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Exponential potential versus dark matter

Donald H. Eckhardt
Phillips Laboratory, Hanscom AFB, Massachusetts 01731
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A two parameter exponential potential explains the anomalous kinematics of galaxies and galaxy clusters without need for the myriad *ad hoc* dark matter models currently in vogue. It also explains much about the scales and structures of galaxies and galaxy clusters while being quite negligible on the scale of the solar system.

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I. INTRODUCTION

Using conventional physics, we can explain the sizes and shapes of stars but we cannot explain the sizes and shapes of galaxies. Observed motions of stars in our Galaxy are inconsistent with observed and inferred mass distributions. Observed rotations of other galaxies are also inconsistent with independent mass estimates. Moreover, applying the virial theorem to galaxy cluster data leads to mass estimates that are grossly inconsistent with estimates from mass to luminosity ratios [1].

Hypotheses to explain these inconsistencies are only hypotheses, and they are incomplete. Complicated dark matter models have been advanced to explain stellar motions perpendicular to the Galactic plane as well as anomalously high rotation rates about the Galactic axis, but nobody has ever detected any dark matter. Modifying Newtonian gravity leads to contrived and awkward alternatives, modified nonrelativistic dynamics [2] being a prime example. Still, neither dark matter nor modified Newtonian gravity explains the sizes and shapes of galaxies and clusters.

One can hypothesize additional forces that are consistent with special relativity and only significant at galactic scales. These forces are applicable at galactic scales because the Compton wavelengths of their bosons have galactic scales. Pairing attractive and repulsive forces derived from Yukawa potentials that have identical coupling constants (except for sign) and slightly different boson masses can result in a net attractive force that is derivable from an exponential potential [3]. Because the coupling constants have identical magnitudes, the net force is negligible at less than galactic dimensions.

II. THE EXPONENTIAL POTENTIAL

Assume that an exponential potential arises from two massive bosons that have the identical coupling constant αG but opposite signs. The potential due to the massless graviton is $V_N = -GM/r$, and the Yukawa potentials due to the massive bosons are $V_A = -\alpha GMe^{-\mu_A r}/r$ (attractive) and $V_R = \alpha GMe^{-\mu_R r}/r$ (repulsive). Let $\mu_R > \mu_A$, and define $\lambda^{-1} = \mu = (\mu_A + \mu_R)/2$, $\mu_R = \mu + \delta\mu/2$, and $\mu_A = \mu - \delta\mu/2$. The net potential is

$$\begin{aligned} V_E = V_A + V_R &= -[\alpha GM][e^{-(\mu-\delta\mu/2)r} - e^{-(\mu+\delta\mu/2)r}] \\ &= -\alpha GMe^{-\mu r}(2/r)\sinh(\delta\mu r/2) \\ &= -\alpha\delta\mu GMe^{-\mu r} \text{ for } r\delta\mu \ll 1. \end{aligned}$$

Define the dimensionless parameter $\gamma = \alpha\delta\mu/\mu$ and set $\xi = \mu r$. Then $V_E = -\gamma\mu GMe^{-\xi}$ is the exponential potential.

Using very credible assumptions, Jagannathan and Singh [4] proved that, in general, the static forces between like charges are attractive for even spin fields and repulsive for odd spin fields. This would suggest that the V_R field is mediated by a vector boson, but such a possibility (as well as other scenarios [5] that involve vector fields) has been ruled out experimentally [6]. An essential condition in the above-cited proof is that each static free-field has positive energy. Because the V_A and V_R fields always occur in superposition with the identical coupling constant (except for sign), the condition might reasonably be relaxed to the requirement that only the net static free-field must have positive energy. In this case, the V_A and V_R fields could both be mediated by scalar or tensor bosons just as long as their net effect is an attractive force.

For a point source, the inward specific forces are $\partial V_N/\partial r = GM/r^2$ and $\partial V_E/\partial r = \gamma\mu^2 GMe^{-\xi}$, and their ratio is $(\partial V_E/\partial r)/(\partial V_N/\partial r) = \gamma\xi^2 e^{-\xi}$. This ratio could be greater or less than 1, depending on the values of the parameters. Constraints on the maximum value of the ratio come from laboratory experiments, solar system kinematics, and the tracking of deep space probes. The deep space probes present the tightest constraints: at $r = 35$ AU $= 1.7 \times 10^{-7}$ kpc where the gravitational acceleration due to the Sun is 500 mGal, the anomalous acceleration is less than 5 mGal [7]. For $\xi \ll 1$, $e^{-\xi} \approx 1$ and $0.01 > (\partial V_E/\partial r)/(\partial V_N/\partial r) \approx \gamma\xi^2 = 3 \times 10^{-14}\gamma\mu^2$ where the unit μ is kpc^{-1}. Thus

$$\lambda > 1.7 \times 10^{-3}\sqrt{\gamma} \text{ pc} . \quad (1)$$

If GM is set to unity, $\lambda\partial V_N/\partial\xi = \xi^{-2}$ and $\lambda\partial V_E/\partial\xi = \gamma e^{-\xi}$. Figure 1 compares $\lambda\partial V_N/\partial\xi$ and $\lambda\partial V_E/\partial\xi$ for various values of γ. For $\gamma > 1.85$, there is a region in the vicinity of $r = \lambda$ where $(\partial V_E/\partial r)/(\partial V_N/\partial r) > 1$ and, therefore, the exponential force dominates; moving away from this region, either toward or away from the source,

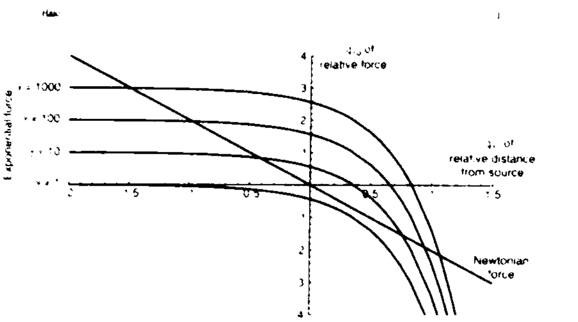

FIG. 1. This log-log plot of $\lambda\partial V_E/\partial\xi = \gamma e^{-\xi}$ (for $\gamma = 1, 10, 100$, and 1000) and of $\lambda\partial V_N/\partial\xi = \xi^{-2}$ shows the region near $r = \lambda$ ($\xi = 1$) where $\partial V_E/\partial r > \partial V_N/\partial r$ and the exponential force dominates the Newtonian force.

eventually $(\partial V_E/\partial r)/(\partial V_N/\partial r) < 1$ and the Newtonian force dominates.

III. GALACTIC ROTATION RATES

The potential V_E can be used to account for otherwise anomalously high galactic rotation rates. Consider a galaxy model in which the major fraction of the mass is in its nucleus and the remaining mass is distributed in a thin layer on the galactic plane in such a way that the density is a function only of the distance from the galactic center. At the distance r from the (point) mass M of the galactic nucleus, the inward specific force on a star is $\mu^2 GM[\xi^{-2} + \gamma e^{-\xi}]$ if the mass of the disk can be neglected. If the star is in a circular orbit about M, its velocity is $v = \sqrt{\mu G M}[\xi^{-1} + \gamma\xi e^{-\xi}]^{1/2}$. A flat velocity curve then occurs if the function $f(\xi,\gamma) = \xi^{-1} + \gamma\xi e^{-\xi}$ has an inflection point determined by $df(\xi,\gamma)/d\xi = 0$ and $d^2 f(\xi,\gamma)/d\xi^2 = 0$. The solution for $\gamma > 0$ is that the inflection point is at $\xi = x = 2 - \sqrt{2} = 0.586$ if $\gamma = e^x/x^2(1-x) = 12.6$. Figure 2 shows a plot of the resulting function $[\xi^{-1} + 12.6\xi e^{-\xi}]^{1/2}$, which is proportional to velocity (for a point mass nucleus), superposed

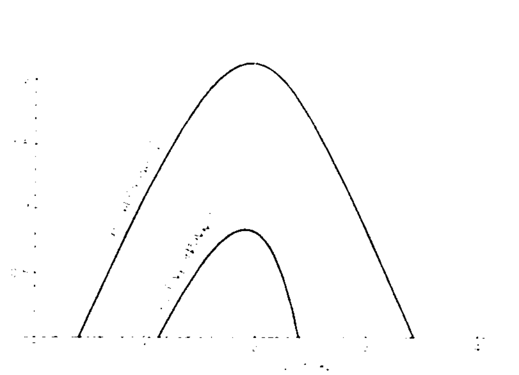

FIG. 3. The lower curve, a plot of $\log_{10}|\partial V_E/\partial r|/\log_{10}|\partial V_N/\partial r|$, shows the region where $\partial V_E/\partial r > \partial V_N/\partial r$ for a point source. The upper curve, a plot of $\log_{10}|\mu\partial U_E/\partial\mu|/\log_{10}|U_N|$, shows the region where the magnitude of the exponential potential term exceeds that of the gravitational term in applying the virial theorem for a uniform density spherical cluster of galaxies. The abscissa is labeled with the logarithm of the range r or the radius of the cluster r, in kpc.

with $\xi^{-1/2}$ and $[12.6\xi e^{-\xi}]^{1/2}$. Comparing it with rotation curves of paradigmatic galaxies [8], a reasonably good fit outside the nucleus is achieved for $\lambda = 40$ kpc. A nominal (but not unique) model is accordingly adopted with the parameters $\gamma = 12.6$ and $\lambda = 40$ kpc; these parameters easily conform with Inequality 1. The mean mass of the massive bosons of this model is $m = \hbar/\lambda c = 1.6 \times 10^{-37}$ GeV/c^2; that is 2×10^{-39} times the mean mass of Z and W bosons. The appearance of the reciprocal of Dirac's large dimensionless number adds some appeal to this model [9]. The lower curve of Fig. 3 is a plot of the ratio of the forces for the model over the range 13 kpc $< r <$ 250 kpc where the force due to a point source exponential potential exceeds that of gravity.

Corrections, which are generally small except within the nucleus, should be made to account for the disk and for the fact that the nucleus is not a point source. Consider the exponential potential at a point that is at the distance r from the center of a spherical shell of radius a, thickness da, and mass dM. The geometry is shown in Fig. 4. The exponential potential of the shell is

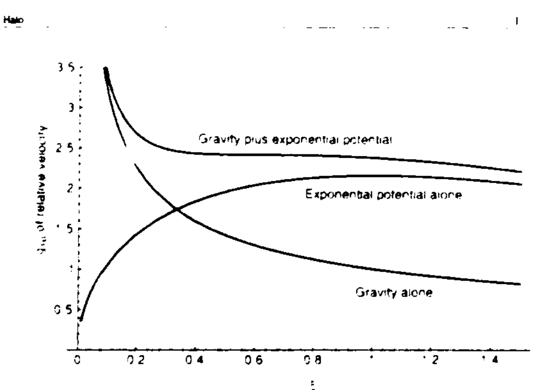

FIG. 2. For a point source at $\xi = 0$, these are the relative velocity curves for gravity alone (pole at origin), the exponential potential alone (zero at origin), and the two combined.

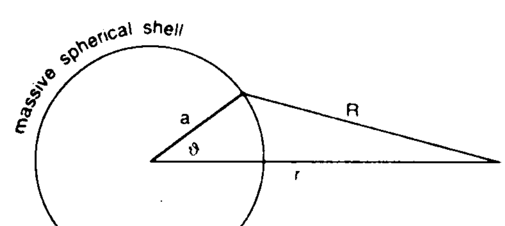

FIG. 4. The mass source for the exponential potential is a thin uniform density spherical shell of radius a. The potential is evaluated at distance r from the center of the shell. In general, r can be less than, equal to, or greater than a.

$dV_E = \gamma\mu G P_S(r,a,\mu) dM$ where

$$P_S(r,a,\mu) = -\frac{1}{2}\int_0^\pi e^{-\mu R}\sin\vartheta\, d\vartheta$$
$$= -\frac{1}{2ar}\int_{|r-a|}^{r+a} e^{-\mu R} R\, dR$$
$$= \{[1+\mu(r+a)]e^{-\mu(r+a)} - [1+\mu|r-a|]e^{-\mu|r-a|}\}/(2\mu^2 ar).$$

The inward specific force due to the shell is $\gamma\mu^2 G Q_S(r,a,\mu)$ where

$$Q_S(r,a,\mu) = dP_S(r,a,\mu)/d\xi$$
$$= \{[1+\mu|r-a| + \mu^2 r(r-a)]e^{-\mu|r-a|} - [1+\mu(r+a)+\mu^2 r(r+a)]e^{-\mu(r+a)}\}/(2\mu^3 ar^2).$$

Because $a \ll \lambda = 40$ kpc, suitable approximations are

$$Q_S(r,a,\mu) \approx [1-(a/r)^2/3]e^{-\mu r}, \quad r \geq a, \tag{2}$$
$$Q_S(r,a,\mu) \approx (2r/3a)e^{-\mu a}, \quad r \leq a. \tag{3}$$

For a spherical nucleus with density $\rho = \rho(a)$, the specific force on a star at radius r is

$$4\pi\gamma\mu^2 G\int_0^{a_{\max}} Q_S(r,a,\mu)\rho(a)a^2 da.$$

From (2) and (3) it is clear that Q_S is always positive, even when $r < a$: all concentric shells of a spherical nucleus contribute to a centripetal force, even at points inside the nucleus.

IV. THE DEFLECTION OF LIGHT

On the scale of the solar system, $V_E/V_N = \gamma\xi e^{-\xi} \approx \gamma\xi \ll 1$, so the deflection of light near the Sun and the perihelion precession of Mercury are the same as in general relativity. On the scale of a galaxy, however, V_E cannot be ignored, so a simple artifice is used to estimate the deflection of light passing through a galaxy. Let ABC in Fig. 5 be a thin straight rod of length $2z$ with uniform linear density ρ, and consider the gravitational field at a point O that is distance r from the center of the rod B and is equidistant from A and C. Let the angle AOC be 2Θ; other symbols are as shown in Fig. 5. The specific force at O, projected toward B, is

$$G\rho\int_{-z}^{z}\frac{\cos\vartheta}{R^2}d\zeta = \frac{G\rho}{r}\int_{-\Theta}^{\Theta}\cos\vartheta\, d\vartheta = \frac{2G\rho}{r}\sin\Theta$$

and the deflection of light passing through O in a plane perpendicular to the rod is

$$\frac{4G\rho}{c^2}\int_{-z}^{z}\frac{\cos\vartheta}{R}d\zeta = \frac{4G\rho}{c^2}\int_{-\Theta}^{\Theta}d\vartheta = \frac{8G\rho\Theta}{c^2},$$

so the ratio of the deflection to the specific force is $4rc^{-2}\Theta/\sin\Theta$. Because the specific force is v^2/r where v is the circular velocity of a body orbiting in the orthogonally bisecting plane at distance r from the rod, the deflection is $4(v/c)^2\Theta/\sin\Theta$. For small Θ, the force varies as r^{-2} and the deflection is approximately $4(v/c)^2$. For $\Theta \approx \pi/2$, the force varies as r^{-1} and the deflection is approximately $2\pi(v/c)^2$. For a galaxy with flat rotation curves, outside the nucleus the specific force generally varies as r^{-1} where r is the distance from the center of the galaxy. Whether the cause is a dark matter halo or the exponential potential, the net field in all directions can be modeled by the Newtonian field of a rod in the plane that orthogonally bisects the rod, so the deflection toward the center of the galaxy is approximately $2\pi(v/c)^2$. The most rapidly rotating disk galaxy known, for which $v = 500$ km s^{-1}, is UGC 12591 [10], so light passing through that galaxy would be deflected by ap-

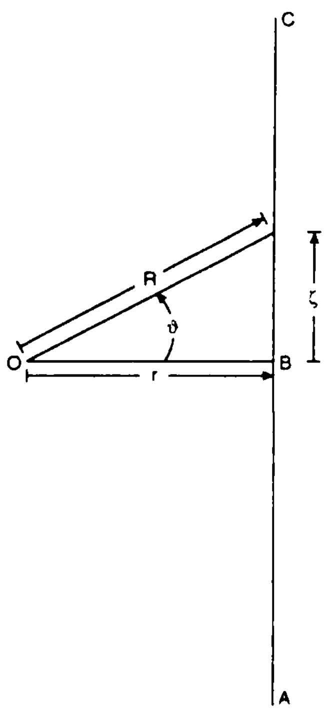

FIG. 5. Point O lies in the bisecting plane orthogonal to the thin massive rod ABC where $AB = BC = z$ and $OB = r$. If $z \gg r$, the gravitational field at O falls off as $1/r$.

proximately 3.6 arcsec. If such a galaxy is a lens for a source, say a quasar, that is relatively much farther from the Earth than the galaxy, the separation of the image would be as high as 7.2 arcsec which is close to the maximum gravitational lens separation (7 arcsec) observed [11].

V. THE DENSITY OF MATTER NEAR THE SUN

The Poisson equation and collisionless Boltzmann equation relate the potential V to the mass densities and velocities of stars and interstellar matter for components of their trajectories that are orthogonal to the Galactic disk (parallel to the z axis); taken together for a variety of isothermal components, they constitute the combined Poisson-Boltzmann (or Poisson-Vlasov) equation. Published solutions have indicated that there are inconsistencies between the motions of tracer stars and observed densities of the disk which are reconciled by postulating dark matter [12,13]. The exponential potential of the disk can be ignored, so the appropriate Poisson equation on or near the disk is

$$\nabla^2 V = \partial^2 V/\partial z^2 - \omega^2 = 4\pi G\rho, \tag{4}$$

where the tidal term $\omega^2 = \omega^2(r)$ is the square of the angular rotation rate of a disk star in a circular orbit about the nucleus, and $\rho = \rho(z)$ is the density of all matter in the disk. Any halo, visible (e.g., globular clusters) or invisible (if any), would perturb ω^2, so its effect is automatically included in (4), but if there is an exponential potential, a halo is not necessary to explain Galactic rotation, so the effect of the halo is then presumably negligible. If (4) is substituted for Bahcall's Eq. (1) (paper I [12]) and combined with his Eq. (2) (and its first derivative with respect to z), the following equation for the density of K giant tracer stars can be derived:

$$\sigma_K^2 \rho_K \frac{d^2\rho_K}{dz^2} - \sigma_K^2 \left[\frac{d\rho_K}{dz}\right]^2 + (4\pi G\rho + \omega^2)\rho_K^2 = 0, \tag{5}$$

where σ_K^2 is the K giant z velocity variance. Setting $\rho_K(z) = \rho_K(0)e^{-u(z)}$, (5) becomes $\sigma_K^2 d^2u/dz^2 = 4\pi G\rho + \omega^2$. Suppose that the density is the sum of N Gaussian distributions:

$$\rho(z) = \sum_{n=0}^{N} \rho_N(0) \exp\left[-\frac{\pi z^2}{4H_n^2}\right],$$

each with half-thickness H_n. With $\zeta_n = \sqrt{\pi}z/(2H_n)$, the solution to (5) is then

$$ln[\rho_K(z)/\rho_K(0)] = -\frac{\omega^2 z^2}{2\sigma_K^2} - \sum_{n=0}^{N} \frac{8H_n^2 G\rho_n(0)}{\sigma_K^2}[\sqrt{\pi}\zeta_n \operatorname{erf} \zeta_n + \exp(-\zeta_n^2) - 1]. \tag{6}$$

Use $\sigma_K = 20$ km s^{-1} [13], and determine ω^2 from the Oort constants, $A = 16.9 \pm 0.9$ km s^{-1} kpc^{-1} and $B = -9.0 \pm 1.5$ km s^{-1} kpc^{-1} [14], adjusted proportionally to their uncertainties until $A = -B = 13.9$ km s^{-1} kpc^{-1} ($A = -B$ is a requirement for a flat velocity curve near the sun); then $\omega^2 = A^2/4 = B^2/4 = 8.1 \times 10^{-31}$ s^{-2}. From the standard Galaxy model of Bahcall and Soneira [15], a simple two component model of the density is constructed with $\rho_1(0) = 0.052\mathcal{M}_\odot/\text{pc}^3$ and $H_1 = 125$ pc (interstellar dust and gas, and $M_V \leq 4$ stars); and $\rho_2(0) = 0.044\mathcal{M}_\odot/\text{pc}^3$ and $H_2 = 325$ pc ($M_V > 4$ stars). The solution for this model is labeled in Fig. 6 as curve 1. Also shown at characteristic distances from the Galactic plane are relative K giant densities of Hill and Oort [16] (circles) and Upgren [17] (triangles), as adjusted by Bahcall (Table 3 [13]), and further adjusted here, using Table 2 [13], to discount spheroid K giants. The model does not fit either data set well. One approach toward resolving the discrepancy is to double the mass of the $M_V > 4$ stars—adding, say, brown dwarfs [18], black holes, or dark matter. This solution is labeled as curve 2; the fit is somewhat better for $z < 700$ pc, but it still is mediocre. Another approach toward resolving the discrepancy is to examine more critically the Hill-Oort and Upgren densities. The first hint that these densities might be wrong is that the two sets only crudely agree with each other. Next, it is obvious that the Upgren densities very nearly fall on a straight line in Fig. 6, implying that the density falls off exponentially with the distance from the origin. The density distribution would have a (near) discontinuity in its first derivative at the origin; that is, there would be a sharp peak in the density on the Galactic plane. Any dark matter model [high $\rho_n(0)$, low

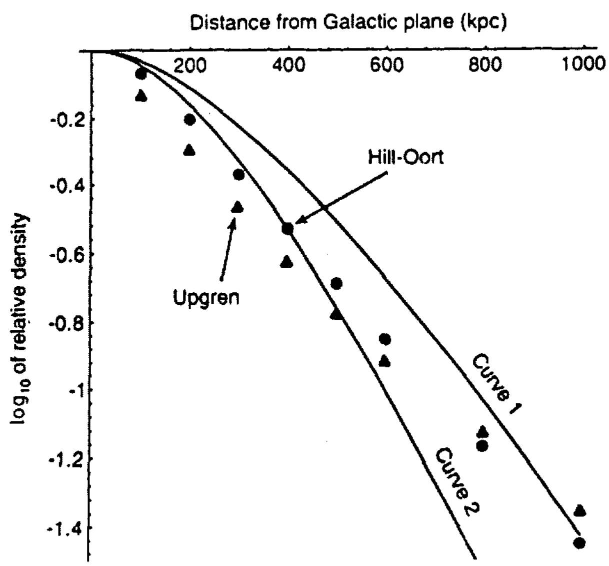

FIG. 6. An analytic solution for the densities of K giant tracer stars (curve 1) is compared with K giant densities of Hill-Oort (circles) and Upgren (triangles), adjusted as described in the text. Adding some "missing matter" to the analytic model results in curve 2. A reconciliation of the patent discrepancies is effected by rejecting both the Hill-Oort and Upgren models because of internal and external inconsistencies in the models.

H_n] that could be in accord with Upgren's K giant density at $z = 100$ pc would not be in accord with his densities beyond. Because the kink is physically unrealizable, the Upgren densities are rejected. The Hill-Oort densities do not seem to have this problem, for their distribution appears more Gaussian near the origin. The shape of the Hill-Oort distribution within about 600 pc of the Galactic plane was calculated from van Rhijn's tabulation [19] by spectral class, visual magnitude ($M_V < 9.44$), and Galactic latitude (0° to ±20°, ±20° to ±40°, ±40° to ±90°) of stars in the Henry Draper Catalogue [20]. Hill used the K stars in the ±40° to ±90° range, and considered them to have an "average latitude" of 59°. However, by van Rhijn's Table I, the K stars in the ±20° to ±40° range have almost the same magnitude distribution as those in ±40° to ±90° range, so if Hill had used the lower latitude stars, he would have had about half as large an "average latitude" and his K star log density curve would have fallen off almost twice as steeply out to about 350 pc. (The difference in interstellar absorption is slight.) The van Rhijn data are inconsistent with Hill's technique, so the Hill-Oort densities are rejected. There remains no acceptable evidence that dark matter is required to explain the kinematics of stars in the vicinity of the Sun.

VI. GALAXY CLUSTERS AND THE VIRIAL THEOREM

Because the exponential force is larger than that of gravitation out to a distance of 250 kpc, it should be important in the dynamics of clusters of galaxies. Let T be the kinetic energy of a cluster of N galaxies (each galaxy considered to be a point mass); let U_N be the total gravitational energy of the cluster; and let $\mathbf{F}_n$ be the exponential force on galaxy n while $\mathbf{r}_n$ is its position vector. Then the virial theorem states that

$$2\overline{T} + \overline{U}_N + \sum_{n=1}^{N} \overline{\mathbf{r}_n \cdot \mathbf{F}_n} = 0,$$

where the overbar denotes the time average. The total energy due to the exponential potential is

$$U_E = -\gamma\mu G \sum_{m\neq n} M_m M_n e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|},$$

so

$$\mathbf{r}_n \cdot \mathbf{F}_n = -\mathbf{r}_n \cdot \partial U_n/\partial \mathbf{r}_n = - \gamma G \sum_{m\neq n} M_m M_n \mathbf{r}_n \cdot \frac{\mathbf{r}_n - \mathbf{r}_m}{|\mathbf{r}_n - \mathbf{r}_m|}\mu^2 e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|}.$$

For every term

$$-\gamma G M_m M_n \mathbf{r}_n \cdot \frac{\mathbf{r}_n - \mathbf{r}_m}{|\mathbf{r}_n - \mathbf{r}_m|}\mu^2 e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|}$$

in $\mathbf{r}_n \cdot \mathbf{F}_n$, there is a corresponding term

$$-\gamma G M_m M_n \mathbf{r}_m \cdot \frac{\mathbf{r}_m - \mathbf{r}_n}{|\mathbf{r}_n - \mathbf{r}_m|}\mu^2 e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|}$$

in $\mathbf{r}_m \cdot \mathbf{F}_m$, and their sum is

$$-\gamma G M_m M_n \mu^2 |\mathbf{r}_n - \mathbf{r}_m| e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|}$$

which (remember that $\gamma\mu = \text{const}$) is also equal to

$$-\mu \frac{\partial}{\partial\mu}[-\gamma\mu G M_m M_n e^{-\mu|\mathbf{r}_n - \mathbf{r}_m|}].$$

Therefore,

$$\sum_{n=1}^{N} \mathbf{r}_n \cdot \mathbf{F}_n = -\mu \frac{\partial U_E}{\partial\mu},$$

and the virial theorem is

$$2\overline{T} + \overline{U}_N - \mu \overline{\frac{\partial U_E}{\partial\mu}} = 0. \quad (7)$$

The estimation of U_N and U_E for a spherical cluster can be approached by constructing the cluster by integrating outward over successive shells, like putting an onion back together from the inside out. As each subsequent shell is added, the decrease in energy is calculated, so the total energy can be calculated by integration. The change in energy caused by adding a shell of radius r, thickness dr, and density $\rho(r)$ is just $4\pi\rho(r)r^2dr$ times the change in energy caused by a unit point mass at radius r. For the simple model of a uniform density sphere of radius r_c, the solutions for U_N and $-\mu(\partial U_E/\partial\mu)$ are

$$\begin{aligned} U_N &= -16\pi^2 G\rho^2 \int_0^{r_c}\int_0^{r} a^2 da\, r dr \\ &= -\tfrac{16}{15}\pi^2 r_c^5 G\rho^2 \end{aligned} \quad (8)$$

and

$$\begin{aligned} -\mu(\partial U_E/\partial\mu) &= -16\pi^2\gamma\mu^2 G\rho^2 \frac{\partial}{\partial\mu}\int_0^{r_c}\int_0^{r} P_S(r,a,\mu)a^2 da\, r^2 dr \\ &= -8\pi^2\gamma\mu^{-4}G\rho^2[(15 - 6\eta^2 + 2\eta^3) \\ &\quad -(15 + 30\eta + 24\eta^2 + 10\eta^3 + 2\eta^4)e^{-2\eta}], \end{aligned} \quad (9)$$

where $\eta = \mu r_c$. The ratio (9)/(8) is

$$\begin{aligned} -\mu\frac{\partial U_E}{\partial\mu}U_N^{-1} &= \frac{15}{2}\gamma\mu[15\eta^{-5} - 6\eta^{-3} + 2\eta^{-2} \\ &\quad -(15\eta^{-5} + 30\eta^{-4} + 24\eta^{-3} \\ &\quad +10\eta^{-2} + 2\eta^{-1})e^{-2\eta}]. \end{aligned} \quad (10)$$

[The terms in (9) and (10) with the factor $e^{-2\eta}$ are negligible, even for small clusters.] This ratio, which is plotted as the upper curve in Fig. 3, attains its peak value 135 at $r_c = 90$ kpc; it equals or exceeds 10 over the range 9 kpc $\leq r_c \leq$ 800 kpc; and it equals or exceeds unity over the range 2.5 kpc $\leq r_c \leq$ 2.7 Mpc.

A uniform density cluster is not especially realistic, but taking the ratio of the $-\mu(\partial U_E/\partial\mu)$ and U_N terms somewhat ameliorates this shortcoming. Whatever the model, however, it is clear that U_E often swamps U_N in importance in applying the virial theorem for galaxy clusters. Indeed, at a sufficient number of "skin depths" ($\lambda \sim 40$ kpc) inside the boundary of a uniform density cluster, the force due to U_E is negligible compared with its force near the surface. The exponential force is a surface force at scales larger than about 100 kpc. If there are density gradients of this scale, this force tends to accelerate matter from lower density toward higher density regions. It is perhaps much more important than gravity in contributing to the instability of density fluctuations that, with time and inflation, have led to a foamy universe with immense voids [21].

VII. SUMMARY

With an exponential potential of galactic scale, there is no need for any dark matter models, so the so-called "conspiracy" [22] in the relationships between visible and dark matter can be dispatched. The exponential potential seems to be able to account for the distinctively different dimensions that are typical of galaxies and galaxy clusters. The leading competing explanation for galactic kinematics calls for the existence of dark matter that is distributed according to very special models, and a model of dark matter that explains one phenomenon is not robust enough to explain another. Moreover, the amount of dark matter required to hold rich galaxy clusters together is "astronomical." Finally, there is scant direct experimental evidence for dark matter. The hypothesis of an exponential potential which conforms consistently with all of the evidence available appears at present to be a viable alternative to the hypothesis of dark matter. The theory presented demands tests that use ample astronomical data in more detailed models than the crude nominal models of this paper so that either its two parameters can be refined from the nominal estimates or the theory can be falsified.

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