EMC2 0191-03

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KA-91-02
January 1991
1/20/91

ION LOSS BY COLLISIONS OUTSIDE THE CORE

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4/24143

92-S-4487

1. INTRODUCTION

In this note, we extend the calculation of ion loss due to collisional upscattering in the core of the Polywell™/SCIF device to include effects of ion-ion scattering in the rest of the device.

As the ions transit through the device, collisional processes, such as upscatter or deflection, can alter their velocity distributions, leading to Maxwellization of initially monoenergetic distributions, or isotropization of initially anisotropic distributions. These effects can contribute to ion loss mechanisms. In a previous note,[1] we considered the effect of upscatter due to core collisions on the ion "loss" rate. The ions which are upscattered to higher radial velocity make a larger radial excursion into regions of higher magnetic field, and may be deflected by the B-field to the extent that they will no longer converge to the center of the device. In that sense, they are "lost" to the dense core which produces the bulk of the fusion reactions in the device.

In this note, we consider the effects of ion-ion collisions outside the core and estimate the loss rates due to associated loss mechanisms. We consider the scattering in azimuthal velocity of ions in the bulk of the device, where the radial ion energies are much larger than the azimuthal energies. Perpendicular deflection and conversion of radial to azimuthal velocity by collisions could lead to a degradation of the ion focus. We also consider the effects of thermalization in the edge region of the device, where the ion energy is low and of the order of the ion birth energy.

2. COLLISION FREQUENCY PROFILE

The spatial profile of the ion-ion collision frequency in the Polywell™/SCIF device is discussed in this section. We first summarize the model results for the ion energy and density profiles.[2-4]

As a first approximation for the motion of a test ion in the device, we assume the ion undergoes conservative motion in a one-dimensional (in the radial direction, denoted by r) nonlinear electrostatic potential well (see N. A. Krall, 89-18R
Ref. 2). The ion is born with an energy E_o at a location r_o. The total energy of the particle is conserved:

$$E_i = \frac{1}{2} m_i \left(v_r^2 + v_\perp^2\right) + U \quad . \tag{1}$$

Here E_i is the total energy, v_r and $v_\perp$ are the radial and azimuthal ion velocities, and U is the electrostatic potential energy, given by

$$U = -e[\phi(r) - \phi(r_o)] \quad . \tag{2}$$

We assume the electrostatic potential has the following spatial form:

$$\phi(r) = \phi_{max} \left(\frac{r}{R}\right)^p \quad . \tag{3}$$

Here ϕ_{max} (r_c) is the value of the potential at $r = r_c$, where r_c is the radial dimension of the dense core. [The magnetic field, which is in the z-direction in this slab-like model, enters in this approximation only in the determination of the integer p, with typically p = m,[5] where $B = B(R)(r/R)^m$.] The particle angular momentum is also conserved; $rv_\perp(r)$ = constant.

The test ion radial energy profile is then approximately (for $r < r_o$)

$$E_r = e\phi_{max}\left[\left[\frac{r_o}{R}\right]^p - \left(\frac{r}{R}\right)^p\right] + E_o \tag{4}$$

Can't be so

$$= e\phi_{max}\left[1 - \left(\frac{r}{R}\right)^p\right] + E_o \qquad \text{for } r_o = R$$

As $E_{tot}(r) \overset{m}{=}$ constant

$= E_\perp(r) + E_\omega(r) = E_{\perp 0}\left(\frac{R}{r}\right)^2 + E_\omega(r)$

The ion azimuthal energy profile follows from conservation of angular momentum

NO; IS OK FOR m=3=p

$$E_\perp = \frac{1}{2} m_i v_\perp^2 = \frac{1}{2} m_i v_{\perp o}^2 \left[\frac{r_o}{r}\right]^2 \tag{5}$$

$$= \frac{1}{2} m_i v_{\perp o}^2 \left(\frac{R}{r}\right)^2 \qquad \text{for } r_o = R \quad ,$$

where $v_{\perp o}(r)$ is the ion's azimuthal velocity at birth.

As a model for the radial dependence of the ion density in the Polywell™/SCIF device we use the following expressions given in Section 3 of N. A. Krall, Ref. 3 (here it is assumed that $r_o = R$)

$$n_i = n_o \left[\frac{\phi_{max}}{\phi(r)}\right]^{1/2} \left(\frac{R}{2r}\right)^2 \qquad r_{edge} > r > r_c \tag{6}$$

$$\frac{n_0}{2}\frac{1}{r_c^2} = n_c$$

$$= \frac{1}{2}\, n_o \left(\frac{e\phi}{E_o}\right)^{1/2} \left[\frac{e\phi_{max}}{E_o}\right]^{1/2} \qquad r < r_c \tag{7}$$

$$n_{ed} = \frac{n_0}{3}\frac{1}{r_c}$$

$$= \frac{1}{3}\, n_o \left[\frac{e\phi_{max}}{E_o}\right]^{1/2} \qquad r > r_{edge} \tag{8}$$

$$r_c = R(E_o/e\phi_{max})^{1/2} \ll R \tag{9}$$

$$r_{edge} \sim R - \frac{E_o}{\left.\frac{ed\phi}{dr}\right|_{r=R}} \cong R \tag{10}$$

$$\frac{n_{\phi K}}{n_c} \approx \frac{2 n_0 (R/2r)^2}{\frac{n_0}{2}(R/r_c)^2}$$

where $n_o = 3n_i(r = R)(E_o/e\phi_{max})^{1/2}$.

$$\frac{n_{\phi K}}{n_c} = \frac{2}{4}\left(\frac{r_c}{r}\right)^2$$

2.1 COLLISION FREQUENCY PROFILES

A. CORE

The core region, of radial dimension r_c, is defined as that region in which $e\phi_{max} = E_{\perp o}\,(R/r_c)^2$. From Eqs. (4) and (5), the radial and perpendicular energies in the core are comparable, of the order of $e\phi_{max}$. The core density is fairly flat, of the order of $n_c \sim (3/2)\, n_i$ (r at R)$(e\phi_{max}/E_o)^{1/2}$. Since the ions in the core region are fairly isotropic, ion-ion collisions ~~could~~ will lead to upscatter of total particle energy, while ~~both the radial~~ and azimuthal velocities distributions will be affected by. ~~Using the above representations for the density and energy~~, the ion-ion collision ~~frequency~~ time (i.e. frequency) is[6]

IN ABSENCE OF VIRTUAL NODE

? $(3/2\ E_i = 1/2\ m_i v_i^2)$

NOT NECESSARILY SO

(COLLISIONAL-RADIALIZATION WHICH, IN TURN, IS SET BY INITIAL ENERGY DISTRIBUTION.

$$\tau_{iic} = \frac{\sqrt{3}\ 6\pi\epsilon_o^2 m_i^{1/2} E_i^{3/2}}{Z^4 e^4 n_c \ln\Lambda_c} \quad , \quad ? \tag{11}$$

NUMERIC COEFF IS SAME AS SPITZ. OR BOOK BY FACTOR OF CA. 110-120 (TOO HIGH HERE)!

which in this case, using Maxwellian m_i E distribution, can be written

UNLESS $\epsilon_o \simeq \frac{1}{10}$?

ϵ_o SEEMS to be $\simeq \frac{1}{10}$ IN UNITS HERE (?)

$$\tau_{iic} = \frac{12\pi}{\sqrt{3}} \frac{\epsilon_o^2 m_i^{1/2} (e\phi_{max}) E_o^{1/2}}{e^4 n_i(r = R) \ln \Lambda_c} \quad , \tag{12}$$

where $\ln \Lambda_c \approx 7 + 2.3 \log_{10}[E_i^{3/2}(\text{eV})/(n_c/10^{14}\text{cm}^3)^{1/2}]$ is the Coulomb logarithm for the core region.

B. BULK

The bulk region of the device is defined as that region in which r is greater than the core r_c but less than the edge radius r_{edge} where the ion energy slows down to a value of the order of its birth energy. In this region, the ion radial energy E_r is given by $E_r = e\phi_{max}[1 - (r/R)^p] = e\phi_{max}[1 - (r/R)^3]$ for $p = m = 3$. The ion perpendicular energy is given by $E_\perp \sim e\phi_{max}(r_c/r)^2$, and thus decreases radially outward much faster than E_r. In the bulk of the device, therefore, the ion energies are anisotropic, and the effect of collisions might be to isotropize the energy distribution, converting radial to perpendicular energy. From Eq. (6), the ion density in the bulk region of the device essentially decreases outward as $(r_c/r)^2$, with a weak additional r dependence from the functional form of the electrostatic potential, which tends to make the density distribution flatter than $1/r^2$ beyond about the R/2 point of the device. The ion-ion collision frequency in the bulk of the device is then

?

$$\tau_{iibk} \approx \frac{8\sqrt{3}\,\pi\,\epsilon_0^2 m_i^{1/2}(e\phi_{max})^2}{\ln \Lambda_{bk} e^4 n_i(r = R) E_0^{1/2}} \left(\frac{r}{R}\right)^2 \left[1 - \left(\frac{r}{R}\right)^3\right]^2 \quad , \qquad (13)\ \text{A}$$

for p = 3.

The ratio of the collision time in the bulk of the device to the collision time in the core is

$$\frac{\tau_{iibk}}{\tau_{iic}} = 2\left(\frac{e\phi_{max}}{E_0}\right)\left(\frac{r}{R}\right)^2 \left[1 - \left(\frac{r}{R}\right)^3\right]^2 \left[\frac{\ln \Lambda_c}{\ln \Lambda_{bk}}\right] \qquad (14)$$

RATIOS ARE OK - THE NUMERICS CANCEL OUT!

pretty good for az ... E_0 = birth energy

$$= \frac{1}{2}\left(\frac{e\phi_{max}}{E_0}\right) \quad \text{for } r = R/2 \quad .$$

13 B

$$= \frac{2}{9}\left(1 + \frac{\ln(1/r_c)}{\cancel{2.30}\ 3}\right)\frac{1}{r_c^2} = \frac{0.556}{r_c^2} \text{ FOR } r_c = 10^{-2}$$

C. EDGE

Defining the edge region of the device as that region in which the ion radial KE slows down to a value of the order of the its birth energy, that is, $E_r \approx E_0 = e\phi_{max}[1 - ((R - r_{edge})/R)^p]$, the thickness of the region, $R - r_{edge} = \Delta r_e$, is given by $\Delta r_e = (1/p)(E_0/e\phi_{max})R$. In this region, the radial and azimuthal energies are comparable, so that the energy distribution is again nearly isotropic. The effect of ion-ion collisions ~~might be~~ is to thermalize the distribution at the edge.

$$\Delta r_e \approx \frac{1}{p} R(r_c)$$

$$6 \quad \Delta r_e \neq \frac{r_c}{p}(r_c) = r_c \frac{(r_c)}{m} = \frac{V_c}{m}(r_c)$$

FOR $E_w \approx E\left(\frac{r}{R}\right)^m$. BUT NOTE THAT REAL CASE FALLS OFF SO WILL BE FLATTER

8 KLUGE

ONLY INCREASED δ_{PE} MAKES

The ratio of the collision time in the edge of the device to the collision time in the core is

$$\frac{\tau_{iie}}{\tau_{iic}} \sim \frac{3}{2}\left(\frac{E_0}{e\phi_{max}}\right)\left(\frac{\ln \Lambda_c}{\ln \Lambda_{edge}}\right) . \tag{15}$$

NC. ≃ 2.0

2.2 COLLISION DENSITIES

A figure of merit for assessing the relative importance of ion-ion collisions in different regions of the device is the ratio of the ion transit time t_{tr} through the region to the collision time τ_{ii} in the region. This essentially gives the number of collisions, N_{coll}, that the test ion undergoes in a particular region of the device per transit. The transit time is given by $\Delta r/v_r$ for a particular region, of dimension Δr, where $v_r = (2E_r/m)^{1/2}$ for that region.

The number of collisions in the core per pass is then

$$(N_{coll})_c \sim \frac{2R}{\tau_{iic}}\left(\frac{E_0}{e\phi_{max}}\right)^{1/2}\left(\frac{m_i}{2e\phi_{max}}\right)^{1/2} . \tag{16}$$

The ratio of the number of collisions in the edge region to the number of collisions in the core per pass is

$$\frac{(N_{coll})_{edge}}{(N_{coll})_c} \sim \frac{1}{p}\,\frac{2}{3}\left(\frac{e\phi_{max}}{E_0}\right)\left(\frac{\ln \Lambda_{edge}}{\ln \Lambda_c}\right) . \tag{17}$$

$\frac{3}{4} \cdot 10^4 \sim 10^3$

As representative values for the bulk region, we take $r = R/2$. Then the ratio of the number of collisions in the bulk region to the number of collisions in the core per pass is approximately

$$\frac{(N_{coll})_{bk}}{(N_{coll})_{c}} = \frac{R}{r_c} \left(\frac{E_o}{e\phi_{max}}\right)\left(\frac{\ln \Lambda_{bk}}{\ln \Lambda_c}\right) \quad . \tag{18}$$

From the above, we see that the number of collisions is highest at the edge region, where the low energy ion distribution is relatively isotropic. The number of collisions is lowest in the bulk of the device, where the radial ion energy is high, and where the energy distribution is highly anisotropic, with $E_r \gg E_\perp$.

3. PERPENDICULAR DIFFUSION IN THE BULK OF THE DEVICE

Perpendicular deflection due to scattering in the bulk of the device could lead to isotropization of the anisotropic ion distribution. From conservation of angular momentum (without scattering), $rv_\perp(r)$ = constant. An increase in the azimuthal ion velocity, $\Delta v_\perp$, is related to an increase in the core convergence radius, Δr_c, by[4] – ref. to old [illegible] – KA 90-45

$$\frac{\Delta v_\perp(r)}{v_\perp(r)} = \frac{\Delta r_c}{r_c} \quad . \tag{19}$$

Thus isotropization in the bulk region of the device could degrade the ion focus. In this section, we estimate the rate at which ions are "lost" to the dense core in the sense that they converge to a ~~significantly~~ larger core radius. editorial

Since the change in ion velocity in a single pass through the device is small, we treat the perpendicular deflection as a diffusion process in velocity space, as in our previous note.[1] We use a continuity equation in velocity space as the basis for the loss process

NOT SENSIBLE IN [illegible] DYNAMIC [illegible]: [illegible] [illegible] [illegible] [illegible]. BUT [illegible] [illegible] [illegible] [illegible] [illegible].

n_{bk} n_{cor} n_{edg}

1 10^2 10^5

$$\frac{\partial n}{dt} = D_{\perp eff}\,\frac{\partial^2 n}{dv_\perp^2} \quad , \tag{20}$$

where $D_{\perp eff}$ is the effective perpendicular diffusion coefficient in the device. The ion "loss" rate is then estimated from

THIS IS OK ONLY WITH NO COUNTER GRADIENT EFFECTS

$$\tau_{loss} \sim \frac{(\Delta v_\perp)^2}{D_{\perp eff}} \quad , \tag{21}$$

THIS IS OK ONLY IF $t_{trans}/_{bk}$ IS >> $t_{col}/_{bk}$. IF $t_{col}/_{bk}$ IS LONGER THAN $t_{trans}/_{bk}$ [illegible] MUST BE TREATED where $\Delta v_\perp$ is the increase in the test ion azimuthal velocity which is related to the increase in the core radius by Eq. (19) above.

As in our previous note, we approximate the components of the diffusion tensor with the expression obtained when the background ions, denoted by a subscript 2, are isotropic and have a Maxwellian distribution. The test ion is denoted by a subscript 1. The perpendicular [to the direction of the initial velocity (before the collision) of the test ion] component of the diffusion tensor is then[6]

$$D_\perp = \frac{e_1^2 e_2^2 n_2 \ln \Lambda}{8\pi\epsilon_o^2 m_i^2 v_1}\left[\Phi\left(\frac{v_1}{v_2}\right) - \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2}\right] \quad , \qquad (22)$$

where Φ is the error function, defined by

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\zeta^2)d\zeta \quad , \qquad (23)$$

and Φ_1 is defined by

AS BEFORE

$$\Phi_1(x) = \Phi(x) - \frac{2x}{\sqrt{\pi}}\exp(-x^2) = \Phi(x) - x\frac{d\Phi}{dx} \quad . \qquad (24)$$

For $x = 1$,[7]

$$\Phi_1(x)/2x^2 = 0.214$$

$$\Phi(x) - \Phi_1(x)/2x^2 = 0.629 \quad . \tag{25}$$

We assume that the background ions in the bulk of the device are described by a Maxwellian distribution with a temperature T_2 corresponding to the radial energy of these energetic ions. The effective perpendicular diffusion coefficient in the device is obtained from Eq. (22) by multiplying it by the fractional time that an ion spends in transiting the bulk region of the device.

THIS IS ERRONEOUS, MUCH WORSE!

As a representative value for the loss rate due to deflection in the bulk of the device, we evaluate the loss rate using $r = R/2$. We assume the background ions in the bulk region have a uniform density n_i $(r = R/2)$ given by

NOT A GOOD REF.

$$n_i\ (r = R/2) = 3n_i(r = R)\left(\frac{E_0}{e\phi_{max}}\right)^{1/2} \quad , \tag{26}$$

and a radial energy given by $E_r \simeq e\phi_{max}$. The transit time for a test ion in the bulk region is then of order of 1/2 its transit time through the entire device. The effective perpendicular diffusion coefficient is then

$$D_{\perp eff} \approx \frac{1}{8}\,\frac{1}{\tau_{iibk}(r = R/2)}\,v_2^2\left(\frac{v_2}{v_1}\right)\left[\Phi\left(\frac{v_1}{v_2}\right) - \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2}\right] \quad . \tag{27}$$

Writing τ_{iibk} evaluated at $r = R/2$ in terms of the collision time in the core of the device, using Eq. (14),

$$D_{\perp eff} \simeq \frac{1}{4}\left(\frac{E_0}{e\phi_{max}}\right)\frac{1}{\tau_{iic}}\,v_2^2\left(\frac{v_2}{v_1}\right)\left[\Phi\left(\frac{v_1}{v_2}\right) - \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2}\right] \quad . \tag{28}$$

We $\boxed{\text{estimate}}$ the ion "loss" time, in the sense of ion focus degradation, as the time for $(\Delta v_\perp / v_\perp)$, and thus $(\Delta r_c / r_c)$ to become of order q, where q is a number comparable to or greater than 1. Then $\Delta v_\perp \sim q v_\perp \sim q v_{\perp o}(R/r) = 2 q v_{\perp o}$ at $r = R/2$, and $\Delta v_\perp \sim 2q(E_o/e\phi_{max})^{1/2} v_2$. ~~The~~ This ion "loss" rate can then be estimated from

$$\tau_{loss} \sim \frac{(\Delta v_\perp)^2}{D_{\perp eff}} \sim 16\, q^2\, \tau_{iic} \left(\frac{v_1}{v_2}\right) \frac{1}{\left[\Phi\left(\frac{v_1}{v_2}\right) - \frac{\Phi_1(v_1/v_2)}{2(v_1/v_2)^2}\right]} \quad , \qquad (29a)$$

0.219 0.629

$$\sim 27\, q^2\, \tau_{iic} \qquad \text{for } v_1 = v_2 \quad . \qquad (29b)$$

Now

$$\tau_{iic} \sim 1\ \text{sec} \cdot \frac{[e\phi_{max}(10\ \text{keV})]^{3/2}}{n_c(10^{12}\text{cm}^{-3})} \; \frac{23}{7 + 2.3\ \log_{10}[e\phi_{max}^{3/2}(\text{eV})/n_c^{1/2}(10^{14}\text{cm}^{-3})]} \quad . \qquad (30)$$

$\frac{\Delta r}{r_c} \sim 1$ $\quad q \sim 1 \quad (q = f)$

For experimental parameters such that $e\phi_{max} \sim 10$ keV, $n_c \sim 10^{12}$ cm^{-3}, ~~the~~ this loss ~~rate~~ time is of order 30 q^2 sec. For reactor grade parameters such that $e\phi_{max} \sim 100$ keV, $n_c \sim 10^{18}$ cm^{-3}, the ion loss ~~rate~~ time due to this collisional process is of the order of

$$\tau_{loss} \sim 30\ q^2\ 3 \times 10^{-5}\ \mathrm{sec} \sim q^2\ \mathrm{ms} \quad , \tag{31}$$

$$\sim 1\ \mathrm{ms\ for}\ q = 1 \quad .$$

We note that since the loss rate is proportional to $(\Delta v_{\perp})^2$, which is proportional to $(\Delta r_c)^2$, an increase of a factor q leads to an increase in the loss time by a factor q^2. OBVIOUS

4. THERMALIZATION BY COLLISIONS AT THE EDGE

A test ion spends approximately the same amount of time transiting the core region as it does transiting the thinner edge region. That is,

$\pi/2$ is better here

$$(t_{tr})_c \sim 2r_c/v_c \sim 2R \left(\frac{E_o}{e\phi_{max}}\right)^{1/2} \left(\frac{m_i}{2e\phi_{max}}\right)^{1/2} \quad , \tag{32a}$$

$= 2 r_c/v_{ic}$; $r_c = \langle r_c \rangle R$; $\langle r_c \rangle = \sqrt{\frac{E_o}{2\phi_m}}$; $v_{ic} = \sqrt{\frac{2e\phi_m}{m_i}}$

$$(t_{tr})_{edge} \sim \frac{2\Delta r_e}{v_o} \sim \frac{2R}{p} \left(\frac{E_o}{e\phi_{max}}\right) \left(\frac{m_i}{2E_o}\right)^{1/2} \sim \frac{1}{p} (t_{tr})_c \quad . \tag{32b}$$

From Eq. (15) the ion-ion collision frequency is larger at the edge than in the core, viz.,

$$\frac{\tau_{iie}}{\tau_{iic}} \sim \left(\frac{E_o}{e\phi_{max}}\right) \left(\frac{\ln \Lambda_c}{\ln \Lambda_{edge}}\right) \approx 1.5\text{-}2$$

In this section we estimate whether the ion distribution would thermalize in the edge region. The Maxwellization time scale is of the order of the ion-ion self collision time τ_{iie}. The amount of time that a test ion spends in the edge region is the transit time through the region $(t_{tr})_e$ times the number of passes through the device.

For experimental parameters such that $n_c \sim 10^{12}$ cm^{-3}, $E_o \sim 5$ eV, $e\phi_{max} \sim$ 10 keV, $\tau_{iie} \sim 1$ sec $\cdot$ $(E_o/e\phi_{max}) \sim 5 \times 10^{-4}$ sec. The ion transit time through the edge region would be of the order of 10 ns for R = 100 cm. Assuming 10^4 transits through the device, a test ion would spend about 10^{-4} sec in the edge region, somewhat less than the ion-ion collision time.

~5x(!!)

COMPUTE SINGLE PASS IN SLOWEST COLL RATE REGIONS (I.E. ANOMALIES)

INCORRECT. CAN'T ANALYZE EACH PROCESS AS SEPARATE. MUST INCLUDE ALL COMPETING EFFECTS FOR UNO

For reactor-grade parameters, however, such that $n_c \sim 10^{18}\ cm^{-3}$, $e\phi_{max} \sim 100$ keV, $E_0 \sim 5$ eV, $\tau_{iie} \sim (3 \times 10^{-5}\ sec) \cdot (E_0/e\phi_{max}) \sim 1.5$ ns. The ion transit time through the edge region would be in this case of the order of 1 ns for R = 100 cm. Thus in several passes through the device, the ion distribution at the edge might be thermalized.

MUST DO THIS RIGHT, WITH ALL COMPETING EFFECTS PLAYING ——— —

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