Structural Acoustics: A General Form of Reciprocity Principles in Acoustics

MITRE

Structural Acoustics: A General Form of Reciprocity Principles in Acoustics

K. Case

Accesion For		
NTIS CRA&I	X	
DTIC TAB	☐	
Unannounced	☐	
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and / or Special	
A-1		

January 1993

...ECTED 1

JSR-92-193

JASON
The MITRE Corporation
7525 Colshire Drive
McLean, Virginia 22102-3481
(703) 883-6997

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
January 14, 1993

3. REPORT TYPE AND DATES COVERED

4. TITLE AND SUBTITLE
Structural Acoustics: A General Form of Reciprocity Principles in Acoustics

5. FUNDING NUMBERS
PR - 8503Z

6. AUTHOR(S)
K. Case

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The MITRE Corporation
JASON Program Office A020
7525 Colshire Drive
McLean, VA 22102

8. PERFORMING ORGANIZATION REPORT NUMBER
JSR-91-193

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
DARPA/TIO
3701 North Fairfax Drive
Arlington, Virginia 22203-1714

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
JSR-91-193

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approcved for public release. Distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A generalized Reciprocity Principle for Acoustics is obtained. By specialization, various principles which appear in the literature are obtained.

14. SUBJECT TERMS
Rayleigh, mass conservation equation, green's thereom

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

Abstract

A generalized Reciprocity Principle for Acoustics is obtained. By specialization, various principles which appear in the literature are obtained.

1 INTRODUCTION

According to reference [1], Russian measurements on the acoustic response of submarines to internal and external excitation make extensive use of the "Reciprocity Principle." While this can hardly be anything but an application of Green's theorem, the way the theorem is stated seems to be slightly different than that in the usual literature — and no derivation is given. Accordingly, it may be useful to see what forms of a "Reciprocity Principle" one might have.

The standard literature is a little vague. Rayleigh's [2] form relates the pressure at a point B due to a source at A to the pressure at A with the same source at B. The derivation assumes a bounded region with rigid boundaries. It is indicated that the results hold for an infinite region with rigid finite boundaries and a radiation condition at infinity.

Landau and Lifshitz [3][a] discuss only an infinite region with no finite boundaries. The statement is the same as Rayleigh but there is a generalization to inhomogeneous media. Addition [3][b] there is a problem given which relates velocities due to dipole sources. Morse and Ingard [4] give the same result as Rayleigh but for both rigid and compliant boundary conditions. The result implied in [1] seems to relate the pressure at A due to a localized force at B to the velocity at B due to a mass source at A.

Below we give a very general relation which requires only an impedance type boundary condition on finite boundaries. By specializing, we immediately obtain the three kinds of reciprocity principles discussed above. Clearly other specializations will yield additional principles. We leave it to the reader to determine these — and their usefulness.

2 BASIC EQUATIONS

These are:

a) The Euler Equation

$$\rho\left\{\frac{\partial\tilde{v}}{\partial t}+(\tilde{v}\cdot\nabla)\tilde{v}\right\}=-\nabla P$$

b) The Mass Conservation Equation

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\tilde{v}=0$$

c) An Equation of State

$$P=P(\rho).$$

For acoustics we linearize putting

$$\rho=\rho_0+\rho'_1,\quad \tilde{v}=\tilde{v}_0+\tilde{v}'.$$

Here, as compared to Landau and Lifshitz [3], we choose the homogeneous case ρ_o = constant and $\tilde{v}_0=0$. The linearized equations are (after dropping the primes)

$$\begin{aligned}\rho_0\frac{\partial\tilde{v}}{\partial t} &= -\nabla P\\ \frac{\partial\rho}{\partial t}+\rho_0\,\nabla\cdot\tilde{v} &= 0.\end{aligned}$$

Now $\frac{\partial\rho}{\partial t}=\frac{\partial\rho}{\partial P}\,\frac{\partial P}{\partial t}\equiv\frac{1}{c^2}\,\frac{\partial P}{\partial t}$. The resulting equations are

$$\rho_0\frac{\partial\tilde{v}}{\partial t}=-\nabla P$$

$$\frac{1}{c^2}\frac{\partial P}{\partial t} + \rho_0 \nabla \cdot \tilde{v} = 0.$$

Assuming simple harmonic motion, i.e., all quantities $\sim e^{-i\omega t}$ yields.

$$-i\omega\rho_o\tilde{v} = -\nabla P$$

$$\frac{-i\omega}{c^2}P + \rho_0 \nabla \cdot \tilde{v} = 0.$$

Let us introduce forcing terms into both equations. Then

$$-i\omega\rho_0\tilde{v} = -\nabla P + \tilde{F}(\tilde{r} - \tilde{r}_1) \qquad (2-1)$$

$$\frac{-i\omega P}{c^2} + \rho_o\nabla \cdot \tilde{v} = M(\tilde{r} - \tilde{r}_2). \qquad (2-2)$$

Eliminating $\tilde{v}$ we obtain

$$(\nabla^2 + k^2)P = \nabla \cdot \tilde{F} + i\omega M, \qquad (2-3)$$

where $k^2 = \omega^2/c^2$.

We want to put this in a form to apply Green's Theorem. We consider pressures P_1 (corresponding to an $\tilde{F}_1, M_1$) and P_2 (corresponding to a $\tilde{F}_2, M_2$). Thus, we have

$$(\nabla'^2 + k^2)P_1(\tilde{r}') = \nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}'_1) + i\omega M_1(\tilde{r}' - \tilde{r}_1) \qquad (2\text{-}4)$$

$$(\nabla'^2 + k^2)P_2(\tilde{r}') = \nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}'_2) + i\omega M_2.(\tilde{r}' - \tilde{r}_2) \qquad (2\text{-}5)$$

Multiply Equation (2-4) by $P_2(\tilde{r}')$ and Equation (2-5) by $P_1(\tilde{r}')$. Subtract and integrate over the fluid. The result is:

$$\int d^3\tilde{r}' \left\{P_2(\tilde{r}')\nabla'^2\, P_1(\tilde{r}') - P_1(\tilde{r}')\nabla'^2 P_2(\tilde{r}')\right\}$$

$$
\begin{aligned}
&= \int d^3r' \left\{ P_2(\tilde{r}')\nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}_1') + i\omega P_2(\tilde{r}')M_1(\tilde{r}' - r_1) \right\} \qquad (2\text{-}6) \\
&- \int d^3r' \left\{ P_1(\tilde{r}')\nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}_2') + i\omega P_1(\tilde{r}')M_2(\tilde{r}' - r_2) \right\}
\end{aligned}
$$

Using Green's Theorem and assuming impedance boundary conditions (i.e. P and $\frac{\partial P}{\partial n}$ are proportional on the boundary) we see that the left side of Equation (2-7) is zero.

Thus

$$
\begin{aligned}
&\int d^3r' \left\{ P_2(\tilde{r}')\nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}_1') + i\omega P_2(\tilde{r}')M_1(\tilde{r}' - r_1) \right\} \\
&= \int d^3r' \left\{ P_1(\tilde{r}')\nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}_2')| + i\omega P_1(\tilde{r}')M_2(\tilde{r}' - r_2) \right\}. \qquad (2-7)
\end{aligned}
$$

This is our general reciprocity principle. Specializing the $\tilde{F}$s and Ms will give the specific principles mentioned in the introduction.

3 SPECIAL CASES

Here we will only consider the situation when the Fs and Ms are constants time δ functions. To keep things straight, it is convenient to introduce a special notation. Thus

P_1 which corresponds to $\tilde{F}_1(\tilde{r} - \tilde{r}'_1\ )$ equal to $\tilde{F}_1\delta(\tilde{r} - \tilde{r}'_1)$ ($\tilde{F}_1$ now a constant vector) and $M_1(\tilde{r} - \tilde{r}_1)$ equal to $M_1\delta(\tilde{r} - \tilde{r}_2)$ will be denoted as

$$P(\tilde{F}_1, \tilde{r}'_1, M_1, \tilde{r}_1; \tilde{r}).$$

a) Let $\tilde{F}_1 = \tilde{F}_2 = 0$.

$$\begin{aligned} M_1(\tilde{r}' - \tilde{r}_1) &= M\delta(\tilde{r}' - \tilde{r}_1) \\ M_2(\tilde{r}' - r_2) &= M\delta(\tilde{r}' - \tilde{r}_2) \end{aligned}$$

(M a constant).

Then Equation (2-7) reads

$$P(\tilde{F} = 0, M, \tilde{r}_1; \tilde{r}_2) = P(\tilde{F} = 0, M, \tilde{r}_2; \tilde{r}_1). \qquad (3-1)$$

Since both sides are proportional to M, we can take this equal to 1 and Equation (3-1) is the statement that

$$P(\tilde{r}_1; \tilde{r}_2) = P(\tilde{r}_2; \tilde{r}_1). \qquad (3-2)$$

This is the standard Rayleigh form of the reciprocity principle. In words: the pressure at $\tilde{r}_1$ due to a unit source at $\tilde{r}_2$ is the same as the pressure at $\tilde{r}_2$ due to a unit source at $\tilde{r}_1$.

b) Let $M_1 = M_2 = 0$.

$$\begin{aligned}\tilde{F}_1(\tilde{r}' - \tilde{r}_1') &= \tilde{F}_1\delta(\tilde{r}' - \tilde{r}_1')\\ \tilde{F}_2(\tilde{r}' - \tilde{r}_2') &= \tilde{F}_2\delta(\tilde{r}' - \tilde{r}_2').\end{aligned}$$

Equation (2-7) becomes

$$\tilde{F}_1 \cdot \nabla_1' P(\tilde{F}_2, \tilde{r}_2'; \tilde{r}_1') = \tilde{F}_2 \cdot \nabla_2' P(\tilde{F}_1 \tilde{r}_1'; \tilde{r}_2'). \qquad (3-3)$$

Since the gradients are proportional to the velocities we can rewrite Equation (3-3) as

$$\tilde{F}_1 \cdot \tilde{v}(\tilde{F}_2, \tilde{r}_2'; \tilde{r}_1') = \tilde{F}_2 \cdot \tilde{v}(\tilde{F}_1, r_1'; \tilde{r}_2') \qquad (3-4)$$

Now both sides are proportional to the magnitudes of both $\tilde{F}_1$ and $\tilde{F}_2$. We can then choose both to be of magnitude 1. We choose $\tilde{F}_1$ and $\tilde{F}_2$ separately to be 1 of 3 orthogonal unit vectors $\tilde{e}_1, \tilde{e}_2, \tilde{e}_3$. This yields the relations

$$\tilde{v}_i(e_i, \tilde{r}_2'; r_1') = \tilde{v}_i(\tilde{e}_i, \tilde{r}_1'; \tilde{r}_2') \qquad (3-5)$$

$$(i = 1, 2, 3).$$

Thus the velocity is the ith direction at $\tilde{r}_1'$ due to a force in the ith direction at $\tilde{r}_2'$ is the same as the velocity in the ith direction at r_2' due to a force in the ith direction at $\tilde{r}_1'$.

If we choose $\tilde{F}_1$ and $\tilde{F}_2$ orthogonal, we get the 6 relations

$$\tilde{v}_i(\tilde{e}_j, \tilde{r}_2'; \tilde{r}_1') = \tilde{v}_j(\tilde{e}_i, \tilde{r}_1{}'; \tilde{r}_2') \qquad (3-6)$$

$$i \neq j.$$

Thus the velocity in the ith direction at $\tilde{r}_1'$ due to a force in the jth direction at $\tilde{r}_2'$ is equal to the velocity in the jth direction at $\tilde{r}_2'$ due to a force in the ith direction at $\tilde{r}_1'$.

c) Let $\tilde{F}_1 = 0,\ M_1 = M\delta(\tilde{r}' - \tilde{r}_1)$,

$$\tilde{F}_2 = \tilde{F}_2\delta(\tilde{r}' - \tilde{r}_2'),\ M_2 = 0.$$

Then $P_1(\tilde{r}') = P(\tilde{F}_1 = 0, M, \tilde{r}_2; \cdot\tilde{r}')$

$$P_2 = P(\tilde{F}_2, 0, \tilde{r}_2'; r').$$

Equation (2-7) becomes (on replacing pressure gradients by velocities)

$$i\omega M P(\tilde{F}_2, 0, r_2'; \tilde{r}_1) = -\frac{\tilde{F}_2}{i\omega\rho_o} \cdot \tilde{v}(\tilde{F}_1 = 0, M, \tilde{r}_1; \tilde{r}_2').$$

Since both sides are $\sim$ M and $(\tilde{F}_2)$, we can take $M = 1$, and $\tilde{F}_2 = e_i$ where i = 1, 2, 3 yields 3 orthogonal unit vectors.

Here we have the three relations

$$\omega^2\rho_0 P(\tilde{e}_i, 0, r_2'; \tilde{r}_1) = v_i(\tilde{F}_1 = 0, 1, \tilde{r}_1; \tilde{r}_2') \qquad i = 1, 2, 3 \qquad (3-7)$$

In other words this says: The pressure at $\tilde{r}_1$ due to a force at $\tilde{r}_2'$ in the $\tilde{e}_i$ direction is proportional to the velocity in the ith direction due to a source at $\tilde{r}_1$ at $\tilde{r}_2'$.

We hazard the guess that this is the reciprocity principle referred to in reference [1].

References

[1] Donskoy D., 1991 "Soviet R&D in Low-Frequency Underwater Acoustics," published by Delphic Associates Inc., 7700 Leesburg Pike, No. 250, Falls Church, Va 22043 .

[2] Rayleigh, J. W. S., 1945, "The Theory of Sound", Vol II, p. 145, Dover Publications, NY.

[3] Landau, L. D. and E. M. Lifshitz, 1959, "Fluid Mechanics," Pergamon Press.

a) No. 74

b) Problem on p.291

[4] Morse, P. M. and K. U. Ingrad,(1986), "Theoretical Acoustics", Princeton University Press, Princeton, NJ, p.134, 312, 320, 342, 405 (1986).

DISTRIBUTION LIST

CMDR & Program Executive Officer
U S Army/CSSD-ZA
Strategic Defense Command
PO Box 15280
Arlington, VA 22215-0150

Mr John M Bachkosky
Deputy DDR&E
The Pentagon
Room 3E114
Washington, DC 20301

Dr Joseph Ball
Central Intelligence Agency
Washington, DC 20505

Dr Arthur E Bisson
DASWD (OASN/RD&A)
The Pentagon
Room 5C675
Washington, DC 20350-1000

Dr Albert Brandenstein
Chief Scientist
Office of Nat'l Drug Control Policy
Executive Office of the President
Washington, DC 20500

Mr. Edward Brown
Assistant Director
DARPA/NMRO
3701 North Fairfax Drive
Arlington, VA 22203

Dr H Lee Buchanan, I I I
Director
DARPA/DSO
3701 North Fairfax Drive
Arlington, VA 22203-1714

Dr Curtis G Callan Jr
Physics Department
PO Box 708
Princeton University
Princeton, NJ 08544

Dr Kenneth M Case
1429 Calle Altura
La Jolla, CA 92037

Dr Ferdinand N Cirillo Jr
Central Intelligence Agency
Washington, DC 20505

Brig Gen Stephen P Condon
Deputy Assistant Secretary
Management Policy &
Program Integration
The Pentagon, Room 4E969
Washington, DC 20330-1000

Ambassador Henry F Cooper
Director/SDIO-D
Room 1E1081
The Pentagon
Washington, DC 20301-7100

D A R P A Library
3701 North Fairfax Drive
Arlington, VA 22209-2308

D T I C [2]
Cameron Station
Alexandria, VA 22314

Mr John Darrah
Senior Scientist and Technical Advisor
HQAF SPACOM/CN
Peterson AFB, CO 80914-5001

Dr Gary L Denman
Director
DARPA/DIRO
3701 North Fairfax Drive
Arlington, VA 22203-1714

Dr Nancy Dowdy
USACDA
320 21st Street NW
Washington, DC 20451

DISTRIBUTION LIST

Mr John N Entzminger
Chief, Advance Technology
DARPA/ASTO
3701 North Fairfax Drive
Arlington, VA 22203-1714

Capt Kirk Evans
Director Undersea Warfare
Space & Naval Warfare Sys Cmd
Code PD-80
Department of the Navy
Washington, DC 20363-5100

Dr S William Gouse
Sr Vice President and
General Manager
The MITRE Corporation
Mail Stop Z605
7525 Colshire Drive
McLean, VA 22102

Col Randall Gressang
DARPA/DIRO
3701 North Fairfax Drive
Arlington, VA 22203-1714

Mr. Thomas H Handel
Office of Naval Intelligence
The Pentagon
Room 5D660
Washington, DC 20350-2000

Maj Gen Donald G Hard
Director of Space and SDI Programs
Code SAF/AQS
The Pentagon
Washington, DC 20330-1000

Dr Robert G Henderson
Director
JASON Program Office
The MITRE Corporation
7525 Colshire Drive
Mailstop Z561
McLean, VA 22102

Dr Barry Horowitz
President and Chief Exec Officer
The MITRE Corporation
202 Burlington Road
Bedford, MA 01730-1420

Dr William E Howard I I I [2]
Director for Space and Strategic Technology
Office/Assistant Secretary of the Army
The Pentagon Room 3E474
Washington, DC 20310-0103

Dr Gerald J Iafrate
U S Army Research Office
PO Box 12211
4330 South Miami Boulevard
Research Triangle NC 27709-2211

J A S O N Library [5]
The MITRE Corporation
Mail Stop W002
7525 Colshire Drive
McLean, VA 22102

Dr George Jordy [25]
Director for Program Analysis
U S Department of Energy
ER30
OER
Washington, DC 20585

Dr O' Dean P Judd
Los Alamos National Lab
Mail Stop A-110
Los Alamos, NM 87545

Dr Bobby R Junker
Office of Naval Research
Code 412
800 North Quincy Street
Arlington, VA 22217

DISTRIBUTION LIST

Mr Robert Madden [2]
Department of Defense
National Security Agency
Attn R-9 (Mr. Madden)
Ft George G Meade, MD 20755-6000

Dr Arthur F Manfredi Jr [10]
OSWR
Central Intelligence Agency
Washington, DC 20505

Mr Joe Martin
Director
OUSD(A)/TWP/NW&M
Room 3D1048
The Pentagon
Washington, DC 20301

Mr Ronald Murphy
DARPA/ASTO
3701 North Fairfax Drive
Arlington, VA 22203-1714

Dr Julian C Nall
Institute for Defense Analyses
1801 North Beauregard Street
Alexandria, VA 22311

Dr Gordon C Oehler
Central Intelligence Agency
Washington, DC 20505

Dr Peter G Pappas
Chief Scientist
U S Army Strategic Defense Command
PO Box 15280
Arlington, VA 22215-0280

Dr Ari Patrinos
Director
Environmental Sciences Division
ER74/GTN
US Department of Energy
Washington, DC 20585

Dr Bruce Pierce
USD(A)D S
Room 3D136
The Pentagon
Washington, DC 20301-3090

Mr John Rausch [2]
Division Head 06 Department
NAVOPINTCEN
4301 Suitland Road
Washington, DC 20390

Records Resource
The MITRE Corporation
Mailstop W115
7525 Colshire Drive
McLean, VA 22102

Dr Fred E Saalfeld
Director
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217-5000

Dr John Schuster
Technical Director of Submarine and SSBN Security Program
Department of the Navy OP-02T
The Pentagon Room 4D534
Washington, DC 20350-2000

Dr Barbara Seiders
Chief of Research
Office of Chief Science Advisor
Arms Control & Disarmament Agency
320 21st Street NW
Washington, DC 20451

Dr Philip A Selwyn [2]
Director
Office of Naval Technology
Room 907
800 North Quincy Street
Arlington, VA 22217-5000

DISTRIBUTION LIST

Superintendent
Code 1424
Attn Documents Librarian
Naval Postgraduate School
Monterey, CA 93943

Dr George W Ullrich [3]
Deputy Director
Defense Nuclear Agency
6801 Telegraph Road
Alexandria, VA 22310

Ms Michelle Van Cleave
Asst Dir/National Security Affairs
Office/Science and Technology Policy
New Executive Office Building
17th and Pennsylvania Avenue
Washington, DC 20506

Mr Richard Vitali
Director of Corporate Laboratory
US Army Laboratory Command
2800 Powder Mill Road
Adelphi, MD 20783-1145

Dr Edward C Whitman
Dep Assistant Secretary of the Navy
C3I Electronic Warfare & Space
Department of the Navy
The Pentagon 4D745
Washington, DC 20350-5000

Mr Donald J Yockey
U/Secretary of Defense For Acquisition
The Pentagon Room 3E9333
Washington, DC 20301-3000

Dr Linda Zall
Central Intelligence Agency
Washington, DC 20505

Mr Charles A Zraket
Trustee
The MITRE Corporation
Mail Stop A130
202 Burlington Road
Bedford, MA 01730