

AD-A265 920



AEROSPACE REPORT NO.
TR-0091(6945-08)-1

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Light Scattering from a Spherical Particle on a Conducting Plane: Part I. Normal Incidence

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1 May 1993

Prepared for

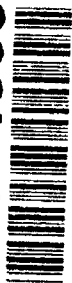
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El Segundo, California

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93-13660



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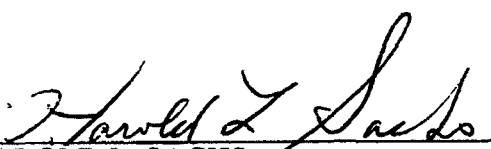
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This report was submitted by The Aerospace Corporation, El Segundo, CA 90245-4691, under Contract No. F04701-88-C-0089 with the Space and Missile Systems Center, P. O. Box 92960, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by A. B. Christensen, Principal Director, Space and Environment Technology Center.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.


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 25 MAR 93
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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) TR-0091(6945-08)-1		5. MONITORING ORGANIZATION REPORT NUMBER(S) SMC-TR-93-30	
6a. NAME OF PERFORMING ORGANIZATION The Aerospace Corporation Technology Operations	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Space and Missile Systems Center	
6c. ADDRESS (City, State, and ZIP Code) El Segundo, CA 90245-4691		7b. ADDRESS (City, State, and ZIP Code) Los Angeles Air Force Base Los Angeles, CA 90009-2960	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F04701-88-C-0089	
8c. ADDRESS (City, State, and ZIP Code)		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO.	TASK NO.
		PROJECT NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Light Scattering from a Spherical Particle on a Conducting Plane: Part I. Normal Incidence			
12. PERSONAL AUTHOR(S) Johnson, Bernard R.			
13a. TYPE OF REPORT	13b. TIME COVERED FROM _____ TO _____	14. DATE OF REPORT (Year, Month, Day) 1993 May 1	15. PAGE COUNT 50
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	BRDF	
		Electromagnetic-wave	
		Light Scattering	
		Mirror	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
<p>A new procedure is developed for calculating light scattering from a spherical particle on a conducting plane. The discussion is limited to problems in which the incident light beam is perpendicular to the plane. A general proof is given for the method of images, which is used to convert the problem of a particle on a conducting plane to an equivalent two-particle problem. The scattering solution for the two-particle problem is then obtained by the multipole expansion method. Mirror symmetry is exploited to simplify the calculation. Formulas are developed for the extinction and absorption cross sections. Several example problems are calculated, and the exact solutions are compared to the results from two commonly used approximation procedures.</p>			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT		21. ABSTRACT SECURITY CLASSIFICATION	
<input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS		Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL		22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

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DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
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Availability Codes	
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I. INTRODUCTION

A new theoretical procedure is developed for calculating light scattering by a spherical particle that is on or near a conducting plane surface (mirror). This computational capability is useful for both scientific investigations and engineering design studies. The scattered light, for example, can be used to detect and measure particles on a flat reflecting surface. This is relevant for certain manufacturing processes where surfaces must be maintained in an uncontaminated condition.¹ The present work originated in the problem of modeling stray radiation in an optical system due to light that is scattered from particles contaminating the surfaces of mirrors. Such stray radiation can seriously degrade performance of the system.² In addition, if the radiation is intense, the energy absorbed by the particle can cause local heating, which may damage the mirror or otherwise cause problems. Another possible application of the theory is to the study of morphology dependent resonances (MDR) of small particles.³ In recent years many studies have been carried out that exploit resonance effects in isolated particles.^{4,5} Interesting new resonance effects may be possible if the particle is on or near a conducting surface.

Previous work^{2,6,7} on light scattering from a particle on a mirror has included both experimental and theoretical studies. Prior theoretical calculations have been based on approximation methods. The method developed in this report is not an approximation, but rather it provides a numerically exact solution to Maxwell's equations.

Section II begins with a general discussion of the scattering problem and a derivation of the method of images. This method allows one to replace the system consisting of the sphere and the conducting plane with an equivalent system consisting of the sphere and an image sphere. The equivalent two-particle system is then solved by the multipole expansion method.^{8,9} Mirror symmetry is exploited to simplify the calculation. The section concludes with a derivation of formulas for the extinction and absorption cross sections. Except for the discussion of the method of images, the procedures developed in this section are limited to cases in which the incident radiation propagates normal to the plane of the mirror. This limitation is not fundamental to the method and will be removed in a future work. Section III reviews two simple approximation techniques that have been used in previous studies of light scattering from a particle on a mirror. These methods are based on Mie theory. It is important to test the accuracy of these approximations by comparing them to exact results. In section IV, we solve several test cases and compare the exact results to the results obtained by the two approximation procedures discussed in section III. Section V ends the report with a few concluding remarks.

II. THEORY

Consider the system shown in Fig. 1a. This system is more general than the special case to be considered in this report. We begin with this system because the method of images holds for the general case and we did not want to limit the discussion to a specialized case.

The $z = 0$ plane is assumed to be the surface of a perfect conductor (infinite conductivity). One or more particles are located on or above this plane in the region $z > 0$. The particles can have arbitrary shapes and nonuniform composition. The electrical characteristics of the system are completely described by specifying the dielectric constant $\epsilon(\mathbf{r})$, the conductivity $\sigma(\mathbf{r})$, and the magnetic permeability $\mu(\mathbf{r})$ as functions of the position coordinate \mathbf{r} throughout the region $z > 0$. In the region of space not occupied by the particles, it is assumed that $\epsilon(\mathbf{r}) = 1$, $\mu(\mathbf{r}) = 1$, and $\sigma(\mathbf{r}) = 0$ (Gaussian units). Inside the particle, these functions take on the values characteristic of the particle. On the $z = 0$ plane, the conductivity is assumed to be infinite. A plane wave, \mathbf{E}_{inc} , is incident on the system. This gives rise to a reflected plane wave, \mathbf{E}_{ref} , and a scattered wave, \mathbf{E}_{scat} . In the region outside of the particles, the total electric field is the sum of these three contributions, $\mathbf{E} = \mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{ref}} + \mathbf{E}_{\text{scat}}$.

The electromagnetic field has a harmonic time dependence given by $\exp(-i\omega t)$. In the region above the reflecting plane, it must satisfy the time harmonic form of Maxwell's equations

$$\begin{aligned}\nabla \times \mathbf{H} + ik\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)\mathbf{E} &= 0 \\ \nabla \times \mathbf{E} - ik\mu\mathbf{H} &= 0 \\ \nabla \cdot (\epsilon\mathbf{E}) &= 0 \\ \nabla \cdot (\mu\mathbf{H}) &= 0\end{aligned}\tag{1}$$

where $k = \omega/c$, with c equal to the velocity of light. The field must also satisfy the following boundary conditions on the $z = 0$ plane:

$$\begin{aligned}\hat{\mathbf{n}} \times \mathbf{E} &= 0 \\ \hat{\mathbf{n}} \cdot \mathbf{H} &= 0\end{aligned}\tag{2}$$

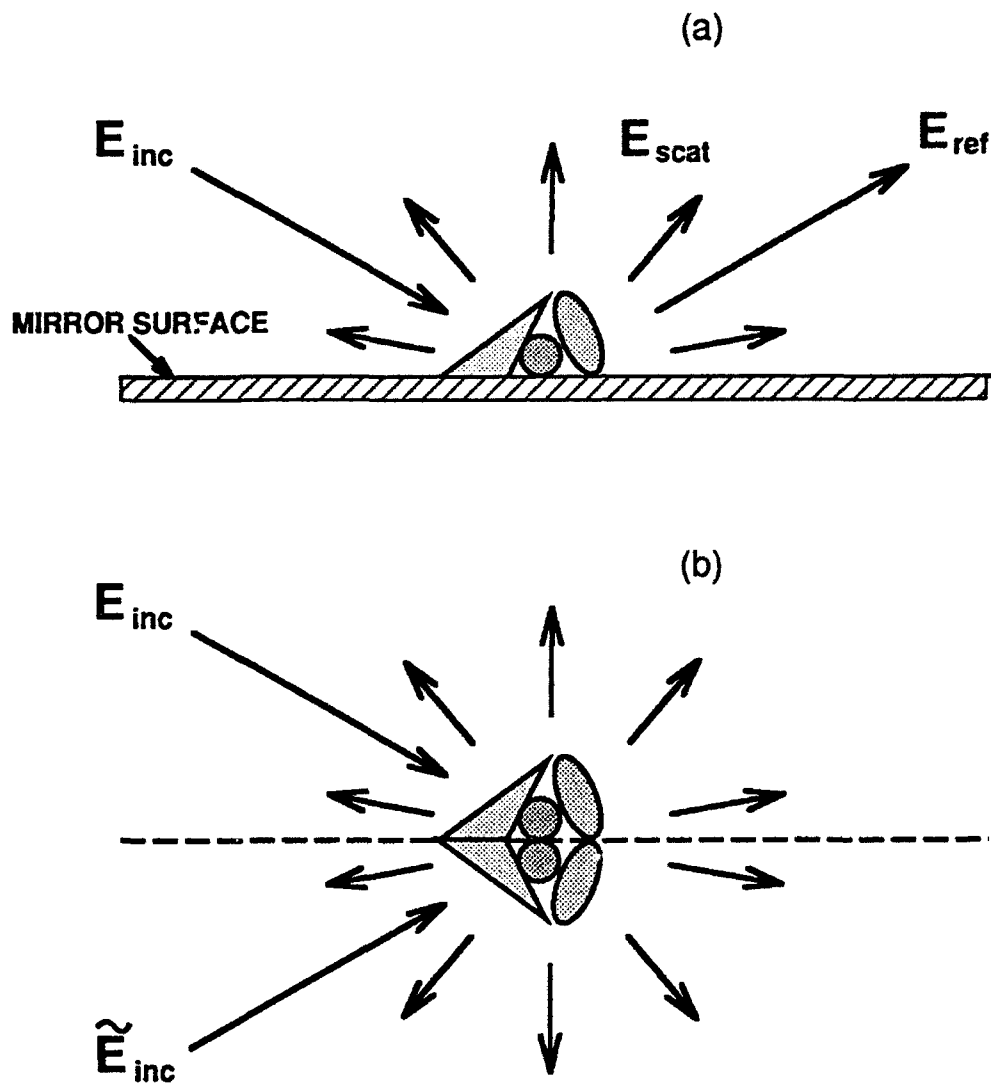


Fig. 1. Illustration of the method of images. (a) Physical system, (b) Equivalent "method of images" system.

where \hat{n} is the unit vector normal to the plane. These conditions, simply stated, say that there can be no tangential component to the electric field and no normal component to the magnetic field at $z = 0$.

The solution to this problem, stated in its present form, is quite difficult to solve. A technique very similar to the method of images in electrostatic problems can be used to reformulate the problem in a much simpler form without explicit boundary conditions at the surface of the mirror.

A. METHOD OF IMAGES

The method of images is intuitively obvious. An observer looking down on the system will see the particles and mirror images of the particles and also the source of the incident wave (e.g., a laser) and a mirror image of the source. The image of the source is the apparent origin of the reflected wave. This apparent picture is turned into reality by creating the system shown in Fig. 1b, where the mirror has been removed, the images of the particles have been replaced by real particles, and the image of the source has been replaced by a real source. The image source is the origin of the plane wave $\tilde{\mathbf{E}}_{inc}$. By definition, $\tilde{\mathbf{E}}_{inc} = \mathbf{E}_{ref}$ in the region $z > 0$. The functions ϵ , σ , and μ , in this model, are symmetric with respect to the z coordinate; i.e., $\epsilon(x,y,z) = \epsilon(x,y,-z)$, $\sigma(x,y,z) = \sigma(x,y,-z)$ and $\mu(x,y,z) = \mu(x,y,-z)$.

By the principle of linear independence, each of the incident waves, \mathbf{E}_{inc} and $\tilde{\mathbf{E}}_{inc}$, can be considered separately. \mathbf{E}_{inc} gives rise to the scattered wave \mathbf{E}_{scat} , and $\tilde{\mathbf{E}}_{inc}$ gives rise to $\tilde{\mathbf{E}}_{scat}$. In each of these separate scattering problems, a plane wave is scattered by a multiparticle system. It is obviously not necessary to solve both of these problems since the solutions are related by symmetry. Assume that we have solved the scattering problem for the field $\mathbf{E} = \mathbf{E}_{inc} + \mathbf{E}_{scat}$. Write the solution in Cartesian coordinates

$$\mathbf{E}(x,y,z) = E_x(x,y,z)\hat{e}_x + E_y(x,y,z)\hat{e}_y + E_z(x,y,z)\hat{e}_z \quad (3a)$$

where \hat{e}_x , \hat{e}_y , and \hat{e}_z are unit vectors along the x , y , and z axes, respectively. The accompanying magnetic field is written similarly

$$\mathbf{H}(x,y,z) = H_x(x,y,z)\hat{e}_x + H_y(x,y,z)\hat{e}_y + H_z(x,y,z)\hat{e}_z \quad (3b)$$

These fields are a solution to the scattering problem and therefore must satisfy Maxwell's equations. We now claim that the field $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\text{inc}} + \tilde{\mathbf{E}}_{\text{scat}}$ and its accompanying magnetic field $\tilde{\mathbf{H}}$ are related to \mathbf{E} and \mathbf{H} by the expressions

$$\begin{aligned}\tilde{\mathbf{E}}(x,y,z) &= -E_x(x,y,-z)\hat{\mathbf{e}}_x - E_y(x,y,-z)\hat{\mathbf{e}}_y + E_z(x,y,-z)\hat{\mathbf{e}}_z \\ \tilde{\mathbf{H}}(x,y,z) &= H_x(x,y,-z)\hat{\mathbf{e}}_x + H_y(x,y,-z)\hat{\mathbf{e}}_y - H_z(x,y,-z)\hat{\mathbf{e}}_z\end{aligned}\tag{4}$$

This claim is justified by the following argument. Insert the above expressions for $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ into Maxwell's equations and use the symmetry of the functions $\epsilon(x,y,z)$, $\sigma(x,y,z)$, and $\mu(x,y,z)$ with respect to z . From this analysis, we see that if $\mathbf{E}(x,y,z)$ and $\mathbf{H}(x,y,z)$ satisfy Maxwell's equations, then $\tilde{\mathbf{E}}(x,y,z)$ and $\tilde{\mathbf{H}}(x,y,z)$ will also satisfy these equations. The total electric and magnetic fields for the scattering problem shown in Fig. 1b are obtained by summing the two linearly independent contributions

$$\begin{aligned}\mathbf{E}_{\text{total}} &= \mathbf{E} + \tilde{\mathbf{E}} \\ \mathbf{H}_{\text{total}} &= \mathbf{H} + \tilde{\mathbf{H}}\end{aligned}\tag{5}$$

It is obvious from Eqs. (3) and (4) that the total fields $\mathbf{E}_{\text{total}}$ and $\mathbf{H}_{\text{total}}$ satisfy the boundary conditions given by Eq. (2). They also satisfy Maxwell's equations. Therefore, in the region $z > 0$, these fields are a solution to the scattering problem shown in Fig. 1a.

B. MULTIPOLE EXPANSION SOLUTION

The physical problem to be considered in this report is illustrated in Fig. 2. The center of a spherical particle of radius R is located a distance d (where $d \geq R$) above a perfectly conducting plane surface. The center is assumed to lie on the z axis and the conducting surface to coincide with the $z = 0$ plane. A circularly polarized plane wave traveling in the negative z direction, $\mathbf{E}_{\text{inc}} = -(\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y) e^{-ikz}$, is incident on this system. By the method of images, this physical problem is converted to the two-particle scattering problem shown in Fig. 3. In addition to the

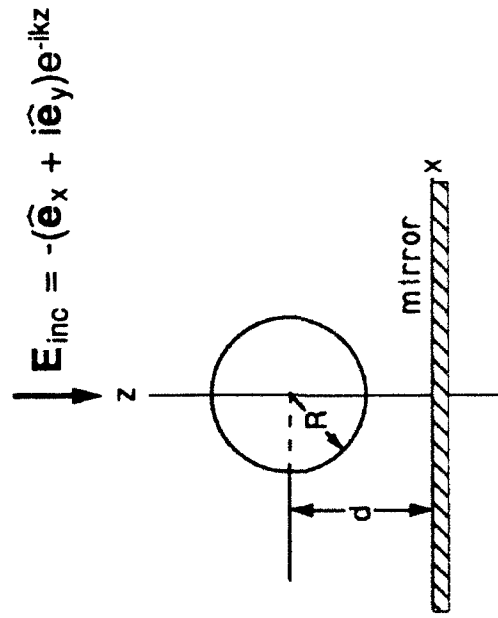


Fig. 2. Geometry of the particle on a mirror problem.

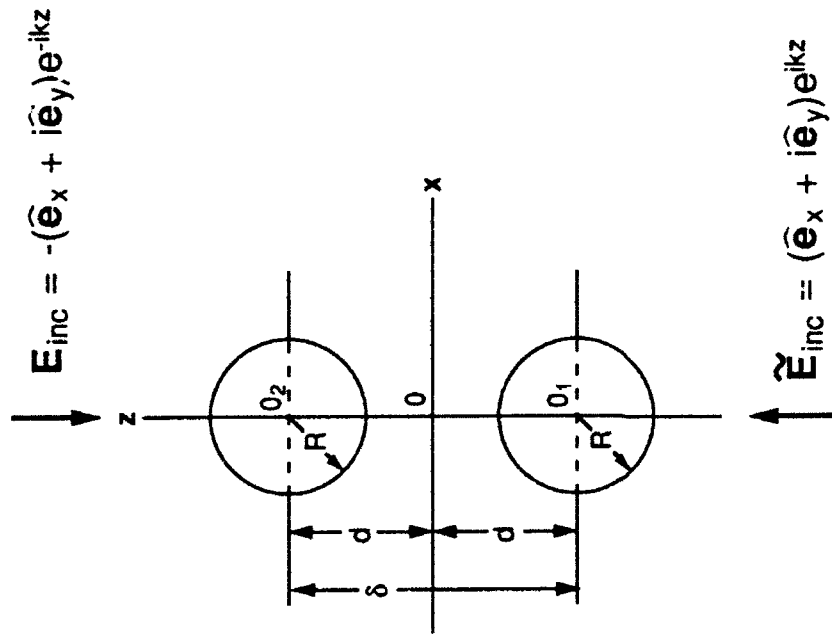


Fig. 3. Geometry of the two-particle problem that is equivalent, by the method of images, to the problem shown in Fig. 2.

incident wave, an image wave, traveling in the positive z direction, $\tilde{E}_{inc} = (\hat{e}_x + i\hat{e}_y) e^{ikz}$, is also present. The total incident wave for the two-particle problem is the sum of these two waves

$$E_i = (\hat{e}_x + i\hat{e}_y) (e^{ikz} - e^{-ikz}) \quad (6)$$

This wave satisfies the boundary conditions given by Eq. (2) on the $z = 0$ plane.

In the region outside the spheres, the total electric field is conveniently written as the sum of the incident wave and two contributions to the scattered wave

$$E = E_i + E_s^{(1)} + E_s^{(2)} \quad (7)$$

The two scattered waves, $E_s^{(1)}$ and $E_s^{(2)}$, propagate radially outward from the centers of the two spheres, O_1 and O_2 , respectively (see Fig. 3).

The electric fields can be expanded in terms of the vector spherical wave functions

$$M_{n,m}^{(j)} = z_n^{(j)}(kr) e^{im\phi} X_{n,m}(\theta)$$

and

$$N_{n,m}^{(j)} = \frac{e^{im\phi}}{kr} \left\{ \frac{\partial}{\partial r} [r z_n^{(j)}(kr)] Y_{n,m}(\theta) + z_n^{(j)}(kr) Z_{nm}(\theta) \right\} \quad (8)$$

where $z_n^{(j)}$ is a spherical Bessel function of type $j_n, h_n^{(1)}$ for $j = 1, 3$ respectively. The vector functions X , Y , and Z are defined by

$$X_{n,m}(\theta) = i \pi_{n,m}(\theta) \hat{e}_\theta - \tau_{n,m}(\theta) \hat{e}_\phi$$

$$Y_{n,m}(\theta) = \tau_{n,m}(\theta) \hat{e}_\theta + i \pi_{n,m}(\theta) \hat{e}_\phi \quad (9)$$

$$Z_{n,m}(\theta) = n(n+1) P_n^m(\cos\theta) \hat{e}_r$$

where

$$\pi_{n,m}(\theta) = \frac{m}{\sin(\theta)} P_n^m(\cos \theta)$$

$$\tau_{n,m}(\theta) = \frac{\partial}{\partial \theta} P_n^m(\cos \theta) \quad (10)$$

The function $P_n^m(x)$ is the associated Legendre polynomial, and $(\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi)$ are unit orthogonal vectors associated with the spherical coordinates r, θ, ϕ .

In the analysis that follows, only the functions $M_{n,m}$ and $N_{n,m}$ with $m = 1$ will be needed. Therefore, to simplify the notation, the subscript $m = 1$ will not be explicitly written. It will be understood that $M_n \equiv M_{n,1}$ and $N_n \equiv N_{n,1}$. Using the standard formulas for the expansion of a linearly polarized plane wave,^{10,11} we can derive the following formula for the expansion of a circularly polarized plane wave

$$(\hat{e}_x + i \hat{e}_y) e^{\pm i k z} = \mp \sum_{n=1}^{\infty} (\pm i)^{n+1} \frac{2n+1}{n(n+1)} [M_n^{(1)}(r, \theta, \phi) \pm N_n^{(1)}(r, \theta, \phi)] \quad (11)$$

In the discussions that follow, the coordinates r, θ, ϕ are defined with respect to coordinate axes with origin at 0 (see Fig. 3). A point in this coordinate space will also be represented by the boldfaced position vector \mathbf{r} . In addition, it will be useful to introduce two other coordinate systems with origins, O_1 and O_2 , located at the centers of the two spheres. The coordinates in these systems will be identified with the subscripts 1 and 2, respectively [i.e., \mathbf{r}_j or (r_j, θ_j, ϕ_j) ; $j = 1, 2$]. These three sets of coordinates are related to each other by translations along the z axis.

The incident wave, defined by Eq. (6), can be expanded in terms of spherical wave functions centered at O_1

$$\mathbf{E}_i(\mathbf{r}_1) = \sum_{n=1}^{\infty} [p_n M_n^{(1)}(\mathbf{r}_1) + q_n N_n^{(1)}(\mathbf{r}_1)] \quad (12)$$

The expansion coefficients in this equation can be obtained with the aid of Eq. (11). They are

$$\begin{aligned} p_n &= -i^{n+1} \frac{2n+1}{n(n+1)} [e^{-ikd} - (-1)^n e^{ikd}] \\ q_n &= -i^{n+1} \frac{2n+1}{n(n+1)} [e^{-ikd} + (-1)^n e^{ikd}] \end{aligned} \quad (13)$$

where d is the displacement of the center of the sphere from origin 0 (see Fig. 3). In addition, the

scattered waves $E_S^{(1)}$ and $E_S^{(2)}$ can be expanded around centers O_1 and O_2 , respectively,

$$E_S^{(j)}(r_j) = \sum_{n=1}^{\infty} [a_n^{(j)} M_n^{(3)}(r_j) + b_n^{(j)} N_n^{(3)}(r_j)]; \quad j = 1, 2 \quad (14)$$

The scattering coefficients $a_n^{(1)}$, $b_n^{(1)}$, $a_n^{(2)}$, and $b_n^{(2)}$ are to be determined by the calculation. This task is made somewhat easier by exploiting the mirror symmetry of the system and the boundary conditions on the $z = 0$ plane. These conditions imply the following relations between the scattering coefficients (see appendix A):

$$a_n^{(1)} = -(-1)^n a_n^{(2)} \quad (15)$$

$$b_n^{(1)} = (-1)^n b_n^{(2)}$$

To carry out the calculations of the scattering coefficients, it is convenient to expand the fields in the coordinate system with origin at O_1 . The fields E_i and $E_S^{(1)}$ are already in this form. $E_S^{(2)}$ is not, but can be converted to this form by means of the translation-addition theorem for vector spherical wave functions. A general formulation and discussion of this theorem is given in references 8 and 9. The special form of this theorem, needed for the present problem, is the following:

$$M_n^{(3)}(r_2) = \sum_{n'=1}^{\infty} [A_{n,n'} M_{n'}^{(1)}(r_1) + B_{n,n'} N_{n'}^{(1)}(r_1)] \quad (16)$$

$$N_n^{(3)}(r_2) = \sum_{n'=1}^{\infty} [A_{n,n'} N_{n'}^{(1)}(r_1) + B_{n,n'} M_{n'}^{(1)}(r_1)]$$

The formulas for the expansion coefficients $A_{n,n'}$ and $B_{n,n'}$ and a discussion of the method for calculating these quantities are presented in appendix B.

The electric field in the region outside the spheres is given by Eq. (7). The multipole expansion of this field around origin O_1 can be derived with the aid of Eqs. (12), (14), and (16). The result is

$$\begin{aligned}
E = \sum_n \{ [p_n + \sum_{n'} (A_{n',n} a_{n'}^{(2)} + B_{n',n} b_{n'}^{(2)})] M_n^{(1)}(r_1) + a_n^{(1)} M_n^{(3)}(r_1) \\
+ [q_n + \sum_{n'} (B_{n',n} a_{n'}^{(2)} + A_{n',n} b_{n'}^{(2)})] N_n^{(1)}(r_1) + b_n^{(1)} N_n^{(3)}(r_1) \}
\end{aligned} \tag{17}$$

This equation is of the form

$$E = \sum_n (\alpha_n M_n^{(1)} + a_n^{(1)} M_n^{(3)} + \beta_n N_n^{(1)} + b_n^{(1)} N_n^{(3)}) \tag{18}$$

The ratios of the amplitudes of the scattered wave multipoles ($M^{(3)}$ and $N^{(3)}$) to the incident wave multipoles ($M^{(1)}$ and $N^{(1)}$) are

$$\begin{aligned}
u_n &= \frac{a_n^{(1)}}{\alpha_n} \\
v_n &= \frac{b_n^{(1)}}{\beta_n}
\end{aligned} \tag{19}$$

The quantities u_n and v_n are elements of the so-called scattering T matrix¹² for the particle. For a homogeneous sphere, u_n and v_n are the Mie theory coefficients for the TE and TM scattering modes, respectively. The formulas for u_n and v_n , for the homogeneous dielectric sphere, are given in appendix C. [It should be noted that it is not necessary for the particle to be homogeneous. If the index of refraction is a function of the radius, u_n and v_n can still be calculated by an appropriate theory. For example, the theory of Aden and Kerker¹³ or Bhandari¹⁴ can be used for a layered sphere, or the theory of Wyatt¹⁵ can be used for a sphere with continuous radially varying index of refraction.]

Assume now that the Mie theory coefficients, u_n and v_n , have been calculated for the particle. Substitute the expressions for α_n and β_n , obtained from Eq. (17), into Eq. (19) to obtain

$$\begin{aligned}
a_n^{(1)} &= u_n \left[p_n + \sum_{n'} (A_{n',n} a_{n'}^{(2)} + B_{n',n} b_{n'}^{(2)}) \right] \\
b_n^{(1)} &= v_n \left[q_n + \sum_{n'} (B_{n',n} a_{n'}^{(2)} + A_{n',n} b_{n'}^{(2)}) \right]
\end{aligned} \tag{20}$$

The sums over n' in Eq. (20) converge and can be truncated to the range $1 \leq n' \leq N$, where N is the number of modes required for convergence of a Mie theory calculation for scattering from the isolated spherical particle.

These equations can be cast in a much more elegant and easy-to-use form by expressing all the quantities as either square matrices or column vectors. The quantities $a_n^{(1)}$, $b_n^{(1)}$, $a_n^{(2)}$, $b_n^{(2)}$, p_n , and q_n are elements of the (length N) column vectors: $\mathbf{a}^{(1)}$, $\mathbf{b}^{(1)}$, $\mathbf{a}^{(2)}$, $\mathbf{b}^{(2)}$, \mathbf{p} , and \mathbf{q} . The quantities $A_{n',n}$ and $B_{n',n}$ are elements of the N by N square matrices \mathbf{A} and \mathbf{B} . The scattering coefficients u_n and v_n are elements of the diagonal matrices $\mathbf{u} = [u_n \delta_{n,n}]$ and $\mathbf{v} = [v_n \delta_{n,n}]$. It is also useful to define the diagonal matrix $\mathbf{g} = [(-1)^n \delta_{n,n}]$. The set of $2N$ linear equations, Eq. (20), can now be written in the super matrix form

$$\begin{pmatrix} \mathbf{A}^T + \mathbf{g}\mathbf{u}^{-1} & \mathbf{B}^T \\ \mathbf{B}^T & \mathbf{A}^T - \mathbf{g}\mathbf{v}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{a}^{(2)} \\ \mathbf{b}^{(2)} \end{pmatrix} = - \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix} \quad (21)$$

where we have taken account of Eq. (15) to eliminate $\mathbf{a}^{(1)} = -\mathbf{g}\mathbf{a}^{(2)}$ and $\mathbf{b}^{(1)} = \mathbf{g}\mathbf{b}^{(2)}$ from the equations. The superscript T indicates the transpose matrix. Standard linear equation-solving subroutines are available on most computers to solve this matrix equation for the scattering coefficients $\mathbf{a}^{(2)}$ and $\mathbf{b}^{(2)}$.

C. CROSS SECTIONS

The scattered wave is the sum of the two scattering components

$$\mathbf{E}_{\text{scat}} = \mathbf{E}_s^{(1)} + \mathbf{E}_s^{(2)} \quad (22)$$

The asymptotic form of this wave can be evaluated in the limit $r \rightarrow \infty$ with the aid of Eq. (14) and the following asymptotic formulas for the spherical wave functions:

$$\begin{aligned} \mathbf{M}_n^{(3)} &= (-i)^n \frac{e^{ikr}}{ikr} \mathbf{X}_n(\theta) e^{i\phi} \\ \mathbf{N}_n^{(3)} &= i(-i)^n \frac{e^{ikr}}{ikr} \mathbf{Y}_n(\theta) e^{i\phi} \end{aligned} \quad (23)$$

The result is

$$\begin{aligned} \mathbf{E}_{\text{scat}} = & \frac{e^{ikr_1}}{ikr_1} e^{i\phi_1} \sum_{n=1}^N (-i)^n \left[a_n^{(1)} \mathbf{X}_n(\theta_1) + i b_n^{(1)} \mathbf{Y}_n(\theta_1) \right] \\ & + \frac{e^{ikr_2}}{ikr_2} e^{i\phi_2} \sum_{n=1}^N (-i)^n \left[a_n^{(2)} \mathbf{X}_n(\theta_2) + i b_n^{(2)} \mathbf{Y}_n(\theta_2) \right] \end{aligned} \quad (24)$$

This formula is expressed in terms of the two coordinates systems, (r_1, θ_1, ϕ_1) and (r_2, θ_2, ϕ_2) . To be useful, it must be expressed in terms of a single set of coordinates (r, θ, ϕ) . The asymptotic transformation relations between the coordinates, in the limit $r \rightarrow \infty$, are

$$\begin{aligned} r_1 &= r + d \cos(\theta) \\ r_2 &= r - d \cos(\theta) \\ \theta_1 &= \theta_2 = \theta \\ \phi_1 &= \phi_2 = \phi \end{aligned} \quad (25)$$

Substitute these transformation relations into Eq. (24) to obtain the scattered wave in terms of r, θ and ϕ . This wave can be written in the form

$$\mathbf{E}_{\text{scat}} = \frac{e^{ikr}}{ikr} e^{i\phi} \left[S_\theta(\theta) \hat{\mathbf{e}}_\theta + i S_\phi(\theta) \hat{\mathbf{e}}_\phi \right] \quad (26)$$

where the vector components of the scattering amplitude are given by

$$\begin{aligned} S_\theta(\theta) &= e^{ikd\cos(\theta)} S_\theta^{(1)}(\theta) + e^{-ikd\cos(\theta)} S_\theta^{(2)}(\theta) \\ S_\phi(\theta) &= e^{ikd\cos(\theta)} S_\phi^{(1)}(\theta) + e^{-ikd\cos(\theta)} S_\phi^{(2)}(\theta) \end{aligned} \quad (27)$$

and where

$$\begin{aligned} S_\theta^{(j)}(\theta) &= - \sum_{n=1}^N (-i)^{n+1} \left[a_n^{(j)} \pi_n(\theta) + b_n^{(j)} \tau_n(\theta) \right] \\ S_\phi^{(j)}(\theta) &= - \sum_{n=1}^N (-i)^{n+1} \left[a_n^{(j)} \tau_n(\theta) + b_n^{(j)} \pi_n(\theta) \right] \end{aligned} \quad (28)$$

for $j = 1, 2$.

The formulas for the differential scattering cross section can be taken directly from the similar formulas obtained in Mie theory.^{10,11} If the incident beam is right or left circularly polarized or if it is unpolarized, the differential scattering cross section is given by

$$\sigma(\theta) = \frac{1}{2k^2} \left[|S_\theta(\theta)|^2 + |S_\phi(\theta)|^2 \right] \quad (29)$$

If the incident beam is plane polarized, with the electric field vector parallel to the x axis, the differential cross section is given by

$$\sigma(\theta) = \frac{1}{k^2} \left[|S_\theta(\theta)|^2 \cos^2(\phi) + |S_\phi(\theta)|^2 \sin^2(\phi) \right] \quad (30)$$

If the electric field is polarized parallel to the y axis, the factors $\cos^2(\phi)$ and $\sin^2(\phi)$ in the above formula are interchanged. The angle θ in these formulas is restricted to the region above the reflecting plane, i.e., $0 \leq \theta \leq \pi/2$.

The total scattering cross section is the integral of the differential cross section over the 2π steradian solid angle above the $z = 0$ plane. For either of the differential cross section formulas, given in Eqs. (29) or (30), the result is

$$C_{\text{scat}} = \frac{\pi}{k^2} \int_0^{\pi/2} \left[|S_\theta(\theta)|^2 + |S_\phi(\theta)|^2 \right] \sin(\theta) d\theta \quad (31)$$

In Mie theory, it is possible to substitute the series expansions for $S_\theta(\theta)$ and $S_\phi(\theta)$ into Eq. (31) and then use the orthogonality properties of the functions $\pi_n(\theta)$ and $\tau_n(\theta)$ to obtain a simple formula for the total scattering cross section. This approach is not feasible in the present case because the functions $S_\theta(\theta)$ and $S_\phi(\theta)$ are much more complicated than in Mie theory. Instead, the scattering cross section is calculated using the general relation

$$C_{\text{scat}} = C_{\text{ext}} - C_{\text{abs}} \quad (32)$$

where C_{ext} is the extinction cross section and C_{abs} is the absorption cross section.

The extinction cross section can be computed with the aid of the optical theorem¹¹

$$C_{\text{ext}} = -\frac{4\pi}{k^2} \text{Re } S(0) \quad (33)$$

where $S(0) = S_\theta(0) = S_\phi(0)$ is the scattering amplitude in the forward direction $\theta = 0$. Use the relations $\pi_n(0) = \tau_n(0) = n(n+1)/2$ and Eqs. (27) and (28) to obtain

$$C_{\text{ext}} = \frac{2\pi}{k^2} \text{Im} \sum_{n=1}^N (-i)^n n(n+1) [e^{ikd} (a_n^{(1)} + b_n^{(1)}) + e^{-ikd} (a_n^{(2)} + b_n^{(2)})] \quad (34)$$

The absorption cross section is calculated by utilizing formulas for power absorption developed in Mie theory. The Mie theory formula for the power absorbed by spherical particle scattering the plane wave $(\hat{e}_x + i \hat{e}_y) e^{ikz}$ is

$$P = F \left\{ -\frac{2\pi}{k^2} \sum_{n=1}^N (2n+1) [(|u_n|^2 + \text{Re } u_n) + (|v_n|^2 + \text{Re } v_n)] \right\} \quad (35)$$

where F is the energy flux of the plane wave and the quantity in braces is the absorption cross section. We now interpret this formula in terms of the contributions of each of the multipole components of the plane wave to the total power absorbed by the particle. Define $P_n^{(M)}$ and $P_n^{(N)}$ to be the power absorbed from the n 'th TE and TM multipole components, respectively, of the incident plane wave

$$\begin{aligned} P_n^{(M)} &= F \left[-\frac{2\pi}{k^2} (2n+1) (|u_n|^2 + \text{Re } u_n) \right] \\ P_n^{(N)} &= F \left[-\frac{2\pi}{k^2} (2n+1) (|v_n|^2 + \text{Re } v_n) \right] \end{aligned} \quad (36)$$

The expansion of the plane wave is given by Eq. (11). The amplitude of each of the multipole components of this wave is

$$f_n = - \left[i^{n+1} \frac{2n+1}{n(n+1)} \right] \quad (37)$$

The power absorbed from each multipole component is proportional to the square of the amplitude f_n . Therefore, to obtain the power absorbed from a multipole of unit amplitude, we must divide the results given in Eq. (36) by $|f_n|^2$. The result is

$$\hat{P}_n^{(M)} = \left[\frac{n(n+1)}{2n+1} \right]^2 P_n^{(M)} \quad (38)$$

$$\hat{P}_n^{(N)} = \left[\frac{n(n+1)}{2n+1} \right]^2 P_n^{(N)}$$

The wave, which is incident on particle 1, has a multipole expansion that can be deduced from Eq. (18):

$$E = \sum_n (\alpha_n M_n^{(1)} + \beta_n N_n^{(1)}) \quad (39)$$

The expansion coefficients, obtained from Eq. (19), are

$$\alpha_n = \frac{a_n^{(1)}}{u_n} \quad (40)$$

$$\beta_n = \frac{b_n^{(1)}}{v_n}$$

The total energy absorbed by particle 1 is obtained by multiplying $\hat{P}_n^{(M)}$ and $\hat{P}_n^{(N)}$ [defined in Eq. (38)] by the squares of the amplitudes $|\alpha_n|^2$ and $|\beta_n|^2$ and summing over all the terms. The absorption cross section is then obtained by dividing this result by the energy flux F . The final result is

$$C_{\text{abs}} = -\frac{2\pi}{k^2} \sum_{n=1}^N \frac{[n(n+1)]^2}{2n+1} \left[\left| \frac{a_n^{(1)}}{u_n} \right|^2 (|u_n|^2 + \text{Re } u_n) + \left| \frac{b_n^{(1)}}{v_n} \right|^2 (|v_n|^2 + \text{Re } v_n) \right] \quad (41)$$

These formulas were tested by comparing the scattering cross section obtained by numerically integrating the differential cross section, given by Eq. (31), with the results obtained using Eqs. (32), (34), and (41). The tests were carried out using both real and complex values for the index of refraction. The results were in complete agreement, within the numerical accuracy of the calculation.

III. APPROXIMATION METHODS

Approximation methods have been used in previous studies to calculate the differential scattering cross section for a particle on a conducting plane.^{2,6} Two of these approximations will be reviewed in this section. In addition, a technique known as the order of scattering (OS) method will also be presented, and the relationship of this method to one of the approximations will be discussed.

A. FORWARD SCATTER MIE APPROXIMATION

This method is illustrated in Fig. 4. Mie theory is used to calculate the scattering from the spherical particle. The forward scattered rays are reflected from the mirror back to the detector. The back scattered rays are usually much weaker than the forward scattered rays and are therefore neglected. The mirror is assumed to have no effect on the Mie scattering cross section. The only effect of the mirror is to reflect the forward scattered rays. Young² developed this approximation and obtained good agreement with experimental results. Nahm and Wolfe⁶ also used this method, which they refer to as the unobstructed reflection model.

B. SINGLE SCATTER APPROXIMATION

The single scatter approximation (SSA) method is also based on Mie theory. This method takes account of all of the ways in which a ray in the incident beam can interact only once with the particle and be deflected to the detector. The four ways this can happen are illustrated in Fig. 5. This method neglects all processes in which a ray is scattered two or three or more times by the particle. This would happen, for example, if a ray were deflected by the particle toward the mirror, then were reflected back and struck the particle a second time, and then were deflected to the detector. Such multiple scattering processes are ignored in this approximation. The amplitudes of the four scattered rays (waves) shown in Fig. 5 are added, taking account of the phase differences due to the different distances that the incident rays must travel to reach the particle, the different distances that the scattered rays must travel from the particle to the detector, and the phase shift of π radians caused by reflection from the mirror. Only a single Mie theory calculation is required to carry out this calculation; therefore, the method is very efficient. This method is almost identical to the double interaction model (DIM) of Nahm and Wolfe.⁶ The DIM includes an "area factor," which is not included in the SSA model.

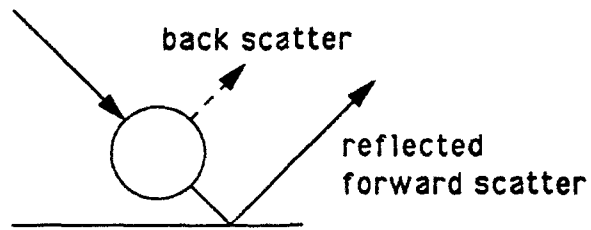


Fig. 4. Forward scatter Mie approximation.
Backscatter is ignored.

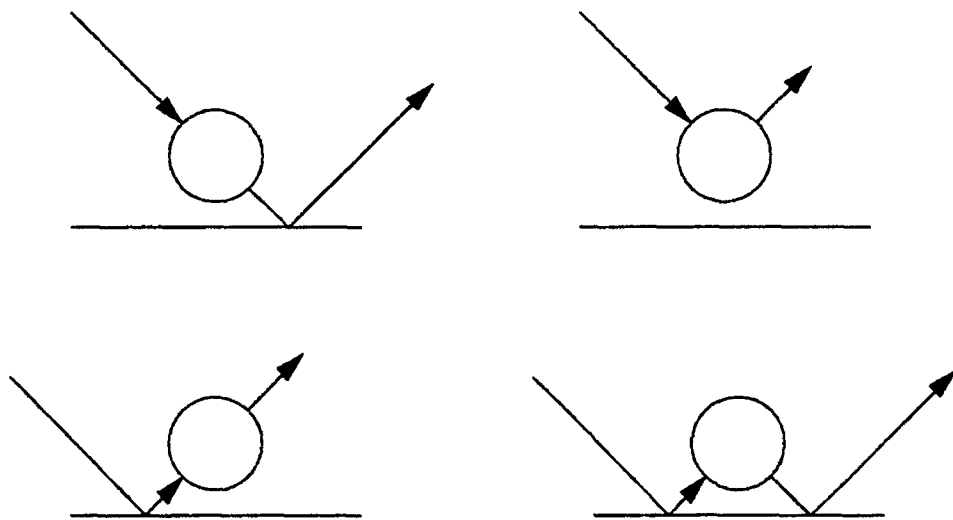


Fig. 5. Single scatter approximation (SSA). The amplitudes of these four single scatter events are added, taking account of phase differences due to the different path lengths.

The SSA can be considered to be the first term of an infinite series expansion in which the second term takes account of all double scattering events, the third term takes account of all triple scattering events and so on. The general technique of summing these multiple scattering events is known as the OS method. This method and its relationship to the exact theory will be discussed next.

C. ORDER OF SCATTERING METHOD

Most of the computational effort in solving the exact scattering problem is due to Eq. (21). This matrix equation is of the form

$$(U^{-1} - C)F = Q \quad (42)$$

where

$$C = \begin{pmatrix} A^T & B^T \\ B^T & A^T \end{pmatrix} \quad (43)$$

$$U = \begin{pmatrix} -gu & 0 \\ 0 & gv \end{pmatrix} \quad (44)$$

$$F = \begin{pmatrix} a^{(2)} \\ b^{(2)} \end{pmatrix} \quad (45)$$

and

$$Q = \begin{pmatrix} p \\ q \end{pmatrix} \quad (46)$$

The formal solution to Eq. (42) is

$$F = (U^{-1} - C)^{-1} Q \quad (47)$$

The inverse matrix in this equation can be evaluated by the following infinite series expansion

$$(U^{-1} - C)^{-1} = U + UCU + UCUCU + \dots \quad (48)$$

If this series converges rapidly, it is a useful procedure for solving Eq. (42). This technique was developed by Fuller and Kattawer¹⁶ and is used to calculate scattering from an ensemble of spheres. The first term of the series represents the single scattering contribution, the second term is the double scattering contribution, and so on. If we retain only the first term, the result is the SSA method. The scattering coefficients for this case can be easily calculated:

$$\begin{aligned} a_n^{(1)} &= u_n p_n \\ b_n^{(1)} &= v_n q_n \end{aligned} \quad (49)$$

This requires a single Mie theory calculation to obtain u_n and v_n .

Based on the simple ray optics picture of the scattering process, one would expect the probability of double scattering events to be small relative to single scattering events whenever the scattering cross section is small (i.e., for small particles) or when the distance d of the particle from the mirror is large. Under these circumstances, the single scatter approximation is expected to be accurate and should converge to the exact result in the limit that the particle size approaches zero or the distance d approaches infinity. In the next section, we will see that these expectations are apparently borne out.

IV. RESULTS

The results of several test calculations, which compare the predictions of the exact theory with the predictions of the approximation methods (the forward scatter Mie approximation and the single scatter approximation method), are presented. These results give a preliminary estimate of the accuracy of the approximation methods. The cross sections are given in reduced units of area. The reduced unit of length, in this system, is defined to be the wavelength of the scattered light (i.e., in these units, $\lambda = 1$). Results for particles of three different sizes are shown in Figs. 6, 7, and 8. The radii of these particles, in reduced units, are $R = 0.2, 1.0$, and 2.5 . The particles are resting on the surface of the mirror: therefore, the distance parameters are given by $d = R$. The index of refraction is $n = 1.46$. As anticipated, the SSA results are fairly good for the small particle size.

In each of the cases shown in Figs. 6-8, the Mie theory approximation gives cross sections for small angle scattering ($0 \leq \theta \leq 20^\circ$) that are smaller than the exact cross sections. To dispel any notion that this might always be true, we show Fig. 9, where the index of refraction has been changed to $n = 1.3$. The other parameters are the same as in Fig. 7, i.e., $R = 1.0$ and $d = 1.0$. The Mie theory results are now much larger than the exact results for low angle scattering.

Figure 10 shows results for a particle that is suspended above the plane of the mirror. The parameters for this case are $R = 1.0$, $n = 1.3$, and $d = 5.0$ (i.e., the center of the sphere is 5.0 units above the mirror). The SSA method, as expected, is a fairly good approximation for this case.

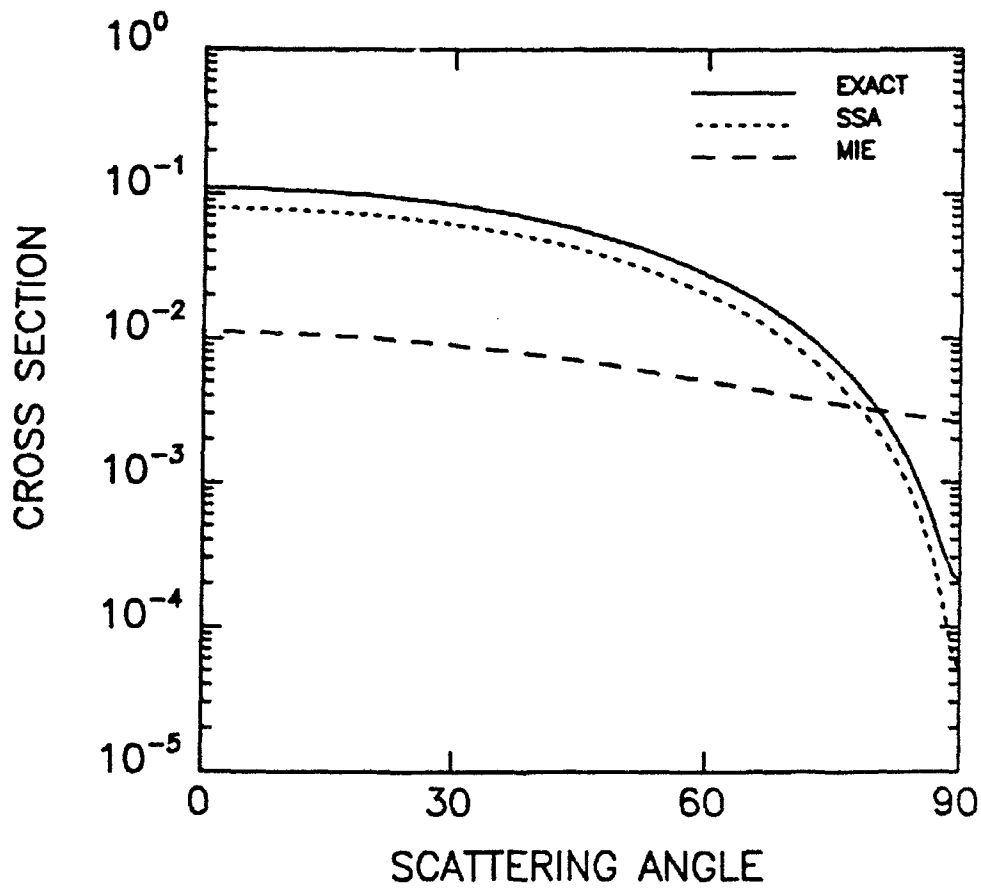


Fig. 6. Cross section for light scattering from a particle on a mirror. $R = 0.2$, $d = 0.2$, $n \approx 1.46$. Comparison of exact solution with the forward scatter Mie approximation (Mie) and the single scatter approximation (SSA).

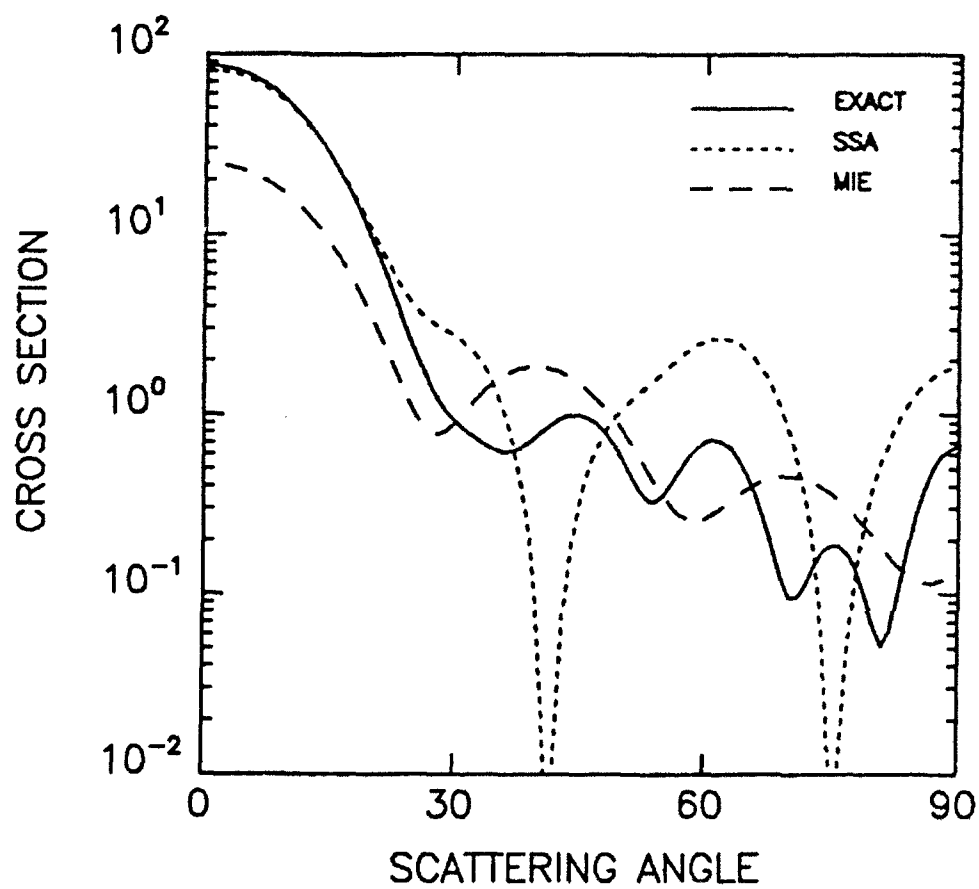


Fig. 7. Cross section for light scattering from a particle on a mirror. $R = 1.0$, $d = 1.0$, $n = 1.46$. Comparison of exact solution with the forward scatter Mie approximation (Mie) and the single scatter approximation (SSA).

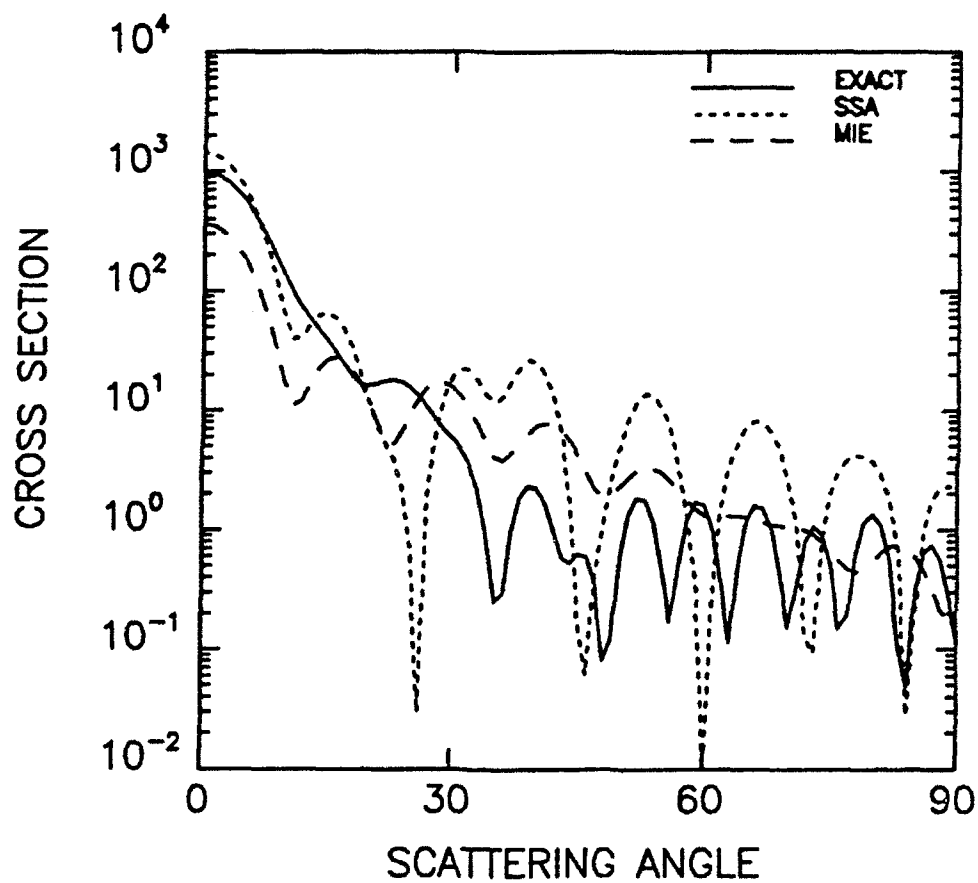


Fig. 8. Cross section for light scattering from a particle on a mirror. $R = 2.5$, $d = 2.5$, $n = 1.46$. Comparison of exact solution with the forward scatter Mie approximation (Mie) and the single scatter approximation (SSA).

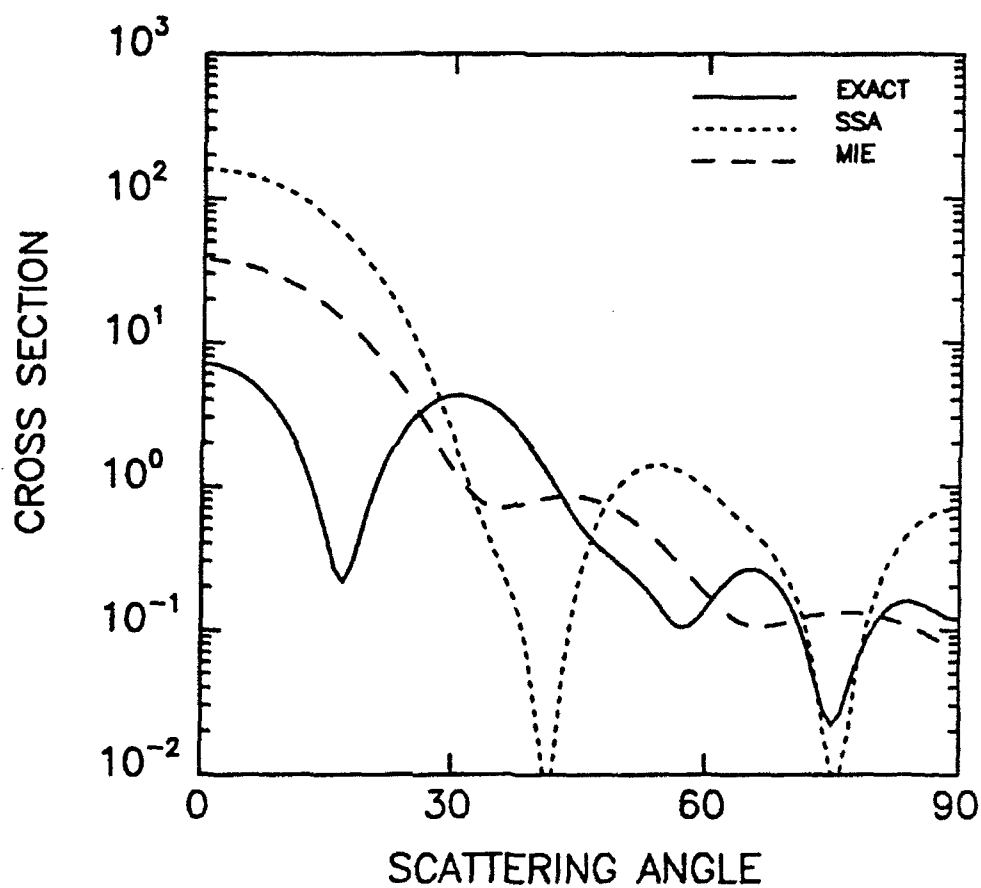


Fig. 9. Cross section for light scattering from a particle on a mirror. $R = 1.0$, $d = 1.0$, $n = 1.3$. Comparison of exact solution with the forward scatter Mie approximation (Mie) and the single scatter approximation (SSA).

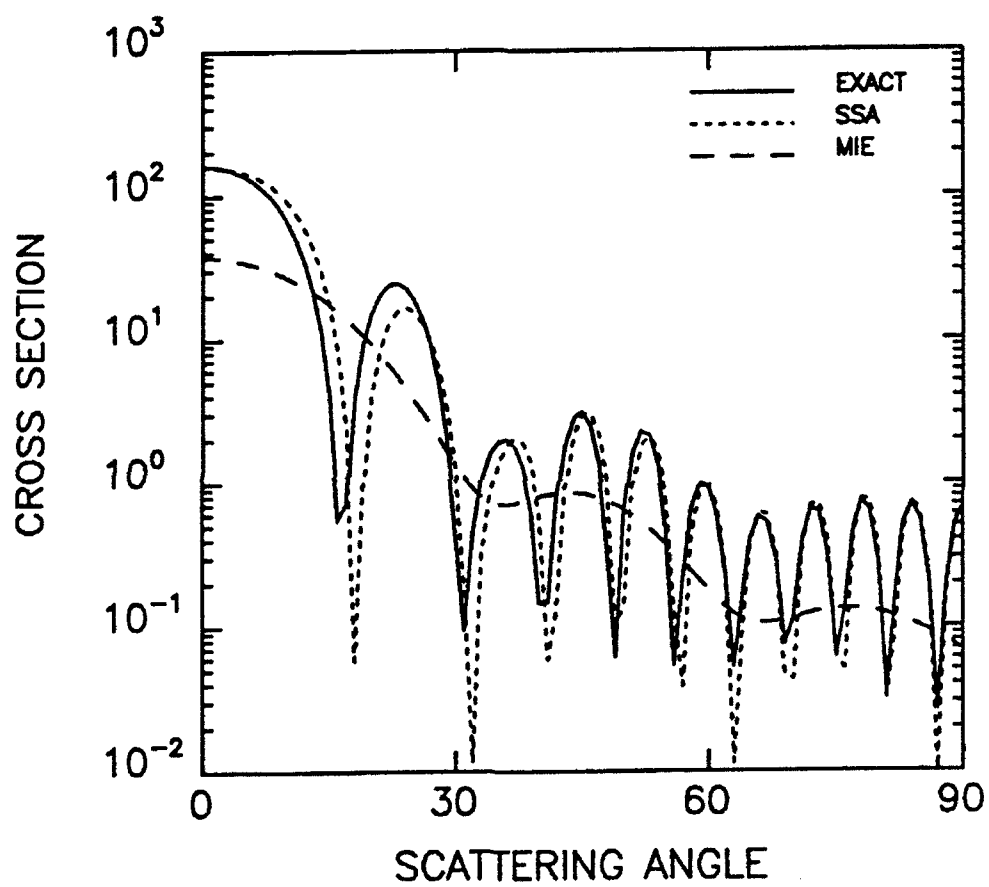


Fig. 10. Cross section for light scattering from a particle near a mirror. $R = 1.0$, $d = 5.0$, $n = 1.3$. Comparison of exact solution with the forward scatter Mie approximation (Mie) and the single scatter approximation (SSA).

V. CONCLUDING REMARKS

This report describes a practical, efficient computational method for calculating light scattering from a spherical particle on a conducting surface. The computational efficiency of this method makes it possible to eliminate, for the most part, the need to use approximation methods. We are presently using this procedure to calculate mirror Bidirectional Reflectance Distribution Functions (BRDF)² due to various distributions of particulate contamination on the surface of a mirror.

The present form of the theory is restricted to the case in which the incident light propagates in a direction normal to the surface of the mirror. This limitation will be removed in future work.

The listing of a Fortran computer code that implements the procedures outlined in this report is presented in Appendix D .

APPENDIX A

SCATTERING COEFFICIENT SYMMETRY RELATIONS

The symmetry relations expressed by Eq. (15) in the text are a consequence of the symmetry of the system and the boundary conditions on the $z = 0$ plane. The boundary conditions imply that at any point on this plane, the component of the electric field that is parallel to the plane must vanish. This applies to the total field given by Eq. (7). Since the incident field, E_i , satisfies this condition separately, it follows that the scattered field, $E_{\text{scat}} = E_s^{(1)} + E_s^{(2)}$, must also satisfy the condition separately.

To carry out the analysis, we choose an arbitrary point P on the $z = 0$ plane. Assume that this point has coordinates r_1, θ_1, ϕ_1 with respect to the coordinate system centered at O_1 . Then, the coordinates of P with respect to the O_2 coordinate system are given by

$$\begin{aligned} r_2 &= r_1 \\ \theta_2 &= \pi - \theta_1 \\ \phi_2 &= \phi_1 \end{aligned} \tag{A1}$$

At point P, the unit vectors \hat{e}_ϕ , \hat{e}_{ϕ_1} and \hat{e}_{ϕ_2} are collinear and lie in the $z = 0$ plane. Thus, the \hat{e}_ϕ component of the total scattered field is obtained by algebraically adding the \hat{e}_{ϕ_1} and \hat{e}_{ϕ_2} components of the $E_s^{(1)}$ and $E_s^{(2)}$ fields. The \hat{e}_{ϕ_1} and \hat{e}_{ϕ_2} components of the scattered field can be obtained with aid of Eqs. (8), (9), and (14). The result is

$$[E_s^{(j)}]_{\phi_j} = - \sum_{n=1}^N \exp(i\phi_j) \left(z_n^{(3)}(kr_j) [a_n^{(j)} \tau_n(\theta_j)] - i \left[\frac{1}{kr_j} \frac{\partial}{\partial r_j} [r_j z_n^{(3)}(kr_j)] \right] [b_n^{(j)} \pi_n(\theta_j)] \right) \tag{A2}$$

where $j = 1, 2$. The boundary conditions at P require that

$$[E_s^{(1)}]_{\phi_1} + [E_s^{(2)}]_{\phi_2} = 0 \tag{A3}$$

This relation can only be true if it holds for each individual multipole component of the field. This implies that

$$a_n^{(1)} \tau_n(\theta_1) + a_n^{(2)} \tau_n(\theta_2) = 0$$

and

$$b_n^{(1)} \pi_n(\theta_1) + b_n^{(2)} \pi_n(\theta_2) = 0 \tag{A4}$$

Use Eq. (A1) and the following symmetry relations for the π_n and τ_n functions

$$\pi_n(\pi-\theta) = -(-1)^n \pi_n(\theta) \tag{A5}$$

$$\tau_n(\pi-\theta) = (-1)^n \tau_n(\theta)$$

to obtain the desired results given by Eq. (15) in the text.

APPENDIX B

THE TRANSLATION-ADDITION THEOREM

The special form of the translation-addition theorem used in this report is given by Eq. (16) in the text. This formula expresses the vector spherical wavefunctions $M_n^{(3)}(\mathbf{r}_2)$ and $N_n^{(3)}(\mathbf{r}_2)$ about origin O_2 in terms of wave functions $M_n^{(1)}(\mathbf{r}_1)$ and $N_n^{(1)}(\mathbf{r}_1)$ about origin O_1 , where O_2 is displaced a distance $\delta = 2d$ along the z axis from O_1 (see Fig. 3). The expansion coefficients, $A_{n,n'}$ and $B_{n,n'}$, which appear in Eq. (16), are obtained from the general formulas given in the appendix of reference 6:

$$A_{n,n'} = -i^{n'-n} \frac{2n'+1}{2n'(n'+1)} \sum_v i^{-v} [n(n+1) + n'(n'+1) - v(v+1)] a(n,n';v) h_v^{(1)}(k\delta) \quad (B1)$$

$$B_{n,n'} = i^{n'-n} \frac{2n'+1}{2n'(n'+1)} \sum_v i^{-v} (2ik\delta) a(n,n';v) h_v^{(1)}(k\delta) \quad (B2)$$

where $h_v^{(1)}(k\delta)$ is the spherical Hankel function of the first kind and $a(n,n';v)$ is special form of the Guant coefficient¹⁷, defined by the following product of two 3-j symbols

$$a(n,n';v) = (2v+1) \left[\frac{n(n+1)}{n'(n'+1)} \right]^{1/2} \begin{pmatrix} n & n' & v \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} n & n' & v \\ 1 & -1 & 0 \end{pmatrix} \quad (B3)$$

The definition of the 3-j symbol involves the summation of many factorial terms. As a result, a straightforward evaluation of $a(n,n';v)$ is very inefficient. Bruning and Lo⁹ found a way to overcome this difficulty by developing a recurrence relation for $a(n,n';v)$ that cycles on the index v .

We did not use this method. Instead, we found it more convenient to use a procedure developed by Schulten and Gordon¹⁸ for calculating the 3-j symbols. This procedure is also based on a recurrence relation that cycles on the index v . It was more convenient because we could make use of an existing computer subroutine available to us to do the calculations. We found this approach to be very efficient.

APPENDIX C

FORMULAS FOR THE u_n AND v_n COEFFICIENTS

The u_n and v_n coefficients are defined by Eq. (19) in the text. For the case of a homogeneous sphere, they are identical to the Mie theory " a_n and b_n " coefficients for the TE and TM scattering modes, respectively. Since there are many different definitions of these quantities in the literature, it seems advisable to give the formulas for these quantities that are consistent with their usage in this report. The definitions we use are the same as those defined by Stratton¹⁰ (Section 9.25). If we take the permeability, μ , of the particle and the surrounding medium to be the same, these coefficients are given by

$$u_n = - \frac{\psi_n(mx) \psi'_n(x) - m \psi_n(x) \psi'_n(mx)}{\psi_n(mx) \xi'_n(x) - m \xi_n(x) \psi'_n(mx)} \quad (C1)$$

and

$$v_n = - \frac{\psi_n(x) \psi'_n(mx) - m \psi_n(mx) \psi'_n(x)}{\xi_n(x) \psi'_n(mx) - m \psi_n(mx) \xi'_n(x)} \quad (C2)$$

where m is the complex index of refraction; $x = kR$ is the size parameter of the particle, the prime means to take the derivative with respect to the argument of the function; and $\psi_n(x)$ and $\xi_n(x)$ are Riccati-Bessel functions defined by $\psi_n(x) = x j_n(x)$ and $\xi_n(x) = x h_n^{(1)}(x)$.

APPENDIX D

FORTRAN COMPUTER CODE

On the following pages is a listing of the Fortran computer code that implements the procedures outlined in this report. The program runs interactively and prompts the user for input data. The code is efficient and can quite reasonably be run on a personal computer. The output consists of the differential scattering cross section as well as the extinction, absorption, and total scattering cross sections for both the "spherical particle on a conducting plane" and for the "isolated sphere." The scattering results for the "isolated sphere" (Mie theory) are a natural by-product of the calculation. The results for the isolated sphere are, by definition, the "forward scatter Mie approximation" cross sections.

Following the program listing is a sample output that was calculated using the default input parameters. The cross sections in this example output correspond to the "exact" (and also the "forward scatter Mie") cross sections shown in Fig. 7. (However, many more points were calculated for the graph than are shown in the sample output.)

The code is self-contained except for the IMSL Library subroutine LECT1C, which appears in the subroutine FSOLVE. LECT1C is a complex linear equation solver that solves Eq. (21) in the text. The user must either supply this subroutine or an equivalent subroutine.

```

PROGRAM MIRMX(input,output,tape5=input,tape6=output)
COMMON/RMIE/RAD,RFR,RFI,WL,KAY,LMAX
COMMON/AB/ A(60,60),B(60,60)
COMMON/CMIE/U(100),V(100)
COMMON/GFMAT/G(120,120),F(120)
COMMON/DIFF/NANG,THETA(400),DIFF(400),DIFS(400),QEXT,QSCA,QABS
COMMON/CRS2/Q2EXT,Q2ABS
DIMENSION TWRT(10)
COMPLEX A,B,U,V,G,F,CSUM,UA,UB,VA,VB
REAL KAY

C
C THIS PROGRAM CALCULATES SCATTERING FROM A SPHERICAL
C PARTICLE ON A MIRROR. IT USES THE METHOD OF IMAGES TO
C CONVERT TO AN EQUIVALENT 2 PARTICLE PROBLEM AND THEN USES
C THE METHOD OF BRUNING AND LO TO SOLVE THIS EQUIVALENT
C SCATTERING PROBLEM. (B.R. JOHNSON, MAY. 1990)
C MAJOR CORRECTIONS AND REVISIONS OCT. 1990)
C INTERACTIVE INPUT VERSION DEC. 1990
C
*****
C PARAMETER DEFINITIONS
C WL = WAVELENGTH
C RAD = PARTICLE RADIUS
C DELANG = INTERVAL BETWEEN ANGLES OF DIFFERENTIAL CROSS SECTIONS.
C NANG = NUMBER OF DIFFERENTIAL CROSS SECTIONS CALCULATED.
C RFR = "N" = REAL PART OF INDEX OF REFRACTION.
C RFI = "K" = IMAGINARY PART OF INDEX OF REFRACTION
C DISP = DISTANCE OF CENTER OF SPHERE ABOVE MIRROR.
C LMAX = NUMBER OF MODES IN CALCULATION.
C C2EXT = EXTINCTION CROSS SECTION FOR PARTICLE ON MIRROR.
C C2ABS = ABSORPTION CROSS SECTION FOR PARTICLE ON MIRROR.
C C2SCA = SCATTERING CROSS SECTION FOR PARTICLE ON MIRROR.
C CEXT = EXTINCTION CROSS SECTION FOR ISOLATED PARTICLE.
C CABS = ABSORPTION CROSS SECTION FOR ISOLATED PARTICLE.
C CSCA = SCATTERING CROSS SECTION FOR ISOLATED PARTICLE.
C CDIF = DIFFERENTIAL CROSS SECTION FOR PARTICLE ON MIRROR.
C SDIF = DIFFERENTIAL CROSS SECTION FOR ISOLATED PARTICLE.
*****
C DEFAULT VALUES OF INPUT PARAMETERS
  WL=1.0
  RAD=1.0
  DELANG=5.
  NANG=19
  RFR=1.46
  RFI=0.
  DISP=RAD
C FIXED INPUT PARAMETERS
  999 LMAX=0
  ACRCY=1.E2
  LDIM=60
C READ INPUT VARIABLES
  CALL INPVAR(WL,RAD,DELANG,NANG,RFR,RFI,DISP)
  IF(WL.LE.0.) GO TO 2000
  PI=ACOS(-1.0)
  KAY=2.*PI/WL
  SZP=RAD*KAY
  AREA=PI*RAD*RAD
  CON=PI/180.
  DO 50 I=1,NANG
  50 THETA(I)=FLOAT(I-1)*DELANG*CON
  IF(LMAX.LE.0) CALL LMAXX(SZP,LDIM,ACRCY,LMAX)
C WRITE THE INPUT VARIABLES
  WRITE(6,65)
  65 FORMAT(/' PROGRAM MIRMX'/)
  WRITE(6,100) WL,RAD,SZP,DISP,LMAX,RFR,RFI

```

```

100 FORMAT(' WL = ',1PE12.3/' RAD = ',1PE12.3/' SZP = ',1PE12.3/
+' DISP = ',1PE11.3/' LMAX = ',I4/' RFR = ',1PE12.3/
+' RFI = ',1PE12.3/)
CALL ABMATR(LMAX,KAY,DISP)
CALL MIEUV
CALL GFMTX(LMAX,DISP,KAY,1)
CALL FSOLVE(LMAX)
CALL CRSEC(SZP,DISP,KAY,LMAX)
C PRINT THE CROSS SECTIONS
Q2SCA=Q2EXT-Q2ABS
C2EXT=AREA*Q2EXT
C2SCA=AREA*Q2SCA
C2ABS=AREA*Q2ABS
WRITE(6,600) C2EXT,C2SCA,C2ABS
600 FORMAT(//' PARTICLE ON A MIRROR CROSS SECTIONS'/
+' C2EXT = ',1PE12.5,5X,'C2SCA = ',1PE12.5,5X,'C2ABS = ',1PE12.5/)
CEXT=AREA*QEXT
CSCA=AREA*QSCA
CABS=AREA*QABS
WRITE(6,1050) CEXT,CSCA,CABS
1050 FORMAT(//' SINGLE PARTICLE CROSS SECTIONS'/
+' CEXT = ',1PE12.5,5X,'CSCA = ',1PE12.5,5X,'CABS = ',1PE12.5/)
IF(NANG.EQ.0) GO TO 800
WRITE(6,998)
998 FORMAT(///34X,'SINGLE'/34X,'PARTICLE')
WRITE(6,1000)
1000 FORMAT(5X,'ANGLE',6X,'CROSS SEC',9X,'CROSS SEC'/)
DO 1010 I=1,NANG
CDIF=AREA*DIFF(I)
SDIF=AREA*DIFS(I)
THETD=THETA(I)/CON
WRITE(6,1020) THETD,CDIF,SDIF
1020 FORMAT(F10.3,1PE15.3,3X,1PE15.3)
1010 CONTINUE
800 WRITE(6,1030)
1030 FORMAT(1H '/' *****'/)
GO TO 999
2000 STOP
END
*****
SUBROUTINE LMAXX(X,LDIM,ACRCY,LMAX)
C THIS SUBROUTINE CALCULATES THE MAXIMUM VALUE OF L TO BE USED IN MIE
C SCATTERING FOR A PARTICLE OF SIZE PARAMETER X AND ACCURACY
C PARAMETER ACRCY. LMAX CANNOT EXCEED LDIM.
L=INT(X)
ZR=(2*L+1)/X
TZR=2./X
PROD=1.
RJI=1.
100 L=L+1
ZR=ZR+TZR
RJI=ZR-1./RJI
PROD=PROD*RJI
IF(L.GT.LDIM) GO TO 200
IF(PROD.LT.ACRCY) GO TO 100
LMAX=L
RETURN
200 WRITE(6,400)
400 FORMAT(' ERROR - LMAX IS LARGER THAN THE DIMENSIONED ARRAYS'/
+9X,' (CHANGE LDIM AND THE CORRESPONDING ARRAY DIMENSIONS)'/
+9X,' (OR - CHECK FOR ERRORS IN THE INPUT VARIABLES)'/)
STOP
END
*****
SUBROUTINE ABMATR(LMAX,KAY,DISP)
C THIS SUBROUTINE CLACULATES THE TRANSFORMATION MATRICES A AND B FOR

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C TRANSLATION OF THE VECTOR SPHERICAL WAVE FUNCTIONS ALONG THE Z-AXIS.
C [SEE APPENDIX OF IEEE AP-19 PAGE 378 (1971)]
  COMPLEX A,B,RBH,KD,SUMA,SUMB,COEF,IM
  REAL KAY
  DIMENSION RBH(0:400),FZ(400),FM(400)
  COMMON/AB/ A(60,60), B(60,60)
  COMMON/PRM/XMIN,XMAX,NOJ
  IM=(0.,1.0)
  D=2.*DISP
  LMX2=2*LMAX+1
  KD=KAY*ABS(D)
  CALL RBESH(KD,LMX2,RBH)
C GENERATE A AND B MATRICES
  DO 100 LL=1,LMAX
    XLL=LL
    DO 100 LM=1,LMAX
      XLM=LM
      FLL=LL*(LL+1)
      FLIM=LM*(LM+1)
      FFLM=FLL+FLIM
      SUMA=(0.,0.)
      SUMB=(0.,0.)
      CALL THREEJ(XLL,XLM,0.,0.,FZ)
      CALL THREEJ(XLL,XLM,1.,-1.,FM)
      LMIN1=INT(XMIN+0.001)-1
      DO 200 J=1,NOJ,2
        L=LMIN1+J
        FL=L*(L+1)
        TL1=2*L+1
        COEF=(IM**(-L))*TL1*RBH(L)
        SUMA=SUMA+COEF*(FFLM-FL)*FZ(J)*FM(J)
        SUMB=SUMB+COEF*FZ(J)*FM(J)
      200 CONTINUE
      CF=FLOAT(2*LM+1)/FLM*SQRT(FLL/FLM)/2.
      COEF=CF*(IM**(LM-LL))
      A(LL,LM)=-COEF*SUMA
      B(LL,LM)=COEF*SUMB*(2.*D)*(KAY*IM)
    100 CONTINUE
  RETURN
END

```

```

  SUBROUTINE MIEUV
C THIS SUBROUTINE EVALUATES THE MIE COEFFICIENTS U AND V
C (WHERE U AND V ARE THE COEFFICIENTS THAT ARE USUALLY
C REFERED TO AS THE MIE A AND B COEFFICIENTS. THEY
C ARE DEFINED AS IN STRATTON'S "ELECTROMAGNETIC THEORY".
  COMPLEX U,V,A,B,Q,D,G,N,ONE,IMG,RJ,RH,Z
  COMMON/RMIE/RAD,RFR,RFI,WL,KAY,LMAX
  COMMON/CMIE/U(100),V(100)
  DIMENSION A(100),B(100),RJ(100),RH(100)
  REAL KAY
  ONE=(1.,0.)
  IMG=(0.,1.)
C CALCULATE A AND B
  N=CMPLX(RFR,RFI)
  Z=N*KAY*RAD
  CALL RATRBJ(Z,LMAX,RJ)
  DO 100 L=1,LMAX
    D=(L+1)/Z-RJ(L)
    A(L)=D*N
  100 B(L)=D/N
C CALCULATE U AND V COEFFICIENTS
  Z=RAD*KAY
  CALL RATRBJ(Z,LMAX,RJ)
  CALL RATRBH(Z,LMAX,RH)
  Q=- (CSIN(Z)/Z-CCOS(Z))*CEXP(-IMG*Z)/(IMG/Z+ONE)

```

```

DO 500 L=1,LMAX
D=(L+1)/Z-RJ(L)
G=(L+1)/Z-RH(L)
U(L)=-Q*(A(L)-D)/(A(L)-G)
V(L)=-Q*(B(L)-D)/(B(L)-G)
500 Q=Q*RJ(L)/RH(L)
RETURN
END
*****
SUBROUTINE GFMTX(LMAX,D,KAY)
C CALCULATE THE SUPERMATRICES G AND F FROM THE SUBMATRICES A,B AND U,V
COMMON/AB/A(60,60),B(60,60)
COMMON/CMIE/U(100),V(100)
COMMON/GFMAT/G(120,120),F(120)
COMPLEX A,B,G,F,KIMD,U,V,EPS,EPSTAR,EPE,EME
COMPLEX UA,UB,VA,VB,RHO,IM
REAL KAY
DO 100 I=1,LMAX
I2=I+LMAX
DO 100 J=1,LMAX
J2=J+LMAX
G(I,J)=A(J,I)
G(I,J2)=B(J,I)
G(I2,J)=B(J,I)
100 G(I2,J2)=A(J,I)
SGN=1.0
DO 200 I=1,LMAX
SGN=-SGN
I2=I+LMAX
G(I,I)=G(I,I)+SGN/U(I)
200 G(I2,I2)=G(I2,I2)-SGN/V(I)
C
C CALCULATE F VECTOR
C
IM=(0.,1.)
KIMD=IM*KAY*D
EPS=CEXP(-KIMD)
EPSTAR=CONJG(EPS)
SGN=1.0
DO 300 I=1,LMAX
SGN=-SGN
EME=EPS-SGN*EPSTAR
EPE=EPS+SGN*EPSTAR
FL=FLOAT(2*I+1)/FLOAT(I*(I+1))
RHO=FL*(IM**(I+1))
F(I)=EME*RHO
300 F(I+LMAX)=EPE*RHO
RETURN
END
*****
SUBROUTINE FSOLVE(LMAX)
COMMON/GFMAT/G(120,120),F(120)
COMPLEX G,F
DIMENSION WA(120)
LMX2=2*LMAX
CALL LEQT1C(G,LMX2,120,F,1,120,0,WA,IER)
RETURN
END
*****
SUBROUTINE CRSEC(X,DISP,KAY,LMAX)
C CALCULATE CROSS SECTIONS
COMMON/GFMAT/G(120,120),F(120)
COMMON/DIFF/NANG,THETA(400),DIFF(400),DIFS(400),QEXT,QSCA,QABS
COMMON/CRS2/Q2EXT,Q2ABS
COMMON/CMIE/U(100),V(100)
COMPLEX G,F,IM,IKD,ST1,ST2,SP1,SP2,ST,SP,A1,B1,A2,B2,U,V,EXPS,EXMN

```



```

COMPLEX IL, IFL, ZERO
REAL KAY
CONST=1.0/(2.*ACOS(-1.)*X*X)
LMX2=2*MAX
IM=(0.,1.)
ZERO=(0.,0.)
IKD=IM*KAY*DISP
C CALCULATE SINGLE PARTICLE TOTAL CROSS SECTIONS
SUMEX=0.0
SUMSC=0.0
DO 300 L=1, LMAX
  FL=FLOAT(2*L+1)
  PEX=-FL*(REAL(U(L))+REAL(V(L)))
  PSC=FL*(U(L)*CONJG(U(L))+V(L)*CONJG(V(L)))
  SUMEX=SUMEX+PEX
300 SUMSC=SUMSC+PSC
  X2=2./(X*X)
  QEXT=X2*SUMEX
  QSCA=X2*SUMSC
  QABS=QEXT-QSCA
C CALCULATE 2-PARTICLE EXTINCTION CROSS SECTION
ST1=ZERO
ST2=ZERO
SGN=1.0
DO 400 L=1, LMAX
  SGN=-SGN
  A2=F(L)
  B2=F(L+LMAX)
  A1=-SGN*A2
  B1=SGN*B2
  IFL=FLOAT(L*(L+1))*((-IM)**L)
  ST1=ST1+IFL*(A1+B1)
400 ST2=ST2+IFL*(A2+B2)
  EXPS=CEXP(IKD)
  EXMN=CONJG(EXPS)
  Q2EXT=X2*AIMAG(EXPS*ST1+EXMN*ST2)
C CALCULATE 2-PARTICLE ABSORPTION CROSS SECTION
Q2ABS=0.
DO 500 L=1, LMAX
  L2=L+LMAX
  AS=F(L)*CONJG(F(L))
  BS=F(L2)*CONJG(F(L2))
  US=U(L)*CONJG(U(L))
  VS=V(L)*CONJG(V(L))
  FL=FLOAT((L*(L+1))**2)/FLOAT(2*L+1)
500 Q2ABS=Q2ABS+FL*(AS/US*(US+REAL(U(L))
  ++BS/VS*(VS+REAL(V(L))))
  Q2ABS=-X2*Q2ABS
  IF(NANG.EQ.0) RETURN
  DO 100 I=1, NANG
    Z=COS(THETA(I))
    ST1=ZERO
    SP1=ZERO
    ST2=ZERO
    SP2=ZERO
    ST=ZERO
    SP=ZERO
    PIM=0.0
    PI=1.0
    SGN=1.0
    DO 200 L=1, LMAX
      SGN=-SGN
      A2=F(L)
      B2=F(L+LMAX)
      A1=-SGN*A2
      B1=SGN*B2

```

```

      FL=L
      S=Z*PI
      T=S-PIM
      PIP=S+(FL+1.)/FL*T
      TAU=FL*T-PIM
      FLT=FLOAT(2*L+1)/FLOAT(L*(L+1))
      IL=(-IM)**L
      ST1=ST1+IL*(A1*PI+B1*TAU)
      SP1=SP1+IL*(A1*TAU+B1*PI)
      ST2=ST2+IL*(A2*PI+B2*TAU)
      SP2=SP2+IL*(A2*TAU+B2*PI)
      ST=ST-FLT*(U(L)*PI+V(L)*TAU)
      SP=SP-FLT*(V(L)*PI+U(L)*TAU)
      PIM=PI
200  PI=PIP
      ST1=IM*ST1
      ST2=IM*ST2
      SP1=-SP1
      SP2=-SP2
      DIFS(I)=CONST*(ST*CONJG(ST)+SP*CONJG(SP))
      EXPS=CEXP(IKD*Z)
      EXMN=CONJG(EXPS)
      ST=EXPS*ST1+EXMN*ST2
      SP=EXPS*SP1+EXMN*SP2
100  DIFF(I)=CONST*(ST*CONJG(ST)+SP*CONJG(SP))
      RETURN
      END
*****
      SUBROUTINE RBESH(Z,LMAX,RBH)
C THIS SUBROUTINE COMPUTES THE SPHERICAL HANKEL FUNCTION H1
C (SEPT,25,1989)
      DIMENSION RBH(0:1)
      COMPLEX Z,RBH,RBH1,RBH2,U,IM
      U=(1.,0.)
      IM=(0.,1.)
      CALL RATXBH(Z,LMAX,RBH)
      RBH1=-IM/Z*CEXP(IM*Z)
      DO 100 L=0,LMAX
      RBH2=RBH1*RBH(L)
      RBH(L)=RBH1
100  RBH1=RBH2
      RBH(LMAX+1)=RBH2
      RETURN
      END
*****
      SUBROUTINE RATXBH(Z,LMAX,RH)
C THIS SUBROUTINE COMPUTES THE RATIO(L+1)/L OF RICCATI
C HANKEL FUNCTIONS UP TO ORDER LMAX AND COMPLEX ARGUMENT
C Z BY UPWARD RECCURANCE. (SEPT. 22, 1989)
      COMPLEX Z,ZR,TZR U,IM,RHL,RH
      DIMENSION RH(0:1)
      U=(1.0,0.0)
      IM=(0.0,1.0)
      RHL=U/Z-IM
      RH(0)=RHL
      ZR=U/Z
      TZR=2.*ZR
      L=0
100  L=L+1
      ZR=ZR+TZR
      RHL=ZR-U/RHL
      RH(L)=RHL
      IF(L.LT.LMAX) GO TO 100
      RETURN
      END
*****

```

```

      SUBROUTINE RATRBJ(Z,LMAX,RJ)
C THIS SUBROUTINE COMPUTES THE RATIO (L+1)/L OF RICCATI
C BESSEL FUNCTIONS "J" UP TO ORDER LMAX AND COMPLEX ARGUMENT Z
C BY DOWNWARD RECURRANCE. (SEPT 22,1989)
      COMPLEX Z,ZR,TZR,U,RJL,RJ,PROD,ERR
      DIMENSION RJ(1)
      U=(1.0,0.0)
C CALCULATE INITIAL L VALUE FOR DOWNWARD RECURSION
      RJL=U
      ZR=U/Z
      TZR=2.*ZR
      PROD=U
      ZR=(2*LMAX+1)*ZR
      L=LMAX
100  L=L+1
      ZR=ZR+TZR
      RJL=ZR-U/RJL
      PROD=PROD*RJL
      E=SQRT(REAL(PROD)**2+AIMAG(PROD)**2)
      IF(L.GT.5000) GO TO 200
      IF(E.LT.1.E8) GO TO 100
C DOWNWARD RECURSION
      RJL=U/(ZR+TZR)
300  RJL=ZR-U/RJL
      L=L-1
      ZR=ZR-TZR
      IF(L.LE.LMAX) RJ(L)=U/RJL
      IF(L.GT.1) GO TO 300
      RETURN
200  WRITE(6,400)
400  FORMAT(' L EXCEED 5000 IN RATSBJ')
      STOP
      END

```

```

*****
      SUBROUTINE RATRBH(Z,LMAX,RH)
C THIS SUBROUTINE COMPUTES THE RATIO(L+1)/L OF RICCATI
C HANKEL FUNCTIONS UP TO ORDER LMAX AND COMPLEX ARGUMENT
C Z BY UPWARD RECURRANCE. (SEPT. 22, 1989)
      COMPLEX Z,ZR,TZR U,IM,RHL,RH
      DIMENSION RH(1)
      U=(1.0,0.0)
      IM=(0.0,1.0)
      RHL=U/Z-IM
      ZR=U/Z
      TZR=2.*ZR
      L=0
100  L=L+1
      ZR=ZR+TZR
      RHL=ZR-U/RHL
      RH(L)=RHL
      IF(L.LT.LMAX) GO TO 100
      RETURN
      END

```

```

*****
      SUBROUTINE THREEJ(XJ2,XJ3,XM2,XM3,F)
C
C THIS SUBROUTINE COMPUTES THE 3-J SYMBOLS. IT IS BASED ON
C THE METHOD OF SCHULTEN AND GORDON J.MATH PHYS. VOL.16, P.1961.
C
      DIMENSION F(500),S(500),T(500)
      COMMON/PRM/XMIN,XMAX,NOJ
C
C TEST FOR BAD INPUT DATA
C
      TST1=AMOD(XJ3+ABS(XM3),1.)
      TST2=AMOD(XJ2+ABS(XM2),1.)

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```

      IF ((TST1.GT.0.).OR.(TST2.GT.0.)) GO TO 990
      IF (ABS(XM3).GT.XJ3) GO TO 990
      IF (ABS(XM2).GT.XJ2) GO TO 990
C
      EPS=1.E-25
      EPSLN=1.E-10
      XM1=-(XM2+XM3)
      XMAX=XJ2+XJ3
      XMIN=AMAX1 (ABS (XJ3-XJ2),ABS (XM1))
      NOJ=INT (XMAX-XMIN+1.+EPSLN)
      IF (NOJ.EQ.1) GO TO 900
      C1=(XJ2-XJ3)**2
      C2=(XJ2+XJ3+1.)**2
      C3=(XJ2*(XJ2+1.)-XJ3*(XJ3+1.))*XM1
      C4=XM1*XM1
      C5=XM3-XM2
      IF ((XM2.EQ.0.).AND.(XM3.EQ.0.)) GO TO 800
      IF ((XM2.EQ.XM3).AND.(XJ2.EQ.XJ3)) GO TO 800
      JCZ=(NOJ+1)/2
      NL=JCZ-1
      NR=NOJ-JCZ
C
C UPWARD ITERATION
C
      S(1)=0.
      NLP1=NL+1
      DO 100 I=1,NLP1
      XJ=XMIN+FLOAT(I-1)
      XJS=XJ*XJ
      A=SQRT (ABS ((XJS-C1)*(C2-XJS)*(XJS-C4)))
      B=-(2.*XJ+1.)*(C3-XJ*(XJ+1.)*C5)
      YJ=XJ+1.
      YJS=YJ*YJ
      A1=SQRT (ABS ((YJS-C1)*(C2-YJS)*(YJS-C4)))
      DEN=B+YJ*A*S(I)
      IF (DEN.EQ.0.) DEN=EPS
      IF (XJ.NE.0.) S(I+1)=-XJ*A1/DEN
      IF (XJ.EQ.0.) S(I+1)=-A1/C5
100 CONTINUE
C
C DOWNWARD ITERATION
C
      T(NOJ)=0.
      DO 200 I=1,NR
      XJ=XMAX-FLOAT(I-1)
      XJS=XJ*XJ
      K=NOJ+1-I
      A=SQRT (ABS ((XJS-C1)*(C2-XJS)*(XJS-C4)))
      B=-(2.*XJ+1.)*(C3-XJ*(XJ+1.)*C5)
      YJ=XJ+1.
      YJS=YJ*YJ
      A1=SQRT (ABS ((YJS-C1)*(C2-YJS)*(YJS-C4)))
      DEN=B+XJ*A1*T(K)
      IF (DEN.EQ.0.) DEN=EPS
200 T(K-1)=-YJ*A/DEN
C
C CALCULATE F
C
      F(JCZ)=1.
      IF (NL.EQ.0) GO TO 350
      DO 300 I=1,NL
      J=JCZ-I+1
300 F(J-1)=S(J)*F(J)
350 DO 400 I=1,NR
      J=JCZ+I-1
400 F(J+1)=T(J)*F(J)

```

```

C
C NORMALIZE THE RESULTS
C
    SUM=0.
    DO 500 I=1,NOJ
    XJ=XMIN+FLOAT(I-1)
500 SUM=SUM+(2.*XJ+1.)*F(I)*F(I)
    SUM=1./SQRT(SUM)
    DO 600 I=1,NOJ
600 F(I)=SUM*F(I)
    GO TO 700

C
C SPECIAL CASES: (M1=M2=M3=0) OR (M2=M3 AND J2=J3)
C
800 DO 810 I=1,NOJ
810 F(I)=0.
    F(1)=1.
    SUM=2.*XMIN+1.
    DO 820 I=3,NOJ,2
    XJ=XMIN+FLOAT(I-1)
    XJM1=XJ-1.
    XJS=XJ*XJ
    XJS1=XJM1*XJM1
    AJ=SQRT(ABS((XJS-C1)*(C2-XJS)*(XJS-C4)))
    AJ1=SQRT(ABS((XJS1-C1)*(C2-XJS1)*(XJS1-C4)))
    F(I)=-XJ*AJ1/(XJM1*AJ)*F(I-2)
820 SUM = SUM+(2.*XJ+1.)*F(I)*F(I)
    SUM=1./SQRT(SUM)
    DO 830 I=1,NOJ,2
830 F(I)=SUM*F(I)
    GO TO 700

C
C SPECIAL CASE NOJ=1
C
900 F(1)=1./SQRT(2.*XMAX+1.)

C
C DETERMINE THE CORRECT SIGN
C
700 K=INT(XJ2-XJ3-XM1+EPSLN)
    S1=(-1.)**K
    S2=SIGN(1.,F(NOJ))
    SG=S1*S2
    DO 710 I=1,NOJ
    IF (ABS(F(I)).LE.EPS) F(I)=0.
710 F(I)=SG*F(I)
    RETURN
990 WRITE(6,995)
995 FORMAT(//' ----- BAD INPUT DATA -----'/)
    STOP
    END
*****
    SUBROUTINE INPVAR(WL,RAD,DELANG,NANG,RFR,RFI,DISP)
C THIS SUBROUTINE READS (OR GENERATES) THE INPUT VARIABLES.
C
C READ THE INPUT VARIABLES
    WRITE(6,700)
    WRITE(6,770)
    WRITE(6,701) WL
    READ(5,*) WL
    IF (WL.LE.0.) GO TO 1000
    WRITE(6,702) RAD
    READ(5,*) RAD
    DISP=RAD
    WRITE(6,703)
    WRITE(6,705) DELANG
    READ(5,*) DELANG

```

```

WRITE(6,706) NANG
READ(5,*) NANG
WRITE(6,707)
WRITE(6,708) RFR
READ(5,*) RFR
WRITE(6,709) RFI
READ(5,*) RFI
WRITE(6,714)
WRITE(6,712) DISP
READ(5,*) DISP
WRITE(6,710)
770 FORMAT(4X,'TYPE A COMMA TO REPEAT THE PREVIOUS VALUE'/
+4X,'IF NO RESPONSE,TYPE A SECOND COMMA')
700 FORMAT(4X,'ENTER WAVELENGTH AND PARTICLE RADIUS IN SAME UNITS (USE
+ WL=0 TO END)')
701 FORMAT(F10.3,8X,'WL=')
702 FORMAT(F10.3,8X,'RAD=')
703 FORMAT(4X,'ENTER INITIAL SCATTERING ANGLE(DEG), ANGLE INCREMENT(DE
+G) AND NUMBER OF'/4X,'ANGLES(MAX=1000). USE NANG=0 FOR NO ANGULAR
+CALCULATIONS')
705 FORMAT(F10.3,8X,'DELANG=')
706 FORMAT(I10,8X,'NANG=')
707 FORMAT(4X,'ENTER INDEX OF REFRACTION(M=N+I*K)')
708 FORMAT(F10.3,8X,'N=')
709 FORMAT(F10.3,8X,'K=')
712 FORMAT(F10.3,8X,'DISP=')
714 FORMAT(4X,'ENTER DISPLACEMENT FROM MIRROR')
710 FORMAT(///)
1000 RETURN
END

```

PROGRAM MIRMX

WL = 1.000E+00
 RAD = 1.000E+00
 SZP = 6.283E+00
 DISP = 1.000E+00
 LMAX = 12
 RFR = 1.460E+00
 RFI = 0.000E+00

PARTICLE ON A MIRROR CROSS SECTIONS

C2EXT = 1.79407E+01 C2SCA = 1.79407E+01 C2ABS = -2.13531E-12

SINGLE PARTICLE CROSS SECTIONS

CEXT = 9.22655E+00 CSCA = 9.22655E+00 CABS = -7.58960E-13

ANGLE	CROSS SEC	SINGLE PARTICLE CROSS SEC
.000	8.686E+01	2.484E+01
5.000	7.719E+01	2.237E+01
10.000	5.333E+01	1.615E+01
15.000	2.731E+01	8.984E+00
20.000	9.610E+00	3.575E+00
25.000	2.426E+00	1.077E+00
30.000	9.241E-01	8.918E-01
35.000	6.348E-01	1.546E+00
40.000	7.631E-01	1.848E+00
45.000	9.904E-01	1.481E+00
50.000	5.704E-01	8.184E-01
55.000	3.518E-01	3.527E-01
60.000	6.999E-01	2.627E-01
65.000	4.685E-01	3.819E-01
70.000	9.596E-02	4.515E-01
75.000	1.861E-01	3.653E-01
80.000	6.593E-02	2.091E-01
85.000	2.744E-01	1.191E-01
90.000	6.653E-01	1.329E-01

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