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FINAL TECHNICAL REPORT

ON

ONR GRANT NO. N00014-89-J-1836

# COMPUTATION OF BROADBAND MIXING NOISE FROM TURBOMACHINERY

PERIOD COVERED BY THIS REPORT

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## 1. STATUS OF RESEARCH WORK

This final technical report on ONR Grant No. N00014-89-J-1836 covers the period of March 1, 1989 to February 28, 1993. The original objectives of the investigation were:

(1) Development of computational aeroacoustics methodology for direct numerical solution of acoustic propagation and scattering problems associated with turbomachinery mixing noise.

(2) Development of a (computation oriented) theory for the prediction of turbulent mixing noise.

Both of the goals, by and large, have been accomplished. An accurate explicit time marching high order finite difference scheme specifically designed for computational aeroacoustics simulations has now been developed (see highlights below). In addition the mathematical and computational framework of a turbulent mixing noise theory has now been formulated. Numerical testing and comparison between the calculated results of the turbulent mixing noise theory and experimental measurements have yet to be carried out due to the premature funding cutoff of this effort by ONR (project funded for three years only). This work is of utmost importance to aeroacoustics. NASA support of this work has now been secured and the research will continue as planned.

## 2. HIGHLIGHTS OF RESEARCH RESULTS

*a.* ***Development of a Dispersion-Relation-Preserving finite difference scheme for computational aeroacoustics.***

Most present day computational fluid dynamics (CFD) finite difference schemes are not only dispersive but have a considerable amount of intrinsic numerical damping. These characteristics are detrimental to accurate acoustic wave computation. Moreover, most CFD schemes were designed for solving time-independent problems. Although superficially they were time marching schemes yet time marching was intended only as a form of iteration. Thus these schemes are not meant for time accurate aeroacoustics computations. One general flaw of these schemes is that they do not simulate wave propagation speed well.

As a part of the present investigation a new way of formulating finite difference scheme is developed. The traditional means of deriving finite difference approximations for derivatives by truncated Taylor series is abandoned. Instead the stencils are formed in the Fourier transform space with optimized coefficients.

Acoustics problems are governed by the linearized Euler equations. According to wave propagation theory, the number of wave modes and their wave propagation characteristics are all encoded in the dispersion relations of the governing equations. Thus one is assured that the numerical solutions of a high order finite difference scheme will have the same number of wave modes (namely, the acoustic, vorticity and entropy waves), the same wave propagation characteristics (namely, nondispersive, nondissipative and isotropic) and the same wave speeds as those of the solutions of the Euler equations if both systems of equations have the same dispersion relations. Finite difference schemes which have the same dispersion relations as the original partial differential equations are referred to as Dispersion-Relation-Preserving (DRP) schemes. A way to construct time marching DRP schemes by optimizing the finite difference approximations of the space and time derivatives in the wave number and frequency space has now been developed. The stability of these schemes is analyzed and a sufficient condition for numerical stability is established. A preliminary paper on the DRP scheme was presented at the 14th AIAA Aeroacoustics Conference (May, 1992) as AIAA Paper 92-02-033. The full article has been accepted for publication in the *Journal of Computational Physics.*

***b. Development of radiation, inflow and outflow boundary conditions.***

Any computation must, invariably, be carried out in a finite domain. To assure that outgoing acoustic waves are not reflected back into the computation domain and thus ruining the computation a set of radiation boundary conditions must be specified at the boundary of the domain. These radiation boundary conditions are to allow the waves to exit the computation domain smoothly.

A set of radiation and outflow boundary conditions compatible with the DRP schemes has been constructed. These conditions are derived from the asymptotic solutions of the governing equations. The asymptotic solutions are found by the use of Fourier-Laplace transforms and the method of stationary phase. A sequence of numerical simulations has been carried out. These simulations are designed to test the effectiveness of the DRP schemes and the radiation and outflow boundary conditions. The computed solutions agree very favorably with the exact solutions. The radiation boundary conditions perform satisfactorily causing little acoustic reflections. The outflow boundary conditions are found to be quite transparent to outgoing disturbances even when the disturbances are made up of a combination of acoustic, vorticity and entropy waves.

*c. Method of artificial selective damping.*

The order of the finite difference equations of a numerical scheme is usually higher than that of the original partial differential equations. As a result the numerical scheme supports wave solutions that are not solutions of the partial differential equations. These spurious wave solutions, often called the parasite waves, generally have short wavelengths. They manifest themselves as grid-to-grid oscillations. They are pollutants of direct numerical simulations. They can be generated by non-smooth initial conditions, at the boundary of the computation domain, solid wall surfaces, non-smooth grids and interfaces and all kinds of inhomogeneities. They must be removed quickly before rendering damage to the computed solution.

Under the support of this grant we have developed a method to add artificial damping to the numerical scheme to selectively damp out the short wave component. The damping terms are so designed that they have almost no effect on the long waves (acoustic waves). The development of such a selective damping method is an important contribution in computational aeroacoustics.

***d. Solid wall boundary conditions for high order finite difference scheme.***

High order finite difference schemes are less dispersive and dissipative, at the same time, more isotropic than low order schemes. They are well suited for solving computational acoustics problems. High order finite difference equations, however, support extraneous wave solutions which bear no resemblance to the exact solution of the original partial differential equations. These extraneous wave solutions which invariably degrade the quality of the numerical solutions are usually generated when solid wall boundary conditions are imposed. A set of numerical boundary conditions simulating the presence of a solid wall for high order finite difference schemes using a minimum number of ghost values has been constructed. The effectiveness of the numerical boundary conditions in producing quality solutions has been demonstrated by comparing the results of direct numerical simulations and exact solutions.

*e. Mixing noise theory based on turbulence modeling.*

One of the objectives of the present investigation is to develop a computation oriented mixing noise theory. This has now been done. The theory is parallel to the familiar $k$-$\varepsilon$ turbulence modeling theory. It is based on the premises that the unsteady turbulence kinetic energy is the source of noise. This is consistent with almost all present day turbulence modeling theories which consider turbulence energy to be the only quantity characterizing fine scale turbulence. The radiated noise is modified by the mean flow. This so called mean flow effect is taken into account by means of the Green's function. An efficient way of determining the Green's function computationally has been developed. In addition, the theory automatically incorporates the effect of source convection. This effect becomes more and more pronounced as the speed of the flow increases. Extensive numerical testing and comparisons between predictions and experimental measurements have yet to be done. This effort will continue under NASA sponsorship.

## 3. PUBLICATIONS

The following is a list of publications on work supported by the Grant.

1. *Discretization errors inherent in finite difference solution of propeller noise problems*. Tam, C.K.W. AIAA Journal, vol. 30, 304-311, 1992.

2. *A study of the short wave components in computational acoustics* Tam, C.K.W., Webb, J.C. and Dong, Z., Journal of Computational Acoustics (to appear 1993).

3. *Dispersion-relation-preserving finite difference scheme for computational acoustics* Tam, C.K.W. and Webb, J.C., (accepted for publication) J. Computational Physics (to appear) 1993.

4. *Radiation boundary condition and anisotropic correction for finite difference solutions of the Helmholtz equation.* Tam, C.K.W. and Webb, J.C. (accepted for publication) J. Computational Physics (1993).

5. *The short waves in computational acoustics.* Tam, C.K.W., Webb, J.C. and Dong, Z. Proceeding of the ICASE/NASA LaRC Workshop on Computational Aeroacoustics. To appear 1993.

6. *Solid wall boundary conditions for computational aeroacoustics.* Tam, C.K.W. and Dong, Z. Proceeding of the ASME Fluids Engineering Conference, Washington, D.C., 1993.