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TIME SERIES ANALYSIS AND MULTIVARIATE ANALYSIS

FINAL REPORT

Theodore W. Anderson
Principal Investigator

Jan. 15, 1989, to July 14, 1992

U. S. Army Research Office
Contract DAAG29-89-K-0033

DEPARTMENT OF STATISTICS
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Research was carried out mainly in areas of time series analysis and multivariate analysis. The most important results in the first area apply to regression, autoregression, moving averages. and more generally, stationary stochastic processes. In the second area the results have to do with elliptically contoured distributions.

A simple condition has been derived for a numerical-valued Markov chain to be first-order autoregressive. The stationary probabilities of a general finite chain can be efficiently estimated by the mean of the corresponding vector-valued process.

Statistical inference about regression models and autoregression models is often based on the estimators of the coefficients being approximately normally distributed – at least in large samples. The applicability of inference based on asymptotic theory has been substantially extended; a Lindeberg-type condition is sufficient to justify this. The disturbance vectors need not have a common covariance matrix nor do they need to be independent; they can be martingale increments. The condition is implied by a strongly uniformly integrable condition.

A very general asymptotic distribution of a (finite) set of autocorrelations has been obtained based on the theory for autoregression coefficients. The results for autocorrelations are valid for linear processes with martingale difference disturbances.

Goodness-of-fit tests for time series models based on standardized spectral distributions have been developed for testing patterns of dependence in stationary processes. Tests can be conducted on the basis of the difference between the sample standardized spectral distribution and hypothesized process standardized spectral distribution. Considered as a stochastic process with frequency as the "time" parameter, this difference multiplied by the square root of the sample size converges weakly to a Gaussian stochastic procession on $[0, \pi]$ with expected value 0. Because the process depends on autocorrelations, the conditions for the limit results are very weak. For a linear process (that is, $y_t = \sum_{s=0}^{\infty} \gamma_s r_{t-s}$) Anderson has shown the limiting distribution of the autocorrelations for the disturbances being martingale differences.

The Gaussian stochastic process is characterized by its covariance function. After a suitable monotonic transformation of the frequency parameter, the covariance function of the transformed process is a constant times $\min(u, v) - uv + q(u)q(v), 0 \leq u, v \leq 1$, where $q(u)$ depends on the hypothetical spectral distribution. Note that

$\min(u, v) - uv$ is the covariance function of the Brownian bridge.

Tests are based on functionals of the process, such as the Cramér-von Mises statistic, the Kolmogorov-Smirnov statistic, and the Anderson-Darling statistic. The distribution of the Cramér-von Mises or Anderson-Darling statistic depends on the eigenvalues of an integral equation involving the covariance function of the limiting process; the integral equation can be converted to a second-order differential equation. A formal solution to this problem for standardized spectral distributions has been found, although it is feasible only in certain cases. The distribution of the Kolmogorov-Smirnov criterion satisfies a partial differential equation; a numerical solution is provided by solving recursively an approximating difference equation.

This asymptotic theory has been justified under very general conditions because the distributions of the criteria depend on the asymptotic distributions of autocorrelations.

The principal investigator with Raùl P. Mentz studied iterative methods for estimating the parameters of a Gaussian moving average process of order 1. These procedures yield the exact maximum likelihood estimates. Efficient calculation is based on the calculation of quadratic forms of relevant matrices in the time domain and the traces of related matrices in the frequency domain. In iterative procedures for exact maximum likelihood estimation in first-order Gaussian moving average model "Woodbury's formula" gives an efficient method for evaluating quadratic forms in the inverse of a covariance matrix. The computation of the trace of the inverse of a covariance matrix can be expressed as an infinite series in the autocorrelation. A very accurate approximation has been obtained.

The other topic of research is inference in elliptically contoured distributions. It was shown that a scale-invariant vector-valued function has the same asymptotic distribution as in the normal case except for a factor depending on the kurtosis of the distribution; this factor can be consistently estimated. A scale-invariant scalar-valued function with value and first derivatives 0 at the true parameter value has the same asymptotic distribution as in the normal case except for the kurtosis factor. In most cases this asymptotic distribution is χ^2.

Scientific Personnel

T. W. Anderson, Principal Investigator

Rebecca Betensky

Enapadam Venkatraman

Douglas Haney

Qizhi Wang

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23. Asymptotic robustness in Regression and Autoregression Based on Lindeberg Conditions, by T. W. Anderson and Naoto Kunitomo, June 1989.
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25. Iterative Procedures for Exact Maximum Likelihood Estimation in the First-Order Gaussian Moving Average Model, by T. W. Anderson and R. P. Mentz, November 1990.
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