# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

(2)

AD-A256 521

...on is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, ...leting and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this ...ducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson ... and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

**2. REPORT DATE**
September 1992

**3. REPORT TYPE AND DATES COVERED**

Multi-periodic Waves in Shallow Water

**5. FUNDING NUMBERS**
DAAL03-89-K-0150

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**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
ARO 26972.2-GS

92-28004

**11. SUPPLEMENTARY NOTES**
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Approved for public release; distribution unlimited.

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 words)*
Nonlinear ocean waves with two-dimensional surface patterns in shallow water are studied analytically and experimentally. The analytical model is a family of periodic solutions of the Kadomtsev-Petviashuili equation. The experiments demonstrate the accuracy of these solutions. When these two-dimensional waves shoal on a planar beach, they quickly generate an array of periodic rip currents. Mach reflections of cnoidal waves are also modelled by these KP solutions.



**14. SUBJECT TERMS**
Ocean waves, two-dimensional, nonlinear, rip currents

**15. NUMBER OF PAGES**
3

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | UL |

# MULTI-PERIODIC WAVES IN SHALLOW WATER

**FINAL REPORT**

by

J. Hammack †

**U.S. ARMY RESEARCH OFFICE**
**DAAL03-89-K-0150**

(10/01/89 - 03/31/92)

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## 1. - Problem statement

This research project on ocean waves is based on the following two observations: the ocean's surface is *two-dimensional* and the dynamics of waves on the ocean's surface is *nonlinear*. Both of these factors become especially important in modelling waves near-shore where the water is shallow and waves ultimately strike the shoreline. Mathematical models of waves in shallow water that include two-dimensional and nonlinear effects and that are analytically tractable are rare. This project examined one of these mathematical models—the Kadomtsev-Petviashvili (KP) equation. The KP equation describes the evolution of weakly nonlinear, weakly two-dimensional waves on water of uniform depth. It has many remarkable mathematical properties, e.g. it is completely integrable and it has an infinite hierarchy of periodic and quasi-periodic solutions. We have examined one family of these periodic solutions—KP solutions of genus 2—that are genuinely two-dimensional (short-crested) and propagate with permanent form. Laboratory experiments were performed to (1) test the applicability of KP solutions of genus 2 as models of water waves, (2) test the stability of these water waves, and (3) examine the shoaling and runup behavior of these water waves on a planar beach.

## 2. - Major results

Our major results are as follows. Experiments in water of uniform depth demonstrate the existence of a family of genuinely two-dimensional, shallow-water waves that are

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fully periodic in two spatial directions and time. These waves are easy to generate and are extremely robust, maintaining their form up to breaking amplitudes. They propagate with practically no change in form for distances up to 25 wavelengths (the basin length), even in the presence depth perturbations comparable to wave amplitudes. Moreover, the KP model of these waves is remarkably accurate. This accuracy persists even for experimental wavetrains that are well outside the putative range of validity of the KP equation.

When a symmetrical subset of the experimental wavetrains in water of uniform depth that are described accurately by symmetrical KP solutions of genus 2 are incident on a wide planar beach, they retain their hexagonal surface patterns up to and subsequent to breaking. In fact, the hexagonal patterns remain detectable in the runup (swash) zones on the beach faces. In addition, they quickly generate periodic *rip currents* along the beach with a spacing of one-half the longshore wavelength of the incident waves. KP theory for the incident waves in the uniform depth region offshore provides a plausible explanation and prediction for the narrow widths of the rip currents. An estimate of the rip-current widths is one-half the cross-shore wavelength of the incident waves. When an asymmetrical subset of the experimental wavetrains in water of uniform depth that are described accurately by asymmetrical KP solutions of genus 2 are incident on a wide planar beach, they also retain their surface patterns up to and subsequent to breaking. In addition, they quickly generate periodic rip currents that *migrate* along the beach.

When one-dimensional cnoidal wavetrains (KP solutions of genus 1) are obliquely reflected by a rigid barrier, the incident and reflected waves interact nonlinearly to form a two-dimensional wave pattern in a wedge-shaped region along the barrier. This pattern includes a wave crest that is perpendicular to the reflecting barrier known as a Mach or stem wave. (This nonlinear reflection is termed Mach reflection in gas dynamics.) Symmetric KP solutions of genus 2 are shown to model the two-dimensional surface waves near the reflecting barrier. The KP model indicates that both the heights and crestlengths of the stem waves at the barrier attain steady-state values.

**3. - Publications**

*Two-dimensional periodic waves in shallow water* (with Norman Scheffner & Harvey Segur). **Journal of Fluid Mechanics 209**, 567-589, 1989.

*The Kadomtsev-Petviashvili equation and water waves* (with Harvey Segur & Norman Scheffner). **Proceedings, Chaos and Order**, Canberra, Australia, 1990.

*A note on the intensity of periodic rip currents* (with Norman Scheffner & Harvey Segur). **Journal of Geophysical Research - Oceans 96**(C3), 4909-4914, March 15, 1991.

*Periodic waves in shallow water* (with Norman Scheffner & Harvey Segur). **International School of Physics ENRICO FERMI, Course CXI: Nonlinear Topics in Ocean Physics**, (ed. by A. Osborne) North-Holland, 891-914, 1991.

Discussion of *Stem waves along breakwater* by S.B. Yoon & P.L.-F. Liu. (with Norman Scheffner & Harvey Segur). **Journal of Waterway Port Coastal & Ocean Engineering, ASCE 117**(5), 542-543, 1991.

*Applications of Genus-2 solutions of the Kadomtsev-Petviashvili Equation* (with Norman Scheffner & Harvey Segur). **Proceedings, Nonlinear Water Waves Workshop**, University of Bristol, England, 1991.

*Resonant interactions among surface water waves* (with Diane Henderson). **Annual Reviews of Fluid Mechanics 25**, 55-97, (to appear) 1993.

**3. - Personnel**

This research project was a collaboration among J. Hammack, formerly at the University of Florida: Norman Scheffner at the Coastal Engineering Research Center (CERC), U.S. Army Engineering Waterways Experiment Station in Vicksburg MS; and Harvey Segur at the University of Colorado.