Fundamental Studies on Hydrology, Hydraulics and Geometry of River Networks

FINAL REPORT

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1. OBJECTIVES

The proposed research had two broad objectives. These objectives, as stated in the proposal, were:

(i) **NETWORK HYDRAULIC-GEOMETRY AND RUNOFF PRODUCTION** To seek analytical generalizations of the current theory of channel networks which include hydraulic-geometric variables, e.g., channel widths, depths, gradients, velocities, streamflows, etc. This will require theoretical studies of channel networks in three-dimensional space, taking into account both elevation and horizontal spatial dimensions.

(ii) **MESOSCALE CLIMATE** To undertake analytical and empirical studies of intermittency and 'statistical self-similarity' in space-time rainfall intensity at natural basin scales.

2. A SUMMARY OF MAIN FINDINGS AND RESULTS

The most important theme which provided the focus for our entire research effort on both of the above objectives is an understanding of the scaling invariance property in spatial variability of channel network geometry, river runoff and rainfall. We will first explain our main findings involving scaling invariance on both of these objectives. Then as an application of this research we will briefly describe some preliminary results on the prediction of major floods from ungauged basins. A solution of this problem is important to the Army, and is the main reason for the support of our basic research by ARO since 1983 .

OBJECTIVE I

Scaling in Channel Network Geometry

Our research on this objective was directed toward a mathematical understanding of the empirical main channel length-area relationship for river networks, called the *Hack's law*, based on the two postulates of the random model. The most significant analytical finding was that for large network magnitude m the average main channel length L varies as the square root of m, or the drainage area A since $A = 2m - 1$, i.e.,

$$L = \mu\, 2\sqrt{\pi}\sqrt{m} \tag{1}$$

where μ, as the scale factor, denotes the mean link length. It has been rigorously proved for constant link lengths [Waymire, 1989] and for exponentially distributed link lengths [Gupta et al., 1990]. The results in Gupta et al. [1990] suggested that Eq. (1) should be true under broad mathematical conditions on link length probability distributions regardless of the parametric form of these distributions. Indeed, for the class of distributions having finite variance this suggestion was verified rigorously in the article by Durrett, Kesten, and Waymire [1991]. In this sense Eq. (1) exhibits universality. This result also solved a long standing problem in theoretical fluvial geomorphology.

The empirical finding of Hack however shows that the exponent of the network magnitude is about .57 rather than .5 as predicted above by the random model. An explanation of this discrepancy was explored by us in different directions. For example, an unpublished calculation by one of our graduate students, Scott Peckham, based on the empirical observation that the average meandering frequency and amplitude of rivers vary systematically as one goes from the drainage divide to the outlet, suggests an exponent larger than .5 in the Hack's law. However, a theoretical demonstration of Hack's law in channel networks requires that this systematic behavior in the river geometry be translated into an appropriate assumption on link lengths. We think that this assumption is tied to a scaling invariance property in link lengths. Since scaling invariance is one the major focus of our research, we are continuing to explore this line of investigation to see if it can theoretically explain the observed departure from .5 in the empirical Hack's law.

In a recent article by us [Gupta and Waymire, 1989], the hypothesis of *simple scaling* behavior in link heights was formulated and shown to give good predictions of the empirical observations. However, the same predictions also suggested a need for generalization of the simple scaling hypothesis in link heights to *multiscaling*. The notion of multiscaling was introduced by Gupta and Waymire [1990]. We expanded a considerable effort in obtaining a closed form solution of the (non-linear) prediction equation employed in testing for simple scaling in link heights [Waymire, 1991]. Even though this equation was solved numerically in Gupta and Waymire [1989], the numerical approach has serious limitations. Consequently, a closed form or semi-analytical solution is required for carrying out tests of multiscaling in link heights. Unfortunately, the article by Waymire [1991] on this problem showed that the main tools (Laplace transforms) employed in solving the special (linear) case of this equation does not go very far in solving the general nonlinear equation. Therefore, this problem has turned out to be much more difficult than what we had anticipated earlier. We are now exploring other avenues to test for multiscaling in link heights.

Scaling in Annual Peak Flows

River basins span a broad range of spatial scales ranging from about 10^{-1} to 10^{6} km^2. So from the view point of predictions from ungauged basins it is necessary to understand and model the spatial variability in rainfall and river flows over this range of scales. In our earlier work [Cadavid, 1988] it was found that the instantaneous annual peak flows from the Southeast United States and the Appalachian Mountains do not obey statistical self-similarity or simple scaling. In fact, it was first shown in a paper by Dawdy [1961] that the regional peak flows violate the assumption of index-flood method, which is tantamount to the demonstration by Cadavid [1988] that they do not obey simple scaling. However, theoretical research on regional flood frequency over the last three decades has not been able to abandon the index-flood method. Therefore it has not had much impact on field applications in so far as the current procedures to predicting regional peak flows continue to be dominated by pure empiricism. Likewise, arguments have been made against self-similarity in spatial rainfall in the recent literature, because it contradicts many of the empirically observed features in data; see e.g., Gupta and Waymire [1992]. In order to accommodate the empirical scaling features in regional peak flows and spatial rainfall, we are exploring the new theoretical framework of multiscaling [Gupta and Waymire, 1990]. This framework is currently being developed further as well as being tested by us on peak flow observations from different regions of the United States.

OBJECTIVE II

Scaling in Spatial Rainfall

A physical understanding of the structure of annual peak flows across a broad range of scales is intimately tied to the nature of spatial variability and extremes in spatial rainfall. Theoretical attempts to model space-time rainfall during the last fifteen years have evolved along two separate lines. The first group of approaches is based on an assumed hierarchy of scales in spatial rainfall as noted by numerous empirical interpretations of remotely sensed observations. The second group of approaches rests on the assumption of self-similarity, or simple scaling in the probability distributions of spatial rainfall. The current theoretical developments involve a common modification of each of these assumptions. It is based on the notion of spatial random mass distribution or *random multifractal measures* generated by random cascades. These measures are highly singular and can capture both the *scaling* as well as the *extreme variability* and *intermittency* observed in spatial rainfall. The current mathematical foundations of the theory of random cascades in the literature have mostly focused on the *ensemble* properties of these measures. In a comprehensive article we investigated the mathematical foundations of multifractal measures with regard to their *spatial sample average* properties [Holly and Waymire, 1992]. In this paper it was shown that the ensemble properties and the spatial sample average properties are not the same for random cascades, because the spatial law of large numbers does not hold due to strong correlations. Tests of these results were carried out by us on the space-time rainfall data set from GATE-I [Gupta and Waymire, 1992]. In particular we carried out the space-time analysis of fractional wetted area of GATE-I rainfall, since this quantity is of basic significance in understanding the production of runoff in space as well as in a variety of other hydrometeorologic and hydroclimatologic studies. These preliminary tests show that the fractional wetted area exhibits the spatial scaling as predicted by the theory. This analysis also shows evidence of the breakdown of the spatial law of large numbers as required by the theory. These findings are quite exiting and we are currently exploring testing this theory on other rainfall data sets.

PRELIMINARY RESULTS ON PREDICTION OF FLOODS FROM UNGAUGED BASINS

Suppose that peak flows obey multiscaling in space. Let the scaling exponent of the n-th statistical moment be denoted by β_n, $n = 1, 2, \cdots$. Moreover, suppose that link slopes in a river network obey statistical self-similarity with a scaling exponent θ [Gupta and Waymire, 1989]. The problem is to predict θ in terms of the scaling exponents β_n describing flows and to test these predictions against the value of θ computed directly from the link slope data in channel networks. The first steps towards solving this problem were taken by Kapoor [1990]. He formulated a notion of *uniform power distribution* over a network in dynamic equilibrium and applied this concept to predicting the exponent θ as

$$\theta = \beta_2 - \beta_1 \tag{2}$$

Using flow data from Brandywine Creek in Pennsylvania he found that the predicted value of θ lies in the same range as the values of θ computed by Gupta and Waymire [1989] for four river basins; the value of θ for the Brandywine Creek was not available to him at the time Kapoor published his findings. This line of investigation also provides the first major step towards linking network geometry with channel forming peak flows.

This line of investigation will play an important role in solving the problem of prediction from ungaged basins. There one can compute θ from the maps and can then apply Eq. (2) to predict the β's. However, this result is only preliminary and much more foundational work remains to be done on solving this important problem.

3. LIST OF PUBLICATIONS

3.1 In Refereed Journals

Durrett, R., H. Kesten, and E. Waymire, 1991: On weighted heights of random trees, *Jour. Theor. Prob.*, 4(1), 223-237.

Gupta, V. K., and E. C. Waymire, 1992: Statistical analysis of mesoscale rainfall as a random cascade, *J. Appl. Meteorology*, (in press).

Gupta, V. K., and E. Waymire, 1990: Multiscaling Properties of Spatial Rainfall and River Flow Distributions, *J. Geophys. Res.*, 95(D3), p. 1999-2009.

Gupta, V. K., and E. Waymire, 1991: Spatial statistics of random networks and a problem in river basin hydrology, *Inst. Math Stat. Lecture Notes - Monograph Series*, v. 20, 103-111.

Gupta, V. K., O. J. Mesa, and E. Waymire, 1990: Tree dependent extreme values: The exponential case, *J. Appl. Prob.*, 27, p. 124-133.

Holley, R., and E. Waymire, 1992: Multifractal dimensions and scaling exponents for strongly bounded random cascades, *Annals Appl. Prob.* (in press).

Kapoor, V., 1990: Spatial uniformity of power and the altitudinal geometry of river networks, *Water Resour. Res.*, 26(10), p. 2303-2310.

Peckham, S., and E. Waymire, 1992: On a symmetry of turbulence, *Comm. Math. Phys.*, (in press).

Wang, S. X., and E. C. Waymire, 1991: A large deviation rate and central limit theorems for Horton ratios, *SIAM Jour. Discrete Math.*, 4(4): 575-588.

Waymire, E., 1991: On network structure function computations, *Proc. of the Institute for Mathematics and its Applications*, Springer-Verlag, New York.

3.2 Theses and Dissertations for Advanced Degrees

Peckham, S., 1990: Stochastic geometry with applications to river networks, *MS Thesis*, Department of Mathematics, Oregon State University.

Subramaniam, C., 1992: On estimation of the multifractal spectra of the mesoscale rainfall, *MS Thesis*, Department of Civil Engineering, University of Colorado-Boulder.

4. LIST OF SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT

V. K. Gupta	Principal Investigator
E. C. Waymire	Co-Principal Investigator
Tom Over	Graduate Research Assistant, University of Colorado
* Scott Peckham	Graduate Research Assistant, Oregon State University and University of Colorado
* Chandran Subramaniam	Graduate Research Assistant, University of Colorado
* Wang Xi	Graduate Research Assistant, Oregon State University

* Earned M.S. degree while employed on the project

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Dawdy, D. R., 1961: Variation of flood rations with size of drainage area, *U. S. Geological Survey Research*, C36.

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Gupta V. K., and E. Waymire, 1989: Statistical self-similarity in river networks parameterized by elevation, *Water Resour. Res.*, 25(3), p. 463-476.

Gupta, V. K., and E. C. Waymire, 1992: Statistical analysis of mesoscale rainfall as a random cascade, *J. Appl. Meteorology*, (in press).

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Gupta, V. K., O. J. Mesa, and E. Waymire, 1990: Tree dependent extreme values: The exponential case, *J. Appl. Prob.*, 27, p. 124-133.

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Kapoor, V., 1990: Spatial uniformity of power and the altitudinal geometry of river networks, *Water Resour. Res.*, 26(10), p. 2303-2310.

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