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Carderock Division, Naval Surface Warfare Center

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Survivability, Structures and Materials Directorate

Research and Development Report

TWO VISUALLY MEANINGFUL CORRELATION MEASURES FOR COMPARING CALCULATED AND MEASURED RESPONSE HISTORIES

by

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and

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CARDEROCKDIV-U-SSM-67-93/15 TWO VISUALLY MEANINGFUL CORRELATION MEASURES FOR COMPARING CALCULATED AND MEASURED RESPONSE HISTORIES

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Two visually meaningful correlation measures are proposed for comparing calculated and measured response histories. One is an error index which is a simplification of RSS (root-sum-square) error factor, and the other is an inequality index which is a simplification of Theil's inequality coefficient. The first compares the difference between the calculated and the measured histories to the measured history. The second compares the difference between the two histories to the sum of the two. The proposed correlation measures are compared to other existing measures, namely, Geer's Error Factors, RSS Error Factor, and Theil's Inequality Coefficient for ease of interpretation and visualization.

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ABSTRACT

Two visually meaningful correlation measures are proposed for comparing calculated and measured response histories. One is an error index which is a simplification of RSS (root-sum-square) error factor, and the other is an inequality index which is a simplification of Theil's inequality coefficient. The first compares the difference between the calculated and the measured histories to the measured history. The second compares the difference between the two histories to the sum of the two. The proposed correlation measures are compared to other existing measures, namely, Geers' Error Factors, RSS Error Factor, and Theil's Inequality Coefficient for ease of interpretation and visualization.

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INTRODUCTION

For over a decade, the Navy's shock community has been using Geers' error factors (magnitude, phase, and comprehensive)[1] as a comparison tool to judge the "goodness" of calculated response histories against the measured. While the Geers' pioneering work has served its purpose and will continue to serve as a comparison tool, a concern raised by the senior author during a recent SEAWOLF MPU/CSA review meeting that some of the Geers' error factors somehow "did not look right" led to this work.

In comparing any two response histories (both calculated, both measured, or one calculated and the other measured), a common practice is to assume one of the two to be "true" (exact or accurate) and the other "approximate," and any discrepancy or deviation from the true is associated with the term "error". For example, when calculated values are compared to measured values, it may be "scientifically correct" to assume the measured to be "true". Very often, however, the measured values can be as uncertain as the calculated values, and thus there would be no justification for favoring the measured over the calculated, in which case any difference between the two is associated with the term "inequality", as opposed to "error".

To accommodate both of the above cases, i.e., "error" and "inequality", two correlation measures are proposed herein: Zilliacus' <u>error index</u> and Whang's <u>inequality index</u>. The proposed error index compares the difference between two histories to the one assumed to be "true", and for convenience the assumed "true" values will be called the "measured", m(t), and the other the "calculated", c(t). The error index is a simplification of the well-known RSS (root-sum-square) error factor. The proposed inequality index, on the other hand, compares the difference between two histories to the sum of the two, without assuming one of the two to be "true". The two histories can be any combination of calculated and measured; however, in this report, one will be called the calculated, c(t), and the other the measured, m(t), for convenience. The inequality index is a simplification of Theil's inequality coefficient[2,3].

The report first presents Geers' error factors and RSS error factor, followed by the proposed error index. The report then presents Theil's inequality coefficient and the proposed inequality index. Discussions of each of the correlation measures and their comparisons then follow.

CORRELATION MEASURES

In what follows, c_i are the calculated values, and m_i are the measured values.

GEERS' Error Factors (M, P, C):

a) Magnitude Error Factor (M):

$$M = \frac{\sqrt{\sum c_i^2}}{\sqrt{\sum m_i^2}} - 1 \qquad (1)$$

Since the first term can be less than 1, M can be negative, and since the first term can be greater than 2, M can exceed 1.

b) Phase Error Factor (P):

$$P = 1 - \frac{\sqrt{\left|\sum c_i m_i\right|}}{\sqrt{\sqrt{\sum c_i^2}\ \sqrt{\sum m_i^2}}} \qquad (2)$$

Since the second term cannot exceed 1, P is bounded between 0 and 1.

c) Comprehensive Error Factor (C):

$$C = \sqrt{M^2 + P^2} \qquad (3)$$

C is the vectorial sum of its orthogonal components M and P, and since M can exceed 1, C can exceed 1.

RSS Error Factor (R):

$$R = \frac{\sqrt{\sum (c_i - m_i)^2}}{\sqrt{\sum m_i^2}} \quad (4)$$

R is the RSS of the differences between c_i and m_i divided by the RSS of m_i. Obviously, R can exceed 1.

ZILLIACUS' Error Index (Z):

$$Z = \frac{\sum |c_i - m_i|}{\sum |m_i|} \quad (5)$$

Z is the area of the residual $(c_i - m_i)$ divided by the area of the measured, and it can exceed 1.

THEIL's Inequality Coefficient (T):

$$T = \frac{\sqrt{\sum (c_i - m_i)^2}}{\sqrt{\sum c_i^2} + \sqrt{\sum m_i^2}} \quad (6)$$

T is the RSS of the differences between c_i and m_i divided by the sum of the RSS of c_i and the RSS of m_i. T is bounded between 0 and 1.[2]

WHANG's Inequality Index (W):

$$W = \frac{\sum |c_i - m_i|}{\sum |c_i| + \sum |m_i|} \quad (7)$$

W is the area of the residual $(c_i - m_i)$ divided by the sum of the areas of the calculated and measured. W is bounded between 0 and 1.

DISCUSSION

Figures 1(a), 2(a), and 3(a) are directly from Reference 1, and they indicate Geers' Magnitude (e_m), Phase (e_p), and Comprehensive (e_c) error factors along with RSS (e_r) error factor for three different sets of comparisons. The starting times for the calculated c(t) and the measured m(t) are different because Geers' error measures require m(t) (or c(t)) to be adjusted horizontally until e_c is minimized. This adjustment was deemed necessary in fairness to the analysts because the starting time (t=0) for m(t) is usually not well known. In the future, before any comparison is made, a common reference time, such as the time from detonation, should be clearly indicated on each history (calculated and measured) so that the adjustment of starting times would not be necessary.

Figures 1(b), 2(b), and 3(b) correspond to their respective Figures without time adjustments, and in each Figure, M, P, C, and R are shown. While C's are greater than e_c's, the differences are not drastic. In Reference 1, Geers points out that the e_r's in Figures 2(a) and 3(a) seem to be unacceptably/too large. However, since the upper bound of e_r is limitless (not bounded by 100%) there is no basis for saying these values appear too large. In fact, e_c is not bounded either. At least, e_r=100% can be understood to be the case when the RSS of m_i is equal to the RSS of (c_i-m_i), though not easily visualized. The case where e_c=100% is beyond visualization.

The most troublesome of these Figures is Figure 2 which shows e_m=0% (or M=0.009) and e_c=4% (or C=0.139). Most observers would agree that the magnitude errors of these sets are nowhere near zero.

In Figure 4, the previous Figures 1(b), 2(b) and 3(b) are repeated, and Z, T, and W are added for comparison. Z and R are consistently similar to each other as one might expect, but the advantage of Z over R is that Z is easier to visualize than R. Both Z and R can be greater than 1. As pointed out earlier, Z can be visualized as the ratio of the area of the residual (c_i-m_i) to the area of the measured. (See Figures A-1 through A-3 in Appendix A.)

The difference between R and T is that the denominator of T has an additional term, the square root of the sum of the squares of c_i. Conceptually, T compares the RSS of the residuals to the <u>sum</u> of the RSS's of the measured and the calculated; therefore, when (c_i-m_i) are small, T tends to be about half of R. T can never be greater than 1, while R is unbounded.

The difference between Z and W is that the denominator of W has an additional term, the sum of the absolute values of c_i; therefore, when (c_i-m_i) are small, W tends to be about half of Z. W can never be greater than 1, while Z is unbounded.

T and W are consistently similar to each other as one might expect, but the advantage of W over T is that W is easier to visualize than T. As pointed out earlier, W can be visualized as the ratio of the area of the residual $(c_i - m_i)$ to the <u>sum</u> of the areas of the calculated and the measured. (See Figures A-1 through A-3 in Appendix A.)

Figures 5 and 6 show what happens when the sign of one of the two curves is reversed. As pointed out by Dawson[4], M, P, and C are insensitive to the sign reversal. (Also, note that in these cases R and Z are greater than 1, while T and W are not.) Recently, to remedy this problem, Geers[5] proposed a revision to P which in turn affects C. The new Geers' error factors are as follows: (M is unchanged.)

$$\text{Pnew} = 1 - \frac{\sum c_i m_i}{\sqrt{\sum c_i^2}\,\sqrt{\sum m_i^2}} \tag{8}$$

$$\text{Cnew} = \sqrt{M^2 + (\text{Pnew})^2} \tag{9}$$

Since the second term in Pnew is bounded between +1 and -1, Pnew is now bounded between 0 and 2. Cnew can still exceed 1. As shown in Figures 5(b) and 6(b), Pnew and therefore Cnew are sensitive to the sign reversal. In fact, Pnew=2.0 is an indication of sign reversal or of being completely out of phase.

The fact that Z can exceed 1 is meaningful since the "residual" can be greater than the "measured". Similarly, the case of W=1.0 is meaningful since that happens when the "residual" is equal to the "sum". However, since C(or Cnew) is the vectorial sum of M and P(or Pnew), the meaning of C(or Cnew)=1.0 is not clear.

Figure 7 shows the effect of reversing c(t) and m(t). M is sensitive to the reversal as pointed out by Dawson[4], while P and Pnew are not. Also, as expected, R and Z are sensitive to the reversal, while T and W are not.

Figure 8 is not an example of common occurrence but is presented here to show the need to distinguish between "early" time comparison and "late" time comparison. The integration limits have been arbitrarily selected as 0 to 1.0, 1.0 to 2.0, and 0 to 2.0.

Figures 9, 10, 11 are related to a common case involving strain records showing a permanent set. Figure 9(a) shows that when c(t) and m(t) are very close to each other, all of the measures are reasonable. In Figure 9(b), R, Z, T, and W appear to be reasonable, while M, P, C, Pnew and Cnew appear to be a bit low. Figure 10 is an example of the cases where M, C, Cnew, R, and Z are close to each other and where T and W are <u>not</u> about half of R and Z, respectively. Figure 11 shows the cases where c(t) shows no sign of permanent set.

Figure 12 is presented here to show that when W=1.0, i.e., when the "difference" is equal to the "sum", the measure does not distinguish the degree of badness, i.e., W indicates that the Figures 12(a) and 12(b) are "equally bad". On the other hand, C values show Figure 12(b) to be worse than Figure 12(a), while R and Z values show the reverse. Pnew does not distinguish the degree of badness when two curves are completely out of phase.

The error measures and inequality measures discussed above are summarized on the next page for the reader's convenience.

GEERS' ERROR FACTORS:

Magnitude

$$M = \frac{\sqrt{\sum c_i^2}}{\sqrt{\sum m_i^2}} - 1$$

Phase

$$P = 1 - \frac{\sqrt{\left|\sum c_i m_i\right|}}{\sqrt{\sqrt{\sum c_i^2}\ \sqrt{\sum m_i^2}}} \qquad \text{Pnew} = 1 - \frac{\sum c_i m_i}{\sqrt{\sum c_i^2}\ \sqrt{\sum m_i^2}}$$

Comprehensive

$$C = \sqrt{M^2 + P^2} \qquad \text{Cnew} = \sqrt{M^2 + (\text{Pnew})^2}$$

RSS ERROR FACTOR:

$$R = \frac{\sqrt{\sum (c_i - m_i)^2}}{\sqrt{\sum m_i^2}}$$

ZILLIACUS' ERROR INDEX:

$$Z = \frac{\sum \left| c_i - m_i \right|}{\sum \left| m_i \right|}$$

THEIL'S INEQUALITY COEFFICIENT:

$$T = \frac{\sqrt{\sum (c_i - m_i)^2}}{\sqrt{\sum c_i^2} + \sqrt{\sum m_i^2}}$$

WHANG'S INEQUALITY INDEX:

$$W = \frac{\sum \left| c_i - m_i \right|}{\sum \left| c_i \right| + \sum \left| m_i \right|}$$

VISUALIZATION

For a correlation measure to be visually meaningful, it must be simple and consistent with "eyeballing", i.e., the mental process of human eyes to compare two response histories. One possible process is to pick off a vertical distance (absolute value) of the difference between two curves at a particular time and divide that distance by another vertical distance at that particular time. These ratios can be obtained at various times, and with relative ease, they can be averaged mentally in an approximate way.

The above process is expressed mathematically as follows:

$$A = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|c_i - m_i\right|}{\left|m_i\right|} \qquad \text{or} \qquad B = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|c_i - m_i\right|}{\left|c_i\right| + \left|m_i\right|} \tag{10}$$

'A' resembles Z, while 'B' resembles W. 'A' can be visualized by most people, but can 'B' be visualized? To answer that question, an unscientific/informal survey was taken among a number of colleagues at Carderock Division. The result of the survey is tabulated in Figure 13. For each set of curves, three numbers were given, and the participants were asked to estimate the value of 'B' visually, and select one number closest to that value. The percentages for each set show the result of the survey. For example, Figure 13(b) shows that 17% of the participants selected 0.770 to be the value of 'B', while 75% thought that 0.388 was the value of 'B'. The letters in parentheses next to "%" indicate which values they actually were. In each of these cases, most of the participants picked W values to be the 'B' values, demonstrating that the 'B' values can be visualized and that they are similar to W values. However, the problem with 'A' and 'B' is that when c(t) and m(t) intersect simultaneously on the time axis, i.e., when $|c_i - m_i| = 0$ and $|m_i| = 0$ or when $|c_i - m_i| = 0$ and $|c_i| + |m_i| = 0$, the condition of indeterminacy occurs. (See Figure 4(a), for example.) This problem has been avoided in Z and W by summing in the numerator and in the denominator separately. This enables one to associate Z and W as the ratios of areas. It should be pointed out here that visualizing or mentally estimating M, P, C, Pnew, Cnew, R, and T is not natural for most people because it involves taking the square root of the sum of squares of many numbers.

In Reference 5, Geers used a simple case of $c_i = km_i$, where k is a constant, to point out correctly that W is not symmetric about k=1.* For example, if one analysis produced $c_i = 0.5m_i$, W would be 0.333 (=0.5/1.5), while if another analysis produced $c_i = 1.5m_i$, its W would be 0.200 (=0.5/2.5). And since each differed from the measured by $0.5m_i$, Geers states that associating two different values of W for these two analyses is "not proper". It is true that W is not

----

* In this example,
$M = |k| - 1$, $P = 0$, $C = ||k| - 1|$, $R = Z = |k - 1|$, and $T = W = |k - 1|/(|k| + 1)$.[5]

symmetric about k=1 as shown in Figure 14 for this simple case. (C is <u>locally</u> symmetric about k=1, and <u>globally</u> symmetric about k=0. Z is symmetric about k=1.) However, the way human eyes compare two curves is <u>not</u> symmetric about k=1. Using the Geers' example of $c_i = km_i$, Figure 15 shows the asymmetric nature of "eyeballing". Figure 15(a) has the appearance of better correlation than Figure 15(b), even though both c(t)'s differ from m(t) by 50%! Figures 15(c) and 15(d) make the point more strikingly.

Figure 16 shows further the asymmetric nature of "eyeballing" for exponentially decaying sine waves for k values of 0.5, 1.5, 2.0 and 3.0. Figure 16(b) appears to have better correlation than Figure 16(a), even though they both differ from the measured by 50% as shown by C=Z=0.500 in both cases. (In this example, P=Pnew=0; therefore C=Cnew.) Figures 16(a) and 16(c) have the same W, but they have different C and Z. Figures 16(c) and 16(d) show that Z=1.0 in general means the "calculated" on the whole is twice the "measured" and that Z=2.0 means the "calculated" on the whole is three times the "measured", etc. (See Figures A-15 through A-18 in Appendix A.)

To complete the discussion on visualization, Appendix A is provided to show how Z and W can be interpreted. In each Figure, under the original c(t) and m(t) (with a reference to the original Figure number), the rectified (absolute-valued) residual [c(t)-m(t)] is compared to the rectified m(t) for Z, and below that, the rectified residual is compared to the sum of the rectified c(t) and m(t) for W. In almost all of the cases shown, the Z-curves intersect, while in all cases, the W-curves never intersect, although at some points they are coincident, which happens when the "difference" is equal to the "sum".

One might recognize that both Z and W are the ratios of means: Z is the ratio of the mean of the rectified residual (shaded area) to the mean of the rectified measured, and W is the ratio of the mean of the rectified residual (shaded area) to the mean of the sum of the rectified calculated and the rectified measured*. However, visually comparing the areas is more appealing than averaging. After all, visually comparing the areas is the whole idea behind "pie charts".

It is not necessary to compare every experimental record to analysis in this graphical fashion. These graphs are presented here to show that Z and W are consistent with the way one might visualize or interpret these values.

Several computer programs (ERROR, EQGEN and RECTIFY) were written to generate these curves and to calculate their corresponding M, P, C, Pnew, Cnew, R, Z, T, and W values. The programs are

---

* Comparing Equations (5) and (7), Z and W can be mathematically combined in the form of $(p + q) \Sigma|c_i - m_i| \;/\; (p \Sigma|c_i| + q \Sigma|m_i|)$ where p and q may be interpreted as weighting factors or some probability coefficients related to uncertainties of c(t) and m(t). However, by doing so, the simplicity and the visual meaning of Z and W would be lost.

available upon request, and even if they are not requested, they are still available.

CONCLUSIONS AND RECOMMENDATIONS

Two visually meaningful correlation measures have been proposed for comparing "calculated" and "measured" response histories: Zilliacus' error index and Whang's inequality index. The error index (Z) is appropriate when there is justification for favoring the "measured" over the "calculated". The inequality index (W) is appropriate when there is no justification for favoring one over the other. However, whether there is justification or not, both of the proposed measures, Z and W, may be used to supplement Geers' Cnew without adjusting starting times as long as what they are comparing and <u>what they are comparing to</u> are kept in mind.

As the community gains experience in associating the values of Cnew, Z, and W with their corresponding plots, a consensus may be reached on qualitative words to go with certain ranges of these values. For example, "Excellent" may be assigned for W values less than 0.1, "Good" to "Fair" for W values between 0.1 and 0.4, etc., and similar words for Cnew and Z. These words are clearly subjective, and the range of values for each word would change as both the computational and experimental technologies improve. In the meantime, it is hoped that the proposed correlation measures would help in making the subjective judgment of whether an analysis on the whole is acceptable or not acceptable.

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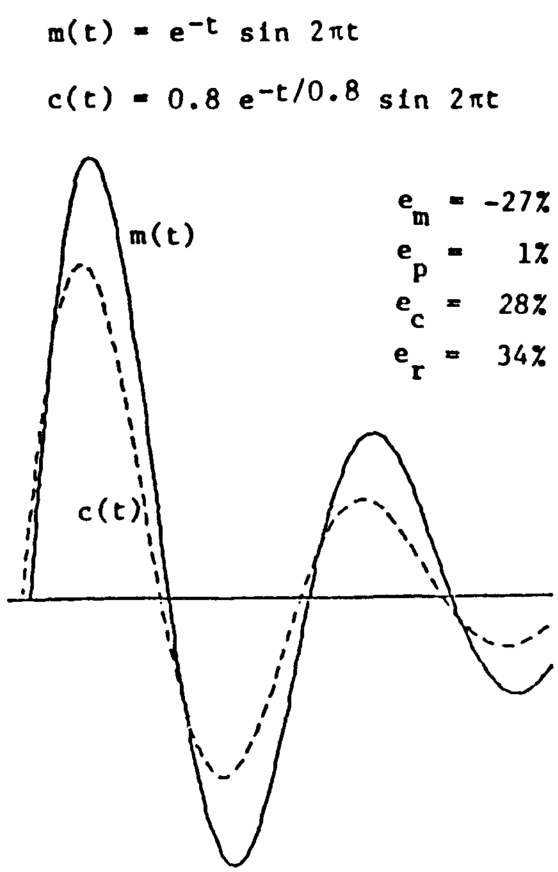
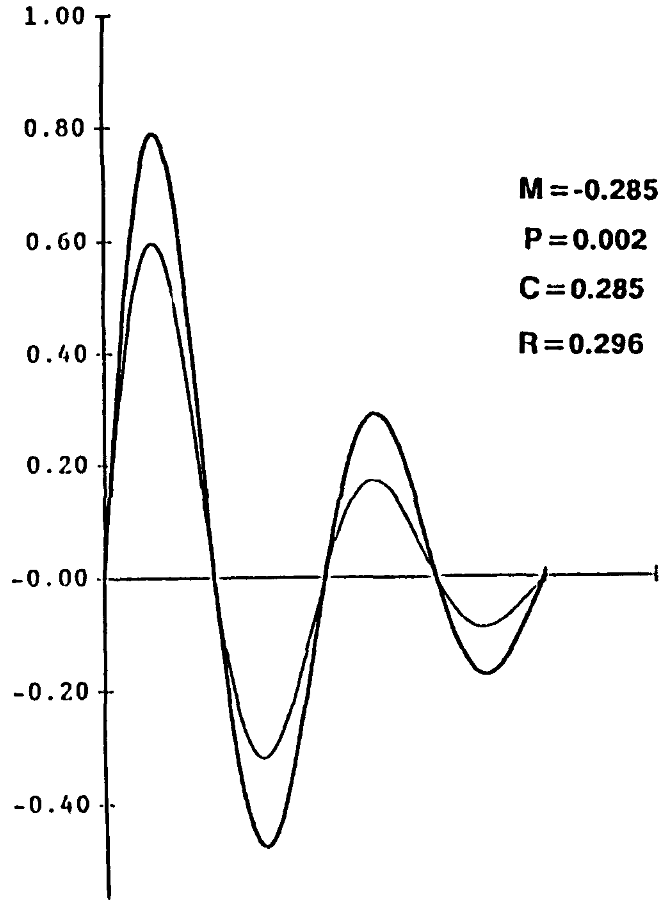

$m(t) = e^{-t} \sin 2\pi t$
$c(t) = 0.8\, e^{-t/0.8} \sin 2\pi t$
m(t)
c(t)
$e_m = -27\%$
$e_p = 1\%$
$e_c = 28\%$
$e_r = 34\%$

(a)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0.80
0.60
0.40
0.20
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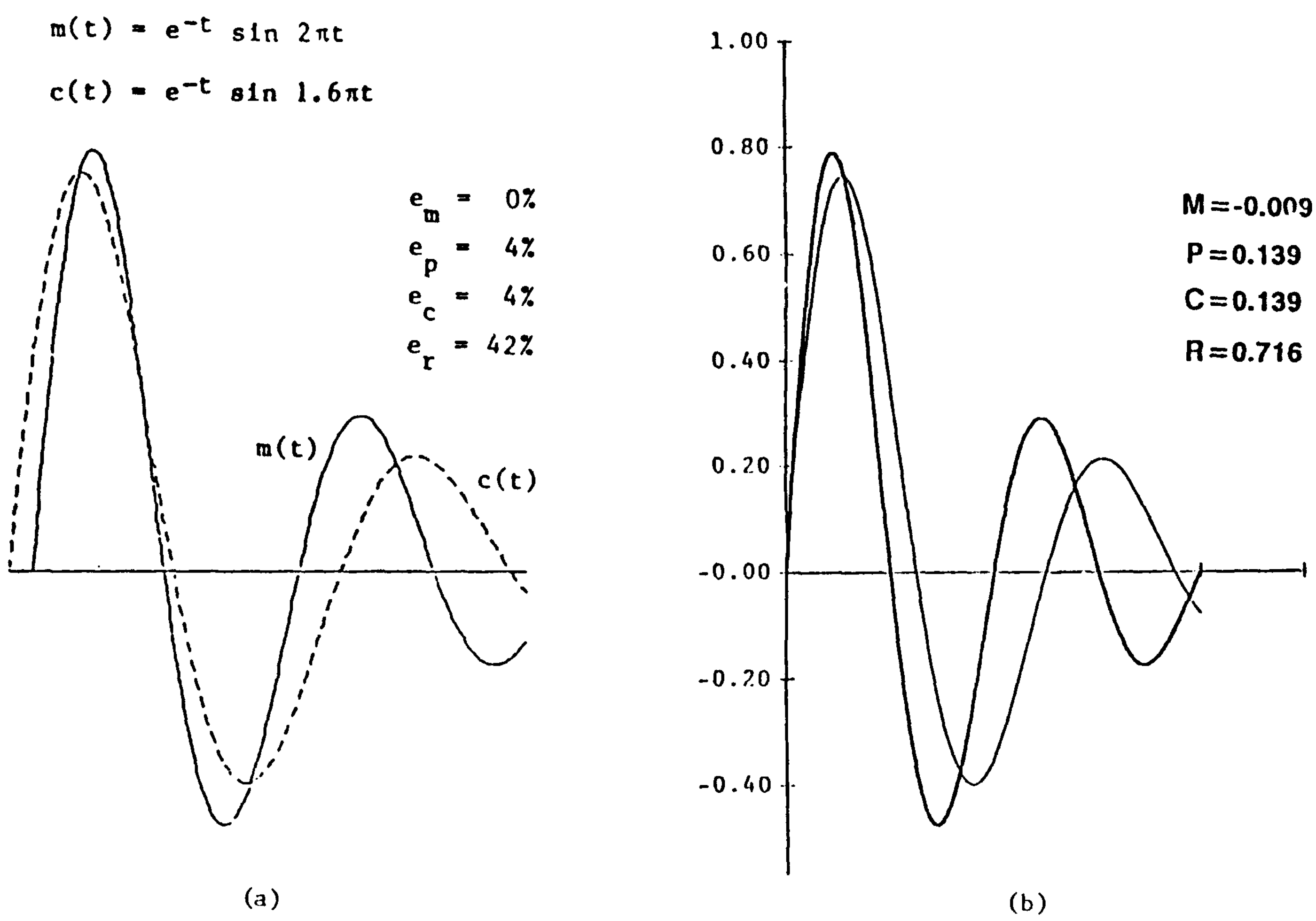
P = 0.002
C = 0.285
R = 0.296

(b)

Figure 1.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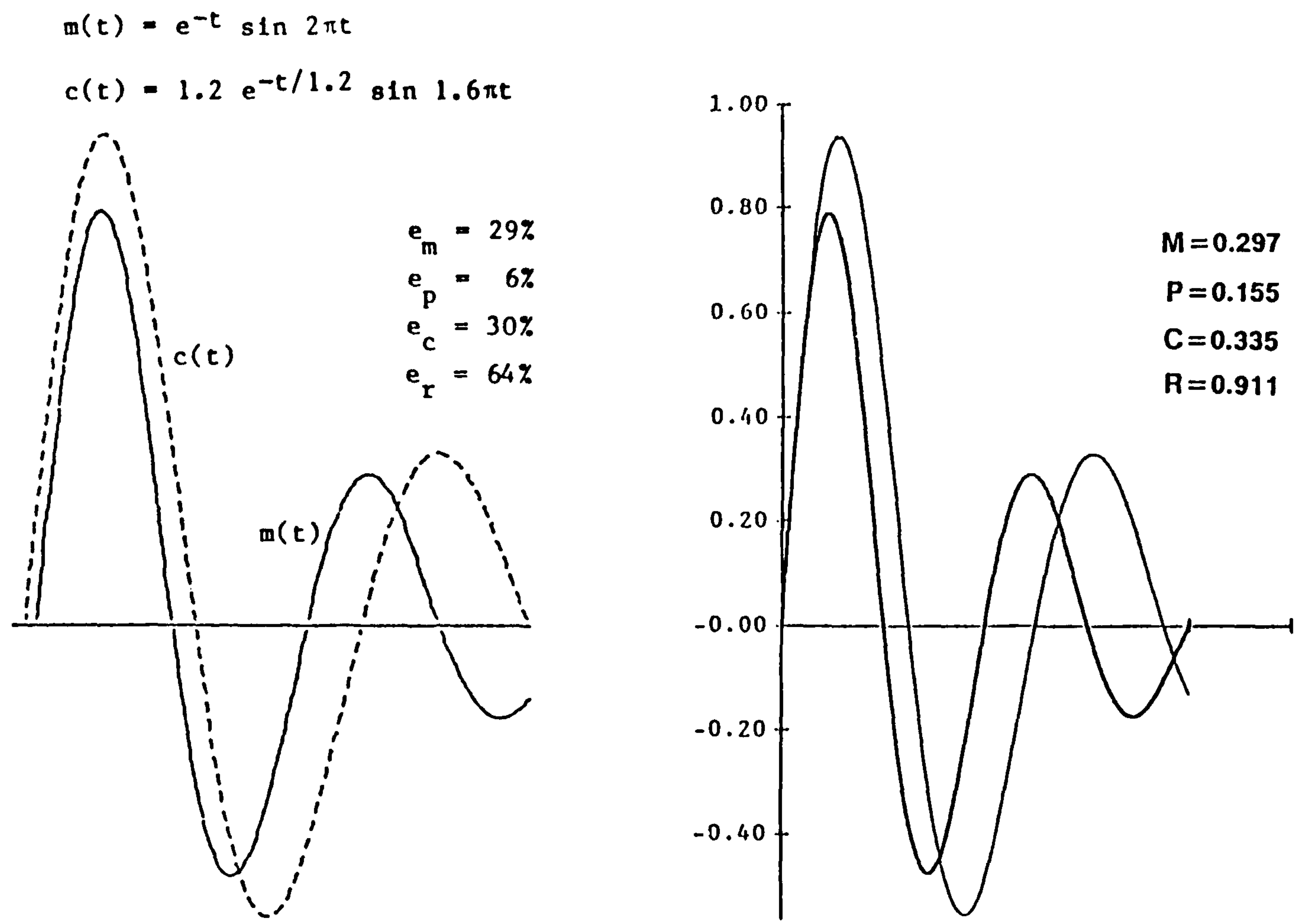
c(t) = e^-t sin 1.6πt
e_m = 0%
e_p = 4%
e_c = 4%
e_r = 42%
m(t)
c(t)
(a)
1.00
0.80
0.60
0.40
0.20
-0.00
-0.20
-0.40
M=-0.009
P=0.139
C=0.139
R=0.716
(b)

Figure 2.


m(t) = e^{-t} sin 2πt
c(t) = 1.2 e^{-t/1.2} sin 1.6πt
c(t)
m(t)
e_m = 29%
e_p = 6%
e_c = 30%
e_r = 64%
1.00
0.80
0.60
0.40
0.20
-0.00
-0.20
-0.40
M=0.297
P=0.155
C=0.335
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(a)

(b)

Figure 3.

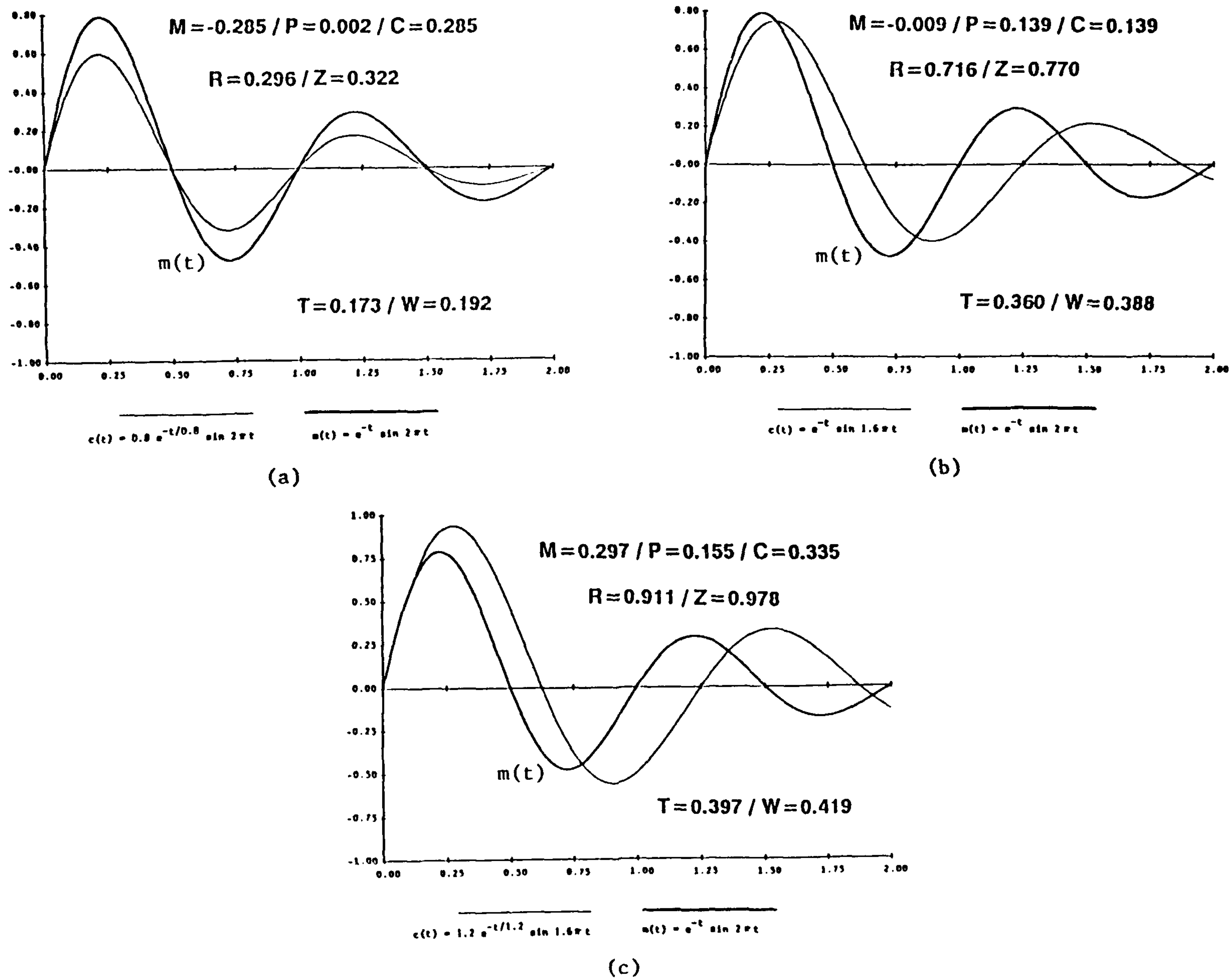

M=-0.285 / P=0.002 / C=0.285
R=0.296 / Z=0.322
m(t)
T=0.173 / W=0.192
c(t) = 0.8 e^-t/0.8 sin 2πt
m(t) = e^-t sin 2πt
(a)
M=-0.009 / P=0.139 / C=0.139
R=0.716 / Z=0.770
m(t)
T=0.360 / W=0.388
c(t) = e^-t sin 1.6πt
m(t) = e^-t sin 2πt
(b)
M=0.297 / P=0.155 / C=0.335
R=0.911 / Z=0.978
m(t)
T=0.397 / W=0.419
c(t) = 1.2 e^-t/1.2 sin 1.6πt
m(t) = e^-t sin 2πt
(c)

Figure 4.

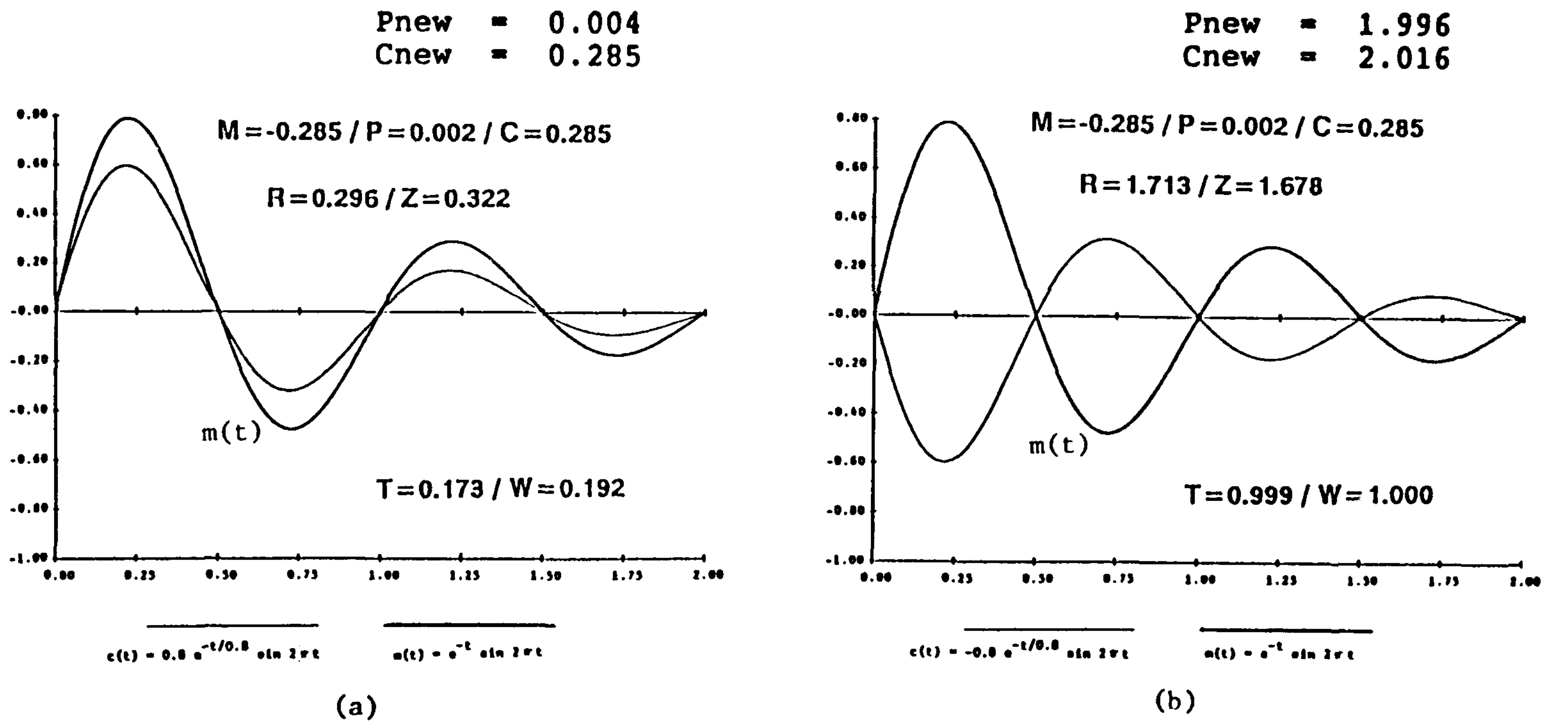
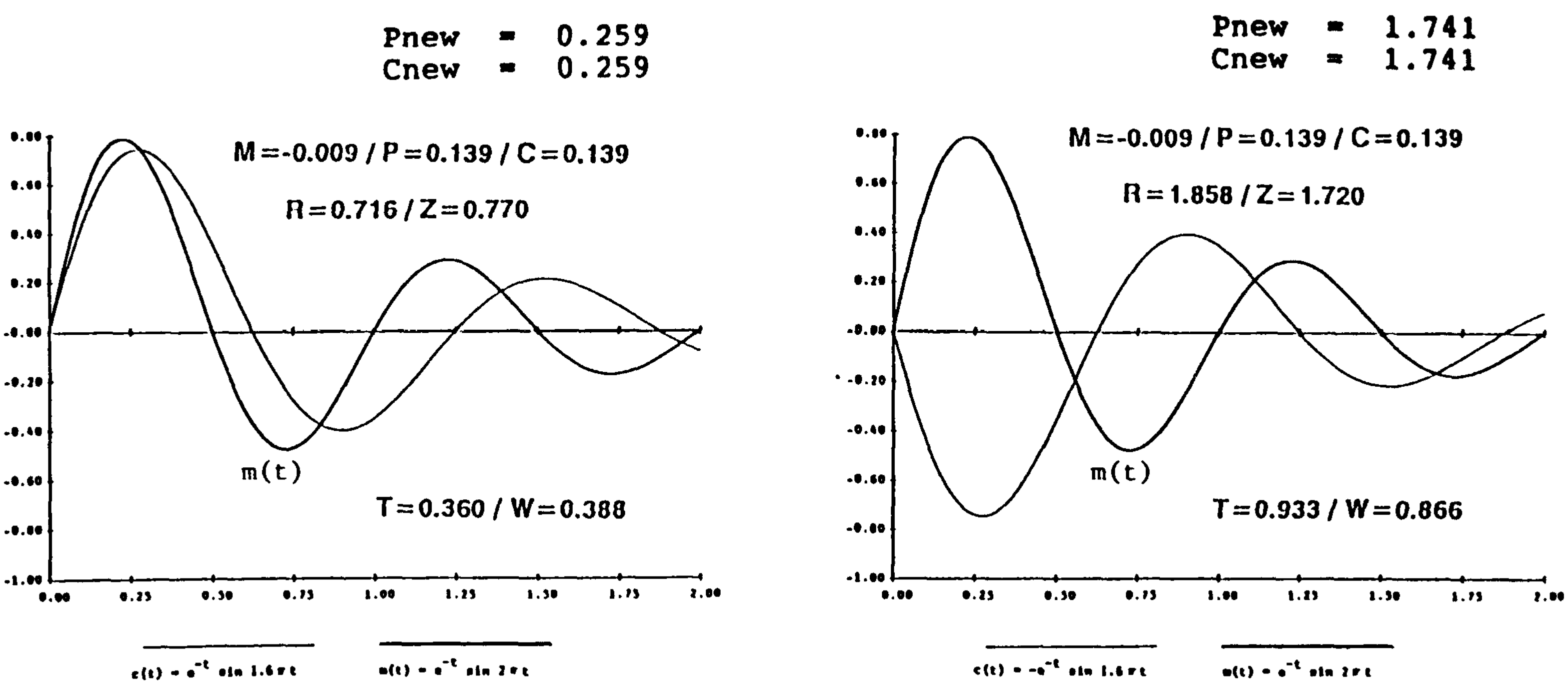

Pnew = 0.004
Cnew = 0.285
M=-0.285 / P=0.002 / C=0.285
R=0.296 / Z=0.322
m(t)
T=0.173 / W=0.192
c(t) = 0.8 e^-t/0.8 sin 2πt
m(t) = e^-t sin 2πt
(a)
Pnew = 1.996
Cnew = 2.016
M=-0.285 / P=0.002 / C=0.285
R=1.713 / Z=1.678
m(t)
T=0.999 / W=1.000
c(t) = -0.8 e^-t/0.8 sin 2πt
m(t) = e^-t sin 2πt
(b)

Figure 5.



Pnew = 0.259
Cnew = 0.259
M=-0.009 / P=0.139 / C=0.139
R=0.716 / Z=0.770
m(t)
T=0.360 / W=0.388
Pnew = 1.741
Cnew = 1.741
M=-0.009 / P=0.139 / C=0.139
R=1.858 / Z=1.720
m(t)
T=0.933 / W=0.866

(a) (b)

Figure 6.

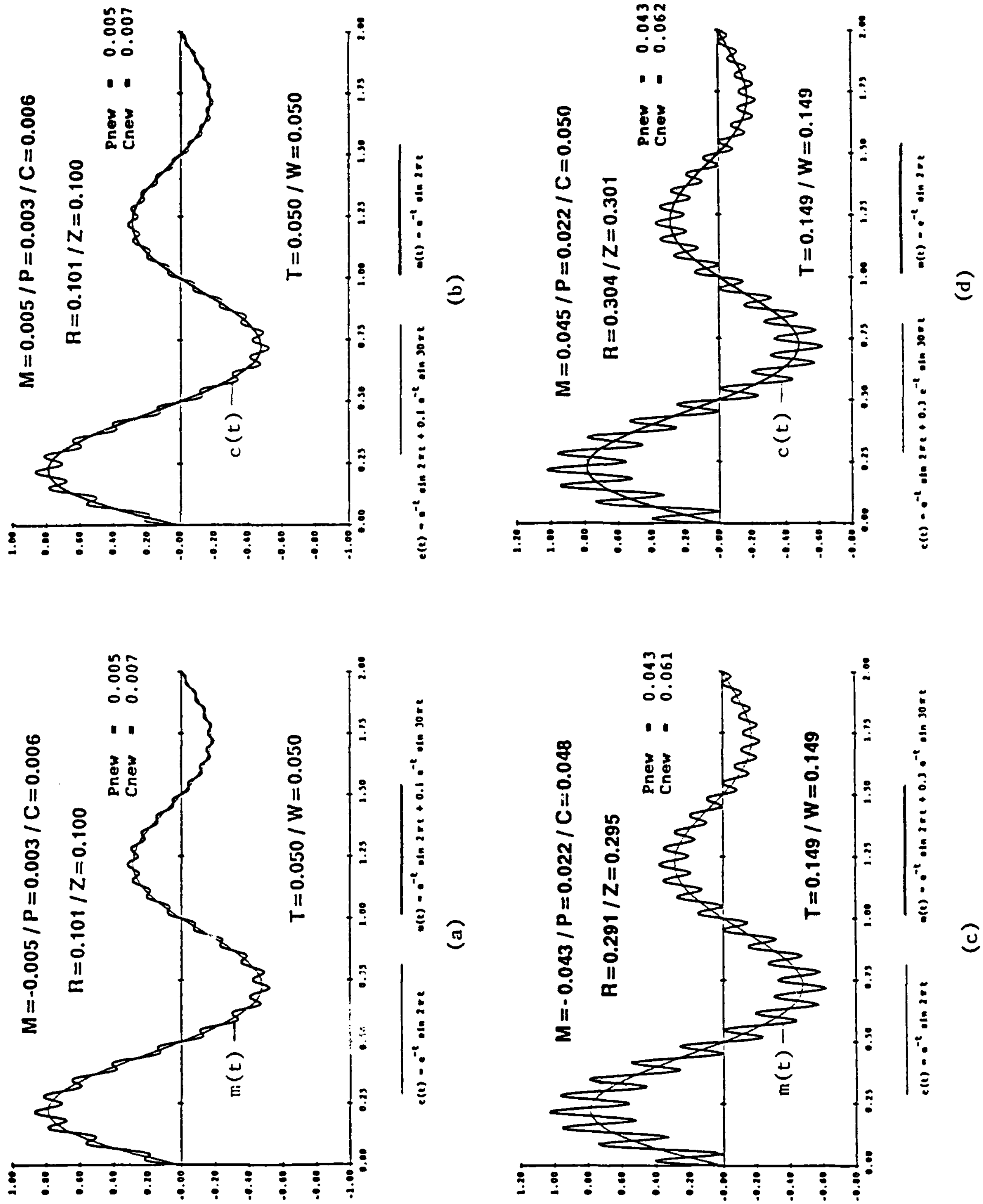

M=-0.005 / P=0.003 / C=0.006
R=0.101 / Z=0.100
Pnew = 0.005
Cnew = 0.007
m(t)
T=0.050 / W=0.050
(a)
M=0.005 / P=0.003 / C=0.006
R=0.101 / Z=0.100
Pnew = 0.005
Cnew = 0.007
c(t)
T=0.050 / W=0.050
(b)
M=-0.043 / P=0.022 / C=0.048
R=0.291 / Z=0.295
Pnew = 0.043
Cnew = 0.061
m(t)
T=0.149 / W=0.149
(c)
M=0.045 / P=0.022 / C=0.050
R=0.304 / Z=0.301
Pnew = 0.043
Cnew = 0.062
c(t)
T=0.149 / W=0.149
(d)

Figure 7.

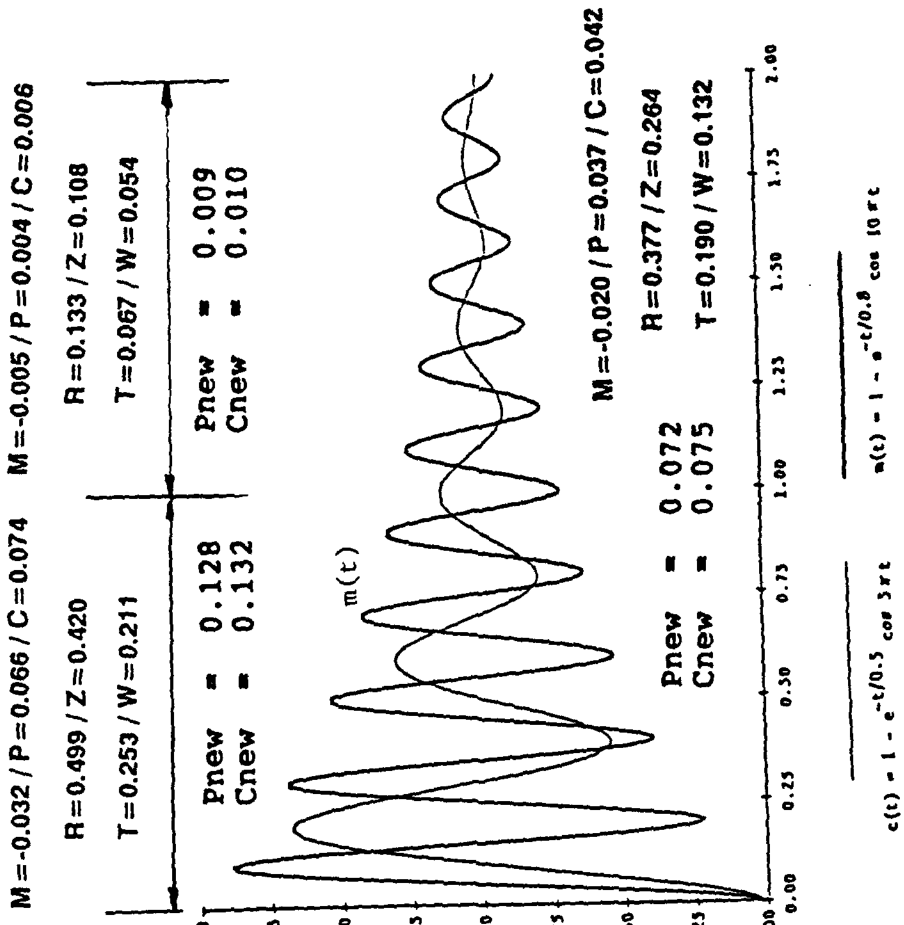
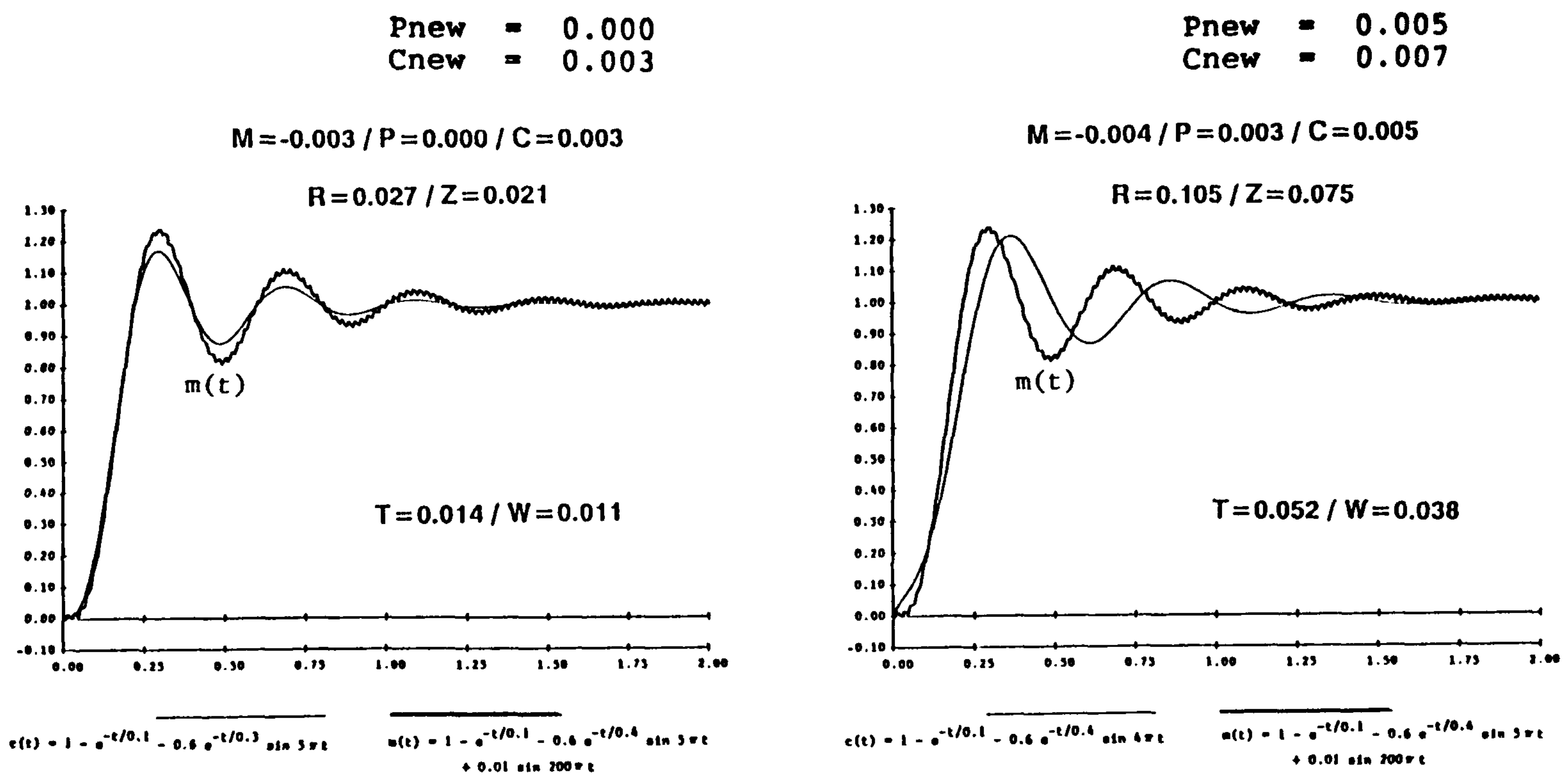

M=-0.032 / P=0.066 / C=0.074
R=0.499 / Z=0.420
T=0.253 / W=0.211
M=-0.005 / P=0.004 / C=0.006
R=0.133 / Z=0.108
T=0.067 / W=0.054
Pnew = 0.128
Cnew = 0.132
Pnew = 0.009
Cnew = 0.010
m(t)
M=-0.020 / P=0.037 / C=0.042
R=0.377 / Z=0.264
T=0.190 / W=0.132
Pnew = 0.072
Cnew = 0.075
c(t) = 1 - e^{-t/0.5} cos 3πt
m(t) = 1 - e^{-t/0.8} cos 10πt

Figure 8.


Pnew = 0.000
Cnew = 0.003
M=-0.003 / P=0.000 / C=0.003
R=0.027 / Z=0.021
m(t)
T=0.014 / W=0.011
Pnew = 0.005
Cnew = 0.007
M=-0.004 / P=0.003 / C=0.005
R=0.105 / Z=0.075
m(t)
T=0.052 / W=0.038

(a) (b)

Figure 9.

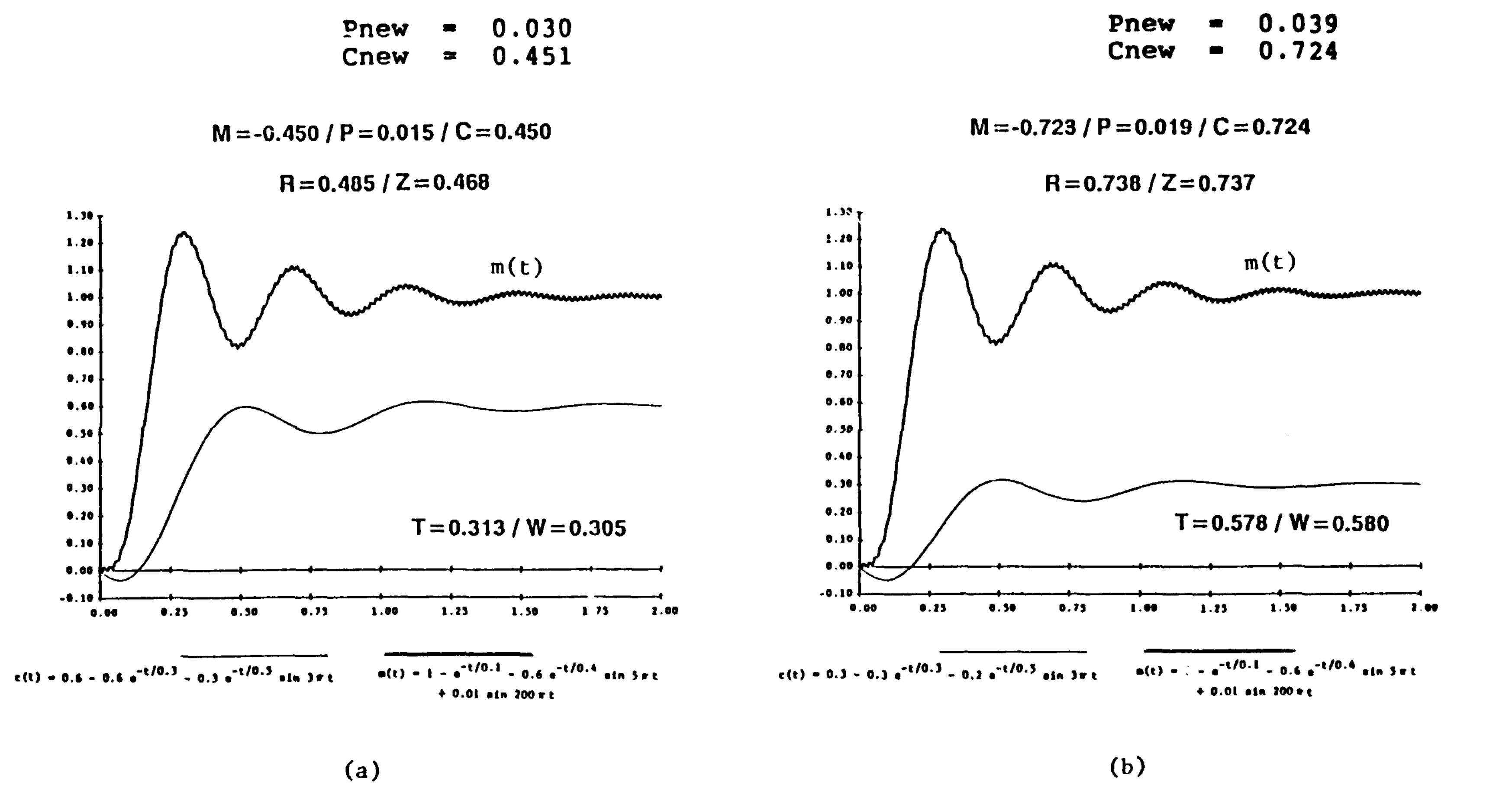
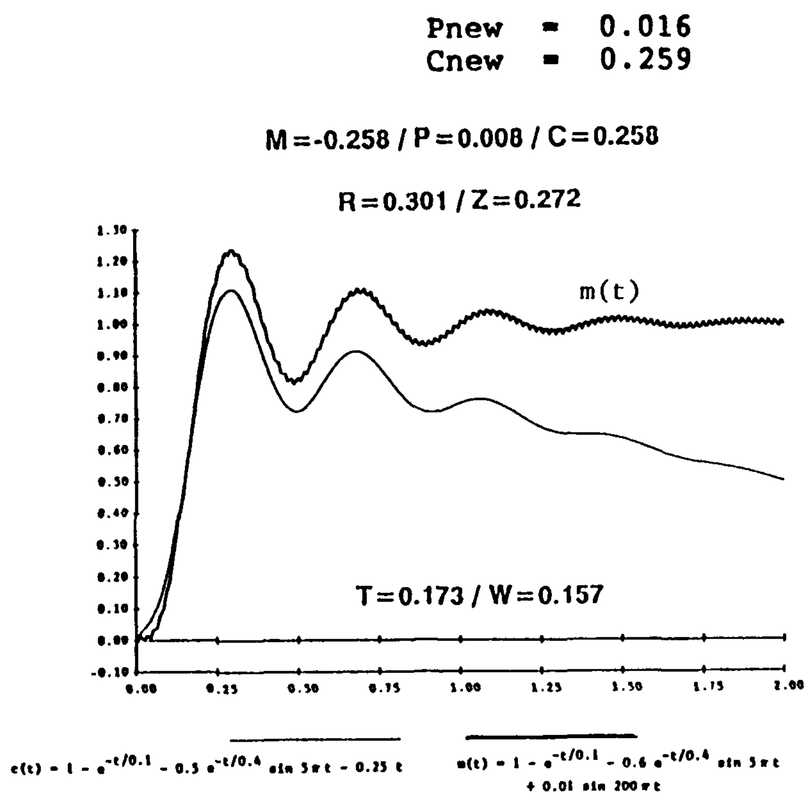
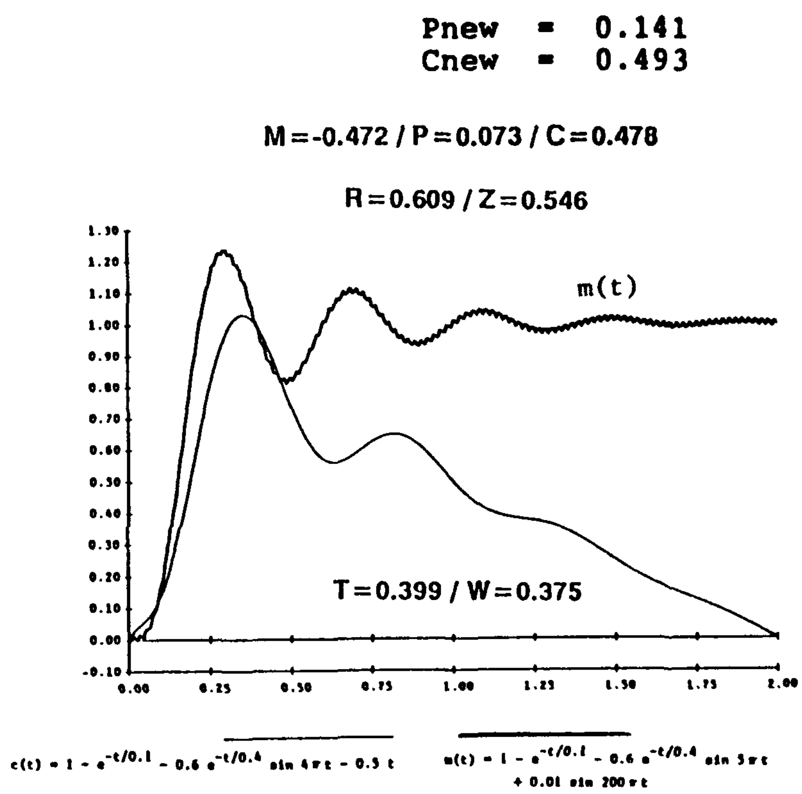

Pnew = 0.030
Cnew = 0.451
M=-0.450 / P=0.015 / C=0.450
R=0.485 / Z=0.468
m(t)
T=0.313 / W=0.305
c(t) = 0.6 - 0.6 e^-t/0.3 - 0.3 e^-t/0.5 sin 3πt
m(t) = 1 - e^-t/0.1 - 0.6 e^-t/0.4 sin 5πt + 0.01 sin 200πt
(a)
Pnew = 0.039
Cnew = 0.724
M=-0.723 / P=0.019 / C=0.724
R=0.738 / Z=0.737
m(t)
T=0.578 / W=0.580
c(t) = 0.3 - 0.3 e^-t/0.3 - 0.2 e^-t/0.5 sin 3πt
m(t) = 1 - e^-t/0.1 - 0.6 e^-t/0.4 sin 5πt + 0.01 sin 200πt
(b)

Figure 10.

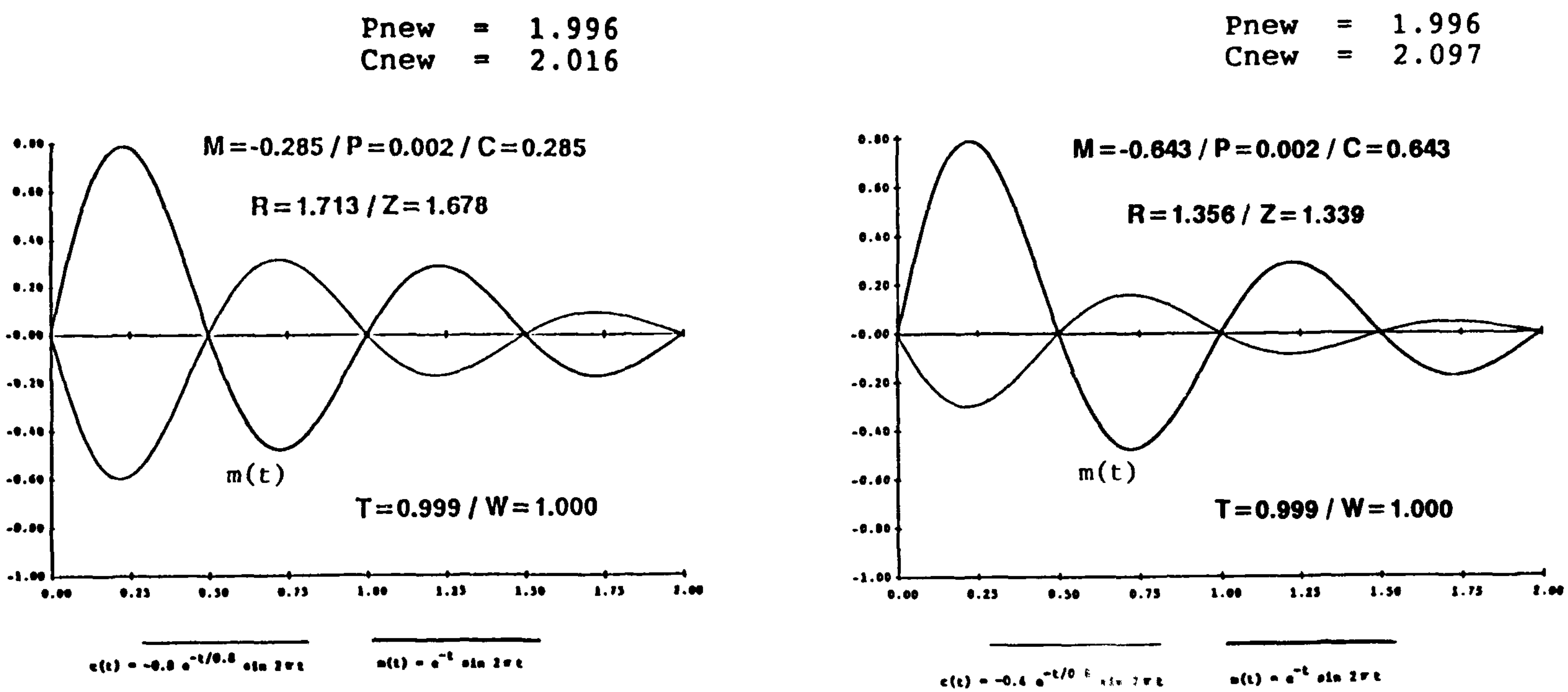

Pnew = 0.016
Cnew = 0.259
M=-0.258 / P=0.008 / C=0.258
R=0.301 / Z=0.272
m(t)
T=0.173 / W=0.157
1.30 1.20 1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0.00 -0.10
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
$c(t) = 1 - e^{-t/0.1} - 0.5\, e^{-t/0.4} \sin 5\pi t - 0.25\, t$
$m(t) = 1 - e^{-t/0.1} - 0.6\, e^{-t/0.4} \sin 5\pi t + 0.01 \sin 200\pi t$

(a)


Pnew = 0.141
Cnew = 0.493
M=-0.472 / P=0.073 / C=0.478
R=0.609 / Z=0.546
m(t)
T=0.399 / W=0.375
1.30 1.20 1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0.00 -0.10
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
$c(t) = 1 - e^{-t/0.1} - 0.6\, e^{-t/0.4} \sin 4\pi t - 0.5\, t$
$m(t) = 1 - e^{-t/0.1} - 0.6\, e^{-t/0.4} \sin 5\pi t + 0.01 \sin 200\pi t$

(b)

Figure 11.


Pnew = 1.996
Cnew = 2.016
M=-0.285 / P=0.002 / C=0.285
R=1.713 / Z=1.678
m(t)
T=0.999 / W=1.000
m(t) = e^-t sin 2πt
Pnew = 1.996
Cnew = 2.097
M=-0.643 / P=0.002 / C=0.643
R=1.356 / Z=1.339
m(t)
T=0.999 / W=1.000
m(t) = e^-t sin 2πt

(a)

(b)

Figure 12.

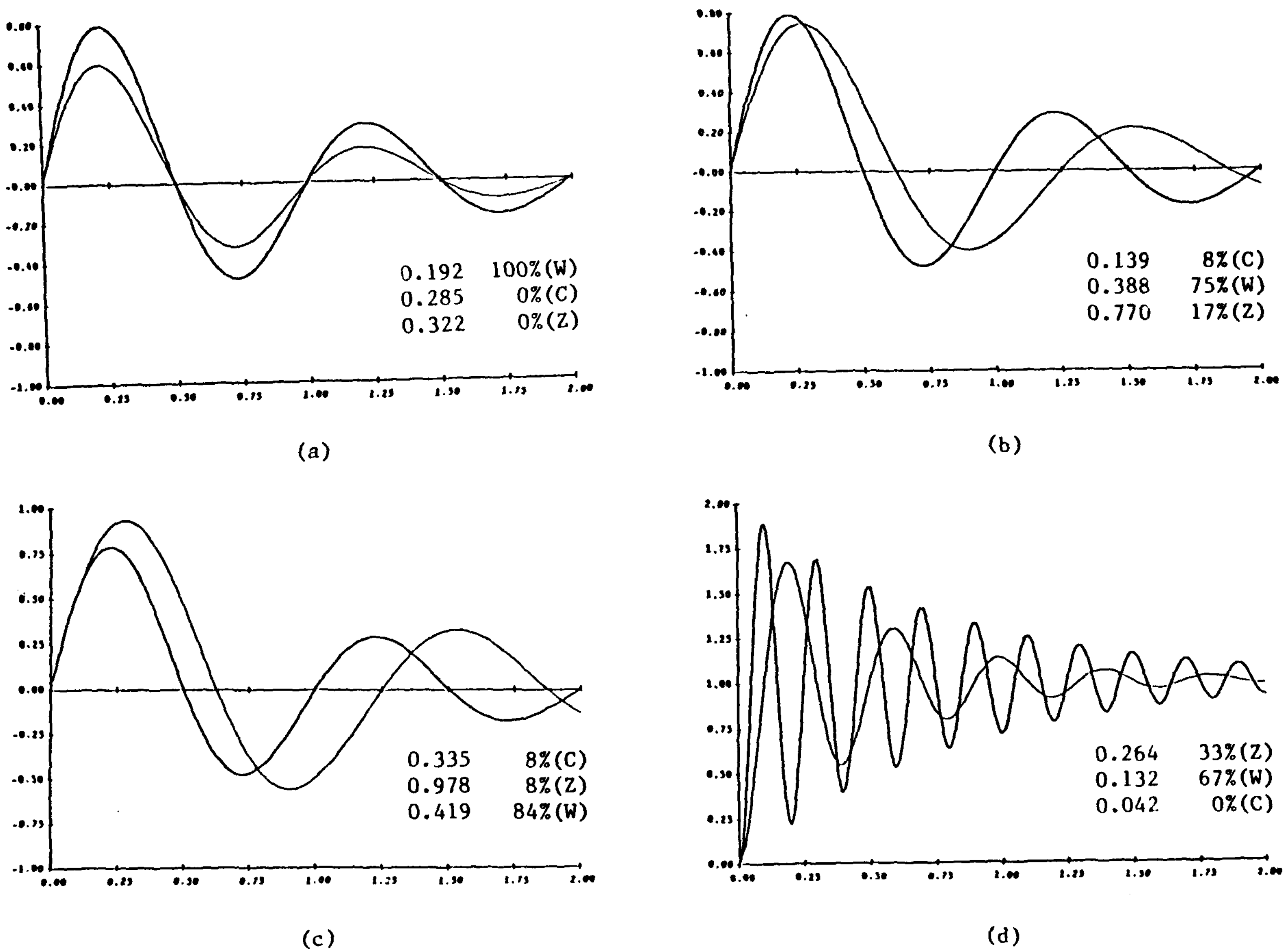

0.192 100%(W)
0.285 0%(C)
0.322 0%(Z)
(a)
0.139 8%(C)
0.388 75%(W)
0.770 17%(Z)
(b)
0.335 8%(C)
0.978 8%(Z)
0.419 84%(W)
(c)
0.264 33%(Z)
0.132 67%(W)
0.042 0%(C)
(d)

Figure 13.

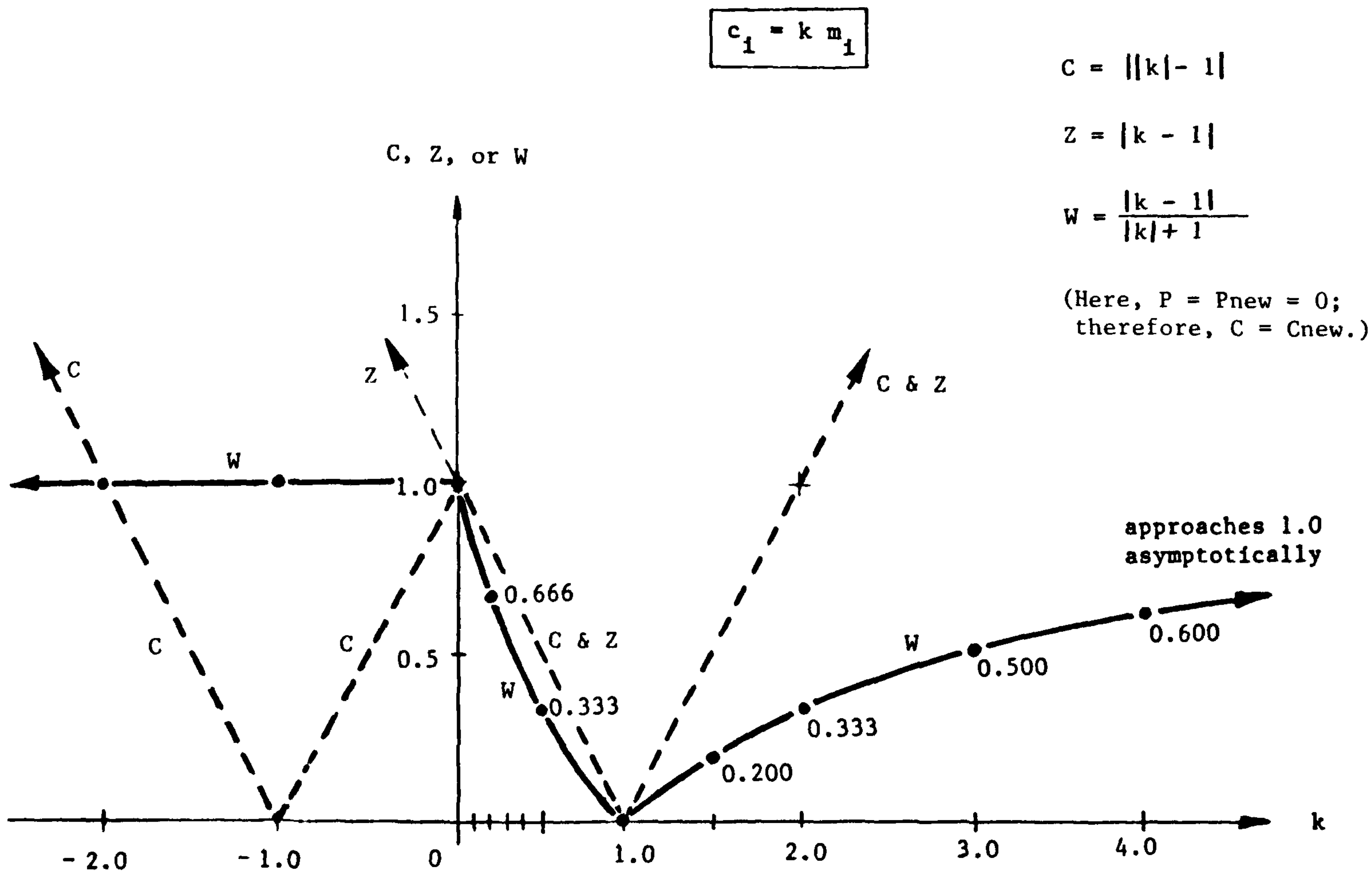

$c_i = k\,m_i$
C = ||k| - 1|
Z = |k - 1|
W = |k - 1| / (|k| + 1)
(Here, P = Pnew = 0;
therefore, C = Cnew.)
C, Z, or W
1.5
1.0
0.5
C
Z
W
C & Z
0.666
0.333
0.200
0.333
0.500
0.600
approaches 1.0
asymptotically
k
- 2.0
- 1.0
0
1.0
2.0
3.0
4.0

Figure 14.

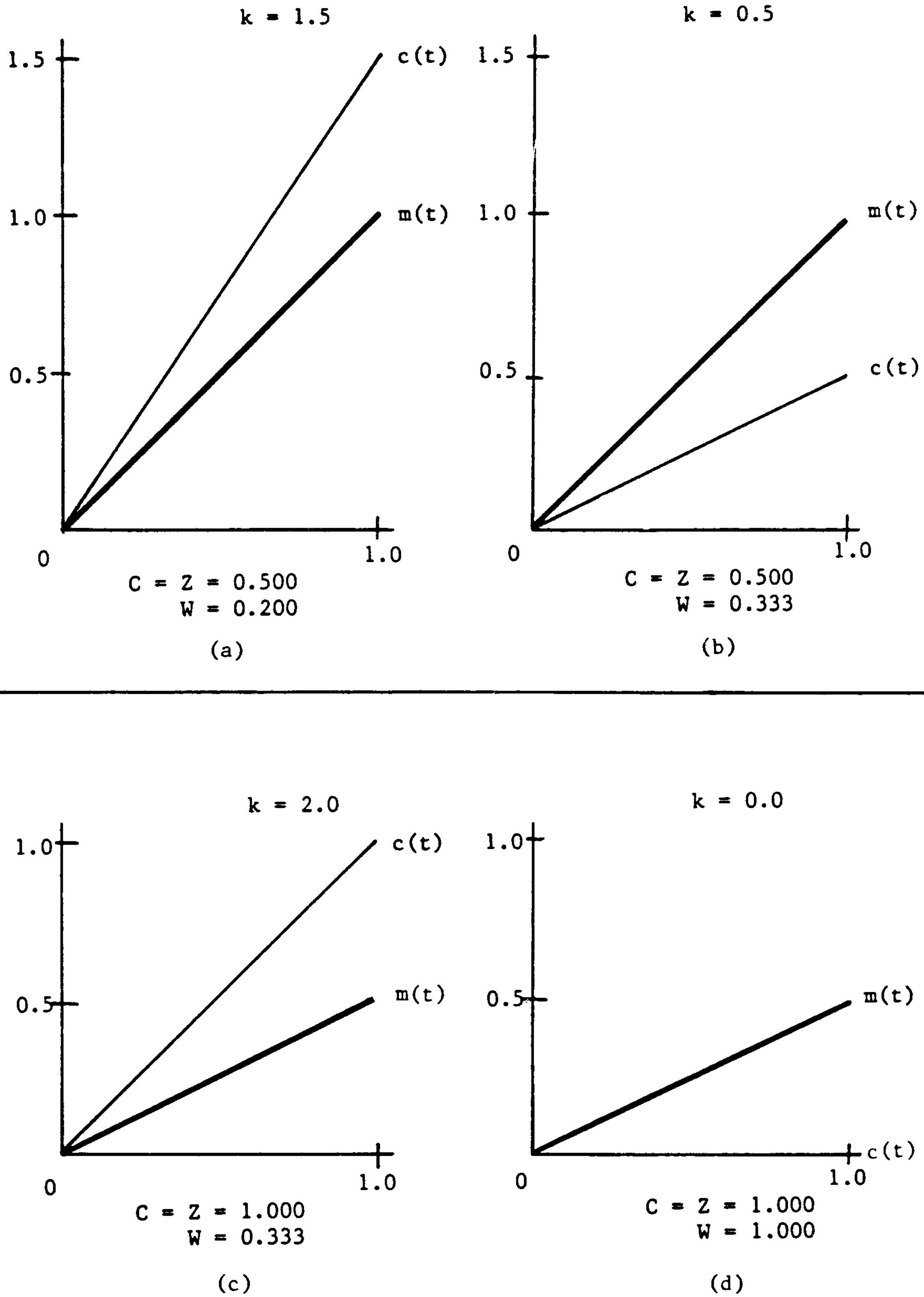
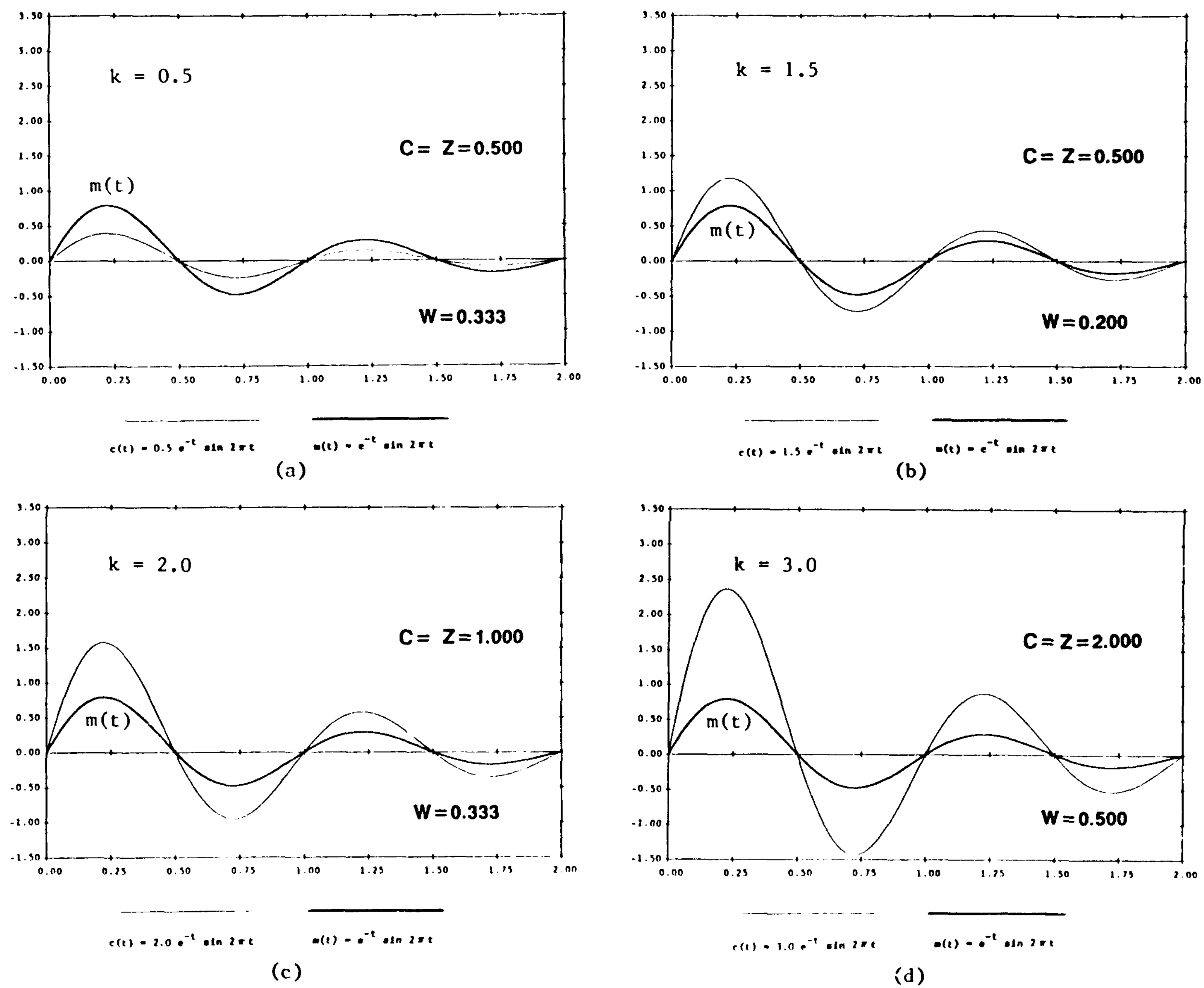

k = 1.5
c(t)
m(t)
1.5
1.0
0.5
0
1.0
C = Z = 0.500
W = 0.200
(a)
k = 0.5
m(t)
c(t)
1.5
1.0
0.5
0
1.0
C = Z = 0.500
W = 0.333
(b)
k = 2.0
c(t)
m(t)
1.0
0.5
0
1.0
C = Z = 1.000
W = 0.333
(c)
k = 0.0
m(t)
c(t)
1.0
0.5
0
1.0
C = Z = 1.000
W = 1.000
(d)

Figure 15.



k = 0.5
m(t)
C= Z=0.500
W=0.333
c(t) = 0.5 e^-t sin 2πt
m(t) = e^-t sin 2πt
(a)
k = 1.5
m(t)
C= Z=0.500
W=0.200
c(t) = 1.5 e^-t sin 2πt
m(t) = e^-t sin 2πt
(b)
k = 2.0
m(t)
C= Z=1.000
W=0.333
c(t) = 2.0 e^-t sin 2πt
m(t) = e^-t sin 2πt
(c)
k = 3.0
m(t)
C= Z=2.000
W=0.500
c(t) = 3.0 e^-t sin 2πt
m(t) = e^-t sin 2πt
(d)

Figure 16.

APPENDIX A

RECTIFIED RESIDUALS FOR Z AND W

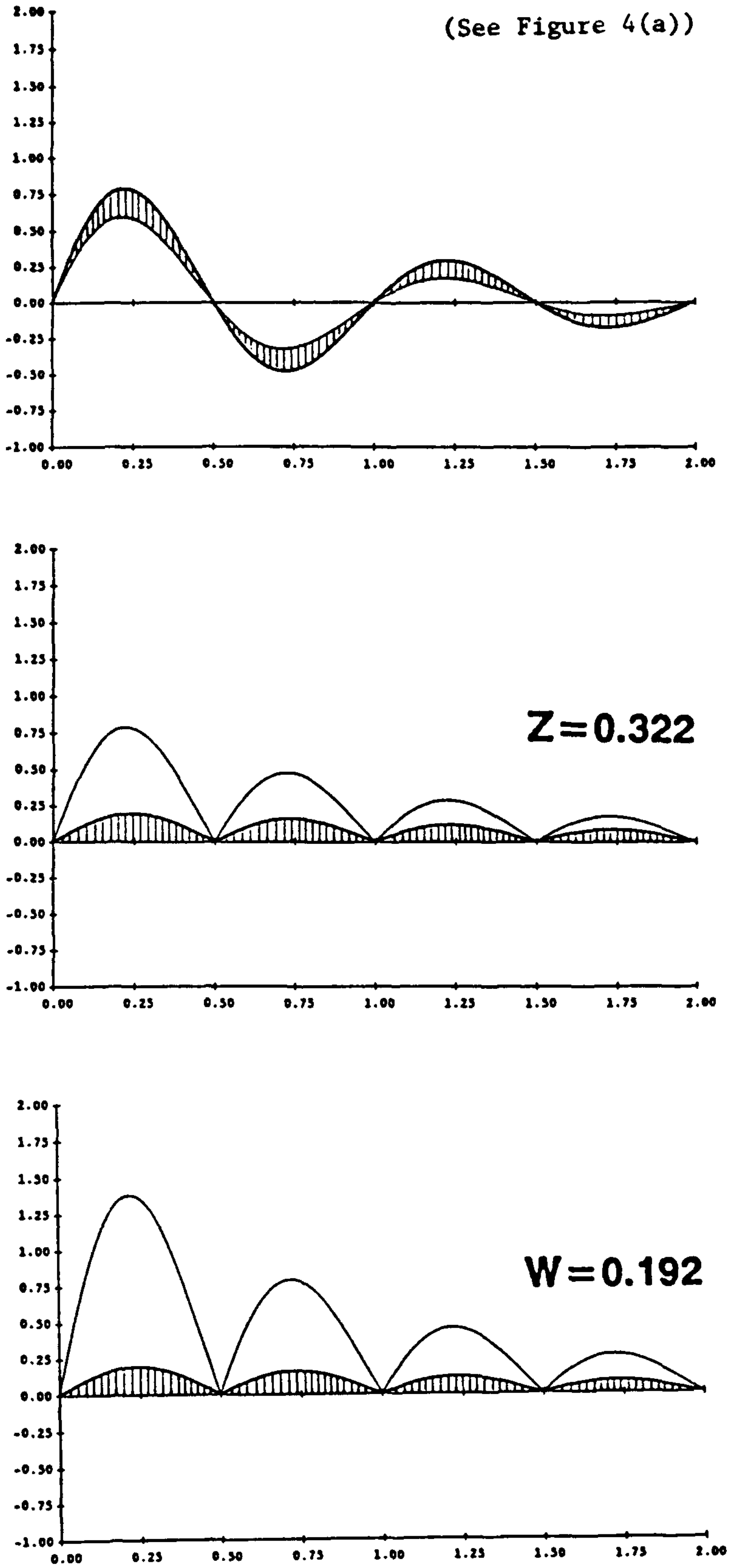


Figure A-1.

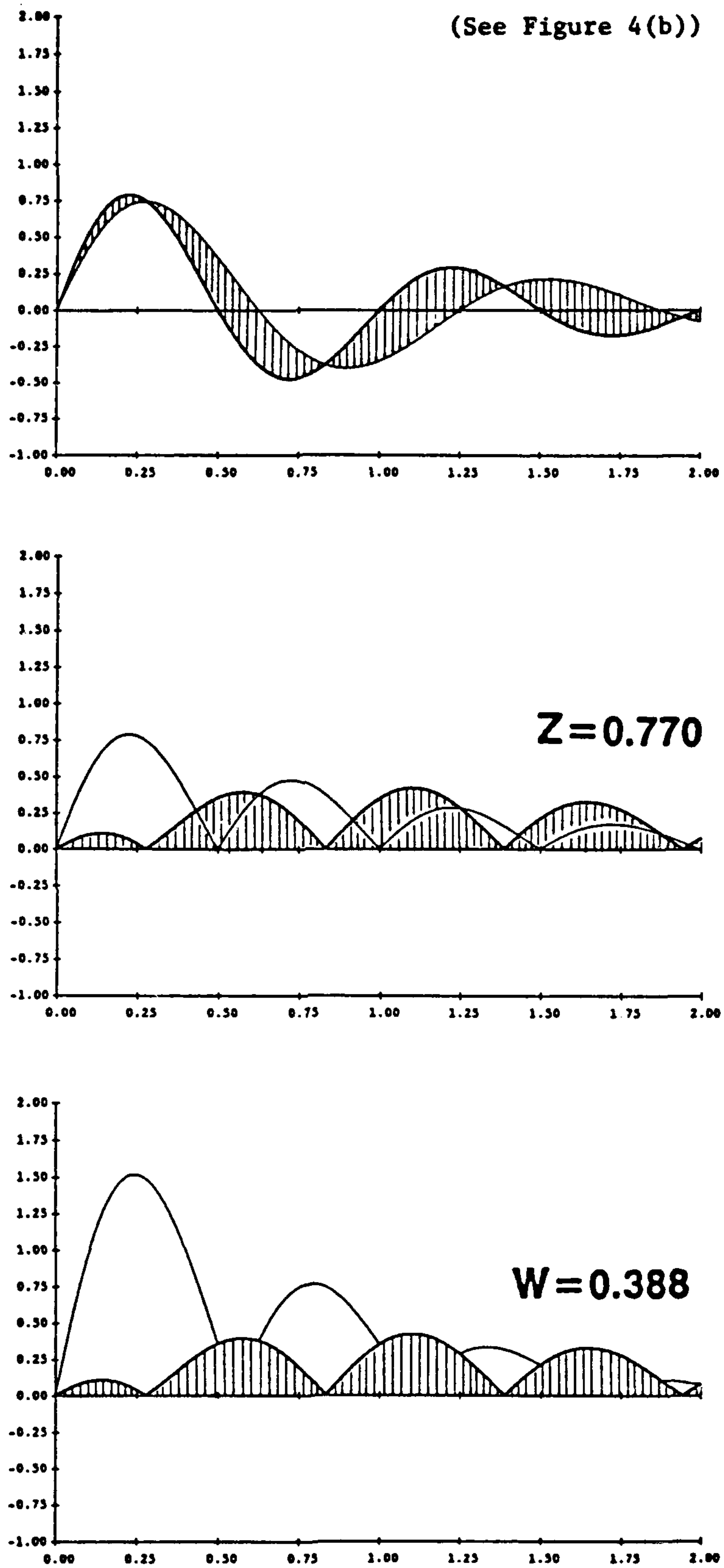

(See Figure 4(b))
Z=0.770
W=0.388
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00

Figure A-2.

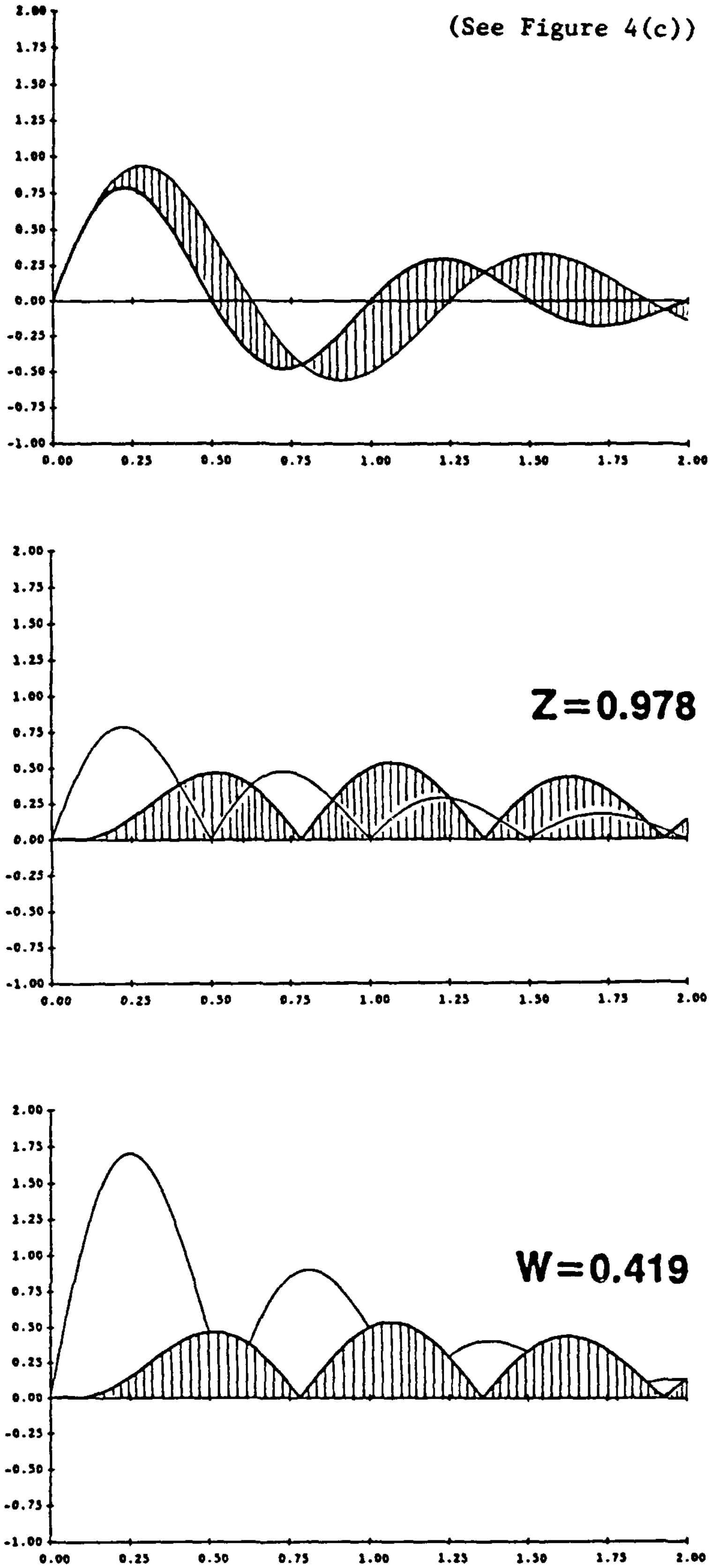


Figure A-3.

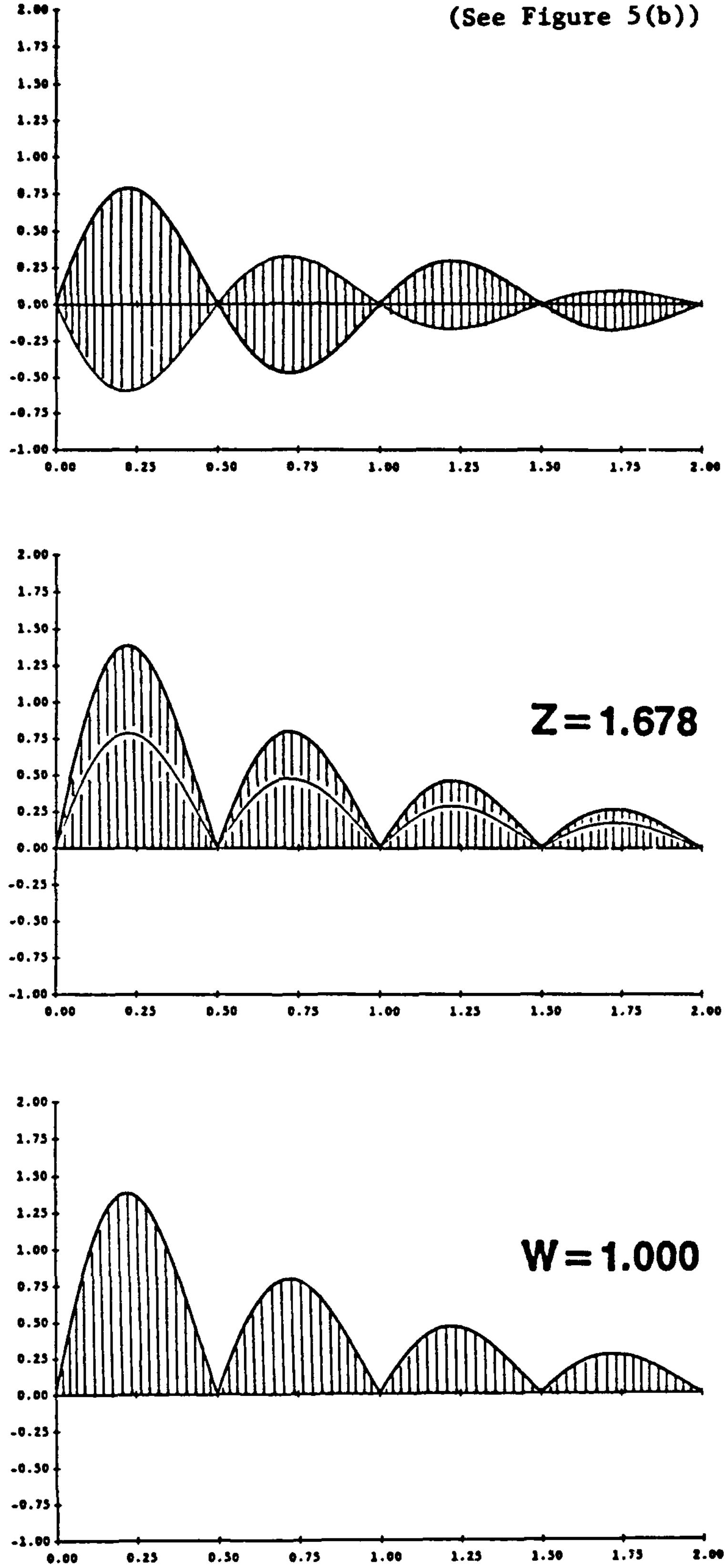


Figure A-4.

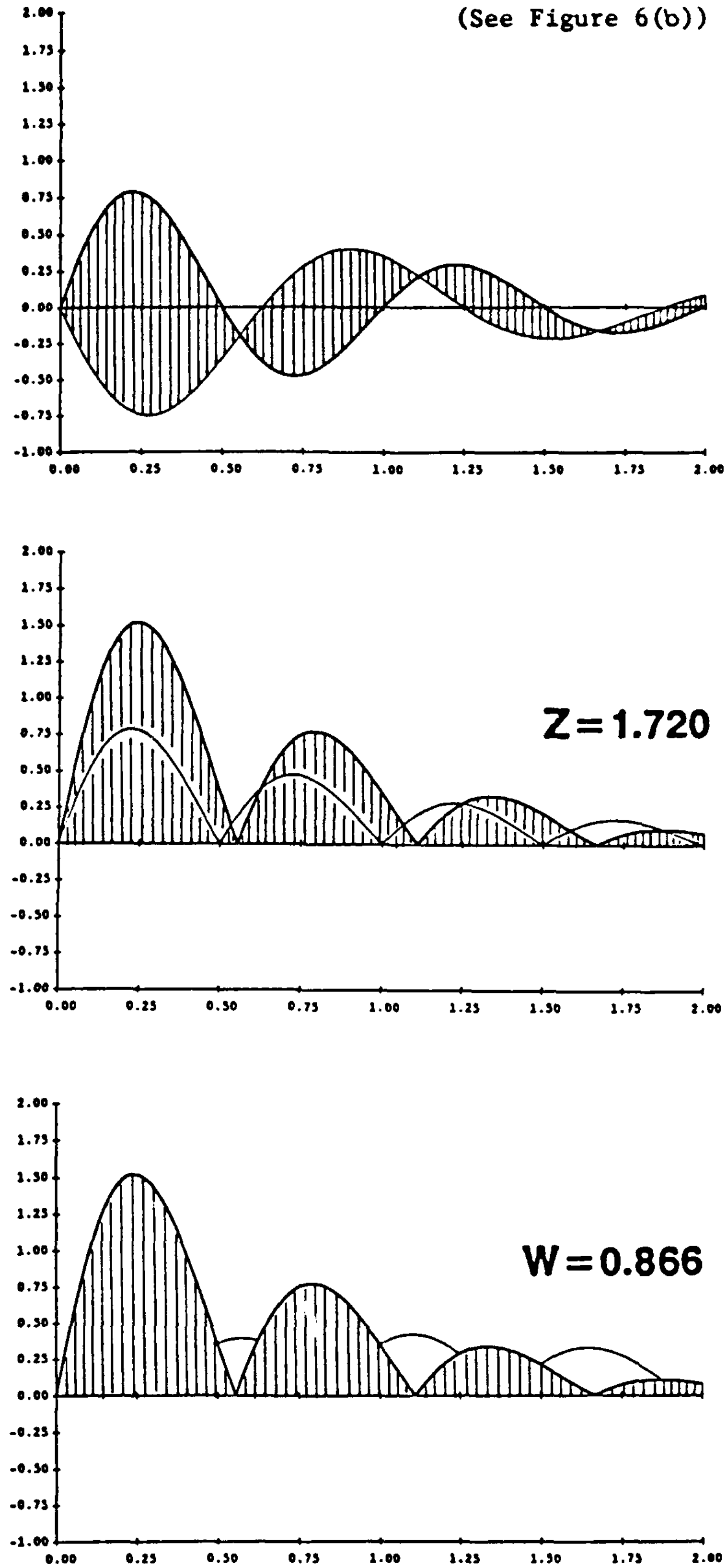

(See Figure 6(b))
Z=1.720
W=0.866
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Figure A-5.

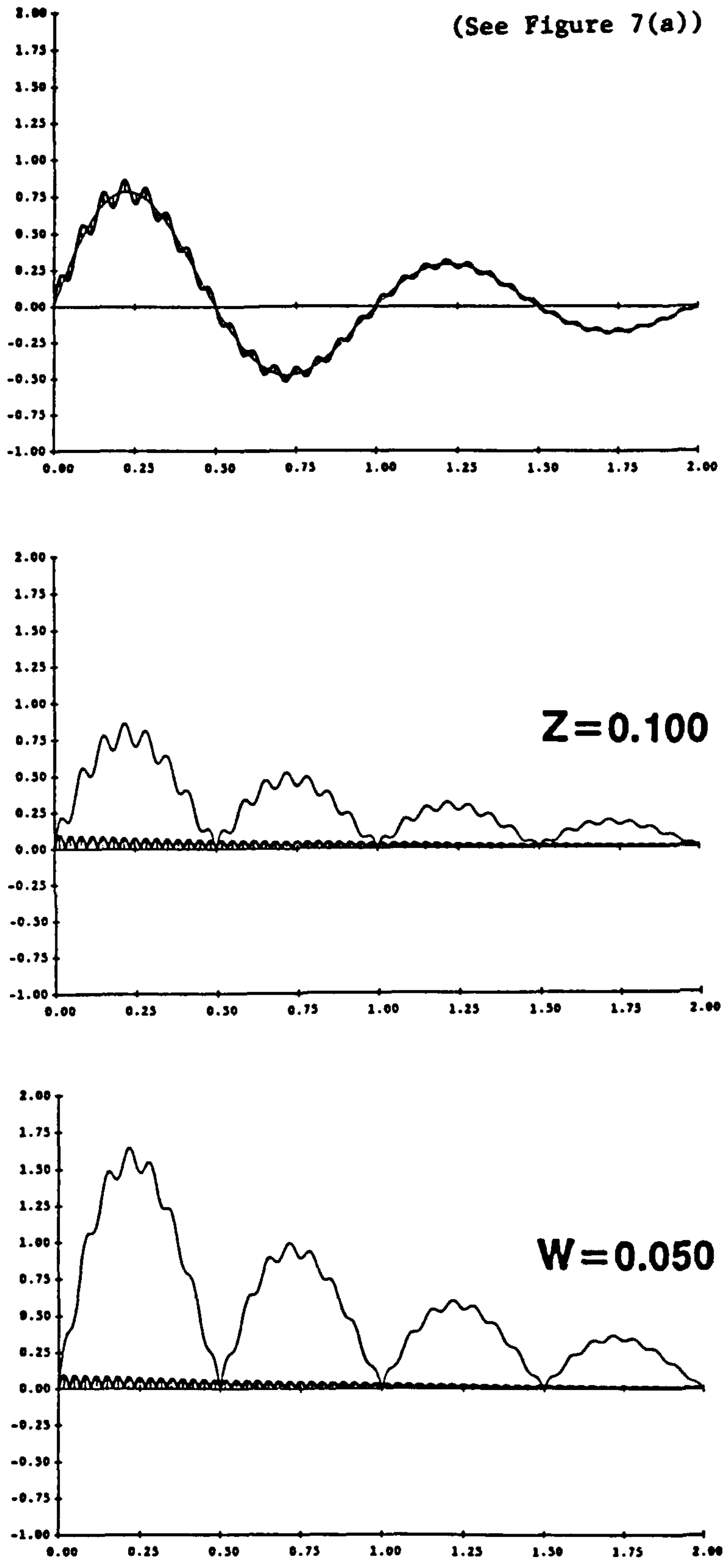


Figure A-6.

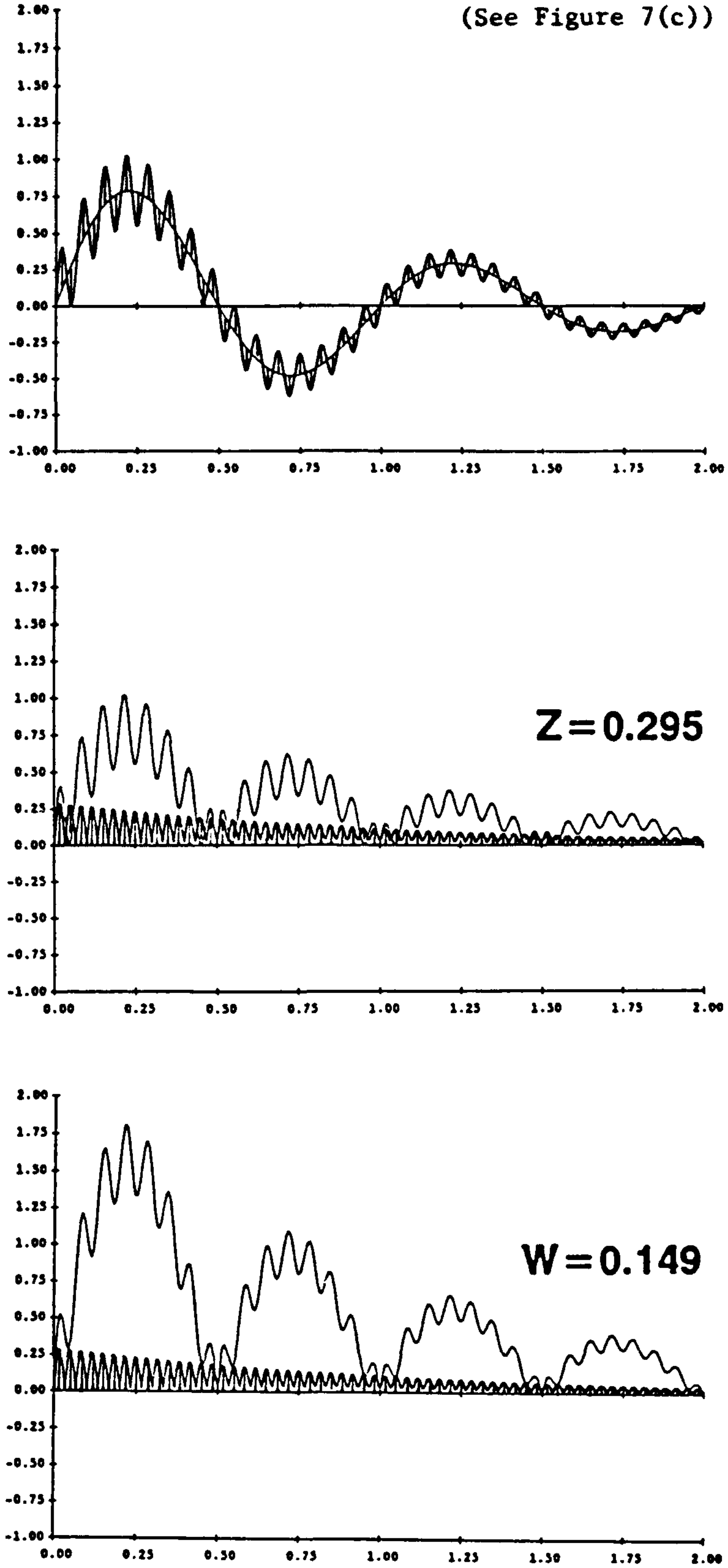

(See Figure 7(c))
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Z=0.295
W=0.149

Figure A-7.

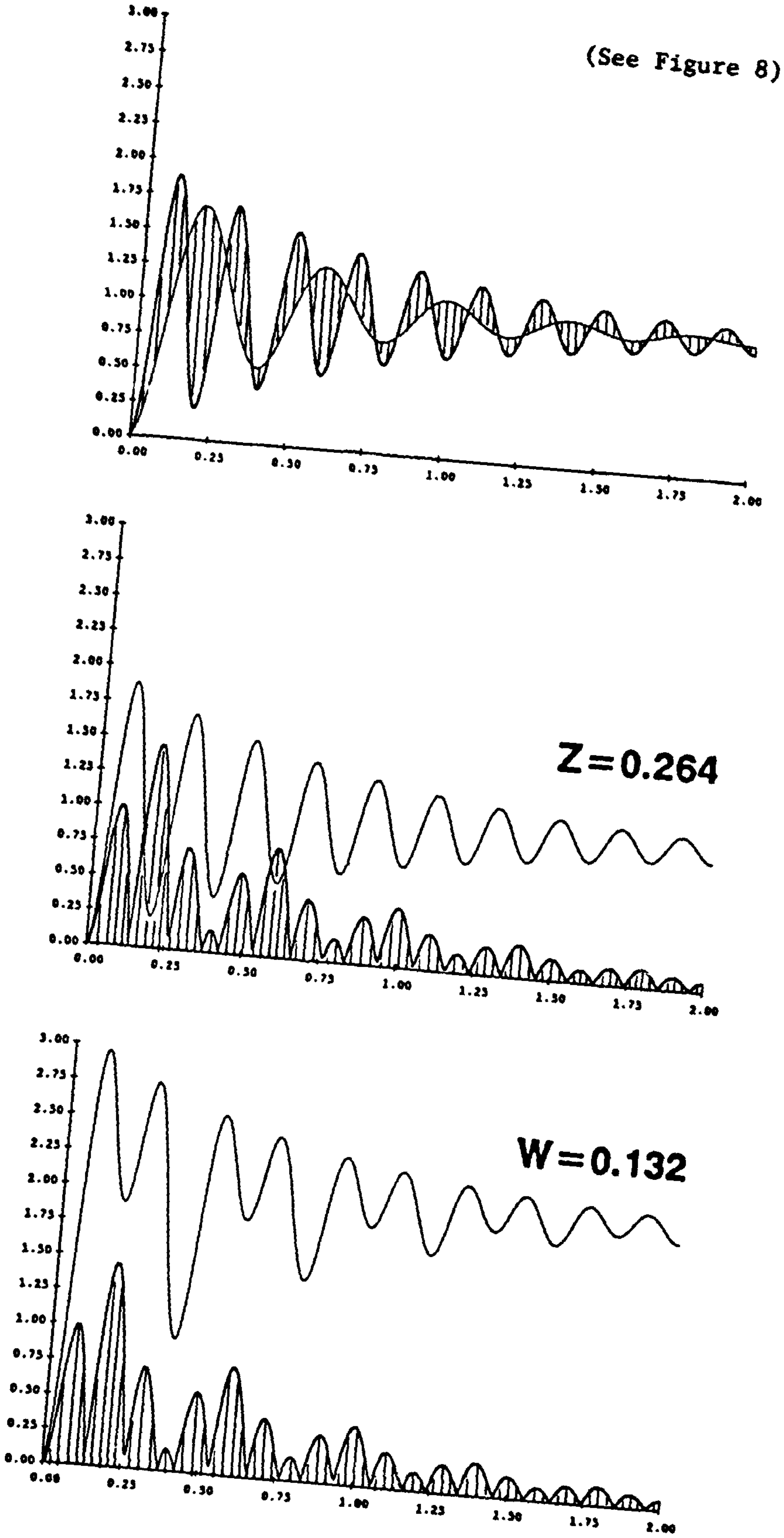


Figure A-8.

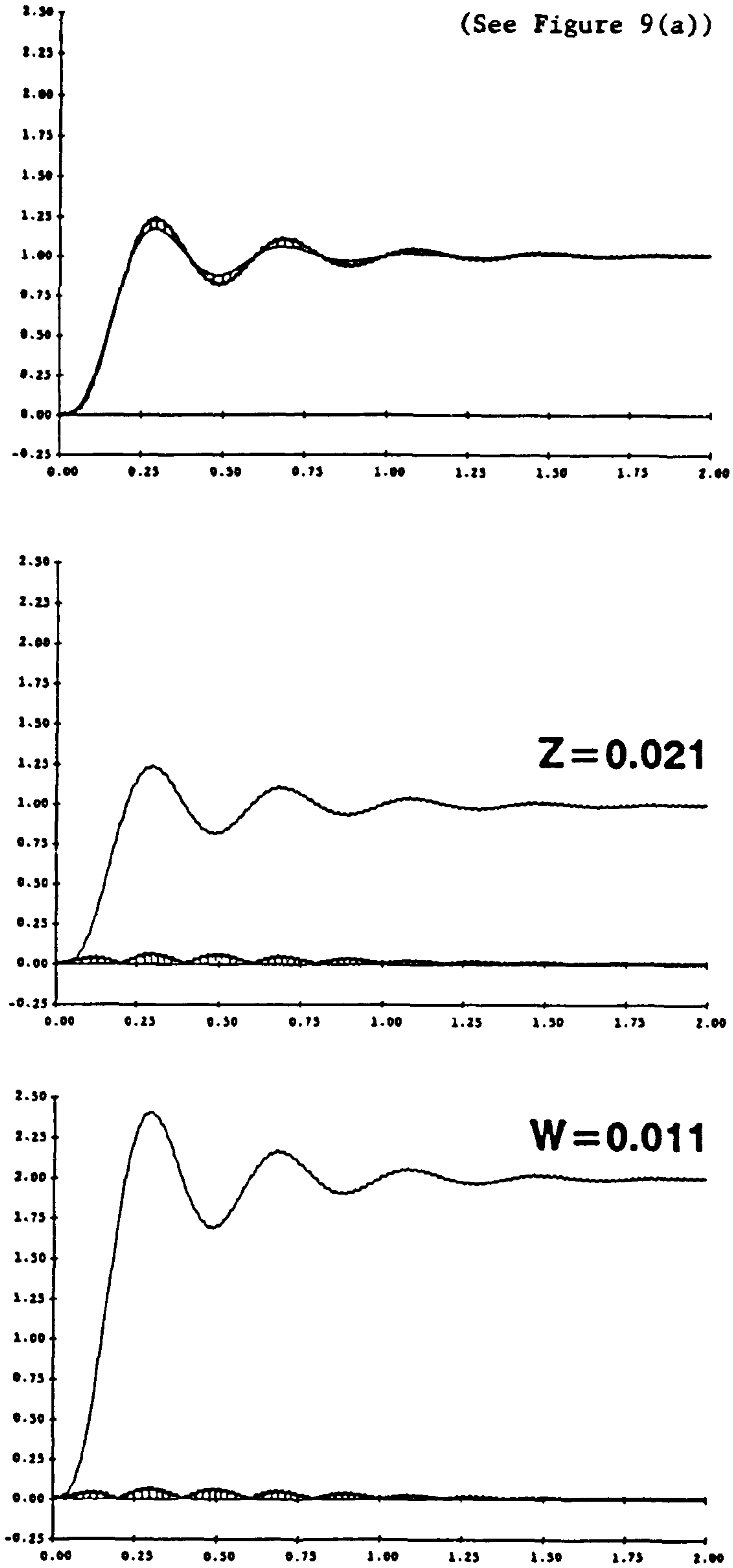

(See Figure 9(a))
Z=0.021
W=0.011
-0.25 0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00

Figure A-9.

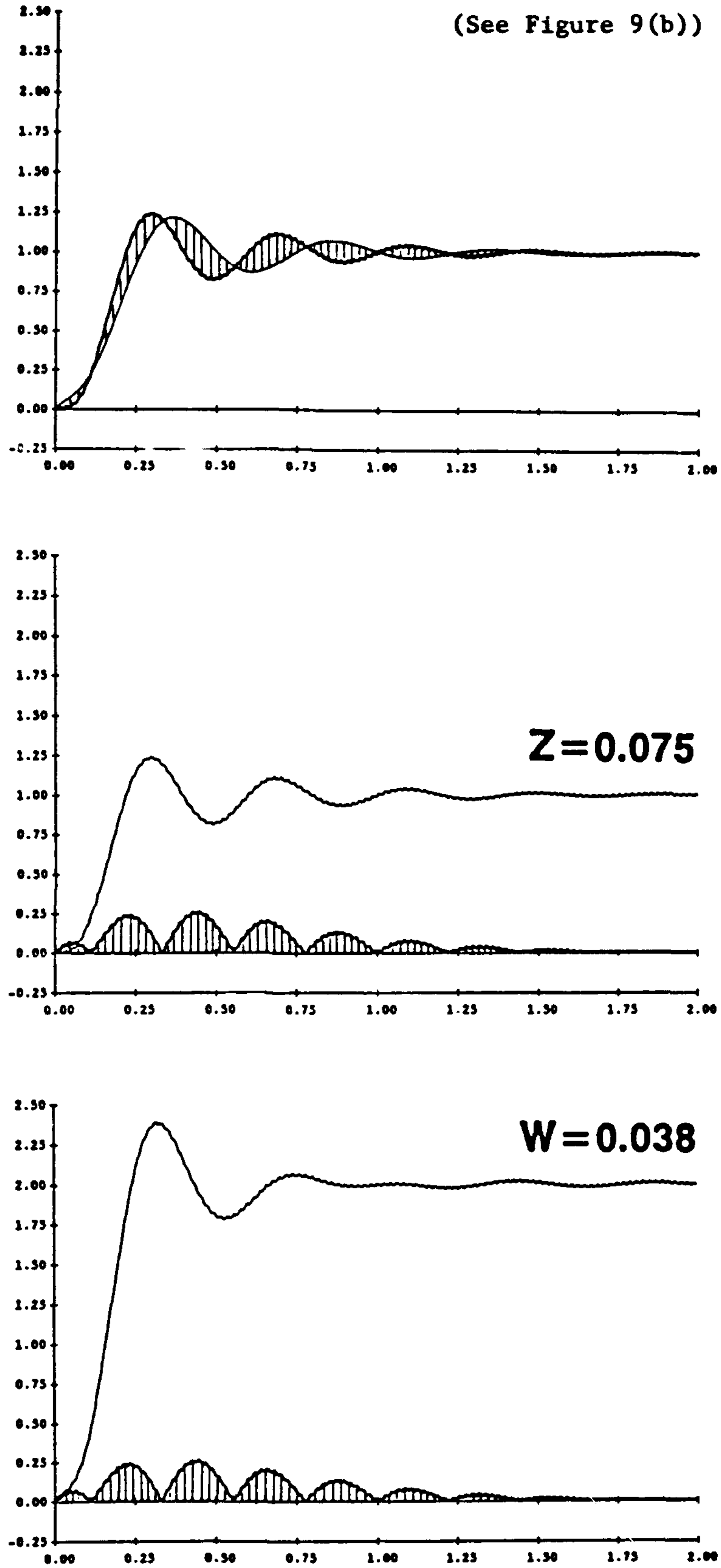

(See Figure 9(b))
Z=0.075
W=0.038
2.50 2.25 2.00 1.75 1.50 1.25 1.00 0.75 0.50 0.25 0.00 -0.25
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00

Figure A-10.

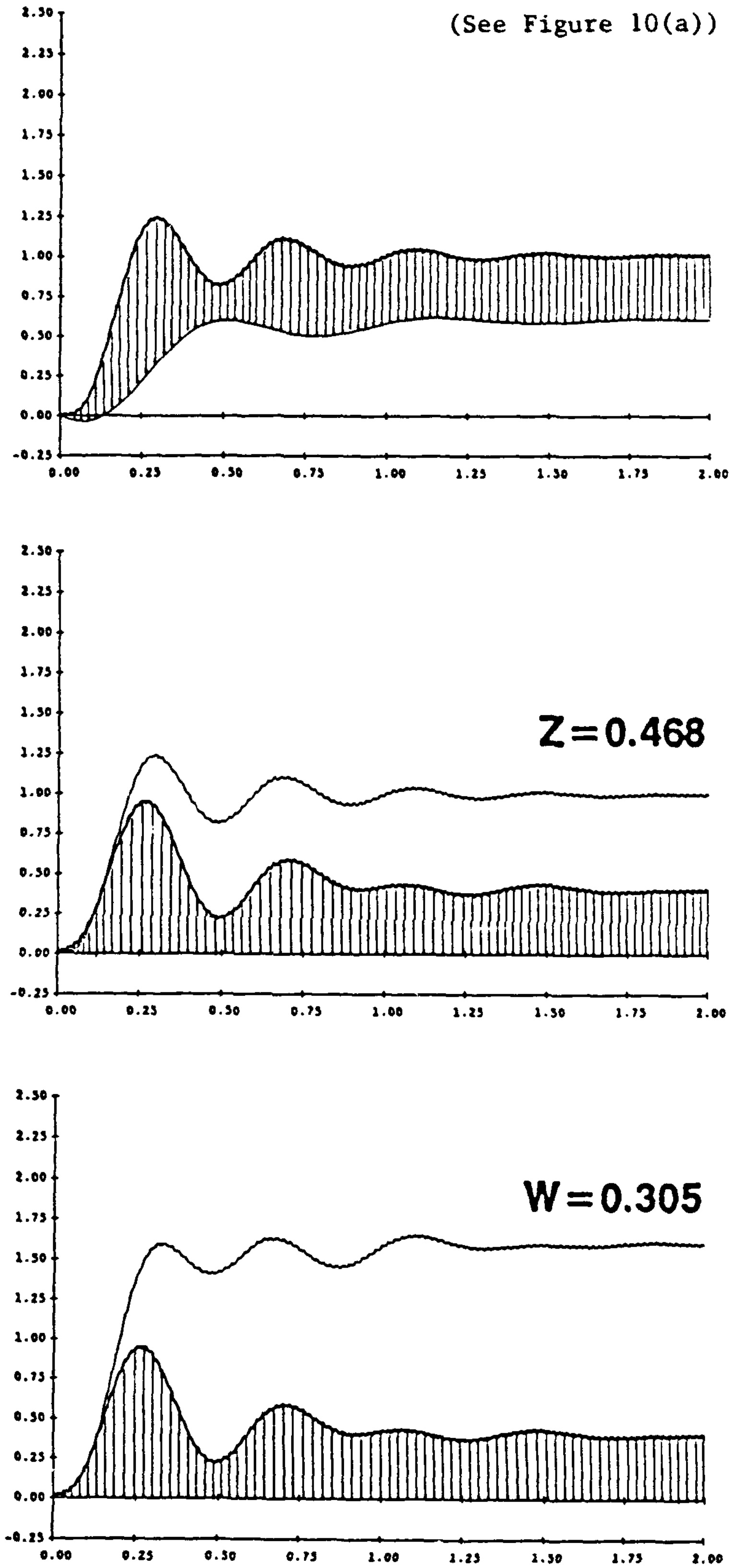


Figure A-11.

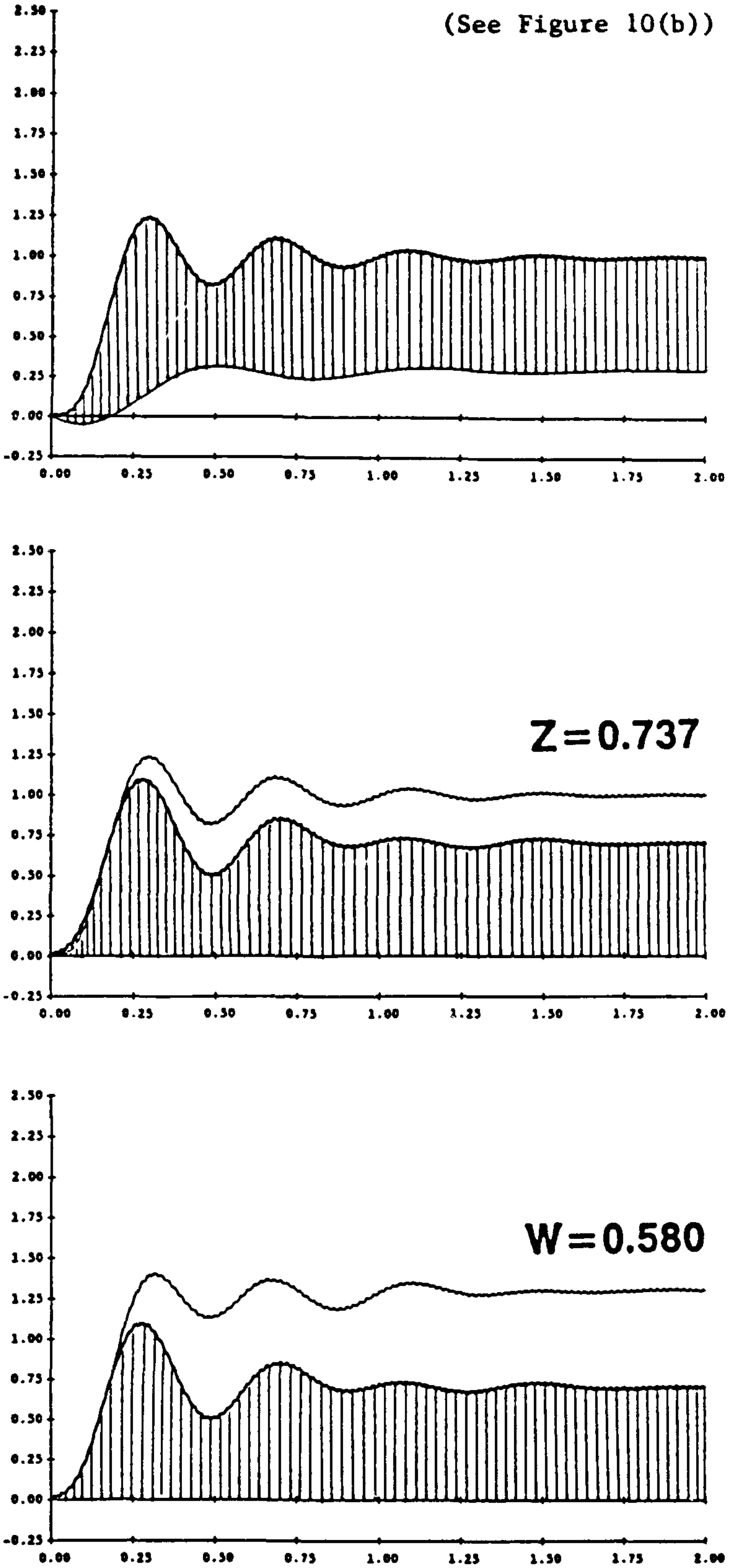

(See Figure 10(b))
Z=0.737
W=0.580
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Figure A-12.

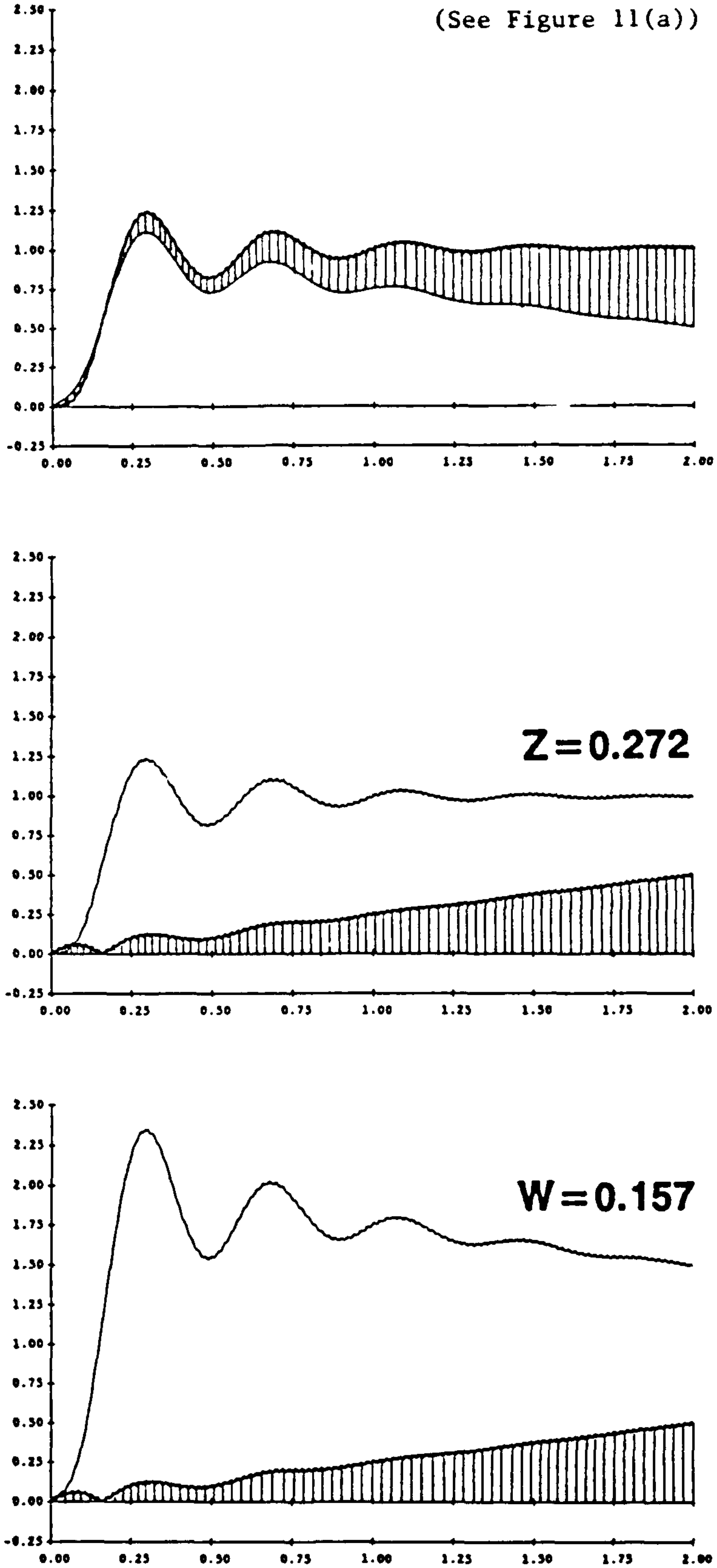


Figure A-13.

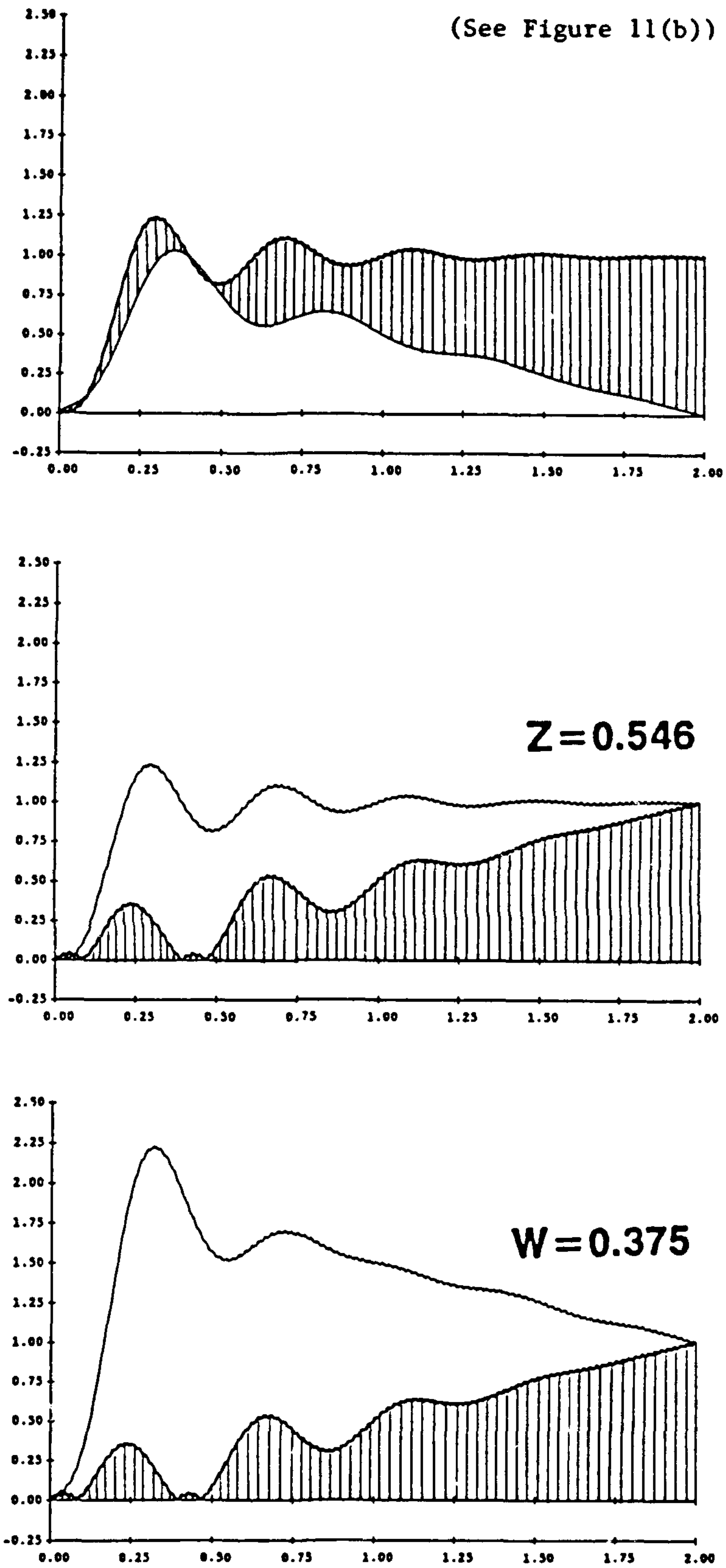


Figure A-14.

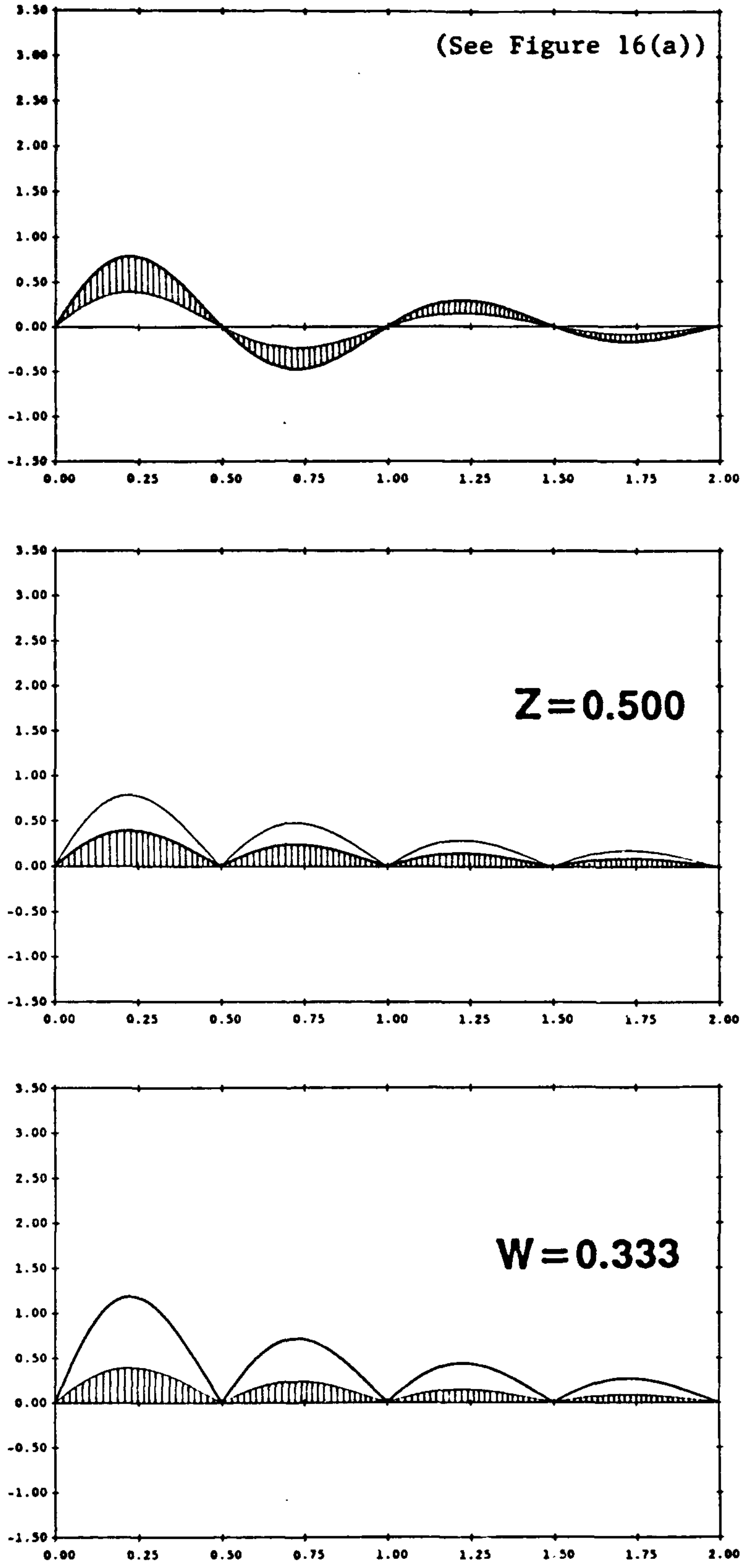


Figure A-15.

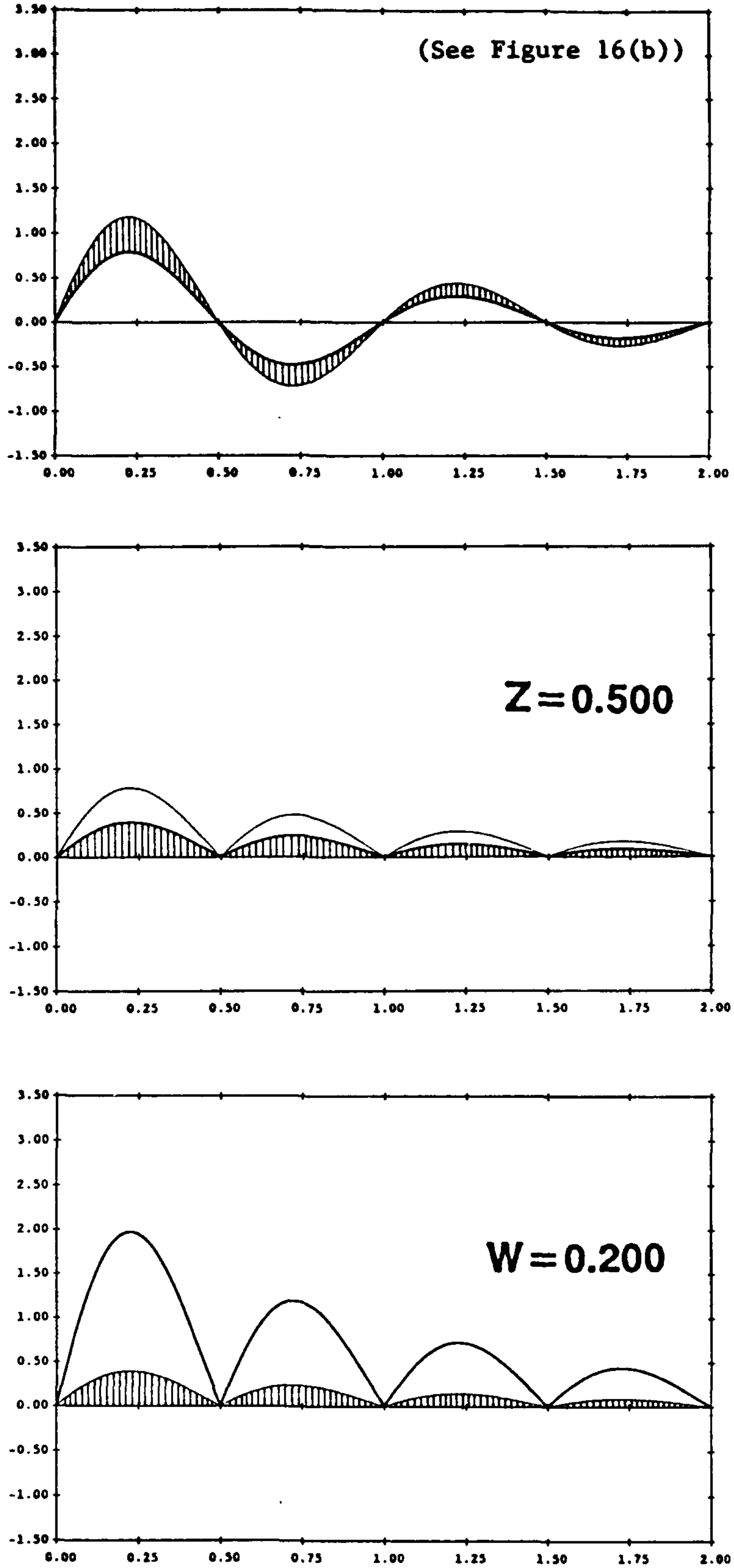


Figure A-16.

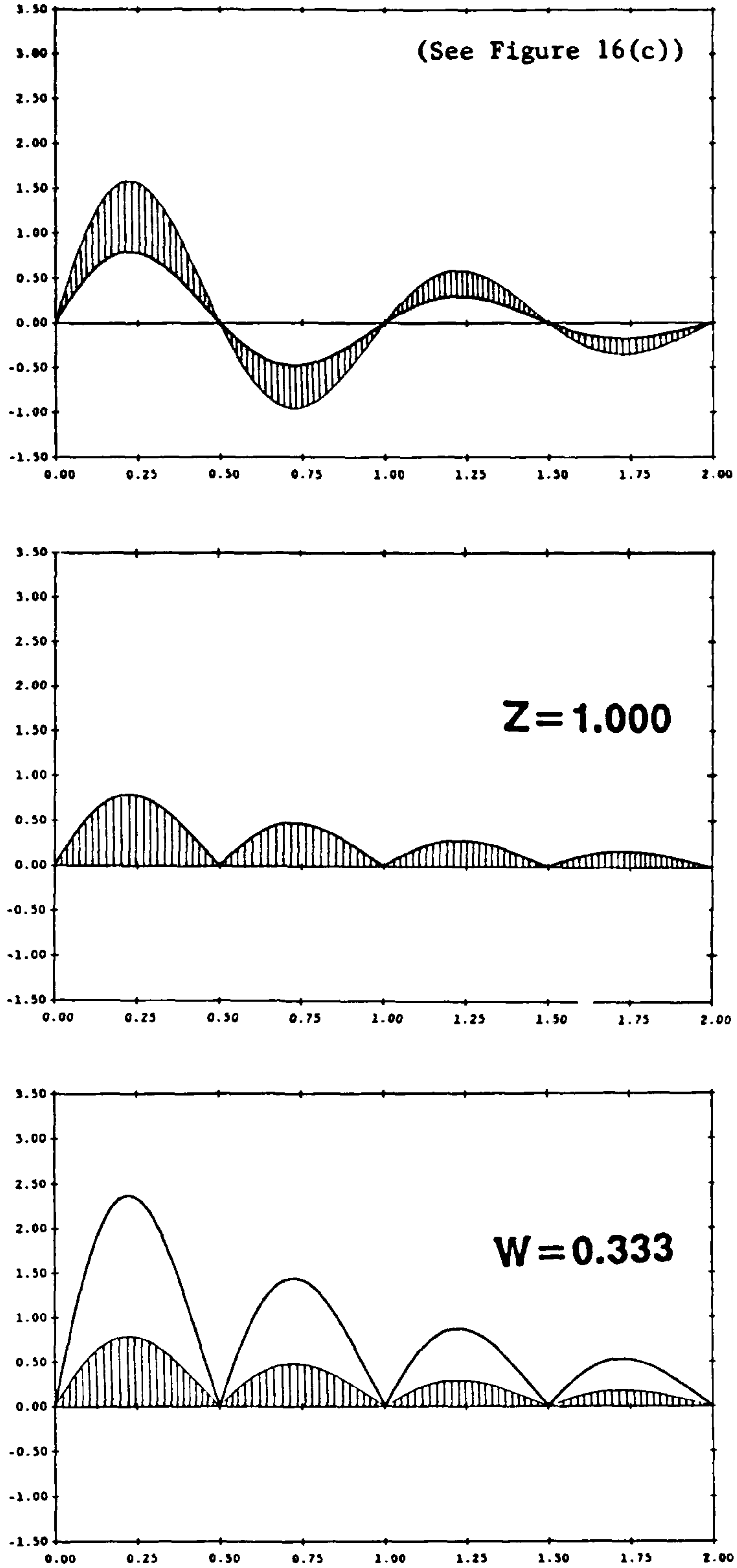


Figure A-17.

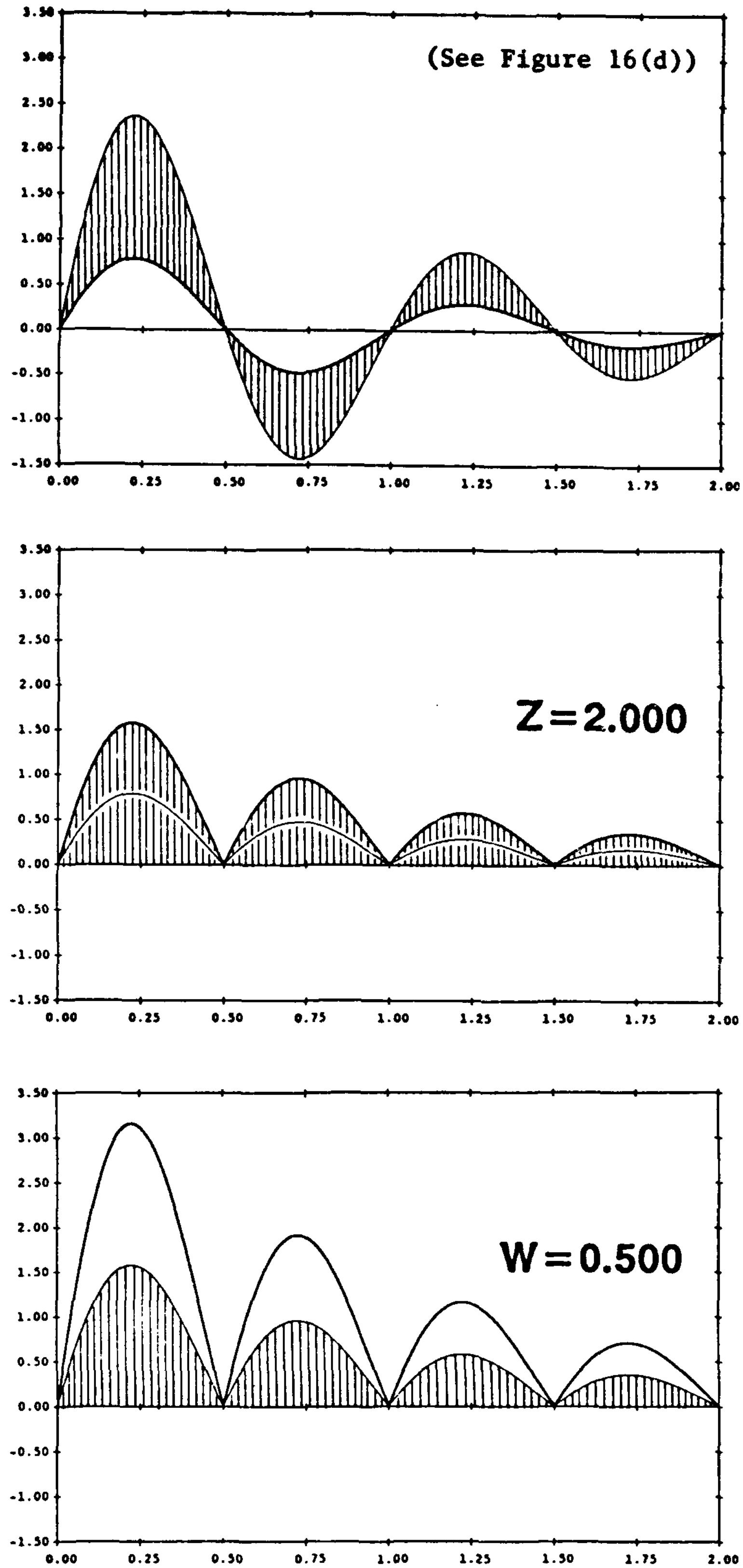

(See Figure 16(d))
Z=2.000
W=0.500
3.50 3.00 2.50 2.00 1.50 1.00 0.50 0.00 -0.50 -1.00 -1.50
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00

Figure A-18.

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