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13. ABSTRACT *(Maximum 200 words)*

The technical objectives of this project are the design and demonstration of methods for the optimal management of sensor systems. As a specific application, we consider the nonlinear filtering of a vector diffusion process, with several noisy vector observations. Any number of sensors can be utilized in the signal processing performed by the nonlinear filter. The problem considered is the optimal selection of a schedule of these sensors from the available set, so as to optimally estimate a function of the state at some given time. Solution of the optimal schedule is derived from solving a system of quasi-linear inequalities (QVIs). The schedule derived from these methods is optimal with respect to predetermined, but possibly varying, running and switching costs for the sensors, as well as some cost functional of the estimate of the dynamics. Methods include precise strategies for adapting the sensor configuration to the engagement at hand, including optimal timing of decisions to communicate or activate active sensors which reveal position. These strategies have been developed by TSI in the form of feedback control laws which are computationally efficient and simple to implement.

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REALTIME CONTROL OF MULTIPLE SENSOR SYSTEMS

Final Report

TSI Report ARO-93.1

Contract: DAAL03-92-C-0025

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This contract was competitively awarded.

1 Introduction

This is the final report for the Phase I project "Realtime Control of Multiple Sensor Systems," (DAAL03-92-C-0025). In Phase I we defined and developed an analytical framework for solution of a class of *sensor scheduling problems*. We developed prototype software tools for their numerical analysis. We also developed special solution techniques for sensor scheduling in linear Gaussian systems and for Gaussian signals observed through nonlinear functions. These results establish the feasibility of sensor scheduling methodologies for a class of sensor and signal processing models.

1.1 Sensor Scheduling and Signal Processing

Sensor technology for military (and civilian) applications has undergone rapid development during the past decade. Improvements and new developments include focal plane electro-optical arrays, electronically scanned arrays, multistatic operational modes, active-spread spectrum-waveforms, and automatic target recognition systems. Conventional systems like radar have been improved, and new technologies have been placed into operation like Infrared Search and Track (IRST), Electro-Optical (EO) sensors, and various type of Electronic Support Measures (ESM) with Automatic Target Recognition capability. Countermeasures and counter-countermeasures have also been improved and the operational environment has become increasingly demanding.

The increased use of multiple sensor systems on platforms has led to changes in the way sensors implemenations and operations are designed and used. In traditional single sensor systems the decision processes (detection, estimation, tracking, classification, etc.) could be located within the processing unit of the sensor. In effect, the decision process is part of the sensor system, and it works with the low dimensional signals generated by the sensor signal processing.

In contrast, in a multi-sensor system, each sensor is only a contributer to a composite decision process. This *data fusion* process lies outside the sensor's processing capability. It brings a new element to the design of sensor systems in which the operational performance of each individual sensor is important only as an element of the whole [17]. *Sensor fusion* is the aggregation of the information from all the sensors. As currently used, sensor fusion does not involve the active "control" of the sensor elements.

As used in our work *sensor scheduling and control* involves the simultaneous selection, based on quantitative performance measures, of a configuration of sensors (from a collection of sensors) to collect data and associated signal processing (detection/estimation) schemes for the individual sensors in the active configuration. Sensor management in this sense is a key concern in design and operation of multiple sensor platforms and distributed sensor networks in both military and commercial applications. Platforms having multiple sensors (e.g., satellites or aircraft with radar, infrared, video detectors, ESM, comm links, etc.) must manage the sensor configuration and coordinate ("fuse") the data obtained from the sensors in use at any time. The data may vary in quality as a function of the system and target state.

Scheduling and manipulating (controlling the parameters of) the sensor suite as a function of time and state is necessary to obtain the best overall performance (detection, estimation, and identification). Given a sensor configuration, algorithms are required for allocating confidence or making decisions based on data collected from different types of sensors – in real time. *These are the two main issues addressed in this research.*

For example, radar sensors are more accurate than infrared sensors for range tracking, but infrared generally supplies better bearing data. Detecting a target and classifying it using the two types of sensors involves decisions to activate the radar, to control its parameters (sweep space, range gate, etc.), and to accept or reject hypotheses in real time based on the two types of information being provided by the sensors. This process involves not only *sensor data fusion* in the conventional sense; but also *dynamic feedback control* of the sensor system itself, including on-off activation of the active (emissions) sensor, and optimal control of its parameters while active. There can also be continuous control in the signal processing algorithms (e.g., automatic gain control). Constraints on emissions to limit self-exposure must be taken into account in using the active sensor(s). Limiting the search space can improve the detection of targets in that space, but hinder the detection of targets elsewhere.

In sensor networks (e.g., satellite surveillance networks or underwater sonar arrays) one needs to coordinate data collected from many sensors distributed over a large geographical area, and in some cases manage the communications among the sensors. Conflicts must be resolved and a preferred set of sensors selected over finite (short) time intervals, and utilized in detection, estimation, or control decisions. Similarly, large scale industrial (e.g., chemical) processes include an information network for collecting and processing data, and making the results available to operators and automatic controllers for actions. The need to collect and coordinate this information in a systematic way is critical to effective and efficient operation of the plant.

In these problems the dynamic management and scheduling of sensors may, in principle, be based on optimization of certain performance measures. These performance measures should include terms allocating penalties for errors in detection and/or estimation. They must also include terms for costs associated with operating the sensors; e.g., turning sensors on or off, and for switching from one sensor configuration to another. For example, turning on an active radar sensor increases the detectability of the platform, and this should be reflected as a switching cost. In certain applications, using a more accurate, more complex sensor, may require higher bandwidth communications and/or more computational resources allocated to that sensor. If quantified, these costs can be included in the control of the sensor systems.

In distributed networks sensor scheduling may mean the physical movement of a platform (such as a helicopter or airplane) to a particular geographical location. In large scale systems use of several (often hundreds) sensors for decision making may provide better average performance, but may reduce the response time of the system to changing conditions; and increase computational and communication costs both in terms of hardware and software. The latter are obviously evident in large computer/communication networks. The values of the operating and switching costs can depend the values of the state vector. For example, certain types of sensors have accuracy or noise characteristics that vary as a function of the values being measured. There are costs associated with the transfer of information, or tracking records, when there are changes in the set of sensors used. These costs can also depend on the state process.

1.2 Sensor Scheduling and Sensor Fusion

Sensor fusion generally refers to the aggregation of information from several sensors to produce statistical determinations (detection, estimation, etc.) superior to the determinations achieved from any single operational sensor. Sensor fusion, interpreted in this way, is a component of the sensor scheduling formalism.

In its most classical form sensor fusion involves an aggregate hypothesis testing procedure (see, e.g., in [18, 38, 40]). Several sensors receive and process data from some phenonmenon, each performing statistical evaluations of the data. For example, each sensor might implement a Neyman-Pearson (NP) test on its data stream to decide between two hypotheses (H_0, H_1). The decisions are transmitted to a "fusion center" where further processing is done to arrive at the "best decision." For example, the fusion center could implement a NP test on the sensor decisions. Using this formulation, it is possible to show, for example, that a configuration of N similar sensors, each characterized by the same probability of false alarm, probability of detection pair (P_F, P_D), can achieve aggregate (P_F, P_D) superior to the best individual sensor if $N \geq 3$ [40]. If the sensors transmit "quality of estimate" (confidence level) information together with the decisions to the fusion center, then performance can be further improved [17, 40].

Defined in this way, the sensor fusion process may be regarded as providing information used in sensor management as a dynamical process. For example, the decision to activate additional sensors could be based on the failure of the aggregate NP test at the fusion center to provide a decision with a sufficiently high confidence level using available information. Similarly, sensors with controllable parameters (sweep rate, sweep volume) could be commanded to change those parameters to conduct a more effective surveillance. If sensor activation or control involves exposure through emission of radiation or limits the detection of new targets by concentration of activity, then these "costs" can be weighed against the detection and estimation improvements gained by active control.

Sensor fusion algorithms are generally developed based on static information (as described above) which may include all the information gathered to present time. While the signal processing algorithms used at the sensors might be recursive (e.g., Kalman filtering), the processing at the fusion center is static; it generally does not update the likelihood (NP) test recursively.[1]

[1] In our terminology a recursive process is one which updates its state at any time based

Figure 1.1: Sensor fusion, scheduling, and control.

In contrast, our formulation of sensor scheduling is essentially a dynamical process leading to a feedback control strategy to manage the sensor configuration. The diagram in Figure (1.1) illustrates the relationship between these views of sensor scheduling and sensor fusion.

on the previous state and the new information available. It does not use all past information, except as summarized in the "state variables." The Kalman filter is a recursive algorithm, the Wiener filter is not. Recursive algorithms require fixed memory size (the state dimension); non-recursive algorithms require "growing memory."

1.3 Sensor Scheduling and Man-in-the-Loop Systems

In many operational environments a human decision maker interacts with the sensor systems on a continous basis; for example, the Radar Intercept Officer in tactical aircraft. The human's function includes subjective fusion of the information presented by the sensors to make determinations about the state of the operating environment. We shall briefly comment on an example of the design of interactive multi-sensor management systems (the KOALAS architecture) to illustrate the role played by a sensor scheduling subsystem in the man-machine interaction. This is important for some applications of multiple sensor systems like ground based control surveillance networks (of satellites); and in civilian systems like criminal entry surveillance systems.

In the work described in [6, 23, 24] the interactive architectures developed by Barrett and his colleagues [5] for real time decision support system have been applied to the development of sensor utilization systems for tactical aircraft. In this architecture, the human operator plays a key role in the sensor management process. Based processed data from the sensors (a subset of the total), the operator can inject "hypotheses" into the system, altering the sensor configuration to evaluate specific possibilities.

Thus, if the infrared search and track sensor (IRST) indicates a target on a heading, the radar can be activated to provide a range measurement, or if already active, its sweep volume can be revised to focus along the bearing indicated by IRST. In the implementations discussed in [6, 23] when a hypothesis is entered by the operator, the sensor configuration is selected (by a rule based expert system) to "best" evaluate this hypothesis. The expert system contains knowledge about sensor use provided by subject matter experts. The overall architecture for this interactive decision support process is called *KOALAS* (Knowledgeable, Observation Analysis-Linked Advisory System).

Our formulation of sensor scheduling could embedded naturally into the KOALAS architecture (or other man-in-the-loop sensor fusion systems with similar function). The scheduling algorithm plays a role analogous to the expert system in selecting the best sensor suite configuration to evaluate a hypothesis. The selection is based on quantitative models and performance criteria. Clearly, in situations where such models are not available, or in those where subjective factors (threat intent) dominate the engagement, quantitative algorithms may not be appropriate. However, if signal and target models are available and useful, the sensor scheduling is useful. For autonomous platforms it is perhaps the best choice.

In Figure (1.2) we show the KOALAS architecture modified to incorporate a sensor scheduling block. In the diagram all elements inside the dashed line are automatic. The operator interaction takes place through the interface. The scheduler functions as an automatic controller to manage the sensor system in respond to inputs from the operator and feedback signals from the signal processing subsystem. In the absence of operator inputs it would control the sensor system "automatically" using the algorithms described in the next section.

An operator input could serve as a manual control, or as a constraint on the scheduling controller. Alternately, the operator input could determine the specific cost structure used to determine the optimal sensor configuration; e.g., weigh radar performance 10 times more heavily than IRST performance as components of the overall sensor system performance index. Given the performance index (weightings) the scheduler would compute and implement the optimal configuration.

This same philosphy could be used for autonomous vehicles in certain operational environments. For example, *a priori* knowledge about anticipated operational scenarios could be stored in the system. When a particular scenario is detected (e.g., atmospheric versus extra-atmospheric or deep versus

Figure 1.2: Sensor scheduling in a man-in-the-loop system.

shallow water operations), then the sensor performance index could be modified to reflect the scenario. The scheduling algorithm then solves for the best configuration, given the performance index, and indirectly, the operational scenario.

In summary, the theme in our formulation is management of a system of sensors providing data (for signal processing), including information of widely varying quality about parameters or variables of interest, for control, detection, estimation, and information fusion etc. The goal of the research is to develop systematic conceptual, analytical, and numerical methods for their treatment.

Our specific objective is to quantify the scheduling/management procedures and develop algorithms for sensor schedulng and control. In Phase I, we demonstrated how this may be done for certain types of sensors and signal processing algorithms. We showed that the optimal sensor scheduling problem can be described as an *impulse control* problem with partial observations in a finite dimensional state space. We then showed that the partially observed (stochastic control) problem can be transformed into a linear control problem with perfect observations evolving on an infinite dimensional state space. We showed that the transformed optimal impulse control problem can be solved by a system of *quasi-variational inequalities* (QVIs); the analog of dynamic programming for control problems with discrete valued controls and switching costs.

In Phase I we implemented prototype numerical algorithms to solve these QVIs. Solution of the QVIs leads not only to the optimal cost functions; but also to the optimal switching schedule expressed in terms of *switching* and *continuation sets* in the state space. We also developed a simplified scheduling strategy for (linear) sensor systems operating on Gaussian signals.

2 Summary of Phase I Work

2.1 A Model for Sensor Scheduling for Diffusion Processes

The basic model used in our formulation of optimal sensor scheduling is as follows: A signal (or state) process $x(\cdot)$ is given, modeled by the vector valued, nonlinear diffusion process

$$dx(t) = f(x(t))dt + g(x(t))dw(t), \quad x(0) = \xi, \quad t \in [0,T] \tag{2.1}$$

in $\mathbb{R}^n$. We consider M noisy observations of $x(\cdot)$, described by

$$dy^i(t) = h^i(x(t))dt + R_i^{1/2}dv^i(t), \quad y^i(0) = 0 \tag{2.2}$$

with values in $\mathbb{R}^{d_i}$. Here $w(\cdot)$, $v^i(\cdot)$ are independent Wiener processes and $R_i = R_i^T > 0$ are $d_i \times d_i$ matrices.

For linear dynamics, these equations take the form

$$dx(t) = A(t)x(t)dt + B(t)dw(t), \quad x(0) = \xi, \quad t \in [0,T] \tag{1'}$$

and

$$dy^i(t) = H^i(t)x(t)dt + R_i^{1/2}dv^i(t), \quad y^i(0) = 0. \tag{2'}$$

Thus, due to the linearity of the problem, the state $x(t)$ and observations $y(t)$ will be Gaussian.

The sensor scheduling problem is to determine an *optimal utilization schedule* for the available sensors, so as to *simultaneously minimize* the cost of errors in estimating a function of $x(\cdot)$ and the costs of using and switching between various sensors. We need to specify these costs. Let $c_i(x)$ denote the cost per unit time when using sensor i, and the state of the system is

x; let $k_{io}(x)$, $k_{oi}(x)$ denote the cost for turning off, respectively on, the i^{th} sensor when the state of the system is x. The signal processing objective is to compute, at time T, an estimate $\hat{\phi}(T)$ of a given function $\phi(x(T))$ of the state.[2] The estimation error (cost) is defined by

$$E\{c_e(\phi(x(T)) - \hat{\phi}(T))\} := E\{|\phi(x(T)) - \hat{\phi}(T)|^2\} \tag{2.3}$$

We use $\mathcal{N}$ to denote the set of all possible *sensor activation configurations.* An element $\nu \in \mathcal{N}$ is a *word* of length M from the alphabet $\{0, 1\}$. If the ℓ^{th} position in ν is occupied by an 1, the ℓ^{th} sensor is activated (used), if by a 0 the ℓ^{th} sensor is off. There are $N = 2^M$ elements in $\mathcal{N}$. A *sensor schedule is a piecewise constant function* $u(\cdot) : [0,T] \to \mathcal{N}$.

We let $\tau_j \in [0,T]$ denote the switching times in the schedule; i.e., the times when at least one sensor is turned on or off. Suppose the schedule before a switch is $\nu \in \mathcal{N}$, and $\nu' \in \mathcal{N}$ is the schedule after the switch. Then the *associated switching cost* is

$$k_{\nu\nu'}(x) := \sum_{\{i\in\nu\}\{i\notin\nu'\}} k_{io}(x) + \sum_{\{j\notin\nu\}\{j\in\nu'\}} k_{oj}(x). \tag{2.4}$$

The *total operating cost, associated* with use of the sensor schedule $\nu \in \mathcal{N}$ is

$$c_\nu(x) := \sum_{\{j\in\nu\}} c_j(x) \tag{2.5}$$

In (2.4), (2.5), *the symbol* $\{i \in \nu\}$ *denotes the set of all indices (from the set* $\{1, 2, \ldots, M\}$ *which are occupied by a* 1 in ν (i.e., the indices corresponding to the sensors which are on); similarly the symbol $\{i \notin \nu\}$ denotes the set of indices of sensors that are off.

[2] State estimation is discussed here as a generic application of sensor signal processing. Detection and hypothesis testing could be treated by methods similar to those discussed.

The observations, *under sensor schedule* $u(\cdot)$ are

$$dy(t,u(t)) := h(x(t),u(t))dt + r(u(t))dv(t), \tag{2.6}$$

where it is apparent that the available observations depend explicitly on the sensor schedule $u(\cdot)$. In (2.7), for $x \in I\!R^n$, $\nu \in \mathcal{N}$,

$$h(x,\nu) := \begin{bmatrix} h^1(x)\chi_{\{\nu\}}(1) \\ \vdots \\ h^i(x)\chi_{\{\nu\}}(i) \\ \vdots \\ h^M(x)\chi_{\{\nu\}}(M) \end{bmatrix}, \tag{2.7}$$

a block column vector, where in standard notation

$$\chi_{\{\nu\}}(i) := \left\{ \begin{array}{ll} 1, & \text{if the } i^{th} \text{ position in the word } \nu \text{ is occupied by an 1} \\ 0, & \text{otherwise} \end{array} \right\} \tag{2.8}$$

Similarly for $\nu \in \mathcal{N}$

$$r(\nu) := \text{Block diagonal}\{R_i^{1/2}\chi_{\{\nu\}}(i)\}, \tag{2.9}$$

where R_i are the symmetric, positive matrices defined above. In (2.7)

$$v(t) := \begin{bmatrix} v^1(t) \\ \vdots \\ v^M(t) \end{bmatrix} \tag{2.10}$$

is a Wiener process. Based (2.8), for all $\nu \in \mathcal{N}$

$$h(\cdot,\nu) : I\!R^n \to I\!R^D, \quad r(\nu) : I\!R^D \to I\!R^D, \quad D = d_1 + d_2 + \cdots + d_M. \tag{2.11}$$

As an example, consider the case $M = 2, N = 4$. Then $\mathcal{N} = \{00, 01, 10, 11\}$ and

$$h(x,00) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad h(x,01) = \begin{bmatrix} 0 \\ h^2(x) \end{bmatrix},$$
$$h(x,10) = \begin{bmatrix} h^1(x) \\ 0 \end{bmatrix}, \quad h(x,11) = \begin{bmatrix} h^1(x) \\ h^2(x) \end{bmatrix}, \tag{2.12}$$

while

$$r(00) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad r(10) = \begin{bmatrix} R_1^{1/2} & 0 \\ 0 & 0 \end{bmatrix},$$
$$r(01) = \begin{bmatrix} 0 & 0 \\ 0 & R_2^{1/2} \end{bmatrix}, \quad r(11) = \begin{bmatrix} R_1^{1/2} & 0 \\ 0 & R_2^{1/2} \end{bmatrix}. \tag{2.13}$$

Clearly the dimension of the range space of $y(\cdot, \nu)$ is $D_\nu := \sum_{i=1}^{M} d_i \, \chi_{\{\nu\}}(i)$.. Also, for all ν, $y(t,\nu) \in I\!R^D$.

Following established terminology for switching (impulse) control problems (c.f. [10]), we see that a sensor scheduling strategy is defined by an increasing sequence of switching times $\tau_j \in [0,T]$ and the corresponding sequence $\nu_j \in \mathcal{N}$ of sensor activation configurations. We shall denote such a strategy by $u(\cdot)$, where

$$\nu_j, \;\; t \in [\tau_j, \tau_{j+1}); \quad j = 1, 2, \ldots \tag{2.14}$$

The sensor scheduling problem involves the *simultaneous* minimization of costs due to estimation errors and sensor system operation. The estimation and scheduling processes are interrelated, and we must therefore consider *joint estimation and sensor scheduling strategies.* Such a strategy consists of two inseparable parts: the sensor scheduling strategy u (see (2.15)) and the state estimator $\hat{\phi}$. The set of admissible strategies U_{ad} is the customary set of strategies adapted to the sequence of observations defined in terms of the σ-algebras

$$\mathcal{F}_t^{y(\cdot,u(\cdot))} := \sigma\{y(s,u(\cdot)),\ s \leq t\}. \tag{2.15}$$

That is, we consider *strict sense* admissible controls in the sense of [19]. This statement must be interpreted carefully. First, as indicated in (2.15), the observation data depends (as is evident from (2.7) - (2.10)) strongly on the sensor schedule $u(\cdot)$. This dependence is *non-standard*, since the dimensions of the observation vector and the noise covariance change drastically at each switching time τ_i. In standard stochastic control formulations [19, 12], the dependence of observations y on controls $u(\cdot)$ is generally implicit. This is a key difficulty of the sensor scheduling problem which limits the use of standard techniques (e.g., Girsanov transformations). Second (2.15) means that the switching times τ_i and the sensor configurations ν_i, which define a schedule $u(\cdot)$, must be adapted to the observation data $\mathcal{F}_t^{y(\cdot,u(\cdot))}$, *which itself depends essentially* on the values of τ_i and ν_i. This is analogous to "free-boundary" problems in mathematical physics where the location of the boundary conditions depend on the solution. As we shall see later, the (QVI) conditions which define the optimal switching cost involve free boundaries also.

Given a sensor schedule $u(\cdot)$, the corresponding cost is

$$J(u(\cdot),\hat{\phi}) := E\{|\phi(x(T))-\hat{\phi}(T)|^2+\int_0^T c(x(t),u(t))dt+\sum_j k(x(t),u(\tau_{j-1}),u(\tau_j))\}. \tag{2.16}$$

The three terms correspond to the estimation error, the operational cost of the sensors, and the cost of switching among sensor configurations.

Here for $x \in I\!R^n$, $\nu,\nu' \in \mathcal{N}$ $c(x,\nu) := c_\nu(x)$, and $k(x,\nu,\nu') = k_{\nu,\nu'}(x)$,.

In summary, the optimal sensor scheduling including nonlinear filtering is therefore the determination of a strategy achieving

$$\inf_{u(\cdot),\hat{\phi}} \quad J(u(\cdot),\hat{\phi}) \tag{2.17}$$

among all admissible strategies.

This problem involves the simultaneous estimation and optimal control/-scheduling of a system with partial observations. Using recent research in the control theory of stochastic systems with partial observations, it is possible to convert the problem (2.17) into an optimal control problem with perfect observations of the conditional density of the signals. The conditional density functions evolve on an infinite dimensional space, which increases the difficulty in solving the system; however, they satisfy a linear (Zakai) equation. The conversion is described below.

To simplify the presentation, we order the elements of $\mathcal{N}$ according to the numbers they represent in binary form. For example in the case M = 2, N = 4 we replace $\mathcal{N} = \{00, 01, 10, 11\}$ by the set of integers {1, 2, 3, 4}. That is, the one-one correspondence between $\mathcal{N}$ and $\{1, 2, \dots, N\}$ is described by

$$\begin{aligned} \nu &\longmapsto \text{(integer represented by } \nu) + 1 \\ k &\longmapsto \text{ binary representation of } (k-1). \end{aligned} \tag{2.18}$$

This permits us to replace all the ν, ν' by the corresponding integers from $\{1, 2, \dots, N\}$.

2.1.1 Transformation of the Control Problem

The transformation is given two steps: (i) reformulate the scheduling problem as a "standard" impulse control problem; (ii) use a probability (Girsanov) transformation to rewrite the cost function in terms of the conditional density which becomes the state for the control/scheduling problem.

Reformulation as an Impulse Control Problem: Consider an *impulsive control* defined as follows: There is a sequence $\tau_1 < \tau_2 \ldots < \tau_k < \ldots$ of increasing stopping times. To each time τ_i we associate an $\mathcal{F}_{\tau_i}$-measurable random variable u_i with values in the set of integers { 1, 2, ..., N}.[3] We define

$$u(t) = u_i, \quad \tau_i \le t < \tau_{i+1}, \quad i = 0, 1, 2, \ldots \tag{2.19}$$

and set $\tau_0 = 0$. We require that $\tau_i \uparrow T$ as $i \uparrow \infty$, with $\tau_k = T$ possible for some finite k.

Let ν_i be the element of $\mathcal{N}$, corresponding to u_i via (2.18). Then define

$$h(x, u(t)) := h(x, \nu_i), \qquad \tau_i \le t < \tau_{i+1}, \tag{2.20}$$

where $h(x, \nu)$ is defined by (2.8), in terms of the given functions $h^i(\cdot)$. Clearly $h(\cdot, u(t))$ maps $I\!R^n$ into $I\!R^D$ for all sensor schedules $u(\cdot)$. and is obviously bounded and Hölder continuous in x. Define also

$$r(u(t)) := r(\nu_i), \qquad \tau_i \le t < \tau_{i+1}, \tag{2.21}$$

where $r(\cdot)$ is defined by (2.10), in terms of the given matrices R_i, $i = 1, 2, \ldots, M$. Clearly $r(u(t))$ maps $I\!R^D$ into $I\!R^D$ for all sensor schedules $u(\cdot)$ but it is *singular*. Define $\tilde{h}(x, \nu)$ to be the vector valued function

[3] Recall that $N = 2^M$ and the binary representation of each integer $1, 2, \ldots, N$ determines a sensor activation configuration by (2.18).

$$\tilde{h}(x,\nu) := \begin{bmatrix} R_1^{-1/2}h^1(x)\chi_{\{\nu\}}(1) \\ \vdots \\ R_i^{-1/2}h^i(x)\chi_{\{\nu\}}(i) \\ \vdots \\ R_M^{-1/2}h^M(x)\chi_{\{\nu\}}(M) \end{bmatrix} \tag{2.22}$$

with $\chi_{\{\nu\}}(i)$ defined as in (2.9). Let

$$\tilde{h}(x,u(t)) \;:=\; \tilde{h}(x,\nu_i), \qquad \tau_i \le t < \tau_{i+1}. \tag{2.23}$$

Clearly $\tilde{h}(\cdot,u(t))$ maps $I\!R^n$ into $I\!R^D$ for all sensor schedules $u(\cdot)$. We refer to $u(\cdot)$ as the *impulsive control*. It defines the decision to select one of the functions $h(\cdot,k)$, $k \in \{1,2,\ldots,N\}$ at a sequence of decision times. This provides a mathematical formulation of the sensor selection decision process.

To simplify the notation we take $\phi(x) = x$. For this choice the selection of the optimal estimator $\hat{\phi}(T)$ is the conditional mean

$$\hat{\phi}(T) = E^{u(\cdot)}\{x(T) \mid \mathcal{F}_T^{y(\cdot,u(\cdot))}\}, \tag{2.24}$$

where $E^{u(\cdot)}$ denotes expectation with respect to $P^{u(\cdot)}$, the probability defined by the sensor schedule. Let $\mu(u,t)$ denote the conditional probability measure of $x(t)$, given $\mathcal{F}_t^{y(\cdot,u(\cdot))}$, on $I\!R^n$. It is convenient to express (2.24) as a vector valued functional of $\mu(u,t)$

$$\hat{\phi}(T) = \Phi(\mu(u,T)) = \int_{I\!R^n} x\,d\mu(u,T). \tag{2.25}$$

This transformation permits us to rewrite the cost as a function of the impulsive control $u(\cdot)$ only (i.e., the selection of the estimator $\hat{\phi}(\cdot)$ has been eliminated):

$$J(u(\cdot)) \;=\; E^{u(\cdot)}\{\|x(T) - \Phi(\mu(u,T))\|^2 + \int_0^T c(x(t),u(t))dt$$

$$+\sum_{j=1}^{\infty} k(x(\tau_j), u(\tau_{j-1}, u(\tau_j)\chi_{\tau_j<T}\}, \tag{2.26}$$

where $\chi_{\tau_i<T}$ is the characteristic function of the Ω-set $\{\omega; \tau_i(\omega) < T\}$.

To prevent zero cost switching cycles, we assume that the switching costs are uniformly bounded below

$$k(x,i,j) \geq k_o, \quad x \in I\!R^n, \ i, \ j \in \{1,\ldots,N\} \tag{2.27}$$

with k_0 a positive constant. As a consequence of this assumption, for T finite the optimal policy will exhibit a finite number of sensor switchings.

The optimal sensor selection problem can now be stated precisely as the optimization problem: $\mathcal{P}$: *Find an admissible impulsive control $u^*(\cdot)$ such that*

$$J(u^*(\cdot)) = \inf_{u(\cdot) \in U_{ad}} J(u(\cdot)), \tag{2.28}$$

where U_{ad} are all impulsive control strategies adapted to $\mathcal{F}^{y(\cdot,u(\cdot))}$. Problem $\mathcal{P}$ is a *non-standard* (because the dimension of the state changes) stochastic control problem of a partially observed diffusion.

The equivalent fully observed problem: Next we transform the impulse control problem, to a fully observed stochastic control problem, by introducing an evolution equation for the conditional density. Based on the theory of nonlinear filtering, consider the operator

$$p(u(\cdot),t)(\psi) = E\{\zeta(t)\psi(x(t)) \mid \mathcal{F}_t^{y(\cdot,u(\cdot))}\} \tag{2.29}$$

for each impulsive control $u(\cdot)$ where the functional ζ is defined by

$$\zeta(t) = \exp\{\int_0^t \tilde{h}(x(s),u(s))^T dz(s) - \frac{1}{2}\int_0^t \|\tilde{h}(x(s),u(s))\|^2 ds\}$$

where ^{T} denotes transpose, $\|\cdot\|$ is the $\mathbb{R}^D$ norm.

The notation is chosen so as to emphasize the dependence on $u(\cdot)$. The operator $p(u(\cdot),t)$ is the *unnormalized conditional probability measure* of $x(t)$ given $\mathcal{F}_t^{y(\cdot,u(\cdot))}$, [26]. As we shall see below, it can be written in terms of a conditional density which satisfies a linear stochastic PDE.

A straightforward calculation [1] implies that

$$E\{\|x(T)-\Phi(\mu(u,T)\|^2\} = E\{\Psi(p(u(\cdot),T))\}$$

where Ψ is a functional on finite measures of $\mathbb{R}^n$ defined by

$$\Psi(\mu) = \mu(\chi^2) - \frac{\|\mu(\chi)\|^2}{\mu(1)} \tag{2.30}$$

where $\chi^2(x) = \|x\|^2$, $x \in \mathbb{R}^n$, and μ is any finite measure on $\mathbb{R}^n$ such that the quantities $\mu(\chi^2)$ and $\mu(\chi)$ make sense.

Using these definitions, we can rewrite the cost corresponding to schedule $u(\cdot)$

$$\begin{aligned} J(u(\cdot)) &= E\{\Psi(p(u(\cdot),T)) + \int_0^T < p(u(\cdot),t), C(u(t)) > dt \\ &+ \sum_{i=1}^{\infty} < p(u(\cdot),\tau_i), K(u_{i-1},u_i) > \chi_{\tau_i<T}\}. \end{aligned} \tag{2.31}$$

where $< p(u(\cdot),t),\psi >$ denotes inner product in $L^2(\mathbb{R}^n)$, and $p(u(\cdot),\cdot)$ unnormalized conditional density which is the "information" state of the equivalent *fully observed* stochastic control problem.

The evolution equation for $p(u(\cdot),\cdot)$ is the Zakai equation from nonlinear filtering theory.

Let

$$L^* = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} a_{ij}(x) \frac{\partial}{\partial x_j} + \sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_i, \tag{2.32}$$

Introducing the notation

$$\delta(x,\nu) = \begin{bmatrix} R_1^{-1}h^1(x)\chi_{\{\nu\}}(1) \\ \vdots \\ R_i^{-1}h^i(x)\chi_{\{\nu\}}(i) \\ \vdots \\ R_M^{-1}h^M(x)\chi_{\{\nu\}}(M) \end{bmatrix} \tag{2.33}$$

Then

$$\begin{aligned} dp(u(\cdot),t) &= L^*p(u(\cdot),t)dt + p(u(\cdot),t)\delta(\cdot,u(t))^T dy(t,u(\cdot)) \\ p(u(\cdot),0) &= p_0, \end{aligned} \tag{2.34}$$

where $y(t,u(t))$ is the observation process as a function of the sensor schedule.

Thus, the "state" of the optimal sensor scheduling problem satisfies a stochastic partial differential equation forced by the "controlled" observations process $dy(t,u(\cdot))$. The "controls" $u(\cdot)$ are the selections of the sensor configurations as a function of time. This formulation of the optimization problem involves complete observations of the (infinite dimensional) state $p((u(\cdot),t)$. Using it, we can define optimality conditions using an extension of the classical theory of impulse control problems [10].

2.1.2 Linear Gaussian Systems

For the linear case, as modeled in Phase I, the Zakai equation reduces to the standard Kalman filter. Thus if the state $x(t)$ and observations $y(t)$ are described by equations (1') and (2'), and if we take $\phi(x) = x$ in the cost function, then the estimate $\hat{x}(t)$ (the conditional mean of $x(t)$ given the observations) and the error covariance (matrix) $\sigma(t)$ are given by

$$d\hat{x}(t) = (A - \sigma H'R^{-1}H)\hat{x}dt + \sigma H'R^{-1}dy(t) \quad \hat{x}(0) = E[x], \tag{2.35}$$

$$\dot{\sigma}(t) = (BB' - \sigma H'R^{-1}H\sigma) + A\sigma + \sigma A' \quad \sigma(0) = \text{cov}(x). \tag{2.36}$$

Hence, $p(u(\cdot),t)$ is Gaussian and is given by

$$p(z,u(\cdot),t) = \frac{(\det\sigma(u(\cdot),t))^{-\frac{1}{2}}}{(2\pi)^{\frac{n}{2}}}\exp(-\frac{1}{2}(z-\hat{x}(u(\cdot),t))'\sigma^{-1}(u(\cdot),t)(z-\hat{x}(u(\cdot),t))) \tag{2.37}$$

where n =dimension of the state vector $x(t)$.

2.2 Quasi-Variational Inequalities for the Optimal Schedule

Consider (2.34) with fixed schedule $u(t) = j$, and let p_j denote the corresponding density $p(\cdot,j)$. Then for $j \in \{1,2,\ldots,N\}$

$$dp_j = L^* p_j\, dt + p_j \tilde{h}^{j^T} dz(t),\ p_j(0) = \pi, \tag{2.38}$$

where

$$\tilde{h}^j := \tilde{h}(\cdot,j). \tag{2.39}$$

We set

$$\Phi_j(t)(F)(\pi) = E\{F(p_{j,\pi}(t))\}, \tag{2.40}$$

where $P_{j,\pi}$ indicates the solution of (2.38) with initial value π.

To simplify the statement and analysis of the quasi-variational inequalities that solve the optimization problem, we consider the case $N = 2$, that is, a single sensor which is scheduled to turn on or off. Consider the cost functions

$$\begin{aligned} C_i &:= C(i,\cdot), \quad i = 1,2, \\ K_1 &:= K(1,2) \\ K_2 &:= K(2,1). \end{aligned} \tag{2.41}$$

and the notation for the expected cost

$$C_1(\pi) = (C_1,\pi) \tag{2.42}$$

Consider now the set of functionals $U_1(\pi,t), U_2(\pi,t)$ such that

$$\begin{aligned} U_1, U_2 &\in C(0,T;\mathcal{C}_1) \\ U_1(\cdot,t) \geq 0, &\quad U_2(\cdot,t) \geq 0 \\ U_1(\pi,T) &= U_2(\pi,T) = \Psi(\pi) \end{aligned} \tag{2.43}$$

$$\begin{aligned} U_1(\pi,t) &\leq \Phi_1(s-t)U_1(s)(\pi) + \int_t^s \Phi_1(\lambda - t)C_1(\pi)d\lambda \\ U_2(\pi,t) &\leq \Phi_2(s-t)U_2(s)(\pi) + \int_t^s \Phi_2(\lambda - t)C_2(\pi)d\lambda \\ \forall s &\geq t \end{aligned} \tag{2.44}$$

$$\begin{aligned} U_1(\pi,t) &\leq K_1(\pi) + U_2(\pi,t) \\ U_2(\pi,t) &\leq K_2(\pi) + U_1(\pi,t). \end{aligned} \tag{2.45}$$

We shall refer to (2.43) as the system of *quasi-variational inequalities* (QVI). In writing this system we have used the notation $U_i(s)(\pi) = U_i(\pi,s), \quad i = 1,2$.

This system defines the "optimal cost" for the sensor scheduling problem in much the same way that the Hamilton-Jacobi-Bellman dynamic programming equation defines the optimal cost in a conventional optimal (stochastic) control problem. Solution of the QVI's not only gives the optimal cost, it also provides an optimal schedule. This is analogous to the case in conventional control problems in which the optimal control is (roughly) the gradient of the optimal cost function.

To understand how the conditions define the cost, consider the two sets of inequalities (2.44) (2.45). At any time in the course of the interval $[0,T]$ it is either optimal to use the existing sensor configuration, or it is optimal to switch to another configuration. If either member of the first set of inequalities (2.44) holds with equality, then the corresponding sensor configuration is optimal; that is, use the one which holds with equality. This corresponds to optimal estimation with a given sensor. Equality in one of the members of (2.44) means that the "cost to go" $U_j(\pi, t)$ is equivalent to the Bellman dynamic programming condition.

Alternatively, if equality holds in one member of (2.45), then it is optimal to switch. For example, suppose optimality holds in the first member, then it is optimal to switch from sensor 1 to sensor 2, incurring switching cost $K_1(\pi)$ and continue using sensor 2, incurring operating cost $U_2(\pi, t)$. In this case inequality will hold in (2.44).

2.2.1 QVIs for the Linear Gaussian Problem

Now consider our Phase I prototype problem of the linear case where the evolving conditional density is given by (2.37) with $u(t) = j$. Note that in this case the problem has been reduced to finite dimensions since we need only determine the vectors $\hat{x}_t$ and σ_t, and not the infinite dimensional "state" $p_t(\pi, t)$. In this case the initial density π is parameterized by the initial vector (x, ρ) of the conditional mean and variance. The QVIs then become, for $i = 1, 2$,

$$\begin{aligned}
&U_1(\cdot,\cdot,t) \geq 0, \qquad U_2(\cdot,\cdot,t) \geq 0 \\
U_1(x,\rho,T) &= U_2(x,\rho,T) = \Psi(x,\rho) = E[x^2] - E[x]^2 = \rho
\end{aligned} \tag{2.46}$$

$$\begin{aligned}
U_1(x,\rho,t) &\leq \Phi_1(s-t)U_1(s)(x,\rho) + \int_t^s \Phi_1(\lambda - t)C_1(x,\rho)d\lambda \\
U_2(x,\rho,t) &\leq \Phi_2(s-t)U_2(s)(x,\rho) + \int_t^s \Phi_2(\lambda - t)C_2(x,\rho)d\lambda \\
\forall s &\geq t
\end{aligned}$$

$$(2.47)$$

$$\begin{aligned} U_1(x,\rho,t) &\leq K_1(x,\rho) + U_2(x,\rho,t) \\ U_2(x,\rho,t) &\leq K_2(x,\rho) + U_1(x,\rho,t). \end{aligned} \tag{2.48}$$

To solve the QVIs numerically it is necessary to rewrite them in differential form (see, for example, [10] or [11] for details). First expand $U_i(x + d\hat{x}, \rho + d\sigma, t + dt)$ about (x, ρ, t) to obtain

$$\begin{aligned} U_i(x + d\hat{x},\rho + d\sigma,t + dt) = U_i(x,\rho,t) + \left.\frac{\partial U_i}{\partial x}\right|_{(x,\rho,t)} d\hat{x}_t + \left.\frac{\partial U_i}{\partial \rho}\right|_{(x,\rho,t)} d\sigma_t + \\ \left.\frac{\partial U_i}{\partial t}\right|_{(x,\rho,t)} dt + \frac{1}{2}\left.\frac{\partial^2 U_i}{\partial x^2}\right|_{(x,\rho,t)} d\hat{x}_t^{\,2} + O(dt) \end{aligned}$$

where $d\hat{x}_t$ and $d\sigma_t$ may be obtained from the Kalman filtering and Riccati equations (2.35) and (2.36). Applying $\Psi_i(\cdot)$ to these expressions and substituting into the QVIs gives

$$-\frac{\partial U_i}{\partial t}dt - \frac{\partial U_i}{\partial x}E[d\hat{x}_t] - \frac{\partial U_i}{\partial \rho}d\sigma_t - \frac{1}{2}\frac{\partial^2 U_i}{\partial x^2}E\left[d\hat{x}_t^2\right] + O(dt) \leq C_i(x,\rho). \tag{2.49}$$

To simplify the notation somewhat, consider a version of the Phase I prototype problem with dynamics and observations given by

$$dx_t = -ax_t dt + dw_t$$

$$dy_t^i = h^i x_t dt + d\nu_t$$

where $a > 0$, w_t and ν_t are zero mean unit variance Gaussian (Wiener) processes, and

$$h_t^i = \begin{cases} 0 & i = 1 \\ c & i = 2 \end{cases}.$$

By varying the constants a and c, the "signal to noise" ratio of the dynamics and the observations may be changed.

To complete writing the QVIs in differential form, the conditional expectations of $d\hat{x}$ and $d\hat{x}^2$, these being the only stochastic terms (depending on the observations) in the expansion, need to be calculated.

First from the Kalman filtering equation,

$$\begin{aligned} E\left[d\hat{x}_t^i\right] &= E\left[-A_t^i\hat{x}_t^i\,dt + h^i\sigma_t^i\,dy_t^i\right] \\ &= -A_t^i\,dt\,E\left[\hat{x}_t^i\right] + h^i\sigma_t^i E\left[dy_t^i\right] \end{aligned},$$

where

$$A_t^i = \begin{cases} a & \text{for } i = 1, \\ a + c\sigma_t^{(2)} & \text{for } i = 2. \end{cases}$$

Taking as the initial condition $x(t) = x$, then $E[\hat{x}_t] = x$. From the observation equation

$$E[dy_t^i] = E[h^i x_t dt + d\nu_t] = h^i x_t dt,$$

since ν_t has zero mean.

Secondly,

$$E[(dx_t^i)^2] = E[(A_t^i\hat{x}_t dt)^2 - 2h^i\sigma_t^i A_t^i dt dy_t^i + (h^i\sigma_t^i dy_t^i)^2.]$$

All but the last term have order greater than dt. Hence,

$$\begin{aligned} E\left[\left(dx_t^i\right)^2\right] &= E\left[\left(h^i\sigma_t^i\,dy_t^i\right)^2\right] + O(dt) \\ &= \left(h^i\sigma_t^i\right)^2 E\left[\left(dy_t^i\right)^2\right] + O(dt) \\ &= \left(h^i\sigma_t^i\right)^2 E\left[\left(d\nu_t^2\right)\right] + O(dt) \\ &= h^i\sigma_t^i\,dt + O(dt) \end{aligned}.$$

Substituting into the QVI formulation (2.49) gives the differential formulation for the Gaussian case as

$$-\frac{\partial U_1}{\partial t} + ax\frac{\partial U_1}{\partial x} - \left(1 - 2a\sigma_t^{(1)}\right)\frac{\partial U_1}{\partial \rho} \leq C_1(x,\rho)$$

$$\begin{aligned} -\frac{\partial U_2}{\partial t} &+ \left(a + c\sigma_t^{(2)} - \left(c\sigma_t^{(2)}\right)^2\right)x\frac{\partial U_2}{\partial x} - \left(1 - 2a\sigma_t^{(2)} - \left(c\sigma_t^{(2)}\right)^2\right)\frac{\partial U_1}{\partial \rho} \\ &-\frac{1}{2}\left(c\sigma_t^{(2)}\right)^2\frac{\partial^2 U_2}{\partial x^2} \leq C_2(x,\rho). \end{aligned} \tag{2.50}$$

2.2.2 Numerical Solution of the QVIs

In general, the QVIs cannot be solved analytically, so numerical approximations must be sought. One method of doing this has been formulated by Rofman and Gonzalez [22]. They show that the solution to the QVIs for optimal control problems with stopping times, and continuous and impulse controls, is given as the maximum element of the set of subsolutions (that is, none of the inequalities need be solved as equalities). Rofman and Gonzalez then present a discretization procedure for finding these subsolutions in both the stationary and non-stationary cases. A summary of their algorithm is given in Figure (2.1).

To adapt the Rofman-Gonzalez algorithm to our problem (a non-stationary *terminal* cost optimal control problem[4]) two minor (and related) modifications must be made.

First, the stopping time cost must be replaced by the terminal cost condition

$$U_i(\pi, T) = \Psi(\pi) \quad \forall i$$

[4]Recall our problem is to generate the best estimate of state at time T by optimally using some configuration of sensors under defined cost conditions.

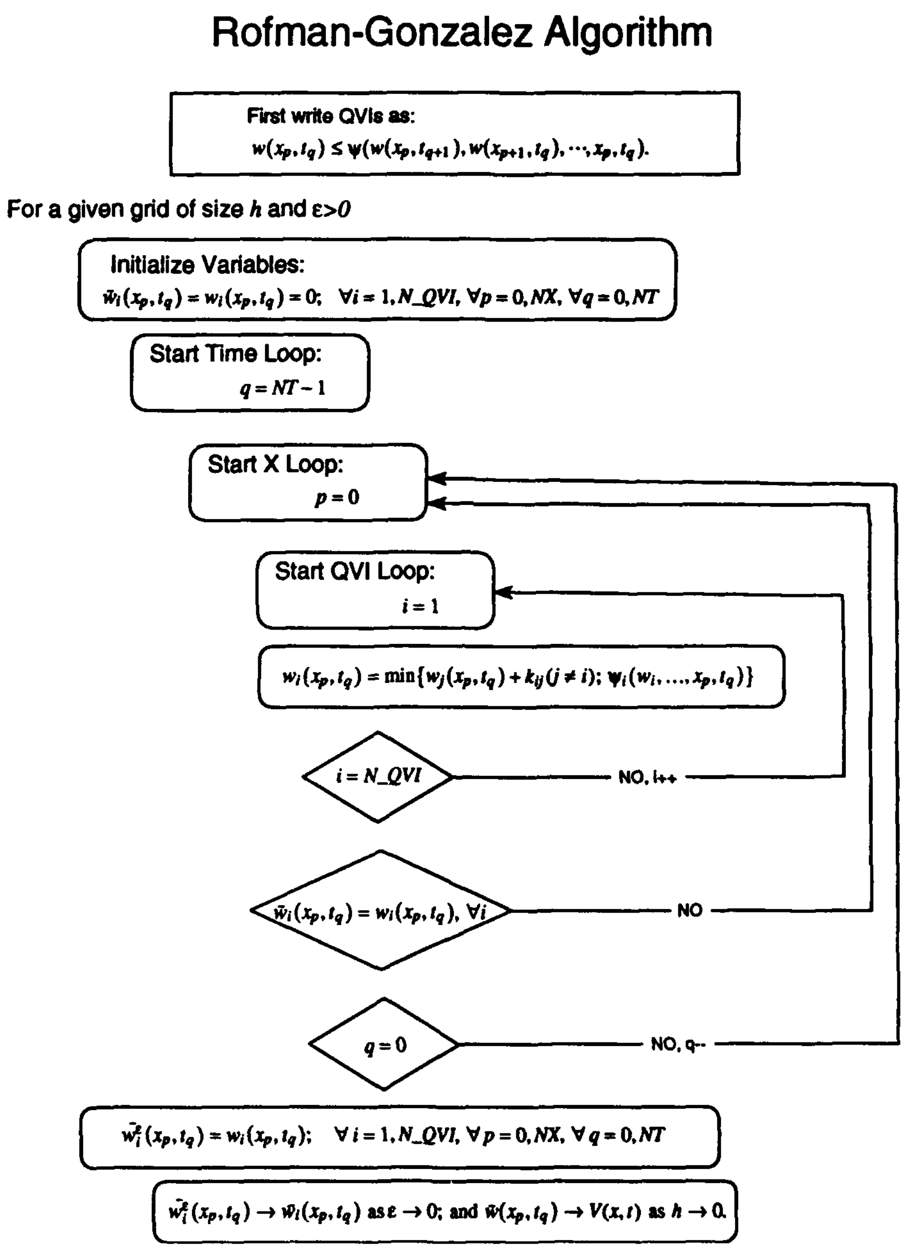

Figure 2.1: Flow diagram of the Rofman-Gonzalez algorithm for solving QVIs of two independent variables.

If, for example, π is a probability measure and we are estimating the state (as opposed to a given function of the state), then $\Psi(\pi)$ is the variance with respect to π.

The second modification is related to the first. The Rofman-Gonzalez algorithms must be initialized by a known subsolution of the QVIs. For the stopping time problems the trivial solution

$$U_i(x,t) = 0 \ \ \forall x, t, i$$

is always a subsolution. For the terminal cost problem, this trivial solution does not usually satisfy the terminal cost condition. To modify the algorithm it is necessary to find a suitable initial solution. In general, this will not be difficult, but will at times introduce minor restrictions on our problems.

For example, returning to the linear Gaussian prototype problem, the QVIs in differential form are given by (2.50) and the boundary conditions

$$U_i(x,\rho,t) \geq 0,$$

$$U_i(x,\rho,T) = \Psi(x,\rho),$$

$$U_i(x,\rho,t) \leq k_{ij} + U_j(x,\rho,t) \ \ \text{for } j \neq i.$$

For this case, $\Psi(x,\rho) = var(\pi(x,\rho)) = \rho$. Thus $U_i \equiv 0$ is not a solution if $\rho \neq 0$. Thus the derivative $\frac{\partial U_i}{\partial \rho} = 1$, which leads to the condition that the coefficients (in the QVIs) of $\frac{\partial U_i}{\partial \rho}$ should be bounded by the running cost. Noting that these coefficients are just the (negative) right hand side of the Riccati equations which implies

$$-\frac{\partial \sigma_t^{(i)}}{\partial t} \leq C_i.$$

That is, any decrease of the overall cost gained by the sensor lowering the variance (error) must be less than any cost incurred running the sensor. Thus this "restriction" is simply a consistency condition which is satisfied by *any* solution. Hence this condition states that an initial condition must in fact be a solution.

2.2.3 Computation of the Optimal Schedule

Given the solutions $U_j(\pi, t), j = 1, 2, t \in [0, T]$, then the optimal sensor schedule is constructed in terms of the *continuation* and *switching sets* associated with the solution. That is, at any given time $t \in [0, T]$ it is optimal to "continue" with the existing sensor – equality holds in one of (2.44), and inequality holds in both of (2.45). At a point t where equality holds in one of (2.45), it is optimal to switch. Bearing in mind that the trajectories followed by the costs $U_j(\pi, t), j = 1, 2, t \in [0, T]$ depend on the underlying state $p(u(\cdot), t)$, the points in the state space at which the members of (2.45) hold with equality define the boundaries of the continuation sets. The boundaries are themselves the switching sets. When the state $p(u(\cdot), t)$ intersects a boundary, it is optimal to switch.[5]

To see that this is the case, suppose that $u(0) = 1$ (we start out using sensor 1). Then define

$$\tau_1^* = \inf_{t \leq T} \{U_1(p_1(t), t) = K_1(p_1(t)) + U_2(p_1(t), t)\} \tag{2.51}$$

This is the first time at which it is optimal to switch (from 1 to 2). We write

$$p^*(t) = p_1(t), \qquad t \in [0, \tau_1^*]. \tag{2.52}$$

which is the state (conditional density) during the (initial) interval that sensor 1 is in use.

[5]The paper [27] contains the explicit computation of switching and continuation sets for a class of simple QVI's.

Next define

$$\tau_2^* = \inf_{\tau_1^* \leq t \leq T} \{U_2(p_2(t), t) = K_2(p_2(t)) + U_1(p_2(t))\} \tag{2.53}$$

This is the first time at which it is optimal to switch (back to 1 from 2).

Continuing this process, we construct a sequence of stopping times $\tau_1^* < \tau_2^* < \tau_3^* < \cdots$ which defines an optimal sensor schedule.

For example, returning to our Phase I prototype problem, Figure (2.2) shows the solutions $U_i(x, 0.5, t)$. These solutions represent U_i with the initial condition of starting with the sensor either off ($i = 0$) or on ($i = 1$) for $\rho = 0.5$, which is the steady state error when no sensor is used (sensor is off). For these solutions, there was no cost incurred to shut off the sensor, and running and switching costs were considered low. The sensor schedule is as expected. That is, regardless of whether the sensor was off or on at the start of the interval, it is off (white region) until the end of the interval at which point it is turned on (black region[6]). The sensor is turned on at the end of the interval for a time interval that will lower the error (variance) to approximately the steady error (variance) of the sensor's on state balance by the sensor's running and switching costs. For example, if these costs are too high, the sensor will remain off.

In higher dimensions the problems and solutions become more complicated since either individual sensors or combinations of sensors may be on. A simple example taken from the Phase I work is given in Figure (2.3) for two sensors. The four diagrams represent sensor schedules which initially start in one of the four configuration ($i = 1$ both sensors off to $i = 4$ both sensors on[7]). In this example, sensor two has five times the gain of sensor

[6] Throughout these graphs, there is *not* a consistency of color representation of the sensors. This is an artifact of the plotting software used. Please see the text of the interpretation of the schedules.

[7] Again note that colors may *not* be consistent between diagrams.

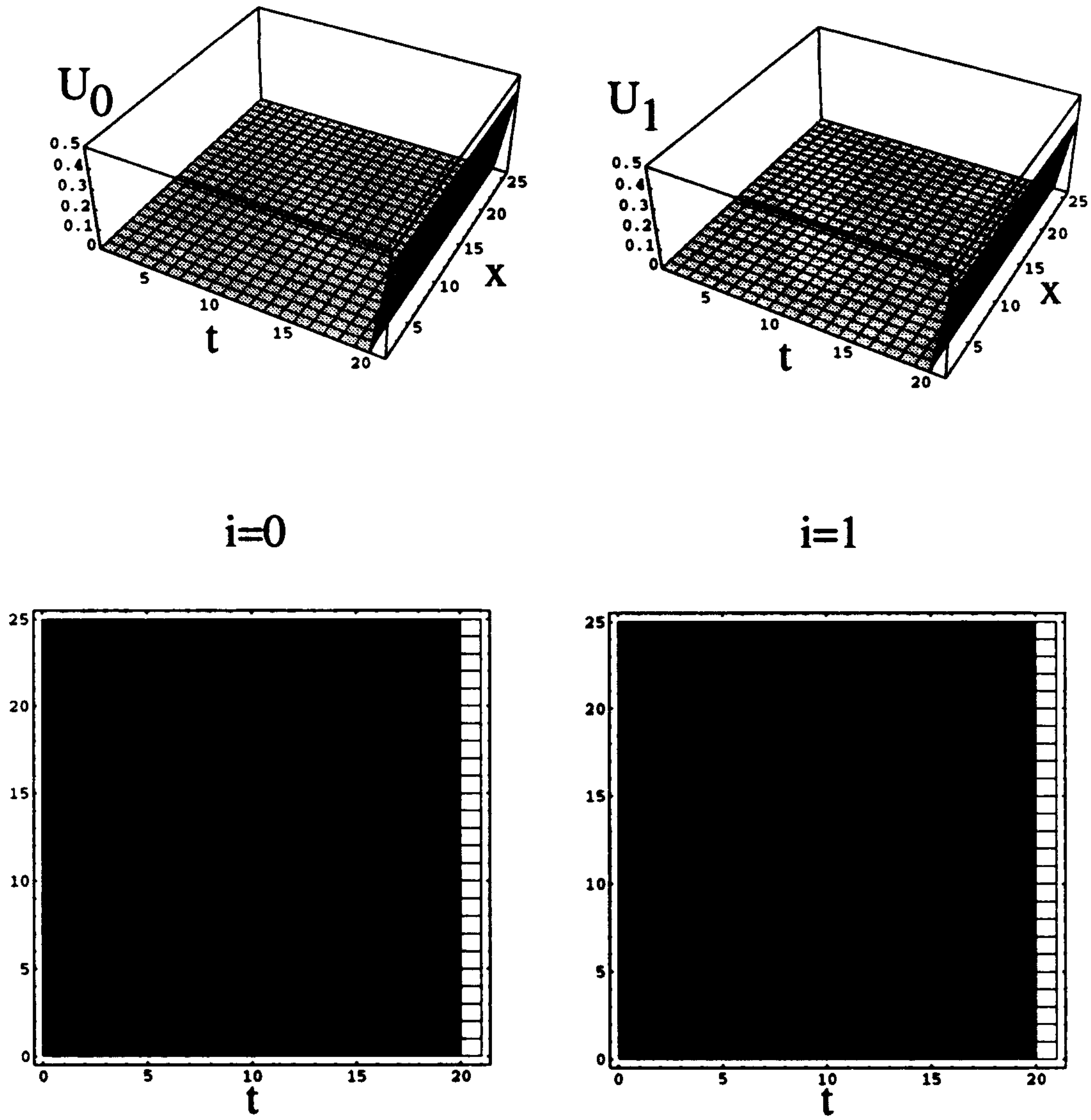

Figure 2.2: Solution and sensor schedule for a single sensor.

one but costs two orders of magnitude greater to run. Switching costs for both sensors are moderate with no cost incurred for turning a sensor off. But note that the steady state solutions of the Riccati equations for all cases are very close, making the sensitivity to the switching cocts greater.

For $i = 1$ (sensor initially off), the sensor generally stays off (black region) except for a small (white) region where the more powerful, and more costly, sensor is turned on for a limited time when the initial state variable (x) is low and likely to be lost is noise. The same behavior occurs for $i = 2$ where the more powerful sensor is initially on. In this case the white region (sensor one on) is slightly large since no "on" switching cost is incurred.

For the other two cases, there is a dramatic change. For $i = 3$ (the white region) and $i = 4$ (the gray region) the second (weak but low cost sensor) is always on. That is, if no switching cost is incurred for turning this sensor on it will continue to run due to its low running cost. For $i = 4$, where the first sensor is also initially on, it will stay on for a short period to lower estimate error where needed but quickly shuts down due to its large running cost. This example indicates the importance of defining good cost functions.

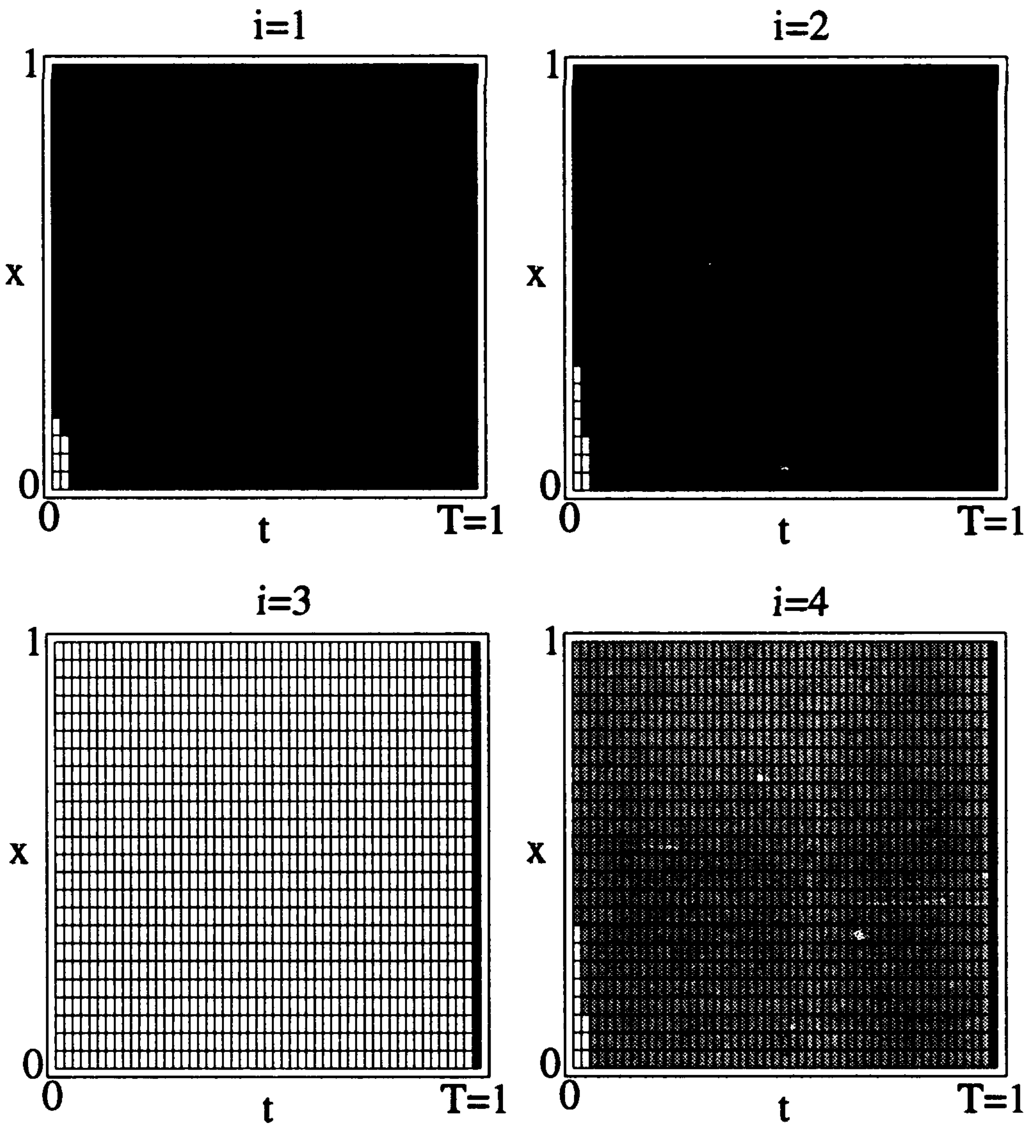

Figure 2.3: Solution and sensor schedule for two sensors.

2.3 Simplified Algorithms for Gaussian State and Sensor Models: An Alternate Approach

In this section we solve the sensor scheduling problem when the state and sensor observations satisfy linear gaussian models using an alternate formulation which may be used to validate the QVI formulation. As before, the optimal state estimate is obtained via Kalman Filtering. We assume that the running cost associated with each sensor is a constant times the duration the sensor is used and that a constant cost is associated with turning on the sensor. Under these assumptions, the optimal sensor schedule is a *single interval sensor schedule policy.* In fact, the scheduling of the sensors reduces to the scheduling of sensor turn-on times, since once a sensor is turned on, it remains on until the terminal time T.

Scalar State and Sensor Models: Assume that the state and two sensor observation models obey the following scalar equations:

$$\begin{array}{ll} \text{State} & dx = a\,x\,dt + b\,dw \\ \text{Sensor 1} & dy_1 = c_1\,x\,dt + dv \\ \text{Sensor 2} & dy_2 = c_2\,x\,dt + dv. \end{array}$$

Here we have assumed both observation noises have unit variance. We do this because the effect of increased or decreased observation noise can be modeled by an appropriate change in the observation coefficient c_i.

A *sensor schedule* consists of two sets of intervals $I_i = \{(a_1^i, b_1^i), \ldots, (a_{n_i}^i, b_{n_i}^i)\}$, $i = 1, 2$ where sensor i is turned on at time a_j^i and turned off at time b_j^i for $j = 1, \ldots, n_i$. (We assume that the intervals are ordered such that $0 \leq a_1^i < b_1^i \leq a_2^i < b_2^i \cdots a_{n_i}^i < b_{n_i}^i \leq T$.)

The sensor scheduling problem seeks to find a sensor schedule which minimizes the cost function

$$J(I_1, I_2) = E((x(T) - \hat{x}(T))^2) + k_1 \sum_{j=1}^{n_1}(b_j^1 - a_j^1) + k_2 \sum_{j=1}^{n_2}(b_j^2 - a_j^2) + n_1 s_1 + n_2 s_2.$$

Here k_i is cost for running sensor i one time unit, $n_i\, s_i$ is the cost for switching sensor i on n_i times, and $\hat{x}(T)$ is the conditional expectation of $x(T)$ given the observations up to T.

The first term in the cost function is the conditional covariance of the state, at time T, given the observations. It is well known that the conditional covariance of x given the observations up to time t satisfies a scalar Riccati equation

$$(R) \qquad \begin{cases} \dot{p}_i(t, t_0; p_0) = 2\, a\, p(t, t_0; p_0) + b^2 - c_i^2\, p^2(t, t_0; p_0) \\ p_i(t_0, t_0; p_0) = p_0. \end{cases}$$

If both sensors are on, the conditional covariance obeys equation (R) with parameters $\{a,\ b, \sqrt{c_1^2 + c_2^2}\}$. We let $p_3(t, t_0; p_0)$ denote the conditional covariance when both sensors are on.

The search for the optimal sensor schedule over the class of all possible sensor schedules can be reduced to the search of the optimal sensor schedule over a much smaller set of schedules. In fact, it can be shown that the search space for the optimal sensor schedule problem can be restricted to a search over the set of single interval sensor schedules.

A *single interval sensor schedule* is a sensor schedule where $n_i = 1$, $i = 1, 2$. That is, a schedule where each sensor is on at most one time interval.

We now state an important result used to find optimal sensor schedules for this case.

Fact: *Assume that the state and observations for each sensor satisfy the equations given above. Given any sensor schedule,* $S = \{I_1, I_2\}$ *with* $n_i \neq 0$, *there exists a single interval sensor schedule,* $\bar{S} = \{\bar{I}_1, \bar{I}_2\}$, *which has cost no* *greater that $\mathcal{S}$, i.e.,*

$$J(\bar{I}_1, \bar{I}_2) \leq J(I_1, I_2).$$

To see how this develops, let $l_j^i = (b_j^i - a_j^i)$ and define

$$\bar{I}_i = \{(T - \sum_{j=0}^{n_i} l_j^i, T)\}.$$

It is clear the $\bar{\mathcal{S}}$ incurs the same running cost as $\mathcal{S}$ since both have their sensors on for the same amount of time. It is also clear that $\bar{\mathcal{S}}$ has no greater switching cost than $\mathcal{S}$ since $\mathcal{S}$ switches at least once for each sensor. It remains to be shown that $\bar{\mathcal{S}}$ has no greater conditional covariance. The complete proof relies on properties of solutions to the Riccati equation.

Finding the Optimal Sensor Schedule: We know that if a sensor is switched on it will be switched off at time T. Hence determining the optimal single interval sensor schedule reduces to finding the optimal sensor switch on times.

The scalar Riccati equation can be solved by separation of variables. The solution is given by

$$p_i(t; p_0) = \begin{cases} (A_i \coth(A_i t + w_i) + a)/c_i^2 & p_0 > (A_i + a)/c_i^2 \\ \\ (A_i \tanh(A_i t + v_i) + a)/c_i^2 & \text{otherwise} \end{cases}$$

where

$$A_i = \sqrt{a^2 + b^2 c_i^2}$$
$$w_i = \coth^{-1}(\frac{c_i^2 p_0 - a}{A_i})$$
$$v_i = \tanh^{-1}(\frac{c_i^2 p_0 - a}{A_i}).$$

In order to compute some of the optimal sensor schedules we will need to compute the derivative of $p_i(t;p_0)$ with respect tc p_0

$$\begin{aligned}\frac{dp_i(t;p_0)}{dp_0} &= \frac{\dot{p}_i}{A_i}\left(\frac{1}{1-\left(\frac{c_i^2 p_0 - a}{A_i}\right)^2}\right)\left(\frac{c_i^2}{A_i}\right) \\ &= \dot{p}_i\left(\frac{c_i^2}{A_i^2-(c_i^2 p_0 - a)^2}\right) \\ &= \dot{p}_i\left(\frac{1}{2\,a\,p_0 + b^2 - c^2 p_0^2}\right).\end{aligned}$$

Let t_1 be the switch-on time for Sensor 1 and t_2 be the switch-on time for Sensor 2. Let $p_0 = -b^2/2a$. The optimal sensor schedule is found by enumerating the six possible switching cases, computing there corresponding optimal costs and picking the case with the lowest overall cost. The six switching cases are (1) $t_1 < T$, $t_2 = T$; (2) $t_1 = T$, $t_2 < T$; (3a) $0 < t_1 < t_2 < T$, (3b) $t_1 = 0$, $t_2 < T$; (4a) $t_2 < t_1 < T$, (4b) $t_1 < T$, $t_2 = 0$; (5) $t_1 = t_2 < T$; (6) $t_1 = t_2 = T$, no switching.

Cases (1) and (2)

Let $\Delta t_i = T - t_i$, then the cost is given by

$$J_i(\Delta t_i) = p_i(\Delta t_i; p_0) + \Delta t_i \, k_i + s_i$$

setting the derivative with respect to Δt_i equal to zero yields

$$\dot{p}_i(\Delta t_i; p_0) = -k_i.$$

Cases (3a) and 4(a)

Let $p_3(t; p_0)$ be the solution to the Riccati equation when both sensors are operating (i.e., $c = \sqrt{c_1 + c_2}$). We will compute the solution for Case (3a), a change in notation gives the solution for Case (4a).

Let $\Delta t_{12} = t_2 - t_1$ and $\Delta t_2 = T - t_2$. The cost to minimize is given by

$$J(\Delta t_{12}, \Delta t_2) = p_3(\Delta t_2; p_1(\Delta t_{12}; p_0)) + \Delta t_{12} k_1 + \Delta t_2 k_2 + s_1 + s_2.$$

Taking partial derivatives of J with respect to Δt_{12} and Δt_2 and setting them to zero yields the two equations:

$$\begin{aligned} \dot{p}_1(\Delta t_{12}; p_0) &= \frac{k_1 \, (c_1^2 + c_2^2) \, p_1^2(\Delta t_{12}; p_0)}{\dot{p}_3(\Delta t_2; p_1(\Delta t_{12}; p_0))} \\ \dot{p}_3(\Delta t_2; p_1(\Delta t_{12}; p_0)) &= -(k_1 + k_2) \end{aligned}$$

Cases (3b) and (4b)

In this case the cost is given as a function of a single time t since the other sensor is always on. Again, we will work out case (3b). The other case is obtained by the appropriate notation changes.

Let $\Delta t_2 = T - t_2$. The cost to minimize is given by

$$J(\Delta t_2) = p_3(\Delta t_2; p_1(T - \Delta t_2)) + T \, k_1 + \Delta t_2 \, k_2 + s_1 + s_2$$

Taking the derivative of J with respect to Δt_2 and setting the result equal to zero yields

$$\dot{p}_3 + \frac{\dot{p}_3}{2p_1 + b^2 - (c_1^2 + c_2^2) p_1^2} (-\dot{p}_1) = k_2$$

with $p_3 = p_3(\Delta t; p_1(T - \Delta t; p_0))$ and $p_1 = p_1(T - \Delta t; p_0)$.

Case 5

Let $p_3(t; p_0)$ be as defined above. Let $\Delta t = T - t$. The cost to minimize is given by

$$J(\Delta t) = P_3(\Delta t; p_0) + \Delta t(k_1 + k_2) + s_1 + s_2$$

Taking derivative of J with respect to Δt and setting the result equal to zero yields

$$\dot{p}_3(\Delta t; p_0) = -(k_1 + k_2).$$

Again, to solve find the optimal sensor schedule, solve for the optimal switch-on times in each of the cases above, compute the corresponding cost and pick the times which result in the lowest overall cost.

Example: Let $a = -1$, $b = 1$, $c_1 = 1$, and $c_2 = 5$. The optimal sensor schedule depends on the values of the running costs k_1, k_2 and the switching costs s_1, s_2. A graph of the optimal schedule switching curves is given in Figure (2.4). In this example there is no switching cost for either sensor and the running cost for each sensor range from exp(−8) to exp(4). The graph is shown in log scale. Regions with the same sensor policy, i.e., $t_2 < t_1 < T$, are shaded with the same color.

We see from the graph that when the running cost for each sensor is low, relative to the conditional covariance, then both sensors are turned on. We also see that as the running cost for a sensor is increased its likelihood of being used is decreased, eventually the sensor is too expensive and hence not used.

These results have an interesting interpretation. Not only do they tell how to schedule the available sensors in order to make the optimal decision in T time units, but they also indicate the minimum desirable duration for the optimal schedule. That is, the smallest time necessary to obtain the optimal state estimate.

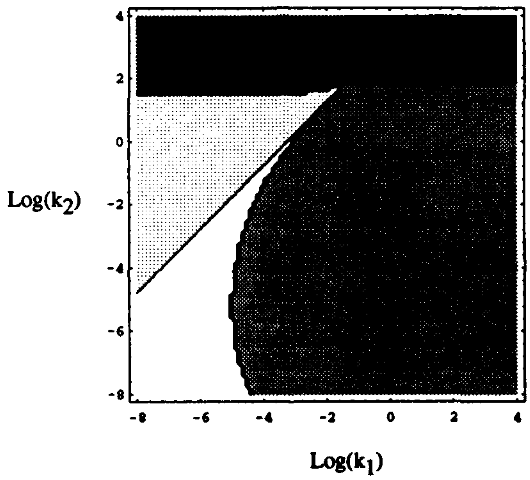

Figure 2.4: Switching Curves for Example 1.

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