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A Retrieval Error Analysis Technique for Passive Infrared Atmospheric Sounders

J.P. Kerekes

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Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

Gary Tutungian

Gary Tutungian
Administrative Contracting Officer
Directorate of Contracted Support Management

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

A RETRIEVAL ERROR ANALYSIS TECHNIQUE FOR PASSIVE INFRARED ATMOSPHERIC SOUNDERS

J.P. KEREKES
Group 96

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ABSTRACT

To support the design and analysis of passive infrared atmospheric sounding instruments, an analytical error analysis technique has been implemented. This technique is based on a linear approximation to the radiative transfer equation and uses a minimum variance estimation approach to atmospheric profile retrieval. As a result, once the proper matrices have been constructed an estimate of the retrieval error is computed through a single linear matrix equation. The solution vector is written with temperature and water vapor explicitly defined, thus producing the error vector for both simultaneously. Examples showing typical results are presented for a high spectral resolution sounder as well as for a lower resolution, filter-wheel type of instrument.

ACKNOWLEDGMENT

Henry E. Fleming is acknowledged for his contributions to the science of satellite atmospheric sounding, including the publication of the retrieval error algorithm on which this work is based.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

1. INTRODUCTION

1.1 MOTIVATION

The science of atmospheric temperature and water vapor profile retrieval from space-based passive infrared and microwave sensors has been developing since the 1960s [1,2] and is routinely applied to satellite measurements for use in weather-forecasting [3] and climate-monitoring [4] applications. However, the accuracy and resolution of the retrieved profiles from current generation instruments are such that the use of their data leads to mixed results [5] and falls short of what is desired from future instruments [6]. These next-generation sounders must be designed to collect data that will result in significant improvements in the quality of the retrieved profiles.

The analysis of the expected quality of retrievals from an instrument design is in general a difficult task. Whereas the raw data quality from an instrument (e.g., signal-to-noise ratio, spatial, and spectral resolution) can be objectively calculated given a specific design, the retrieval process depends on algorithms and sources of data external to the instrument. Retrieval algorithms vary from those specifically based on a statistical regression between collocated in-situ measurements and satellite soundings to those which use a radiative transfer model along with linear estimation theory and iterate from a first guess. The quality of these retrievals depend in a large part on the a priori knowledge of the state of the observed atmosphere. These variabilities, along with the fact that data collected are used for other applications such as trace gas monitoring and the calculation of cloud heights and amounts, challenge the characterization of "performance" for atmospheric sounding instruments.

Given these challenges, however, it still remains an important task to evaluate the expected performance of a sounding instrument both to justify the instrument design requirements as well as to judge the impact of design modifications that may become necessary during instrument development. The most accurate way of predicting on-orbit performance is a full simulation of a variety of atmospheric situations and the application of the operational retrieval algorithm to the resulting data. This prediction would have to be done for every point in the design trade space and would not be desirable given finite resources.

Alternatively, a simplified approach to retrieval error analysis based on linear estimation theory can be implemented as described in this report. This algorithm, based on previous work [7], uses a linear retrieval model to estimate the variance of the error in the retrieved temperature and water vapor profiles. Since the algorithm can be described with an analytical equation, parameter sensitivity studies can be run quickly and aid in the development of a sounding instrument.

1.2 CONTEXT AND LIMITATIONS

The application and results of the methodology documented in this report should be considered in light of its limitations. First, it is not a full retrieval analysis. While the basic retrieval operator derived could be applied to simulated or measured data, this report focuses on its use in the estimation of expected retrieval error. Thus, the algorithm does not require sounding data but rather a description of the instrument including the number, bandwidth, noise level, and spectral location of the channels. No spatial effects are considered including off-axis sounding or the impact of clouds. Some of these limitations could be remedied with extensions to the work presented.

Fundamentally, the error analysis algorithm involves a linear expansion around the known solution to the nonlinear retrieval problem. This formulation has the impact of underestimating the retrieval error in most situations. The results should be considered to be a lower bound on the error. This characteristic of the results leads naturally to the question: if the error levels predicted are optimistic compared to full retrievals, what can be said about the sensitivity to instrument parameters? This issue has been considered and while no guarantee can be provided, the impact on retrieval error of instrument parameter variations can be evaluated in a relative sense with the general trends expected to hold.

In addition to the above limitations of this approach, it should be noted that only temperature and water vapor are explicitly considered in the retrieval analysis. The performance of the instrument design in other sounding applications, such as trace gas monitoring, has not been included.

The most appropriate application of this methodology is in the relative comparison of various design options during a top-level system analysis study. The ease of implementation and use as well as speed in computation allow a large number of design options to be considered. After a design has been constrained, final design choices can be made using performance predictions based on full simulations and retrievals.

1.3 REPORT OVERVIEW

The main goal of this report is to document in detail the algorithm used to estimate the error in retrieved atmospheric temperature and water vapor profiles given an instrument description. This algorithm has been implemented at Lincoln Laboratory and has formed the basis for several sounding system studies. Section 2 provides the details behind the theory and implementation of the technique. Section 3 presents mean vectors and covariance matrices of atmospheric temperature and water vapor profiles obtained from NOAA. These statistics constitute the a priori information used in the error analysis technique. Section 4 presents some example results of the technique applied to two different sensors. Section 5 concludes the report with a summary.

2. THEORY AND IMPLEMENTATION

2.1 RADIATIVE TRANSFER EQUATION

The fundamental equation for atmospheric sounding from a satellite is the radiative transfer equation (RTE) shown in Equation (2.1).

$$R = B_s \tau_s + \int_{\tau_s}^{1} B \, d\tau \quad , \tag{2.1}$$

where s indicates surface conditions and

R = upwelling radiance seen by satellite in a particular spectral channel in $mW/(m^2\text{-}Sr)$,

B = radiance emitted by a given layer of the atmosphere in a particular channel, and

τ = transmittance from given layer of atmosphere to top of atmosphere for a particular channel.

The upwelling radiance R is in general a function of temperature, pressure, the concentrations of various atmospheric constituents as well as the central wavenumber and spectral response of the channel. In this analysis the concentration of all atmospheric constituents except water vapor is assumed to be known and constant. Also, sensor noise will be neglected for now but will be added in later.

Considering variability of atmospheric temperature and water vapor profiles, a mean state can be defined as in Equation (2.2).

$$\overline{R} = \overline{B_s \tau_s} + \overline{\int_{\tau_s}^{1} B \, d\tau} \quad . \tag{2.2}$$

Assuming that the radiance and the transmittance are uncorrelated (not entirely true, but a practical assumption), Equation (2.2) can be rewritten as follows.

$$\overline{R} = \overline{B_s} \, \overline{\tau_s} + \int_{\tau_s}^{1} \overline{B} \, \overline{d\tau} \quad . \tag{2.3}$$

Now, defining the difference quantity to be the mean subtracted from the true value (i.e., $\Delta x = x - \overline{x}$) subtracting Equation (2.3) from Equation (2.1) results in Equation (2.4).

$$
\begin{aligned}
\Delta R &= B_s\tau_s - \overline{B_s}\,\overline{\tau_s} + \int_{\tau_s}^{1} B\,d\tau - \int_{\tau_s}^{1} \overline{B}\,\overline{d\tau} \\
&= B_s\tau_s - \overline{B_s}\tau_s + \overline{B_s}\tau_s - \overline{B_s}\,\overline{\tau_s} \\
&\quad + \int_{\tau_s}^{1} \left(B\,d\tau - \overline{B}\,d\tau + \overline{B}\,d\tau - \overline{B}\,\overline{d\tau}\right) \\
&= \Delta B_s\tau_s + \overline{B_s}\Delta\tau_s + \int_{\tau_s}^{1} \Delta B\,d\tau + \int_{\tau_s}^{1} \overline{B}\Delta d\tau \quad . \qquad (2.4)
\end{aligned}
$$

By integrating the last term of Equation (2.4) by parts, the following equation results.

$$
\begin{aligned}
\int_{\tau_s}^{1} \overline{B}\Delta d\tau &= \overline{B}\Delta\tau\Big|_{\tau_s}^{1} - \int_{1}^{40} \Delta\tau \frac{d\overline{B}}{dX}dX \\
&= 0 - \overline{B_s}\Delta\tau_s - \int_{1}^{40} \Delta\tau \frac{d\overline{B}}{dX}dX \quad . \qquad (2.5)
\end{aligned}
$$

Here, an auxiliary variable X is used which represents the layers of the atmosphere. The pressure levels at the bottom of each of the 40 layers used in the analysis are defined in Section 3. Substituting Equation (2.5) into Equation (2.4) results in Equation (2.6).

$$
\Delta R = \Delta B_s\tau_s + \int_{1}^{40} \Delta B \frac{d\tau}{dX}dX - \int_{1}^{40} \Delta\tau \frac{d\overline{B}}{dX}dX \quad . \qquad (2.6)
$$

The radiance of a layer depends on the layer's temperature through the nonlinear Planck function, but one can approximate the radiance difference factor ΔB by the first term of a Taylor's expansion about the mean. This expansion is shown in Equation (2.7) where Δt is the difference between the true temperature of that layer and its mean temperature.

$$
\Delta B = \frac{\overline{dB}}{dt}\Delta t \quad . \qquad (2.7)
$$

Substituting Equation (2.7) for the ΔB terms in Equation (2.6) yields Equation (2.8).

$$
\Delta R = \frac{\overline{dB_s}}{dt}\Delta t_s\tau_s + \int_{1}^{40} \Delta t \frac{\overline{dB}}{dt}\frac{d\tau}{dX}dX - \int_{1}^{40} \Delta\tau \frac{\overline{dB}}{dX}dX \quad . \qquad (2.8)
$$

So far, only the effect of temperature has been considered explicitly. The effect of water vapor on the upwelling radiance is included by expanding the $\Delta\tau$ in the last term of Equation (2.8) about the mean value of precipitable water $\bar{u}$ measured in centimeters.

$$\begin{aligned}
\Delta\tau &= \left.\frac{d\tau}{du}\right|_{u=\bar{u}} \Delta u \\
&= \frac{d\left(\tau_D \tau_W\right)}{du}\Delta u \\
&= \left[\tau_W \frac{d\tau_D}{du} + \tau_D \frac{d\tau_W}{du}\right]\Delta u \\
&= \left[0 + \tau_D \frac{d\tau_W}{du}\right]\Delta u \\
&= \left[\frac{\tau}{\tau_W}\frac{d\tau_W}{du}\right]\Delta u \quad . \qquad (2.9)
\end{aligned}$$

Here, $\tau = \tau_D \tau_W$ and

τ_D = transmittance of dry atmosphere with no precipitable water, and

τ_W = transmittance of wet atmosphere with only precipitable water.

Equation (2.9) can then be substituted into Equation (2.8) to result in Equation (2.10), which explicitly shows the upwelling radiance as a function of atmospheric temperature and precipitable water.

$$\Delta R = \overline{\frac{dB_s}{dt}}\tau_s \Delta t_s + \int_1^{40} \Delta t \overline{\frac{dB}{dt}}\frac{d\tau}{dX}dX - \int_1^{15} \frac{\tau}{\tau_W}\frac{d\tau_W}{du}\overline{\frac{dB}{dX}}\Delta u\, dX \quad . \qquad (2.10)$$

Note that precipitable water is defined to be 0 above the 15th layer in the atmosphere.

2.2 NUMERICAL IMPLEMENTATION

2.2.1 Numerical Quadrature

The integrals in Equation (2.10) can be evaluated numerically by a variety of quadrature rules. In this analysis, the following equation is derived for two arbitrary functions $f(X)$ and $g(X)$.

$$
\begin{aligned}
\int_1^{40} f(X)\,g(X)\frac{d\tau}{dX}\,dX &= \sum_{j=1}^{40}\int_j^{j+1} f(X)\,g(X)\frac{d\tau}{dx}\,dX \\
&\approx \sum_{j=1}^{40}\overline{f_j g_j}\int_j^{j+1}\frac{d\tau}{dx}\,dX \\
&= \sum_{j=1}^{40}\frac{f_j\,g_j + f_{j+1}\,g_{j+1}}{2}\left[\tau_{j+1}-\tau_j\right] \\
&= \frac{1}{2}\sum_{j=1}^{40} f_j\,g_j\left[\tau_{j+1}-\tau_j\right] + \frac{1}{2}\sum_{j=2}^{41} f_j\,g_j\left[\tau_j-\tau_{j-1}\right] \\
&= f_1\,g_1\left[\frac{\tau_2-\tau_1}{2}\right] + \sum_{j=2}^{40} f_j\,g_j\left[\frac{\tau_{j+1}-\tau_{j-1}}{2}\right] \\
&\quad + f_{41}\,g_{41}\left[\frac{\tau_{41}-\tau_{40}}{2}\right] \quad . \qquad (2.11)
\end{aligned}
$$

Here, note that the pressure levels are defined as in Table 1. Also, $\tau_{41} = 1$ and $f_{41}\,g_{41} = f_{40}\,g_{40}$ since the 41st layer is above the atmosphere. Thus, using Equation (2.11), Equation (2.10) can be rewritten in a discrete form as follows.

$$
\Delta R_i = b_{si}\,\Delta t_s + \sum_{j=1}^{40} b_{ij}\,\Delta t_j - \sum_{j=1}^{15} c_{ij}\,\Delta u_j \quad . \qquad (2.12)
$$

The index i refers to the spectral channel. The coefficients are defined as follows.

$$b_{si} = \left(\frac{\overline{dB_s}}{dt}\right)_i \tau_{si}$$

$$b_{ij} = \begin{cases} \left(\frac{\overline{dB}}{dt}\right)_{i1}\left[\frac{\tau_{i2}-\tau_{i1}}{2}\right] & \text{for } j=1 \\ \left(\frac{\overline{dB}}{dt}\right)_{ij}\left[\frac{\tau_{i,j+1}-\tau_{i,j-1}}{2}\right] & \text{for } 2 \le j \le 39 \\ \left(\frac{\overline{dB}}{dt}\right)_{i40}\left[\frac{2-\tau_{i40}-\tau_{i39}}{2}\right] & \text{for } j=40 \end{cases}$$

$$c_{ij} = \begin{cases} \left(\frac{\tau}{\tau_W}\frac{d\tau_W}{du}\right)_{il}\left[\frac{\overline{B_{i2}}-\overline{B_{i1}}}{2}\right] & \text{for } j=1 \\ \left(\frac{\tau}{\tau_W}\frac{d\tau_W}{du}\right)_{ij}\left[\frac{\overline{B_{i,j+1}}-\overline{B_{i,j-1}}}{2}\right] & \text{for } j=2 \text{ to } 15 \end{cases} .$$

The computation of $d\tau_W/du$ is shown later in Section 2.2.4.

2.2.2 Conversion From Precipitable Water *u* to Mixing Ratio *q*

The water vapor statistics that are used in this analysis are given in terms of water vapor mixing ratio q (g/kg) rather than precipitable water u. Thus, a change of variable for the last term in Equation (2.12) is necessary. The amount of precipitable water u in the atmosphere above a pressure level p is given by Equation (2.13).

$$u(p) = \frac{1}{g}\int_0^p q(p')\,dp' \quad . \tag{2.13}$$

Here, $g = 980$ g/(cm-sec^2) is the gravity constant. By taking delta quantities, reversing the limits of integration, and applying the quadrature result of Equation (2.11), the following equation results.

$$
\begin{aligned}
\Delta u_j &= -\frac{1}{g}\int_{p_j}^{0} \Delta q(p')\,dp' \\
&= -\sum_{k=j}^{15} \frac{1}{g}\int_{p_k}^{p_{k+1}} \Delta q(p')\,dp' \\
&= -\frac{1}{g}\sum_{k=j}^{15}\left(\frac{\Delta q_{k+1}+\Delta q_k}{2}\right)\left(p_{k+1}-p_k\right) \\
&= \frac{1}{2g}\left[\sum_{k=j+1}^{16} \Delta q_k\left(p_{k-1}-p_k\right)+\sum_{k=j}^{15} \Delta q_k\left(p_k-p_{k+1}\right)\right] \\
&= \frac{1}{2g}\Delta q_j\left(p_j-p_{j+1}\right)+\frac{1}{2g}\sum_{k=j+1}^{15} \Delta q_k\left(p_{k-1}-p_{k+1}\right) \\
&= \frac{1}{2g}\sum_{k=j}^{15} \Delta q_k\,\Delta p_k \quad .
\end{aligned}
\tag{2.14}
$$

where

$$
\begin{aligned}
\Delta p_k &= p_k - p_{k+1} && \text{for } k = j \\
&= p_{k-1} - p_{k+1} && \text{for } k > j \quad .
\end{aligned}
$$

Using Equation (2.14), the last term of Equation (2.12) can be replaced as follows.

$$
\begin{aligned}
\sum_{j=1}^{15} c_{ij}\Delta u_j &= \sum_{j=1}^{15} \frac{c_{ij}}{2g}\left[\sum_{k=j}^{15} \Delta q_k\,\Delta p_k\right] \\
&= \frac{1}{2g}\sum_{k=1}^{15} \Delta q_k\left[\sum_{j=1}^{k} c_{ij}\,\Delta p_k\right] \quad .
\end{aligned}
\tag{2.15}
$$

Let

$$
d_{ik} = \frac{1}{2g}\sum_{j=1}^{k} c_{ij}\,\Delta p_k \quad ,
\tag{2.16}
$$

where Δp_k is as defined above.

Then, Equation (2.12) can now be written as in Equation (2.17).

$$\Delta R_i = b_{si}\,\Delta t_s + \sum_{j=1}^{40} b_{ij}\,\Delta t_j - \sum_{j=1}^{15} d_{ij}\,\Delta q_j \quad . \tag{2.17}$$

This shows the explicit dependence on temperature and water vapor in the forward radiative transfer equation.

2.2.3 Matrix Solution to RTE

Equation (2.17) can be written in matrix form as Equation (2.18).

$$\boldsymbol{b} = \boldsymbol{A}\,\boldsymbol{v} \quad . \tag{2.18}$$

Here, $\boldsymbol{b}$ is the radiance difference vector, $\boldsymbol{A}$ is the radiative transfer matrix, and $\boldsymbol{v}$ is the solution difference vector as below. N is the number of spectral channels.

$$\boldsymbol{b} = \begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \bullet \\ \bullet \\ \Delta R_N \end{bmatrix} \qquad \boldsymbol{v} = \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \bullet \\ \bullet \\ \Delta t_{40} \\ \Delta t_s \\ \Delta q_1 \\ \Delta q_2 \\ \bullet \\ \bullet \\ \Delta q_{15} \end{bmatrix}$$

$$\boldsymbol{A} = \left[\begin{array}{cccc|c|cccc} b_{1,1} & \bullet & \bullet & b_{1,40} & b_{s,1} & d_{1,1} & \bullet & \bullet & d_{1,15} \\ \bullet & & & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & & & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & & & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & & & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & & & \bullet & \bullet & \bullet & & & \bullet \\ b_{N,1} & \bullet & \bullet & b_{N,40} & b_{s,N} & d_{N,1} & \bullet & \bullet & d_{N,15} \end{array}\right]$$

This represents a linearized version of the forward RTE. Section 2.3 discusses solutions to this equation given measured radiances as well as the computation of the covariance of the retrieval estimates.

2.2.4 Calculation of Channel Radiances and Transmittances

The channel radiances and transmittances used above include the effect of the instrument channel spectral response. This section details their computation.

Channel Radiances. The spectral radiance B_ν emitted by a layer with temperature T (Kelvin) at wavenumber ν is assumed to have a blackbody spectrum with unit emissivity and thus is computed with the Planck function.

$$B_\nu = \frac{C_1 \nu^3}{e^{\frac{C_2 \nu}{T}} - 1} \quad \frac{\mathrm{mW}}{\mathrm{m}^2 - \mathrm{Sr} - \mathrm{cm}^{-1}} \quad , \tag{2.19}$$

where

$$C_1 = 1.191062 \times 10^{-5} \frac{\mathrm{mW}}{\mathrm{m}^2 - \mathrm{Sr} - \left(\mathrm{cm}^{-1}\right)^4} \quad \text{and}$$

$$C_2 = 1.438786 \frac{\mathrm{K}}{\mathrm{cm}^{-1}}$$

are the appropriate constants.

The derivative of the Planck spectral radiance function with respect to temperature is shown in Equation (2.20).

$$\frac{dB_\nu}{dT} = \frac{C_1 C_2 \nu^4 e^{\frac{C_2 \nu}{T}}}{T^2 \left(e^{\frac{C_2 \nu}{T}} - 1 \right)^2} \quad \frac{\mathrm{mW}}{\mathrm{m}^2 - \mathrm{Sr} - \mathrm{cm}^{-1} - \mathrm{K}} \quad . \tag{2.20}$$

The bandwidth of a channel is assumed to be small enough that the Planck function and its derivative can be assumed to be constant; thus, the in-channel quantities of the spectral radiance and its derivative are found by multiplying them by the bandwidth of the channel. The radiance factors used in Equation (2.12) are computed for channel i with central wavenumber n_i and bandwidth Δn_i at pressure level j with mean temperature T_j as in Equations (2.21) and (2.22).

$$\overline{B_{ij}} = \Delta \nu_i \, B_\nu \Big|_{T=T_j, \nu=\nu_i} \quad . \tag{2.21}$$

$$\left(\frac{\overline{dB}}{dt} \right)_{ij} = \Delta \nu_i \frac{dB_\nu}{dt} \Big|_{T=T_j, \nu=\nu_i} \quad . \tag{2.22}$$

Channel Transmittances. The channel transmittances are computed by applying the channel spectral response $a_i(\nu)$ to high-resolution transmittances $\tau_j(\nu)$.

$$\tau_{ij} = \int_{-\infty}^{+\infty} a_i(\nu)\tau_j(\nu)d\nu \quad . \tag{2.23}$$

The high-resolution transmittances were computed for all wavenumbers at each pressure level j in wavenumber steps of Δn = 0.01 cm^{-1} using FASCOD3 [8]. A user-defined atmosphere was created for FASCOD3 using the pressure levels and mean temperature/water vapor profiles specified in Section 3 for the selected season/latitude combination. The summer midlatitude case was chosen as the default. FASCOD3 was run with the seven major atmospheric constituents including: H_2O, CO_2, O_3, N_2O, CO, CH_4, and O_2. The default FASCOD3 profiles of these constituents for the season/latitude combination were chosen, except for O_2 which was artificially set low. The reduction of O_2 level was found to result in better behaved transmittance functions. The details of the FASCOD3 input are shown in Appendix A, which contains the program used to write the input TAPE5 file.

The channel spectral responses were discretized based on their desired shape and bandwidth to match the 0.01-cm^{-1} step transmittances. Figure 1 shows an example for the case of a triangular-shaped response function with a full width half maximum (FWHM) bandwidth of 0.05 cm^{-1}.

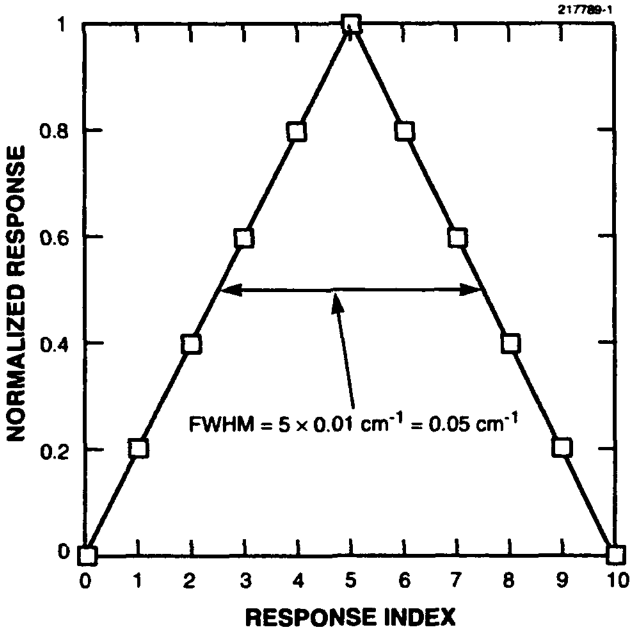

Figure 1. Triangular spectral response array for case with 0.05-cm^{-1} bandwidth.

With the spectral response of channel i defined by a_i normalized to a peak value of one with $K+1$ indices, the channel i transmittance for pressure level j is found by Equation (2.24). The transmittances are centered at the central wavenumber ν_i of the channel and summed with the weighting of the spectral response coefficients. They are then normalized by the sum of the coefficients to maintain the proper amplitude between 0 and 1.

$$\tau_{ij} = \frac{1}{\sum_{k=0}^{K} a_{ik}} \sum_{k=0}^{K} a_{ik}\, \tau_j\left[\nu_i + \Delta\nu(k - K/2)\right] \quad . \tag{2.24}$$

Computation of $d\tau_W/du$. The derivative of transmittance of a wet atmosphere with respect to precipitable water was computed by running FASCOD3 twice — once for an atmosphere with water vapor only, and a second time with 10% more water vapor. (In practice, FASCOD3 will not run with only one molecule, so a small amount of O_2 was added to the atmosphere.) The derivative was then approximated as in Equation (2.25) for all of the pressure levels and wavenumbers.

$$\frac{d\tau_W}{du} = \frac{\tau_{1.1W} - \tau_W}{1.1\bar{u} - \bar{u}} \quad . \tag{2.25}$$

2.3 ESTIMATION OF RETRIEVAL OPERATOR AND ERROR COVARIANCE

2.3.1 Retrieval Operator

While Equation (2.18) defines an ideal linear forward radiative transfer problem, a real measurement will have a noise term corrupting the radiance. Thus, we will define the measured radiance vector of the N channels to be $\boldsymbol{b}_o$,

$$\boldsymbol{b}_o = \boldsymbol{A\nu} + \boldsymbol{\varepsilon} \quad , \tag{2.26}$$

where $\boldsymbol{\varepsilon}$ is the noise term assumed to have zero mean and covariance matrix $\boldsymbol{N}$. The linear retrieval operator then is defined to be $\boldsymbol{C}$ which is used to compute an estimate $\hat{\boldsymbol{\nu}}$ of the temperature/water vapor difference profile based on the measurement.

$$\hat{\boldsymbol{\nu}} = \boldsymbol{C}\boldsymbol{b}_o \quad . \tag{2.27}$$

Several techniques can be used to find a suitable retrieval operator. For this report, a minimum variance (MV) approach is used which solves for $\boldsymbol{C}$ such that the variance of the estimate is minimized. Since the mean value is assumed to be zero, this is also a linear mean square error (LMSE) estimate. Thus, we want to minimize Equation (2.28) with respect to $\boldsymbol{C}$.

$$\sigma_{\hat{v}}^2 = E\left\{\left(v - C b_o\right)^T \left(v - C b_o\right)\right\}$$
$$= E\left\{v v^T - v^T C b_o - b_o^T C^T v + b_o^T C^T C b_o\right\} \quad . \tag{2.28}$$

Taking the derivative with respect to C and setting equal to 0 yields

$$0 = -2 E\left\{v b_o^T\right\} + 2 C E\left\{b_o b_o^T\right\} \quad .$$

Solving for C,

$$C = E\left\{v b_o^T\right\} E\left\{b_o b_o^T\right\}^{-1}$$
$$= E\left\{v (A v + \varepsilon)^T\right\} E\left\{(A v + \varepsilon)(A v + \varepsilon)^T\right\}^{-1}$$
$$= E\left\{v v^T A^T + v \varepsilon^T\right\} E\left\{A v v^T A^T + A v \varepsilon^T + \varepsilon v^T A^T + \varepsilon \varepsilon^T\right\}^{-1} \quad .$$

Since the noise term and the solution difference vector v are uncorrelated, and both have zero mean, the retrieval operator becomes as in Equation (2.29).

$$C = S A^T \left(A S A^T + N\right)^{-1} \quad . \tag{2.29}$$

Here,

$$S = E\left\{v v^T\right\} \quad .$$

To avoid taking the inverse of a large matrix (the dimension of which would be the number of channels which may be in the thousands), the following matrix identity is applied.

$$S A^T \left(A S A^T + N\right)^{-1} = \left(A^T N^{-1} A + S^{-1}\right)^{-1} A^T N^{-1} \quad . \tag{2.30}$$

Thus, the final form for the retrieval operator is shown in Equation (2.31).

$$C = \left(A^T N^{-1} A + S^{-1}\right)^{-1} A^T N^{-1} \quad . \tag{2.31}$$

2.3.2 Error Covariance

The error in the estimate of the temperature/water vapor difference vector v is described by the covariance matrix U, which is computed as follows.

$$
\begin{aligned}
U &= E\left\{(v-\hat{v})(v-\hat{v})^T\right\} = E\left\{\left(v - C b_o\right)\left(v - C b_o\right)^T\right\} \\
&= E\left\{v v^T - C b_o v^T - v b_o^T C^T + C b_o b_o^T C^T\right\} \\
&= S - C A S - S A^T C^T + C\left(A S A^T + N\right) C^T \\
&= S - C A S - S A^T \left(A S A^T + N\right)^{-1} A S^T \\
&\quad + S A^T \left(A S A^T + N\right)^{-1} \left(A S A^T + N\right) \left(A S A^T + N\right)^{-1} A S^T \\
&= S - C A S \qquad . \qquad (2.32)
\end{aligned}
$$

This can be simplified through the use of Equation (2.31) and the following matrix manipulations.

$$
\begin{aligned}
U &= S - \left(A^T N^{-1} A + S^{-1}\right)^{-1} A^T N^{-1} A S \\
&= \left[I - \left(A^T N^{-1} A + S^{-1}\right)^{-1} A^T N^{-1} A\right] S \\
&= \left[\left(A^T N^{-1} A + S^{-1}\right)^{-1} \left(A^T N^{-1} A + S^{-1}\right) - \left(A^T N^{-1} A + S^{-1}\right)^{-1} A^T N^{-1} A\right] S \\
&= \left(A^T N^{-1} A + S^{-1}\right)^{-1} \left[A^T N^{-1} A + S^{-1} - A^T N^{-1} A\right] S \qquad . \qquad (2.33)
\end{aligned}
$$

$$
U = \left(A^T N^{-1} A + S^{-1}\right)^{-1} \qquad . \qquad (2.34)
$$

The vector e of retrieval rms errors is then

$$
e = \sqrt{\mathrm{diag}(U)} \qquad (2.35)
$$

Note that throughout the analysis, the mean bias of the estimate is assumed to be zero; thus, the expected error associated with an instrument and given a priori statistics is completely described by the error covariance.

3. A PRIORI TEMPERATURE AND WATER VAPOR STATISTICS

The a priori statistics used in the retrieval error analysis were obtained from NOAA. They were derived from a database of radiosonde measurements [9]. Table 1 shows the 40 pressure levels at which the values were measured. Temperature measurements were obtained at all 40 levels plus the surface, while the water vapor measurements were obtained only up to 300 mbar. Above that, the water vapor concentration was assumed to be zero.

TABLE 1

Pressure Levels Used in Analysis (mbar)

Level	Pressure	Level	Pressure	Level	Pressure	Level	Pressure
1	1000	11	475	21	100	31	7.0
2	950	12	430	22	85	32	5.0
3	920	13	400	23	70	33	4.0
4	850	14	350	24	60	34	3.0
5	780	15	300	25	50	35	2.0
6	700	16	250	26	30	36	1.5
7	670	17	200	27	25	37	1.0
8	620	18	150	28	20	38	0.5
9	570	19	135	29	15	39	0.2
10	500	20	115	30	10	40	0.1

The statistics were obtained for six different latitude and month combinations, as listed in Table 2. Figures 2 and 3 show the mean temperature and water vapor profiles, while Figures 4 and 5 present their standard deviations.

TABLE 2

Month and Latitude Statistics

Month	Latitude Region
January	0 – 25 °N
January	25 – 55 °N
January	55 – 90 °N
August	0 – 25 °N
August	25 – 55 °N
August	55 – 90 °N

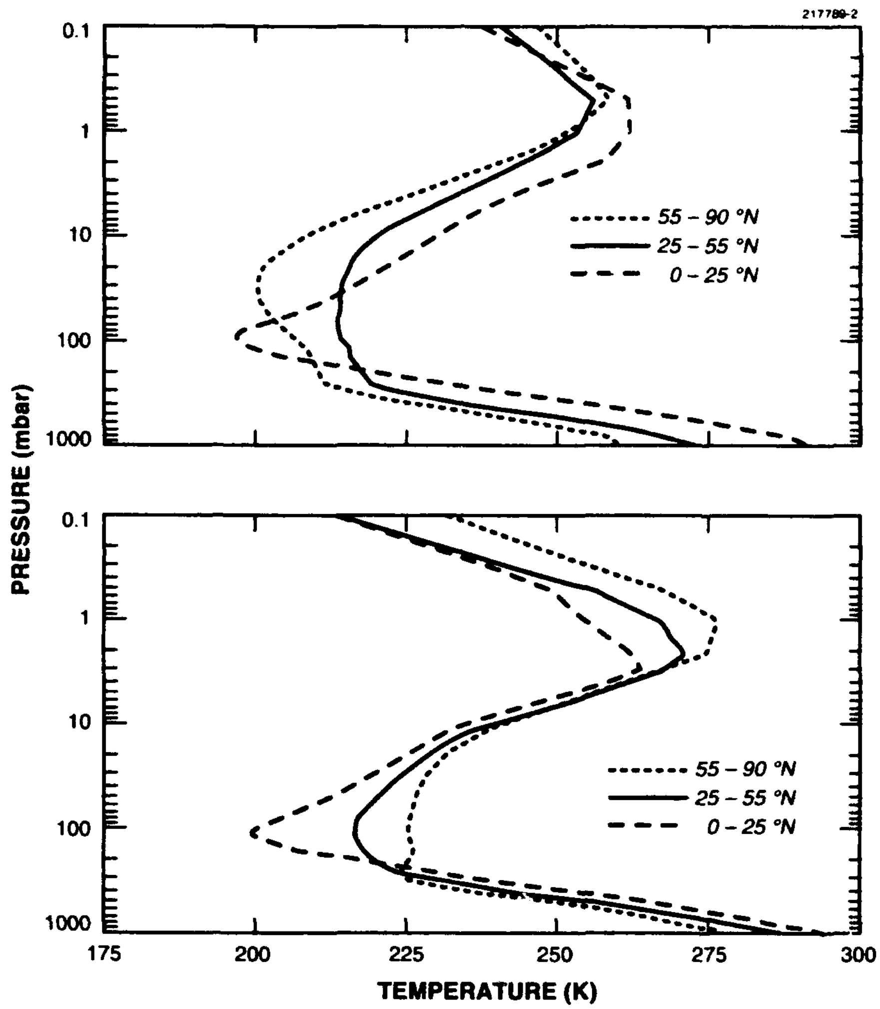

Figure 2. Mean temperature profiles for January (top) and August (bottom).

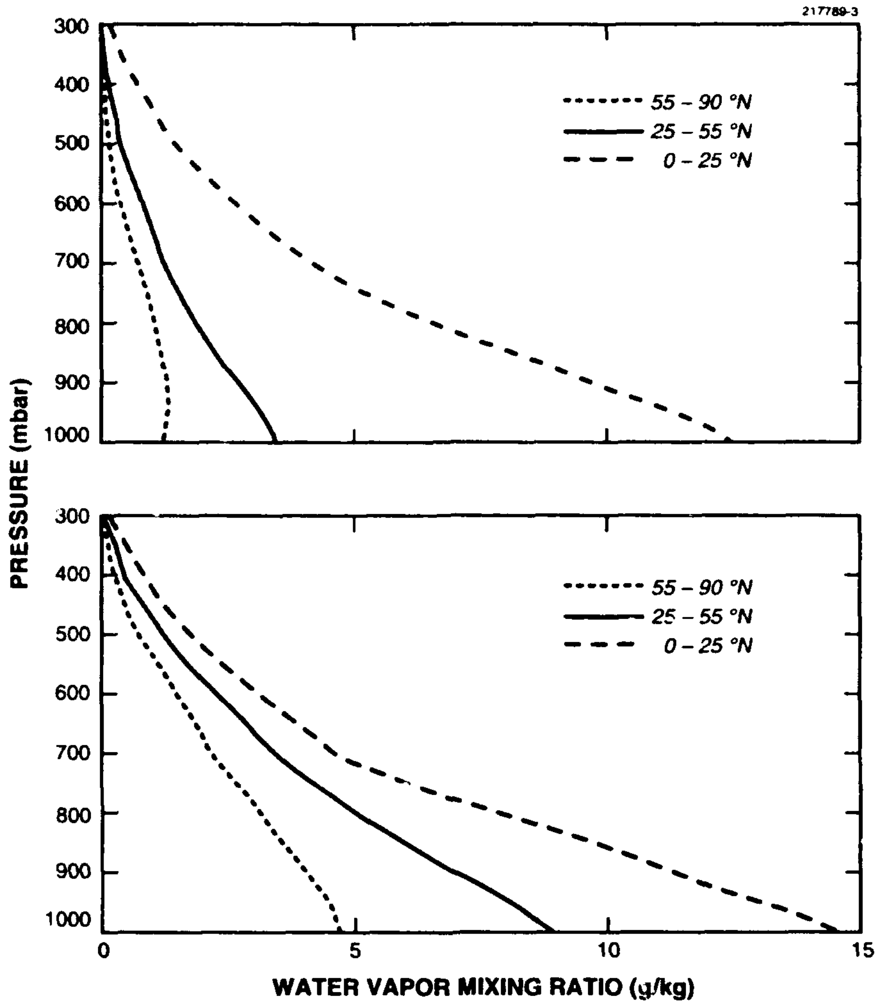

Figure 3. Mean water vapor profiles for January (top) and August (bottom).

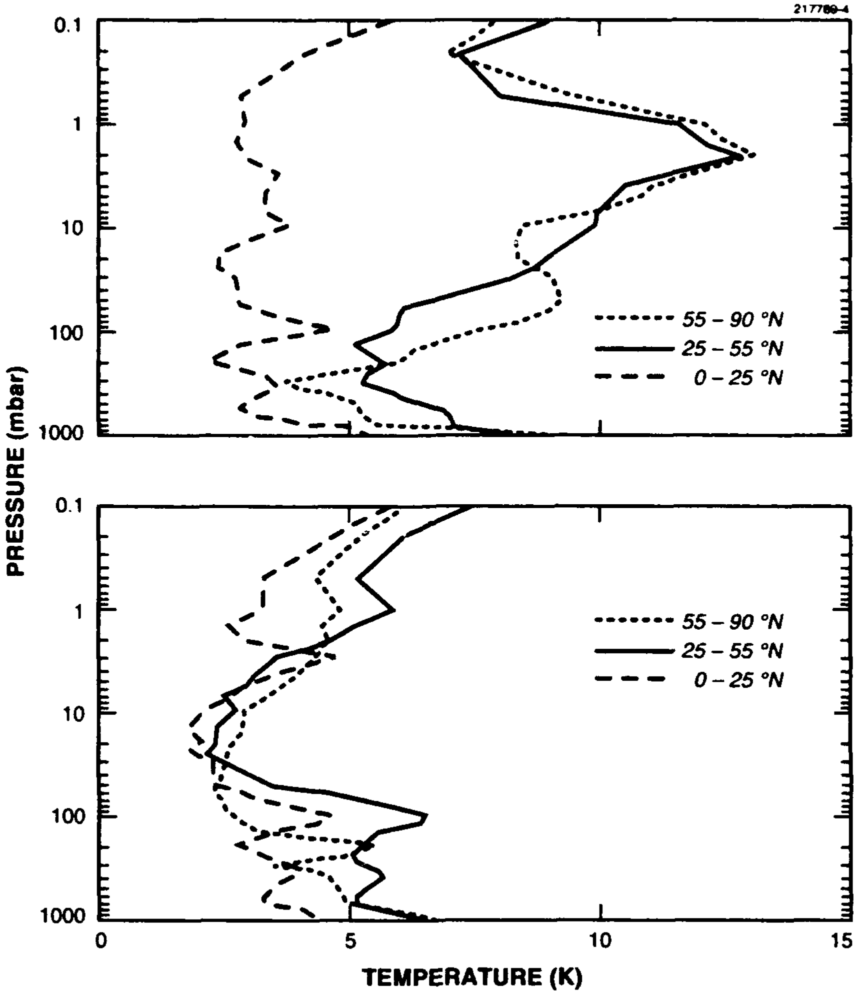

Figure 4. Standard deviation of temperature for January (top) and August (bottom).

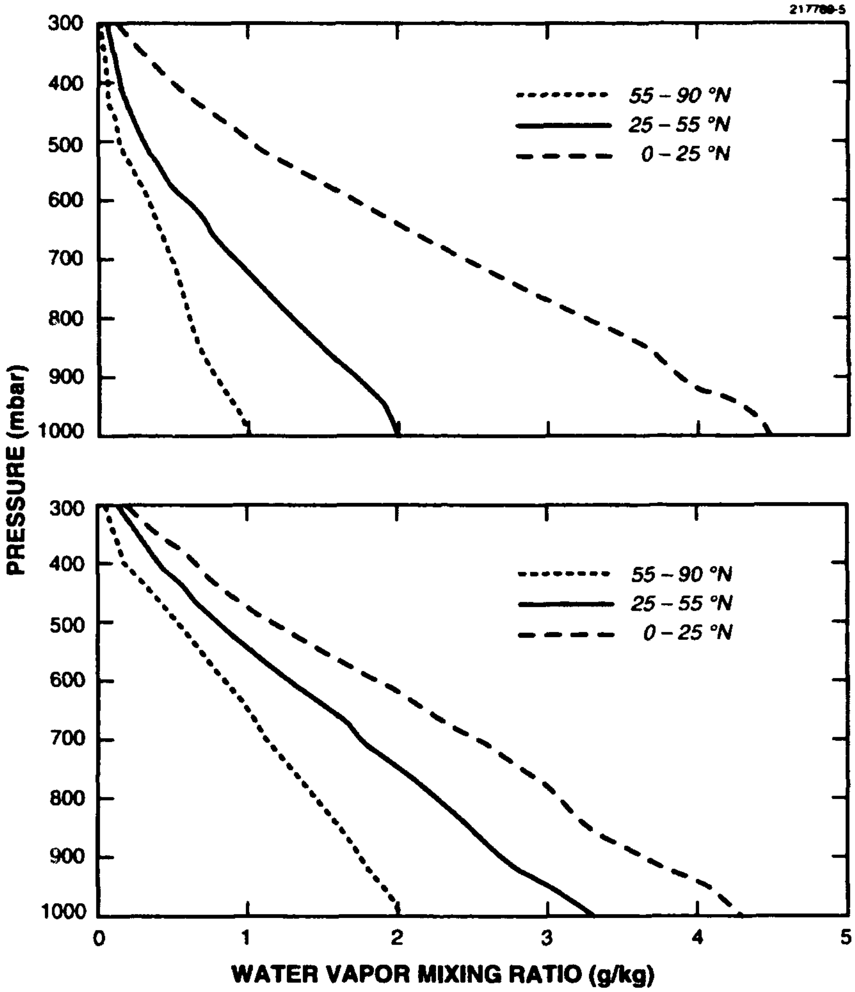

Figure 5. Standard deviation of water vapor for January (top) and August (bottom).

4. EXAMPLE RETRIEVAL ERROR ANALYSES

4.1 INTRODUCTION

This section presents two examples of how the retrieval error analysis technique can be applied to study the expected retrieval error from infrared sounding instruments. The first example examines the performance of the proposed Atmospheric Infrared Sounder (AIRS) instrument as an example of the technique applied to a high spectral resolution instrument with a large number of channels. The second example shows the application of the technique to the GOES-I sounder, a lower resolution filter-wheel type of instrument. Plots showing the expected error are presented as example results of the analyses.

A few comments on the computational implementation are in order. The algorithm was implemented in the IDL programming environment to take advantage of the built-in mathematical and matrix operations as well as to allow easy plotting and data visualization. A single parameter input file was used to define conditions for each analysis run. Also, an intermediate file was written to disk containing the A matrix for each instrument. This file allowed subsequent runs for the same instrument but with different noise levels to be completed quickly. Computations to create the A matrix generally took 1/2 to 1 h on the Silicon Graphics Indigo R4000, while the N matrix formation and error computation required only 15 s.

4.2 AIRS

The proposed AIRS is planned for deployment aboard NASA's Earth Observing System (EOS) PM satellite, which is to be launched in 2000. The AIRS instrument is currently undergoing design changes, but for illustrative purposes an instrument model based on the original specifications [10] has been implemented using the error analysis methodology.

The analysis was performed using the August midlatitude a priori statistics and an A matrix formed for the AIRS instrument using the information shown in Table 3. The specific channel locations were taken from a memo by NOAA scientist David Wark [11]. All channels were implemented except those in the ozone band (1000–1100 cm^{-1}) and those above 2400 cm^{-1}. The channel bandwidths for the AIRS instrument are specified to be $n/1200$, where n is the wavenumber. The error analysis software, however, requires the bandwidths be constant for a given spectral region. To avoid defining 1229 regions, the total spectral bandwidth was divided into ten regions, as shown in Table 4, with each channel within a region assigned the average bandwidth for that region.

TABLE 3

Model AIRS Instrument Specifications

Number of channels	1229
Channel set	WARK
Spectral response shape	Rectangle
Number of spectral regions	10

TABLE 4

Model AIRS Spectral Regions (cm^{-1})

Spectral Region	Lowest	Highest	Bandwidth
1	650	750	0.58
2	750	875	0.68
3	875	1000	0.78
4	1100	1300	1.00
5	1300	1500	1.00
6	1500	1700	1.33
7	1700	1900	1.50
8	1900	2100	1.67
9	2100	2300	1.83
10	2300	2400	1.96

The noise matrix N was formed as a diagonal matrix with each entry being the variance of the noise equivalent radiance (NEN) for that channel. The standard deviation of the NEN was calculated assuming an NEΔT = 0.25 K for a unit emissivity blackbody at 250 K using Equation (4.1).

$$\sigma_{\text{NEN}} = \Delta\nu(0.25)\frac{dB_\nu}{dT}\bigg|_{T=250\,\text{K}} , \qquad (4.1)$$

where $\Delta\nu$ is the appropriate channel bandwidth and dB_ν/dT is the derivative of the Planck spectral radiance function, as shown in Equation (2.20).

After selecting the August midlatitude case for the a priori covariance matrix S and forming the required instrument-based matrices A and N, the computation of the error covariance matrix was performed using Equation (2.34). The resulting standard deviations of the estimated profiles are shown in Figure 6 for the temperature and the water vapor mixing ratio.

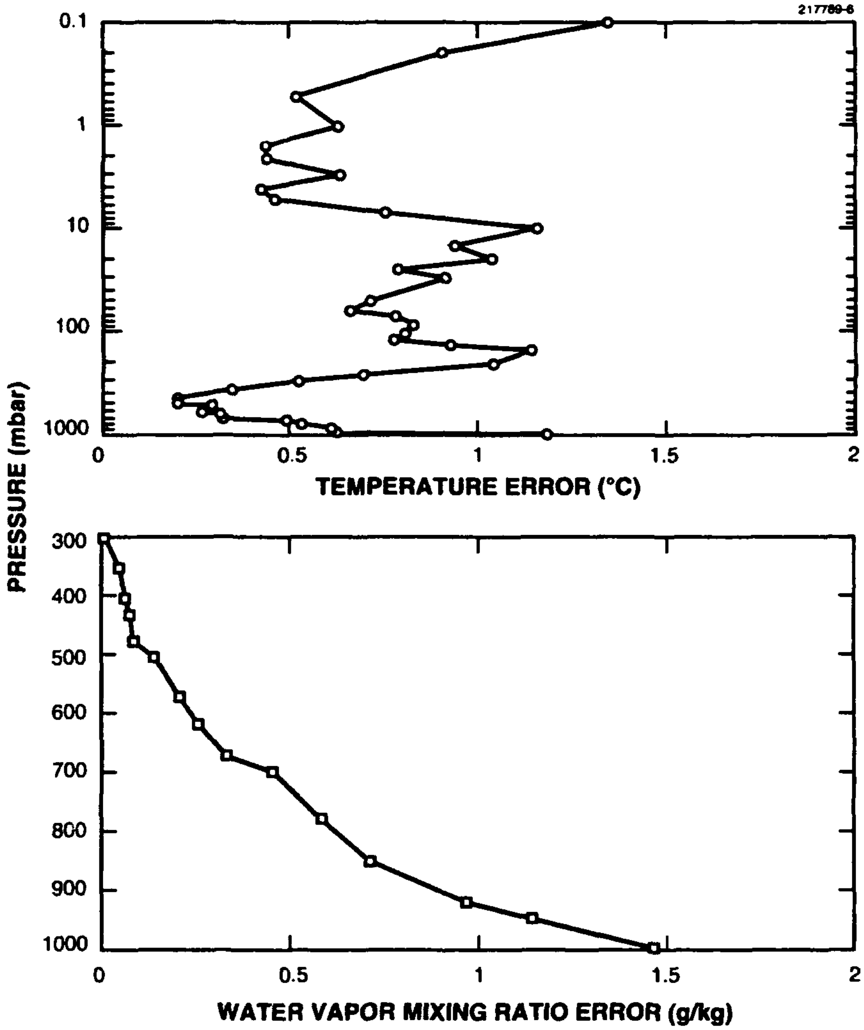

Figure 6. AIRS retrieval error for temperature (top) and water vapor (bottom).

4.3 GOES-I SOUNDER

The GOES-I satellite [12] to be launched in 1994 will carry an infrared sounder derived from the HIRS instrument on the NOAA polar satellite. This 18-channel filter-wheel instrument is expected to provide data that will result in soundings comparable to those from the polar satellite, but which will be available much more often because of the geostationary orbit.

Table 5 shows the general instrument model as implemented for this example, while Table 6 presents the specifications of the individual channels. The central wavenumbers and bandwidths have units of cm^{-1}, while the noise equivalent spectral radiance (NEΔN) is given in units of $mW/(m^2\text{-}Sr\text{-}cm^{-1})$.

Figure 7 shows the resulting errors for the temperature and for the water vapor mixing ratio when using the GOES-I sounder model. These errors are significantly higher than those estimated for the high spectral resolution sounder AIRS. This type of relative comparison of predicted errors is a useful application of this error analysis technique.

TABLE 5

Model GOES-I Sounder Specifications

Number of channels	18
Spectral response shape	Rectangle

TABLE 6

Model GOES-I Sounder Channels

Channel Number	Central Wavenumber	Bandwidth	NEΔN
1	680	13	0.66
2	696	13	0.58
3	711	13	0.[illegible]4
4	733	16	0.45
5	748	16	0.44
6	790	30	0.25
7	832	50	0.16
8	907	50	0.16
9	1030	25	0.33
10	1345	55	0.16
11	1425	80	0.12
12	1535	60	0.15
13	2188	23	0.013
14	2210	23	0.013
15	2245	23	0.013
16	2420	40	0.008
17	2513	40	0.0082
18	2671	100	0.0036

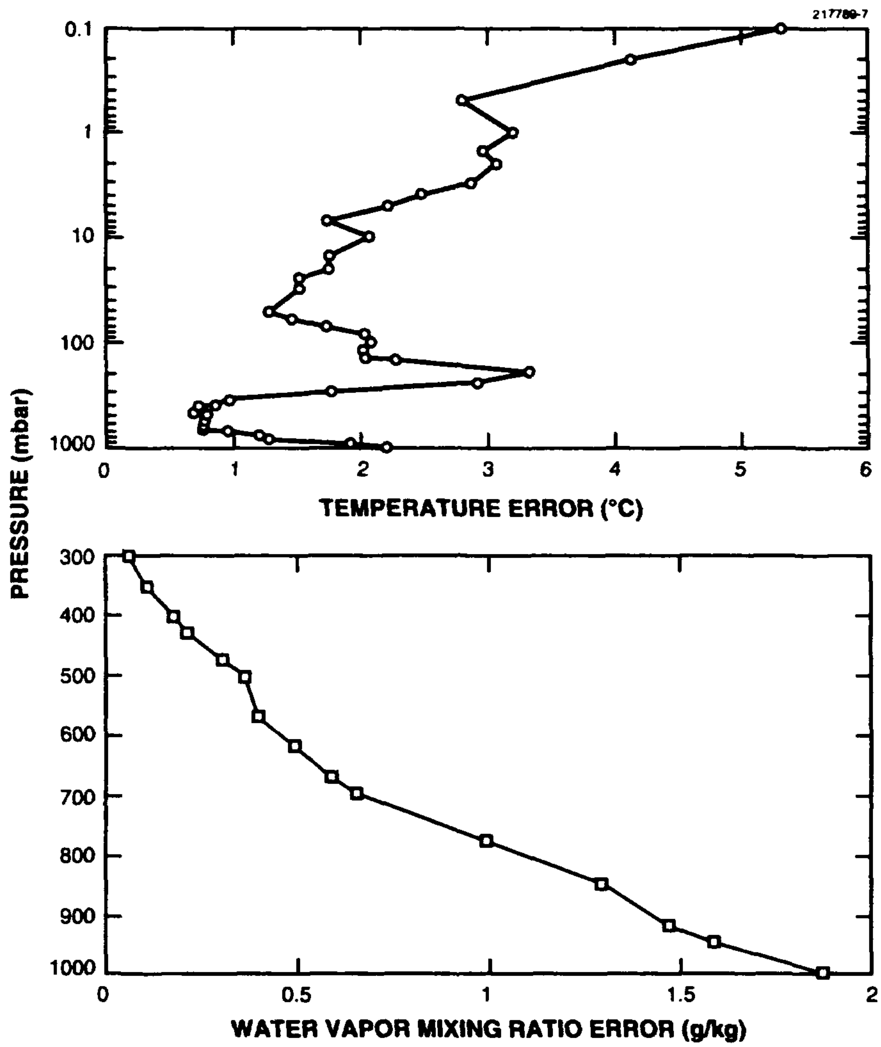

Figure 7. GOES-I sounder retrieval error for temperature (top) and water vapor (bottom).

5. SUMMARY

The derivation and implementation of a retrieval error analysis scheme for passive infrared atmospheric sounding instruments has been described. This technique is based on a linearization of the radiative transfer equation with terms for temperature and water vapor explicitly written. After construction of an instrument matrix containing factors for Planck radiance, numerical quadrature weights, and atmospheric layer transmittances, the estimated retrieval error is computed in one matrix operation. Subsequent analyses for various instrument noise levels can be computed quickly with simple replacement of the entries in the noise covariance matrix.

Examples were presented of the technique applied to a high spectral resolution sounder with a large number of channels and to a low resolution filter-wheel sounder. Predicted retrieval rms errors of temperature and water vapor for each of these instruments were presented. The estimated error for the filter-wheel sounder was significantly higher than for the high resolution sounder showing a relative comparison application of the error analysis technique.

The retrieval error analysis technique presented in this report has formed the basis for several system studies conducted at Lincoln Laboratory, and it has been found to be a useful tool in the relative comparison of various proposed sounding instruments and design options.

APPENDIX A: FASCOD3 INPUT

The following FORTRAN program was used to generate the FASCOD3 input TAPE5 file which was then used to generate the high spectral resolution layer transmittances used in the retrieval error analysis. The default conditions were a midlatitude summer atmosphere with the seven major atmospheric constituents present. The input TAPE5 file was modified slightly to produce the transmittances for the water-vapor-only situation by zeroing the levels of all constituents except the water vapor. In this case, the level of oxygen was set artificially small as it was found that FASCOD3 would not execute with only one molecule present.

The program below uses two input files: TWCM which contains the mean and covariance statistics of the temperature and water vapor profiles and SOUNDP1 which listed the 40 pressure levels corresponding to the profile entries in TWCM.

The resulting output of FASCOD3 consists of the transmittance for all wavenumbers for each of the layers in separate files. These layer transmittances then were combined to produce the transmittance from each layer to the top of the atmosphere, as was needed for the error analysis software.

```
c*****************************************************************************
c       SOUNDFAS
c
c       J. Kerekes    July 22, 1992
c
c       APPLICATION:
c
c       Program to write a TAPE5 input file for FASCODE
c       using a defined atmospheric profile with FSCATM.
c       Set up for generating 0.01 cm^-1 steps from
c       user prompted range.
c       MODEL 2 Midlatitude summer defaults
c       August 25-55 N data set.
c
c
c       INPUT FILES:
c
c       SOUNDP1         vector of pressure levels
c       TWCM            temperature and water vapor means and covariance
c                       matrices
c
c
c       OUTPUT FILES:
c
c       TAPE5           input file for FASCOD3 to generate layer by
c                       layer atmospheric transmittances over desired
c                       spectral range
c
c
```

```
c     LOCAL VARIABLES:
c
c     airm           mass of air
c     dvpar          wavenumber step size for interpolated output
c     gconst         gravity acceleration constant
c     iseaslat       index selecting season latitude combination
c                    of TWCM file
c                    = 1 January 25 - 55 N
c                    = 2 January 55 - 90 N
c                    = 3 August 55 - 90 N
c                    = 4 August 25 - 55 N
c                    = 5 January 0 - 25 N
c                    = 6 August 0 - 25 N
c     ibuf           integer buffer used in reading TWCM
c     itempsurf      index in vbar of surface temperature
c     maxlevel       maximum number of atmospheric levels
c     maxmol         maximum number of molecules in atmosphere
c     midstart       initial unit number of interim file sequence
c     numlevel       number of atmospheric levels used
c     nummol         number of molecules used in atmosphere
c     numtemp        number of atmospheric temperature layers
c     numv           total number of indices in vbar
c     numwv          number of atmospheric water vapor layers
c     outstart       initial unit number for output tau's
c     pm             array of pressure levels
c     q              water vapor mixing ratio
c     rconst         constant used in hypsometric equation
c     scov           temp/wv covariance matrices from TWCM
c     startnu        starting (lowest) wavenumber for spectral range
c     stopnu         stopping (highest) wavenumber for spectral range
c     tbar           mean temperature profile for selected iseaslat
c     tstar          tbar adjusted by mixing ratio
c     vbar           temp/wv mean vectors from TWCM
c     zdiff          altitude difference between atmospheric layers
c
c
c     Other variables defined in FASCOD3P User's Guide
c
c
      program soundfas
      parameter(maxmol=32,maxlevel=67,midstart=20,outstart=60,
     +       numlevel=41,iseaslat=4,
     +       numtemp=40,numwv=15,numv=56,itempsurf=41,nummol=7,
     +       dvpar=0.01)
      character*80 cxid
      character*24 hmod
      character*2 cmrg
      character*1 jcharp,jchart,jchar(maxlevel,maxmol)
      real zbnd(maxlevel),zm(maxlevel),pm(maxlevel),tm(maxlevel)
      real vmol(maxlevel,maxmol)
      integer*4 ibuf(3200)
      real*4 vbar(numv,6),scov(numv,numv,6)
```

```
c
c      Prompt for input frequency range
c
       write(6,*)'Enter starting wavenumber in cm^-1'
       read(5,*)startnu
       write(6,*)'Enter ending wavenumber in cm^-1'
       read(5,*)stopnu
       write(6,'(a,2f8.2)')' Range is ',startnu,stopnu
c
c      Open temperature/water vapor statistics file
c      and read in arrays
c
       open(unit=11,file='TWCM',status='unknown')
       rewind(11)
       do 30 i=1,6
          read(11,*) (ibuf(j),j=1,3200)
          do 10 j=1,56
              vbar(j,i)=ibuf(j)*1e-6
10            continue
          incr=56
          do 20 j=1,56
          do 20 k=1,56
              incr=incr+1
              scov(j,k,i)=ibuf(incr)*1e-6
20            continue
30        continue
       close(11)
c
c      Open and read in pressure profile
c
       open(unit=12,file='SOUNDP1',status='unknown')
       rewind(12)
       do 15 l=1,numlevel
          read(12,*)pm(l)
15        continue
       close(12)
c
c      Open TAPE5 file
c
       open(unit=10,file='TAPE5',status='unknown')
       rewind(10)
c
c      Prompt for user identification label
c      Record 1.1
c
       write(6,*)'Please enter user label (starting with $)'
       read(5,'(a80)')cxid
       write(10,'(a80)')cxid
c
c      Record 1.2
c
```

```
      ihirac=1
      ilblf4=2
      icntnm=1
      iaersl=0
      iemit=1
      iscan=2
      ifiltr=0
      iplot=1
      itest=0
      iatm=1
      cmrg='01'
      ilas=0
      ims=0
      ixsect=0
      irad=0
      mpts=-1
      npts=-1
      write(10,102)ihirac,ilblf4,icntnm,iaersl,iemit,iscan,ifiltr
     +          ,iplot,itest,iatm,cmrg,ilas,ims,ixsect,irad,
     +          mpts,npts
102   format(10(4x,i1),3x,a2,3(4x,i1),i1,i4,1x,i4)
c
c     Record 1.2.1 is omitted since ims=0
c
c
c     Record 1.3
c
      v1=startnu
      v2=stopnu
      sample=4
      dvset=0.0
      alfao=0.08
      avmass=36.0
      dptmin=0.0
      dptfac=0.001
      write(10,103)v1,v2,sample,dvset,alfao,avmass,dptmin,dptfac
103   format(8f10.3)
c
c     Record 1.4
c
      tbound=vbar(itempsurf,iseaslat)
      sremis1=0.95
      sremis2=0.0
      sremis3=0.0
      srrefl1=1.0-sremis1
      srrefl2=0.0
      srrefl3=0.0
      write(10,104)tbound,sremis1,sremis2,sremis3,srrefl1,srrefl2
     +          ,srrefl3
104   format(7f10.3)
c
c     Skip records 2 (they are not used here)
```

```
c
c
c       Record 3.1 (modified for user defined profile)
c
        model=0
        itype=2
        ibmax=numlevel
        nozero=0
        noprnt=1
        nmol=nummol
        ipunch=1
        re=0.0
        hspace=100.0
        freqbar=0.0
        co2mx=0.0
        write(10,301)model,itype,ibmax,nozero,noprnt,nmol,ipunch,
     +            re,hspace,freqbar,co2mx
301     format(7i5,5x,4f10.3)
c
c       Record 3.2
c
        h1=100.0
        h2=0.0
        angle=180.0
        range=0.0
        beta=0.0
        len=0
        write(10,302)h1,h2,angle,range,beta,len
302     format(5f10.3,i5)
c
c       Record 3.3b
c       zbnd(l) is the height of level l
c
c       Level 1 is at surface, Level 41 is top of atmosphere (0.01 mbar)
c       Use Hypsometric equation from General Meteorology by Byers
c
        rconst=8.3143e7
        airm=28.9
        gconst=980.665
c
c       For levels 1 through 15 use water vapor mixing ratio
c
        zbnd(1)=0.0
        do 23 l=2,15
           tbar=vbar(itempsurf-l,iseaslat)
           q=0.001*vbar(numv-l+1,iseaslat)
           tstar=tbar/(1.0-0.6*q)
           zdiff=(1e-5)*((rconst*tstar)/(airm*gconst))*
     +            (alog(pm(l-1))-alog(pm(l)))
           zbnd(l)=zbnd(l-1)+zdiff
23         continue
c
```

```
c       For levels 16 through 41 assume water vapor is zero
c
        do 25 l=16,numlevel-1
           tstar=vbar(itempsurf-l,iseaslat)
           zdiff=(1e-5)*((rconst*tstar)/(airm*gconst))*
     +             (alog(pm(l-1))-alog(pm(l)))
           zbnd(l)=zbnd(l-1)+zdiff
25         continue
c
c       Do level 41 using temperature of level 40 plus t39-t40
c
        tstar=vbar(1,iseaslat)-(vbar(2,iseaslat)-vbar(1,iseaslat))
        zdiff=(1e-5)*((rconst*tstar)/(airm*gconst))*
     +          (alog(pm(40))-alog(pm(41)))
        zbnd(41)=zbnd(40)+zdiff
        write(10,303)(zbnd(l),l=1,8)
        write(10,303)(zbnd(l),l=9,16)
        write(10,303)(zbnd(l),l=17,24)
        write(10,303)(zbnd(l),l=25,32)
        write(10,303)(zbnd(l),l=33,40)
        write(10,303)(zbnd(l),l=41,numlevel)
303     format(8f10.3)
c
c       Record 3.4, User comments
c
        inmax=ibmax
        hmod=' Sounder Study 4 Sum Mid'
        write(10,304)inmax,hmod
304     format(i5,a24)
c
c       Record 3.5, 3.6 repeat for each layer
c
c       Get water vapor in mixing ratio (jchar(*,1)=C)
c       H20 is molecule number 1
c
        do 33 l=1,numwv
           vmol(l,1)=vbar(numv-l+1,iseaslat)
33         continue
        do 32 l=numwv+1,numlevel
           vmol(l,1)=0.0
32         continue
c
c       Get altitudes and temperatures
c
        do 34 l=1,inmax-1
              zm(l)=zbnd(l)
              tm(l)=vbar(itempsurf-l,iseaslat)
34            continue
        zm(41)=zbnd(41)
        tm(41)=vbar(1,iseaslat)-(vbar(2,iseaslat)-vbar(1,iseaslat))
        jcharp='A'
        jchart='A'
```

```
c
c     Set up models to use for other molecules
c     ***************model=6 is US Standard
c     ***************model=2 is summer midlatitude
c
      do 37 l=1,numlevel
         jchar(l,1)='C'
         do 35 k=2,nmol
             jchar(l,k)='2'
             vmol(l,k)=0.0
35           continue
c
c     MODIFICATION FOR O2 HERE
c     (reduced O2 level led to smoother tau's)
c
         jchar(l,7)='C'
        vmol(l,7)=0.001
37       continue
      do 36 l=1,inmax
             write(10,305)zm(l),pm(l),tm(l),
     +       jcharp,jchart,(jchar(l,k),k=1,nmol)
             write(10,306)(vmol(l,k),k=1,nmol)
36           continue
305   format(3f10.3,5x,2a1,3x,28a1)
306   format(8f10.3)
c
c     Record 9.1 (repeat for all layers, end with negative dv)
c
      dv=dvpar
      v1=v1
      v2=v2
      jemit=0
      i4pt=0
      iunit=10
      nfils=1
      npts=-1
      do 91 ifilst=1,inmax-1
         junit=midstart+ifilst-1
         write(10,901)dv,v1,v2,jemit,i4pt,iunit,ifilst,nfils,
     +                junit,npts
91       continue
c
c     end with negative dv
c
      dv=(-1.0)*dv
      write(10,901)dv,v1,v2,jemit,i4pt,iunit,ifilst,nifils,
     +          junit,npts
901   format(3f10.3,2i5,15x,5i5)
c
c     Record 11.1
c
```

```
      write(10,'(a30)')cxid
c
c     Record 11.2a (repeat 11.2a and 11.3a for number of layers,
c              end with negative v1)
c
c     Records 11.2a and 11.3a are used to produce ascii files o
c     the transmittances. Other "plotting" variables are arbitrary.
c
      xsize=9.0
      delv=5.0
      numsbx=5
      noendx=1
      lskipf=0
      scale=1.0
      iopt=0
      i4p=0
      ixdec=0
c
c     Record 11.3a
c
      ymin=0.0
      ymax=0.9
      ysize=6.5
      dely=0.1
      numsby=5
      noendy=0
      idec=2
      jemit=0
      jplot=0
      logplt=0
      jhdr=1
      jdummy=0
      jout=3
      do 12 l=1,inmax-1
         lfile=midstart+l-1
         write(10,112)v1,v2,xsize,delv,numsbx,noendx,lfile,
     +             lskipf,scale,iopt,i4p,ixdec
         jpltfl=outstart+l-1
         write(10,113)ymin,ymax,ysize,dely,numsby,noendy,idec,jemit,
     +          jplot,logplt,jhdr,jdummy,jout,jpltfl
12       continue
112   format(4f10.4,4i5,f10.3,i2,i3,i5)
113   format(2f10.4,2f10.3,6i5,i2,i3,i2,i3)
c
c     End with negative v1
c
      v1=(-1.0)*v1
      write(10,112)v1,v2,xsize,delv,numsbx,noendx,lfile,
     +          lskipf,scale,iopt,i4p,ixdec
c
```

```
c       End record
c
        write(10,'(a5)')'% End'
        close(10)
        stop
        end
```

LIST OF SYMBOLS

A	Radiative transfer matrix
C	Retrieval operator matrix
N	Sensor noise covariance matrix
S	A priori covariance matrix of temperatures and water vapor mixing ratios
U	Retrieval error covariance matrix
$E\{\bullet\}$	Expectation operator
B	Radiance emitted by layer of atmosphere in a particular spectral channel
B_v	Spectral radiance emitted by layer of atmosphere
C_1	Planck function constant = 1.191062×10^{-5} mW/m^2-Sr-(cm^{-1})^4
C_2	Planck function constant = 1.438786 K/cm^{-1}
R	Upwelling radiance received by satellite in a particular spectral channel
T,t	Temperature of layer of atmosphere (Kelvin)
X	Layer in model atmosphere
b	Vector of radiance differences between mean and true value
b_o	Vector of radiance differences between mean and measured value
e	Vector of retrieval error standard deviations
v	Solution vector of temperature and water vapor differences between mean and true value
$\hat{v}$	Estimated vector of temperature and water vapor differences between mean and measured value
ε	Vector of sensor noise radiance
$a_i(\nu)$	Channel i spectral response function
a_{ik}	Channel i spectral response at index k
p	Atmospheric pressure (mbar)
q	Water vapor mixing ratio (g/kg)
u	Precipitable water (cm)
ν	Wavenumber (cm^{-1})
τ	Transmittance from a particular layer to the top of the atmosphere
τ_D	Transmittance to top of atmosphere with no water vapor present
τ_W	Transmittance to top of atmosphere with only water vapor present

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John P. Kerekes

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Lincoln Laboratory, MIT
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To support the design and analysis of passive infrared atmospheric sounding instruments, an analytical error analysis technique has been implemented. This technique is based on a linear approximation to the radiative transfer equation and uses a minimum variance estimation approach to atmospheric profile retrieval. As a result, once the proper matrices have been constructed an estimate on the retrieval error is computed through a single linear matrix equation. The solution vector is written with temperature and water vapor explicitly defined, thus producing the error vector for both simultaneously. Examples showing typical results are presented for a high spectral resolution sounder as well as for a lower resolution, filter wheel type of instrument.

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