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Final Report

Professor Griff L. Bilbro

November 7, 1993

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Fusion of Range and Luminance Data from Laser Radar Systems

Griff L. Bilbro

November 8, 1993

Abstract

In this project, a technique was developed for computing an optimal description of some physical scene by using global optimisation and Bayesian techniques to combine correlated outputs of diverse nonlinear sensors. Two laser range/intensity imagers were considered, but the major application was to obtain the best known restorations of very noisy PD-, T1-, and T2-weighted Magnetic Resonance Images by fusing the information in all the input images.

1 Foreword

Sensor fusion is an automatic technique for describing some single thing or event by combining relevant data from several sensors. Sensor fusion is becoming more important as more sensors are integrated with computers. The general objectives of sensor fusion are three:

- 1. To reduce noise,
- 2. To increase the extent or detail of the information, or
- 3. To tolerate faulty or missing data.

This report is restricted to an imaging application, but our formulation was consciously developed to be more general as will be discussed after imaging.

1.1 Imaging Application

For imaging, the objective is to is to usefully combine several nonlinearly related, incomplete, distorted, or noisy signals to obtain an improved restoration or reconstruction of the true scene. This imaging problem is easiest for pixel registered images, which in this case is multiple image restoration to remove noise without degrading sharp edges. We considered three imaging applications for sensor fusion.

- 1. The Tri-Services Laser Radar, a time-of-flight imaging ranger with five channels: coarse range, fine range, intensity, doppler, and passive IR.
- 2. The Odetics range camera, a triangulation imaging ranger with two channels: range and luminance.
- 3. Commercial magnetic resonance imaging system, a radio-frequency imager in three dimensions controlled by user-specified echo time TE and repetition time TR, with three channels: proton density, single nuclear relaxation time T1, and nuclear pair relaxation time T2.

We formally addressed each problem in the first year, but settled on the MRI application, item 3, because of the widespread use of these diagnostic medical systems and the resulting availability of data.

1.2 General Formulation of Sensor Fusion

We developed a general formulation of low-level sensor fusion as minimization problem. Find the optimal estimate of an unknown measurement, scene, or dataset f, given a prior expectation V[f] of f and several incomplete, noisy, or distorted observered images $G_m[f]$ of f. We model f as a 2D or 3D array of scalars or vectors. Image formation for channel G_m is modeled as a functional of the unknown f. We considered several noise models, including Poisson, Rayleigh, and additive zero-mean Gaussian, although the following report is restricted to additive zero-mean Gaussian, which we model as a random image n_m for each channel.

We model V as a functional of the unknown f. We considered a smoothness prior in terms of the Laplacian $\sum_{\sigma} (\nabla_{\sigma}^2 f)^2$ as well as the quadratic variation. We considered a flatness prior in terms of the sobel $\sum_{\sigma} (\nabla_{\sigma} f)^2$. We also considered priors that limit the search to a specified interval of values. The following report restricts itself to the last two.

The resulting problem is then to minimize the objective

$$H[f] = \sum_{m} \frac{1}{\sigma_{m}^{2}} ||g_{m} - G_{m}(f)||^{2} + \sum_{p} V_{p}[f]$$

over all possible values of the measurement, scene, or dataset f. The following report restricts itself to the problem of Magnetic Resonance Image Restoration.

2 Statement of the Problem Studied

We studied the specific problem of optimally combining several nuclear Magnetic Resonance Images (MR images) to obtain the best possible description of human tissue. We formulated this multiple image optimization problem mathematically as follows.

Let G be a measured vector-set of images

....

$$\mathbf{G} = \{\mathbf{g}_c\}_{c=1,d} \tag{1}$$
$$\mathbf{g}_c = [g_{c,i}]_{i=1,n}$$

where d is the number of channels in the vector-set, and where $g_{c,i}$ represents the c-th channel value associated with the i-th pixel.

Using similar notation, let S(F) represent the undegraded ideal images as a deterministic function of F where F are the undegraded ideal basis images, and let N represent additive noise such that G = S + N. Note that

$$\mathbf{F} = \{\mathbf{f}_{\psi}\}_{\psi=1,\varphi}$$
(2)
$$\mathbf{f}_{\psi} = [f_{\psi,i}]_{i=1,n}$$

where p is the number of basis images in the vector-set, and where $f_{\psi,i}$ represents the value associated with the *i*-th pixel of the ψ -th basis image.

2.1 Bayesian Model

In Bayesian restoration, the most acceptable result is the result with the highest probability of occurrence. Let $\hat{\mathbf{F}}$ be an estimate of \mathbf{F} . Bayes' rule gives the posterior distribution [8] of $\hat{\mathbf{F}}$ given the data \mathbf{G} as

$$P(\hat{\mathbf{F}}|\mathbf{G}) = \frac{P(\mathbf{G}|\hat{\mathbf{F}})P(\hat{\mathbf{F}})}{P(\mathbf{G})}.$$
 (3)

That is, the conditional probability of occurrence of a specific restoration $\hat{\mathbf{F}}$ given the data G is equal to the conditional probability of occurrence of the data G given the specific restoration $\hat{\mathbf{F}}$ times the probability of the occurrence of the specific restoration $\hat{\mathbf{F}}$ divided by the probability of the occurrence of the data G. We refer to $P(\mathbf{G}|\hat{\mathbf{F}})$ as the "noise term", and it describes the noise distribution. $P(\hat{\mathbf{F}})$ is called the "prior term" and it describes the *a priori* distribution which can be chosen using *a priori* knowledge about \mathbf{F} . Obviously $P(\mathbf{G})$ is constant and independent of $\hat{\mathbf{F}}$, so in order to maximize the posterior distribution, we need only maximize $P(\mathbf{G}|\hat{\mathbf{F}})P(\hat{\mathbf{F}})$.

2.2 Physical Model of MR Images

The function S(F) is given by the physical model. In this work, one simplified nonlinear image formation model [11] is used.

$$s_{c,i} = \rho_i \exp\left(-\mathrm{TE}_c/T_{2i}\right) \{1 - \exp\left(-\mathrm{TR}_c/T_{1i}\right)\}$$
(4)

where ρ , T₂ and T₁ are basis images of f_{ψ} where $\psi = 1, 2, 3$, respectively. TE_c and TR_c represent the echo time and relaxation time used during acquisition of the c-th data image. T₁ and T₂ are nuclear relaxation times, and ρ represents proton density, contributions due to proton flow, and MRI system gain. Most brain tissue is perfuse with slowly moving blood, hence the data should not be subject to large variations in proton flow. This work does not address the effect of proton flow. Our data was acquired with MRI system gain held constant for all scans.

Note that this physical model is undefined and exhibits singularities in the gradient when T_1 or T_2 equals zero. T_1 and T_2 are real, positive and bounded below in time, but using a noninfinitesimal step size during gradient descent requires that T_1 and T_2 be constrained in code, otherwise negative values of T_1 or T_2 might occur, causing numerical overflow. Because of this, a constrained optimization technique is required to find a global solution in the minimization process.

2.3 Global Optimization with Mean Field Annealing

Mean Field Annealing (MFA) is based on Simulated Annealing (SA) and derives its power and generality from that popular optimization procedure. MFA differs from SA by analytically approximating the relevant Gibbs distribution rather than stochastically simulating it. SA works by gradually cooling an on-going stochastic simulation of a Gibbs distribution. Mean field theory provides a deterministic approximation to a Gibbs distribution which also can be cooled in the same way to produce a Mean Field Annealing (MFA) algorithm. Many SA algorithms can be converted to analogous MFA algorithms that run in 1/50 the time required by the SA version[13, 1, 9, 2]. However because it is an approximation, MFA does not inherit any guarantee of convergence even when the analogous SA does converge.

In earlier work we[1, 9, 2] showed that simulated annealing could be accelerated with the mean field approximation. In this approach the important structure of P is approximated with a more convenient distribution P_0 for a sequence of falling values of T. In this section we provide an informationtheoretic procedure for studying a given difficult P using an essentially arbitrary easy P_0 by minimizing the entropy of P_0 relative to P, or equivalently, the cross-entropy or Kullback-Leibler[10] distance between P_0 and P. This information-theoretic procedure leads to our previously successful approach based on the theoretical tools of statistical physics.

Assume we have another positive but otherwise arbitrary distribution $P_0[s,m]$. It is useful to choose P_0 to be easily analyzed and to have adjustable parameters represented by some vector m. We rewrite P_0

$$P_0[s,m] = \frac{1}{Z_0} \exp(-U_0[s,m]/T), \qquad (5)$$

where $Z_0 = \int ds \exp(-U_0[s,m]/T)$ which in general depends on m through U_0 .

The entropy of P_0 relative to P is

$$R = \int ds P_0[s,m] \ln \frac{P_0[s,m]}{P[s]}, \qquad (6)$$

where we have suppressed the dependence of R on the vector of adjustable parameters m. Using Equation 5, we rewrite Equation 6 as

$$R = \int ds \frac{\exp(-U_0/T)}{Z_0} (-U_0/T - \ln Z_0 + U/T + \ln Z).$$
 (7)

We define the average with respect to P_0 of a function $\phi[s]$ as

$$\langle \phi
angle = \int ds \phi \exp(-U_0/T)/Z_0$$

and obtain

$$R = -\frac{1}{T} \langle U_0 - U \rangle - \ln Z_0 + \ln Z.$$
(8)

We define $F_0 = -T \ln Z_0$ and $F = -T \ln Z$ and obtain

$$R = \frac{1}{T}(F_0 - F + \langle U - U_0 \rangle), \qquad (9)$$

It is known[10] that $R[m] \ge 0$ with equality holding if and only if $P_0 = P$. Here T is also positive so that

$$F \leq F_0 + \langle U - U_0 \rangle, \tag{10}$$

which is the basis of our mean field approximations to discrete, continuous, and even problems with both discrete and continuous variables.

The mean field approximation is obtained by minimizing Equation 9 with respect to m to find the tightest bound in Equation 10; mean field annealing involves tracking the minimum from high to low values of T[12]. In the case of discrete s_i as in graph coloring or binary image restoration[4] it is useful to choose

$$U_0 = -\sum_i m_i s_i, \tag{11}$$

but in the present context of problems with continuous s_i the simplest useful choice[9, 2] is

$$U_0 = \frac{1}{2} \sum_i (x_i - m_i)^2.$$
 (12)

In either case the m_i are real.

3 Summary of the Most Important Results

We have discussed three important results

- 1. The formulation of low-level sensor fusion as an optimization problem that admits prior expectation in a rigorous Bayesian sense
- 2. The application of Mean Field Annealing (MFA) to the problem of sensor fusion
- 3. The development of a specific algorithm with produces the best known restorations of Magnetic Resonance Images

4 **Publications and Technical Reports**

This project has resulted in four publications.

- A journal article reporting on MFA for as a general technique and for image optimization problems specifically[3]: Griff L. Bilbro, Wesley E. Snyder, Steven J. Garnier, and James W. Gault. Mean field annealing: A formalism for constructing GNC-like algorithms. *IEEE Transactions on Neural Networks*, 3(1), 1992.
- An conference presentation and subsequent proceedings publication of the specific MRI problem studied[7]: Steven J. Garnier, Griff L. Bilbro, James W. Gault, Wesley E. Snyder, and Y. S. Han. Magnetic resonance image analysis. In SPIE Proceedings Vol. 1904: The SPIE and IST Conference on Image Modeling, 1993.
- 3. An journal publication that formulates the specific MRI problem as a sensor fusion and global optimization problem and reports state-of-theart[6]:

Steven J. Garnier, Griff L. Bilbro, James W. Gault, and Wesley E. Snyder. Magnetic resonance image restoration. *Journal of Mathematical Imaging and Vision*, 1993. Accepted for publication.

4. A second journal article (currently in review) discussing refinements of the technique[5]:

Steven J. Garnier, Griff L. Bilbro, James W. Gault, and Wesley E. Snyder. The effects of various basis image priors on mr image map restoration. *Journal of Mathematical Imaging and Vision*, 1993. In review.

In addition we are preparing a clinical article for the medical literature in conjunction with radiologists at Bowman Gray School of Medicine.

5 Participating Scientific Personnel

Mr. Stephen F. Garnier Mr. Michael McCormick Dr. Griff L. Bilbro Dr. James W. Gault Dr. Wesley E. Snyder

S. J. Garnier earned the Masters degree.

S. J. Garnier has finished his PhD thesis research and expects to earn his PhD degree early in 1994.

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