Boundary Layer Coherent Structures
(MBL ARI)
$360,000 (January 1, 1993 – December 31, 1995)
and
AASERT Supplement
$208,393 (June 1, 1993 – May 31, 1996)

CY 1993 Report

Hampton N. Shirer and George S. Young, co-Principal Investigators
Penn State University
Department of Meteorology
503 Walker Bldg
University Park, PA 16802

(814) 863-1992 (HNS)
(814) 863-4228 (GSY)
(814) 865-3663 (FAX)

HNS@PSUVM.PSU.EDU (HNS)
G.YOUNG/OMNET (GSY)
YOUNG@EMS.PSU.EDU (GSY)

Key words describing research

chaotic time series, coherent structures, correlation dimension, marine boundary layer, principal component analysis

Key words describing technologies impacted

time series analysis

Number of post-docs supported **(1).**

Number of graduate students supported (including AASERT) **(4).**

Papers submitted and/or published in refereed journals **(0).**

***Papers to be submitted in refereed journals* (2).**

Fosmire, C. J., H. N. Shirer and R. Wells, 1994: Estimating the correlation dimension of boundary layer winds using the generalized Takens algorithm. *J. Atmos. Sci.*

No Abstract Available

Wells, R., H. N. Shirer and C. J. Fosmire, 1994: Generalization and convergence of the Takens estimators for the correlation dimension. *Physica D.*

Abstract

The correlation dimension D is used commonly to quantify the chaotic structure of an attractor of a smooth dynamical system. The standard algorithm for estimating the value of D is based on finding the mean slope of the curve obtained by plotting $\ln C(r)$ versus $\ln r$, where $C(r)$ is the correlation integral and r is the distance between points on the attractor. The alternative, probabilistic method proposed by Takens (1985) is based on finding the sample means of the random variable $\ln(r/\rho)$, expressed as the conditional expected value $E\left(\ln(r/\rho) : r < \rho\right)$.

In this article, this first Takens estimator is extended and generalized using the expected value $E\left((r/\rho)^p \left|\ln(r/\rho)\right|^k : r < \rho\right)$, where k is an integer, to provide an infinite number of independent estimates $D_{app}(\rho)$ of the correlation dimension D. Convergence of these estimates is proved rigorously for $C(r)$ slowing varying and having $C(r)/r^D$ bounded away from zero and infinity. In addition, an empirical criterion for slow variation is introduced and justified.

The sensitivity of the mean slope method and of the generalized estimators is studied in detail for two *ad hoc* correlation integrals, the first representing the effects of noise at small distances and the second capturing periodic lacunarity in the correlation integral itself. All the generalized estimators give results that are superior to that produced by the mean slope method. Moreover, the various estimators exhibit much different behavior in the two *ad hoc* cases—noise-contaminated signals are best diagnosed using $p = k = 1$, and lacunar signals are best studied using $p = 0$ and k as large as possible in magnitude. Therefore, by using a wide range of values of p and k, one can infer whether noise or lacunarity most likely dominates the time series being studied and hence can decide which of the estimates most likely approximates the correlation dimension for the series.

Finally, these ideas are applied to relatively coarse samplings of the Henon and Lorenz attractors. As expected, lacunarity appears to dominate the Henon results, and the best estimate of D, $D_{app} = 1.21$, is given by the case $p = 0$ and $k = 3$. In contrast, undersampling or noise appears to affect the Lorenz results, with the best estimate of D, $D_{app} = 2.06$, given by the case $p = k = 1$.

***Papers published or accepted in nonrefereed journals* (1).**

Henderson, H. W. and D. W. Thomson, 1994. **Fractal dimensions of remotely sensed** atmospheric signals. *Proc. of the Second Experimental Chaos Conference*, Oct. 6-8, 1993, Arlington, VA. World Scientific Publishing Company, in press.

Abstract

Meteorological measurements of turbulence variables and measurements of acoustic and electromagnetic signals are used to study the properties of atmospheric structures. Determination of the fractal dimensions of these structures may allow better quantification of their time- and height-dependent structure. However, much work still must be done to find the best algorithm for estimating this dimension.

The method discussed here is one proposed by Higuchi in 1988. This technique directly measures the fractal dimension of a time series graph. As shown by Higuchi, this method also provides an alternative way to interpret the power spectra of the series. The conversion of the line length dimension d (given by the Higuchi algorithm) to spectral slope s is $d=(5-s)/2$ where $1<s<3$. The algorithm has some features that may make the technique more desirable than the power spectral approach in data analysis. The method is unusually robust in the analysis of the highest frequencies, an area of the spectrum that is not known for its robustness through power spectra analysis. Also, the method is particularly capable of identifying periodicities in the series. Here, meteorological examples are shown to illustrate both the strengths and weaknesses of the technique.

***Significant presentations* (1).**

Henderson, H.W. and D.W. Thomson, 1993. Fractal dimensions of remotely sensed atmospheric signals. *Proc. of the Second Experimental Chaos Conference*, Oct. 6-8, 1993, Arlington, VA.

The method discussed in this presentation is the one proposed by Higuchi in 1988. This technique directly measures the fractal dimension of a time series graph. As shown by Higuchi, this method also provides an alternative way to interpret the power spectra of the series. The conversion of the line length dimension d (given by the Higuchi algorithm) to spectral slope s is $d=(5-s)/2$ where $1<s<3$. The algorithm has some features that may make the technique more desirable than the power spectral approach in data analysis. The method is unusually robust in the analysis of the highest frequencies, an area of the spectrum that is not known for its robustness through power spectra analysis. Also, the method is particularly capable of identifying periodicities in the series.

Major Accomplishments

Objective 1: To improve the correlation dimension algorithm for use with MBL datasets

We have extended the correlation dimension algorithm of Takens (1985) to one that in principle produces an infinite number of estimates of the correlation dimension (Wells *et al.*, 1994). We have tested our new algorithm extensively using *ad hoc* cases to determine the sensitivity of the results to noise or undersampling and to a phenomenon known as lacunarity (Theiler 1988). Based on these tests, we believe we are able to identify the estimate that is most likely to provide the best estimate for a given time series. This approach is applied in an analysis of boundary layer winds by Fosmire *et al.* (1994).

Objective 2. To implement and test the multiscale algorithm of Higuchi (1988)

The multiscale algorithm was coded and tested, both on time series of known properties and on several archived datasets. The results indicated that there were benefits obtained from the approach that went beyond those discussed in the original paper. For example, we found that the approach was a sensitive indicator of imposed periodic signals, and that it was able to detect such signals even when the signal-to-noise ratio was as low as 1 to 1000. We also found that usable estimates are obtainable from datasets having ~10,000 points, and that this accuracy is achieved at the highest frequencies.

Objective 3. To implement and test the obliquely rotated PCA algorithm

The idealized data tests of the PCA algorithm have shown that the method is able to distinguish multiple coherent structure types under several realistic conditions, including: Different coherent structure types not linearly independent, different coherent structure types overlapping in space and time, and random noise of amplitude equal to that of the signal. In a blind test on such "worst case" data, PCA was able to quantify the profiles associated with each coherent structure type. This test provides proof that PCA can yield valid results without the *a priori* conceptual model required of previous (conditional sampling) methods.

***Technical reports* (0).**

***Books published* (0).**

***Book chapters published* (0).**

***Patent applications* (0).**

***Honors and awards* (0).**

Significant transitions

None

Impact of research

None

References

Fosmire, C. J., H. N. Shirer and R. Wells, 1994: Estimating the correlation dimension of boundary layer winds using the generalized Takens algorithm. *J. Atmos. Sci.* (Manuscript in preparation)

Henderson, H.W. and D.W. Thomson, 1994. Fractal dimensions of remotely sensed atmospheric signals. *Proc. of the Second Experimental Chaos Conference*, Oct. 6-8, 1993, Arlington, VA. World Scientific Publishing Company, in press.

Higuchi, T., 1988: Approach to an irregular time series on the basis of the fractal theory. *Physica D*, **31**, 277-283.

Takens, F., 1985: On the numerical determination of the dimension of an attractor. In *Dynamical Systems and Bifurcations, Proceedings, Groningen 1984*, pp. 99-106, B. L. J. Braaksma, H. W. Broer, and F. Takens (Eds.). *Lecture Notes in Mathematics*, **1125**, Springer-Verlag, Berlin.

Theiler, J., 1988: Lacunarity in a best estimator of fractal dimension. *Phys. Lett. A*, **133**, 195-200.

Wells, R., H. N. Shirer and C. J. Fosmire, 1994: Generalization and convergence of the Takens estimators for the correlation dimension. *Physica D* (Manuscript in preparation).