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PROPERTIES OF READINGS OF PERIODICALLY CORRELATED RANDOM PROCESSES

by

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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PROPERTIES OF READINGS OF PERIODICALLY CORRELATED RANDOM PROCESSES

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Periodically correlated random processes, which serve as models for natural rhythmic phenomena [2], are a generalization of amplitude modulation. It is noted in work [4] that the operation of modulation is not invariant, since the phase of the carrier introduces a specific time structure, but it is invariant in the case of carries which are multiples of the period of the carriers. Therefore there is interest in the investigation of the properties of readings of a periodically correlated random process (PKSP) in points which are multiples of the period of the correlated state.

Theorem 1. Readings of PKSP over a period form a sequence which is stationary in a wide sense.

Actually, if the function of covariation [3] $b(t, u)=r(t+u, t) = E\overset{\circ}{\xi}(t+u)\overset{\circ}{\xi}(t)$ is introduced, where $\overset{\circ}{\xi}(t)=\xi(t)-m_{\xi}(t)$ - centered process, then the coefficient of correlation of readings with the initial phase t_0 $r_{kl}=E\overset{\circ}{\xi}(t_0+kT)\overset{\circ}{\xi}(t_0+lT)=b(t_0+kT, (l-k)T)$. On the basis of the property of the covariation function of PKSP

$$b(t+T, u)=b(t, u) \quad (1)$$

we obtain $r_{kl}=b(t_0, (l-k)T)=R_{l-k}$, i.e., the coefficient of correlation of readings depends only on the difference of their indexes (in the case of fixed T and t_0). Since as a result of the periodic nature of mathematical expectation of PKSP the mathematical expectation of the readings $m_k=E\xi(t_0+kT)=m_{\xi}(t_0+kT)=m_{\xi}(t_0)$ is constant for all k, then the theorem can be considered proven.

Theorem 2. The statistics

$$m_\xi^N(t_0) = \frac{1}{N}\sum_{k=0}^{N-1} \xi(t_0 + kT) \qquad (2)$$

are an asymptotically consistent unbiased estimate of the mathematical expectation of PKSP at the moment t_0.

The unbiasedness is evident from the equation

$$Em_\xi^N(t_0) = \frac{1}{N}\sum_{k=0}^{N-1} m_\xi(t_0 + kT) = m_\xi(t_0),$$

and the consistency follows from the fact that $\frac{1}{N}\sum_{k=0}^{N-1} \xi_k$ is an asymptotically consistent estimate of an ergodic stationary sequence $\{\xi_k\}$.

Definition 1. A stationary random sequence which is ergodic relative to mathematical expectation we will give the name I-ergodic.

Definition 2. Stationary random sequences $\{\xi_k\}$ and $\{\eta_k\}$, the estimates of the mathematical expectation of which are asymptotically correlated, i.e., $\lim_{N\to\infty} Em^N_{(\xi_k)}m^N_{(\eta_k)} = 0$, we will call I-ergodically bound.

Definition 3. The vector stationary process possesses the property E_1 if its components are I-ergodic and I-ergodically bound.

Theorem 3. For ergodicity of the reading sequence of PKSP it is necessary and sufficient that the vector process of its stationary components possess the property E_1.

For proof we will calculate the dispersion of the estimate of mathematical expectation of the reading sequence

$$D_{\hat{m}_\xi}(t_0) = E|\hat{m}_\xi(t_0) - m_\xi(t_0)|^2 = E\left|\frac{1}{N}\sum_{k=0}^{N-1} \xi(t_0 + kT) - m_\xi(t_0)\right|^2.$$

which with a calculation of the periodicity of mathematical expectation can be written as

$$D_{\hat{m}_\xi}(t_0) = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{j=0}^{N-1} r(t_0 + kT, t_0 + jT) = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{j=0}^{N-1} b(t_0, (j-k)T).$$

The last equality is written on the basis of correlation (1). Taking into account the readily checked identity

$$\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_{l-k} = \sum_{m=0}^{N-1}\sum_{l=-m}^{m} a_l = \sum_{l=-N+1}^{N-1}(N-|l|)a_l, \tag{3}$$

dispersion we present in the form

$$D_{\hat{m}_\xi}(t_0) = \frac{1}{N}\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)b(t_0, kT). \tag{4}$$

On the other hand [3],

$$b(t, u) = \sum_{l=-\infty}^{\infty} e^{il\Lambda t}\sum_{j=-\infty}^{\infty} r_{l,j-l}(u)e^{ij\Lambda u},$$

where $\Lambda = \frac{2\pi}{T}$, T -period of the correlated state of the process, $r_{jl}(u)$ - mutual covariation of the j- and l-th stationary components.

From here, considering that $e^{2\pi i} = 1$, we obtain

$$b(t_0, kT) = \sum_{l=-\infty}^{\infty} e^{il\Lambda t}\sum_{l=-\infty}^{\infty} r_{l,j-l}(kT).$$

Substituting the resulting expression into correlation (4) and changing the order of summation of the convergent series, we find

$$D_{\hat{m}_\xi}(t_0) = \sum_{l=-\infty}^{\infty} e^{il\Lambda t}\sum_{l=-\infty}^{\infty}\left[\frac{1}{N}\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)r_{l,j-l}(kT)\right]. \tag{5}$$

Since the dispersion of the estimate $\hat{m}_\xi = \frac{1}{N}\sum_{k=0}^{N-1}\xi(kT)$ of the reading sequence of a stationary process has the following form [1]:

$$D_{m_\xi} = \frac{1}{N}\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)r(kT) = \frac{2}{N}\sum_{k=1}^{N-1}\left(1-\frac{|k|}{N}\right)r(kT) + \frac{\sigma^2}{N}.$$

and the necessary and sufficient conditions of I-ergodic connectedness of random sequences is the Cesaro summability of their mutual covariation

$$\lim_{N\to\infty}\sum_{k=-N+1}^{N-1}\left(1-\frac{|k|}{N}\right)r_{(\xi_j)(\eta_j)}(kT)=o(T).$$

which is proven easily by application of the identity (3) to the correlation

$$E\hat{m}^N_{(\xi_j)}\hat{m}^N_{(\eta_j)}=\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}r_{(\xi_j)(\eta_j)}((l-k)T),$$

then from expression (5) follows directly the validity of the theorem.

Corollary 1. If the covariations and mutual covariations of reading sequences of PKSP components satisfy the Slutsky condition $\frac{1}{N}\sum_{k=0}^{N-1}r(kT)\to 0$, then its reading sequence will be I-ergodic.

Theorem 4. The reading sequences with different initial phases form a two-dimensional stationary sequence.

The function of mutual covariation of sequences with phases t_0 and $t_1=t_0+\tau$ will be

$$r^{(1,2)}_{kl}=r_\xi(t_0+kT,\ t_0+\tau+lT)=b_\xi(t_0,\ \tau+(l-k)T),$$

i.e., in the case of a fixed shift τ depends only on the difference of the indexes l-k. Then from theorem 1 follows the validity of the assertion. Using these results, it is easy to show that the corollaries are valid.

Corollary 2. The statistic

$$\hat{b}_\xi(t_0,\tau)=\frac{1}{N}\sum_{k=0}^{N-1}\overset{0}{\xi}(t_0+kT)\overline{\overset{0}{\xi}(t_0+\tau+kT)} \qquad (6)$$

is the consistent unbiased estimate of the function of covariation in points t_0 and τ.

Corollary 3. The highest harmonic of function $m_\xi(\cdot)$ is the M-th, and for an estimate with a sufficient reliability of the value of the mathematical expectation in point t_0 according to statistic (2) N values are necessary, then the estimate $\hat{m}_\xi(t)$, $t \in [0, T]$, is found based on N(2M+1) values of readings $\xi\left(t^0 + j\frac{T}{2M+1} + kT\right)$, where t_0 - initial phase of the readings.

This assertion is proven by using the Cauchy theorem of readings [5] for a periodic function $x(\cdot)$:

$$x(t) = \sum_{j=0}^{2M} x\left(\frac{kT}{2M+1}\right) \frac{\sin (2M+1)\frac{\pi}{T}\left(t - \frac{kT}{2M+1}\right)}{(2M+1)\sin\frac{\pi}{T}\left(t - \frac{kT}{2M+1}\right)},$$

from which it follows that such a function is determined uniquely by its own 2M+1 values $x\left(\frac{kT}{2M+1}\right)$, $k = \overline{0,2M}$.

Corollary 4. If under analogous conditions the spectrum of function $b(t_0, \cdot)$ is limited by the frequency Λ, then for determining the estimate $\hat{b}(t_0, \tau)$ with a change in the parameter τ on the segment (0, L) it is necessary to have approximately 2NL(2M+1) readings $\xi\left(t_0 + \frac{jT}{2M+1} + \frac{m}{2\Lambda} + kT\right)$, $j = \overline{0, 2M}$, $m = \overline{0, E(2\Lambda L)}$, where $E(\cdot)$ - whole part of the number. The proof of this assertion is analogous to the proof of corollary 3.

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