## REPORT DOCUMENTATION PAGE

**AD-A255 984**

Form Approved OMB No. 0704-0188



| 1. AGENCY USE ONLY *(Leave blank)* | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | | FINAL 1 Apr 90 - 31 Jul 92 |

**4. TITLE AND SUBTITLE**
"MACROSCOPIC PROPERTIES OF RANDOM & QUASIPERIODIC MEDIA"(U)

**5. FUNDING NUMBERS**
61102F
2304/A4

**6. AUTHOR(S)**
Dr. Kenneth Golden

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
Princeton University
Department of Mathematics
Princeton NJ 08544

**8. PERFORMING ORGANIZATION REPORT NUMBER**
AFOSR-TR- 92 09

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
AFOSR-90-0203



**11. SUPPLEMENTARY NOTES**
N/A

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**
Approved for public release;
Distribution unlimited

**12b. DISTRIBUTION CODE**
UL

**13. ABSTRACT** *(Maximum 200 words)*

In a series of papers, we have proven new, fundamental rigorous results about the critical behavior of percolation models. In the discrete case, for a hierarchical model of the conducting backbone, we have proven inequalities on the critical exponent for the conductivity of the random resistor network. Our inequality less than or equal to 2 in three dimensions rules out roughly one fourth of the numerical estimates published over the last 25 years.



**14. SUBJECT TERMS**

**15. NUMBER OF PAGES**
2

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | SAR |

## Final Report

**Grant:** AFOSR - 90 - 0203

**Title:** Macroscopic Properties of Random and Quasiperiodic Media

**Period:** 4/1/90 to 7/31/92

**PI:** Kenneth Golden, Princeton University
(Currently at U. of Utah)

RESULTS

Percolation models play a fundamental role in the analysis of the transport properties of a wide variety of both natural and man-made materials in science and engineering. Models such as the discrete random resistor network and the continuous random checkerboard capture the essential features of many systems where percolation effects dominate, yet lend themselves to rigorous mathematical analysis. Media represented by these models include semiconductors, cermets, thermistors, thick-film resistors, superconducting composities, and porous media.

In a series of papers, we have proven new, fundamental rigorous results about the critical behavior of these models. In the discrete case, for a hierarchical model of the conducting backbone, we have proven inequalities on the critical exponent $t$ for the conductivity of the random resistor network [1,2]. Our inequality $t \leq 2$ in three dimensions rules out roughly one fourth of the numerical estimates published over the last 25 years. For the classical continuous random checkerboard model in two dimensions with conductors 1 and $\delta$ in proportions $p$ and $1-p$, it has been known since the 1960's via Keller duality that the effective conductivity $\sigma^*$ obeys $\sigma^* = \sqrt{\delta}$ at $p = 1/2$. We have discovered the surprising result [3] that this $\sqrt{\delta}$ is exact to leading order as $\delta \to 0$ for $p$ throughout the interval $(1-p_c, p_c)$, where $p_c$ is the site percolation probability, i.e., $\sigma^* = \sqrt{\delta} + 0(\delta)$ as $\delta \to 0$ for $p \in (1-p_c, p_c)$. Finally, we have developed a generalization of this checkerboard model which represents the behavior of porous media with complex, self similar microstructure, or of certain types of superconducting composites, and obtained rigorous results on its transport properties near criticality [4].

Currently we are in the process of writing an invited article to review the above work on percolating systems [5].

In other work, we wrote a review article on transport in quasiperiodic media, in which we applied some of our previous results on classical transport to quantum transport problems in such media [6]. This past summer, while visiting the University of Toulon, we began discussing further applications of these ideas in the quantum domain. In the area of bounds on complex transport coefficients for multicomponent materials we found new symmetric representation formulas for the coefficients [7], and this past summer began trying to extend this work with my Ph.D. student.

## REFERENCES


1. Convexity and Exponent Inequalities for Conduction near Percolation, Phys. Rev. Lett., **65**(24), 1990, 2923–2926.

2. Exponent Inequalities for the Bulk Conductivity of a Hierarchical Model, Comm. Math. Phys., **143**, 1992, 467-499.

3. Exact Result for the Effective Conductivity of a Continuum Percolation Model (with L. Berlyand), sumitted.

4. Percolation Analysis for the Effective Permeability of Porous Media (with S. M. Kozlov), submitted.

5. Critical Behavior of Transport in Percolating Systems (invited review article for Intl. J. of Mod. Phys. B), in preparation.

6. Classical Transport in Quasiperiodic Media, in Proceedings of AMS-SIAM Summer Seminar on the Mathematics of Random Media, Blacksburg, Va., June 1989, W. Kohler and B. White, eds., 1991, Amer. Math. Soc., 359–373.

7. Representations for the Conductivity Functions of Multicomponent Composites (with G.W. Milton), Comm. Pure and Appl. Math., **43**, 1990, 657–671.