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The research described in this report deals with the development of (1) efficient, consistent discretization techniques for use in semi-infinite optimization, optimal control algorithms that solve problems with dynamics governed by partial differential equations, (2) superlinearly converging semi-finite optimization algorithms, (3) various techniques for finding design parameters satisfying specifications, (4) optimal control algorithms for discrete and distributed systems with control, state and shape constraints, and (5) efficient numerical procedures for the integrated design of flexible structures and their control systems.

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NONSMOOTH OPTIMIZATION ALGORITHMS, SYSTEM THEORY, AND SOFTWARE TOOLS

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1. SUMMARY

The research described in this report deals with the development of *(i)* efficient, consistent discretization techniques for use in semi-infinite optimization, optimal control, and optimal shape design algorithms that solve problems with dynamics governed by partial differential equations, *(ii)* superlinearly converging semi-infinite optimization algorithms for the integrated design of flexible structures and their feedback-control systems, as well as for shape optimization, *(iii)* global techniques for finding design parameters satisfying specifications, *(iv)* optimal control algorithms for discrete and distributed systems with control, state and shape constraints, *(v)* efficient numerical procedures for the integrated design of flexible structures and their control systems.

The most significant accomplishments are *(i)* the development of a theory of consistent discretizations that offers a framework for the numerical solution, by means of nonlinear programming and discrete optimal control algorithms, of semi-infinite optimization optimal control, and shape optimization problems, *(ii)* the development of new tools for the study of rate of convergence of optimization algorithms in both conceptual and implementable form, *(iii)* the construction of algorithms for the solution of optimal control problems with plants described either by ODE's or PDE's (in evolution equation form), with control and state space constraints, *(iv)* the completion of about two thirds of a 500 page advanced level book: "Computational Methods in Optimization: A Unified Approach", to be published by Academic Press. This book will be unique in optimization literature because it deals with computational methods for optimization in a unified framework, and hence includes algorithms for the solution of nonlinear programming, semi-infinite optimization, and optimal control problems with control and state space constraints. The algorithms for semi-infinite optimization, and optimal control problems are developed within the framework of the theory of consistent approximations that was developed with AFOSR support. A MATLAB library of programs is being developed to supplement the theory.

Five students who were partially supported by this project have received their PhD degrees: E. J. Higgins, L. He, Y. P. Harn, J. E. Wiest, and T. H. Yang.

2. TECHNICAL REPORT

(i) Research Objectives

The broad research objective of this project was the development of optimization algorithms and related system-theoretic aspects for the integrated design of flexible structures and their control systems, and shape design. The specific aspects under consideration are *(i)* efficient, consistent discretization techniques for use in semi-infinite optimization, optimal control, and optimal shape design algorithms that solve problems with dynamics governed by partial differential equations, *(ii)* superlinearly converging semi-infinite optimization algorithms for the integrated design of flexible structures and their feedback-control systems, as well as for shape optimization, *(iii)* global techniques for finding design parameters satisfying specifications, *(iv)* optimal control algorithms for discrete and distributed systems with control, state and shape constraints, *(v)* techniques for nonlinear feedback-system stabilization via open-loop optimal control, *(vi)* efficient numerical procedures for the integrated design of flexible structures and their control systems, and *(vii)* interactive software for optimization-based control system design.

(ii) Accomplishments

***(a)* Discretization Strategies for Semi-infinite Optimization and Optimal Control.** Discretization strategies play a crucial role in the implementation of semi-infinite optimization and optimal control algorithms. We feel that we have made truly significant contributions in this area, as follows.

In [32] we present a theory of consistent approximations that enables one to determine whether discretization techniques, that may be proposed for the solution of a semi-infinite optimization, optimal control, or shape optimization problem, result in consistent approximations or not. Furthermore, it leads to a set of techniques for using highly polished nonlinear programming algorithms in the solution of such problems. In this theory, we always consider a pair consisting of an optimization problem and its optimality function ($\mathbf{P}$,θ), and define consistency of approximating problem-optimality function pairs, ($\mathbf{P}_N$,θ_N) to ($\mathbf{P}$,θ), in terms of the epigraphical convergence of the $\mathbf{P}_N$ to $\mathbf{P}$, and the hypographical convergence of the optimality functions θ_N to θ. In [32], we show that standard discretization techniques decompose semi-infinite optimization and optimal control problems into families of finite dimensional problems, which, together with associated optimality functions, are consistent discretizations to the original problems. More recently, we have shown that standard discretization techniques decompose various optimal beam design problems into families of finite dimensional consistent approximations. In addition, we present in [32] two types of techniques for using consistent approximations in obtaining an approximate solution of the original problems. The first is a "filter" type technique, similar to that used in conjunction with penalty functions, the second one is an adaptive discretization technique that can be viewed as an implementation of a conceptual algorithm for solving the original problems.

In [6, 14] we formulated the following optimal discretization problem: given a need to reduce the cost error by a factor α, find the number of stages, and the corresponding numbers of discretization points and iterations to be performed at each stage by a linearly converging algorithm, so as to minimize the total computing effort. Although this problem is not solvable exactly, we showed that it is possible to obtain approximate solutions.

In [11, 28, 30] We have shown that it is possible to construct discretization strategies which preserve for the overall solution process the rates of linearly and superlinearly converging algorithms which are used on the finite dimensional approximations. The results in [30] are particularly elegant and important. First a very effective new technique is presented for the establishment of superlinear convergence of semi-infinite optimization algorithms under much weaker assumptions than in the current literature. Second, Newton's method is extended to a superlinearly converging (with rate 3/2) implementable version for the solution of semi-infinite minimax problems.

(b) **Barrier Function Methods for Minimax Problems.** Currently barrier function (interior point) methods are very popular for linear programming. In [19, 29] we show that they also lead to very effective minimax algorithms, partly because they tend to improve problem conditioning, and partly because they are much simpler to implement than other minimax algorithms.

(c) **Algorithms for the Solution of Semi-infinite Constraint Satisfaction Problems.** It is often necessary to find a design that satisfies specifications. In economics this problem is usually referred to as the *satisfycing problem.* In [24, 22] we present two alternative approaches based on local methods for solving semi-infinite inequalities in a finite number of iterations.

In [15, 21] we present a global, multistart technique with the novel feature that it estimates the rate of convergence of the local algorithm being used and local minimum value to which it is converging, and incorporates the resulting information in its stopping rule. We show that on difficult problems our scheme can be about four times faster than an ordinary multistart method.

(d) **Algorithms for the Solution of Optimal Control Problems with ODE and PDE Dynamics, Control and State Constraints.** We have made a considerable amount of progress in the development of optimal control algorithms. In [7], we present an implementable algorithm for the solution of fixed and free time optimal slewing problems involving an Euler-Bernoulli beam, subject to control, deformation and end point constraints. In [12, 33] we present a very effective optimal control algorithm based on barrier functions, for problems with ODE dynamics, control and state space constraints. This algorithm should be extendible to PDE dynamics.

(e) **New Subprocedures for Semi-infinite Optimization.** The computation of search directions is a major part of any semi-infinite optimization algorithm. In [9] we present a very effective algorithm which can be used in solving a number of standard search direction finding problems.

(f) **Rate of Convergence of Semi-infinite Optimization Algorithms.** In [18] we present a new phase I phase II method of feasible directions and establish its linear convergence properties. This method is distinguished by the fact that it is the only known constrained semi-infinite optimization algorithm for which we were able to construct a linearly converging implementation that does not rely on strong assumptions.

In [5, 10, 20] we have studied the rate of convergence of the Pshenichnyi-Pironneau-Polak minimax algorithm and introduced a variable metric technique for the automatic rescaling of a class of semi-infinite optimization problems. The rescaling improves rate of convergence.

In [31] we show that a phase I - phase II feasible directions algorithm proposed by Polak 20 years ago, but never used because it was not clear how to implement it, can be easily implemented, and that its rate of convergence compares favorably with that of competing algorithms in its class.

(g) **Tools for the Design of Control Systems for Flexible Structures.** In [2] we develop a constrained, semi-infinite optimization approach to the design of control systems in H^∞ spaces.

In [1, 23] we present a new stability test and show that it can be used in conjunction with semi-infinite optimization to design stabilizing controllers for flexible structures described by evolution equations.

In [17] we show that it is possible to construct proportional - plus - integral stabilizing controllers for a class of MIMO linear systems described by evolution equations. Such controllers are interesting because they result in system that can follow polynomial inputs.

In [34] we present an evaluation of the potential improvements in seismic disturbance rejection to be obtained by using active variable damping control in a structure. Using the responses to seismic excitation of an optimally controlled variable structure and of a minimax optimally designed fixed structure, we obtain an upper bound on the achievable performance and a lower bound on the acceptability of a control system for a variable damping structure. Our numerical experiments lead to the conclusion that: the gap between the upper and lower bounds is rather small, which makes designing a superior feedback law very difficult. The best choice for a feedback law appears to be continuous moving horizon control, of the type described in [24-26].

(h) **Moving Horizon Control.** In [25-27] we propose and explore the properties of a moving horizon feedback control law, based on constrained optimal control algorithms, for linear and nonlinear plants with input saturation. Our moving horizon feedback control law results in a a nonconventional sampled-data system: its sampling periods vary from sampling instant to sampling instant, and the control during the sampling time is not constant, but determined by the solution of an open loop optimal control problem. We show that the resulting feedback system is robustly stable, whether the state of the plant is measured or estimated, that it is capable of following a class of reference inputs, and of suppressing a class of disturbances. Experimental results show that the behavior of the moving horizon control system is superior to that resulting from some alternative control laws.

(i) **Survey Papers and Various Presentations.** In [4, 5, 34-41] we report a number of survey papers and various oral presentations, including one plenary address, in which we brought our results to the attention of fellow researchers as well as to a broader audience of semi-infinite algorithm users.

(iii) Publications

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