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APPROACHES TO MODELING WIDTH ADJUSTMENT IN CURVED ALLUVIAL CHANNELS

Final Report

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SYMBOLS

$c(z)$	suspended sediment concentration at vertical coordinate z (ppm)
c'	bank material cohesion (Pa)
d	mean channel depth (m)
d_i	median sediment diameter of the ith bed material size-density class
d_{50}	median bed material diameter (m)
e	dimensionless bank erodibility coefficient
g	acceleration due to gravity (ms^{-2})
h	flow depth (m)
h_f	height of bank exposed above water surface (m)
i	bank slope angle (degrees)
m	dimensionless exponent in mixing length models
m'	dimensionless empirical exponent in Rais model
n^*	dimensionless lateral location of streamwise velocity maximum
p	pressure (Pa)
q_n	is the total lateral sediment flux per unit channel width (m^2s^{-1})
q_s	is the total longitudinal sediment flux per unit channel width (m^2s^{-1})
q_{si}	potential sediment flux per unit channel width (m^2s^{-1})
q_{si}^*	actual sediment flux per unit channel width (m^2s^{-1})
r	dimensionless radius of curvature
r^*	ratio of lift to drag force
t	time coordinate (s)
$\langle u \rangle$	dimensionless depth-averaged streamwise velocity
u	streamwise velocity component (s direction)
v	transverse velocity component (n direction)
w	vertical velocity component (z direction)
x	longitudinal coordinate (cartesian system)
y	lateral coordinate (cartesian system)
z	vertical coordinate (all coordinate systems)
y'	tension crack depth (m)
A	dimensionless eddy viscosity coefficient
A'	coefficient in equation (3.57)
$A1$	function in equation (3.44B)
$A2$	function in equation (3.44B)
B	dimensionless constant of integration in Rais model
BW	failure block width (m)

C_f	Chezy friction factor
CONC	pore fluid salt concentration
D_{mix}	mixed layer depth (m)
F	function in equation (3.36)
F_b	lateral bedload flux per unit channel width (m^2s^{-1})
F_{bank}	total (reach scale) bank material flux (m^3s^{-1})
F_n	transverse friction term
F_s	streamwise friction term
F_z	vertical friction term
F_D	driving force acting on failure block (KN)
F_R	resisting force acting on failure block (KN)
H	total bank height (m)
H'	height of uneroded bank face (m)
H_o	mean water depth (m)
I_o	bed slope
K	dimensionless tension crack index
K'	dimensionless coefficient in bedload function
K(z)	dimensionless velocity distribution function
K_2	constant in equation (3.35)
K_3	constant in equation (3.35)
K_4	constant in equation (3.35)
L	mixing length (m)
LE	lateral erosion rate ($mmin^{-1}$)
P(c')	probability of occurrence of soil cohesion, c'
$P(\phi)$	probability of occurrence of soil friction angle, ϕ
$P(\gamma)$	probability of occurrence of soil unit weight, γ
P_I	probability of occurrence of individual combination of soil properties
$P_I(FS<1)$	probability of occurrence of failure for combination of soil properties
$P_R(FS<1)$	probability of occurrence of mass failure
R	initial rate of lateral erosion ($mmin^{-1}$)
R_c	centreline radius of curvature (m)
Re	Reynolds number
SAR	sodium adsorption ratio
T	function in equation (3.64)
T4	function in equation (3.37)
U	reach-averaged streamwise flow velocity (ms^{-1})
U_b	near bank depth-averaged streamwise flow velocity (ms^{-1})
U_{max}	maximum value of dimensionless streamwise depth-averaged velocity

VB	unit failure block volume (m^3s^{-1})
V_s	dimensionless streamwise velocity in cylindrical coordinate system
V_r	dimensionless transverse velocity in cylindrical coordinate system
V_z	dimensionless vertical velocity in cylindrical coordinate system
W	mean channel width (m)
Wt	weight of failure block (KN)
X	dimensionless difference between depth and depth of velocity maximum
Z	bed elevation (m)
Z'	dimensionless flow depth in Rais model
Z_m	dimensionless depth of the core of streamwise velocity maximum
β	failure plane angle (degrees)
β_i	fraction of ith bed material size-density class present in the mixed layer
χ	river bank migration rate (ms^{-1})
χ_i	inner bank migration rate (ms^{-1})
χ_o	outer bank migration rate (ms^{-1})
δ	angle between fluid stress and down stream direction (degrees)
$\Delta\varepsilon$	dielectric dispersion coefficient of bank material
ε	eddy viscosity coefficient (m^2s^{-1})
η	bed elevation (m)
π	pi = 3.14157
ϕ	internal angle of friction of bank material (degrees)
Φ	internal angle of friction of dispersed bank material clasts (degrees)
Φ^*	function in equation (3.64)
γ	unit weight of bank material (KNm^{-3})
γ'	ratio of flow depth to radius of curvature
k	Von-Karman constant (0.4)
λ	porosity of the bed material
μ	dynamic coefficient of Coulomb friction
μ_k	static coefficient of Coulomb friction
ω	side slope angle (degrees)
Ω	half width to depth ratio
ρ	fluid density (Kgm^{-3})
ρ_s	sediment density (Kgm^{-3})
τ	bed shear stress (dynes cm^{-2})
τ_c^*	dimensionless critical Shield stress
τ^*	dimensionless mean Shields stress
Υ	average transverse slope of the bank (degrees)
Γ	ratio of half width to centreline radius of curvature

ς	water surface elevation (m)
ζ_1	dimensionless streamwise derivative of streamwise velocity
ζ_2	dimensionless streamwise derivative of streamwise velocity
ζ_3	function in equation (3.44A)

ABSTRACT

The application of many existing numerical models of river channel morphology is limited by their inability to account for bank erosion and changing channel width. In this report, methods of numerically modelling river channel width adjustment in curved streams are investigated in order to determine the feasibility of developing an effective, rational method for predicting dynamic river channel response to modification in control variables. The main scope of the report is concerned with seeking approaches to modelling width adjustment in relatively large, sand-bed, curved alluvial channels with cohesive bank material, though an approach for modelling width adjustment in non-cohesive channels is also outlined.

Width adjustment can be modelled by determining the migration rates of both river banks, using the concept of basal endpoint control. Bank retreat proceeds by combinations of both direct fluvial entrainment of bank material and mass failure under gravity. Mass failure occurs when a critical threshold of stability is exceeded due to steepening by lateral erosion or by near bank degradation increasing the height of the bank. Conversely, bank accretion and bank line advance occurs when more sediment is supplied to the near bank zone by fluvial transport and mass wasting mechanisms than can be removed by the flow. So, in the long term bank line retreat and advance is controlled by the dynamic sediment balance in the basal area adjacent to the bank. It is, therefore, necessary to model the flow, sediment transport and bank stability processes within the near bank zone in order to simulate width adjustment rationally. The current ability to model these processes is the subject of this report.

In the case of channels with cohesive bank material, valid algorithms for predicting both streamwise and transverse sediment transport fluxes are presently available. It is also currently possible to predict the stability of cohesive riverbanks, and to combine the bank stability and sediment transport algorithms in order to maintain the continuity of both the bed and the bank material.

A review of the validity of approaches to modelling bend flow hydraulics for applications within the near bank zone is also presented. While considerable progress has been made in developing increasingly sophisticated 2 and 3 dimensional hydraulic models, the assumptions used in these models limits means that they are invalid within the outer bank zone. Research on a previous attempt to model the flow in the outer bank region using a

combined empirical/analytical method is presented, but this approach was found to be unsuitable. It was concluded that no model is currently able to simulate the flow within the pivotal outer bank zone, so that it is not presently possible to model width adjustment in curved channels using an approach based on a rational determination of the flow within the near bank zones.

A potential method of modelling width adjustment in non-cohesive channels using a kinematic bank migration model is also developed. The kinematic bank migration model is based on relating the bank migration rate to the streamwise near bank velocity via an "erodibility" coefficient. This approach has the advantage that the use of a near bank velocity means that the flow does not have to be calculated within the near bank boundary layer. For this approach sophisticated flow models are presently available. However, it was concluded that present methods of calculating the erodibility coefficient are inadequate, so that more research is also required in order to further develop the potential of this approach for modelling width adjustment in curved non-cohesive river channels.

1. INTRODUCTION

The application of many existing numerical models of river channel morphology is limited by their inability to account for bank erosion and dynamic adjustment of the channel width. The principal objective of the work reported here was to investigate methods of numerically modelling river channel width adjustment in curved channels in order to determine the feasibility of developing an effective, rational method for predicting dynamic river channel response to modifications in control variables and, thus, identifying stream form relationships for a range of bed and bank materials and imposed hydrologic and sedimentologic regimes. The aim was, therefore, to develop an exploratory framework for approaches to modelling width adjustment in curved river channels.

In the proposal for this research, which was conducted at the Waterways Experiment Station in the summer months of 1992, the opportunity to build upon ongoing research on width adjustment in straight channels at the University of Nottingham was recognised. The Nottingham University research has highlighted the importance of coupling hydraulics, sediment transport and bank stability algorithms in the pivotally important near bank zone in order to predict the channel width. This coupling is achieved by applying the framework concept of "basal endpoint control" to maintain continuity of sediment of both the bed and bank material. The primary aim of this study was to investigate the potential for coupling existing meander bend hydraulic models with the sediment transport and bank stability algorithms developed in the Nottingham University research, within the near bank zone, in order to use these modules to build a numerical model valid in curved channels. Secondary aims were to assess the validity of the sediment transport and bank stability algorithms developed at Nottingham, and to refine these models, or develop alternatives to them, where necessary. The main scope of the report is concerned with seeking approaches to modelling width adjustment in relatively large, sand-bed, curved alluvial river channels with cohesive banks. These are the types of rivers typically associated with the large watercourses of the continental United States. However, an approach to modelling width adjustment in channels with non-cohesive bed and bank materials is also suggested in this report.

The approach taken stresses the importance of having the ability to successfully model the flow in the pivotal near bank zone. An explicit aim of this research was to assess and further develop a model of near bank flows using a method (Rais, 1985) formulated at

Colorado State University by a former graduate student of the principal investigator, as well as conducting a critical literature review of existing approaches to numerical modelling of near bank flows. The literature review was supplemented by visiting a number of researchers located throughout the United States, in an attempt to develop a truly state-of-the-art review.

This report contains a chapter on the conceptual basis for modelling width adjustment in curved channels which establishes the elements required to construct a model of width adjustment. The following chapters then attempt to provide a more detailed review of these individual, technical modules to assess the strengths and weaknesses of these technologies, particularly with respect to applying them in the near bank zone of curved channels. These chapters include a critical literature review of methods of modelling the hydraulics in the bank zones of meandering channels, together with a report of the results of research on modelling near bank flows using the Rais method. Then, chapters on the sediment transport and bank stability algorithms (valid for channels with non-cohesive bed material and cohesive bank material) developed at Nottingham University are presented. Finally, a chapter of conclusions and recommendations attempts to summarise the current ability to numerically model width adjustment in curved channels. While the primary aim of this report is to establish an approach to modelling width adjustment in channels with cohesive banks, a section on an approach to modelling width adjustment in channels with non-cohesive bank material is also included. In total, this report attempts to provide a blueprint for developing approaches to width adjustment modelling. Two appendices are included that lay out the data requirements and logic structure that would be required to code and implement a numerical model of width adjustment using the algorithms reported/developed here.

2. CONCEPTUAL FRAMEWORK FOR MODELLING WIDTH ADJUSTMENT IN NATURAL RIVER CHANNELS

Bank top width adjustment occurs when there is a relative difference in the rate of retreat or advance of the channel banks. Bank retreat proceeds by combinations of both direct fluvial entrainment of bank material and mass failure. Mass failure occurs when a critical threshold of stability is exceeded due to steepening of the bank angle by lateral erosion, or by near bank degradation increasing the height of the bank. Conversely, bank accretion and bank line advance occurs when more sediment is supplied to the near bank zone by fluvial transport and mass wasting mechanisms than can be removed by the flow. Consequently, the long term bank line retreat and advance is controlled by the dynamic sediment balance in the basal area adjacent to the bank (Figure 2.1).This theory of "basal endpoint control" (Carson & Kirkby, 1972), outlined above, provides the conceptual framework for the research reported here.

Figure 2.1 schematically summarizes the theory of basal endpoint control. Essentially, predicting width adjustment is viewed as a sediment budgeting problem. If more sediment is removed from the bank zone than enters, the bank zone sediment store is depleted and bank retreat occurs. If the reverse occurs, the bank will advance. Figure 2.1 shows that, in a natural river channel, the sediment balance in the near bank region is controlled by the sediment fluxes into and out of the near bank zone. If the sediment flux field can be modelled in the way suggested by Figure 2.1 (that is both bed and bank material fluxes), then the morphological evolution of the channel can be modelled by specifying these fluxes and applying the sediment continuity equation across the full width of the channel:

$$\frac{\delta Z}{\delta t} = \frac{1}{1-\lambda}\left(\frac{\delta q_s}{\delta x} + \frac{\delta q_n}{\delta y}\right) \tag{2.1}$$

In equation (2.1), Z is the bed elevation (m), t is the time (s), λ is the porosity of the bed material, q_s is the longitudinal sediment flux per unit channel width (m^2s^{-1}) and q_n is the lateral sediment transport flux per unit channel width (m^2s^{-1}).

It is important to recognise that in applying the basal endpoint control framework the focus is sharply on the near bank basal region as the critical part of the channel with respect to river bank stability and width adjustment processes (Andrews, 1982). Within the near bank zone the concept of basal endpoint control provides the pivotal link between bank stability and fluvial processes that is necessary in order to explain the dynamics of width adjustment in terms of a response to either changes over time in the water and sediment discharges, or changes in the bed and bank boundary materials. The theory of basal endpoint control suggests that three major components are required to model width adjustment: first, a suitable model of the hydraulics, second, a model for the sediment flux field, and third a bank stability algorithm to predict the bank fluxes shown in Figure 2.1. It is again worth stressing that these components must be applied in the near bank zone if width adjustment is to be successfully modelled using this approach. These modules are now reviewed in more detail.

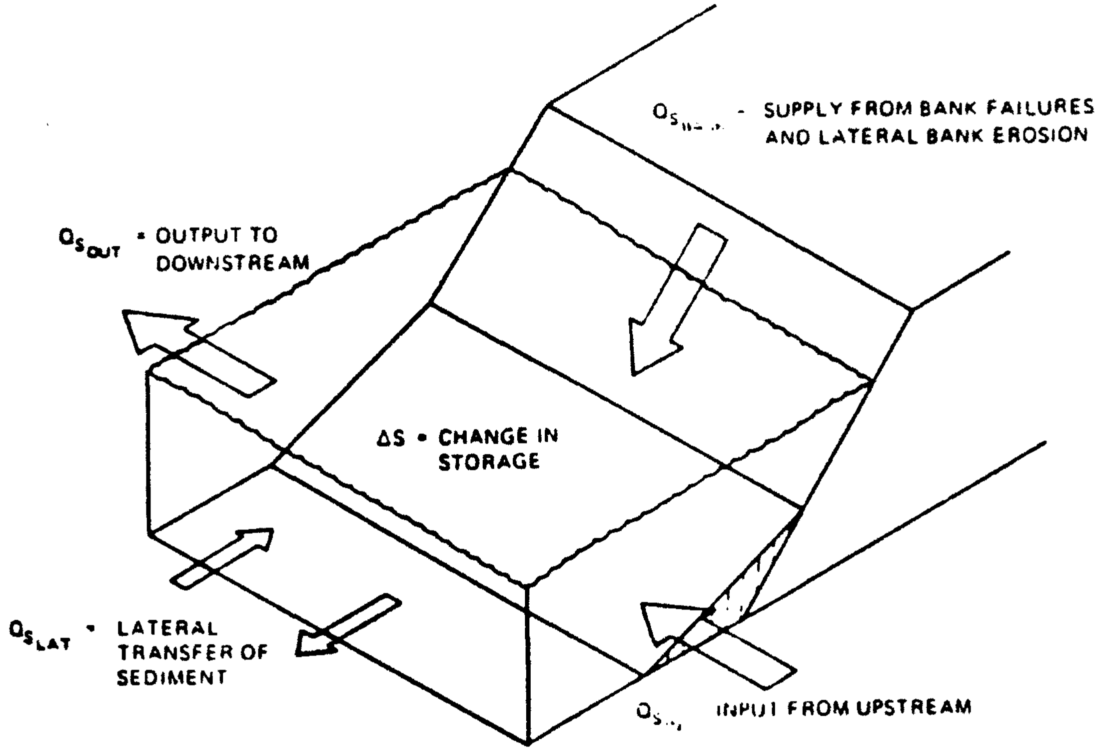

Figure 2.1 Sediment fluxes in the near bank zone

3. APPROACHES TO MODELLING BEND FLOW FOR WIDTH ADJUSTMENT MODELLING

In the previous section, it was suggested that in order to "solve" the problem of width adjustment, it is necessary to specify all of the sediment fluxes in the near bank zone. In order to achieve this goal, it is apparent that width adjustment models using the concept of basal endpoint control must be able to simulate realistically the hydraulics across the full width of the bendway, including the near bank zone. Hence, it is the theory of basal endpoint control that generates the requirement that the flow be accurately modelled in the near bank zone. Olesen (1987) correctly points out that the application of a refined flow model does not necessarily result in a more reliable sediment transport prediction. There are two reasons for this: first, because the flow distribution strongly depends on the alluvial roughness distribution, which cannot be predicted very accurately: and second, because many sediment transport models are not very reliable in any case. Such a refined model is ultimately sought in this study to improve the physical basis of the model, and in recognition of the likelihood that roughness and sediment transport predictions are likely to continue to improve in the future. The accurate prediction of the hydraulics in the near bank zone is viewed as having fundamental importance to the predictive ability of any width adjustment model on the basis of the concept of basal endpoint control (Darby & Thorne, 1992).

In this chapter, methods of modelling the hydraulics across the full width of curved channels are reviewed. To be useful, the analytical and numerical models of bend flow reviewed later must be able to replicate at least these gross features of curved channel flow. It is, therefore, appropriate to begin with an introduction to bend flow theory and a description of the typical gross structure of flow in the bends of natural river channels.

3.1 Characteristics Of Flow Through River Bends

The morphology and evolution of meander bends depends on the velocity distribution, large scale flow structures, the transfer of momentum within the flow, and the boundary shear stress distribution (Markham, 1990). The gross features of bend flow - the overall flow structure and velocity and shear stress distribution - have been frequently observed and documented, and although the principles of fluid mechanics relating to these phenomena are relatively well understood in simple situations, Markham (1990) notes that the assumptions necessary to obtain solutions to the governing equations of flow are not always applicable to complex natural channels.

Thomson (1876) was among the first to analyse the flow of fluid through a river bend way. In a river bend, the fluid is not only subject to the gravitational and frictional forces, but a centrifugal force also acts outward on the water as it flows through the curve. The centrifugal force is proportional to the flow velocity squared, and since the flow velocity varies with depth, the centrifugal force also varies non-uniformly with depth. Moreover, since the outward directed centrifugal force tends to push the water toward the outer bank region, the resulting water and momentum flux causes a build up of water adjacent to the bank, and a raising of the free surface known as superelevation (Markham, 1990). The resultant transverse water surface slope causes an inward acting pressure gradient force (p.g.f), which has a uniform magnitude through the flow depth. This results in a situation where, near the water surface, the centrifugal force exceeds the p.g.f, while near the bed the p.g.f exceeds the centrifugal force. The net result is a circulation in the transverse plane, which is usually termed a "secondary circulation".

While the nature of this main secondary cell at a bend has long been understood, observations have identified an additional secondary cell located in the outer bank zone. Einstein & Harder (1954) and Rozovskii (1957) observed the outer bank cell in flume studies while Hey & Thorne (1975), Bridge & Jarvis (1977) and deVriend & Geldof (1979) confirmed its existence in natural river channels. The outer bank cell has a reverse circulation to that of the main skew-induced cell, and can extend one or two water depths away from the outer bank where that bank is steep (Markham, 1990).

Until recently, the generally accepted model of secondary flow structure at a bend envisaged a single skew induced helix occupying the full width of the channel (Figure 3.1a). Observations of the outer bank cell modified this model to include this extra cell in the near bank zone (Figure 3.1b). But, Dietrich & Smith (1983) argued that in river channels with natural bed topography there is a net outward flow of water over the point bar in channels with varying curvature (Figure 3.1c). Dietrich & Smith argued that water shoaling over the point bar caused convective accelerations which lead to a resultant decrease in the downstream water surface slope along the inner bank. In turn, this leads to a decrease in the cross-stream p.g.f. so that centrifugal force dominates above the point bar, resulting in outward flow throughout the flow depth at the inside of the bend (Figure 3.1c). Empirical support for the revised model of the secondary flow structure in natural river bends (Figure 3.1c) is further provided by Hickin (1978) and Thorne *et al.,* (1985).

The significance of the secondary flow for channel morphology lies mainly in its impact upon the distribution of velocity and boundary shear stress. In a classic study, Ippen & Drinker (1962) found the secondary flow to be an important control on both the magnitude and distribution of boundary shear stress in a smooth, trapezoidal laboratory channel. Locations of the shear stress maxima were found to be associated with the core of maximum velocity. In gently curving bends, the high shear stress zone migrated from near the inside bank in the curve to near the outside bank downstream of the curve exit, as the core of maximum velocity similarly migrated outwards. Numerous studies have similarly identified that the core of maximum velocity shifts from the the inner to the outer bank as it flows around the bend, both in laboratory (Yen, 1970; Hooke, 1975; Varshney & Garde, 1975; Kikkawa *et al.,* 1976; Chen & Shen, 1983), but also in natural, channels (Bridge & Jarvis, 1977; Bridge & Jarvis, 1982; Bathurst *et al.,* 1979; Dietrich & Smith, 1983; Whiting & Dietrich, 1991). Moreover, Bathurst *et al.* (1979) found shear stress peaks not only occurred beneath the velocity maxima, but also at the junction of the outer bank and main secondary cells, where the plunging secondary currents compressed isovels and

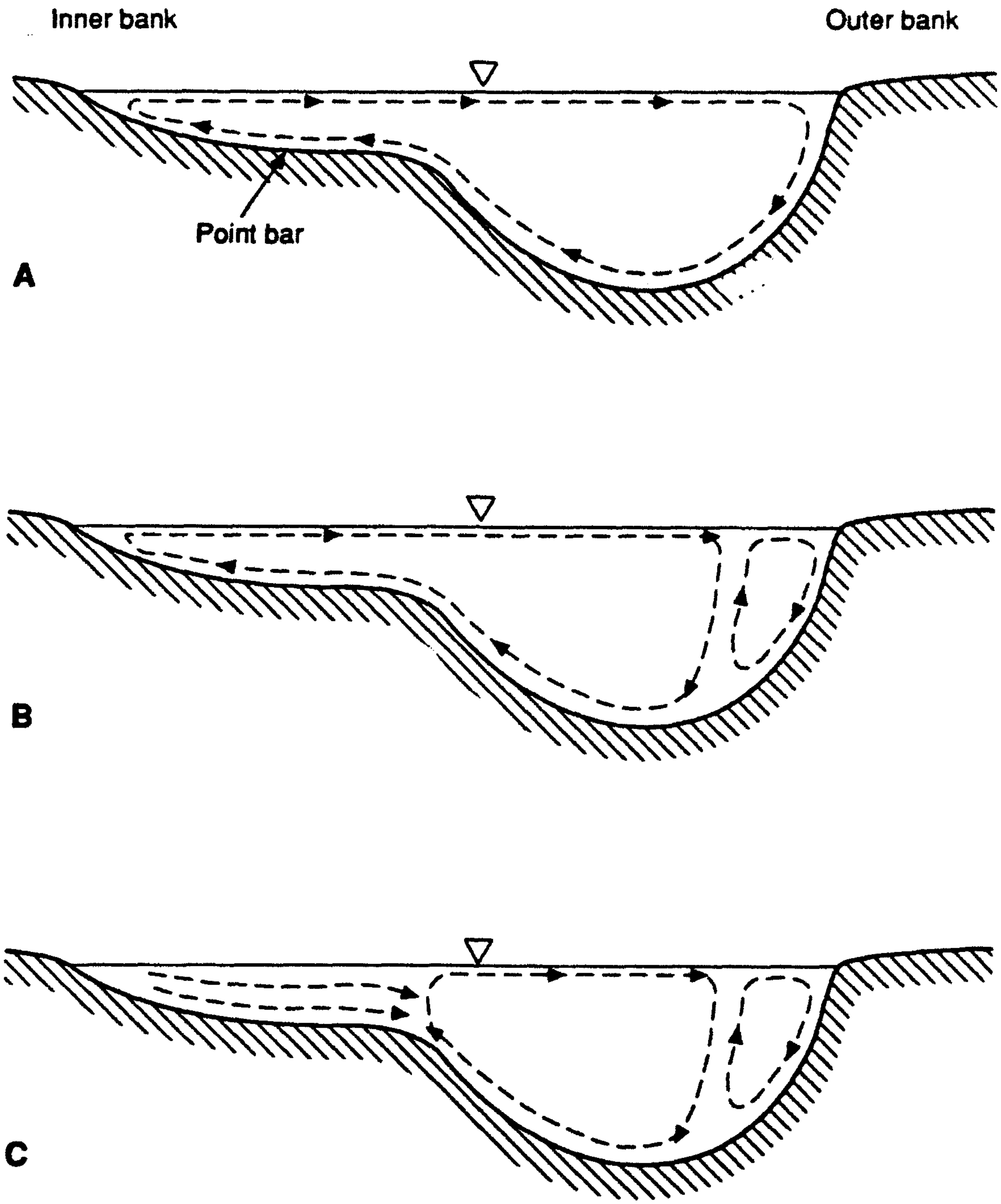

Figure 3.1 Models of secondary flow structures in curved natural river channels

steepened velocity gradients. This highlights the potential significance of the outer bank cell in driving bank migration processes. Not only will the patterns of secondary flow directly impact the bank stability through influencing the flow patterns that govern the near bank sediment flux field and evolution of the near bank basal toe scour, but the near outer bank cell may also directly influence fluid shear stresses on the side wall and thus impact processes of direct lateral erosion of the cohesive bank material. A number of authors have debated the role of the near bank secondary flow in controlling the importance of the various bank erosion mechanisms (Thorne, 1978; LaPointe & Carson, 1986; Kitanidis & Kennedy, 1984), with no firm conclusions emerging. The development and application of a suitable near bank secondary flow model, within the context of a bank stability or width adjustment model, would undoubtedly shed considerable light on this issue. For the moment it is only necessary to stress the significance of the outer bank cell with respect to width adjustment modelling. A method to predict the flow near the outer bank where there may be an outer bank cell, was developed by Rais (1985) and is evaluated later in this chapter.

To summarise, while the main features of the bend flow appear to be a migration of velocity and shear stress maxima from the inner to the outer bank, due to the convection of primary flow momentum by the secondary flow (e.g Johannesson & Parker, 1989abc), the work of Dietrich & Smith (1983) has also highlighted the importance of convective accelerations induced by bed topographical features (in particular the point bar typically found at the inner bank) on the shear stress distribution and secondary flow structure, and how this may influence the morphological evolution of inner bank point bars. The outer bank cell also apparently exerts considerable control on the shear stress distribution. For the purposes of width adjustment modelling, analytical and numerical bend flow models must be able to predict these features accurately.

3.2 Bend Flow Hydraulic Models

3.2.1 Governing Equations of Flow

All bend flow models start with the Navier-Stokes equations of motion for fluid flow, in some form of coordinate system. Most often a channel fitted curvilinear coordinate system is employed (Figure 3.2). Three dynamical equations (one for each direction) and one equation representing the conservation of mass (continuity equation) are required. In a curvilinear coordinate system with s the streamwise direction, n the transverse coordinate perpendicular to the s-axis, and z the vertical coordinate directed upwards normal from the bed, these equations can be written:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} + w\frac{\partial u}{\partial z} + \frac{u\,v}{r} = \frac{-1}{\rho}\frac{\partial p}{\partial s} + F_s \tag{3.1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial s} + v\frac{\partial v}{\partial n} + w\frac{\partial v}{\partial z} - \frac{u^2}{r} = \frac{-1}{\rho}\frac{\partial p}{\partial n} + F_n \tag{3.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial s} + v\frac{\partial w}{\partial n} + w\frac{\partial w}{\partial z} + g = \frac{-1}{\rho}\frac{\partial p}{\partial z} + F_z \tag{3.3}$$

$$\frac{\partial u}{\partial s} + \frac{1}{r}\frac{\partial (v\,r)}{\partial n} + \frac{\partial w}{\partial z} = 0 \tag{3.4}$$

where the friction terms, F, are:

$$F_s = \varepsilon\left(\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial n^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{3.5}$$

$$F_n = \varepsilon\left(\frac{\partial^2 v}{\partial s^2} + \frac{\partial^2 v}{\partial n^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{3.6}$$

$$F_z = \varepsilon\left(\frac{\partial^2 w}{\partial s^2} + \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{3.7}$$

where ε is the eddy viscosity coefficient, u, v and w are the velocity components (ms^{-1}) in the s, n and z directions, respectively, g is the acceleration due to gravity (ms^{-2}), r is the local radius of curvature (m), p is the pressure (Pa), t the time coordinate (s), ρ is the fluid density (Kgm^{-3}) and F_s, F_n and F_z are the friction terms in the s, n and z directions.

Term 1 in each of the dynamical equations (3.1) through (3.3) represents the prognostic local acceleration term, while terms 2 through 4 are the convective acceleration terms. The remaining terms in the equations of motion describe the principal cross-stream and downstream force balances between the centrifugal and p.g.f forces in the cross-stream direction, and the downstream balance between gravitational and frictional forces

(Markham, 1990). The final terms in each of the dynamical equations represent the frictional forces in each direction, and are very complex. These friction terms are usually modelled using eddy viscosity and empirical roughness "laws". In order to solve the set of partial differential equations, appropriate initial and boundary conditions must be specified. At this stage it is also worth noting that the dynamical equations illustrate that, in reality, each directional component of the flow interacts and is inter-related. This is mentioned here to dispel any idea, perhaps engendered by the necessarily classifacatory approach used in the previous section, that the primary and secondary flow are independent of each other.

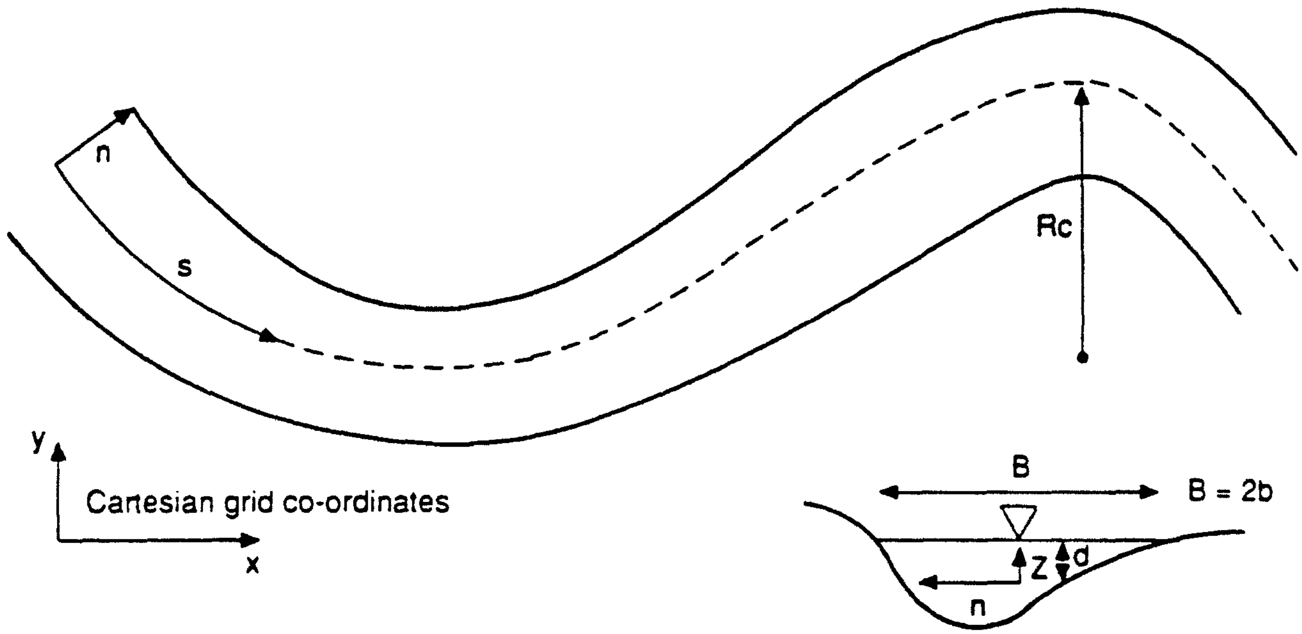

Figure 3.2 Coordinate system definition diagram

The various models of bend flow hydraulics usually introduce simplifying approximations to these full equations of motion in order to facilitate their solution. However, in doing so, the physical basis of the equation set is reduced and their validity and predictive ability is reduced. It is the aim of this chapter to review the validity of the assumptions used to develop solutions to the Navier-Stokes equations in some of the more common bend flow models, especially with regard to the pivotal near bank zone.

3.2.2 2-Dimensional Bend Flow Models

In this section a number of two dimensional approaches to modelling the flow through river channels are reviewed critically in order to assess their validity for modelling applications within the near bank zone. Among these "bend flow" models are included a number of models whose aim is to simulate the bed topography of river bends. The principle difference between the fixed and mobile bed approaches lies in a problem associated with the 2D approach. Depth averaged fixed bed models necessarily exclude the effect of the secondary currents on the flow, while the mobile bed depth averaged models attempt to model the influence of the secondary flows by taking account of the resulting adjustment of the bed topography. However, the essential theory constituting either of these approaches is broadly the same, so both are reviewed here. Despite the limitations described above, both approaches have had some success in modelling the features of bend flow and bed topography.

Among the first river researchers to attempt to solve the equations of motion was Rozovskii (1957). It is useful to review his approach in detail, since his approximations and assumptions have been used since by many other hydraulic modellers.He analysed steady flows (term 1 equals zero) using an orthogonal cartesian coordinate system. Rozovskii introduced a rigid lid approximation for the water surface boundary condition (Olesen, 1987), in order to facilitate solution of the mathematical model. This implies that the water surface is considered a rigid impermeable and shear stress free plate only with normal stresses (pressure). The error introduced by this approximation is small when the deviation between the local water surface level and the "average" water surface level (i.e. the level of the lid (Olesen, 1987)), is small. This requires the restriction that the Froude number and ratio of waterdepth to radius of curvature (h/R) both be small.

Approximations are also necessary in order to simplify the frictional terms (term 7) in equations 6 to 8. Rozovskii showed that, neglecting wall effects, the frictional terms can be divided into terms of relative magnitude of unity and of $(h/R)^2$. Since h/R is assumed small, these latter terms may be neglected, so that the following approximations are used:

$$F_s = \frac{1}{\rho}\frac{\partial \tau_{sz}}{\partial z} \quad ; \quad \frac{\tau_{sz}}{\rho} = \varepsilon \frac{\partial u}{\partial z} \tag{3.8}$$

$$F_n = \frac{1}{\rho}\frac{\partial \tau_{nz}}{\partial z} \quad ; \quad \frac{\tau_{nz}}{\rho} = \varepsilon \frac{\partial v}{\partial z} \tag{3.9}$$

$$F_z = \frac{1}{\rho}\frac{\partial \tau_{zz}}{\partial z} \quad ; \quad \frac{\tau_{zz}}{\rho} = \varepsilon \frac{\partial w}{\partial z} \tag{3.10}$$

where τ is the shear stress. It is important to note that this approximation implies that all lateral exchanges of momentum due to friction in the fluid are neglected, limiting the validity of the model to the central region of wide channels, where the influence of sidewall friction is negligible. It is this limitation that is particularly problematic in applying these bendflow models in the near bank zones (where lateral shear due to bank friction is significant) for the purposes of width adjustment modelling. Further problems arise in assuming a hydrostatic distribution of pressure. Again, in the central portion of the channel, this is a reasonable approximation, but this (static approximation) rapidly breaks down when there are significant (dynamic) vertical velocities, as there often are in the outer bank zone where the central and outer bank cells meet (Figure 3.1). Further assumptions concerning the eddy viscosity distribution and a number of boundary conditions are required in order to close the model. Although Rozovskii's approach can be used to give good predictions of the flow in the central portion of wide, mildly curved river channels with low Froude number, it is clear that within the crucial near bank zones, the model is invalid.

Since Rozovskii's pioneering work, many other bend flow models have been introduced. Often, the aim of these models has not only been to model the flow through a fixed bed channel, but to investigate the interaction between the flow and bed topography. An important contribution was made by Engelund (1974), who used the analysis of fluid motion developed by Rozovskii to produce a model for flow and bed topography. Engelund's contribution was twofold. First, he claimed that the vertical velocity profile could be described best in natural rivers by the velocity defect law. Second, following this assumption, he first approximated two-dimensional bend flow, and subsequently included some second order calculations to take into account the effect on the flow field of radial variations of depth and velocity (Markham, 1990). In a series of papers, Bridge and his co-workers have used Engelund's approach as the foundation of their attempts to model the flow and bed topography in river bends (Bridge, 1977, 1982, 1984; Bridge & Jarvis, 1982).

However, in addition to the usual restrictions outlined in Rozovskii's approach and also present in these models, there are a number of problems with the Engelund-Bridge approach. The primary concern is that these models should not treat the convective acceleration terms as second order. In fact scaling arguments used by Smith & Mclean (1984) and Nelson (1988) strongly indicate that the convective acceleration terms should be considered as first order effects (Nelson & Smith, 1989a; Dietrich & Whiting, 1989). Moreover, these theoretical arguments are supported by both field (Dietrich & Smith, 1983) and laboratory data (Yen & Yen, 1971). These data sets were used by Dietrich & Smith (1983) to show evidence for a "substantial topographically induced alteration in the cross stream flow pattern relative to that for analagous constant bottom cases" in channels with natural bed topography. Apparently, shoaling over the point bar in the upstream part of the bend forces the high velocity filament of the flow towards the pool, first a decrease in the cross stream water surface slope caused by the convective acceleration and second, dominance of the vertically averaged centrifugal force (Dietrich & Smith, 1983). The data also showed that forces arising from topographically-induced convective accelerations are of the same order of magnitude as the downstream boundary shear stress and water surface slope components, and must be modelled as first order effects, not as second order effects as handled in the Engelund-Bridge models.

Odgaard & Bergs (1988) replicated Dietrich & Smith's study and similarly found that convective accelerations, induced as a result of change in channel curvature at the bend entrance, had a significant effect on the flow processes in their laboratory bend. However, Odgaard & Bergs argue that while the downstream accelerations are important in this regard, they found the cross channel component to be negligible, conflicting with Dietrich & Smith's results. Dietrich & Whiting (1989) argue that Odgaard & Bergs' results are inconclusive, however, since first, they incorrectly formulate a critical term in their analysis and second, they use a measurement procedure which tends to reduce the critical cross-stream terms in question. In light of the apparent conflict, though, Dietrich & Whiting (1989) and Whiting & Dietrich (1991) further investigated the magnitude of the terms in the force balance equation, in the latter case over an alternate bar topography in a straight channel rather than in a meander bend. Again, they found that topographically-induced convective accelerations, both downstream and cross-stream, are sufficiently large that their influence on the force balance is of the same order as the p.g.f and the boundary shear stress. It is clear from both the theoretical scaling arguments of Smith & Mclean and Nelson and the field and laboratory analyses reviewed above that these terms should be included in the equations of motion for the purposes of modelling the flow in channels with downstream varying bed topography (point or alternate bars) and curvature, both of these are characteristics of many natural river channels.

In order to simplify the governing equations, the approaches used by Engelund and Bridge, Odgaard (1986ab) and DeVriend (1977) either ignored the convective accelerations induced by downstream varying curvature and bed topography, or employed mathematically inconsistent expansions based on the assumption of fully developed flow (Nelson & Smith, 1989a). Nelson & Smith (1989a) note that models of this kind will only be accurate when the momentum fluxes associated with bed topography and changes in curvature are much smaller than the pressure gradient and bottom stress terms in the vertically averaged downstream momentum equations. They argue that a valid model of flow in curved channels with naturally occurring bed topography must include these convective acceleration terms.

Smith & Mclean (1984) used a regular perturbation expansion about a zero order solution that included the critical convective acceleration terms described above to develop a model of 2D depth averaged flow in meandering streams. This study is noteworthy as one of the first attempts to derive the depth averaged 2D equations in an orthogonal curvilinear coordinate system, and to include the convective acceleration terms in the derivation. They also showed that in deriving the horizontal equations of motion for an appropriate curvilinear coordinate system, a metrical coefficient to account for streamwise changing radius of curvature must be included. Although Smith & Mclean successfully validated their model using the data of Hooke (1975), their model still excludes vertical velocities and lateral momentum exchange, so it is inapplicable within the near bank zones.

Nelson & Smith (1989ab) further generalized and tested the meander flow model of Smith & Mclean (1984). They first developed a meander model as a precursor to the development of a meander flow model, in which the technique of solution was based on a perturbation expansion about a 3D zero order velocity field comprise of a solution to an approximate set of vertically averaged equations in conjunction with a properly scaled similarity vertical structure function (Nelson & Smith, 1989b). Simplification of the vertically averaged 2D equations was made using careful scaling arguments specifically valid for natural meander bends. But in order to construct a model of general validity, the scaling constraints employed in the meander model are altered in the flow model (Nelson & Smith, 1989b), though most of the physical assumptions used are common to both models:

1. u >> v >> w. This may not apply close to steep banks, where vertical and transverse velocity components may both be of similar orders of magnitude to the streamwise velocity.

2. Hydrostatic pressure field. This does not apply near steep banks where vertical velocity components may be large.

3. Fully turbulent flow is assumed. Turbulent flow is, however, anisotropic and difficult to parameterize near banks.

4. Effects of lateral friction are neglected. Clearly, this is inadequate close to the bank regions.

5. Lowest order structure of the velocity field is assumed to be characterized by strong shear near the boundary and relatively weak shear (near constant velocity) in the region away from the boundary, so that a downstream velocity is assumed nearly equal to the vertically averaged velocity throughout most of the flow depth. This is important, since it leads to the assumption that the vertically averaged convective accelerations in the downstream momentum equations are reasonable approximations to the full convective acceleration terms (Nelson & Smith, 1989b). This assumption breaks down near the bed, but here the accelerations are small compared to the stress divergence. Nelson & Smith (1989ab) showed that their model reproduced both laboratory (Hooke, 1975) and field (Dietrich & Smith, 1983) conditions quite well.

Similar models to that proposed by Smith & Mclean and Nelson & Smith have been proposed by Shimizu *et al.* (1987), Shimizu *et al.* (1990), Shimizu & Itakura (1989) and Shimizu & Itakura (1986); again, comparisons of model predictions and observed conditions for a variety of meandering streams demonstrate the powerful predictive ability of this approach, at least in the central and point bar regions of natural meandering streams.

Odgaard (1986ab) presented a 2D model for simulating flow and bed topography in meandering alluvial channels. He also stressed that an understanding of the role of developing flow is crucial if erosion and deposition is to be understood fully. He proposed a model, based on a solution to the equations for conservation of mass, conservation of momentum, and lateral stability of the bed, that he claimed accounted for both the effects of developing flow, and the convective accelerations. The model was applied and found to give adequate predictions of the bed topography in natural meanders. However, the usual assumptions are again employed in this study, invalidating this model in the near bank zones. Moreover, both Nelson & Smith (1989a) and Dietrich & Whiting (1989) criticise the Odgaard model for omitting the important cross-stream convective acceleration term. Odgaard himself justifies this by referring to the (faulty) data of Odgaard & Bergs (1988) (Dietrich & Whiting, 1989).

A number of other meander flow models have originated from researchers based at Delft Hydraulics and Delft University in the Netherlands. DeVriend (1977) formulated a model of steady flow in curved shallow channels in order to derive the vertical distribution of the secondary flow. DeVriend used a cartesian coordinate system and simplified the 3D set of equations using the traditional assumptions, so that this model also does not apply in the near bank zones. DeVriend closed the model by choosing an eddy viscosity model based on the mixing length hypothesis such that a logarithmic streamwise velocity profile was obtained. The assumption that such a logarithmic distribution is a reasonable approximation of the conditions actually experienced in a river bend, while tenable in the central part of the channel, is questionable in the outer bank zone (Rais, 1985), where the convection of primary flow momentum by the descending secondary flow may result in a depression of the core of maximum velocity well below the water surface. DeVriend (1977) used a perturbation method to yield, after depth averaging, a first order approximation of the complete solution. He then transformed the solution into a curvilinear coordinate system to aid his physical analysis of the equations. He found that his model yielded satisfactory vertical distributions of the streamwise velocity component, but it is clear that the model is invalid in the near bank zones. Nevertheless, the work is noteworthy since it represents one of the first attempts to include the effects of realistic channel geometry, and uses depth averaged equations in a curvilinear, rather than cylindrical, coordinate system. In this respect, this, and later papers (e.g. Kalkwijk & DeVriend, 1980; DeVriend & Geldof, 1983), the work of DeVriend is similar to the important work of Smith & Mclean (1984).

Olesen (1987) presented a model of flow in meander bends in order to obtain a bed shear stress distribution for his morphological model computations. Olesen used a curvilinear coordinate system and notes that although the 3D mathematical description of the flow is very comprehensive, it can be considerably simplified if only rivers of constant width are considered. In that case the n - axis is straight and, as a consequence, a number of small inertia and friction terms vanish. However, this assumption of uniform width, employed in all the models reviewed in this chapter, is a further limitation of these models when applied to the problem of modelling width adjustment. Not only is the assumption of uniform width in any case unrealistic as an initial condition but also, since the flow and sediment transport and bank stability processes vary throughout the bendway, it is reasonable to expect rates of width adjustment to vary through the length of the bend, so that the bend is likely to evolve away from a uniform width. This means that width adjustment models in curved channels may be restricted to cases where the streamwise variation in width is small. Although Kalkwijk *et al.* (1980) have given a 3D mathematical description of the flow in a curvilinear coordinate system in which both horizontal coordinate axes are curved, they avoid the necessary comprehensive description of the friction terms.

To simplify the governing equations, Olesen introduces the traditional simplifying assumptions of rigid lid approximatio- and the approximation that lateral momentum exchange can be omitted, together with the hydrostatic approximation. Only steady flow was considered. Despite these limitations with respect to width adjustment modelling, Olesen's work is interesting since he analysed the suitability of 3 different mixing length models used to formulate the eddy viscosity closure. Since the friction terms are given by:

$$\frac{\tau_s}{\rho} = A\frac{\partial u}{\partial z} \qquad (3.11)$$

$$\frac{\tau_n}{\rho} = A\frac{\partial v}{\partial z} \qquad (3.12)$$

for the streamwise and transverse flows, respectively, the choice of the eddy viscosity coefficient, A, is vital. Introducing the Prandtl mixing length hypothesis:

$$A = L^2\frac{\partial\sqrt{u^2 + v^2 + w^2}}{\partial z} = L^2\frac{\partial u}{\partial z} \qquad (3.13)$$

where L is a mixing length, which is some function of z. Olesen notes that the applied formulation implies the eddy viscosity is assumed isotropic. Olesen applied 3 mixing length models:

$$L = k\,z\sqrt{1-z}\;h \tag{3.14}$$

$$L = k\,z^{1-\frac{1}{m}}\sqrt{1-z}\;h \tag{3.15}$$

$$L = 2\,k\,(1-\sqrt{1-z})\,\sqrt{1-z}\;h \tag{3.16}$$

where k is the von-Karman constant, m is a factor depending on bed roughness and $z = z/h$ is the dimensionless vertical coordinate. Equation (3.14) results in the well known logarithmic profile in straight uniform flow, equation (3.15) is a power law profile and equation (3.16) is the von-Karman velocity profile. Equations (3.14) and (3.16) represent the two most frequently used mixing length models for river flows (Olesen, 1987). Olesen conducted sensitivity tests to show the influence of the eddy viscosity closure model on predictions of flow velocity, and the results are shown in Figure 3.3. This figure shows that the eddy viscosity model can significantly influence the predictions obtained. While Olesen argues that equations (3.14) and (3.16) have the most realistic form on physical grounds, the eddy viscosity closure model remains one of the greatest problems in hydraulics, particularly in the near bank zones, since most models are developed for straight, uniform shear flow in the central part of the channel.

Struiksma (1985), Struiksma *et al.* (1985) and Struiksma & Crosato (1989) have reported the development of a 2D bed topography computer model which aims to simulate the 2D bed topography evolution in rivers. Again, assuming constant, uniform width, using the hydrostatic approximation, assuming steady flow and neglecting lateral momentum exchange, the 2D depth averaged momentum and continuity equations can be derived from the fully 3D set of equations. They coupled the flow equations together with a sediment transport equation in order to predict the evolution of bed topography. Flume experiments were used to give a reasonably good validation of the bed topography model, suggesting the hydraulics model gives adequate predictions, at least in the central part of the channel. Detailed data of the model performance in the near bank zone are not available.

3.2.3 3-Dimensional Bend Flow Models

Depth-averaged 2D models have been widely used to model shallow water flows, as discussed in the previous section, even though genuinely 2 dimensional flows are uncommon in hydraulics. A problem with such 2D models is that they are unable to account for all of the effects of helicoidal flow, which is a 3D phenomenon which cannot be represented by conventional depth averaging (Bernard & Schneider, 1992). This problem is greatest within the near bank zones of meandering rivers, where significant vertical velocities are frequently observed, and the flow is truly 3 dimensional. Yet it is precisely this near bank zone which is of greatest interest to width adjustment modellers. It may be that 3 dimensional models offer the best hope for modelling the near bank hydraulics for width adjustment modelling. It is the purpose of this section to review some of the existing 3D models to assess their suitability for predicting the near bank flows in meandering channels.

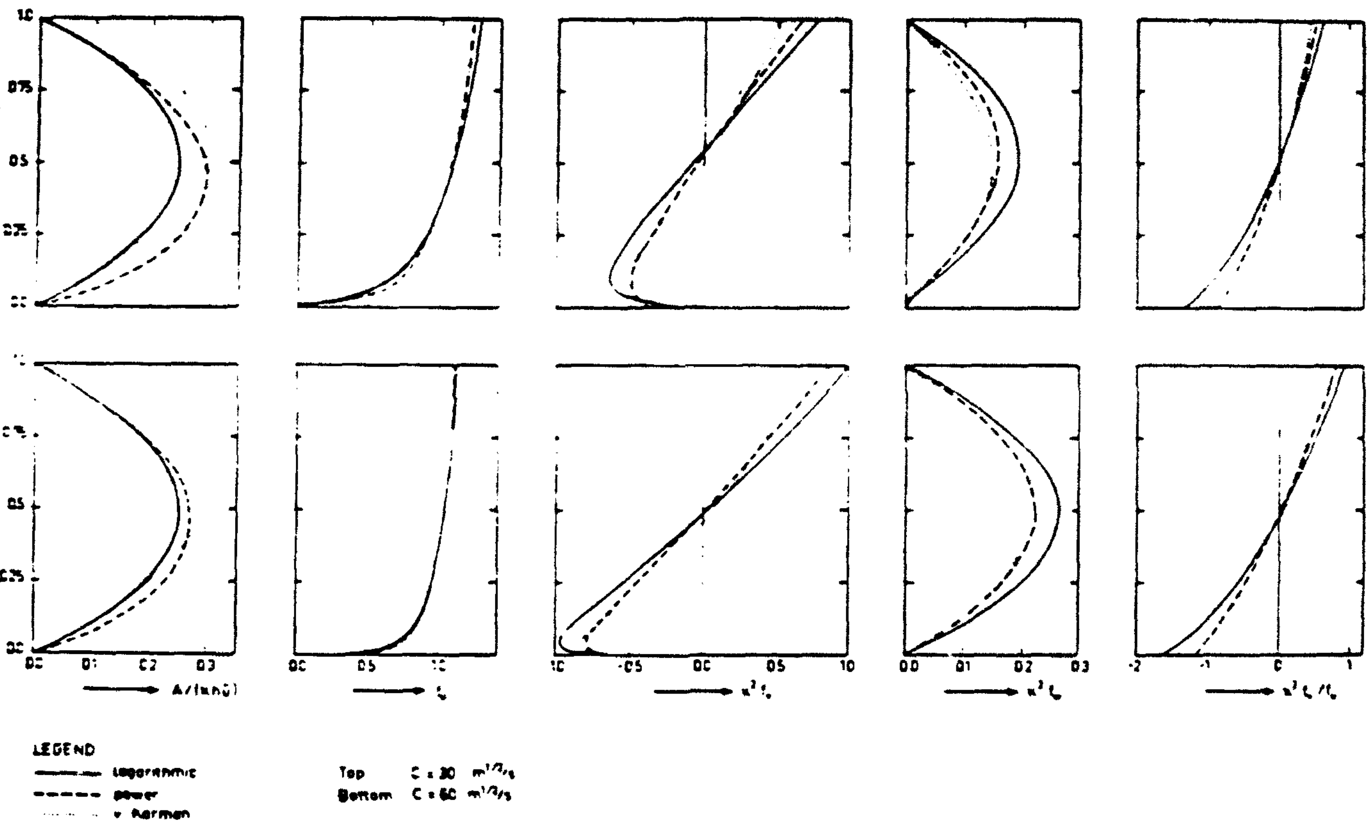

Figure 3.3 Vertical distributions of eddy viscosity, flow velocity and flow direction for 3 eddy viscosity models (after Olesen, 1987)

Among the first to attempt to model bend flow processes 3 dimensionally were Leschziner & Rodi (1979). They modelled the idealized case where a steady and stable meandering stream is represented by a series of straight and curved channel sections having either rectangular or semi-circular cross-sections. Leschziner & Rodi recognised this was a considerable approximation to the natural geometry of meander bends, but their stated purpose was to demonstrate the innovative calculation method of Pratap & Spalding (1975) for modelling flow in closed conduits. Leschziner & Rodi considered the flow equations in a cylindrical coordinate system, but in deriving the equations they used the restriction that flow separation was absent. In consequence, there is a negligible diffusive transport of momentum in the streamwise direction. A kinetic energy dissipation model of eddy viscosity was used to close the model (Launder & Spalding, 1974). In spite of the gross simplifications, Leschziner & Rodi found good quantitative agreement with experimental data, but this was for a relatively simple test case.

Since this pioneering study, a number of other attempts to model bend flow in 3 dimensions have been made by, for example, DeVriend (1981), Tamai & Ikeya (1985) and by Shimizu *et al.* (1990). DeVriend modelled the primary velocity redistribution in steady flow through curved conduits of shallow rectangular cross-sections, so again this study is concerned with a rather restrictive channel geometry. Perhaps the most impressive 3D studies to date have been conducted by Tamai & Ikeya (1985) who attempted to model the flow over an alternating bar bed topography, and Shimizu *et al.* (1990), who used a 3D computation of both flow and bed deformation. Previous studies considered only the fixed bed case.

Shimizu *et al.* (1990) developed the 3D model of flow and bed deformation for the explicit purpose of improving upon the results of a 2D model developed by Itakura & Shimizu (1986). This is a useful study, since it is concerned directly with evaluating the benefits of a 3D approach. In constructing the 3D model, a number of simplifications were used by Shimizu *et al.* The main approximations are the assumptions of a hydrostatic pressure distribution and a logarithmic velocity profile. As discussed previously, both of these assumptions are frequently invalid in the near outer bank zone of meandering channels. Thus, while Shimizu *et al.* did indeed find that the 3D model gave a definite improvement over the 2D model, even the 3D approach is unable to predict the near bank flows with any certainty.

3.2.4 Summary

To summarize, great progress has been made in understanding and modelling many of the features of bend flows, both with 2D and 3D models, even though a number of simplifying assumptions are necessary in order to obtain tractable solutions to the governing equations. However, these assumptions result in the hydraulic models having a validity limited to the central part of the channel. Thus, while sophisticated models of bend flow hydraulics, both in 2 and 3 dimensions, are available, it is debatable that these models can be applied to modelling the hydraulics in the near bank zones for the purposes of width adjustment modelling.

Perhaps the greatest progress made in the last 10 to 15 years in understanding and modelling bendflows has been the recognition of the important role of the growth of the point bar at the inner bank on the bed topography in meandering streams, and the significance of the resulting convective accelerations that act to force the flow toward the opposite bank. The work of Dietrich and Smith, and other workers from the University of Washington, has clearly demonstrated the importance of these terms both in explaining equilibrium point bars at the inner bank in river bends, and alternate bars in straight sections (Nelson, 1988; Dietrich & Whiting, 1989). It is now clear that to be valid physically bend flow hydraulic models should include these terms in the governing equations. This is especially true if the flow model is to be applied to width adjustment problems, since the growth of the point bar at the inner bank is undoubtedly a vital width determining mechanism (e.g. Mosselman, 1992). Indeed, it is possible to argue that currently available flow models not only give good predictions valid in the central part of the channel, but are also reasonably valid in the inner bank zone.

The basis for this argument lies in recognising the assumptions that limit the validity of the hydraulic models to the central part of the channel. These are:

1. The hydrostatic pressure distribution (small vertical velocities);
2. The neglect of lateral momentum exchange due to bank friction, and:
3. The assumption of a logarithmic vertical velocity profile (again this requires vertical velocities to be small, see Rais (1985)).

But, from Figure 3.1C, it will be noticed that while vertical velocities are expected to be significant close to the steep, eroding outer bank, the gently sloping inner bank region (the site of the point bar) helps to restrict vertical velocities. Similarly, the influence of bank friction on lateral momentum exchange is likely to be much greater in the outer bank zone due to the presence of a relatively steep, high bank in this zone. It is, therefore, reasonable to assume that hydraulic models that include the convective acceleration terms are probably able to model the flow in both the central and inner bank regions quite well. It is only the outer bank region where these models are invalid. In the central and inner bank zones, probably the best available flow models are the 2D models of Nelson and Smith and Shimizu, and the 3D model of Shimizu and his co-workers. Future research should be directed at including vertical velocities and bank boundary layer effects into these models, in order to extend their validity to the outer bank also.

The argument that these models are invalid only in the outer bank zone raises the possibility of using some kind of adaptive grid approach to modelling the flow in meander bends, where two models, one valid for the outer bank zone, the other valid for the remainder of the channel, can be joined together in order to construct a suitable hydraulics algorithm as the front end of a width adjustment model. While the Nelson-Smith or Shimizu models could provide predictions for the inner bank and central regions, in order for this type of approach to be effective, there must also be a suitable model in the outer bank zone. Moreover, both models must be consistent in the physical processes they account for, and must have similar predictive abilities. If two models are to be joined using some kind of adaptive grid technology, this can only be done if one of the models is used to generate the boundary conditions for the other, so that predictions are identical at the "seam". If the models have different predictive abilities, or are physically inconsistent, the better model would inevitably become polluted by the boundary condition generated by the poorer model. This factor needs to be taken into account when deriving a suitable outer bank flow model, which is the subject of the next section of this chapter.

3.3 Model Of Flow Near The Outer Bank Of A River Bend

Very few studies have concerned themselves with attempting to model the flow close to the outer bank of a river bend. Studies explicitly concerned with the outer bank zone have usually been qualitative or semi quantitative empirical studies of some aspects of the flow in this zone (e.g. Hicks *et al.*, 1990; Jin *et al.*, 1990). The exception to this rule is the PhD dissertation of Samira Rais (Rais, 1985), a graduate student under the supervision

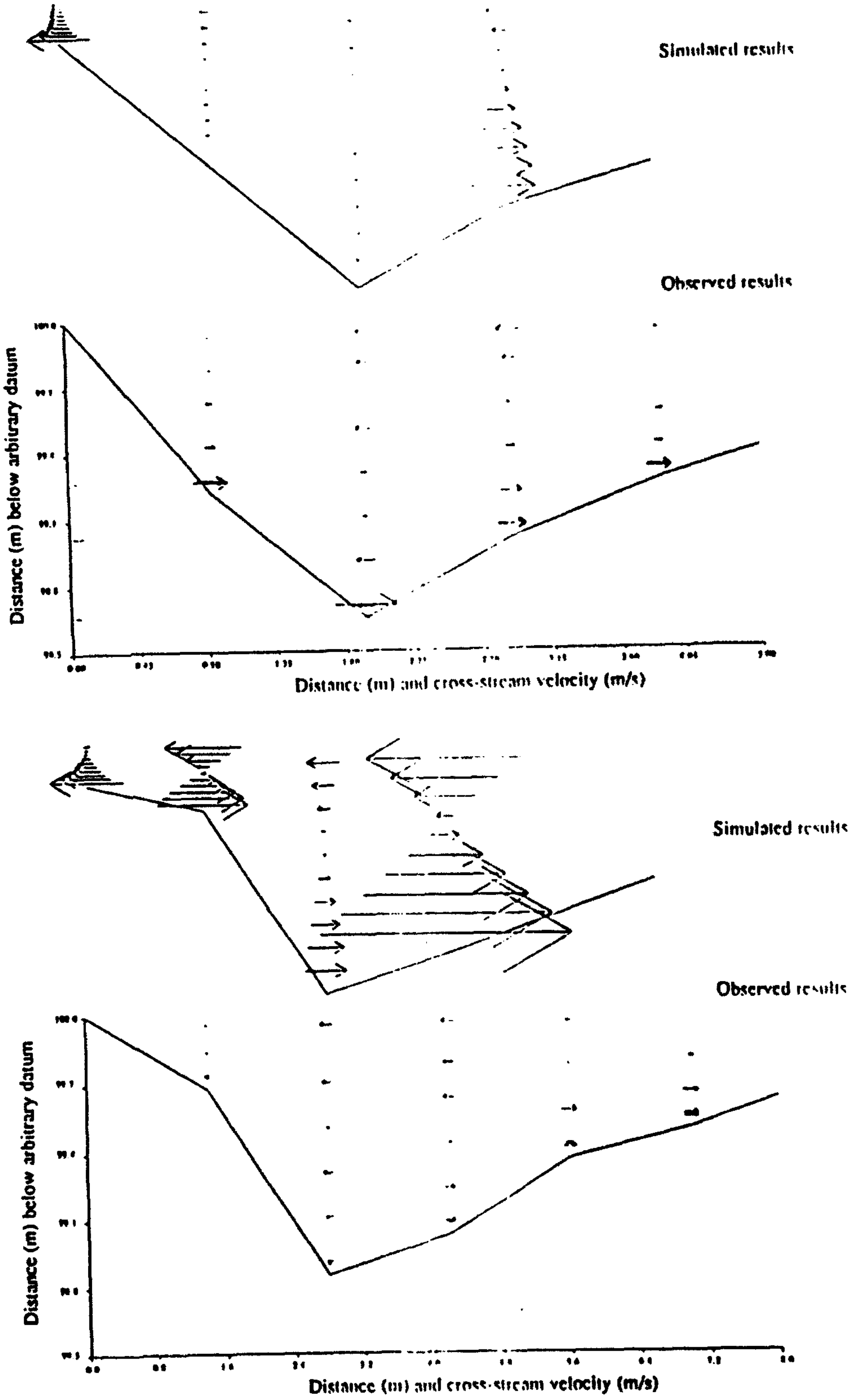

Figure 3.4 Predictive ability of Rais model (after Markham, 1990)

of the principal investigator of this project. She attempted to predict the 3 components of the flow close to the outer bank using a mixed empirical-analytical solution of the governing equations of flow close to the outer bank. It was recognised at the outset of this project that this approach, if valid, would be of great importance to the potential development of any width adjustment model. Although Rais (1985) did not conduct any rigourous test of the validity of her model, Markham (1990) did conduct an analysis of the predictive ability of the Rais model, using data that he collected from the Fall River in Colorado (Figure 3.4). The results of these tests suggested that despite the relative simplicity of the Rais model, the model was able to predict the gross structure of the flow close to the outer bank, but also that the model proved to be unstable and prone to large and unpredictable errors. In view of the importance of attempting to model the flow close to the outer bank for width adjustment modelling, it was decided that a more thorough appraisal of the Rais model was required, together with any developmental work necessary to improve the model. This work was a primary aim of the research reported here. The results of this appraisal/developmental work are reported later in this section. First, a brief review of the original Rais flow model is presented.

The starting point of the Rais approach to modelling flow close to the outer bank is to consider the governing equations of flow. Rais presented these equations (3.1 through 3.7) in a cylindrical coordinate system, a disadvantage since the ultimate aim is to apply the model together with a flow model applicable to the remainder of the channel, and these latter models are usually presented in the more convenient curvilinear coordinate system. Still, this does not influence the predictive ability of the model. Rais (1985) then made a number of assumptions to simplify the solution. She considered steady, turbulent flow where coriolis effects are assumed negligible. However, she also considered only fully developed flow, equating the derivative of any quantity in the longitudinal direction to zero, and also assumed the vertical pressure distribution is to be hydrostatic. The latter assumption, while it may be inaccurate, is very hard to circumvent and is, therefore, considered justifiable, as are in fact the first three assumptions. However, the neglection of terms with downstream derivatives, while helping to considerable simplify the problem, leads to the neglection of many of the important convective acceleration terms discussed in the previous section. Rais also justified neglecting some of the diffusion terms in these equations on the basis of the results of a scaling analysis. Using these assumptions, and non-dimensionalizing, the equations of motion were written:

$$V_r \frac{\partial V_s}{\partial r} + \frac{1}{\gamma'} V_z \frac{\partial V_s}{\partial z} + \frac{V_s V_r}{r} = \frac{A}{Re\,\gamma'} \frac{\partial^2 V_s}{\partial z^2} \tag{3.17}$$

$$V_r \frac{\partial V_r}{\partial r} + \frac{1}{\gamma'} V_z \frac{\partial V_r}{\partial z} - \frac{V_s^2}{r} = -\frac{\partial p}{\partial r} + \frac{A}{Re\,\gamma'} \frac{\partial^2 V_r}{\partial z^2} \tag{3.18}$$

$$V_r \frac{\partial V_z}{\partial r} + \frac{1}{\gamma'} V_z \frac{\partial V_z}{\partial z} = \frac{A}{Re\,\gamma'} \frac{\partial^2 V_z}{\partial z^2} \tag{3.19}$$

where V_s, V_r and V_z are the dimensionless longitudinal, transverse and vertical velocity components, respectively, A is the dimensionless eddy viscosity, Re is the Reynolds number and γ the ratio of flow depth to radius of curvature. By introducing an empirical function for the longitudinal velocity component, this set of equations was reduced to three unknowns with three equations. Evaluating the derivatives of the longitudinal velocity, and substituting back these evaluated values and rearranging, the radial velocity was expressed as a function of the known quantities in equation (3.17). This expression was then substituted into (3.19) to obtain a partial differential equation in V_z only. This equation was approximately solved using a perturbation analysis to get an analytical expression for the vertical velocity component, which on substitution back into (3.17) allows, with the application of the correct boundary conditions, solution for the transverse velocity, which Rais (1985) expressed as:

$$V_r = \frac{2}{\gamma'\,(c+1)} \left(\frac{\alpha 3\, r}{[Z_m{}^2 - (Z' - Z_m)^2]} - B\, r^{-c}\, (Z' - Z_m) \right) \tag{3.20}$$

where B is a dimensionless constant of integration determined from the boundary conditions, c is a dimensionless empirical exponent (see equation 3.24), Z_m is the dimensionless depth of the core of the longitudinal velocity maximum, and $\alpha 3 = \frac{A}{Re}$.

The predictive ability of (3.20) is summarized in Figure 3.4 as previously stated. In fact, although this model predicts the gross features of the flow, it is clearly inadequate for the purposes of a width adjustment model. Indeed, on closer inspection, several errors were noted in the original derivation by Rais (1985). These errors were associated with incorrect evaluation of the derivatives of the empirical longitudinal flow velocity model.

Rais (1985) suggested that the dimensionless longitudinal velocity could be expressed by a function of the form:

$$V_s = K(z)\, r^{-c} \tag{3.21}$$

with r the dimensionless radius of curvature, c a positive coefficient and K(z) a velocity distribution function to express the vertical velocity profile. Rais (1985) empirically derived expressions for K(z) and c. Concerning the evaluation of K(z) Rais argued that a fundamental feature of the flow near the outer bank is the depression of the longitudinal velocity maximum below the surface of the water, due to the convection of primary flow momentum by the plunging secondary flow at the outer bank. Further arguing that the magnitude of the velocity depression was a function of the strength of the plunging secondary flow, in turn related to the steepness of the outer bank, Rais used data that she collected from the Fall river in Colorado to derive the expression:

$$K(z) = 11\,(Z_m)^{-0.75}\,\left(Z_m^{\,2} - (Z' - Z_m)^2\right) \tag{3.22}$$

where Z_m is the dimensionless depth of the core of the maximum longitudinal velocity and Z' is the dimensionless depth. A further empirical expression for Z_m as a function of the steepness of the outer bank was presented:

$$Z_m = 0.55\,(\sin i)^{-0.8} \tag{3.23}$$

with i the angle of the outer bank. Rais further derived an empirical expression for the dimensionless exponent, c, in equation (3.21), such that:

$$c = 1500\left(\frac{R_c}{W}\right)^{-0.1}\left(\frac{d}{W}\right)^{0.3} Re^{-0.3} \tag{3.24}$$

where R_c is the centreline radius of curvature (m), W is the mean channel width (m), d is the mean channel depth (m) and Re is the Reynolds number.

However this empirical model appears to be somewhat unrealistic. Firstly, it can be seen that equation (3.22) predicts that the longitudinal velocity at the point of the velocity maximum in the vertical may be up to 11 times the depth averaged value, which is unlikely. This suggests that the formulation for K(z) is incorrect. Secondly, the form of equation

(3.21) also appears to be incorrect. While the negative exponent on the radius of curvature ensures that the velocity increases from the bank towards the centreline, the shape of this curve is incorrect. The Rais model predicts an exponential decline in velocity from channel centreline to the bank, while experience suggests in river bends the longitudinal velocity increases from its centreline value to some maximum value close to the outer bank, then decreases to zero at the bank due to the influence of the bank boundary layer. The effect of this error is to lead to systematic discrepancies in the transverse derivatives of velocity in the equations of motion.These facts, together with the rather sweeping assumptions used by Rais were hypothesized as the factors responsible for the poor predictive ability of her model. It was, therefore, decided to completely re-derive this equation using the approach described by Rais (1985) in an attempt to improve the predictive ability of the model. The starting point of this re-derivation was to rewrite the governing equations of flow (equations 3.1 through 3.7) in a curvilinear coordinate system, so that the model would be compatible with the other bend flow models discussed in the previous sections. The assumptions concerning steady, turbulent flow with negligible coriolis effects were retained, together with the hydrostatic approximation. However, attempts to retain the dominant convective acceleration terms were made. Analysis of the vorticity equations suggests that the derivatives of the streamwise velocity in the streamwise direction in generating streamwise vorticity are more significant than either the streamwise derivatives of the transverse or vertical velocities. In this derivation, therefore, the streamwise derivatives of the transverse and vertical velocities are neglected while streamwise derivatives of the streamwise velocity are retained, in order to retain this important physical mechanism. Introducing these approximations, omitting terms of order of magnitude $(h/r)^2$ and non-dimensionalizing the equations in preparation for substitution of non-dimensionalized empirical streamwise velocity functions, the equations of motion can be written:

$$u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial n}+\Omega\, w\frac{\partial u}{\partial z}+\Gamma\frac{uv}{r}=\frac{\varepsilon\,\Omega}{Re}\frac{\partial^2 u}{\partial z^2} \tag{3.25}$$

$$v\frac{\partial v}{\partial n}+\Omega\, w\frac{\partial v}{\partial z}-\Gamma\frac{u^2}{r}+\frac{1}{\rho}\frac{\partial p}{\partial n}=\frac{\varepsilon\,\Omega}{Re}\frac{\partial^2 v}{\partial z^2} \tag{3.26}$$

$$\frac{1}{\Omega}v\frac{\partial w}{\partial n}+w\frac{\partial w}{\partial z}=\frac{\varepsilon}{Re}\frac{\partial^2 w}{\partial z^2} \tag{3.27}$$

where Ω is the half width to depth ratio and Γ is the half width to centreline radius of curvature ratio.

Following Rais, a solution to these equations is possible by introducing a non-dimensional empirical relationship for the primary velocity, u, and evaluating the derivatives of this function in all 3 dimensions. Unlike the Rais approach (where u is a function of n and z only), the dimensionless streamwise velocity must be evaluated as a function of the non-dimensionalized s, n and z. Following the approach of Rais (1985), the form of this relationship is expressed as:

$$u = K(z) \langle u \rangle \tag{3.28}$$

where the dimensionless depth-averaged streamwise velocity, $\langle u \rangle$, is a function of s and n. In the latter half of a bendway, the depth-averaged streamwise velocity tends to increase from the centreline to some maximum at the edge of the bank boundary layer, close to the outer bank. At the outer bank itself, the streamwise velocity must be equal to zero. Further, as the water flows around the bend, the core of maximum velocity tends to migrate from the inner to the outer bank. It may be assumed that at any cross-section in the bend the streamwise depth-averaged velocity varies from its reach averaged value, U, at the centreline to a maximum, U_{max}, close to the outer bank, as described previously. It may also be assumed that the velocity varies linearly with transverse position from its reach averaged, centreline value to some unknown maximum value close to the outer bank. Such a form is supported empirically by, among others, Nelson & Smith (1989), Odgaard (1986), DeVriend & Geldof (1983) and Kikkawa *et al.* (1976). Hence, the depth-averaged streamwise velocity distribution is hypothesized to have the form shown in Figure 3.5. By further assuming that the decrease of the velocity from its maximum value at n^* to zero at the outer bank can be described by a parabolic function, the depth averaged velocity can be completely determined throughout the bendway if the non-dimensional values U_{max} and n^* are empirically specified. Using laboratory data provided by Dr. Stephen Maynord from the Rip Rap Test Facility (RRTF) at WES, together with data from Muddy Creek, Wyoming, (Nelson & Smith, 1989), the following empirical non-dimensional relationships were obtained for these parameters:

$$U_{max} = 1.0 + 0.01968\,s \qquad (R^2 = 0.998) \tag{3.29}$$

$$n^* = 0.55241 + 0.267 \log(s) \qquad (R^2 = 0.975) \tag{3.30}$$

These formulations allow the dimensionless depth-averaged streamwise velocities to be expressed as:

$$\langle u \rangle = 1.0 + (U_{max} - 1)\frac{n}{n^*} \qquad n \le n^* \qquad (3.31)$$

$$\langle u \rangle = U_{max} - \left(U_{max}\frac{(n - n^*)^2}{(1 - n^*)^2}\right) \qquad n > n^* \qquad (3.32)$$

To complete the empirical formulation for the streamwise velocity, the function K(z) also needs to be calculated: that is the vertical velocity distribution function must be specified. While Rais (1985) correctly emphasised the importance of being able to predict the depression of the streamwise velocity maximum below the water surface, her formulation is incorrect. The empirical formulation presented by Rais (1985) for the depth at which the velocity maximum occurs (equation 3.23) does appear to be reasonable, giving good results when tested against Markham's (1990) Fall River data, the formulation for K(z) (equation 3.22) as presented by Rais appears to give gross overestimates of the streamwise velocity for certain combinations of Z' and Z_m. Repeating her regression analysis, it was found that the equation as presented by Rais (1985) was in error. A corrected version of the Rais function, retaining the formulation for Z_m (equation 3.23) presented by Rais, was thus written:

$$K(z) = (19 - 15 Z_m)\ (Z_m^2 - (Z' - Z_m)) \qquad (3.33)$$

so that the empirical specification of the streamwise velocity is given by equations (3.28) through (3.33), together with equation (3.23).

Having re-defined the empirical streamwise velocity function, the solution of the simplified, non-dimensional equations of motion for the transverse and vertical velocities follows the method originally presented by Rais. However, since there now are two solution domains on either side of the location of the velocity maximum, n*, this process must be repeated for each of the empirical formulations in these domains. First, the derivatives of the streamwise velocity are evaluated. These derivatives are somewhat more complex than those derived by Rais, due to the introduction of more terms in the empirical velocity formulation. Substituting the evaluated expressions for the derivatives of the streamwise velocities back into the equations of motion and rearranging (3.25) to get an expression for v in terms of all the other known terms and the unknown w, and substituting this expression into (3.27), a partial differential equation in w only is found:

$$\frac{v}{\Omega}\frac{\partial w}{\partial n} + w\frac{\partial w}{\partial z} = \frac{\varepsilon}{Re}\frac{\partial^2 w}{\partial z^2} \tag{3.34}$$

where, from (3.25), v is of form:

$$v = K_2 - (K_3 - wK_4) \tag{3.35}$$

and where the K terms are constants.

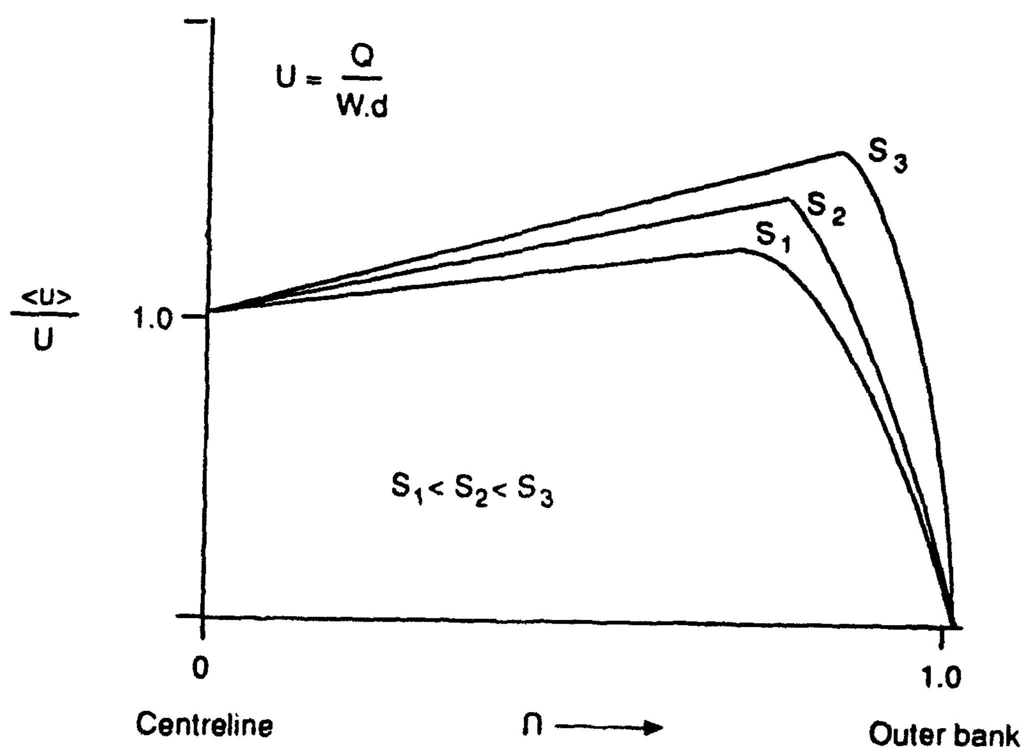

Figure 3.5 Proposed model of lateral distribution of depth-averaged streamwise velocity

Following Rais, an approximate analytical solution to this equation is found by means of a perturbation technique, so that it can be shown that the vertical velocity is given by:

$$w = B\,F^{-2}\left(Z_m^{\,2} - (Z' - Z_m)^2\right) \qquad (3.36)$$

where F is a function given by:

$$F = \frac{-2\langle u\rangle}{T4 + \dfrac{\Gamma\langle u\rangle}{r}} \qquad (3.37)$$

with:

$$T4 = \frac{U_{max} - 1}{n^*} \qquad n \le n^* \qquad (3.38)$$

$$T4 = -2U_{max}\,\frac{(n - n^*)}{(1 - n^*)^2} \qquad n > n^* \qquad (3.39)$$

where B is a constant of integration determined by the boundary conditions. In this case, the boundary conditions are:

1. $w = 0$ at $z = 0$
2. $w = 0$ at $n = 1$
3. $\tau_{nz} = 0$ at $z = 1$ (shear stress at free surface is zero)

In dimensionless terms, τ_{nz} can be expressed:

$$\tau_{nz} = \varepsilon\left(\frac{\partial v}{\partial z} + \frac{1}{\Gamma}\frac{\partial w}{\partial n}\right) \qquad (3.40)$$

From these boundary conditions it is possible to evaluate the integration constants, B, in each of the solution domains which may then be substituted into (3.36) to solve for w. These expressions for the vertical velocities are then substituted back into the rearranged equation (3.25) to give the following solutions for the transverse velocity:

$$v = \frac{<u>\left(\frac{-2\varepsilon\Gamma}{Re} - ((Z_m{}^2 - X^2)\,\zeta_1 - 2\,w\,\Omega\,X)\right)}{\left(\frac{U_{max} - 1}{n^*} + \frac{\Gamma<u>}{r}\right)(Z_m{}^2 - X^2)} \qquad n \le n^* \qquad (3.41)$$

$$v = \frac{<u>\left(\frac{-2\varepsilon\Gamma}{Re} - ((Z_m{}^2 - X^2)\,\zeta_2 - 2\,w\,\Omega\,X)\right)}{\left(-2U_{max}\frac{(n - n^*)}{(1 - n^*)^2} + \frac{\Gamma<u>}{r}\right)(Z_m{}^2 - X^2)} \qquad n > n^* \qquad (3.42)$$

where $X = (Z' - Z_m)$ and

$$\zeta_1 = \frac{\partial u}{\partial s} = K(z)\, n\frac{0.0085 + 0.0023 \ln s}{(0.55 + 0.116 \ln s)^2} \qquad n \le n^* \qquad (3.43)$$

$$\zeta_2 = \frac{\partial u}{\partial s} = 0.0197 - \zeta_3 \qquad n > n^* \qquad (3.44A)$$

where:

$$\zeta_3 = 2U_{max}\frac{(n - n^*)}{(1 - n^*)}\,(A1 + A2) \qquad (3.44B)$$

$$A1 = \frac{(0.0197n - 0.0197nn^*) - \frac{0.1183}{s}}{(1 - n^*)^2} \qquad (3.44C)$$

$$A2 = \frac{0.0108 - \frac{0.1291n^*}{s} - \frac{0.116U_{max}(n^*-n)}{s}}{(1 - n^*)^2} \qquad (3.44D)$$

An attempt to assess the predictive ability of equations (3.41 and 3.42) was made by coding these equations into a numerical model. Using data collected by Markham (1990) from the Fall River, Colorado and the River Roding, Britain, the numerical model was used to generate predicted transverse velocities for the Fall River sites. These predictions were then compared to the observed velocities at the Fall River sites. In the data selection process, care was taken to select field sites that best matched the conditions imposed by the model assumptions. Thus, differences in observed and predicted velocities should be due only to inability of the model to correctly predict those velocities. In order to evaluate if the

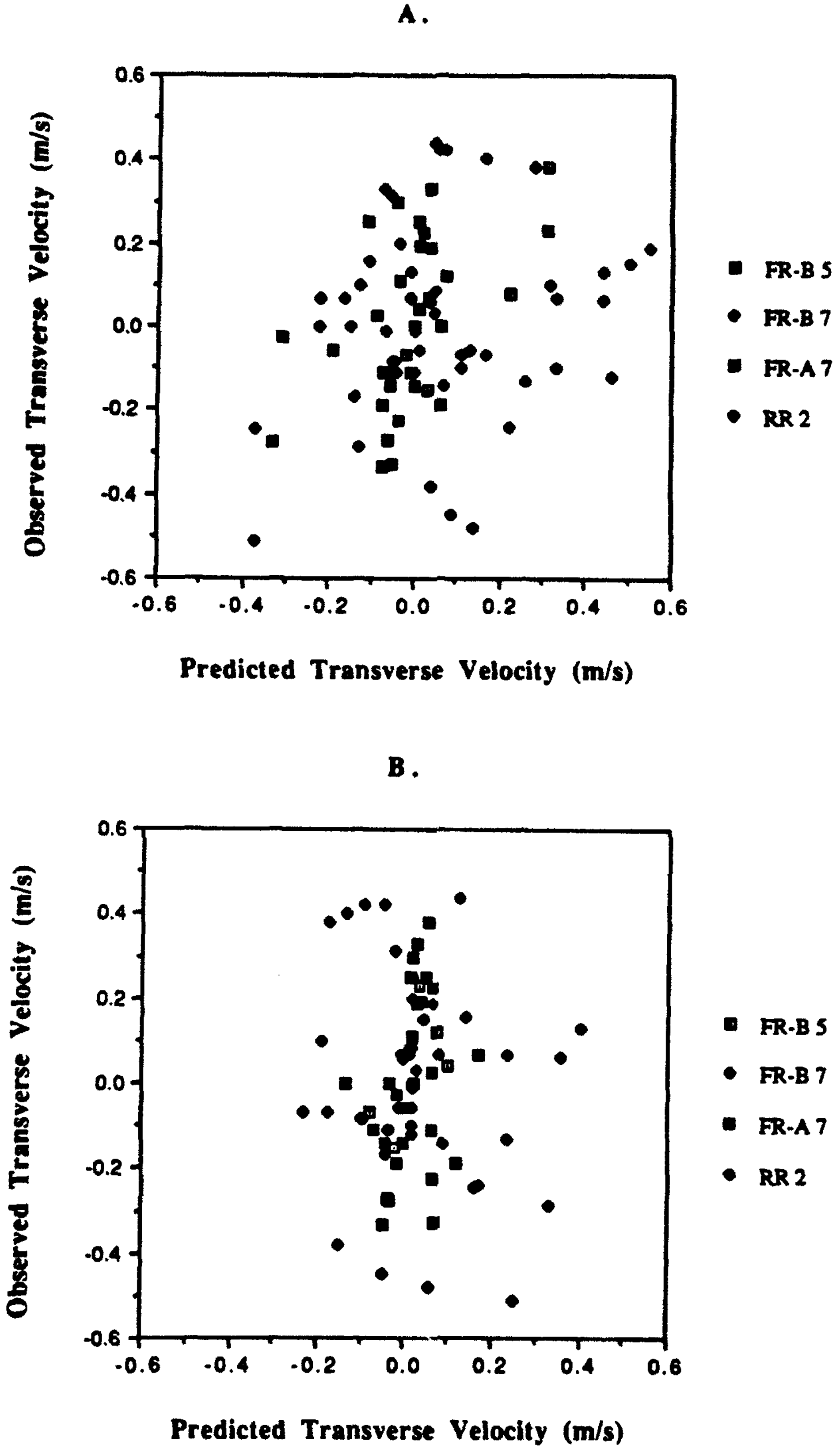

Figure 3.6 Predictive ability of original (A) and revised (B) Rais models

re-derived model resulted in improved predictions, the original Rais (1985) model was also used to generate predicted transverse velocities for the Markham data.

The results of the comparison are summarised in Figure 3.6. It is apparent that neither the original, nor the modified, Rais models are able to predict the secondary flows observed by Markham, though there is some less scatter in the modified model. It is clear from this comparison that the Rais approach is unable to predict near bank flows, even when a thorough and accurate derivation is conducted. The reason for this is that it appears that the approach is in fact unsuitable. It appears that any velocity distribution with a velocity maximum depressed below the surface will result in the overall gross features of the outer bank cell (i.e. correct rotation of cells) being replicated (Nelson, pers. comm, 1992, see Figure 3.5), but that the approach is not capable of correctly predicting the magnitudes of these velocities, nor even systematically miscalculating them. This is because, firstly the approximate analytical solution, in conjunction with the empirical entry to the problem, means that uncertainty is in any case high. But, more important is the decoupling of the solution of the streamwise velocity from the solutions for the transverse and vertical velocities, when in reality these velocities are intimately and mutually interrelated. While the Rais method provides an *estimate* of the three velocity components, then, it does not take into account the constraints of the continuity equation, or attempt to correct (iterate) the solution in light of the first estimate. So, to conclude, the Rais approach is fundamentally flawed and has little or no ability to predict the flows near the outer banks of river bends. It appears that the state of the art is that no method currently exists that is able to model the flow in the crucial zone adjacent to the outer bank.

3.4 Kinematic Approach To Modelling Width Adjustment Based On Predicting Near Bank Primary Flow Velocity

An important class of river meander models attempts to relate bank migration rates to an erodibility coefficient and near bank flow intensity. It can be hypothesized that these approaches may be modified and applied to width adjustment prediction. It is, therefore, appropriate to review these models with special regard to the possibility of developing this approach to modelling width adjustment. Even though the primary aim of these models is to predict bank migration rates, via an erodibility coefficient, it is also necessary to predict a near bank flow intensity parameter, and so this approach is presented in the hydraulics chapter. This recognises the concept of basal endpoint control, in which the primary control on bank migration is seen as the near bank flow intensity.

It is possible to formulate a simple model of river bank migration (Parker *et al.*, 1983; Parker *et al.*, 1983, Ikeda *et al.*, 1981) such that:

$$\chi = e\,(U_b - U) \tag{3.45}$$

where χ is the bank migration rate (ms^{-1}), U_b is the near bank streamwise flow velocity (ms^{-1}), U the reach averaged streamwise velocity (ms^{-1}) and e is a dimensionless erosion coefficient which represents the cumulative influences of soil type, vegetation, and other variables which, in addition to the near bank flow, determine the rate of bank migration (Pizzuto & Meckelnburg, 1989). In studies of meander bend migration, (3.45) is usually applied to only the eroding outer bank, the meander migration rate being equated to the bank erosion rate via the assumption that the channel width is constant through time (outer bank erosion is matched by inner bank accretion). However, it is possible to formulate a width adjustment model based on applying (3.45) at both banks, since by definition the time rate of width adjustment is equal to the sum of the bank migration rates at the inner and outer banks. Thus:

$$\frac{\partial W}{\partial t} = \chi_i + \chi_o \tag{3.46}$$

In equation (3.46), W is the channel width (m), χ_i and χ_o are the inner and outer bank migration rates, respectively; these are positive when the bank is eroding.

If an approach to width adjustment based on (3.45) and (3.46) is to be successful, it is necessary to show that the form of equation (3.45) is valid, that the near bank streamwise velocities may be accurately modelled and that the erodibility coefficient can be rationally derived to take into account all of the factors influencing bank erosion and accretion, both at the inner and outer banks.

3.4. 1 Bank Migration Model

Using equation (3.45) Parker and his co-workers (Parker *et al.*, 1982; Parker *et al.*, 1983; Ikeda *et al.*, 1981) presented a convincing explanation for the symmetry of low amplitude meander bends and asymmetry of high amplitude bends. Odgaard (1987) also used the migration model in order to predict patterns of migration along 2 streams in Iowa, with reasonable success. Pizzuto & Meckelnburg (1989) hypothesized that these results suggested that, despite its simplicity, equation (3.45) provides a reasonable method for predicting rates of bank migration. They attempted to evaluate this bank migration model using data collected from Brandywine Creek near Chadds Ford, Pennsylvania. Their results confirm that bank erosion rates along the Brandywine Creek are strongly correlated with near bank velocity, supporting the hypothesis of a linear equation relating bank migration rates and the excess near bank velocity.

An important point to note is that Pizzuto & Meckelnburg (1989) analyzed the migration equation over both a 2 year and 44 year period, and found that the correlation between the near bank velocity and migration rate was different for the two time scales considered. The short term data indicate that finite erosion rates occur even at velocities lower than the reach-averaged velocity, U, while the longer term data suggest that erosion rates are essentially zero when velocities are less than U. However, this does not invalidate the use of (3.45) at shorter time scales, since this result was attributed to differences in the definitions of "near bank" velocity used in each study. The short-term analysis was based on velocity measurements collected in the field, which were likely to be taken within the bank boundary layer, while the longer term analysis used "near bank" velocities generated by the model of Ikeda *et al.* (1981). These latter velocities are defined as the maximum velocity at a cross-section and, therefore, provide an estimate of the near bank velocity just outside the bank boundary layer. The short term analysis probably underestimates the near bank velocity so defined, explaining the difference in the results. However, this formulation still requires the assumption that the reach averaged velocity is equal to the critical velocity for entrainment of bank material (see Mosselman, 1992), which seems unreasonable. It is tentatively suggested that the bank migration model should omit the reach-averaged velocity variable, U. Bank migration rates would then still be correlated with near bank primary flow velocity.

While there is empirical evidence to suggest that the form of equation (3.45) is valid, results of other studies are less encouraging. LaPointe & Carson (1986) emphasize that the direction of the near bank flow is important in evacuating sediment from the toe and destabilising the bank. The rate of meander migration will also depend on factors such as the mechanics of failure (Pizzuto, 1984; Ullrich *et al.*, 1986; Osman & Thorne, 1988), and the grain size of the bed sediment at the toe (Nanson & Hickin, 1986). While the form of equation (3.45) may very well be valid, it is clear that the model will have no predictive ability unless the values of the near bank velocity and erodibility coefficient can accurately and rationally be predicted using methods that take into account all the mechanisms responsible for the bank migration and width adjustment process.

3.4.2 Prediction Of Near Bank Velocity

Ikeda *et al.* (1981) coupled the kinematic bank erosion model (3.45) with a dynamical bend flow model to predict the near bank primary velocity in an attempt to develop a "Bend Theory" of river meander deformation. The analysis of the flow was based on the de St. Venant equations of shallow water flow in a sinuous channel. By making the traditional assumptions of steady flow in a constant width channel with small ratio of width to centreline radius of curvature, the equations of motion were written:

$$u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} + \frac{u\ v}{R_c} = -g\frac{\partial \varsigma}{\partial s} - \frac{\tau_s}{\rho h} \qquad (3.47)$$

$$u\frac{\partial v}{\partial s} + v\frac{\partial v}{\partial n} - \frac{u^2}{R_c} = -g\frac{\partial \varsigma}{\partial n} - \frac{\tau_n}{\rho h} \qquad (3.48)$$

$$\frac{v\ h}{R_c} + \frac{\partial\ vh}{\partial n} + \frac{\partial\ uh}{\partial s} = 0 \qquad (3.49)$$

where R_c is centreline curvature (m), h is the local flow depth (m) and ς the water surface elevation (m). The bed elevation (m) is denoted by η. Bed stresses were calculated using the Chezy friction factor, C_f, so:

$$\tau_s = \rho\ C_f\ V_u \qquad (3.50)$$

$$\tau_n = \rho\ C_f\ V_v \qquad (3.51)$$

where:

$$V = \sqrt{u^2 + v^2} \qquad (3.52)$$

Ikeda *et al.* (1981) linearized the equations using a perturbation analysis up to $O(h/R_c)^2$, corresponding to Engelund's (1974) second approximation, where local flows are defined in terms of perturbations such that $U' = U + u'$, $H = \varsigma - \eta$, $\varsigma = \varsigma_o + \varsigma'$, $\eta = \eta_o + \eta'$, where subscripts denote reach averaged values, to yield:

$$U\frac{\partial U'}{\partial s} = - g\frac{\partial \varsigma}{\partial s} - C_f\frac{U^2}{H}\left(\frac{2U'}{U} - \frac{\varsigma'}{H} + \frac{\eta'}{H}\right) \tag{3.53}$$

$$H\left(\frac{\partial U'}{\partial s} + \frac{\partial v'}{\partial n}\right) + U\frac{\partial H}{\partial s} = 0 \tag{3.54}$$

Noting that (3.47 & 3.49) do not contribute at $O(h/R_c)$, at $O(h/R_c)$ (3.48) yields:

$$u^2\, C' = g\frac{\partial \varsigma'}{\partial n} \tag{3.55}$$

where $C' = (1/R_c)'$, and (3.55) may be integrated to yield:

$$\varsigma' = \frac{1}{g}C'\, u^2\, n \tag{3.56}$$

which must be supplemented by a relation for η'. However, the St. Venant equations do not allow for a treatment of secondary flow in bends, and the resulting lateral variation in bed elevation (Ikeda *et al.* 1981), so that they used the analyses of Engelund (1974), Kikkawa *et al.* (1976) and Zimmerman & Kennedy (1978) to yield, for $O(h/R_c)$:

$$\frac{\eta'}{H} = - A'\, C'\, n \tag{3.57}$$

Engelund (1974) used the fact that this relationship is not changed at $O(h/R_c)^2$ to obtain an $O(h/R_c)^2$ description of tangential velocity variation, U', and bed topography. Ikeda *et al.* (1981) determined the near bank velocity $U_b' = (U')_{n=\pm b}$ by substituting (3.56) and (3.57) into (3.53) and evaluating at n=b, where b is the channel half width (m). This gives:

$$U\frac{\partial U'_b}{\partial s} + \frac{2U}{H}C_f\, U'_b = b\left(- U^2\frac{\partial(1/R_c)}{\partial s} + \frac{C_f}{R_c}\left(\frac{U^4}{gH^2} + \frac{A'U^2}{H}\right)\right) \tag{3.58}$$

While Ikeda *et al.* (1981) only evaluated the near outer bank velocity, the near inner bank velocity can also be evaluated using (3.58) by substituting the value -b. Ikeda *et al.* successfully applied the approach in subsequent papers (Parker *et al.*, 1983; Parker, 1983; Parker *et al.*, 1982). The approach, or a variation of it, has also been used to predict channel shift by, among others, Beck *et al.* (1983), Johannesson (1985), Odgaard (1987, 1989ab) and Crosato (1987, 1990). However, there are a number of limitations with equation (3.58). Firstly, uncertainty surrounds the specification of the precise value of A' in (3.57) and (3.58). Engelund (1974) suggested a constant value of A of about 4, and while Kikkawa *et al.* (1976) and Zimmerman & Kennedy (1978) predict similar values, both of these theories predict that A' varies with U. Moreover, since the original work of Ikeda *et al.* (1981) several researchers (e.g. Kalkwijk & DeVriend, 1980; DeVriend, 1981; DeVriend & Geldof, 1983) have emphasized that an important cause of primary flow velocity redistribution in meandering rivers is the convective transport of primary flow momentum by the secondary flow.

While the importance of this convective transport mechanism has been clearly demonstrated by Johannesson & Parker (1987), it was neglected both by Engelund (1974) and Ikeda *et al.* (1981), and also by Blondeaux & Seminara (1985), Struiksma *et al.* (1985) and Odgaard (1986). Recently, the flow field model of Ikeda *et al.* has been completely re-derived by Johannesson & Parker (1989ab) to take into account the convective transport of primary flow momentum by the secondary flow. The Johannesson-Parker model also accounts for the phase lag between the secondary flow and channel curvature (Kitanidis & Kennedy, 1984; Ikeda & Nishimura, 1986) induced by the downstream convective acceleration of the secondary flow. This effect is also neglected by Engelund (1974) and Ikeda *et al.* (1981).

A further deficiency of the Ikeda *et al.* (1981) approach is that the bed topography is assumed to be a function only of local channel curvature (equation 3.57), rather than being calculated under the restriction imposed by the continuity equation of sediment transport (Johannesson & Parker, 1989c). The importance of coupling between the flow field, sediment transport and bed topography is emphasized by Blondeaux & Seminara (1985) and Struiksma *et al.* (1985). Johannesson & Parker (1989c) generalized their previous models to also include an erodible bed, so that they have effectively entirely re-derived the Ikeda *et al.* flow model in order to eliminate the deficiencies described above. Johannesson & Parker (1989c) provide a full derivation. It is interesting to note that Johannesson & Parker used their model together with the bank erosion model of Ikeda *et* *al.* (1981) to predict wavelengths of river meanders that are in general agreement with both laboratory and field data (Figure 3.7). Moreover, they found that the agreement obtained was better than that obtained using the original flow model of Ikeda *et al.* (1981), and that the scatter in Figure 3.7 was also shown to be primarily due to sensitivity to the streamwise sediment transport relation used, rather than a deficiency in the flow model *per se*.

Other workers have also attempted to re-derive the Ikeda *et al.* approach. Crosato (1990) formulated a model for flow and channel bed prediction based on the linear analysis of the 2D model for riverbed topography developed by Koch & Flokstra (1980), Olesen (1983) and Struiksma *et al.* (1985), in which the governing equations are obtained by fully coupling flow field, sediment transport and bed topography, but the influence of the secondary flow momentum convection is only roughly taken into account by weighting the transverse bed friction (Crosato, 1990). Odgaard (1989ab) also recognised the importance of coupling the flow field and bed topography, but neglected the effect of the convective transport of primary flow momentum by the secondary current. Bernard & Schneider (1992) modified a 2D depth averaged code to take into account of this convection mechanism. But, it is clear that the Johannesson-Parker model represents the most complete rederivation of the Ikeda *et al.* approach, and is, therefore, most likely to give good predictions of the transverse distribution of the primary flow.

Finally, it should be noted that in the original Ikeda *et al.* (1981) model, the near bank velocity is determined at n=b, that is at the outer bank where, in fact, the velocity should be zero. Actually, the near bank velocity should be defined as being located just outside the bank boundary layer (Pizzuto & Meckelnburg, 1989), which is frequently defined as extending for approximately one or two bank heights away from the bank. This is an important point because, since the flow model is not applied within the near bank zone, the problems of invalidity in the near bank zone discussed in previous sections are inapplicable. It appears that, if correctly applied, the Johannesson-Parker approach is capable of giving sophisticated, reliable and accurate predictions of the near bank primary flows for use in the bank migration model of equation (3.45).

3.4.3 Prediction Of Migration Coefficient

The "erodibility" coefficient, e, in equation (3.45) must represent the combined influence of all the factors influencing bank erosion and accretion. If the width adjustment model (3.46) is to give any physical insight, the coefficient, e, must be predicted using

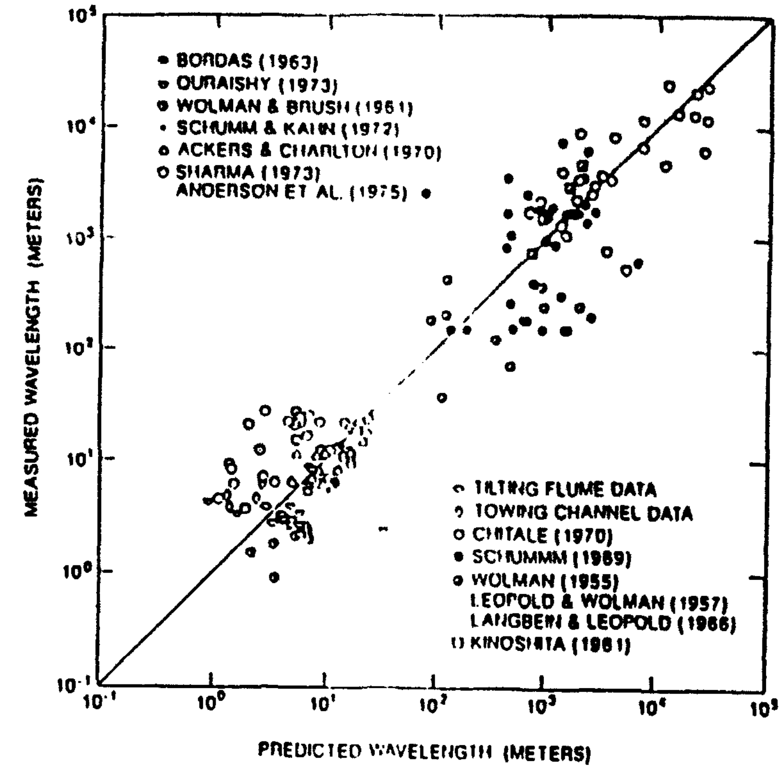

Figure 3.7 Predicted and measured wavelengths of river meanders (after Johannesson & Parker, 1989c)

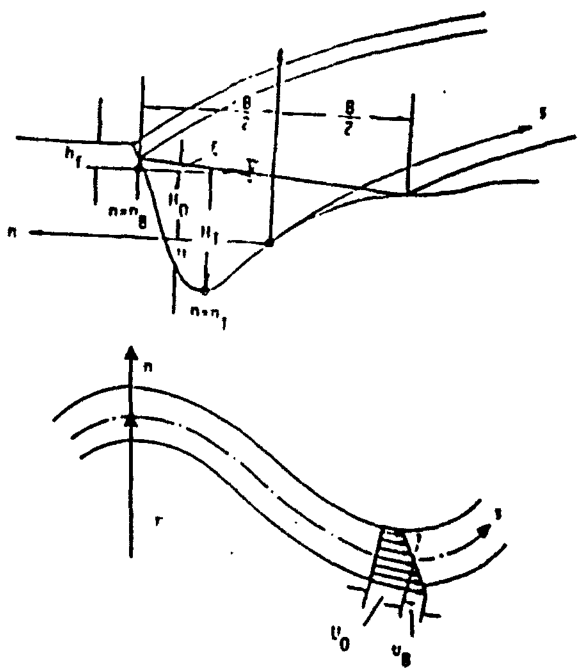

Figure 3.8 Definition sketch for Hasegawa's (1989) analysis

methods that take account of the mechanisms of bank erosion and accretion, and their interaction with the hydraulic conditions in the channel. In effect, e, must be derived using first principles in the conceptual framework provided by basal endpoint control. In this case the coefficient will represent the coupling between the flow and bank erosion processes that determine the bank migration rate. This coefficient must also be calculated for both the inner and outer banks, and include the possibility of either erosion or accretion. It is, therefore, inappropriate to label this variable as an "erodibility" coefficient, so it is here termed a migration coefficient.

In fact a rational method, based on applying the sediment continuity equation across the near bank zone, for calculating the migration coefficient, e, has been presented by Hasegawa (1989ab). While Hasegawa was concerned with calculating an erodibility coefficient only at the outer bank, the basic principle of his method can be applied to both the outer and inner banks. Mosselman (1992) has shown that by introducing a similarity assumption such that the channel is assumed to approximately maintain its cross-sectional shape as it shifts, bank migration equations can be written for the outer bank:

$$\chi_o = \frac{\partial n_b}{\partial t} = \frac{-1}{\tan i}\frac{\partial Z_b}{\partial t} \qquad \frac{\partial Z_b}{\partial t} \leq 0 \tag{3.59}$$

$$\chi_o = \frac{\partial n_b}{\partial t} = 0 \qquad \frac{\partial Z_b}{\partial t} > 0 \tag{3.60}$$

and for the inner bank:

$$\chi_i = \frac{\partial n_b}{\partial t} = \frac{1}{\tan i}\frac{\partial Z_b}{\partial t} \tag{3.61}$$

But, according to the definition diagram (Figure 3.8), the equation of continuity of sediment can be written (Hasegawa, 1989a), in a curvilinear coordinate system:

$$\frac{\partial Z_b}{\partial t} = \frac{1}{1-\lambda}\left(\frac{\partial q_s}{\partial s} + \frac{\partial q_n}{\partial n} + \frac{q_n}{r+n}\right) \tag{3.62}$$

The starting point of Hasegawa's analysis is to attempt to integrate (3.62) from the deepest point of the channel to the water margin at the outer bank (in his study). Hasegawa neglected the 3rd term of (3.62) and assumed, as described above, that the channel approximately maintains its shape as it migrates. This latter assumption is reasonable for channels migrating in dynamic equilibrium, but is not valid in channels that are actively adjusting their width. The consequences of this problem are discussed below. By applying the bank migration relations (3.59) through (3.61), together with appropriate boundary conditions for the sediment inflow at the water margins, Hasegawa obtained a relation for the bank migration rate:

$$\frac{\partial n_b}{\partial t} = \frac{1}{(1 - \lambda)\,[H_o + \eta_{nt} + h_f]} \left(\int_{nt}^{n_b} \frac{\partial q_s}{\partial s} \partial n - q_n(nt) \right) \tag{3.63}$$

where H_o is the mean channel water depth (m), h_f is the height of the bank above the water surface (m), η_{nt} is the elevation of the deepest point in the section and $q_n(nt)$ is the lateral sediment flux at the deepest point in the channel. In effect, equation (3.63) provides a quantification of the theory of basal endpoint control, where the bank migration rate is determined by the balance between the rate of supply of debris by fluvial transport from upstream and local bank failure, and the rate of removal of that material by the transverse and downstream flow.

Hasegawa substituted transport relations for the streamwise and transverse fluxes, and related those rates to the flow velocity. He then quantified the deviations from sectional means that are assumed to control bank erosion in terms of perturbations about velocity and depth, and substituted all these relations back into (3.63). Finally, by applying a scaling analysis to exclude smaller terms, a relation for the rate of bank migration in terms of the near bank flow was formulated by Hasegawa (1989a):

$$\chi_o = \sqrt{C_f}\, I_o \left(\frac{3\ K'\ T \tan \Upsilon}{(1 - \lambda) \left(\frac{\rho}{\rho_s} - 1 \right) \sqrt{\Phi^*}} \right) U'_b \tag{3.64}$$

where I_o is the bed slope, K' the coefficient in a bed load function, Υ is the average transverse slope of the bank, $T = \sqrt{\tau^*_c/(\mu\mu_k\tau^*)}$, $\Phi^* = \tau^*/(\tau^* - \tau^*_c)$ and ρ_s is the density of the bed material. τ^*_c and τ^* are the critical and cross-sectional mean Shields stresses, respectively, while μ and μ_k are the dynamic and static Coulomb friction factors for the bed material, respectively.The migration coefficient was thus determined (from 3.45) as:

$$e = \sqrt{C_f}\, I_o \left(\frac{3\ K\ T \tan \Upsilon}{(1 - \lambda)\left(\frac{\rho}{\rho_s} - 1\right)\sqrt{\Phi^*}} \right) \tag{3.65}$$

which is, as expected, a function of both hydraulic and sedimentological factors.

However, the validity of the migration coefficient, as determined by Hasegawa, is open to question. Mosselman & Crosato (1990) indicate that a number of additional assumptions, not stated by Hasegawa, have to be made in the analysis. In particular, problems occur because the influence of the secondary flow on transverse sediment transport is excluded, and because equations (3.59) through (3.61) imply that the bank erosion rate is fully determined by bed degradation. While this is probably realistic for non-cohesive banks, the processes for cohesive banks are much more complicated (Osman & Thorne, 1988; Mosselman, 1989). While Hasegawa (1990) argues that, over the long term, equations (3.59) through (3.61) are valid, even for cohesive banks, it is clear that the approach is much more suitably applied to the case of non-cohesive bank material. Nevertheless, Hasegawa's work is of great value, since it demonstrates the approach to be taken to calculate the migration coefficient.

It is clear that this approach should be limited to non-cohesive channels, but more research is required to properly account for all of the appropriate terms in the derivation. A particular difficulty lies in specifying the boundary condition of sediment inflow at the margins of the channel in non-cohesive channels, but significant progress is shortly expected to be made on this problem as the results of current research at the St. Anthony Falls Hydraulics Laboratory, Minneapolis, become available.

But, perhaps the biggest problem currently preventing the use of a rationally derived migration coefficient approach in a width adjustment model is the similarity assumption that must be used to integrate (3.62). This assumption implies that the cross-sectional shape remains constant, which is clearly unrealistic in channels adjusting their width. This precludes integrating equation (3.46) through time in order to solve for width. Until a means to circumvent this problem is found, it is only possible to diagnose an instantaneous width adjustment rate given a specified, fixed, cross-sectional morphology.

This problem is related to the original assumption of Ikeda *et al.* (1981) that, for their bank erosion model to be valid, the channel must be in dynamic equilibrium (what they termed "grade"). It is the challenge of future research to relax this assumption of equilibrium.

3.4.4 Summary

To summarize, it is possible to formulate a valid width adjustment model based on a kinematic bank migration model (3.45). For this model to give physical insight into the process of width adjustment, it is important that the near bank velocity and migration coefficients be realistic physically and predicted accurately at both the inner and outer banks. The model of Ikeda *et al.* (1981), used to predict the near bank velocity, has been re-derived by Johannesson & Parker (1989abc) to the extent that a valid, sophisticated model of near bank flow is currently available. However, while recent research by Hasegawa has demonstrated the possibility of rationally deriving the migration coefficient, this approach is not yet developed to the point where it can be applied in the kinematic width adjustment model. The current availability of an apparently valid kinematic width adjustment model and a flow model means that the time is now right for significant progress to be made by combining these two models. It is highly likely that future research could build upon Hasegawa's approach to allow the construction of a width adjustment model applicable to at least non-cohesive bank material, even if cohesive bank, models needed further work to account for mass instability.

4. SEDIMENT TRANSPORT ALGORITHM

To predict the sediment transport fluxes shown in Figure 2.1, appropriate sediment transport technology must be applied. While the mechanics of streamwise sediment transport are relatively well understood, and the advantages and limitations of individual theories of sediment transport have been well documented (e.g. Yang & Wan, 1992; Stevens & Yang, 1989; White *et al.*, 1975), Figure 2.1 suggests that to specify completely the sediment fluxes entering and leaving the near bank zone, both streamwise and transverse sediment transport fluxes must be formulated. Darby and Thorne (1992) have shown that, in the case of straight channels, even though lateral sediment transport fluxes are orders of magnitude smaller than streamwise transport fluxes, predictions of near bank aggradation or degradation are still moderately sensitive to the incorporation of these lateral exchange mechanisms. This is because the (small) morphological changes due to lateral sediment transfer feedback to influence the streamwise flux via the hydraulics.

In curved channels, secondary flow circulations are also generally much stronger than in their straight channel counterparts, suggesting lateral sediment transfer may be more significant in curved than in straight channels. It is clear that both longitudinal and lateral sediment transfer mechanisms must be incorporated into a curved channel width adjustment model. Formulations for both streamwise and transverse sediment transport mechanisms are now considered.

4.1 Streamwise Sediment Transport Flux

While many sediment transport theories which claim to be able to predict streamwise transport of sediment in sand bed streams are available, it is beyond the scope of this study to review the advantages and limitations of these theories. In any case, such reviews have previously been made by, amongst others, White *et al.*. (1975), Stevens & Yang (1989) and Yang & Wan (1992). In fact, even physically based sediment transport theories invariably contain some empirical elements in their derivation, so that the predictive ability of any individual transport function varies according to the type of constraints associated with the environment in which it is applied (Yang & Wan, 1992). The selection of a streamwise sediment transport theory will ultimately depend on the type of channel for which the model simulation is to be conducted.

4.2 Lateral Sediment Transport Fluxes

It is possible to hypothesize that there are a number of mechanisms of lateral sediment transport in natural river channels. In the case of straight channels, Parker (1978) suggested that a lateral component of streamwise bedload is generated due to the downslope gravitational pull acting on sediment grains moving along the side slope in the bank region, whereas lateral transport of suspended load is effected by the mechanism of turbulent diffusion. Parker argued that in straight channels secondary currents are weak, allowing him to assume that convective lateral sediment transport could be neglected. However, in curved channels, secondary flows are not insignificant, and this mechanism of transport may be important in determining the net lateral exchange of sediment. In fact, it is possible to hypothesize that in curved channels, turbulent diffusion of suspended sediment is probably much less significant than either of the lateral gravitational or convective transport mechanisms. Darby & Thorne (1992) found that, even in straight channels with weak secondary flow, the diffusive flux could be 2 orders of magnitude smaller than the gravitational flux and an order of magnitude smaller than the convective flux. Following Olesen (1987) and Talmon (1989), the lateral diffusive transport of suspended sediment in bendways is neglected in this study. Formulations for the lateral transport of bed and suspended load fluxes due to the gravitational and convective transport mechanisms are now considered.

4.2.1 Lateral Bedload Flux

Lateral bedload transport in curved channels is hypothesized to be due to the influence of 2 forces acting on the bed material , fluid drag due to the secondary flow, and gravitational drag due to the component of the sediment grains weight directed down a side slope. A suitable physically-based model of lateral bed load transport under these conditions has been developed by Ikeda and Parker (Ikeda (1982, 1989); Parker (1984); Parker & Andrews (1985); Sekine & Parker (1992), Sekine & Kikkawa (1992)). Full details of the derivation of the lateral bedload flux may be found in Parker & Andrews (1985). They give the lateral bedload flux per unit channel width, F_b (m^2s^{-1}), as:

$$F_b = q_s\left(\tan \delta + \frac{1 + r^*\mu}{\mu}\left(\frac{\tau_c^*}{\tau^*}\right)^{0.5} \tan \omega\right) \tag{4.1}$$

where q_s is the streamwise sediment transport flux per unit channel width (m^2s^{-1}), δ is the angle between the direction of fluid bed stress and the down stream direction (degrees), r^* is the ratio of lift to drag force, μ is the dynamic Coulomb friction coefficient, ω is the angle of the lateral inclination of the bed (degrees), and τ_c^* and τ^* are the dimensionless critical Shields stress and Shields stress, respectively. It is worth to mentioning that, in deriving equation (4.1), the assumptions that the lateral slope is small and that the streamwise slope is almost zero are made in order to obtain a linearized relation for the transverse bedload transport rate. However, the effect of the transverse gravitational component is not in general linearly dependent on transverse bed slope. Although for most cases this limitation is not a problem, since side slopes are generally small, errors may occur in calculating the lateral flux in the near bank zone, since the lateral side slope at the water margin is typically large. This may not be a large problem in channels with cohesive banks, since the region of non-cohesive bed material transport is often relatively flat. But, in non-cohesive channels, this non-linear effect must be taken into account. This will be the case in approaches to width adjustment modelling based on rational calculation of the migration coefficient through a solution of the sediment continuity equation, as discussed in section 3. Reformulating the lateral sediment transport equations to account for this effect is a topic of current research at the St. Anthony Falls Hydraulics Laboratory.

4.2.2 Lateral Suspended Load Flux

Applying the assumption that in curved channels, convective transport of suspended load is dominant over the turbulent diffusion mechanism, the lateral suspended load flux is given by Olesen's (1987) formulation of the convective suspended load flux:

$$F_S = v(z)\ c(z) \qquad (4.2)$$

where F_S is the lateral suspended load flux and v(z) and c(z) are the lateral velocity and suspended sediment concentration at the vertical coordinate, z, respectively.

5. BANK STABILITY ALGORITHM

Following the calculation of the streamwise and transverse sediment transport fluxes, the only remaining flux to be specified (Figure 2.1) is the bank material inflow flux. In fact, there are 3 subtopics associated with this problem.

First, bank stability theory must be applied in order to predict:

1. The onset of instability;
2. The total volumetric inflow of bank material associated with both direct fluvial entrainment and mass failure, and;
3. The bank top widening associated with the mass failure.

In essence, this first topic is concerned with predicting the failure block geometry accurately.

However, in order to apply the bank material flux in the sediment continuity equation meaningfully, it is also necessary to predict how the bank material failure products are distributed spatially across the near bank zone, in order to calculate the lateral bank are distributed spatially across the near bank zone, in order to calculate the lateral bank material inflow flux field. Furthermore, it is necessary to predict the physical properties of the failed bank materials. This is because it is the interaction between the hydraulics and the physical properties of the bank and bed material mixture that determines the rate of removal of these sediments from the near bank zone and, consequently, the sediment status of the near bank basal sediment balance. Thus, the third and final subtopic is concerned with calculating the streamwise and transverse sediment fluxes following bank material inflow, when the bed material consists of mixtures of bed and bank material characterised by widely varying physical properties.

While the first subtopic is concerned with applying bank stability theory in order to predict the timing and magnitude of the total volumetric influx of bank material, the latter two topics are used to distribute this failed bank material both spatially across the channel, and fractionally into its various transport modes following lateral erosion or mass failure. Conceptually, the latter two steps are necessary in order to maintain continuity of the bed *and* the bank material mixture following mass failure, and so thereby allow the necessary coupling of sediment transport and bank stability mechanisms required under the framework of basal endpoint control. All three of the subtopics outlined above are now considered in turn.

5.1 Prediction Of Volumetric Inflow Of Bank Material

Figure 2.1 shows that inflows of bank material occur in response to both direct fluvial entrainment and mass failure under gravity. It is, therefore, appropriate to consider each of these mechanisms in turn.

5.1.1 Inflow Due To Lateral Erosion

Considering the geometry typical of eroding riverbanks depicted in Figure 5.1, it is apparent that the volume of bank material inflow due to lateral erosion is controlled by the lateral erosion increment and the geometry of the bank. It is, therefore, necessary to predict the lateral erosion of cohesive bank material. A method to calculate the rate and amount of lateral erosion of a cohesive bank material was developed during laboratory work at WES by Arulanandan *et al.*. (1980), as reported in Osman & Thorne (1988). They found that the rate of lateral erosion (mmin^{-1}), LE, is given by a linear surplus shear stress formulation once the entrainment threshold is exceeded, so that:

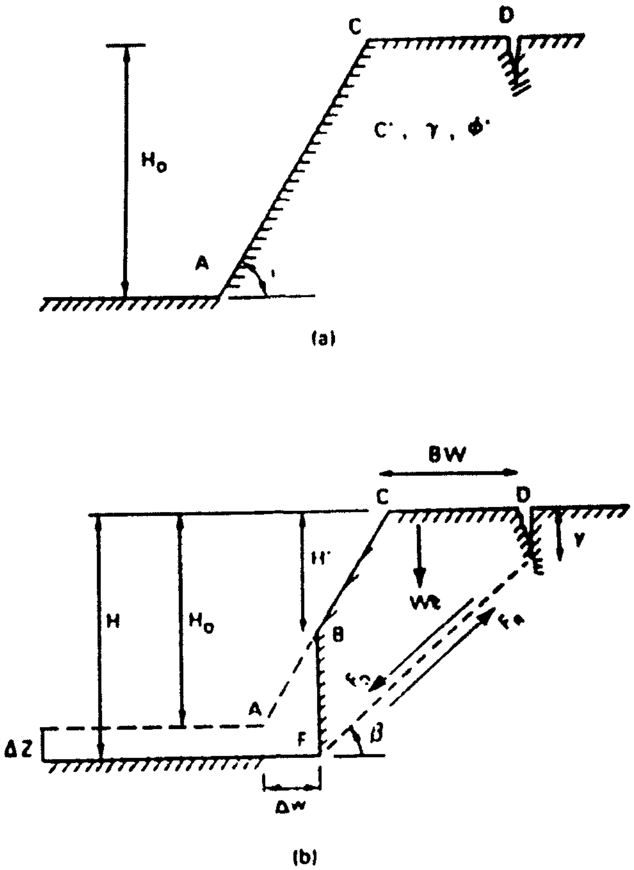

Figure 5.1 Definition diagram for riverbank stability analysis (after Osman & Thorne, 1988)

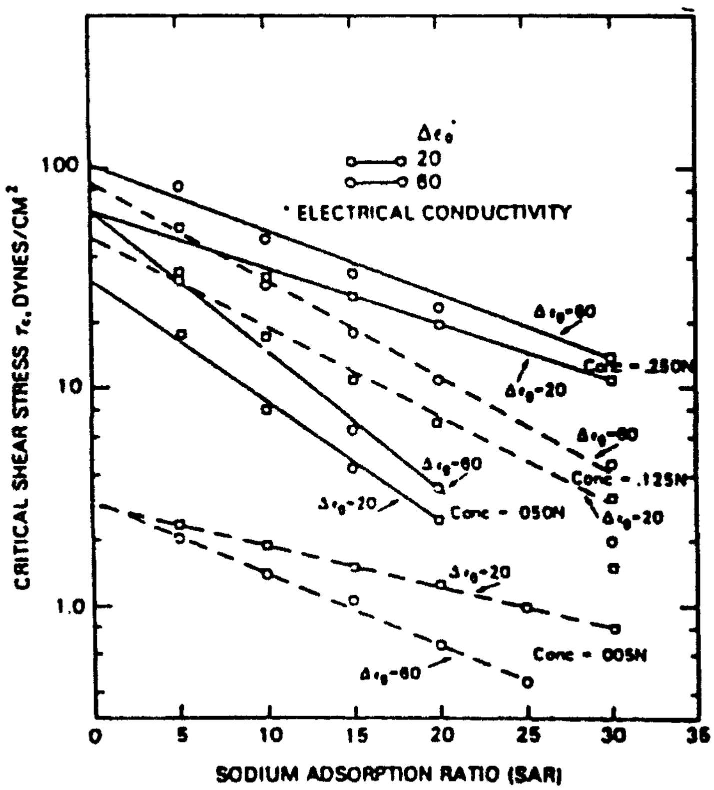

Figure 5.2 Critical shear stress versus SAR for different soil salt concentrations and die'ectric dispersion values (after Arulanandan *et al.*, 1980)

$$LE = \frac{R}{\gamma}\left(\frac{\tau - \tau_c}{\tau_c}\right) \tag{5.1}$$

where τ and τ_c are the applied fluid and critical entrainment shear stresses (dynescm^{-2}), respectively, γ is the soil unit weight (KNm^{-3}) and R is the initial rate of soil erosion (gmcm^{-2}min^{-1}), which Arulanandan *et al.*. empirically determined as:

$$R = 0.0223\ \tau_c \exp(-0.13\tau_c) \tag{5.2}$$

While the formulation of equation (5.1) appears to be reasonable on physical grounds, problems in predicting the lateral erosion rate arise firstly due to uncertainties in predicting the applied fluid shear stress *at the wall*, stressing the importance of modelling the near bank flow accurately, and secondly due to uncertainty in predicting the entrainment threshold for cohesive bank material, a notoriously difficult problem. Arulanandan *et al.* (1980) presented a method to estimate this critical stress as a function of sodium adsorption ratio (SAR), pore fluid salt concentration (CONC), and the dielectric dispersion ($\Delta\varepsilon$) (Figure 5.2). While this approach makes physical sense, since the resistance of cohesive soils to fluid shear has been shown to depend largely upon the physical and chemical makeup of the soil, and the types and amounts of salts in the pore and eroding fluids (Arulanandan *et al.*, 1980), the high data requirements and intrinsic uncertainty associated with this empirical approach combine to suggest that a direct measurement of the critical shear stress (e.g. using erosion pin monitoring) may be both more convenient and more accurate than the "predictive" method outlined above.

5.1.2 Inflow Due To Mass Failure

Prediction of the volumetric inflow due to mass failure under gravity involves firstly calculating the gross stability (factor of safety) of the riverbank, to predict when the bank fails, and secondly predicting the magnitude of the failure block. An additional factor is that, depending on the bank geometry and soil properties, riverbanks fail by a variety of mechanisms, with a separate analysis required for each mechanism. On large-scale rivers, although cantilever failure mechanisms (*e.g.* Thorne & Tovey, 1981) are very common, they may be assumed to be relatively small and tertiary in nature and can, therefore, be neglected. Piping and sapping type failures (*e.g.* Hagerty, 1991) are also excluded from consideration. Although such water-driven failures are common, they are problematic to analyse and in any case the continued instability of banks subject to piping and sapping processes will also be constrained by the interaction between mass wasting and fluvial processes. In this study, only rotational and wedge (Culman) type failure mechanisms are considered, but this is not a significant limitation for many natural river channels.

For each of the failure mechanisms of interest there are a large number of published stability analyses available to choose from. While it is beyond the scope of this report to review the quality of these studies, it is important to note that most of these studies are limited because they fail to take into account the influence of both toe scour and direct lateral erosion on the geometry and stability of the riverbank. Osman (1985) and Osman & Thorne (1988) presented analyses for the stability of rotational slip and wedge type failures, respectively, which do take explicit account of combinations of toe scour and lateral erosion on the geometry of eroding riverbanks. These analyses are, therefore, recommended to be used as the framework for calculating bank stability in width adjustment modelling, and it is appropriate to briefly review them here.

Osman (1985) applied Bishop's method of slices in order to calculate the factor of safety with respect to failure under gravity by the rotational slip mechanism. In order to simplify the calculations, it is necessary to make assumptions about the shape of the failure surface. In this case, the failure surface is assumed circular, and is constrained to pass through the toe of the bank. This is thought to be a reasonable approximation. The shape of the failure surface is important, since it determines, together with the bank profile, the volume of the failure block, VB (m^3m^{-1}), and the increment of bank top widening, BW (m), caused by the rotational slip. These values are given by:

$$BW = H - \frac{H'}{\tan i} \tag{5.3}$$

$$VB = \pi H^2 - \frac{H'^2}{2 \tan i} \tag{5.4}$$

where H is the overall height of the bank (m) , H' is the height of the upper part of the bank (m) and i is the bank angle (degrees).

Figure 5.1 shows the geometry of an eroding riverbank, and the forces acting along a potential wedge-type failure surface. In order to predict the stability of the riverbank, it is necessary to define a factor of safety (FS) as the ratio of the resisting forces and driving forces acting on the block. Failure is predicted to occur when the factor of safety falls below unity. The driving, F_D, and resisting, F_R, forces are functions of the failure block geometry and soil properties so that:

$$F_D = Wt \sin\beta = \frac{\gamma}{2}\left(\frac{H^2 - y'^2}{\tan\beta} - \frac{H'^2}{\tan i}\right)\sin\beta \tag{5.5}$$

$$F_R = \frac{(H - y')\,c'}{\sin\beta} + \frac{\gamma}{2}\left(\frac{H^2 - y'^2}{\tan\beta} - \frac{H'^2}{\tan i}\right)\cos\beta\tan\phi \tag{5.6}$$

where Wt is the weight of the failure block (KN), H is the overall bank height (m), H' is the uneroded bank face height (m), y' is the depth of the tension crack (m), i is the bank angle, β is the failure plane angle (degrees) and c', ϕ and γ are the soil cohesion (KPa), friction angle (degrees) and unit weights (KNm^{-3}), respectively. Osman & Thorne's (1988) contribution was to formulate the above equations for the geometry depicted in Figure 5.1. Since the soil properties and bank profile are given as input data, in order to predict the failure block geometry it is necessary to predict the location of the tension crack and the failure plane angle. Taylor (1948) and Spangler & Handy (1973) showed that the failure plane angle corresponds to the plane of fully developed cohesion, on which the stability number is a maximum. In this case the failure plane angle can be found by equating the first derivative of c' with respect to β in equation (5.6), to zero (Osman &Thorne, 1988). This leads to:

$$\beta = \frac{1}{2}\{\tan^{-1}[\frac{H}{H'}(1 - K^2)\tan i] + \phi\} \tag{5.7}$$

where K is the tension crack index, the ratio of tension crack depth to bank height. Osman & Thorne (1988) suggest the tension crack index may be given as user specified input data, or it is possible to predict the depth of the tension crack (e.g. Lohnes & Handy, 1968). Simon (pers. comm, 1992) and Simon *et al.* (1990) stress that the assumptions used in the Osman-Thorne analysis, and others like it, constrains the failure surface to pass through the toe of the bank, which may not be realistic for many types of failure. He has developed a procedure which allows this constraint to be relaxed (Simon *et al.*, 1990), but it is not incorporated into this study, due to difficulties associated with implementing the approach under the probabilistic framework developed below. However, it is clear that the Simon method should be implemented where possible, suggesting that research is required to develop this approach in the framework described below.

Once failure is predicted, the magnitude of the failure block is determined by the failure block geometry:

$$BW = \left(\frac{H - y'}{\tan \beta} - \frac{H'}{\tan i}\right) \tag{5.8}$$

$$VB = \frac{1}{2} \left(\frac{H^2 - y'^2}{\tan \beta} - \frac{H'^2}{\tan i}\right) \tag{5.9}$$

where BW is the width of floodplain bank retreat (m) and VB is the volume of failed bank material per unit channel length ($m^3 m^{-1}$). It will be apparent that the prediction of the bank retreat increment and volumetric inflow due to mass failure is sensitive to the prediction of the failure plane angle and the specification of the tension crack index, as described above. This is because the block width is determined indirectly when a tension crack *depth* is specified, since the failure block is defined by moving the tension crack along the floodplain until it intersects the failure plane, rather than by directly estimating the location of the tension crack on the floodplain. The problem of over-sensitive control of failure block geometry by tension crack depth is exacerbated by the poor predictive ability of tension crack models. Darby & Thorne (1993) have recently presented an improved method of directly predicting the location of the tension crack and hence the failure block geometry, based on estimating the tensile stresses generated in a failing bank material block. This has the potential to considerably improve predictions of failure block magnitude, but this technique remains to be tested.

In addition to these concerns there remains a much more serious problem in the prediction of the volumetric inflow of bank material due to mass failure that arises when using the Osman-Thorne bank stability equations, or similar analyses, for the purposes of morphological modelling. In a numerical model of width adjustment that utilises the concept of basal endpoint control, the aim is to predict all of the fluxes in Figure 2.1 and then solve the sediment continuity equation through time in order to predict the evolution of channel morphology. Unfortunately, the bank stability theory reviewed above becomes difficult to apply to the problem of estimating a bank material inflow flux under this kind of solution framework. This is because these bank stability models are threshold models -

failure instantaneously occurs when the factor of safety falls below some critical value, so that the delivery of bank material to the reach by mass failure is viewed as a discrete, rather than continuous, process. It is, therefore, impossible to calculate a meaningful time rate of bank material inflow, leading to potential problems in solving the sediment continuity equation numerically.

A second, somewhat related, problem is that in a discretized model solution domain, the bank stability is necessarily calculated at only a finite number of computational nodes (model cross-sections), with the predicted bank stability value at that *point* assumed representative of the *reach* that the point is supposed to represent. The consequence of this assumption is that in order to calculate a volumetric inflow due to mass failure to the reach, it is necessary to integrate the 2 dimensional bank stability calculations described above along the length of the reach. Hence, when failure is predicted at a computational point, failure along the entire reach is assumed to occur instantaneously, resulting in a sudden and large sediment input to the reach which may not only shock the computations and cause numerical instability, but is in any case unrealistic. In reality, the frequency of riverbank failures along a reach progressively increases as the reach is destabilized by fluvial erosion. It is clear that some kind of modifying function needs to be applied to the Osman-Thorne bank stability theory described above in order to reduce the prediction of reach length failure block magnitude from its "potential" value, so that the true reach scale time averaged inflow rate may be better defined, if the Osman-Thorne theory is to be used in a model of width adjustment.

Research on width adjustment in straight channels at Nottingham University has proposed that this aim may be achieved by modifying the Osman-Thorne bank stability theories from an entirely deterministic to a more probabilistic framework. This applies to both wedge type and rotational slip failures. Using the observation that the alluvial soil properties c', ϕ, γ, which, for a given bank geometry, determine the bank stability, vary according to some frequency distribution at the scale of the river reach (Figure 5.3), it may be hypothesized that the variation in bank stability observed along a river reach is entirely due to this observed soil property variation. This assumption implies that the reach geometry is assumed uniform, as is the case when using a numerical model cross-section to characterize a reach.

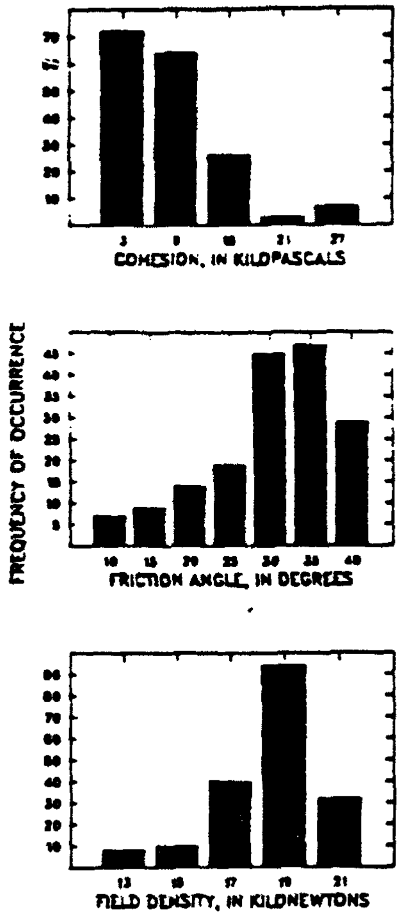

Figure 5.3 Frequency distributions of soil mechanics data for west Tennessee streams (after Simon, 1989)

Using this assumption, the probability of failure of a riverbank along a reach is then defined as being equal to the probability of the reach factor of safety being less than unity. This latter value is calculated by computing all factors of safety possible for the given constraints of the bank geometry, and soil property frequency distributions. In effect, since the geometry is assumed uniform along the reach, the factor of safety is calculated for each possible individual combination of c', ϕ and γ. Furthermore, the probability of each individual factor of safety can be equated to the probability of occurrence of that soil property combination, as determined from the observed soil property frequency distribution, so that:

$$P_I = P(c') * P(\phi) * P(\gamma) \qquad (5.10)$$

where P_I is the probability of occurrence of an individual soil property combination, and $P(c')$, $P(\phi)$ and $P(\gamma)$ are the probabilities of occurrence of an individual value of c', ϕ, or γ, respectively. If the individual factor of safety calculated is less than unity (P_I (FS<1)), it is stored while computations proceed through all the other possible combinations. Finally, the probability of the reach scale factor of safety being less than unity is then found from the sum of all the probabilities of occurrence of individual factors of safety being less than unity:

$$P_R\,(FS<1) = \sum_{I=1}^{I=I} P_I\,(FS<1) \qquad (5.11)$$

where P_R (FS<1) is the reach scale probability of failure, and P_I (FS<1) is the probability of an individual factor of safety being less than unity. It is important to note that this method requires no assumptions concerning the frequency distribution of the soil properties since at the scale of the river reach, these distributions are measurable.

The probability of failure occurring along the reach may be assumed equal to the fraction of the reach that actually fails, so that the "potential" volumetric inflow determined from equations (5.4 and 5.9) can now be modified to give the more realistic value:

$$F_{bank} = P_R(FS<1) * V_B \qquad (5.12)$$

where F_{bank} is the total mass failure bank material inflow along the reach (m^3). Following the prediction of a non-zero probability of failure, the bank geometry must be updated using a reach averaged updating scheme, so that the assumption of uniform reach scale geometry can be maintained.

An important implication of the probabilistic method follows from the inability of the method to specify precisely where and when failure occurs within the reach during a computational time step, except at a statistical level. Because the inflow is expressed as a statistical phenomenon, it is possible conceptually to transform the inflow of sediment from each failure along a reach from a discrete value in space and time to a continuous time averaged inflow rate for the reach as a whole - that is the bank material inflow is expressed as a true bank material sediment flux.

5.2 Disposition Of Failed Bank Material In The Near Bank Zone

During mass failure, bank material accelerates downslope and is translated down the failure plane before coming to rest at the base of the slope, which may or may not be below the surface of the water in the near bank zone. Even if the failed bank material enters the river channel, observations suggest that it is reasonable to assume that all of the material is deposited downslope and along the failure plane. Thus, in the time between the bank material failing and coming to rest at the base of the slope, the near bank zone is subject to a lateral bank material inflow flux field. However, the methods outlined in the previous section only allow the total inflow flux to be determined in any given time step. These methods do not allow for the computation of the disposition of the failed bank material in the time steps after failure, nor do they allow for the calculation of the distribution of those failure products across the near bank zone in order to accurately update the morphology of the near bank zone. It is the aim of this section to demonstrate how the mass failure mechanism and geometry of the failure surface and near bank zone determine how, and in what form, mass failure products are distributed across the channel.

It can be hypothesized that the failure mechanism and failure geometry exert the primary control on the disposition and distribution of failure products following mass failure. Observations indicate that rotational slip type failures are characterised by translation of the failure block around an approximately circular failure surface. As a consequence, the failure block is usually not subject to major external forces and typically the failure block retains its cohesion and remains intact. However, in the case of wedge type failures, the failure surface is short, steep and planar, and the block may topple violently, slide rapidly or otherwise collapse into the channel in an almost vertical fall. While the potential energy associated with the failure block in the case of rotational slip is largely dissipated in overcoming the frictional resistance offered by the long, shallow failure plane, the potential energy possessed by the block failing by a wedge type mechanism will be dissipated mainly in the impact of the failure block with the base of the slope (the channel bed). Field observations of bank failures by the principal investigators suggest the failure block associated with wedge type failures often lose cohesion and internal structure during the shock of this impact, resulting in the dispersion of the failure block into a mass of aggregates. Moreover, observations of the dispersion of blocks of cohesive bank material dropped vertically in order to simulate the wedge type "failure block-drop" suggest that these aggregates are usually well graded, the size of the clasts perhaps being controlled by any internal aggregate or crumb structures (see Thorne, 1978) present in the cohesive soil, so that these aggregates may be well characterised by a single representative clast size. If this clast size is indeed scaled on the crumb structure, it may in fact be possible to predict the size of the bank material aggregates following failure. However, this is a topic that needs to be addressed by future research. A further observation leads to the assumption that while the aggregates are composed of cohesive bank material, the aggregates themselves are large enough to behave as cohesionless sediment clasts. This is a very important assumption, since it implies that, following dispersion, cohesive bank material is effectively supplied to the near bank zone as a cohesionless sediment, allowing the application of conventional sediment transport theory to the problem of the transport of both the bed material and failed bank material mixture away from the near bank zone. This point is developed further below.

To predict the disposition of the bank material products inflown after mass failure, it is assumed that the failure block may either remain intact and come to rest as a block of bank material at some point down the failure plane (case of rotational slip), or it may disperse (case of wedge type failure), resulting in an inflow of bank material in the form of cohesionless aggregates of cohesive bank material. In the former case, while bank top widening and a decrease in bank slope still occur, there is effectively no bank material inflow, and the failure block is conceptually treated as still being bank material (Simon *et al.*, 1990). In the case of wedge failure, on the other hand, there is a bank material inflow flux, and the bank material is conceptually transformed to the status of bed material (Simon *et al.*, 1990).

In the case of wedge type failures it is necessary to predict the distribution of the dispersed bank material inflow products across the near bank zone, in order to calculate the bank material inflow flux field. (No such flux field is assumed to exist in the case of rotational slip, since the failure block remains as bank material.) In the research on width adjustment undertaken at Nottingham University, the bank material products are assumed to be uniformly distributed within a near bank zone defined to extend for a distance of two bank heights away from the bank. However, since the bank material failure products are likely to significantly influence the evolution of the morphology of the bank profile following mass failure (Hupp & Simon, 1991; Simon & Hupp, 1992), this process is expected to influence the bank stability and dynamics of width adjustment strongly. The importance of correctly modelling the bank morphology evolution warrants a rationally-based solution of the sediment continuity equation in the near bank zone, and this includes accurately predicting the bank material flux field in the near bank zone (Simon, pers. comm., 1992).

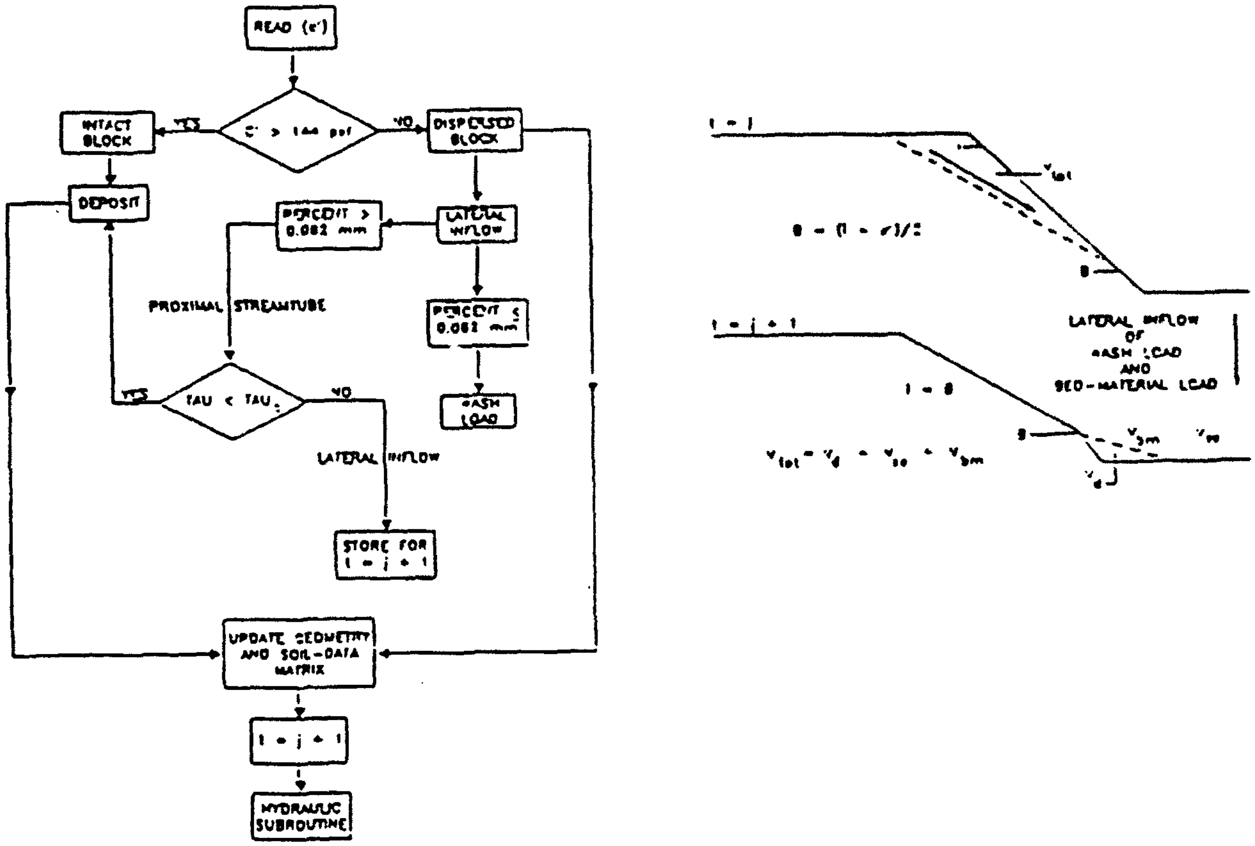

Figure 5.4 Distribution of bank material failure products across near bank zone

In practice a relatively simple model is all that is required to explain the distribution of the bank material products across the near bank zone following mass failure (Figure 5.4). Following a wedge type mass failure, the failure block accelerates down the failure plane angle, dispersing into a mass of cohesionless aggregates in the process. At the base of the failure plane, in the transitional zone where the bed and bank meet, the lateral slope is small in comparison with the failure plane angle, so that the dispersed bank material products decelerate and come to rest. By assuming that the failure products come to rest at their angle of repose, Φ (degrees), it is possible to calculate the distribution of the bank material products across the near bank zone, since the volume of the bank material inflow is also known. This is demonstrated in Figure 5.4. Using this model, it is relatively straightforward to update the geometry of the bed and bank in the near bank zone following mass failure.

This simple model is applicable to the idealized case in which only gravitational forces act on the dispersing failure block, so that all of the dispersed sediment mass comes to rest at the base of the bank. In natural river channels, the failure products may fall into the channel and be subjected to fluid forces before coming to rest, so that the model may not predict the true distribution (Simon, pers. comm., 1992). The magnitude of this effect is unknown at present, though Simon *et al.* (1990) note that bank failures may often occur during rapid drawdown at low flow stages, which, together with the fact that near bank flows are generally subject to bank friction, suggests the magnitude of the fluid forces to which the dispersing block is subject may typically be small. It is clear, that while the techniques described above are logical and based on assumptions made using combinations of physical reasoning and field observations, and represent an improvement on existing methods, more research is required (in this relatively unglamourous field!) in order to derive rigorous versions of these formulations from first principles and to validate them using experimental data collected in the field.

5.3 Transport Of Bed And Bank Material Mixtures

In the previous section it was argued that the disposition of the bank material immediately following failure is determined by the failure mechanism. In the case of rotational slip, the failure block remains intact, there is no sediment inflow, and the bank material is updated as bank material. In the case of wedge type failures, it is assumed that the block disperses, delivering cohesionless aggregates of the cohesive bank material to the basal area at the toe of the bank. Thus, for this failure mechanism, the bank material is conceptually transfered to bed material. Observations suggest that these assumptions are reasonable approximations of the natural disposition of bank material immediately post failure.

Simon *et al.* (1990) proposed a conceptual model under which the updating procedure for the failed bank material is determined by the physical properties of the failed material and the hydraulic properties of the flow (Figure 5.5). By allowing the given inflow volume of bank material to be updated as either bank material, bed material, bed material load or wash load, their model is able to transfer the bank material between a mass wasting and sediment transport algorithms, thereby achieving the coupling between mass wasting and sediment transport mechanisms envisaged by basal endpoint control. The conceptual framework proposed by Simon *et al.* recognises the need to take account of the influence of the physical properties of the failed bank material and bed material mixture on the sediment transport mechanics at the base of the bank. While this represents an improvement over existing models of meander migration (e.g. Crosato (1990)) and width adjustment (e.g. Alonso & Combs (1986); Borah & Bordoloi (1988)) which simply assume that cohesive bank material is transfered to wash load following failure, only broad criteria are suggested by Simon *et al.* to transfer bank material to the appropriate sediment transport mode. It is, however, the transport of the the failed bank material and bed material mixture away from the basal area that is the pivotal link in maintaining the continuity of the bank material. Consequently, it is vital to take explicit account of the impact of mixtures of bed and bank material with widely varying physical properties on the near bank sediment transport flux.

The problem of transport of mixtures of sediments with widely varying physical properties (cohesion, size, density) can be treated using a mixed, or 'active', layer theory. A number of schemes have been developed by a variety of authors (e.g. Bennett & Nordin, 1977; Thomas, 1982; Karim & Kennedy, 1981; Borah *et al.*, 1982; Holly & Karim, 1986; Rahuel *et al.*, 1989; Niekerk *et al.*, 1992). The motivation for these studies has been nc only the need to predict the transport of widely graded sediments, but also to predict the transport of mixtures of sediment of different densities. Since the failed bank material has been assumed to consist of dispersed, cohesionless aggregates of cohesive bank material, suitable schemes may be directly applied to calculate transport of bed and bank material mixtures. Only minor modifications are required in order to apply these theories to the special conditions of the near bank environment.

The essential method of handling the sediment mixture is to assume that the sediment mixture can be discretized into any number of size-density classes, with a single size-density value representing that class. It is then assumed that, during any given time increment, the flow is only to transport the bed material to a finite depth below the surface of the bed, so that a control volume of sediment termed the mixed, or active, layer equal to that finite depth can be established at each model computational point. The sediment in the mixed layer is assumed to be mixed evenly throughout this time step, so that each sediment size-density class distributed throughout the surface layer is equally susceptible to entrainment by the flow (Figure 5.6). Usually the downstream migration of bedforms is invoked as the physical mechanism responsible for the mixing (Figure 5.6), with the depth of the mixing equal to the depth of the mixed layer, D_{mix} (m). Various formulations are available that allow the mixed layer depth to be predicted using some measure of flow intensity. A simple, yet physically plausible model is presented by Karim & Kennedy

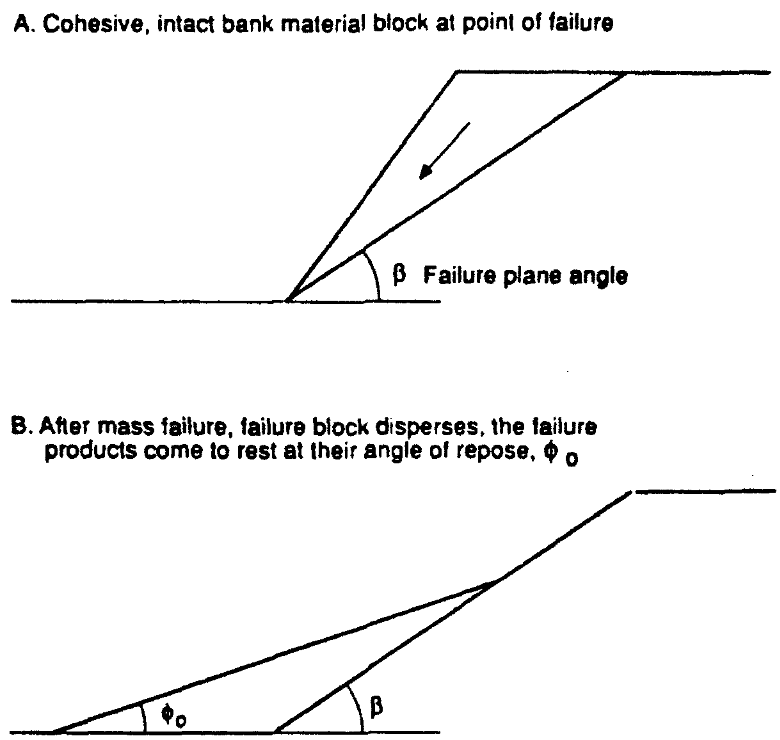

Figure 5.5 Model of bank and bed material disposition in time steps following failure (after Simon *et al.*, 1990)

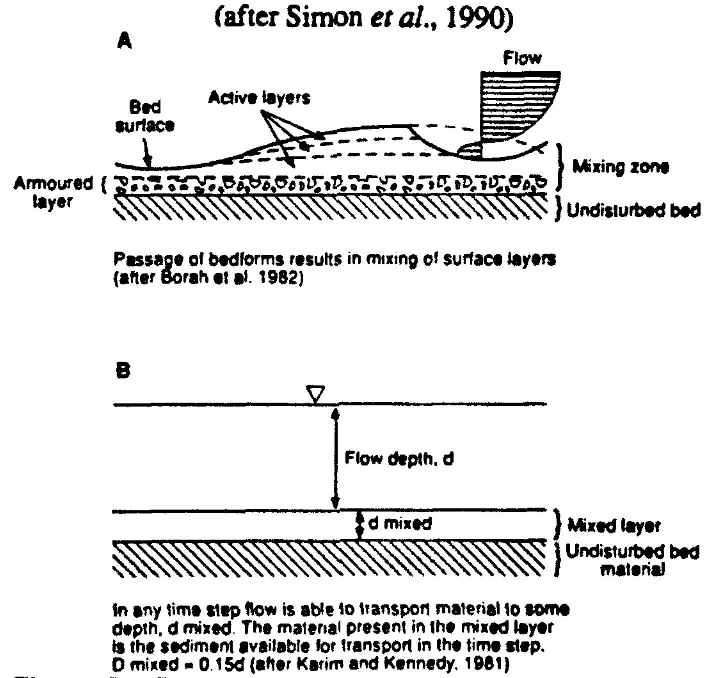

Figure 5.6 Conceptual illustrations of mixed layer theory

(1981), who simply relate the mixed layer depth to the flow depth:

$$D_{mix} = 0.15\,h \tag{5.13}$$

Regardless of the precise formulation of the mixed layer depth model, the important point is that the mixing hypothesis allows the transport of each size-density class to be treated as an independent process. The transport rate of the sediment mixture can, therefore, be expressed in the form:

$$q_s = \sum_{i=1}^{i=i} q_{si}^* \tag{5.14}$$

In equation (5.14), the total sediment discharge is expressed as the sum of all the actual sediment transport rates, q_{si}^*, of each of the i independent bed material size-density classes. However, while the *potential* sediment transport rate, q_{si}, in each sediment size-density class is calculated using the sediment transport theory described in Part 4 of this report (both for longitudinal and lateral fluxes), which applies to uniformly graded sediments where supply limitation is not an issue, the *actual* value of each sediment size-density class must be modified from its potential value in order to take into account the availability for transport of sediment in that class in the mixed layer. Additionally, there is a further physical process that needs to be taken into account. This is the influence of the variation on sediment sizes on the entrainment process.

In sediment mixtures, small grains tend to fall into the voids between large grains, creating a "hiding" effect, while the larger grains tend to protrude into the flow, so that the drag force on these grains is preferentially increased. The overall effect is that the mobility of the large and small grains *tends* to "equalize" although truly equal mobility probably does not occur. Hence, it is necessary to add an additional hiding factor to express this effect. The actual sediment transport rate of each individual size-density sediment class is then expressed in the form:

$$q_{si}^* = q_{si}\ \beta_i \left(\frac{d_i}{d_{50}}\right)^{0.85} \tag{5.15}$$

where β_i is the fractional composition of the i th size-density class present in the mixed layer, d_i is the median sediment diameter (m) of the i th sediment class and d_{50} is the median diameter (m) of the mixture. The value of the exponent on the hiding factor expresses the degree of "equal mobility" observed in the sediment transport process, and its value has been a subject of intense debate (Parker *et al.*, 1982; Andrews, 1983; Komar, 1987, 1989; Ashworth & Ferguson, 1989). The value is influenced by a variety of sediment properties. Rahuel *et al.* (1989) suggest that the exponent takes a value of 0.85. It is apparent that the remaining unknown in the mixed layer calculations is β_i, the fractional composition of the mixed layer. In order to apply this method it is, therefore, necessary to track this parameter throughout the model simulation.

In fact the fractional composition of the sediment size-density classes may be calculated using a budgeting approach for each of the fractions within the mixed layer. Since the composition of the bed material mixture is a known at the start of the model simulation (e.g. form the sediment gradation curve), it is possible to apply the sediment continuity equation to each of the individual size-density classes in each time step, in order to determine the depth of scour or deposition for each individual size-density class. Summation gives the total scour or deposition at that section. It should be noted that the depth of scour of each size-density class is limited in each time step to the depth of material present in the mixed layer during that time step. This information, together with the mixed layer depth during that time step (which is already known from equation (5.13)), allows the fractional compositions of the sediment classes to be updated at the end of each time step. This also allows the recalculation of the median sediment size in the mixture, necessary as input to equation (5.15), to be made. At the start of the next time step, the new mixed layer depth allows the new fractional compositions to be calculated, and the calculations are repeated. This procedure for calculating the fractional composition of the sediment size-density classes is illustrated in Figure 5.7.

Figure (5.7) shows that, firstly, the change in composition due to scour and fill is calculated at the end of the time step, and then at the beginning of the next time step, following the insertion of the newly calculated mixed layer depth, the composition for the next time step is updated. Explicit account is, therefore, taken of the influence of the mixed layer depth on the composition of the mixed layer. This is a subtle, yet important, difference from many mixed layer schemes. This minor modification is necessary in regard to the application of this method in the near bank zone. It is seen that this scheme allows for the tracking of any new stratigraphic layer created following deposition. This is important in the near bank zone, since cycles of aggradation and degradation may be expected to occur in response to fluctuations in bank material inflow rates as the stability of the banks varies. This, in turn, will result in cycles of creation and destruction of stratigraphic layers, and the entrainment of sediments from almost any existing stratigraphic layer appears possible. If proper account of the composition of stratigraphic layers into which the current mixed layer may extend in future time steps is not made, significant errors in determining the fractional composition of the sediment classes will result.

The scheme described above allows the bed and bank material mixture to be updated either as bed material or bed material load, depending on the hydraulic and sedimentological conditions, and so enables the continuity of both the bed and bank material to be maintained. However, the scheme is incomplete. Although the bank material clasts in the near bank basal sediment mixture behave as cohesionless aggregates of bank material, these clasts nevertheless are made up of cohesive bank material particles, and tend to be much less dense and also softer than bed material. Although cohesive bank material usually is made up of predominantly very fine grained materials, fractions of sand may also be contained within the bank material. It may be assumed that following entrainment, the (relatively soft) clasts of bank material very rapidly break down to their constituent particles by the process of attrition. It will, therefore, be necessary to track these particles, since once broken down the very fine particles will become washload and play no further part in the sediment transfer process, whilst the sand fraction of the bank material may be updated as either bed material or bed material load. In fact, by assuming that the rapid breakdown of the bank material clasts occurs instantaneously following entrainment and by specifying the fraction of the bank material that is coarser than very fine sand as input data, it is possible to track these constituents as sediment classes in the mixed layer compositions, and so maintain the continuity of all the bank and bed material sediments, as envisaged by Simon *et al.* (1990). In effect, rather than simply updating the bank material to wash load or bed material fractions immediately following failure, it is the assumptions that the bank material disperses into clasts following mass failure, and that these clasts then attrit instantaneously on entrainment, that allow this scheme to take into account the effect of storing the wash load fraction in the bank material clasts at the base of the bank on the transport of the bed and bank material mixture from the near bank basal zone.

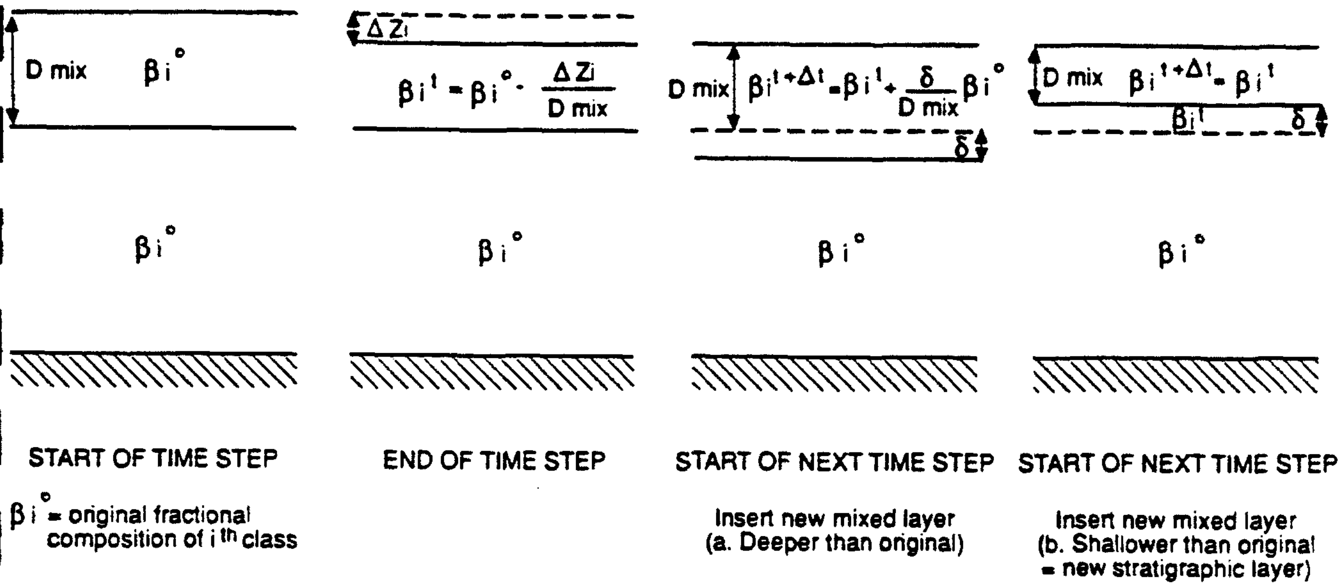

Figure 5.7 Procedure for updating composition of mixed layer

5.4 Summary

The methods presented above allow the bank material inflow flux field, and bank top widening associated with mass failures, to be determined using approaches to bank stability that are based upon first principles, for wedge type failures and also for rotational slip type failures. These analyses have also been specifically developed to be applicable for the geometry typically associated with eroding riverbanks. By determining the near bank bank material inflow flux field and the transport rate of the resulting mixture of bed and bank material, the necessary steps to couple bank stability and sediment transport theories, as envisaged by the concept of basal endpoint control, have been accomplished. To conclude, procedures to couple sediment transport and bank stability theories in the near bank zone, under the framework of basal endpoint control have been developed in research on width adjustment in straight channels at the University of Nottingham, and these approaches may also be applied in a numerical model of width adjustment in curved alluvial channels. However, further research is required to further develop and test these new procedures.

6. CONCLUSION AND RECOMMENDATIONS

The research reported here has outlined two possible approaches, one applicable to cohesive bank material the other to non-cohesive bank material, to the problem of developing a numerical model of width adjustment in curved river channels. Both of these approaches are based upon the concept of basal endpoint control, but while the cohesive bank material model is a dynamical model based upon attempting to describe the mechanics of the flow, sediment transport and bank stability in the near bank zone, the non-cohesive bank material model utilises a kinematic model of bank migration, in which the rate of bank migration is related to near bank flows via a calibration coefficient. It is useful to summarise the main advantages and limitations of each approach.

In the case of the cohesive bank material model there is one main difficulty that stands in the way of the goal of developing a numerical model of width adjustment. This is that the detailed, process-based approach outlined in this report demands that a rigourous solution of the governing equations for the flow be made across the full width of the channel. Although considerable progress has been made in understanding the mechanics of flow in curved channels, and in building increasingly sophisticated and accurate numerical models of these processes, particularly in natural river channels, none of these models are applicable in the crucial near bank zone. A second problem is that a number of largely untested assumptions have to be made in order to formulate a solution scheme for the transport of mixtures of bed and bank material in the near bank zone, in order to maintain the continuity of sediment in this near bank zone. These problems are linked to the main advantage of the fully mechanistic approach, which lies in the potential a detailed model offers for being able to describe a number of very complex interacting processes, and this should ultimately be reflected in the predictive ability of the model. On the positive side, even though the bank stability algorithms outlined above remain untested, they do have the potential to provide accurate descriptions of the bank stability processes at work in channels with cohesive banks. Further, detailed sediment transport theories are now available to calculate both longitudinal and lateral bed material transport fluxes and a data set suitable for validating the width adjustment model is also available (see Appendix 2). Hence, it can be concluded that most of the elements required to formulate and apply a numerical model of width adjustment in curved channels with cohesive banks are presently available, with the exception of perhaps the most important element - the hydraulic model for the outer bank zone of curved channels. It is to be expected that, as research on flow modelling continues, this element will become available in the future.

In contrast to the fully dynamical model, the kinematic approach applicable to non-cohesive bank materials does not require a detailed description of the hydraulics within the bank boundary layer, since it utilises a less detailed formulation based on near bank primary flows alone. Currently available models of the lateral distribution of the primary flow are able to model these flows quite well. However, in the case of the kinematic approach, the element that is currently lacking is a rational formulation of the migration rate coefficient (bank erosion coefficient) in both the bank zones. It is doubly frustrating that a bank stability algorithm is available for the dynamical model which does not have a hydraulics module available, but not for the kinematic approach, where a suitable hydraulic model is available! This problem can probably be resolved, since only a relatively small amount of research time would need to be spent to develop a suitable formulation for this migration coefficient. A model of width adjustment for non-cohesive channels based on this approach is closer to being reality than a fully process-based dynamical model. But this is related to the main disadvantage of this approach, which is that this much less detailed kinematic approach may not be able to give as good a quality predictions of width adjustment, or to improve our understanding of the dynamics of width adjustment to the extent that the dynamical approach could As Pizzuto (1990) notes, these methods provide no detailed information about the interaction between bank stability and the channel's bed topography and planform. In the absence of cost-benefit estimates for each of the two modelling approaches, it is impossible to recommend pursuing either one of these approaches, and perhaps this would in any case be undesirable, since the approaches are applicable to different morphological types of river channel.

In the light of the remarks above, it is possible to conclude by making the following recommendations:

1. Further research should be directed to both of the approaches to modelling width adjustment in curved channels outlined in this report.

2. In the case of the kinematic model, this research should be directed towards rationally formulating the migration coefficients for both the inner and outer banks, using an

approach based on Hasegawa's (1989) method. A suitable data set also needs to be compiled in order to validate the model (*e.g.* experimental flume data).

3. In the case of the dynamical model, research needs to be undertaken in a number of areas. The most important of these is the hydraulic modelling in the near bank zone. As a minimum, this would involve including a representation of the effects of bank friction on the flow pattern. The flow model of Nelson & Smith (1989ab) would be very useful with such a modification. Ideally, not only should the lateral shear be accounted for, but provision should also be made for adding vertical velocities, non-logarithmic primary velocity profiles, and non-hydrostatic effects into the hydraulic model. These modifications would have to be made within the framework of a 3-dimensional modelling approach. Further research is also required to further refine and validate the bank stability algorithm reported here, and extending the range of failure mechanisms and processes covered by this algorithm. It is envisaged that this bank stability research would be conducted at a lower priority than the hydraulics research.

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APPENDIX II: MODEL DATA REQUIREMENTS

The data requirements of a numerical model for width adjustment in curved channels with cohesive riverbanks, developed using the approach outlined in this report, are summarised below.

MORPHOLOGICAL DATA

- Initial cross-section data along study reach.
- Planform data.
- Bank morphology data (Bank heights and angles).

SEDIMENTOLOGICAL DATA

- Bed material gradation curves.
- Bed material density data.
- Bank material soil property frequency distributions (cohesion, friction angle, unit weight).
- Bank material critical shear stress values.
- Gradation curves of dispersed bank material aggregates.
- Gradation curves of intact bank material.

ROUGHNESS VALUES

- Distribution of roughness around cross-section boundaries.

FLOW AND SEDIMENT LOAD DATA

- Water inflow hydrograph for period of study.
- Stage/discharge relationship at downstream boundary.
- Sedigraph at upstream boundary.

It is useful to record here that a suitable data set for the application, calibration and validation of any developed model based on the approach outlined in this report is in existence. Data recording the evolution of the Philip Bayou realignment on the Red River at

Alexandria, La., were collected by the Vicksburg District of the U.S. Army Corps of Engineers.

APPENDIX 3: WIDTH ADJUSTMENT ALGORITHM