AD-A273 515

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED FINAL/01 JAN 90 TO 31 AUG 93

4. TITLE AND SUBTITLE CENTER FOR ANALYSIS OF HETEROGENEOUS AND NONLINEAR MEDIA (U)	5. FUNDING NUMBERS 3484/A5 AFOSR-90-0090
6. AUTHOR(S) Professor Robert Kohn	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) New York University Department of Mathematical Sciences New York, NY 10003	8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR- 93 0867
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM 110 DUNCAN AVE, SUITE B115 BOLLING AFB DC 20332-0001	10. SPONSORING/MONITORING AGENCY REPORT NUMBER AFOSR-90-0090
11. SUPPLEMENTARY NOTES	
12a. DISTRIBUTION/AVAILABILITY STATEMENT APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED	12b. DISTRIBUTION CODE UL

DTIC ELECTE DEC 07 1993 S A

13. ABSTRACT (Maximum 200 words)

The researcher had many publications and refereed journals under this grant. Kohn worked with G. Allaire on numerical shape optimization for minimum compliance in plane stress. This work makes use of a "relaxed" formulation. seeking an optimal design made from perforated composite materials. The latter surveys recent progress on optimal bounds. Topics addressed include the optimal energy bounds for elastic composites (proved using the Hashin-Shtrikman principle), and optimal bounds on the effective moduli of polycrystals (proven by means of the translation method).

14. SUBJECT TERMS			15. NUMBER OF PAGES 6
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR(SAME AS REPORT)

CENTER FOR ANALYSIS OF HETEROGENEOUS AND NONLINEAR MEDIA

Courant Institute of Mathematical Sciences

Supported by a University Research Initiative grant through the Air Force Office of Scientific Research, No. 90-0090

Robert V. Kohn, Principal Investigator

FINAL TECHNICAL REPORT FOR 1 JAN 90 - 31 AUG 93 (per AN)

The Center's principal research directions are materials science and fluid dynamics. Its projects in the first direction include bounding the effective moduli of composite materials, and modelling the microstructures which arise from coherent phase transitions. Its projects in the second area include modelling the effective diffusivity due to turbulent transport. Here are some highlights of the Center's scientific achievements:

(a) Robert Kohn and Stefan Muller studied the twinning that occurs in a martensitic phase transition. Elastic energy considerations explain gross features such as the directions of the twins and the relative volume fraction of each phase variant. However, finer features such as the length scale of twinning require a more detailed model, incorporating surface energy effects. Kohn and Muller have developed a mathematical model which captures the essence of this phenomenon. It turns out that there are two distinct regimes. In one, the twin planes are exactly parallel, and the twin width w is related to the grain size L by the law $w=L^{1/2}$. In the second, the twins branch self-similarly as the approach the austenite, and w is related to the distance x to the austenite by the law $w=x^{2/3}$. Previous analyses -- of which there have been many in the physics and materials literatures -- all missed the second regime, though something similar has been noted in the context of ferroelectric domains.

(b) Graeme Milton and Gilles Francfort have been investigating the relation between sequential lamination and bounds on effective moduli. Their long-range goal is to explain why sequential lamination always seems sufficient for constructing composites with extremal effective behavior. This goal has not yet been achieved, but considerable progress has been made. Milton and Francfort have studied what it means for a set L of effective tensors to be stable under lamination. They have shown the following property: for each point x on the boundary of L there is an associated quadratic form τ_x, depending only on the second order behavior of the boundary at x, with the property that τ_x is quasiconvex. Moreover, associated with τ_x there is a set of effective moduli which is stable under homogenization and includes x on its boundary. Milton's idea is that as x varies along the boundary of L, these stable sets should wrap around L. If this could be proved, then it would follow that lamination-closed sets are in fact homogenization-closed. While this grand objective has not yet

been achieved, some other important properties of G-closures have been established. In particular, the work of Milton and Francfort has shown that for characterizing the set of possible effective moduli obtainable by mixing two materials in fixed volume fractions, it is sufficient to consider bounds on sums of energies and complementary energies.

(c) Marco Avellaneda and Andrew Majda have been working on the passive advection of a scalar quantity by a stationary, incompressible velocity field. The goal is to identify the appropriate long-distance, large-time scaling, and a suitable effective equation. The scientific importance of this work lies in its link to recent theories for modelling eddy diffusivity in turbulent flow. In prior work Avellaneda and Majda introduced a class of "exactly renormalizable" models, whose effective behavior can be computed explicitly. Their recent work includes an extension to the transport of *interfaces* rather than individual particles. Rigorous results have been obtained concerning the fractal dimension of an evolving interface, and these can be linked to experimental work in the turbulence literature. The work of Avellaneda and Majda has also explored the sense in which their "exactly renormalizable models" provide an analytical test of the RNG ("renormalization group") method, introduced by Yakhot and Orszag for estimating eddy diffusivity in fully developed turbulence. There are a limited number of possible effective behaviors, which can be viewed as distinct types or "phases" of turbulent behavior. The RNG method misses some of the phases completely, because it is perturbative in character. Where it applies, however, its results are remarkably accurate. Avellaneda and Majda have shown that the phase diagram is not specific to the details of the "exactly renormalizable" examples. The same picture applies for any random, isotropic, incompressible velocity field in a neighborhood of the Kolmogorov-Obukhov regime.

(d) Marco Avellaneda and Yves Achdou studied the permeability of porous rock (the constant in D'Arcy's law), and more generally the dynamical permeability (describing the response to an oscillatory pressure gradient). Prior work in the physics literature had proposed empirical or "approximate" laws for these quantities in terms of the electrical behavior of the same rock. These empirical laws were derived by means of a certain "effective medium" approximation. The work of Achdou and Avellaneda showed that pore size dispersion and pore roughness effects can lead to systematic deviations from the empirical laws. They used computer simulations to explore how the breakdown of the effective medium approximation depends on various features of the microstructure.

(e) Graeme Milton and Leonid Gibiansky investigated the extension to viscoelasticity of the well-known Hashin-Shtrikman bounds on the effective bulk modulus. This effective parameter controls the propagation of acoustic pressure waves. Milton and Gibiansky showed that the complex bulk modulus of a two component composite lies in a certain lens shaped region of the complex plane. Moreover special points on the boundary of the lens can be identified with the specific geometries. This work is interesting not only for its result, but also for its method: it offers the first specific application of a new variational principle for complex effective moduli, formulated a few years ago by Cherkaev, Gibiansky, and Milton.

(f) Robert Kohn and Gregoire Allaire considered optimal bounds on the elastic energy of a mixture of two materials, when the average strain is fixed. Such bounds are useful for structural optimization, and also in the modelling of coherent phase transitions. Prior work of Avellaneda, Allaire, and Kohn showed how to derive optimal energy bounds in considerable generality, provided that the elasticity tensors of the two components are "well-ordered." The new work of Allaire and Kohn extends the optimal lower bound on elastic energy to the case of two *non-well-ordered* but isotropic

materials. Actually, two different arguments are presented: the first extends an argument used by Walpole to bound the effective bulk modulus; the second is based on the translation method. Besides the intrinsic interest of the result, this work helps to demonstrate the power of the translation method, and it puts the work of Walpole in an entirely new light.

1. Publications submitted to refereed journals

Y. Achdou and M. Avellaneda, "Influence of pore roughness and pore-size dispersion in estimating the permeability of a porous medium from electrical measurements", Physics of Fluids A, to appear 12/92.

Y. Achdou and O. Pironneau, "Integral equations for the generalized Stokes operator with applications to boundary layer -- Euler matching,"

Y. Achdou, "Integral equations for the generalized Stokes operator: applications to high Reynolds number flows," Comm. Pure Appl. Math, to appear.

S. Alama, M. Avellaneda, P. Deift and R. Hempel, "On the existence of eigenvalues of a divergence-form operator $A+\lambda B$ in the gap of $\sigma(A)$". Asymptotic Analysis, to appear.

G. Allaire and R. Kohn, "Optimal design for minimum weight and compliance in plane stress using extremal microstructures," European J. Mechanics A/Solids, submitted.

G. Allaire and R. Kohn, "Optimal lower bounds on the elastic energy of a composite made from two non well-ordered isotropic materials," Quart. Appl. Math., to appear.

M. Avellaneda and A. J. Majda "Renormalization theory for eddy-diffusivity in turbulent transport". Phys. Rev. Lett. 68, 1992, 3028-3031.

M. Avellaneda and A. Majda, "Superdiffusion in Nearly Stratified Flows", J. Stat. Phys. 69, 1992. 689-729.

M. Avellaneda and A. Majda, "Simple examples with features of renormalization for turbulent transport", Proc. Cambridge Philosophical Soc., to appear.

M. Avellaneda and T. Olson, "Effective medium theories and effective electromechanical coupling factors of piezoelectric composites, J. Intell. Mater. Struct., to appear.

J. Axell, "Bounds for field fluctuations in two-phase materials," J. Appl. Phys. 72, 1992, 1217-1220.

J.T. Beale, T. Hou, and J. Lowengrub, "Growth rates for the linearized motion of fluid interfaces away from equilibrium," Comm. Pure Appl. Math, submitted.

J. Berryman and G. Milton, "Exact results in linear thermomechanics of fluid-saturated porous media." Appl. Phys. Lett., to appear.

K. Bhattacharya, "Comparison of the geometrically nonlinear and linear theories of martensitic transformation," Continuum Mechanics and Thermodynamics, submitted.

R. Caflisch, N. Ercolani and T. Y. Hou, "Multi-Valued Solutions and Branch Point Singularities for Nonlinear Hyperbolic Systems," Comm. Pure Appl. Math., to appear.

A. Cherkaev, K. Lurie, and G. Milton, "Invariant properties of the stress in plane elasticity and equivalence classes of composites," Proc. Roy. Soc. London A., to appear.

K. Clark and G. Milton, "Modelling the effective conductivity function of an arbitrary two-dimensional polycrystal using sequential laminates," Proc. Roy. Soc. Edinburgh Ser. A, submitted.

G. Francfort and G. Milton, "Sets of conductivity and elasticity tensors stable under lamination." Comm. Pure Appl. Math., submitted.

L. Gibiansky and G. Milton, "On the effective viscoelastic moduli of two-phase media: I. Rigorous bounds on the complex bulk modulus," Proc. Roy Soc. London A, to appear.

T. Y. Hou and Ph. Le Floch, "Why Nonconservative Schemes Converge to Wrong Solutions: Error Analysis," Experimental Mathematics, to appear.

R. Kohn and S. Muller, "Branching of twins near an austenite/twinned-martensite interface," Phil. Mag. A, to appear.

R. Kohn and S. Muller, "Surface energy and microstructure in coherent phase transitions," Comm. Pure Appl. Math., submitted.

G. Milton, "A link between sets of tensors stable under lamination and quasiconvexity," Comm. Pure Appl. Math., submitted.

M. Mineev, "Stability of two-dimensional needle growth," Phys. Rev. A, to appear.

T. Olson and M. Avellaneda: "Effective dielectric and elastic constants of piezoelectric polycrystals", J. Appl. Phys. 71, 1992, 4455-4464.

2. Book chapters and conference proceedings

Y. Achdou and M. Avellaneda, "Permeability of a porous medium: electrical and diffusional estimators", to appear in Proceedings of the ASME Winter Meeting (Workshop on Heterogeneous Media), Anaheim CA, Nov. 1992

G. Allaire and R. Kohn, "Topology optimization and optimal shape design using homogenization," to appear in Topology Design of Structures, M. Bendsoe and C. Mota Soares, eds., Kluwer (Amsterdam).

M. Avellaneda and T. Olson: "Effective medium theories for piezoelectric composite materials", to appear in Recent Advances in Adaptive and Sensory Materials and Their Applications", C. Rogers ed., Technomic Publish. Co., Lancaster, PA, 1992, p. 71ff.

R. Kohn and G. Milton, "Extremal microstructures for composite materials," to appear in Synergism of Mechanics, Mathematics and Materials, S.-C. Chou, ed., (Army Symposium on Solid Mechanics, Plymouth, MA November 1991).

R. Kohn and S. Muller, "Surface energy and microstructure," to appear in Shape Memory Materials and Phenomena -- Fundamental Aspects and Applications, C. T. Liu et al. eds., Materials Research Society Proceedings Vol. 246.

R. Kohn and S. Muller, "Surface energy and the length scale of twinning in martensite," to appear in proceedings of ICOMAT-92, J. Perkins, ed.

S. Torquato and M. Avellaneda, "Cross-property relations for transport in porous media: rigorous link between fluid permeability, conductivity and relaxation times", in Multiphase Transport in Porous Media, R.R. Eaton et al. eds., ASME Conf. Proc. 73-81, 1991.

3. Graduate students supported.

Jiangbo Lu, Ken McLaughlin, Sunder Sethuraman, Chris Apelian, Min Chen, Uri Keich, Hongwei Cheng.

4. Postdoctoral and senior visitors:

Jorgen Axell, Kaushik Bhattacharya, Yves Achdou, Mark Mineev, Eric Siggia

5. Honors

No major honors such as scientific society awards.