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AUXETIC MATERIALS: AN ANNOTATED BIBLIOGRAPHY OF MATERIALS WITH NEGATIVE POISSON'S RATIO

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13. ABSTRACT *(Maximum 200 words)*
The purpose of this study was to evaluate the potential of materials with negative Poisson's ratio ("auxetic" materials) for application in hydroacoustics. The results of the literature search were collected in this report in order to facilitate the task of anyone who would like to become familiar with this subject. For an isotropic material, the upper bound of Poisson's ratio ν is 1/2. This bound is closely approximated by soft elastomers. It is energetically possible for a isotrophic material to have a negative Poisson's ratio, with a lower bound of -1. K.E. Evans has proposed the designation "auxetic" for such materials, derived from the Greek word <u>auxesis</u>, which in biology is used for "increase of cell size without cell division". The bibliography is preceded by comments that are divided into sections parallel to the headings in the list of works. The categorized listing of the literature follows an alphabetic listing of the complete set. It does not appear that thus far auxetic materials have found applications in hydroacoustics that are drastically different from those obtained by means of the more common materials.

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AUXETIC MATERIALS: AN ANNOTATED BIBLIOGRAPHY OF MATERIALS WITH NEGATIVE POISSON'S RATIO

BACKGROUND

The purpose of this literature search was to evaluate the potential of materials with negative Poisson's ratio ("auxetic" materials) for application in hydroacoustics. Few direct references to acoustic applications were found. It appeared worthwhile to collect the results of the search, in order to facilitate the task of anyone who would like to become familiar with this literature.

The comments below are divided into sections by categories. The first section serves to introduce the reader to the main concepts, and thus relies strongly on tutorial and review papers. The reference section includes the complete set of literature sources arranged alphabetically. References cited in the text refer to the alphabetic listing. The review paper by Evans (1991) is a good general introduction and some of its figures are included here. A parallel bibliography of references arranged by categories is included in the Appendix.

INTRODUCTION

When a differential volume of a solid is subjected to a uniaxial (normal) stress in a given direction, strains will be produced as a consequence, both in that same direction as well as in a plane perpendicular to the direction of the stress axis. For the great majority of materials, these strains are opposite in sign, and thus it is natural that Poisson's ratio ν is defined as the negative of the ratio of the lateral to the longitudinal strain. It follows from the expression for the total strain energy that the value of ν is

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of that energy (Fung, 1965). For an isotropic material, the upper bound of ν is 1/2; this bound is closely approximated by soft elastomers. It is energetically possible for an isotropic material to have a negative Poisson's ratio with a lower bound of -1. Evans (1991) has proposed the designation "auxetic" [derived from the Greek word auxesis, which in biology is used for "increase of cell size without cell division" (Webster's New Collegiate Dictionary, G.& C. Merriam Co, 1977)] for such materials.

The prevalent intuitive aversion to acceptance of a negative ν appears to be connected with the notion that it does not conserve volume. But, first, there is no general law of conservation of volume; and second, the volume of the sample increases upon application of a longitudinal stress for the whole range of allowed values of ν (Evans, 1991). The relative change of volume θ is related to the longitudinal stress σ by the formula $\theta=(1-2\nu)\sigma/E$, where E is Young's modulus. Thus θ varies monotonically from 0 to $3\sigma/E$ when ν varies from 1/2 to -1. Therefore, insofar as the change in volume is concerned, a negative value of Poisson's ratio does not embody a special phenomenon and should not be considered as counterintuitive. Even so, relatively few materials with this property are known at this time.

It is well known that an isotropic material is characterized by two (complex) moduli. As a first step in acquiring a feel for the auxetic property, one may study Table 1, condensed from Lipsett and Beltzer (1988).

Table 1. Relations Between Elastic Moduli and Poisson's Ratio

ν	E	G	K	relation
1/2	finite	finite	∞	E = 3 G
1/2	0	0	finite	
-1	finite	∞	finite	E = 9 K
-1	0	finite	0	

The entries in Table 1 follow from the relations between elastic constants. For an isotropic material two of the constants are sufficient for the computation of the other ones. As an example, one can express the bulk modulus K and Young's modulus E in terms of the shear modulus G and Poisson's ratio ν by $K = 2/3\ G\ (1+\nu)/(1-2\nu)$ and $E = 2\ (1+\nu)\ G$, respectively. These expressions show that $\nu = 0.5$ and $\nu = -1$ are special cases. Specifying ν alone does not characterize a material. The above table lists only the extreme (physically unrealistic) cases of one or the other variable being zero or infinity. One obtains a better insight if one considers these words zero and infinity as shorthand for physical realizations that lead to different values for the elastic moduli.

Soft elastomers have a value of Poisson's ratio close to 1/2. One often reads that these materials are almost incompressible. This is incorrect; ν=0.5 implies only that upon applying an uniaxial stress the volume is preserved, not that the bulk modulus (related to all-sided compression) is large. The only conclusion to be drawn when ν is close to 1/2 is that the shear and Young's moduli are small compared with the bulk modulus, and that, approximately, the relation E=3G holds. In fact, the bulk modulus of soft elastomers is quite small relative to that of metals.

Similarly, if ν is close to -1, one should not submit to the impression that necessarily G would become very large, as some authors have suggested. For example, the deflection δ of the center of a circular plate under a point force F at its center is given by $\delta = 3Fa^2(1-\nu^2)/(16Ed^3)$, where a is the radius and d the thickness of the plate, of a material with Young's modulus E and Poisson's ratio ν. For $\nu \approx -1$ one cannot conclude that δ becomes small, since its limit will obviously depend on the limit for E.

Instead of reaching a conclusion on absolute values of the elastic moduli, one can only conclude that, when ν is close to -1, the values of E and K will be small compared with the value of G, and that, approximately, the relation E=9K holds. In fact, Fig. 1 (from Evans 1991) shows that the auxetic materials realized thus far have little stiffness. It is obvious that this will be the case for reentrant elastomer foams (discussed below). No complete set of explicit data on the moduli of various auxetic materials was found while preparing this report; thus, it was not possible to judge how far progress was made in obtaining materials with a large value of G (compared with conventional stiff materials). Attaining high values of G appears to be an important impetus for developing auxetic materials, according to Evans (1991).

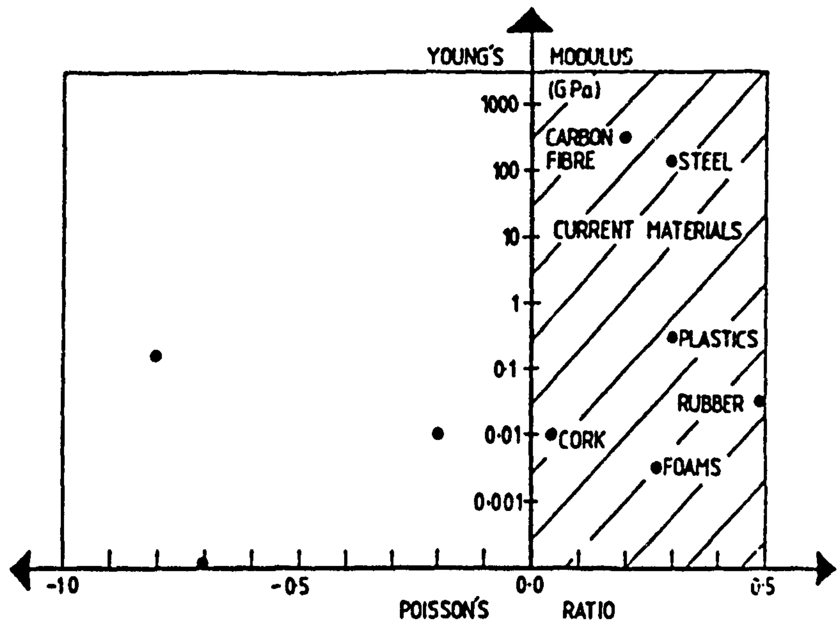

Fig. 1 - The elastic performance of isotropic materials can be characterised by this diagram showing the possible range of Young's moduli and Poisson's ratios. The hatched region on the right shows the current range of materials with a negative Poisson's ratio. The few examples so far fabricated are also illustrated. The aim is to make a material in the top left hand corner! [Figure and caption taken from K.E. Evans, "Auxetic polymers-a new range of materials," *Endeavour* 15, 170-174 (1991), with permission of the author.]

In a short review article, Lakes (1992a) refers to recent work by Milton (1992).

In his introduction, G. Wei (1992) gives an overview of the history of negative Poisson's ratio concepts. He presents a review of recent papers not mentioned in this bibliography and, therefore, his article may be used as a supplement to the references listed here.

The introduction in an article by G.W. Milton (1992) critically reviews various mechanical models leading to negative Poisson's ratio materials (indicated by the term "dilatational"). He describes a family of laminates with chevron structure that result in negative values of Poisson's ratio. His list of references should also be used as a supplement to this bibliography.

THEORY OF CELLULAR STRUCTURE

R. Lakes is usually credited with being the first to produce an elastomer foam with negative Poisson's ratio, based on his publication in Science (1987a). Starting from the Kelvin minimum-area tetrakaidecahedron described by Lanceley et al.(1966), he introduces the concept of a reentrant foam, where the ribs bend inwards. This is shown in Figs. 2 and 3 (from Friis et al., 1988).

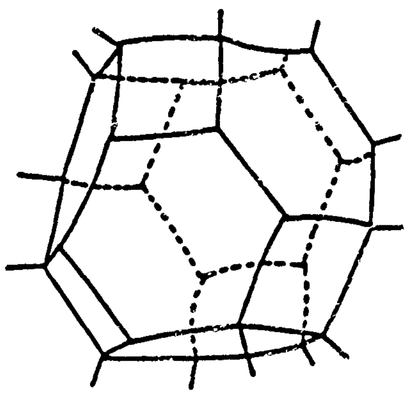

Fig. 2 - Idealized unit cell of conventional foam. [Figure and caption taken from E.A. Friis, et al., "Negative Poisson's ratio polymeric and metallic foams," *J. Mater. Sci.* **23**, 4406-4414 (1988), with permission of Dr. R.S. Lakes.]

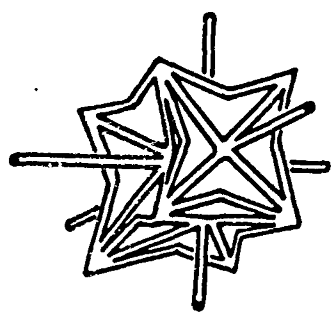

Fig. 3 - Idealized unit cell of reentrant foam. [Figure and caption taken from E.A. Friis, et al., "Negative Poisson's ratio polymeric and metallic foams," *J. Mater. Sci.* **23**, 4406-4414 (1988), with permission of Dr. R.S. Lakes.]

In his ASME paper (1991c), Lakes places such foams within the context of modeling solids as Cosserat continua. This theory, also known as micropolar elasticity, follows from dropping the assumption advanced in the usual continuum theory that the stress tensor is symmetric.

The article by Lakes in the Journal of Materials Science (1991a) contains an important overview of various proposed microstructures (as contrasted to a macroscopic foam). He reaches the following conclusions:

1. Negative Poisson's ratios can be obtained by a non-affine deformation geometry alone.
2. Negative Poisson's ratios can result from a combination of non-central forces and prestrain.
3. Negative Poisson's ratios can result from a chiral structure with rotational degrees of freedom combined with non-central force or non-affine deformation.

These conclusions follow some examples that may be useful in interpreting the physical meaning of the terminology. The examples are given for a two-dimensional structure and are macroscopic in nature; the realization of these structures in the microscopic domain is not always clear.

As an example of a non-affine deformation, Lakes refers to the reentrant honeycomb shown in Fig.4 (from Evans 1991). This configuration, as well as the regular honeycomb, undergoes a non-affine deformation under stretching; i.e., the relative displacement of points of the structure is not the same for all the point pairs. (Affine geometry is that geometry in which there is exactly one line through a point parallel to a given line. Algebraically it is characterized by a linear point transformation.) Lakes' first conclusion states that non-affine deformation by itself may lead to producing a negative Poisson's ratio.

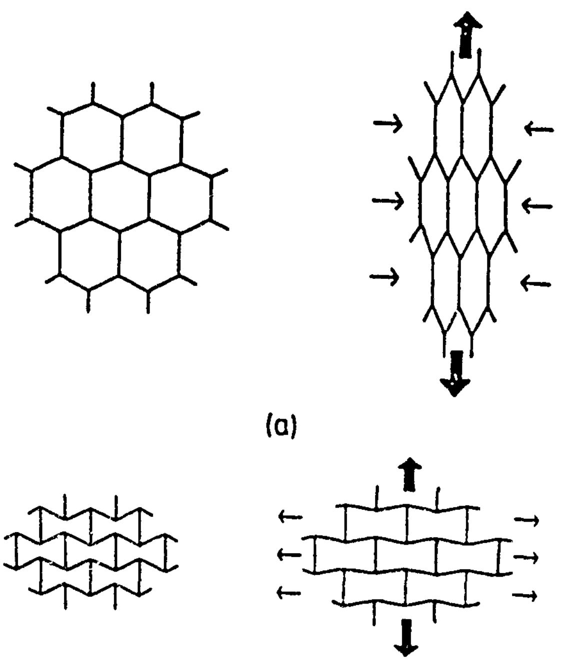

Fig.4. a) A schematic diagram of a conventional hexagonal structure and how it deforms when stretched, producing a conventional positive Poisson's ratio. b) A reentrant honeycomb producing a negative Poisson's ratio. [Figure and caption taken from K.E. Evans, "Auxetic polymers-a new range of materials," *Endeavour* 15, 170-174 (1991), with permission of the author.]

ɔ!

The situation of non-central forces is modeled by a lattice of rotatable "nodes" (disks) linked by elastic ligaments (Fig.5). In order to obtain a negative Poisson's ratio, it is necessary to prestrain the ligaments represented by the springs k_1 and k_3 in the figure. Lakes points out the similarity of this structure to that described by Bathurst and Rothenburg (1988) and Rothenburg et. al. (1991), discussed below.

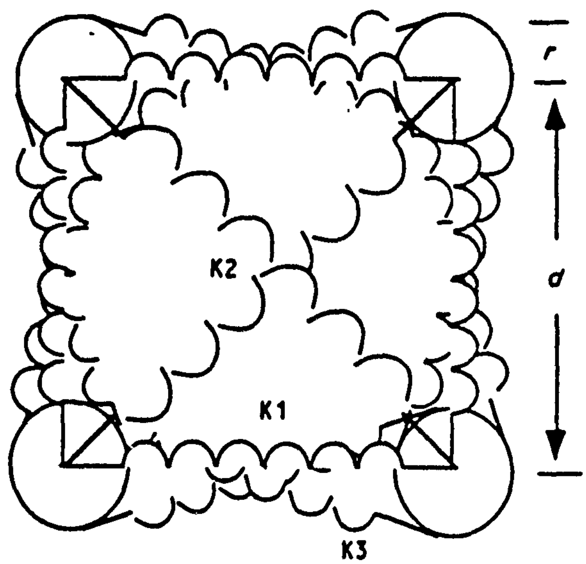

Fig. 5 - Lattice of rigid rotatable nodes linked by elastic ligaments. [Figure and caption taken from R. Lakes, "Deformation mechanisms in negative Poisson's ratio materials: structural aspects," *J. Mater. Sci.* 26, 2287-2292 (1991), with permission of the author.]

An unusual type of anisotropy occurs when a structure is not equivalent to its mirror image and, thus, does not possess a center of symmetry; this is described by the terms "non-centrosymmetric," "hemitropic," or "chiral" (Lakes, 1991a). Figure 6 (from Lakes 1991a) shows this concept; the centers of the nodes (the disks in the figure) move in an affine way, and the resulting Poisson's ratio is negative. This is an example of Lakes' third conclusion. Lakes refers to the work of Wojchiechowski (see below) for a "molecular" example; there the original disks are centrosymmetric, but the center of symmetry disappears when a preferential tilt is given to the disks.

In the papers by Bathurst and Rothenburg (1988) and by Rothenburg et. al. (1991), a study is undertaken of the microstructure of isotropic materials that would lead to negative ν, in contrast to macroscopic reentrant foams.

Their investigation starts from Poisson's theory relating ν to the microstructure, in which a material is considered to consist of randomly packed smooth spherical particles under central forces (this results in ν=1/4). These authors add the assumed presence of frictional forces perpendicular to the line connecting the centers of the particles. The resulting expression for ν contains the parameter λ (not to be confused with the Lamè factor λ) equal to the ratio of tangential to normal stiffness. It is given as $\nu=(1-\lambda)/(4+\lambda)$. This expression leads to negative ν for $\lambda>1$. The paper (Rothenburg et. al. 1991) does not give practical examples of this type of structure.

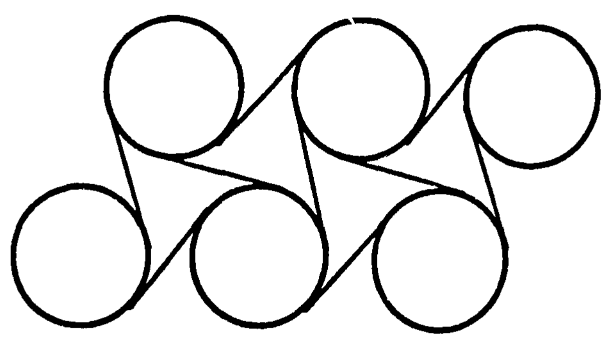

Fig. 6 - Chiral (noncentrosymmetric) hexagonal microstructure of rotatable nodes bendable ligaments. Poisson's ratio is negative. [Figure and caption taken from R. Lakes, "Deformation mechanisms in negative Poisson's ratio materials: structural aspects," *J. Mater. Sci.* 26, 2287-2292 (1991), with permission of the author.]

Rothenburg et al. (1991) cite the work by Wojciechowski. In the representative paper cited here, Wojciechowski and Brańka (1989) develop a model for a two-dimensional isotropic solid based on a system of hard hexamer molecules (i.e. molecules consisting of six equidiameter, disk-shaped "atoms"). This system displays a negative Poisson's ratio in a high-density crystalline phase, where the disks are tilted.

The investigations by Evans and his group are concentrated on expanded polytetrafluoroethylene (PTFE, or Teflon®). Pictures of the assumed mechanism are shown in Fig. 7 (from Evans 1991). Several papers by this group are given in the literature list (Caddock and Evans 1989; Evans 1989; Evans and Caddock 1989; Evans 1990; Evans 1991; Evans et. al. 1991).

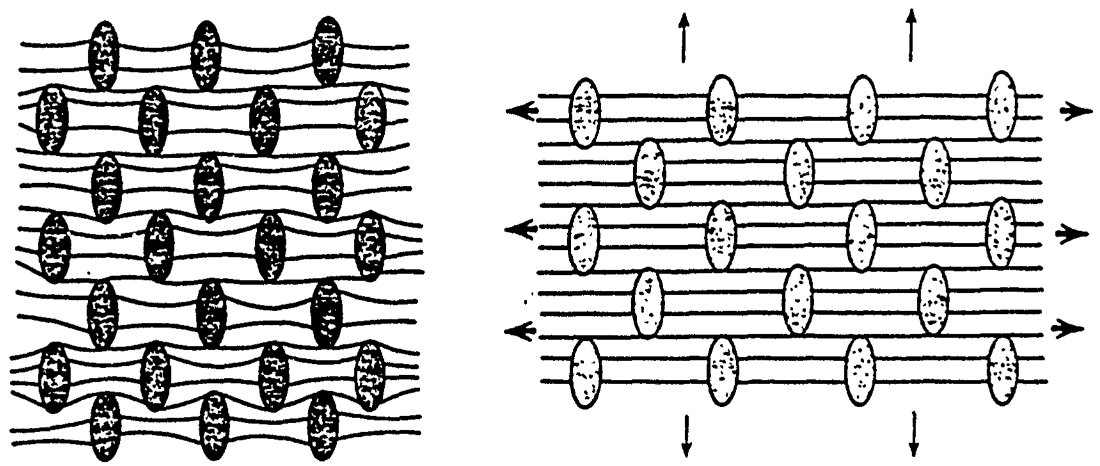

Fig. 7 - Schematic representation of the polymeric microstructure producing a negative Poisson's ratio. Typically, the nodules and fibrils are about 20 microns long. [Figure and caption taken from K.E. Evans, "Auxetic polymers-a new range of materials," *Endeavour* 15, 170-174 (1991), with permission of the author.]

The two groups, those of Evans and of Lakes, refer to the earlier work by Gibson and others in two articles (Gibson et al. 1982; Gibson and Ashby 1982).

The works of T.L.Warren (1989) and W.E. Warren and A.M. Kraynik (1988) have a common approach. They derive the value of Poisson's ratio by analyzing the mechanics of a reentrant honeycomb structure (see Fig.4) and a reentrant foam, respectively. In 1985, R.F. Almgren had already published a design for a structure consisting of rods, hinges, and springs which realized a reentrant honeycomb structure, with Poisson's ratio of -1.

The book Foams by J.J. Bikerman (1973), referenced by Warren and Kraynik (1988), has been added to the list of monographs since it contains a good introduction to the morphology of liquid foams. For the subject of solid foams, Bikerman refers to the monograph by Benning (1969).

The article by Warren and Kraynik (1988) contains models that relate the relative Young's modulus of a foam to the square of its relative density with a proportionality constant that depends on the shape of the ligaments' cross section. This theoretical relation agrees well with values of E for foamed aluminum measured by Dubbelday (1992).

The paper by G. Wei (1992) extends the work of the above authors. Quoting Wei:

> "This paper is intended as a complete generalization of the Warren-Kraynik-Warren approach and the associated microstructures applied to both existing and a novel class of polymeric networks, the latter of which are now under experimental investigations to show negative Poisson ratio effects. It must be emphasized that the theoretical approach adopted here is significantly different from those of Gibson and Ashby and Evans et al. in that polymeric networks containing certain microstructures are assumed to be spatially nonperiodic." (Wei, 1992, pp. 3226-3227)

A computation and graphic representation paper by Wei and Edwards, devoted entirely to the tailoring of materials with negative Poisson ratios, has been submitted for publication in Computational Polymer Science.

FABRICATION AND MEASUREMENTS

A discussion of the fabrication of auxetic foams is given in Friis et al. (1988), both for elastomer foams and for metallic foams. The desired effect is the bending inward of the ribs of the foam cells, which normally bend outward. In the case of plastic foams, this reentrant feature is obtained by all-sided compression at elevated temperature, whereby one uses the fact that the elastomer material becomes gradually softer at higher temperatures. Applied to a polyester urethane foam, a permanent volumetric compression ratio of 3.4 was obtained. In the case of metal foams, the transition to liquid is too sharp, and therefore one relies on plastic deformation at high pressure. The metal is treated by successive applications of small increments of plastic deformation in three orthogonal directions. The resulting material has a permanent volumetric compression ratio of 1.73.

Results of mechanical tests are given by Friis et al. (1988), but it is difficult to extract a complete description of the elastic properties from the data. This work was continued by an investigation of loss properties by Chen and Lakes (1989) and by a holographic study by Chen and Lakes (1991).

In a recent paper, Alderson and Evans (1992) report on the development of a novel thermoforming processing route to produce a microporous form of ultra high molecular weight polyethylene.

Although not directly related to auxetic foams, the paper by Davies and Zhen (1983) is a good review of fabrication techniques of metal foams.

A paper by Hirotsu (1990) is only marginally related to the present interest, but it is nonetheless interesting reading.

MECHANICAL CONSEQUENCES AND DYNAMIC EFFECTS

In two articles to be published in the Journal of Mechanical Design (1992c), and the Journal of Applied Mechanics (1992b), R.S.Lakes discusses the consequences of using negative Poisson's ratio materials in mechanical design. Stress concentrations in solids do not depend on Poisson's ratio in many two-dimensional situations, but they may do so in three dimensions. A negative Poisson's ratio may be important in applying Saint Venant's principle concerning end effects in solids.

Three papers by Freedman (1990a-c) form an impressive expansion of the work by Mindlin on the spectrum of Lamb modes in a plate, to include the case of negative Poisson's ratio. It would take considerable time to follow the discourse in detail, but one does not get the impression that novel acoustic effects are revealed.

The study by Lipsett and Beltzer (1988) reconsiders some dynamic elasticity problems for the case where Poisson's ratio is negative: reflection from a free surface, propagation of Rayleigh waves, and lateral vibrations of beams and plates. It is interesting reading, but does not appear to predict any unusual results of importance to practical hydroacoustics.

APPLICATIONS

Choi and Lakes (1991) describe the design of a fastener that is based on negative Poisson's ratio foam:

> "The fastener consists of a hollow circular cylindrical core, made of negative Poisson's ratio material, and a socket with a corresponding cylindrical hole. The fastener is made bigger than the hole by a tolerance t in the radial dimension. As it is inserted the longitudinal force of insertion causes its diameter (transverse dimension) to become smaller by virtue of the negative Poisson's ratio. With sufficient force, the core diameter becomes equal to the hole diameter and the core can be inserted into the hole. Axial force applied to remove the fastener causes the core to expand laterally, generating an increased frictional force which resists removal." (Choi and Lakes, 1991, p. 206)

They offer an example of the problem referred to in the introduction, namely that the foam is lacking in strength: "The observed maximum removal force was limited by the strength of the copper foam..." (Choi and Lakes, 1991, p. 211). Lakes points out that the negative Poisson ratio would be important for the holding power of fibers in a matrix similar to the fastener in a hole.

The report by Howell et al.(1991) is the only publication found that has the word "acoustic(s)" in the title. The acoustical experiments are done in air. Their most important conclusion was that "Foams with a negative Poisson's ratio were shown to be better acoustic absorbers over the entire frequency range 100 to 1600 Hz when compared with unconverted materials." (Howell et al., 1991, p. 9)

Narang and Nigam (1992) gave an (invited) presentation on acoustic properties of materials with reduced Poisson ratio at the 123rd meeting of the Acoustical Society of America. The abstract does not provide much information beyond an enumeration of the subjects to be covered. Among other issues, the oral presentation discussed the behavior of liquid crystals in a soft matrix and the large negative Poisson's ratio that may occur in anisotropic materials (Dr. A.J. Rudgers, private communication).

CONCLUSIONS

The main conclusion of this literature review is that one should be aware that, by itself, a negative Poisson's ratio does not make a material suitable for specified hydroacoustic or other applications. For an isotropic material, one needs one elastic modulus in addition to ν plus the density to characterize the material in order to predict its behavior.

The development of auxetic materials is an interesting subject. It would appear worthwhile to monitor further development of these materials, although at this time it is not clear what their applications to hydroacoustics might be. Thus far, the number of existing materials and the range of material properties is quite limited.

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