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PENETRABLE WEDGE ANALYSIS
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I. PHYSICAL IMPEDANCE BOUNDARY CONDITION

We can now confidently state that the pair of coupled difference equations, that arose [61] in the Kontorovich-Lebedev analysis of the actual Leontovich-boundary-condition wedge, are a more complicated and therefore inferior version of William's [7] formulae. Although the literature has concentrated on the formulation of Maliuzhinets [6], an asymptotic evaluation of the Williams result to extract high frequency diffraction mechanisms would ultimately duplicate the work of Tiberio, et al [10], since the Williams and Maliuzhinets results are necessarily equivalent. Therefore, we will not be pursuing the physical impedance boundary wedge anymore during the remainder of this grant.

II. INHOMOGENEOUS OR LINEARLY-VARYING IMPEDANCE BOUNDARY

Felsen's 1959 (!) paper [66] presents a rather complete analysis and discussion of this curious problem that I re-discovered in 1992 [61]. A thorough dissection and understanding of Felsen's work is required before any meaningful extensions or extrapolations can be made. This has not been done to date, owing to our attention to the:

III. GENERAL PENETRABLE WEDGE

Since our previous quarterly report of 3 August 1993, and motivated partly by the findings above for both cases of impedance boundaries, we have returned (again!) to the truly penetrable wedge scatterer. A fresh, and in retrospect, logical approach starting with a fundamental application of Green's theorem, has yielded a surface integral equation for a single unknown surface distribution. The kernel is specific to the wedge geometry; the free-space Green's function is not used. The radiation condition is invoked in *both* regions, ensuring uniqueness of the integral equation solution. The unboundedness of the penetrable scatterer is thusly the very feature of the physical problem that permits this

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formulation, while rendering integral equations developed [2-5,18-22,28,59] for finite bodies inefficient and effectively nonapplicable.

To extract the physical solution from this exciting and mathematically sound formulation of the boundary value problem, we are concentrating on a creative and physically-motivated approach to these issues:

(1) *Choice of a suitable basis in which to expand the surface distribution.* Convergence of the solution, as well as physical interpretation and utility, is greatly enhanced through an expansion that anticipates the actual physical behavior. In this problem of a semi-infinite domain, the expected far ($r \rightarrow \infty$) behavior on the wedge surface should be that of the

(2) *Line-source excitation of the Sommerfeld half-space problem.* All wave mechanisms that we can asymptotically pull out of this simpler geometry and account for in the wedge problem are valuable both computationally and conceptually.

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The continuation of this wedge scattering research is proceeding as proposed, with several significant accomplishments and findings to date. The nature of this rigorous mathematical approach to scattering problems is such that original formulation is very important and usually very slow to evolve. For example, the previous work summarized in the report [61] had several abandoned and potentially useful starts, while the finished product is substantial.

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