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TRADE-OFFS IN MASS AND EFFECTIVENESS IN SATELLITE SHIELDING: A DESIGN APPROACH

Bryan H. Fortson

August 1992

Final Report

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PHILLIPS LABORATORY
Directorate of Space Technologies Directorate
AIR FORCE MATERIAL COMMAND
KIRTLAND AIR FORCE BASE, NM 87117-6008

PL-TR-92-1020

This final report was prepared by the Phillips Laboratory, Kirtland Air Force Base, New Mexico, under Job Order 3763AE01. The Laboratory Project Officer-in-Charge was Capt Bryan H. Fortson (WSSD).

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
August 1992

3. REPORT TYPE AND DATES COVERED
Final, 24 May 91 - 31 Dec 91

4. TITLE AND SUBTITLE
TRADE-OFFS IN MASS AND EFFECTIVENESS IN SATELLITE SHIELDING: A DESIGN APPROACH

5. FUNDING NUMBERS
PE: 64711F
PR: 3763
TA: AE
WU: 01

6. AUTHOR(S)
Bryan H. Fortson

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Phillips Laboratory
Kirtland AFB, NM 87117-6008

8. PERFORMING ORGANIZATION REPORT NUMBER
PL-TR--92-1020

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
This research was funded by Wright Laboratory.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
The optimal design for a debris shield for a spacecraft is discussed. A shield 3-ft thick would provide excellent protection, but would be prohibitively expensive to deploy. Conversely, suing no shield at all would be highly economical, but would provide poor protection from impacts. Somewhere between these two extremes is a shielding design that combines effectiveness with practicality. A rationale for determining this optimal configuration is outlined. Consider the case for which the thickness of a shield determines its mass and its debris-stopping effectiveness. A shield is considered to have failed when its ballistic limit is exceeded, that is, when it is penetrated. Using this criterion, along with existing models of the orbital debris environment, computer simulations can determine the probability that a given shield will be penetrated by debris over a 1-yr time span. This probability can be expressed as a function of the shield thickness. The shield thickness corresponding to the maximum acceptable probability of failure is the optimum thickness. This report outlines an experimental plan for applying this rationale.

14. SUBJECT TERMS
Space Debris, Spacecraft Shielding, Ballistic Limit, Hypervelocity Impact

15. NUMBER OF PAGES
30

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

PL-TR--92-1020

PREFACE

This study was conducted by personnel of the Air Force Systems Command (AFSC), Phillips Laboratory Space Kinetic Impact and Debris Branch (PL/WSSD), under the supervision of Dr. Firooz Allahdadi, Chief, WSSD and was sponsored by the AFSC Wright Laboratory (WL). The WL project officer was Capt. Marc Chiminiello, WL/MLPJ.

The report was reviewed by a technical committee consisting of Dr. Allahdadi, Dr. Charles Stein (PL/WSMD), Dr. Lalit Chhabildas (Sandia National Laboratories), and Dr. Theodore Nicholas (WL/MLLN).

CONTENTS

1.0 INTRODUCTION

Optimization of spacecraft shield design must consider factors of both mass and cost, and how they affect the protection afforded by the shield. An arbitrarily massive shield could provide impenetrable protection for a spacecraft, but would be unacceptably expensive to deploy. At the other extreme, having no shield at all is highly satisfactory from a cost and mass penalty standpoint, but affords no protection to the spacecraft. Clearly, then, the optimum design lies somewhere between these extremes. But what criteria should be used to determine when the optimal design is obtained? This report outlines one approach to answering this question, an approach designed to develop a valid criterion at minimum cost.

First, a rationale for determining the optimum spacecraft design is suggested. Consideration of this rationale leads to a set of experiments that will put it into practice. Detailed descriptions of the resulting experimental plan are offered.

2.0 APPROACH

2.1 KILL CRITERION

At the heart of this program is the development of a semi-empirical relationship between the mass of a shield and the probability of a kill, P_k, by a given projectile against that shield. The shield mass should be easy to compute for a known configuration, which leaves only P_k to be calculated. Calculating P_k will require some simplifying assumptions. Assume that a kill occurs when a projectile passes completely through the shield, that is, when the projectile velocity exceeds the shield's ballistic limit. (This ballistic limit, or critical impact velocity, is "a velocity below which an object will fail to penetrate a barrier or some type of protective device."[Ref.1]). One can certainly argue with this assumption. A projectile, having penetrated the shield, may have too little energy left to render the satellite inoperable. Or, it may simply fail to make contact with a vital region of the satellite. However, a thorough consideration of these factors requires detailed knowledge of the satellite under consideration, and is beyond the scope of a simple semi-empirical model. The pros and cons of using a more sophisticated approach to finding P_k are discussed at length in Appendix A; this report recommends the simple criterion described above.

2.2 MATHEMATICAL STATEMENT OF KILL CRITERION

This criterion can be defined mathematically: a kill occurs when

$$V_p > V_l \tag{1}$$

where

V_p = the velocity of the projectile relative to the target
V_l = the ballistic limit

Solving Equation 1 analytically requires an expression for V_l. The best such expression will depend upon the material selected for the shield. Options range from monolithic plates to Whipple bumpers to multiple layers of advanced ceramic fibers (ACFs)(Ref. 2). Two illustrative examples follow. The first considers the simplest possible configuration, a monolithic plate. The second, which appears in Section 2.4, shows an approach to a Whipple bumper, which could be generalized to multiple-layer shields.

One semi-empirical expression for V_1, which assumes a homogeneous, single-plate shield, can be found in Zukas et al. (Ref. 1, pp 196-201):

$$V_1 = \sqrt{\alpha\left(\frac{L}{D}\right)^c f(z)\,\frac{D^3}{M}} \tag{2}$$

where

α = an empirical constant
c = an empirical constant
k = an empirical constant
L = the projectile length
D = the projectile diameter
M = the projectile mass
f(z) = a function as defined below:

$$f(z) = z+e^{-z}-1 \tag{3}$$

where

$$z = \frac{T\sec^k\theta}{D} \tag{4}$$

The variable z can be thought of as the ratio of the target thickness T to the projectile diameter, corrected for the obliquity of the impact, represented by the angle of incidence θ.

Equation 2 is based on the work of Lambert (Ref. 3), who developed it for the case of long-rod penetrators impacting single-plate targets of rolled homogeneous armor (RHA). It is important to note that, in the derivation of

Equation 2, the ballistic limit is defined in terms of the residual velocity, that is, the velocity that the projectile will have after passing through the target. In Equation 2, the ballistic limit is that projectile velocity that produces a residual velocity of zero after impact. Several other definitions are in use, and some will be referred to in this report. These definitions contain subtle but significant differences.

Performing regression on an extensive limit-velocity data base, Lambert arrived at the following values for the empirical constants:

$$\begin{Bmatrix} \alpha \\ c \\ k \end{Bmatrix} = \begin{Bmatrix} 1.6\times10^{7} \\ 0.3 \\ 0.75 \end{Bmatrix} \qquad (5)$$

However, these constants are dependent on the materials used for the target and the projectile. Thus, they may not be applicable to all situations of interest, and their accuracy would have to be confirmed for the present work.

It would be more convenient, in Equation 4, to isolate the variable T. If Equation 3 is substituted into Equation 2, and all terms containing z are isolated on the right-hand side of the equation, the result is

$$1+V_I^2\left(\frac{D}{L}\right)^c\frac{M}{D^3\alpha} = z+e^{-z} \qquad (6)$$

The exponential term on the right-hand side can be neglected if z is assumed to be large, say, >3. However, a great deal would be lost by making such an assumption.

For example, consider a case in which the shield is struck normally by a projectile with a diameter equal to the shield thickness. For this case, z = 1. The right-hand side of Equation 6 is equal to 1.368 if the exponential term is kept, but it is equal to 1.000 if the exponential term is neglected. An error of 26.9 percent has been introduced. Thus, neglecting the exponential term leads to unacceptable errors for a situation that may well arise. However, keeping the exponential term makes it difficult to obtain an analytical expression for z. A compromise can be effected as follows. Since

$$0 \leq e^{-z} \leq 1 \tag{7}$$

for all nonnegative z, and since Equation 4 shows that z will always be nonnegative, write

$$z = 1-\epsilon+V_1^2\left(\frac{D}{L}\right)^c \frac{M}{D^3\alpha} \tag{8}$$

where ϵ will later be assumed to be either 0 or 1, whichever makes the model conservative. Plugging Equation 4 into Equation 8 gives an expression for T:

$$T = \frac{D}{\sec^k\theta}\left[1-\epsilon+V_1^2\left(\frac{D}{L}\right)^c \frac{M}{D^3\alpha}\right] \tag{9}$$

Equation 2 allowed determination of the speed at which a given projectile would have to be propelled in order completely to penetrate a given plate. Equation 9 allows determination of how thick a plate of a given material has to be, in order to stop a projectile of given size, mass, and speed. Once the empirical constants are determined for the relevant target and projectile materials, Equation 9 can be used to determine whether a given impact will penetrate a given target, by comparing the required "stopping thickness" to the actual thickness of the target. The independent variables are M, L, D, θ, and the projectile velocity V, which is substituted for V_1. At this point, it is seen that assuming

$$\epsilon = 0 \tag{10}$$

results in a conservative criterion (one that may predict that a shield will be penetrated when it will actually survive, but will not make the opposite error). Use of Equation 10 gives a value for ϵ that will always be too small, making the right-hand side of Equation 9 too large, resulting in an overestimate of T. So the final expression for T becomes

$$T = \frac{D}{\sec^{k}\theta}\left[1 + V^{2}\left(\frac{D}{L}\right)^{c}\frac{M}{D^{3}\alpha}\right] \tag{11}$$

Note that, conversely, if it is assumed that

$$e = 1 \tag{12}$$

a nonconservative criterion (one that may predict that a shield will survive when it will actually fail, but will not make the opposite error) is obtained. The corresponding expression for T is

$$T = \frac{D}{\sec^{k}\theta}\left[V^{2}\left(\frac{D}{L}\right)^{c}\frac{M}{D^{3}\alpha}\right] \tag{13}$$

2.3 SIMULATION TECHNIQUES

Equation 11 will predict more kills than will actually take place (P_k overestimated), and Equation 13 will predict fewer kills than will actually take place (P_k underestimated). Thus, if Equations 11 and 13 are used as the failure criteria in two independent Monte Carlo simulations, the results will define a range in which P_k must fall.

Approximate probability distributions for the independent variables in Equations 11 and 13 are available, but inserting them into Equations 11 and 13 to derive analytical probability distributions for T is an intractable problem. This can be illustrated by considering the expression for the mean value of T (Ref. 4):

$$\overline{T} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{V^{2}MD^{c-2}}{L^{c}\alpha\sec^{k}\theta}\,P_{D}P_{\theta}P_{V}P_{L}P_{M}\,dD\,d\theta\,dV\,dL\,dM \tag{14}$$

where

$\bar{T}$ = the mean value of T

P_D = the probability density function for D

P_θ = the probability density function for θ

P_V = the probability density function for V

P_L = the probability density function for L

P_M = the probability density function for M

It is readily seen that Equation 14 can only be integrated for a highly fortuitous combination of probability density functions. And Equation 14 only describes the mean value of T; finding the probability density function for T would be more difficult.

A Monte Carlo simulation would be more promising. In fact, if closed-form equations for the distributions of the independent variables could be obtained, an extremely simple Monte Carlo computer program could be written, which would give P_k as a function of T for a given material.

Consider this illustrative example: a Monte Carlo simulation is set up to describe a satellite with a shield 1-cm thick, orbiting for 1 year. This simulation is run 10,000 times, and 100 times the satellite is destroyed. According to this simulation, $P_k = 0.01$ for a shield thickness of 1 cm. This simulation can be repeated for different values of T, giving a plot of P_k as a function of shield thickness.

2.4 SUMMARY

The recommended approach is:

TASK 1: Select a material of interest.

TASK 2: Determine the empirical constants for Equation 2, and confirm that this equation is applicable to the cases of interest. This task may not be necessary if a sufficiently well-characterized shielding design is selected in Task 1.

TASK 3: Obtain probability distributions for the independent variables in Equations 11 and 13.

TASK 4: Perform a Monte Carlo simulation to get P_k as a function of T. Figure 1 is a schematic representation of the resulting plot.

TASK 5: Define a maximum acceptable P_k. The thickness T corresponding to this value of P_k, as shown in Figure 1, is the optimum design.

TASK 6: Produce a final report that summarizes the results of the research.

Some caveats are necessary here. First, recall that this derivation is for a single-plate, homogeneous shield, where varying the shield thickness is the only way to change the weight. Thus, this approach would be of little or no application to bumpers, two-layer shields, shields with material gradients, or other, more complex designs. For such designs, it would be necessary to replace Equation 2 with an empirical expression.

An example of such an empirical expression is provided by Cour-Palais (Ref. 5). For a two-layer shield, designed so as to provide optimal protection from a projectile of known mass, velocity, and material, the thickness of the second layer of the shield necessary to prevent spall is

$$t_b = \frac{C \times m \times V}{S^2} \tag{15}$$

where

- t_b = the thickness of the second layer
- C = a constant that depends on the shield material and failure mechanism
- m = the projectile mass
- V = the projectile velocity
- S = the spacing between the first and second layers of the shield

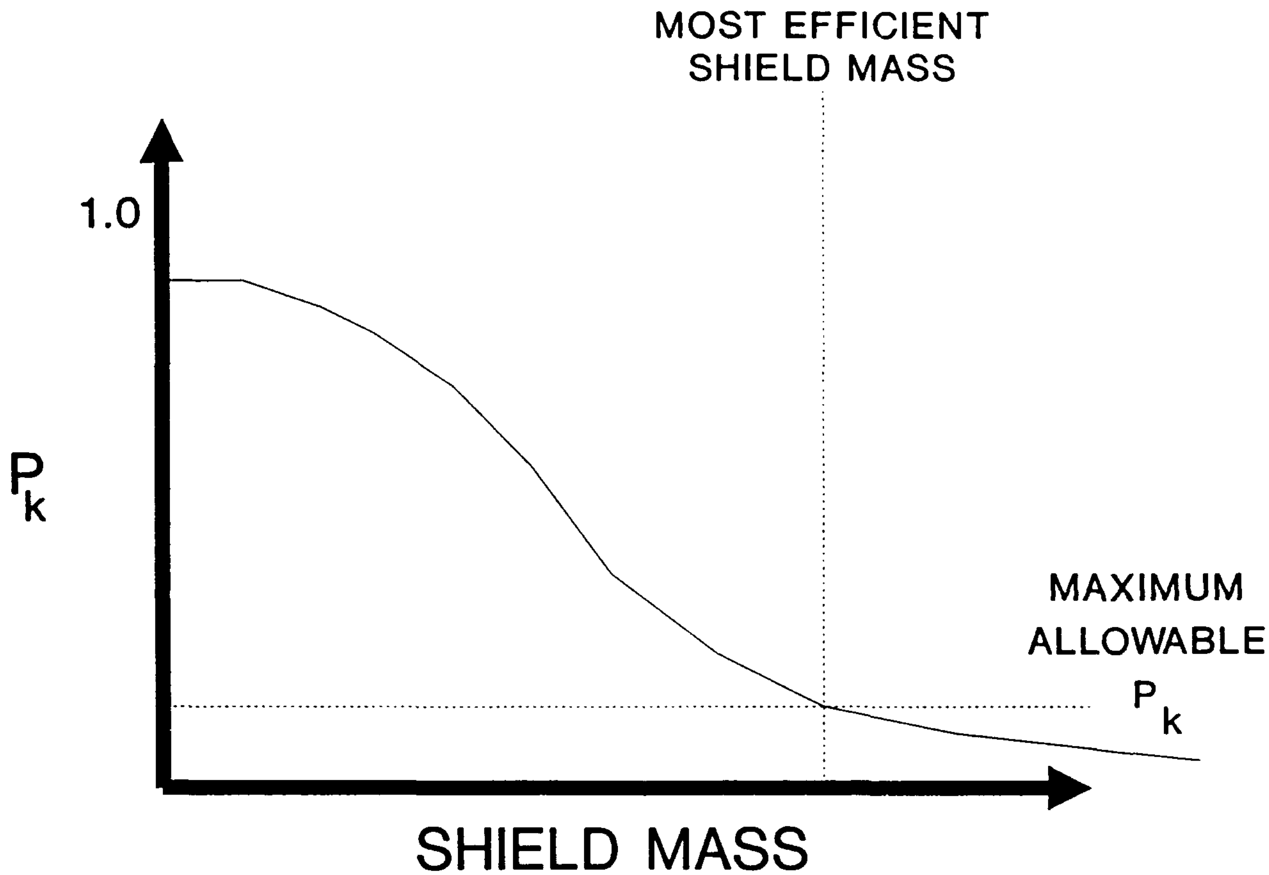

Figure 1. Optimization of shield mass.

Since the shield cannot be varied to provide optimal protection from projectiles of all sizes, the expression for t_b in the case of nonoptimal protection is more relevant, and is given in Reference 5:

$$t_b = \frac{C_1 m^{1/3} V}{S^{1/2}} \times \left(\frac{C_2}{\sigma_y} \right)^{1/2} \qquad (16)$$

where

C_1 = an empirical material constant
C_2 = an empirical material constant
σ_y = the yield strength of the target material

Equations 14 and 15 indicate that it may be desirable to vary the interlayer spacing and the second-layer thickness of the shield, and perhaps the first-layer thickness as well. This would result in a larger matrix of experiments, to determine the effect of these three variables on the ballistic limit of the overall shielding design. Depending on the exact material selected, it may be possible to select a ballistic limit expression based upon previous research.

The second caveat is that tasks 2 and 3 above are hardly trivial. Two recent papers by Fish* make one thing clear: it may not be possible to get a reliable ballistic limit equation off the shelf. And getting the necessary probability distributions may be equally difficult.

*Fish, D., "Ballistic Tests of Two Sheet Structures," and "Ballistic Limit of 6061 T6 Aluminum and Threat to Surface Coatings for Use with Orbiting Space Station Space Suit Materials," unpublished research for NASA Ames Research Center, 1986.

3.0 EXPERIMENTAL PROGRAM

3.1 TEST MATRIX

Of the five tasks defined in the preceding section, only Task 2 involves actual experimentation, so this section will be focussed upon Task 2.

Consider Equation 2. The ballistic limit V_l is expressed in terms of the five independent variables L, D, T, θ, and M. A series of impact tests is necessary to find the best values of the empirical constants α, c, and k for each material. At least two different values of each independent variable should be considered, so an exhaustive test matrix would involve $2^5 = 32$ ballistic limit tests. The number can be reduced to 16 if the projectile geometry and material are held constant, as M is then determined by L and D. If the aspect ratio L/D is assumed to be constant, then one more variable disappears: L and D can be replaced by

$$M = c_1 \rho L^3 \tag{17}$$

where

c_1 = a constant that depends on the assumed geometry
ρ = the projectile density

(A discussion of the effect of aspect ratio on ballistic limit is included as Appendix B.) This assumption permits reduction of the test matrix to eight ballistic limit tests. While some generality has been lost, the importance of this reduction of the test matrix, in terms of keeping the program to a reasonable cost, will become apparent.

Since only two values of each independent variable are considered, they should span the range of possible values of the variables. Call these two values "low" and "high." They would be determined by a survey of the shield configurations of interest. Thus, the test matrix would be as shown in Table 1. Candidate low and high values for each variable are given in Table 2. The values for θ and T are based upon engineering judgment. The values of M are based upon spherical tantalum projectiles with diameters

Table 1. Proposed test matrix for ballistic limit experiments.

Test #	M	θ	T
1	LOW	LOW	LOW
2	LOW	LOW	HIGH
3	LOW	HIGH	LOW
4	LOW	HIGH	HIGH
5	HIGH	LOW	LOW
6	HIGH	LOW	HIGH
7	HIGH	HIGH	LOW
8	HIGH	HIGH	HIGH

Table 2. Low and high values.

Variable	Low Value	High Value
M	0.005 g	2 kg
θ	15 degrees	90 degrees
T	0.1 cm	2.0 cm

between 1 mm and 10 cm, the size range of most interest to the Air Force's Space Debris Program.*

After these eight tests have been performed, the adequacy of Equation 2 can be assessed. If empirical coefficients can be found that will give a good fit to the data, then Equation 2 can be used as discussed in the preceding section. If not, it will be necessary to find another expression for the ballistic limit, probably as a result of curve-fitting. It should be emphasized that there is a large body of empirical equations, such as Equation 2, for both conventional and hypervelocity impacts. Zukas (Ref. 6) provides a long list of references for such equations, one of which may prove useful for the material selected for this program.

3.2 BALLISTIC LIMIT DETERMINATION

An important issue is how the ballistic limit will be assessed for each one of these eight tests. Zukas (Ref. 1, pp. 170-183) notes that there are two approaches to determining the ballistic limit; these approaches might be called the deterministic and the experimental. The deterministic approach attempts to find the ballistic limit on the basis of first principles. However, because of the complexity of the resulting equations, it is frequently necessary to perform experiments to determine a few constants. In the simplest experimental approach, a series of shots is fired at a target in order to determine the quantity V_{50}, the velocity for which there is a 50 percent chance of perforating the target. This quantity is the average of the velocities of six shots: the three slowest shots that completely penetrated the target, and the three fastest shots that resulted in only partial penetration. Since the uncertainty in the value of V_{50} increases with the range of the six values used, convention dictates that the range of these six velocities must be 46 m/s or less. Note that the definition of V_{50} differs from the definition of the ballistic limit used in the illustrative examples of Section 2.0.

*Private communication with Capt. Albert Reinhardt of Phillips Laboratory, November, 1991.

The determination and use of V_{50} is recommended for this program. Although analytical models are more sophisticated, they must include simplifications in order to remain tractable, and experiments must eventually be performed in order to calibrate the theories. If V_{50} is determined, it will be possible to proceed directly to the experiments. Another significant advantage is that V_{50} can be determined in a consistent fashion for any shielding design, whereas separate penetration theories must be developed for homogeneous plates, inhomogeneous plates, multiple-bumper designs, et cetera.

If V_{50} is to be determined in an economical way, it is important to begin with good estimates of what the ballistic limit might be. These estimates can be gained by modelling the impacts with hydrocodes prior to actual testing. Such modelling of impacts has been successfully performed at PL/WSSD, using the gridless Lagrangian hydrocode Smoothed Particle Hydrodynamics.*

In addition, it is recommended that the simplest definition of complete penetration be employed. Under this definition, a plate is penetrated when it transmits light through the point of impact (Ref. 1, p. 174). Other criteria either do not fit the type of testing to be done in this program, or add complexity to the experiment without noticeable benefits.

If the customer is primarily interested in a more complicated design for the shield, such as a multi-layer design, another alternative is to obtain an empirical expression for the ballistic limit by curve-fitting. The customer and the performing agency should define the design(s) of interest before any experimentation is planned.

It is recommended that at the end of Task 2, the experimenter pause to determine whether the experimentally-obtained ballistic limit expression is accurate enough to warrant continuation of the research. If the ballistic limit expression is inadequate, the remainder of the effort will be wasted.

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*Petschek, A.G. and Libersky, L.D., "Cylindrical Smoothed Particle Hydrodynamics," and Libersky, L., Allahdadi, F., and Carney, T.C., "Simulating Hypervelocity Impact Effects on Structures Using the Smoothed Particle Hydrodynamics Code MAGI," both accepted for publication in the Journal of Computational Physics at this writing.

4.0 CONCLUSION

This report has demonstrated that it is possible to optimize the design of a space debris shield, using currently available technology. While a more advanced and analytical approach may be available some day, it is possible to obtain a workable solution now. Such design optimization would require only a definition of acceptable risk, and a decision to proceed with the necessary research. Such research would contribute to the mitigation ofthe debris threat to U.S. space assets.

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APPENDIX A

SELECTION OF CRITERIA FOR DEFINING A KILL

The kill criterion defined in Equation 1 is admittedly simplistic. Other, more detailed approaches to determining whether a satellite has been killed are available—for example, the KAPP semi-empirical penetration code* and the KASP semi-empirical structural response code**, both developed by Kaman Sciences. Why, then, should Equation 1 be used?

One reason is that this simple criterion is conservative. Assuming that the purpose of this program is to defend United States space assets, use of Equation 1 will result in overprotecting these assets, rather than underprotecting them. A safety factor is thus built in. If the results of the program show that this safety factor is not affordable, this valuable information will have been gained at relatively little cost.

Another advantage is that Equation 1 is easily verifiable. By the time that the first phase of this program is completed, a ballistic limit equation will be available that is supported by experimental data. Semi-empirical models, while providing more detailed analyses of the failure mechanisms of specific satellites, lack supporting experimental data for many configurations.

Additionally, Equation 1 provides a rough universal estimate of the effectiveness of a shielding design, independent of the type of satellite being used. If the probability of a kill is defined entirely in terms of the shield, then it will not be necessary to recalculate P_k for every new space asset configuration that is encountered.

Finally, use of Equation 1 allows the researcher to focus on the particular problem at hand. Semi-empirical codes attempt to model a wide range of test conditions. If the total number of shielding designs to be considered is not

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*Greer, Rodger and Simmerman, Tom, KAPP User's Manual, Report K-90-27U(R), Kaman Sciences Corporation, Colorado Springs, Colorado, July 31, 1990.

**Mr. Jeff Elder, Kaman Sciences Corporation, Huntsville, AL, private communication, October 1990.

large, it is more accurate to characterize the behavior of each design individually.

In summary, Equation 1 presents a simple, effective criterion for defining the failure of a shielding design. There is no need to use more complicated approaches.

APPENDIX B

SENSITIVITY STUDY FOR BALLISTIC LIMIT

Consider Equation 2 for the ballistic limit. Let the aspect ratio be defined as

$$\eta = \frac{L}{D} \tag{B-1}$$

and rewrite Equation 17 as

$$\begin{aligned} M &= c_1 \rho D^2 L \\ &= c_1 \rho \eta D^3 \end{aligned} \tag{B-2}$$

Equation 2 can then be rewritten as

$$\begin{aligned} V_1 &= \sqrt{\alpha \eta^{c} f(z) \frac{D^3}{M}} \\ &= \sqrt{\frac{\alpha \eta^{c} f(z)}{c_1 \rho \eta}} \\ &= \sqrt{\frac{\alpha}{c_1 \rho}} \times \eta^{\frac{c-1}{2}} \times \sqrt{f(z)} \\ &\equiv c_2 \eta^{\frac{c-1}{2}} \times \sqrt{f(z)} \end{aligned} \tag{B-3}$$

Since the variable D is included in z, some way must be found to express it in terms of the aspect ratio. One practical and useful approach is to hold the mass of the projectile constant. Since

$$D^3 = \frac{M}{c_1 \rho \eta} \tag{B-4}$$

it follows that

$$D = \left(\frac{m}{c_1 \rho}\right)^{1/3} \eta^{-1/3} \equiv c_3 \eta^{-1/3} \tag{B-5}$$

Thus,

$$z = \frac{T \sec^k \theta}{c_3 \eta^{-1/3}} \equiv c_4 \eta^{1/3} \tag{B-6}$$

and

$$f(z) = c_4 \eta^{1/3} + e^{-c_4 \eta^{1/3}} - 1 \tag{B-7}$$

So the following ballistic limit expression is obtained:

$$V_l = c_2 \eta^{\frac{c-1}{2}} \left(c_4 \eta^{1/3} + e^{-c_4 \eta^{1/3}} - 1\right) \tag{B-8}$$

It is seen that c, c_2, and c_4 cannot be bracketed tightly enough to permit broad generalizations about V_l based on Equation B-8. One way to get a feel for the effect of the aspect ratio on the ballistic limit is to plot the relationship for a few representative cases. Consider the situations described in Table B-1. Figure B-1 shows plots of the ballistic limit as a

Table B-1. Some example impacts.

Variable	Case 1	Case 2	Case 3	Case 4
M, grams	1.0	0.005	1.0	0.005
T, cm	1.0	0.1	0.1	1.0
θ, degrees	75.0	75.0	75.0	75.0
c_1	1.0	1.0	1.0	1.0
ρ, g/ml	16.6	16.6	16 6	16.6
c	0.3	0.3	0.3	0.3
α	1.6×10^7	1.6×10^7	1.6×10^7	1.6×10^7
k	0.75	0.75	0.75	0.75

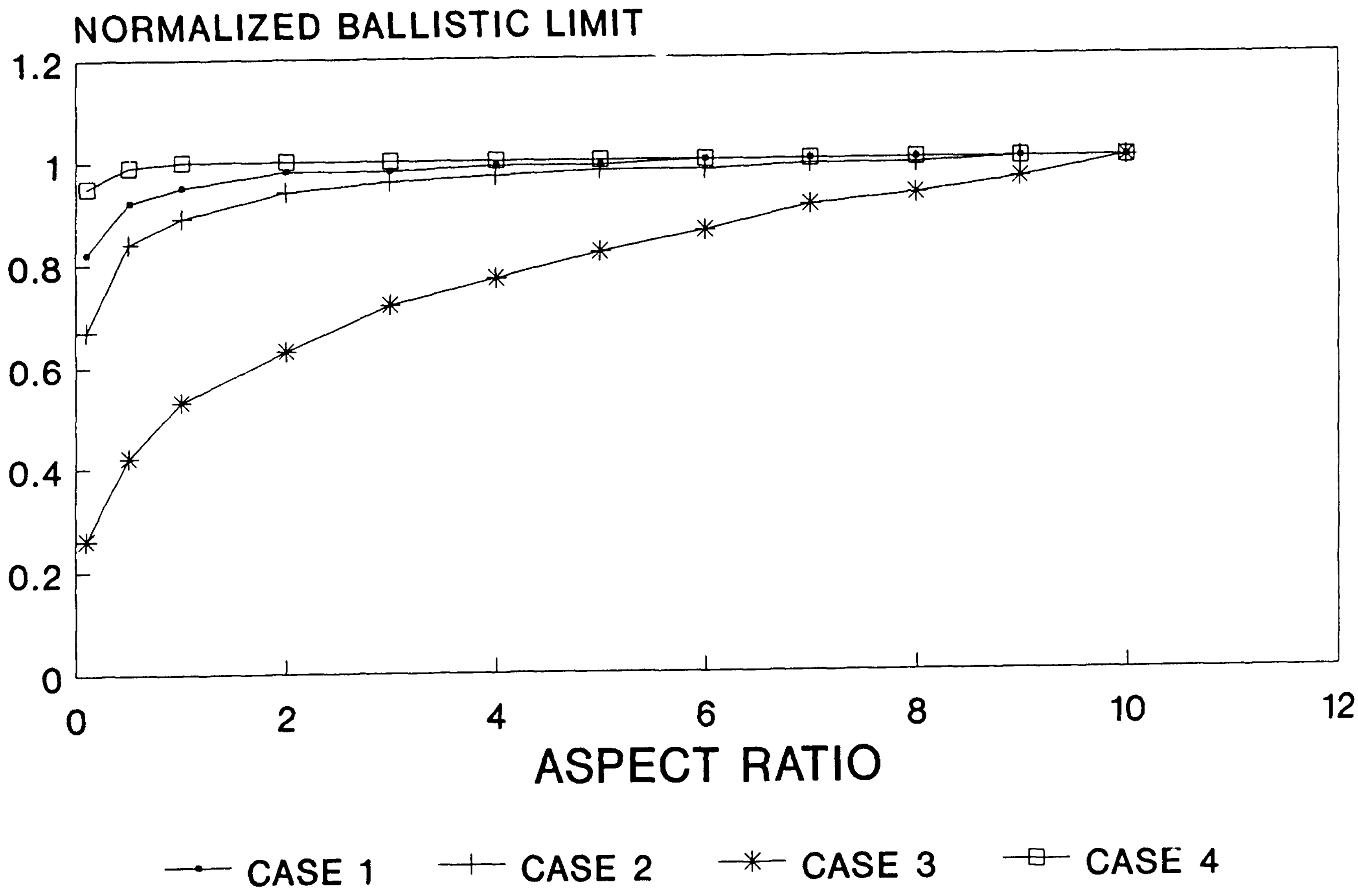

Figure B-1. Ballistic limit versus aspect ratio.

function of aspect ratio for these four cases, based upon Equation B-8. To allow plotting the four curves on the same axes, the ballistic limit data are normalized to the value at an aspect ratio of 10, that is,

$$V_{l,\ normalized} = \frac{V_l}{V_l\big|_{\eta=10}} \tag{B-9}$$

It is seen that, for three of the four cases considered, the curve levels off at fairly low values of the aspect ratio. The exception is case 3, which concerns a relatively large projectile penetrating a thin shield. The ballistic limits for this case were very low, < 60 m/s.

The levelling off of the curves means that the behavior of a projectile with an aspect ratio of unity is similar to the behavior of projectiles with larger aspect ratios. In other words, little generality of the solution is lost by holding the aspect ratio fixed. For case 3, with its very low ballistic limits, this does not seem to hold. However, such low ballistic limits are of little application to this program.

While Equation B-8 cannot be evaluated for an aspect ratio of zero, calculations with very small aspect ratios suggest that

$$\lim_{\eta \to 0} V_l = 0 \tag{B-10}$$

which makes no physical sense. It is important to remember that Equation 2 is a semi-empirical relationship developed for long-rod penetrators.

In summary, this examination has shown that the ballistic limit is relatively insensitive to aspect ratio for projectiles of constant mass and aspect ratios greater than unity. Thus, the aspect ratio can be held constant in the test matrix, without generating results that are irrelevant to other aspect ratios.