AD-A256 410

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY

3. REPORT TYPE AND DATES COVERED
FINAL ! Sep 89 - 31 Aug 92

(2)

4. TITLE AND SUBTITLE
"SPATIO-TEMPORAL COMPLEXITY & LARGE SCALE STRUCTURES IN PROBLEMS OF CONTINUUM MECHANICS" (U)

5. FUNDING NUMBERS
61103D
3484/D7 (URI)

6. AUTHOR(S)
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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-89-0507

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
Distribution unlimited

12b. DISTRIBUTION CODE
UL

13. ABSTRACT (*Maximum 200 words*)

We have investigated some difficulties in estimating dynamics from time-delay embeddings of experimental data that can be characterized as low-dimensional. A new procedure is developed to reduce noise by exploiting the properties of saddle periodic orbits on the reconstructed attractor. Most of these methods involve the estimation of a derivative form the data or in some way require a least squares estimate of the location of some portion of the attractor. Our work addresses some of the problems inherent in the estimation of dynamics from data, regardless of the type of model used to approximate the dynamics. These difficulties may arise from the fractal structure of the attractor and errors in all the observations. The problems persist regardless of the amount of available data and affect one's ability to determine an accurate local model of the dynamics, even when an accurate model should be obtainable in principle. Many of these problems can be circumvented by using as much dynamical information as possible in the formulation of the statistical relationship between the observations. Our attempt to do this involves the use of recurrent orbits to derive an accurate linear model of the dynamics in the vicinity of saddle periodic orbits on the attractor. We have applied our method to two experimental data sets from Taylor-Couette flows.

401878

14. SUBJECT TERMS

15. NUMBER OF PAGES
17

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

FINAL

Progress Report, FY 1991
A.F.O.S.R. U.R.I.

on

Spatio-Temporal Complexity and Large-Scale Structures in Problems of Continuum Mechanics

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1. SYNOPSIS

The AFOSR University Research Initiative at A.S.U. on "Spatio-Temporal Complexity and Large-Scale Structures in Problem of Continuum Mechanics" focusses on effective low-dimensional behavior in such systems. In FY91, we have further expanded our research on:

- "Inertial Sets" or "Exponential Attractors": these are fractal enlargements of global attractors for dissipative systems; they capture not only the ultimate asymptotic states, but also all the slow transient dynamics;
- Intermittent Coherent Structures in some moderately turbulent Navier-Stokes Flows, which are ruled by a low-dimensional dynamical "skeleton structure"; symmetry invariances and symmetry breaking shadow the turbulent dynamics far beyond the critical transitions;
- Explicit Symmetry-breaking in the Taylor-Couette problem;
- Problems of estimating low-dimensional dynamics from experimental data.

2. INFINITE DIMENSIONAL DYNAMICAL SYSTEMS: INERTIAL SETS VERSUS INERTIAL MANIFOLDS

In many fluid mechanical flows, the dissipative effects due to diffusion, friction or damping are reflected in the global, infinite-dimensional, phase picture of the partial differential equations modelling these flows. Such equations (with appropriate boundary conditions) are called dissipative if they define a forward regularizing flow in an adequate phase space containing an absorbing set. In general an ***absorbing set*** is a bounded set that attracts all bounded solutions in finite time, at an exponential rate [3,4].

The basic property of dissipative Partial Differential Equations (P.D.E.'s) is that their global asymptotic behavior is controlled by a finite number of parameters. A ***global attractor*** X exists, which is the largest compact set both positively and negatively invariant under the flow, and *uniformly* attracting all bounded sets in the phase space and is unique. One of the properties that makes the attractors an important object to study is the fact that often they have finite dimension. Various notions of dimensions have been considered in conjunction with attractors, among them the Hausdorff and fractal dimensions seem to stand up.

Once the importance of the attractors for the long time behavior of dissipative PDE's is established, the next stage is to unravel and/or compute global bifurcations of the attractor. This turns out to be quite a difficult task. Since the attractor is finite dimensional, it is natural to expect that it can be recovered by solving a large enough system of ODE's, that is, the solutions on the attractor satisfy a system of ODE's. An indirect way of obtaining such a system is to imbed the attractor into a finite dimensional smooth manifold. An ***inertial manifold*** is an exponentially attracting, finite dimensional, Lipschitz manifold that is invariant under the forward flow [4]. Clearly, an inertial manifold when it exists will contain the attractor and when constructed properly will satisfy the desired conditions mentioned above. Within the context of ODE's, an inertial manifold resembles a global central unstable manifold and can even be constructed by using similar tools. (Note, however, that the two notions do not coincide.)

Many methods have been devised to construct inertial manifolds [4]. No matter which method is chosen, a variation of the same type of condition appears; this condition is called a *spectral gap condition.* It requires the linear part of the PDE to have large enough gaps. The spectral gap condition remains the Achilles heel in this theory. Even for the simplest linear partial differential operators in, say, two-space dimension, it is not possible to obtain an explicit control over the gaps of consecutive eigenvalues; the simplest such example is the 2D Laplace operator for which the eigenvalues are known explicitly, yet the gap condition is only satisfied by a suitably chosen subsequence. Returning back to our quintessential equation, the 2D Navier-Stokes equations (N-S), the existence of a true inertial manifold is still an open problem.

Fundamentally, we do not know the optimal rate of convergence for the trajectories of a dissipative PDE to its global attractor X. It need not be exponential. As we cannot yet construct a true inertial manifold for the N-S equations, we can still address the following problem: what is the smallest compact set which is forward invariant under the flow and which attracts at a *uniform exponential rate* all bounded trajectories? Can one define a generalized (i.e., with noncontinuous coefficients) system of ODE's on such a set? The principal investigator and his collaborators have (partially) resolved these questions through construction of *Inertial Sets* [3,4,8]; an Inertial Set (also called *Global Exponential Attractor*):

1) is compact and forward invariant under the flow; hence it contains the attractor X;

2) has a finite fractal dimension

3) it attracts at a uniform exponential rate all trajectories which start in a bounded initial ball.

We have constructed such sets for 2-D N.S. equations (periodic boundary conditions), damped hyperbolic systems (Sine-Gordon, Klein-Gordon, Compressible Van der Waals gases with Korteweg capillarity phase change models) and many other dissipative equations (see below) [3,4,5,6,7]. Like inertial manifolds, inertial sets are not unique.

Inertial sets possess a deeper and more practical property: they remain ***more robust under perturbations and numerical approximations*** than global attractors. We elaborate on this point, since the literature sometimes gives the wrong impression that attractors are robust under perturbations. One can only establish upper-semi-continuity of attractors for approximations of semigroups and partial differential equations. Specifically, if X_ϵ is the approximation to the attractor X, and the functional space is equipped with a norm $|\cdot|$:

$$\max_{x\in X_\epsilon}\left\{\min_{a\in X}|x-a|\right\}\leq\epsilon,$$

that is, there exists a spherical ϵ-neighborhood of X which contains the approximate X_ϵ. The reverse is not true.* Similar problems plague the many constructions of approximate inertial manifolds: these are in the vicinity of the exact attractor only in the sense of upper-semicontinuity. Whereas, we prove, at least for classical Galerkin approximations, denoting by X and $\mathcal{M}$ the exact global attractor and inertial set; by X_ϵ and $\mathcal{M}_\epsilon$ the approximate ones:

$$\max_{u\in X}\left\{\min_{u_\epsilon\in\mathcal{M}_\epsilon}|u-u_\epsilon|\right\}\leq\epsilon$$

(that is, the exact attractor X is within an ϵ-spherical neighborhood of the approximate inertial set $\mathcal{M}_\epsilon$). Moreover:

$$\max_{u_\epsilon\in X_\epsilon}\left\{\min_{u\in\mathcal{M}}|u-u_\epsilon|\right\}\leq\epsilon$$

(that is, the approximate attractor X_ϵ is within an ϵ-spherical neighborhood of the exact inertial set $\mathcal{M}$). Essentially, we also prove that approximate and exact inertial sets are continuous with respect to the Hausdorff distance, modulo a time-shift (ϵ-dependent), at least for classical Galerkin approximations [3,4].

What we effectively measure or compute are trajectories on inertial sets. The latter contain the slow transients as well as the global attractor. In the theory of dynamical

* Note that the above expression is not the Hausdorff distance between two compact sets, but rather one of the two asymmetric pseudo-distances necessary for the exact definition of the Hausdorff distance.

systems, the slow transients correspond to slowly converging stable manifolds. Numerical simulations of infinite-dimensional dynamical systems often capture both the slow transients and parts of the attractor. After a large but finite time, the state of the system obtained from the numerical calculation may often be at a finite distance from the global attractor but at an infinitesimal distance to the inertial set. In this sense, we also call the inertial set an *Exponential Attractor* [8] to be consistent with the physical intuition. Specifically, after a short transient time the infinite-dimensional system is arbitrarily close to an Exponential Attractor whenever the latter exists.

The intrinsic interest of the exponential attractor for N-S turbulence lies in the fact that there is no natural single time scale for the N-S global attractor. In addition, the exponential attractor allows for a study of large intermittent deviations at small scales. This phenomenon of turbulent intermittency has been pre-excluded in the theory of inertial manifolds since, in the latter, small scales are assumed to be globally slaved to large scales, which implies no amplification of disturbances propagating towards small scales. Other time scales also appear in the analysis of the equations that leads to the existence of exponential attractors; these might or might not be related with the above mentioned scale. In this sense, the study of exponential attractors might shed light to those physically observed and thus relevant behaviors, no matter whether they are on the global attractor or not [8].

We are preparing a book on "Inertial Sets for Dissipative Partial Differential Equations." During FY91, two-thirds of the book has been completed and is available to the scientific community as an IMA (U. of Minnesota) preprint Series #812. The contents (135 pages) give a good idea of the range of our applications:

5. Applications:
   1. Kuramoto-Sivashinsky Equation
   2. Kolmogorov-Sivashinsky-Spiegel Equation
   3. 2D Navier-Stokes Equations
   4. 3D Navier-Stokes Equations
   5. Non-local Burgers Equations
   6. Chaffee-Infante Equations in n-dimensions
6. Exponential Attractors for Second Order Evolution Equations with Damping and Applications
7. Inertial Manifolds, Review and Comparison

We have also expanded our construction to cover classes of weakly dissipative problems whose semi-flows are not compact and only asymptotically regularizing [5–7]. Indeed, any dynamical system for which some weak squeezing property can be established is a good candidate for applying our construction. Our treatment depends heavily on the Lipschitzianity of the nonlinearity under different norms. We cover general classes of damped semilinear wave equations: damped Sine-Gordon (in R^N, $N \leq 4$). Klein Gordon (in R^N, $N \leq 3$) systems of Sine-Gordon equations [6], and compressible (viscous) gas dynamics with a nonconvex Van der Waals equation of state and a capillarity Korteweg dispersive term modelling phase changes (in 1-D) [7] and extensible beams equations [5]. The results are significant since there are well-known examples of damped wave equations that have global attractors but generically fail to admit C^1-inertial manifolds. This adds weight to our claim that the *exponential attractors* are as common as attractors.

The compressible gas problem highlights the nature of difficulties for hyperbolic damped equations. An elastic compressible fluid with pressure given by the non-convex Van der Waals equation of state generates complex propagating phase boundaries. Typically, stable liquid and gas domains can coexist with an unstable spinoidal decomposition domain; the latter corresponds to the monotone increasing part of the Van der Waals isotherm curve (pressure versus specific volume). New mathematical issues appear which are not present in the classical hyperbolic equations of inviscid compressible fluid dynamics; indeed, the initial value problem is mixed hyperbolic-elliptic, with the Hadamard instability of solutions lying in the unstable elliptic regime. Regularizing the problem with standard viscosity terms is unsatisfactory, as it rules out certain physically observed propagating phase boundaries [7]. One way to circumvent this difficulty is to introduce a higher order correction term given by Korteweg's theory of capillarity [7]. Interfacial capillarity effects become important precisely for the initial value problem in the elliptic domain. For such a model, the conservation laws have the form:

$$w_t = u_x; \quad u_t + p(w)_x - \eta u_{xx} + \delta w_{xxx} = 0, \tag{1}$$

where $\eta, \delta > 0$ are small coefficients describing, respectively, dissipative and dispersive (capillarity) effects, and w: specific volume, u: lagrangian velocity, θ: temperature, p: pressure, with the specific pressure term

$$p(w) = \frac{R\theta}{w-b} - \frac{B}{w^2}, \tag{2}$$

and R, b, B and $\theta > 0$. For the above model, the fluid is immersed in a heat bath, forcing the temperature to remain constant. For a fluid initially in the elliptic region, Marsden and Slemrod have exhibited homoclinic chaos as a result of small time periodic temperature fluctuations. Initial conditions in the unstable spinoidal region clearly generate complex dynamics. In [7], we have established, for (1) with periodic boundary conditions, the existence of:

- a global compact attractor, following Hale's α-contraction theory,
- an exponential attractor,
- an inertial manifold (with much larger dimension than the exponential attractor).

The major difficulty is not only the non-convexity of the energy functional, but its very unboundedness as $w \to b$, $w \geq b$. The existence of an attractor in the mixed hyperbolic-elliptic regime is delicate. Numerical simulations [7] suggest a wealth of steady states in the spinoidal region.

3. COHERENT STRUCTURES VERSUS SYMMETRY BREAKING

Inertial sets are an appropriate tool for a deeper mathematical understanding of complex spatio-temporal structures in chaotic and turbulent flows. For Reynolds numbers (Re) substantially beyond the first transitions to instability, to what extent does the random occurrence of coherent large-scale events reflect intermittent dynamics on much lower dimensional manifolds? Generally, a most intriguing problem in the theory of hydrodynamic turbulence is the formation of large-scale structures in a flow performing random turbulent motion at small scales.

The dynamics of Coherent Structures (C.S.) in developed 2-D turbulence are better understood (than 3-D) from a phenomenological point of view. They correspond to a condensation of the vorticity field, and confine the major part of enstrophy. Such coherent vortex structures reflect a rate of rotation which dominates their rate of deformation. They survive on time scales *much larger* than the mean eddy turnover time, and yet are observed along the *whole scale of the "inertial" range* (of the Energy Spectrum).

Spatio-temporal intermittency seems to be linked with the spatially intermittent localization of *quasi-singular* (cusped) C.S.; and the temporal intermittency of events when such structures undergo strong interactions. Temporal intermittency indeed seems to be correlated with strongly nonlinear interactions between C.S. on a short time scale. Such strong interactions sharply localized both in time and space are regions of much more intense dynamics, vested with *local Reynolds numbers* much larger than those encountered in the remainder of the flow. Fluid mechanists conjecture that intermittency stems from such a mixing between regions which are spatio-temporally active (at arbitrarily small scale) and much more stable and dynamically passive regions. Indeed, one may conjecture that these are the most fundamental characteristics of turbulent flows.

Recently, we have extensively investigated 2D turbulent bursting flows which strikingly fit the phenomenological C.S. picture outlined above. These are generalized Kolmogorov flows with spatially periodic forcing [1,11-13]. Moreover, there is recent renewed interest in the actual experimental realization of these 2D flows, via electric or magnetic fields. The classical two-dimensional Kolmogorov flow is the solution of the 2D Navier-Stokes equation with a unidirectional force $\mathbf{f} = (\nu k_f^3 \sin k_f y, 0)$. It was introduced by Kolmogorov in the late fifties as an example on which to study transition to turbulence. For large enough viscosity, ν, the only stable flow is a plane parallel periodic shear flow $\mathbf{u}_0 = (\nu k_f \sin k_f y, 0)$, usually called the "basic Kolmogorov flow." The macroscopic Reynolds number of the basic flow is easily found to be $1/\nu$; this will be used later as a free parameter to define the bifurcation sequence. It was shown by Meshalkin and Sinai that large-scale instabilities are present for Reynolds numbers exceeding a critical value, $\sqrt{2}$. In a 2π-periodic box, the equations are:

$$\begin{gathered}\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \nu \nabla^2 \mathbf{u} + \mathbf{f}, \qquad \nabla \cdot \mathbf{u} = 0, \\ \mathbf{f} = (\nu k_f^3 \sin k_f y, 0), \qquad 0 \leq x, \quad y \leq 2\pi.\end{gathered} \tag{3}$$

Sequences of bifurcations have been investigated by She. But the most interesting transitions occur at even higher Reynolds number; they lead to sparsely distributed bursts in time for a fairly large range of Reynolds number above a certain threshold, i.e., Re $\approx$ 20.8 for k_f = 8 [11-13]. The most striking feature of this transition is that the bursts generate substantial spatial disorder and drive developed turbulence. We have also investigated such bursting regimes in *generalized Kolmogorov flows*, where the *force is an eigenfunction of the linearized Stoke's operator*. For instance, a forcing stream function of the type $\cos(k_f x)\cos(k_f y)$ generates a basic flow of square eddies. Generally, quiescent states associated to large scale coherent vortices dominate the dynamics between the bursts. The burst corresponds first to weak, then to strong interactions between these C.S., with turbulent dynamics at *much smaller spatial scales*. Cusped, near singular vortices are sharply localized both in space and in time, during the bursts [11-13]. As the Reynolds number increases beyond 100, the ***same dynamical regime persists***, with a striking role reversal: "bursting" regimes now become prevalent (with homogeneous shear turbulence at smaller scales) whereas the intermittencies now reflect brief reorganization of the flow around the larger-scale, more symmetric coherent vortices: we have "bursts of reordering"; remarkably, dynamics around the C.S. are now turbulent, not laminar (as for the lower Re). We have established that symmetry-breaking mechanisms are the prime engine behind such ***persistent*** (in Re) dynamics [11-13].

We establish both computationally and analytically that the Kolmogorov bursting regimes are linked with symmetry-breaking heteroclinic connections which generate persistent (in Re) homoclinic cycles [11-13]. The coherent vortices are invariant under isotropy subgroups of symmetries of the Kolmogorov flow. The heteroclinic connections correspond to invariant submanifolds with further reduced symmetries; they exchange the slow-stable and unstable manifolds of the coherent vortices. Similar phenomena have first been unravelled for the 1D Kuramoto-Sivashinsky equations. The lattices of isotropy subgroups play a key role in the analytic theory of these heteroclinic-homoclinic cycles and substantial literature has appeared on this subject, for symmetries simpler than the Kolmogorov flow's (e.g., $O(2)$, $O(3)$, D_4, etc.).

For the Kolmogorov flows, small-scale dynamics prevail in a neighborhood of the heteroclinic connections, whereas large-scale dynamics are linked with slow-stable manifolds of the hyperbolic Tori (C.S.) [1, 11-13]. This is another paradigm of transient dynamics on an inertial set. The ideal global attractor is made of the hyperbolic tori and their multiple heteroclinic connections. Theoretical dynamics should get closer and closer to the ideal geometric connections, and pseudo-periods between bursts should increase monotonically to infinity. This is not observed in practice, where bursts occur randomly, and (numerical) noise throws the trajectories onto larger slow manifolds of the hyperbolic tori. So, for a very long time, (computationally, forever) dynamics fluctuate within a tubular neighborhood of the ideal geometric heteroclinic connections. These results might suggest slow transients on an inertial set. Even more, for larger Reynolds numbers, the heteroclinic connections are probably not strictly attracting (for $t \to +\infty$).

A suitable tool to extract phase-space information out of large scale PDE simulations seems to be the proper orthogonal decomposition (POD) also known as Karhunen-Loeve decomposition [1]. Simulations for Kolmogorov flow by [11] clearly give the impression of a finite, in fact very low dimensional dynamics for this regime. Consequently, our long term goal is to come up with a reasonable low dimensional dynamical system derived via a proper orthogonal decomposition and amenable to traditional phase space analysis. While we have not yet *synthesized* such a system, we have *analyzed* the simulations. We think our preliminary results give very interesting partial answers and raise some new and crucial questions about the nature of turbulence [1].

Using POD we extract the dominant features of the data as spatial eigenfunctions of the covariance matrix and project the data onto the first few eigenfunctions. Doing this for a time sequence, we can reconstruct the time evolution of the flow using a suitable number of these eigenmodes. Specifically, we are reconstructing two regions of the simulations: The region believed to be predominantly laminar and the region containing a burst, respectively. While the PDE numerical code integrates a stream function, turbulence theory prefers to deal with vorticity. Performing a POD on both, we get the surprising result that the dimensions derived from an energy criterion on the POD and visual comparison of reconstructed and original data, as well as those derived from an embedding algorithm [1], do not coincide for stream-function and vorticity data. We offer interpretations and speculations on the reasons for that discrepancy in terms of low-dimensional phase-space dynamics versus enstrophy cascade.

4. EXPLICIT SYMMETRY BREAKING IN THE TAYLOR-COUETTE PROBLEM

There are two types of symmetry breaking events that often occur in fluid dynamics. They are: (i) solution symmetry breaking and (ii) system symmetry breaking. There exists a vast literature on solution symmetry breaking bifurcations. Much less is known about system symmetry breaking. In the latter case, the whole system reduces its symmetry. A number of examples of system symmetry breaking in Hamiltonian and dissipative situations are discussed in Marsden (1991), Guckenheimer and Mahalov (1991).

We consider an infinitely long Taylor-Couette problem which is translationally and reflectionally symmetric along the cylinders. We investigate the bifurcation to Taylor vortices [2] when the reflection symmetry in the axial direction is broken in two ways: (i) by applying a constant pressure gradient in the axial direction, (ii) by sliding cylinders relative to each other. We calculate the effect of these symmetry breaking perturbations and find in both cases a slow drifting of the Taylor vortices along the axial direction. We discuss a total symmetry breaking of the translational and reflectional symmetry along the axial direction [2]. This forces the system either to choose a state from a circle of states of the unperturbed system or into an inhomogeneous drifting state. The efforts of these symmetry breaking perturbations on the system are studied using normal forms [2].

5. LOW DIMENSIONAL DYNAMICS

5.1. Problems in Estimating Dynamics from Data

We have investigated some difficulties in estimating dynamics from time-delay embeddings of experimental data that can be characterized as low-dimensional. A new procedure is developed to reduce noise by exploiting the properties of saddle periodic orbits on the reconstructed attractor [9].

There are several different approaches to noise reduction in time series data whose underlying dynamics can be described as low dimensional. Kostelich and Yorke outlined a procedure that uses an approach originally suggested by Eckmann and Ruelle for computing Lyapunov exponents. Farmer and Sidorowich describe another method for use when the dynamics are known. Schreiber and Grasshberger describe a simple time-series based approach. Cawley and Hsu have suggested an approach based on projecting trajectories onto planes that locally approximate the manifold containing the attractor.

Most of these methods involve the estimation of a derivative from the data or in some way require a least squares estimate of the location of some portion of the attractor. Our work [9] addresses some of the problems inherent in the estimation of dynamics from data, regardless of the type of model used to approximate the dynamics. These difficulties may arise from the fractal structure of the attractor and errors in all the observations. The problems persist regardless of the amount of available data and affect one's ability to determine an accurate local model of the dynamics, even when an accurate model should be obtainable in principle.

Many of these problems can be circumvented by using as much dynamical information as possible in the formulation of the statistical relationship between the observations. Our attempt to do this involves the use of recurrent orbits to derive an accurate linear model of the dynamics in the vicinity of saddle periodic orbits on the attractor. We have applied our method to two experimental data sets from Taylor-Couette flows [9].

5.2. Calculating Low Dimensional Stable and Unstable Manifolds

A numerical procedure is constructed [10] for computing the successive images of a curve under a diffeomorphism of R^N. Given a tolerance, ϵ, we show how to rigorously guarantee that each point on the computed curve lies no further than a distance ϵ from the "true" image curve. In particular, if ϵ is the distance between adjacent points (pixels) on a computer screen, then a plot of the computed curve coincides with the true curve within the resolution of the display. A second procedure is developed [10] to minimize the amount of computation of parts of the curve that lie outside a region of interest. We apply the method to compute the one-dimensional stable and unstable manifolds of the Hénon and Ikeda maps, as well as a Poincaré map for the forced damped pendulum.

Thus, we have developed a procedure to calculate the stable and unstable manifolds of fixed points of diffeomorphisms to a prespecified precision, provided that the manifolds are one-dimensional. If a Lipschitz constant can be determined for the map and its derivatives, and if a lockout region can be identified, then the procedure produces a rigorous plot of a portion of the stable and unstable manifolds within a region of interest around the fixed point. In other words, the manifolds can be computed to an accuracy that is smaller than the resolution of the plotting device. Moreover, the procedure guarantees that no point is omitted that should be plotted, and no extra point is plotted that does not belong. A simpler method can also be used if suitable Lipschitz constants cannot be determined easily. Although the resulting plots are not rigorous, numerical evidence suggests that they are still very accurate, and they can be generated in a few minutes on a workstation computer.

6. PUBLICATIONS IN FY 91

[1] D. Armbruster, R. Heiland, E. Kostelich and B. Nicolaenko, *Phase-Space Analysis of Bursting Behavior in Kolmogorov Flow*, Physica D, to appear.

[2] D. Armbruster and A. Mahalov, "On the Explicit Symmetry Breaking in the Taylor-Couette Problem," submitted.

[3] A. Eden, C. Foias, B. Nicolaenko and R. Temam, C.R. Acad. Sci. Paris, t. 310, ser. I (1990), 559–562.

[4] A. Eden, C. Foias, B. Nicolaenko and R. Temam, *Inertial sets for dissipative evolution equations: Part I: Construction and application*, IMA preprint 812 (1991).

[5] A. Eden and A.J. Milani, "Exponential attractors for extensible beam equations," preprint (1991).

[6] A. Eden, A.J. Milani and B. Nicolaenko, "Finite dimensional exponential attractors for semilinear wave equations with damping," IMA preprint 693 (1990), to appear in *JMAA*.

[7] A. Eden, A.J. Milani and B. Nicolaenko, "Exponential attractors for models of phase change for compressible gas dynamics," preprint (1991).

[8] A. Eden, C. Foias, B. Nicolaenko and Z.S. She, "Exponential Attractors and their Relevance to Fluid Mechanics Systems," preprint (1991).

[9] E. Kostelich, "Problems in Estimating Dynamics from Data," to appear, Physica D.

[10] E.J. Kostelich, J.A. Yorke and Z. You, "Calculating Stable and Unstable Manifolds," Int. J. of Bifurcation and Chaos, **1**, 3 (1991), 605–623.

[11] B. Nicolaenko and Z.S. She, "Temporal Intermittency and Turbulence Production in the Kolmogorov Flow," *Topological Fluid Dynamics*, II, K. Moffatt Ed., Cambridge U. Pres (1990), 265–277.

[12] B. Nicolaenko and Z.S. She, *Symmetry Breaking homoclinic chaos and vorticity bursts in periodic Navier-Stokes flows*, Eur. J. Mech. **10**, no. 2 (1991), 67–74.

[13] B. Nicolaenko and Z.S. She, *Turbulent Bursts, Inertial Sets and Symmetry Breaking Homoclinic Cycles in Periodic Navier-Stokes Flows*, Proc. IMA Conf. on Turbulence, IMA Series, Springer-Verlag Publ., to appear.

7. Conference and Seminar Presentations

A) *Dieter Armbruster*

1) **The Karhunen Loéve decomposition as a tool to analyze PDE data.**

Applied Dynamics and Bifurcations, Oberwolfad 1/92

Arizona Days, Los Alamos 2/92

Metz Days, Metz, France 7/92

Seminar, Institut for Information Science, Universität Tubingen 7/92

Workshop on 'Reactive Turbulence', Los Alamos 8/92

2) **Topological constraints for explicit symmetry breaking**

Int. Conf. on "Bifurcations in differentiable dynamics," Limburg, Belgium 6/92

3) *Same lecture* presented at:
AMS Summer Seminar "Exploiting symmetry in Applied and Numerical Analysis," Ft. Collins, Colorado 7/92

Workshop on "Reactive Turublence," Los Alamos 8/92

4) **Structurally stable heteroclinic orbits**

Appl. Math Seminar, Colorado State University, Ft. Collins, CO 3/92

B) *Eric Kostelich*

1) Naval Research Laboratory, Seminar, Washington, D.C. 7/16/92
2) International Union of Theoretical and Applied Mechanics Symposium on Interpretation of Time Series from Nonlinear Mechanical Systems (also a NATO conference), invited lecture, University of Warwick, Coventry, England 8/28/91.
3) First experimental chaos conference, invited lecture, sponsored by the Office of Naval Research, Arlington, Virginia 10/1-3/91
4) University of Maryland, Instructional Television System, one-hour lecture in nationally televised, closed-circuit short course on chaotic dynamics 3/5/92
5) American Geophysical Union, Annual Meeting, invited workshop lecture, Montreal, Canada 5/12/92
6) American Geophysical Union, Annual Meeting, contributed presentation, Montreal, Canada 5/13/92
7) Santa Fe Institute, workshop on prediction in nonlinear time series, contributed lecture, Santa Fe, NM 5/15/92
8) SPIE, annual meeting, contributed lecture, San Diego, CA 7/23/92
9) Second workshop on Dynamical Measures of Complexity and Chaos, invited lecture, Bryn Mawr, PA 8/14/92

C) *Basil Nicolaenko*

1) "Phase Space Analysis of Bursting Behavior in Kolmogorov Flow," IUTAM Symposium on Interpretation of Time Series from Nonlinear Mechanics Systems, U. of Warwick (U.K.) 8/91

2) "Turbulent Bursts in Kolmogorov Flows," Colloquium, Dept. of Mechanical Engineering, U.C. Berkeley 2/92

3) "Exponential Attractors and Inertial Sets," Colloquium, U. of Paris, Orsay 10/91

4) *Same presentation* at:

   Colloquium, U. of Bordeaux (France) 12/92

5) "Non Trivial Dynamics for the Kolmogorov Navier-Stokes Equations," International Conf. on Nonlinear Analysis, Tampa, Florida 8/92

8. Graduate Students Advised

R. Hedges
R. Heiland
A. Lopez
K. Madsen
A. Palacios
Weijie Qian
N. Smaoui
P. Vaz
Sharon Walker
Qi Zhao