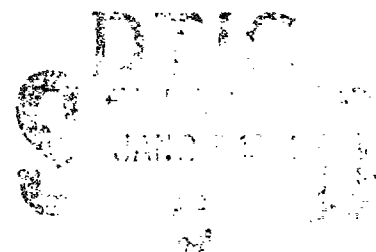


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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California

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### THESIS

SPURS IN DIGITAL RADIO FREQUENCY MEMORY  
AND APPLICATIONS OF DRFM

by

B.V. NITYANANDA

September, 1993

Thesis Advisor:

G.S. Gill

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Spurs in digital radio frequency memory  
and applications of DRFM

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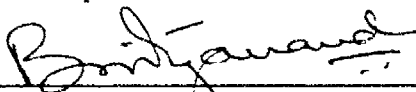
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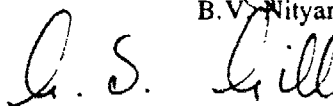
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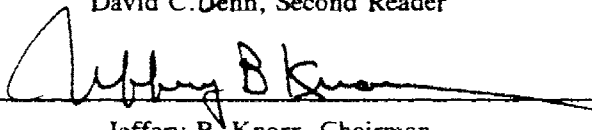
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## ABSTRACT

This thesis presents a method to calculate the power level of harmonics generated in DRFM due to sampling and quantization process. The Fourier series analysis is used for the calculation of the harmonic levels. The quantization process will generate the harmonics of the fundamental signal, which will be folded into the DRFM instantaneous bandwidth by the sampling process. Power level of the harmonics is dependent on the number of quantization bits of the analog-to-digital converter. Further, it is also dependent on the ratio of the signal frequency to sampling frequency. MATLAB programs for computation of harmonic power levels and plots of the harmonic power levels of the multibit DRFM are included in the thesis. Some applications of DRFM like deception jammer, broadband frequency source and radar simulator are also discussed.

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## I INTRODUCTION

### A. BACKGROUND

The early generation radars were based on non-coherent detection techniques and therefore were susceptible to noise jamming. However, modern radars utilize coherent waveforms and sophisticated signal processing techniques such as pulse compression, doppler processing and digital correlators etc. These techniques provide additional processing gain to radar to discriminate unwanted signals including noise jamming. The jamming waveform can achieve the maximum processing gain if the waveform has similar characteristics as that of victim radar waveform. Comparable processing gain can be achieved by receiving, storing and retransmitting the victim radar's waveform with some suitable modulation.

In the recent past, waveform storage was implemented with frequency memory loops (FML). FML is an analog device using delay lines. Also, FML is generally not able to maintain coherence with the received waveform over a long storage period. Further FML is limited in terms of variety of deception techniques that could be implemented.

The digital radio frequency memory (DRFM) is used for waveform storage. In DRFM, the incoming RF signal is sampled and stored in random access memory (RAM). The RF signal is

regenerated before retransmitting to the victim radar. Since DRFM employs the digital techniques for waveform storage, it can maintain comparatively high coherence over a long storage period. Thus DRFM eliminates the signal fidelity deterioration with delay time. Because a complete pulse can be stored, pulse compression and phase coded signals with intra-pulse modulations can be stored and replicated with DRFM. When digital techniques are used, the flexibility to generate a variety of deception techniques is increased. Also, reprogrammability to meet scenario changes is an additional advantage.

Some first generation DRFMs have been produced by several manufacturers. These first generation DRFMs are limited to single bit quantization. The single bit DRFMs have comparatively high harmonic levels. Thus useful jamming power is wasted in harmonics, reducing effective jamming power as well as providing a clue to the radar that it is being jammed. However, there are multi-bit, high performance DRFMs under development or in production.

## **B. OBJECTIVE**

One disadvantage of DRFMs is that a sampling and quantization process generates the unwanted spurious/harmonics in the output. The objective of this thesis is to present a method for calculating the multi-bit DRFM harmonics/spurious signals. The operation of DRFM and its important

specifications are also discussed. Finally state of current technology, and some applications of the technology are given.

#### C. RELATED WORK

The information on DRFM technology is generally difficult to obtain. The reasons are

- the technology is state-of-the-art and there is competition among manufacturers/contractors due to economics involved,
- the technology is classified because of the military nature of the applications.

Limited amount of literature is available in trade magazines and has been referred to in this thesis.

#### D. OVERVIEW

This thesis consists of five chapters and three appendices. Chapter I gives an introduction to the thesis. Chapter II describes the DRFM specifications. Chapter III provides the method for calculating the harmonics/spurious signals generated due to the sampling and quantization process. Chapter IV gives some of the applications for the DRFM technology, the current technology and specifications of a representative DRFM. Chapter V presents the conclusions and recommendations.

Appendix A contains the MATLAB program for calculation of multi-bit quantized signal and plots of harmonic power levels.

Appendix B gives the MATLAB program for computation of harmonics of a squared error minimized quantized signal and the plots for the same. Finally Appendix C presents the MATLAB program for estimation of harmonic power levels of sampled and quantized signals and plots of harmonic power vs normalized input frequency.

## II DRFM DESCRIPTION

### A. INTRODUCTION

This chapter explains the functional organization of the DRFM and the major characteristics and requirements of the DRFM.

### B. DRFM OPERATION

The simplified functional block diagram of the DRFM is shown in Figure 2.1. The RF channelizer downconverts the RF into a baseband signal for digitization. In single bit DRFMs the baseband signal is amplitude hard limited to obtain proper amplitude level for the digitizer. However, in multi-bit amplitude DRFMs, the baseband signal has to be processed through a complicated AGC circuit to control the proper amplitude levels required by the digitizer circuits. The multi-bit phase encoded DRFMs do not need the complicated AGC circuits because amplitude is not used for encoding.

The sampler/digitizer is an analog-to-digital converter, which may encode either the amplitude or the phase. The clock frequency of the analog-to-digital converter is selected such that it is twice the maximum baseband frequency. The clock frequency is limited by the existing technology [Ref. 1]. Since memories cannot operate at the high clock frequencies, the multiplexer and demultiplexer circuits are required to

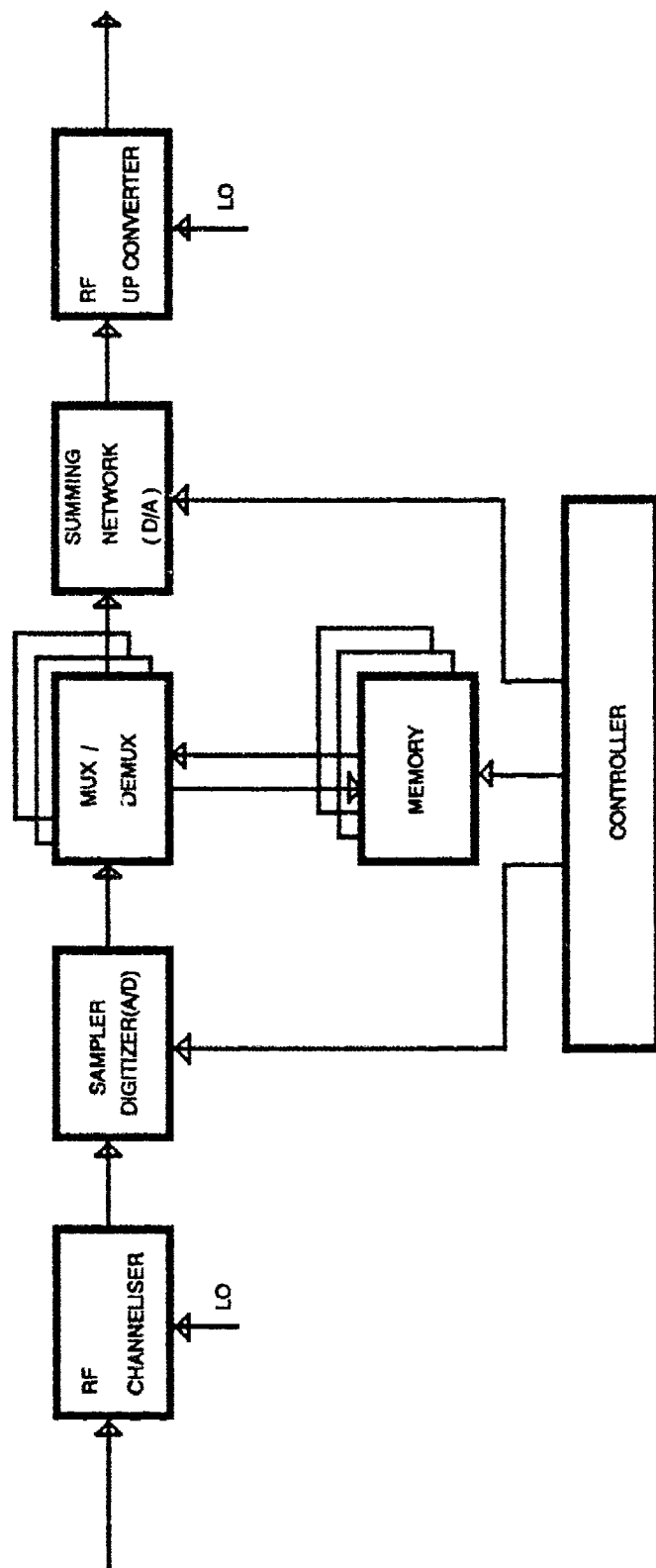


Figure 2.1 Functional organization of DRFM

match the high data rate of the sampler to the read and write speed of the memory. The multiplexer circuits are parallel-to-serial converters and demultiplexer circuits are serial-to-parallel converters. The control circuits allow DRFM to generate the various deception waveforms. Control circuits also maintain the synchronous time reference for the proper operation of the DRFM as a whole.

Since the basic function of the DRFM is to repeat the radar signals, the important requirement is to maintain sufficient signal fidelity at the output. Also DRFMs must be capable of meeting future needs for different deception techniques through software modifications.

The following paragraphs discuss some of the major characteristics and requirements of DRFM.

### **C. INSTANTANEOUS BANDWIDTH (IBW)**

The instantaneous bandwidth (IBW) is defined as the range of RF input signal frequencies at any one instant of time over which the DRFM system shall digitize, memorize, and recall the input signal without switching or tuning. The IBW of the DRFM determines the sampling clock rate of the system and also the memory size.

Due to limitations of digital technology, the analog-to-digital converter and associated circuits can operate at maximum clock speeds on the order 1 GHz. Current DRFMs have an IBW on the order of 500 MHz. The IBW in the



range of 1 GHz - 2 GHz have been predicted for GaAs circuitry [Ref. 1].

#### D. FREQUENCY TUNING RANGE

If the achievable IBW is less than the total bandwidth of expected threats, then the DRFM may have to be steered to the threat frequency. This requires the system to position the IBW anywhere in the desired threat spectrum using suitable local oscillators. Depending on the conversion hardware, it is feasible to cover an octave or possibly more than octave bandwidth. DRFM can be used to jam several threats simultaneously. Also, the unwanted signals in the IBW can be filtered out using a tunable bandstop filter. Signal gating in the time domain can also be used to select a threat from among several signals for storage in DRFM.

#### E. LOCAL OSCILLATOR REQUIREMENTS

The type of local oscillator required is dependent on

- rate at which IBW must be stepped and settled,
- the frequency accuracy of the DRFM output and
- maximum storage time for pulse

The stepping of the IBW depends on the slew rate of the local oscillator and its settling time at the desired frequency. Similarly, frequency accuracy of the DRFM output is dependent on the short term frequency stability. The maximum storage time of the pulse is dependent on the long term frequency

stability of the local oscillator. The type of local oscillators that meet the above requirements are fixed frequency oscillators like dielectric stabilized oscillator (DSO), digitally tuned voltage controlled oscillator (VCO), crystal oscillator-comb generator, or a frequency synthesizer.

The highly accurate replicas are required for deception jamming of pulse doppler radar. This leads to the requirement that residual FM on the local oscillator should be low. Also, the FM on local oscillator should not generate the inter-modulation products in the required pulse doppler bandwidth of victim radar. A synthesized source appears to be good choice in this regard, but it comes at the expense of settling speed.

#### **F. DRFM SENSITIVITY**

As DRFM utilizes down conversion hardware similar to the superheterodyne receiver, it is subjected to similar constraints involving sensitivity, bandwidth and false alarm. Proper RF and IF amplification are to be used to meet the required minimum signal-to-noise ratio. The sensitivity of the DRFM can be calculated by computing the signal-to-noise ratio (SNR) required to hold the false alarm rates to reasonable values. Figure 2.2 shows the plot of instantaneous bandwidth vs sensitivity for an SNR of 13 dB.

As shown in the plot, the sensitivity is dependent not only on IBW, but also the noise figure of the system. So, it

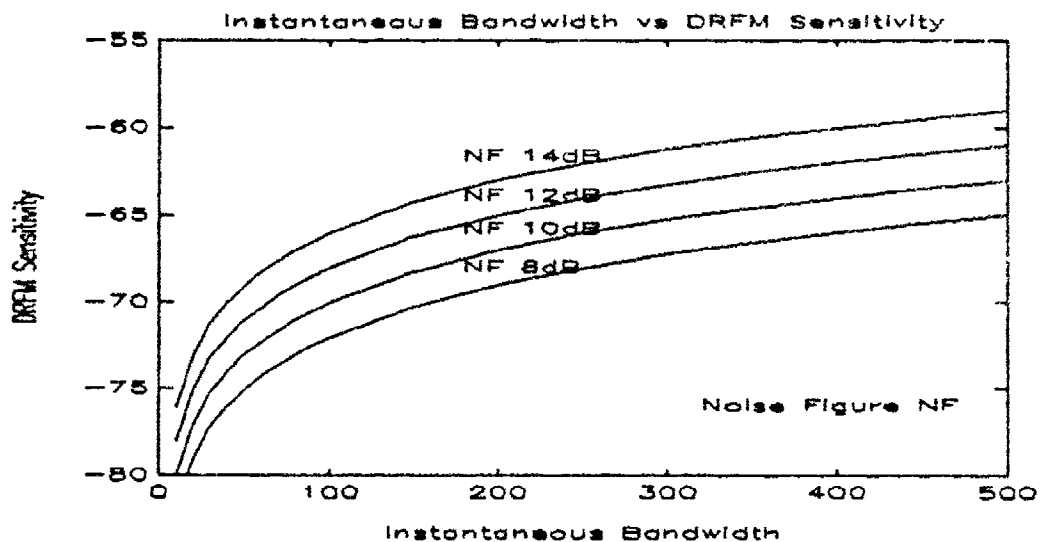


Figure 2.2 DRFM sensitivity vs instantaneous bandwidth

is important that RF-IF implementation incorporates low noise components to achieve the required sensitivity.

#### G. DYNAMIC RANGE

The dynamic range of the DRFM is dependent on the number of bits used in the quantization. Single bit DRFM can operate over a large dynamic range compared to multi-bit DRFM without additional circuitry. However, the multi-bit DRFMs can be made to operate over the required dynamic range with additional hardware, like instantaneous automatic gain control circuits, to provide proper input for the analog-to-digital converters. The input to the analog-to-digital converter has

to be maintained at the required level for spurious suppression at the output of the DRFM.

#### H. SAMPLING REQUIREMENTS

The sampling rate is the limiting factor in the design of DRFM. From the Nyquist sampling theorem, the signal has to be sampled at a frequency equal to or greater than twice the highest frequency component of the signal or twice the bandwidth of the signal. Hence in the case of DRFM, the sampling rate should be at least twice the IBW. If this requirement is met, it is ensured that proper reconstruction of the signal is possible. For example, if the IBW required is 500 MHz, the sampling rate should be greater than or equal to 1000 MHz.

The IBW can be effectively increased by quadrature sampling [Ref. 2]. A block diagram of quadrature sampling is given in Figure 2.3. In quadrature sampling scheme, one sample per cycle is taken from both I and Q channels. This results in taking two samples per cycle using two analog-to-digital converters without increasing the sampling rate of analog-to-digital converters. Thus the sampling rate will be same as the IBW instead of twice the IBW. This is an important design consideration since the same sampling rate will achieve twice the IBW.

However the IQ channel DRFM has some disadvantages. Due to the gain and phase mismatch between the I and Q channels the

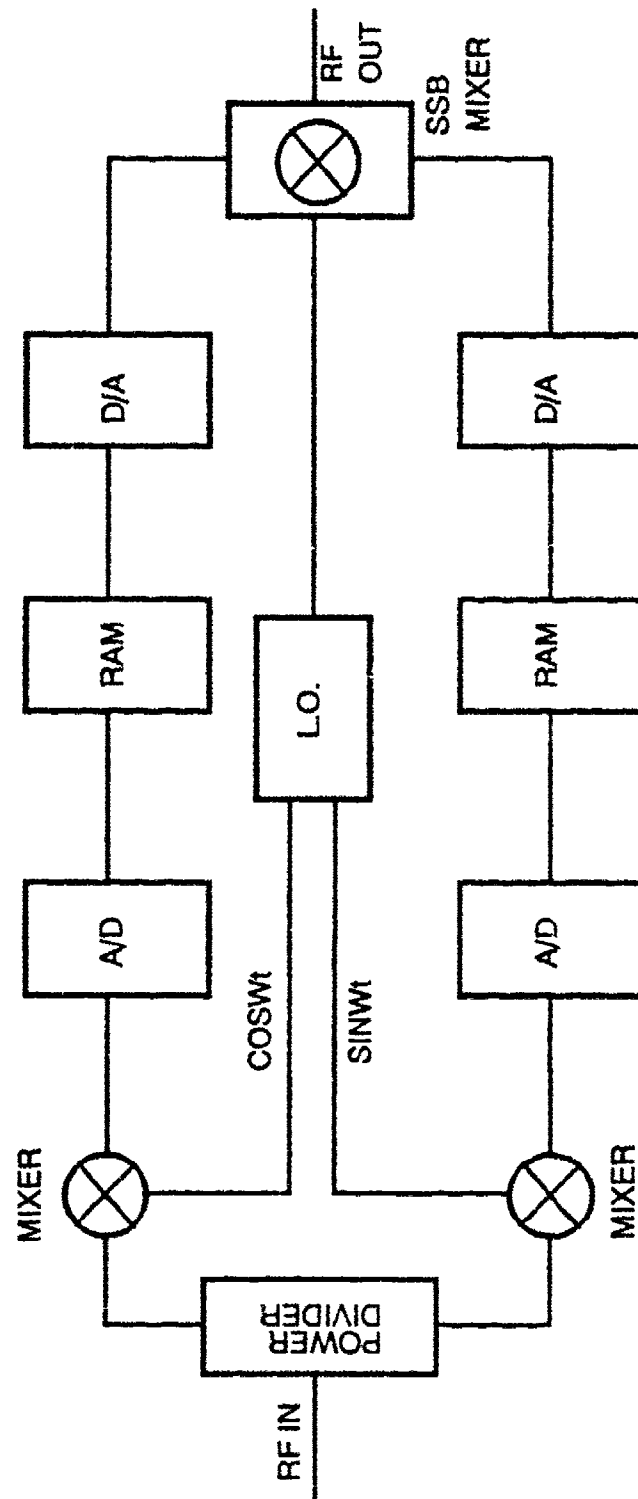


Figure 2.3 Quadrature sampled DRFM block diagram

output signal distortion may be increased. Another drawback is increased circuit complexity due to doubling of channels.

#### I. QUANTIZATION REQUIREMENTS

The sampler/ quantizer can be either an amplitude or a phase quantization analog-to-digital converter. The analog-to-digital converter should operate at a clock speed compatible with instantaneous bandwidth of the DRFM. Also the clock speed should be compatible with memory devices. Quantization is the most crucial function in terms of the achieving the waveform fidelity of the DRFM. The level of harmonic suppression required dictates the selection of the sampling rate and also the number of bits of quantization. For a given sampling rate, the harmonic signal level is reduced as the number of quantization bits are increased. Harmonic levels of multi-bit amplitude encoded DRFM are discussed in Chapter III.

#### J. STORAGE REQUIREMENTS

The memory size of the DRFM is determined by the instantaneous bandwidth and the maximum signal length to be stored (which is equal to longest pulse width). The total number of bits to be stored (N) is given by

$$N = f_s \cdot N_s \cdot N_b$$

where

$f_s$  = sampling rate =  $2 \cdot \text{IBW}$

$N_s$  = maximum pulse width of the threat

$N_b$  = number of bits per sample

Assuming that one bit per sample is collected from the A/D and that the sampling rate is 50 MHz, one bit will be available at every 20 nanoseconds. If the longest pulse width is to be stored is 100 microseconds, the storage required will be 5000 bits.

Due to technology limitations, the IBW required to meet some types of threats is still difficult to achieve. However, multiplexing schemes can be used to provide higher IBW with achievable sampling rates. There are two multiplexing schemes available known as frequency multiplexing and time multiplexing.

#### **1. Frequency Multiplexing**

In frequency multiplexing, the signal is divided into a number of channels of reduced bandwidth. Each channel is sampled and converted to digital bit stream and stored. To reconstruct the signal, each channel is then recalled and an analog signal is reconstructed. By proper mixing and filtering, the composite threat signal is regenerated.

#### **2. Time multiplexing**

In time multiplexing, the signal is sampled with several time-phased clocks to produce several data streams which are then stored. The data streams are interleaved in an appropriate time sequence to reproduce the original signal. Duplex and quadruplex time multiplexing is possible to achieve higher sampling rates. The frequency multiplexing

is comparatively difficult process and hence the time multiplexing scheme is preferred.

In this chapter, we have discussed the major characteristics and requirements of the DRFM. In the next chapter, we will calculate the levels of harmonic signals generated due to the quantization process.



### III SPURIOUS SIGNAL ANALYSIS

#### A. INTRODUCTION

The spurious signal is generated in the DRFM due to quantization and sampling. The spurious levels are of importance since they may create the recognizable signature of a jammer using DRFM. Also the power in spurious is not useful in the jamming. The theoretical analysis of the spurious signals generated in DRFM is elaborated in this chapter. Also a method to calculate the spurious signal level generated due to quantization is given.

#### B. WAVEFORM SAMPLING AND QUANTIZATION

The radar waveforms are continuous in amplitude and time. Therefore, the waveforms are to be converted to digital format for storing the radar waveform in the memory. The conversion

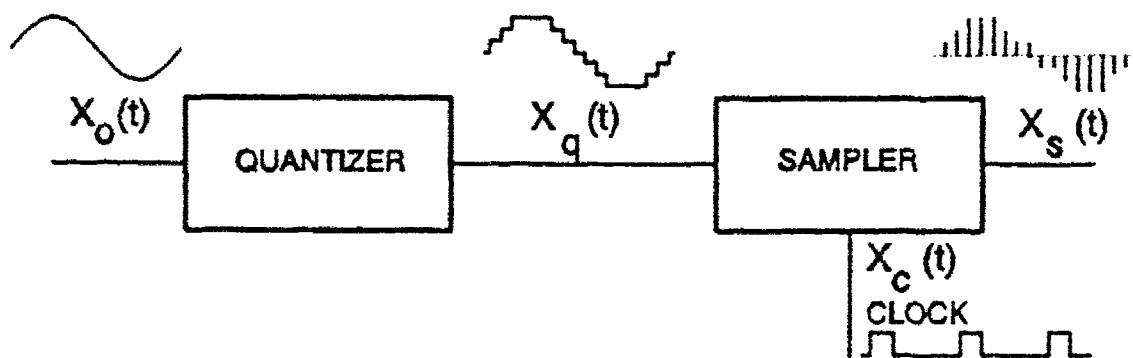


Figure 3.1 Analog to digital converter

process involves two operations: sampling and quantization. In principle, operation of sampling and quantization can be performed in any order. However, in practice, conversion process is performed by the analog-to-digital converter and sampling operation is performed first. The analog-to-digital converter can be modeled as quantizer and sampler as shown in Figure 3.1.

### 1. Quantization of the signal

The quantization process can be considered to result from the application of signal to a staircase transducer. A staircase transducer converts the input voltage into binary levels representing the amplitude of the input signal. Assuming the input signal is sinusoid, the quantizer maps the amplitude of the sinusoidal signal to discrete levels represented by the binary number, i.e., the real valued instantaneous amplitude 'a' of the signal is mapped to a number k by the transducer. Number k is a binary number where  $0 < k < N$ ,  $N = 2^m$  where m is the number of bits of the analog-to-digital converter. Therefore the output of the quantizer is a discrete level signal instead of continuously varying signal. Because of the nonlinear characteristics of the quantization process, the harmonic distortion is expected in the output signal of the quantizer. Assuming that the sinusoid is quantized to one bit, that is to two levels representing positive and negative half cycles of the

sinusoid, the sinusoid input signal is converted to a square wave at the output of the quantizer. The harmonic content of the square wave signal is shown in Figure 3.2. It can be seen that the power of the each harmonic reduces monotonically. A method for calculating the harmonic content of the quantized signal is presented in the next section.

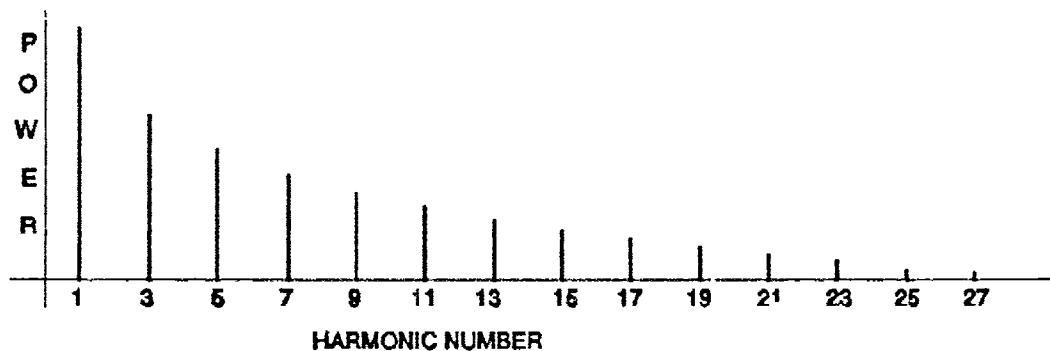


Figure 3.2 Harmonics of the square wave

## 2. Sampling of the signal

The Nyquist sampling theorem states that an analog signal should be sampled at a rate greater than twice its highest frequency component. If this condition is satisfied, the continuous signal can be theoretically reconstructed completely from its samples as per sampling theorem.

The sampling theorem, in its simple form, assumes infinite quantization. However, due to circuit limitations, the infinite quantization is physically not possible. So, the

harmonic distortion is inevitable in the DRFM sampling and quantization process.

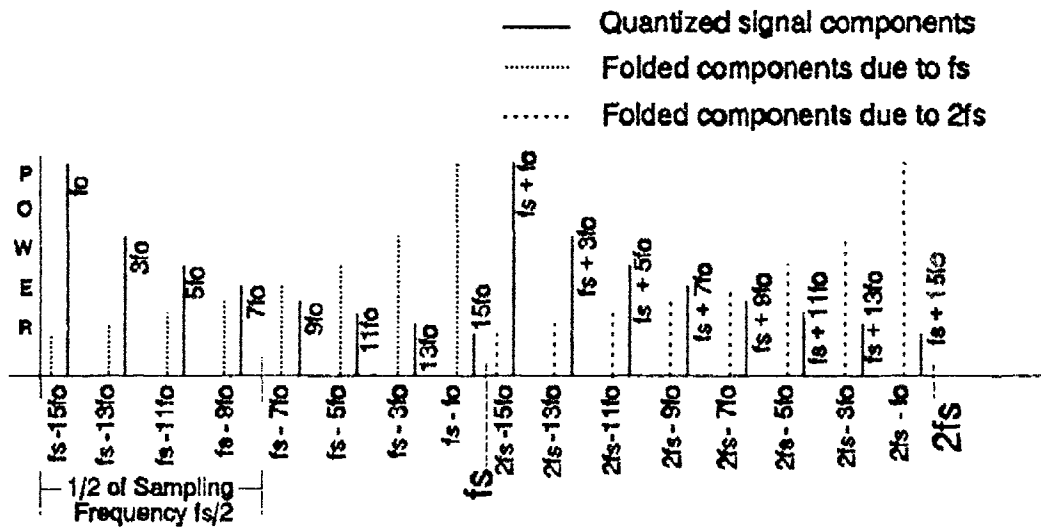


Figure 3.3 Harmonics due to quantization and ideal sampling

The sampling theorem also assumes that sampling width is infinitesimally small as represented by Dirac delta function  $\delta(t)$ . As the sampling function is periodic in time, the input spectrum is replicated around the integer multiples of sampling frequency in the frequency domain. In Figure 3.3, the input signal spectrum has frequency components  $f_0, 3f_0, \dots$  generated by the quantization process. (Only up to 15th harmonic is shown in the figure for convenience). The input spectrum is replicated around sampling frequency  $f_s$  as  $(f_s - f_0), (f_s - 3f_0)$  and so on, and also  $(f_s + f_0), (f_s + 3f_0)$  and so on. Similarly, the replication around twice the sampling frequency  $2f_s$  will be  $(2f_s - f_0), (2f_s - 3f_0)$ , so on. As the band of interest

is half the sampling frequency  $f_s/2$ , it can be seen from Figure 3.3 that the harmonic signals  $(f_s-15f_0)$ ,  $(f_s-13f_0)$ ,  $(f_s-11f_0)$ ,  $(f_s-9f_0)$  are folded into the signal bandwidth due to sampling. Also, it is to be noted that the spectral lines due to integer multiples of sampling frequency  $3f_s$ ,  $4f_s$ , ... etc will also be folded into the signal bandwidth, but will be of much smaller magnitude.

### 3. Effects due to finite width sampling

In practice, the sampler is clocked with a periodic clock signal of finite width  $\tau_c$  and hence the samples are of finite width  $\tau_c$  repeating at the clock period of  $T_c$ . The effect of quantization and finite width sampling on input sinusoid, quantized to single bit, is shown in Figure 3.4. As can be seen from the Figure 3.4, the harmonics of the fundamental generated by the quantization process are folded into the passband of DRFM (which is d.c. to half the sampling frequency).

As in case of infinitesimally small width sampling, the spectral lines generated by the integer multiples of the sampling frequency are also folded into the DRFM passband. However, the effect of finite width sampling will be to modify the spectral levels by the sinc function as shown in the Figure 3.4. The harmonic distortion will be proportional to ratio of the signal frequency to sampling frequency. If the

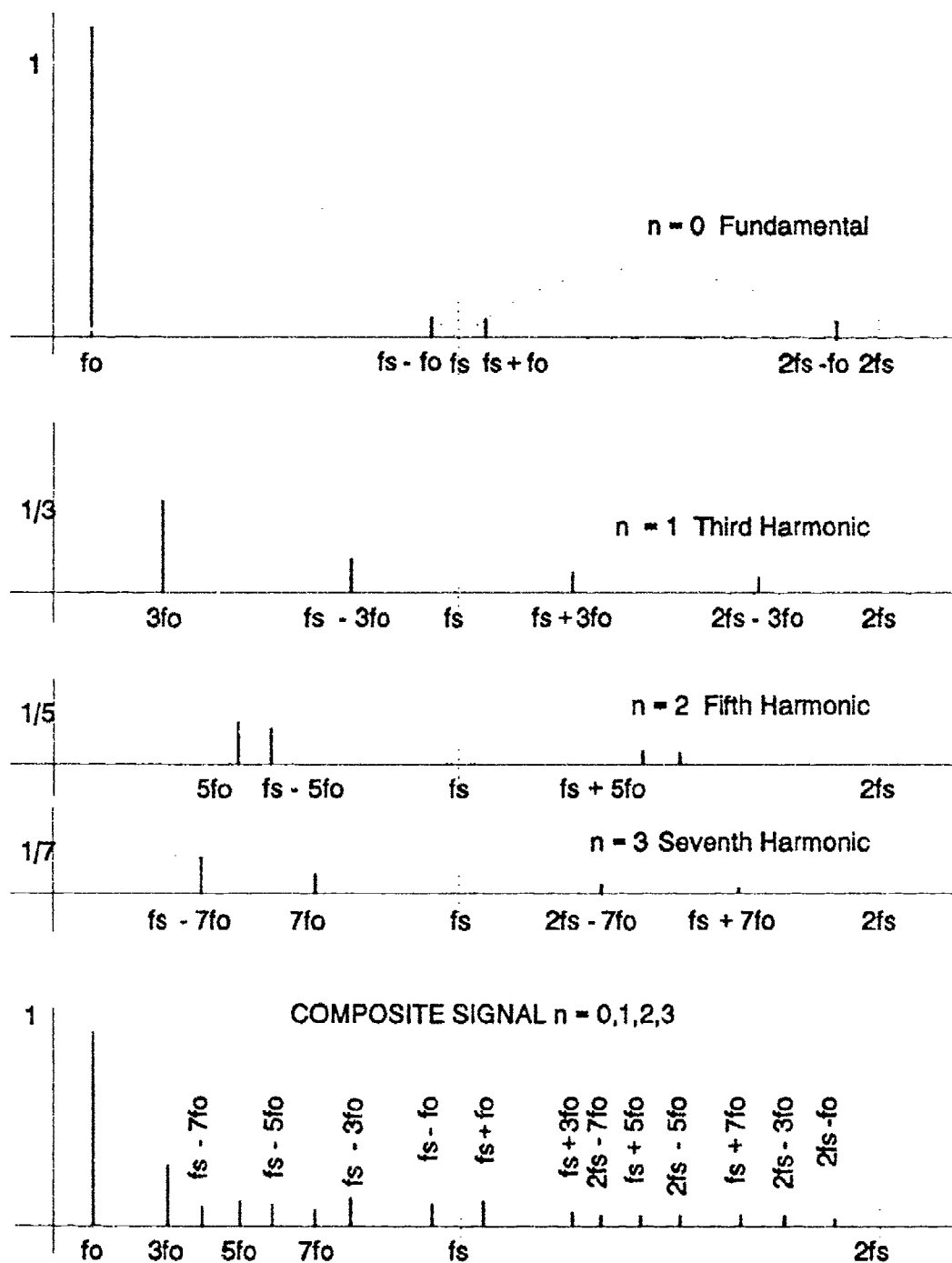


Figure 3.4 Harmonics due to quantization and finite width sampling

ratio of the signal frequency to sampling frequency is low, harmonic distortion will also be low.

Though the input signal is band-limited to frequency less than half the sampling frequency, the quantization process generates the harmonics of the fundamental signal and the sampling process folds them into the passband. If the input signal has wider spectrum, it is to be expected that the quantization process will generate harmonics of each of the spectral line in the input signal. Further, the sampling process will fold the intermodulation products of harmonics and sampling frequency into the instantaneous bandwidth of the DRFM. As the sampling and quantization processes are essential, the requirement will be to reduce the harmonic levels due to quantization such that overall spurious outputs of the DRFM is within acceptable levels.

### C. COMPUTATION OF HARMONICS DUE TO QUANTIZER

This section presents a method to calculate the harmonic signal levels generated by the quantizer.

In order to calculate the harmonic distortion introduced by the quantizing process, we consider a sinusoidal signal  $x(t) = A_0 \sin \omega_0 t$ . A quantized sinusoidal signal can be represented by the quantized signal  $x_q(t)$  as shown Figure 3.5. If  $m$  is number of bits of quantization, then each half cycle of the sinusoid is quantized into  $N$  steps where  $N = 2^{m-1}$ . The quantized signal  $x_q(t)$  can be considered

as superposition of  $N$  pairs of rectangular step signals of period  $T_0$  equal to the period of the sinusoid. The amplitude of each rectangular step is  $A_0/N$ . The width of step  $k$  is  $t_k$  in each half cycle, and is given by

$$t_k = \frac{T_0}{2} - \frac{2}{\omega_0} \sin^{-1} \frac{(k-1)}{N} \quad \dots (3.1)$$

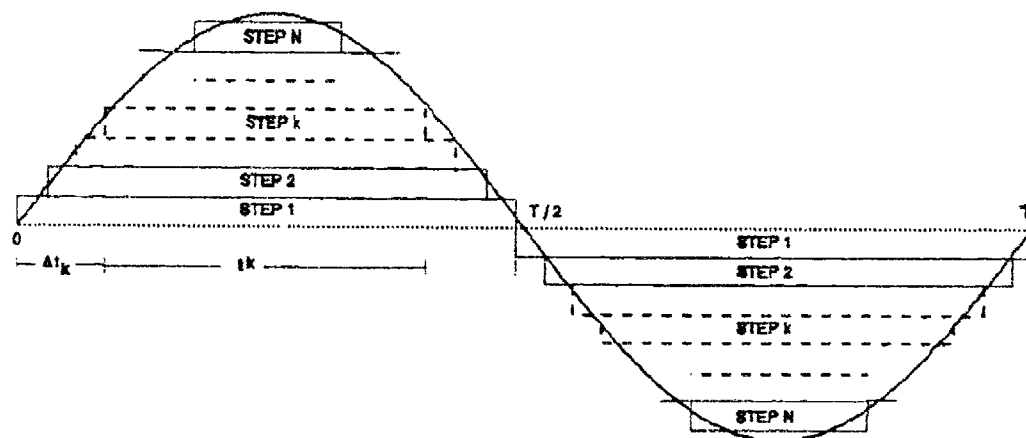


Figure 3.5 Quantized signal step pairs

The equation 3.1 is equivalent to

$$\begin{aligned} t_k &= \frac{T_0}{2} - \frac{2}{\omega_0} \sin^{-1} \frac{(k-1)}{N} \\ &= \frac{T_0}{2} - \frac{2}{2\pi f_0} \sin^{-1} \frac{(k-1)}{N} \\ &= \frac{T_0}{2} - \frac{T_0}{\pi} \sin^{-1} \frac{(k-1)}{N} \end{aligned}$$



For two bit amplitude quantization, number of pairs  $N = 2$ , amplitude step  $= A_o/2$  and  $k = 1, 2$ . Then for  $N = 2$ ,  $k = 1$ , the width  $t_k$  is given by

$$t_k = t_1 = \frac{T_o}{2}$$

The waveform for  $N = 2$ ,  $k = 1$  is as in Figure 3.6

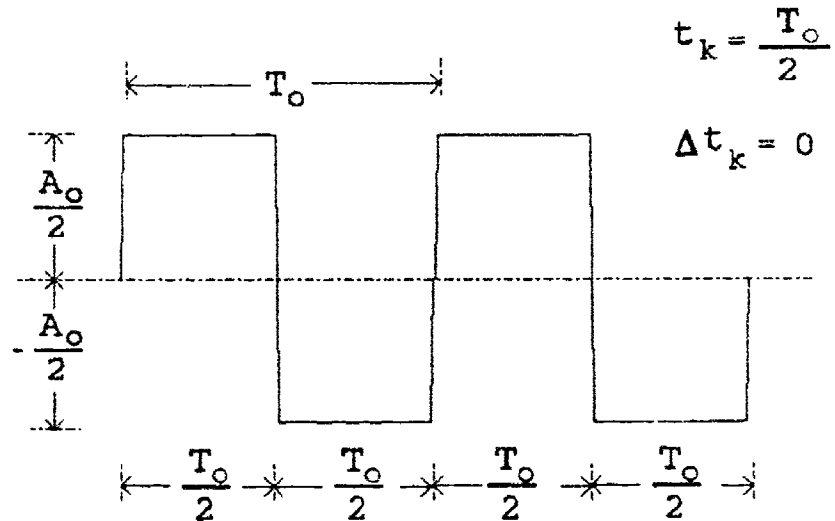


Figure 3.6 Quantization step pair  $N = 2$ ,  $k = 1$

and for  $N = 2$ ,  $k = 2$ , the width  $t_k$  is given by

$$t_k = t_2 = \frac{T_o}{2} - \frac{T_o}{\pi} \sin^{-1} \frac{1}{2}$$

$$= \frac{T_o}{3}$$

The waveform for  $N = 2, k = 2$  is as in Figure 3.7

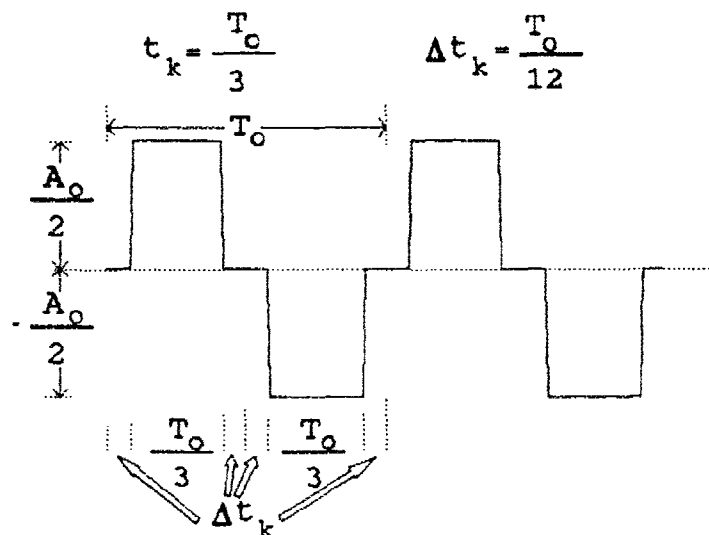


Figure 3.7 Quantization step pair  $N = 2, k = 2$

The quantized waveform of the 2 bit amplitude quantized sinusoid is given in Figure 3.8.

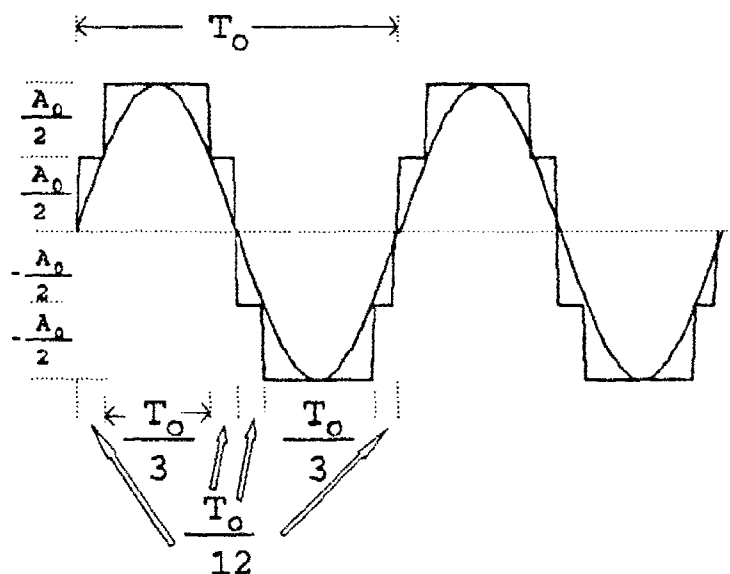


Figure 3.8 Quantized combined waveform for  $N = 2, k = 1, 2$

The k'th rectangular step pair may be represented by the Fourier series as

$$x_k(t) = \sum_{n=1}^{\infty} B_{n,k} \sin \omega_0 t \quad \dots (3.2)$$

where  $B_{n,k}$  is given by

$$B_{n,k} = \frac{A_0}{N} \frac{\omega_0}{\pi} \int_{\Delta t_k}^{\frac{T}{2} - \Delta t_k} \sin n \omega_0 t dt \quad n=1,3,5,7\dots \quad \dots (3.3)$$

where  $\Delta t_k$ , offset of the of k'th step is

$$\Delta t_k = \frac{1}{\omega_0} \sin^{-1} \left( \frac{(k-1)}{N} \right) \quad \dots (3.4)$$

Then  $B_{n,k}$  is

$$B_{n,k} = \frac{4A}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{(k-1)}{N} \right) \right] \quad \dots (3.5)$$

Hence the Fourier series of the k'th step pair is

$$x_k(t) = \sum_{n=0}^{\infty} \frac{4A}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{(k-1)}{N} \right) \right] \cdot \sin(2n+1) \omega_0 t \quad \dots (3.6)$$

For N pair of rectangular pulses, we obtain Fourier series of the quantized signal  $x_q(t)$  as

$$\begin{aligned} x_q(t) &= \sum_{k=1}^N x_k t \\ &= \sum_{k=1}^N \sum_{n=0}^{\infty} \frac{4A}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{(k-1)}{N} \right) \right] \cdot \sin(2n+1) \omega_0 t \quad \dots (3.7) \end{aligned}$$

$$= \sum_{n=0}^{\infty} A_{2n+1} \sin(2n+1) \omega_0 t \quad \dots (3.8)$$

where  $A_{2n+1}$  is the amplitude of the  $(2n+1)$ 'th harmonic and is given by

$$A_{2n+1} = \frac{4A}{\pi N(2n+1)} \sum_{k=1}^N \cos[(2n+1) \sin^{-1}(\frac{k-1}{N})] \quad \dots (3.9)$$

The signal power in each of the harmonics, given by  $(A_{2n+1})^2$ , can be calculated using equation 3.9. The harmonic power for each harmonic is shown in Appendix A in Figures A.1 through A.18. It can be seen from the figures that

- the third harmonic is predominant
- the total harmonic power decreases as the number of bits of quantization is increased.

Table 3.1 shows the maximum harmonic power in dB relative to fundamental and total harmonic power relative to fundamental for different quantization levels.

As the third harmonic is predominant and it will be folded back into the IBW due to sampling, it is advisable to move the strongest harmonic from the third to a higher harmonic.

#### D. REDUCTION OF HARMONIC POWER

As discussed in the last section, the harmonic power level in the third harmonic is maximum for all the quantization bit levels. Also this harmonic will be folded into the passband by the sampling process. Hence, it is necessary that the harmonic signal with maximum harmonic power level be as far from the fundamental as possible so as not to fold into the

**TABLE 3.1**  
COMPARISON OF QUANTIZATION BITS AND HARMONIC POWER LEVEL

Number of bits	Third harmonic power level	Total harmonic power in 99 harmonics
1	- 9.5 dBr	- 6.4 dBr
2	- 14.9 dBr	- 12.1 dBr
3	- 21.3 dBr	- 18.0 dBr
4	- 28.9 dBr	- 24.1 dBr
5	- 34.7 dBr	- 30.6 dBr
6	- 41.3 dBr	- 38.2 dBr

passband. The following paragraphs present certain methods to reduce the harmonic power in the output of the DRFM.

**1. Harmonic reduction in single bit quantization DRFMs**

The spur reduction in single bit DRFM is based on the relationship between the level of odd harmonics of the rectangular wave and its duty cycle [Ref. 3]. By adjusting the duty cycle to  $1/N$ , the harmonic spur  $N$  can be eliminated at the expense of generating even harmonics. If the duty cycle is varied from less than 0.5 to greater than 0.5, the energy is transferred from odd harmonics to even harmonics, but due to reverse duty cycle, the even harmonics are phased out.

It is also possible to reduce the harmonics by use of controlled time jitter in the sampling clock [Ref. 4]. The

time jitter  $t_n$  of the clock should be controlled such that  $t_n$  is an identically distributed independent random variable with a uniform probability density function over  $[-nT_0/2, nT_0/2]$ , where  $0 < n \leq 1$ , and  $T_0$  is the time period of sampled signal. It is stated that power level of the fundamental signal is also reduced to some extent. It is to be noted that this method can also be used in case of multi-bit quantization.

## 2. Harmonic reduction multi-bit quantization DRFMS

Consider the input sinusoid signal quantized to  $2^{m-1}$  amplitude levels where  $m$  is the number bits of the analog to digital converter in each of the half cycles as shown in Figure 3.9. The amplitude step of quantization is 'a' and the transition angle at the quantization point is  $\theta$  as shown.

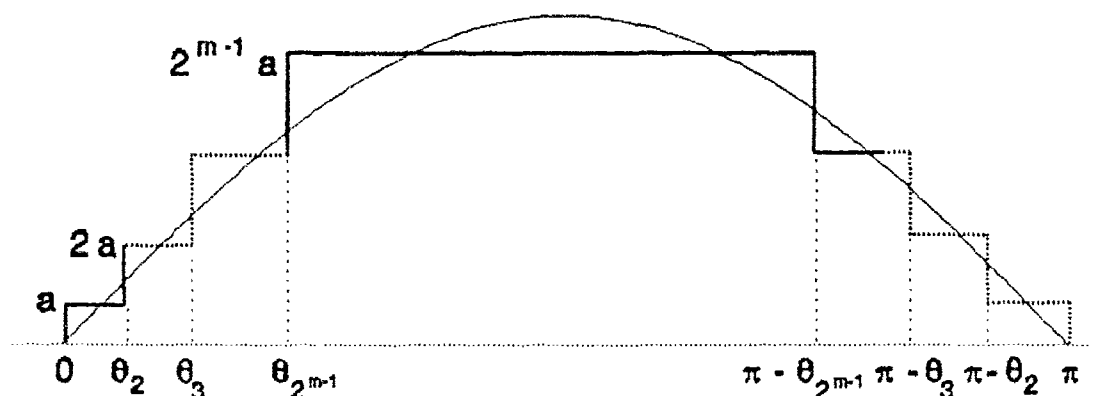


Figure 3.9 The quantization levels and angles of signal

Assuming positive and negative half cycle symmetry in the quantized signal, the Fourier series of the  $2^{m-1}$  level quantized signal is given by

$$x_o(t) = \sum_{n=0}^{\infty} b_n \sin n\omega_o t \quad \dots (3.10)$$

where  $b_n$  is given by

$$b_n = \frac{2}{\pi} \left[ \int_0^{\theta_2} a \sin nx \, dx + \int_{\theta_2}^{\theta_1} 2a \sin nx \, dx + \int_{\theta_1}^{\theta_4} 3a \sin nx \, dx + \dots \right. \\ \left. + \int_{\theta_{2^{m-1}}}^{\pi - \theta_{2^{m-1}}} 2^{m-1} a \sin nx \, dx + \dots + \int_{\pi - \theta_2}^{\pi} a \sin nx \, dx \right] \quad \dots (3.11)$$

i.e.,

$$b_n = \frac{2a}{(2n+1)\pi} (1 - \cos((2n+1)\pi)) \\ \cdot [1 + \cos(2n+1)\theta_2 + \cos(2n+1)\theta_3 \\ + \dots \cos(2n+1)\theta_{2^{m-1}}] \quad \dots (3.12)$$

For two bit DRFM, the harmonic amplitude levels are given by

$$b_n = \frac{4a}{(2n+1)\pi} [1 + \cos(2n+1)\theta_2] \quad \dots (3.13)$$

For three bit DRFM, the harmonic amplitude levels are given by

$$b_n = \frac{4a}{(2n+1)\pi} [1 + \cos(2n+1)\theta_2 + \cos(2n+1)\theta_3 + \cos(2n+1)\theta_4] \quad \dots (3.14)$$

The step amplitude level 'a' and transition angle  $\theta_s$ ,  $s = 1, 2, \dots$  can be selected such that staircase output is best fit to the input sinusoid. To determine the best fit step amplitude 'a' and transition angles  $\theta_s$ , we proceed as follows.

As shown in Figure 3.9, the error in the different intervals is as given below.

Transition angle interval	Error in the interval
$0 < \theta < \theta_2$	$a - \sin \theta$
$\theta_2 < \theta < \theta_3$	$2a - \sin \theta$
$\theta_3 < \theta < \theta_4$	$3a - \sin \theta$
-----	-----
$\theta_{2^{m-1}} < \theta < \pi/2$	$2^{m-1}a - \sin \theta$

So the squared error over the complete cycle is given by

$$\begin{aligned} \sum e^2 = & 4 \int_0^{\theta_2} (a - \sin \theta)^2 d\theta + \int_{\theta_2}^{\theta_3} (2a - \sin \theta)^2 d\theta + \dots \\ & + \int_{\theta_{2^{m-1}}}^{\frac{\pi}{2}} (2^{m-1}a - \sin \theta)^2 d\theta \quad \dots (3.15) \end{aligned}$$

For minimum error in the output signal with respect to input signal, the partial derivatives of the above equation with respect to  $\theta_2, \theta_3, \dots, \theta_{m-1}$ , and step amplitude 'a' must be equal to zero. Solving for step amplitude 'a' and  $\theta_2, \theta_3, \dots, \theta_{m-1}$  that meets the above requirement, we get following relations

$$\begin{aligned} \sin \theta_2 &= 3a/2 \\ \sin \theta_3 &= 5a/2 \\ &\dots \\ \sin \theta_{2^{m-1}} &= (2^m - 1)a/2 \quad \dots (3.16) \end{aligned}$$

and the step amplitude 'a' as below.

Number of Bits	Amplitude step
1	0.6366198
2	0.4188161
3	0.2332685
4	0.1214300
5	0.0616995



The harmonic signal levels can be calculated based on the above values for angle  $\theta_s$  and amplitude 'a' in the equation 3.12. For two bit DRFM, the equation 3.12 reduces to

$$b_n = \frac{4a}{n\pi} [1 + \cos(n\theta_2)]$$

$$= \frac{0.53325}{n} [1 + \cos(38.92 * n)] \quad \dots (3.17)$$

The harmonic power in the higher bit DRFM can be calculated accordingly by using the above values in equation 3.12. The plot of harmonic power in multi bit DRFMs calculated as per equation 3.12 is shown in Figures B.1 through B.8 in Appendix B. However, it is to be noted that the harmonic power as shown is dependent on the sampling point in the RF cycle and amplitude at the sampling point. In a high frequency sampling system, the point of sampling becomes critical and hence calls for a high stability clock for sampling. The plots in Figures B.1 through B.8 show that

- the third harmonic is not the worst offender
- the harmonic power is not decreasing with increasing harmonic number

Table 3.2 gives the harmonic power level and total harmonic power for versus number of bits.

The results in the Table 3.2 are more conservative because the following factors contribute towards reduction of the harmonic content in actual design [Ref. 5].

- rounding of the quantized waveform due to stray capacitances in the circuit.
- generation of even harmonics due to nonsymmetric quantized waveform will reduce the odd harmonic power,
- drift in the L.O. frequency spreads the harmonics over the IBW.

**TABLE 3.2**  
NUMBER OF QUANTIZATION BITS AND HARMONIC POWER

Number of bits	Worst Harmonic	Power level of worst harmonic	Total Harmonic power w.r.t fundamental
2	9	- 18.1 dBr	5.44 %
3	19	- 25.5 dBr	1.17 %
4	43	- 34.3 dBr	0.27 %
5	45	- 45.9 dBr	0.63 %

Also, note that the results in the Table 3.2 are more optimistic than actual as the harmonic power level dependency on the relation between signal frequency and sampling frequency is not considered.

In the next section, we discuss the dependence of harmonic power level with respect to signal frequency and sampling frequency.

### E. EXPRESSION FOR SAMPLED AND QUANTIZED SIGNAL

The quantized signal  $x_q(t)$  is sampled with the clock signal  $x_c(t)$ . Thus the sampled quantized signal  $x_s(t)$  is given by

$$x_s(t) = x_q(t) \cdot x_c(t) \quad \dots(3.18)$$

where  $x_q(t)$  is the quantized signal as derived before. The clock signal  $x_c(t)$  is a signal as shown in Figure 3.10.

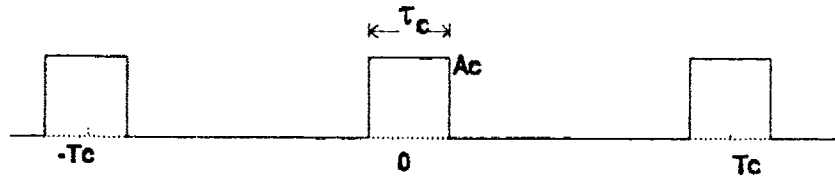


Figure 3.10 Clock signal

The Fourier series of the clock signal  $x_c(t)$  is given by

$$x_c(t) = \frac{A_c \tau_c}{T_c} \left[ 1 + 2 \sum_{m=1}^{\infty} \frac{\sin \frac{m \omega_c \tau_c}{2}}{\frac{m \omega_c \tau_c}{2}} \cos m \omega_c t \right] \quad \dots(3.19)$$

Replacing clock signal  $x_c(t)$  and quantized signal  $x_q(t)$  with their Fourier series expansions, we get  $x_s(t)$  as

$$\begin{aligned} x_s(t) = & A_c \frac{\tau_c}{T_c} \left[ 1 + 2 \sum_{m=1}^{\infty} \frac{\sin \frac{m \omega_c \tau_c}{2}}{\frac{m \omega_c \tau_c}{2}} \cos m \omega_c t \right] \\ & \cdot \left[ \sum_{k=1}^N \sum_{n=0}^{\infty} \frac{4A_o}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{(k-1)}{N} \right) \right] \right. \\ & \left. \cdot \sin(2n+1) \omega_o t \right] \quad \dots(3.20) \end{aligned}$$

By expansion and trigonometric manipulation we have

$$\begin{aligned}
 x_s(t) = & \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{k-1}{N} \right) \right] \\
 & \cdot \sin(2n+1) \omega_o t \\
 & + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m \omega_c \tau_c}{2}}{\frac{m \omega_c \tau_c}{2}} \\
 & \cdot \cos \left[ (2n+1) \sin^{-1} \left( \frac{k-1}{N} \right) \right] \sin[m \omega_c t + (2n+1) \omega_o t] \\
 & + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m \omega_c \tau_c}{2}}{\frac{m \omega_c \tau_c}{2}} \\
 & \cdot \cos \left[ (2n+1) \sin^{-1} \left( \frac{k-1}{N} \right) \right] \sin[m \omega_c t - (2n+1) \omega_o t] \\
 & \dots (3.21)
 \end{aligned}$$

The first term in the above equation

$$\begin{aligned}
 & \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \cos \left[ (2n+1) \sin^{-1} \left( \frac{k-1}{N} \right) \right] \\
 & \cdot \sin[(2n+1) \omega_o t] \dots (3.22)
 \end{aligned}$$

gives all the harmonics of the fundamental frequency  $\omega_o$ . The harmonic frequencies beyond the filter bandwidth  $f_c/2$  are rejected by the filtering process. Hence only the frequency terms  $(2n+1)\omega_o$  less than  $\omega_c/2$  are to be considered.

The second term in the above equation

$$\begin{aligned}
 & \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m \omega_c \tau_c}{2}}{\frac{m \omega_c \tau_c}{2}} \cos \left[ (2n+1) \sin^{-1} \left( \frac{k-1}{N} \right) \right] \\
 & \cdot \sin[m \omega_c t + (2n+1) \omega_o t] \dots (3.23)
 \end{aligned}$$

does not contribute as higher order frequencies  $[(2n+1)\omega_o + m\omega_c]$  generated by this term are rejected by the filtering process because they are above  $\omega_c/2$ . So the second term in equation (3.21) can be neglected.

The third term in equation (3.21) is

$$\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m\omega_c \tau_c}{2}}{\frac{m\omega_c \tau_c}{2}} \cos[(2n+1) \sin^{-1}(\frac{(k-1)}{N})] \cdot \sin[m\omega_c t - (2n+1)\omega_o t] \quad \dots (3.24)$$

and it generates the inter-modulation products as in a mixing operation and contributes to the harmonic power if the  $[2(n+1)\omega_o - m\omega_c]$  is less than  $\omega_c/2$ .

Finally, the sampled signal  $x_s(t)$  can be written as

$$x_s(t) = \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \cos[(2n+1) \sin^{-1} \frac{(k-1)}{N}] \cdot \sin(2n+1)\omega_o t$$

$$+ \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m\omega_c \tau_c}{2}}{\frac{m\omega_c \tau_c}{2}} \cdot \cos[(2n+1) \sin^{-1}(\frac{(k-1)}{N})] \cdot \sin[m\omega_c t - (2n+1)\omega_o t] \quad \dots (3.25)$$

The equation (3.25) shows that the inter-modulation products of the harmonics of the signal frequency and

harmonics of the clock frequency will contribute to the spurs.

It is convenient to rewrite the equation (3.25) as

$$x_d(t) = \sum_{n=0}^{\infty} \sum_{k=1}^W P_{n,k} \sin(2n+1)\omega_o t + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^W Q_{m,n,k} \sin[m\omega_c t - (2n+1)\omega_o t] \quad \dots (3.26)$$

where  $P_{n,k}$  is defined as

$$P_{n,k} = \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \cos[(2n+1) \sin^{-1} \frac{(k-1)}{N}] \quad \dots (3.27)$$

and  $Q_{m,n,k}$  is defined as

$$Q_{m,n,k} = \frac{A_c \tau_c}{T_c} \frac{4A_o}{\pi N(2n+1)} \frac{\sin \frac{m\omega_c \tau_c}{2}}{\frac{m\omega_c \tau_c}{2}} \cos[(2n+1) \sin^{-1} \frac{(k-1)}{N}] \quad \dots (3.28)$$

The plots of harmonic power in dB vs frequency as per equation 3.26 is given in Appendix C. The Figures C.1 through C.16 show the plot for 1, 2, 3 and 4 bit amplitude quantized sampled signals. As can be seen from the plots, the harmonic power folded into the instantaneous bandwidth is dependent on the relation between input signal and the sampling frequency. If the ratio of the signal frequency to sampling frequency is low, the harmonic power relative to the fundamental is also low. Table 3.3 shows the relationship of normalized input frequency to harmonic power for 1 through 4 bits sampled quantized signal.

As can be seen from Table 3.3, the third harmonic level of quantized signal will be approximately equal to quantized

sampled signal only if the sampling frequency is high compared to signal frequency. As the ratio of signal frequency to sampling frequency increases, the harmonic power also increases. Tables C.1 through C.4 list the harmonic power of 1 through 4 bit sampled quantized signals.

**TABLE 3.3**  
HARMONIC POWER LEVEL OF QUANTIZED AND SAMPLED SIGNAL

Normalized Input Frequency	Third Harmonic power level (dB) Quantization Bits			
	1	2	3	4
Quantized only	- 9.54	- 14.96	- 21.33	- 28.00
0.005	- 9.45	- 14.76	- 20.86	- 26.98
0.025	- 8.86	- 13.44	- 18.28	- 22.66
0.05	- 8.23	- 12.18	- 16.34	- 19.10
0.1	- 7.01	- 9.94	- 12.96	- 15.23

## F. EXPRESSION FOR DRFM OUTPUT

The stored signal is repeated with pulse repetition period  $T_r$  (PRF of  $f_r$ ), pulse width of  $\tau_r$  and amplitude of  $A_r$ . The Fourier series of such a waveform is

$$x_r(t) = A_r \frac{\tau_r}{T_r} \left[ 1 + 2 \sum_{l=1}^{\infty} \frac{\sin \frac{l\omega_r \tau_r}{2}}{l\omega_r \frac{\tau_r}{2}} \cos l\omega_r t \right] \quad \dots (3.29)$$

So, the output of the DRFM is

$$x_d(t) = x_s(t) \cdot x_r(t) \quad \dots (3.30)$$

Replacing the Fourier series expansion for  $x_s(t)$  and  $x_r(t)$  we have  $x_d(t)$  as

$$\begin{aligned} x_d(t) = & \left[ \sum_{n=0}^{\infty} \sum_{k=1}^N P_{n,k} \sin(2n+1)\omega_o t \right. \\ & \left. + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N Q_{m,n,k} \sin[m\omega_c t - (2n+1)\omega_o t] \right] \\ & \cdot \left[ \frac{A_r \tau_r}{T_r} + \sum_{l=1}^{\infty} \frac{2A_r \tau_r}{T_r} \frac{\sin \frac{l\omega_r \tau_r}{2}}{\frac{l\omega_r \tau_r}{2}} \cos(l\omega_r t) \right] \quad \dots (3.31) \end{aligned}$$



By expansion and trigonometric manipulations

$$\begin{aligned}
 x_d(t) = & \sum_{n=0}^{\infty} \sum_{k=1}^N p_{n,k} \frac{A_r \tau_r}{T_r} \sin((2n+1)\omega_o t) \\
 & + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N Q_{m,n,k} \frac{A_r \tau_r}{T_r} \sin(m\omega_c - (2n+1)\omega_o) t \\
 & + \sum_{l=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N p_{n,k} \frac{A_r \tau_r}{T_r} \frac{\sin \frac{l\omega_r \tau_r}{2}}{\frac{l\omega_r \tau_r}{2}} \\
 & \quad \cdot \sin((2n+1)\omega_o + l\omega_r) t \\
 & + \sum_{l=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N p_{n,k} \frac{A_r \tau_r}{T_r} \frac{\sin \frac{l\omega_r \tau_r}{2}}{\frac{l\omega_r \tau_r}{2}} \\
 & \quad \cdot \sin((2n+1)\omega_o - l\omega_r) t \\
 & + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N Q_{m,n,k} \frac{A_r \tau_r}{T_r} \frac{\sin \frac{l\omega_r \tau_r}{2}}{\frac{l\omega_r \tau_r}{2}} \\
 & \quad \cdot \sin(m\omega_c - (2n+1)\omega_o - l\omega_r) t \\
 & + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{k=1}^N Q_{m,n,k} \frac{A_r \tau_r}{T_r} \frac{\sin \frac{l\omega_r \tau_r}{2}}{\frac{l\omega_r \tau_r}{2}} \\
 & \quad \cdot \sin(m\omega_c - (2n+1)\omega_o + l\omega_r) t \quad \dots (3.33)
 \end{aligned}$$

The above equation defines the output of the DRFM completely. As can be seen from the equation, the effect of quantization and sampling the band-limited signal is to fold the harmonics produced by the quantization process into the signal passband. The harmonics outside the instantaneous bandwidth can be removed by placing an appropriate bandpass filter after the conversion to analog signal. But the harmonics folded into the instantaneous bandwidth of the DRFM can not be filtered out easily.

#### IV DRFM APPLICATIONS

This chapter describes some of the applications of DRFM such as deception jammer, broadband frequency source, signal analyzer and simulator. Some ECM techniques using DRFM are also described here.

##### A. ECM SYSTEM USING DRFM

This section will describe a DRFM based system which will capture radar signals in a band covering 8 to 16 GHz. The block schematic of the system is shown in Figure 4.1. The DRFM used in this system has an instantaneous bandwidth of 500 MHz. As the frequency range required by the system is 8 GHz, it is necessary to steer the instantaneous band (500 MHz) of DRFM to the threat signal frequency. Instantaneous frequency measuring (IFM) receiver is used for measuring the incoming signal frequency. The measured frequency is used to choose a local oscillator from the local oscillator bank. The local oscillator bank contains a set of fixed frequency oscillators as given in Table 4.1. The delay line in front of the DRFM will provide necessary time delay required for setting the local oscillator frequency.

The incoming signal is first down-converted to a 500 MHz band centered at 1 GHz by selecting an appropriate local oscillator. The resulting down-converted signal is quadrature

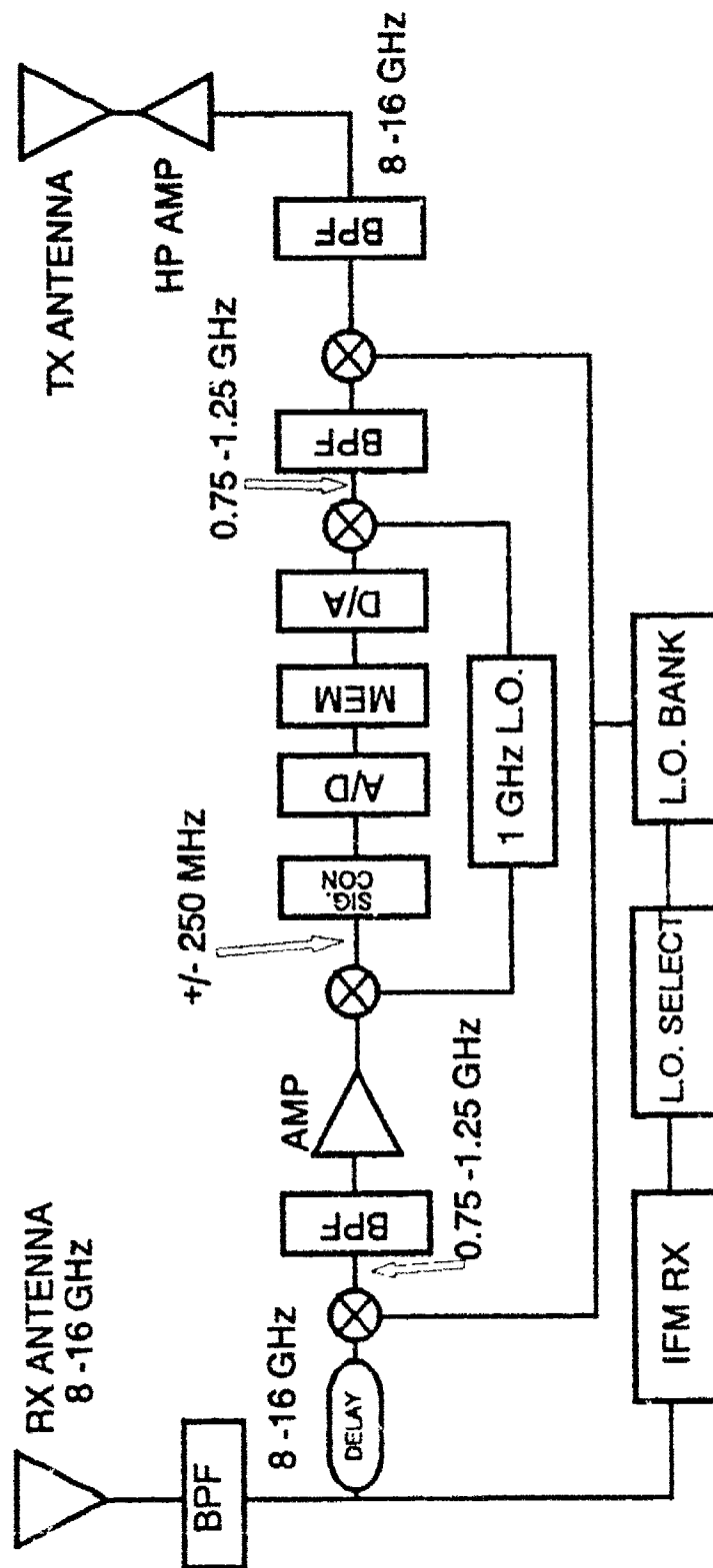


Figure 4.1 Frequency Steered DRFM block schematic

mixed against 1 GHz local oscillator to provide +/- 250 MHz

**TABLE 4.1** SELECTION LIST OF LOCAL OSCILLATOR  
FREQUENCY, IF FREQUENCY BAND FOR EACH SUB-BAND

Freq range (GHz)	L.O. (GHz)	IF (GHz)
8.0-8.5	9.25	1.25 -0.75
8.5-9.0	9.75	1.25 -0.75
9.0-9.5	10.25	1.25 -0.75
9.5-10.0	10.75	1.25 -0.75
10.0-10.5	11.25	1.25 -0.75
10.5-11.0	11.75	1.25 -0.75
11.0-11.5	10.25	0.75-1.25
11.5-12.0	10.75	0.75-1.25
12.0-12.5	11.25	0.75-1.25
12.5-13.0	13.75	1.25 -0.75
13.0-13.5	14.25	1.25 -0.75
13.5-14.0	14.75	1.25 -0.75
14.0-14.5	13.25	0.75-1.25
14.5-15.0	13.75	0.75-1.25
15.0-15.5	14.25	0.75-1.25
15.5-16.0	14.75	0.75-1.25

baseband in-phase (I) and quadrature (Q) signals. The I and Q signals are passed through signal conditioner circuits. Each of the conditioned I and Q signals are sampled and quantized with 500 MHz clock in analog-to-digital converter

(A/D). The samples are then stored in a random access memory (RAM) for later signal reconstruction. To replicate the signal, the inverse process is implemented. The stored digital samples are clocked out at 500 MHz rate to the digital-to-analog converter (D/A) for generation of an analog baseband signal. The regenerated baseband signal is upconverted with the same local oscillators which were used in the down conversion process. Thus DRFM will replicate the instantaneous frequency of the received signal at the output.

## **B. ECM TECHNIQUES**

Basic ECM techniques such as range gate stealing, velocity gate stealing, cover pulse noise jamming, false target generation, etc., can be implemented with DRFM.

### **1. Range Gate Stealer (RGS)**

Range gate stealing is an effective ECM technique against the tracking radars. The range deception is accomplished by transmitting the stored signal to capture the radar automatic gain control circuits (AGC). After the capture of the AGC, the stored signal is transmitted with a successively increased delay to generate the required range gate pull-off. At the end of maximum delay, the cycle of range gate stealing is repeated. The increment of range delay can be made to imitate any pattern required to simulate different types of platform maneuvers.

Due to storage of complete pulse width by the DRFM, the phase coded and chirped waveforms can be coherently reproduced with controlled pulse width and phase. Due to long storage times available, it is feasible to generate range gate pull-in for generating approaching targets.

## **2. Velocity Gate stealer (VGS)**

The velocity deception or velocity gate stealing is effective ECM against the Doppler sensing radars like CW radar and pulse Doppler radars. The VGS is accomplished by repeating a frequency shifted replica of the victim radar's signal. The frequency shift is initially programmed so that repeated signal is with-in the passband of the Doppler filter containing the target return. This allows the jammer to capture the radar's AGC circuits. The repeated signal is then slowly walked off in frequency to the expected maximum Doppler frequency of the radar. The repeated signal is then removed, forcing the radar to reacquire the target.

In DRFM, the frequency of the signal can be shifted, by introducing the required frequency change in the upconverting local oscillator with respect to down converting local oscillator. The Doppler frequency added to the upconverting local oscillator can be generated digitally to the required accuracy. As the stability of the of DRFM output is determined by the stability of the LO and sampling clock, short term stability of few Hz can be easily achieved. The

multi-bit DRFMs become a necessity if the system is to be used for velocity gate stealing since the harmonic signal and spurious signal levels decrease as the number of quantization bits are increased.

### **3. Coordinated RGS/VGS**

RGS or VGS, by themselves, may not deceive the pulse Doppler radar. The pulse Doppler radar may check the consistency of target velocity obtained by differentiating range data with that obtained from target Doppler data. Hence, to deceive pulse Doppler radar, RGS and VGS are to be coordinated so as to provide a realistic target motion.

The ability of DRFM to coordinate is excellent as digital techniques are used for signal storage and reconstruction. Precise RGS is achieved due to timed readout of memory. VGS is coordinated by selecting an appropriate Doppler corresponding to the RGS selected. Thus it will be difficult for pulse Doppler radar to reject the false target.

### **4. Multiple False Target Generation**

As the readout from the memory is nondestructive, it is possible to regenerate any number of false targets. The false targets can also be generated at any range since the storage time is unlimited. The memory available can be segmented for use to generate different false targets. As the multiple targets are generated sequentially, it is possible to

toggle between different false targets in a coordinated fashion. However, care should be taken not to exceed the duty cycle limitations of the high power section.

#### **C. DRFM AS A FREQUENCY SOURCE**

The wide band signals required in radar systems can be generated by DRFMs. Digital data, required by DRFM, can be either from computer simulated data or from a real world digitized signal. Wide bandwidth signals such as phase coded or frequency chirped can be easily generated by this process. In radar designs, DRFM technology has been used to enhance the function of exciter of the radar by providing ready made and more versatile waveforms for various specialized functions [Ref. 6]. The main advantage of using DRFM technology will be that the signal waveform can be easily manipulated by changing the digital data in the DRFM memory.

#### **D. SIGNAL ANALYSIS USING DRFM**

The DRFM technology can also be used in receiving systems. The digital techniques are available for determining signal characteristics such as amplitude, phase and frequency of the signal. The block diagram of the digital receiver is given in Figure 4.2.

Unlike conventional receivers, digital receivers can store the digitized signal information for long periods of time without degradation. The digital demodulation techniques can



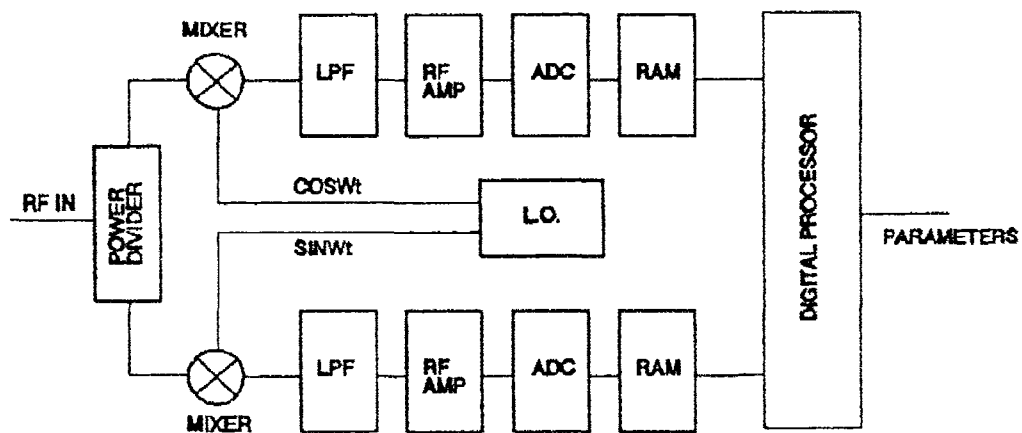


Figure 4.2 Digital Receiver

be applied to the stored signal to obtain information like amplitude, phase and frequency [Ref. 7]. If the incoming signal  $a(t)\cos[w_0t + p(t)]$  is quadrature sampled, it can be represented as

$$\begin{aligned} I(t) &= a(t) \cos(p(t)) \\ Q(t) &= a(t) \sin(p(t)) \end{aligned}$$

The amplitude of the signal can be obtained by

$$a(t) = \sqrt{I(t)^2 + Q(t)^2}$$

The phase information of the signal can be obtained by

$$\begin{aligned} \phi(t) &= \tan^{-1} \left[ \frac{Q(t)}{I(t)} \right] \\ &= \tan^{-1} \left[ \frac{a(t) \sin(p(t))}{a(t) \cos(p(t))} \right] \\ &= p(t) \end{aligned}$$

The  $p(t)$  contains both the carrier phase offset and the modulation frequency phase slope. The other signals, like FM

or phase coded, can be processed in a similar fashion as the pulsed signals. However, due to technology limitations, the real time analysis of the signals is not yet feasible over very wide bandwidths.

Also the input data stored in the memory could be processed several times to obtain different signal information. For example, if a data set contains both strong and weak signals, it is possible to obtain information on a strong signal and then subtracting strong signal data from the stored data, the weak signal can be detected. This is useful in improving the dynamic range of the receiver.

#### **E. DRFM AS SIMULATOR**

The DRFM has been used in simulation for testing of radar. The DRFM can be used for simulation of targets, clutter, and ECM waveforms. The main advantages in this type of application are

- a) The digitized real world signal or the digital words of signal waveform simulated on computer can be stored in DRFM to generate the required signal.
- b) The system is generic in application; a single system can be used against different radars by modifying the data in DRFM memory.
- c) Due to digital techniques involved, the simulations are repeatable to high accuracy.
- d) Only a short reprogramming time is required to react to changes in scenarios.

Such systems have been integrated into certain ground based fire control radars for test purposes.

#### F. SPECIFICATIONS OF DRFM

Following is the typical specification of a representative DRFM [Ref. 8].

1	Frequency of operation	5.4 - 5.8 GHz
2.	Instantaneous BandWidth	400 MHz
3.	Dynamic Range	30 dB
4.	Memory length	100 ns - 27 micro seconds
5.	Output frequency	Input +/- 1 KHz
6.	Output pulse width	Input +/- 40 ns
7.	Output power levels	- 12 +/- 4 dBm
8.	Operating Modes	
	a. RGPO	
	Dwell	0.5 - 5.0 seconds
	Walk off time	0.5 - 20.0 seconds
	Delay range	1 - 30 micro seconds
	Minimum delay	60 nanoseconds
	Resolution	40 nanoseconds
	Sweep	parabolic
	Hold mode	1 - 15 seconds
	b. False Targets (Preset replicas)	
	Number of false targets	2, 4, 8, 16 spaced 26 microseconds apart
	Continuous wave (CW)	continuous readout of stored pulse
	External trigger	1 replica per trigger

## G. TECHNOLOGY TRENDS

The DRFM technology has developed in the past few years as a versatile means for effective ECM systems against coherent radars. DRFMs are being currently produced by several companies including Whittaker and Raytheon. Table 4.2 gives the list of DRFMs manufactured by different vendors with their brief specifications [Ref. 1].

The future developments in very high speed digital integrated circuits, analog-to-digital converters and digital-to-analog converters will further improve the performance of the DRFMs. The future trend appears to be the use of MICs and ASICs to reduce the weight and size the DRFMs. ITT Avionics has developed a DRFM that consumes 15 watts of power and is only 20 cubic inches in size. This is an improvement over the earlier 70 watt and 70 cubic inches version.

With high speed intelligent processing built into the DRFMs, it will be possible to generate complex signals required for ECM applications. Efforts to design and fabricate GaAs/ECL DRFMs for higher circuit integration are reported. The sampling speed appears to be the significant factor driving the DRFM technology. DRFMs with sampling clock rates better than 1 GHz and IBW better than 1 GHz have been reported [Ref. 1].

It has been reported that the Air Force Institute of Technology (AFIT) and Wright Laboratory have plan to build a

DRFM onto a chip [Ref. 9]. The AFIT has used Hilbert transform to accomplish the amplitude and phase modulation. Both amplitude and phase of the signal is controlled on a sample by sample basis by this technique. The ultimate goal is to place all of the required converters, memory and control circuits on a chip. As noted earlier, the capabilities of the DRFMs will be enhanced further because of these efforts.

**TABLE 4.2**  
**LIST OF DRFMS WITH THEIR BRIEF SPECIFICATIONS**

Manufacturer and Model Number	RF Freq Range (GHz) and (1BW)	A/D Bits and type	Memory length in (micro sec)	RF Output		
				Power out dBm	Modulation type	Delay available (min - max)
Anaren 45010	7.0 - 17.0 (500)	3, Phase	200	+4	Doppler, Amplitude, Frequency	15 ns - 3.07 ms
CAL TCM-875	7.0 - 11.0 (1000)	4, Amplitude	340	+40	Amplitude	10 ns - 1.09 ms
KOR 1030	3.0 - 3.5 (500)	1, Phase	182	+10	Doppler, Frequency	<95 ns - 2.9 ms
WHITTAKER MIP 830	6 - 18 (220)	8, Amplitude	1048	+3	***	10 ns - 2.048 ms

## V CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

This thesis presents a method to calculate the level of harmonics generated in the DRFM due to the sampling and quantization process. Fourier series analysis is used for the calculation of the harmonic levels. The quantization process will generate the harmonics of the fundamental signal. The sampling process will fold these harmonics into the baseband (IBW) of the DRFM as inter-modulation products of the sampling frequency (clock frequency of the A/D) and the signal frequency. The level of the harmonics is dependent on the number of quantization bits of the analog-to-digital converter. Further, it is also dependent on the ratio of the input signal frequency and the sampling frequency (normalized signal frequency). If the normalized frequency is low, the study indicated that the harmonic levels in the output are low and correspondingly, if the normalized frequency is high, the harmonic level is high.

The results obtained here are more conservative since the model used does not consider circuit limitations like

- circuit stray capacitances which round off the quantized waveform
- generation of even harmonics due to variations in components values, thus reducing odd harmonics

- harmonic power spread due to local oscillator drifts

The reduction of harmonic levels in single bit and multi bit DRFMs is possible. This can be accomplished by introducing time jitter in the sampling signal. Another method is to vary the duty cycle of the quantized output waveform.

The harmonic computation programs written in MATLAB can accept number of bits as input and plot the harmonic power level vs harmonic number to a maximum of 99 harmonics. The total harmonic power is also computed by the program. The sampled and quantized signal harmonic power computation programs accept normalized input frequency with respect to sampling frequency. The plot and data for each harmonic level is generated. The user of these programs can determine spur levels of DRFM having any number of bits.

In system applications, DRFMs offer many advantages over FMLs as deception jamming devices. The DRFM technology can be utilized to build broad bandwidth frequency sources for radar and communication applications. Also this technology can be used to store signals which can be analyzed either in real time or off-line. The DRFMs have been used in radar simulators for generation of different types of signals like target return, clutter signal, and jamming signal.

## B. RECOMMENDATIONS

The DRFM is a new area of interest to the EW community. The reduction in harmonic levels due to sampling time jitter and duty cycle modulation should be investigated further. Phase encoded DRFMs and their characteristics have to be investigated as they offer some advantages over amplitude encoded DRFMs. Studies can also be conducted in the area of output frequency accuracy with respect to stored pulse length and sampling rate. The digital methods for imparting modulation to the acquired signal is another area of interest. Investigations are also suggested in the time multiplexed and frequency multiplexed techniques in DRFM.



#### LIST OF REFERENCES

1. Sheldon B. Herskovitz, "A sample of digital RF memories," Journal of Electronic Defense, Vol. 15, No. 2, pp. 43-47, February 1992.
2. Brian G Sasaki, Broadband I/O demodulation techniques, Federal Systems Division, Watkins-Johnson Company.
3. W.J. Schneider, "Digital countermeasures memories : New techniques possible," International Countermeasures Handbook, Cardiff Publishing, 11th Edition, pp. 367-373, 1986.
4. D. Gold and H. Ur, "Method for reduction of harmonics caused by coarse quantization suitable for digital radio frequency memory," Electronic Letters, Vol. 29, No. 4, pp. 411-418, February 1993.
5. William Kerins, "Spur levels in multiple-bit DRFMs," Journal of Electronic Defense, Vol 14, No. 1, pp. 49-54, January 1991.
6. Dr. Oscar Lowenschuss, "Coherent digital rf memory -- A new signal processing component," IEEE National Aerospace and Electronics Conference 1980 Proceedings, Vol. 3, pp. 1188-1194, 1980.
7. Robert Armitano and Roland Hassun, "Digital demodulation verifies accuracy of advanced signal simulators: motivation and theory," Microwave Journal, Vol. 33, No. 6, pp. 183-194, June 1990.
8. D. Curtis Schleher, Introduction to Electronic Warfare, Aertech House, 1986.
9. Darryl T. Gehly, "Business as usual," Journal of Electronic Defense, Vol 14, No. 9, pp. 60-66, September 91.

## APPENDIX A      PLOTS OF MULTIBIT QUANTIZED SIGNAL

The Appendix A contains the MATLAB program to calculate the harmonic of the multi bit quantized signal and the plots of harmonic power vs harmonic number for one to six bits quantized signal. The comparison of the harmonic power level of one to six bit amplitude quantized signal is given in Table A.7.

# ONE BIT AMPLITUDE QUANTIZED SIGNAL

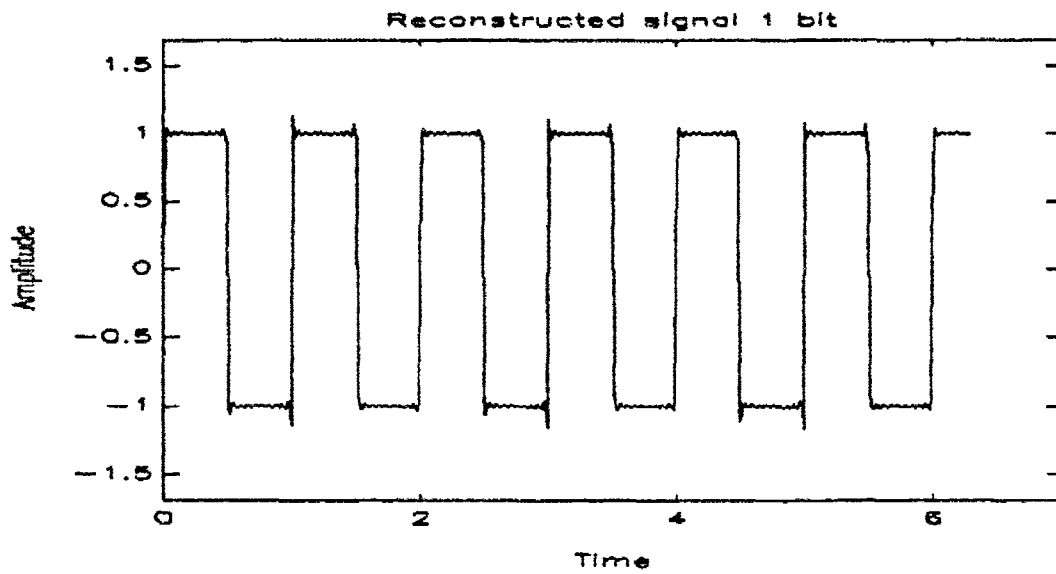


Figure A.1 One bit amplitude quantized signal

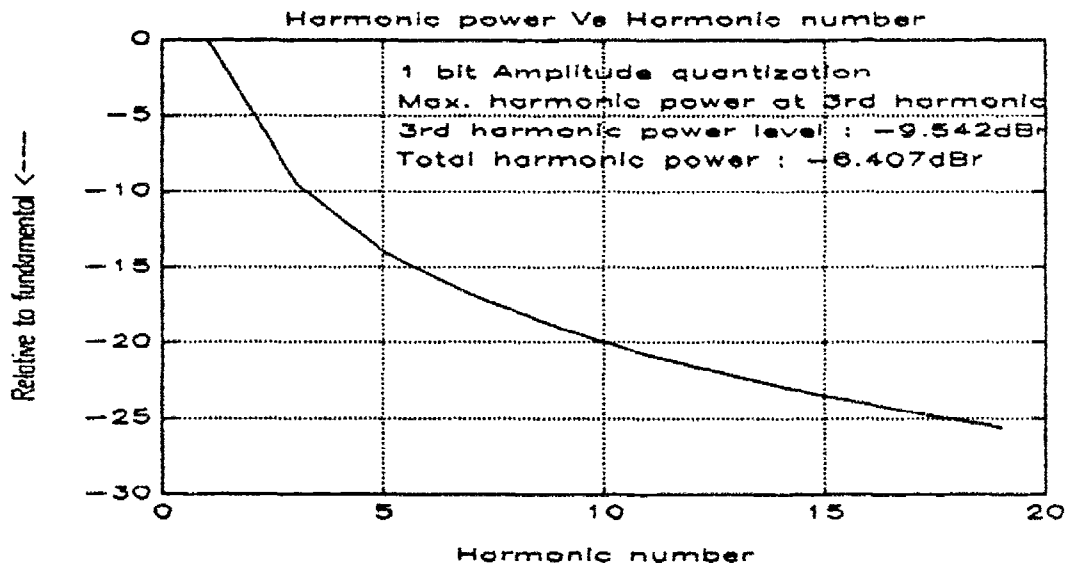


Figure A.2 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of one bit amplitude quantized signal (expanded view, harmonics 1 to 19)

# ONE BIT AMPLITUDE QUANTIZED SIGNAL

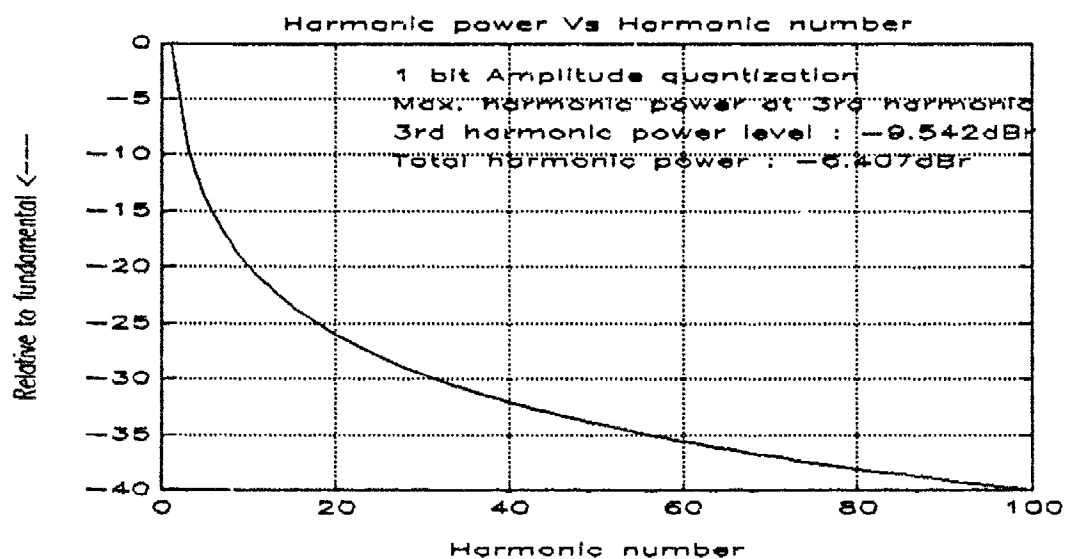


Figure A.3 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of one bit amplitude quantized signal (harmonics 1 to 99)

TABLE A.1 HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF ONE BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level	Harmonic number	Power level	Harmonic number	Power level
1	0.0	13	- 22.28	25	- 27.96
3	- 9.54	15	- 23.52	27	- 28.63
5	- 13.98	17	- 24.61	29	- 29.25
7	- 16.90	19	- 25.58	31	- 29.83
9	- 19.08	21	- 26.44	33	- 30.37
11	- 20.83	23	- 27.23	35	- 30.88

## TWO BIT AMPLITUDE QUANTIZED SIGNAL

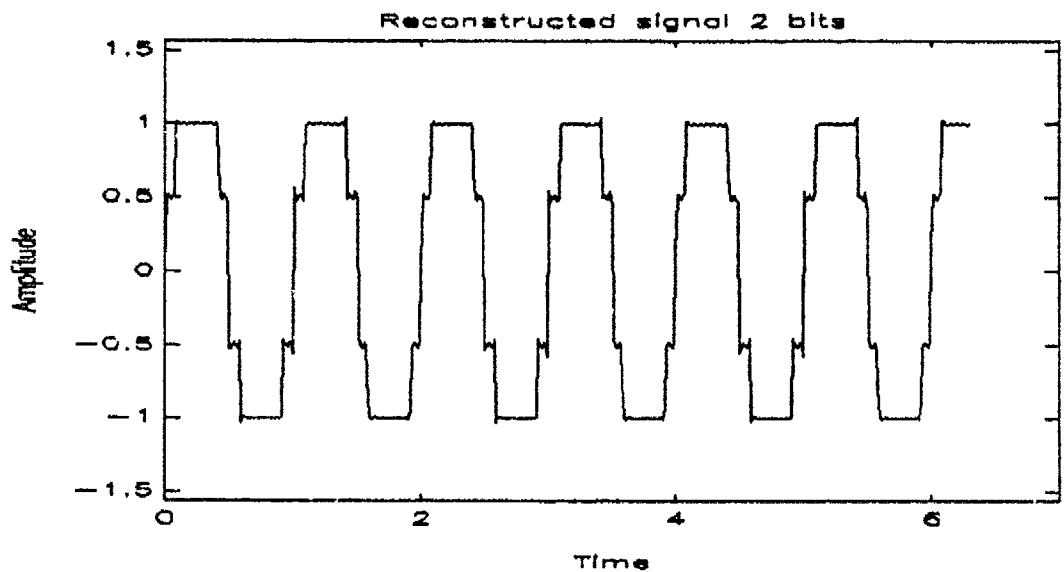


Figure A.4 Two bit amplitude quantized signal

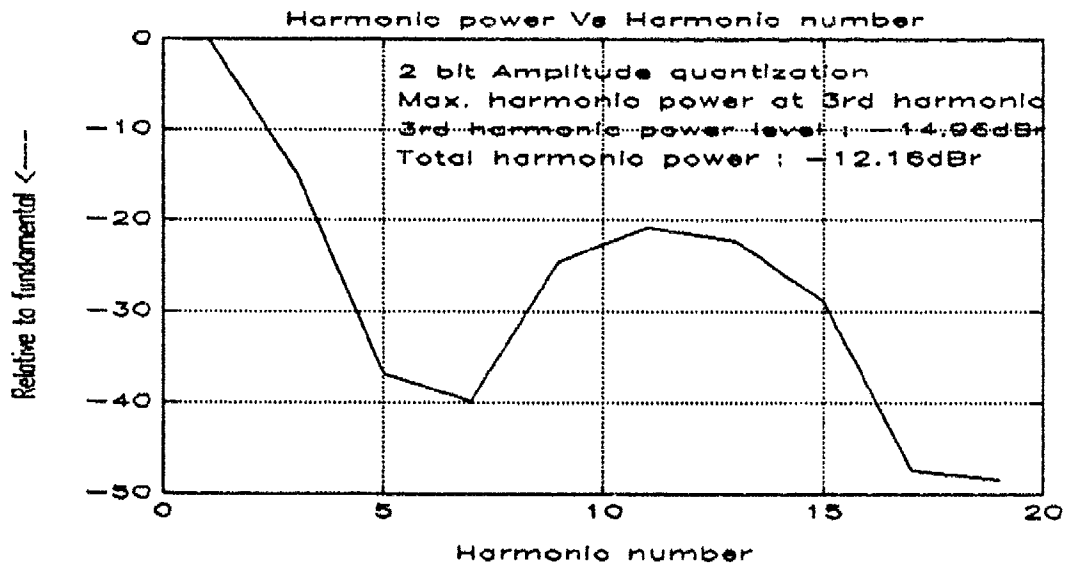
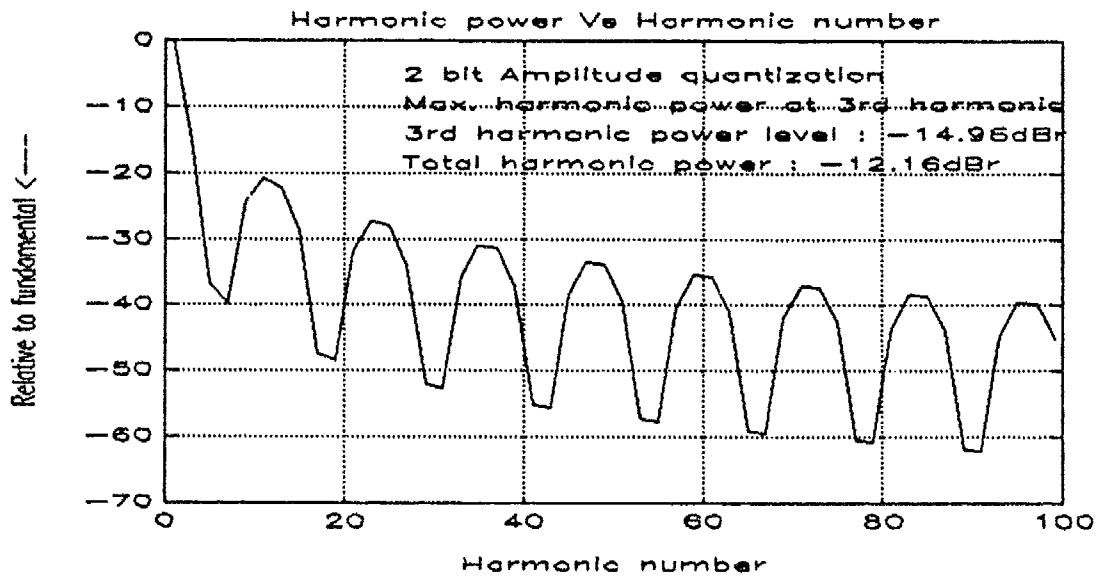


Figure A.5 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of two bit amplitude quantized signal (expanded view, harmonics 1 to 19)

## TWO BIT AMPLITUDE QUANTIZED SIGNAL



**Figure A.6** Plot of harmonic power (in dB) relative to fundamental vs harmonic number of two bit amplitude quantized signal (harmonics 1 to 99)

**TABLE A.2** HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF TWO BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level (dB)	Harmonic number	Power level (dB)	Harmonic number	Power level (dB)
1	0	13	- 22.28	25	- 27.96
3	- 14.96	15	- 28.94	27	- 34.05
5	- 36.86	17	- 47.49	29	- 52.13
7	- 39.78	19	- 48.45	31	- 52.71
9	- 24.50	21	- 31.86	33	- 35.79
11	- 20.83	23	- 27.23	35	- 30.88

# THREE BIT AMPLITUDE QUANTIZED SIGNAL

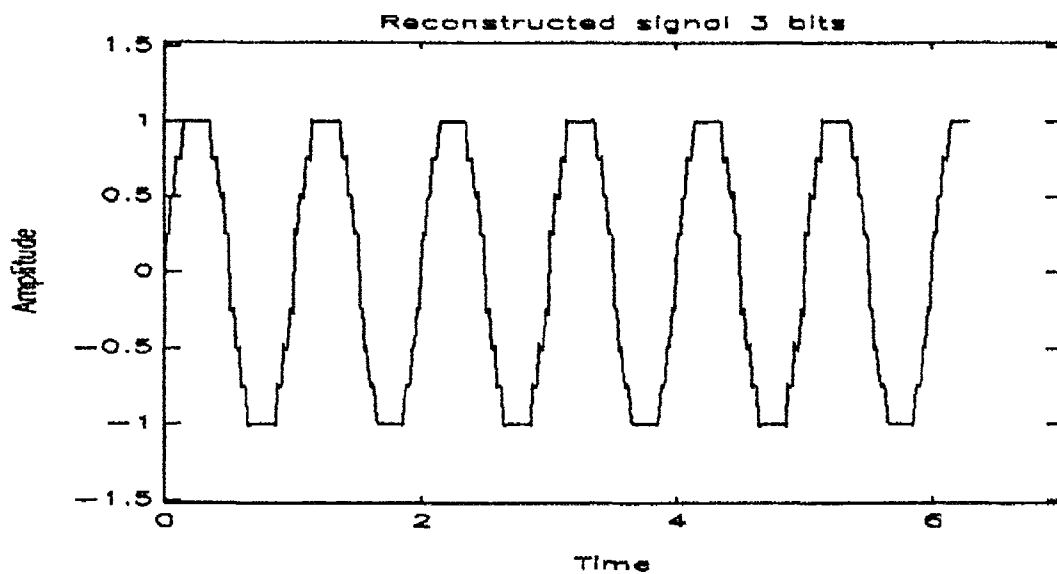


Figure A.7 Three bit amplitude quantized signal

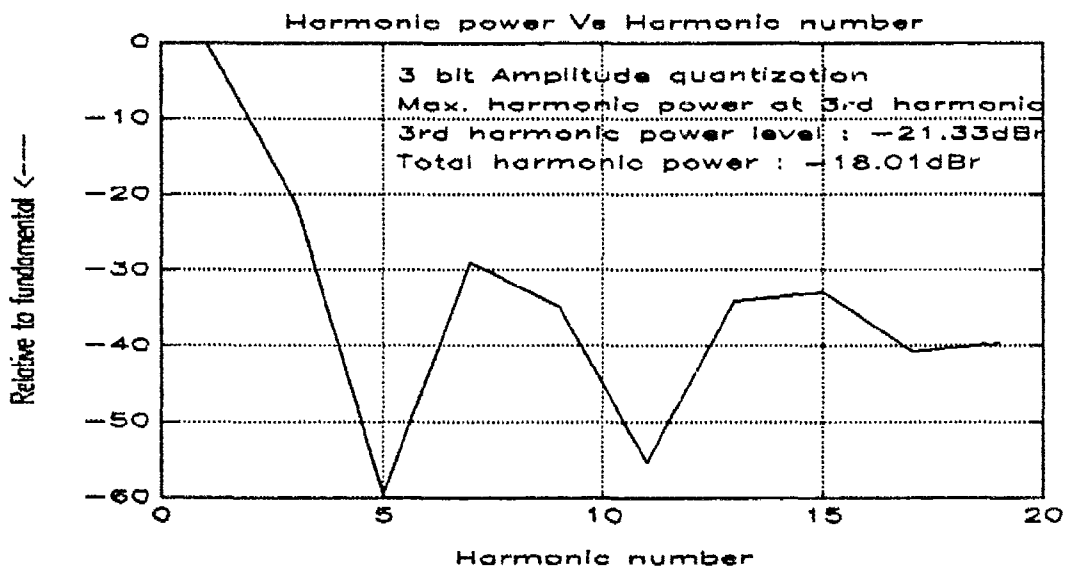


Figure A.8 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of three bit amplitude quantized signal (expanded view, harmonics 1 to 19)

### THREE BIT AMPLITUDE QUANTIZED SIGNAL

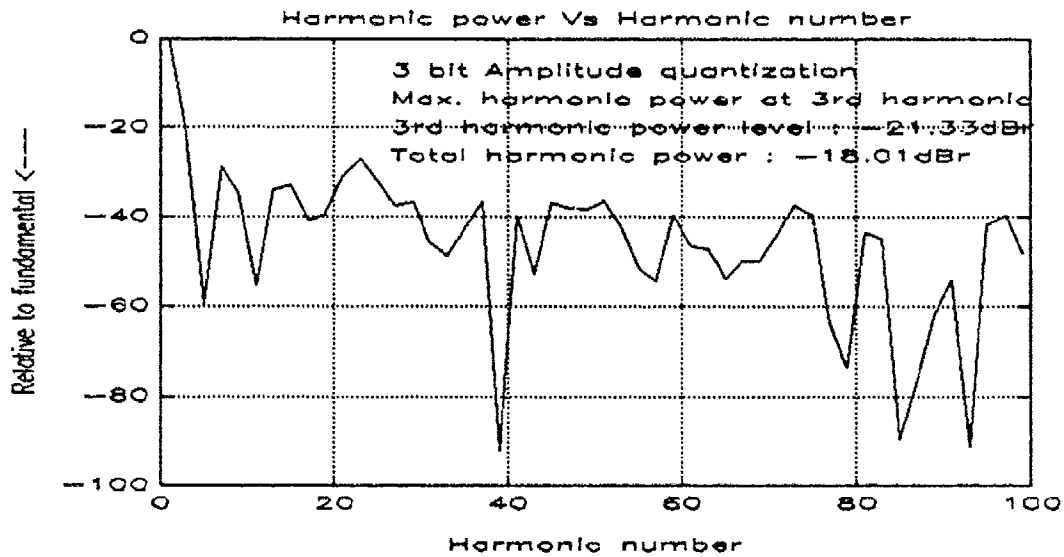


Figure A.9 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of three bit amplitude quantized signal (harmonics 1 to 99)

TABLE A.3 HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF THREE BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level (dB)	Harmonic number	Power level (dB)	Harmonic number	Power level (dB)
1	0.0	13	- 34.01	25	- 32.14
3	- 21.33	15	- 32.87	27	- 37.62
5	- 59.65	17	- 40.72	29	- 36.63
7	- 28.90	19	- 39.59	31	- 45.78
9	- 34.79	21	- 31.01	33	- 48.68
11	- 55.51	23	- 27.10	35	- 42.98



# FOUR BIT AMPLITUDE QUANTIZED SIGNAL

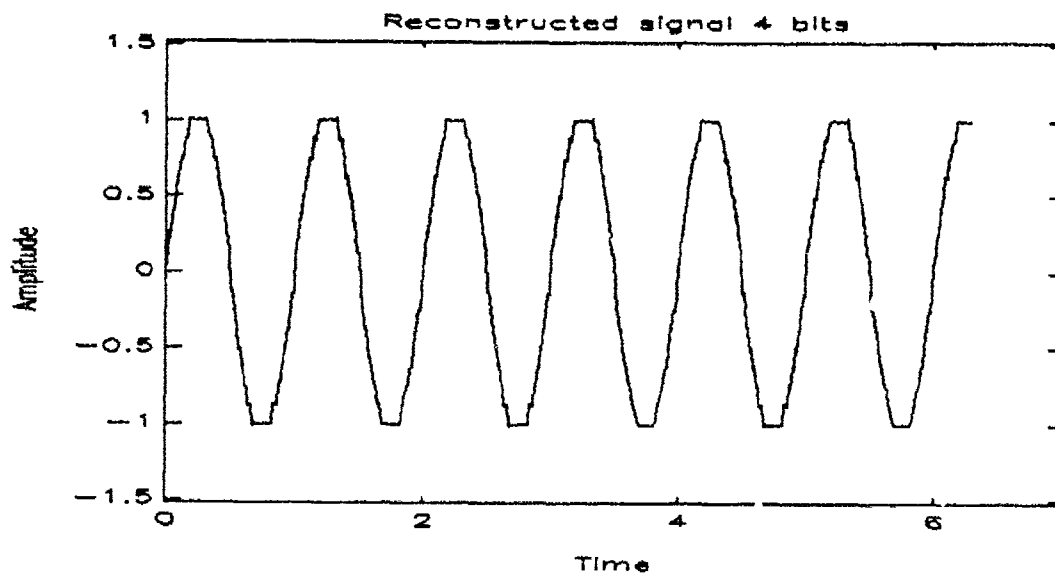


Figure A.10 Four bit amplitude quantized signal

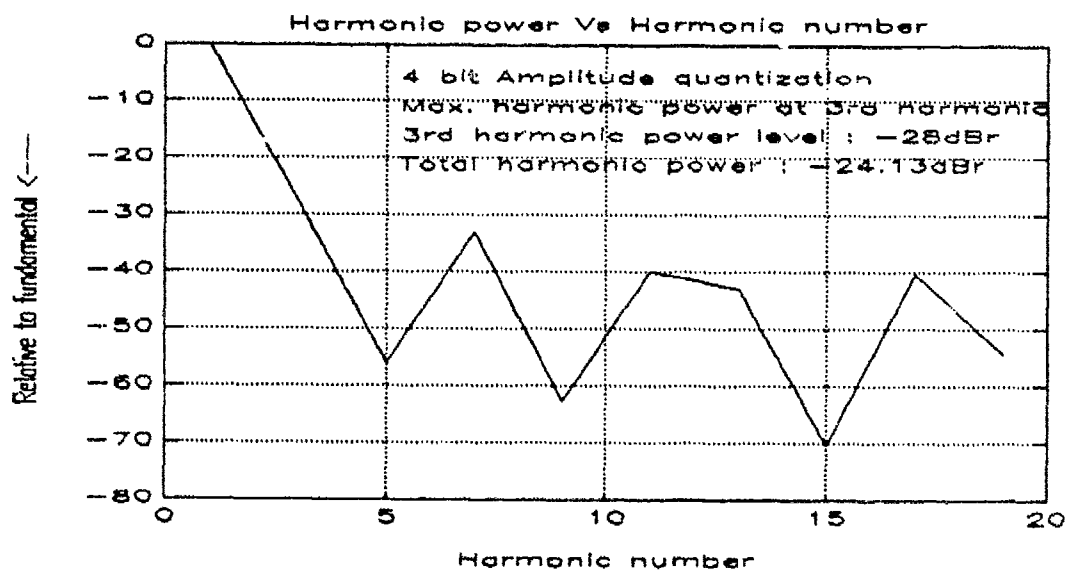
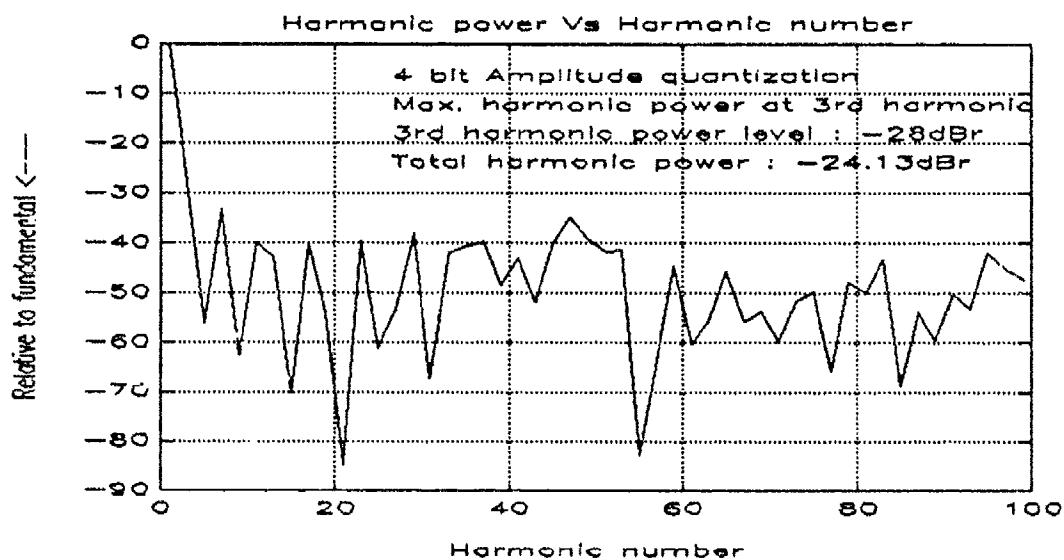


Figure A.11 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of four bit amplitude quantized signal (expanded view, harmonics 1 to 19)

# FOUR BIT AMPLITUDE QUANTIZED SIGNAL



**Figure A.12** Plot of harmonic power (in dB) relative to fundamental vs harmonic number of four bit amplitude quantized signal (harmonics 1 to 99)

**TABLE A.4** HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF FOUR BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level (dB)	Harmonic number	Power level (dB)	Harmonic number	Power level (dB)
1	0.0	13	- 43.02	25	- 61.30
3	- 28.00	15	- 70.51	27	- 53.63
5	- 56.18	17	- 40.32	29	- 38.21
7	- 33.26	19	- 54.32	31	- 67.44
9	- 62.79	21	- 84.92	33	- 41.96
11	- 39.96	23	- 39.79	35	- 40.67

# FIVE BIT AMPLITUDE QUANTIZED SIGNAL

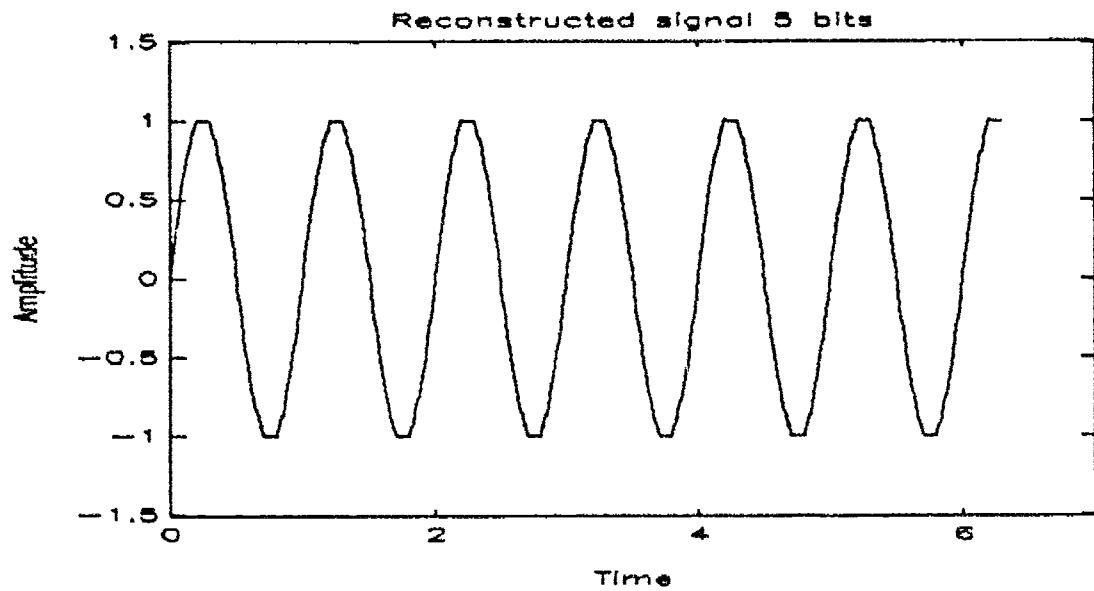


Figure A.13 Five amplitude reconstructed signal

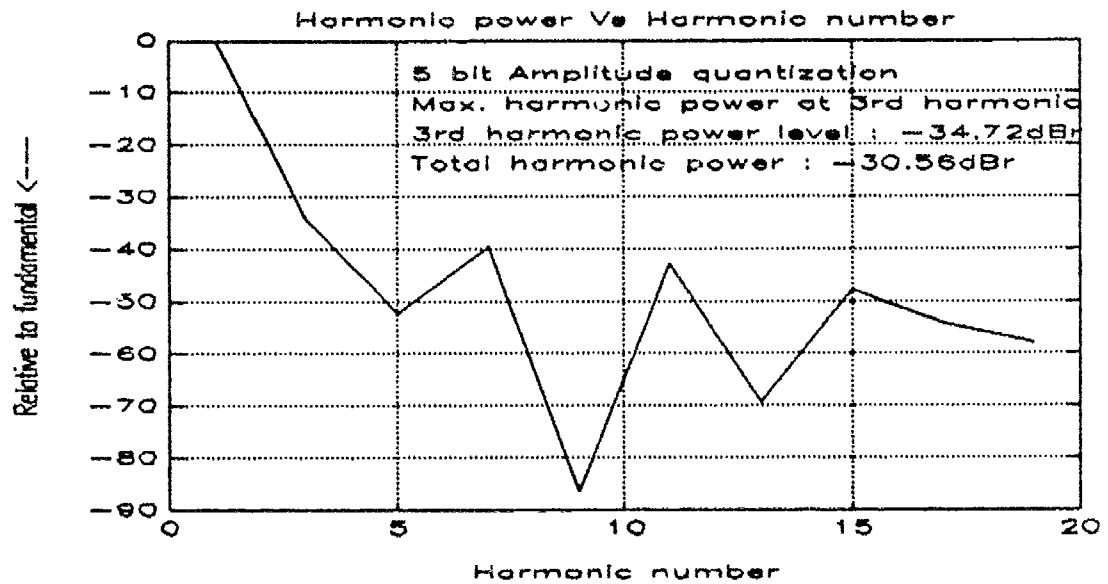
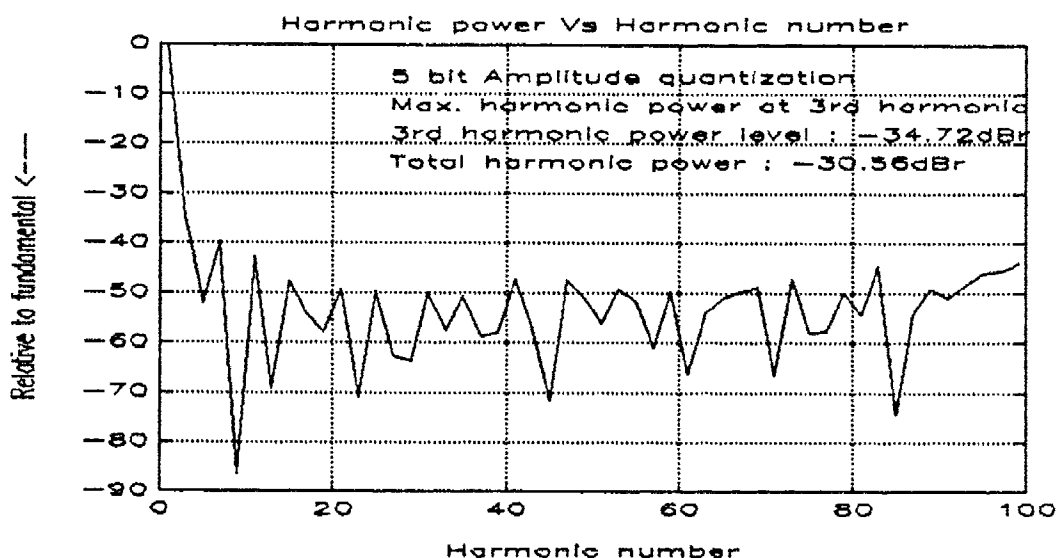


Figure A.14 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of five bit amplitude quantized signal (expanded view, harmonics 1 to 19)

# FIVE BIT AMPLITUDE QUANTIZED SIGNAL



**Figure A.15** Plot of harmonic power (in dB) relative to fundamental vs harmonic number of five bit amplitude quantized signal, (harmonics 1 to 99)

**TABLE A.5** HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF FIVE BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level (dB)	Harmonic number	Power level (dB)	Harmonic number	Power level (dB)
1	0.00	13	- 69.32	25	- 49.24
3	- 34.72	15	- 47.79	27	- 62.70
5	- 52.35	17	- 54.30	29	- 63.65
7	- 39.67	19	- 58.01	31	- 49.97
9	- 86.66	21	- 49.28	33	- 57.84
11	- 43.02	23	- 71.22	35	- 50.92

# SIX BIT AMPLITUDE QUANTIZED SIGNAL

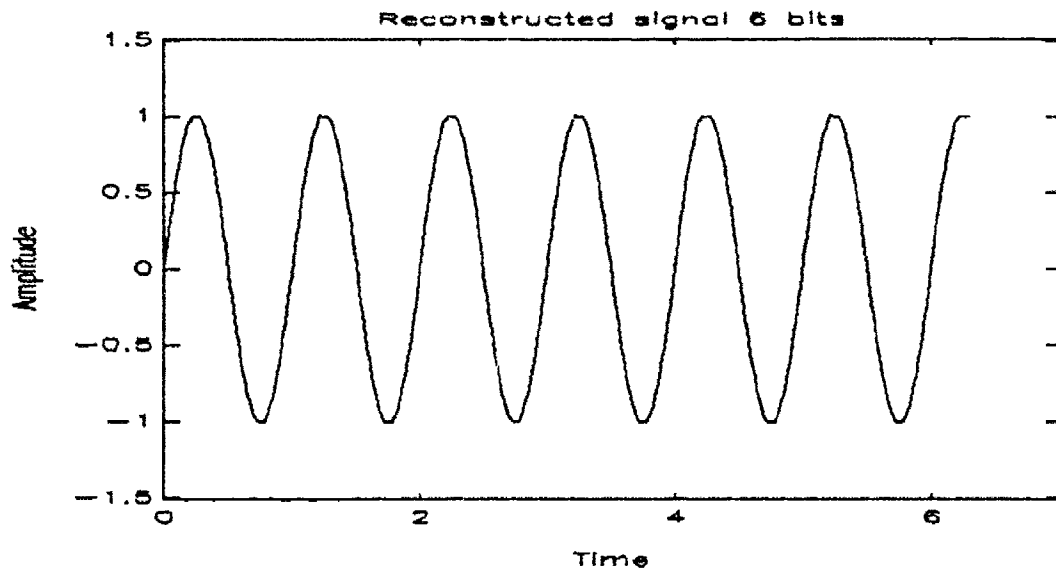


Figure A.16 Six bit amplitude quantized signal

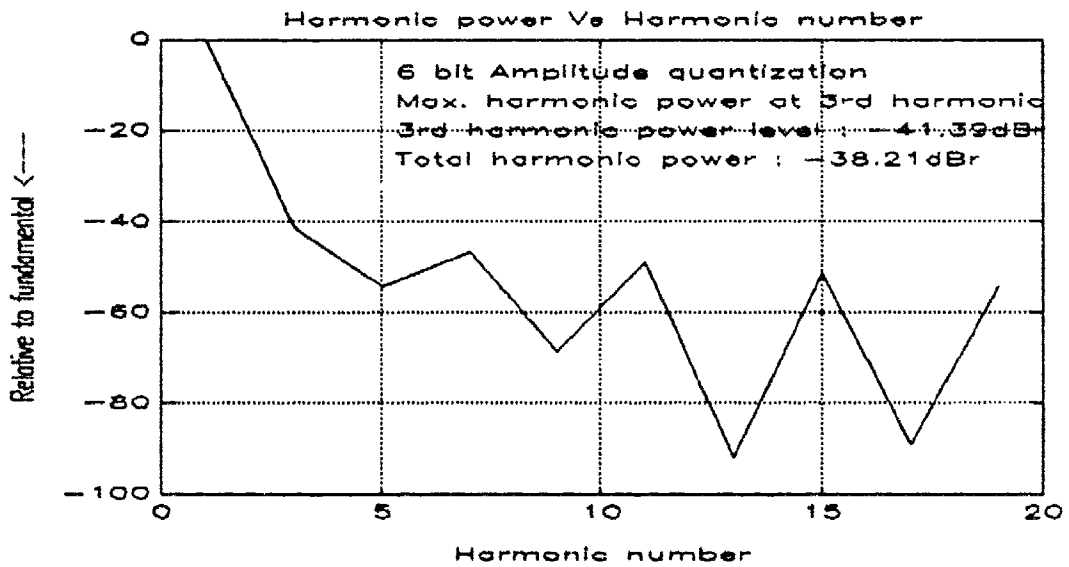


Figure A.17 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of six bit amplitude quantized signal (expanded view, harmonics 1 to 19)

# SIX BIT AMPLITUDE QUANTIZED SIGNAL

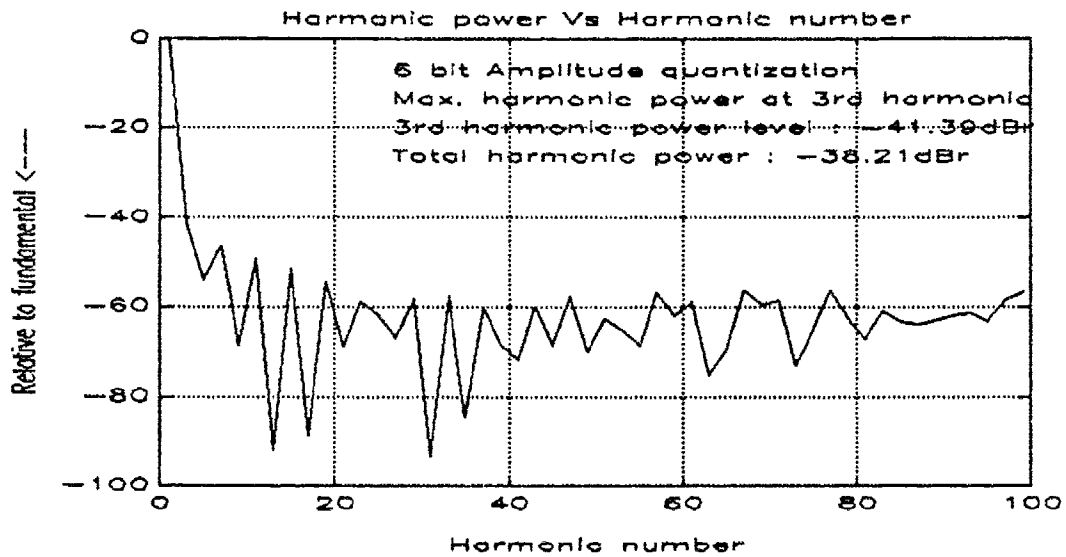


Figure A.18 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of six bit amplitude quantized signal (harmonics 1 to 99)

TABLE A.6. HARMONIC POWER (IN DB) RELATIVE TO FUNDAMENTAL VS HARMONIC NUMBER OF SIX BIT AMPLITUDE QUANTIZED SIGNAL

Harmonic Number	Power level (dB)	Harmonic number	Power level (dB)	Harmonic number	Power level (dB)
1	0.00	13	- 92.20	25	- 61.96
3	- 41.39	15	- 51.56	27	- 67.19
5	- 54.23	17	- 89.04	29	- 58.55
7	- 46.57	19	- 54.49	31	- 93.48
9	- 68.74	21	- 69.00	33	- 57.82
11	- 49.22	23	- 58.95	35	- 84.77

**TABLE A.7 HARMONIC POWER (DB) VS HARMONIC NUMBER**

Harmonic Number	Power level in dB of the Harmonics relative to the fundamental (Quantization bits)					
	1 bit	2 bit	3 bit	4 bit	5 bit	6 bit
1	0.00	0.00	0.00	0.00	0.00	0.00
3	- 9.54	- 14.96	- 21.33	- 28.00	- 34.72	- 41.39
5	- 13.98	- 36.86	- 59.65	- 56.18	- 52.35	- 54.23
7	- 16.90	- 39.78	- 28.90	- 33.26	- 39.67	- 46.57
9	- 19.08	- 24.50	- 34.79	- 62.79	- 86.66	- 68.74
11	- 20.83	- 20.83	- 55.51	- 39.96	- 43.02	- 49.22
13	- 22.28	- 22.28	-34.01	- 43.02	- 69.32	- 92.20
15	- 23.52	- 28.94	- 32.87	- 70.51	- 47.79	- 51.56
17	- 24.61	- 47.49	- 40.72	- 40.32	- 54.30	- 89.04
19	- 25.58	- 48.45	- 39.59	- 54.32	- 58.01	- 54.49
21	- 26.44	- 31.86	- 31.01	- 84.92	- 49.28	- 69.00
23	- 27.23	- 27.23	- 27.10	- 39.79	- 71.22	- 58.95
25	- 27.96	- 27.96	- 32.14	- 61.30	- 49.94	- 61.96
27	- 28.63	- 34.05	- 37.62	- 53.63	- 62.70	- 67.19
29	- 29.25	- 52.13	- 36.63	- 38.21	- 63.65	- 58.55
31	- 29.83	- 52.71	- 45.78	- 67.44	- 49.97	- 93.48
33	- 30.37	- 35.79	- 48.68	- 41.96	- 57.84	- 57.82
29	- 30.88	- 30.88	- 42.98	- 40.67	- 50.92	- 84.77

**A. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER IN MULTI BIT QUANTIZATION.**

```
% FILE NAME           : DRQUA.M
% OPERATING SYSTEM    : DOS 5.0
% SYSTEM              : PC 486/33
%
% M BIT  NOOFFLEVELS = (2^M)/2 WHERE M IS NO OF BITS

!del ampl1.met
!del ampl2.met
!del ampl3.met

!del ampl1.pic
!del ampl2.pic
!del ampl3.pic

clear
axis('square')
bits = input('Enter number of bits 1 to 8 = '); % no of bits
N = (2^bits)/2;          % no of levels
A0 = 1;                  % amplitude of the sinusoid

Maxhar = 50;              % calculation is done up to maximum
                          % of 99 harmonics;

A2 = 4*A0/(pi*N);

% HARMONIC CONTENT CALCULATION
for k = 1:N;               % STEP NO IN HALF CYCLE
    for n = 1:Maxhar,
        s(n) = cos ( (2*(n-1)+1) * asin ( (k-1) / (N) ) );
        hno(n) = 2*(n-1)+1;
        f1(n,k) = A2/(2*(n-1)+1)*s(n);
        theta(n) = (2*(n-1)+1)*(2*pi);
    end,
end,
disp('Harmonic calculation complete')

pnk=zeros(1:hno);
for k = 1:N
    pnk = pnk + f1(:,k);
end,
```



```

disp('calculating the log')
pk = pnk .* pnk;
p = pk / max(pk);
pp = 10*log10(p);

% HARMONIC POWER CALCULATIONS
disp('calculating max harmonic')
horpow = 0;
for i = 2:Maxhar,
    horpow = horpow + 10^(pp(i)/10);
end,
horpow4bit = 10*log10(horpow);

text1 = ['kf21/nityan/dramp.m'];
text2 = [num2str(bits) ' bit Amplitude quantization'];
text3 = ['Total harmonic power : '];
text3 = [text3 num2str(horpow4bit) 'dBr'];
text4 = ['Maximum harmonic power at '];
text4 = [text4 num2str(hno(2)) 'rd harmonic'];
text5 = [num2str(hno(2)) 'rd harmonic power level : '];
text5 = [text5 num2str(pp(2)) 'dBr'];
text6 = ['Harmonic power Vs Harmonic number'];
text7 = ['Relative to fundamental <---'];
text8 = ['Harmonic number'];
text81 = ['Normalized frequency'];

% TIME PLOT OF THE RECONSTRUCTED SIGNAL
disp('calculating reconstructed signal')t=0:pi/256:2*pi;
for z= 1:length(t),
    fz= pnk .* sin(theta' * t(z));
    v=0;
    for u= 1:length(fz)
        v=v+fz(u);
    end,
    ss(z) = v;
end,
axis([0 7 -1.5*max(ss) 1.5*max(ss)]);
plot(t,ss)

```

```

if bits == 1
    title(['Reconstructed signal ' num2str(bits) ' bit'])
else
    title(['Reconstructed signal ' num2str(bits) ' bits'])
end
xlabel('Time')
ylabel('Amplitude')
meta ampl1

% PLOT THE DATA
axis
plot(hno(1:10),pp(1:10))
title(text6)
ylabel(text7)
xlabel(text8)
grid

text(0.4,0.85, text2, 'sc')
text(0.4,0.80, text4, 'sc')
text(0.4,0.75, text5, 'sc')
text(0.4,0.70, text3, 'sc')
meta ampl2

plot(hno,pp)
title(text6)
ylabel(text7)
xlabel(text8)
grid

text(0.4,0.85, text2, 'sc')
text(0.4,0.80, text4, 'sc')
text(0.4,0.75, text5, 'sc')
text(0.4,0.70, text3, 'sc')
meta ampl3

disp('Writing data to the file QUA.dat')
filename = 'qua.dat';
noofbits = ['    No of bits = ' num2str(bits) '\n'];
fprintf(filename, noofbits);
fprintf( filename, '  Harmonic           Power \n');
fprintf( filename, '  number           level \n');

```

```
for k = 1:20,  
    fprintf(filename, '%6.0f %12.2f \n', hno(k), pp(k))  
end,
```

```
!gpp amp11 /dhppl /famp11.pic
```

```
!gpp amp12 /dhppl /famp12.pic
```

```
!gpp amp13 /dhppl /famp13.pic
```

## APPENDIX B PLOTS OF ERROR MINIMIZED QUANTIZED SIGNAL

Appendix B contains the MATLAB programs to calculate the harmonic power of error minimized 2, 3 ,4 and 5 bit amplitude quantized signal. The plots of harmonic power vs harmonic number for error minimized quantized signal are also presented in this appendix.

# TWO BIT AMPLITUDE QUANTIZED SIGNAL (ERROR MINIMIZED)

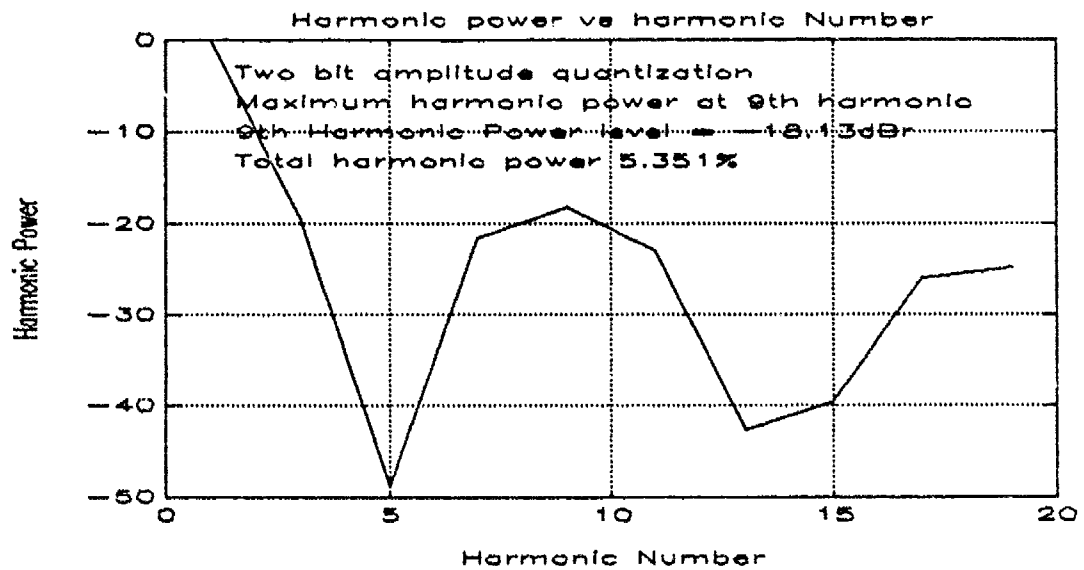


Figure B.1 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of two bit amplitude quantized signal (squared error minimized, expanded view, harmonics 1 to 19)

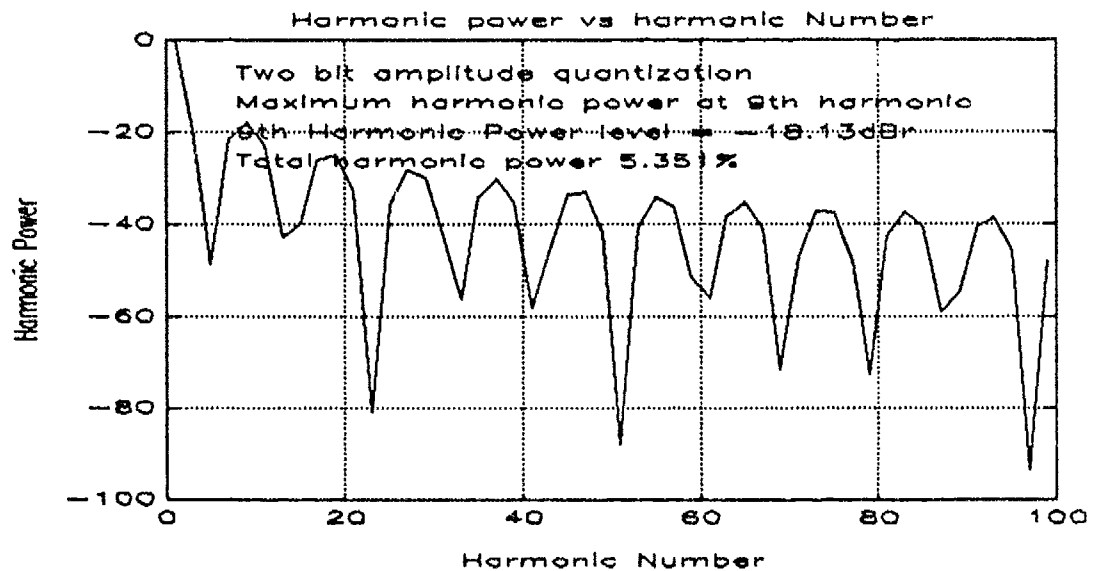


Figure B.2 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of two bit amplitude quantized signal (squared error minimized, harmonics 1 to 99)

# THREE BIT AMPLITUDE QUANTIZED SIGNAL (ERROR MINIMIZED)

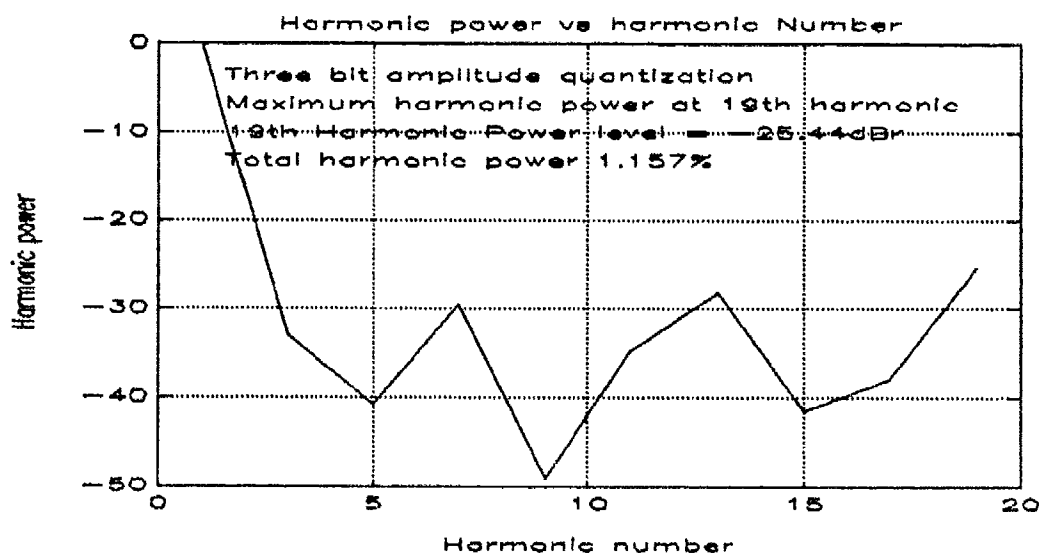


Figure B.3 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of three bit amplitude quantized signal (squared error minimized, expanded view, harmonics 1 to 19)

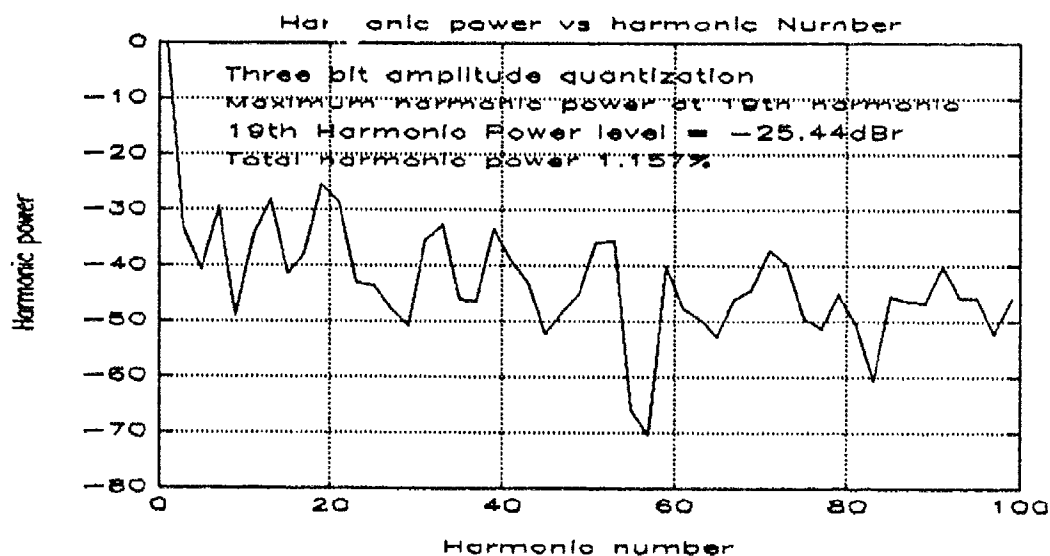


Figure B.4 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of three bit amplitude quantized signal (squared error minimized, harmonics 1 to 99)

# FOUR BIT AMPLITUDE QUANTIZED SIGNAL (ERROR MINIMIZED)

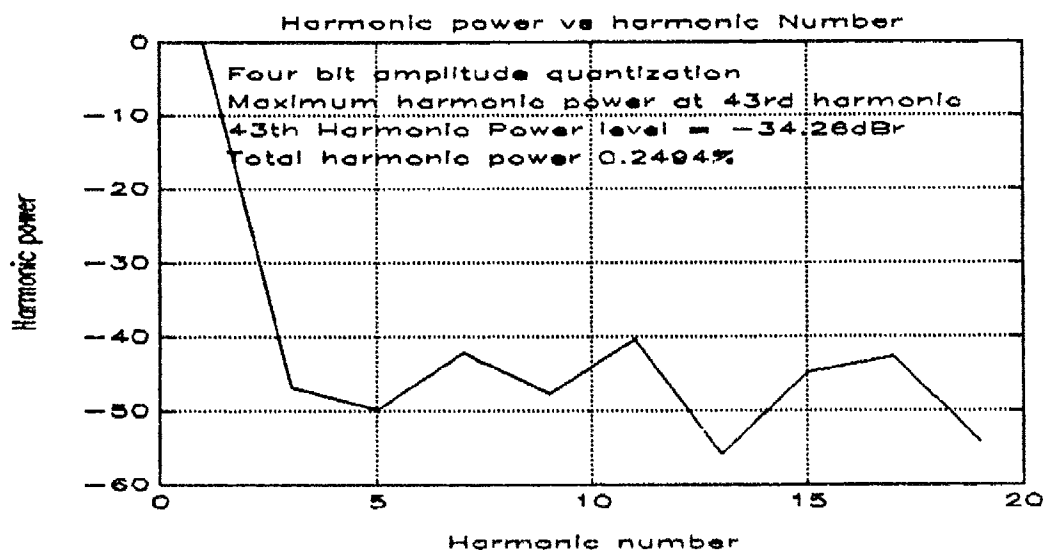


Figure B.5 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of four bit amplitude quantized signal (squared error minimized, expanded view, harmonics 1 to 19)

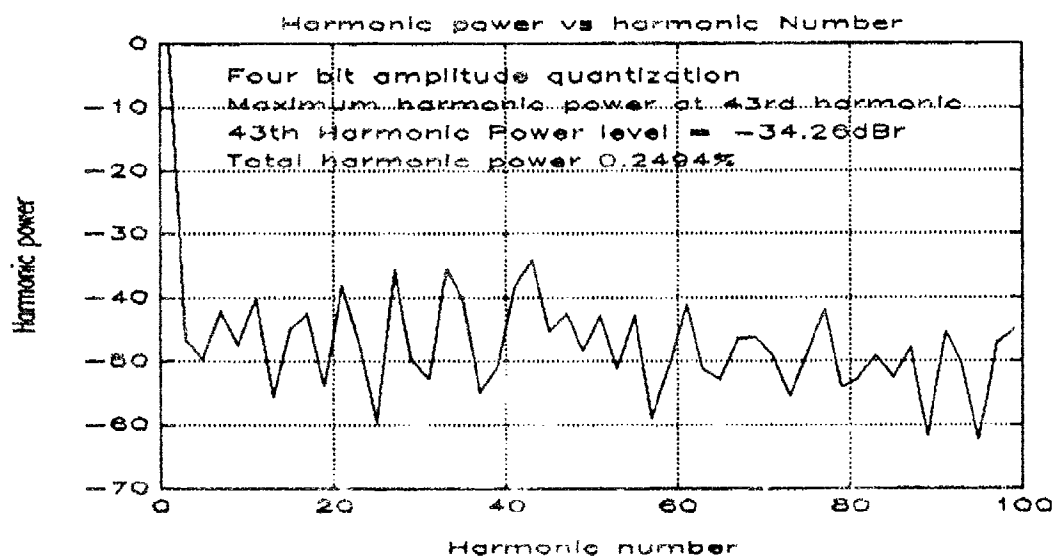


Figure B.6 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of four bit amplitude quantized signal (squared error minimized, harmonics 1 to 99)

# FIVE BIT AMPLITUDE QUANTIZED SIGNAL (ERROR MINIMIZED)

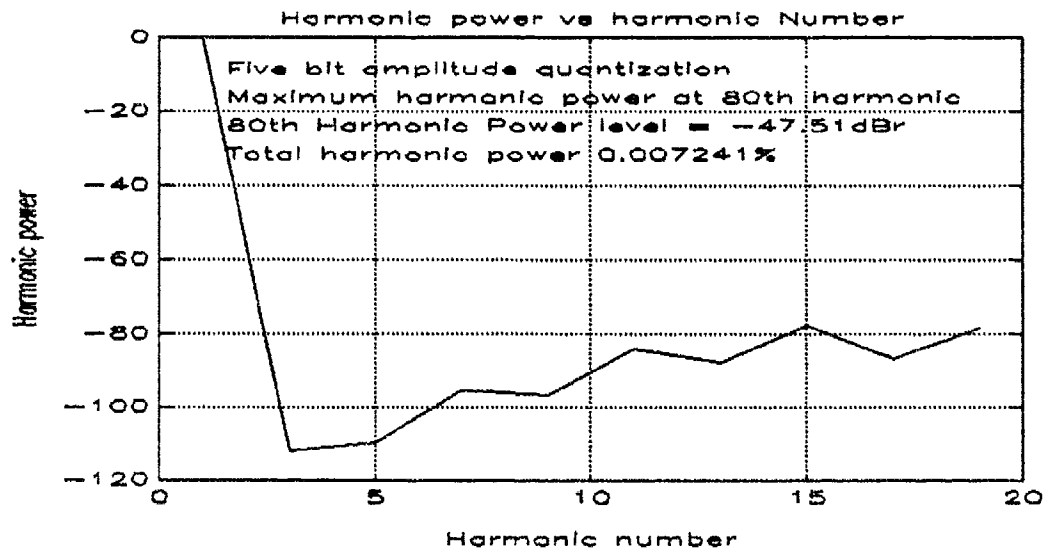


Figure B.7 Plot of harmonic power (in dB) relative to fundamental vs harmonic number of five bit amplitude quantized signal (squared error minimized, expanded view, harmonics 1 to 19)

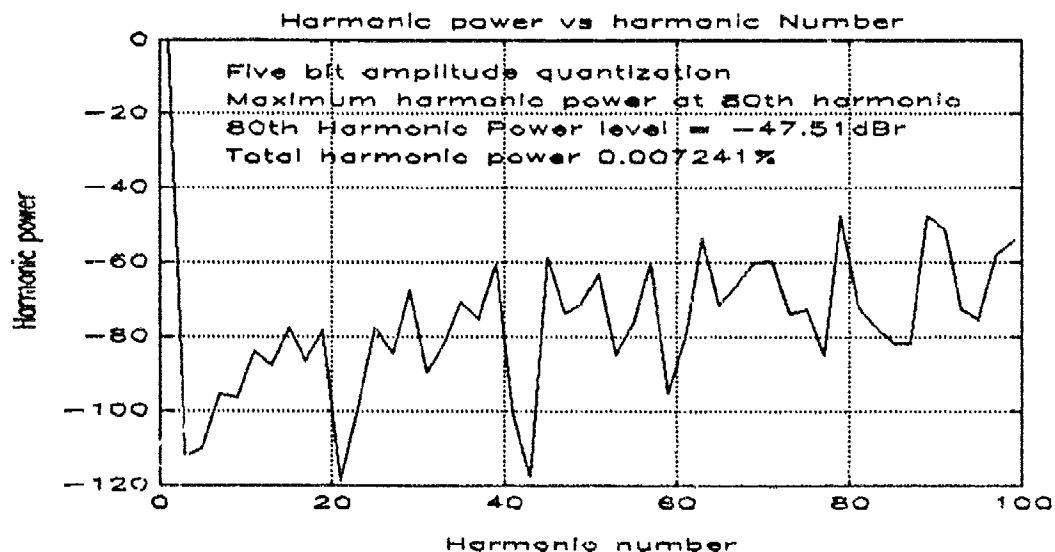


Figure B.8. Plot of harmonic power (in dB) relative to fundamental vs harmonic number of five bit amplitude quantized signal (squared error minimized, harmonics 1 to 99 plotted)



**A. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER IN 2 BIT  
AMPLITUDE QUANTIZED (ERROR MINIMIZED) SIGNAL**

```
% FILE NAME           : MODBIT2.M
% OPERATING SYSTEM    : DOS 5.0
% SYSTEM              : PC 486/33

clear
!del modbit21.met
!del modbit22.met

% SET THE AMPLITUDE STEP VALUE AND TRANSITION ANGLES
a = 0.4188161;
t2 = asin(3*a/2);

% TWO BIT HARMONIC CALCULATION
i=0:100;
n=2*i+1;
for i = 1:length(n),
    a2 = cos(n(i)*t2);
    s(i) = 4*a/(n(i)*pi) * (1+a2);
end
p = 10*log10(s .* s) - 10*log10(s(1) * s(1));

% CALCULATION OF TOTAL HARMONIC POWER
harpower = 0;
for i = 2:length(p)
    harpower = harpower +10^(p(i)/10);
end,
harpow = harpower*100;

% PLOT THE DATA
ttext = 'Harmonic power vs harmonic Number';
xtext = 'Harmonic Number'; ytext = 'Harmonic Power';
text2 = 'Two bit amplitude quantization';
text3 = 'Maximum harmonic power at 9th harmonic';
```

```

text4 = ['9th Harmonic Power level = ', num2str(p(5)), 'dBr'];
text5 = ['Total harmonic power ', num2str(harpow), '%'];

plot(n(1:10), p(1:10)); title(tttext);
xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text2, 'sc');
text(0.3,0.80, text3, 'sc');
text(0.3,0.75, text4, 'sc');
text(0.3,0.70, text5, 'sc')
meta modbit21; pause;
!gpp modbit21 /dhppl /fmodbit21.pic

plot(n(1:50), p(1:50)); title(tttext);
xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text2, 'sc'); text(0.3,0.80, text3, 'sc')
text(0.3,0.75, text4, 'sc'); text(0.3,0.70, text5, 'sc')
meta modbit22; pause;
!gpp modbit22 /dhppl /fmodbit22.pic

```

B. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER IN 3 BIT  
AMPLITUDE QUANTIZED (ERROR MINIMIZED) SIGNAL

```
% FILE NAME : MODBIT3.M
% OPERATING SYSTEM : DOS 5.0
% SYSTEM : PC 486/33

clear
!del modbit31.met
!del modbit32.met

% AMPLITUDE STEP a
a = 0.2332685;
t2 = asin(3*a/2);
t3 = asin(5*a/2);
t4 = asin(7*a/2);

% THREE BIT HARMONIC CALCULATIONS
i=0:100;
n=2*i+1;
for i = 1:length(n),
    a2 = cos(n(i)*t2);
    a3 = cos(n(i)*t3);
    a4 = cos(n(i)*t4);
    s(i) = 4*a/(n(i)*pi) * (1+a2+a3+a4);
end

p = 10*log10(s .* s) - 10*log10(s(1) * s(1));

% CALCULATION OF THE TOTAL HARMONIC POWER
harpower = 0;
for i = 2:length(p)
    harpower = harpower +10^(p(i)/10);
end,
harpow = harpower*100;
```

```

% PLOT THE DATA
ttext = 'Harmonic power vs harmonic Number';
xtext = 'Harmonic number';
ytext = 'Harmonic power';
text1 = 'Three bit amplitude quantization';
text2 = 'Maximum harmonic power at 19th harmonic';
text3 = '19th Harmonic Power level = ';
text3 = [text3 num2str(p(10)) 'dBr'];
text4 = ['Total harmonic power ', num2str(harpow), '%'];

plot(n(1:10), p(1:10))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc');
text(0.3,0.80, text2, 'sc');
text(0.3,0.75, text3, 'sc');
text(0.3,0.70, text4, 'sc')
meta modbit31; pause

plot(n(1:50), p(1:50))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc')
text(0.3,0.80, text2, 'sc')
text(0.3,0.75, text3, 'sc')
text(0.3,0.70, text4, 'sc')
meta modbit32

!gpp modbit31 /dhppl /fmodbit31.pic
!gpp modbit32 /dhppl /fmodbit32.pic

```

C. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER IN 4 BIT  
AMPLITUDE QUANTIZED (ERROR MINIMIZED) SIGNAL

```
% FILE NAME           : MODBIT4.M
% OPERATING SYSTEM    : DOS 5.0
% SYSTEM              : PC486/33

clear
!del modbit41.met
!del modbit42.met

% SET AMPLITUDE STEP VALUE AND TRANSITION ANGLES
a = 0.12139;
t2 = asin(3*a/2);
t3 = asin(5*a/2);
t4 = asin(7*a/2);
t5 = asin(9*a/2);
t6 = asin(11*a/2);
t7 = asin(13*a/2);
t8 = asin(15*a/2);

% THREE BIT HARMONICS CALCULATIONS
i=0:100;
n=2*i+1;
for i = 1:length(n),
    a2 = cos(n(i)*t2);
    a3 = cos(n(i)*t3);
    a4 = cos(n(i)*t4);
    a5 = cos(n(i)*t5);
    a6 = cos(n(i)*t6);
    a7 = cos(n(i)*t7);
    a8 = cos(n(i)*t8);
    s(i) = 4*a/(n(i)*pi) * (1+a2+a3+a4+a5+a6+a7+a8);
end

p = 10*log10(s .* s) - 10*log10(s(1) * s(1));
```

```

% CALCULATION OF TOTAL HARMONIC POWER
harpower = 0;
for i = 2:length(p)
    harpower = harpower +10^(p(i)/10);
end,
harpow = harpower*100;

% PLOT THE DATA
ttext = 'Harmonic power vs harmonic Number';
xtext = 'Harmonic number';
ytext = 'Harmonic power';
text1 = 'Four bit amplitude quantization';
text2 = 'Maximum harmonic power at 43rd harmonic';
text3 = ['43th Harmonic Power level = ' num2str(p(22)) 'dBr'];
text4 = ['Total harmonic power ',num2str(harpow), '%'];

plot(n(1:10), p(1:10))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc'); text(0.3,0.80, text2, 'sc')
text(0.3,0.75, text3, 'sc'); text(0.3,0.70, text4, 'sc')
meta modbit41; pause

plot(n(1:50), p(1:50))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc'); text(0.3,0.80, text2, 'sc')
text(0.3,0.75, text3, 'sc'); text(0.3,0.70, text4, 'sc')
meta modbit42

!gpp modbit41 /dhppl /fmodbit41.pic
!gpp modbit42 /dhppl /fmodbit42.pic

```

D. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER IN 5 BIT  
AMPLITUDE QUANTIZED (ERROR MINIMIZED) SIGNAL

```
% FILE NAME      : MODBIT5.M
% OPERATING SYSTEM : DOS 5.0
% SYSTEM         : PC486/33
```

```
!del modbit51.met
!del modbit52.met
```

```
clear
```

```
%SET THE AMPLITUDE STEP AND TRANSITION ANGLE VALUES
```

```
a = 0.0616995;
t2 = asin(3*a/2);
t3 = asin(5*a/2);
t4 = asin(7*a/2);
t5 = asin(9*a/2);
t6 = asin(11*a/2);
t7 = asin(13*a/2);
t8 = asin(15*a/2);
t9 = asin(17*a/2);
t10 = asin(19*a/2);
t11 = asin(21*a/2);
t12 = asin(23*a/2);
t13 = asin(25*a/2);
t14 = asin(27*a/2);
t15 = asin(29*a/2);
t16 = asin(31*a/2);
```

```

% FIVE BIT BIT HARMONIC CALCULATIONS
i=0:100;
n=2*i+1;
for i = 1:length(n),
    a2 = cos(n(i)*t2);
    a3 = cos(n(i)*t3);
    a4 = cos(n(i)*t4);
    a5 = cos(n(i)*t5);
    a6 = cos(n(i)*t6);
    a7 = cos(n(i)*t7);
    a8 = cos(n(i)*t8);
    a9 = cos(n(i)*t9);
    a10 = cos(n(i)*t10);
    a11 = cos(n(i)*t11);
    a12 = cos(n(i)*t12);
    a13 = cos(n(i)*t13);
    a14 = cos(n(i)*t14);
    a15 = cos(n(i)*t15);
    a16 = cos(n(i)*t16);
    a = 1+a2+a3+a4+a5+a6+a7+a8+a9;
    a = a+a10+a11+a12+a13+a14+a15+a16;
    s(i) = 4*a/(n(i)*pi)*(a);
end

p = 10*log10(s .* s) - 10*log10(s(1) * s(1));

% CALCULATION OF TOTAL HARMONIC POWER
harpower = 0;
for i = 2:length(p)
    harpower = harpower +10^(p(i)/10);
end,
harpow = harpower*100;

```



```

% PLOT THE DATA

ttext = 'Harmonic power vs harmonic Number';
xtext = 'Harmonic number';
ytext = 'Harmonic power';
text1 = 'Five bit amplitude quantization';
text2 = 'Maximum harmonic power at 80th harmonic';
text3 = ['80th Harmonic Power level = '];
text3 = [text3 num2str(p(40)) 'dBr'];
text4 = ['Total harmonic power ', num2str(harpow), '%'];

plot(n(1:10), p(1:10))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc'); text(0.3,0.80, text2, 'sc')
text(0.3,0.75, text3, 'sc'); text(0.3,0.70, text4, 'sc')
meta modbit51; pause

plot(n(1:50), p(1:50))
title(ttext); xlabel(xtext); ylabel(ytext); grid
text(0.3,0.85, text1, 'sc'); text(0.3,0.80, text2, 'sc')
text(0.3,0.75, text3, 'sc'); text(0.3,0.70, text4, 'sc')
meta modbit52;

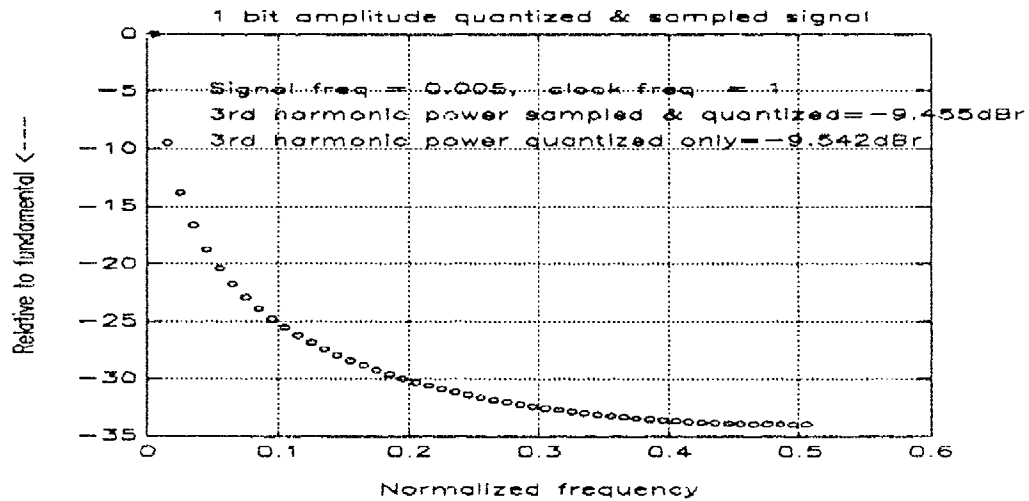
!gpp modbit51 /dhppl /fmodbit51.pic
!gpp modbit52 /dhppl /fmodbit52.pic

```

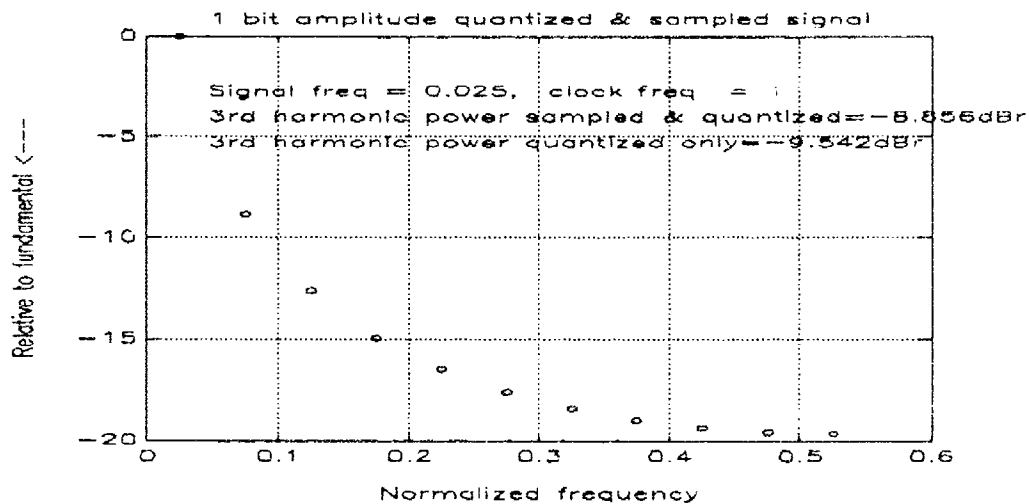
## APPENDIX C      PLOTS OF SAMPLED AND QUANTIZED SIGNAL

Appendix C contains all the plots of harmonic power vs normalized signal frequency for one to four bit of quantized and sampled signal. The MATLAB program for calculating the harmonic power level is also included in this appendix.

# ONE BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL

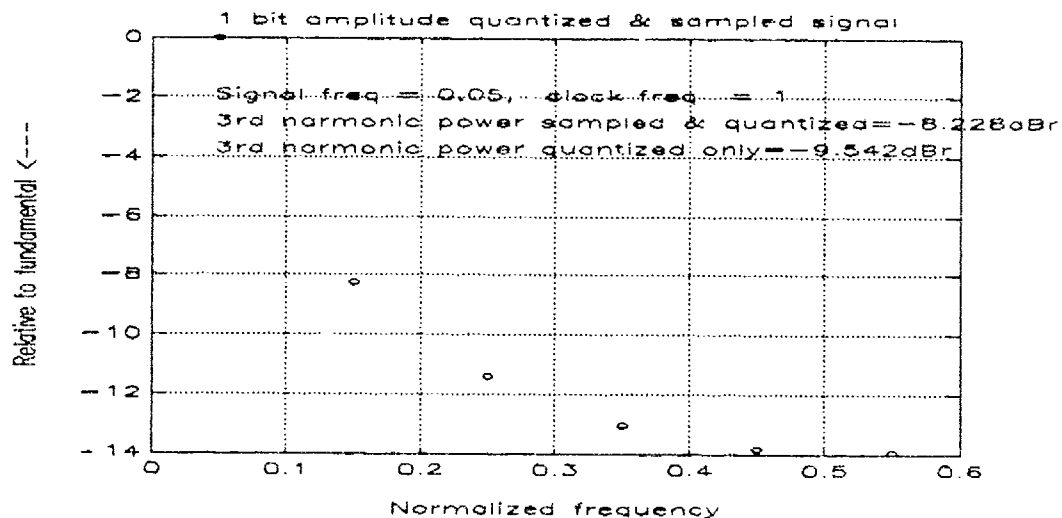


**Figure C.1** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of one bit amplitude quantized sampled signal. Normalized input signal frequency = 0.005. (Plot from dc to half the sampling frequency)

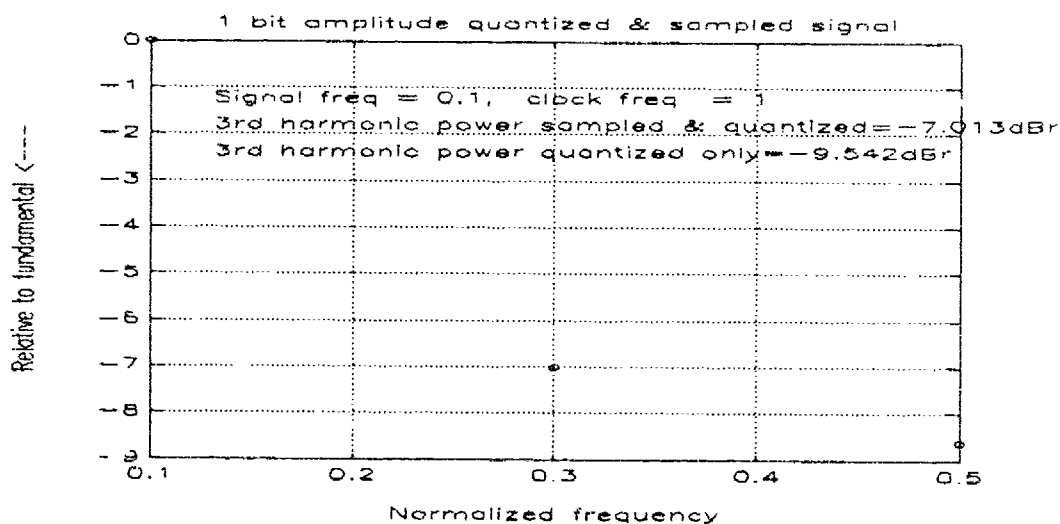


**Figure C.2** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of one bit amplitude quantized sampled signal. Normalized input signal frequency = 0.025. (Plot from dc to half the sampling frequency)

# ONE BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL



**Figure C.3** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of one bit amplitude quantized sampled signal. Normalized input signal frequency = 0.05. (Plot from dc to half the sampling frequency)



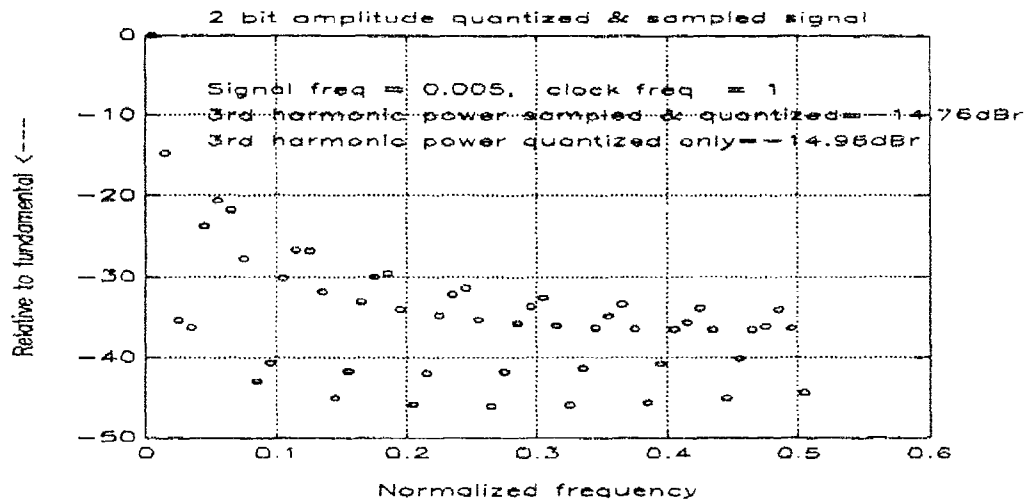
**Figure C.4** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of one bit amplitude quantized sampled signal. Normalized input signal frequency = 0.1. (Plot from dc to half the sampling frequency)

**TABLE C.1 HARMONIC POWER W.R.T FUNDAMENTAL VS HARMONIC NUMBER OF ONE BIT SAMPLED AND QUANTIZED SIGNALS**

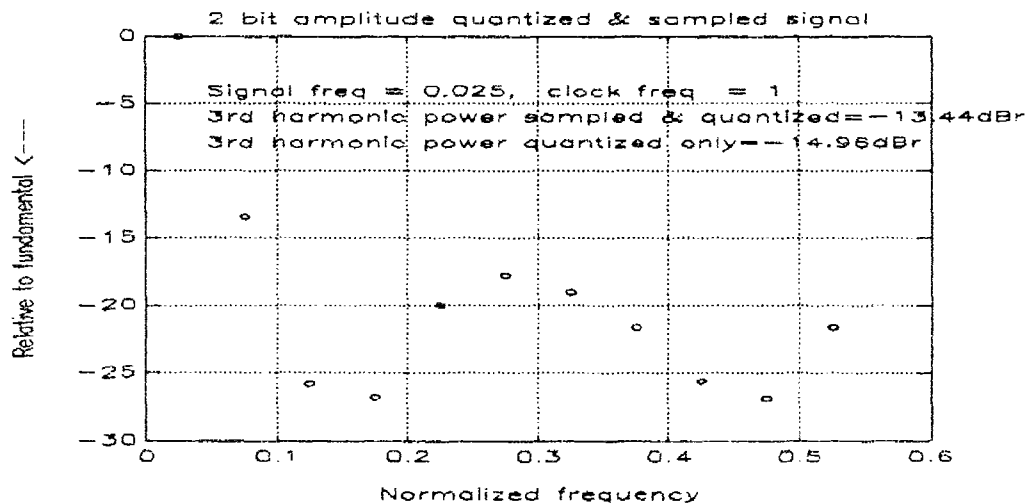
Harmonic Number	Power level relative to fundamental in dB				
	Sampled and quantized signal of normalized frequency				Quantized signal only (frequency independent)
	Freq = 0.005	Freq = 0.025	Freq = 0.05	Freq = 0.1	
1	0.00	0.00	0.00	0.00	0.00
3	- 9.45	- 8.86	- 8.23	- 7.01	- 9.54
5	- 13.80	- 12.64	- 11.41	*****	- 13.98
7	- 16.64	- 14.93	- 13.05	*****	- 16.90
9	- 18.73	- 16.49	- 13.83	*****	- 19.08
11	- 20.38	- 17.61	*****	*****	- 20.83
13	- 21.74	- 18.42	*****	*****	- 22.28
15	- 22.89	- 19.00	*****	*****	- 23.52
17	- 23.88	- 19.39	*****	*****	- 24.61
19	- 24.75	- 19.61	*****	*****	- 25.58

NOTE: \*\*\*\*\* INDICATES THAT THE HARMONIC IS BEYOND HALF THE SAMPLING FREQUENCY AND HENCE NEGLECTED.

## TWO BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL

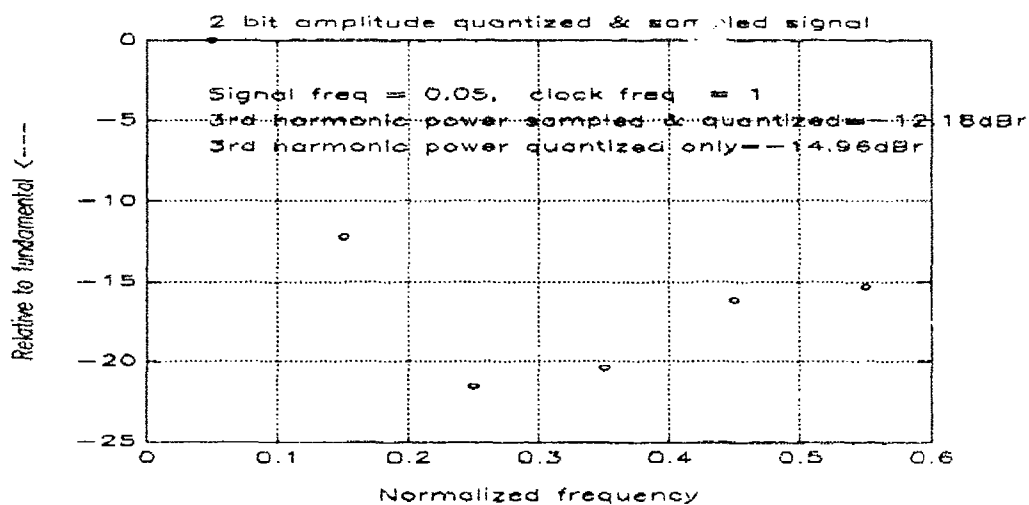


**Figure C.5** Plot of Harmonic power (in dB) relative to fundamental vs normalized frequency of two bit amplitude quantized sampled signal. Normalized input signal frequency = 0.005. (Plot from dc to half the sampling frequency)

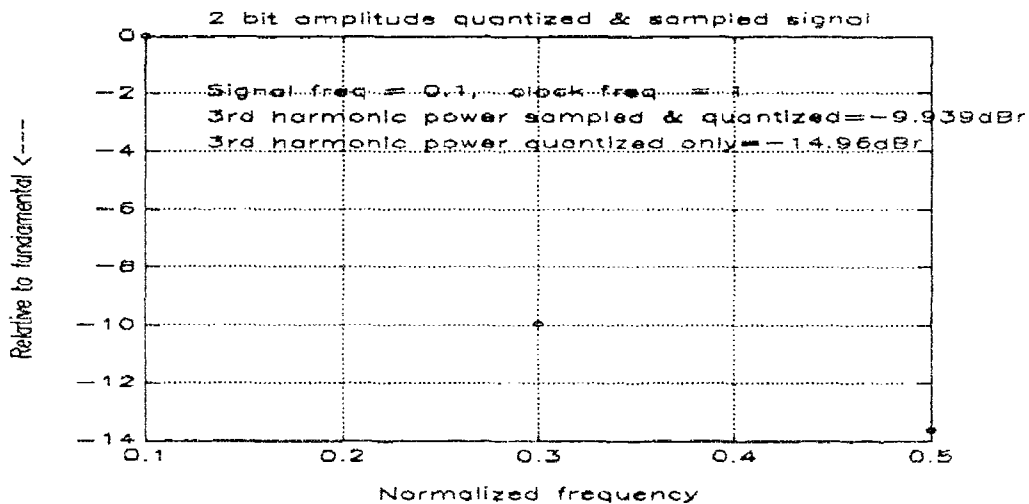


**Figure C.6** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of two bit amplitude quantized sampled signal. Normalized input signal frequency = 0.025. (Plot from dc to half the sampling frequency)

## TWO BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL



**Figure C.7** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of two bit amplitude quantized sampled signal. Normalized input signal frequency = 0.05. (Plot from dc to half the sampling frequency)



**Figure C.8** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of two bit amplitude quantized sampled signal. Normalized input signal frequency = 0.1. (Plot from dc to half the sampling frequency)

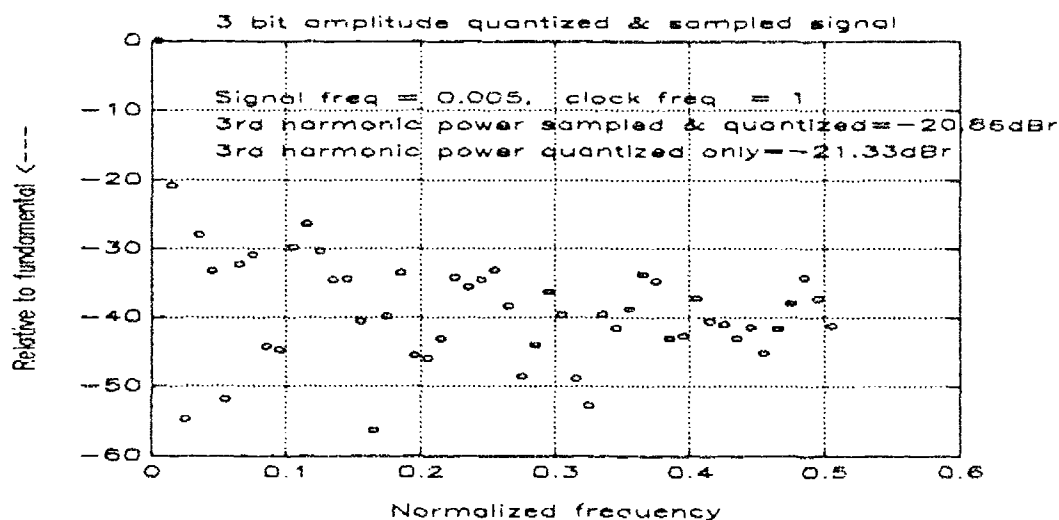
**TABLE C.2 HARMONIC POWER W.R.T FUNDAMENTAL VS HARMONIC NUMBER OF TWO BIT SAMPLED AND QUANTIZED SIGNALS**

Harmonic Number	Power level relative to fundamental in dB				
	Sampled and quantized signal of normalized frequency				Quantized signal only (frequency independent)
	Freq = 0.005	Freq = 0.025	Freq = 0.05	Freq = 0.1	
1	0.00	0.00	0.00	0.00	0.00
3	- 14.76	- 13.44	- 12.18	- 9.94	- 14.96
5	- 35.38	- 25.80	- 21.55	*****	- 36.86
7	- 36.27	- 26.82	- 20.41	*****	- 39.78
9	- 23.81	- 19.99	- 16.13	*****	- 24.50
11	- 20.60	- 17.79	*****	*****	- 20.83
13	- 21.74	- 19.05	*****	*****	- 22.28
15	- 27.76	- 21.69	*****	*****	- 28.94
17	- 42.96	- 25.61	*****	*****	- 47.49
19	- 40.67	- 26.89	*****	*****	- 48.45

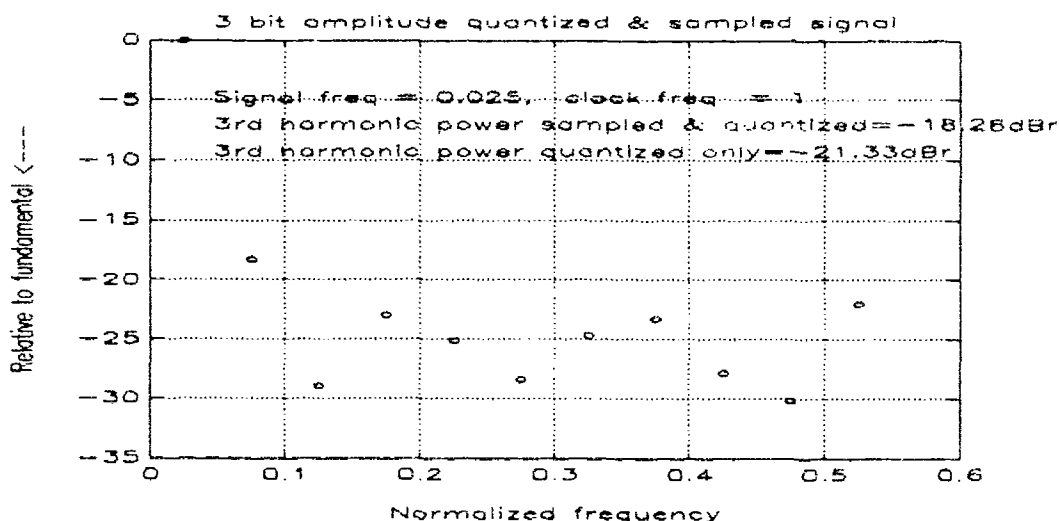
NOTE: \*\*\*\*\* INDICATES THAT THE HARMONIC IS BEYOND HALF THE SAMPLING FREQUENCY AND HENCE NEGLECTED.



### THREE BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL

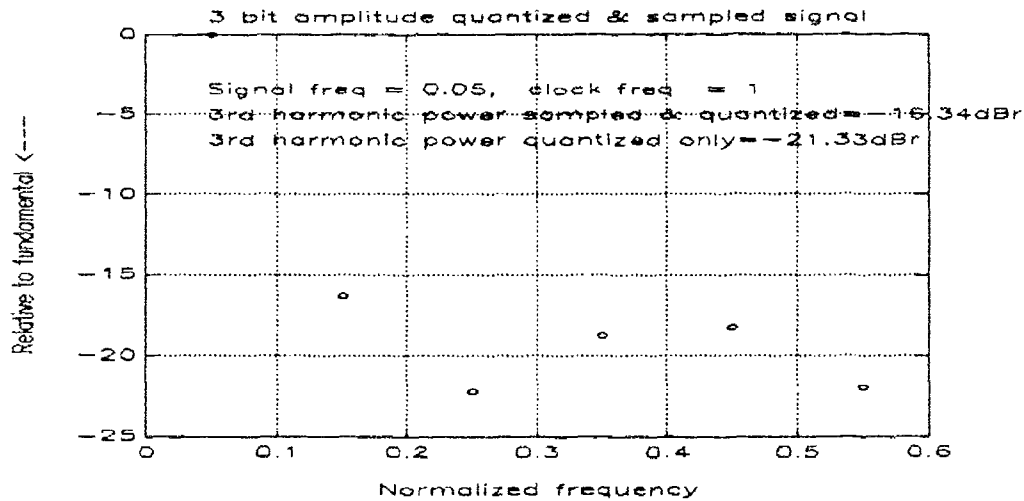


**Figure C.9** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of three bit amplitude quantized sampled signal. Normalized input signal frequency = 0.005. (Plot from dc to half the sampling frequency)

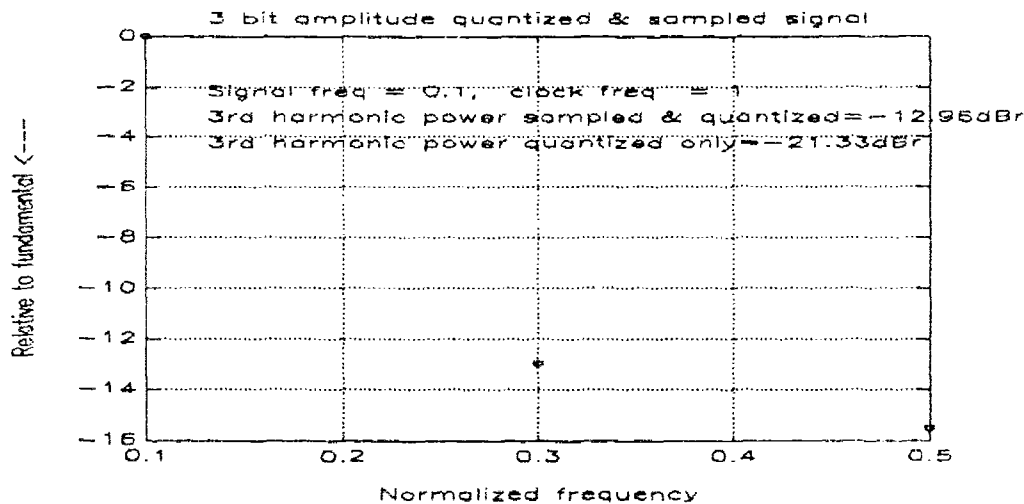


**Figure C.10** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of three bit amplitude quantized sampled signal. Normalized input signal frequency = 0.025. (Plot from dc to half the sampling frequency)

### THREE BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL



**Figure C.11** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of three bit amplitude quantized sampled signal. Normalized input signal frequency = 0.05. (Plot from dc to half the sampling frequency)



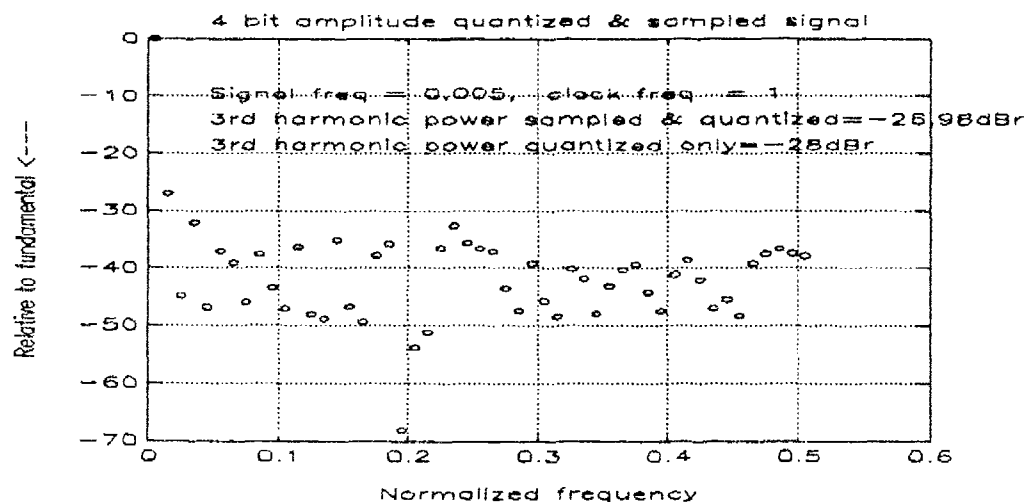
**Figure C.12** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of three bit amplitude quantized sampled signal. Normalized input signal frequency = 0.1. (Plot from dc to half the sampling frequency)

**TABLE C.3 HARMONIC POWER W.R.T FUNDAMENTAL VS HARMONIC NUMBER OF THREE BIT SAMPLED AND AMPLITUDE QUANTIZED SIGNALS**

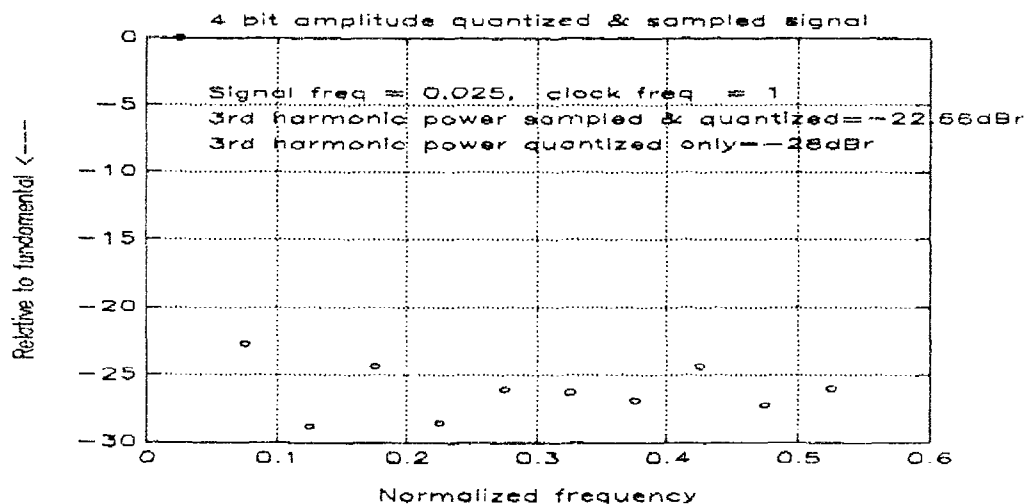
Harmonic Number	Power level relative to fundamental in dB				
	Sampled and quantized signal of normalized frequency				Quantized signal only (frequency independent)
	Freq = 0.005	Freq = 0.025	Freq = 0.05	Freq = 0.1	
1	0.00	0.00	0.00	0.00	0.00
3	- 20.86	- 18.28	- 16.34	- 12.96	- 21.33
5	- 54.70	- 28.97	- 22.23	*****	- 59.65
7	- 27.97	- 22.95	- 18.77	*****	- 28.90
9	- 33.23	- 25.09	- 18.25	*****	- 34.79
11	- 51.85	- 28.42	*****	*****	- 55.51
13	- 32.30	- 24.68	*****	*****	- 34.01
15	- 30.93	- 23.23	*****	*****	- 32.87
17	- 44.24	- 27.85	*****	*****	- 40.72
19	- 44.74	- 30.18	*****	*****	- 39.59

NOTE: \*\*\*\*\* INDICATES THAT THE HARMONIC IS BEYOND HALF THE SAMPLING FREQUENCY AND HENCE NEGLECTED.

# FOUR BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL

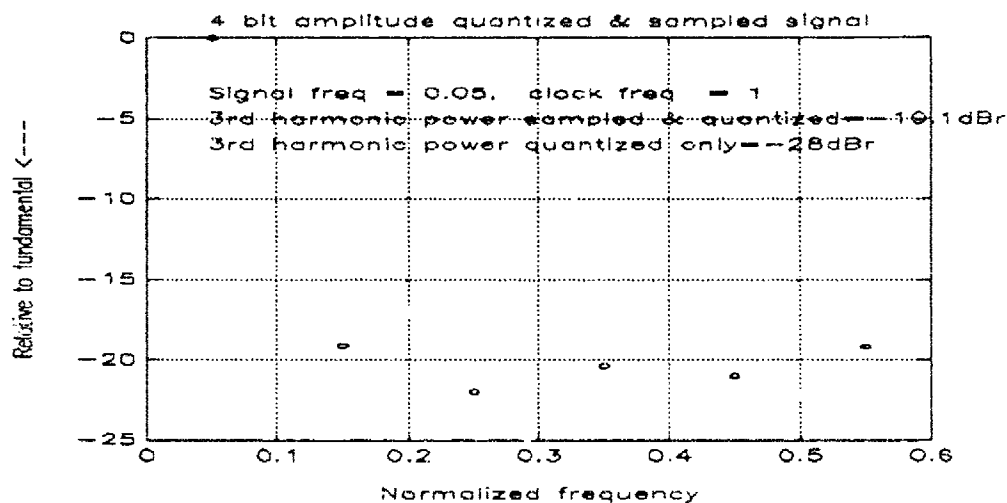


**Figure C.13** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of four bit amplitude quantized sampled signal. Normalized input signal frequency = 0.005. (Plot from dc to half the sampling frequency)

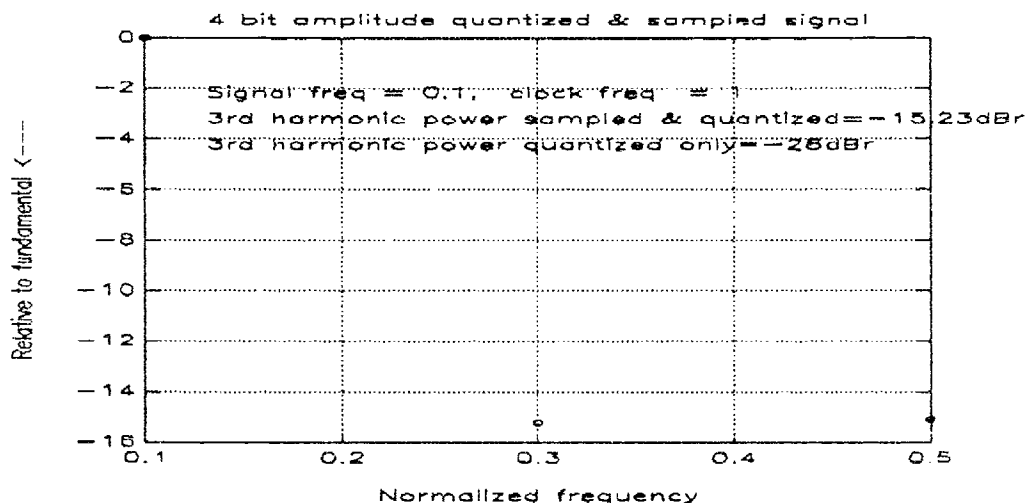


**Figure C.14** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of four bit amplitude quantized sampled signal. Normalized input signal frequency = 0.025. (Plot from dc to half the sampling frequency)

# FOUR BIT AMPLITUDE QUANTIZED SAMPLED SIGNAL



**Figure C.15** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of four bit amplitude quantized sampled signal. Normalized input signal frequency = 0.05. (Plot from dc to half the sampling frequency)



**Figure C.16** Plot of harmonic power (in dB) relative to fundamental vs normalized frequency of four bit amplitude quantized sampled signal. Normalized input signal frequency = 0.1. (Plot from dc to half the sampling frequency)

**TABLE C.4 HARMONIC POWER W.R.T FUNDAMENTAL VS HARMONIC NUMBER OF FOUR BIT SAMPLED AND QUANTIZED SIGNALS**

Harmonic Number	Power level relative to fundamental in dB				
	Sampled and quantized signal of normalized frequency				Quantized signal only (frequency independent)
	Freq = 0.005	Freq = 0.025	Freq = 0.05	Freq = 0.1	
1	0.00	0.00	0.00	0.00	0.00
3	- 26.98	- 22.66	- 19.10	- 15.23	- 28.00
5	- 44.79	- 28.81	- 21.99	*****	- 56.18
7	- 32.12	- 24.33	- 20.35	*****	- 33.26
9	- 46.72	- 28.54	- 20.99	*****	- 62.79
11	- 37.03	- 26.07	*****	*****	- 39.96
13	- 39.12	- 26.28	*****	*****	- 43.02
15	- 45.84	- 26.88	*****	*****	- 70.51
17	- 37.62	- 24.35	*****	*****	- 40.32
19	- 43.36	- 27.22	*****	*****	- 54.32

NOTE: \*\*\*\*\* INDICATES THAT THE HARMONIC IS BEYOND HALF THE SAMPLING FREQUENCY AND HENCE NEGLECTED.

**A. MATLAB PROGRAM FOR CALCULATION OF HARMONIC POWER OF QUANTIZED SAMPLED SIGNAL.**

```
% FILENAME           : DRAMP.M
% OPERATING SYSTEM   : DOS 5.0
% SYSTEM              : PC 486/33
% M BIT NOOFLEVELS = (2^M)/2 WHERE M IS NO OF BITS

clear
!del dr1.met
bits = input(' No of bits of quantization (1 to 8) = ');

N = (2^bits)/2;           % no of levels
A0 = 1;
fo= input('Input signal frequency = ');
To = 1/fo;
wo = 2*pi*fo;

Ac = 1;
fc = 1;
Tc = 1/fc;
wc = 2*pi*fc;
tc = 0.001*Tc;

Maxhar = fix(fc/fo);      % CALCULATION IS DONE UP TO MAXIMUM
                           % HARMONICS;
harp = Maxhar/2;

A1 = Ac*tc/Tc;
A2 = 4*A0/(pi*N);

text1 = ['kf21/nityan/dramp.m'];
text2 = [num2str(bits) 'bit Amplitude quantization'];

% HARMONIC CONTENT CALCULATION
for k = 1:N;               % step no in half cycle
    for n = 1:Maxhar,
        s(n) = cos ( (2*(n-1)+1) * asin ( (k-1) /(N) ) );
        hno(n) = 2*(n-1)+1;
        fl(n,k) = A1*A2/(2*(n-1)+1)*s(n);
    end
end
```

```

        theta(n) = (2*(n-1)+1)*(2*pi*fo);
    end,
end,
disp('Harmonic calculation complete')

pnk=zeros(1:hno);
for k = 1:N
    pnk = pnk + f1(:,k);
end,

disp('calculating the log')
pk = pnk .* pnk;
p = pk / max(pk);
pp = 10*log10(p);

% HARMONIC POWER CALCULATIONS
disp('calculating max harmonic')
harpow = 0;
for i = 2:Maxhar,
    harpow = harpow + 10^(pp(i)/10);
end,
harpow4bit = 10*log10(harpow);

text3 = ['Total Harmonic power : '];
text3 = [text3 num2str(harpow4bit) 'dBr'];
text4 = ['Max harmonic power at '];
text4 = [text4 num2str(hno(2)) 'rd harmonic'];
text5 = [num2str(hno(2)) 'rd Harmonic power level : '];
text5 = [text5 num2str(pp(2)) 'dBr'];
text6 = ['Harmonic power Vs Harmonic number'];
text7 = ['Relative to fundamental <---'];
text8 = ['Harmonic Number'];
text81 = ['Normalized frequency'];
textsp = ' ';
text10 = ['3rd harmonic power sampled and quantized = '];
text10 = [text10 num2str(LPP(2)) ' dBr'];
text11 = ['Signal freq = ' num2str(fo) ', clock freq = '];
text11 = [text11 num2str(fc)];
text13 = ['3rd harmonic power quantized only = '];
text13 = [text13 num2str(pp(2)) ' dBr'];

```



```

f2 = zeros(Maxhar, N);
disp ('calculating the q')
for m = 1:5,
    A3 = m*tc*wc/2;
    sinc(m) = sin(A3)/A3;
    for n = 1:100,
        freq = m*fc - (2*(n-1)+1)* fo;
        if abs(freq/fo) <= Maxhar
            harno = fix(abs(freq/fo)/2)+1;
            for k = 1:N,
                sincc(harno) = sinc(m);
                hhno(harno) = freq/fo;
                s(n) = cos((2*(n-1)+1)*asin((k-1)/(N)));
                hno(n) = 2*(n-1)+1;
                q(n,k) = A1* A2/(2*(n-1)+1)* sinc(m)*s(n);
                f = f2(harno, k) .* f2(harno,k);
                f2(harno,k) = (f + q(n,k).*q(n,k)).^0.5;
            end,
        end,
    end,
end,

disp('calculating QK')
qmnk = zeros(1:Maxhar);
for k = 1:N,
    qmnk = qmnk + f2(:,k)';
end,

PP = (pnk'+qmnk)/(pnk(1) +qmnk(1));
LPP = 10*log10(PP .* PP);
normfreq = hno*fo;

plot(normfreq(1:harp/2+1), LPP(1:harp/2+1), 'o')
title([num2str(bits) ' bit amplitude quantized sampled
signal'])
xlabel(text81)
ylabel(text7)
grid

text(0.2, 0.75, text10, 'sc')
text(0.2, 0.8, text11, 'sc')

```

```

text(0.2, 0.7, text13, 'sc')
meta dr1

% FOR PRINTING THE PLOT
%!gpp dr1

%FOR CONVERTING THE PLOT COMPATIBLE WITH WORDPERFECT
!gpp dr1 /dhppl /fdr1.pic
disp('pic file is DR1.pic')

filename = 'SAM.dat';
disp('Writing data to file SAM.dat')
freqt = ['Normalized input signal frequency = ' ];
freqt = [freqt num2str(fo) '\n\n\n'];
pw = ['Clock pulsewidth to period ratio = ' num2str(tc) '\n'];
noofbits = ['Number of bits of quantization = ' ];
noofbits = [noofbits num2str(bits) '\n'];
fprintf(filename, noofbits);
fprintf(filename, pw);
fprintf(filename, freqt);
ptext1 = '                Power level relative to'
ptext1 = [ptext1 ' fundamental\n'];
fprintf( filename, ptext1);
ptext2 = 'Harmonic          Sampled          Quantized \n';
fprintf( filename, ptext2);
fprintf( filename, 'number          quantized          only\n');
ptext3 = '                signal          signal \n\n';
fprintf( filename, ptext3)
ptext4 = '                dBr          dBr          \n\n\n';
fprintf( filename, ptext4);

% MAXIMUM OF 19 HARMONICS FOR FILE DATA
if harp/2 >= 10,
    kmax = 10;
else
    kmax = harp/2;
end,
ptext5 = '%4.0f %18.2f %16.2f \n'
for k = 1:kmax
    fprintf(filename, ptext5, hno(k), LPP(k), pp(k))
end,

```

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