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This report gives a brief discussion of the different problem areas under which research papers have been published during the term of the grant as well as a short description of the main results. In this research finite element methods were applied to a simple model of shear band formation. They were also used for elliptic and parabolic problems that arise in optimal control and for equations that simulate phase transitions. These finite element schemes were analyzed and implemented. Also, studies of space-time schemes where the Galerkin technique is used to discretize time as well as space were the focus of several papers. Finally, a last paper contains a thorough analysis of an adaptive time discretization technique.

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Final Research Report
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This report gives a brief discussion of the different problem areas under which research papers have been published during the term of the grant *Numerical Analysis and Computation of Nonlinear Partial Differential Equations from Applied Mathematics* (January 1, 1991 – September 30, 1993) as well as a short description of the main results. In this research finite element methods were applied to a simple model of shear band formation in [FG] and [ARO2]. They were also used for elliptic and parabolic problems that arise in optimal control in [FK1] and [FK2] and for equations that simulate phase transitions in [FJ1] and [ARO1]. These finite element schemes were analyzed and implemented. Also, studies of space-time schemes where the Galerkin technique is used to discretize time as well as space were the focus of the papers [F], [FJ2], and [FP]. Finally, the paper [EF] contains a thorough analysis of an adaptive time discretization technique.

Shear bands are thin regions in a thermoplastic material where the strain rate is very high due to the applied stresses. We have begun to study, through scientific computa ions, the qualitative aspects of a time-dependent model in one and two dimensions as well as analyzing the numerical schemes needed to compute the approximate solutions. Thus far we have considered the spatially discrete time continuous method and have obtained optimal order error estimates for this scheme, [FG]. Some sample two dimensional computations were included in this work. The report [ARO2] contains our first attempts at computing blowup in one dimension. We confirm several known theoretical results and begin to compute approximations in cases where there is no theory.

Problems with limited regularity frequently arise in optimization since the desired control is often not a smooth function. With J.T. King of the University of Cincinnati a parabolic initial/boundary value problem was considered that arises in optimal control problems (see [FK2]). The boundary data was nonhomogeneous and since it typically represents a "bang-bang" control it has limited regularity. We believe it is important to use schemes that are robust – handle the *rough* cases as well as the *smooth* ones. Methods analyzed in the literature require the approximation to be zero on the boundary which restricts the possible accuracy for the scheme in the smooth case. We analyzed a numerical scheme consisting of piecewise linear finite elements in space and a backward Euler method in time. We showed the approximate solution from this scheme converged to the true solution in an optimal manner in both the rough and smooth data cases. We also analyzed an elliptic optimal control problem, [FK1].

The use of finite elements to obtain time discretizations for evolution problems has become more important in recent years. C. Johnson and others have demonstrated the usefulness of the discontinuous Galerkin method for parabolic problems (see [J]). We have considered the continuous

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time Galerkin (CTG) method which has the advantage of preserving energy properties for the time-dependent problems it is approximating (see [FS]). An advantage of this attribute is that it allows us to analyze the qualitative properties of the approximation.

To demonstrate this we studied fully discrete approximations which used the CTG method applied to two specific evolution problems (see [FJ1] and [ARO1]). Both problems modeled phase transitions, the first one involved antiplane shear deformations of a viscoelastic solid and the second was the well-known viscous Cahn-Hilliard equation. With S. Jensen, the long-time behavior of the approximation scheme was analyzed and we showed that the numerical solution converged to a solution of the discrete steady state equation as $t \to \infty$. We also used these techniques to study the long time behavior of discontinuous Galerkin approximations of a reaction-diffusion equation (see [FJ2]).

With T.E. Peterson several optimal order error estimates for the CTG method applied to the linear wave equation were derived (see [FP]). This was the first step in a more extensive program which involves the use of finite element method techniques to analyze time discretizations.

A finite element scheme which allows unstructured meshes in the space-time domain was also implemented and analyzed in [F]. This approach would allow local mesh refinement around a moving singularity.

Adaptive techniques will be a crucial part of our study of shear bands (discussed above). With D. Estep the approximation of a system of ordinary differential equations by an adaptive version of the CTG method was considered, [EF]. We showed, rigorously, that if certain *a posteriori* quantities are sufficiently small then the approximation will be accurate.

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