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PHYSICAL OPTICS APPROXIMATIONS TO FORWARD SCATTERING BY ELASTIC SPHERICAL SHELLS AND RIGID/SOFT SPHERES

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Abstract

Numerical results demonstrate that both Fraunhofer and Fresnel diffractions provide good approximations to forward scattering by elastic spherical shells and rigid/soft spheres, for non-dimensional frequency values $5 \leqslant ka \leqslant 40$, and scattering angles $0° \leqslant \theta \leqslant 10°$. The calculations were done for the far field, with incident plane waves.

Introduction

Forward scattering has been used in particle sizing in optical [1] and acoustic [2] measurements. Both Fraunhofer diffraction and Fresnel diffraction [3] have been used as predictive tools in calculations of scattering from objects of suitably small sizes. Since acoustic waves penetrate elastic objects, but the Fraunhofer and Fresnel diffractions do not include elasticity, the validity of these physical optics approaches, even in forward scattering, may be questioned. In this report we compare predictions of forward scattering from elastic spherical shells and rigid/soft spheres, using known exact formulas [4], to Fraunhofer and Fresnel predictions.

The methods of physical optics have been applied to acoustic scattering problems by many authors [2, 5–12] with varying degrees of success. The best known of these techniques is the Kirchhoff approximation [5] which assumes that the incident field and its normal derivative are unperturbed in an open aperture, and are zero on the shadow side of a scatterer. The application of these assumptions to backscattering from semi-infinite plates [5], and from spheres and cylinders [6–8] has shown that the Kirchhoff method is unreliable in these instances.

Application of the Kirchhoff method to forward scattering from a semi-infinite plate has been discussed by Officer [9]. Pierce and Hadden find [10, 11] that for small forward angles, the Kirchhoff method provides a good approximation to scattering by a wedge. Hunter, Lee, and Waag [2] have experimentally measured the forward scattering patterns of single nylon filaments. They demonstrated that the patterns were well explained by a Huygens construction, calculated numerically. The Fraunhofer and Fresnel calculations [3] discussed below are subsets of the Huygens construction. Kirchhoff predictions for forward scattering by an infinite rigid cylinder have been shown by one of the authors [12] to agree with exact predictions [13]. For forward scattering, the Kirchhoff method is identical to Fresnel diffraction [3, 9]. Since both Fresnel and Fraunhofer diffractions are special cases of a unified theory based on the Fresnel-Kirchhoff integral [3], we prefer to retain the Fresnel label instead of the Kirchhoff label. We show that in the examples considered here, both Fraunhofer and Fresnel diffractions provide good approximations to forward scattering by elastic spherical shells and rigid/soft spheres [14].

1. Exact formulas

The well-known exact formulas [4] for spherical elastic and rigid/soft scatterers are summarized in the present section. For spherical scatterers, the complex pressure of the incident plane wave can be written as

$$p_{inc} = \exp(i k r \cos\theta), \tag{1}$$

where k is the wave number, r is the distance of the observation point from the center of the sphere, and θ is the spherical polar angle, with θ = 0 for the forward direction.

Similarly, for the scattered signal

$$p_{scatt} = -\left(\frac{i}{kr}\right)\exp(ikr)\sum_{n=0}^{\infty}(2n+1)A_n P_n(\cos\theta) = \left(\frac{1}{r}\right)\exp(ikr)f(\Omega) \ . \quad (2)$$

Here $P_n(\cos\theta)$ are Legendre polynomials, and the coefficients A_n are determined from the elastic boundary conditions [4]. The formulas are for the asymptotic case $kr \gg 1$. For rigid/soft boundary conditions [4]

$$A_n(\text{rigid}) = -j_n'(ka)/h_n'(ka), \text{ and } A_n(\text{soft}) = -j_n(ka)/h_n(ka) \ . \quad (3)$$

Here the j_n are spherical Bessel functions, and the h_n are spherical Hankel functions of the first kind. The total field at any point is given by the coherent sum of the incident and scattered fields as

$$p_{tot} = p_{inc} + p_{scatt} \ . \quad (4)$$

2. Physical optics formulas

In the physical optics approach to the forward scattering problem, we replace the scattering object (sphere) with an opaque screen having the same area of cross section. Consider the case when the wavefront due to the incident signal coincides with the screen. The complex pressure at any forward point is then given by the Fresnel-Kirchhoff integral [3]

$$p_{tot} = -i\frac{k}{2\pi}\left[\frac{\exp(iks')}{s'}\right]\iint_{\text{insonified}} \exp[ikf(\xi,\eta)d\xi d\eta] \quad (5)$$

for incident plane waves of unit strength and small forward scattering angles. Here k is the wave number, and (ξ,η) denote the two-dimensional cartesian co-ordinates of a point Q on the opaque screen (Figure 1). Let the point P′ denote the source (at infinity), the point P denote the receiver (observation point), and the point O denote the intersection of the straight line P′P with the opaque screen. Let $s' = OP$, and $s = QP$. Then the function $f(\xi,\eta) = s - s'$. The integral in Eq.(5) runs over the entire insonified region, viz. the wavefront extending to $\pm\infty$ in ξ and η, but excluding the opaque screen.

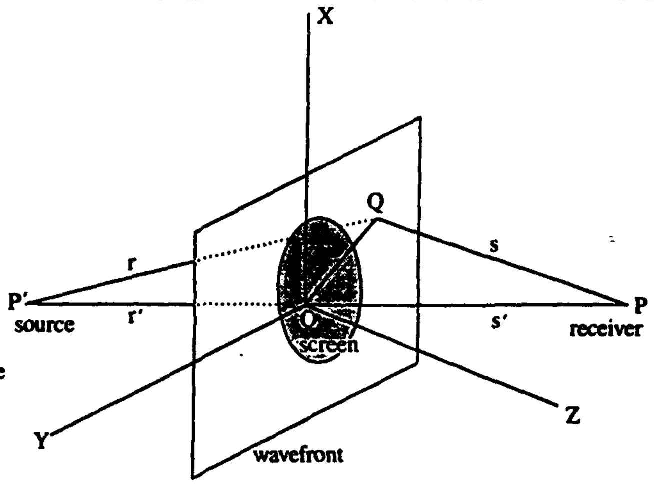

Figure 1.
Source, receiver, co-ordinate axes, and the position of the wavefront as it coincides with the screen

For better physical insight, let us rewrite Eq.(5) as

$$p_{tot} = C'\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[ikf(\xi,\eta)]d\xi d\eta - C'\iint_{\text{disc}}\exp[ikf(\xi,\eta)]d\xi d\eta \equiv p_{inc} - p_{disc} \ , \quad (6)$$

where the integral labeled 'disc' runs over the opaque screen. Equation (6) can be understood as a result of Babinet's principle [3]. The first term p_{inc} in Eq.(6) represents the field due to the entire infinite wavefront, i.e., the field in the absence of any scatterer. The second term p_{disc} in Eq.(6) represents the field due to the complementary geometry, i.e., an insonified opening in an infinite opaque screen. The terms p_{tot} and p_{inc} in Eq.(6) have exactly the same meaning as the corresponding quantities in Eq.(4).

Within the Fraunhofer approximations, using some change of variables and straightforward integration, we obtain [3]

$$p_{disc}(\text{Fraunhofer}) = -\frac{i}{2}\left[\frac{\exp(iks')}{ks'}\right](ka)^2\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]. \tag{7}$$

Here θ is the angular separation of a point on the diffraction pattern measured from the forward direction, and is the same as the angle θ in the previous section.

To calculate p_{disc} using the Fresnel approximation, Born and Wolf [3] choose a co-ordinate system such that the linear terns in $f(\xi,\eta)$ are identically zero. Defining two new variables $u = \xi(k/\pi s')^{1/2}$, and $v = \eta(k/\pi s')^{1/2}$, we can write [3]

$$p_{disc}(\text{Fresnel}) = -\frac{i}{2}\exp(iks')[C_{uv}(\text{disc}) + iS_{uv}(\text{disc})] , \tag{8}$$

where

$$C_{uv}(\text{disc}) = 2\int_{\text{disc}} \cos\left(\frac{\pi}{2}u^2\right)C[v_{max}(u)]du - 2\int_{\text{disc}} \sin\left(\frac{\pi}{2}u^2\right)S[v_{max}(u)]du \tag{9}$$

$$S_{uv}(\text{disc}) = 2\int_{\text{disc}} \sin\left(\frac{\pi}{2}u^2\right)C[v_{max}(u)]du + 2\int_{\text{disc}} \cos\left(\frac{\pi}{2}u^2\right)S[v_{max}(u)]du \tag{10}$$

$$v_{max} = \left[a'^2 - (u+u_0)^2\right]^{1/2}, \qquad a' = \left[\frac{k}{\pi}\left(\frac{1}{r'}+\frac{1}{s'}\right)\right]^{1/2} a \tag{11}$$

and u_0 is the (positive) distance in the u-v plane between the origin of the co-ordinate system and the center of the opaque disc. This distance is also equal to the distance in the observation plane (also measured in u-v space) of a point on the diffraction pattern from the center of the pattern. The quantities C and S on the right hand sides of Eqs.(9) and (10) are the well known Fresnel integrals [3, 15]. We calculated C_{uv}(disc) and S_{uv}(disc) of Eqs.(8)−(10) by numerical integration using Simpson's rule.

The incident field p_{inc} is a plane wave of unit strength, given by

$$p_{inc}(\text{Fraunhofer}) = \exp(iks'\cos\theta) \tag{12}$$

in polar co-ordinates (s',θ), used in the Fraunhofer calculation. As mentioned above, the Fresnel calculation uses a special co-ordinate system [3] in which the diffracted field is calculated on a plane perpendicular to the incident signal direction, at a distance s' from the scatterer. For this, the incident signal is given by

$$p_{inc}(\text{Fresnel}) = \exp(iks') . \tag{13}$$

3. Numerical results

Here we present the results of our calculation of the total field based on the Fraunhofer and the Fresnel approximations. These results are compared with corresponding results obtained using exact formulas [4] for spherical steel shells and rigid/soft spheres. In all examples, we have calculated the diffraction pattern in the far field, with $(s'/\lambda) = 1000$, for incident plane waves. In the Fresnel calculation, the increment for numerical integration in u-space was set at $\Delta u = 0.001$; the fields were calculated at u-space intervals of $\Delta u = 0.04$. This generated 198 plotting points in the angular range $0° \leq \theta \leq 10°$. In the Fraunhofer and exact elastic/rigid/soft calculations, the fields were calculated at angular intervals of $\Delta\theta = 0.05°$. For the external fluid medium, water, sound speed = 1482.5m/s; density = 1000kg/m^3. For steel, longitudinal sound speed = 5950m/s; shear speed = 3240m/s; density = 7700kg/m^3. For air inside the steel shell, sound speed = 330m/s; density = 1.3kg/m^3.

Figures 2−5 show the squared magnitude of the total field p_{tot} plotted vs. the forward scattering angle θ, for ka = 5 and 40. Since signal penetration into the scatterer would be greater at larger wavelengths, the choice of ka = 5 (wavelength = 1.26 × sphere radius) is intended to demonstrate that the physical optics predictions are valid even at such relatively large wavelengths. The Fraunhofer and Fresnel predictions are compared with rigid/soft results in Figures 2 and 3. Exact predictions for steel shells of 1%, 5%, and 10%

thicknesses are compared with the Fresnel results in Figs. 4 and 5. Angular positions of the diffraction crests and troughs predicted by the four approaches, viz. Fraunhofer, Fresnel, steel shell, and rigid/soft show good mutual agreement in the angular positions of the crests and troughs. In all cases the asymptotic value (as θ increases) is 0dB, consistent with the normalization of the incident signal strength to unity, and the fact that the scattered field strength falls off rapidly for increasing θ. The unit normalization of the incident signal also explains why the oscillations are centered at 0dB.

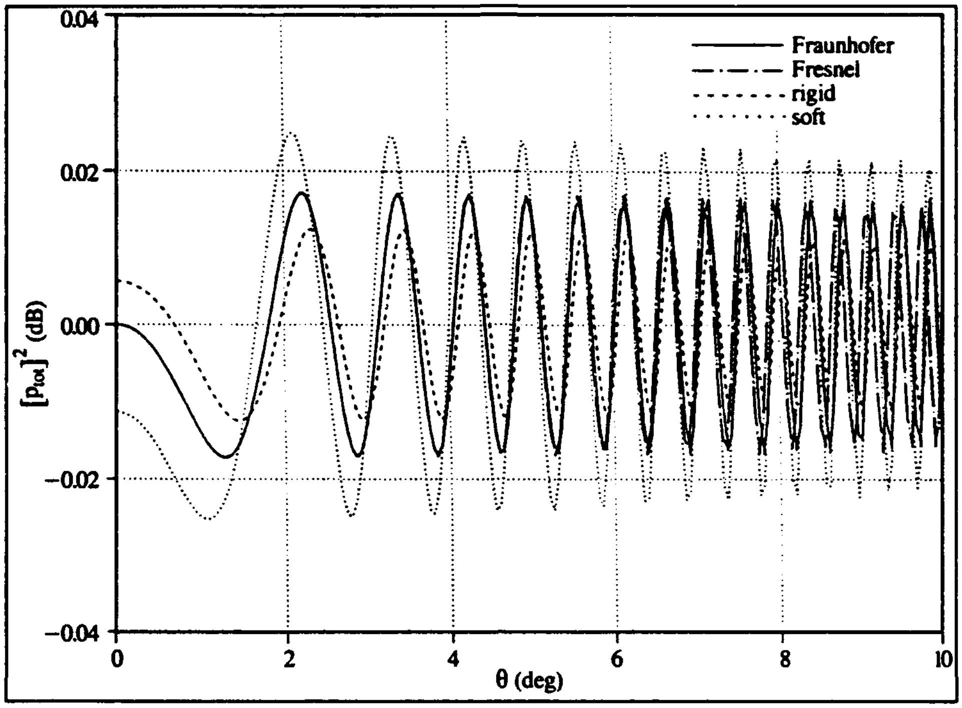

Figure 2. Variation of the total acoustic field strength with forward scattering angle θ for rigid/soft spheres, compared with Fraunhofer and Fresnel diffractions, ka = 5.

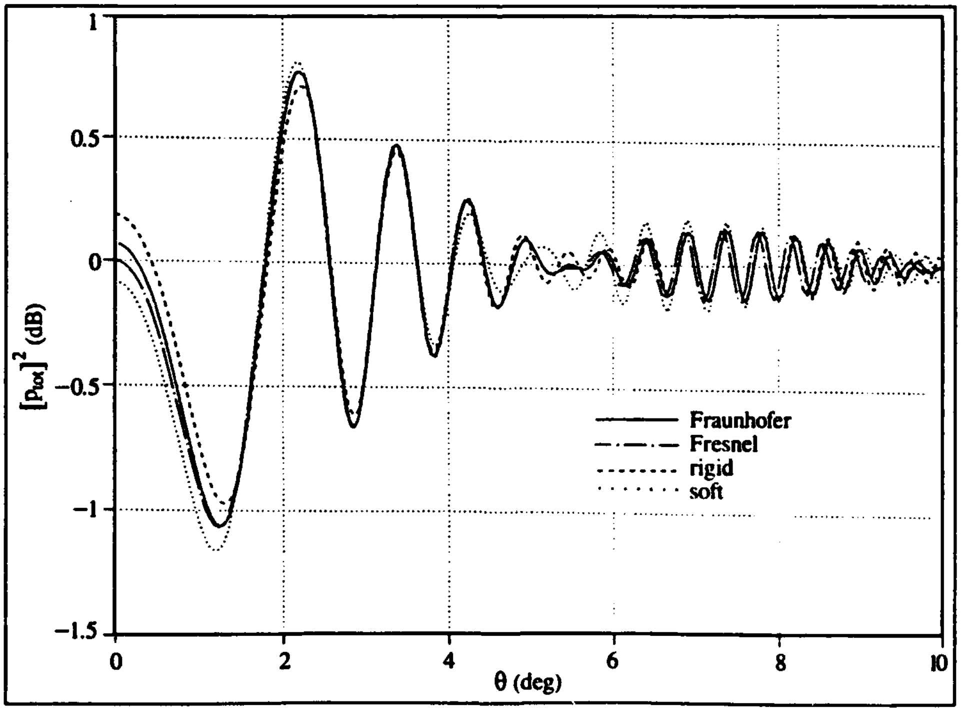

Figure 3. Similar to Figure 2, for ka = 40.

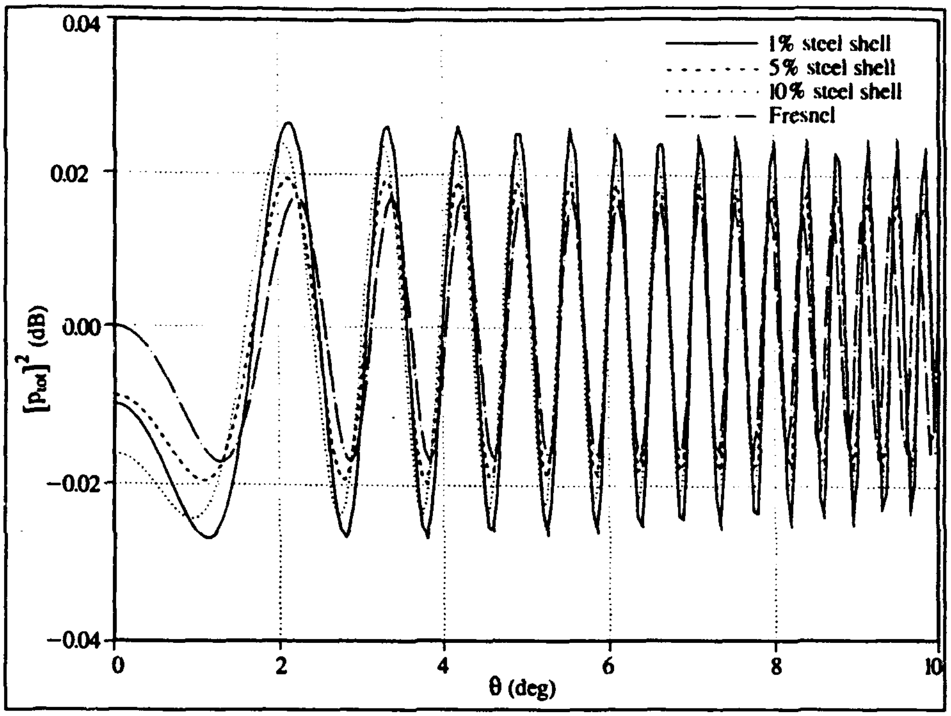

Figure 4. Variation of the total acoustic field strength with forward scattering angle θ for 1%, 5% and 10% steel shells compared with Fresnel diffraction, ka = 5.

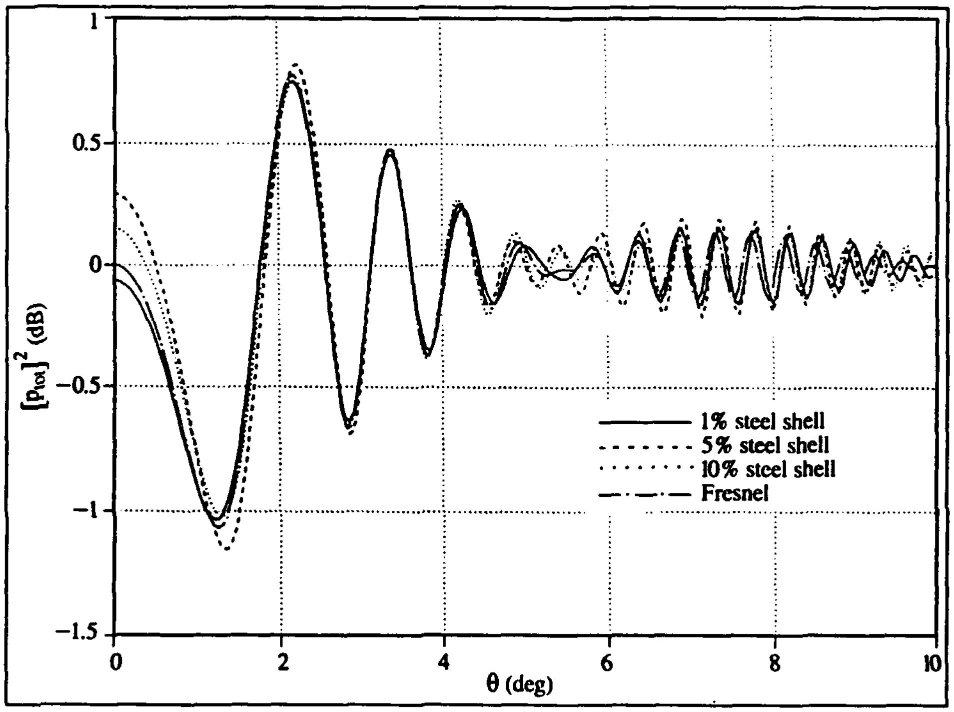

Figure 5. Similar to Figure 4, for ka = 40.

4. Conclusions

Both Fraunhofer and Fresnel diffractions have been demonstrated to provide good approximations to forward scattering by elastic spherical shells and rigid/soft spheres, for non-dimensional frequency values $5 < ka < 40$, and scattering angles $0° < \theta < 10°$. Calculations were done for the far field, for incident plane waves. Angular positions of the diffraction crests and troughs predicted by the four approaches, viz. Fraunhofer, Fresnel, elastic shell, and rigid/soft are in good mutual agreement.

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