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Mathematical Model of Frost Heave and Thaw Settlement in Pavements

Gary L. Guymon, Richard L. Berg and Theodore V. Hromadka

April 1993

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Abstract

Since 1975 the U.S. Army Corps of Engineers, the Federal Highway Administration and the Federal Aviation Administration have been working cooperatively to develop a mathematical model to estimate frost heave and thaw weakening under various environmental conditions and for various pavement designs. A model has been developed. It is a one-dimensional representation of vertical heat and moisture flux, is based on a numerical solution technique termed the nodal domain integration method, and estimates frost heave and frost penetration reasonably well for a variety of situations. The model is now ready for additional field evaluation and implementation in appropriate cases. The main objectives of this report are: 1) to describe the model, FROST, including modeling uncertainties and errors; 2) to summarize recent comparisons between measured and computed values for frost heave and frost penetration; and 3) to describe parameters necessary for input into the model.

Cover: Instrumentation at Albany County Airport.

For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380-89a, *Standard Practice for Use of the International System of Units,* published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

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US Army Corps of Engineers
Cold Regions Research & Engineering Laboratory

Mathematical Model of Frost Heave and Thaw Settlement in Pavements

Gary L. Guymon, Richard L. Berg and Theodore V. Hromadka

April 1993

Prepared for
U.S. DEPARTMENT OF TRANSPORTATION
and
OFFICE OF THE CHIEF OF ENGINEERS

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PREFACE

This report was prepared by Gary L. Guymon, Professor, Department of Civil Engineering, University of California, Irvine, Dr. Richard L. Berg, Research Civil Engineer, Civil and Geotechnical Engineering Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory, and Theodore V. Hromadka II, Professor, Department of Mathematics, University of California, Fullerton.

Funding for this work was provided by the Federal Highway Administration, the Office of the Chief of Engineers and the Federal Aviation Administration. The authors thank them for their confidence that a working frost heave model could be developed. None had existed before, and there was and still is a lack of complete knowledge of the mechanics of freezing soil at ice segregation points.

While the authors have full responsibility for any shortcomings of the model, they are indebted to the advisory committee on this project who devoted much time to reviewing the work. They are Professor D. Fredlund, University of Saskatchewan; Professor M. Harr, Purdue University; Professor Emeritus R. Miller, Cornell University; E. Penner, retired, National Research Council of Canada; and Professor M. Witczak, University of Maryland. Finally, over the years numerous graduate students at the University of California, Irvine (UCI), and staff at CRREL have contributed to the overall modeling effort. In particular, the authors thank J. Ingersoll, retired, of CRREL, who did such a masterful job in the laboratory.

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SELECTED CONVERSION FACTORS

Length:	1 ft = 30.48 cm = 0.3048 m
	1 in. = 2.54 cm
Volume:	1 ft^3 = 7.48 gal (U.S.) = 0.02832 m^3 = 28.32 L
Mass:	1 lbm = 453.59 g
	1 kg = 2.2046 lbm
Pressure:	1 kPa = 0.14504 lbf/$in.^2$ (psi)
	1 atm = 101.3 kPa = 1.013 bars
Energy:	1 Btu = 252 cal
Power:	1 Btu/s = 1055 W = 252 cal/s
Specific heat:	1 Btu/lbm °R = 1000 cal/kg K = 1 cal/g K
Speed:	1 ft/s = 30.48 cm/s
Temperature:	°F = 1.8 (°C) + 32
	°C = (°F −32)/1.8
Heat transfer:	1 Btu/ft^2 s = 1.136 × 10^4 W m^2 = 0.27 cal/cm^2 s
Hydraulic conductivity:	1 cm/hr = 0.79 ft/day = 5.89 gal/ft^2 day

NOMENCLATURE

A_w, a	Gardner fit coefficients for soil moisture characteristics
A_k, b	Gardner fit coefficients for hydraulic conductivity function
C_m	volumetric heat capacity of soil–liquid-water–ice mixture
C_i	volumetric heat capacity of ice
C_w	volumetric heat capacity of water
C_s	volumetric heat capacity of mineral soil
E	phenomenological calibration factor for partly frozen soil
g	gravitational constant
h	total hydraulic head ($h = h_p + h_e$)
h_e	elevation head ($h_e = -x$)
h_o	vertical total stress expressed as hydraulic head
h_L	column bottom hydraulic head
h_p	pressure head ($h_p = u/\gamma_w$)
k_s	saturated hydraulic conductivity (unfrozen soil)
K_F	hydraulic conductivity of partly frozen soil
K_H	hydraulic conductivity (unfrozen soil) [$K_H = K_H(h_p)$]
K_T	thermal conductivity of soil–liquid-water–ice mixture
K_i	thermal conductivity of ice
K_w	thermal conductivity of water
K_s	thermal conductivity of mineral soil
ℓ_e	element length
L	latent heat of fusion of water
m_v	coefficient of volume compressibility
N_o	Corps of Engineers *n*-factor
P_o	surcharge pressure
P_L	lower pore pressure head
Q	heat flux
S	degree of saturation
t	time
T	temperature
T_f	freezing point depression of water
T_L	column bottom boundary temperatures
T_o	air temperature
T_u	column top boundary temperature
u	pore fluid pressure
v	liquid water velocity flux
x	coordinate (positive downward)
y	frost heave
θ_i	volumetric ice content
θ_n	volumetric unfrozen water content factor for frozen soil
θ_o	porosity
θ_s	volumetric segregated ice content
θ_u	volumetric water content (unfrozen)
γ	unit weight of soil, water and ice
γ_w	unit weight of water ($\gamma_w = g\rho_w$)
ρ_i	density of ice
ρ_s	density of soil
ρ_w	density of water
σ'	vertical effective stress
σ_o	vertical total stress

Mathematical Model of Frost Heave and Thaw Settlement in Pavements

GARY L. GUYMON, RICHARD L. BERG AND THEODORE V. HROMADKA

INTRODUCTION

Agencies responsible for pavement design and maintenance have a large investment in their pavement systems. In frost areas, these agencies generally manage their existing pavements and design new pavements to provide a reasonable degree of protection against the detrimental effects of frost action. To date, unfortunately, rigorous methods have not been developed for evaluating various alternative designs with respect both to the amount of frost heave each would experience and to the vulnerability of each to accelerated damage caused by thaw weakening.

Investigation background

Since 1975 the U.S. Army Corps of Engineers, the Federal Highway Administration and the Federal Aviation Administration have been working cooperatively to develop a mathematical model to estimate frost heave and thaw weakening under various environmental conditions and for various pavement designs. The study, conducted by CRREL, consists of the following eight research and verification phases:

1. Development of frost heave model:
   Select research team.
   Develop mathematical model.
   Test model.
2. Development of work plan for field studies.
3. Determination of frost-susceptibility:
   Review laboratory test methods.
   Conduct laboratory tests.
4. Mathematical modeling of frost action:
   Refine frost heave model.
   Develop a thaw-weakening model.
5. Development and use of laboratory soil column device:
   Design and construct equipment.
   Characterize soils.
   Analyze results.
6. Development of thaw-weakening index of subgrade soils:
   Conduct laboratory tests.
   Conduct field tests.
7. Investigations at field test sites:
   Select sites.
   Measure important parameters.
8. Analysis and verification:
   Make recommendations.
   Outline guidelines for design and construction

Phases 1 and 2, including initial development of the model, were completed in early 1979 and are documented by Berg et al. (1980a). The model was refined and frost heaves computed by the model were compared with observations of heave in laboratory samples and in full-scale field test sections as part of phases 3–7 (Berg et al. 1980a, Guymon et al. 1980, Guymon et al. 1981a,b, Guymon et al. 1983). Parts of phase 8 are contained in the reports and articles listed above; others are in Chamberlain (1987), Johnson et al. (1986a,b,c) and Cole et al. (1986, 1987)

Objectives

Comparisons cited above and those contained in this report indicate that the mathematical model estimates frost heave and frost penetration reasonably well for a variety of situations. The model is now ready for additional field evaluation and implementation in appropriate cases. The main objectives of this report are: 1) to describe the model, FROST, including modeling uncertainties and errors; 2) to summarize recent comparisons between measured and computed values for frost heave and frost penetration; and 3) to describe parameters necessary for input into the model.

Description of model

The model is a one-dimensional representation of vertical heat and moisture flux and is based on a numerical solution technique termed the nodal domain integration method. Initial model development (Berg et al. 1980a) used the finite element method, but recently we have adopted the nodal domain integration method because it allows use of the same computer program to solve a problem by the finite element method, the integrated finite difference method or any other mass lumping numerical method.

Several mathematical models that calculate simultaneous heat and moisture flux have appeared in the literature (e.g., Harlan 1973, Guymon and Luthin 1974, Sheppard et al. 1978, O'Neill and Miller 1980, Taylor and Luthin 1978, Hopke 1980). Some models use a finite difference method and others a finite element method, but all of the models solve the same basic equations. The major differences among the models are in simulating processes within the freezing zone. Although this zone may be only a few millimeters thick, it controls the volume of moisture movement within the entire system. Unfortunately, the physical, chemical and mechanical processes taking place in the freezing zone are not well understood, nor does agreement exist on the interrelationships among the processes. We believe that the model described here simulates phenomena in the freezing zone adequately for our present purpose, and that it will meet the needs of practicing pavement engineers for estimating frost heave and some of the parameters influencing thaw weakening of pavements. More complex models await a more complete understanding and formulation of processes in the freezing zone.

The model presented in this report has primarily been developed and tested for noncohesive frost-susceptible soils with grain sizes ranging from silts to dirty gravels. The model has been used for cohesive soils—e.g., clays—but the results have not been as thoroughly validated.

The scientist or engineer who may not be familiar with the processes of ice segregation in soil may wish to review the Polar Research Board (1984) report on *Ice Segregation and Frost Heaving*, which also contains an extensive bibliography of the important literature in this area to the early 1980's. Penner et al. (1983) describe various aspects of the phenomena in *Frost Heave and Ice Segregation*. Anderson et al. (1984), who contributed to the Polar Research Board report, discuss the principles of ice segregation. Chamberlain and Gaskin (1984) discuss the various state and regional methods for classifying frost-susceptible soils, the soils of interest in this report. Kay and Perfect (1988) review current understanding of heat and mass transfer in freezing soils.

MODEL

This section describes the manner in which the mathematical model has been constructed. At this time, the model is intended for use with noncohesive soils, although it has been applied to cohesive soils. The model is intended for use with seasonally freezing and thawing soils below pavements where the maximum frost penetration is above the water table. The model is intended for use where surcharge effects are not large (usually less than 60 kPa).

The strategy employed recognizes that the zone in which the most crucial processes take place is normally very thin by comparison with the depth of soil beneath a pavement. During downward freezing of a uniform or horizontally stratified soil, the soil profile can be viewed as having three zones. The uppermost zone is "fully frozen." The lowermost zone is "fully unfrozen." Between them is a descending "zone of freezing," which, in effect, is importing fully unfrozen soil and exporting fully frozen soil. To the extent that the volume of soil being exported exceeds the volume being imported, the soil is "heaving."

The numerical solution scheme used requires that the soil be divided into horizontal "elements" by appropriately spaced "nodes." Time must be subdivided into discrete increments required for accurate solutions of the model. During each period, elements being frozen gain a certain amount of liquid water and sensible heat if both are moving upward through the lower boundary. Meanwhile, elements lose a certain amount of sensible heat that diffuses upward through the upper boundary. Knowing the initial and final temperatures of the elements, the initial water contents and the final ice contents (including segregated ice), one can arrive at the net export of thermal energy from the elements during the time elapsed. Knowing the initial water content and the influx of water from below, one can arrive at the final ice content. To the extent that the final ice content exceeds the initial pore volume of the element, the element must have expanded, producing a corresponding increment of heave.

The model reconciles, over time, net exports of thermal energy from the moving zone of freezing with thermal boundary conditions of the system, while at the same time it reconciles the flow of water and accumulation of pore ice and segregated ice with hydraulic boundary conditions and load to be heaved.

To generate the required information, one must stipulate some mechanism, real or hypothetical, within elements being traversed by the zone of freezing. The mechanism devised for use in this model actually embraces separate mechanisms that operate in series over an element as a device for separating processes that in real soils involve series–parallel mechanisms operating in a much narrower zone of freezing. To achieve this separation of functions, the freezing element is treated as if it were a "short circuit" for thermal diffusion during solution of the thermal problem and is therefore represented as being isothermal. This tactic allows simultaneous solutions of the heat and water flow problems using conventional numerical methods in each case, decoupled by the series connection between the processes in the two layers but recoupled by the release of latent heat at the common boundary.

Solution of the hydraulic problem is based on an assumed characteristic value of (negative) water pressure at the top of a freezing element. This characteristic value, however, is systematically displaced toward zero water pressure by an amount corresponding to the current weight of overlying material (including any surface load) per unit area. This has the effect of reducing the calculated rate of frost heave, whereupon the solution of the thermal problem demands an increase in the rate of traverse of the element by the zone of freezing, i.e., an increase in the rate of penetration of the frost line for the stipulated boundary conditions. This procedure involves finding a suitable constant value of unfrozen water content in the overlying frozen element.

Within the fully unfrozen soil, the hydraulic conductivity is assumed to be a function of the pore water pressure, as would be determined during a drying process for the unfrozen soil. In the zone of freezing, however, the hydraulic conductivity is taken to be the same function of pore water pressure, except it is reduced by an empirical exponential function of ice content and unfrozen saturated hydraulic conductivity.

Model uncertainty and particularly uncertain parameters are evaluated using a universal probability function that was developed by using a two-point probability method, applied to a number of numerical simulations of frost heave. Model simulation results are presented in terms of confidence limits as well as deterministic results.

The number of materials upon which the model has been tested is relatively small and all of these are noncohesive soils. Accordingly, there is no way of knowing at this time whether performance of the model in the case of cohesive soils will approach its apparently excellent performance with the noncohesive soils involved in most tests to date.

Main features and assumptions

The main assumptions embodied in the model are as follows:

1. Moisture transport in the unfrozen zone is governed by the unsaturated flow equation based upon continuity and Darcy's law.
2. Moisture flow is by way of liquid movement and vapor flow is negligible.
3. Moisture flow in the frozen zone is negligible and there is no moisture escape or addition at the frozen soil surface.
4. Soil deformations in the unfrozen zone are negligible.
5. Soil pore water pressures in the freezing zone are governed by an unfrozen water content factor.
6. All processes are single valued, i.e., there is no hysteresis.
7. Heat transport in the entire soil column is governed by the sensible heat transport equation, including an advective term.
8. Salt exclusion processes are negligible, i.e., the unfrozen water content is constant with respect to temperature.
9. Phase change effects and moisture effects can be modeled as decoupled processes.
10. Freezing or thawing can be approximated as an isothermal phase change process.
11. During thawing, settlement in the thaw zone is dominant and consolidation effects are negligible.
12. Constant parameters are invariant with respect to time.
13. All parameter and model uncertainty can be incorporated into a universal probability model applicable to a specific class of soils.

Mathematical basis

A number of investigators have sought ways to model the complex frost heave process. Hopke (1980), Guymon et al. (1980) and O'Neill (1983) review these attempts, which generally include solution of the coupled heat and moisture transport problem. There are considerable differences in approaches taken to model ice segregation processes and incorporate overburden effects. Most investigators model phase change effects by using the apparent heat capacity concept (e.g., Nakano and Brown 1971), which yields satisfactory results when one is considering heat transport alone in freezing and thawing soils. However, Hromadka et al. (1981a) show that, when considering the coupled heat and moisture transport problem for freezing or thawing soils, there are undesirable restraints on the apparent heat capacity parameter when thermal or moisture content gradients are approximately linear in the frequency region. They suggest an isothermal phase change approximation, which is used in our model. Additionally, there are certain numerical efficiency advantages to this approach. Mu and Ladanyi (1987) developed a numerical model of coupled heat and moisture movement in freezing soil and accounted for the effects of stress on pore water pressures in the freezing zone. These effects are accounted for in our model.

The model developed here does not include the effects of solutes. Cary's (1987) frost heave model included solute effects; he concluded that the increasing salt content decreases heave. Our model is intended primarily for cases where solute concentrations in soil water are low.

Another significant difference in models is the manner in which overburden effects are considered, if at all. Most theories of frost heave, such as those of Everett (1961) and Penner (1957), rely on the so-called "capillary theory." Stresses on film ice are related to pore water pressures and ice/water interface tensions. Although earlier versions of our model adopted this theory (Berg et al. 1980a), our current version computes pore water pressures (neutral stresses) from total overburden and surcharge stresses in a finite freezing volume, provided that there is ice segregation at the freezing front. If segregated ice is not present, FROST assumes that the soil matrix is supporting the total overburden and surcharge stresses.

Most investigators use finite difference methods to solve the partial differential equations of state. As will be shown later, the model adopted here incorporates the nodal domain integration method (Hromadka et al. 1982), which was an outgrowth of the research reported here. This method actually includes integrated finite difference methods with other domain methods, such as Galerkin finite element methods.

Another difference among various modeling approaches is that the so-called "convective" or "advective" term of the heat equation is eliminated in most models to make numerical computations more stable. Taylor and Luthin (1978) suggest that this term is negligible when evaluating heat flow. We have found, however, that the exclusion of this term in the freezing process may introduce significant errors in estimates of frost heave, at least for our model, and we therefore incorporate this term into the model. In this same regard, many investigators eliminate the gravity term in the moisture transport equation. We include the gravity term by solving for total energy head, avoiding possible numerical difficulties in the solution of the moisture transport equation. There are a number of problems associated with very moist soils or situations when ice-rich soils are thawing where the gravity term would be significant.

The model calculates moisture movement in the unfrozen portion of a soil column by assuming that the soil is nondeformable. It is assumed that such soils range from silt to "dirty" small gravel sizes, and that all consolidation has occurred during some previous period. Thus, consolidation is negligible. Moisture movement in fully frozen zones is assumed to be negligible over the annual freezing and thawing cycles for which the model was developed. Moisture movement and thaw settlement in thawing or thawed zones at the top of a soil column will be dealt with subsequently.

Since the model is primarily intended for use in situations where the water table is well below a pavement and base course, and below the maximum depth of frost penetration, unsaturated flow is occurring to produce measurable heave. The model assumes that such moisture flux is primarily in the form of connected liquid water films driven by a hydraulic gradient; vapor flow is assumed to be negligible.

An appropriate equation describing soil moisture flow that is consistent with the above assumptions can readily be derived by substituting the extended Darcy's law into the one-dimensional continuity equation for an incompressible fluid and porous media, i.e.

$$\frac{\partial}{\partial x}\left[K_{\mathrm{H}}\,\partial h/\partial x\right] = \frac{\partial \theta_u}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \tag{1}$$

where the total hydraulic head h equals the sum of the pore pressure head ($h_p = u/\gamma_w$) and the elevation head ($h_e = -x$). The vertical coordinate x is oriented downward and t is time. The coefficient of hydraulic conductivity K_H is a function of pore pressure head in the unfrozen soil zones. The volumetric unfrozen water content is θ_u and the volumetric ice content is θ_i. The densities of ice and water are ρ_i and ρ_w respectively. The ice sink term $\rho_i \partial\theta_i / \rho_w \partial t$ only exists in a freezing or thawing zone, and in these zones eq 1 is coupled to the heat transport equation. The model assumes that θ_i is a continuous function of time.

Equation 1 requires a known relationship between total hydraulic energy head h and volumetric unfrozen water content θ_u. Such a relationship is provided by the so-called "soil-water characteristics." Thus, if such a single valued continuous function is available, the temporal water content term of eq 1 may be replaced as follows

$$\frac{\partial\theta_u}{\partial t} = \frac{\partial\theta_u}{\partial h_p}\frac{\partial h}{\partial t} \qquad (2)$$

where the $\partial\theta_u / \partial h_p$ quantity may be determined from the soil-water characteristics. It is computationally convenient to represent the soil-water characteristics as a known or assumed function, relating pore water pressure and volumetric water content. This can be done by determining point values of θ_u and h_p in the laboratory and by least squares fitting of an assumed function to their data. CRREL has done this for a large number of soils and has found that Gardner's (1958) function fits these soils well, i.e.

$$\theta_u = \frac{\theta_o}{A_w |h_p|^a + 1} \qquad (3)$$

where a and A_w are best fit parameters determined for different soils and θ_o is the soil porosity.

Similarly, it is computationally convenient to represent the coefficient of permeability function for unsaturated soils as a known or assumed function. This function can be obtained from laboratory data by determining point values of K_H and h_p for different soils and by least squares fitting of an assumed function to these data. Again, CRREL has done this for a large number of soils and has found that Gardner's function fits these data well, i.e.

$$K_H = \frac{k_s}{A_K |h_p|^b + 1} \qquad (4)$$

where k_s is the saturated hydraulic conductivity and A_K and b are best fit parameters determined for different soils.

Appendix A contains a comprehensive list of soils studied in the laboratory to determine soil moisture characteristics and hydraulic conductivity functions. Data on easily obtained soil parameters such as porosity and particle size may be used to estimate Gardner's coefficients where the required parameters are unknown.

Because eq 1 is also applied to thawing or freezing zones, an empirical phenomenological relationship is assumed for adjusting the unfrozen coefficient of hydraulic conductivity to represent conditions where ice may be partly blocking soil pores, reducing hydraulic conductivity. We assume that

$$K_F = K_H(h_p) \cdot 10^{-E\theta_i}, \qquad E\theta_i \geq 0 \qquad (5)$$

where E is a parameter to be determined from freezing tests on different soils. Both Taylor and Luthin (1978) and Jame (1978) use a somewhat similar concept to reduce hydraulic conductivity in the freezing zone. Nakano et al. (1982) demonstrated that the presence of ice in soil pores reduces hydraulic conductivity in an exponential fashion, and Nakano (1990) concluded from a mathematical analysis that the transport equation of water in the frozen fringe was the major factor determining a condition of steady growth of segregated ice. Most studies suggest that soil-water diffusivity in a frozen soil is a function of some power of water content. Lundin (1990) has studied various impedance functions that are used to decrease unfrozen unsaturated hydraulic conductivity, including the form advocated here. He demonstrates that such an approach is essential to models of frozen soil. A rigorous theoretical principle describing this phenomenon has not yet been advanced; consequently, we have adopted the empirical phenomenological relationship above.

As part of the research reported here, numerous empirical studies were conducted to determine a suitable function to describe hydraulic conductivity in the freezing zone. In the cases we studied, a freezing zone is defined as a finite area that generally is larger than the true freezing zone. Hence, our results are determined on a macro-scale. A number of functions, including Washburn's (1924) use of the Clausius–Clapeyron equation, were tried. Although investigators using our model at Texas A&M (Lytton et al. 1990) reported success using Washburn's method for estimating pressures in the frozen zone, coupled with the use of Gardner's equation for hydraulic conductivity, our results using this approach generally under-predicted observed frost heave by a significant amount. From our empirical investigation, it is clear that some form of macro-scale relationship, such as eq 5, is required to accurately simulate frost heave.

It is possible, based upon empirical calibration of the model to observed frost heave, to replace the empirical E-factor in eq 5 with a function based upon saturated hydraulic conductivity k_s. Based upon nine different non-cohesive soils, the E-factor may be determined by

$$E = \frac{5}{4}(k_s - 3)^2 + 6 \tag{6}$$

where k_s is in centimeters/hour.

The computer model allows the user to either apply eq 6 or to specify an E-factor that can be determined by calibrating the model against observed frost heave, i.e., in a laboratory column or from field studies.

The well known one-dimensional heat transport equation for a freezing or thawing soil column is given by

$$\frac{\partial}{\partial x}\left[K_T \partial T/\partial x\right] - v\, C_w \frac{\partial T}{\partial x} = C_m \frac{\partial T}{\partial t} - L\, \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} \tag{7}$$

The model assumes the DeVries (1966) relationship for computing thermal parameters in eq 7, i.e.

$$C_m = C_w \theta_u + C_i\, \theta_i + C_s(1 - \theta_o) \tag{8}$$

and

$$K_T = K_w \theta_u + K_i\, \theta_i + K_s(1 - \theta_o) \tag{9}$$

where C_m = volumetric heat capacity
K_T = thermal conductivity of the soil–water–ice mixture
C_w = volumetric heat capacity of water
C_i = volumetric heat capacity of ice
C_s = volumetric heat capacity of soil
K_w = thermal conductivity of water
K_i = thermal conductivity of ice
K_s = thermal conductivity of soil.

DeVries' relationship for thermal conductivity includes a correction factor for mineral soil contact area, which is not included here since we are dealing with fine-grained soils where contact area correction factors are unnecessary. Therefore, DeVries' effective thermal conductivity of soil–water–ice is somewhat different from that computed from eq 9. Velocity flux is computed by Darcy's law

$$v = -K_H \frac{\partial h}{\partial x}. \tag{10}$$

The dominating phase change process is modeled by an isothermal approach that decouples the source–sink terms of eq 1 and 7. During a computation time step, a freezing or thawing element is considered to be isothermal and have a temperature equal to the freezing point depression of water T_f. Fully frozen zones have a below-freezing temperature and fully thawed zones have an above-freezing temperature. Temperatures in these freezing or thawing zones are computationally continuously reset to T_f until the latent heat of fusion is satisfied in freezing or thawing zones. The amount of heat extracted in a computation time step Δt in a unit volume of soil is calculated by

$$\Delta Q_1 = C_m\left(T^{t+\Delta t} - T_f\right). \tag{11}$$

This quantity is compared to the amount of heat left to be extracted in a unit volume of soil before there can be complete freezing

$$\Delta Q_2 = L\,(\theta_u - \theta_n) \tag{12}$$

where θ_n is the minimum volumetric unfrozen water content, which is regarded as a constant in this model provided ice segregation is not taking place. It can be determined from the soil freezing characteristics, such as discussed by Anderson et al. (1973) and elsewhere. The latent heat coefficient is regarded as a constant equal to the value for bulk water. If $\Delta Q_1 \geq \Delta Q_2$, computed temperatures are set to T_f. If $\Delta Q_1 \leq \Delta Q_2$, computed temperatures are negative and remain so. The reverse process is for thawing. Thus, in eq 1 and 7

$$\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} = \frac{1}{L}\frac{\Delta Q}{\Delta t}. \tag{13}$$

In a freezing zone, eq 13 is used to correct computed pore water pressure head in eq 1, which, in effect, sets pore water pressure head in the frozen zone to $h_p = h_p(\theta_n)$ and for all practical purposes sets velocity flux to zero in this zone.

Overburden is modeled by adding together the weights of soil, water, ice and surcharge and converting this weight to an equivalent head of water h_o. This head is set to zero if

$$\theta_i < \theta_o - \theta_n \tag{14}$$

i.e., there is no ice segregation and the overburden weight is supported by the soil matrix. At any point where

$$\theta_i \geq \theta_o - \theta_n \tag{15}$$

i.e., the volume of ice is greater than the available pore ice space, there is ice segregation and the model assumes that liquid films on ice lenses support the entire overburden. Hence h_o is added to $h_p(\theta_n)$ and a revised θ_n is computed from eq 3

$$\theta_n' = \frac{\theta_o}{A_w |h_p(\theta_n) + h_o|^a + 1} \tag{16}$$

Since $h_o > 0$, the effective pore pressure is increased (less negative), decreasing the hydraulic energy gradient toward the freezing zone.

Frost heave is estimated as a lumped quantity that is equal to the total ice segregation in the frozen zone

$$\theta_s = \theta_i - (\theta_o - \theta_n). \tag{17}$$

If $\theta_s > 0$, there has been ice segregation and a frost heave is computed. Thaw settlement from ice melting is the reverse process.

Appendix B contains thermal parameters for water, ice and some soils. Typically, published soil thermal parameters are for bulk soil, including unfrozen or frozen moisture. It should be noted that the model developed here requires heat capacity and thermal conductivity for dry mineral soil alone.

Figure 1 illustrates the solution of a freezing problem at a certain time. The θ_n parameter establishes the initial negative pore water pressure at the freezing front for the solution of the moisture transport equation. As indicated in Figure 1, the uppermost element has been frozen and the surface moisture boundary condition has been set to the zero flux condition. The lower moisture flow boundary condition is usually the water table, i.e., where $h_p = 0$. The surface temperature, which is below freezing, and the lower temperature boundary conditions are specified. In Figure 1, vertical stresses σ_o on a lumped ice lense are the sum of mineral soil, water and ice overburden pressures and surcharge pressure P_o. The pore pressure head at the lumped freezing front is adjusted by adding the vertical stress head, thereby decreasing the moisture energy gradient and decreasing the rate at the same location where water is drawn into the freezing element.

Thaw settlement

The thaw settlement portion of the model is separately discussed because of the importance of this submodel to determining thaw weakening of pavements, a major objective of this project. The concepts advanced by Morgenstern and Nixon (1971) provide the framework for the thaw settlement and pore water pressure algorithm presented here. Historically, limited quality laboratory data

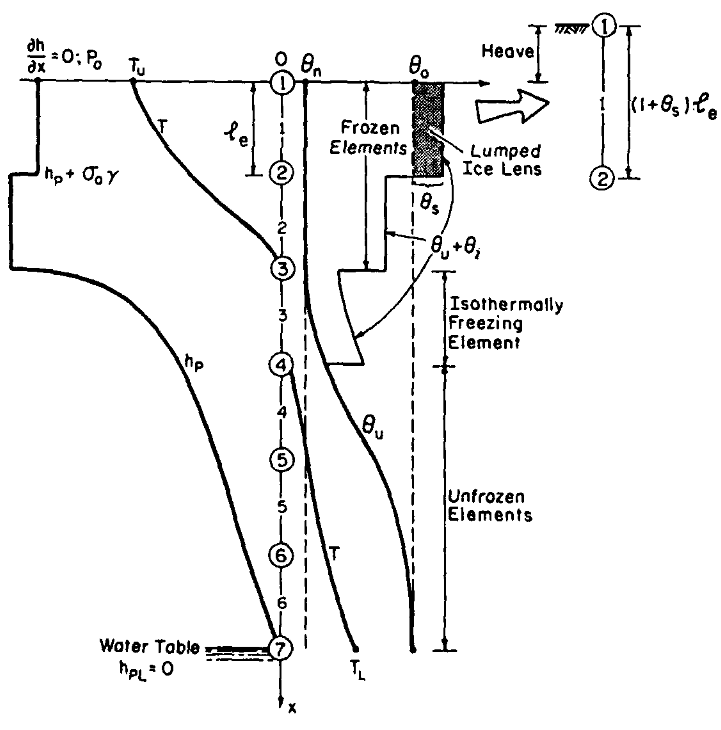


Figure 1. Solution of a soil freezing problem.

seem to have somewhat inhibited the development of accurate and tested thaw settlement models. Additional data were collected during this study using the CRREL soil column to test the thaw settlement algorithm that we adopted.

The Morgenstern and Nixon algorithm is based upon well-known theories of heat conduction and of linear consolidation of compressible soils. Terzaghi's one-dimensional consolidation theory is applied to develop a moving boundary solution applicable to permafrost soils that thaw and consolidate under the application of a "first time" load. A closed form solution was obtained.

The application envisioned here is for engineered soils and noncohesive soils having an overlying pavement. Consequently, consolidation effects will normally be minimal, since engineered soils will have been consolidated as they were placed. Frost action will normally be confined to winter heaving of subsurface soils and spring *thaw settlement*, with little net change in pavement elevation over a sequence of several years of freeze–thaw action.

A second departure from the Morgenstern and Nixon model is that our algorithm can solve the linear governing equation of excess pore water pressure (Terzaghi's equation) numerically, rather than analytically, where specific constraining boundary conditions need to be assumed. The numerical code already exists in the frost heave model, as was described previously, and which will again be described. Rather than incorporating the moving boundary condition solution proposed by Morgenstern and Nixon, the ice source–sink term is already accounted for in the model, and eq 1 physically describes the thawing process. Additionally, more flexibility is available in handling the boundary condition imposed by the soil surface pore water pressure. It is possible with a general numerical procedure to include positive pore water pressure at the soil surface, simulating ponding effects.

A final advantage of the method proposed here is use of the general heat transport equation (eq 7). Thus, the need to employ the limiting Stefan solution is avoided and more general numerical solutions can be achieved.

It can be shown that eq 1 for a deforming soil is modified to include a temporal void ratio term (Lambe and Whitman 1979) as follows

$$K_H \frac{\partial^2 h}{\partial x^2} = \frac{\partial \theta_u}{\partial t} - S\, m_v \frac{\partial \sigma'}{\partial t} + \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} \qquad (18)$$

where the new variables introduced are S, the degree of saturation, m_v, the coefficient of volume compressibility, and σ', the effective stress. If we assume that the total stress is constant with respect to time, i.e.

$$\sigma' + u = \text{constant with respect to time, i.e., } \frac{\partial \sigma}{\partial t} < 0$$

where $u = \rho_w g h_p = \gamma_w h_p$, then

$$\frac{\partial \sigma'}{\partial t} = \frac{\partial u}{\partial t} = -\gamma_w \frac{\partial h_p}{\partial t}$$

where $\frac{\partial h}{\partial t} = \frac{\partial h_p}{\partial t}$ (recall that $h = h_p - x$).

Substituting this result into eq 18 yields

$$K_H \frac{\partial^2 h}{\partial x^2} = \frac{\partial \theta_u}{\partial t} - S\, m_v\, \gamma_w \frac{\partial h}{\partial t} + \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t}. \qquad (19)$$

If the soil is saturated, $\partial\theta_u/\partial t$ equals zero, and if the soil is thawed, the ice source term is zero; thus, eq 19 reduces to the well known Terzaghi one-dimensional consolidation equation.

Equation 19 is the basis of the thaw settlement and thaw pore water pressure estimation algorithm. When soil surface temperatures are above freezing and the upper element is fully saturated, soil surface pore water pressures are set to a specified value, which is usually atmospheric pressure. However, the model is not able to apply a specified positive pressure representing a slowly leaking pavement overlying the soil subbase material. When the upper soil element becomes partly drained, i.e., $S < 100\%$, or when the surface element refreezes, the soil surface boundary condition for the moisture equation is reset to a no-flux boundary condition.

As thawing progresses downward, each discrete soil element is checked to determine the degree of saturation. If excess pore water pressure exists, water in excess of the porosity is treated as a source, forcing an upward drainage of water. Underlying fully frozen zones are assumed to be essentially impermeable.

During thawing the total stress equation has to be satisfied. If the computed pore water pressure exceeds the total stress, i.e., the weight of overlying soil, water and surcharge per unit area, effective stress is set to zero and the total stress is set to the computed excess pore water pressure value.

As mentioned previously, consolidation effects are assumed negligible, and soil deformation during thawing is assumed to be the result of thaw settlement, i.e., settlement equals the volume of ice per unit area that is melted.

When the soil column is thawing and excess pore water pressure develops, drainage is vertically upward and it is assumed that water seeping from the soil surface flows off horizontally. When the soil column is completely thawed, there is free downward drainage in accordance with eq 1.

Numerical approach

Numerical solution of the governing equations discussed above, subject to their respective boundary and initial conditions, is by the nodal domain integration method (Hromadka et al. 1982). The one-dimensional solution domain is divided into a number of variable length "finite elements," where parameters are assumed temporarily constant for a Δt time step, but may vary from element to element. Figure 2 illustrates the division of a vertical column into elements and nodes. The state variable in each element is assumed to be described by a linear basis function, such that the state variable is continuous throughout the solution domain. The time domain solution is either by the well-known Crank–Nicholson method or the fully implicit method.

In this section, we review the nodal domain integration numerical method. By using the subdomain version of the weighted residuals methods defined on subsets of a finite element discretization (to divide up into smaller connected lengths) (nodal domains), we derive an element matrix system that is similar to the element matrix system developed for a Galerkin finite element analog. The nodal domain integration element matrix system is found

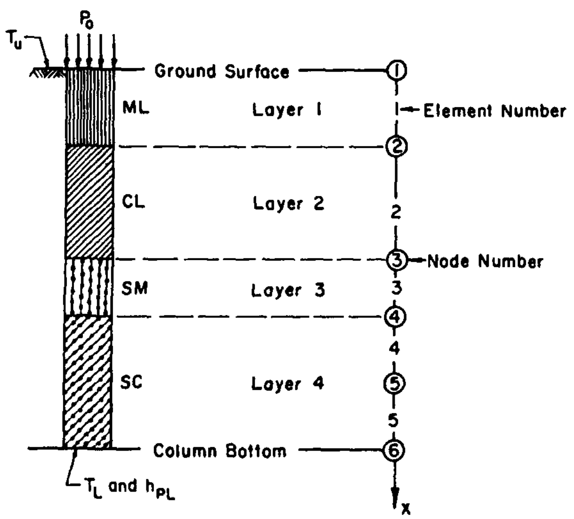


Figure 2. Nonuniform soil profile divided into elements.

to be a function of a single parameter, where the Galerkin finite element, subdomain integration and finite difference methods are represented as special cases.

The governing heat and soil-water flow equations can be written in the operator relationship

$$A(C) - f = \frac{\partial}{\partial x}\left(k_1 \frac{\partial C}{\partial x}\right) - \frac{\partial}{\partial x}(k_2 C) - k_3 \frac{\partial C}{\partial t} \qquad (20)$$

where, for the heat flow process

k_1 = thermal conductivity
k_2 = $C_w v$
k_3 = C_m
C = temperature T.

For the soil-water flow equation

k_1 = K_H
k_2 = 0
k_3 = $\partial\theta_u/\partial\theta_p$
C = total hydraulic head h.

The ice content terms of both flow processes are not needed in eq 20 because of the isothermal phase change approximation used. Therefore, eq 17 is solved for heat and soil-water flow processes during a small time step Δt; then the computed values of unfrozen water content, ice content and temperature are recalculated to accommodate isothermal phase change of available soil water.

Numerical solution is achieved by setting an appropriate weighting function orthogonal to eq 20

$$\int \left(A(c) - f\right) w_j dx = 0 \qquad (21)$$

where eq 21 is defined over appropriate domains. A n-nodal point distribution can be defined such that an approximation $\hat{C}$ for C is defined

$$\hat{C} = \sum_{j=1}^{n} N_j(x) C_j \qquad (22)$$

where $N_j(x)$ are linearly independent global shape functions, and C_j are values of the state variable C at nodal points j. Equations 20 and 22 are substituted into eq 21 yielding for element e

$$-\frac{k_\ell}{\ell_e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}C_e\\ C_{e+1}\end{Bmatrix}$$

$$+\frac{k_2}{2}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}\begin{Bmatrix}C_e\\ C_{e+1}\end{Bmatrix}$$

$$=\frac{\ell_e k_3}{2(\eta+1)}\begin{bmatrix}\eta & 1\\ 1 & \eta\end{bmatrix}\begin{Bmatrix}\frac{\partial C_e}{\partial t}\\ \frac{\partial C_{e+1}}{\partial t}\end{Bmatrix} \quad (23)$$

where $\eta = (2,3,\infty)$ gives the Galerkin finite element, subdomain integration and finite difference models respectively. In eq 23, the nonlinear parameters (k_1, k_2, k_3) are assumed constant for a small duration of time Δt and ℓ_e is the length of finite element *e*.

Element equations (eq 23) are assembled into a matrix system for the entire solution domain, giving

$$\underset{\approx}{G}\,\underset{\sim}{C} + \underset{\approx}{H}\,\dot{\underset{\sim}{C}} = \underset{\sim}{F} \quad (24)$$

where $\underset{\approx}{G}$ = banded square matrix incorporating the diffusion and advective terms of eq 20

$\underset{\approx}{H}$ = banded square matrix of the capacitance term of eq 20

$\underset{\sim}{F}$ = vector of boundary conditions

$\underset{\sim}{C}$ and $\dot{\underset{\sim}{C}}$ = vectors of unknown state variable values.

The dot indicates the time derivative. This system of ordinary equations is solved by the Crank–Nicholson method

$$\left(\underset{\approx}{G} + \frac{2}{\Delta t}\underset{\approx}{H}\right)\underset{\sim}{C}^{t+\Delta t} = \left(\frac{2}{\Delta t}\underset{\approx}{H} - \underset{\approx}{G}\right)\underset{\sim}{C}^{t} + 2\underset{\sim}{F} \quad (25)$$

where the nonlinear parameters in $\underset{\approx}{G}$ and $\underset{\approx}{H}$ are held constant for time step Δt. Equation 25 is applicable to situations that involve a soil column that is unsaturated everywhere. Where it is necessary to solve problems in which a water table exists in the soil column and unsaturated and saturated zones exist, it is necessary to use the fully implicit time solution method, where eq 24 is rewritten as

$$\left(\underset{\approx}{G} + \underset{\approx}{H}/\Delta t\right)\underset{\sim}{C}^{t+\Delta t} - \underset{\approx}{H}\,\underset{\sim}{C}^{t}/\Delta t = \underset{\sim}{F}^{t+\Delta t}. \quad (26)$$

The computer code allows the selection of either time domain solution method. Computation is initiated by given initial conditions and the solution is advanced in time. At specified times, called here "update frequency," nonlinear parameters are updated. Iteration of nonlinear parameters is not necessary because soil systems are highly damped.

Boundary conditions

The model requires auxiliary conditions as follows:

1. Initial conditions for pore pressure head, ice content and temperature.
2. Soil surface boundary conditions for pore pressure and temperatures (may vary with time).
3. Lower boundary conditions for pressure head and temperature (may vary with time).

While there is a large variety of possibilities for incorporating boundary conditions into the model, depending upon specific applications, the current version of the model has the features discussed below. Figure 3 illustrates the format of boundary conditions used in the current program version.

The upper pore water pressure head boundary is either a fixed constant value with respect to time or, if the surface temperature is below freezing, $\partial h/\partial x$ is set to zero, which means that velocity flux across this boundary is zero. If the top temperature is greater than T_f and there are frozen regions remaining in the soil column, a specified constant upper boundary pore pressure head is used (i.e., 0, P_o/γ_w, or an intermediate value). This boundary condition simulates pressures generated while thawing takes place below a pavement. After the column is completely thawed and downward vertical drainage occurs, the surface pore water pressure head boundary condition is modeled as a no-flux boundary.

The lower pore pressure head boundary condition is usually a water table condition or known pore water pressure head condition. Time variable boundary conditions are specified such that a set of discrete pore water pressure heads (tensions) at specific times are input to the model. Intermediate times and pore water pressure heads are linearly interpolated.

The upper temperature boundary condition consists of a set of specified step functions, such as mean daily air temperatures. These values can be multiplied by a factor to represent soil surface temperatures, such as is done in the Corps of Engineers *n*-factor approach.

Bottom temperature boundary conditions consist of a set of times and temperatures where intermediate times and temperatures are linearly interpolated.

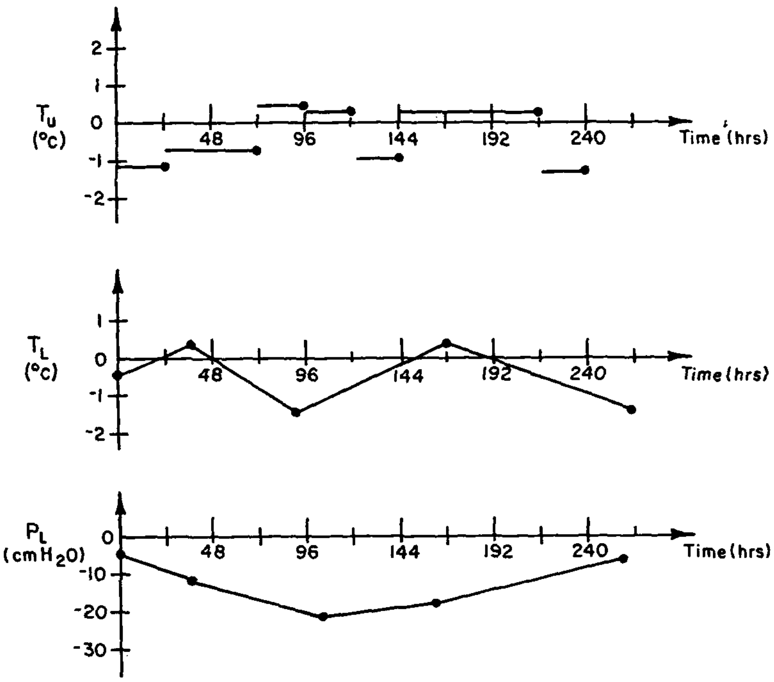


Figure 3. Format of boundary conditions for the CRREL version of FROST.

Other forms of boundary conditions may be easily incorporated into the model. For example, Lytton et al. (1990) integrated FROST into a comprehensive model of climatic effects on pavements using an energy balance surface boundary condition algorithm. Their computer code is written in an easy to follow modular form, permitting alternate boundary conditions to be easily inserted.

Probabilistic concepts

Figure 4 is one approach to viewing the modeling process. The prototype system $\underline{S}$, e.g., a laboratory soil column, is subject to excitations x (or inputs), which are spatially and temporarily distributed. Then there are spatially and temporally distributed outputs. Inputs such as boundary conditions may be subfreezing temperatures, water table location and surface surcharge (overburden). Outputs may be frost heave y or soil pore pressure head, temperatures or ice content. Because it usually is impossible to measure x exactly, subsystem $\underline{X}$ indicates a model process to determine an index x' of x, which has some error. In our case we are generally lumping x in space but are preserving as much as possible any dynamic characteristics of x. Since the deterministic model $\underline{M}$ is based upon the continuum assumption, certain parameters arise in the model derivation that purport to characterize $\underline{S}$, e.g., thermal conductivity or hydraulic conductivity. Subsystem $\underline{P}$ indicates this modeling or sampling process, which yields imperfectly known parameters p_i. Model outputs y' will therefore be imprecise but may be compared to imperfect observations of y for some bounded time period to determine model uncertainty $\varepsilon(t)$, where

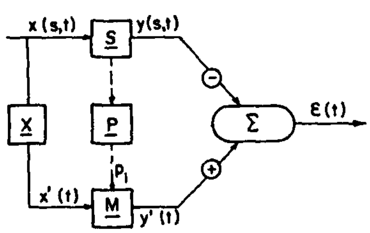


Figure 4. Schematic of modeling uncertainty.

$$\varepsilon(t) = y'(t) - y(t). \quad (27)$$

We are considering y as lumped to make this computation. Modeling uncertainty is arbitrarily grouped into four general areas:

1. Errors, α_1, attributable to the choice of $\underline{M}$, which include the choice of a numerical analog.

2. Errors, α_2, attributable to the spatial and temporal discretization and averaging.

3. Errors, α_3, attributable to boundary condi-

tions (i.e., choice of $\underline{X}$) and ascribable to choice of initial conditions.

4. Errors, α_4, attributable to the selection of p_i, i.e., choice of $\underline{P}$.

The total model uncertainty is some function of the α_i errors

$$\varepsilon(t) = \varepsilon(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \quad (28)$$

where the α_i errors may be interrelated and ε may be non-stationary. We hope that ε will be reasonably bounded, which is the reason we adopted the conceptual physics-based approach in the first place. However, because of approximations necessarily incorporated into the model, there obviously will be some error or uncertainty in model predictions.

Errors due to the choice of a model are probably not determinable in a strictly analytical way. Such questions are probably best left to experience with the model in a great number of applications. However, errors associated with the choice of a numerical analog are readily examined. These will be explored in the following section. Also, errors associated with spatial and temporal discretization are readily defined by conducting numerous simulations with the model. These errors will also be explored in the following section of the report.

Errors associated with boundary conditions and particularly with parameters will require special attention owing to the probabilistic nature of these variables. For this reason, a probabilistic theory is required to deal with this problem.

Freeze (1975) among others has investigated the combination of stochastic and deterministic models. In particular, Freeze considers the problem of groundwater flow in a nonuniform, one-dimensional, homogeneous medium. On the basis of his study, Freeze had "doubts about the presumed accuracy of the deterministic conceptual models that are so widely used in groundwater hydrology." If he has doubts about a similar but simpler system, considerable pessimism might be expressed about deterministic models of the more complex porous media processes considered here. Freeze (1975) had only a few parameters to concern himself with, while there are ten inexact parameters required in the frost heave model. The heat capacity, thermal conductivity, density and latent heat capacity of water and ice are assumed nearly exact as given by standard tables.

Freeze's (1975) stochastic analysis was based upon the well known Monte Carlo technique, which requires an assumption of the statistical distribution of the stochastic variables. Freeze assumed that porosity had a normal distribution and that saturated hydraulic conductivity had a log–normal distribution. Freeze used 500 Monte Carlo simulations for each parameter that was randomly generated from an assumed probability distribution and was applied to a deterministic model.

Typically, most investigations of this nature use a large number of deterministic model simulations, i.e., 500 or even thousands (Harr 1987). Because of the apparent need for many Monte Carlo simulations, this type of stochastic analysis can be somewhat expensive, particularly if the variance is non-stationary for the type of dynamic problems considered and if the variance is significantly different for different soil types.

An alternative approach to the Monte Carlo method is based upon Rosenblueth's point probability estimation method, which is developed in Guymon et al. (1981b) and further refined in Yen and Guymon (1990). Let y' be simulated frost heave or thaw settlement where

$$y' = f\left(\bar{p}_i \pm S_{p_i}, \ldots\right) \quad (29)$$

where $\bar{p}_i$ is the mean of the i^{th} parameter and S_{p_i} is the standard deviation (i.e., the positive square root of the variance) of the parameter. If it is assumed the p_i are uncorrelated, Rosenblueth deduced the general relationship for the N^{th} moment of y'

$$E\left[(y')^N\right] = \frac{1}{2^m}\left[\left(y'_{+++\ldots m}\right)^N + \left(y'_{-++\ldots m}\right)^N + \left(y'_{---\ldots m}\right)^N\right] \quad (30)$$

where there are m parameters to be considered, and N is the exponent (moment) of y'. The notation $y'_{---\ldots m}$ indicates the use of all sign permutations of

$$y' = f\left(\bar{p}_1 \pm S_{p1}, \bar{p}_2 \pm S_{p2}, \cdots, \bar{p}_m \pm S_{pm}\right) \quad (31)$$

where $\bar{p}_i$ is the mean of the i^{th} parameter and S_{p_i} is the standard deviation of the parameter. The subscript sign is determined by the sign of S_p. The mean $\bar{y}'$ and variance $V_{y'}$ of y' are computed in the usual fashion

$$\bar{y}' = E\left(y'\right) = \frac{1}{N}\sum_{0}^{N} y'_i \quad (32)$$

and

$$V_y = E[(y')^2] - [E\ (y')]^2$$

$$= \frac{1}{N} \sum_{0}^{N} (y_i' - \bar{y}')^2. \quad (33)$$

Usually, for a given soil the coefficient of variation is known (Harr 1987) or readily assumed for a given parameter such as porosity. The coefficient of variation is defined as

$$CV = \frac{|\sqrt{V_{y'}}|}{\bar{y}'} \quad (34)$$

where the positive square root of the variance is called the standard deviation.

Now, if some or all of the p_i are correlated (sometimes called "auto correlation"), Rosenblueth's method can be modified using the covariance (cov) statistic (Harr 1977, 1987) as follows

$$\rho_{r,n} = \rho_{n,r} = \frac{\text{cov}\,(\rho_r, \rho_n)}{S_{p_r}\, S_{p_n}} \quad (35)$$

where subscripts denote that there are m random variables (parameters) that are correlated a pair at a time. Now we define a q-function such that there will be M of these functions given by

$$q_{ij\cdots m} = 1 + \sum_{\substack{r=1\\ n=1}}^{M} \frac{|rn|}{rn}\, \delta_{r,n}\, \rho_{r,n} \quad (36)$$

$$\delta_{r,n} = \begin{cases} 0, |r| \geq |n| \\ 1, |n| < |n| \end{cases}$$

where the $i,j...m$ are all the permutations of the signs of the standard deviation of each parameter, where each sign is attached to the subscript. The moments of y' are defined as

$$E\left[(y')^N\right] = \frac{1}{2^m} \sum_{0}^{M} (q_{ij\cdots m})\, (y_{ij\cdots m})^N \quad (37)$$

and the first and second moments are computed as in eq 32 and 33.

Rosenblueth's method is a powerful tool that is ideally suited to the type of problem being considered. No prior assumptions are required concerning the probability distribution of the parameter variables. Only an estimate of parameter mean and coefficient of variation is required. This method requires the specification of a functional relationship between y' and x', i.e., the deterministic model. The method is completely general, however, and is applicable to any deterministic model. Instead of the many costly simulations required by the commonly employed Monte Carlo method, only exactly 2^m simulations are required using Rosenblueth's method.

We extend our capability by first supposing that we know nothing about the distribution of frost heave y and that Chebeshev's inequality applies as follows

$$p\left[\bar{y} - zS_y \leq y \leq \bar{y} + zS_y\right] \geq 1 - \frac{1}{z^2}\,. \quad (38)$$

For example, if two standard deviations are used ($z = 2$), the probability that y is bounded by $2S_y$ is greater than or equal to 75%. Now, if we assume that y is symmetrically distributed, Gauss' inequality applies

$$p\left[\bar{y} - zS_y \leq y \leq \bar{y} + zS_y\right] \geq 1 - \frac{4}{9z^2} \quad (39)$$

which says that for $z = 2$ there is a greater or equal probability of 89% that y is so bounded. Finally, if we are willing to assume that we know everything about the distribution of y, we can further narrow our uncertainty. An ideal distribution to assume is the beta distribution, which can fit many distributions. This distribution is given as (Harr 1977)

$$f\,(y) = \frac{\alpha!\ \beta!\ (b-a)^{\alpha+\beta+1}}{\alpha+\beta+1}\,(y-a)^{\alpha}\,(b-y)^{\beta} \quad (40)$$

where, to find the α and β parameters, all we need to know are y, S_y and a and b, the lower and upper bounds of the distribution. The parameters y and S_y are generated by Rosenblueth's method. The a and b parameters are estimated by field or laboratory data. Once a beta distribution is determined (Harr 1977, 1987), confidence limits and other desired statistical properties of $f(y)$ can be estimated.

Questions yet to be resolved include the question of stationarity: how will the statistical properties of $f(y)$ vary with time? The second question concerns the nature of $f(y)$ for various soils. Can we find a single beta distribution that is applicable to a class of soils such as the so-called "frost-susceptible soils?" If this were possible, we could avoid a substantial amount of computation with the model. We would only need to conduct 2^m computations once, using the same results for all other problems considered.

Limitations

The above discussed model is specifically developed for frost-susceptible soils that range from silts to silty sands and silty gravels. Generally, clay soils have a very low hydraulic conductivity so that moisture cannot move fast enough relative to heat extraction to produce appreciable frost heave. Similarly, clean sands and gravels do not exhibit appreciable frost heave in most cases. In the case of such soils, pore pressures at the freezing front are relatively high and thus hydraulic gradients are not sufficiently developed to promote moisture flow relative to heat extraction rates. While there are no known theoretical reasons not to apply the model to clay and coarse-grained soils, we do not recommend its application to such soils. The primary reason for this is that we have not explored the model's sensitivity to such parameters. Furthermore, where overburden and surcharge conditions are significant, the model may not properly simulate such conditions for coarse-grained soils. The algorithm that accounts for overburden and surcharge appears to work well for silts. To be applicable to coarser soils, some form of stress partition factor or function may be required.

Another limitation is the manner in which unfrozen water content is estimated. A constant factor is used when the real soil system is characterized by a functional relationship between unfrozen water content and subfreezing temperature. While such relationships could be accommodated in the model, a constant unfrozen water content factor appears to work reasonably well. The primary reason for not including a functional relationship is that such relationships are not routinely determined in most laboratories. However, a constant unfrozen water content factor must be estimated to use the model. At this time the best way to do this appears to be by assuming pressures in the freezing zone and calculating θ_n from the soil moisture characteristic curve.

A final limitation is the use of an empirical phenomenological function to decrease hydraulic conductivity in freezing zones. The *E*-factor included in this function must be assumed or be based upon calibration with actual heave data.

MODEL UNCERTAINTY AND ERRORS

This section of the report deals with model uncertainty or model errors, and will present guidelines for reducing or predicting modeling errors.

Errors caused by choice of model

There is no clear cut analytic methodology for determining the quality of a conceptual model, i.e., the governing partial differential equations embodied in this model. The classical approach is to demonstrate the validity of such models by comparing solutions with prototype data. Unfortunately, other errors, as we have discussed, mask the solution results so that it is difficult to determine the source of error, i.e., model errors or parameter errors.

Many investigators use a verification technique consisting of making the equations of state linear and comparing them to analytical solutions that may readily be obtained for a number of one-dimensional heat transport (e.g., the classical Stefan problem) or moisture diffusion problems. Because, for nonlinear problems, boundary conditions interact with nonlinear aspects of the problem, this technique is not a valid verification, particularly where coupling exists. The only real value of such a procedure is to check for coding errors for specific segments of the computer program. Additionally, some insight into convergence characteristics may be obtained. A substantial amount of this type of analysis was undertaken with the computer model. Much of this work was reported by Berg et al. (1980b).

It is, however, possible to evaluate analytical errors attributable to the choice of a numerical analog of the governing partial differential equations, provided a unifying concept of numerical methods is available. Hromadka et al. (1982) investigated errors associated with the choice of a numerical algorithm and associated with discretization. Such a unifying numerical method, nodal domain integration, was presented in the previous section.

We evaluated errors by comparing simulation results with frost heave measured in an instrumented soil column in the laboratory. Fairbanks silt was used in the soil column and the required model parameters were determined for this soil. The model was subjected to measured boundary conditions imposed on the laboratory column and model parameters were slightly calibrated so that simulated frost heave closely approximated measured frost heave. Next, spatial and temporal discretization errors were evaluated to determine an optimum time step size and spatial element (see next section). Arbitrarily, we used a temporal discretization that produced the worst results to study numerical analog effects. Other parameters were not adjusted. We concluded that there is little advantage of one numerical technique over another. Most of our simulations were conducted with η in eq 23 set at 1000.

Discretization errors

Errors caused by spatial and temporal discretization can be readily determined. As mentioned, simulated frost heave in Fairbanks silt was compared to laboratory measurements of frost heave. The results indicated that there is little sensitivity to spatial discretization, while there is marked sensitivity to temporal discretization, i.e., the choice of Δt to advance the solution in time.

The primary temporal variable to control in the model is parameter update frequency, which should be on the order of 1 hour. Numerous simulations have suggested for most silts and sandy silts a time step size of 0.2 hours and an update frequency of 1 hour. Thus, five time steps are taken before nonlinear parameters are updated. For coarse-grained soils, it may be necessary to use a smaller time step because a relatively large advective term in the heat equation will lead to instability.

Parameter errors

As was discussed in the previous section, a new theory was developed to assess parameter variability errors in the model. There are several aspects of this problem that will be addressed here.

First, the sensitivity of the model to all parameters can be evaluated by using the above-mentioned laboratory tests. Parameters were first measured and then calibrated by comparing simulated results to measured frost heave. Next, we varied individual parameters while holding all other parameters at their calibrated value and simulated frost heave.

Although a substantial variation in the thermal conductivity of mineral soil showed some sensitivity, we concluded that thermal parameters would have a minor effect on frost heave simulation results for Fairbanks silt under the conditions of the laboratory tests because phase change processes overshadow sensible heat processes in a freezing soil. Variation of thermal parameters for Fairbanks silt had an insignificant effect on simulated frost penetration, which very closely approximated measured frost penetration. Simulated frost heave showed marked sensitivity to hydraulic parameter variations. Consequently, these parameters were selected for a more detailed analysis using Rosenblueth's method. The most sensitive parameters are porosity, unfrozen water content factor and volumetric unfrozen hydraulic conductivity.

Oftentimes, layered or heterogeneous systems are evaluated by assuming a uniform soil profile. Average parameters are assumed or determined using relatively standard procedures. A nonuniform soil profile situation was examined to demonstrate the feasibility of modeling a layered soil profile as an averaged uniform profile.

First, we assumed that the soil profile had, from top down, a 5-cm layer of sandy soil, a 5-cm layer of silty soil, a 5-cm layer of clayey silt soil and finally a 30-cm layer of silty soil. Representative hydraulic parameters were applied, and frost heave simulated for 30 days, real time. The resulting heave was compared to a similar simulation using exactly the same boundary conditions but assuming a uniform soil profile with hydraulic parameters about equal to the average of those used in the layer simulation. The simulated frost depth at the end of the simulation was over 17 cm below the original ground surface, so that freezing had completely penetrated through the first three layers of the soil profile. Surprisingly, both results were almost identical. Consequently, we concluded on the basis of this test and other simulations that lumping of soil profile conditions is permissible if done with care.

A review of the literature concerning parameter variability reveals a paucity of data. Harr (1977, 1987), Schultze (1972) and Nielsen et al. (1973) present information on soil parameter variability. Parameter variations for laboratory test cases seem to be more prevalent than data on the variation of in-situ field soils of the same type and in the same locality. Obviously, there are differences in parameter variations, depending upon the care taken in measuring them or the level of ignorance of in-situ field parameters. Table 1 suggests general guidelines for parameter variations for porosity, hydraulic conductivity and volumetric unfrozen water

Table 1. Suggested coefficients of variation (%) for porosity, unsaturated hydraulic conductivity and unfrozen water content factor.

	Parameter		
	θ_o	$K_H(h_p)$	θ_u
Laboratory tests (remolded soils)	10	30–100	15
Uniform field soils (limited remolded tests)	20	100–400	20
Uniform field soils (assumed from gradation curves)	25	200–500	25
Nonuniform field soils (evaluated as uniform case)	30	400–500	30

Table 2. Simulated frost heave statistics using Rosenblueth's method and an assumed beta distribution for unrestrained Fairbanks silt, Chena Hot Springs silt and West Lebanon gravel.

	Parameter coefficient of variation				*Normalized simulated frost heave*							
Soil	θ_o	θ_n	E	$K_H(h_p)$	CV	*Min*	*Max*	a/y	b/y	α	β	$P[y-2S_y \le y \le y+2S_y]$
Fairbanks silt	13.3	15	10	30	11	0.81	1.18	0.67	1.44	3.7	5.3	97
Fairbanks silt	20	20	20	50	17	0.76	1.24	0.48	1.66	3.6	5.5	97
Fairbanks silt	13.3	15	10	100	97	0.0	2.16	0*	6.79*	3.7	5.3	97
Fairbanks silt	13.3	15	10	200	97	0.0	2.16	0*	6.79*	3.7	5.3	97
Chena Hot Springs silt	13.3	15	10	30	9	0.85	1.15	0.73	1.36	3.7	5.3	96
Chena Hot Springs silt	3.3	15	10	100	95	0.02	2.13	0*	6.67*	3.7	5.3	97
Chena Hot Springs silt	13.3	15	10	200	96	0.02	2.13	0*	6.72*	3.7	5.3	97
West Lebanon gravel†	13.3	15	10	30	23	0.61	1.39	0.31	1.92	3.7	5.3	97
West Lebanon gravel**	13.3	15	10	30	107	0.0	2.91	0*	7.49*	3.4	5.2	97
West Lebanon gravel†	13.3	15	10	100	103	0.0	2.53	0*	7.21*	3.7	5.3	97
West Lebanon gravel†	13.3	15	10	200	103	0.0	2.51	0*	7.21*	3.7	5.3	97

* Limits shifted so that lower bound is positive.
† 0.5 lb/in.^{2} (3.45 kPa) surcharge.
** 5.0 lb/in.^{2} (34-5 kPa) surcharge.

Notation

- CV = coefficient of variation in percent
- y = mean frost heave in cm
- a = lower beta-distribution bound
- b = upper beta-distribution bound
- α = beta-distribution parameter
- β = beta-distribution parameter
- θ_o = porosity
- θ_h = volumetric unfrozen water content factor
- E = frozen soil hydraulic conductivity correction factor
- $K_H(h_p)$ = unfrozen hydraulic conductivity relationship
- S_y = standard deviation of frost heave

content factor. These suggested variations also account for hysteresis effects and to some extent changes in parameters because of freeze–thaw cycles. These effects are not accounted for in the model.

The volumetric unfrozen water content factor controls the available space for pore ice to develop before ice segregation occurs. And in the deterministic model, this parameter also establishes the pore pressure head at the bottom of the frozen zone, thereby determining the hydraulic gradient and the rate at which water is drawn into the freezing zone. The balance between the rate of heat extraction and water importation to this zone is the controlling factor in the ice segregation processes, as the deterministic model is conceived.

The hydraulic conductivity of the soil system is obviously, for this reason, an important, if not the most significant, parameter. Unfortunately, this parameter is difficult to measure accurately for unsaturated fine-grained soils and is subject to considerable uncertainty. Very little work has been done on measuring hydraulic conductivity for partly frozen soils in the range of temperatures found in field soils under winter conditions.

Because some correlation between parameters, e.g., porosity and hydraulic conductivity, may be expected, preliminary investigations were undertaken using the data from Appendix A. We found no clear relationship among the hydraulic parameters used in the model. Consequently, the covariance statistic may be assumed to be essentially zero.

We conducted a number of simulations using Rosenblueth's method for Fairbanks silt, Chena

Hot Springs silt and West Lebanon gravel (a dirty gravel), considering both restrained and unrestrained cases. The coefficient of variation of simulated frost heave proved to be stationary with respect to time and is a function of the coefficient of variation of the parameters that were varied: porosity, unfrozen water content factor, unfrozen hydraulic conductivity and *E*-factor (Guymon et al. 1981b). These data were fit to the two-parameter beta distribution by assuming that the beta-distribution lower bound *a* equaled the deterministic mean minus three standard deviations, and that the beta-distribution upper bound *b* equaled the deterministic mean plus four standard deviations. The results of this analysis are shown in Table 2. As can be seen, nearly the same α and β parameters were obtained in each case. Consequently, we concluded that a universal beta distribution can be used for frost heave in soils similar to those tested. We also concluded that the coefficient of variation of simulated frost heave was stationary in time.

MODEL VERIFICATION WITH FIELD AND LABORATORY DATA

We have been continually verifying and refining the model since we completed early work on formulating it (Berg et al. 1980a). The older report of Berg et al. contained early verification of decoupled components of the model (e.g., sensible heat transport) against analytical solutions using linear computer simulations. As verification work progressed, we found it necessary to refine the computer code to more accurately simulate pore pressures, temperatures and frost heave. Guymon et al. (1980) further reported on verification efforts using laboratory tests on Fairbanks silt as a test case. Subsequently, Guymon et al. (1981a, 1983) presented in much greater detail verification of the model against laboratory and field data, while Guymon et al. (1981b) described additional laboratory verification of the overburden assumptions.

This report contains additional verification efforts, which are summarized together with previously reported results. Verification is divided into four subsections: *Soil Column Data; Tomakomi, Japan, Data; Winchendon, Massachusetts, Data;* and *Albany County Airport, New York, Data.*

Soil column data

Soil column data are obtained in two steps: first, frost heave, pore water pressures, soil temperatures and other data are measured in a frost heave column, and second, remolded soil parameters are measured using standard techniques or special techniques as required. Ingersoll and Berg (1982) and Berg et al. (1980b) describe the frost heave column and associated soil tests, and Ingersoll (1981) describes some of the techniques for determining hydraulic conductivity and soil moisture characteristics. Three soils have been tested in the soil column: Fairbanks silt, Chena Hot Springs silt and West Lebanon gravel. Tests on these soils are summarized by Ingersoll and Berg (1982) and are included in Appendix C.

Figure 5 shows an isometric view of the frost heave test column. The soil column test device is an open system that also permits an unsaturated soil column. Soil was molded within the 100-cm-long, circular cylinder, having a diameter of about 14 cm.

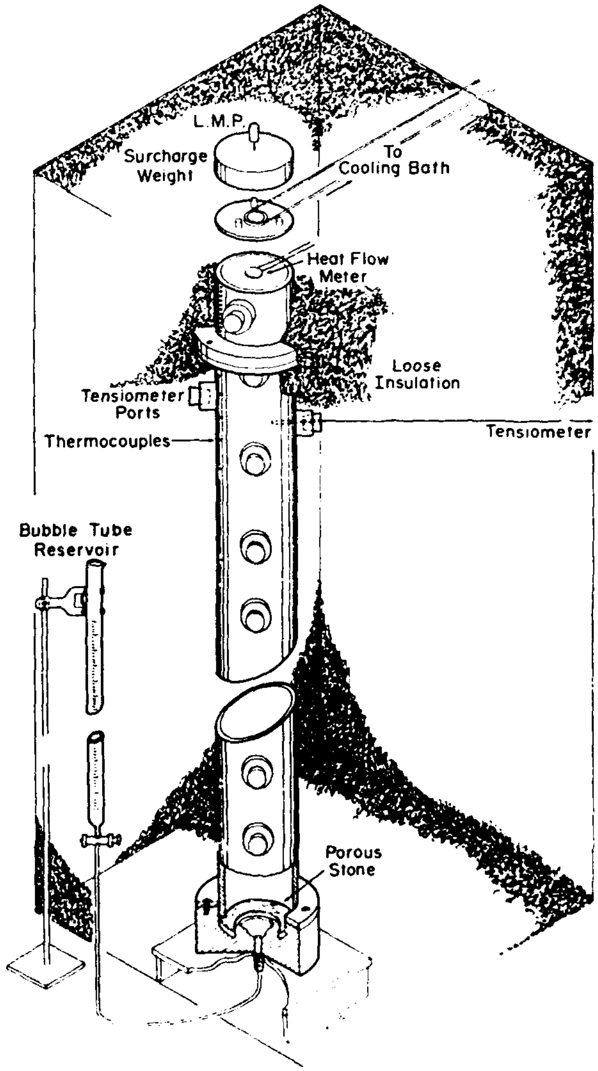


Figure 5. CRREL soil column.

The inside of the upper 15 cm of the cylinder is tapered outward slightly and was lined with Teflon tape to minimize sidewall resistance to heaving. The top portion of the cylinder is detachable from the lower portion.

Thermocouples were inserted through the cylinder walls and into the soil at intervals of 1 cm in the upper portion and at intervals of 2.5 to 10 cm in the lower portion. Tensiometers were placed at 1.5- to 20-cm intervals, depending on the test and location of the column. In early tests the uppermost tensiometer was 18 cm below the top of the column, while later tests had tensiometers at the 5- and 10-cm depths. Additional thermocouples were installed adjacent to the 5- and 10-cm tensiometers.

A Linear Motion Potentiometer (LMP) and a dial gauge were used to measure vertical movement of the sample surface. Water absorption by the soil was monitored by a graduated constant head reservoir. The reservoir was also used to control the free water level in the column. Electrical resistivity gauges were placed within the upper 15 cm to locate the solidly frozen soil. We created a surcharge on the soil by placing lead weights on a pedestal attached to the surface plate. A heat flow meter was recessed into the bottom of the surface plate contacting the soil. Data from the thermocouples, LMP and heat flow meter were monitored hourly by a digital data collection system.

Electrical pressure transducers were attached to most tensiometers to allow monitoring by the data collection system and to minimize the amount of fluid movement to and from the soil. Negative pressure dial gauges were attached to the tensiometers without transducers. The tensiometers with dial gauges were placed near the bottom of the column and were read daily. Tensiometers within the zone to be frozen were filled with a 30% ethylene glycol and water solution.

Copper electrical resistivity probes were used in most of the tests to delineate the solidly frozen zone. These probes were spaced at 1- to 2-cm intervals from the surface of the column to the 16-cm depth. Resistivity probes were read manually once per day with an oscillator and a digital multimeter. The resistance probes were later omitted as they probably retarded heaving of the soil.

Loose cork insulation was placed around the upper 17 cm of the column for the three tests using Fairbanks silt, and to the 50-cm depth for the remainder of the tests. Only the top surface was exposed to subfreezing temperatures, allowing one-dimensional freezing. In early tests this was accomplished by cold air circulation, later by use of a refrigerated surface plate. The ambient temperature of the room that housed the soil column was maintained at about 4.5°C.

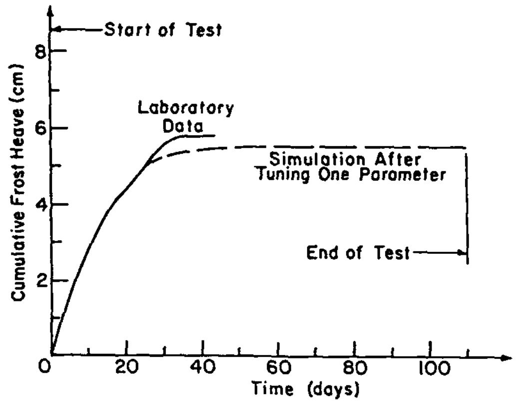
Table 3. Comparison of simulated and measured frost heave for Fairbanks silt with a 3.4-kPa surcharge.

	Time (days)		
	5	10	15
Laboratory data			
Frost heave (cm)	1.6	2.8	4.0
0°C Isotherm depth (cm)	6	11	11
Moisture tension at 24-cm depth (cm of water)	≈ 200	≈ 200	≈ 200
Simulated data			
Frost heave (cm)	2.5	2.9	3.9
0°C isotherm depth (cm)	4.5–7.5	7–10	10–12.5
Moisture tension at 24-cm depth (cm of water)	100	130	150

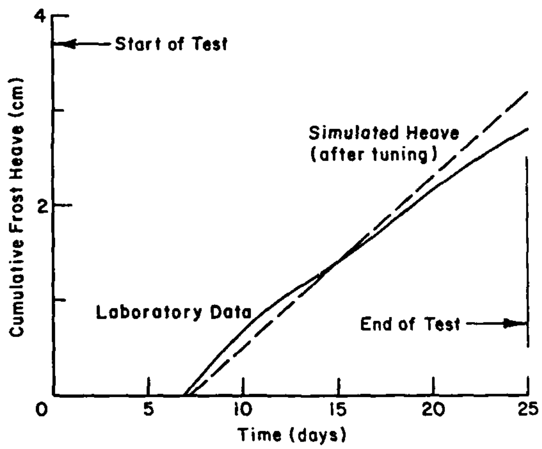
Verification of the frost heave model against the frost heave column data consists of applying measured or assumed soil parameters to the model and using measured initial and boundary conditions. Generally, soil density, hydraulic conductivity, moisture characteristics and porosity were measured from remolded samples of the same soils used in the frost heave column. These data are summarized in Appendix A. Generally, thermal parameters were assumed from Kersten (1949) or Haynes et al. (1980). Surface boundary conditions for soil temperature and surcharge were closely approximated in the model. Column-bottom boundary conditions of pore water pressure and temperature were also closely approximated by the model. Simulated frost heave was compared to measured frost heave as well as to other variables.

The first simulation is for Fairbanks silt. A comparison of frost heave, pore water tensions and soil temperatures is shown in Table 3, where simulated values closely approximate laboratory results. However, to achieve this comparison the soil moisture characteristics parameter A_w in Gardner's relationship, eq 3, was slightly adjusted. The need for calibration of the model is present in all tests and will be elaborated upon at the conclusion of this section.

Figure 6a shows a comparison of simulated and measured frost heave for another Fairbanks silt test case. To achieve these results, the Gardner A_w parameter was slightly adjusted. We did this early test to verify that the model could simulate lengthy tests without becoming mathematically unstable.



a. Surcharge of 3.45 kPa.



b. Surcharge of 34.5 kPa.

Figure 6. Simulated vs measured frost heave in a vertical column of Fairbanks silt.

As can be seen, over 100 days of real time are simulated without apparent instability problems. Laboratory data were available for only about 40 days. This is usually the case with the frost heave column because it was difficult to maintain specified cold side temperatures for a long period without a breakdown in equipment. This was particularly true during early tests when the column was being improved.

Figure 6b shows one of our first efforts at verification of the surcharge algorithm. Simulated versus measured restrained frost heave (34.5-kPa surcharge) is shown for Fairbanks silt. Tuning of Gardner's parameter A_w gave us these results.

Generally, the Fairbanks silt comparisons yielded promising results. In each case boundary conditions used in laboratory experiments were closely approximated in our model simulations and surface temperature boundary conditions were usually held constant through time. The need for calibration or fine tuning of the model is evident. Parameters selected for calibration were somewhat arbitrary; similar results could have been obtained by adjusting hydraulic conductivity or the unfrozen water content factor θ_n. One of the difficulties in the Fairbanks silt test cases was that the lower part of the soil column was not insulated nor was the water table depth in the soil column accurately maintained. For this reason more detailed study was not warranted. Tests on the other soils were more carefully controlled and, hence, more detailed study was undertaken.

Figure 7 compares measured and simulated frost heave and frost penetration for Chena Hot

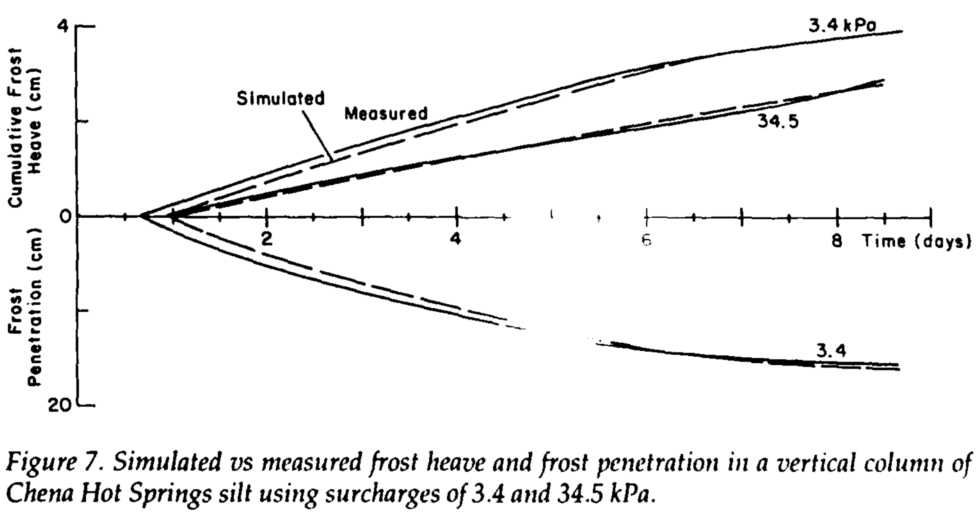
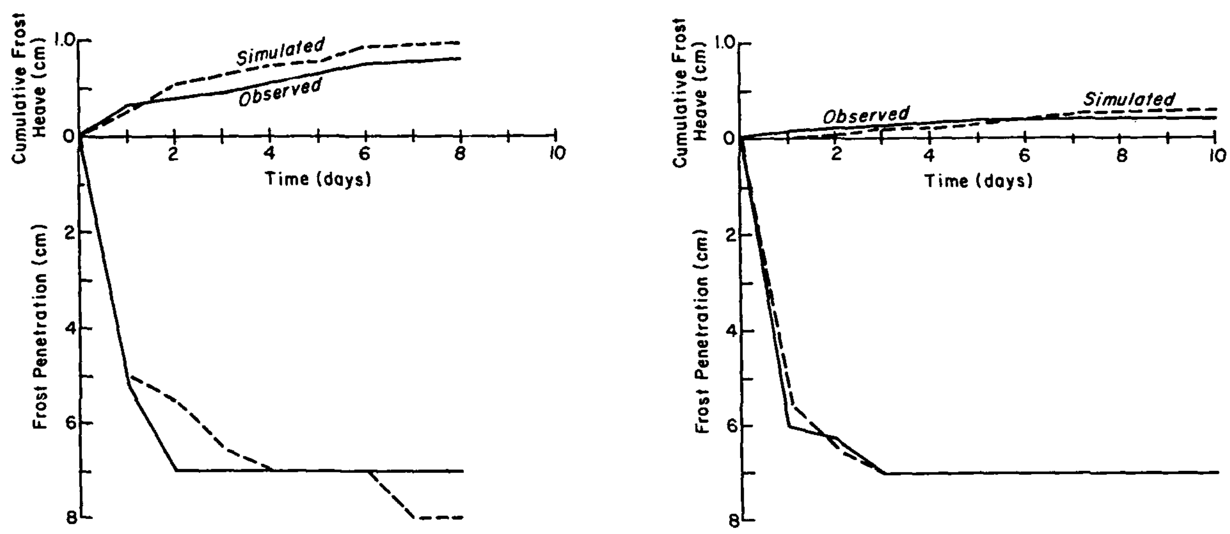


Figure 7. Simulated vs measured frost heave and frost penetration in a vertical column of Chena Hot Springs silt using surcharges of 3.4 and 34.5 kPa.



a. Surcharge of 3.45 kPa. *b. Surcharge of 34.5 kPa.*

Figure 8. Simulated vs measured frost heave and frost penetration in a vertical column of West Lebanon gravel.

Springs silt, showing both a restrained and unrestrained case. Soil surface boundary conditions on the laboratory column were controlled by a plate with a circulating bath. Temperatures were imposed to closely approximate a ramp function beginning at 0°C at time zero and gradually dropping to about −5°C at about 8 days. A water table was maintained at about 50 cm below the column top where soil temperatures were about 7°C. The restrained and unrestrained laboratory tests were conducted with essentially the same imposed boundary conditions. Simulation consisted of applying these same boundary conditions as closely as possible and using measured hydraulic parameters and assumed thermal parameters. Only the frozen soil hydraulic conductivity correction factor E was varied to calibrate the model. By selection of only one parameter to calibrate, a more systematic calibration procedure can be developed. The E-factor was calibrated for the 3.4-kPa surcharge case. As can be seen, the magnitude and rate of measured frost heave and frost penetration are accurately simulated. The slight lag in simulated heave may be attributable to too coarse a computational mesh size near the column top; the column was divided into uniform 1-cm elements. Without further calibration, a 34.5-kPa surcharge boundary condition was applied to the model. As can be seen, measured frost heave for this case was closely simulated. This simulation case gives some indication of the validity of the overburden algorithm, at least for relatively small surcharges.

Figure 8 shows comparisons of measured and simulated frost heave and frost penetration in West Lebanon gravel for 3.45- and 34.5-kPa surcharges. Soil surface temperatures were maintained at a constant −2°C during both tests, and a water table was maintained at 15 cm below the column top. Parameters measured in the laboratory or assumed were left unchanged for both simulations. Only the E-factor was calibrated for the 3.45-kPa surcharge case. The 34.5-kPa surcharge case was simulated correctly without further calibration. This study further verified the model and the validity of the approach used to simulate surcharge effects.

Table 4 summarizes soil parameters for the three soils considered. On the basis of these verification studies, we conclude that the model can accurately simulate frost heave and frost penetration for highly frost-susceptible silts, and for marginally frost-susceptible silty (or dirty) gravels. Furthermore, relatively light surcharge effects can be accurately modeled. A calibration procedure based on tuning the E-factor, a phenomenological parameter incorporated into the model, appears to be a practical approach.

The thawing algorithms' accuracy in estimating thaw settlement and thaw pore water pressures was evaluated from two tests, both using Graves silty sand from the Winchendon, Massachusetts,

Table 4. Soil parameters for remolded Fairbanks silt, Chena Hot Springs silt and West Lebanon gravel.

Parameter	*Fairbanks silt*	*Chena Hot Springs silt*	*West Lebanon gravel*	*Method of determination*
Soil density (g/cm^3)	1.60	1.62	1.99	Standard methods
Soil porosity (cm^3/cm^3)	0.425	0.416	0.260	Standard methods
Soil-water freezing point depression (°C)	0	0	0	Assumed
Volumetric heat capacity of mineral soil (cal/cm^3 °C)	0.3	0.2	0.2	Assumed
Thermal conductivity of mineral soil (cal/cm·hr·°C)	17.0	5.0	3.0	Assumed
Unfrozen water content factor (cm^3/cm^3)	0.15	0.30	0.09	Assumed
Soil-water characteristics				
A_w	0.004	0.00000607	0.123	Curve fit to
a	1.14	1.736	0.453	laboratory data
Saturated hydraulic conductivity (cm/hr)	0.04	0.625	0.42	Laboratory*
Frozen soil hydraulic conductivity factor (E)	8.0	12.0	20.0	Calibration with model

* Complete $K_H(h_p)$ data included in Appendix A.

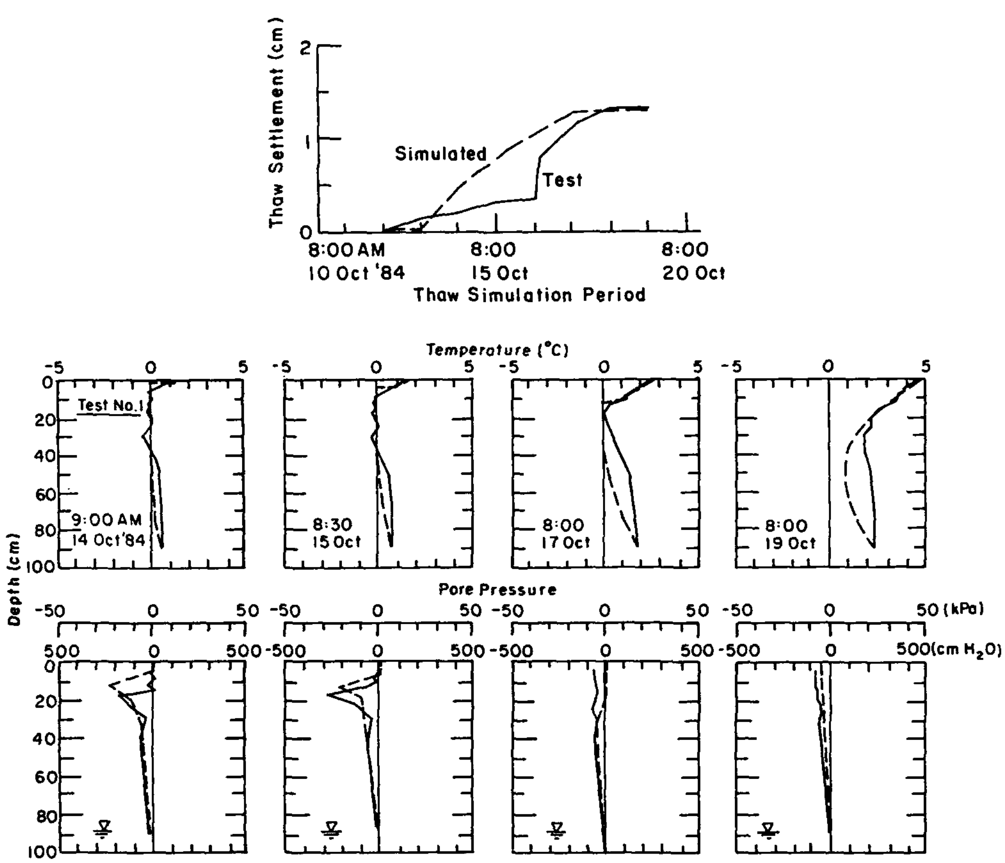
field test site (to be described subsequently). Of particular importance here is the verification of thaw pore water pressures, which largely determine the strength of pavements during the thawing process.

Both thawing tests were conducted in a similar manner. A sample of remolded soil, 15 cm in length, was first frozen using a ring freezing device developed in another phase of this project (Chamberlain 1986). A 0.5-lb/in.^{2} (3.54-kPa) surcharge was used in each case and a positive water pressure was provided on the warm side of the freezing column. The cold side temperature was essentially a ramp function going from 0 to −4°C over 100 hours. Frost heaves recorded, about 1.8 cm, were used to determine initial ice contents. These samples were then placed in the column, described above, and positive surface temperatures were applied to the soil surface while the water table was maintained at about 1 m below the sample top.

Tensiometers and thermistors were used to measure pore water pressures and soil temperatures during thawing. Hydraulic parameters determined in the laboratory or calibrated from field tests were used in the model, as were measured boundary conditions for temperature and pore water pressures.

Results from test 1 are shown in Figure 9a. Variations in temperature between simulated and measured data may also be caused by heat leakage through sides of the soil column. Assuming such leakage, we increased simulated soil surface temperatures by 10% to account for the possible additional heating. The effects of the isothermal assumption in the model are clearly evident when we compare measured and simulated temperatures. While measured temperatures show a tendency for the frozen part of the soil to reach isothermal conditions, the model exaggerates this. Simulated positive temperatures lag measured temperatures by several hours. If this lagging effect is ignored, simulated temperatures are quite accurate. Simulated pore water temperatures depend upon the simulated temperatures. Hence, pore water pressures also lag those actually measured. Nevertheless, the pattern of simulated pore water pressures is very reasonable. The model developed excess pore water pressures in about the same magnitude as was measured. Computed excess pore water pressures persist longer than measured values because of the lag in melting through the frozen layer. In this regard, the model is conservative. Measured and simulated thaw settlements compare favorably, as is shown in Figure 9a.

Results from test 2 are shown in Figure 9b and are similar to those described for test 1. Much more care was taken in setting up this test because of experience gained from test 1. Again, there is a tendency for a lag in simulated results, possibly because of errors inherent in the isothermal as-



a. Test 1.

Figure 9. Comparison of measured (solid lines) and simulated (dashed lines) thaw settlement, temperature and pore water pressure head.

sumption used in the model. In this case, however, there is a much closer correlation of simulated results with measured data.

Tomakomi, Japan, data

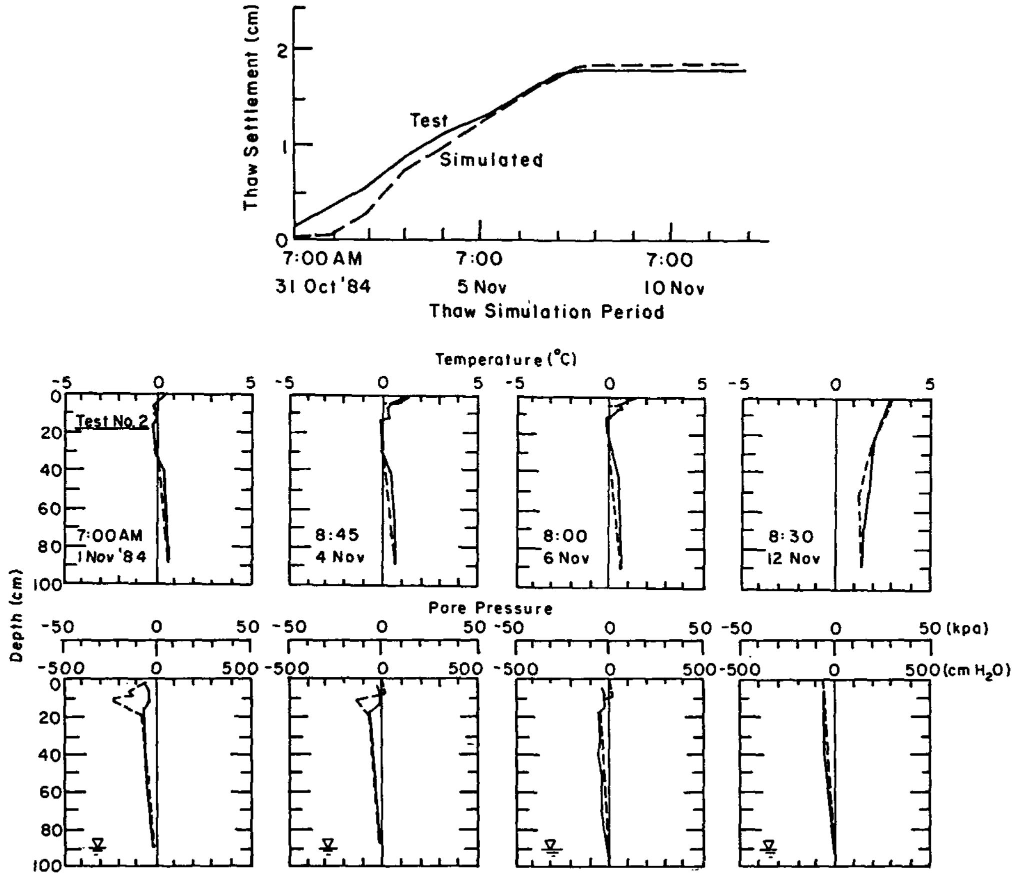
This test case used data developed by Kinosita et al. (1978). Frost heave, soil temperatures, water levels and other data were measured for soils in outdoor concrete tanks at the Tomakomi research site, Hokkiado, Japan. Soil parameters were provided by Kinosita* and a sample of soil was furnished to develop soil moisture characteristics and hydraulic conductivity relationships. Freezing was by natural means.

Figure 10 shows the comparison of measured and simulated frost heave and frost penetration for the 1977–78 winter. These results were achieved by calibrating the E-factor alone. More detailed study of this case was not undertaken because of uncertainty concerning the surface temperature boundary condition. Relatively good data are available at depth (Kinosita et al. 1978).

Data used in the simulation are $A_w = 0.037$, $a = 0.411$, $\rho_s = 1.5$ g/cm^3, $\theta_n = 0.36$, $\theta_o = 0.59$, $C_s = 0.3$ cal/cm^3 °C, $K_s = 15.48$ cm/hr, $E = 5$ and $k_s = 0.00063$ cm/hr. Complete moisture characteristics and hydraulic conductivity data were developed in the laboratory. Kinosita† provided physical and thermal parameters, while other parameters were assumed or calibrated. This case generally had a water table depth of 3 to 4 m. A 50-cm soil column was used for simulations where elements were 1 cm in length, $\Delta t = 0.2$ hours and parameters were updated at 1-hour intervals.

* Personal communication with Professor Kinosita, University of Hokkiado, 1979.

† Personal communication with Professor Kinosita, University of Hokkiado, 1979.



b Test 2.

Figure 9 (cont'd).

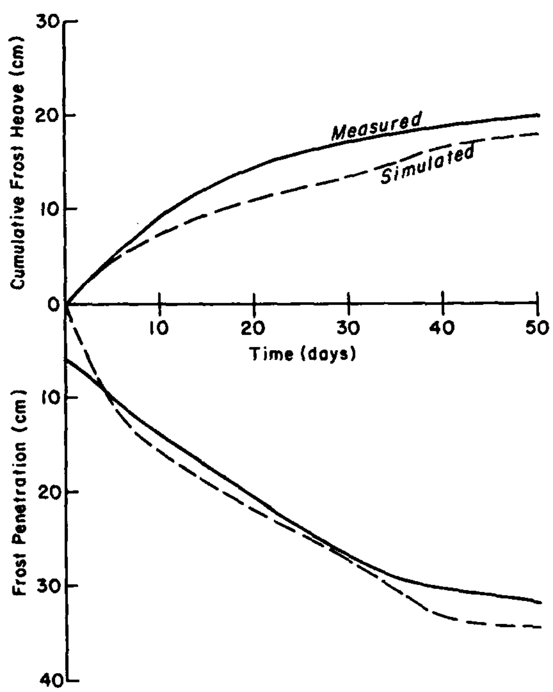


Figure 10. Simulated vs measured frost heave and frost penetration for an instrumented field tank containing Tomakomi silt.

Winchendon, Massachusetts, data

The Winchendon test site is about 5 miles (8 km) south of the New Hampshire border and about 20 miles (32 km) east of the Connecticut River in Massachusetts. The test site consists of 26 AC pavement sections over different soil types. Figure 11 shows photographs of two of the pavement sections. Climatic data, groundwater levels, soil temperatures and soil pore water pressures were collected, and undisturbed and remolded samples were evaluated in the laboratory to determine physical, hydraulic and mechanical properties of the different soil materials. Observations of frost heave, frost depth and soil moisture tension were obtained for the following six materials during the 1978–79 winter: Ikalanian silt, Graves silty sand, Hart Brothers sand, Sibley till, Hyannis sand and Dense-graded stone. In general, the groundwater depth at these sections was about 1.5 m below the pavement surface.

Figure 12 shows mean daily air temperature beginning 10 December 1978 and extending through 15 March 1979. These data are derived from the average of the maximum and minimum daily temperatures taken from a thermograph at the test site. As can be seen, there are several major freeze–thaw cycles. Because of diurnal temperature variations, there are also numerous daily freeze–thaw cycles during the winter. Soil surface temperatures were measured or estimated using the Corps of Engineers n-factor method. A constant surface diurnal temperature amplitude of 7°C was used in some calibrations.

A soil column length of 1 m was assumed for all soils except Sibley till, where a 1.3-m column was used. The soil column was divided into 50 elements of different lengths, ranging from 0.5 cm at the column top to 10 cm at the column bottom. Time increments were 0.2 hours and parameters were updated every 1.0 hours. Column bottom boundary conditions were estimated from recorded data. Mean daily surface temperature conditions were estimated from pavement surface temperature data where available and air temperature data using the Corps of Engineers n-factor method (described in next section) when soil surface temperature data were unavailable. We assumed that mean daily surface temperatures varied diurnally, following a constant sine function with a 7°C amplitude. Parameters were assumed, measured in the laboratory or calibrated. Table 5 summarizes parameter values for each soil.

Results of simulation studies are shown in Figure 13. Also shown in this figure are the results when mean daily surface temperatures are used without a diurnal variation. In general, errors introduced to simulated heave and thaw

Figure 11. Two pavement sections at Winchendon, Massachusetts.

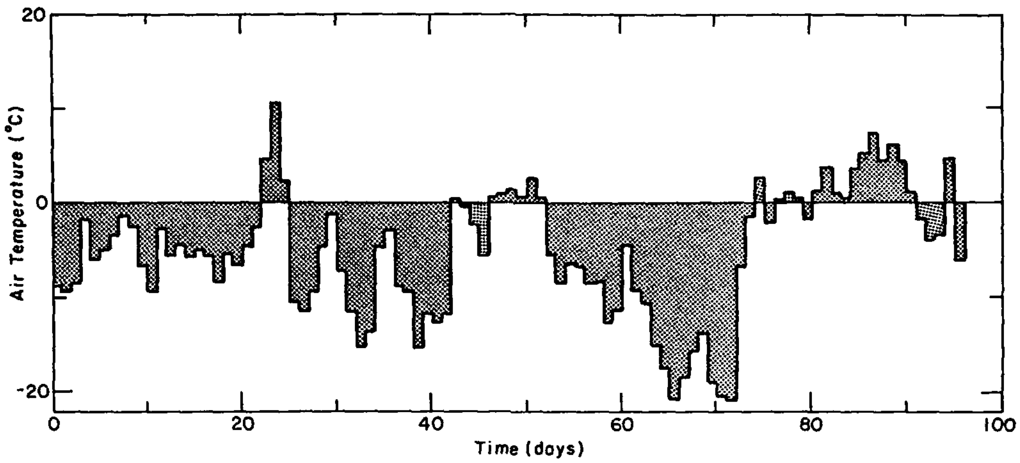


Figure 12. Mean daily air temperature, Winchendon, Massachusetts, 10 December–15 March 1979.

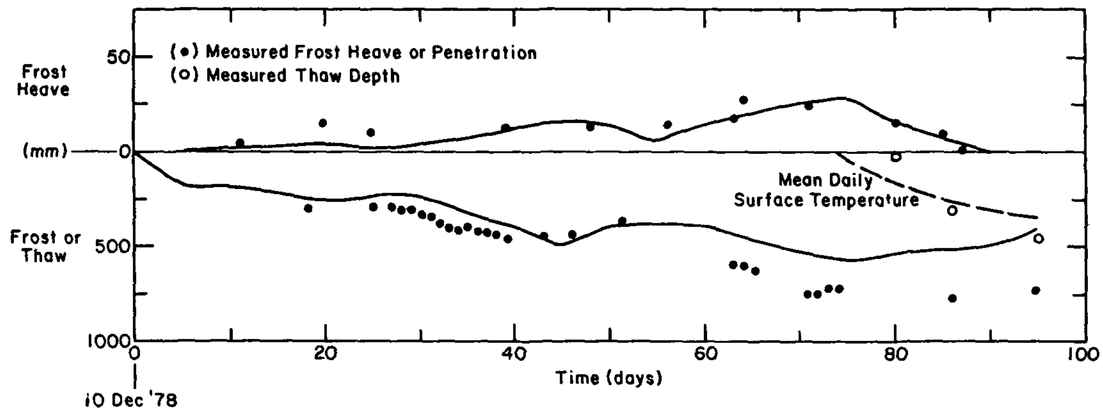
Table 5. Soil parameters for remolded Winchendon, Massachusetts, test site soils.

Parameter	*Ikalanian silt*	*Graves silty sand*	*Hart Brothers sand*	*Sibley till*	*Hyannis sand*	*Dense-graded stone*
Soil density (g/cm^3)	1.70	1.49	1.69	1.97	1.69	1.87
Soil porosity (cm^3/cm^3)	0.370	0.460	0.282	0.282	0.367	0.334
Soil-water freezing point depression (°C)	0	0	0	0	0	0
Volumetric heat capacity of mineral soil (cal/cm^3 °C)	0.2	0.2	0.2	0.2	0.2	0.2
Thermal conductivity of mineral soil (cal/cm·hr·°C)	17.0	17.0	17.0	20.0	17.0	17.0
Unfrozen water content factor (cm^3/cm^3)	0.03	0.12	0.12	0.15	0.01	0.1
Soil-water characteristics						
A_w	0.000546	0.00560	0.022	0.062	0.00154	0.053
a	1.500	0.900	0.867	3.45	1.806	0.462
Saturated hydraulic conductivity (cm/hr)*	0.37	1.92	4.08	0.36	1.23	5.54
Frozen soil hydraulic conductivity factor (unitless) (E)	16.0	4.5	5.0	8.0	15.0	15.0

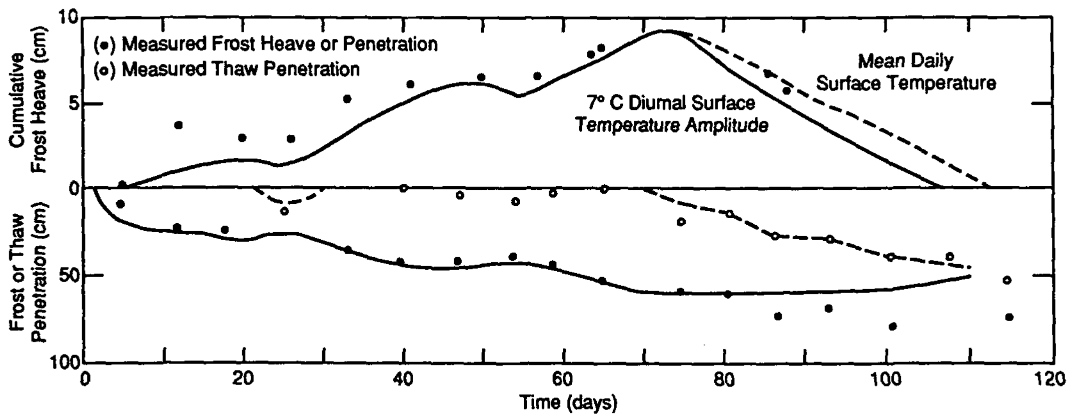
* See Appendix A for complete $K_H(h_p)$ data.

consolidation by using mean daily surface temperatures were negligible. The most significant difference observed was for Graves silty sand (Fig. 13b). In all cases, the use of mean daily surface temperatures predicted thaw penetration better than when a 7°C amplitude diurnal variation was superimposed over the mean daily temperature values. The reason for this is that the model assumes an isothermal freezing process and, when soils are alternately frozen and thawed during a day, a small amount of ice is present in the upper soil profile. It is thus difficult to detect a real or simulated thaw depth from the model output results. It is also possible that there is some error in field measurements, which are taken at certain times during the day. Soil surface freezing resulting from low nighttime temperatures would not be detected if observations were made in the afternoon, which was the case in most instances.

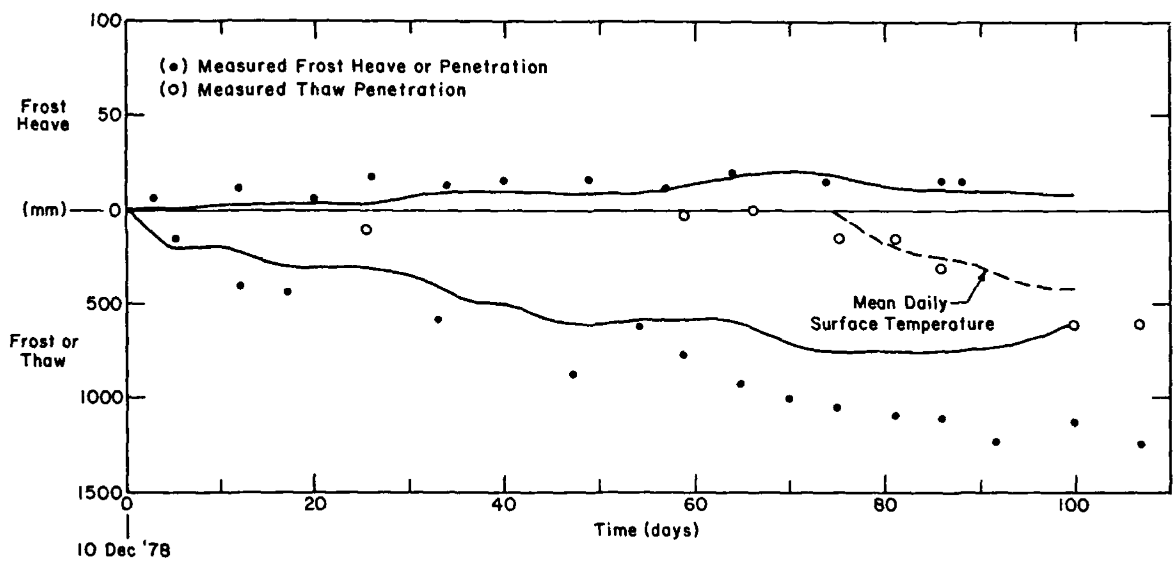
Additional field verification simulations were conducted for the Winchendon site materials—Ikalanian silt, Graves silty sand, Hart Brothers sand and Sibley till—with data from the 1979–80 winter. Unfortunately, much fewer field data were avail-



a. Ikalanian silt.

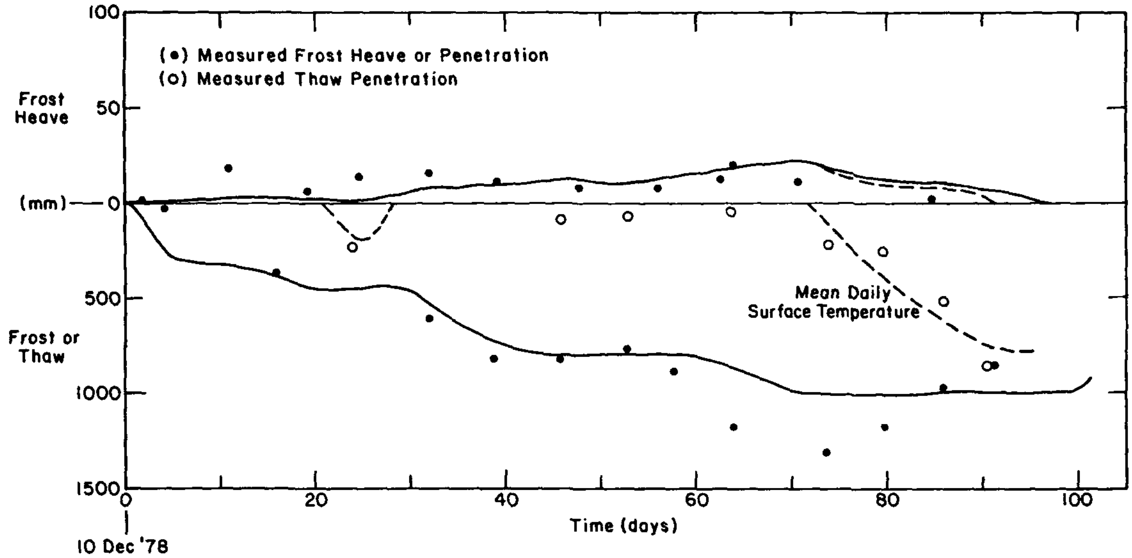


b. Graves silty sand.

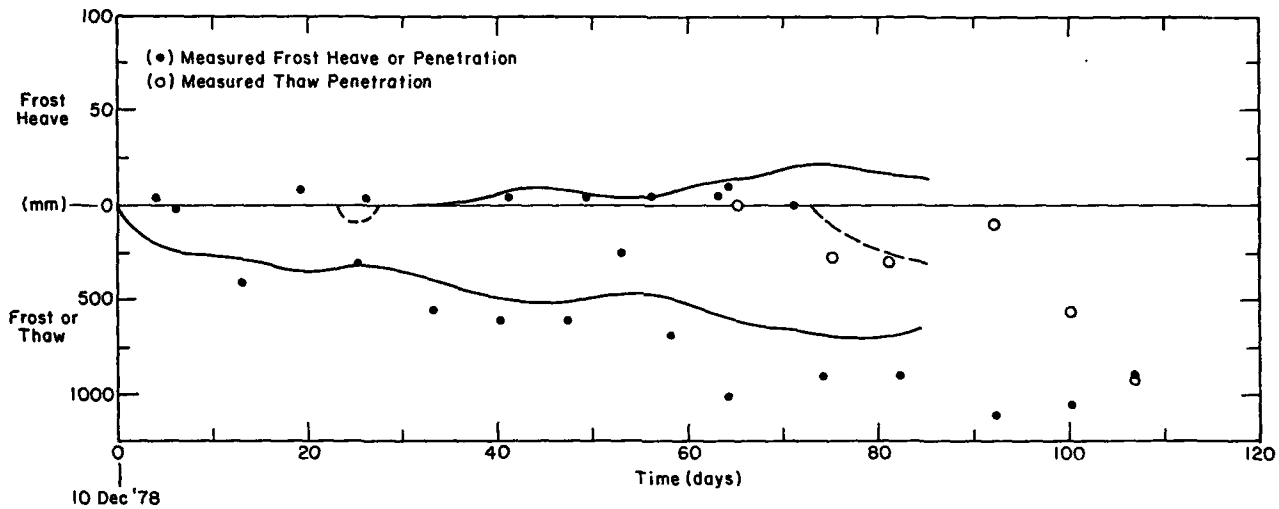


c. Hart Brothers sand.

Figure 13. Simulated frost heave, thaw settlement, frost penetration and thaw penetration, 1978–79.



d. Sibley till.



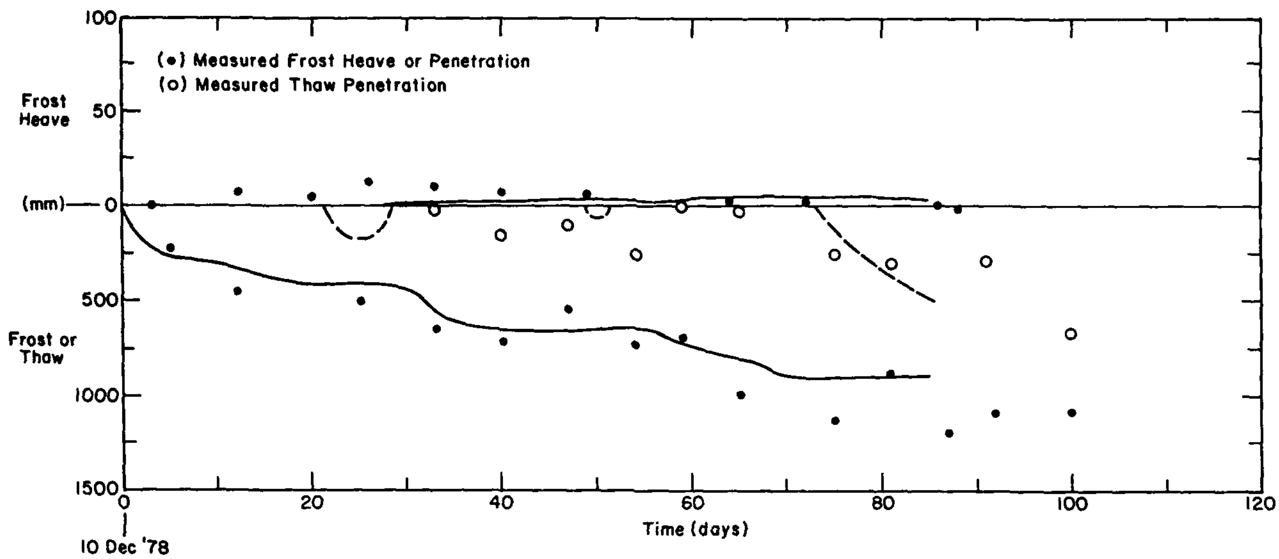
e. Hyannis sand.

Figure 13 (cont'd).

able for the 1979–80 winter, and as a consequence simulation results are less precise.

Results are shown in Figure 14 and are similar to those obtained for the 1978–79 winter data. To achieve a slightly better fit for maximum frost heave for the 1979–80 winter data, the *E*-factors are modified somewhat. *E*-factors are, respectively, for the 1979–80 winter simulations 10.3, 5.0, 9.0 and 8.5 for Ikalanian silt, Graves silty sand, Hart Brothers sand and Sibley till. In Figure 14a, Graves sandy silt also shows a simulation using $E = 4.5$, which was used for the 1978–79 simulation. Generally, this *E*-value gave an overall better fit, only the maximum frost heave is somewhat greater than measured. Overall, the results indicate the validity of using a calibrated model for simulating frost heave in soils.

In most cases, it was difficult to accurately predict frost penetration during the end of the season. Measured frost depths, which are subject to some error, are generally deeper than those simulated with the model. In some cases, such as for the Hart Brothers sand (Fig. 13c), the effective thermal conductivity value for mineral soil may have been too low. Another problem that may cause this appar-



f. Dense-graded stone.

Figure 13 (cont'd). Simulated frost heave, thaw settlement, frost penetration and thaw penetration, 1978–79.

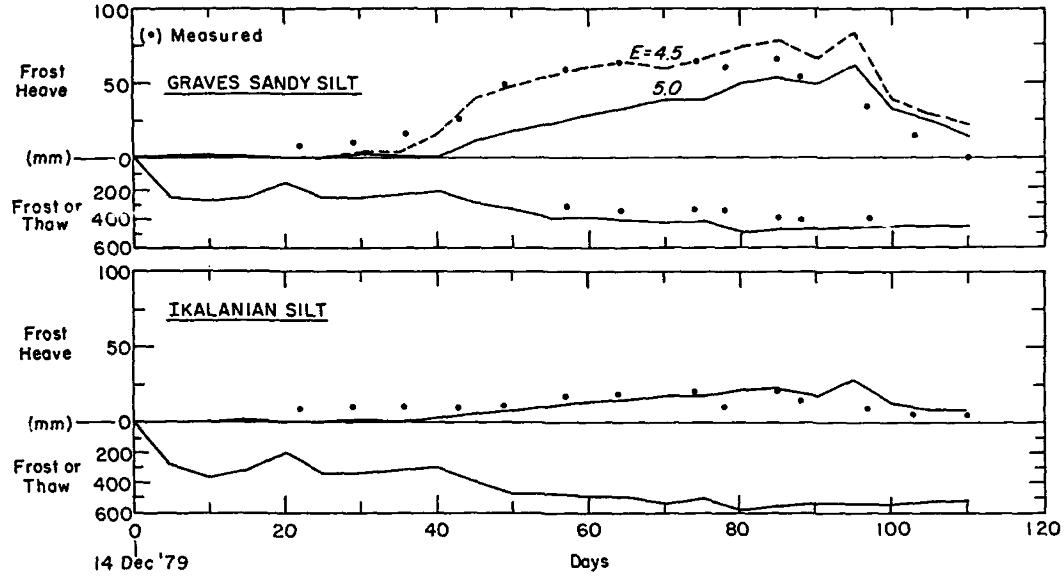
ent error is that rather large elements, 10 cm, are assigned to the column bottom, while small elements are assigned to the column top. This problem is probably not related to boundary condition effects.

In all cases, it was difficult to calibrate the model so that frost heave was accurately predicted at the beginning and end of the season. The only parameter calibrated was the frozen hydraulic conductivity correction factor, and adjusting other parameters such as the soil water characteristics might have yielded better overall results. However, this type of calibration is probably not a wise procedure since errors in the model might be masked. The difficulty in modeling the entire season may stem from three sources: 1) a surface moisture flux boundary condition error, 2) soil parameter variations that may be caused by freeze–thaw cycles and 3) pavement surface temperatures being used instead of soil surface temperatures.

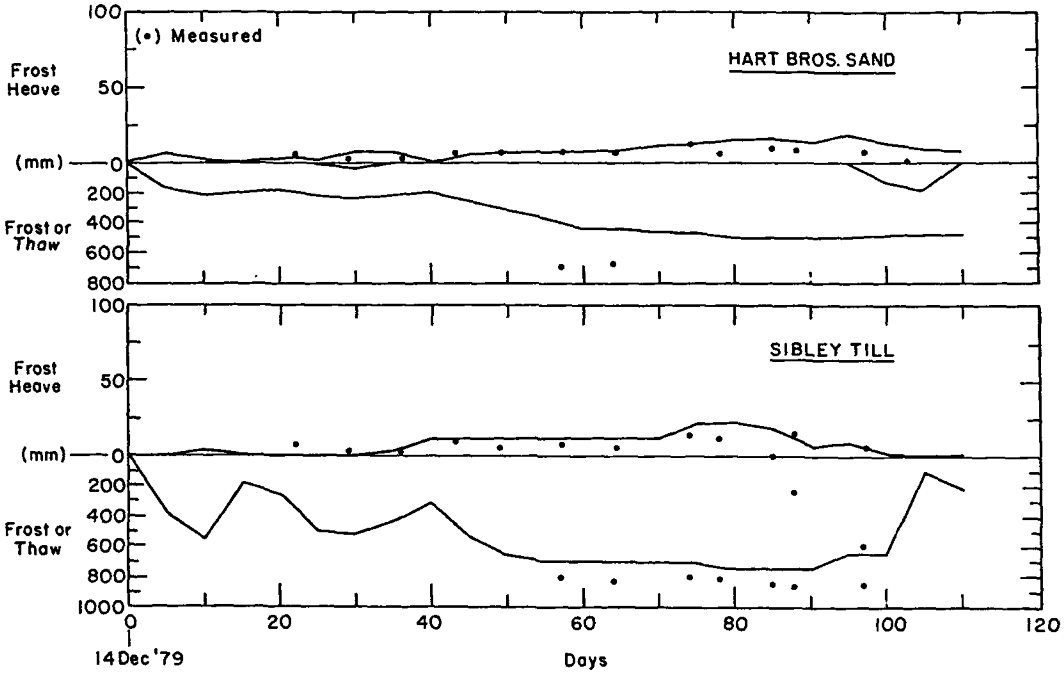
Chamberlain (1980) showed that freeze–thaw cycles drastically altered the saturated hydraulic conductivity of clay; it was increased almost two orders of magnitude by repeated freezing and thawing. Logsdail and Webber (1960) found that alternate freezing and thawing of clay caused a significant disaggregation, while Benoit (1973) found that alternate freezing and thawing might increase or decrease saturated hydraulic conductivity, depending upon initial soil moisture and particle size. While most of the above cited work was for clays, silts could exhibit some of the same features owing to freeze–thaw cycles.

It seems appropriate that any complete model of frost heave should include an analog that would account for changes in parameters, such as hydraulic conductivity, caused by alternate freezing and thawing, as would be the case in most field prototype situations represented by the Winchendon, Massachusetts, test data. However, in this case, the main location of ice segregation is probably at the frost penetration front and soil in this region is being frozen more or less monotonically downward. Alternate freezing and thawing is happening near the soil surface. While the properties of the soil surface are certainly being modified by alternate freezing and thawing, this is not a factor in heave prediction by the model as it is now conceived.

For the simulations of the Winchendon soils, we assumed the soil surface moisture boundary to be a zero flux condition. It is generally believed that moisture movement in a fully frozen soil is by liquid water films. This movement is very slow at low temperatures and more rapid at near thawing temperatures. It is, however, possible for moisture to exit from a frozen soil to the atmosphere or to a snow pack. The mechanism for this is probably liquid water vaporizing at the soil surface so that water vapor can move away from the soil surface. For a relatively warm, slightly freezing soil, there may be appreciable water loss from the soil in this manner, which tends to desiccate the soil surface.



a. Graves sandy silt and Ikalanian silt.



b. Hart Brothers sand and Sibley till.

Figure 14. Simulated frost heave, thaw settlement, frost penetration and thaw penetration, 1979–80.

Also, water can infiltrate into the soil profile. If the soil surface region is thawed, snowmelt or rainfall could infiltrate and be partially or almost totally trapped above a frozen zone. This water would be available during a subsequent freezing cycle to produce even more ice segregation than was produced during previous freezing periods. Winchendon test site data are collected for soils covered by asphalt concrete, which is probably relatively impermeable. When simulating frost heave below pavements, a surface moisture boundary condition other than a zero flux condition would probably not be required, except for cracked or highly porous pavement surfaces.

The most likely problem in simulating the Winchendon soils is that the pavement surface temperature was used as a boundary condition. More accurate results would have been possible if soil surface temperatures below the pavement were used. However, this model is intended to be used to evaluate pavement performance; therefore, a pavement surface over a granular material is a realistic simulation.

Albany County Airport, New York, data

Two taxiways at the Albany County Airport, New York, were instrumented and frost heave measured. Since frost heave was negligible at taxiway A, only the taxiway B data, for the 1980–81 winter, are evaluated. Figure 15 shows a photograph of the study area at taxiway B.

The soil profile consists of a 3-in. (7.6-cm) layer of asphalt concrete underlain by 4 in. (10.2 cm) of asphalt-penetrated gravel, then a 5-in. (12.7-cm) layer of clean gravel, underlain by a silty sand subgrade soil.

Measurements taken include air temperatures at the Albany County Airport National Weather Service station, soil temperatures, water table and pore water pressures. Samples from various soil layers were evaluated in the laboratory to determine physical and hydraulic parameters (included in Appendix A). The E-factor was determined by calibration, and thermal parameters are assumed. Frost heave was measured at 39 points in a regular grid. Because the reference point used to survey these grid points may have heaved, there is some uncertainty about the heave data. Figure 16 shows cumulative average frost heave for all points surveyed on days of measurement. The standard deviation of the measured data is also plotted for each measurement day.

We used a soil column length of 1 m for simulation, with uniform 0.2-cm elements. Each time step size was 0.2 hours and parameters were updated each hour. Boundary conditions measured in the field for the column bottom were closely approximated. Generally, the water table depth was from 1 to 1.5 m, and soil temperatures at this depth were about 2°C. Mean daily soil surface temperatures were estimated from soil thermistors or from air temperature data using the n-facto- -pproach (see next section).

Also plotted in Figure 16 are the results of the simulation for comparison with measured data.

Discussion

The results presented in this section demonstrate that, for different soils, ranging from silts to relatively coarse-grained and marginally frost-susceptible soils, good results can be obtained with the deterministic model. Moreover, these results have been demonstrated with carefully controlled laboratory data as well as with less precise field data.

To achieve such results, however, good estimates of hydraulic parameters are required. As was discussed in the *Model Uncertainty and Errors*

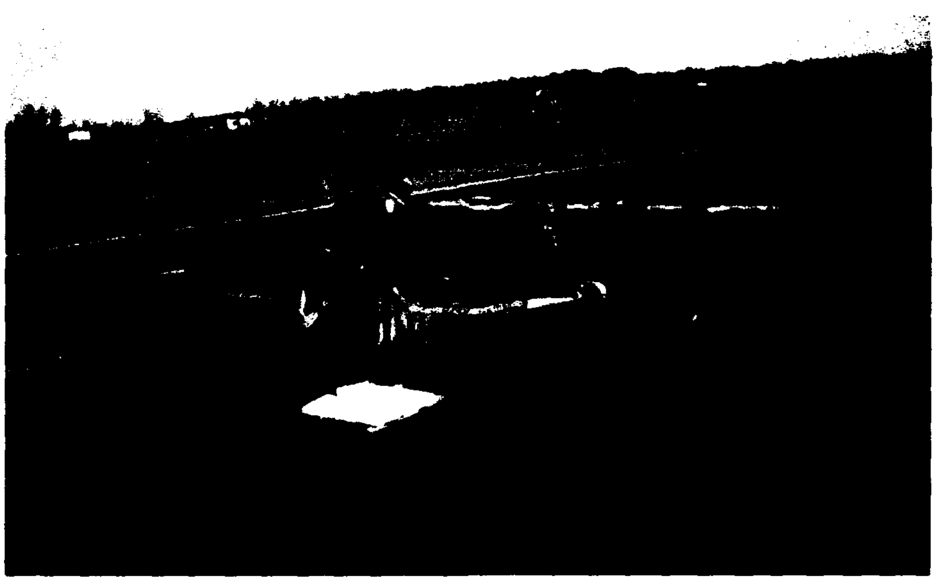
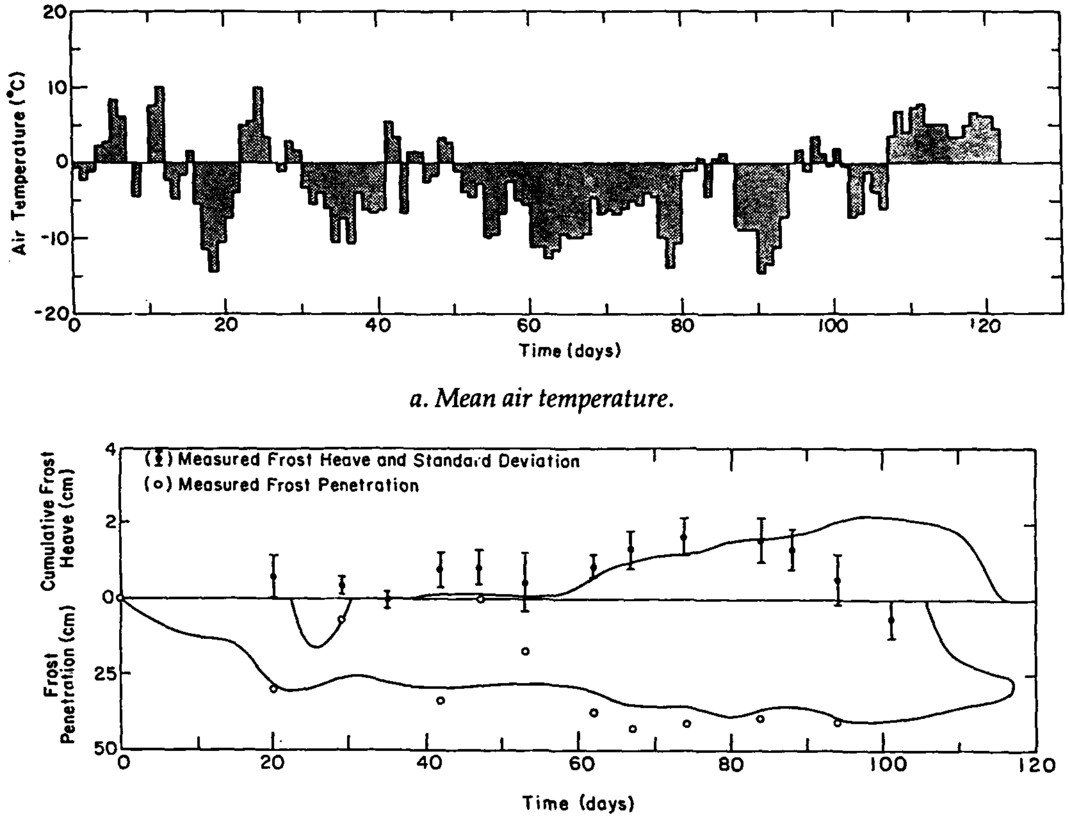


Figure 15. Study area on taxiway B, Albany County Airport.



a. Mean air temperature.

b. Simulated frost heave, thaw settlement, frost penetration and thaw penetration.

Figure 16. Data and results from taxiway B, Albany County Airport, 1979–80.

section, there are large errors in the most carefully measured soil parameters, particularly unsaturated hydraulic conductivity.

Modeling of freezing soil requires calibration of the *E*-factor, which corrects for freezing soil hydraulic conductivity, or estimation of this parameter based upon reported tests. We used the *E*-factor as the primary calibration parameter to achieve the results presented here. On the basis of these calibration tests, eq 5 was developed as a guide to determining the *E*-factor.

Even if more precise scientific knowledge were available for the hydraulic conductivity function during freezing, calibration would still be required for precise results. There is no model in existence for porous media flow processes that does not require calibration to achieve acceptable results. Hypothetical solutions of such problems given assumed parameters have a considerable error, which for some porous media problems may be tolerable for engineering analysis. Usually, human judgment and experience are exercised to draw inferences on the level of certainty of such computations. This need is evident in the problem considered here.

Finally, it is important to recognize that the model presented here is a tool to examine different responses of a soil thermal system subjected to different environmental conditions and parameters. Models of porous media flow processes are not precise for predicting a specific state of the system but are excellent tools for evaluating differences in response to imposed boundary conditions and parameters.

BOUNDARY CONDITION EFFECTS

This section examines boundary condition effects on the prediction of frost heave, thaw consolidation, frost penetration and thaw penetration. The associated problem of soil strength or the loss of strength during the thaw weakening phase is also examined. Laboratory data on frost heave from an experimental soil column were used to calibrate FROST, which is used with a variety of different boundary conditions to examine both systematic and random errors and to examine boundary condition effects. The soils used are Chena Hot Springs silt, a frost-susceptible soil similar to Fairbanks silt, and West Lebanon gravel, a marginally frost-susceptible dirty gravel. Also, one of the Winchendon, Massachusetts, test site soils, Graves silty sand, is used to evaluate surface temperature errors.

Frost heave, thaw consolidation, frost or thaw penetration and the associated problem of soil strength depend on soil properties and environmental conditions. Historically, these dependencies have been examined through laboratory experiments on so-called "frost-susceptible soils." Unfortunately, laboratory experiments are costly and sometimes yield conflicting results, depending on similarities when comparing laboratory experiments. Because only one or a few experiments are conducted at one time, it is difficult to form unifying concepts of how soil physical properties and environmental conditions interact. Comprehensive models are a tool to study such effects and one of the central objectives of the modeling exercise is to be able to evaluate environmental or boundary condition effects. Additionally, modeling errors introduced by errors in boundary condition specifications are important to evaluate.

The model requires a soil surface temperature T_u, a column-bottom soil temperature T_L and pore water pressure head h_L, each of which may be a function of time. Although a soil surface pore water pressure can be specified, the model assumes that no liquid water moves across the soil surface during the freezing or thawing process. Total overburden effects at ice segregation fronts are the sum of the weights of all materials above the freezing front and surcharge pressure P_o.

Although there is a vast variety of boundary condition forms that could have been used, we chose fairly simple and easily obtainable field boundary condition forms. An example of model boundary conditions is shown in Figure 3.

The column bottom requires temperature and pore pressure head boundary conditions, such as a water table. These conditions must be measured or estimated. We will subsequently show that predicted frost heave and thaw consolidation are relatively insensitive to the column-bottom boundary conditions for relatively fine-grained soils. The location of the water table is, however, important for relatively coarse-grained, marginally frost-susceptible soils. We will also show that initial conditions for temperature and water content are relatively unimportant to predictions, provided that the soil is completely unfrozen at the initiation of a simulation. The soil surface temperature boundary condition was highly important for all cases studied. The following subsection investigates the surface temperature sensitivity in some detail.

Soil surface temperature

The objective of any predictive model is to forecast what will happen given certain parameters and given certain environmental conditions that may be, for instance, related to design criteria. For field applications, these environmental conditions—e.g., surface soil temperatures—must be readily obtainable for a wide variety of climate, terrain or vegetative areas to make the model useful. This usefulness will, however, be impaired, depending on the approximation level incorporated into the boundary conditions and the errors introduced into predictions by boundary condition uncertainty.

It is generally assumed that the energy budget technique is the most precise method of estimating soil, water or snow surface temperature or heat flux. Berg (1974a) presents a detailed form of the heat budget equation for any surface interface with air

$$0 = Q_s - Q_r + Q_w - Q_e \pm Q_c \pm Q_\ell \pm Q_u \pm Q_m \pm Q_g \pm Q_i \quad (41)$$

where individual heat fluxes are

Q_s = incident shortwave radiation
Q_r = reflected shortwave radiation
Q_w = longwave radiation emitted by the atmosphere
Q_e = longwave radiation emitted by the earth
Q_c = convection
Q_ℓ = evaporation, condensation, sublimation and evapotranspiration
Q_u = conduction into air
Q_m = mass flow to surface
Q_g = conduction into ground
Q_i = infiltration of moisture into ground.

Units are heat/area per time. Components carrying heat toward the surface are positive, those carrying heat away from the surface are negative, and those that may flow in either direction are shown with both signs. Depending on the type of surface considered, some of these heat flow quantities are neglected. For instance, Berg (1974a) considered energy balance on a paved surface and was able to specify $Q_u = Q_m = Q_i = 0$. One of the primary surfaces that we are concerned with is pavement, although we envision application of the frost heave model to soil surfaces (e.g., gravel roads). The various quantities in eq 41 are evaluated from ancillary relationships involving quasi-theoretical considerations, actual measurements or empirical relationships, or all three. Heat flow into the ground surface may be directly estimated or surface temperature may be estimated from the ancillary relationships used to compute one or more of the heat flow quantities. The most comprehensive computations usually rely on nonlinear relationships so that iterative techniques are required to determine surface temperature or heat flow. Application of the heat budget technique generally requires a substantial amount of meteorological data that is only available for a few sites in the U.S. Because of both of these problems, a more simplistic, although more approximate, method is desirable.

Scott (1957) and Berg (1974a) both investigate the use of heat-transfer coefficients that primarily rely on air temperature and other data such as wind speed. We propose semi-empirical relationships for determining heat-transfer coefficient so that surface temperatures may be estimated.

It would be ideal if soil or pavement surface temperatures could be estimated with sufficient precision using air temperatures alone. Air temperatures measured at standard U.S. Weather Service installations (about 1.5 m above the ground surface) are the most widely available meteorological data. Furthermore, the most common air temperature data are daily means (usually computed from maximum and minimum daily temperatures). Figure 3 shows the use of mean daily soil temperatures as input data to the frost heave model.

The Corps of Engineers has used a simple empirical relationship (sometimes called the "*n*-factor approach") based upon air temperature T_o or freezing index and soil surface temperature T_u or soil surface freezing index (Berg 1974b). Average *n*-factors relating soil surface and air freezing indices in degrees Celsius, where

$$T_u = N_o T_o \qquad (42)$$

are given by Berg (1974b) for freezing conditions (Table 6). The *n*-factor increases with increasing latitude and wind speed. Other factors such as rainfall and evaporation will also influence the *n*-factor. Berg (1974b) suggests that the *n*-factor is about double for thawing processes, and he cautions that, for design applications in a specific locality, actual air temperatures and surface temperatures should be measured for several seasons to develop a reliable relationship.

Table 6. Average *n*-factors.

Surface type	*n-factor* (N_o)
Snow	1.0
Pavement	0.9
Sand and gravel	0.9
Turf	0.5

We demonstrated the feasibility of using the approach of Berg (1974b) for analysis of frost heave data from the Winchendon, Massachusetts, test site using air and soil temperature data for 1978–79. These data consisted of maximum and minimum air temperatures measured at the standard height of 1.5 m from 10 November 1978 to 26 March 1979, and incomplete soil surface temperatures for several different soils, with the most complete data for January and March. We computed mean daily air temperatures from maximum and minimum air temperatures, and we estimated average mean daily soil surface temperatures on the basis of maximum and minimum soil surface temperatures for four soils: Ikalanian silt, Hart Brothers sand, Graves silty sand and Sibley till. Average mean daily soil temperatures had a coefficient of variation of about 70%, which is probably attributable to albedo and evaporation differences, as well as measurement errors. We performed a standard regression upon the data to obtain a regression coefficient assuming the functional relationship in eq 42 between air and soil temperature as shown in Table 7. N_o-factors for the Corps of Engineers relationship, for both predominantly freezing and thawing, are similar to the values given by Berg (1974b). The version of the FROST model presented here uses the *n*-factor approach to relate air temperatures to soil surface temperatures.

Table 7. Regressions of air and soil surface temperatures at Winchendon, Massachusetts, 1978–1989, for the Corps of Engineers *n*-factor.

Case	N_o	R*	*RMS* error (%)*
Predominantly freezing	0.594	0.91	2.77
Predominantly thawing	0.976	0.74	5.74
All data combined	0.645	0.91	8.44

* *R* = coefficient of correlation; RMS = root mean square.

Table 8. Diurnal temperature variations at the Winchendon test site, 1978–1979.

Temperature location	Mean daily amplitudes (°C)			Maximum daily amplitudes (°C)			Minimum daily amplitudes (°C)			Mean daily amplitude coefficient of variation (%)		
	Jan	*Feb*	*Mar*	*Jan*	*Feb*	*Mar*	*Jan*	*Feb*	*Mar*	*Jan*	*Feb*	*Mar*
Air*	4.7	7.4	7.3	10.3	14.4	18.3	0.6	0.7	1.4	51	50	58
Soil surface												
Ikalanian silt	4.6	9.2	7.4	8.5	11.4	16.6	0.6	6.6	2.2	50	20	58
Graves sandy silt	3.2	7.4	5.8	5.0	10.3	17.0	0.1	5.4	1.1	50	30	78
Hart sand	3.1	7.1	8.1	5.8	10.6	17.4	0.2	1.4	2.0	48	42	58
Sibley till	4.6	10.4	5.9	10.6	14.0	13.2	0.3	5.8	1.8	61	28	69

* Approximately 1.5 m above the ground surface.

One of the possible problems with using mean daily soil surface temperatures, particularly when these temperatures are near the freezing point depression of water, is what Outcalt and Goodwin (1979) refer to as the "high frequency cut-off effect." Diurnal effects may be important to frost heave, thaw settlement and frost and thaw penetration predictions. Lunardini (1981) found that for a simplified freezing problem a sinusoidal and step change surface temperature produced about the same freeze distance but significantly different freeze rates. The freezing rate is very important to the ice segregation process since the interaction of freeze rate and water flux will influence the amount of ice segregation (frost heave).

Diurnal temperature variations of both air and soil were evaluated for the Winchendon, Massachusetts, test site (Table 8). Variations between soils may in part be ascribable to differences in albedo and evaporation (i.e., soil surface wetness). The presence or absence of shade from nearby trees may also be a factor in noted variations. The use of average monthly air temperature amplitude analysis to represent, say, a sine curve diurnal variation is subject to at least a 50% error for the 1978–79 data.

Initial condition effects

Simulations require initial conditions for pore water pressures, temperatures and ice content as a function of depth. We examined initial condition effects by altering the initial conditions from those specified in calibration simulations and comparing predicted and measured frost heave and frost penetration. For Chena Hot Springs silt and West Lebanon gravel, four-fold variations in pore pressures and temperatures resulted in negligible differences in predicted frost heave and frost penetration after one day in the freezing process. Initial ice content conditions are significant if the ice content approaches the pore ice space, which is defined in the model as follows

$$(\theta_o - \theta_n)$$

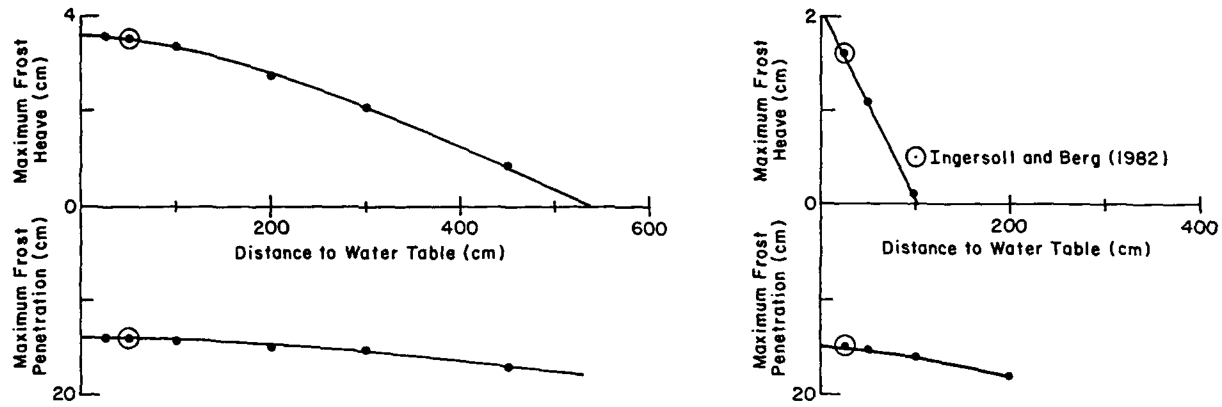
where these variables have been previously defined. The error in predicted frost heave is directly proportional to the error in initial pore ice specification when pore ice approaches the above relationship.

Boundary condition effects

We evaluated boundary condition effects by using calibrated parameters for Chena Hot Springs silt and West Lebanon gravel (Table 4). In each case, we used a 50-cm column of uniform soil, which is divided into 1-cm elements. Time-steps were advanced each 0.2 hours and parameters were updated every hour. Generally, each simulation consisted of a 9-day freezing period followed by a 9-day thawing period. We varied one boundary condition while holding all others unchanged.

Column-bottom temperature effects

Under the conditions assumed in the simulations conducted, column-bottom boundary temperature variations had a negligible effect. Column-bottom temperatures were held at 5°C for each of the 18-day simulations. A ±50% variation of this temperature had little effect on simulated frost heave, thaw settlement, frost penetration or thaw penetration. The reason for this is that a 50-cm column was used for simulations and frost penetrated to a maximum depth of only 10 to 20 cm. Because the lower boundary condition is somewhat removed from the frost penetration depth, variations in lower boundary condition tempera-



a. Chena Hot Springs silt.

b. West Lebanon gravel.

Figure 17. Effects of water table depth on simulated frost heave and frost penetration. Open circles indicate laboratory measurements.

tures had little effect on the thermal gradient in the vicinity of the freezing fringe.

For a shorter column or for deeper frost penetration, the specification of a column bottom boundary temperature should become more critical to prediction precision. If in a given field application of the model there is considerable uncertainty in subsurface soil temperatures, we suggest that an appropriate modeling strategy to minimize bottom boundary temperature effects is to choose a column length about twice as deep as the expected maximum frost penetration.

Column-bottom water table effects

Water table effects were studied by holding column-bottom temperatures at 5°C and soil surface temperatures at -3°C. Maximum heave and frost penetration at the end of 9 days were evaluated in terms of water table depth below the original ground surface elevation. The results for Chena Hot Springs silt are shown in Figure 17a, and for West Lebanon gravel in Figure 17b, for relatively shallow freezing (less than 20 cm depth). For deeper freezing, the results for West Lebanon gravel (Fig. 17b) would be particularly altered. In both cases, the position of the water table had some, but not great, effect on the depth of frost penetration. This is because there is a different amount of ice freezing for each case and a resulting difference in phase change heat, depending on water table position.

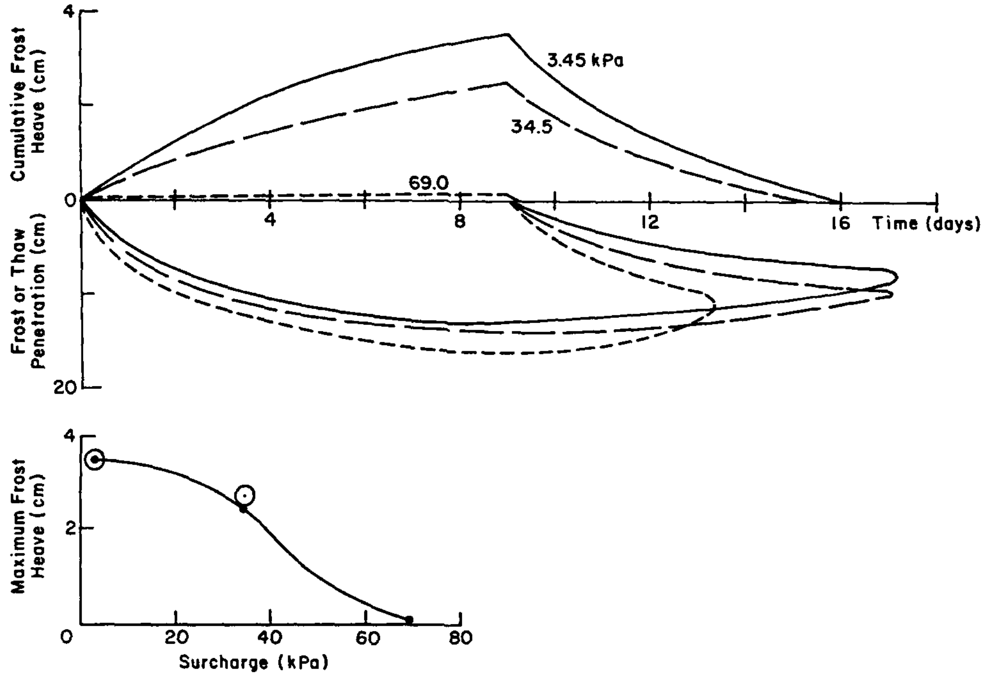
On the basis of Figure 17a, the model predicts that a water table depth of 5 to 6 m will eliminate frost heave of Chena Hot Springs silt, provided frost penetration is relatively shallow. This result is reasonable because materials similar to the Chena Hot Springs silt have a so-called "capillary fringe" on this order of magnitude. For the 50-cm column simulations conducted here, a 100% variation in water table depth produced only a small change in frost heave and frost penetration predictions. Thus, when simulating a shallow, unsaturated soil column of silt soils with relatively high water table conditions, simulations are relatively insensitive to the column-bottom pore pressure boundary condition. This would become even more pronounced for finer-grained soils.

Figure 17b indicates that water table position is highly important when assessing frost heave for relatively coarse-grained, marginally frost-susceptible soils. For the shallow freezing case considered here, the model predicts a steep, almost linear, decrease in frost heave with increase in water table depth. This behavior of the model is generally borne out by experience. Laboratory results from two tests indicate that the model somewhat overpredicts the effect of water table depth on frost heave of West Lebanon gravel.

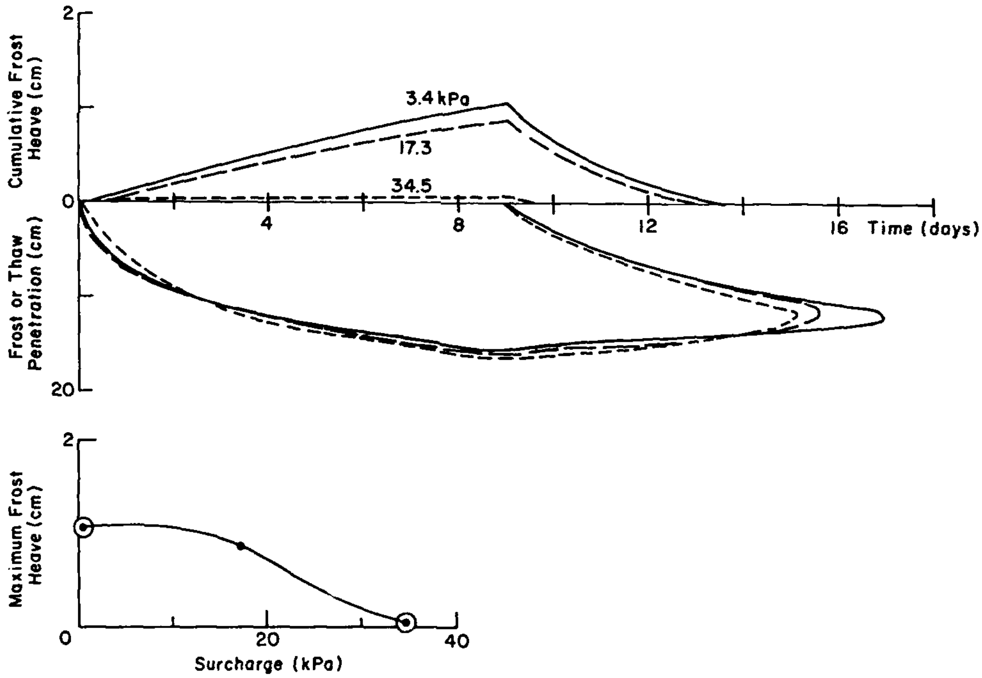
Surcharge effects

We studied surcharge effects by varying the column soil surface surcharge boundary condition while holding the water table at the bottom of a 50-cm simulation column. Soil temperatures at the column bottom were maintained at 5°C, while the soil surface boundary temperature condition was maintained at -3°C for the first 9 days of simulation and 2°C for the final 9 days of simulation.

Figure 18a shows the results for Chena Hot Springs silt. Two laboratory results are available to verify the total heave versus surcharge simula-



a. Chena Hot Springs silt.



b. West Lebanon gravel.

Figure 18. Effects of surcharge on simulated frost heave, thaw consolidation, frost penetration and thaw penetration for a freeze cycle with the soil surface temperature at −3°C and a thaw cycle with the soil surface at 2°C. Solid dots indicate simulated points and open circles indicate laboratory verification.

tions. Penner (1981) conducted tests examining frost heave rate versus the ratio of surcharge to cold side temperature for an apparently saturated silt, similar to the one used here. His results suggest that the total heave versus surcharge relationship should more or less asymptotically approach the surcharge axis. The model probably somewhat under-predicts frost heave at high surcharge levels. Further calibration of the model would eliminate this discrepancy; however, the model should be regarded as primarily applicable to light surcharge situations. We conducted one simulation using a 3.45-kPa surcharge for the 9-day freezing period and then applied a 34.5-kPa surcharge during the following 9-day thaw period. Thaw consolidation was about 10% more during the initial thaw period than is indicated for the 3.45-kPa case in Figure 18a. The lengths of the thaw consolidation period was about the same as is shown in Figure 18a for the 3.45-kPa case.

Figure 18a also demonstrates the effect that surcharge has on frost penetration and thaw depth. Relatively moderate surcharge on soils with large percentages of silt-sized particles will significantly alter the total depth of frost penetration and the rate and duration of thaw penetration. This latter point is significant for the degree and duration of thaw weakening problems. The obvious reason for this behavior is that surcharge impedes the growth of ice in the soil, which requires less phase-change heat, and the soil can thus freeze deeper during the freezing stage. During the thawing stage there is less ice to thaw and the thawing process is much more rapid than when no surcharge is applied.

Figure 18b shows the results for West Lebanon gravel. The total heave versus surcharge simulation results are substantially verified by data obtained from the freezing laboratory experiments. However, similar to the case discussed above, it is expected that the relationship should more or less asymptotically approach the surcharge axis.

Figure 18b shows that for marginally frost-susceptible soils, small to moderate surcharges will have only marginal effect on simulated frost heave, frost penetration and thaw penetration. The reason for this is that the coarser-grained texture of such soils tends to promote more support by the soil matrix, i.e., effective stresses are higher than for finer-grained soils. The effect is more pronounced on frost heave because, unlike highly frost-susceptible soils, there is little tendency to form lens ice and thus the thermal regime of the soil profile is only marginally altered by surcharge effects.

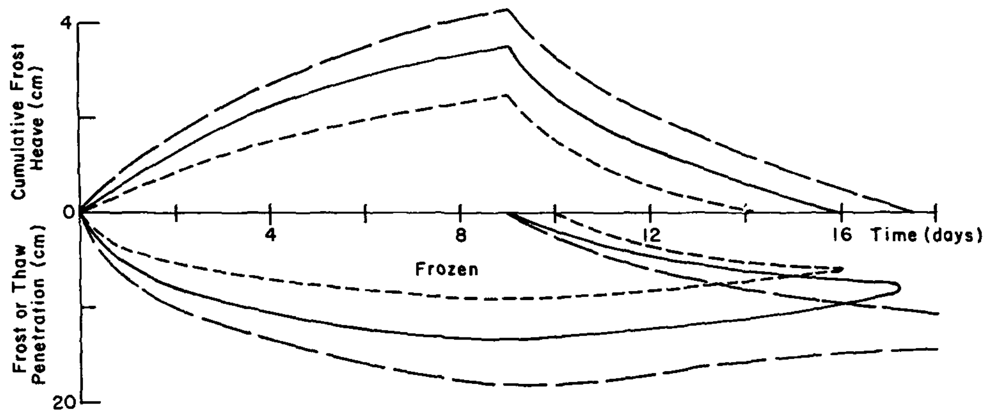
Mean daily soil surface temperature effects

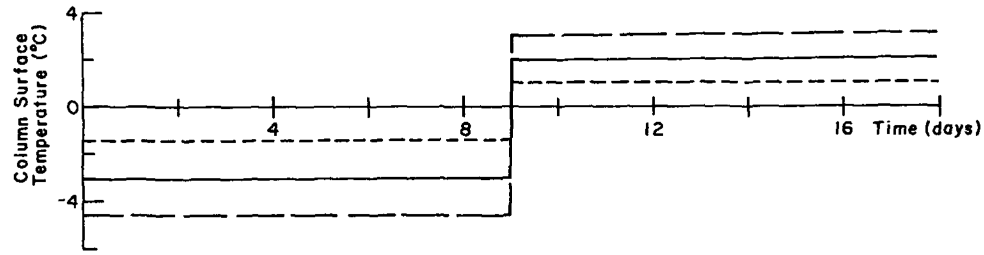
Column-bottom temperatures were held at 5°C and pore water pressures were held at 0 for all simulations attempted. Figure 19a shows the results for Chena Hot Springs silt, and Figure 19b shows the results for West Lebanon gravel. Boundary conditions indicated by the dashed lines represent a ±50% variation in soil surface temperatures. If the n-factor method of estimating soil surface temperatures at field sites from mean daily air temperature data were used, there would be at least as much temperature variation as was used in the simulations presented in Figure 19. A ±50% variation in soil freezing temperatures results in a simulation coefficient of variation for frost heave of about 100%, a rather significant effect of systematic errors in specification of soil surface temperatures. Recall that in Table 7, significant errors in estimating surface temperatures are possible when air temperature data are used. The errors introduced to the positive thawing temperatures are less pronounced; however, there is considerable variation in thawing regimes because of the errors in freezing processes. To accurately predict thaw weakening phenomena apparently will require a high degree of precision in estimating freezing effects. Prediction of soil surface temperatures during thawing is somewhat less important.

Diurnal soil surface temperature effects

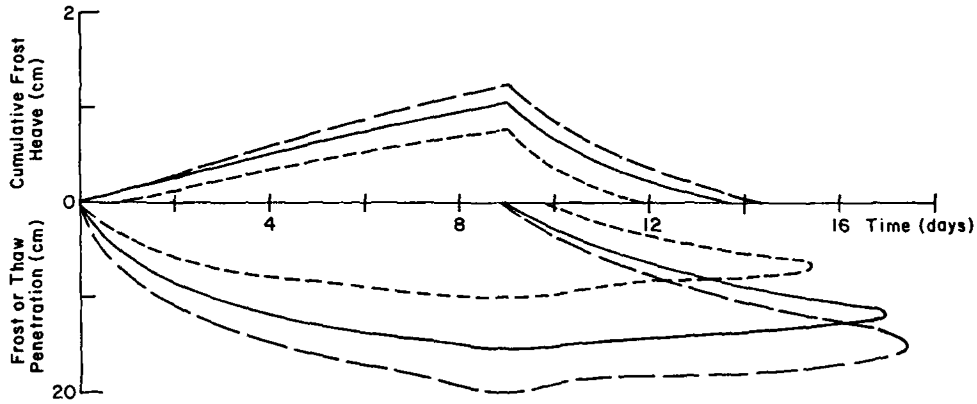
We evaluated diurnal effects by using the same study cases described earlier, i.e., column-bottom temperature and pore pressure were held at 5°C and 0 respectively. Soil surface temperature trends are −3°C for the first 9 days and 2°C for the final 9 days. Previously, the Winchendon, Massachusetts, test site showed that average diurnal variations range from about 3°C to about 10°C, with a coefficient of variation of about 50%. An "average" sinusoidal diurnal amplitude of 6°C was used as one study case. This diurnal variation results in alternate daily freeze–thaw cycles during the 18-day simulation period. Also, a sine curve diurnal temperature amplitude of 2°C was used so that during the freezing period there would be no thaw and during the thawing period there would be no freeze.

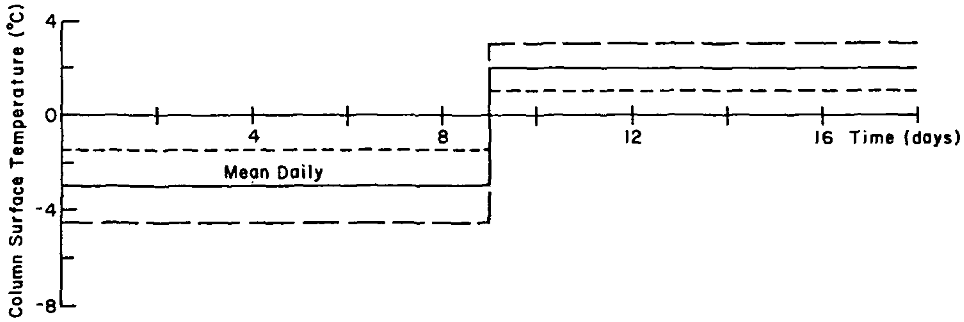
The results of this analysis are shown in Figure 20. The 2°C amplitude of the diurnal temperature variation caused only a minor effect in both cases. Larger variations might also produce minor variations, provided mean daily temperatures were suf-





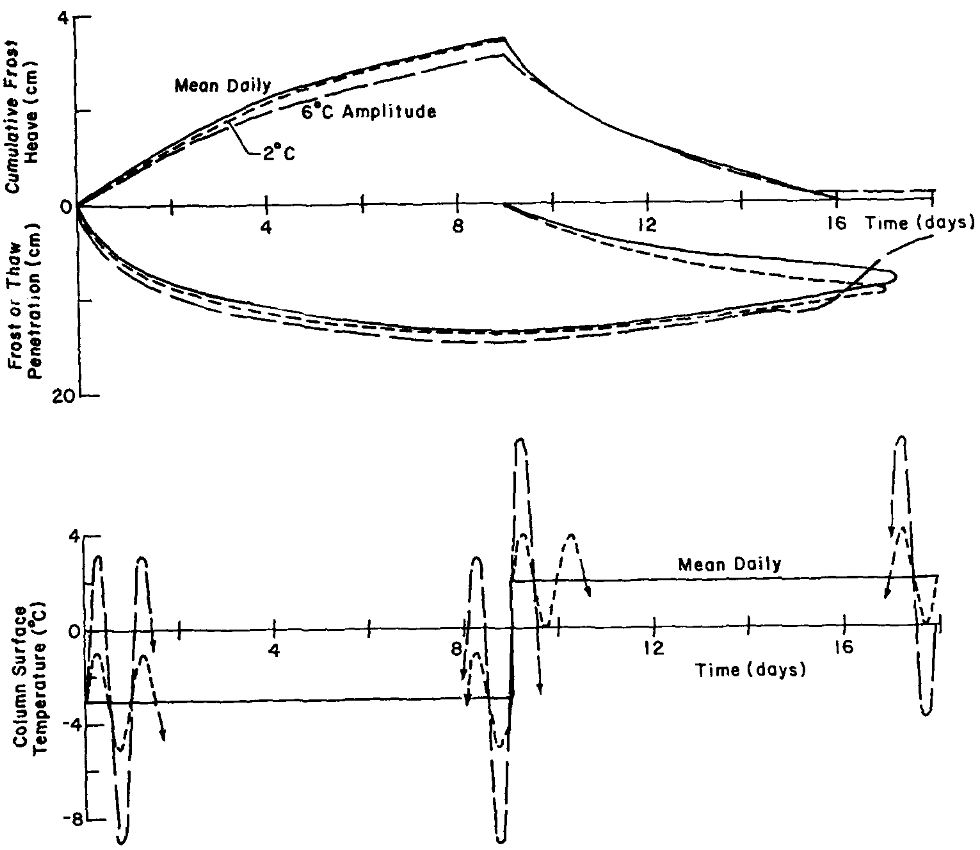
a. Chena Hot Springs silt.



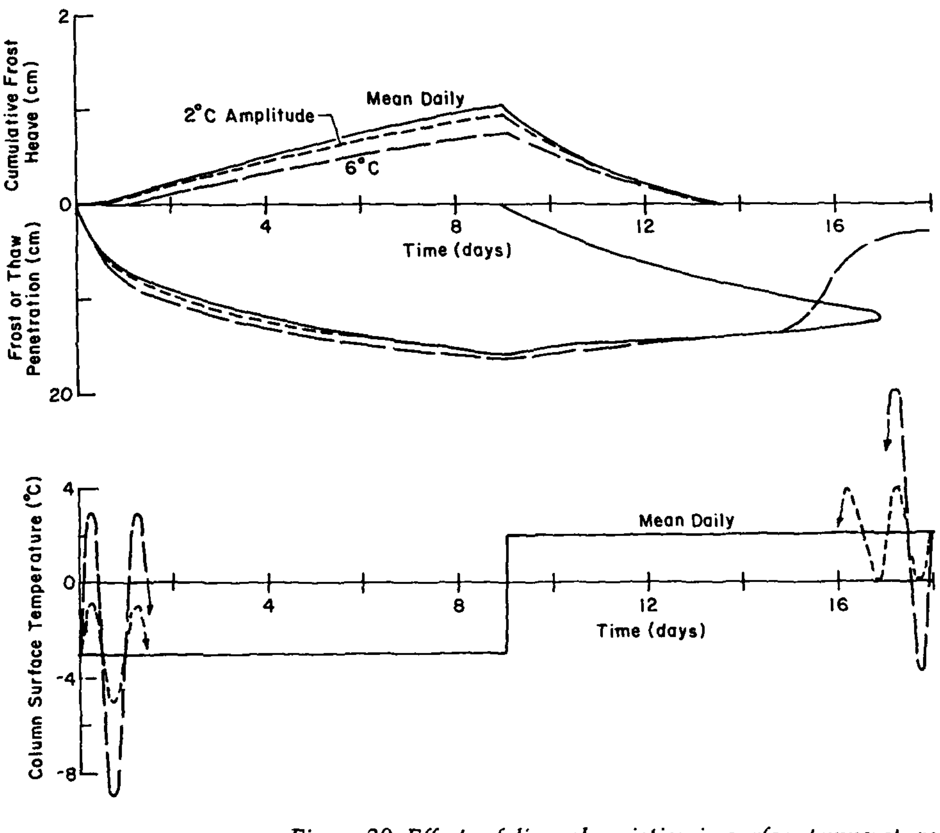


b. West Lebanon gravel.

Figure 19. Effects of surface temperature boundary condition.



a. Chena Hot Springs silt.



b. West Lebanon gravel.

Figure 20. Effects of diurnal variation in surface temperature.

ficiently different from 0°C. The 6°C amplitude of the diurnal variation resulted in a significant variation in results for Chena Hot Springs silt, and still more in the case of West Lebanon gravel. When there are alternate daily freeze–thaw cycles, it is important to use diurnal temperatures. This is particularly true in the thaw settlement and thaw weakening phase. A slightly subfreezing temperature of silty soil during the thaw weakening stage will produce a markedly different soil strength, depending on unfrozen water and ice content (Johnson et al. 1978).

USING THE MODEL

Our emphasis here has been to describe the basis of FROST and to give some insight into the modeling process, particularly modeling uncertainty. To present a totally user-friendly PC computer code is beyond our scope; however, we will discuss the structure of the computer code and procedures for implementing the model.

A user-friendly computer code consists of three elements: the basic analysis algorithm, a user-friendly "front-end" data loader and data editor, and a "back-end" display (usually graphical). This report presents the basic analysis algorithm.

Increasingly, there is a wide variety of software being marketed for displaying data or computer-generated output on PC color monitors. Many agencies and engineering firms have one or more software packages that allow the user a wide variety of output formats. We suggest that existing commercially available software be adopted by the user to graphically display FROST output for their specific applications. Nevertheless, FROST has readily interpreted digital output formats if the user wants. This output format will be discussed later.

People who would like a copy of FROST and an example data file may call or write to CRREL,* who will furnish a floppy diskette compatible with DOS-based PC's containing an executable version of the program. Upon request, CRREL will provide a list of private firms who market user-friendly versions of FROST.

Preliminary concepts

Various levels of FROST use may be required. For example, some projects may only require a "rough" estimate of frost effects and there is no justification for detailed geotechnical exploration or laboratory analysis. In such cases "traditional" techniques such as the use of frost-susceptibility index test data might be the most appropriate procedure. In other projects, the study of the effects of a variety of environmental conditions upon frost action may be required, justifying detailed geotechnical exploration and laboratory testing. It is at this level of effort that the mathematical model would be most useful. To a large extent, the degree of effort expended in obtaining soil parameters or environmental conditions for use with the model will depend on the different needs of a variety of potential model users. Different levels of use will depend on whether the user's objective is basically analysis or design through the synthesis of hypothetical frost action. Analysis must yield a unique solution, while design is characterized by generic solutions. The certainty, or more appropriately the uncertainty, of a solution will depend on the level of effort expended in the analysis or design project.

Models such as the one presented here are best used to determine derivatives of behavior, i.e., the difference in response to manipulated parameters. For example, one might want to explore the effects of water table elevation relative to roadbed elevation to see if water table control would materially reduce frost heave or the extent of thaw weakening. In most cases, it will probably be uneconomical to conduct detailed geotechnical tests and it will be more practical to reasonably infer the numerous parameters required in FROST using the data presented in this report.

Problem setup

The first step in a modeling problem is to describe the soil column, which will be based upon geotechnical borings or other logs or may be partially or totally assumed. The length of the column will depend upon the depth of known or assumed column bottom boundary conditions, which may vary with time. Two types of boundary conditions are required: soil temperatures and pore water pressures. The length of the column will also depend upon the anticipated maximum frost penetration depth. It is necessary that the column bottom be below the maximum anticipated frost penetration. We suggest that the column length be at least twice the anticipated total frost penetration depth; this criterion will ensure that column bottom boundary condition errors have only a small contribution to model solution errors.

After deciding upon a column length, it is neces-

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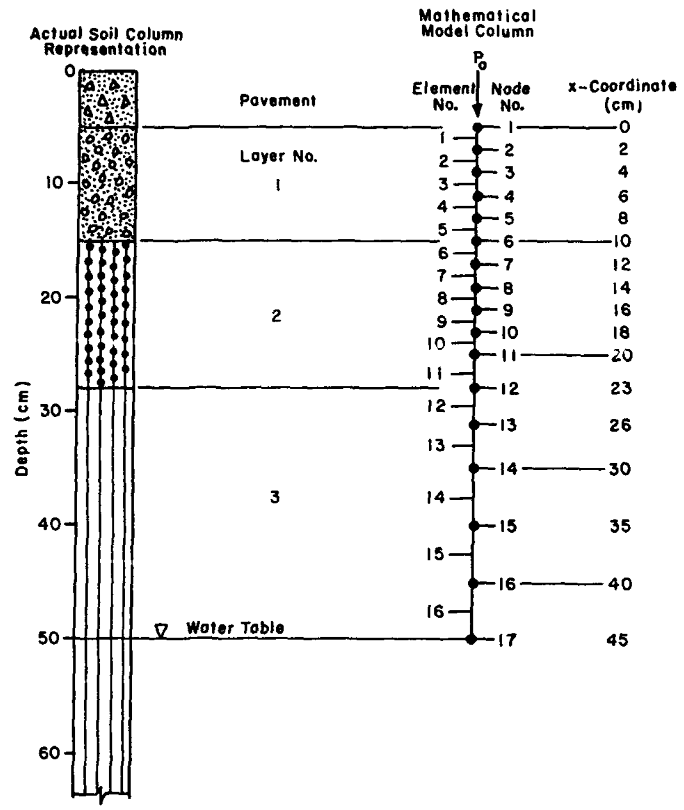

Figure 21. Example soil profile divided into finite elements.

sary to decide on how the column will be divided into finite elements (subdomains). This decision will partly depend upon how much is known about the soil profile and to some extent upon the precision desired in the solution. If a uniform soil profile actually exists, or if a nonuniform soil profile is analyzed as a uniform profile using average parameters, the easiest approach is to divide the column into uniform element lengths. If a nonuniform profile solution is desired, and there are sufficient data on parameters for each layer or the engineer is willing to assume parameters, nodes must be located at each material interface. Next, each layer is usually divided into elements of uniform length. Generally, it is advisable to have element lengths on the order of 1 to 2 cm in the zone that is expected to be frozen. Element lengths may approach 10 cm at greater depths below the anticipated maximum frost penetration without undue loss of accuracy.

Figure 21 illustrates the process of selecting a column length and dividing a nonuniform (layered) soil profile into finite elements and nodes. Such a structure is modeled by specifying a surcharge P_o on the column top, as shown in Figure 21.

In this example, a known water table exists at 50 cm below the pavement surface, at the beginning of the simulation, and thus $h_p = 0$ at node 17, where $x = 45$ cm. Also, a temperature T_L is assumed known at the column bottom. Column bottom temperatures T_L and pore water pressure heads $(h_p)_L$ must be specified for the duration of simulation in the format shown in Figure 22. Both boundary conditions vary in time. The length of the simulation will depend upon the analysis objective. In the example given in Figure 22, 10 days is assumed.

The surface soil temperature T_u must be provided for the length of simulation period as shown in Figure 22. Surface pore water pressure head is a constant for freezing soil and is computed internally for all other conditions, as was previously described. The best way of estimating the freezing pore-water pressure head is to compute $(h_p)_u$ from eq 3 and known or assumed parameters A_w and a and the unfrozen water content factor θ_n, i.e.

$$(h_p)_u = -[(\theta_o/\theta_n - 1)/A_w]^{1/a} .$$

The values of the upper surface pore water pressure head during freezing should be between –200 and –1000 cm of water; we normally use a value of –800 cm of water.

Surface temperature data required for the CRREL version of the model are a sequence of three data points consisting of {temperature in degrees Celsius, hours past initial time, n-factor}. Column-bottom pore water pressures consist of a sequence of data pairs {pore pressure head in centimeters of water, hours past initial time}. Column-bottom temperatures consist of a sequence of data pairs {temperature in degrees Celsius, hours past initial time}. The program also requires the amplitude of a sine curve of diurnal temperature, which may for convenience be set to zero. An example is shown in Figure 22.

Generally, a minor phase of problem setup consists of determining initial conditions for pore water pressures, temperatures and ice contents. If the problem involves an initially unfrozen soil, these conditions can be assumed without introducing appreciable error. Because they are usually assumed, it is best to assume that they are constant with depth. In the event ice may be present in a soil profile at the initial simulation time desired, accurate data on spatial ice content are required. These

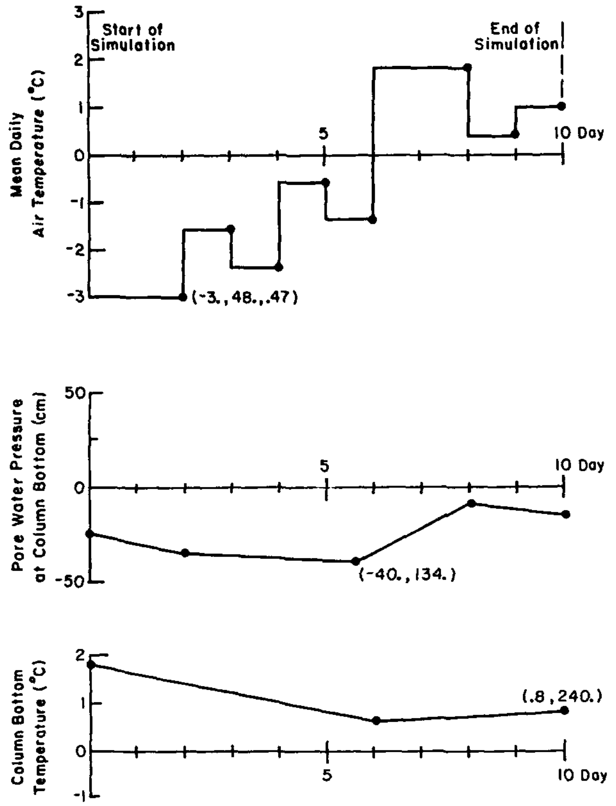

Figure 22. Example boundary conditions for a 50-cm soil column.

can only be developed by means of a boring and careful measurement of ice content. Often, soil moisture contents are routinely obtained as part of subsurface exploration programs. If such data are available, it is relatively easy to obtain initial pore water pressure conditions by using eq 3. Sufficient detail may be available so that the engineer may wish to specify different initial conditions with depth. If so, then it must be specified for each node.

The next important aspect of problem setup is to obtain the required soil parameters for each layer of material in the soil profile. These are:

1. Physical parameters
   a. Porosity, θ_o.
   b. Soil density, ρ_s.
2. Hydraulic parameters
   a. Moisture characteristics for drying curve (Gardner's A_w and a).
   b. Unsaturated hydraulic conductivity function (k_s, A_k, and b).
   c. A multiplier factor for hydraulic conductivity (usually 1.0).
   d. Phenomenological correction factor E for freezing soil, which may be internally computed if requested or input as a calibrated E-factor based on soil freezing tests.
3. Thermal parameters
   a. Volumetric heat capacity of mineral soil, C_s.
   b. Thermal conductivity of mineral soil, K_s.
   c. Freezing point depression of soil water, T_f.
   d. Unfrozen water content factor, θ_n.

Hydraulic parameters may be assumed using the data in Appendix A as a guide or may be developed from laboratory data. Thermal parameters for the soil may be developed from laboratory data or other sources or may be assumed based upon data presented in Appendix B.

Data input file structure

The data file for FROST uses open formats, i.e., floating point or integer numbers separated by commas. The first line is an alphanumeric string and all following lines are numerical. The following is the general structure of the individual input lines:

1. 80 characters of any alphanumeric data (description or title of simulation).
2. Numerical solution methods.
3. Switches for controlling form of data input and computation flow.
4. Number of nodes and number of layers with different soil parameters.
5. Boundary condition form controls.
6. Length of elements (1 to 100 lines).
7. Time step, parameter update frequency, output times and length of simulation.
8. Surcharge, freezing point depression and modifier for pore pressure during thaw.
9. Soil layer parameters (1 to 10 lines).
   a. Gardener's A_w and a and θ_o.
   b. Soil heat capacity, thermal conductivity, hydraulic conductivity multiplier (usually 1.0), soil density, and θ_n.
   c. Saturated hydraulic conductivity, Gardner's A_k and b, E-factor (if to be input otherwise omitted) and modifier to the E-factor during thaw.
10. Lower node number of each layer and layer number (a pointer array) (1 to 10 lines).
11. Coefficient of variation of hydraulic conductivity of the subgrade.
12. Initial conditions for pore pressure head, temperature and volumetric ice content for each node (1 to 100 lines).
13. Upper pore water pressure head.

14. Number of boundary condition data points for upper surface temperature, lower pore water pressure head, and lower boundary temperature and diurnal temperature variation amplitude.

15. Upper air temperature, hour and *n*-factor for each data point (1 to 300 lines).

16. Lower boundary temperature and hour (1 to 300 lines).

The computer code for FROST is included in Appendix D and an example input file for FROST is shown in Appendix E. Additionally, Appendix F is an example work sheet to set up a input data file for FROST.

Output

An example of output from FROST is also included in Appendix E. Generally, all input controls and parameters are output in a digital format. Two choices of output are available: 1) an expanded output that prints all pore water pressure heads, temperatures and volumetric ice contents for each node for each output time period and a summary of frost heave, thaw depth, frost depth and confidence limits for each specified output level, and 2) the summary only. The example included in Appendix E is for an expanded output.

Other information may be output depending upon the application. For example, an application to determine thaw weakening of pavements, another phase of the overall project discussed in the *Introduction,* requires corrected bulk density and porosity of frozen soil. Some results of this work were reported by Guymon et al. (1986). While these data are not output in the version of FROST presented in this report, they are calculated and stored in two separate arrays.

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APPENDIX A. PHYSICAL AND HYDRAULIC PARAMETERS FOR SOILS

Tables A1 and A2 summarize results from laboratory tests that have been conducted on a variety of soils by CRREL. Table A1 contains grain-size distribution, density, void ratio and other pertinent information about various soils that have been tested. Table A2 contains values of some hydraulic properties for each of the soils listed in Table A1.

Data from the tables can be used to estimate hydraulic properties of a soil; however, we recommend that hydraulic parameters be determined in the laboratory. If this is not possible, data in this appendix can be used to make rough estimates of the hydraulic parameters required by FROST.

The following procedure is used to obtain estimates of the hydraulic properties of a soil: 1) locate a soil in Table A1 that has a grain-size distribution, density and porosity similar to the unknown soil, 2) using the *soil number* from Table A1, go to the same *soil number* in Table A2 to obtain the Gardner coefficients for the moisture characteristic curve (relationship between moisture content and pore water pressure), and for the relationship between pore water pressure and hydraulic conductivity.

Note that variable and parameter symbols may be different from in the text. Symbols used in this appendix are defined at the end of each table.

Table A1. Soil properties with percent passing indicated sieve.

Soil No.	Material & Source	Procedure Used	Max. size (mm)	D60	D10	PERCENT PASSING INDICATED SIEVE 4.6 (mm)	0.42 (mm)	.074 (mm)	.02 (mm)	.01 (mm)	.005 (mm)	CU	G	Unified Soil Symbol	Frost Susc. Class	Frost Group	Dry Dens. (g/cc)	Void Ratio E	Sat. Perm. (cm/hr)
								GRAVELS											
	ALASKA		25	8	0.1	46	21	7	4	3	2.5	80	2.73	GW	L-VH	F-1			
AK-1	DOT #1	P.P.															2.18	.252	2.1
	DENSE GRAD		13	5	.10	56	17	9	6	5	3	50	2.80	GW	L-M	F-1			
DGS-1	STONE	V.P.															1.94	.443	
DGS-2	MASS.	P.P.															1.86	.506	5.5
	JACKMAN ME		50	8	.2	53	16	6	4	3	2	40	2.71	GW	VL-M	S-1			
AP GB	GRAVEL	P.P.															2.07	.311	.34
	JACKMAN ME		50	7	.12	54	18	9	6	4.5	3	58	2.71	GW	VL-M	F-1			
JNR GB	Nichols Base	P.P.															1.79	.514	191
	Mn/ROAD		21	9.5	.43	38	10	4	2	1	.8	22	2.79	GW	N	NFS			
MN-CL6	Class 6 Spec.	P.P.															1.84	.495	4.7
	BASE A CR		40	10	.03	58	26	12	9	7	4	333	2.71	GP	L-H	F-1			
AB-1	STONE NY	P.P.															2.16	.255	1.1
	DIRTY GRAV		19	6	.06	55	25	11	5	3	2	100	2.75	GM	VL-M	F-1			
WLNH-1	LEB. NH	P.P.															1.99	.382	.46
	SIBLEY TILL		4.6	.18	.001	100	79	41	24	19	6	180	2.75	GM					
SBT-TTL	MASS.	P.P.															1.88	.608	1.0
	WISCO.		25	.80	.006	90	50	33	25	16	8	133	2.70	GM	L-M	F-1			
CWA-2	SILTY	P.P.															1.86	.451	.53
	WISCO.		10	0.29	.0055	96	70	40	32	22	10	53	2.70	GM-GC	L-M	F-1			
CWA 1	CLAYEY	P.P.															1.90	.420	.0052

Table A1 (cont'd). Soil properties with percent passing indicated sieve.

SANDS & SILTY-SANDS

	Hamilton, MT		50	6	.1	56	30	7	3	2	1	60	2.83	SW	N-M	PFS			
HMT5-1	Gravelly	P.P.															2.03	.395	.85
	SAND																		
	Jackman, ME		25	1.9	.21	85	19	4	2.5	2.0	1.5	9.0	2.71	SW	N-L	NFS			
JAP SB	AP SAND	P.P.															1.86	.459	5.2
	LEB Airport		40	1	.18	85	28	3	2	2	1	5.6	2.76	SW	NFS	NFS			
LNH-SB	SUB BASE	T.C.															1.78	.551	
	LEBANON		2.0	.17	.07	100	85	11	2	1	.5	2.4	2.73	SW	N-M	0			
LCSS-1	CR. STONE	P.P.															1.64	.664	1.6
	POMPEY PIT		50	2.2	.075	73	25	10	4	2	1	29	2.74	SW	VL-M	S-2			
PP SAND	SAND VT	P.P.															1.82	.506	5.5
	Mn/ROAD		8	.8	.05	92	39	12	7	5	4	16	2.69	SW					
MN-CL3	Class 3 Spec.	P.P.															2.03	.336	4.5
	WRJ		4.8	1.0	.07	81	37	13	3	2	1	14.3	2.73	SW	V-M	F-2			
ALRS-S	VT	P.P.															1.97	.385	4.1
	BANK RUN		2	.18	.07	100	86	12	3	2	1	2.6	2.73	SW-SM	N-M	F-2			
LNHS-1	SAND LEB.	V.P.															1.54	.773	
	FINE SAND		.6	.21	.083	100	95	3	0	0	0	[illegible]	2.69	SP	NFS	NFS			
MFS-1	MANCH. NH	T.C.															1.56	.712	
MFS-2		V.P.															1.55	.736	
MFS-3		P.P.															1.48	.818	18.3
	INIGOK BAR-		.85	.23	.098	100	98	5	1	0	0	2.3	2.66	SP	NFS				
IGK 1-1	ROW ALASKA	T.C.															1.67	.593	
IGK 1-2		T.C.															1.68	.583	
	INIGOK BAR-		.7	.17	.07	100	99	11	3	2	1	2.4	2.66	SP-SM	N-L	F-2			
IGK B-1	ROW ALASKA	T.C.															1.69	.574	
IGK B-2		T.C.															1.68	.583	
	SIBLEY TILL		.6	.45	.33	100	50	0	0	0	0	1.4	2.68	SP					
SBT30/50		P.P.															1.57	.802	35
	SIBLEY TILL		.3	.23	.18	100	100	0	0	0	0	1.3	2.69	SP					
SBT50/100		P.P.															1.61	.772	8.1
	SIBLEY TILL		.15	.12	.08	100	100	0	0	0	0	1.5	2.72	SP					
SBT100/200		P.P.															1.63	.669	1.7
	SPECIAL		13	.35	.017	95	64	32	12	7	2	20.6	2.72	SM	VL-H	F-2			
SPEC-1	TEST SAND	T.C.															1.60	.700	
SPEC-2	HANOVER	T.C.															1.68	.619	
SPEC-3	NH	T.C.															1.76	.546	
SPEC-4		T.C.															1.84	.479	
SPEC-5		T.C.															1.92	.417	
SPEC-6		V.P.															1.84	.479	
	SIBLEY TILL		13	.20	.001	98	78	41	24	20	15	200	2.75	SM	L-H	F-4			
SBT-1	MASS	V.P.															1.97	.396	
SBT-2	GLACIAL TILL	P.P.															1.89	.455	1.5
SBT-3		P.P.															2.07	.328	.24
SBT-TOT		P.P.	41	.18	.001							180	2.75	GM	L-H	F-4	1.88	.608	1.0
	GRAVES		2	.12	.013	100	94	44	14	8	5	9.2	2.73	SM	VL-H	F-2			
GSS-1	SILTY-SAND	V.P.															1.58	.728	
GSS-2	MASS	P.P.															1.49	.832	1.92
	HYANNIS		5	.25	.035	98	76	21	4	1	0	7.1	2.67	SM	M-H	F-2			
HYS-1	SAND	V.P.															1.65	.619	
HYS-2	MASS	P.P.															1.69	.580	1.3
	HART BROS		5	.16	.06	99	91	25	4	3	2	2.7	2.78	SM	VL-M	F-2			
HBS-1	SAND	V.P.															1.76	.580	
HBS-2	MASS	P.P.															1.73	.607	4.0
	SUB-BASE		50	2.8	.04	70	37	11	8	7	5	70	2.72	SM	L-H	F-1			
ASB-A1	ALBANY NY	P.P.															2.16	.259	2.8
	DANVILLE		20	.30	.023	93	65	32	8	5	3	13.0	2.74	SM	VL-H	F-2			
DVT1-0	VT	V.P.															1.25	1.192	

Table A1 (cont'd).

DVT19-24	DANVILLE VT	V.P.	2	.21	.018	100	70	41	12	6	3	11.7	2.75	SM	VL-H	F-2	1.84	.495	
DVT21-9	DANVILLE VT	V.P.	2	.10	.02	100	95	48	10	4	2	5	2.78	SM	VL-H	F-2	1.68	.655	
DVT21-0	DANVILLE VT	V.P.	2	.20	.0	100	89	29	6	3	2	6.7	2.76	SM	N-H	F-2	1.61	.715	
ASG-B1	SUB-GRADE B ALBANY NY	P.P.	2	.15	.06	100	99	14	3	1	0	2.5	2.71	SM	N-M	F-2	1.67	.623	2.4
IKE-1	IKELANIAN SAND MASS	V.P.	3	.15	.032	100	8	34	6	2	1	4.7	2.68	SM	N-H	F-2	1.61	.664	
IKE-2		P.P.															1.70	.577	.77
CH-A	CHARLTON A HANOVER NH	T.C.	5	.15	.006	99	79	47	25	15	8	25	2.63	SM	VL-H	F-3	1.3	1.024	.13
CH-B	CHARLTON B HANOVER NH	T.C.	5	.17	.008	99	74	46	22	13	7	21	2.69	SM	VL-H	F-3	1.30	1.070	2.8
CH-C	CHARLTON C HANOVER NH	T.C.	5	.20	.009	100	42	20	11			22	2.70	SM	VL-H	F-3	1.57	.720	.6
WR-A	WINDSOR A LEBANON NH	T.C.	2	.34	.044	100	69	14	5	3	2	7.7	2.63	SM	N-H	F-2	1.54	.707	.14
WR-B	WINSOR B LEBANON NH	T.C.	2	.40	.05	100	62	15	4	2	1	8	2.69	SM	N-H	F-2	1.47	.831	10
WR-C	WINDSOR C LEBANON NH	T.C.	2	.19	.036	100	82	32	4	2	1	5.3	2.73	SM	N-H	F-2	1.43	.909	18
CTS-1	CHENA TOP SOIL AK	T.C.	15	.18	.012	96	88	36	13	8	3	15	2.65	SM	VL-H	F-2	1.54	.721	
CRG-1	CHENA GRA ALASKA	T.C.	4.8	.35	.056	100	68	13	4	3	2	6.3	2.71	SM	N-H	F-2	1.75	.548	
DV32-23	W DOVER VT	T.C.	30	.15	.028	95	84	41	7	4	2	5.4	2.78	SM	N-H	F-2	1.53	.818	
DV32-33	W DOVER VT	T.C.	25	.22	.03	94	77	27	5	3	2	7.3	2.79	SM	N-H	F-2	1.80	.550	
DV32-16	W DOVER VT	T.C.	7	.13	.018	98	88	44	11	7	4	7.2	2.75	SM	VL-H	F-2	1.29	1.132	
DV32-8	W DOVER VT	T.C.	4.8	.11	.02	99	90	48	10	7	4	5.5	2.59	SM	VL-H	F-2	.81	2.196	
DV31-6-1	W DOVER VT	T.C.	4.8	.12	.016	100	86	46	13	6	2	7.5	2.66	SM	VL-H	F-2	.84	2.16	
DV31-6-2		T.C.															.69	2.85	
LNH-SG	LEB AIRPORT SUBGRADE	T.C.	50	.4	.009	85	62	35	18	10	5	44	2.74	SM	L-H	F-4	1.90	.442	
STS	STERRETT TOP SOIL	V.P.	2	.15	.005	100	95	39	25	16	8	30	2.65	SM-SC	L-H	F-4	1.60	.656	
LCSS-2	LEB. SAND AND SILT	P.P.	2.0	.1	.018	100	91	49	14	4	2	5.6	2.74	SM	VL-H	F-4	1.67	.642	.33
DLV-1	DRY LAKE NEVADA	P.P.	4.6	.33	.008	100	67	27	14	11	9	41	2.62	SM	VL-H	F-2	1.81	.447	5.1
LLFS	Lebanon Landfill Sandy Silt	P.P.	20	.30	.080	90	75	29	9	4	1	3.8	2.79	SM	VL-H	F-2	1.62	.721	1.6
NHSPS5	NH-VT SANDY SILT	P.P.	4.6	.11	.009	100	67	56	30	12	5	12.2	2.75	SM	L-H	F-3	1.84	.468	.00087
FR-M.P.	FT. RILEY KA	P.P.	1.9	.3	.004	93	81	43	16	12	10	75	2.61	SC	N	F-3	1.88	.397	.57

Table A1 (cont'd). Soil properties with percent passing indicated sieve.

SILTS

	MANCHESTER		.15	.025	.006	100	100	98	52	21	8	4.2	2.73	ML	L-VH	F-4			
NHS-1	NH	T.C.															1.36	1.015	
NHS-2	SILT	T.C.															1.44	.902	
NHS-3		T.C.															1.52	.802	
NHS-4		T.C.															1.60	.712	
NHS-5		V.P.															1.30	1.10	
NHS-6		P.P.															1.45	.883	.32
	FAIRBANKS		.47	.038	.0042	100	99	94	33	15	11	9.0	2.73	ML	L-M	F-4			
FBKS-1	SILT	T.C.															1.56	.751	
FBKS-2	FAIRBANKS	V.P.															1.69	.615	
FBKS-3	ALASKA	P.P.															1.62	.686	.042
	MOULTON		.08	.016	.0019	100	100	99	74	40	18	8.4	2.82	ML	L-VH	F-4			
MPS-1	PIT SILT	T.C.															1.33	1.067	
MPS-2	LEB NH	T.C.															1.49	.845	
MPS-3		T.C.															1.55	.774	
MPS-4		V.P.															1.50	.880	
MPS-5		P.P.															1.35	1.037	.28
	DANVILLE		2	.07	.02	100	90	60	10	4	1	3.5	2.69	ML	VL-H	F-4			
DVT19-5	VT	V.P.															1.16	1.139	
	DANVILLE		2	.10	.02	100	92	48	10	2	0	5	2.59	ML-OL	VL-H	F-4			
DVT10-3	VT	V.P.															.95	1.721	
	DANVILLE		2	.10	.02	100	92	51	10	4	2	5	2.82	ML	VL-H	F-4			
DVT10-24	VT	V.P.															1.33	1.119	
	DANVILLE		19	.11	.02	97	82	53	13	8	4	5.5	2.74	ML	VL-H	F-4			
DVT17-18	VT	V.P.															1.63	.689	
	DANVILLE		19	.07	.02	98	89	65	10	7	2	3.5	2.61	ML	VL-H	F-4			
DVT17-0	VT	V.P.															1.02	1.558	
	DANVILLE		9	.08	.02	98	84	57	10	4	3	4	2.71	ML	VL-H	F-4			
DVT17-6	VT	V.P.															1.12	1.421	
	APPLE VAL-		2	.03	.003	100	94	84	42	28	18	10	2.59	OL	L-H	F-4			
AVM-2	LEY MN	V.P.															1.16	1.232	
	CRREL SILT		5	.048	.007	100	94	78	25	13	7	6.9	2.69	ML	L-H	F-4			
CS7-1	HANOVER NH	T.C.															1.43	.880	
CS7-2		T.C.															1.39	.935	
CS7-3		V.P.															1.37	.964	
	CRREL SILT		2	.05	.009	100	96	81	21	12	5	5.6	2.70	ML	L-H	F-4			
CS8-1	HANOVER NH	T.C.															1.48	.825	
CS8-2		T.C.															1.48	.825	
CS8-3		V.P.															1.42	.901	
	CRREL SILT		.047	.01	–	100	96	81	21	10	4	4.7	2.71	ML	L-H	F-4			
CS9-1	HANOVER NH	T.C.															1.45	.869	
CS9-2		T.C.															1.48	.831	
CS9-3		V.P.															1.38	.964	
	CHENA HOT		.2	.027	.005	100	100	92	39	20	12	5.4	2.80	ML	L-VH	F-4			
CHSS-1	SPRINGS	T.C.															1.57	.783	
CHSS-2	AK SILT	V.P.															1.59	.761	
CHSS-3		P.P.															1.62	.279	.017
CHSS-4		P.P.															1.54	.817	.063
CHSS-5		P.P.															1.49	.879	.07
	NW STANDARD		2	.03	.005	100	99	98	38	17	10	6	2.65	ML	L-VH	F-4			
NWS-1	SILT AK	P.P.															1.42	.866	.26
	OTTAWA SAND		.15	.038	.0028	100	100	91	37	25	15	13.6	2.60	ML	L-H	F-4			
OWS-1		T.C.															1.43	.858	
OWS-2		T.C.															1.53	.700	
	HANOVER		.2	.032	.004	100	100	95	34	17	11	8	2.69	ML	L-H	F-4			
HNVS-1	SILT	T.C.															1.30	1.07	
HNVS-2	HANOVER NH	T.C.															1.46	1.07	
HNVS-3		T.C.															1.58	.703	
HNVS-4		T.C.															1.67	.611	
	JENKS		*.85*	*.068*	*.006*	*100*	*99*	*66*	*22*	*13*	*9*	*11*	2.73	ML	VL-H	F-4			
JSS-1	SANDY-SILT	T.C.															1.70	.606	
JSS-2		T.C.															1.61	.695	
JSS-3		T.C.															1.51	.808	
JSS-4		T.C.															1.38	.979	

Table A1 (cont'd).

DV32-12	W DOVER VT	T.C.	5	.08	.018	99	92	57	11	7	4	4.4	2.65	ML	VL-H	F-4	1.15	1.304	
DV32-5	W DOVER VT	T.C.	4.8	.07	.001	100	94	62	26	19	15	70	2.56	ML	L-VH	F-4	.81	2.163	
SSS	STERRETT SUB SOIL	V.P.	.8	.10	.0005	100	97	53	45	37	30	200	2.69	ML-CL	L-VH	F-4	1.59	.692	
AK-8	ALASKA DOT #8 SILT	P.P.	4.0	.17	.025	100	90	22	8	6	4	6.8	2.75	ML	L-H	F-4	1.84	.495	1.3
AK-3	ALASKA DOT #3 SILT	P.P.	2.0	.21	.012	100	92	30	15	8	4	17.5	2.71	ML	L-VH	F-4	1.69	.603	.77
AK-2	ALASKA DOT #2 SILT	P.P.	20	.074	.006	98	95	60	30	17	7	12.3	2.53	ML-OL	L-M	F-4	1.34	.885	5.7
LCSS-3	MOULTON LEBANON	P.P.	.5	.048	.013	100	99	83	20	7	3	3.7	2.75	ML	VL-M	F-4	1.51	.821	.4
ILL-TS	ILL TOPSOIL	P.P.	2.0	.015	<.001	100	99	96	69	50	34	>100	2.56	OL-CL			1.39	.841	.13
FDS B	*FULL DEPTH* SILT BLEND	P.P.	*.08*	*.013*	*.0030*	*100*	97	93	77	46	17	4.3	2.76	ML	M-VH	F-4	1.43	.931	.086
FDS UB	FULL DEPTH SILT -UN - BLENDED	P.P.	.074	.013	.0036	100	100	99	75	45	16	3.6	2.75	ML	M-VH	F-4	1.34	1.053	.22
	JACKMAN ME NICHOLS		10	.05	.004	92	77	65	25	18	11	12.5	2.74	ML	L-H	F-4			
JNR-SG4	(4 ft.)	P.P.															1.87	.466	.070
JNR-SG3	(3 ft.)	P.P.															1.77	.548	.060
JAP SGB	JACKMAN ME Airport Subgrade Silt	P.P.	4.0	.02	.0011	100	94	92	60	42	24	18	2.78	ML	M-VH	F-4	1.78	.563	.014
HMT16-2	Hamilton, MT SILT	P.P.	10	.05	.003	99	95	70	30	23	15	17	2.71	ML	L-H	F-4	1.54	.761	3.2
H.MT-27	Hamilton, MT SILT	P.P.	5	.07	.007	99	97	61	20	12	8	10	2.79	ML	L-H	F-4	1.77	.577	.17
SBT200/400	SIBLEY TILL	P.P.	-.74	.052	.04	100	100	100	0	0	0	1.3	2.72	ML			1.57	.733	.35
SBT400C	SIBLEY TILL	P.P.	.038	.021	<.001	100	100	100	57	46	28	>100	2.71	ML			1.46	.855	.6
SB-400F	SIBLEY TILL	P.P.	.038	.012	<.001	100	100	100	82	57	42	>100	2.78	ML			.155	.792	.57
AK-5	ALASKA DOT #5 SILT	P.P.	2.0	.042	*.006*	*100*	95	73	38	20	7	7.0	2.40	OL	L-VH	F-4	1.17	1.051	.35

CLAYS

BMD12	BELTSVILLE MD	V.P.	.9	.09	.001	100	92	58	37	31	22	90	2.71	ML-CL	L-H	F-4	1.65	.642	
BMD5	BELTSVILLE MD	V.P.	2	.15	.01	100	92	50	15	11	5	15	2.65	ML-CL	L-M	F-4	1.61	.645	
AM-10	APPLE VAL-LEY MN	V.P.	2	.023	.0022	100	96	90	50	30	17	8.7	2.64	CL	M-H	F-4	1.21	1.183	
AVM-24	APPLE VAL-LEY MN	V.P.	5	.045	.0005	100	87	68	40	28	22	90	2.73	CL	L-H	F-4	1.53	.785	
	MORIN CLAY		.04	.0058	–	100	100	100	85	70	53	–	2.80	CL	L-H	F-3			
MCL-1		T.C.															1.74	.621	
MCL-2		P.P.															1.56	.795	.048
SL11-0	ST LOUIS	T.C.	.4	.0045	–	100	100	98	85	73	61	900+	2.71	CL	L-H	F-3	1.57	.727	4.2E-4
SL11-10	ST LOUIS	T.C.	2	.02	–	100	96	77	60	51	43	900+	2.73	CL	L-H	F-3	1.53	.785	.025

Table A1 (cont'd). Soil properties with percent passing indicated sieve.

SL11-24	ST LOUIS	T.C.	2	.035	–	100	93	70	50	44	37	900+	2.72	CL	L-H	F-3	1.69	.61	.026
SL12-24	ST LOUIS	T.C.	2	.02	.0001	100	96	78	60	41	30	200	2.72	CL	L-H	F-3	1.49	.825	.014
SL12-29	ST LOUIS	T.C.	2	.04	.0002	100	93	71	50	40	31	200	2.69	CL	L-H	F-3	1.68	.601	.017
SL12-8	ST LOUIS	T.C.	2	.02	–	100	97	82	60	48	38	900+	2.69	CL	L-H	F-3	1.37	.964	8.8E-4
SL12-13	ST LOUIS	T.C.	2	.006	–	100	99	97	90	70	56	900+	2.73	CL	L-H	F-3	1.44	.897	9.2E-4
SL12-19	ST LOUIS	T.C.	2	.015	–	100	98	87	65	50	37	900+	2.73	CL	L-H	F-3	1.51	.808	.054
DCO-7	DEER CREEK OHIO	T.C.	2	.02	.0006	100	94	80	60	43	29	33	2.71	CL	L-H	F-3	1.67	.623	
DCO-14	DEER CREEK OHIO	T.C.	5	.009	–	100	94	84	72	63	52	900+	2.72	CL	L-H	F-3	1.38	.972	
DCO-0	DEER CREEK OHIO	T.C.	3	.03	.0005	100	86	71	53	38	26	60	2.67	CL	L-H	F-3	1.40	.908	
DCO-3	DEER CREEK OHIO	T.C.	2	.022	.0005	100	92	76	57	41	28	44	2.67	CL	L-H	F-3	1.71	.562	
DCO-6	DEER CREEK OHIO	T.C.	10	.02	.0001	98	93	78	61	52	42	200	2.67	CL	L-H	F-3	1.57	.701	1.3E-4
DCO-14	DEER CREEK OHIO	T.C.	5	.035	.0001	100	84	72	52	44	37	350	2.73	CL	L-H	F-3	1.80	.517	.018
DCO-24	DEER CREEK OHIO	T.C.	9	.09	.0007	95	76	58	41	32	23	128	2.74	CL	L-VH	F-4	1.82	.506	.05
DCO-34	DEER CREEK OHIO	T.C.	10	.065	.0007	96	78	62	45	33	25	93	2.76	CL	L-VH	F-4	1.95	.415	.011
ALRS-SG-1	GONIC "A"	P.P.	2.0	.04	.002	100	92	80	42	31	22	18.8	2.70	CL	M-H	F-3	1.67	.616	.28
ALRS-SG-2		P.P.															1.49	.812	.022
FERF-SG	GONIC "B"	P.P.	2.0	.004	<.001	100	100	95	88	78	65		2.80	CL	M-H	F-3	1.19	1.353	.18
FR-222	FT. RILEY KA	P.P.	.8	.033	<.001	100	98	96	48	39	34	>100	2.70	CL	L-M	F-3	1.37	.972	.0057
FR-F.P.	FT. RILEY KA	P.P.	2.0	.04	.002	100	99	93	34	22	16	20	2.75	CL	M-H	F-3	1.59	.73	.64
FR-C.H.	FT. RILEY KA	P.P.	2.0	.045	.001	100	96	90	43	31	28	45	2.63	CL	L-M	F-3	1.38	.905	.4
RAC-1	RACINE WISCO.	P.P.	10	.06	.0023	87	78	69	39	28	18	26	2.73	CL			1.66	.645	.8
RAC-2	RACINE WISCO.	P.P.	10	.03	.0015	97	83	72	50	33	22	20	2.75	CL			1.68	.637	1.0
FT ED 1	Ft. Edward CLAY	P.P.	2.0	.0043	.002	100	98	97	93	86	75	2.2	2.79	CH	VL	F-3	1.47	.898	.048
FT ED 2		P.P.															1.52	.835	.00073
FT ED 3		P.P.															1.56	.789	.000034
CRL-VC1	CRREL VARVED CLAY	P.P.	.2	.017	.0020	100	100	93	66	42	22	8.5	2.78	CL	M-VH	F-4	1.54	.805	.097
CRL-VC2		P.P.															1.56	.783	.093
MN1232	MINN 1232 CLAY	P.P.	10	.11	.001	98	82	52	32	28	22	110	2.70	CL	M	F-3	1.84	.468	.00087
MN1171	MINN 1171 CLAY	P.P.	20	.2	.001	94	81	48	26	22	18	200	2.70	CL	M	F-3	1.74	.553	.022
MN1206	MINN 1206 CLAY	P.P.	9	.05	.0005	100	90	64	43	38	30	100	2.70	CL	L-M	F-3	1.69	.597	.014

Table A1 (cont'd).

	Owens Valley		5	.1	.001	98	72	55	40	32	26	100	2.68	CL	L-H	F-4			
OVCA 60	CA-CLAY	P.P.															1.62	.656	.14
	Owens Valley		7	.042	.002	98	[illegible]	64	52	42	28	21	2.66	CL	L-H	F-3			
OVCA 90	CA CLAY	P.P.															1.73	.538	.040
	Owens Valley		7	.12	.002	99	80	52	35	29	22	60	2.69	CL	L-VH	F-3			
OVCA120	CA CLAY	P.P.															1.85	.453	.026

NOTES:

G = Specific Gravity of Solids

CU = Uniformity Coefficient, D60/D10, where:

D60 is the Grain Diameter Corresponding to 60% passing
D10 is the Grain Diameter Corresponding to 10% passing

T.C. = Tempe Cell

V.P. = Volumetric Plate Entractor

P.P. = Pressure Cell Permeameter

UNIFIED SOIL SYMBOL: determined from the grain size distribution and visual classification Atterberg Limits. (Not available for most soils).

SATURATED PERMEABILITY: Also called Saturated Hydraulic Conductivity

FROST SUSCEPTIBILITY CLASSIFICATIONS:

NFS = Non-frost susceptibility
N = Negligible frost susceptibility
VL = Very low frost susceptibility
L = Low frost susceptibility
M = Medium frost susceptibility
H = High frost susceptibility
VH = Very high frost susceptibility

Table A2. Soil properties with Gardner's coefficients and exponents.

Soil No.	Material & Source	Procedure Used	Max. size (mm)	D60	D10	CU	G	Unified Soil Symbol	Frost Susc. Class	Frost Group	Dry Dens. (g/cc)	Void Ratio E	Sat. Perm. (cm/hr)	**** GARDNER'S Coefficients **** AWL	XWL	AKL	XKL
								GRAVELS									
AK-1	ALASKA DOT#1	P.P.	25	80	.10	80	2.73	GW	L-VH	F-1	2.18	.252	2.1	0.309	0.319	0.349E-01	2.645
DGS-1	DENSE GRAD STONE MASS.	V.P.	13	5	.10	50	2.80	GW	L-M	F-1	1.94	.443		0.306	0.345		
DGS-2		P.P.									1.86	.506	5.5	0.596	0.318	2.033	1.078
JAP-GB	JACKMAN ME GRAVEL	P.P.	50	8	0.2	40	2.71	GW	VL-M	S-1	2.07	.311	.34	0.567	0.375	0.8824	1.281
JNR-GB	JACKMAN ME Nichols Base	P.P.	50	7	0.12	58	2.71	GW	VL-M	F-1	1.79	.514	191	0.806	0.389	3581.3	0.869
MN-CL6	Mn/ROAD Class 6 Spec.	P.P.	21	9.5	.43	22	2.79	GW	N	NFS	1.84	.495	4.7	1.0001	0.444	0.107E-07	5.895
AB-1	BASE A CR STONE NY	P.P.	40	10	.03	333	2.71	GP	L-H	F-1	2.16	.255	.46	0.065	0.548	0.303E-03	2.627
WLNH-1	DIRTY GRAV LEB. NH	P.P.	19	6	.06	100	2.75	GM	VL-M	F-1	1.99	.382	.42	0.396E-01	0.648	0.369E-03	2.721
SBT-TTL	SIBLEY TILL MASS.	P.P.	4.6	.18	.001	180	2.75	GM	L-H	F-4	1.88	.608	1.0	0.613E-01	0.416	0.490E-05	3.905
CWA-2	WISCO SILTY	P.P.	25	.80	.006	133	2.70	GM	L-M	F-1	1.86	.451	.53	0.561E-01	0.276	0.318E-02	2.081
CWA-1	WISCO. CLAYEY	P.P.	10	.29	.006	53	2.7	GM-GC	L-M	F-1	1.90	.420	.005	0.427E-02	0.635	0.294E-02	1.336
								SANDS & SILTY-SANDS									
HMT05-1	Hamilton, MT Gravelly SAND	P.P	50	6	0.1	60	2.83	SW	N-M	PFS	2.03	.395	0.85	0.362	0.355	0.558E-01	2.187
JAP-SB	Jackman, ME AP SAND	P.P	25	1.9	0.21	9.0	2.71	SW	N-L	NFS	1.86	.458	5.2	0.980	0.398	41.901	1.336
LNH-SB	LEB Airport SUB BASE	T.C.	40	1	.18	5.6	2.76	SW	NFS	NFS	1.78	.551		0.156	0.560		
LCSS 1	LEBANON CR. STONE	P.P.	2.0	.17	.07	2.4	2.73	SW	N-M	0	1.64	.664	1.6	0.279E-04	2.044	0.388E-08	5.204
PPSAND	POMPEY PIT SAND VT	P.P	50	2.2	.075	29	2.74	SW	VL-M	S-2	1.82	.506	5.5	0.371E-02	1.268	0.287E-04	3.806
MN-CL3	Mn/ROAD Class 3 Spec.	P.P	8	.8	.05	16	2.69	SW			2.03	.336	4.5	0.1735	0.324	1647.1	0.721
ALRS-S	WRJ VT	P.P.	4.8	1.0	.07	14.3	2.73	SW	V-M	F-2	1.97	.385	4.1	0.114	0.611	0.292	1.336
LNHS-1	BANK RUN SAND LEB.	V.P.	2	.18	.07	2.6	2.73	SW-SM	N-M	F-2	1.54	.773		0.132E-01	1.061		
MFS-1	FINE SAND MANCH. NH	T.C.	.6	.21	.083	2.5	2.69	SP	NFS	NFS	1.56	.712		0.521E-01	0.797		
MFS-2		V.P.									1.55	.736		0.452E-01	0.885		
MFS-3		P.P.									1.48	.818	18.3	0.418E-01	0.900	0.143E-03	3.485
IGK 1-1	INIGOK Barrow, AK	T.C.	.85	.23	.098	2.3	2.66	SP	NFS	NFS	1.67	.593		0.784E-01	0.660		
IGK 1-2		T.C.									1.68	.583		0.548E-01	0.803		
IGK B-1	INIGOK Barrow, AK	T.C.	.7	.17	.07	2.4	2.66	SP-SM	N-L	F-2	1.69	.574		0.485E-01	0.768		
IGK B-2		T.C.									1.68	.583		0.101	0.603		
SBT30/50	SIBLEY TILL MASS.	P.P.	.6	.45	.33	1.4	2.68	SP			1.57	.802	35.4	0.906E-03	1.722	0.978E-02	3.101

Table A2 (cont'd).

	SIBLEY TILL		.30	.23	.18	1.3@	2.69	SP									
SBT50/100	MASS.	P.P.									1.61	.772	8.1	0.359E-02	1.297	0.413E-07	5.268
	SIBLEY TILL		.15	.12	.08	1.5	2.72	SP									
SBT100/200	MASS.	P.P.									1.63	.669	1.7	0.693E-05	2.210	0.469E-13	6.986
	SPECIAL		13	.35	.017	20.6	2.72	SM	VL-H	F-2							
SPEC-1	TEST SAND	T.C.									1.60	.700		0.149	0.385		
SPEC-2	HANOVER	T.C.									1.68	.619		0.727E-01	0.502		
SPEC-3	NH	T.C.									1.76	.546		0.313E-01	0.628		
SPEC-4		T.C.									1.84	.479		0.669E-02	0.845		
SPEC-5		T.C.									1.92	.417		0.862E-05	1.827		
SPEC-6		V.P.									1.84	.479		0.767E-02	0.756		
	SIBLEY TILL		13	.20	.001	200	2.75	SM	L-H	F-4							
SBT-1	MASS	V.P.									1.97	.396		0.157E-01	0.560		
SBT-2	Glacial Till	P.P.									1.89	.455	1.5	0.642E-01	0.381	0.456E-03	3.060
SBT-3		P.P.									2.07	.328	.24	0.365E-02	0.729	0.495E-04	3.011
SBT-TOT		P.P.	41	.18	.001	180	2.75	GM	L-H	F-4	1.88	.608	1.0	0.433E-01	0.478	0.398E-05	3.840
	GRAVES		2	.12	.013	9.2	2.73	SM	VL-H	F-2							
GSS-1	SILTY-SAND	V.P.									1.58	.728		0.375E-01	0.553		
GSS-2	MASS	P.P.									1.49	.832	1.92	0.152E-01	0.772	0.054E-06	4.238
	HYANNIS		5	.25	.035	7.1	2.67	SM	M-H	F-2							
HYS-1	SAND	V.P.									1.65	.619		0.107E-01	1.012		
HYS-2	MASS	P.P.									1.69	.580	1.3	0.270E-03	1.645	0.672E-06	4.002
	HART BROS		5	.16	.06	2.7	2.78	SM	VL-M	F-2							
HBS-1	SAND	V.P.									1.76	.580		0.849E-01	0.587		
HBS-2	MASS	P.P.									1.73	.607	4.0	0.776E-01	0.650	0.526E-06	4.064
	SUB-BASE		50	2.8	.04	70	2.72	SM	L-H	F-1							
ASB-A1	ALBANY NY	P.P.									2.16	.259	2.8	0.152	0.269	0.658E-04	2.962
	DANVILLE		20	.30	.023	13.0	2.74	SM	VL-H	F-2							
DVT1-0	VT	V.P.									1.25	1.192		0.110	0.338		
	DANVILLE		2	.21	.018	11.7	2.75	SM	VL-H	F-2							
DVT19-24	VT	V.P.									1.84	.495		0.293E-01	0.435		
	DANVILLE		2	.10	.02	5	2.78	SM	VL-H	F-2							
DVT21-9	VT	V.P.									1.68	.655		0.395E-01	0.467		
	DANVILLE		2	.20	.03	6.7	2.76	SM	N-H	F-2							
DVT21-0	VT	V.P.									1.61	.715		0.298E-01	0.608		
	SUB-GRADE B		2	.15	.06	2.5	2.71	SM	N-M	F-2							
ASB-B1	ALBANY NY	P.P.									1.67	.623	2.4	0.146E-01	0.835	0.431E-03	2.903
	IKELANIAN		3	.15	.032	4.7	2.68	SM	N-H	F-2							
IKE-1	SAND MASS	V.P.									1.61	.664		0.101E-01	1.021		
IKE-2		P.P.									1.70	.577	.77	0.132E-03	1.707	0.002E-06	4.873
	CHARLTON A		5	.15	.006	25	2.63	SM	VL-H	F-3							
CH-A	HANOVER NH	T.C.									1.3	1.024	.13	0.106E-01	0.669		
	CHARLTON B		5	.17	.008	21	2.69	SM	VL-H	F-3							
CH-B	HANOVER NH	T.C.									1.30	1.070	2.8	0.129E-01	0.704		
	CHARLTON C		5	.20	.009		2.70	SM	VL-H	F-3							
CH-C	Hanover, NH	T.C.									1.57	.720	.6	0.234E-01	0.546		
	WINSOR A		2	.34	.044	7.7	2.63	SM	N-H	F-2							
WR-A	Lebanon, NH	T.C.									1.54	.707	.14	0.114E-01	0.727		
	WINSOR B		2	.40	.05	8	2.69	SM	N-H	F-2							
WR-B	Lebanon, NH	T.C.									1.47	.831	10	0.212	0.432		
	WINSOR C		2	.19	.036	5.3	2.73	SM	N-H	F-2							
WR-C	Lebanon, NH	T.C.									1.43	.909	18	0.112	0.604		
	CHENA TOP		15	.18	.012	15	2.65	SM	VL-H	F-2							
CTS-1	SOIL AK	T.C.									1.54	.721		0.106E-01	0.828		
	CHENA GRA.		4.8	.35	.056	6.3	2.71	SM	N-H	F-2							
CRG-1	ALASKA	T.C.									1.75	.548		0.150	0.574		
	W DOVER		30	.15	.028	5.4	2.78	SM	N-H	F-2							
DV32-23	VT	T.C.									1.53	.818		0.162E-01	0.826		
	W DOVER		25	.22	.03	7.3	2.79	SM	N-H	F-2							
DV32-33	VT	T.C.									1.80	.550		0.214E-01	0.767		

Table A2 (cont'd). Soil properties with Gardner's coefficients and exponents.

	W DOVER		7	.13	.018	7.2	2.75	SM	VL-H	F-2							
DV32-16	VT	T.C.									1.29	1.132		0.283E-01	0.751		
	W DOVER		4.8	.11	.02	5.5	2.59	SM	VL-H	F-2							
DV32-8	VT	T.C.									.81	2.196		0.895E-01	0.535		
	W DOVER		4.8	.12	.016	7.5	2.66	SM	VL-H	F-2							
DV31-6-1	VT	T.C.									.84	2.16		0.426E-01	0.498		
DV31-6-2		T.C.									.69	2.85		0.267E-01	0.568		
	LEB Airport		50	.4	.009	44	2.74	SM	L-H	F-4							
LNH-SG	SUBGRADE	T.C.									1.90	.442		0.624E-02	0.895		
	STERRETT		2	.15	.005	30	2.65	SM-SC	L-H	F-4							
STS	TOP SOIL	V.P.									1.60	.656		0.126E-01	0.783		
	LEB. SAND		2.0	.10	.018	5.6	2.74	SM	VL-H	F-4							
LCSS 2	AND SILT	P.P.									1.67	.642	.33	0.458E-04	1.760	0.243E-08	4.320
	DRY LAKE		4.6	.33	.008	41	2.62	SM	VL-H	F-2							
DLV 1	NEVADA	P.P.									1.81	.447	5.1	0.180	0.325	0.508E-02	2.775
	Lebanon Landfill		20	0.30	.080	3.8	2.79	SM	VL-H	F-2							
LLFS	Sandy Silt	P.P									1.62	.721	1.6	.196E-04	1.975	.159E-08	4.623
	NH-VT Sandy		4.6	0.11	.009	12.2	2.75	SM	L-H	F-3							
NHSPS5	Silt	P.P									1.84	.495	0.22	.180E-05	2.110	.212E-09	4.215
	FT. RILEY		1.9	.30	.004	75	2.61	SC									
FR-M.P.	KA	P.P.									1.88	.397	.57	0.267E-02	0.884	0.159E-03	2.969

SILTS

	Manchester		15	.025	.006	4.2	2.73	ML	L-VH	F-4							
NHS-1	NH	T.C.									1.36	1.015		0.264E-02	1.044		
NHS-2	SILT	T.C.									1.44	.902		0.148E-02	1.097		
NHS-3		T.C.									1.52	.802		0.918E-03	1.141		
NHS-4		T.C.									1.60	.712		0.896E-03	1.112		
NHS-5		V.P.									1.30	1.10		0.257E-02	1.011		
NHS-6		P.P.									1.45	.883	.32	0.165E-08	3.133	0.288E-7	3.673
	FAIRBANKS		.47	.038	.0042	9.0	2.73	ML	L-M	F-4							
FBKS-1	SILT	T.C.									1.56	.751		0.414E-03	1.135		
FBKS-2	FAIRBANKS	V.P.									1.69	.615		0.957E-02	0.662		
FBKS-3	ALASKA	P.P.									1.62	.686	.042	0.158E-01	0.638	0.646E-04	2.360
	MOULTON		.08	.016	.0019	8.4	2.82	ML	L-VH	F-4							
MPS-1	PIT SILT	T.C.									1.33	1.067		0.374E-04	1.593		
MPS-2	LEB NH	T.C.									1.49	.845		0.309E-04	1.500		
MPS-3		T.C.									1.55	.774		0.285E-04	1.436		
MPS-4		V.P.									1.50	.880		0.160E-08	2.587		
MPS-5		P.P.									1.35	1.037	.28	0.290E-09	3.202	0.691E-10	4.097
	DANVILLE		2	.07	.02	3.5	2.69	ML	VL-H	F-4							
DVT19-5	VT	V.P.									1.16	1.139		0.147E-01	0.728		
	DANVILLE		2	.10	.02	5	2.59	ML-OL	VL-H	F-4							
DVT10-3	VT	V.P.									.95	1.721		0.630E-01	0.537		
	DANVILLE		2	.10	.02	5	2.82	ML	VL-H	F-4							
DVT10-24	VT	V.P.									1.33	1.119		0.325E-01	0.681		
	DANVILLE		19	.11	.02	5.5	2.74	ML	VL-H	F-4							
DVT17-18	VT	V.P.									1.63	.689		0.155E-01	0.537		
	DANVILLE		19	.07	.02	3.5	2.61	ML	VL-H	F-4							
DVT17-0	VT	V.P.									1.02	1.558		0.677E-01	0.404		
	DANVILLE		9	.08	.02	4	2.71	ML	VL-H	F-4							
DVT17-6	VT	V.P.									1.12	1.421		0.548E-01	0.364		
	APPLE VAL-		2	.03	.003	10	2.59	OL	L-H	F-4							
AVM-2	LEY MN	V.P.									1.16	1.232		0.103	0.301		
	CRREL SILT		5	.048	.007	6.9	2.69	ML	L-H	F-4							
CS7-1	Hanover, NH	T.C.									1.43	.880		0.176E-02	1.028		
CS7-2		T.C.									1.39	.935		0.128E-02	1.047		
CS7-3		V.P.									1.37	.964		0.232E-01	0.666		

Table A2 (cont'd).

	CRREL SILT		2	.05	.009	5.6	2.70	ML	L-H	F-4							
CS8-1	Hanover, NH	T.C.									1.48	.825		0.858E-04	1.481		
CS8-2		T.C.									1.48	.825		0.606E-03	1.157		
CS8-3		V.P.									1.42	.901		0.314E-02	0.981		
	CRREL SILT		2	.047	.01	4.7	2.71	ML	L-H	F-4							
CS9-1	Hanover, NH	T.C.									1.45	.869		0.321E-03	1.254		
CS9-2		T.C.									1.48	.831		0.588E-03	1.186		
CS9-3		V.P.									1.38	.964		0.431E-02	0.952		
	CHENA HOT		.2	.027	.005	5.4	2.80	ML	L-VH	F-4							
CHSS-1	SPRINGS	T.C.									1.57	.783		0.118E-01	0.578		
CHSS-2	AK SILT	V.P.									1.59	.761		0.114E-01	0.604		
CHSS-3		P.P.									1.62	.279	.017	0.834E-04	1.304	0.856E-05	2.574
CHSS-4		P.P.									1.54	.817	.063	0.343E-04	1.488	0.421E-05	2.854
CHSS-5		P.P.									1.49	.879	.07	0.123E-02	1.037	0.089E-06	3.626
	NW Standard		2	.03	.005	6	2.65	ML	L-VH	F-4							
NWS-1	SILT AK	P.P.									1.42	.866	.26	0.571E-03	1.141	0.167E-01	1.547
	Ottawa Sand		.15	.038	.0028	13.6	2.60	ML	L-H	F-4							
OWS-1		T.C.									1.43	.858		0.290E-02	0.985		
OWS-2		T.C.									1.53	.700		0.321E-02	0.921		
	HANOVER		.2	.032	.004	8	2.69	ML	L-H	F-4							
HNVS-1	SILT	T.C.									1.30	1.07		0.121E-01	0.824		
HNVS-2	Hanover, NH	T.C.									1.46	1.07		0.583E-02	0.868		
HNVS-3		T.C.									1.58	.703		0.759E-02	0.767		
HNVS-4		T.C.									1.67	.611		0.113E-01	0.650		
	JENKS		.85	.068	.006	11	2.73	ML	VL-H	F-4							
JSS-1	SANDY-SILT	T.C.									1.70	.606		0.232E-01	0.701		
JSS-2		T.C.									1.61	.695		0.211E-01	0.686		
JSS-3		T.C.									1.51	.808		0.146E-01	0.692		
JSS-4		T.C.									1.38	.979		0.214E-01	0.578		
	W DOVER		5	.08	.018	4.4	2.65	ML	VL-H	F-4							
DV32-12	VT	T.C.									1.15	1.304		0.834E-02	0.871		
	W DOVER		4.8	.07	.001	70	2.56	ML	L-VH	F-4							
DV32-5	VT	T.C.									.81	2.163		0.370E-01	0.614		
	STERRETT		.8	.10	.0005	200	2.69	ML-CL	L-VH	F-4							
SSS	SUB SOIL	V.P.									1.59	.692		0.665E-01	0.383		
	Alaska DOT		4.0	0.17	.025	80.0	2.75	ML	L-H	F-4							
AK-8	#8 SILT	P.P.									1.84	.495	1.3	0.580E-01	0.825	0.122E-03	3.220
	Alaska DOT		2.0	.21	.012	17.5	2.71	ML	L-VH	F-4							
AK-3	#3 SILT	P.P.									1.69	.603	.77	0.306E-01	0.722	0.278E-05	3.404
	Alaska DOT		20	.074	.006	12.3	2.53	ML-OL	L-M	F-4							
AK-2	#2 SILT	P.P.									1.34	.885	5.7	0.170E-03	1.362	0.402E-03	2.852
	MOULTON		0.5	.048	.013	3.7	2.75	ML	VL-M	F-4							
LCSS 3	LEBANON	P.P.									1.51	.821	.40	0.608E-05	2.057	0.757E-10	4.836
	FULL DEPTH		0.8	.013	.0030	4.3	2.76	ML	M-VH	F-4							
FDS-B	SILT BLEND	P.P.									1.4	.931	.086	0.343E-10	3.440	0.238E0-5	2.169
	FULL DEPTH		.074	.013	.0036	3.6	2.75	ML	M-VH	F-4							
FDS-UB	SILT UN-BLENDED	P.P.									1.34	1.053	0.22	0.689E-06	2.101	0.771E-06	2.611
	Jackman, ME NICHOLS SILT		10	.05	.004	12.5	2.74	ML	L-H	F-4							
JNR-SG4	(4 FT)	P.P									1.87	.466	.070	0.161E-04	1.661	0.592E-06	3.083
JNR-SG3	(3FT)	P.P.									1.77	.548	.060	0.314E-04	1.469	0.103E-04	2.168
	Jackman, ME		4.0	.02	.0011	18	2.78	ML	M-VH	F-4							
JAP-SGB	AP SUB-GRADE SILT	P.P.									1.78	.563	.014	0.589E-06	1.983	0.258E-04	1.871
	Hamilton, MT		10	.05	.003	17	2.71	ML	L-H	F-4							
HMT16-2	SILT	P.P.									1.54	.761	3.2	0.276E-01	0.244	44.673	0.921
	Hamilton, MT		5	.07	.007	10	2.79	ML	L-H	F-4							
HMT-27	SILT	P.P.									1.77	.577	0.17	0.458E-03	1.157	0.465E-01	1.368
	SIBLEY TILL		.074	.052	.04	1.3	2.72	ML									
SBT200/400	MASS.	P.P.									1.57	.733	0.35	0.179E-06	2.732	0.208E-11	5.841

Table A2 (cont'd). Soil properties with Gardner's coefficients and exponents.

SBT400C	SIBLEY TILL MASS.	P.P.	.038	.021	<.001	>100	2.71	ML			1.46	.855	0.6	0.305E-02	0.880	0.447E-05	3.158
SBT400F	SIBLEY TILL MASS.	P.P.	.038	.012	<.001	>100	2.78	ML			1.55	.792	0.57	0.117E-01	0.402	0.779E-02	2.386
AK-5	Alaska DOT #5 SILT	P.P.	2.0	.042	.006	7.0	2.40	OL	L-VH	F-4	1.17	1.051	.35	0.102E-02	1.022	0.436E-06	3.405

CLAYS

BMD12	BELTSVILLE MD	V.P.	.9	.09	.001	90	2.71	ML-CL	L-H	F-4	1.65	.642		0.131E-01	0.629		
BMD5	BELTSVILLE MD	V.P.	2	.15	.01	15	2.65	ML-CL	L-M	F-4	1.61	.645		0.118E-02	1.044		
AM-10	APPLE VAL-LEY MN	V.P.	2	.023	.0022	8.7	2.64	CL	M-H	F-4	1.21	1.183		0.645E-01	0.420		
AVM-24	APPLE VAL-LEY MN	V.P.	5	.045	.0005	90	2.73	CL	L-H	F-4	1.53	.785		0.262E-01	0.516		
MCL-1	MORIN CLAY	T.C.	.04	.0058	–	–	2.80	CL	L-H	F-3	1.74	.621		0.181E-03	0.989		
MCL-2		P.P.									1.56	.795	.048	0.580E-02	0.665	0.907E-03	1.780
SL11-0	ST LOUIS	T.C.	.4	.0045	–	100	2.71	CL	L-H	F-3	1.57	.727	4.2E-4	0.145E-01	0.415		
SL11-10	ST LOUIS	T.C.	2	.02	–	100	2.73	CL	L-H	F-3	1.53	.785	.025	0.463E-02	0.593		
SL11-24	ST LOUIS	T.C.	2	.035	–	900+	2.72	CL	L-H	F-3	1.69	.61	.026	0.229E-03	0.956		
SL12-24	ST LOUIS	T.C.	2	.02	.0001	200	2.72	CL	L-H	F-3	1.49	.825	.014	0.184E-01	0.519		
SL12-29	ST LOUIS	T.C.	2	.04	.0002	200	2.69	CL	L-H	F-3	1.68	.60	.017	0.220E-06	2.091		
SL12-8	ST LOUIS	T.C.	2	.02	–	100	2.69	CL	L-H	F-3	1.37	.964	8.8E-4	0.236E-07	1.969		
SL12-13	ST LOUIS	T.C.	2	.006	–	900+	2.73	CL	L-H	F-3	1.44	.897	9.2E-4	0.236E-06	1.855		
SL12-19	ST LOUIS	T.C.	2	.015	–	900+	2.73	CL	L-H	F-3	1.51	.808	.054	0.265E-05	1.718		
DCO-7	DEER CREEK OHIO	T.C.	2	.02	.0006	33	2.71	CL	L-H	F-3	1.67	.623		0.489E-02	0.584		
DCO-14	DEER CREEK OHIO	T.C.	5	.009	–	900+	2.72	CL	L-H	F-3	1.38	.972		0.280E-01	0.344		
DCO-0	DEER CREEK OHIO	T.C.	3	.03	.0005	60	2.67	CL	L-H	F-3	1.40	.908		0.825E-01	0.365		
DCO-3	DEER CREEK OHIO	T.C.	2	.022	.0005	44	2.67	CL	L-H	F-3	1.71	.562		0.820E-02	0.574		
DCO-6	DEER CREEK OHIO	T.C.	10	.02	.0001	200	2.67	CL	L-H	F-3	1.57	.701	1.3E-4	0.954E-02	0.424		
DCO-14	DEER CREEK OHIO	T.C.	5	.035	.0001	350	2.73	CL	L-H	F-3	1.80	.517	.018	0.236E-01	0.359		
DCO-24	DEER CREEK OHIO	T.C.	9	.09	.0007	128	2.74	CL	L-VH	F-4	1.82	.506	.05	0.906E-01	0.294		
DCO-34	DEER CREEK OHIO	T.C.	10	.065	.0007	93	2.76	CL	L-VH	F-4	1.95	.415	.011	0.550E-01	0.273		
ALRS-SG-1	GONIC "A	P.P.	2.0	.04	.002	18.8	2.70	CL	M-H	F-3	1.67	.616	.28	0.161E-01	0.471	0.533E-02	1.781
ALRS-SG-2		P.P									1.49	.812	.14	0.237E-02	0.817	0.559E-02	1.772
FERF-SG	GONIC "B"	P.P.	2.0	.004	.001	–	2.80	CL	M-H	F-3	1.19	1.353	0.18	0.832E-02	0.567	0.369E-01	1.508

Table A2 (cont'd).

FR-222	FT. RILEY KA	P.P.	0.8	0.33	.001	100	2.70	CL	L-M	F-3	1.37	.972	.006	0.427E-02	0.475	0.347	0.956
FR-F.P.	FT. RILEY KA	P.P.	2.0	.04	.002	20	2.75	CL	M-H	F-3	1.59	.730	64	0.236E-01	0.472	0.166E-03	2.648
FR-C.H.	FT. RILEY KA	P.P.	2.0	.045	.001	45	2.63	CL	L-M	F-3	1.38	.905	.40	0.325E-01	0.375	0.221E-02	2.514
RAC-1	RACINE WISC.	P.P.	10	.06	.0023	26	2.73	CL			1.66	.645	.8	0.530E-01	0.354	0.193	1.943
RAC-2	RACINE WISC.	P.P.	10	.03	.0015	20	2.75	CL			1.68	.637	1.0	0.930E-01	0.297	0.821	1.608
FTED-1	Ft. Edward CLAY	P.P.	2.0	.0043	.0020	2.2	2.79	CH	VL	F-3	1.47	.898	.048	0.204E-03	0.975	0.834E-03	2.092
FTED-2		P.P.									1.52	.835	.00073	0.459E-02	0.368	1.041	0.307
FTED-3		P.P.									1.56	.789	.00003	0.802E-04	0.917	0.645E-02	1.070
CRLVC-1	CRREL VARVED CLAY	P.P.	0.2	.017	.0020	8.5	2.78	CL	M-VH	F-4	1.54	.805	.097	0.152E-05	1.927	0.969E-07	3.071
CRLVC-2		P.P.									1.56	.783	.093	0.150E-8	3.038	0.468E-03	1.655
MN1232	MINN 1232 CLAY	P.P.	10	0.11	.001	110	2.70	CL	M	F-3	1.8	.468	.00087	0.222E-02	0.677	0.106E-02	1.922
MN1171	MINN 1171 CLAY	P.P.	20	0.2	.001	200	2.70	CL	M	F-3	1.74	.553	.022	0.140E-01	0.457	0.165	1.591
MN1206	MINN 1206 CLAY	P.P.	9	.05	.0005	100	2.70	CL	L-M	F-3	1.69	.597	.014	0.235E-02	0.713	0.571E-03	2.640
OVC-60	Owens Valley CA-CLAY	P.P.	5	0.1	.001	100	2.68	CL	L-H	F-4	1.62	.656	0.14	0.306E-02	0.779	0.403E-03	2.224
OVC-90	Owens Valley CA-CLAY	P.P.	7	.042	.002	21	2.66	CL	L-H	F-3	1.73	.538	.040	0.476E-02	0.624	0.754E-03	2.229
OVC-120	Owens Valley CA-CLAY	P.P.	7	0.12	.002	60	2.69	CL	L-VH	F-3	1.8	.453	.026	0.267E-02	0.703	0.123E-04	3.278

NOTES:

G = Specific Gravity of Solids

CU = Uniformity Coefficient, D60/D10, where:

D60 is the Grain Diameter Corresponding to 60% passing
D10 is the Grain Diameter Corresponding to 10% passing

T.C. = Tempe Cell

V.P. = Volumetric Plate Entractor

P.P. = Pressure Cell Permeameter

UNIFIED SOIL SYMBOL: determined from the grain size distribution and visual classification Atterberg Limits. (Not available for most soils).

SATURATED PERMEABILITY: Also called Saturated Hydraulic Conductivity

FROST SUSCEPTIBILITY CLASSIFICATIONS:

NFS = Non-frost susceptibility
N = Negligible frost susceptibility
VL = Very low frost susceptibility
L = Low frost susceptibility
M = Medium frost susceptibility
H = High frost susceptibility
VH = Very high frost susceptibility

AWL: Multiplier of pore pressure for Gardner's *Moisture Content Function*
XWL: Exponent of pore pressure for Gardner's Moisture Content Function

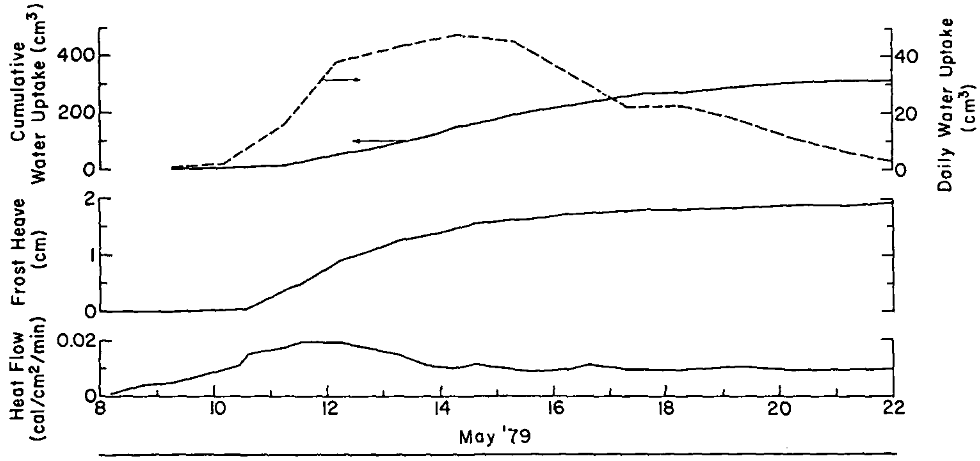
AKL: Multiplier of pore pressure for Gardner's Unsaturated Permeability Function
XKL: Exponent of pore pressure for Gardner's Unsaturated Permeability Function

APPENDIX B. SELECTED THERMAL PARAMETERS
Data taken from many sources and are intended as guidelines only

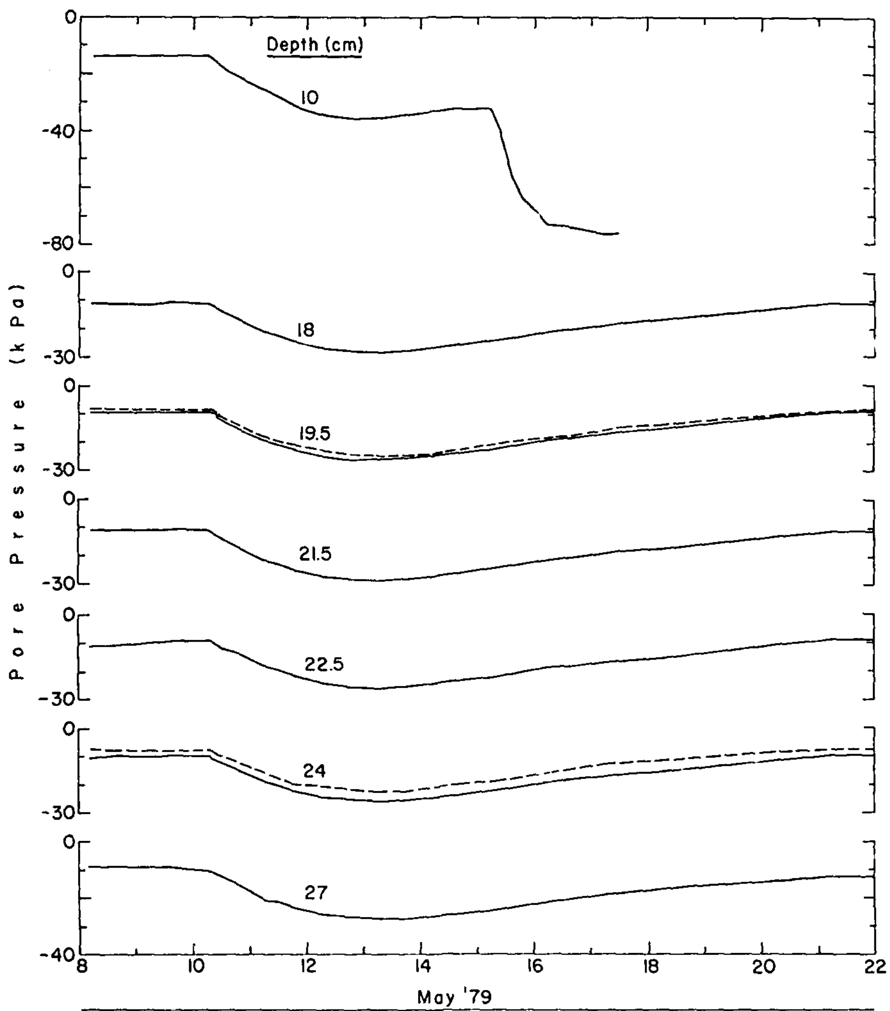
Material	*Specific heat** *(cal/cm³·°C)*	*Thermal conductivity (cal/cm·hr·°C)*
	Water†	
Liquid	1.00	5.0
Ice	0.55	18.0
	Concrete	
Portland cement	0.2	7.2
Asphalt cemcnt	0.4	7.0–12.0
	Soil**	
Clays-clayey soil	0.2–0.3	1.0–7.0
Silts-silty soil	0.3–0.4	12.0–16.0
Sand and gravel	0.4–0.5	20.0–25.0

* Specific heat equal volumetric heat capacity.
† Latent heat of fusion of water is 80 cal/g or 80 cal/cm^3.
** Dry mineral soil solids.

APPENDIX C: LABORATORY SOIL COLUMN TEST RESULTS, CHENA HOT SPRINGS ROAD SILT

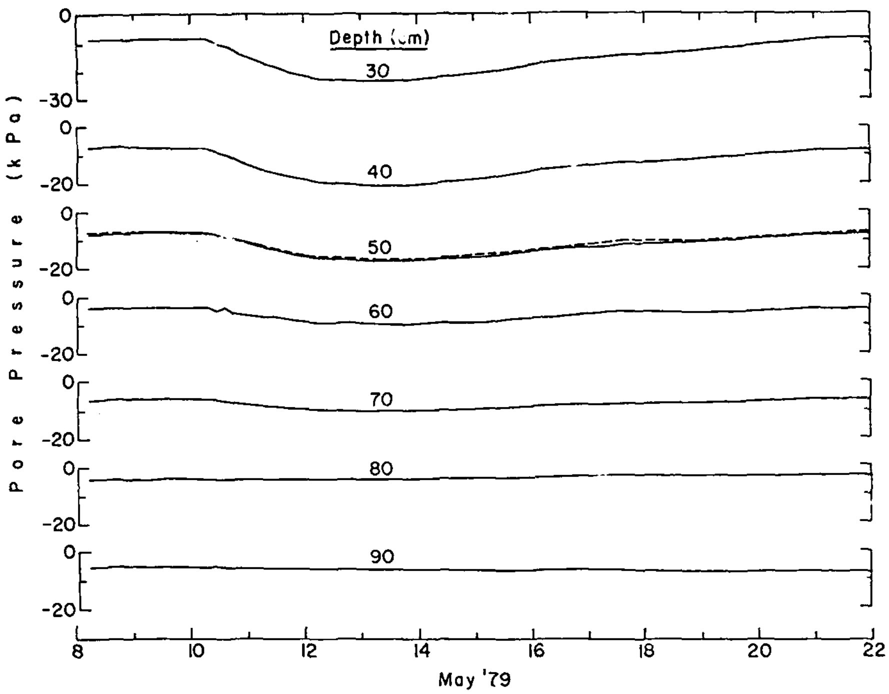


a. Surface heat flux, total frost heave and water uptake.

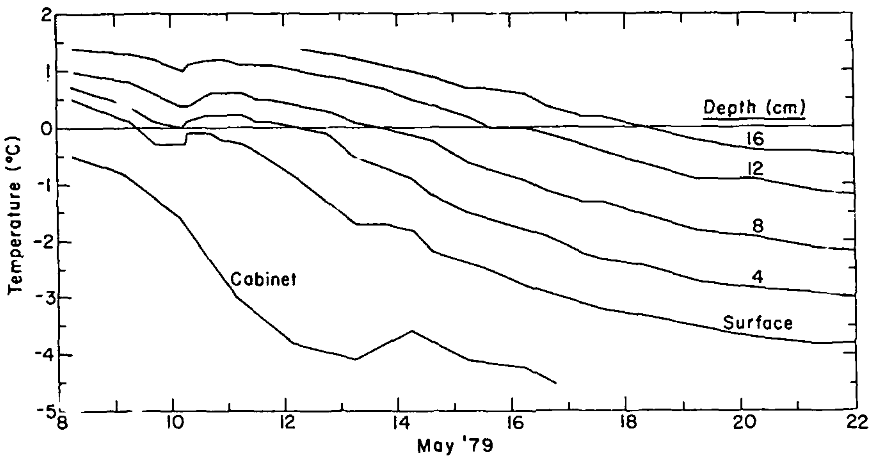


b. Pore water pressures, 10- to 27-cm depths.

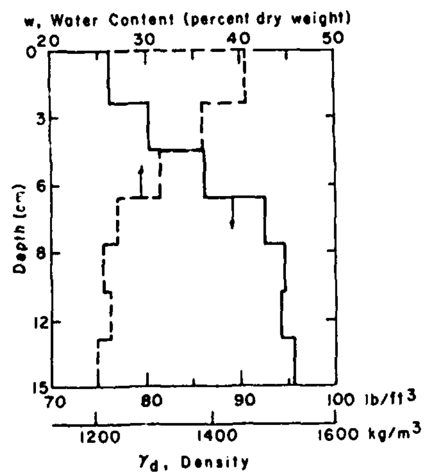
Figure C1. Test 2.



c. Pore water pressures, 30- to 90-cm depths.

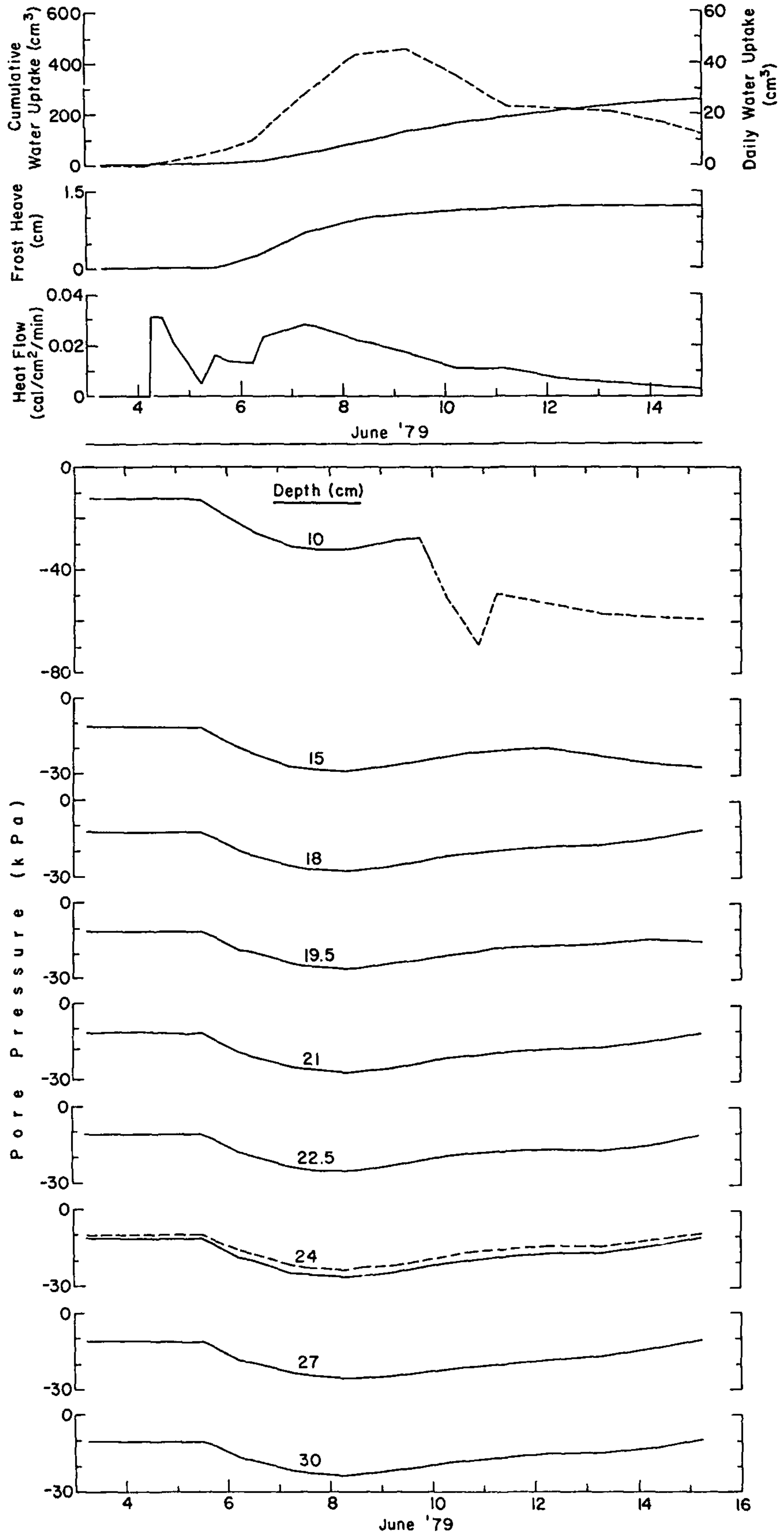


d. Temperatures during the test.



e. Moisture contents and densities after the test.

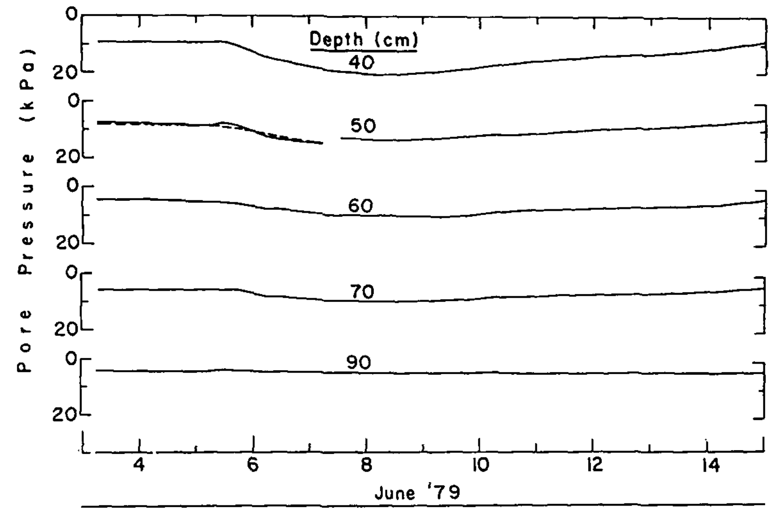
Figure C1 (cont'd). Test 2.



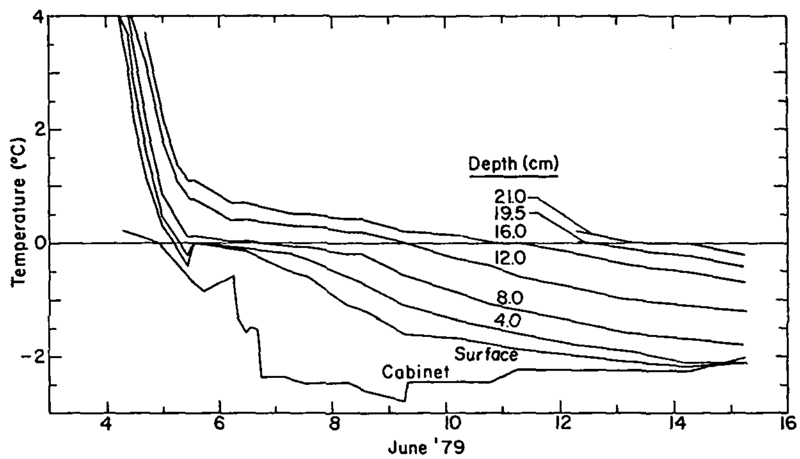
a. Surface heat flux, total frost heave and water uptake.

b. Pore water pressures, 10- to 30-cm depths.

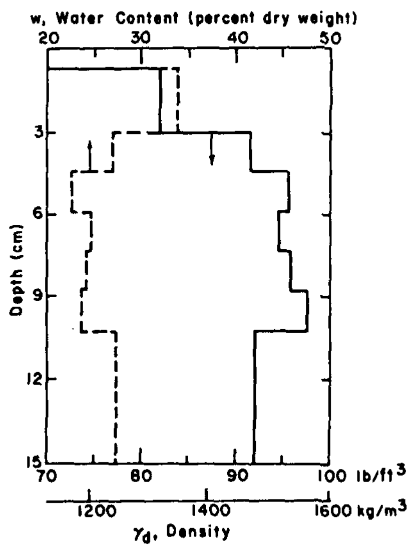
Figure C2. Test 3.



c. Pore water pressures, 40- to 90-cm depths.

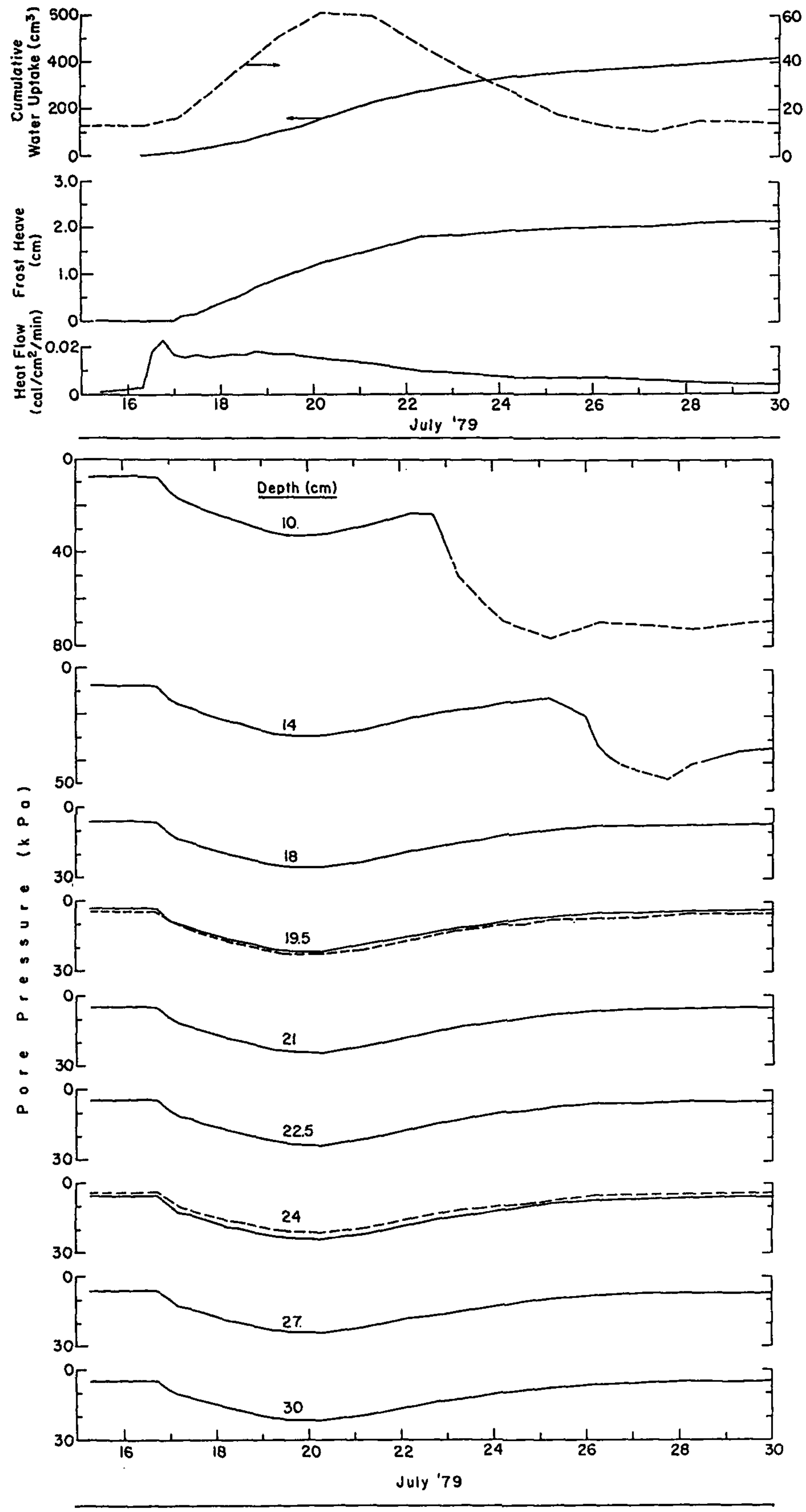


d. Temperatures during the test.



e. Moisture contents and densities after the test.

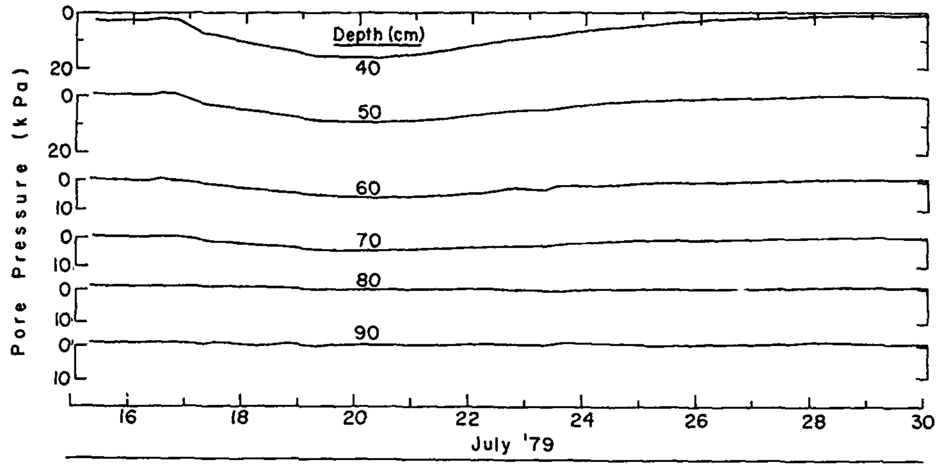
Figure C2 (cont'd). Test 3.



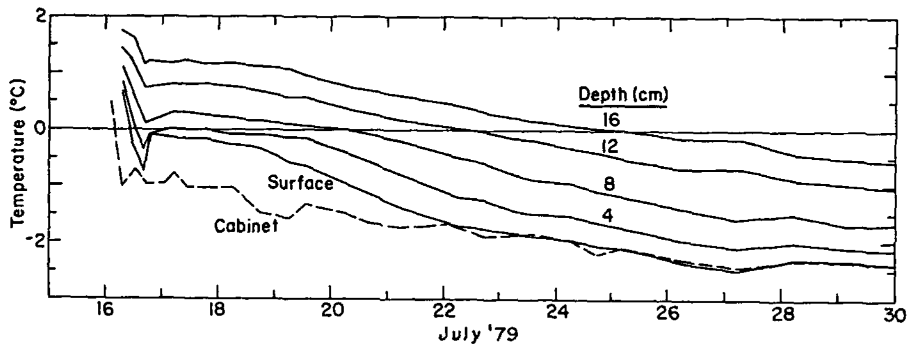
a. Surface heat flux, total frost heave and water uptake.

b. Pore water pressures, 10- to 30-cm depths.

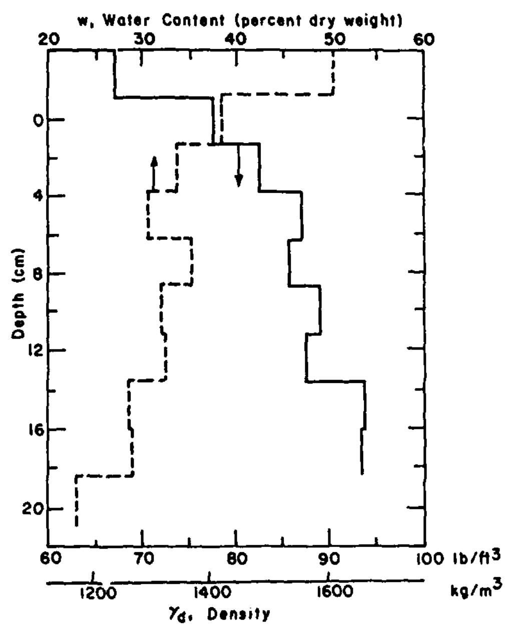
Figure C3. Test 4.



c. Pore water pressures, 40- to 90-cm depths.

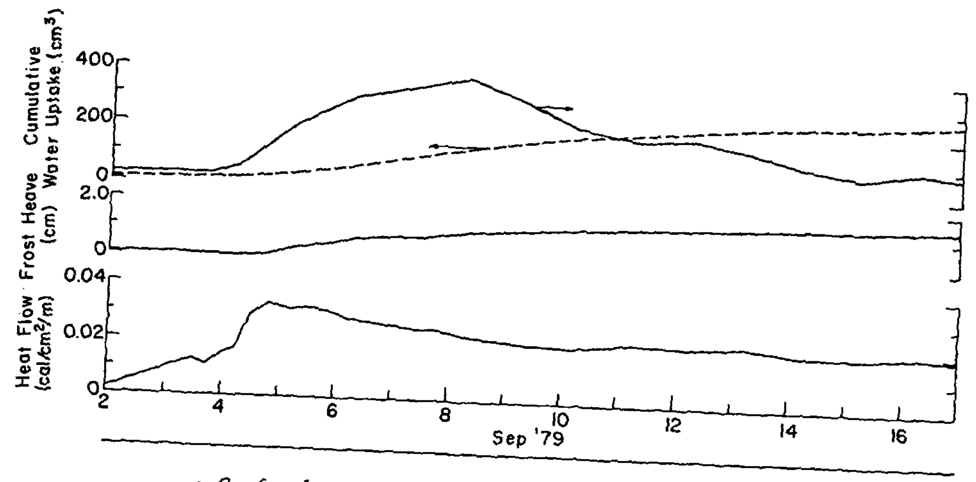


d. Temperatures during the test.

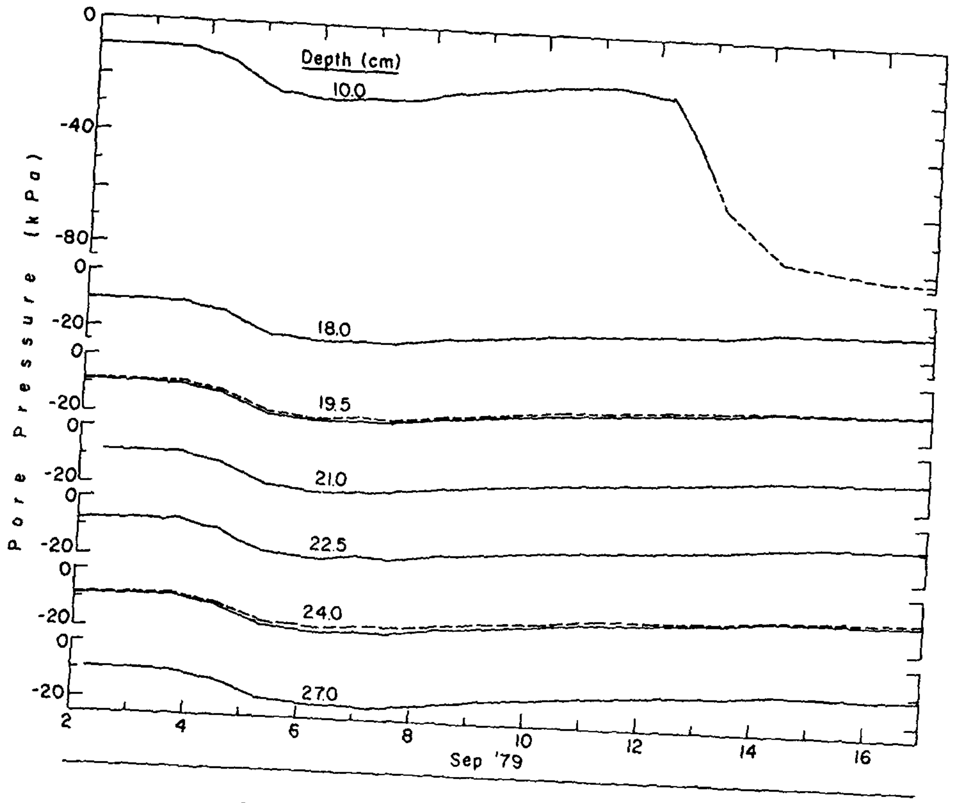


e. Moisture contents and densities after the test.

Figure C3 (cont'd). Test 4.

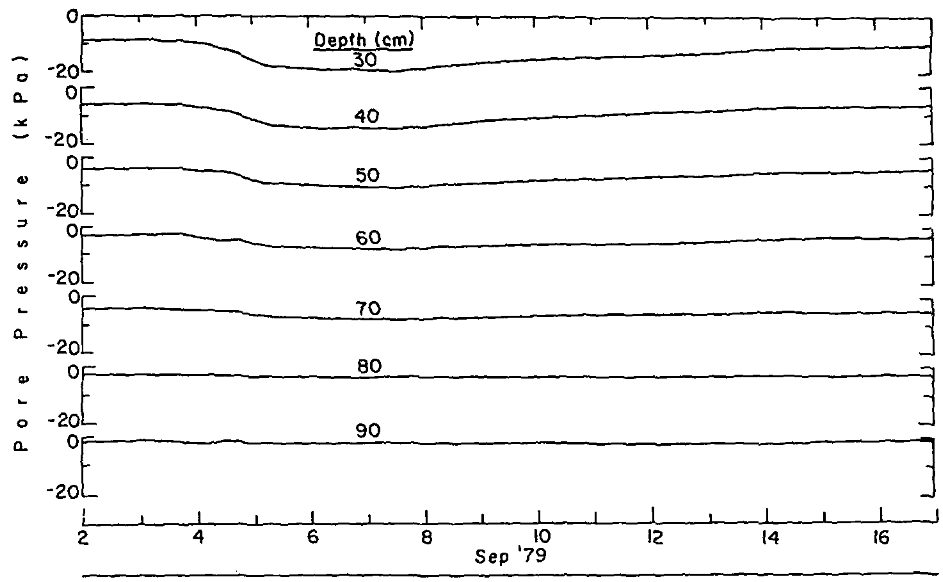


a. Surface heat flux, total frost heave and water uptake.

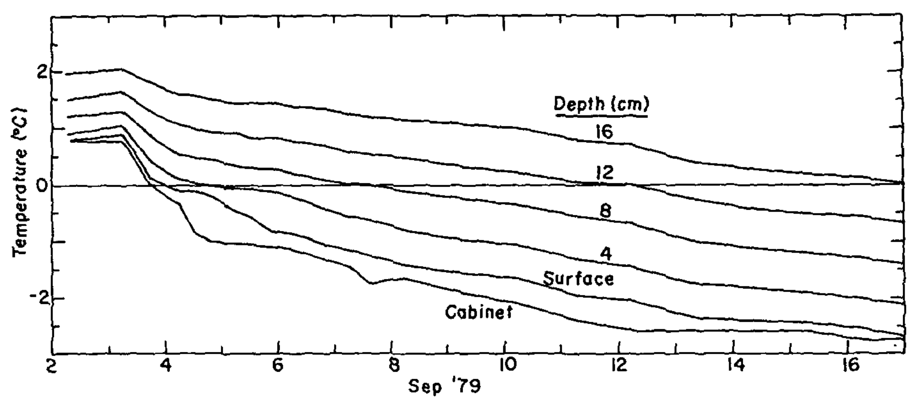


b. Pore water pressures, 10- to 27-cm depths.

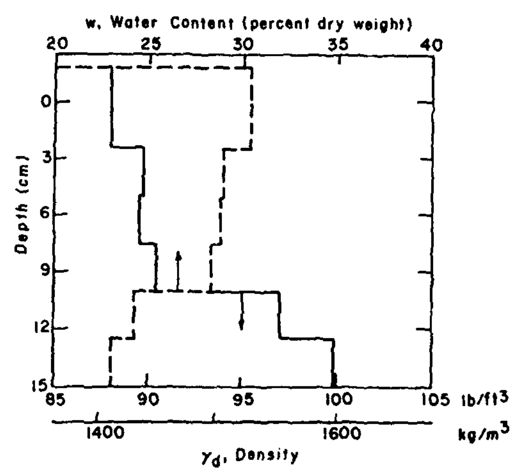
Figure C4. Test 5.



c. Pore water pressures, 30- to 90-cm depths.

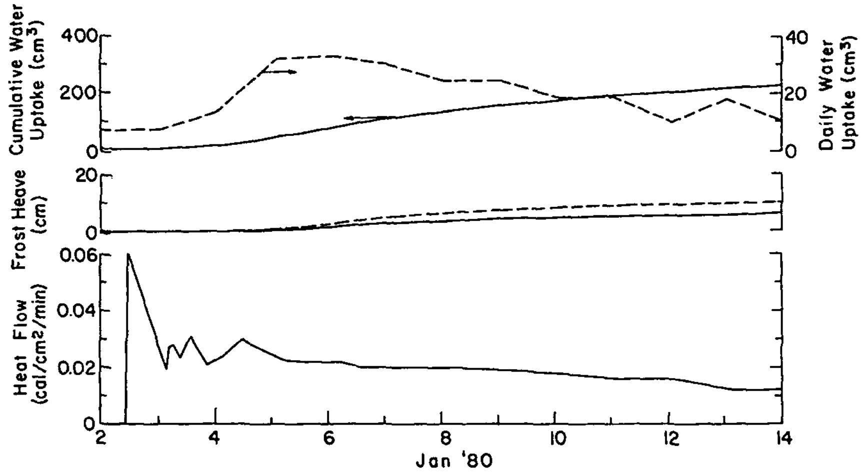


d. Temperatures during the test.

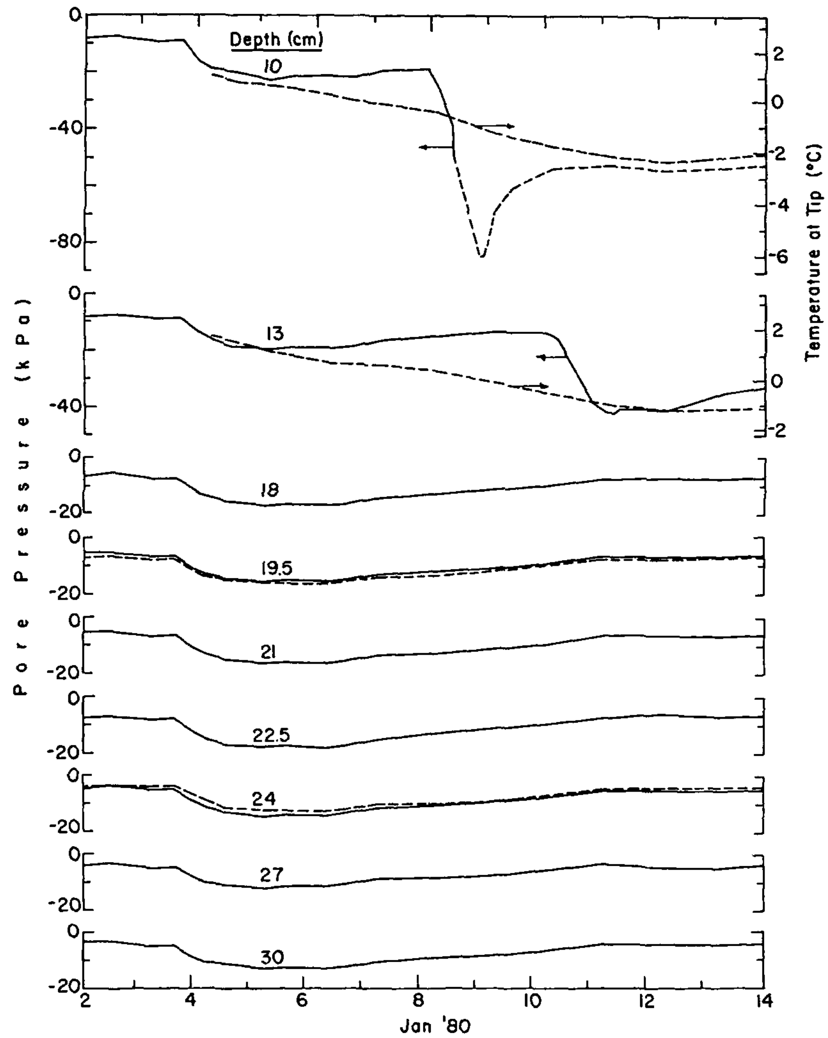


e. Moisture contents and densities after the test.

Figure C4 (cont'd). Test 5.

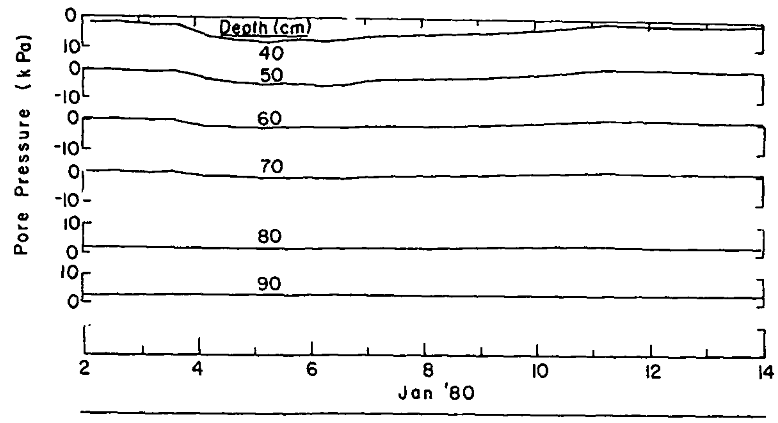


a. Surface heat flux, total frost heave and water uptake.

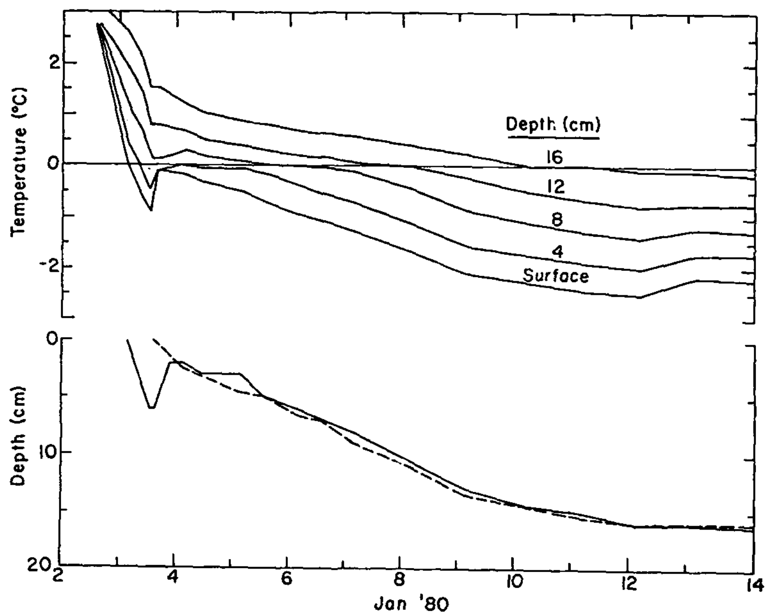


b. Pore water pressures, 10- to 30-cm depths.

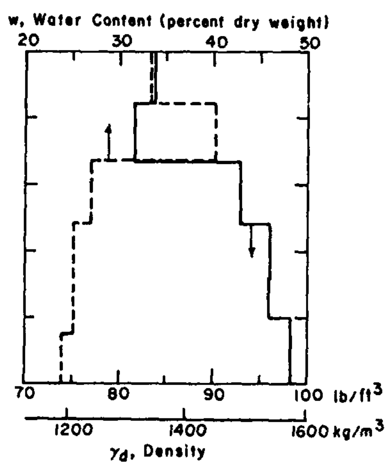
Figure C5. Test 6.



c. Pore water pressures, 40- to 90-cm depths.

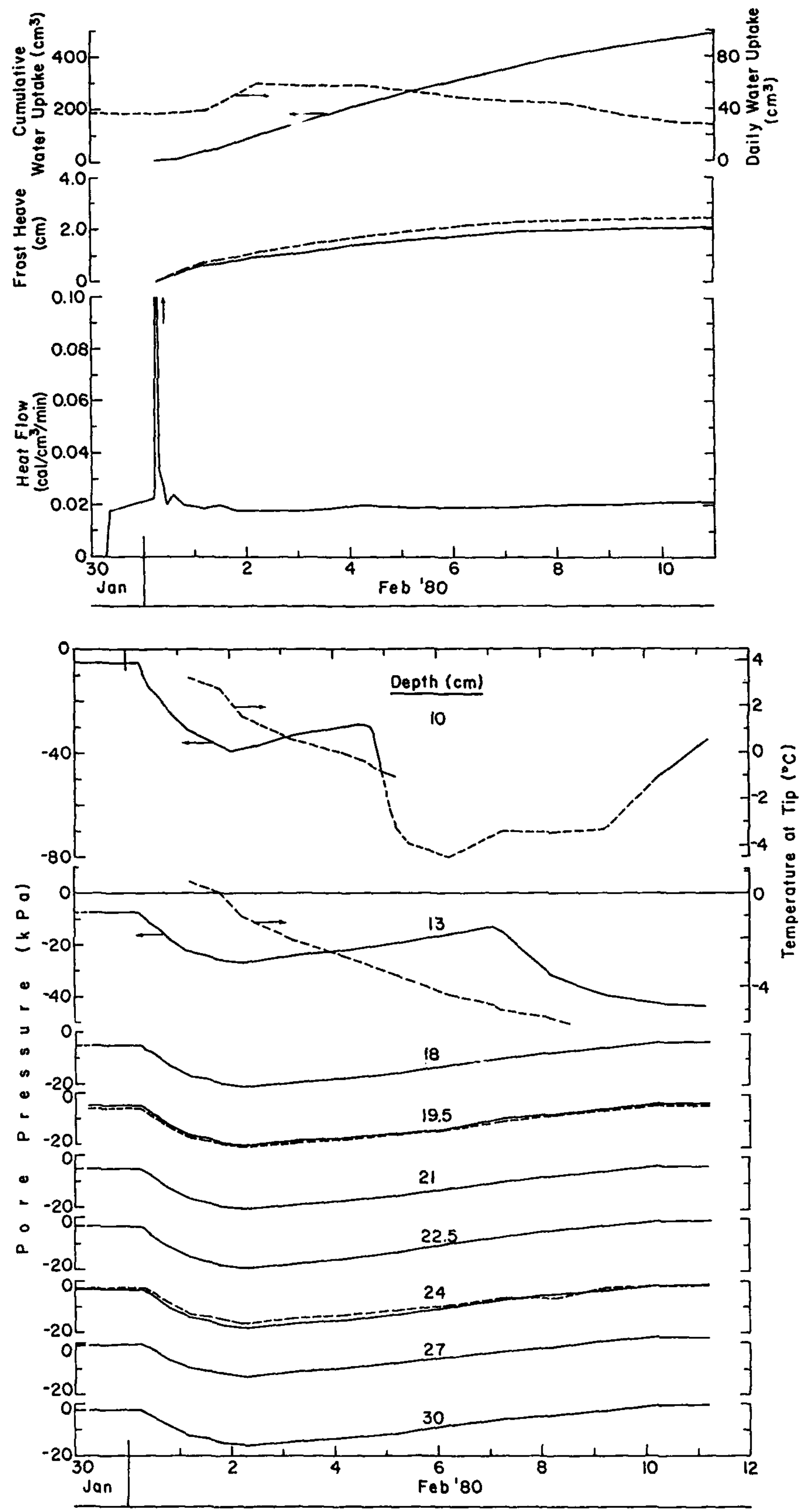


d. Temperatures and frost penetration during the test.



e. Moisture contents and densities after the test.

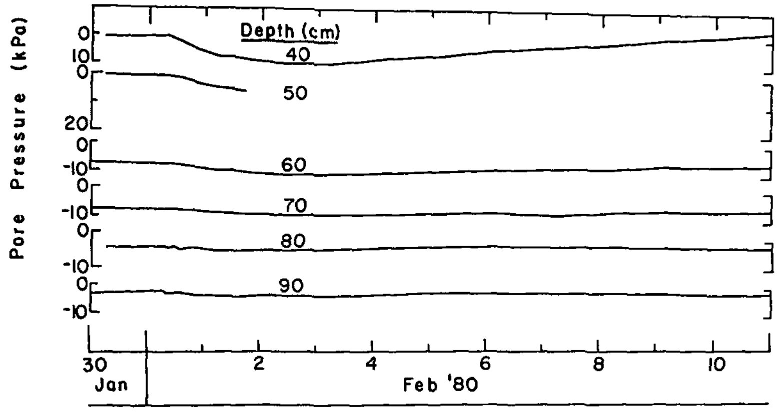
Figure C5 (cont'd). Test 6.



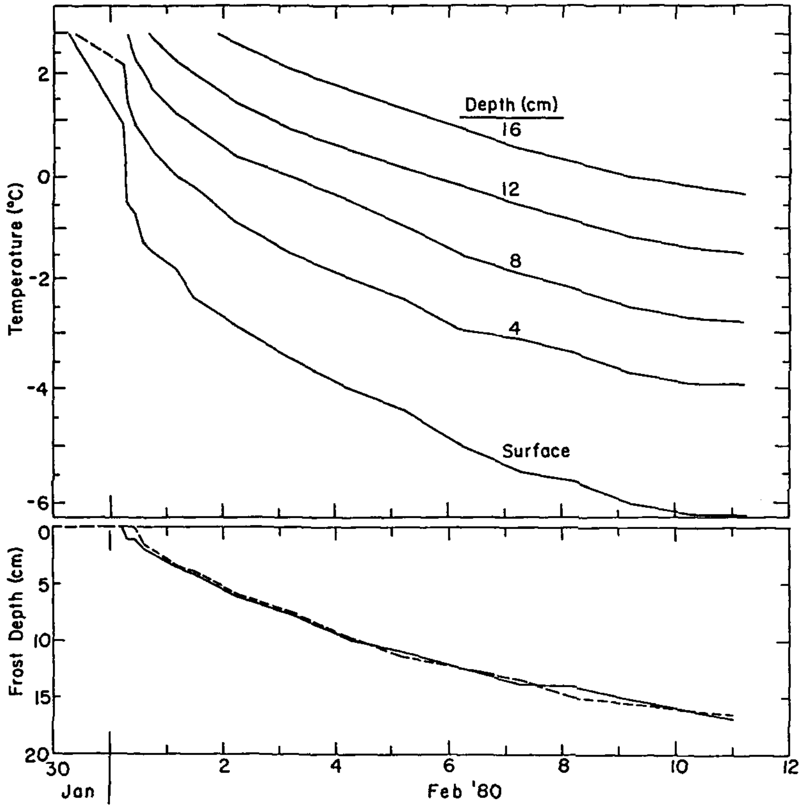
a. Surface heat flux, total frost heave and water uptake.

b. Pore water pressures, 10- to 30-cm depths.

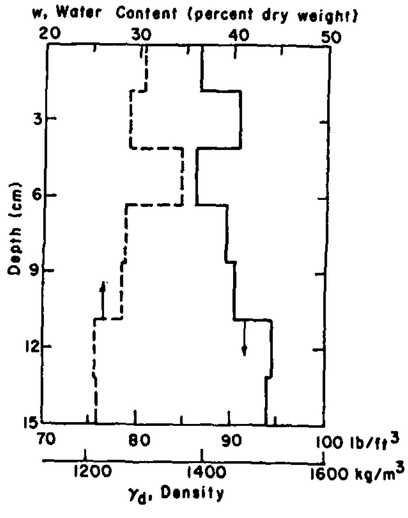
Figure C6. Test 7.



c. *Pore water pressures, 40- to 90-cm depths.*

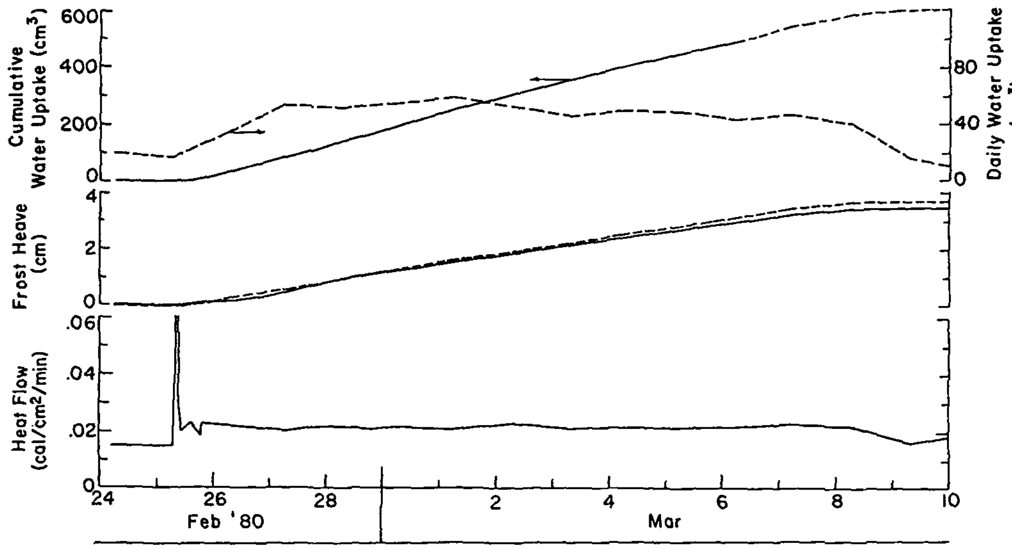


d. *Temperatures and frost penetration during the test.*

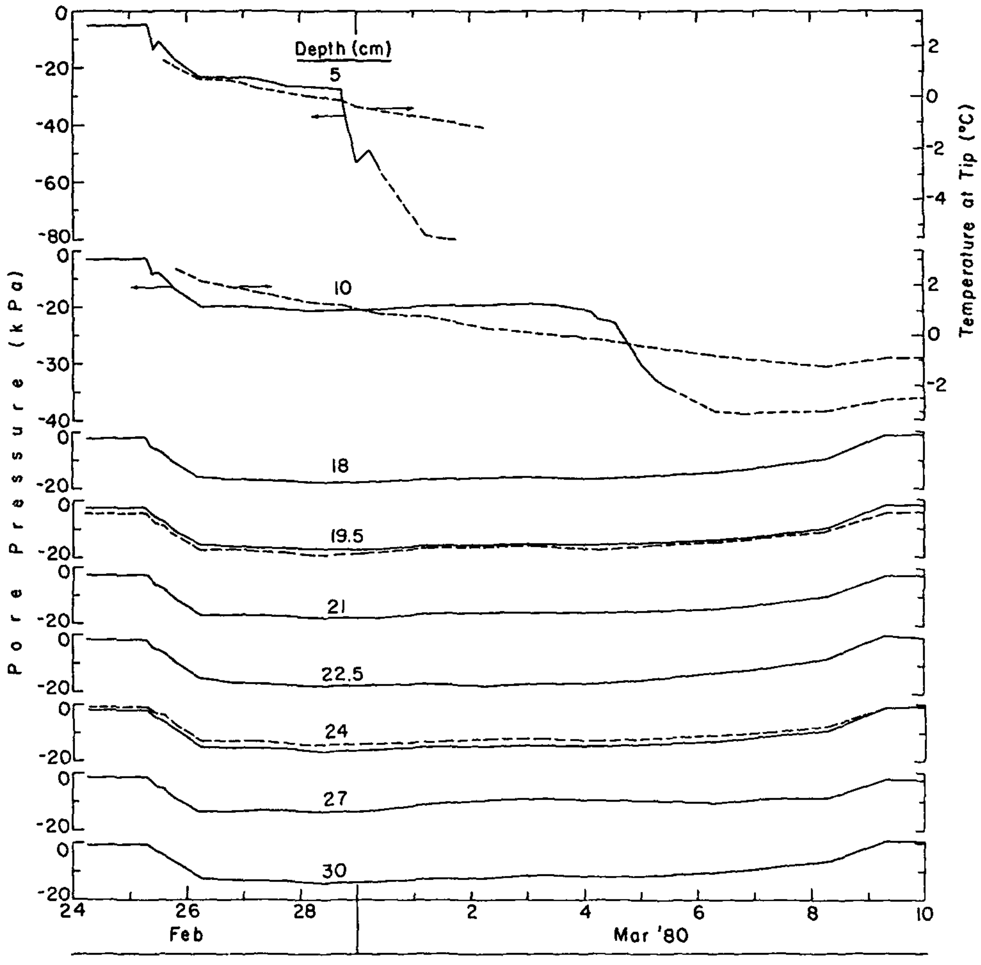


e. *Moisture contents and densities after the test.*

Figure C6 (cont'd). Test 7.

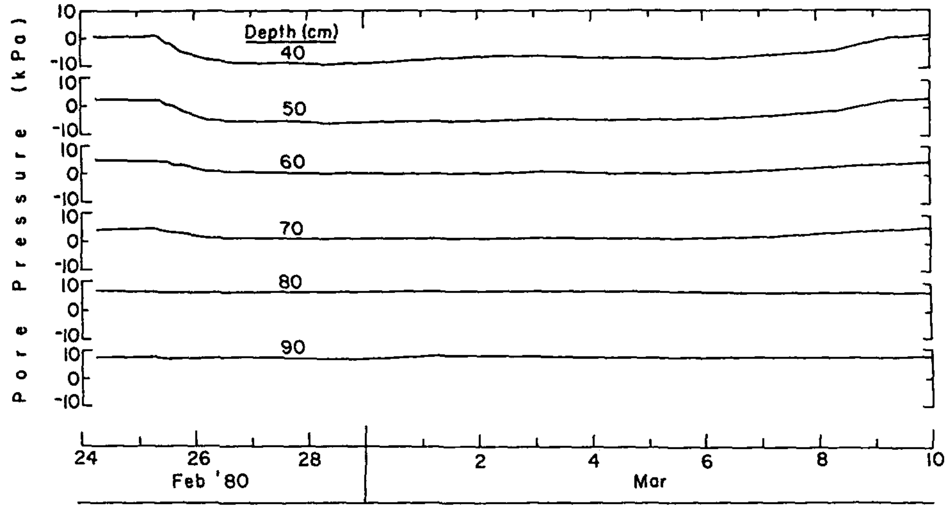


a. Surface heat flux, total frost heave and water uptake.

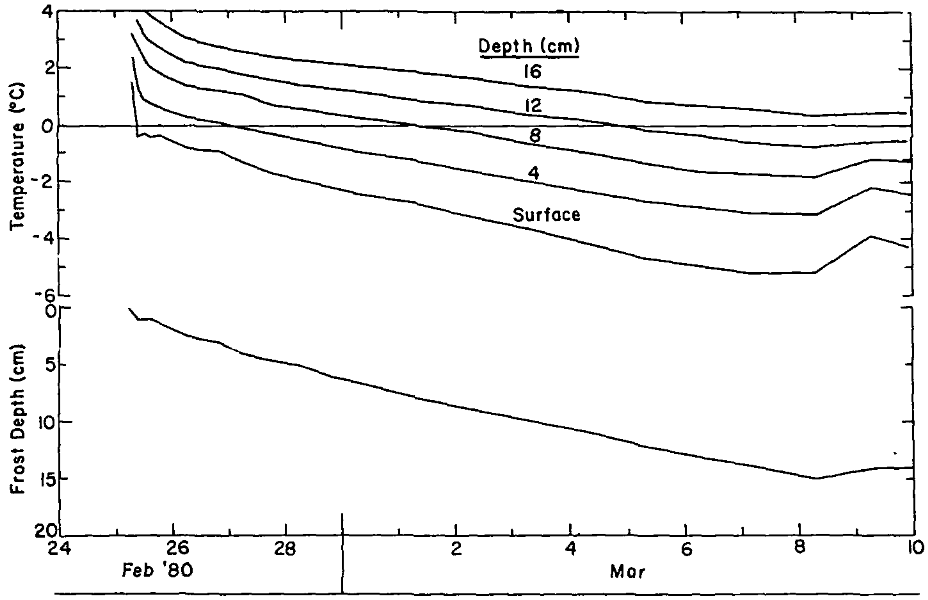


b. Pore water pressures, 10- to 30-cm depths.

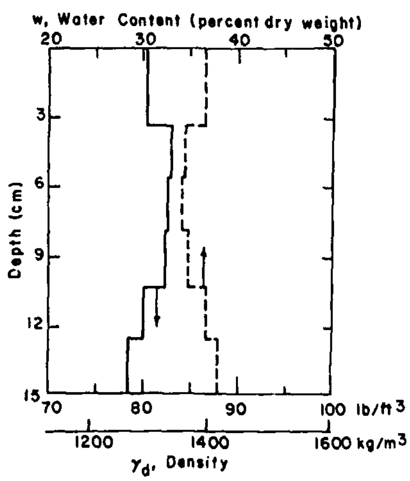
Figure C7. Test 8.



c. Pore water pressures, 40- to 90-cm depths.

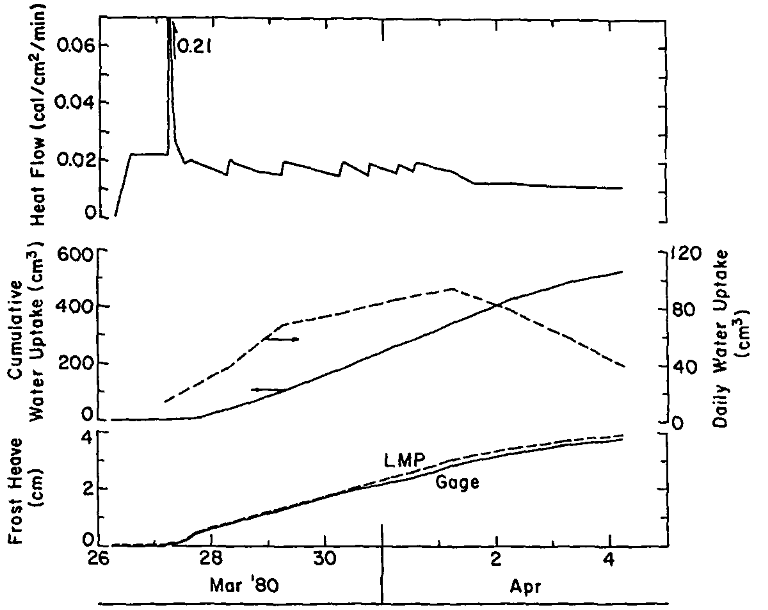


d. Temperatures and frost penetration during the test.

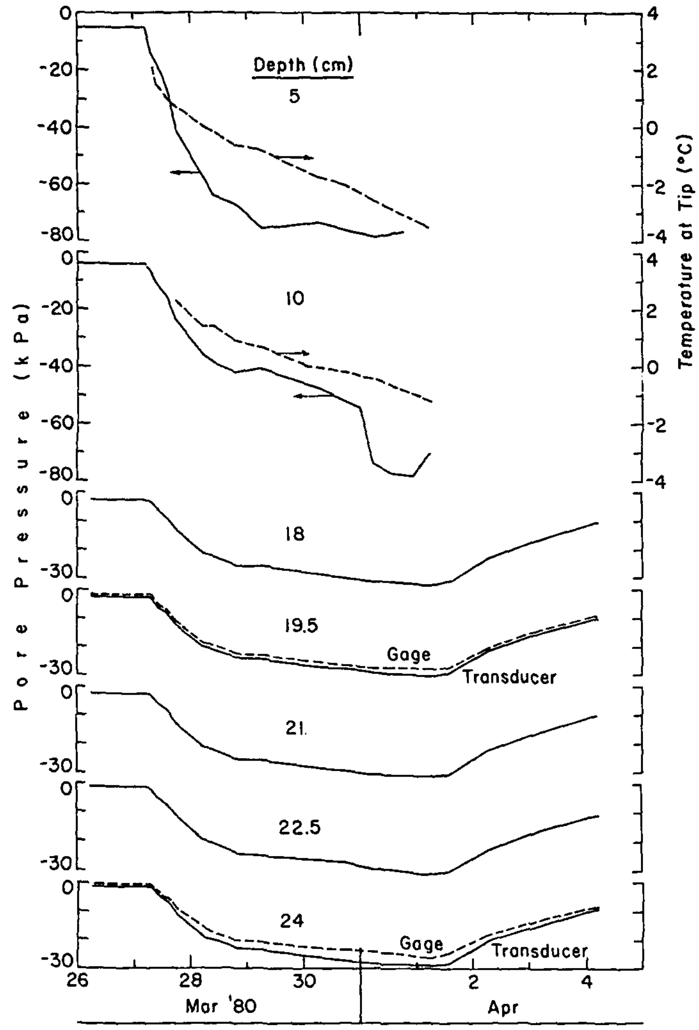


e. Moisture contents and densities after the test.

Figure C7 (cont'd). Test 8.

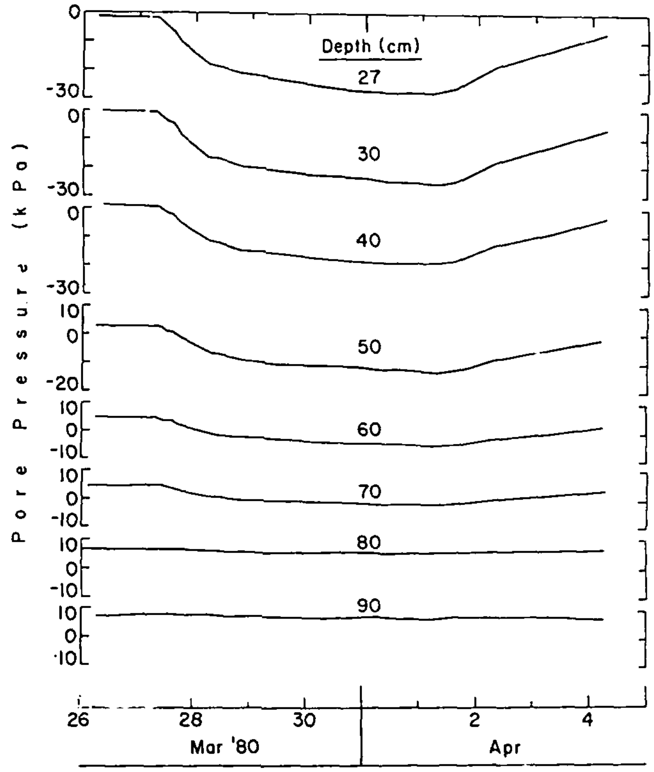


a. Surface heat flux, total frost heave and water uptake.

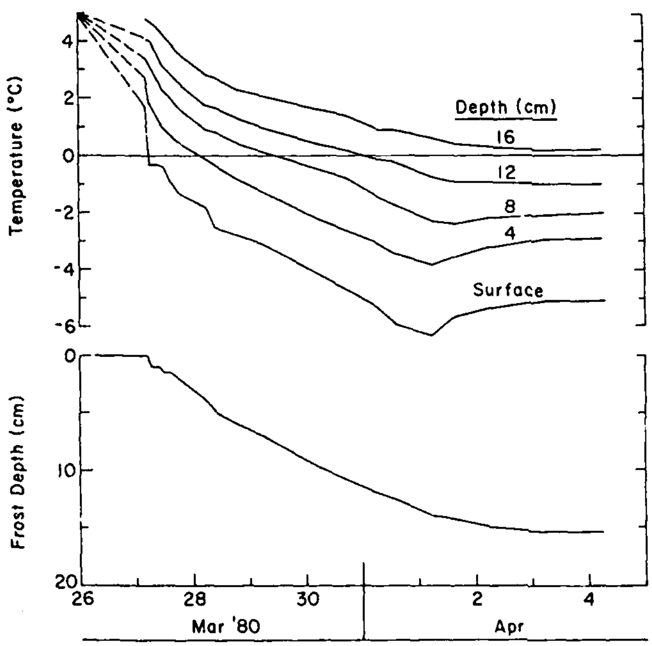


b. Pore water pressures, 5- to 24-cm depths.

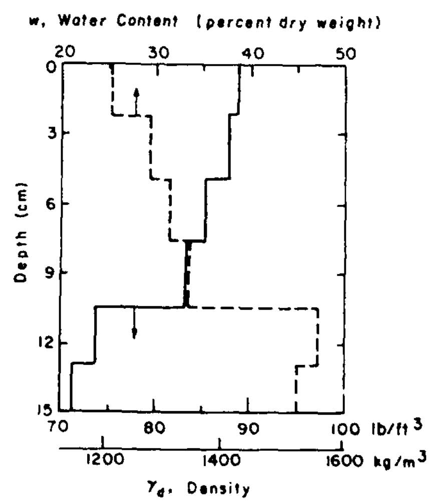
Figure C8. Test 9.



c. Pore water pressures, 27- to 90-cm depths.



d. Temperatures and frost penetration during the test.



e. Moisture contents and densities after the test.

Figure C8 (cont'd). Test 9.

Table C1. Summary table.

Test no.	Dry unit wt. (pcf)	Dry unit wt. (g/cc)	Porosity (n)	Orig. water (% by wt.)	Avg. final water (% by wt.)	Total water uptake (cc)	Surcharge (kPa)	Free water from surf. (cm)	Method of freezing	Max. daily water uptake (cc)	Max. freeze depth (cm)	Avg. freeze rate (cm/day)	Max. freeze rate (cm/day)	Expected heave from water uptake freezing (cm)	Total heave (cm)	Total heave % of frost depth (%)	Max. daily heave rate (cm/day)	Max. tension in freeze zone (kPa)	Depth(1) of max. tension (cm)	Avg. heat flux (cal./cm²/min)	Max. heat flux (after initial surge) (cal./cm²/min)	Avg. ice segregation ratio at end of frost penetration	Remarks on test
FBKS-1	95	1.52	44.3	21.3	38.4	717	3.45	45	CP	29.5	15.0	0.35	1.1	4.6	5.9	39.3	0.42	N.A.	N.A.	N.A.	N.A.	0.282	Top 15 cm only insulated. Did not freeze through tensiometers.
FBKS-2	95	1.52	44.3	20.0	30.0	514	34.50	45	CP	26.0	15.0	0.80	1.8	3.3	3.2	21.3	0.27	N.A.	N.A.	N.A.	N.A.	0.176	Top 15 cm only insulated. Did not freeze through tensiometers.
FBKS-3	95	1.52	44.3	21.0	27.4	410	3.45	45	CP	28.0	15.0+	1.0	2.0	2.7	2.2	14.3	0.90	62.0	15	N.A.	N.A.	0.128	Top 15 cm only insulated.
CHSS-1	99	1.58	43.5	20.1	26.5	120	3.45	100	CP	36.8	Sample went isothermal			0.8	1.1	–	1.0	68.0	18	N.A.	N.A.	N.A.	Poor test—cold trapped in top of cabinet—redesigned.
CHSS-2	95	1.52	45.7	22.0	30.2	315	3.45	100	CP	48.0	18.0	1.0	2.1	2.0	1.9	10.5	0.5	76.0	10	0.012	0.019	0.095	1st test using resistance gauges.
CHSS-3	98	1.57	43.9	19.6	26.2	261	3.45	100	CP	45.5	21.0**	2.1	2.7	1.7	1.8*	8.6	0.56	69.0	10	0.015	0.028	0.079	*Measured at end of test. **Irregular freeze depth 16 to 23 cm.
CHSS-4	97	1.56	44.2	20.1	32.0	426	3.45	50	CP	61.0	19.5**	1.7	1.7	2.8	2.2*	11.3	0.48	76.0 47.0	10 14	0.010	0.018	0.101	*Irregular heave—2.1 & 3.1 cm. **Irregular freeze depth—16.5 & 19.3 cm.
CHSS-5	100	1.60	42.9	19.0	27.3	268	34.50	100	CP	37.0	15.0	1.2	2.3	1.7	1.65*	11.0	0.32	83.3	10	0.026	0.036	0.099	*Measured at end of test.
CHSS-6	99	1.58	43.6	19.5	29.2	232	34.50	50	CP	33.0	16.0	2.0	2.5	1.5	0.99*	6.2	0.24	84.0 42.0	10 13	0.020	0.030	0.058	*Measured at end of test.
CHSS-7	101	1.62	42.2	20.0	28.6	498**	3.45	15	CHF	60.0**	16.5	1.6	2.7	3.2**	2.3*	13.9	0.80	80.0 44.0	10 13	0.020	0.024	0.122	*Measured at end of test. **Base leaked water at start and end of test. Max. uptake O.K.
CHSS-8	104	1.67	40.4	16.4	35.7	610	34.50	15	CHF	59.8	15.0	1.25	1.7	3.9	3.9*	26.0	0.50	80.2 57.0	5 10	0.022	0.024	0.206	No resist. gauges. *Measured at enc of test—uneven
CHSS-9	102	1.63	41.6	19.3	35.3	529	3.45	15	CHF	94.0	15.5	2.2	4.0	3.4	4.1*	26.5	0.60	78.0 78.0	5 10	0.017	0.020	0.209	No resist. gauges—replicate no. 7. *Avg. heave end of test.
CHSS-10	105	1.68	40.0	17.6	N.A.	80	3.45	15	CHF	58.0	14.0	8.5	12.0	0.5	0.5	3.6	0.90	87.0 82.0	5 10	0.040	0.048	0.034	No final density or water profile. Results on 1st 48 hr only.
CHSS-11 Step-1						241				52.0	3.8	1.9	2.5	1.6	1.6	42.0	0.6	N.A.		0.006	0.028	0.296	Temp of surface plate -2.0°C.
Step-2						407				56.0	7.0	0.75	1.5	2.6	2.5	78.1	0.8	62.5	5	0.010	0.022	0.561	Temp of surface plate -3.5°C.
Step-3						310				51.0	10.5	0.56	1.0	2.0	2.5	100.0	0.6	N.A.		0.013	0.026	0.462	Temp of surface plate -5.5°C.
Entire test	100	1.60	42.9	24.0	62.0	958	3.45	1.	SC					6.2	6.6	62.9						0.403	
West Leb Gravel-1																							Entire column insulated.
Step-1						168				41	7.5	3.5	5.0	1.1	0.8	10.6	0.28	N.A.		0.020	0.033	0.096	Temp of surface plate -2.0°C.
Step-2						95				24	12.0	1.0	1.8	0.6	0.4	8.9	0.08	79.5		0.020	0.026	0.082	Temp of surface plate -3.0°C.
Step-3						114				16	14.5	0.3	2.2	0.8	0.5	20.0	0.04	63.0		0.020	0.022	0.170	Temp of surface plate -3.5°C.
Entire test	136	2.16	21.3	9.0	12.8	377	3.45	15	SC					2.5	1.7	11.7						0.105	
West Leb Gravel-2																							Entire column insulated.
Step-1						39				12	5.5	1.4	5.5	0.3	0.6	10.1	0.13	N.A.		0.025	0.030	0.098	Temp of surface plate -2.0°C.
Step-2						85				19	11.0	0.9	4.1	0.6	0.5	9.0	0.11	70.0	5	0.022	0.028	0.083	Temp of surface plate -3.0°C.
Step-3						95				13	13.0	0.3	0.5	0.7	0.4	20.0	0.03	61.5		0.021	0.021	0.170	Temp of surface plate -3.5°C.
Entire test	131	2.10	23.6	9.0	13.9	219	3.45	100	SC					1.6	1.5	11.5							

Standard Proctor–Moisture Density

	Max. density (pcf)	Max. density (g/cc)	Opt. water content (%)
FBKS	106	1.70	17
CHSS	105	1.68	20
[illegible]	[illegible]	2.16	7

CP = Constant frost penetration
CHF = Constant heat flux
SC = Step changes
(1) Depth of measurement

APPENDIX D: FROST CODE

```
C-
C-FROST PROGRAM
C-
C-THIS PROGRAM WAS PREPARED AT THE UNIVERSITY OF CALIFORNIA(IRVINE)
C-AND AT USA-CRREL(HANOVER). ALL RIGHTS TO ITS USE AND DISSEMINATION
C-RESIDE WITH USA-CRREL.
C-
C-THIS VERSION HAS THE ABILITY TO HANDLE VARIABLE BOUNDARY CONDITIONS
C-AND LAYERED SOIL PROFILE.
C-THIS VERSION HAS THE LATEST OVERBURDEN ALGORITHM.
C-THIS VERSION USES GARDNERS FUNCTION TO REPRESENT
C-HYDRAULIC CONDUCTIVITY VS. PORE PRESSURE.
C-
C-SOLUTION OF A ONE DIMENSIONAL SOIL-WATER AND HEAT FLOW PROBLEM
C-WITH ISOTHERMAL SOIL-WATER PHASE CHANGE APPROXIMATION.
C-NUMERIC SOLUTION IS BY NODAL DOMAIN INTEGRATION METHOD.
C-
C-TIME DOMAIN APPROXIMATION CAN BE APPROACHED BY CRANK-NICOLSON
C-SCHEME OR BY FULLY-IMPLICIT SCHEME.
C-
C-THIS VERSION ALLOWS 102 NODES, 300 BOUNDARY CONDITION POINTS
C- AND 10 LAYERS
C-
C-IT SHOULD BE NOTED THAT THIS PROG. IS TO BE COMPILED IN F77.
C-
C-
C---------------------------------------------------------------------
C-
C-ARRAYS
C-
C---------------------------------------------------------------------
C-
C-
      REAL*8 GP(102),GT(102),PX(102),TX(102),WAT(102)
      REAL*8 FZHET(102),CA(102),TK(102)
      REAL*8 ALHET(102),DELX(102)
      REAL*8 S(102,3),P(102,3)
      REAL*8 R(102),EXW(102)
      REAL*8 SP(102,3),V(102)
      REAL*8 THETS(102),D(102)
      REAL*8 X,Y,Z,W
      REAL*8 THEO,AW,XG
      REAL*8 PPA
      COMMON/BLK2/HRTU(300),TUB(300),TUN(300)
      COMMON/BLK3/HRPL(300),PLB(300)
      COMMON/BLK4/HRTL(300),TLB(300)
      COMMON/BLK5/DEEP(102)
      COMMON/BLK10/QI(102)
      REAL*8 WT(102)
      REAL*8 FIS(300)
      REAL*8 AWL(11),XGL(11),THEOL(11),CSL(11),XTAY(11)
      REAL*8 TKSL(11),FHCL(11),DENSL(11),RESL(11)
      INTEGER NODL(11),IDLAY(11),IPNT(112)
      DIMENSION LEADIN(40)
      REAL*8 TDAY(300),HV(300),DHV(300),FDPTH(300),TDPTH(300)
      REAL*8 HKSL(11),AKL(11),XKL(11),XMV(11)
      REAL*8 POROST(102),DENSIT(102)
C-
C---------------------------------------------------------------------
C-
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```
C-FUNCTIONS:
C-
C-FGARD IS GARDNERS MOISTURE CONTENT FUNCTION
C-FOR THE CHARACTERISTIC CURVE.
C-FSTAR COMPUTES THE GUYMON AND LUTHIN RICHARDS EQUATION FUNCTION.
C-E COMPUTES THE E_FACTOR ON THE BASIS OF SATURATED HYDRAULIC CONDUCTIVITY
C-COND IS A VARIABLE REPRESENTING SATURATED HYDRAULIC CONDUCTIVITY
C-
C-------------------------------------------------------------------------
C-
      FGARD(X,Y,Z,W)=X/(Y*ABS(Z)**W+1.)
      FSTAR(X,Y,Z,W)=(W*X*Y*(ABS(Z)**(W-1.)))/((Y*(ABS(Z)**W)+1.)**2)
      E(COND) = 1.25*ABS((COND-3)**2.) + 6.
C-
C---------------------------------------------------------------
C-
C-THE FOLLOWING OPEN'S ARE TO CONSTRUCT INPUT-OUTPUT FILES FOR
C-FOR A SPECIFIC COMPUTER SYSTEM
C-THESE OPEN'S MUST BE REWRITTEN WHEN INSTALLING ON
C-A DIFFERENT COMPUTER SYSTEM
C-
C- ALSO CHECK CLOSE STATEMENTS AT END OF MAIN PROG.
C--------------------------------------------------------------
C-
        NRD=5
        NWT=6
        NPD=7
        OPEN(UNIT=NRD,FILE='FROST1.DAT',STATUS='OLD')
        OPEN(UNIT=NWT,FILE='FROST1.OUT',STATUS='NEW')
C-
C-------------------------------------------------------------------------
C-
C-FORMAT STATEMENTS
C-
C-------------------------------------------------------------------------
C-
C-
  500 FORMAT(T18,40A2,//)
  501 FORMAT(5X,40A2)
  502 FORMAT(9I5)
  503 FORMAT(8F10.0)
  504 FORMAT(2F10.0,3I5)
  505 FORMAT(7F10.0,/,7F10.0)
  506 FORMAT(3F10.0)
  550 FORMAT(/////)
  552 FORMAT('1')
  555 FORMAT(///,38X,'FROST PENETRATION IN CM',44X,'FROST HEAVE IN
     * CM',//)
  560 FORMAT(1X,'100',8X,'90',8X,'80',8X,'70',8X,'60',8X,'50',8X,
     1 '40',8X,'30',8X,'20',8X,'10',8X,'0',9X,'10',8X,'20')
  565 FORMAT(2X,'|',9X,'|',9X,'|',9X,'|',9X,'|',9X,'|',9X,'|',
     1 9X,'|',9X,'|',9X,'|',9X,'|',9X,'|',9X,'|')
  570 FORMAT('+++++++++++++++++++++++++++++++++++++++++++++++++++',
     1 '+++++++++++++++++++++++++++++++++++++++++++++++++',
     2 '++++++++++++++++++++++')
  575 FORMAT(120A1)
  580 FORMAT('+',101X,'+')
  585 FORMAT(//,101X,'DAYS')
  590 FORMAT(80('-'))
  591 FORMAT(80('-'))
```

```
660 FORMAT(/,20X,'FULLY IMPLICIT SCHEME FOR TIME DOMAIN ',
   1'APPROXIMATION IN:')
661 FORMAT(/,20X,'CRANK-NICOLSON SCHEME FOR TIME DOMAIN ',
   1'APPROXIMATION IN:')
662 FORMAT(37X,'HEAT TRANSPORT MODEL.')
663 FORMAT(37X,'MOISTURE TRANSPORT MODEL.')
600 FORMAT(//,20X,' BOUNDARY CONDITIONS',//)
601 FORMAT(1H1,T21,36HCRREL ONE-D FROST HEAVE MODEL BY UCI,//)
602 FORMAT(1H ,20A4/)
603 FORMAT(/,1H0,T20,35H*** UNITS ARE CAL-CM-GM-HR-DEG C***,//)
604 FORMAT(T21,18HNUMBER OF ELEMENTS,T61,I5,/,
   1 T21,14HTIME INCREMENT,T60,F10.3,/,
   2       T21,16HUPDATE FREQUENCY,T58,I8,/,
   3  20X,'TOTAL NUMBER OF UPDATES IN THE SIMULATION',1X,I5,//)
605 FORMAT(T36,'CONSTANT PARAMETERS',/,T21,'HEAT CAPACITIES:',7X,2HCW,
   1 T60,F10.3,/,T44,2HCI,T60,F10.3,/,T21,'THERMAL CONDUCTIVITIES:TKW',
   2 T60,F10.3,/,T44,3HTKI,T60,F10.3,/,T21,
   3 16HOVERBURDEN (PSI),T60,F10.3,/,T21,
   4 32HTFPD (FREEZING POINT DEPRESSION),T60,F10.3,/,
   5 T21,34HOFAT (PORE PRES MODIFIER FOR THAW),T60,F10.3,/)
620 FORMAT(/,T30,34HCONSTANT SOIL HYDRAULIC PARAMETERS,//,
   1 3X,'DEPTH',1X,'LAYER',2X,'KSAT',6X,'AK',7X,'B',6X,'AW'
   2 ,9X,'A',4X,'FHC',4X,'THEO',4X,'E',/)
622 FORMAT(/,T30,32HCONSTANT SOIL THERMAL PARAMETERS,//,
   1 3X,'DEPTH',1X,'LAYER',5X,'CS'2X,'TKS'
   2 ,6X,'DENS',5X,'RES',6X,'MV',/)
621 FORMAT(2X,F6.2,2X,I2,2X,F6.3,2X,E10.3,1X,F6.3,1X,
   1 E10.3,1X,F6.3,
   2 1X,F6.3,1X,F6.3,1X,F8.3)
623 FORMAT(2X,F6.2,2X,I2,2X,F7.3,1X,F6.3,2X,F6.3,F8.3,1X,1PE10.3)
606 FORMAT(/,1H1,T26,18HINITIAL CONDITIONS,//,
   1 T5,4HNODE,9X,8HPRESSURE,7X,11HTEMPERATURE,
   2 5X,11HICE CONTENT,//)
607 FORMAT(I3,T7,1PE10.3,T23,1PE10.3,T37,1PE10.3,
   1 T48,1PE10.3,T61,1PE10.3,T74,1PE10.3,T87,1PE10.3)
608 FORMAT(///,T4,5HTIME=,F8.3,1X,3HHRS,F8.3,1X,4HDAYS,
   1 3X,18HFROST HEAVE EQUALS,F10.2,1X,2HCM///)
610 FORMAT(/,1X,'NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS ',
   1'HAS BEEN SET EQUAL TO ZERO',/)
609 FORMAT(' NODE',T9,'DEPTH',T24,'PRESS',T38,'TEMP',
   1 T49,'WAT.CONT',T62,'ICE CONT',T75,'DENSITY',T88,'POROSITY')
619 FORMAT(5X,'DAY',5X,'UP PRESS BC',5X,'UP TEMP BC'.5X,
   1 'LO PRESS BC',5X,'LO TEMP BC',//)
625 FORMAT(1X,F6.1,F16.3,F15.3,F16.3,F15.3,//)
626 FORMAT(1X,F6.1,F16.3,F15.3,F16.3,F15.3)
611 FORMAT(2X,I4,4X,5F12.7)
612 FORMAT(10X,17HTOO MANY ELEMENTS)
615 FORMAT(4X,I3,7X,1P2E15.3)
617 FORMAT(/,13X,'HYD.COND.',2X,'HEAT COND.',3X,'THETASTAR',
   C3X,'HEAT CAP.',2X,'CONVCT FLUX')
618 FORMAT(4X,I3,5X,F12.3,2F16.3)
629 FORMAT(20X,'NODAL DOMAIN INTEGRATION MATRIX VARIABLE = ',F9.3)
630 FORMAT(//,T20,'PRESS VS. HYDRAULIC CONDUCTIVITY',//)
640 FORMAT(T22,F12.2,2X,1PE10.3)
649 FORMAT(///80('*'),/,80('-'),/,80('*'),//)
650 FORMAT(//,T20,'SUMMARY OF RESULTS',//)
651 FORMAT(4X,3HDAY,4X,'CUMULATIVE HEAVE (CM)',4X,'HEAVE RATE',3X,
   1 ' ISR',3X,'FROST DEPTH',2X,'THAW DEPTH')
652 FORMAT(12X,'MIN    MAX    MEAN',8X,'CM/HR',16X,'CM',10X,'CM')
653 FORMAT(1X,F6.1,3F8.2,F13.3,F10.3,F10.2,F12.2)
```

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  654 FORMAT(//,T5,'THE MAXIMUM COEF OF VARIATION OF ',
     1'SIMULATED HEAVE IS',F10.3,/,T5,'MEAN HEAVE ',
     2'IS WITHIN THE INDICATED BOUNDS WITH',/,T5,
     3' AT LEAST A 95% CONFIDENCE (MIN=MEAN-2*SIGMA OR ZERO',/,
     4T5,'AND MAX=MEAN+2*SIGMA) FOR HYD COND CV OF',F6.3/)
  655 FORMAT(/,T5,'DURING COMPUTATION CONV TERM SET TO ZERO',
     1I6,1X,'TIMES'/)
  656 FORMAT(/,T5,'SURFACE TEMP DIURNAL VARIATION EQUAL',F6.2,' CELCIUS',
     1/)
C-
C-----------------------------------------------------------------------
C-
C-SEGMENT 1-INPUT
C-
C-
C-----------------------------------------------------------------------
C-
C-
C-READ INPUT CONTROLS:
C-
C-LEADIN IS THE TITLE OF DATA FILE
C-ZN IS THE NODAL INTEGRATION CAPACITANCE MATRIX VARIAVBLE
C-NTDH=1 IS FOR FULLY IMPLICIT HEAT TRANSPORT
C-    =2 IS FOR CRANK-NICOLSON HEAT TRANSPORT
C-NTDM=1 IS FOR FULLY IMPLICIT MOISTURE TRANSPORT
C-    =2 IS FOR CRANK-NICOLSON MOISTURE TRANSPORT
C-KODE1=1 IS FOR CONSTANT INITIAL CONDITIONS
C-KODE2=1 WILL SUPPRESS OUTPUT OF NODAL PRESSURES, TEMPS, ETC.
C-KODE3=1 IS FOR CONSTANT ELEMENT LENGTH
C-KODE4=1 IS FOR '45 DEGREE ANGLE ICS', IE PX(I)=PX(1)+DEEP(I)
C-KODE5=1 IS FOR CONVECTIVE HEAT INCLUSION
C-KODE6=1 IS SWITCH FOR COMPUTED PARAMETER OUTPUT
C-KODEE=1 IS SWITCH FOR E-FACTOR INPUT (0 FOR E-FACTOR CALC.)
C-NLAY IS THE NUMBER OF LAYERS
C-KPU,KPL,KTU,KTL=1 IS FOR NATURAL BOUNDARY CONDITIONS
C-NEL IS THE NUMBER OF ELEMENTS
C-NNOD IS THE NUMBER OF NODES
C-----------------------------------------------------------------------
C-
      READ(NRD,501) LEADIN
      READ(NRD,*) ZN,NTDM,NTDH
      ZN1=ZN/(ZN+1.)
      ZN2=1.-ZN1
      READ(NRD,*) KODE1,KODE2,KODE3,KODE4,KODE5,KODE6,KODEE
      READ(NRD,*) NNOD,NLAY
      READ(NRD,*) KPU,KPL,KTU,KTL
      KPPU=KPU
      NEL=NNOD-1
      IF(NEL.GT.101) WRITE(NWT,612)
C-
C-----------------------------------------------------------------------
C-
C-READ ELEMENT LENGTH AND TIME SOLUTION CONTROLS.
C-
C-DELX IS THE LENGTH OF THE ELEMENT
C-DT IS TIMESTEP INCREMENT (IN HOUR).
C-NTSTP IS UPDATE FREQUENCY. NTSTP=NUMBER OF DT TIMESTEPS BETWEEN UPDATES.
C-TOUT IS OUTPUT FREQUENCY. TOUT=NUMBER OF DAYS BETWEEN DATA OUTPUT.
C-TEND IS PROGRAM DURATION. TEND=NUMBER OF DAYS FOR ENTIRE SIMULATION.
C-
```

```
C-----------------------------------------------------------------------
C-
CCC
      IF (KODE3.EQ.1) THEN
         READ (NRD,*) DELX(1)
         DO 1000 M=2,NEL
            DELX(M) = DELX(1)
1000     CONTINUE
      ELSE
         READ (NRD,*) (DELX(M),M=1,NEL)
      ENDIF
CCC
      READ (NRD,*) DT,NTSTP,TOUT,TEND
      IDAZE=TEND
C-
C-----------------------------------------------------------------------
C-
C-READ SOIL MOISTURE PARAMETERS,HEAT CAPACITIES,THERMAL CONDUCTIVITIES,
C-  AND PHYSICAL CHARACTERISTICS.
C-
C-OVER IS OVERBURDEN IN PSI.
C-TFPD IS THE FREEZING POINT DEPRESSION.
C-OFAT IS THE MODIFIER OF THE PORE PRES DURING THAW
C-AWL IS THE MULTIPLIER OF PORE PRESSURE IN GARDNER'S MOISTURE FUNCTION.
C-XGL IS THE EXPONENT OF PORE PRESSURE IN GARDNER'S MOISTURE FUNCTION.
C-THEOL IS THE SOIL POROSITY.
C-CSL IS THE HEAT CAPACITY OF THE SOIL.
C-TKSL IS THE THERMAL CONDUCTIVITY OF THE SOIL.
C-FHCL IS THE MULTIPLIER FOR HYDRAULIC CONDUCTIVITY FUNCTION.
C-DENSL IS THE DENSITY OF THE SOIL.
C-RESL IS THE RESIDUAL WATER CONTENT.
C-HKSL IS THE SATURATED HYDRAULIC CONDUCTIVITY.
C-AKL IS THE MULTIPLIER OF PORE PRESSURE IN GARDNER'S HYDRAULIC CONDUCTIVITY
C-     FUNCTION.
C-XKL IS THE EXPONENT OF PORE PRESSURE IN GARDNER'S HYDRAULIC CONDUCTIVITY
C-     FUNCTION.
C- XTAY IS THE MODIFIER (E) OF HYD. COND. IN FREEZING ZONE
C- XMV IS THE MODIFIER OF (E) DURING THAW
C-----------------------------------------------------------------------
C-
      READ(NRD,*) OVER,TFPD,OFAT
CCC
      READ(NRD,*) (AWL(N),XGL(N),THEOL(N), N=1,NLAY)
      READ(NRD,*) (CSL(N),TKSL(N),FHCL(N),DENSL(N),RESL(N), N=1,NLAY)
        IF(KODEE.EQ.1) THEN
      READ(NRD,*) (HKSL(N),AKL(N),XKL(N),XTAY(N),XMV(N), N=1,NLAY)
        ELSE
      READ(NRD,*) (HKSL(N),AKL(N),XKL(N),XMV(N), N=1,NLAY)
      DO 1210 N=1,NLAY
        XTAY(N)= E(HKSL(N))
 1210 CONTINUE
        ENDIF
CCC
C-
C-------------------------------------------------------------------
C-
C-SETUP POINTER ARRAY FOR LAYER PARAMETERS
C-NODL AND IDLAY EQUAL SET OF NLAY LOWEST
C-  NODE NUMBER AND ASSOCIATE LAYER NUMBER
C-
```

```
C----------------------------------------------------------------------
C-
      DO 1220 I=1,NLAY
      READ(NRD,*) NODL(I),IDLAY(I)
 1220 CONTINUE
      J=NODL(1)
      DO 1230 N=1,J
 1230 IPNT(N)=IDLAY(1)
      IF(J.GE.NNOD) GO TO 1245
      DO 1240 I=2,NLAY
      M=NODL(I-1)+1
      MM=NODL(I)
      DO 1240 N=M,MM
      IPNT(N)=IDLAY(I)
      IF(N.GT.NNOD) GO TO 1245
 1240 CONTINUE
 1245 CONTINUE
C-
C----------------------------------------------------------------------
C-
C-READ THE COEFFICIENT OF VARIATION OF THE HYDRAULIC CONDUCTIVITY
C-
C----------------------------------------------------------------------
C-
      READ(NRD,*) CVK
C-
C--------------------------------------------------------------------------
C-
C-READ INITIAL CONDITIONS FOR PRESSURE, TEMPERATURE AND ICE CONTENT.
C-
C--------------------------------------------------------------------------
C-
CCC
      IF (KODE1.EQ.1) THEN
         READ(NRD,*) PX(1), TX(1), QI(1)
         DO 1001 M=2,NNOD
            PX(M) = PX(1)
            TX(M) = TX(1)
            QI(M) = QI(1)
1001     CONTINUE
      ELSE
         READ(NRD,*) (PX(N),TX(N),QI(N),N=1,NNOD)
      ENDIF
CCC
C-
C--------------------------------------------------------------------------
C-
C-READ TIME VARYING BOUNDARY CONDITIONS AND UPPER PORE PRESSURE HEAD
C-
C-NTU IS NUMBER OF DATA SETS FOR SURFACE TEMPERATURE BOUNDARY CONDITIONS.
C-NPL IS NUMBER OF DATA PAIRS FOR LOWER PORE PRESSURE BOUNDARY CONDITIONS.
C-NTL IS NUMBER OF DATA PAIRS FOR LOWER TEMPERATURE BOUNDARY CONDITIONS.
C-AMPT IS DIURNAL AMPLITUDE IN SURFACE TEMPERATURE.
C-
C-SURFACE TEMPERATURE IS APPROXIMATED BY STEP FUNCTION BETWEEN EACH DATA
C-SETS.
C-LOWER TEMPERATURE AND PORE PRESSURE ARE APPROXIMATED BY LINEAR
C-INTERPOLATION BETWEEN EACH DATA PAIRS.
C-
C--------------------------------------------------------------------------
```

```
C-
      READ(NRD,*) PU
      READ(NRD,*) NTU,NPL,NTL,AMPT
      READ (NRD,*) (TUB(N), HRTU(N), TUN(N),N=1,NTU)
      READ(NRD,*)(PLB(N),HRPL(N),N=1,NPL)
      READ(NRD,*)(TLB(N),HRTL(N),N=1,NTL)
C-
C------------------------------------------------------------------------
C-
C-SET UP STATE VARIABLE ARRAYS.
C-DEEP IS THE DEPTH OF THE SOIL.
C-
C------------------------------------------------------------------------
C-
      DEEP(1)=0.
      SDELX=0.
      DO 1500 M=2,NNOD
      SDELX=SDELX+DELX(M-1)
      DEEP(M)=SDELX
 1500 CONTINUE
      IF(KODE4.NE.1) GO TO 1510
      DO 1520 N=1,NNOD
      PX(N)=PX(1)+DEEP(N)
 1520 CONTINUE
 1510 CONTINUE
C-
C--------------------------------------------------------------
C-
C-COMPUTE CONSTANTS
C-
C--------------------------------------------------------------
C-
      CW=1.0
      CI=0.55
      TKW=5.0
      TKI=18.0
      IOUT=TOUT*24/(DT*NTSTP)     +.001
      IEND=TEND*24/(DT*NTSTP)+.001
      SURC = OVER*1034./14.7
c     ^^^^^^^^^^^^^^^^^^^^^^    JGM 2/4/86  input in cm water
C-
C-
C-------------------------------------------------------------
C-
C-WRITE FIRST PAGE OF INPUT DATA
C-
C----------------------------------------------------------
C-
      WRITE(NWT,500)LEADIN
      WRITE(NWT,603)
      WRITE(NWT,629)ZN
      IF(NTDH.EQ.1 .OR. NTDM.EQ.1)WRITE(NWT,660)
      IF(NTDH.EQ.1)WRITE(NWT,662)
      IF(NTDM.EQ.1)WRITE(NWT,663)
      IF(NTDH.EQ.2 .OR. NTDM.EQ.2)WRITE(NWT,661)
      IF(NTDH.EQ.2)WRITE(NWT,662)
      IF(NTDM.EQ.2)WRITE(NWT,663)
      WRITE(NWT,604) NEL,DT,NTSTP,IEND
      WRITE(NWT,605) CW,CI,TKW,TKI,OVER,TFPD,OFAT
      WRITE(NWT,620)
```

```
      DO 1512 I=1,NNOD
      N=IPNT(I)
      WRITE(NWT,621)DEEP(I),N,HKSL(N),AKL(N),XKL(N),
     1 AWL(N),XGL(N),FHCL(N),THEOL(N), XTAY(N)

 1512 CONTINUE
        WRITE(NWT,622)
        DO 1513 I=1,NNOD
        N=IPNT(I)
        WRITE(NWT,623) DEEP(I),N,CSL(N),TKSL(N),DENSL(N),RESL(N),XMV(N)
1513    CONTINUE
      WRITE(NWT,606)
      IF(KODE1.EQ.1) GO TO 1530
      WRITE(NWT,618) (N,PX(N),TX(N),QI(N),N=1,NNOD)
      GO TO 1540
 1530 N=1
      WRITE(NWT,618) N,PX(1),TX(1),QI(1)
 1540 CONTINUE
      WRITE(NWT,590)
      WRITE(NWT,591)
C-
      IF(KODE2.EQ.1) WRITE(NWT,600)
C-
C-
C-------------------------------------------------------------------------
C-
C-SEGMENT 2-BUILD SYSTEM MATRICES
C-BEGIN K-LOOP WHICH SPANS REMAINDER OF PROGRAM.
C-
C-------------------------------------------------------------------------
C-
      K=0
      INT=0
      MMM=0
      ITIM=0
      KODEP=0
C-
C-----------------------------------------------------------------------
C-
C-INITIALIZE WATER CONTENT FIELD AS A FUNCTION OF THE PORE-PRESSURE
C-FIELD (AS GIVEN BY THE INITIAL CONDITIONS).
C-
C-----------------------------------------------------------------------
C-
      DO 1900 M=1,NNOD
      MM=IPNT(M)
      THEO=THEOL(MM)
      AW=AWL(MM)
      XG=XGL(MM)
      IF(PX(M).GE.0.)GO TO 1910
      WAT(M)=FGARD(THEO,AW,PX(M),XG)
      GO TO 1900
 1910 WAT(M)=THEO
 1900 CONTINUE
 2000 CONTINUE
      K=K+1
      MMM=MMM+1
C-
C-------------------------------------------------------------------------
C-
```

```
C-FIND TIME VARYING BOUNDARY CONDITIONS AND SET BOUNDARY COND INTO
C-STATE VARIABLE VECTORS
C-TU=UPPER TEMP,TN=A MULTIPLIER,AMPT=HALF THE AMPLITUDE OF A
C-SINE VARYING DIURNAL CYCLE,TL=LOWER TEMP,PU=UPPER PRESS,
C-PL=LOWER PRESS
C-TTT=REAL TIME HOURS IN SIMULATION
C-
C---------------------------------------------------------------------
C-
      TTT=DT*(K-1)*NTSTP
      CALL BOUNTU(TTT,TU,TN,NTU)
      TU=(TU+AMPT*SIN(TTT*.2617994))*TN
      CALL BOUNPL(TTT,PL,NPL)
      CALL BOUNTL(TTT,TL,NTL)
      IF(KPU.EQ.0)PX(1)=PU
      IF(KPL.EQ.0)PX(NNOD)=PL
      IF(KTU.EQ.0)TX(1)=TU
      IF(KTL.EQ.0)TX(NNOD)=TL
C-
C---------------------------------------------------------------------
C-
C-COMPUTED PARAMETERS:
C-PP IS THE AVERAGE PRESSURE.
C-WAT IS THE LIQUID WATER CONTENT(APPROXIMATE AVERAGE IN NODAL DOMAIN)
C-THETS IS THE GUYMON AND LUTHIN MOISTURE CONTENT FUNCTION.
C-ALHET IS NODAL DOMAIN LATENT HEAT BUDGET ARRAY
C-FZHET IS THE NODAL DOMAIN ISOTHERMAL PHASE CHANGE HEAT EVOLUTION BUDGET.
C-CA IS HEAT CAPACITY
C-TK IS THE THERMAL CONDUCTIVITY  S-W-I MIX.
C-       D(M) IS HYDRAULIC CONDUCTIVITY
C-       V(M) IS CONVECTED HEAT FLUX AT NODAL DOMAIN BOUNDARY
C-WT IS AN ARRAY OF THE OVERBURDEN INCLUDING SURCHARGE PRESSURE
C-
C-------------------------------------------------------------------
C-
C-PREPARE HEAT BUDGET ARRAYS
C-AND INCORPORATE OVERBURDEN EFFECTS BY ADJUSTING THE UNFROZEN
C-WATER CONT FACTOR WHICH WILL RESULT IN A CORRECTED PORE
C-PRESSUER AT A FREEZING FRONT(S) WHERE ICE SEG IS OCCURING
C-IF ICE SEGREGATION IS NOT OCCURING WT(M)=ZERO
C-
C--------------------------------------------------------------------
C-
      WT(1)=SURC
      IF(KPU .NE. KPPU)WT(1)=SURC*OFAT
      WTS=0.
      WTWI=0.
      DO 2200 M=2,NNOD
      MM=IPNT(M)
      WTS=WTS+DENSL(MM)*(DEEP(M)-DEEP(M-1))
      WTWI=WTWI+((WAT(M)+WAT(M-1))/2.+(QI(M)+QI(M-1))/(2.*1.09))*
     C     (DEEP(M)-DEEP(M-1))
      WT(M)=WTS+WTWI+WT(1)
      QSEG=QI(M)-THEOL(MM)+RESL(MM)
      IF(QSEG.LE.0.) WT(M)=0.
 2200 CONTINUE
      DO 2300 M=1,NNOD
      MM=IPNT(M)
      XXGL=1./XGL(MM)
      PPA=((THEOL(MM)/RESL(MM)-1.)/AWL(MM))**XXGL
```

```
      PPA=PPA-WT(M)
      IF(PPA.LE.0.) PPA=-1.
      RESID=FGARD(THEOL(MM),AWL(MM),PPA,XGL(MM))
      FZHET(M)=0.
      ALHET(M)=80.*(WAT(M)-RESID)
      IF(ALHET(M).LT.0.)ALHET(M)=0.
 2300 CONTINUE
C-
C------------------------------------------------------------------
C-
C-CALCULATE MOISTURE FLOW PARAMETERES
C-UNFROZEN HYD COND CAN BE ADJUSTED BY A CONSTANT FACTOR (FHC)
C-UNFROZEN HYD COND CORRECTED BY (EFAC) FOR PARTIALLY FROZEN SOIL
C-
C------------------------------------------------------------------
C-
      DO 2310 M=1,NEL
      MM=IPNT(M)
      FHC=FHCL(MM)
      PP=(PX(M)+PX(M+1))/2.
      XH=XKL(MM)
      HKS=HKSL(MM)
      AK=AKL(MM)
      XXTAY=XTAY(MM)
        IF (PP.GE.0) GOTO 2301
        XK=HKS/((AK*((ABS(PP)**XH)))+1)
C
C
 2302   EFAC=XXTAY*(QI(M)+QI(M+1))/2.
        IF(EFAC.GT.30) EFAC=30.
        EFAC=10**EFAC
        IF(EFAC.LT.1) EFAC=1
        D(M)=XK*FHC/EFAC
        GO TO 2310
2301       XK=HKS
      IF(QI(M).GT.0.)GO TO 2302
      D(M)=HKS*FHC
2310  CONTINUE
      DO 2320 M=1,NNOD
      MM=IPNT(M)
      THEO=THEOL(MM)
      XG=XGL(MM)
      AW=AWL(MM)
      IF(PX(M).GE.0)GO TO 2311
      THETS(M)=FSTAR(THEO,AW,PX(M),XG)
      GO TO 2320
 2311 THETS(M)=0.
 2320 CONTINUE
      IF(KPU .EQ. KPPU)GO TO 2326
      DO 2325 M=1,NNOD
      IF(THETS(M).GT.0. .OR. THETS(M).LT.0.)GOTO 2326
      MM=IPNT(M)
      THETS(M)=XMV(MM)
 2325 CONTINUE
 2326 CONTINUE
C-
C------------------------------------------------------------
C-
C-CALCULATE HEAT FLOW PARAMETERS
C-
```

```
C-----------------------------------------------------------
C-
      DO 2330 M=1,NEL
      MM=IPNT(M)
      THEO1=1.-THEOL(MM)
      TKS=TKSL(MM)
      TK(M)=(THEO1*TKS+(QI(M+1)+QI(M))/2.*TKI+
     1(WAT(M)+WAT(M+1))/2.*TKW)
 2330 CONTINUE
      DO 2340 M=1,NNOD
      MM=IPNT(M)
      THEO1=1.-THEOL(MM)
      CS=CSL(MM)
      SG=DENSL(MM)/THEO1
      CA(M)=CW*WAT(M)+.917*CI*QI(M)+SG*CS*THEO1
C-USE APPARENT HEAT CAPACITY IN FREEZING ELEMENT
C     IF(QI(M).GT.0.) CA(M)=CA(M) +80.
 2340 CONTINUE
      DO 2350 M=1,NEL
      V(M)=CW*D(M)*((PX(M)-PX(M+1))/DELX(M)+1.)
      IF(KODE5.NE.1)V(M)=0.
 2350 CONTINUE
C-
C-IF V(M) TOO LARGE SET CONVECTIVE TERM TO ZERO AND
C-IDENTIFY FREQUENCY IN OUTPUT
C-PECL=PECLET NO.
C-
      IF(KODE5.NE.1) GO TO 2353
      PMAX=0.
      DO 2351 M=1,NEL
      PECL=V(M)*DELX(M)/TK(M)
      IF(PECL.GT.PMAX) PMAX=PECL
 2351 CONTINUE
      IF(PMAX.LT.1.)GO TO 2353
      KODEP=KODEP+1
      DO 2352 M=1,NEL
 2352 V(M)=0.
 2353 CONTINUE
C-
C-----------------------------------------------------------
C-
C-APPROXIMATE FROST HEAVE EFFECTS
C-BY VARYING TRANSPORT PARAMETERS
C-(DELL=ELEMENT DISTORTION FACTOR)
C-
C-----------------------------------------------------------
C-
      DO 2360 M=1,NEL
      MM=IPNT(M)
      THEO=THEOL(MM)
      DELL=QI(M)+WAT(M)-THEO
      IF(DELL.LE.0)GO TO 2360
      DELL=DELL+1.
      TK(M)=TK(M)/DELL
      D(M)=D(M)/DELL
      THETS(M)=THETS(M)*DELL
      CA(M)=CA(M)*DELL
      V(M)=V(M)/DELL
 2360 CONTINUE
C-
```

```
C--------------------------------------------------------------
C-
C-ADVANCE SOIL-WATER FLOW ENERGY-HEAD FIELD THRU TIME INCREMENT
C-NTSTP
C-
C-CONVERT PORE-PRESSURE FIELD TO ENERGY-HEAD FIELD
C-
C-------------------------------------------------------------
C-
      DO 2400 M=1,NNOD
 2400 PX(M)=PX(M)-DEEP(M)
      DO 2425 M=1,NNOD
      R(M)=0.
      DO 2425 J=1,3
      S(M,J)=0.
 2425 P(M,J)=0.
C-
C-------------------------------------------------------------
C-
C-ACCOMODATE INTERIOR NODAL DOMAINS
C-
C-------------------------------------------------------------
C-
       DO 2450 M=2,NEL
       XEL=(DELX(M)+DELX(M-1))/2.*THETS(M)
       S(M,1)=-D(M-1)/DELX(M-1)
       S(M,2)=D(M)/DELX(M)-S(M,1)
       S(M,3)=-D(M)/DELX(M)
       P(M,1)=XEL*ZN2/2.
       P(M,2)=ZN1*XEL
       P(M,3)=P(M,1)
 2450  CONTINUE
C-
C-------------------------------------------------------------
C-
C-ACCOMODATE BOUNDARY ELEMENTS
C-
C-------------------------------------------------------------
C-
       XEL=DELX(1)/2.*THETS(1)
       S(1,1)=0.
       S(1,2)=D(1)/DELX(1)
       S(1,3)=-S(1,2)
       P(1,1)=0.
       P(1,2)=ZN1*XEL/2.
       P(1,3)=XEL*ZN2/2.
       XEL=DELX(NEL)/2.*THETS(NNOD)
       S(NNOD,1)=-D(NEL)/DELX(NEL)
       S(NNOD,2)=-S(NNOD,1)
       S(NNOD,3)=0.
       P(NNOD,1)=XEL*ZN2/2.
       P(NNOD,2)=ZN1*XEL/2.
       P(NNOD,3)=0.
C-
C------------------------------------------------------------
C-
C-TIME DOMAIN ADVANCEMENT:
C-NTDM = 1 INDICATES FULLY IMPLICIT SCHEME
C-     = 2 INDICATES CRANK-NICOLSON SCHEME
C-
```

```
C---------------------------------------------------------------
C-
      EPS=1.0
      IF(NTDM.EQ.2)EPS=0.5
      DO 9000 I=1,NNOD
      DO 9000 J=1,3
      P(I,J)=P(I,J)/DT-(1.-EPS)*S(I,J)
      S(I,J)=S(I,J)+P(I,J)
9000  CONTINUE
C-
C---------------------------------------------------------------
C-
C-INSERT ENERGY-HEAD BOUNDARY CONDITIONS
C-
C---------------------------------------------------------------
C-
       IF(KPU.EQ.1)GO TO 2600
       R(1)=0.
       R(2)=R(2)-PU*S(2,1)+PU*P(2,1)
       S(1,2)=1.
       S(1,3)=0.
       S(2,1)=0.
       P(1,2)=1.
       P(1,3)=0.
       P(2,1)=0.
 2600  IF(KPL.EQ.1)GO TO 2700
       PL1=PL-DEEP(NNOD)
       R(NNOD)=0.
       R(NEL)=R(NEL)-PL1*S(NEL,3)+PL1*P(NEL,3)
       S(NNOD,2)=1.
       S(NNOD,1)=0.
       S(NEL,3)=0.
       P(NNOD,2)=1.
       P(NNOD,1)=0.
       P(NEL,3)=0.
 2700  CONTINUE
C-
C---------------------------------------------------------------
C-
C-ADVANCE ENERGY-HEAD FIELD THROUGH NTSTP TIME INCREMENT
C-
C---------------------------------------------------------------
C-
       MU=0
       CALL FPRESO(S,SP,NNOD,3)
 2800  CONTINUE
       CALL FCOMB(P,PX,GP,NNOD,3)
       DO 2900 N=1,NNOD
       IF(MU.EQ.0)GP(N)=GP(N)+R(N)+EXW(N)
       IF(MU.NE.0)GP(N)=GP(N)+R(N)
C       GP(N)=GP(N)+R(N)
2900   CONTINUE
       CALL FFINSO(S,GP,SP,NNOD,3)
       MU=MU+1
       IF(MU.GE.NTSTP)GO TO 2950
       DO 2925 N=1,NNOD
 2925  PX(N)=GP(N)
       GO TO 2800
 2950  CONTINUE
C-
```

```
C------------------------------------------------------------
C-
C-RETURN ENERGY-HEAD FIELD TO PORE-PRESSURE FIELD
C-
C------------------------------------------------------------
C-
      DO 2975 N=1,NNOD
 2975 PX(N)=GP(N)+DEEP(N)
C-
C--------------------------------------------------------------
C-
C-ADVANCE TEMPERATURE FIELD THRU TIME INCREMENT NTSTP
C-
C--------------------------------------------------------------
C-
      DO 3000 M=1,NNOD
      R(M)=0.
      DO 3000 J=1,3
      S(M,J)=0.
 3000 P(M,J)=0.
C-
C-------------------------------------------------------------
C-
C-ACCOMODATE INTERIOR NODAL DOMAINS
C-
C-------------------------------------------------------------
C-
      DO 3050 M=2,NEL
      XEL=(DELX(M)+DELX(M-1))/2.*CA(M)
      S(M,1)=-TK(M-1)/DELX(M-1)-V(M-1)/2.
      S(M,2)=TK(M)/DELX(M)+TK(M-1)/DELX(M-1)
      S(M,3)=-TK(M)/DELX(M)+V(M)/2.
      P(M,1)=XEL*ZN2/2.
      P(M,2)=ZN1*XEL
      P(M,3)=P(M,1)
 3050 CONTINUE
C-
C--------------------------------------------------------------
C-
C-ACCOMODATE BOUNDARY ELEMENTS
C-
C--------------------------------------------------------------
C-
      XEL=DELX(1)/2.*CA(1)
      S(1,1)=0.
      S(1,2)=TK(1)/DELX(1)
      S(1,3)=-TK(1)/DELX(1)+V(1)/2.
      P(1,1)=0.
      P(1,2)=ZN1*XEL/2.
      P(1,3)=XEL*ZN2/2.
      XEL=DELX(NEL)/2.*CA(NNOD)
      S(NNOD,1)=-TK(NEL)/DELX(NEL)-V(NEL)/2.
      S(NNOD,2)=TK(NEL)/DELX(NEL)
      S(NNOD,3)=0.
      P(NNOD,1)=XEL*ZN2/2.
      P(NNOD,2)=ZN1*XEL/2.
      P(NNOD,3)=0.
C-
C--------------------------------------------------------------
C-
```

```
C-TIME DOMAIN ADVANCEMENT:
C-NTDH = 1 INDICATES FULLY IMPLICIT SCHEME
C-       2 INDICATES CRANK-NICOLSON SCHEME
C-
C-------------------------------------------------------------------
C-
      EPS=1.0
      IF(NTDH.EQ.2)EPS=0.5
      DO 9100 I=1,NNOD
      DO 9100 J=1,3
      P(I,J)=P(I,J)/DT-(1.-EPS)*S(I,J)
      S(I,J)=S(I,J)+P(I,J)
9100  CONTINUE
C-
C-------------------------------------------------------------------
C-
C-INSERT THERMAL BOUNDARY CONDITIONS
C-
C-------------------------------------------------------------------
C-
       IF(KTU.EQ.1)GO TO 3100
       R(1)=0.
       R(2)=R(2)-TU*S(2,1)+TU*P(2,1)
       S(1,2)=1.
       S(1,3)=0.
       S(2,1)=0.
       P(1,2)=1.
       P(1,3)=0.
       P(2,1)=0.
 3100  IF(KTL.EQ.1)GO TO 3150
       R(NNOD)=0.
       R(NEL)=R(NEL)-TL*S(NEL,3)+TL*P(NEL,3)
       S(NNOD,2)=1.
       S(NNOD,1)=0.
       S(NEL,3)=0.
       P(NNOD,2)=1.
       P(NNOD,1)=0.
       P(NEL,3)=0.
 3150  CONTINUE
C-
C-------------------------------------------------------------------
C-
C-ADVANCE TEMPERATURE FIELD THROUGH NTSTP TIME-INCREMENT
C-
C-------------------------------------------------------------------
C-
       MU=0
       CALL FPRESO(S,SP,NNOD,3)
 3180  CONTINUE
       CALL FCOMB(P,TX,GT,NNOD,3)
       DO 3190 N=1,NNOD
 3190  GT(N)=GT(N)+R(N)
       CALL FFINSO(S,GT,SP,NNOD,3)
       MU=MU+1
       IF(MU.GE.NTSTP)GO TO 3200
       DO 3210 N=1,NNOD
 3210  TX(N)=GT(N)
       GO TO 3180
 3200  CONTINUE
C-
```

```
C-----------------------------------------------------------------------
C-
C-ISOTHERMAL PHASE-CHANGE APPROXIMATION
C-
C-ADJUST TX(1) TO APPROXIMATE MEAN TEMPERATURE IN BOUNDARY NODAL DOMAIN.
C-
C-----------------------------------------------------------------------
C-
      GT(1)=0.75*GT(1)+0.25*GT(2)
C-
C------------------------------------------------------------------
C-
C-NODAL DOMAIN ISOTHERMAL PHASE CHANGE APPROXIMATION
C-
C------------------------------------------------------------------
C-
      DO 3900 M=1,NNOD
      MM=IPNT(M)
      THEO1 = 1-THEOL(MM)
      TEMP=GT(M)
      IF (TEMP-TFPD) 3300,3500,3500
 3300 CONTINUE
C-
C------------------------------------------------------------------
C-
C-SOIL-WATER FREEZING
C-
C------------------------------------------------------------------
C-
      KPU=KPPU
      IF(ALHET(M).LE.0.)GO TO 3700
      FZHET(M)=CA(M)*(TFPD-TEMP)
      IF(FZHET(M).GT.ALHET(M))GO TO 3400
      TX(M)=TFPD
      GO TO 3800
 3400 TX(M)=TFPD-(FZHET(M)-ALHET(M))/CA(M)
      FZHET(M)=ALHET(M)
      GO TO 3800
 3500   CONTINUE
C-
C------------------------------------------------------------------
C-
C-SOIL-ICE THAWING
C-
C------------------------------------------------------------------
C-
      IF(QI(M).LE.0.)GO TO 3700
      FZHET(M)=CA(M)*(TEMP-TFPD)
      HEAT=QI(M)*73.4
      IF(FZHET(M).GT.HEAT)GO TO 3600
      TX(M)=TFPD
      FZHET(M)=- FZHET(M)
      GO TO 3800
 3600 TX(M)=TFPD+(FZHET(M)-HEAT)/CA(M)
      FZHET(M)=-HEAT
      GO TO 3800
 3700 TX(M)=GT(M)
 3800 CONTINUE
 3900 CONTINUE
C-
```

```
C-------------------------------------------------------------
C-
C-MODIFY PORE-PRESSURE FIELD FOR ICE-SINK
C-(ISOTHERMAL) APPROXIMATION
C-
C-ADJUST PX(1) TO APPROXIMATE MEAN PORE-PRESSURE IN
C-BOUNDARY NODAL DOMAIN
C-
C-------------------------------------------------------------
C-
      PX(1)=0.75*PX(1)+0.25*PX(2)
C-
C--------------------------------------------------------------
C-
C-CALCULATE SOIL-WATER SOURCE TERM AND ADJUST PORE-PRESSURE FIELD
C-
C--------------------------------------------------------------
C-
      DO 3950 M=1,NNOD
      EXW(M)=0.
      MM=IPNT(M)
      THEO=THEOL(MM)
      AW=AWL(MM)
      XG=XGL(MM)
      XXG=1./XG
      IF(PX(M).GE.0.)GO TO 3910
C-
C--------------------------------------------------------------
C-
C-UNSATURATED NODAL DOMAIN PRIOR TO ICE-SINK TERM
C-
C--------------------------------------------------------------
C-
        WATX=FGARD(THEO,AW,PX(M),XG)
      GO TO 3920
C-
C--------------------------------------------------------------
C-
C-SATURATED NODAL DOMAIN PRIOR TO ICE-SINK TERM
C-
C--------------------------------------------------------------
C-
 3910 WATX=THEO
 3920 WATX=WATX-FZHET(M)/80.
      IF(WATX.GE.THEO)GO TO 3930
      IF (WATX.LE.0.)WATX=.005
C-
C--------------------------------------------------------------
C-
C-NODAL DOMAIN BECOMES UNSATURATED
C-
C--------------------------------------------------------------
C-
      PPA=((THEO/WATX-1.)/AW)**XXG
      PX(M)=-PPA
      WAT(M)=WATX
      GO TO 3950
C-
C--------------------------------------------------------------
C-
```

```
C-NODAL DOMAIN BECOMES SATURATED:
C-ASSUME SYSTEM RETURNS TO THAWED
C-CONDITION AS APPROXIMATION.
C-ASSUME MOISTURE IN EXCESS OF NON-DEFORMED SOIL POROSITY
C-TO FLOW AWAY FROM SYSTEM, SUCH AS ALONG AN INCLINED GROUND
C-SURFACE.
C-
C------------------------------------------------------------
C-
 3930 CONTINUE
      IF(PX(M).LT.0.)PX(M)=0.
      IF ( M .EQ. 1 ) THEN
      EXW(1)=0.
      KPU=0
      PU=0.
      ELSE
      EXW(M)=.5*(DELX(M)+DELX(M-1))*(WATX-THEO)
      END IF
      WAT(M)=THEO
 3950 CONTINUE
C-
C------------------------------------------------------------
C-
C-RETURN PX(1) AND TX(1) TO BOUNDARY CONDITION VALUE
C-
C------------------------------------------------------------
C-
      PX(1)=(4.*PX(1)-PX(2))/3.
      IF(PX(1).GT.0.)PX(1)=0.
      TX(1)=(4.*TX(1)-TX(2))/3.
      IF(TX(1).GT.TU)TX(1)=TU
C-
C------------------------------------------------------------
C-
C- UPDATE ICE-CONTENT FIELD, QI(M)
C-
C------------------------------------------------------------
C-
      KICE=0
      DO 4200 M=1,NNOD
      QI(M)=QI(M)+FZHET(M)/73.4
      IF(QI(M).LT.0.)QI(M)=0.
      IF((QI(M).GT.-0.000001).AND.(QI(M).LT.0.000001)) KICE=KICE+1
 4200 CONTINUE
      IF(KICE .EQ. NNOD)KPU=KPPU
C-
C------------------------------------------------------
C-
C-FROST-HEAVE APPROXIMATION
C-MODIFIED BULK POROSITIES AND DENSITIES ARE COMPUTED AND STORED
C-IN POROST AND DENSIT (THESE VARIABLES ARE NOT USED IN THIS
C-VERSION)
C-
C------------------------------------------------------
C-
      HEAVE=0.
      THEOIC=THEOL(1)-RESL(1)
        IF(QI(1).LE.THEOIC)GO TO 4225
      HEAVE=(QI(1)-THEOIC)*DELX(1)/2.
        FACTOR=DELX(1)/(DELX(1)+HEAVE)
```

```
      POROST(1)=(QI(1)+RESL(1))*FACTOR
      DENSIT(1)=DENSL(1)*FACTOR
      GO TO 4226
4225  CONTINUE
      POROST(1)=THEOL(1)
      DENSIT(1)=DENSL(1)
4226  CONTINUE
      DO 4250 N=2,NNOD
      MM=IPNT(N)
      THEOIC=THEOL(MM)-RESL(MM)
      IF(QI(N).LE.THEOIC)GO TO 4251
      DELH=(QI(N)-THEOIC)*(DELX(N-1)+DELX(N))/2.
        FACTOR=DELX(N)/(DELX(N)+DELH)
      HEAVE=HEAVE+DELH
      POROST(N)=(QI(N)+RESL(MM)  )*FACTOR
      DENSIT(N)=DENSL(MM)*FACTOR
      GO TO 4250
4251  CONTINUE
      POROST(N)=THEOL(MM)
      DENSIT(N)=DENSL(MM)
 4250 CONTINUE
C-
C----------------------------------------------------------------
C-
C-SEGMENT 5-OUTPUT
C-
C-XFROST IS THE FROST PENETRATION DEPTH(APPROX)
C-FDPTH=XFROST
C-FIS IS THE ICE SEGREGATION RATIO (ISR)
C-DHV IS THE HEAVE RATE
C-
C----------------------------------------------------------------
C-
      IF(MMM.LT.IOUT)GO TO 2000
      MMM=0
      TTT=DT*K*NTSTP
      TTDAY=TTT/24.
      ITIM=ITIM+1
      TDAY(ITIM)=TTDAY
      HV(ITIM)=HEAVE
      CALL FROSTP(NNOD,XFROST)
      CALL THAWP(NNOD,XTHAW)
      TDPTH(ITIM)=XTHAW
      FDPTH(ITIM)=XFROST
      DENOM=FDPTH(ITIM)+HV(ITIM)
      IF(DENOM.GT.-0.0001 .AND. DENOM.LT.0.0001) GO TO 4400
      FIS(ITIM)=HV(ITIM)/DENOM
      GO TO 4450
 4400 FIS(ITIM)=0.
 4450 CONTINUE
C-
C-STRESS ANALYSIS
C-WT IS THE TOTAL WEIGHT OF THE COLUMN ABOVE NODE
C-WT IS NOT USED IN THIS VERSION EXCEPT TO CORRECT PORE PRESSURES
C-IF THE ABOVE SOIL IS SATURATED
C-
      IF(KPU .EQ. KPPU)GO TO 4490
      WTS=0.
      WTWI=0.
      WT(1)=SURC*OFAT
```

```
      DO 4475 M=2,NNOD
      MM=IPNT(M)
      WTS=WTS+DENSL(MM)*(DEEP(M)-DEEP(M-1))*(1.-THEOL(MM))
      WTWI=WTWI+((WAT(M)+WAT(M-1))/2.+(QI(M)+QI(M-1))/(2.*1.09))*
     1     (DEEP(M)-DEEP(M-1))
      WT(M)=WTS+WTWI+WT(1)
      IF(WT(M) .GE. PX(M))GO TO 4475
      PX(M)=WT(M)
4475  CONTINUE
4490  CONTINUE
C-
      WRITE(NWT,608) TTT,TTDAY,HEAVE
      WRITE(NWT,619)
      WRITE(NWT,625) TDAY(ITIM),PX(1),TU,PL,TL
      WRITE(NWT,610)
      WRITE(NWT,609)
      WRITE(NWT,607) (N,DEEP(N),PX(N),TX(N),
     1                WAT(N),QI(N),DENSIT(N),POROST(N),N=1,NNOD)
      IF(KODE6.NE.1)GO TO 5550
      WRITE(NWT,617)
      WRITE(NWT,611) (I,D(I),TK(I),THETS(I),CA(I),V(I),I=1,NEL)
      GO TO 5550
 5500 CONTINUE
      IF(INT.EQ.0) WRITE(NWT,619)
      IF(KODE2.EQ.1) WRITE(NWT,626) TDAY(ITIM),PX(1),TU,PL,TL
      INT=1
 5550 CONTINUE
      IF(K.LT.IEND)GO TO 2000
      DHV(1)=HV(1)/(TDAY(1)*24.)
      DO 5600 I=2,ITIM
      DHV(I)=(HV(I-1)-HV(I))/((TDAY(I-1)-TDAY(I))*24)
 5600 CONTINUE
C-
C-
C-DETERMINE MAXIMUM COEF OF VARIATION OF HEAVE ASSUMIN
C-A BETA DIST WHERE ALPHA=3.5 AND BETA=5.0
C-ASSUME THE LOWER BETA DIST BOUND (ABET)=MEAN - 3*STANDARD DEV
C-
C-CVK IS THE COEF OF VARIATION OF HYD COND
C-CVY IS THE COEF OF VARIATION OF HEAVE
C-
C-
      CVPP=CVK
      CVK=CVK*3.
      CVY=0.
      DO 5700 I=1,ITIM
      ABET=HV(I)*(1.-CVK)
      BMA=(HV(I)-ABET)*2.33
      IF(HV(I).LE.0.) GO TO 5690
      CVYI=(BMA*.15)/HV(I)
 5690 CONTINUE
      IF(CVYI.GT.CVY) CVY=CVYI
 5700 CONTINUE
      WRITE(NWT,649)
      WRITE(NWT,650)
      WRITE(NWT,651)
      WRITE(NWT,652)
      DO 5710 I=1,ITIM
      HMIN=HV(I)*(1.-2.*CVY)
      IF(HMIN.LT.0.) HMIN=0.
```

```
      HMAX=HV(I)*(1.+2.*CVY)
      WRITE(NWT,653) TDAY(I),HMIN,HMAX,HV(I),DHV(I),FIS(I),
     1FDPTH(I),TDPTH(I)
 5710 CONTINUE
      WRITE(NWT,654) CVY,CVPP
      IF(KODEP.GT.0) WRITE(NWT,655) KODEP
      WRITE(NWT,656) AMPT
C-
C-------------------------------------------------------------------
C-
C-CLOSE FILES AND EXIT PROGRAM
C-SEE COMMENT UNDER OPEN FILES AT FRONT OF PROG
C-
C-------------------------------------------------------------------
C-
        CLOSE(UNIT=5)
        CLOSE(UNIT=6)
        STOP
        END
C-
C----------------------------------------------------------------
C----------------------------------------------------------------
C-
C-THIS SUBROUTINE TRIANGULARIZES A NON-SYMMETRIC MATRIX
C-W(NROW,NCOL) IN BAND FORM FOR SOLUTION BY THE GAUSSIAN
C-ELIMINATION METHOD.  FINAL SOLUTION IS BY FINSOL.
C-
C----------------------------------------------------------------
C-
      SUBROUTINE FPRESO(W,ST,NROW,NCOL)
C-
      REAL*8 W(102,3),ST(102,3)
      ICOL2=(NCOL/2)
      IF(ICOL2+1.EQ.2) GO TO 300
      DO 200 I=2,ICOL2
      JJ=ICOL2
      JJJ=JJ-I+2
      DO 200 J=JJJ,ICOL2
      I1=I+J-ICOL2-1
      I2=ICOL2+1
      ST(I,J)=W(I,J)/W(I1,I2)
      W(I,J)=0.
      DO 100 K=1,ICOL2
      I3=J+K
      I4=I3+JJ+I-I2
      W(I,I3)=W(I,I3)-W(I1,I4)*ST(I,J)
  100 CONTINUE
      JJ=JJ-1
  200 CONTINUE
  300 CONTINUE
      I2=ICOL2+1
      DO 500 I=I2,NROW
      JJ=ICOL2
      DO 500 J=1,ICOL2
      I5=I-JJ
      ST(I,J)=W(I,J)/W(I5,I2)
      W(I,J)=0.
      DO 400 K=1,ICOL2
      I3=J+K
      I6=I3+JJ
```

```
      W(I,I3)=W(I,I3)-W(I5,I6)*ST(I,J)
  400 CONTINUE
      JJ=JJ-1
  500 CONTINUE
      RETURN
      END
C-
C-
C-----------------------------------------------------------------
C-----------------------------------------------------------------
C-
C-THIS SUBROUTINE MULTIPLIES THE NON-SYMMETRIC MATRIX S(NROW,NCOL)
C-TIMES THE VECTOR Y(NROW) AND STORES THE RESULT IN Z(NROW).
C-
C-----------------------------------------------------------------
C-
      SUBROUTINE FCOMB(S,Y,Z,NROW,NCOL)
C-
      REAL*8 S(102,3),Y(102),Z(102)
      ICOL2=(NCOL/2)
      NCON=ICOL2
      DO 100 I=1,ICOL2
      II=NROW-I+1
      Z(I)=0.
      Z(II)=0.
      NCON=NCON+1
      DO 100 J=1,NCON
      I1=ICOL2-I+J+1
      JJ=NCON-J+1
      IJ=NROW-J+1
      Z(I)=Z(I)+S(I,I1)*Y(J)
      Z(II)=Z(II)+S(II,JJ)*Y(IJ)
  100 CONTINUE
      N1=ICOL2+1
      N2=NROW-ICOL2
      DO 200 I=N1,N2
      Z(I)=0.
      DO 200 J=1,NCOL
      K=I+J-N1
      Z(I)=Z(I)+S(I,J)*Y(K)
  200 CONTINUE
      RETURN
      END
C-
C-----------------------------------------------------------------
C-----------------------------------------------------------------
C-
C-THIS SUBROUTINE SOLVES A SET OF LINEAR SIMULTANEOUS EQUATIONS
C-WHOSE COEFFICIENT MATRIX, W(NROW,NCOL), HAS BEEN PRE-TRIANGU-
C-LARIZED BY THE GAUSSIAN ELIMINATION METHOD.  THE SYSTEM MATRIX
C-IS IN BAND FORM AND THE SOLUTION IS PLACED IN THE LOAD VECTOR
C-SS(NROW).  ST(NROW,NCOL) IS USED TO REDUCE THE ORIGINAL LOAD
C-VECTOR TO THE LOAD VECTOR OF THE TRIANGULARIZED SET OF
C-SIMULTANEOUS EQUATIONS.
C-
C-----------------------------------------------------------------
C-
      SUBROUTINE FFINSO(W,SS,ST,NROW,NCOL)
C-
      REAL*8 W(102,3),SS(102),ST(102,3)
```

```
C-
C----------------------------------------------------
C-
C-REDUCE THE LOAD VECTOR
C-
C----------------------------------------------------
C-
      N=1
      ICOL2=(NCOL/2)
      DO 200 I=2,NROW
      N=N-ICOL2+1
      DO 200 K=1,ICOL2
      IF(N.LE.0) GO TO 100
      SS(I)=SS(I)-SS(N)*ST(I,K)
  100 CONTINUE
      N=N+1
  200 CONTINUE
C-
C-------------------------------------------------------------
C-
C-   NORMALIZE THE LOAD VECTOR WITH RESPECT TO THE MAIN DIAGONAL
C-   CALCULATE S(NROW)
C-
C-------------------------------------------------------------
C-
      ICON=ICOL2+1
      ICON1=ICON+1
      DO 300 I=1,NROW
      SS(I)=SS(I)/W(I,ICON)
  300 CONTINUE
C-
C-------------------------------------------------------------
C-
C-   BACK SUBSTITUTION
C-
C-------------------------------------------------------------
C-
      DO 400 I=2,NROW
      J=NROW-I+1
      DO 400 K=2,ICON
      L=J+K-1
      IF(L.GT.NROW) GO TO 400
      KK=ICOL2+K
      SS(J)=SS(J)-W(J,KK)*SS(L)/W(J,ICON)
  400 CONTINUE
      RETURN
      END
C-
C--------------------------------------------------------
C--------------- ----------------------------------------
C-
C-THIS SUBROUTINE FINDS THE TU AND TN BOUNDARY CONDITIONS
C-FOR T TIME ASSUMMING DATA IS SERIES OF STEP FUNCTIONS
C-
C--------------------------------------------------------
C-
      SUBROUTINE BOUNTU(T,TU,TN,NT)
C-
      COMMON/BLK2/HRTU(300),TUB(300),TUN(300)
      IF(T.GT.-0.000001 .AND. T.LT.0.000001) GO TO 2000
```

```
      DO 1000 I=1,NT
C     DO 1000 I=2,NT
      IF(T.GE.HRTU(I)) GO TO 1000
      TU=TUB(I)
C     TU=TUB(I-1)+(TUB(I)-TUB(I-1))*(T-HRTU(I-1))/(HRTU(I)-HRTU(I-1))
      TN=TUN(I)
      GO TO 3000
 1000 CONTINUE
 2000 TU=TUB(1)
      TN=TUN(1)
 3000 RETURN
      END
C-
C-------------------------------------------------------------
C-------------------------------------------------------------
C-
C-THIS SUBROUTINE FINDS THE PL BOUNDARY CONDITION AT
C-TIME T BY A LINEAR INTERPOLATION METHOD
C-
C-------------------------------------------------------------
C-
      SUBROUTINE BOUNPL(T,PL,NT)
C-
      COMMON/BLK3/HRPL(300),PLB(300)
      IF(T.GT.-0.000001 .AND. T.LT.0.000001) GO TO 2000
      DO 1000 I=2,NT
      IF(T.GT.HRPL(I)) GO TO 1000
      PL=PLB(I-1)+(PLB(I)-PLB(I-1))*(T-HRPL(I-1))/
     1(HRPL(I)-HRPL(I-1))
      GO TO 3000
 1000 CONTINUE
 2000 PL=PLB(1)
 3000 RETURN
      END
C-
C--------------------------------------------------------------
C--------------------------------------------------------------
C-
C-THIS SUBROUTINE FINDS THE TL BOUNDARY CONDITION
C-AT TIME T USING LINEAR INTERPOLATION
C-
C--------------------------------------------------------------
C-
      SUBROUTINE BOUNTL(T,TL,NT)
C-
      COMMON/BLK4/HRTL(300),TLB(300)
      IF(T.GT.-0.000001 .AND. T.LT.0.000001) GO TO 2000
      DO 1000 I=2,NT
      IF(T.GT.HRTL(I)) GO TO 1000
      TL=TLB(I-1)+(TLB(I)-TLB(I-1))*(T-HRTL(I-1))/
     1(HRTL(I)-HRTL(I-1))
      GO TO 3000
 1000 CONTINUE
 2000 TL=TLB(1)
 3000 RETURN
      END
C-
C-
C-------------------------------------------------------------
C-------------------------------------------------------------
```

```
C-
C-THIS SUBROUTINE DETERMINES THE FROST PENETRATION DEPTH
C-
C-NNOD IS THE NUMBER OF NODES
C-FRSDEP IS THE FROST DEPTH
C-DEEP IS THE DEPTH
C-QI IS THE ICE CONTENT
C-
C-
C-------------------------------------------------------------
C-
      SUBROUTINE FROSTP(NNOD,FRSDEP)
C-
      COMMON/BLK5/DEEP(102)
      COMMON/BLK10/QI(102)
C-
      DO 1000 I=1,NNOD
      N=NNOD-I+1
      IF(QI(N).GT.0.005) GO TO 2000
 1000 CONTINUE
      FRSDEP=0
      GO TO 5000
 2000 NX=N+1
      XL=DEEP(NX)-DEEP(N)
      FRSDEP=DEEP(N)+QI(NX)*XL
 5000 CONTINUE
      RETURN
      END
C-
C-
C----------------------------------------------------------------
C----------------------------------------------------------------
C-
C-THIS SUBROUTINE FINDS THE THAW PENETRATION DEPTH
C-
C----------------------------------------------------------------
C-
      SUBROUTINE THAWP(NNOD,THWDEP)
C-
      COMMON/BLK5/DEEP(102)
      COMMON/BLK10/QI(102)
C-
      THWDEP=0.
      DO 1000 I=1,NNOD
      IF(QI(I).LT..005) GO TO 1000
      GO TO 2000
 1000 CONTINUE
      THWDEP=9999.
      GO TO 3000
 2000 IF(I.EQ.1) GO TO 3000
      XL=DEEP(I)-DEEP(I-1)
      THWDEP=DEEP(I)-XL*QI(I)
 3000 CONTINUE
      RETURN
      END
```

APPENDIX E: EXAMPLE FROST FILES

Input file

```
 ********FBKSNEW 10SEP86 **************
  1000.000     1     2
  1     1     1     0     1     0     0
 46     4
  1     0     0     0
     1.0000
 1.0000      1          1.000       10.000
  5.0000000       .0000000     1.0000000
.93028E-03     1.0712000      .4250000
.92028E-03     1.0612000      .4000000
.93528E-03     1.0812000      .3850000
.93528E-03     1.0662000      .3900000
    .1000000    18.0000000     1.0000000     1.5500000      .1500000
    .0990000    17.0000000     1.0700000     1.5700000      .1600000
    .1000000    19.0000000     1.0500000     1.6000000      .1700000
    .0990000    16.0000000     1.0900000     1.5200000      .1400000
    .0417000   .37975E-03     2.0080000     3.0e-15
    .0418000   .37978E-03     2.0050000     3.1e-15
    .0427000   .38975E-03     2.0040000     3.0e-15
    .0407000   .35975E-03     2.0090000     3.1e-15
 11     1
 22     2
 34     3
 46     4
     .6000
 -50.0000000     5.8000000      .0000000
   -300.0000
 14     2     2     .0000
  6.0000000       .0000000     1.0000000
  4.0000000     24.0000000     1.0000000
  2.0000000     48.0000000     1.0000000
  1.0000000     72.0000000     1.0000000
   .0000000     96.0000000     1.0000000
 -1.2000000    120.0000000     1.0000000
 -2.9000000    192.0000000     1.0000000
 -3.5000000    240.0000000     1.0000000
 -3.9000000    288.0000000     1.0000000
 -5.0000000    336.0000000     1.0000000
 -6.0000000    384.0000000     1.0000000
 -5.5000000    432.0000000     1.0000000
 -5.5000000    480.0000000     1.0000000
 -5.5000000    816.0000000     1.0000000
      .000          .000
      .000       816.000
     8.000          .000
     8.000       816.000
```

Output file

********FBKSNEW 10SEP86 **************

0 *** UNITS ARE CAL-CM-GM-HR-DEG C***

NODAL DOMAIN INTEGRATION MATRIX VARIABLE = 1000.000

FULLY IMPLICIT SCHEME FOR TIME DOMAIN APPROXIMATION IN:
MOISTURE TRANSPORT MODEL.

CRANK-NICOLSON SCHEME FOR TIME DOMAIN APPROXIMATION IN:
HEAT TRANSPORT MODEL.

NUMBER OF ELEMENTS	45
TIME INCREMENT	1.000
UPDATE FREQUENCY	1
TOTAL NUMBER OF UPDATES IN THE SIMULATION	240

CONSTANT PARAMETERS

HEAT CAPACITIES:	CW	1.000
	CI	.550
THERMAL CONDUCTIVITIES:	TKW	5.000
	TKI	18.000
OVERBURDEN (PSI)		5.000
TFPD (FREEZING POINT DEPRESSION)		.000
OFAT (PORE PRES MODIFIER FOR THAW)		1.000

CONSTANT SOIL HYDRAULIC PARAMETERS

DEPTH	LAYER	KSAT	AK	B	AW	A	FHC	THEO	E
.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
1.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
2.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
3.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
4.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
5.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
6.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
7.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
8.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
9.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
10.00	1	.042	.380E-03	2.008	.930E-03	1.071	1.000	.425	16.939
11.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
12.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
13.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
14.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
15.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
16.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
17.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
18.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
19.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
20.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
21.00	2	.042	.380E-03	2.005	.920E-03	1.061	1.070	.400	16.939
22.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
23.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
24.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
25.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932

26.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
27.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
28.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
29.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
30.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
31.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
32.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
33.00	3	.043	.390E-03	2.004	.935E-03	1.081	1.050	.385	16.932
34.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
35.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
36.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
37.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
38.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
39.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
40.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
41.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
42.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
43.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
44.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947
45.00	4	.041	.360E-03	2.009	.935E-03	1.066	1.090	.390	16.947

CONSTANT SOIL THERMAL PARAMETERS

DEPTH	LAYER	CS	TKS	DENS	RES	MV
.00	1	.100	18.000	1.550	.150	3.000E-15
1.00	1	.100	18.000	1.550	.150	3.000E-15
2.00	1	.100	18.000	1.550	.150	3.000E-15
3.00	1	.100	18.000	1.550	.150	3.000E-15
4.00	1	.100	18.000	1.550	.150	3.000E-15
5.00	1	.100	18.000	1.550	.150	3.000E-15
6.00	1	.100	18.000	1.550	.150	3.000E-15
7.00	1	.100	18.000	1.550	.150	3.000E-15
8.00	1	.100	18.000	1.550	.150	3.000E-15
9.00	1	.100	18.000	1.550	.150	3.000E-15
10.00	1	.100	18.000	1.550	.150	3.000E-15
11.00	2	.099	17.000	1.570	.160	3.100E-15
12.00	2	.099	17.000	1.570	.160	3.100E-15
13.00	2	.099	17.000	1.570	.160	3.100E-15
14.00	2	.099	17.000	1.570	.160	3.100E-15
15.00	2	.099	17.000	1.570	.160	3.100E-15
16.00	2	.099	17.000	1.570	.160	3.100E-15
17.00	2	.099	17.000	1.570	.160	3.100E-15
18.00	2	.099	17.000	1.570	.160	3.100E-15
19.00	2	.099	17.000	1.570	.160	3.100E-15
20.00	2	.099	17.000	1.570	.160	3.100E-15
21.00	2	.099	17.000	1.570	.160	3.100E-15
22.00	3	.100	19.000	1.600	.170	3.000E-15
23.00	3	.100	19.000	1.600	.170	3.000E-15
24.00	3	.100	19.000	1.600	.170	3.000E-15
25.00	3	.100	19.000	1.600	.170	3.000E-15
26.00	3	.100	19.000	1.600	.170	3.000E-15
27.00	3	.100	19.000	1.600	.170	3.000E-15
28.00	3	.100	19.000	1.600	.170	3.000E-15
29.00	3	.100	19.000	1.600	.170	3.000E-15
30.00	3	.100	19.000	1.600	.170	3.000E-15
31.00	3	.100	19.000	1.600	.170	3.000E-15
32.00	3	.100	19.000	1.600	.170	3.000E-15
33.00	3	.100	19.000	1.600	.170	3.000E-15
34.00	4	.099	16.000	1.520	.140	3.100E-15
35.00	4	.099	16.000	1.520	.140	3.100E-15
36.00	4	.099	16.000	1.520	.140	3.100E-15
37.00	4	.099	16.000	1.520	.140	3.100E-15
38.00	4	.099	16.000	1.520	.140	3.100E-15

39.00	4	.099	16.000	1.520	.140	3.100E-15
40.00	4	.099	16.000	1.520	.140	3.100E-15
41.00	4	.099	16.000	1.520	.140	3.100E-15
42.00	4	.099	16.000	1.520	.140	3.100E-15
43.00	4	.099	16.000	1.520	.140	3.100E-15
44.00	4	.099	16.000	1.520	.140	3.100E-15
45.00	4	.099	16.000	1.520	.140	3.100E-15

1 INITIAL CONDITIONS

NODE	PRESSURE	TEMPERATURE	ICE CONTENT
1	-50.000	5.800	.000

BOUNDARY CONDITIONS

TIME= 24.000 HRS 1.000 DAYS FROST HEAVE EQUALS .00 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
1.0	-48.063	4.000	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-4.806E+01	4.000E+00	4.015E-01	0.000E+00	1.550E
2	1.000E+00	-4.706E+01	3.974E+00	4.019E-01	0.000E+00	1.550E
3	2.000E+00	-4.605E+01	4.248E+00	4.024E-01	0.000E+00	1.550E
4	3.000E+00	-4.504E+01	4.287E+00	4.029E-01	0.000E+00	1.550E
5	4.000E+00	-4.402E+01	4.356E+00	4.034E-01	0.000E+00	1.550E
6	5.000E+00	-4.299E+01	4.448E+00	4.039E-01	0.000E+00	1.550E
7	6.000E+00	-4.195E+01	4.545E+00	4.044E-01	0.000E+00	1.550E
8	7.000E+00	-4.091E+01	4.638E+00	4.049E-01	0.000E+00	1.550E
9	8.000E+00	-3.987E+01	4.728E+00	4.054E-01	0.000E+00	1.550E
10	9.000E+00	-3.882E+01	4.818E+00	4.060E-01	0.000E+00	1.550E
11	1.000E+01	-3.777E+01	4.908E+00	4.065E-01	0.000E+00	1.550E
12	1.100E+01	-3.671E+01	4.999E+00	3.838E-01	0.000E+00	1.570E
13	1.200E+01	-3.565E+01	5.091E+00	3.843E-01	0.000E+00	1.570E
14	1.300E+01	-3.459E+01	5.183E+00	3.848E-01	0.000E+00	1.570E
15	1.400E+01	-3.353E+01	5.275E+00	3.853E-01	0.000E+00	1.570E
16	1.500E+01	-3.247E+01	5.367E+00	3.857E-01	0.000E+00	1.570E
17	1.600E+01	-3.140E+01	5.459E+00	3.862E-01	0.000E+00	1.570E
18	1.700E+01	-3.033E+01	5.550E+00	3.867E-01	0.000E+00	1.570E
19	1.800E+01	-2.926E+01	5.642E+00	3.872E-01	0.000E+00	1.570E
20	1.900E+01	-2.818E+01	5.733E+00	3.877E-01	0.000E+00	1.570E
21	2.000E+01	-2.711E+01	5.825E+00	3.881E-01	0.000E+00	1.570E
22	2.100E+01	-2.603E+01	5.916E+00	3.886E-01	0.000E+00	1.570E
23	2.200E+01	-2.496E+01	6.007E+00	3.737E-01	0.000E+00	1.600E
24	2.300E+01	-2.388E+01	6.089E+00	3.742E-01	0.000E+00	1.600E
25	2.400E+01	-2.280E+01	6.170E+00	3.747E-01	0.000E+00	1.600E
26	2.500E+01	-2.172E+01	6.251E+00	3.752E-01	0.000E+00	1.600E

27	2.600E+01	-2.063E+01	6.332E+00	3.757E-01	0.000E+00	1.600E
28	2.700E+01	-1.955E+01	6.413E+00	3.762E-01	0.000E+00	1.600E
29	2.800E+01	-1.847E+01	6.494E+00	3.768E-01	0.000E+00	1.600E
30	2.900E+01	-1.738E+01	6.575E+00	3.773E-01	0.000E+00	1.600E
31	3.000E+01	-1.630E+01	6.656E+00	3.778E-01	0.000E+00	1.600E
32	3.100E+01	-1.521E+01	6.737E+00	3.783E-01	0.000E+00	1.600E
33	3.200E+01	-1.413E+01	6.817E+00	3.788E-01	0.000E+00	1.600E
34	3.300E+01	-1.304E+01	6.898E+00	3.793E-01	0.000E+00	1.600E
35	3.400E+01	-1.196E+01	6.978E+00	3.849E-01	0.000E+00	1.520E
36	3.500E+01	-1.087E+01	7.071E+00	3.854E-01	0.000E+00	1.520E
37	3.600E+01	-9.784E+00	7.165E+00	3.859E-01	0.000E+00	1.520E
38	3.700E+01	-8.697E+00	7.259E+00	3.864E-01	0.000E+00	1.520E
39	3.800E+01	-7.610E+00	7.353E+00	3.869E-01	0.000E+00	1.520E
40	3.900E+01	-6.523E+00	7.443E+00	3.873E-01	0.000E+00	1.520E
41	4.000E+01	-5.436E+00	7.528E+00	3.878E-01	0.000E+00	1.520E
42	4.100E+01	-4.349E+00	7.616E+00	3.883E-01	0.000E+00	1.520E
43	4.200E+01	-3.262E+00	7.739E+00	3.887E-01	0.000E+00	1.520E
44	4.300E+01	-2.174E+00	7.913E+00	3.892E-01	0.000E+00	1.520E
45	4.400E+01	-1.087E+00	7.740E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 48.000 HRS 2.000 DAYS FROST HEAVE EQUALS .00 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
2.0	-45.286	2.000	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-4.529E+01	2.000E+00	4.029E-01	0.000E+00	1.550E
2	1.000E+00	-4.429E+01	2.218E+00	4.032E-01	0.000E+00	1.550E
3	2.000E+00	-4.329E+01	2.244E+00	4.037E-01	0.000E+00	1.550E
4	3.000E+00	-4.228E+01	2.406E+00	4.042E-01	0.000E+00	1.550E
5	4.000E+00	-4.128E+01	2.568E+00	4.047E-01	0.000E+00	1.550E
6	5.000E+00	-4.028E+01	2.708E+00	4.052E-01	0.000E+00	1.550E
7	6.000E+00	-3.928E+01	2.844E+00	4.057E-01	0.000E+00	1.550E
8	7.000E+00	-3.827E+01	2.981E+00	4.062E-01	0.000E+00	1.550E
9	8.000E+00	-3.727E+01	3.121E+00	4.068E-01	0.000E+C0	1.550E
10	9.000E+00	-3.626E+01	3.261E+00	4.073E-01	0.000E+00	1.550E
11	1.000E+01	-3.526E+01	3.400E+00	4.078E-01	0.000E+00	1.550E
12	1.100E+01	-3.425E+01	3.540E+00	3.849E-01	0.000E+00	1.570E
13	1.200E+01	-3.325E+01	3.681E+00	3.854E-01	0.000E+00	1.570E
14	1.300E+01	-3.224E+01	3.822E+00	3.858E-01	0.000E+00	1.570E
15	1.400E+01	-3.124E+01	3.962E+00	3.863E-01	0.000E+00	1.570E
16	1.500E+01	-3.023E+01	4.102E+00	3.867E-01	0.000E+00	1.570E
17	1.600E+01	-2.923E+01	4.242E+00	3.872E-01	0.000E+00	1.570E
18	1.700E+01	-2.822E+01	4.382E+00	3.876E-01	0.000E+00	1.570E
19	1.800E+01	-2.721E+01	4.520E+00	3.881E-01	0.000E+00	1.570E
20	1.900E+01	-2.621E+01	4.659E+00	3.886E-01	0.000E+00	1.570E
21	2.000E+01	-2.520E+01	4.797E+00	3.890E-01	0.000E+00	1.570E
22	2.100E+01	-2.419E+01	4.934E+00	3.895E-01	0.000E+00	1.570E
23	2.200E+01	-2.319E+01	5.071E+00	3.745E-01	0.000E+00	1.600E
24	2.300E+01	-2.218E+01	5.194E+00	3.750E-01	0.000E+00	1.600E
25	2.400E+01	-2.117E+01	5.316E+00	3.755E-01	0.000E+00	1.600E
26	2.500E+01	-2.016E+01	5.437E+00	3.760E-01	0.000E+00	1.600E

27	2.600E+01	-1.916E+01	5.558E+00	3.764E-01	0.000E+00	1.600E
28	2.700E+01	-1.815E+01	5.678E+00	3.769E-01	0.000E+00	1.600E
29	2.800E+01	-1.714E+01	5.799E+00	3.774E-01	0.000E+00	1.600E
30	2.900E+01	-1.613E+01	5.918E+00	3.779E-01	0.000E+00	1.600E
31	3.000E+01	-1.512E+01	6.038E+00	3.783E-01	0.000E+00	1.600E
32	3.100E+01	-1.412E+01	6.157E+00	3.788E-01	0.000E+00	1.600E
33	3.200E+01	-1.311E+01	6.275E+00	3.793E-01	0.000E+00	1.600E
34	3.300E+01	-1.210E+01	6.393E+00	3.797E-01	0.000E+00	1.600E
35	3.400E+01	-1.109E+01	6.511E+00	3.853E-01	0.000E+00	1.520E
36	3.500E+01	-1.008E+01	6.648E+00	3.858E-01	0.000E+00	1.520E
37	3.600E+01	-9.075E+00	6.784E+00	3.862E-01	0.000E+00	1.520E
38	3.700E+01	-8.067E+00	6.920E+00	3.867E-01	0.000E+00	1.520E
39	3.800E+01	-7.059E+00	7.056E+00	3.871E-01	0.000E+00	1.520E
40	3.900E+01	-6.050E+00	7.192E+00	3.875E-01	0.000E+00	1.520E
41	4.000E+01	-5.042E+00	7.327E+00	3.880E-01	0.000E+00	1.520E
42	4.100E+01	-4.034E+00	7.459E+00	3.884E-01	0.000E+00	1.520E
43	4.200E+01	-3.025E+00	7.588E+00	3.888E-01	0.000E+00	1.520E
44	4.300E+01	-2.017E+00	7.759E+00	3.892E-01	0.000E+00	1.520E
45	4.400E+01	-1.008E+00	7.835E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 72.000 HRS 3.000 DAYS FROST HEAVE EQUALS .00 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
3.0	-45.027	1.000	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-4.503E+01	1.000E+00	4.030E-01	0.000E+00	1.550E
2	1.000E+00	-4.403E+01	1.223E+00	4.034E-01	0.000E+00	1.550E
3	2.000E+00	-4.303E+01	1.280E+00	4.039E-01	0.000E+00	1.550E
4	3.000E+00	-4.203E+01	1.477E+00	4.044E-01	0.000E+00	1.550E
5	4.000E+00	-4.103E+01	1.646E+00	4.049E-01	0.000E+00	1.550E
6	5.000E+00	-4.003E+01	1.802E+00	4.054E-01	0.000E+00	1.550E
7	6.000E+00	-3.903E+01	1.959E+00	4.059E-01	0.000E+00	1.550E
8	7.000E+00	-3.803E+01	2.118E+00	4.064E-01	0.000E+00	1.550E
9	8.000E+00	-3.703E+01	2.278E+00	4.069E-01	0.000E+00	1.550E
10	9.000E+00	-3.602E+01	2.437E+00	4.074E-01	0.000E+00	1.550E
11	1.000E+01	-3.502E+01	2.596E+00	4.079E-01	0.000E+00	1.550E
12	1.100E+01	-3.402E+01	2.756E+00	3.850E-01	0.000E+00	1.570E
13	1.200E+01	-3.302E+01	2.918E+00	3.855E-01	0.000E+00	1.570E
14	1.300E+01	-3.202E+01	3.079E+00	3.859E-01	0.000E+00	1.570E
15	1.400E+01	-3.102E+01	3.241E+00	3.864E-01	0.000E+00	1.570E
16	1.500E+01	-3.002E+01	3.402E+00	3.868E-01	0.000E+00	1.570E
17	1.600E+01	-2.902E+01	3.563E+00	3.873E-01	0.000E+00	1.570E
18	1.700E+01	-2.802E+01	3.724E+00	3.877E-01	0.000E+00	1.570E
19	1.800E+01	-2.702E+01	3.885E+00	3.882E-01	0.000E+00	1.570E
20	1.900E+01	-2.602E+01	4.045E+00	3.886E-01	0.000E+00	1.570E
21	2.000E+01	-2.502E+01	4.205E+00	3.891E-01	0.000E+00	1.570E
22	2.100E+01	-2.402E+01	4.365E+00	3.895E-01	0.000E+00	1.570E
23	2.200E+01	-2.302E+01	4.525E+00	3.746E-01	0.000E+00	1.600E
24	2.300E+01	-2.202E+01	4.667E+00	3.751E-01	0.000E+00	1.600E
25	2.400E+01	-2.102E+01	4.810E+00	3.755E-01	0.000E+00	1.600E
26	2.500E+01	-2.002E+01	4.952E+00	3.760E-01	0.000E+00	1.600E

27	2.600E+01	-1.901E+01	5.094E+00	3.765E-01	0.000E+00	1.600E
28	2.700E+01	-1.801E+01	5.235E+00	3.770E-01	0.000E+00	1.600E
29	2.800E+01	-1.701E+01	5.377E+00	3.774E-01	0.000E+00	1.600E
30	2.900E+01	-1.601E+01	5.518E+00	3.779E-01	0.000E+00	1.600E
31	3.000E+01	-1.501E+01	5.659E+00	3.784E-01	0.000E+00	1.600E
32	3.100E+01	-1.401E+01	5.800E+00	3.788E-01	0.000E+00	1.600E
33	3.200E+01	-1.301E+01	5.940E+00	3.793E-01	0.000E+00	1.600E
34	3.300E+01	-1.201E+01	6.080E+00	3.798E-01	0.000E+00	1.600E
35	3.400E+01	-1.101E+01	6.220E+00	3.853E-01	0.000E+00	1.520E
36	3.500E+01	-1.001E+01	6.383E+00	3.858E-01	0.000E+00	1.520E
37	3.600E+01	-9.007E+00	6.545E+00	3.862E-01	0.000E+00	1.520E
38	3.700E+01	-8.006E+00	6.707E+00	3.867E-01	0.000E+00	1.520E
39	3.800E+01	-7.005E+00	6.869E+00	3.871E-01	0.000E+00	1.520E
40	3.900E+01	-6.005E+00	7.031E+00	3.875E-01	0.000E+00	1.520E
41	4.000E+01	-5.004E+00	7.193E+00	3.880E-01	0.000E+00	1.520E
42	4.100E+01	-4.003E+00	7.355E+00	3.884E-01	0.000E+00	1.520E
43	4.200E+01	-3.002E+00	7.512E+00	3.888E-01	0.000E+00	1.520E
44	4.300E+01	-2.002E+00	7.685E+00	3.892E-01	0.000E+00	1.520E
45	4.400E+01	-1.001E+00	7.832E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 96.000 HRS 4.000 DAYS FROST HEAVE EQUALS .00 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
4.0	-44.999	.000	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-4.500E+01	0.000E+00	4.030E-01	0.000E+00	1.550E
2	1.000E+00	-4.400E+01	1.365E-01	4.034E-01	0.000E+00	1.550E
3	2.000E+00	-4.300E+01	4.066E-01	4.039E-01	0.000E+00	1.550E
4	3.000E+00	-4.200E+01	5.351E-01	4.044E-01	0.000E+00	1.550E
5	4.000E+00	-4.100E+01	7.244E-01	4.049E-01	0.000E+00	1.550E
6	5.000E+00	-4.000E+01	9.107E-01	4.054E-01	0.000E+00	1.550E
7	6.000E+00	-3.900E+01	1.093E+00	4.059E-01	0.000E+00	1.550E
8	7.000E+00	-3.800E+01	1.274E+00	4.064E-01	0.000E+00	1.550E
9	8.000E+00	-3.700E+01	1.455E+00	4.069E-01	0.000E+00	1.550E
10	9.000E+00	-3.600E+01	1.636E+00	4.074E-01	0.000E+00	1.550E
11	1.000E+01	-3.500E+01	1.817E+00	4.079E-01	0.000E+00	1.550E
12	1.100E+01	-3.400E+01	1.999E+00	3.851E-01	0.000E+00	1.570E
13	1.200E+01	-3.300E+01	2.183E+00	3.855E-01	0.000E+00	1.570E
14	1.300E+01	-3.200E+01	2.368E+00	3.859E-01	0.000E+00	1.570E
15	1.400E+01	-3.100E+01	2.552E+00	3.864E-01	0.000E+00	1.570E
16	1.500E+01	-3.000E+01	2.736E+00	3.868E-01	0.000E+00	1.570E
17	1.600E+01	-2.900E+01	2.920E+00	3.873E-01	0.000E+00	1.570E
18	1.700E+01	-2.800E+01	3.103E+00	3.877E-01	0.000E+00	1.570E
19	1.800E+01	-2.700E+01	3.287E+00	3.882E-01	0.000E+00	1.570E
20	1.900E+01	-2.600E+01	3.470E+00	3.886E-01	0.000E+00	1.570E
21	2.000E+01	-2.500E+01	3.652E+00	3.891E-01	0.000E+00	1.570E
22	2.100E+01	-2.400E+01	3.835E+00	3.895E-01	0.000E+00	1.570E
23	2.200E+01	-2.300E+01	4.017E+00	3.746E-01	0.000E+00	1.600E
24	2.300E+01	-2.200E+01	4.180E+00	3.751E-01	0.000E+00	1.600E
25	2.400E+01	-2.100E+01	4.343E+00	3.756E-01	0.000E+00	1.600E
26	2.500E+01	-2.000E+01	4.506E+00	3.760E-01	0.000E+00	1.600E

27	2.600E+01	-1.900E+01	4.668E+00	3.765E-01	0.000E+00	1.600E
28	2.700E+01	-1.800E+01	4.830E+00	3.770E-01	0.000E+00	1.600E
29	2.800E+01	-1.700E+01	4.992E+00	3.774E-01	0.000E+00	1.600E
30	2.900E+01	-1.600E+01	5.153E+00	3.779E-01	0.000E+00	1.600E
31	3.000E+01	-1.500E+01	5.315E+00	3.784E-01	0.000E+00	1.600E
32	3.100E+01	-1.400E+01	5.476E+00	3.789E-01	0.000E+00	1.600E
33	3.200E+01	-1.300E+01	5.637E+00	3.793E-01	0.000E+00	1.600E
34	3.300E+01	-1.200E+01	5.798E+00	3.798E-01	0.000E+00	1.600E
35	3.400E+01	-1.100E+01	5.958E+00	3.854E-01	0.000E+00	1.520E
36	3.500E+01	-1.000E+01	6.144E+00	3.858E-01	0.000E+00	1.520E
37	3.600E+01	-9.000E+00	6.331E+00	3.862E-01	0.000E+00	1.520E
38	3.700E+01	-8.000E+00	6.517E+00	3.867E-01	0.000E+00	1.520E
39	3.800E+01	-7.000E+00	6.702E+00	3.871E-01	0.000E+00	1.520E
40	3.900E+01	-6.000E+00	6.888E+00	3.876E-01	0.000E+00	1.520E
41	4.000E+01	-5.000E+00	7.074E+00	3.880E-01	0.000E+00	1.520E
42	4.100E+01	-4.000E+00	7.260E+00	3.884E-01	0.000E+00	1.520E
43	4.200E+01	-3.000E+00	7.443E+00	3.888E-01	0.000E+00	1.520E
44	4.300E+01	-2.000E+00	7.632E+00	3.892E-01	0.000E+00	1.520E
45	4.400E+01	-1.000E+00	7.813E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 120.000 HRS 5.000 DAYS FROST HEAVE EQUALS .05 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
5.0	-490.558	-1.200	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-4.906E+02	-1.200E+00	2.322E-01	2.458E-01	1.550E
2	1.000E+00	-8.005E+02	0.000E+00	1.933E-01	3.277E-01	1.472E
3	2.000E+00	-2.604E+02	0.000E+00	3.125E-01	2.234E-01	1.550E
4	3.000E+00	-7.780E+01	0.000E+00	3.868E-01	1.199E-01	1.550E
5	4.000E+00	-6.022E+01	0.000E+00	3.953E-01	3.394E-02	1.550E
6	5.000E+00	-5.384E+01	4.999E-02	3.985E-01	0.000E+00	1.550E
7	6.000E+00	-5.236E+01	2.543E-01	3.992E-01	0.000E+00	1.550E
8	7.000E+00	-5.088E+01	4.637E-01	4.000E-01	0.000E+00	1.550E
9	8.000E+00	-4.941E+01	6.711E-01	4.007E-01	0.000E+00	1.550E
10	9.000E+00	-4.795E+01	8.774E-01	4.014E-01	0.000E+00	1.550E
11	1.000E+01	-4.649E+01	1.083E+00	4.021E-01	0.000E+00	1.550E
12	1.100E+01	-4.504E+01	1.290E+00	3.801E-01	0.000E+00	1.570E
13	1.200E+01	-4.363E+01	1.500E+00	3.807E-01	0.000E+00	1.570E
14	1.300E+01	-4.222E+01	1.709E+00	3.814E-01	0.000E+00	1.570E
15	1.400E+01	-4.082E+01	1.918E+00	3.820E-01	0.000E+00	1.570E
16	1.500E+01	-3.943E+01	2.126E+00	3.826E-01	0.000E+00	1.570E
17	1.600E+01	-3.804E+01	2.334E+00	3.832E-01	0.000E+00	1.570E
18	1.700E+01	-3.666E+01	2.541E+00	3.839E-01	0.000E+00	1.570E
19	1.800E+01	-3.529E+01	2.748E+00	3.845E-01	0.000E+00	1.570E
20	1.900E+01	-3.392E+01	2.954E+00	3.851E-01	0.000E+00	1.570E
21	2.000E+01	-3.256E+01	3.159E+00	3.857E-01	0.000E+00	1.570E
22	2.100E+01	-3.121E+01	3.365E+00	3.863E-01	0.000E+00	1.570E
23	2.200E+01	-2.986E+01	3.570E+00	3.713E-01	0.000E+00	1.600E
24	2.300E+01	-2.852E+01	3.753E+00	3.720E-01	0.000E+00	1.600E
25	2.400E+01	-2.718E+01	3.936E+00	3.726E-01	0.000E+00	1.600E
26	2.500E+01	-2.585E+01	4.118E+00	3.732E-01	0.000E+00	1.600E

27	2.600E+01	-2.452E+01	4.300E+00	3.739E-01	0.000E+00	1.600E
28	2.700E+01	-2.320E+01	4.481E+00	3.745E-01	0.000E+00	1.600E
29	2.800E+01	-2.189E+01	4.662E+00	3.751E-01	0.000E+00	1.600E
30	2.900E+01	-2.058E+01	4.842E+00	3.758E-01	0.000E+00	1.600E
31	3.000E+01	-1.927E+01	5.023E+00	3.764E-01	0.000E+00	1.600E
32	3.100E+01	-1.797E+01	5.202E+00	3.770E-01	0.000E+00	1.600E
33	3.200E+01	-1.667E+01	5.382E+00	3.776E-01	0.000E+00	1.600E
34	3.300E+01	-1.537E+01	5.561E+00	3.782E-01	0.000E+00	1.600E
35	3.400E+01	-1.408E+01	5.740E+00	3.840E-01	0.000E+00	1.520E
36	3.500E+01	-1.279E+01	5.947E+00	3.846E-01	0.000E+00	1.520E
37	3.600E+01	-1.150E+01	6.154E+00	3.851E-01	0.000E+00	1.520E
38	3.700E+01	-1.022E+01	6.360E+00	3.857E-01	0.000E+00	1.520E
39	3.800E+01	-8.938E+00	6.567E+00	3.863E-01	0.000E+00	1.520E
40	3.900E+01	-7.657E+00	6.772E+00	3.868E-01	0.000E+00	1.520E
41	4.000E+01	-6.379E+00	6.978E+00	3.874E-01	0.000E+00	1.520E
42	4.100E+01	-5.101E+00	7.183E+00	3.879E-01	0.000E+00	1.520E
43	4.200E+01	-3.825E+00	7.387E+00	3.885E-01	0.000E+00	1.520E
44	4.300E+01	-2.549E+00	7.593E+00	3.890E-01	0.000E+00	1.520E
45	4.400E+01	-1.275E+00	7.796E+00	3.895E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 144.000 HRS 6.000 DAYS FROST HEAVE EQUALS .34 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
6.0	-839.235	-2.900	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-8.392E+02	-2.900E+00	1.881E-01	2.939E-01	1.535E
2	1.000E+00	-8.365E+02	-2.265E+00	1.883E-01	3.331E-01	1.465E
3	2.000E+00	-8.345E+02	-2.580E+00	1.886E-01	3.594E-01	1.429E
4	3.000E+00	-8.324E+02	-1.549E+00	1.889E-01	3.591E-01	1.430E
5	4.000E+00	-8.303E+02	-1.440E+00	1.892E-01	3.387E-01	1.457E
6	5.000E+00	-6.737E+02	0.000E+00	2.129E-01	3.126E-01	1.494E
7	6.000E+00	-1.961E+02	0.000E+00	3.358E-01	2.007E-01	1.550E
8	7.000E+00	-7.463E+01	0.000E+00	3.883E-01	9.266E-02	1.550E
9	8.000E+00	-5.982E+01	0.000E+00	3.955E-01	2.341E-02	1.550E
10	9.000E+00	-5.214E+01	1.700E-02	3.993E-01	0.000E+00	1.550E
11	1.000E+01	-5.050E+01	2.651E-01	4.001E-01	0.000E+00	1.550E
12	1.100E+01	-4.887E+01	5.007E-01	3.784E-01	0.000E+00	1.570E
13	1.200E+01	-4.730E+01	7.376E-01	3.791E-01	0.000E+00	1.570E
14	1.300E+01	-4.574E+01	9.742E-01	3.798E-01	0.000E+00	1.570E
15	1.400E+01	-4.419E+01	1.211E+00	3.805E-01	0.000E+00	1.570E
16	1.500E+01	-4.264E+01	1.446E+00	3.812E-01	0.000E+00	1.570E
17	1.600E+01	-4.111E+01	1.681E+00	3.819E-01	0.000E+00	1.570E
18	1.700E+01	-3.959E+01	1.916E+00	3.825E-01	0.000E+00	1.570E
19	1.800E+01	-3.807E+01	2.149E+00	3.832E-01	0.000E+00	1.570E
20	1.900E+01	-3.657E+01	2.382E+00	3.839E-01	0.000E+00	1.570E
21	2.000E+01	-3.508E+01	2.614E+00	3.846E-01	0.000E+00	1.570E
22	2.100E+01	-3.360E+01	2.845E+00	3.852E-01	0.000E+00	1.570E
23	2.200E+01	-3.212E+01	3.076E+00	3.703E-01	0.000E+00	1.600E
24	2.300E+01	-3.066E+01	3.282E+00	3.710E-01	0.000E+00	1.600E
25	2.400E+01	-2.920E+01	3.487E+00	3.717E-01	0.000E+00	1.600E
26	2.500E+01	-2.775E+01	3.691E+00	3.723E-01	0.000E+00	1.600E

27	2.600E+01	-2.631E+01	3.895E+00	3.730E-01	0.000E+00	1.600E
28	2.700E+01	-2.488E+01	4.097E+00	3.737E-01	0.000E+00	1.600E
29	2.800E+01	-2.346E+01	4.300E+00	3.744E-01	0.000E+00	1.600E
30	2.900E+01	-2.204E+01	4.501E+00	3.751E-01	0.000E+00	1.600E
31	3.000E+01	-2.063E+01	4.702E+00	3.757E-01	0.000E+00	1.600E
32	3.100E+01	-1.923E+01	4.903E+00	3.764E-01	0.000E+00	1.600E
33	3.200E+01	-1.783E+01	5.102E+00	3.771E-01	0.000E+00	1.600E
34	3.300E+01	-1.644E+01	5.302E+00	3.777E-01	0.000E+00	1.600E
35	3.400E+01	-1.505E+01	5.500E+00	3.835E-01	0.000E+00	1.520E
36	3.500E+01	-1.367E+01	5.730E+00	3.842E-01	0.000E+00	1.520E
37	3.600E+01	-1.229E+01	5.960E+00	3.848E-01	0.000E+00	1.520E
38	3.700E+01	-1.092E+01	6.189E+00	3.854E-01	0.000E+00	1.520E
39	3.800E+01	-9.544E+00	6.417E+00	3.860E-01	0.000E+00	1.520E
40	3.900E+01	-8.175E+00	6.645E+00	3.866E-01	0.000E+00	1.520E
41	4.000E+01	-6.808E+00	6.872E+00	3.872E-01	0.000E+00	1.520E
42	4.100E+01	-5.444E+00	7.098E+00	3.878E-01	0.000E+00	1.520E
43	4.200E+01	-4.082E+00	7.324E+00	3.884E-01	0.000E+00	1.520E
44	4.300E+01	-2.720E+00	7.550E+00	3.889E-01	0.000E+00	1.520E
45	4.400E+01	-1.360E+00	7.775E+00	3.895E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 168.000 HRS 7.000 DAYS FROST HEAVE EQUALS .56 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
7.0	-839.235	-2.900	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-8.392E+02	-2.900E+00	1.881E-01	2.939E-01	1.535E
2	1.000E+00	-8.365E+02	-2.494E+00	1.883E-01	3.331E-01	1.465E
3	2.000E+00	-8.345E+02	-2.436E+00	1.886E-01	3.594E-01	1.429E
4	3.000E+00	-8.324E+02	-1.902E+00	1.889E-01	3.591E-01	1.430E
5	4.000E+00	-8.304E+02	-1.717E+00	1.892E-01	3.387E-01	1.457E
6	5.000E+00	-8.283E+02	-1.227E+00	1.895E-01	3.382E-01	1.458E
7	6.000E+00	-8.262E+02	-1.189E+00	1.897E-01	3.650E-01	1.422E
8	7.000E+00	-7.649E+02	0.000E+00	1.984E-01	3.781E-01	1.405E
9	8.000E+00	-1.039E+02	0.000E+00	3.746E-01	1.835E-01	1.550E
10	9.000E+00	-5.417E+01	0.000E+00	3.983E-01	5.269E-02	1.550E
11	1.000E+01	-4.508E+01	0.000E+00	4.028E-01	6.849E-05	1.550E
12	1.100E+01	-4.362E+01	2.423E-01	3.807E-01	0.000E+00	1.570E
13	1.200E+01	-4.226E+01	4.737E-01	3.814E-01	0.000E+00	1.570E
14	1.300E+01	-4.090E+01	7.128E-01	3.820E-01	0.000E+00	1.570E
15	1.400E+01	-3.955E+01	9.529E-01	3.826E-01	0.000E+00	1.570E
16	1.500E+01	-3.821E+01	1.193E+00	3.832E-01	0.000E+00	1.570E
17	1.600E+01	-3.687E+01	1.432E+00	3.838E-01	0.000E+00	1.570E
18	1.700E+01	-3.554E+01	1.670E+00	3.844E-01	0.000E+00	1.570E
19	1.800E+01	-3.421E+01	1.908E+00	3.850E-01	0.000E+00	1.570E
20	1.900E+01	-3.289E+01	2.146E+00	3.855E-01	0.000E+00	1.570E
21	2.000E+01	-3.158E+01	2.383E+00	3.861E-01	0.000E+00	1.570E
22	2.100E+01	-3.027E+01	2.620E+00	3.867E-01	0.000E+00	1.570E
23	2.200E+01	-2.897E+01	2.857E+00	3.718E-01	0.000E+00	1.600E
24	2.300E+01	-2.767E+01	3.069E+00	3.724E-01	0.000E+00	1.600E
25	2.400E+01	-2.638E+01	3.280E+00	3.730E-01	0.000E+00	1.600E
26	2.500E+01	-2.509E+01	3.491E+00	3.736E-01	0.000E+00	1.600E

27	2.600E+01	-2.381E+01	3.701E+00	3.742E-01	0.000E+00	1.600E
28	2.700E+01	-2.253E+01	3.911E+00	3.748E-01	0.000E+00	1.600E
29	2.800E+01	-2.125E+01	4.121E+00	3.754E-01	0.000E+00	1.600E
30	2.900E+01	-1.998E+01	4.330E+00	3.760E-01	0.000E+00	1.600E
31	3.000E+01	-1.871E+01	4.539E+00	3.766E-01	0.000E+00	1.600E
32	3.100E+01	-1.745E+01	4.747E+00	3.772E-01	0.000E+00	1.600E
33	3.200E+01	-1.619E+01	4.956E+00	3.778E-01	0.000E+00	1.600E
34	3.300E+01	-1.494E+01	5.163E+00	3.784E-01	0.000E+00	1.600E
35	3.400E+01	-1.368E+01	5.371E+00	3.842E-01	0.000E+00	1.520E
36	3.500E+01	-1.243E+01	5.612E+00	3.847E-01	0.000E+00	1.520E
37	3.600E+01	-1.118E+01	5.852E+00	3.853E-01	0.000E+00	1.520E
38	3.700E+01	-9.933E+00	6.092E+00	3.858E-01	0.000E+00	1.520E
39	3.800E+01	-8.687E+00	6.332E+00	3.864E-01	0.000E+00	1.520E
40	3.900E+01	-7.443E+00	6.571E+00	3.869E-01	0.000E+00	1.520E
41	4.000E+01	-6.200E+00	6.810E+00	3.875E-01	0.000E+00	1.520E
42	4.100E+01	-4.959E+00	7.049E+00	3.880E-01	0.000E+00	1.520E
43	4.200E+01	-3.718E+00	7.287E+00	3.885E-01	0.000E+00	1.520E
44	4.300E+01	-2.478E+00	7.525E+00	3.890E-01	0.000E+00	1.520E
45	4.400E+01	-1.239E+00	7.763E+00	3.895E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 192.000 HRS 8.000 DAYS FROST HEAVE EQUALS .70 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
8.0	-839.235	-2.900	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-8.392E+02	-2.900E+00	1.881E-01	2.939E-01	1.535E
2	1.000E+00	-8.365E+02	-2.606E+00	1.883E-01	3.331E-01	1.465E
3	2.000E+00	-8.345E+02	-2.435E+00	1.886E-01	3.594E-01	1.429E
4	3.000E+00	-8.324E+02	-2.091E+00	1.889E-01	3.591E-01	1.430E
5	4.000E+00	-8.304E+02	-1.896E+00	1.892E-01	3.387E-01	1.457E
6	5.000E+00	-8.283E+02	-1.533E+00	1.895E-01	3.382E-01	1.458E
7	6.000E+00	-8.262E+02	-1.270E+00	1.897E-01	3.650E-01	1.422E
8	7.000E+00	-8.242E+02	-9.372E-01	1.900E-01	3.873E-01	1.393E
9	8.000E+00	-8.220E+02	-1.059E+00	1.903E-01	4.053E-01	1.371E
10	9.000E+00	-1.278E+02	0.000E+00	3.639E-01	2.234E-01	1.550E
11	1.000E+01	-5.102E+01	0.000E+00	3.999E-01	5.084E-02	1.550E
12	1.100E+01	-4.196E+01	0.000E+00	3.815E-01	8.668E-04	1.570E
13	1.200E+01	-4.024E+01	2.658E-01	3.823E-01	0.000E+00	1.570E
14	1.300E+01	-3.895E+01	4.917E-01	3.828E-01	0.000E+00	1.570E
15	1.400E+01	-3.767E+01	7.310E-01	3.834E-01	0.000E+00	1.570E
16	1.500E+01	-3.639E+01	9.761E-01	3.840E-01	0.000E+00	1.570E
17	1.600E+01	-3.512E+01	1.224E+00	3.845E-01	0.000F+00	1.570E
18	1.700E+01	-3.386E+01	1.472E+00	3.851E-01	0.000E+00	1.570E
19	1.800E+01	-3.260E+01	1.720E+00	3.857E-01	0.000E+00	1.570E
20	1.900E+01	-3.134E+01	1.967E+00	3.862E-01	0.000E+00	1.570E
21	2.000E+01	-3.010E+01	2.213E+00	3.868E-01	0.000E+00	1.570E
22	2.100E+01	-2.885E+01	2.458E+00	3.874E-01	0.000E+00	1.570E
23	2.200E+01	-2.762E+01	2.704E+00	3.724E-01	0.000E+00	1.600E
24	2.300E+01	-2.638E+01	2.922E+00	3.730E-01	0.000E+00	1.600E
25	2.400E+01	-2.515E+01	3.140E+00	3.736E-01	0.000E+00	1.600E
26	2.500E+01	-2.393E+01	3.358E+00	3.742E-01	0.000E+00	1.600E

27	2.600E+01	-2.271E+01	3.575E+00	3.747E-01	0.000E+00	1.600E
28	2.700E+01	-2.149E+01	3.792E+00	3.753E-01	0.000E+00	1.600E
29	2.800E+01	-2.028E+01	4.008E+00	3.759E-01	0.000E+00	1.600E
30	2.900E+01	-1.907E+01	4.223E+00	3.765E-01	0.000E+00	1.600E
31	3.000E+01	-1.786E+01	4.439E+00	3.770E-01	0.000E+00	1.600E
32	3.100E+01	-1.666E+01	4.653E+00	3.776E-01	0.000E+00	1.600E
33	3.200E+01	-1.546E+01	4.868E+00	3.782E-01	0.000E+00	1.600E
34	3.300E+01	-1.426E+01	5.082E+00	3.787E-01	0.000E+00	1.600E
35	3.400E+01	-1.307E+01	5.295E+00	3.844E-01	0.000E+00	1.520E
36	3.500E+01	-1.187E+01	5.543E+00	3.850E-01	0.000E+00	1.520E
37	3.600E+01	-1.068E+01	5.790E+00	3.855E-01	0.000E+00	1.520E
38	3.700E+01	-9.487E+00	6.037E+00	3.860E-01	0.000E+00	1.520E
39	3.800E+01	-8.298E+00	6.283E+00	3.865E-01	0.000E+00	1.520E
40	3.900E+01	-7.110E+00	6.530E+00	3.871E-01	0.000E+00	1.520E
41	4.000E+01	-5.924E+00	6.775E+00	3.876E-01	0.000E+00	1.520E
42	4.100E+01	-4.738E+00	7.021E+00	3.881E-01	0.000E+00	1.520E
43	4.200E+01	-3.553E+00	7.266E+00	3.886E-01	0.000E+00	1.520E
44	4.300E+01	-2.368E+00	7.511E+00	3.891E-01	0.000E+00	1.520E
45	4.400E+01	-1.184E+00	7.756E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 216.000 HRS 9.000 DAYS FROST HEAVE EQUALS .84 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
9.0	-839.235	-3.500	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-8.392E+02	-3.500E+00	1.881E-01	2.939E-01	1.535E
2	1.000E+00	-8.365E+02	-3.157E+00	1.883E-01	3.331E-01	1.465E
3	2.000E+00	-8.345E+02	-2.970E+00	1.886E-01	3.594E-01	1.429E
4	3.000E+00	-8.324E+02	-2.635E+00	1.889E-01	3.591E-01	1.430E
5	4.000E+00	-8.304E+02	-2.370E+00	1.892E-01	3.387E-01	1.457E
6	5.000E+00	-8.283E+02	-2.057E+00	1.895E-01	3.382E-01	1.458E
7	6.000E+00	-8.262E+02	-1.796E+00	1.897E-01	3.650E-01	1.422E
8	7.000E+00	-8.242E+02	-1.457E+00	1.900E-01	3.873E-01	1.393E
9	8.000E+00	-8.220E+02	-1.265E+00	1.903E-01	4.053E-01	1.371E
10	9.000E+00	-8.199E+02	-9.115E-01	1.906E-01	4.219E-01	1.352E
11	1.000E+01	-2.474E+02	0.000E+00	3.170E-01	2.711E-01	1.550E
12	1.100E+01	-5.514E+01	0.000E+00	3.756E-01	8.409E-02	1.570E
13	1.200E+01	-4.328E+01	0.000E+00	3.809E-01	8.699E-03	1.570E
14	1.300E+01	-3.940E+01	1.693E-01	3.826E-01	0.000E+00	1.570E
15	1.400E+01	-3.812E+01	4.286E-01	3.832E-01	0.000E+00	1.570E
16	1.500E+01	-3.685E+01	6.856E-01	3.838E-01	0.000E+00	1.570E
17	1.600E+01	-3.558E+01	9.417E-01	3.843E-01	0.000E+00	1.570E
18	1.700E+01	-3.431E+01	1.198E+00	3.849E-01	0.000E+00	1.570E
19	1.800E+01	-3.305E+01	1.453E+00	3.855E-01	0.000E+00	1.570E
20	1.900E+01	-3.179E+01	1.709E+00	3.860E-01	0.000E+00	1.570E
21	2.000E+01	-3.053E+01	1.964E+00	3.866E-01	0.000E+00	1.570E
22	2.100E+01	-2.928E+01	2.218E+00	3.872E-01	0.000E+00	1.570E
23	2.200E+01	-2.803E+01	2.473E+00	3.722E-01	0.000E+00	1.600E
24	2.300E+01	-2.679E+01	2.700E+00	3.728E-01	0.000E+00	1.600E
25	2.400E+01	-2.555E+01	2.927E+00	3.734E-01	0.000E+00	1.600E
26	2.500E+01	-2.431E+01	3.153E+00	3.740E-01	0.000E+00	1.600E

27	2.600E+01	-2.307E+01	3.379E+00	3.746E-01	0.000E+00	1.600E
28	2.700E+01	-2.184E+01	3.605E+00	3.752E-01	0.000E+00	1.600E
29	2.800E+01	-2.061E+01	3.830E+00	3.757E-01	0.000E+00	1.600E
30	2.900E+01	-1.938E+01	4.055E+00	3.763E-01	0.000E+00	1.600E
31	3.000E+01	-1.816E+01	4.280E+00	3.769E-01	0.000E+00	1.600E
32	3.100E+01	-1.694E+01	4.504E+00	3.775E-01	0.000E+00	1.600E
33	3.200E+01	-1.572E+01	4.727E+00	3.780E-01	0.000E+00	1.600E
34	3.300E+01	-1.450E+01	4.951E+00	3.786E-01	0.000E+00	1.600E
35	3.400E+01	-1.329E+01	5.174E+00	3.843E-01	0.000E+00	1.520E
36	3.500E+01	-1.207E+01	5.432E+00	3.849E-01	0.000E+00	1.520E
37	3.600E+01	-1.086E+01	5.691E+00	3.854E-01	0.000E+00	1.520E
38	3.700E+01	-9.649E+00	5.949E+00	3.860E-01	0.000E+00	1.520E
39	3.800E+01	-8.440E+00	6.206E+00	3.865E-01	0.000E+00	1.520E
40	3.900E+01	-7.232E+00	6.464E+00	3.870E-01	0.000E+00	1.520E
41	4.000E+01	-6.025E+00	6.721E+00	3.875E-01	0.000E+00	1.520E
42	4.100E+01	-4.819E+00	6.977E+00	3.881E-01	0.000E+00	1.520E
43	4.200E+01	-3.614E+00	7.233E+00	3.886E-01	0.000E+00	1.520E
44	4.300E+01	-2.409E+00	7.489E+00	3.891E-01	0.000E+00	1.520E
45	4.400E+01	-1.204E+00	7.745E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

TIME= 240.000 HRS 10.000 DAYS FROST HEAVE EQUALS 1.02 CM

DAY	UP PRESS BC	UP TEMP BC	LO PRESS BC	LO TEMP BC
10.0	-839.235	-3.500	.000	8.000

NOTE : "*" INDICATES THAT THE EFFECTIVE STRESS HAS BEEN SET EQUAL TO ZERO

NODE	DEPTH	PRESS	TEMP	WAT.CONT	ICE CONT	DENSIT
1	0.000E+00	-8.392E+02	-3.500E+00	1.881E-01	2.939E-01	1.535E
2	1.000E+00	-8.365E+02	-3.219E+00	1.883E-01	3.331E-01	1.465E
3	2.000E+00	-8.345E+02	-2.984E+00	1.886E-01	3.594E-01	1.429E
4	3.000E+00	-8.324E+02	-2.691E+00	1.889E-01	3.591E-01	1.430E
5	4.000E+00	-8.304E+02	-2.430E+00	1.892E-01	3.387E-01	1.457E
6	5.000E+00	-8.283E+02	-2.148E+00	1.895E-01	3.382E-01	1.458E
7	6.000E+00	-8.262E+02	-1.890E+00	1.897E-01	3.650E-01	1.422E
8	7.000E+00	-8.242E+02	-1.600E+00	1.900E-01	3.873E-01	1.393E
9	8.000E+00	-8.220E+02	-1.379E+00	1.903E-01	4.053E-01	1.371E
10	9.000E+00	-8.199E+02	-1.175E+00	1.906E-01	4.219E-01	1.352E
11	1.000E+01	-8.178E+02	-7.206E-01	1.909E-01	4.106E-01	1.365E
12	1.100E+01	-2.440E+02	0.000E+00	3.043E-01	2.805E-01	1.509E
13	1.200E+01	-4.809E+01	0.000E+00	3.788E-01	6.989E-02	1.570E
14	1.300E+01	-3.761E+01	0.000E+00	3.834E-01	3.131E-05	1.570E
15	1.400E+01	-3.636E+01	2.456E-01	3.840E-01	0.000E+00	1.570E
16	1.500E+01	-3.517E+01	5.205E-01	3.845E-01	0.000E+00	1.570E
17	1.600E+01	-3.399E+01	7.831E-01	3.851E-01	0.000E+00	1.570E
18	1.700E+01	-3.280E+01	1.043E+00	3.856E-01	0.000E+00	1.570E
19	1.800E+01	-3.162E+01	1.302E+00	3.861E-01	0.000E+00	1.570E
20	1.900E+01	-3.043E+01	1.562E+00	3.867E-01	0.000E+00	1.570E
21	2.000E+01	-2.925E+01	1.822E+00	3.872E-01	0.000E+00	1.570E
22	2.100E+01	-2.807E+01	2.081E+00	3.877E-01	0.000E+00	1.570E
23	2.200E+01	-2.689E+01	2.341E+00	3.728E-01	0.000E+00	1.600E
24	2.300E+01	-2.570E+01	2.573E+00	3.733E-01	0.000E+00	1.600E
25	2.400E+01	-2.453E+01	2.804E+00	3.739E-01	0.000E+00	1.600E
26	2.500E+01	-2.335E+01	3.035E+00	3.744E-01	0.000E+00	1.600E

27	2.600E+01	-2.217E+01	3.266E+00	3.750E-01	0.000E+00	1.600E
28	2.700E+01	-2.100E+01	3.496E+00	3.756E-01	0.000E+00	1.600E
29	2.800E+01	-1.982E+01	3.726E+00	3.761E-01	0.000E+00	1.600E
30	2.900E+01	-1.865E+01	3.956E+00	3.767E-01	0.000E+00	1.600E
31	3.000E+01	-1.748E+01	4.186E+00	3.772E-01	0.000E+00	1.600E
32	3.100E+01	-1.631E+01	4.415E+00	3.778E-01	0.000E+00	1.600E
33	3.200E+01	-1.514E+01	4.644E+00	3.783E-01	0.000E+00	1.600E
34	3.300E+01	-1.397E+01	4.873E+00	3.789E-01	0.000E+00	1.600E
35	3.400E+01	-1.280E+01	5.101E+00	3.845E-01	0.000E+00	1.520E
36	3.500E+01	-1.163E+01	5.366E+00	3.851E-01	0.000E+00	1.520E
37	3.600E+01	-1.047E+01	5.631E+00	3.856E-01	0.000E+00	1.520E
38	3.700E+01	-9.303E+00	5.895E+00	3.861E-01	0.000E+00	1.520E
39	3.800E+01	-8.139E+00	6.159E+00	3.866E-01	0.000E+00	1.520E
40	3.900E+01	-6.975E+00	6.423E+00	3.871E-01	0.000E+00	1.520E
41	4.000E+01	-5.811E+00	6.687E+00	3.876E-01	0.000E+00	1.520E
42	4.100E+01	-4.648E+00	6.950E+00	3.881E-01	0.000E+00	1.520E
43	4.200E+01	-3.486E+00	7.213E+00	3.886E-01	0.000E+00	1.520E
44	4.300E+01	-2.324E+00	7.475E+00	3.891E-01	0.000E+00	1.520E
45	4.400E+01	-1.162E+00	7.738E+00	3.896E-01	0.000E+00	1.520E
46	4.500E+01	-7.363E-05	8.000E+00	3.900E-01	0.000E+00	1.520E

****

****

SUMMARY OF RESULTS

DAY	CUMULATIVE HEAVE (CM)			HEAVE RATE	ISR	FROST DEPTH	THAW DEPTH
	MIN	MAX	MEAN	CM/HR		CM	CM
1.0	.00	.00	.00	.000	.000	.00	9999.00
2.0	.00	.00	.00	.000	.000	.00	9999.00
3.0	.00	.00	.00	.000	.000	.00	9999.00
4.0	.00	.00	.00	.000	.000	.00	9999.00
5.0	.00	.12	.05	.002	.013	4.00	.00
6.0	.00	.76	.34	.012	.040	8.00	.00
7.0	.00	1.26	.56	.009	.058	9.00	.00
8.0	.00	1.57	.70	.006	.065	10.00	.00
9.0	.00	1.90	.84	.006	.066	12.00	.00
10.0	.00	2.30	1.02	.007	.078	12.00	.00

THE MAXIMUM COEF OF VARIATION OF SIMULATED HEAVE IS .629
MEAN HEAVE IS WITHIN THE INDICATED BOUNDS WITH
AT LEAST A 95% CONFIDENCE (MIN=MEAN-2*SIGMA OR ZERO
AND MAX=MEAN+2*SIGMA) FOR HYD COND CV OF .600

SURFACE TEMP DIURNAL VARIATION EQUAL .00 CELCIUS

APPENDIX F: EXAMPLE WORK SHEET

FROST PROBLEM SETUP

Line

1 Title ____

2 Nodal Domain Method ________, Time Solu. Method Heat ________, Time Solu. Method Moisture ________ (1 = Fully Implicit, 2 = Crank-Nicolson)

3 Const. Initial Conditions (?) ________, suppress some output (?) ________, const. elem. lengths (?) ________, zero __0__, include conv. heat. trans (?) ________, output computed parameters (?) ________, input E-factor (?) ________ (1 = yes, 0 = no)

4 No. of nodes, ________ no of layers ________

5 No flux upper pore pressure B.C. __1__, specified lower press. B.C. (?) ________, specified upper temp. B.C. (?) ________, specified lower temp. B.C. (?) ________ (1 = Natural, 0 = Specified)

Element Geometry (max. 100 elements) (only one line required if third entry line 3=1)

6a Length of first element ________

b Length of second element ________

____
____
____
____
____
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____

7 Time step (hr) ________, update freq. ________, output freq. (days) ________, simulation length (days) ________ (update freq. = number of time steps between updates of computed parameters)

8 Surcharge (psi) ________, freezing point depression ________ pore press. modifier __1__. (Pore pressure modified is for thaw conditions, normally set to 1.0)

Layer Data (max. 10)

	A_w	a	θ_o
9a			
b			
c			
d			
e			
f			
g			
h			
i			
j			

	C_s	K_s	One	ρ_s	θ_a
10a			1.0		
b			1.0		
c			1.0		
d			1.0		
e			1.0		
f			1.0		
g			1.0		
h			1.0		
i			1.0		
j			1.0		

	k_s	A_k	b	E	m_v
11a					
b					
c					
d					
e					
f					
g					
h					
i					
j					

(Omit E if internally calculated)

<u>Line</u>

Line				
12a	Lower node No. of layer	________,	layer number	________
b		________		________
c		________		________
d		________		________
e		________		________
f		________		________
g		________		________
h		________		________
i		________		________
j		________		________

13 Coefficient of variation of hydraulic conductivity ________

<u>Initial Conditions Each Node</u> (only one line required if first entry line 3=1)

	Pore pressure head	Temperature	Ice content
14a	________	________	________
b	________	________	________
c	________	________	________
d	________	________	________
e	________	________	________
f	________	________	________
g	________	________	________
h	________	________	________
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(use additional sheets if required)

Line

Boundary Condition Data (up to 300 data sets)

15 Upper pressure head B.C. during thawing ________

16 No of data sets for surface temp. ________, no. of data sets for lower pore press. head ________, no. of data sets lower temp. ________, amplitude of diurnal temperature variation ________

Surface Temp. Data

	Temperature	Hour	*n*-factor
17a	________	________	________
b	________	________	________
c	________	________	________
d	________	________	________
e	________	________	________
f	________	________	________
g	________	________	________
h	________	________	________

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(use additional sheets if required)

Lower Pore Press. Head

	Pore Press. Head	Hour
18a	________	________
b	________	________
c	________	________
d	________	________
e	________	________
f	________	________
g	________	________
h	________	________
.		
.		
.		
.		

(use additional sheets if required)

Lower Temperature

	Temperature	Hour
19a	________	________
b	________	________
c	________	________
d	________	________
e	________	________
f	________	________
g	________	________
h	________	________
⋮		

(use additional sheets if required)

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Since 1975 the U.S. Army Corps of Engineers, the Federal Highway Administration and the Federal Aviation Administration have been working cooperatively to develop a mathematical model to estimate frost heave and thaw weakening under various environmental conditions and for various pavement designs. A model has been developed. It is a one-dimensional representation of vertical heat and moisture flux, is based on a numerical solution technique termed the nodal domain integration method, and estimates frost heave and frost penetration reasonably well for a variety of situations. The model is now ready for additional field evaluation and implementation in appropriate cases. The main objectives of this report are: 1) to describe the model, FROST, including modeling uncertainties and errors; 2) to summarize recent comparisons between measured and computed values for frost heave and frost penetration; and 3) to describe parameters necessary for input into the model.

14. SUBJECT TERMS
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