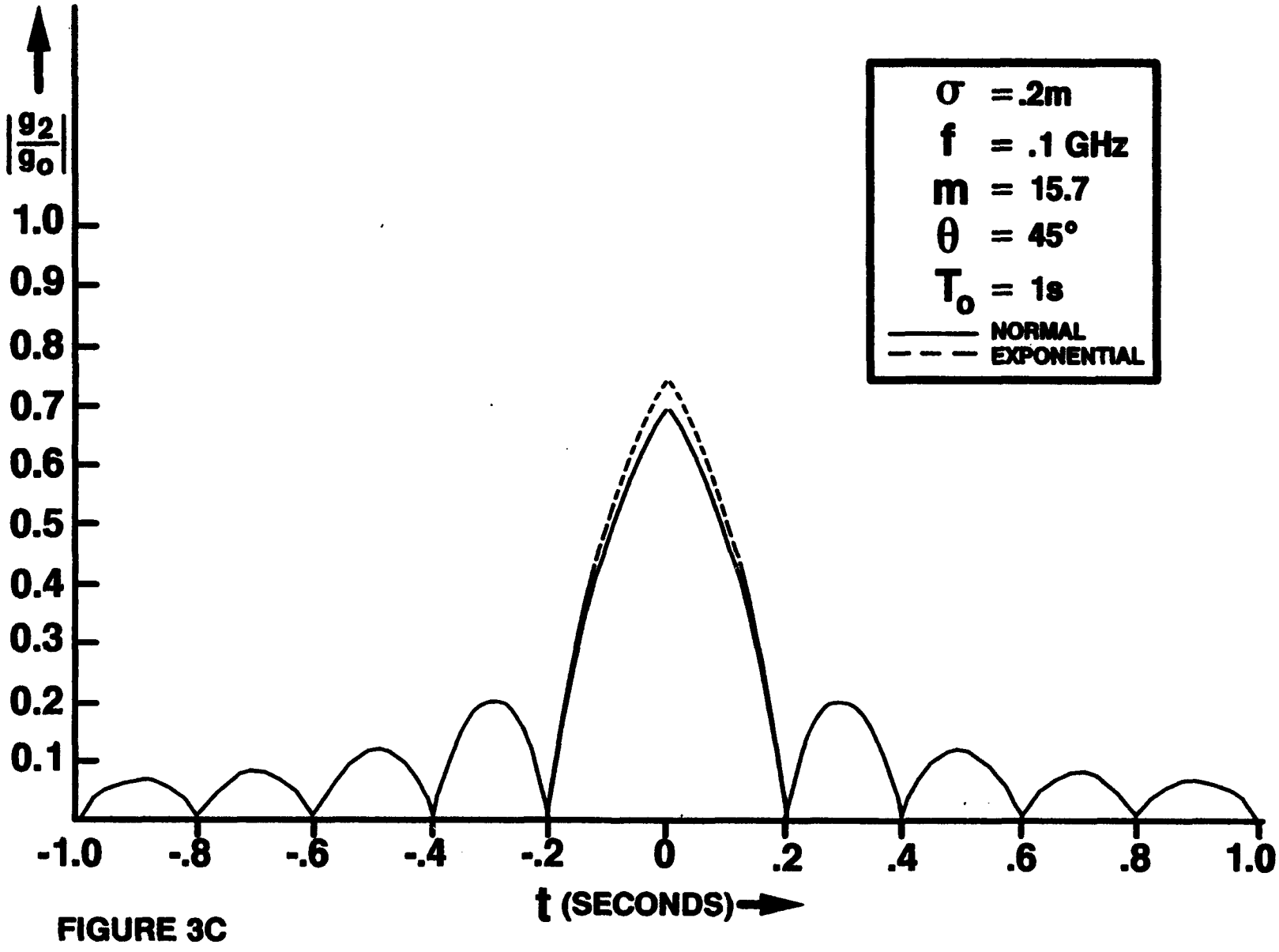
σ = .2m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 3C

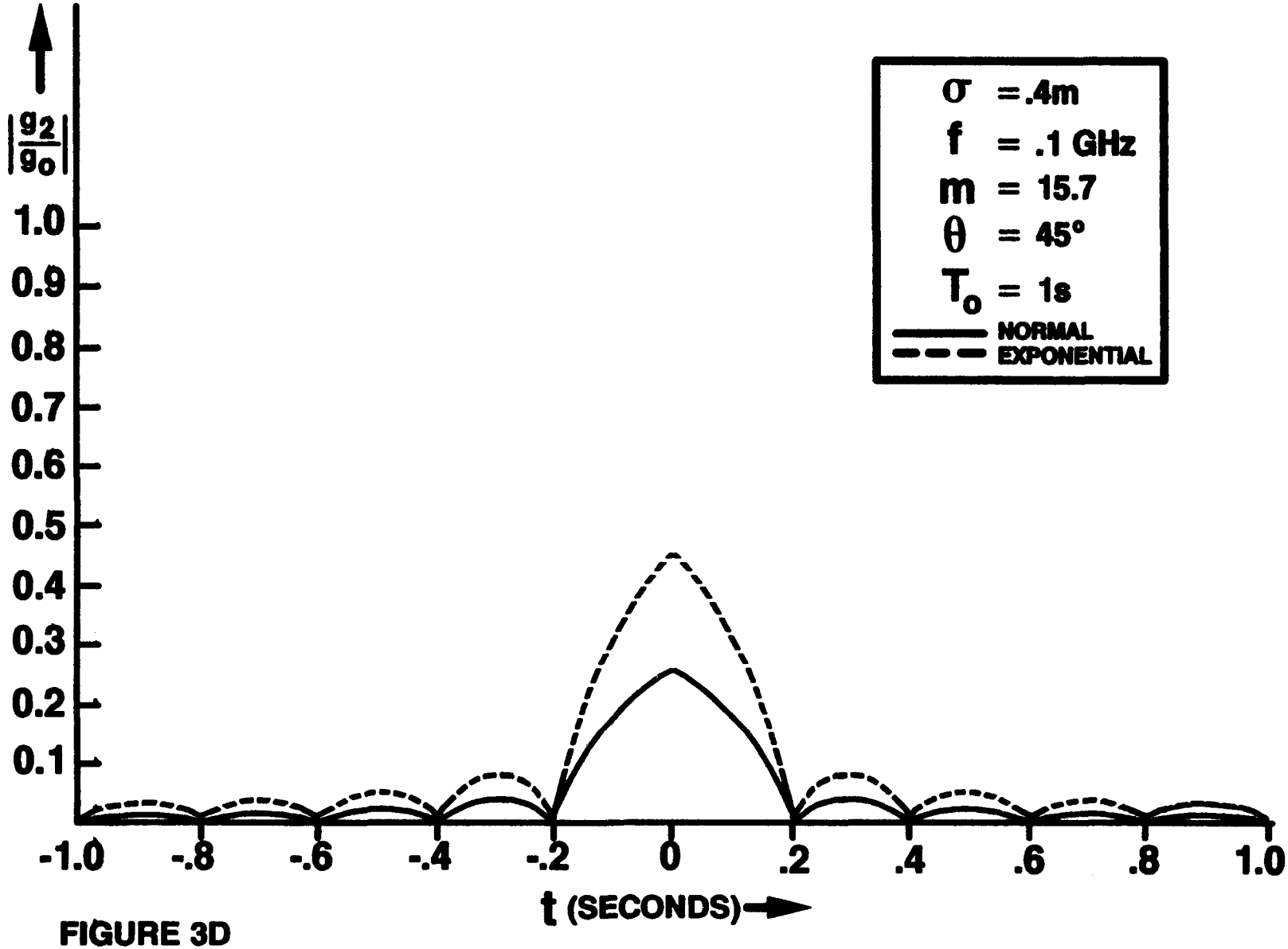
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)
σ = .4m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL

FIGURE 3D

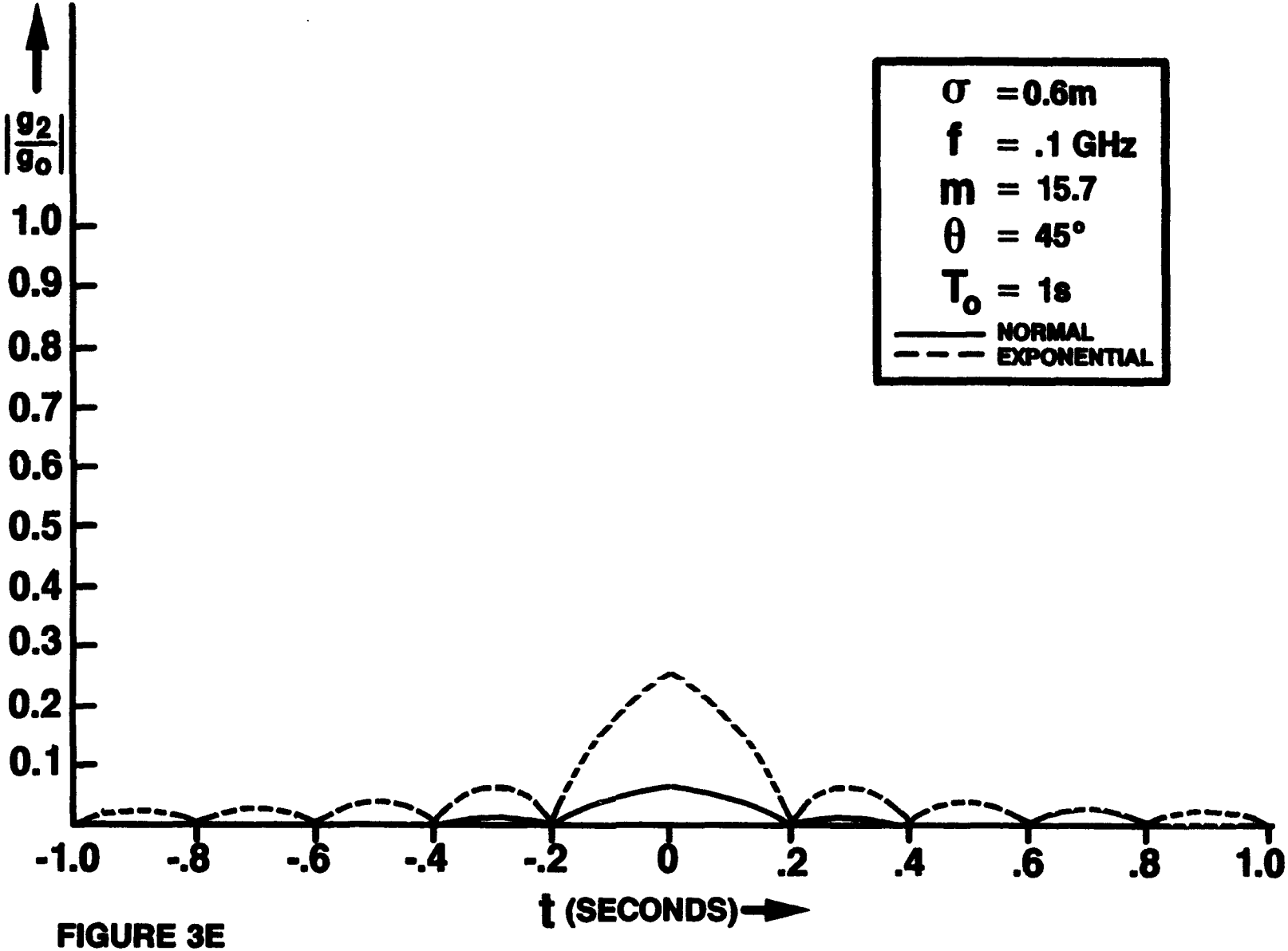
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
σ =0.6m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 3E

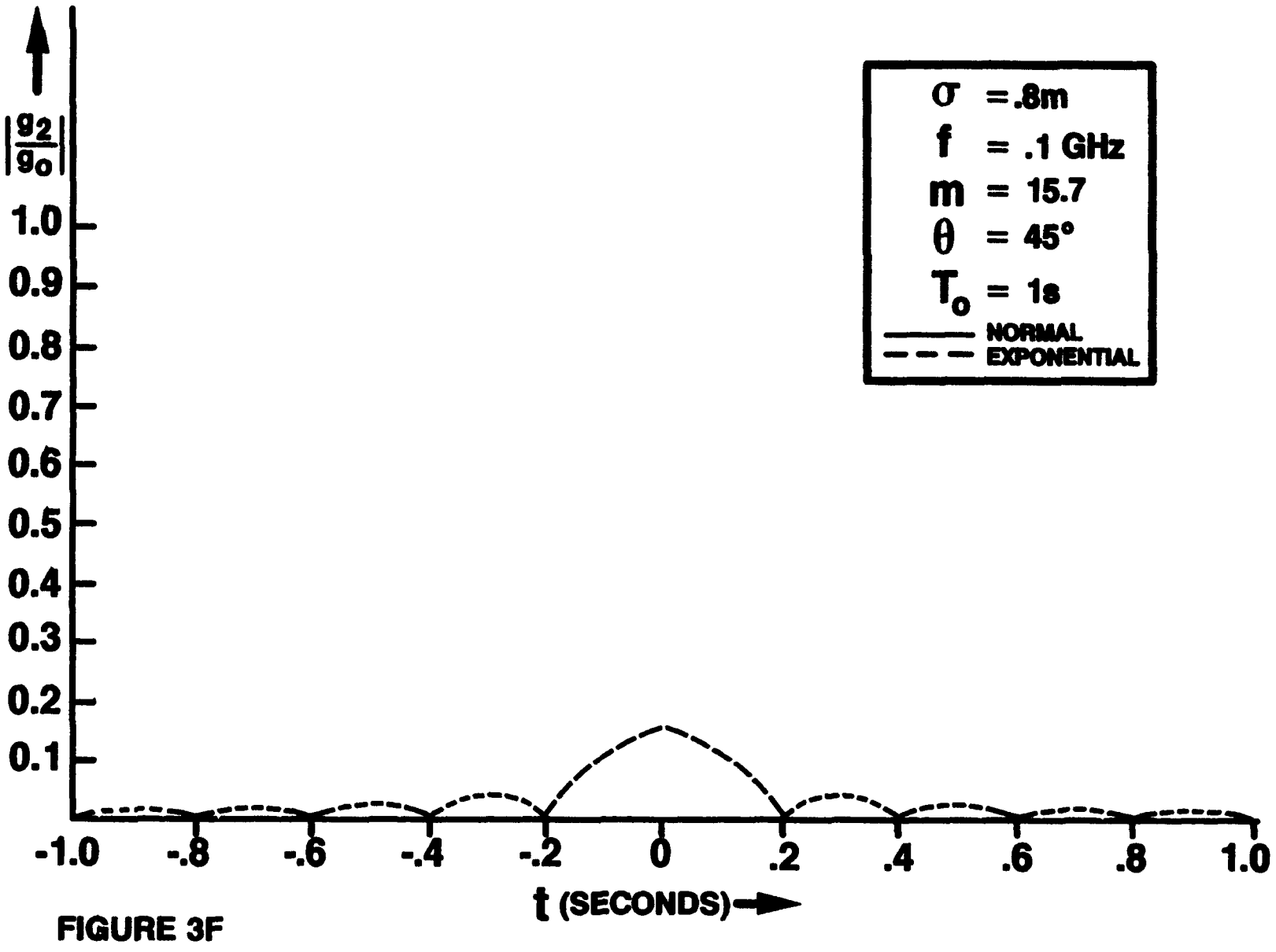
σ = .8m
f = .1 GHz
m = 15.7
θ = 45°
T_o = 1s
NORMAL
EXPONENTIAL
$\left|\frac{g_2}{g_o}\right|$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
t (SECONDS)

FIGURE 3F

7. CONCLUSION

In this report, the effects of ground reflections on frequency modulated signals were studied. On recognizing the detrimental effect on target detection caused by interference due to ground reflections our primary objective is to compute the relative amplitude of the ground reflected signals with respect to the direct signal. Four different surfaces are considered for the analysis of the ground reflected signals. Among them the normal distribution has symmetric characteristics while the other three are asymmetric. Due to the nonlinear nature of the matched filter, the received signals for various types of ground have quite different characteristics. Notice that the coherent reflection coefficients of the four surfaces have distinct frequency dependences. Explicit analytic expressions for the ground reflected signal I are provided for each of the four types of surfaces. For the sake of illustration we have compared the ground reflected signals for exponentially distributed and normally distributed surfaces. It is observed that when the Rayleigh parameter is very small the two results are very close to each other. For larger Rayleigh parameters the results are quite different from each other. This means that the universally accepted Gaussian model may at times lead to significant errors. We conclude therefore that for the study of scattering from rough surfaces a careful choice of the statistical distribution appropriate for the problem is essential.

References

1. Evans, G. C.(1966), Influence of ground reflections on radar target-tracking accuracy, Proc. IEE, **113**:1281-1286.
2. Smith, P. G. and Mrstik, A. V.(1979), Multipath tracking errors in elevation-scanning and monopulse radars, IEEE Trans. Aerosp. Electron. Syst., **AES-15**:765-776.
3. Barton, B. K.(1974), Low-angle radar tracking, Proc. IEEE, **62**:687-704
4. Papa, R. J., Lennon, J. F., and Taylor, R. L.(1983), Multipath effects on an azimuthal monopulse system, IEEE Trans. Aerosp. Electron. Syst., **AES-19**:585-597.
5. Cook, C. E.(1960), Pulse compression, key to more efficient radar, Proc. IRE, **48**:310-316.
6. Cohen, M. W.(1987), Pulse Compression in Radar Systems, in Principles of Modern Radar edited by J. L. Eaves and E. K. Reedy, Van Nostrand, 465-501.
7. Beckmann, P. and Spizzichino, A.(1963), The Scattering of Electromagnetic Waves from Rough Surfaces, Pergamon, Oxford.

Appendix A

Characteristic Functions

In this appendix we consider four statistical distributions and obtain their characteristic functions. The characteristic function $\chi(q)$ is defined as:

$$\chi(q) = \int_{-\infty}^{\infty} d\xi \; p(\xi) \; \exp(iq\xi) \tag{A1}$$

where $p(\xi)$ is the probability density function of the height of the randomly rough surface. The rough surface has zero mean and a root mean square height of α.

A1. NORMAL DISTRIBUTION

$$p(\xi) = \frac{(2\pi)^{-.5}}{\sigma} \exp\left[\frac{-\xi^2}{2\sigma^2}\right] \tag{A2}$$

$$\chi(q) = \exp\left(-.5q^2\sigma^2\right) \tag{A3}$$

A2. EXPONENTIAL DISTRIBUTION

$$p(\xi) = \begin{cases} \frac{1}{\sigma} \exp\left[- (\xi+\sigma)/\sigma \right] & ; \quad \xi \geq -\sigma \\ 0 & ; \quad \xi < -\sigma \end{cases} \tag{A4}$$

$$\chi(q) = \frac{e^{- iq\sigma}}{1 - iq\sigma} \tag{A5}$$

A3. RAYLEIGH DISTRIBUTION

$$p(\xi) = \begin{cases} \frac{2c^2}{\sigma^2} \left[\xi + \frac{\sqrt{\pi}\sigma}{2c} \right] \exp\left\{ - (c/\sigma)^2 \left[\xi + \frac{\sqrt{\pi}\sigma}{2c} \right]^2 \right\} & \text{for } \xi \geq -\frac{\sqrt{\pi}\sigma}{2c} \\ 0 & \text{otherwise} \end{cases} \tag{A6}$$

where

$$c^2 = 1 - \pi/4 \tag{A7}$$

$$\chi(q) = \exp\left[- i \frac{\sqrt{\pi}\sigma q}{2c} \right] \left\{ 1 + i \frac{\sqrt{\pi}\sigma q}{2c} \operatorname{erfc}\left[- i \frac{\sqrt{\pi}\sigma q}{2c} \right] \exp\left[- (.5q\sigma/c)^2 \right] \right. \tag{A8}$$

A4. SKEW-NORMAL DISTRIBUTION

$$p(\xi) = \begin{cases} \frac{1}{\sqrt{(2\pi)}\sigma} \exp\left[-.5 \left((\xi - a)/\sigma_1\right)^2\right] ; \xi \geq a \\ \\ \frac{1}{\sqrt{(2\pi)}\sigma} \exp\left[-.5 \left((\xi - a)/\sigma_2\right)^2\right] ; \xi < a \end{cases} \quad \text{(A9)}$$

where

$$a = (2/\pi)^{.5} (\sigma_2 - \sigma_1) \quad \text{(A10a)}$$

$$\sigma = .5(\sigma_1 + \sigma_2) \quad \text{(A10b)}$$

$$\chi(q) = \frac{1}{2\sigma} \exp(iqa)$$

$$\left\{ \sigma_1 \exp\left(-.5q^2\sigma_1^2\right) \operatorname{erfc}\left[-\frac{i}{\sqrt{2}} q\sigma_1 \right] \right.$$

$$\left. + \sigma_2 \exp\left(-.5q^2\sigma_2^2\right) \operatorname{erfc}\left[-\frac{i}{\sqrt{2}} q\sigma_2 \right] \right\} \quad \text{(A11)}$$

Appendix B

Coherent Reflection Coefficient

The coherent reflection coefficient R is given by Beckmann and Spizzichino [1963][7] as

$$R = \chi(v_z) \tag{B1}$$

where

$$v_z = 2\ (\mu_o \varepsilon_o)^{0.5}\ \omega \cos\theta \tag{B2}$$

Since the characteristic functions for various statistical distributions are given in Appendix A the corresponding reflection coefficients may readily be obtained as follows.

B1. NORMAL DISTRIBUTION

$$R = \exp\left(-0.5\,\alpha^2\omega^2\right) \tag{B3}$$

where

$$\alpha = 2\ (\mu_o \varepsilon_o)^{0.5}\ \sigma \cos\theta \tag{B4}$$

B2. EXPONENTIAL DISTRIBUTION

$$R = \frac{e^{-i\alpha\omega}}{1 - i\alpha\omega} \tag{B5}$$

B3. RAYLEIGH DISTRIBUTION

$$R = \exp\left(- i/\pi\tilde{\alpha}\omega \right)$$

$$\cdot \left[1+i/\pi\tilde{\alpha}\omega \ \mathrm{erfc}(i\tilde{\alpha}\omega) \ \exp\left(- \tilde{\alpha}^2\omega^2 \right) \right] \tag{B6}$$

where

$$\tilde{\alpha} = (\mu_o\varepsilon_o)^{.5} \frac{\sigma}{c} \cos\theta \tag{B7}$$

B4. SKEW-NORMAL DISTRIBUTION

$$R = \frac{1}{2\sigma} \exp\left[i(2/\pi)^{.5} \left(\alpha_1 - \alpha_2 \right) \omega \right]$$

$$\cdot \left\{ \sigma_1 \exp\left(- .5 \ \alpha_1^2 \ \omega^2 \right) \mathrm{erfc} \left[- \frac{i}{\sqrt{2}} \alpha_1\omega \right] \right.$$

$$\left. + \ \sigma_2 \exp\left(- .5 \ \alpha_2^2 \ \omega^2 \right) \mathrm{erfc} \left[- \frac{i}{\sqrt{2}} \alpha_2\omega \right] \right\} \tag{B8}$$

where

$$\alpha_1 = 2 \ (\mu_o\varepsilon_o)^{.5} \ \sigma_1 \cos\theta \tag{B9a}$$

$$\alpha_2 = 2 \ (\mu_o\varepsilon_o)^{.5} \ \sigma_2 \cos\theta \tag{B9b}$$

Nomenclature

χ	characteristic function
R	reflection coefficient
D	delay in time for the ground reflected signal
m	frequency modulation parameter
H	transfer function of the matched filter
ω	angular frequency
ω_c	angular carrier frequency
$2T_o$	pulse width of the transmitted signal
Ψ_1	confluent hypergeometric function
θ	angle of incidence
$g_0(t)$	direct signal
$g_1(t)$	ground reflected signal-I
$g_2(t)$	ground reflected signal-II
σ	rms height of the rough surface
α	$2(\mu_0\varepsilon_0)^{0.5}\sigma \cos\theta$
$\bar{\alpha}$	$0.5\ \alpha\ (1 - 0.25\pi)^{-0.5}$
α_i	$2(\mu_0\varepsilon_0)^{0.5}\sigma_i \cos\theta$: $i = 1, 2$

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