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A RAND NOTE

Time Series Models for Predicting Monthly
Losses of Air Force Enlisted Personnel

Marygail K. Brauner, Kevin L. Lawson,
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N-3167-AF

**Time Series Models for Predicting Monthly
Losses of Air Force Enlisted Personnel**

**Marygail K. Brauner, Kevin L. Lawson,
William T. Mickelson, Joseph Adams,
Jan M. Chaiken**

**Prepared for the
United States Air Force**

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PREFACE

RAND is helping to design an Enlisted Force Management System (EFMS) for the Air Force.¹ The EFMS is a decision support system designed to assist managers of the enlisted force in setting and meeting force targets. It contains computer models that project the force resulting from given management actions, so actions that meet targets can be found. Some of those models analyze separate job specialties (disaggregate models) and others analyze the total enlisted force across all specialties (aggregate models); some models make annual projections (middle-term models) and others make monthly projections.

The Short-Term Aggregate Inventory Projection Model (SAM) is the component of the EFMS that makes monthly projections (for the rest of the current fiscal year) of the aggregate enlisted force.

The overall SAM model contains five modules:

- Module P: Preprocessor.
- Module 1: Separation Projection.
- Module 2: Inventory and Cost Projection.
- Module 3: Computer Aided Design.
- Module 4: Plan Comparison. .

SAM is documented in C. Peter Rydell and Kevin L. Lawson, *Short-Term Aggregate Model for Projecting Air Force Enlisted Personnel (SAM)*, RAND, N-3166-AF, 1991, which gives detailed specifications for modules P and 2 through 4. Module 1 projects monthly loss and reenlistment behavior. The detailed specifications for alternative versions of Module 1 are presented in separate publications. These describe three promising methods of predicting the separations required from Module 1:

- Time series forecasting.

¹For an overview of the EFMS see Grace Carter, Jan Chaiken, Michael Murray, and Warren Walker, *Conceptual Design of an Enlisted Force Management System for the Air Force*, RAND, N-2005-AF, August 1983.

- Robust separation projection.
- Benchmark separation projection.

All three methods predict the monthly losses and reenlistment flows that are needed as inputs to Module 2. They predict "policy-free" flows—the losses and reenlistments that would occur in the absence of early release and early reenlistment programs. (Module 2 accounts for the effect of past and present management actions on losses and reenlistments.) However, in spite of having the same objectives, the three methods differ fundamentally in the way they accomplish those objectives.

The time series forecasting method uses models such as constant rate, regression, autoregressive, and straight line running average. These models are documented in this Note.

The robust separation projection method uses data on past losses and reenlistments to estimate separation rates for a model that predicts loss and reenlistment flows one month at a time for each of a mutually exclusive set of about 500 cohorts. After these flows are predicted for a projection month, the inventory is updated and the models are applied to the updated inventories to predict the flows for the following month. This process is repeated until the inventory for the last month of the fiscal year is projected. Thus, it applies separation rates to a series of different inventories. The robust method is specified in Marygai K. Brauner and Daniel A. Relles, *The Robust Separation Projection Method for Predicting Monthly Losses of Air Force Enlisted Personnel*, RAND, N-3169-AF, 1991.

The benchmark separation projection (BSP) method uses data on past losses and reenlistments to estimate a set of separation rates for each month of the fiscal year for a mutually exclusive set of about 280 "decision groups." Those separation rates are then applied to the current inventory to predict monthly loss and reenlistment flows for the rest of the fiscal year. Thus, the BSP method applies different sets of separation rates to a single inventory (that single inventory is the inventory at the start of the projection period). The BSP method is documented in C. Peter Rydell and Kevin L. Lawson, *The Benchmark Separation Projection Method for Predicting Monthly Losses of Air Force Enlisted Personnel*, RAND, N-3168-AF, 1991.

The names "robust" and "benchmark" are historical artifacts. "Robust" refers to a particular method of averaging past separation rates that is not unduly influenced by

outliers in the historical data. "Benchmark" refers to the method's original purpose: to serve as a standard of comparison for the accuracy, reliability, and run time of alternative methods for Module 1. The benchmark model became an attractive alternative in its own right.

This Note documents RAND's research that led to the mathematical specification for the time series method. It should be of interest to the Air Force members of the EFMP who are building the EFMS. It should also be of interest to modelers and analysts who are involved in manpower and personnel research for the uniformed services. This specification was presented to the Air Force as one possible solution to the problem of predicting the short-term behavior of airmen. The Air Force is using this and other specifications as the point of departure for developing a method for predicting the monthly losses of enlisted personnel in Module 1 of SAM. As a consequence, the version of Module 1 that will be used in the EFMS is likely to differ considerably from that presented in this Note.

The work described here is part of the Enlisted Force Management Project (EFMP), a joint effort of the Air Force (through the Deputy Chief of Staff for Personnel) and RAND. RAND's work falls within the Resource Management Program of Project AIR FORCE. The EFMP is part of a larger body of work in that program concerned with the effective utilization of human resources in the Air Force.

Kevin Lawson and Joseph Adams are Majors in the Air Force. The other authors are RAND staff members. Jan Chaiken is a consultant to RAND who was on the RAND staff at the time the research was performed.

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SUMMARY

The Short-Term Aggregate Inventory Projection Model (SAM) is one component of the Enlisted Force Management System (EFMS). SAM makes monthly projections (for the rest of the current fiscal year) of the aggregate force (the total enlisted force across all specialties). It can be used to analyze the total size, grade composition, and budget cost of the enlisted force during a fiscal year. It supports planning of management actions to achieve user specified end-of-year force levels (known as "end strengths") and user specified end-of-year grade levels (known as "grade strengths").

SAM contains five modules:

- Module P: Preprocessor.
- Module 1: Separation Projection.
- Module 2: Inventory and Cost Projection.
- Module 3: Computer Aided Design.
- Module 4: Plan Comparison.

Module 1 predicts "policy-free" monthly losses and reenlistments of Air Force enlisted personnel for the rest of the current fiscal year. "Policy-free" means that the predictions assume zero early releases and zero early reenlistments caused by actions of enlisted force managers.

Time series models are one way of predicting the separations required from Module 1. In general, five distinct types of separations must be predicted:

- Losses on or close to the expiration of an airman's term of service (ETS losses).
- Losses during the term (attrition losses).
- Losses when an airman is on extension status.
- Retirements.
- Reenlistments.

Different models were used to predict each of these types of losses for each term of an airman's career. To find the model that fitted the historical data best (and was likely to produce the best predictions), four times series modeling approaches were tried:

- Constant rate.
- Regression.
- Autoregressive.
- Straight line running average.

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ACRONYMS AND ABBREVIATIONS

ACF	Autocorrelation function
AFSC	Air Force Specialty Code
AR	Autoregressive
ARIMA	Autoregressive integrated moving average (type of time-series model)
DOS	Date of separation
EFMP	Enlisted Force Management Project
EFMS	Enlisted Force Management System
ETS	Expiration of term of service
FY	Fiscal year (October through September)
HS	High school graduate
HYT	High year of tenure
IPM	Inventory Projection Model
METS	Months to expiration of term of service
MOS	Months of service
NHS	Non-high school graduate
OETS	Original expiration of term of service
OETSYOS	Number of years of service that the airman will have completed by the end of the month of original ETS
PACF	Partial autocorrelation function
PDGL	Promotion/Demotion Gain/Loss file
SAM	Short-term Aggregate Inventory Projection Model
SPD	Separation Program Designator
TOE	Term of enlistment (number of years of enlisted obligation)
YAR	Year-At-Risk file (data file used for analysis)
YOS	Years of service

I. INTRODUCTION

The conceptual design of the Enlisted Force Management System (EFMS) envisioned a number of inventory projection models (IPMs) to predict the future inventory of airmen in various categories (see Carter et al., 1983). The middle-term inventory projection models of the EFMS project the inventory yearly for up to six years into the future. The Short-term Aggregate Inventory Projection Model (SAM) provides monthly projections for the "aggregate" force (total active duty enlisted personnel by grade and year of service across all specialities) for the remaining months in a fiscal year (see Rydell and Lawson, 1991). The projections depend on force behavior as well as on such planned management actions as accession, promotion, and early-release policies. SAM provides monthly inventory and loss projections as well as cost and end-of-fiscal-year data as a function of the size and grade composition of the force.

SAM is designed to capture the most current trends in losses and reenlistments. Recent years have seen several drastic early-out programs and major changes in the reenlistment policy. Since these policies are designed to alter the force structure to meet end-strength targets and to reduce the budget to meet budget constraints, it is imperative to be able to monitor the consequences of such actions.

The short-term component of the EFMS shows the immediate consequences of policy changes, while the middle-term component predicts the ripple effect of actions. It is important to be aware of both kinds of effects. For example, it is tempting to reduce non-prior service accessions to meet an end-strength target or a budget constraint in a given fiscal year. However, such an action may cause staffing difficulties in future years, as the smaller cohort marches through time. The EFMS, with both short-term and middle-term IPMs, will be able to project the effect of such actions on the future structure of the Air Force enlisted force.

In addition to distinguishing loss models by time frame, the conceptual design of the EFMS distinguishes between aggregate and disaggregate loss models. A disaggregate model predicts losses and reenlistments by Air Force Specialty Code (AFSC). An aggregate model predicts losses and reenlistments over all AFSCs. The models documented here are aggregate loss and reenlistment models that had been intended to be used in the EFMP's Short-term Aggregate IPM (SAM). Other models that

have been designed for possible use in SAM have been documented by Brauner and Relles (1991) and by Rydell and Lawson (1991 a,b). The model to be used in the operational EFMS will be chosen by the Air Force after extensive testing and evaluation of the possibilities.

CONTEXT

In general, five types of actions must be predicted:

- Losses on or close to the expiration of an airman's term of service (ETS losses).
- Losses during the term (attrition losses).
- Losses when an airman is on extension status.
- Losses due to retirement.
- Reenlistments.

There are high attrition loss rates at the beginning of the first term of service, as new enlistees participate in basic training and attempt to adjust to military life. The remainder of the first term has lower attrition rates, but higher than in the second and career terms. In the first term, the entire year before ETS is critical for modeling purposes because in addition to attrition losses, there are numerous early releases, extensions, ETS losses, and reenlistments. It is in this last year of the first term that many special release programs are instituted.

The second and career terms also experience a highly active ETS period, but this period is shorter. It begins three months before ETS in the second term and one month before ETS in the career term. Airmen with 20 or more years of service are retirement eligible. These airmen experience very high loss rates when they reach first opportunity to retire. Table 1 lists all the submodels that constitute Module 1 of SAM.

It is important to distinguish between rates and flows. For example, the attrition loss *rate* in the second month of service is the number of airmen lost during that month divided by the number of airmen who began the second month of service. The loss *flow* in the second month of service is the raw number of airmen who were lost during that month. Because more airmen enlist in June than in December, the number of second-month losses in July is higher than the number of second-month losses in January, but the

Table 1
SUBMODELS CONSTITUTING THE SHORT-TERM AGGREGATE LOSS MODEL

Submodel	Phase of Airman's Career
First term attrition	Months of service 1-36 (if TOE = 4) Months of service 1-60 (if TOE = 6)
First term ETS	Months of service 37-48 (if TOE = 4) Months of service 49-60 (if TOE = 6)
First term reenlistment	Last year of first term
First term extension	Months of service 49+ (if TOE = 4) Months of service 60+ (if TOE = 6)
Second term attrition	All but the last three months of second term
Second term ETS	Last three months of second term
Second-term reenlistment	Last year of second term
Second-term extension	Past ETS and on extension
Career-term attrition	All but the last month of the career term
Career-term ETS	Last month of career term
Career-term reenlistment	Last year of career term
Career-term extension	Past ETS and on extension
Retirement	20 or more years of service

second-month loss rate in July is normally about the same as the second-month loss rate in January. One way of interpreting this result is that joining the Air Force is no more risky in June than in December.

Generally, each of the models for short-term loss and reenlistment prediction has two parts. First, the fraction of airmen in the category that leave or reenlist is predicted by month in term. Then the predicted number of losses or reenlistments is distributed by grade. There are several reasons for predicting losses and reenlistments in this fashion:

1. The distribution of grades within terms is changing over time with the implementation of fully qualified promotions in the first term.
2. Grade was found not to be a good predictor of losses or reenlistments.
3. The models are easier to estimate when done in two parts.

DATA SOURCES

Initial fitting of time series loss and reenlistment models required longitudinal information on Air Force enlisted personnel. For the developmental analysis, we used the YAR3 (Year-At-Risk) file, which contained longitudinal data on every airman who was on regular active duty in the Air Force any time between June 30, 1971 and June 30, 1984 (see Murray et al., 1989).¹ The YAR file combines data on individual airmen with economic data on the ratio of military to civilian pay, bonuses, and unemployment rates. The economic data used to create the YAR3 file are documented in Walker and McGary, 1989. Preliminary versions of these models were fitted using an earlier version of the YAR file called the ETS file, which contained information on enlisted personnel who were on regular active duty in the Air Force between June 30, 1971 and June 30, 1980.

The information that is available on each airman in the YAR file includes:

- Initial traits at the time of enlistment (e.g., educational status, term of enlistment, age, mental category, marital status).
- Annual snapshot information taken each June 30th (e.g., grade, time in grade, occupation, marital status, number of dependents).
- Information on transactions—reenlistments, extensions, and losses.
- Economic data (e.g., military/civilian wage ratio, basic pay, unemployment rate, bonuses available). The YAR file is structured so that analysis can be performed on a random sample of airmen. Most of the models were fit to a 30 percent random sample.

EXPLORATORY ANALYSIS

Before specifying any of the submodels we looked at a plot of flows and rates over time. For example, the plots of first term ETS loss rates revealed a substantial decrease in this rate over the time frame of our data.

Originally it was intended that the short-term loss prediction models would use information about stated intentions. "An airman who decides to reenlist informs the Air Force of that decision before the expiration of his term of service (ETS) and frequently

¹The YAR file has recently been updated to contain information on every airman who was on regular active duty in the Air Force any time between June 30, 1971 and June 30, 1990.

months earlier (currently up to a year)" (Carter et al., 1983, p. 37). Unfortunately, the information about intent to reenlist was not available in our data base. However, because of the drastic early-out programs implemented in recent years, it is doubtful that these notices would have proven as useful as first envisioned.

Detailed information on extensions was available on the YAR file. Before analyzing the data, we believed that an airman who committed to an extension before his ETS point would almost certainly remain in the Air Force past his original ETS date. That was not the case. Approximately 495,610 of the airmen in the Air Force between 7407² and 8406 had committed to an extension before ETS; and, of those, approximately 241,480 (48.72 percent) left the force on or before their original ETS date. The loss behavior of these airmen was the same as the behavior of those who had not extended. This led to the decision that, for purposes of modeling, an airman would be considered on extension only if he had committed to an extension and remained in the force after his original ETS date.³

The guidelines for developing EFMS models stated in the conceptual design (Carter et al., 1983, pp. 50-51) were followed when we developed the short-term loss models. First, we examined data to determine historical patterns of losses and reenlistments. An airman's career was then divided into phases for purposes of loss modeling. (In the first term there are separate models for month 1, month 2, month 3, months 4-12, months 13-24, etc.) We explored linear models, logit and probit models, time series models, and running average models and considered lag structures for economic variables and seasonal patterns. In many cases, Box-Jenkins time series models (Box and Jenkins, 1976) were found to be the most accurate for predicting short-term losses. That is, the recent behavior of similar cohorts proved to be the best information for predicting the behavior of the current cohort.

²In this Note we often code a year and month as a four-digit number, yymm, where yy are the last two digits of the year and mm is the month. Thus, 7407 refers to July 1974.

³The definition of extension used in SAM differs from this definition. In SAM, an airman is on extension if he has committed to the extension without respect to his original ETS date. The definition was changed for two reasons. The first has to do with the way the data are collected to run the IPM. When the inventories are counted for the IPM, only two months of an airman's history are considered. We need to record an extension when it occurs because it will not be in the record when the next two months are counted. The second reason involves the necessity of determining which airmen are eligible for such special programs as the "reenlist or get out" program and the "early-out" program. An airman is eligible for these programs based on his date of separation (DOS), which is the same as his original expiration of term of service (OETS) until he extends.

Loss rates averaged over the entire enlisted force, particularly those for attrition and retirement, exhibit seasonal patterns. In the case of the attrition models, division of an airman's career into the phases noted above was successful in eliminating seasonal patterns. The composite seasonal pattern can be attributed to changes in the size and composition of entering cohorts at different times of the year. In the case of retirements, there is a true seasonal pattern, so the retirement model includes seasonality factors.

II. MODELING APPROACHES

All of the short-term aggregate loss models in this Note are time series models. The feature of time series analysis that distinguishes it from other statistical analyses is the explicit recognition that the order of observations, or time when an observation occurred, is important. The data for the short-term models is time specific. In fitting time series models, one computes the correlations of observations spaced one unit apart, two units apart, etc. These correlations are assumed to characterize the dependence in the data. This series of correlations, called the "sample autocorrelation function," estimates a "theoretical autocorrelation function," which in turn points to a "correct" model. But because sample autocorrelations can have large variances, they do not necessarily pin down the theoretical autocorrelation function very well, so finding the "correct" model for the time series depends on some artistic *ad hoc* trial and error procedures as well as on the scientific theory. The types of models explained in this section were chosen as best for at least one phase of an airman's career. The following sections provide insight into how we chose the particular submodels. In most cases we used the SAS Econometric and Time-Series Library (1982) to fit the models.

CONSTANT RATE MODELS

When the observed differences from month to month are the result of random, uncorrelated disturbances to the system, we say that the series is white noise. With such data, knowing what the attrition rate was last month gives no information about what will happen next month. The plot of such data will look like Fig. 1. This time series is best described by a constant rate model of the form

$$r(t) = c + e(t),$$

where $r(t)$ is the attrition rate at time t ,
 $e(t)$ is the error at time t , and
 c is a constant (the mean of the time series).

It is assumed that the $e(t)$ are independent, normally distributed with a mean of zero and a known, nonzero variance.

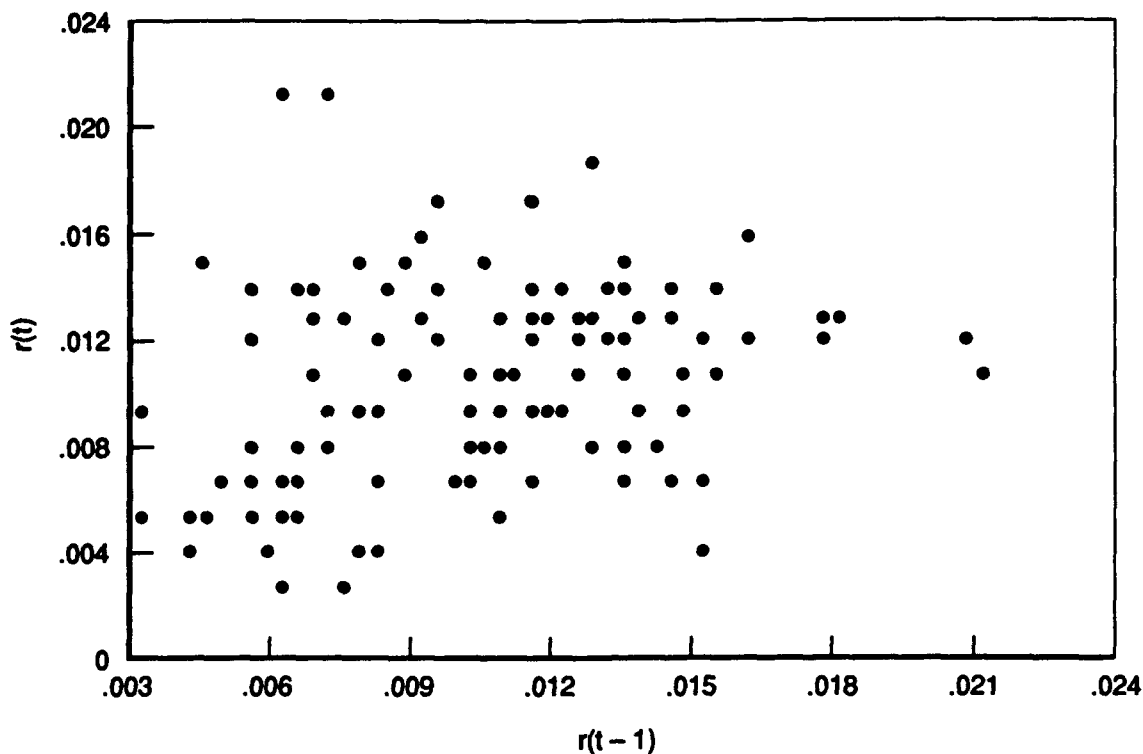


Fig. 1—Attrition rates for high school graduates in their third month of service

REGRESSION MODELS

Some data exhibit a dependence on another set of observable variables; for example the rate of attrition at time t , $r(t)$, might depend on the airman's salary at time t , $s(t)$. If the $r(t)$ are independent and uncorrelated, this relationship could be expressed as the simple linear regression model

$$r(t) = (a) s(t) + c + e(t),$$

where a is the slope of the linear relationship, c is the intercept, and $e(t)$ is the normally independently distributed error-term with mean zero. When $r(t)$ is plotted against $s(t)$, such a relationship looks like a straight line.

If the attrition rate, $r(t)$, depends on more than one variable, such as term of enlistment, $m(t)$, number of years to original ETS, $y(t)$, and grade, $g(t)$, then the relationship might be described by the multiple regression model

$$r(t) = (a1)m(t) + (a2)y(t) + (a3)g(t) + c + e(t),$$

where the constants a_1 , a_2 , and a_3 are the partial regression coefficients; c is a constant; and $e(t)$ is the normally independently distributed error-term with mean zero.

AUTOREGRESSIVE MODELS

When working with time-dependent information (such as in Fig. 2), you need to exploit the relationships among the data. A model that exploits this time dependency in the observations is the autoregressive model. Whereas the regression model expresses a relationship between the pairs $(r(t), s(t))$, the autoregressive model expresses a relationship between $[r(t), r(t-1)]$, or $[r(t-1), r(t-2)]$, etc. In such models the time dependency in successive observations is inferred from plots of autocorrelation functions, then the parameters are fitted to summarize the functional form of these dependencies. In a *first order autoregressive model* the next observation depends only on the last observation and has the form:

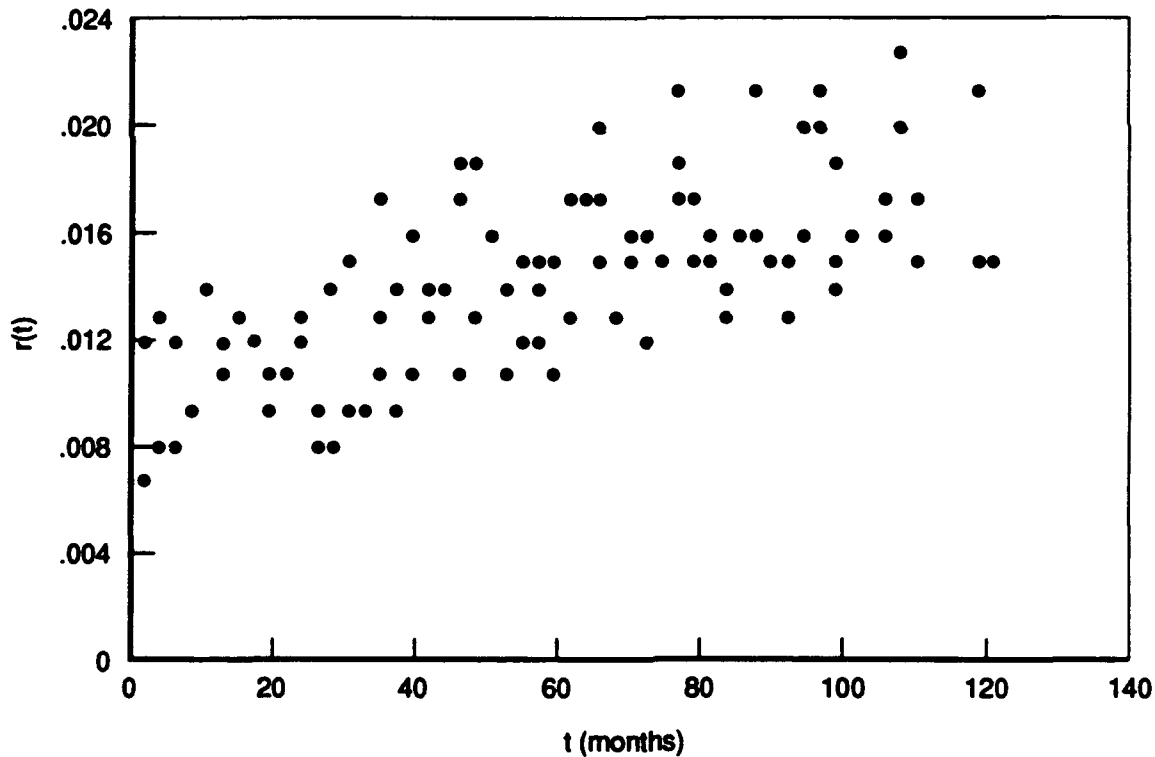


Fig. 2—Autocorrelation

$$r(t) = (a_1)r(t - 1) + c + e(t),$$

where $e(t)$ is the normally independently distributed error-term, often referred as the noise. The specification of c , the constant-term, incorporates the mean of the time series and the autoregressive coefficients. When reporting the functional form of autoregressive models in subsequent sections, we show the mean of the time series for completeness, in addition to the constant-term and the autoregressive coefficients.

Three other autoregressive models need to be mentioned. A *second order autoregressive model* depends on the last two observations and has the form:

$$r(t) = (a_1)r(t - 1) + a_2 * r(t - 2) + c + e(t).$$

A *third order autoregressive model* depends on the last three observations and has the form:

$$r(t) = (a_1)r(t - 1) + (a_2)r(t - 2) + (a_3)r(t - 3) + c + e(t).$$

The above autoregressive models assume that the data are stationary. Sometimes there are trend or seasonal patterns to the time series that must be removed before an autoregressive model can be fitted to the data.

If the time plot of the data reveals a yearly seasonal pattern to the data, a *first order autoregressive model differenced at lag 12 months* could be fitted to the time series. It has the following mathematical form:

$$r(t) - r(t - 12) = (a_1)(r(t - 1) - r(t - 13)) + c + e(t).$$

STRAIGHT LINE RUNNING AVERAGE MODELS

A straight line running average model uses the average of the most recent k monthly flows to predict the attrition rate. Its form is:

$$x(t) = [X(t - 1) + \dots + X(t - k)] / [P(t - 1) + \dots + P(t - k)]$$

where

$x(t)$ = k -month straight line running average

$X(t)$ = Number of losses at time t

$P(t)$ = Population at risk at time t

k = Number of months to include in the average

For example, an attrition rate from a 12-month straight line running average model is found by summing the previous 12 months of losses and dividing that total by the sum of the previous 12 start-of-month inventories (populations at risk). If the loss model is a seasonal model, a seasonal adjustment factor for the given calendar month is multiplied by the loss rate. A 12-month straight line running average model with seasonal adjustment factors has the following form:

$$r(t) = [s(t)][x(t)]$$

where

$r(t)$ = Attrition rate at time t

$s(t)$ = Seasonal adjustment factor for calendar month t

$x(t)$ = 12-month straight line running average

The seasonal adjustment factor for a month is defined to be the ratio of the average loss rate for the calendar month to the overall average loss rate. For example,

$$s(\text{December}) = \frac{[\text{SUM (December Losses)} / \text{SUM (December Inventories)}]}{[\text{SUM (All Losses)} / \text{SUM (All Inventories)}]}$$

RATIO MODELS

These models were developed to add the grade dimension to the loss predictions. The loss models predict the loss rate for a particular time in an airman's career without regard to grade. However, our research indicated that airmen in different grades attrit at different rates. In the first year of service a higher percentage of airmen in grades E-1 and E-2 attrit than in grade E-3. If the losses were to be distributed as grade is distributed, we would predict too many E-3 losses and not enough grade E-1 and E-2 losses. The ratio models are described in detail at the end of Sec. III.

III. FIRST TERM ATTRITION LOSS MODELS

Airmen who leave the service before the end of their commitment (ETS) are classified by the general reason for leaving—attrition, early release, or special programs. Attrition losses in the first-term can substantially affect force structure. As much as 46 percent of *all* losses from the Air Force can be due to first-term attrition, and attrition rates show high variability. Air Force regulations govern all attrition separations, but those regulations can be applied in different ways depending on the civilian and military environment at the time. In times of high civilian unemployment, attritions due to hardship decline. When retention is high and there are large numbers of recruits, quality standards are more stringently applied, so quality attritions increase. The number of first-term airmen who attrited in a given month was divided by the total number of Air Force losses during that month to see if the proportion remained constant. Figure 3 shows this proportion for the time period 7505 to 8406. The lowest proportion (.16) occurred in 7508 and the highest proportion (.47) occurred in 8204. Thus, the number of first-term airmen who attrit is not a constant proportion of the total losses. Seasonal patterns are also apparent. August is always a month for a low attrition rate. This is shown on the graph by the symbol " \Leftarrow ".

The question of seasonality is very important to modeling short-term losses. Figure 2 suggests that seasonality is a factor in the proportion of total Air Force losses due to attrition behavior. It is critical to good modeling to distinguish between losses due to an airman's decision and losses due to Air Force policy. One of the reasons that the fraction of attrition losses is low in August relative to all losses is that August has been a month in which early release programs occurred. Airmen could leave early to return to school, and often the force size needed to be reduced before the end of the fiscal year. The seasonal patterns that are observed in Fig. 2 are eliminated when the loss rates (number of losses divided by number of airmen at risk) by subgroups are plotted.

Figure 4 shows the attrition rate (number of attrition losses in the subgroup divided by total number of airmen in the subgroup) for high school graduates in the third through twelfth months of service. Lines have been drawn through the time plot each July. They should assist the reader in visual examination of annual patterns. There are no apparent cyclic patterns to the data. The fluctuations are random and have no orderly

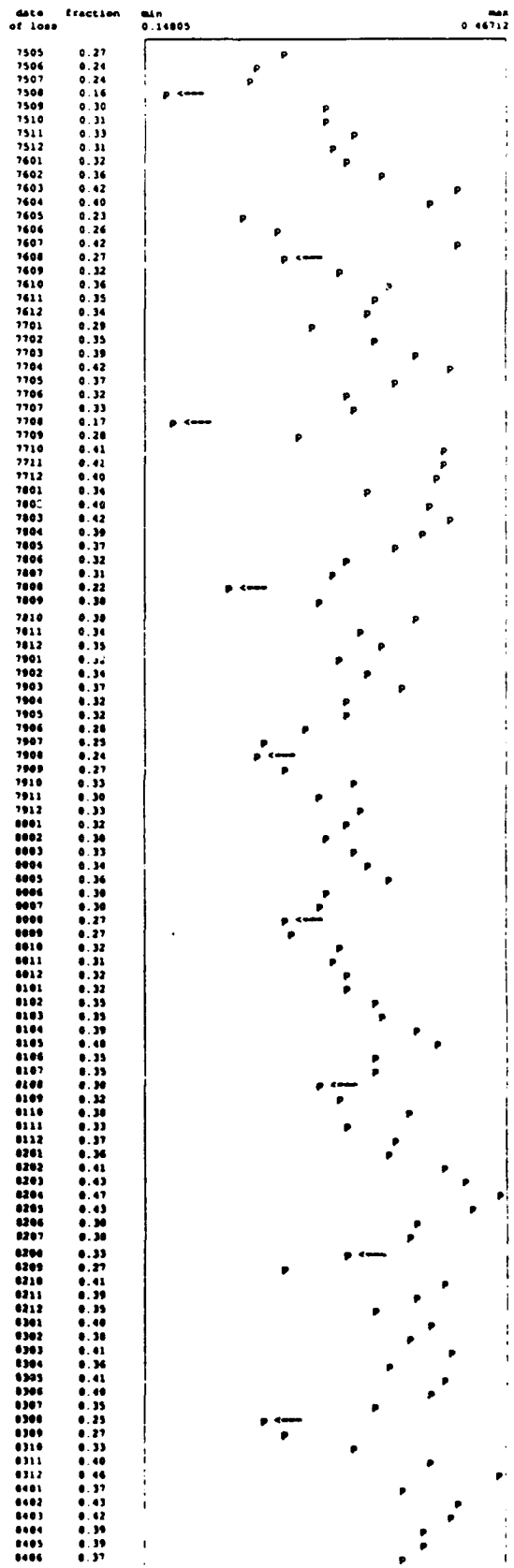


Fig. 3—First term attrition/total
Air Force losses

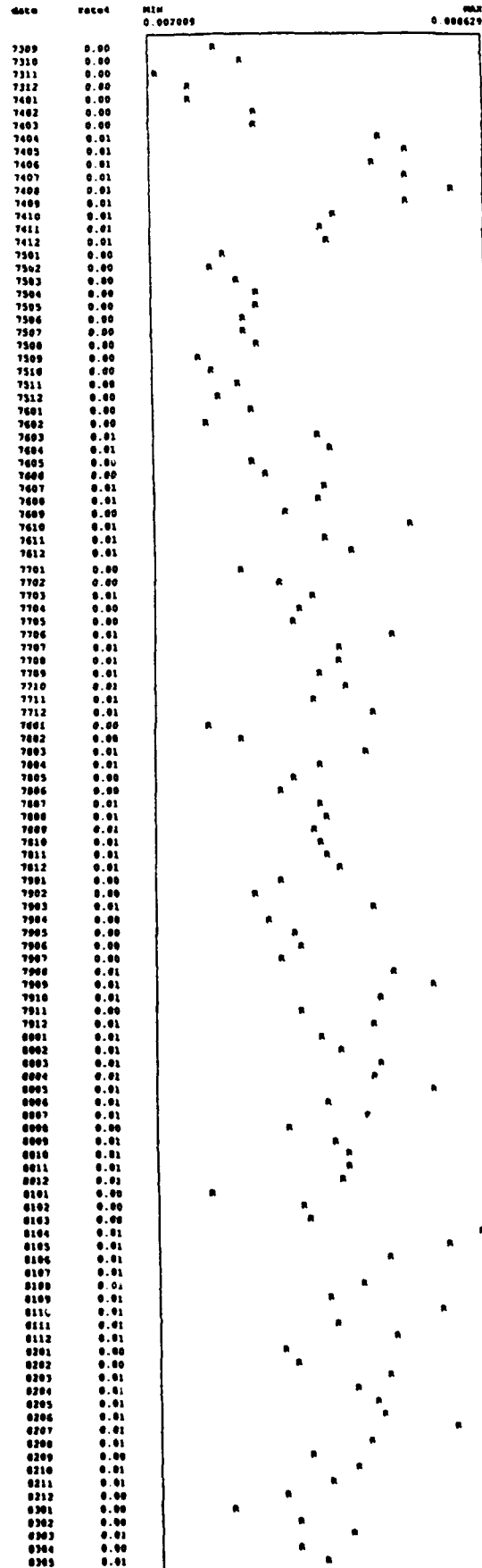


Fig. 4—Attrition rate of high school enlistees with $4 \leq \text{MOS} \leq 12$ (number of attrition losses in subgroup/total in subgroup)

pattern. These visual impressions are confirmed by autocorrelations and partial autocorrelations calculated in the time series analysis.

The initial step in modeling first-term losses was to identify the point in the airman's career when the loss decision could be considered an ETS rather than an attrition loss. In general, the Air Force treats first-term airmen in the year before their ETS differently than it does in the early years of their term. The ETS year is when large early-out or reenlistment programs are implemented. It was natural for the first-term attrition loss model to look at the years before the ETS year and model the ETS year separately. For airmen with TOE = 4, the first-term attrition model covers months of service 1-36, and for airmen with TOE = 6, the model covers months of service 1-60.

The short-term aggregate loss models as initially constructed by Joseph Adams and Jan Chaiken (unpublished RAND research) did not predict losses by grade. There were several reasons for building loss models without the grade dimension. First it was believed that accurate prediction of losses in each time phase of an airman's career was the primary purpose of the loss models. Because timing of promotions and hence grade is dependent on Air Force policy, the presence of grade in the model could add another random error-term and possibly reduce the accuracy of any forecasting model. Also, grade is much more of a group effect, meaning that when promotions are slowed they are slowed for everyone eligible for promotion, and entire cohorts have reduced chances of promotion.

However, losses, particularly attrition losses, are an individual decision. By predicting overall loss rates and then distributing those losses by grade, the models were able to accurately forecast losses by months of service and grade.

DECISION GROUPS AND SUBGROUPS

When modeling first-term attrition, we believed it was desirable to divide the data into subgroups that had fairly stable rates over time. We chose the decision groups after examining how attrition rates varied by month of service and found that airmen who were in basic training (roughly the first three months of service) behaved differently than did the airmen who had completed their training. Airmen in the remainder of their first year of service behaved differently than did airmen in their second year of service, etc. After the first year of service, the decision groups are defined by years of service.

Other factors that affect attrition during the first-term are:

- Length of the term of enlistment (TOE = 4 or 6).
- Educational level, high school diploma (hs) or not (nhs).
- Reason for attrition.

Table 2 shows that first-term airmen without a high school diploma attrit at roughly twice the rate of airmen with a high school degree. In fact over this time frame, the mean monthly attrition rate for all first-term airmen before their ETS year was 0.81 percent for high school graduates and 1.92 percent for non-high school graduates.

Table 3 shows that the number of accessions without a high school diploma has shrunk in recent years to only 1 percent of all accessions. Even so, the first-term attrition and ETS models predict losses separately for high school graduates and non-high school graduates for four-year enlistees. However, we found that the IPM could not operate with inventory disaggregated to that level, so we never directly implemented the models for non-high school graduates with four-year terms of enlistment.

Table 2

ATTRITION OF AN ENTERING COHORT BY LENGTH OF SERVICE
AND AMOUNT OF EDUCATION
(Percent)

FY	Length of service							
	6 Months		12 Months		24 Months		36 Months	
	nhs	hs	nhs	hs	nhs	hs	nhs	hs
77	19.95	9.59	26.65	12.61	40.03	19.29	47.45	24.36
78	18.71	8.46	27.83	11.93	42.71	19.12	49.85	23.84
79	17.64	7.56	27.28	11.15	41.67	17.74	49.36	22.86
80	16.10	6.94	23.97	10.17	38.66	16.68	48.06	21.89
81	17.06	8.74	24.04	11.43	36.82	17.34	46.90	23.04
82	18.05	9.39	22.74	11.67	33.75	17.05	41.93	22.16
83	15.28	8.27	20.12	10.09	29.58	14.46	37.10	18.77
84	18.63	9.34	22.43	11.06	30.75	15.38	37.44	19.08
85	18.01	9.45	22.23	11.32	30.66	15.97		
86	18.07	10.02	21.49	11.73				

nhs = non-high school graduate.
hs = high school graduate.

Table 3

ENTERING COHORT
WITHOUT HIGH
SCHOOL DIPLOMA

FY	Percentage
77	7.68
78	14.43
79	15.74
80	15.54
81	10.54
82	5.40
83	1.61
84	1.10
85	0.96
86	0.99

During the first three years of service, four- and six-year enlistees have similar attrition rates. No distinction is made for TOE in the attrition models until more than 36 months of service. However, there is a distinction by grade. Six-year enlistees have a different grade progression than do four-year enlistees. Six-year enlistees often enter in grade E-3. It is rare for a four-year enlistee to enter at that level. Grade models for the first three years of service spread the losses differently by term of enlistment.

When an airman attrits from the force, the reason for attrition is contained in his Separation Program Designator (SPD) code. To simplify the loss modeling, we aggregated the hundreds of specific attrition codes into five categories: disability, hardship, quality, death, and miscellaneous. Table 4 shows the percent of attrition losses in each category. Attrition due to quality is the largest category. Because the number of losses in the other categories is small when compared with the number of quality losses, the other categories of losses were further combined to form a "nonquality" loss category. Quality and nonquality loss subgroups were defined within several of the decision groups.¹

¹The Air Force maintains a file containing all the transactions for an individual over the course of his career called the Promotion/Demotion Gain/Loss file (PDGL). On this file two variables indicate reasons for separation from the Air Force, SPDTRCD and ADNSPD. When Chaiken and Adams were initially modelling attrition losses, they used the ADNSPD because it was the variable the Air Force used most at that time. Since then, much more attention has been paid to SPDTRCD to insure that it is accurately

Table 4

FIRST TERM ATTRITION LOSSES BY CATEGORY
(Percent)

Category Number	Category Name	Attrition Losses
1	Disability	5.37
2	Hardship	12.43
3	Quality	74.58
4	Miscellaneous	6.50
5	Death	1.13

The reason for an airman's attrition is not known until the actual attrition. To apply the loss rates, the total inventory must be separated into the number of airmen subject to loss for quality reasons versus the number lost for nonquality reasons. For each decision group for which quality/nonquality predictions needed to be made, we calculated the mean fraction of attritions for quality reasons. (See notes to Table 6 below for these numbers.) The formula to predict the number of losses is:

$$\# \text{ losses} = (\text{LRATE}) N = (1 - \text{FRQUAL}) N (\text{LRNONQ}) + (\text{FRQUAL}) N (\text{LRQ}),$$

where

LRATE = Predicted attrition loss rate

FRQUAL = Mean fraction of quality losses

LRNONQ = Loss rate predicted for nonquality losses

LRQ = Loss rate predicted for quality losses

N = Number of airmen in subgroup.

Table 5 summarizes the eight decision groups used for the first-term attrition analysis along with their further subdivisions by high school graduation status and type of attrition. In total, 19 first-term attrition models were fitted to data from October 1976 through June 1983. The purpose of including the mean attrition rate associated with each

coded. The crosswalk between the two codes is not perfect and could lead to some data anomalies. We consider only groupings based on the SPDTRCD variable. As of 1987, SPDTRCD was being blanked out on the PDGL file when the record was indicating a change in AFSC, but ADN SPD was not being blanked out. Neither variable has been continuously suitable for analysis purposes.

Table 5

FIRST TERM ATTRITION DECISION GROUPS AND SUBGROUPS

Phase of Airman's Career	Subgroups	Mean Attrition Rate
Decision Group I (month 1)	1. High school grad.	0.013
	2. Non-high school grad.	0.028
Decision Group II (month 2)	3. High school grad.	0.035
	4. Non-high school grad.	0.077
Decision Group III (month 3)	5. High school grad.	0.010
	6. Non-high school grad.	0.026
Decision Group IV (months 4-12)	7. High school grad. quality	0.005
	8. Non-high school grad. quality	0.014
	9. All other attrition	0.022
Decision Group V (months 13-24)	10. High school grad. quality	0.006
	11. Non-high school grad. quality	0.016
	12. All other attrition	0.033
Decision Group VI (months 25-36)	13. High school grad. quality	0.004
	14. Non-high school grad. quality	0.011
	15. All other attrition	0.044
Decision Group VII (months 37-48, TOE=6)	16. High school grad.	0.005
	17. Non-high school grad.	0.009
Decision Group VIII (months 49-60, TOE=6)	18. High school grad.	0.004
	19. Non-high school grad.	0.008

model is to direct attention to the differences in the mean attrition rate among the subgroups.

Once we established the eight decision groups and 19 subgroups, we took the following steps to fit the attrition model for each subgroup:

- a. Find the monthly loss rate for the subgroup.
- b. Plot the monthly loss rate over time using SAS procedure TIMEPLOT.
- c. Analyze the time plot carefully for patterns and outliers.
- d. If the timeplot showed seasonality or trends, take differences and plot the differenced values over time.
- e. Identify the time series using SAS procedure ARIMA.
- f. Study the plots of autocorrelations and partial autocorrelations to identify the appropriate model.

- g. Estimate the model using loss rates for each month in the time frame 7406–8306 using the SAS procedure ARIMA.
- h. Forecast loss rates for the last year of the data (8307–8406) using the proposed models.
- i. Compare the actual loss rates with the forecast loss rates.

THE LOSS MODELS

Table 6 summarizes the final models, giving the form of the model for each subgroup and the coefficients. (The notation AR1 means first order autoregressive, and similarly AR2 is second order autoregressive, etc.) Constant models were fitted to the data when there were no discernible patterns in the timeplot, the autocorrelations (ACF), or the partial autocorrelations (PACF), and the check for white noise showed that the residuals were in fact random noise. Two of the subgroups (Decision Group I, High School Graduates; Decision Group V, Non-High School Graduates, quality attrition) had time series that were difficult to fit until we replaced outliers with mean attrition values.

Decision Group I (month 1), High School Graduates was a very difficult time series to fit. Because airmen come into the force at different times in the month, they are at risk to attrit for different lengths of time. For example, if in one month all accessions occurred after the 15th of the month and in the next month all accessions occurred before the 15th of the month, the two months could have a very different number of attritions and the same rate of attrition per day. There is no way for our database to capture this phenomenon. However, other data sources were investigated to find months when accessions occurred early and those when accessions occurred later in the month. Using these "rate weights," we modified the number of attritions to capture the "length of time at risk to attrit." However, this exercise did not reveal the true loss model for first month attrition.

The most help for fitting the model came from careful study of the time plots of the loss rates. The plot of attrition rate over time revealed that the attrition rate in 7612 and in 7712 was far higher than in any other month from 7610 through 8306 (see App. A). When these two outlier attrition rates were replaced with the mean attrition rate (0.013), the λ of the time series model became evident in the plots of the autocorrelations and partial autocorrelations.

Table 6

FIRST TERM ATTRITION MODELS

Decision Group	MOS	Subgroup ^a	Type of Prediction Model	Parameters of Model					
				Mean	ar1	ar2	ar3	ar4	Constant
I	1	1. High school grad.	AR1	.0129	.5037	—	—	—	.0054
		2. Non-high school grad.	Constant	.0275	—	—	—	—	—
II	2	3. High school grad.	AR3	.0349	.2398	.1984	.2268	—	.0114
		4. Non-high school grad.	AR1	.0765	.3355	—	—	—	.0495
III	3	5. High school grad.	Constant	.0104	—	—	—	—	—
		6. Non-high school grad.	Constant	.0257	—	—	—	—	—
IV	4-12	7. High school grad. quality	AR1(diff lag 12)	-.0048	.2771	—	—	—	-.0002
		8. Non-high school grad. quality	AR1	.0144	.3921	—	—	—	.0091
		9. All other attrition	AR1	.0221	.1747	—	—	—	.0191
V	13-24	10. High school grad. quality	AR4(diff lag 12)	-.0059	.3562	.1868	.0231	.3126	-.0001
		11. Non-high school grad. quality	Constant	.0155	—	—	—	—	—
		12. All other attrition	AR2	.0033	.3220	.6780	—	—	.0000
VI	25-36	13. High school grad. quality	AR2	.0044	.2624	.2514	—	—	.0021
		14. Non-high school grad. quality	AR2	.0105	.1232	.2737	—	—	.0063
		15. All other attrition	AR2	.0441	.3867	.4228	—	—	.0004
VII TOE=6	37-48	16. High school grad.	Constant	.0054	—	—	—	—	—
		17. Non-high school grad.	Constant	.0086	—	—	—	—	—
VIII TOE=6	49-60	18. High school grad.	Constant	.0043	—	—	—	—	—
		19. Non-high school grad.	Constant	.0076	—	—	—	—	—

^aFraction of losses for quality reasons MOS 4-12: FRQUAL = .819657
MOS 13-24: FRQUAL = .776284
MOS 25-36: FRQUAL = .716992

The ACF and the PACF are shown in Fig. 5. The asterisks represent the value of the autocorrelation. Two standard deviations on either side of zero are marked by dots. The spike in the PACF at lag 1 and the decaying ACF indicate an autoregressive moving average model with coefficient at lag 1. A comparison of the autocorrelation check for white noise before and after we fitted the model revealed little probability that the errors were random before fitting (probability = 0) and that, after fitting, the residuals are random (probability $\gg .1$). If two more outliers were removed, the spikes in the PACF at lag 5 and lag 9 would disappear. However, the model fit would change only slightly. It was decided to alter the data as little as possible to obtain the best fit.

Decision Group V (13-24 months of service) Non-High School Graduates, quality attrition was also a difficult time series to fit. The plot revealed that the attrition rates in 7903 and in 7910 were far higher than in any other month from 7610 through 8306 (see timeplot in App. B). When these two outlier attrition rates were replaced with the mean attrition rate (.016), the form of the time series model became evident in the plots of the autocorrelations and partial autocorrelations.

ADDITION OF GRADE DIMENSION TO FIRST TERM ATTRITION MODELS

The models previously described predict total losses for each month of an airman's career before his last year of commitment. However, what is really needed is losses by grade and time in service. Depending on their enlistment contract, airmen can enter the Air Force in grades E-1, E-2, or E-3. Thus, from the first month of service airmen can be in different grades. As their careers progress, airmen are promoted. Some are outstanding performers and are promoted early; others lag behind their cohort in promotion to higher grades. Thus the losses that are predicted by the models are from different grades.

This section outlines how to distribute the total losses in a particular month of service over the possible grades that an airman could have in that month of service. The YAR file does not contain data showing an airman's grade for each month of service. Hence, the first step in adding grade to the submodels was to add a prediction of an airman's grade at each month of service to the YAR data using information in the snapshot variables and the variable XGRADE (see Murray et al., 1989). This prediction is necessary for analysis only. When the loss models are actually operating, each airman's grade will be known.

LAG	COVARIANCE	CORRELATION	1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
0	0.00002144	1.00000																				
1	.000010737	0.50000																				
2	.000007051	0.32887																				
3	5.888E-06	0.27462																				
4	4.244E-06	0.19797																				
5	-5.234E-07	-0.02441																				
6	-1.190E-06	-0.05549																				
7	-1.400E-06	-0.06531																				
8	-2.048E-07	-0.00955																				
9	3.006E-06	0.14022																				
10	3.116E-06	0.14533																				
11	2.171E-06	0.10127																				
12	4.540E-06	0.21177																				
13	3.689E-06	0.17207																				
14	1.637E-06	0.07636																				
15	1.112E-06	0.05185																				
16	1.330E-06	0.06202																				
17	1.403E-06	0.06542																				
18	6.592E-07	0.03075																				
19	1.741E-06	0.08119																				
20	8.738E-07	0.04076																				
21	2.475E-06	0.11545																				
22	1.003E-06	0.04678																				
23	5.275E-07	0.02461																				
24	-4.771E-07	-0.02225																				

MARKS TWO STANDARD ERRORS

PARTIAL AUTOCORRELATIONS

LAG	CORRELATION	1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	
1	0.50080																					
2	0.10420																					
3	0.10107																					
4	0.01048																					
5	-0.22347																					
6	-0.02049																					
7	-0.01530																					
8	0.10722																					
9	0.25715																					
10	0.01198																					
11	-0.07043																					
12	0.09856																					
13	-0.06988																					
14	-0.00847																					
15	0.04608																					
16	0.02907																					
17	0.10928																					
18	-0.05089																					
19	0.04898																					
20	-0.03310																					
21	0.03005																					
22	-0.07512																					
23	0.00750																					
24	-0.04801																					

AUTOCORRELATION CHECK FOR WHITE NOISE

TO	CHI	AUTOCORRELATIONS																					
LAG	SQUARE	DF	PROB																				
6	40.53	6	0.000	0.501	0.329	0.275	0.198	-0.024	-0.055														
12	50.12	12	0.000	-0.065	-0.010	0.140	0.145	0.101	0.212														
18	34.85	18	0.000	0.172	0.076	0.052	0.062	0.065	0.031														
24	57.62	24	0.000	0.081	0.041	0.115	0.047	0.025	-0.022														

AUTOCORRELATION CHECK OF RESIDUALS

TO	CHI	AUTOCORRELATIONS																					
LAG	SQUARE	DF	PROB																				
6	5.23	4	0.265	-0.055	0.036	0.103	0.163	-0.132	-0.028														
12	11.81	10	0.298	-0.067	-0.057	0.146	0.085	-0.064	0.169														
18	13.13	16	0.663	0.086	-0.030	-0.001	0.018	0.043	-0.051														
24	18.85	22	0.655	0.092	-0.057	0.138	-0.015	0.030	-0.136														

Fig. 5—ARIMA diagnostics for high school graduates in their first month of service

The models for distributing losses by grade in the first-term before the ETS year use the same eight Decision Groups as the first-term attrition loss models. Thus, the time groupings are month 1, month 2, month 3, months 4-12, months 13-24, etc. Airmen attrit at different rates depending on their grade relative to the average grade of their cohort. For example, airmen who are promoted early are less likely to attrit than others in their cohort. Airmen who are slow to promote relative to the rest of their cohort attrit at a higher rate. To predict losses by grade as well as time in service, we calculated the ratio of losses in one grade (the base grade, which has the most losses for the subgroup) to losses in all other grades for each time period. Figure 6 is the plot of the ratio of losses in months of service 4-12 for high school graduates who attritted for quality reasons in grade E-3 against those in grade E-1/E-2. For illustrative purposes, the figure contains only the odd-numbered months from July 1975 through May 1984; however, in the analysis we plotted all months for all subgroups. The plots of the loss ratios exhibit no discernible patterns. We calculated the average loss rate for the entire 99 months of data for each grade/month-of-service group. The mean loss ratios were the average losses rate in one grade divided by the average losses in the base grade. Table 7 contains the mean loss ratios for all first-term attrition decision groups. Data limitations made it difficult to accurately separate airmen in grades E-1 and E-2. For this reason, and because airmen are quickly promoted out of these two grades, the analysis grouped grades E-1 and E-2 together.

In addition to grade, TOE was added to the models because during years 1973-1980 there was a dramatic change in the number and percentage of six-year enlistees, as shown in Fig. 7. Six-year enlistees reach grades E-3 and E-4 much earlier than do four-year enlistees. (As the number of six-year enlistees declines, we may be able to remove this dimension from the model.) To incorporate TOE into the models, we calculated the average loss rate for each grade and TOE and then the mean loss ratios for each subgroup, grade, and TOE combination. The method of calculating both the loss rate ratios and the mean loss ratios, as well as for an example of how to apply the ratios, is given below.

Grade Distributions for First Term Attrition Models

Table 7 summarizes the mean loss ratios for all decision groups and subgroups. In the first three months (Decision Groups I, II, and III) of an airman's career, almost all

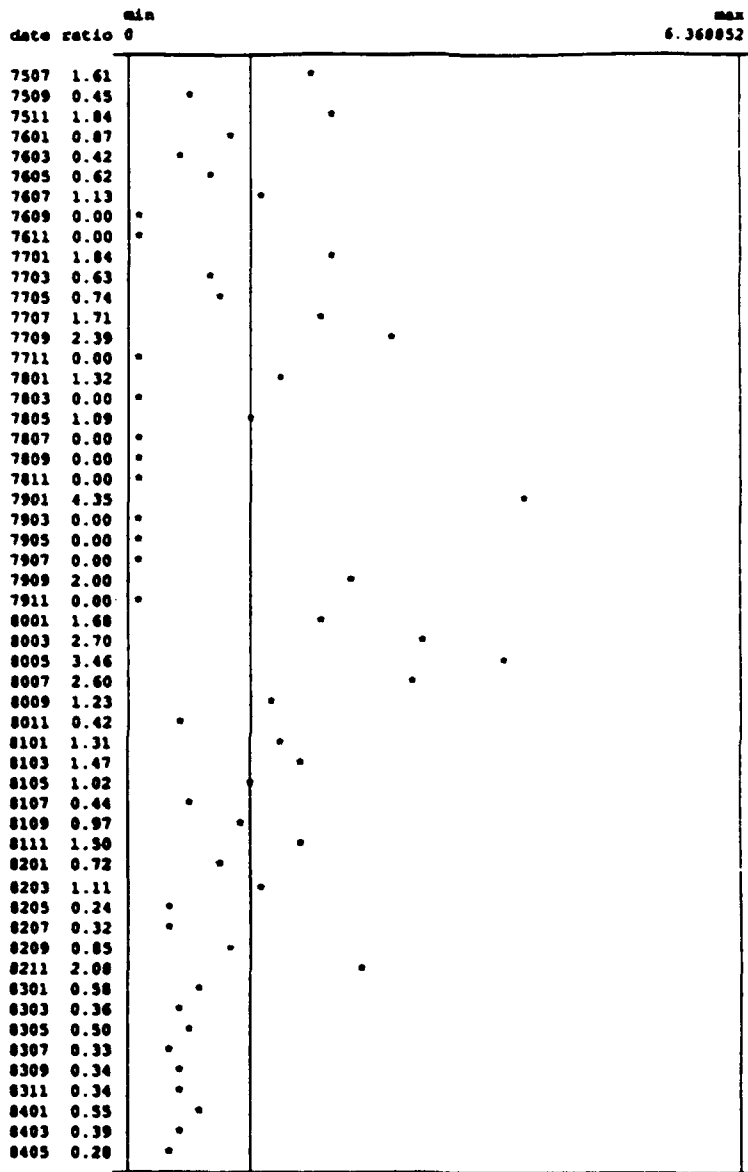


Fig. 6—Ratio of losses in months of service 4-12 for high school graduates, quality attrition in grade E-3 compared with that in grade E-1/E-2

Table 7

MEAN LOSS RATIOS FOR FIRST TERM ATTRITION MODELS

Decision Group	MOS	Subgroup	Grade	Mean Loss Ratio	
I	1	1. High school grad.	E-1/2	1.00	
		2. Non-high school grad.	E-1/2	1.00	
II	2	3. High school grad.	E-1/2	1.00	
		4. Non-high school grad.	E-1/2	1.00	
III	3	5. High school grad.	E-1/2	1.00	
		6. Non-high school grad.	E-1/2	1.00	
IV	4-12	7. High school grad. quality	E-1/2	1.00	
			E-3	.90	
		8. Non-high school grad. quality	E-1/2	1.00	
			E-3	.86	
		9. All other attrition	E-1/2	1.00	
			E-3	.83	
V	13-24	10. High school grad. quality	E-1/E-2	16.87	14.66
			E-3	1.00	1.00
		11. Non-high school grad. quality	E-1/E-2	33.41	26.76
			E-3	1.00	1.00
		12. All other attrition	E-1/E-2	2.66	1.47
			E-3	1.00	1.00
VI	25-36	13. High school grad. quality	E-1/E-2	18.94	30.29
			E-3	1.00	13.03
			E-4	0.22	1.00
		14. Non-high school grad. quality	E-1/E-2	36.91	80.08
			E-3	1.00	20.72
			E-4	0.26	1.00
		15. All other attrition	E-1/E-2	1.16	1.35
			E-3	1.00	2.02
			E-4	1.48	1.00
VII	37-48	16. High school grad.	E-4	1.00	
		17. Non-high school grad.	E-4	1.00	
VIII	49-60	18. High school grad.	E-4	1.00	
		19. Non-high school grad.	E-4	1.00	

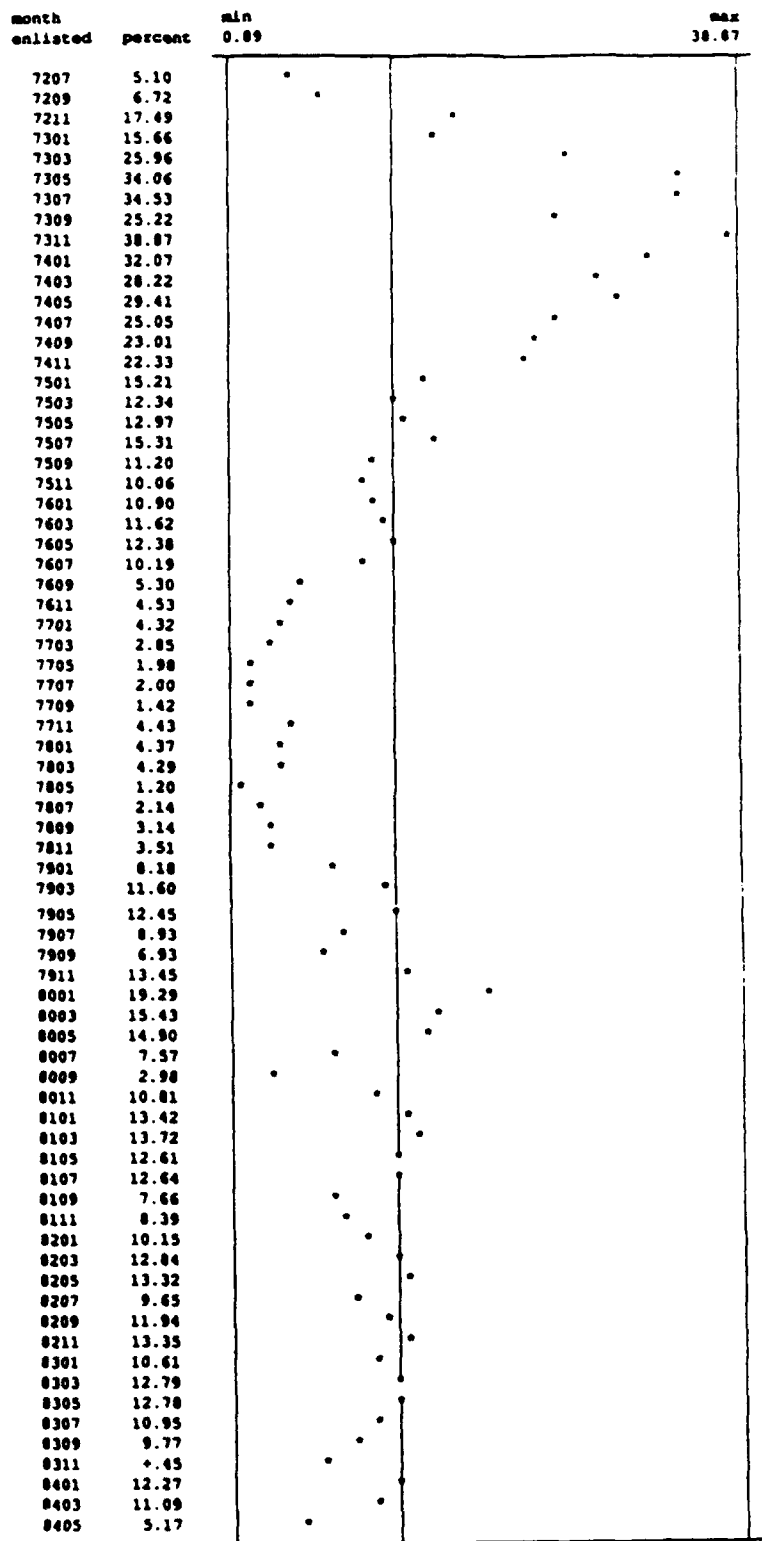


Fig. 7—Percent of entry cohort with six-year term of enlistment

attrition occurs in grades E-1/E-2. The losses predicted using the attrition models discussed above would all be from grades E-1/E-2.

In Decision Group IV (4th-12th months of service) grades E-1/E-2 and grade E-3 have significant attrition. We calculated attrition rates for each subgroup by grade. The base grade (the one with the most airmen) for each subgroup was E-1/E-2. The three subgroups for which attrition models were fitted are shown in Table 7 along with their mean loss ratios.

Decision Group V (13th-24th months of service) also had significant attrition in grades E-1/E-2 and grade E-3. However, in this group the base grade for each of the three subgroups was E-3. Analysis of the data had revealed a difference in attrition behavior between four- and six-year enlistees by the time the airman reached this period in his career. Table 7 therefore lists mean loss ratios for TOE = 4 and TOE = 6.

Significant attrition occurred in four grades, E-1/E-2, E-3, and E-4 for Decision Group VI (25th-36th months of service). When the TOE was four years, the base grade for each of the three subgroups was E-3. However, when the TOE was six years, the base grade was E-4. The mean loss ratios are documented in Table 7.

In both Decision Group VII (37th-48th months of service, six-year enlistees) and Decision Group VIII (49th-60th months of service, six-year enlistees) almost all attrition was in grade E-4. Thus, all losses predicted using the attrition models discussed above are assumed to come from grade E-4.

Calculation Formulas

The formulas for calculating the loss rate ratios, mean loss ratios, and for applying them to determine the loss rates by grade are shown here.

Calculate loss rate ratios. For each subgroup by grade, find the loss rate each month. Call the grade that has the most airmen (on average) the *base grade*. (The base grade may vary from one subgroup to another.) The loss rate ratios for each subgroup by grade group are found by dividing the loss rate of each grade by the loss rate of the base grade.

Calculate mean loss ratio for each subgroup. Mean loss ratio =

$$\frac{(l1 + l2 + \dots + l99) / [n1 + n2 + \dots + n99]}{(b1 + b2 + \dots + b99) / (bn1 + bn2 + \dots + bn99)}$$

The variables are defined in Table 8.

Table 8

VARIABLES IN MEAN LOSS RATIO

Month	Base Group		Other Group	
	Airmen	Losses	Airmen	Losses
1	bn1	bl1	n1	l1
2	bn2	bl2	n2	l2
.
.
99	bn99	bl99	n99	l99

Apply loss rate ratios. It would be tempting to apply the mean loss ratios directly without accounting for the different sizes of the cohorts. For subgroup 7, let L12 denote the predicted losses from grades E-1/2 and L3 denote the predicted losses from grade E-3. Table 7 shows that $L3/L12 = 0.9$.

$$L12 + L3 = \text{Total losses}$$

$$(1.9) L12 = \text{Total losses}$$

thus, $L12 = \text{Total losses}/1.9$ (1)

$$L3 = 0.9 \text{ total losses} / 1.9$$

Although this is mathematically correct, it does not account for the different sizes of the cohorts when identified by grade and months of service. Thus, we used a more complex formula to obtain the number of losses by grade and month of service.

For each subgroup, there will be one or more mean loss ratios by grade (called, for example, ratio(1/2), ratio(3), and ratio(4)), predicted numbers of airmen in each grade (called, for example, inv(1/2), inv(3), and inv(4)), and a predicted overall loss rate from the current version of the loss model (call this lrate). The loss rate for any grade (lrate(g)) can then be found by solving:

$$lrate(g) = \frac{lrate(\sum inv(g))}{\sum (ratio(g)inv(g))} \quad (2)$$

To illustrate why the more complex formula was needed, consider a hypothetical situation using the mean loss ratio from subgroup 7 (see Table 9). If $lrate = .01$ and total inventory in the subgroup is 500, then, in this subgroup, total losses = 5. The columns labeled invA, invB, and invC are three possible inventories for each grade. The column labeled loss (1) calculates the losses for each grade using formula (1). The column labeled loss (2) calculates the losses using formula (2).

Using formula (1) the number of losses in each grade is the same regardless of the inventory. The number of losses is very much dependent on the number of airmen in each grade when using formula (2).

Table 9

COMPARISON OF TWO FIRST TERM ATTRITION LOSS MODELS
(High school graduates, quality, months of service 4-12)

Grade	Mean Loss Ratio	invA			invB			invC		
		Loss (1)	Loss (2)	Loss (1)	Loss (2)	Loss (1)	Loss (2)			
E-1/2	1.00	250	3	3	100	3	1	400	3	4
E-3	.90	250	2	2	400	2	4	100	2	1

IV. FIRST TERM ETS MODELS

FIRST TERM ETS LOSS MODELS

The short-term aggregate ETS loss models that Adams and Chaiken fitted were regression models (unpublished RAND research). They used such variables as the fraction of the cohort lost before the ETS year and fraction of the cohort that extended to predict f12stay (f12stay is the fraction of the ETS cohort that would remain in the Air Force beyond their original ETS). An ETS cohort is all the airmen in the Air Force at ETS - (1 year). The first step in our refitting process was to fit their regression models with additional years of data. We used a 30 percent sample from the YAR data file (see Murray et al., 1989) for the refitting. A time plot of f12stay showed a dramatic change in this fraction over time (see timeplot in App. C).

Since the behavior of draft-induced airmen is different from that of airmen who entered the force after the draft, the updated models were fitted for airmen who entered after June of 1973. Four-year enlistees therefore had ETS greater than 7706 and six-year enlistees had ETS greater than 7906. The last year of data in our YAR files (those with ETS between 8307 and 8406 inclusive) was reserved for validation.

Regression models were first fitted to the data for the four decision groups identified by Adams and Chaiken:

1. High school graduates with TOE = 4.
2. Non-high school graduates with TOE = 4.
3. High school graduates with TOE = 6.
4. Non-high school graduates with TOE = 6.

Plots of the residuals (actual value - fitted value) were not white noise but showed definite patterns (see App. D). Therefore, time series models were fitted to the data. In the 1980s few airmen with TOE = 6 did not have a high school diploma. Their behavior was similar to that of six-year enlistees with high school diplomas. Therefore, we combined groups 3 and 4 and used the following three decision groups for fitting the time series models:

- I. High school graduates with TOE = 4.
- II. Non-high school graduates with TOE = 4.
- III. All airmen with TOE = 6.

The time series models predict:

f##stay Fraction of the original ETS cohort who were present (includes reenlists and early outs) at METS = ## and are still in the force at ETS.

f12reup Fraction of the cohort present at original ETS who have reenlisted.

Decision Group I: High school graduates with TOE = 4

Of the three decision groups, this one contained the largest number of airmen. The model fitted was first order autoregressive. While the means, the coefficient of the autoregressive term, and the constant change little from one month to the next, it is informative to note the trend. The mean and autoregressive coefficient decrease as the months to ETS increases. The constant term is just the opposite.

Decision Group II: Non-high school graduates with TOE = 4

Analysis of the entire time series (7406-8306) showed that loss rates for this group were white noise. Analysis for the time period 8110-8406 also produced loss rates that were white noise. We chose the mean of the latter subset for our loss rates.

Decision Group III: TOE = 6

The loss rates for this subgroup were white noise for the time period 7406-8406. Analysis for the time period 8110-8406 also produced loss rates that were white noise. The loss rate in 8308 was an outlier and was replaced with the average of the loss rates for 8302-8307. The mean reported is for the 30 percent sample 8110-8406.

Table 10 summarizes the loss and reenlistment models giving the form of the model for each decision group and the coefficients.

ADDITION OF GRADE DIMENSION TO FIRST-TERM ETS LOSS MODELS

The addition of new grade variables in YAR3 (S1GRD,S1TG,S2GRD,S2TG) made finding the grade each month for the ETS year much easier than with earlier

Table 10

FIRST-TERM ETS LOSS AND REENLISTMENT MODELS

Model	Decision Group I First Order Autoregressive			Decision Group II Constant Rate	Decision Group III Constant Rate
	mean	ar1	constant	mean	mean
f1stay	.6824	.4856	.3510	.6498	.7271
f2stay	.6797	.4731	.3582	.6435	.7271
f3stay	.6757	.4743	.3552	.6389	.7271
f4stay	.6728	.4547	.3669	.6339	.7253
f5stay	.6690	.4280	.3827	.6315	.7230
f6stay	.6661	.4087	.3939	.6172	.7196
f7stay	.6628	.3938	.4018	.6101	.7155
f8stay	.6593	.3666	.4176	.6091	.7155
f9stay	.6562	.3638	.4175	.6010	.7022
f10stay	.6529	.3601	.4177	.5925	.6987
f11stay	.6491	.3650	.4122	.5877	.6979
f12stay	.6454	.3577	.4145	.5833	.6906
f12reup	.6687	.9705	.0197	.5360	.5978

editions of the YAR or ETS files (see Murray et al., 1989). If the airman was not lost, the new variables enabled us to calculate the exact grade and time in grade for each month of the ETS year. If the airman was lost, and XGRADE (the grade on the loss transaction) was not equal to the last snapshot grade, then the month in which the grade changed was estimated as follows: We computed the time T between the last snapshot and the end of the term and chose a random number D from the uniform distribution (0-T). The time in XGRADE was set equal to D.

The distribution of grades for each month of the ETS year remained stable over the time period 7407-8006. The loss rates by grade also remained stable over time. Most airmen were in grade E-4 in their ETS year and most losses came from grade E-4. Thus, the grade distributions given in Table 11 can be used to assign a grade to first term ETS losses.

The following is an example of how to use the grade distributions in Table 11 to produce ETS loss rates by grade. Table 12 shows a hypothetical distribution of losses for cohorts who are in their 36th, 37th, 38th, and 39th month of service when the prediction begins. The model discussed in the first part of this section forecast the losses through the end of the ETS year for each cohort. With the grade distribution in Table 11, it is

Table 11

FIRST-TERM GRADE DISTRIBUTION BY METS

TOE (yrs)	Grade	METS												
		12	11	10	9	8	7	6	5	4	3	2	1	0
4	E-3	.07	.07	.06	.05	.04	.04	.04	.03	.03	.03	.03	.03	.03
	E-4	.93	.93	.94	.95	.95	.95	.95	.96	.96	.96	.96	.95	.94
	E-5	.00	.00	.00	.00	.01	.01	.01	.01	.01	.01	.01	.01	.02
6	E-3	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.01	.01
	E-4	.64	.63	.62	.60	.58	.57	.57	.56	.55	.53	.52	.53	.53
	E-5	.36	.37	.38	.40	.42	.43	.43	.44	.44	.46	.47	.46	.46

Table 12

PREDICTED NUMBER OF LOSSES BY METS FOR FOUR FIRST-TERM ETS COHORTS

Cohort (months)	METS												
	12	11	10	9	8	7	6	5	4	3	2	1	0
36	14	16	13	17	21	24	32	35	41	53	61	70	119
37		17	15	14	22	23	31	37	43	51	62	74	126
38			16	13	19	25	29	36	42	49	65	72	123
39				15	20	25	30	35	40	50	60	70	120

possible to apportion these losses by grade. Table 13 shows the distribution of the predicted losses by grade for this example. Exercise caution when distributing the losses by grade, because rounding errors can make the totals in Table 13 not sum to the numbers in Table 12.

FIRST-TERM REENLISTMENT MODELS

Because 99 percent of all reenlistments in the first term occur during the ETS year, the short-term reenlistment model covers only the last 12 months of the first term. Table 14 shows how reenlistments have been distributed over these 12 months based on data in the YAR3 file for the period June 1978 through September 1983. The f12reup model (discussed above) predicts the fraction of airmen in the force at ETS who will have reenlisted during the ETS year. To use the model, multiply the forecast value of f12reup

Table 13

PREDICTED NUMBER OF LOSSES BY GRADE AND METS
FOR FOUR FIRST-TERM ETS COHORTS

MOS	Grade	METS												
		12	11	10	9	8	7	6	5	4	3	2	1	0
36	E-3	1	1	1	1	1	1	2	1	2	2	2	2	4
	E-4	13	15	12	16	20	23	30	34	39	51	58	67	112
	E-5	0	0	0	0	0	0	0	0	0	0	1	1	3
37	E-3		1	1	1	1	1	1	2	2	2	3	4	
	E-4		16	14	13	21	22	30	36	41	48	59	70	118
	E-5		0	0	0	0	0	0	0	1	1	1	4	
38	E-3			1	1	1	1	1	1	2	2	2	4	
	E-4			15	12	18	24	28	35	41	47	62	69	116
	E-5			0	0	0	0	0	0	0	1	1	3	
39	E-3				1	1	1	1	1	1	2	2	4	
	E-4				14	19	24	29	34	39	48	57	67	112
	E-5				0	0	0	0	0	1	1	1	4	

Table 14

FIRST-TERM REENLISTMENTS BY METS
(Percent)

METS												
12	11	10	9	8	7	6	5	4	3	2	1	0
4.78	8.00	7.01	6.21	5.74	5.67	5.76	5.59	5.27	9.80	8.60	8.64	18.93

SOURCE: Data in YAR3 file for period June 1978 through September 1983.

by the forecast number of airmen who will be present at ETS. This yields the total number of reenlistments predicted to occur by ETS. Then, by using the Table 14, the reenlistments can be distributed back over the ETS year.

For cohorts that have entered the ETS year at the time forecasting begins, one can predict the number of reenlistments expected in each of the remaining months of the ETS year using the prediction of the total number of reenlistments expected to occur among

the members of the ETS cohort from METS = 12 to the end of METS = 0. Subtracting the reenlistments that have already occurred by the end of METS = k yields the number of reenlistments to be distributed over the remaining months. This is accomplished by conditioning the values in Table 14 by the number of months remaining. For example, if data are available through the end of METS = 2, and 180 predicted reenlistments remain to be distributed, they will be modeled as occurring in METS = 1 and METS = 0 as follows:

$$\text{METS} = 1: 180 [8.64 / (18.93 + 8.64)] = 56$$

$$\text{METS} = 0: 180 [18.93 / (18.93 + 8.64)] = 124$$

Once the monthly number of reenlistments have been predicted, they can be disaggregated by grade using the model presented earlier in this section.

ADDITION OF GRADE DIMENSION TO FIRST-TERM REENLISTMENT MODELS

Analysis showed that almost all reenlistments from four-year terms of enlistment came from grade E-4 during the first eight months of the ETS period. By METS = 0, 8 percent of the reenlistments came from grade E-5 when TOE = 4 (see Table 15). Because airmen with TOE = 6 have been in the force for five years when they reach their ETS year, more of them have been promoted to grade E-5. Hence, we observe many more reenlistments in grade E-5 for TOE = 6. The change in distribution of grade during the ETS year for these airmen is notable. At the beginning of the ETS year, more than

Table 15

DISTRIBUTION OF FIRST TERM REENLISTMENTS BY GRADE AND METS

TOE (yrs)	Grade	METS												
		12	11	10	9	8	7	6	5	4	3	2	1	0
4	E-3	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
	E-4	1.00	.99	.99	.99	.99	.99	.98	.99	.97	.97	.94	.93	.92
	E-5	.00	.01	.01	.01	.01	.01	.02	.01	.03	.03	.06	.07	.08
6	E-3	.00	.00	.01	.00	.01	.00	.00	.00	.00	.00	.00	.00	.00
	E-4	.64	.55	.61	.56	.51	.47	.50	.45	.45	.37	.42	.30	.31
	E-5	.36	.45	.39	.44	.49	.53	.50	.53	.54	.63	.57	.70	.68

half of the reenlistments are in grade E-4; but by METS = 5, more than half of the reenlistments are in grade E-5. Promotion policy has a big influence on these percentages, so these distributions should be examined regularly for possible changes. Unless Air Force regulations change, it is unlikely that there will be many reenlistments from grade E-3, because these airmen are not generally eligible for reenlistment.

V. SECOND-TERM ATTRITION MODELS

From the research conducted while fitting the middle-term loss models (Carter et al., 1987) we learned that second-term attrition losses are small and stable over time. Further analysis revealed that almost all second-term ETS losses occurred within three months of ETS. Thus, the attrition model covers all but the last three months of the term.

We created a monthly loss rate file using the YAR file for the time frame 7407-8406. Analyses of the data in this file showed: (1) attrition loss rates are lowest in the first year of the term and gradually rise during the term (see Table 16); and (2) more E-4s attrit than E-5s (see Table 17), but the attrition loss rate is highest for E-3s and below. Almost all second-term airmen are in grades E-4 and E-5. These tables emphasize the necessity of examining loss rates rather than loss flows. For example, in Table 16 the average losses for the years preceding the ETS year are higher than in the ETS year, yet the loss rate is lower because there are more people in the force before the ETS year.

Models were fitted on the second-term attrition data (7407-8406) for the following groups of airmen:

Table 16

LOSSES BY METS IN THE SECOND TERM

METS	Total Losses 7406-8406	Total Present 7406-8406	Avg. Loss per Month	Avg. Total per Month	lrate
	(1)	(2)	(3)=(1)/85	(4)=(2)/85	(3)/(4)
3	89	11659	1.03	135.57	.0076
4	92	12625	1.07	146.80	.0073
5	66	8975	.77	104.36	.0074
6	71	10200	.83	118.60	.0070
7	73	9338	.85	108.58	.0078
8	58	7352	.67	85.49	.0079
9	62	6466	.72	75.19	.0096
10	60	7954	.70	92.49	.0075
11	55	7650	.64	88.95	.0072
12	68	9206	.79	107.05	.0074
12<METS<25	919	364452	10.69	4237.81	.0025
METS>24	1999	1094319	23.24	12724.64	.0018

Table 17

LOSSES BY GRADE IN THE SECOND TERM

Grade	Total Losses 7406-8406	Total Present 7406-8406	Avg. Loss per Month	Avg. Total per Month	Irate
	(1)	(2)	(3)=(1)/85	(4)=(2)/85	(3)/(4)
E-1/E-2	18	45	.21	.52	.4000
E-3	16	65	.19	.76	.2462
E-4	2456	808909	28.56	9405.92	.0030
E-5	1118	740707	13.00	8612.87	.0015
E-6	4	470	.05	5.47	.0085

METS	Grades
> 24	E-4, E-5
13-23	E-4, E-5
12	E-4, E-5
11	E-4, E-5
10	E-4, E-5
9	E-4, E-5
8	E-4, E-5
7	E-4, E-5
6	E-4, E-5
5	E-4, E-5
4	E-4, E-5
3	E-4, E-5

For months to ETS greater than 24 and grade E-4 a first-order autoregressive lag 2 model fitted the data well.

mean .0023
 ar1 .3181
 ar2 .2523
 constant .0010

All other groups showed white noise. A 12-month straight line moving average model should be used for each of them.

VI. SECOND-TERM ETS MODELS

The original analysis of second-term loss rates in the ETS period was performed by Adams and Chaiken (unpublished RAND research). We kept the form of the original models and refitted them with additional years of data. We also added a grade dimension.

SECOND-TERM ETS LOSS MODELS

The time horizon for the second-term ETS losses models is the last three months of the term. There is a model to cover each of the possible horizons (prediction one, two, and three months in advance). The models predict the fraction of those who are present at the end of month m who will still be present at the end of month n . Figure 8 depicts the six second-term ETS submodels.

In all six cases, retention increased between 7407 and 8406. We attempted to eliminate as many of the extraneous influences on the data as possible. For this reason, the models were fitted to data for only 7610-8306. We chose this time frame for two reasons:

1. The fiscal year ended in September during this time.

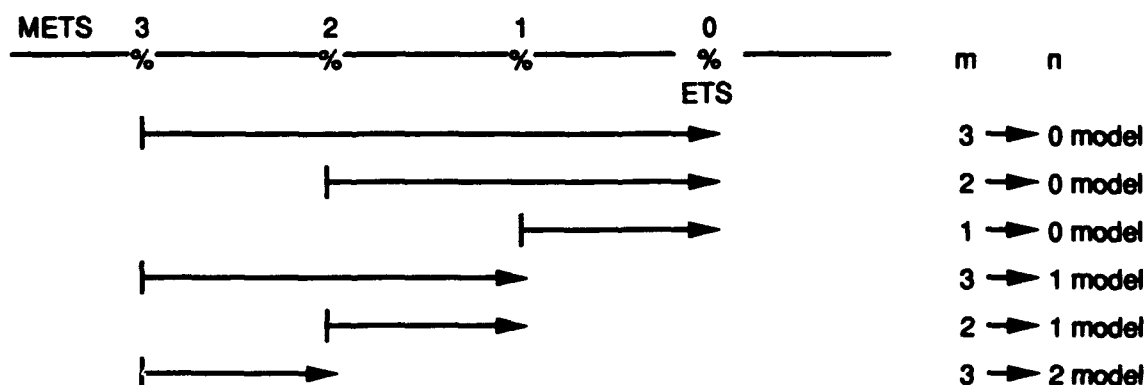


Fig. 8—The six second-term ETS loss models

2. This left one year of data for validating the models.

The variables analyzed were

FRCSTAm Fraction of the ETS cohort who were present at (ETS - m) months and were still in the force at ETS.

FRCm_n Fraction of the ETS cohort who were present at (ETS - m) months and were still in the force at (ETS - n) months (n = 0, 1, 2).

The plots of the fraction staying over time for the three $m \rightarrow 0$ models (FRCSTAm) were remarkably similar. The plot of FRCSTA3 from July 1977 to May 1984 revealed that the retention rate was increasing steadily during this time frame and had no seasonal patterns (see Fig. 9). Compare it with the plot of FRC3_1 (see Fig. 10). The latter plot is white noise with no pattern to the data.

We explored various time series models using PROC ARIMA in SAS. The graphs of the autocorrelations and partial autocorrelations revealed that the best fitting models fell into two types: second order autoregressive and constant rate.

The Autoregressive Models

Months	Mean	a1	a2	Constant
3 → 0	.8897	.1737	.3472	.4262
2 → 0	.8921	.1784	.3522	.4188
1 → 0	.8956	.1948	.2941	.4578

The Constant Rate Models

Months	Constant
2 → 1	.9966
3 → 2	.9970
3 → 1	.9936

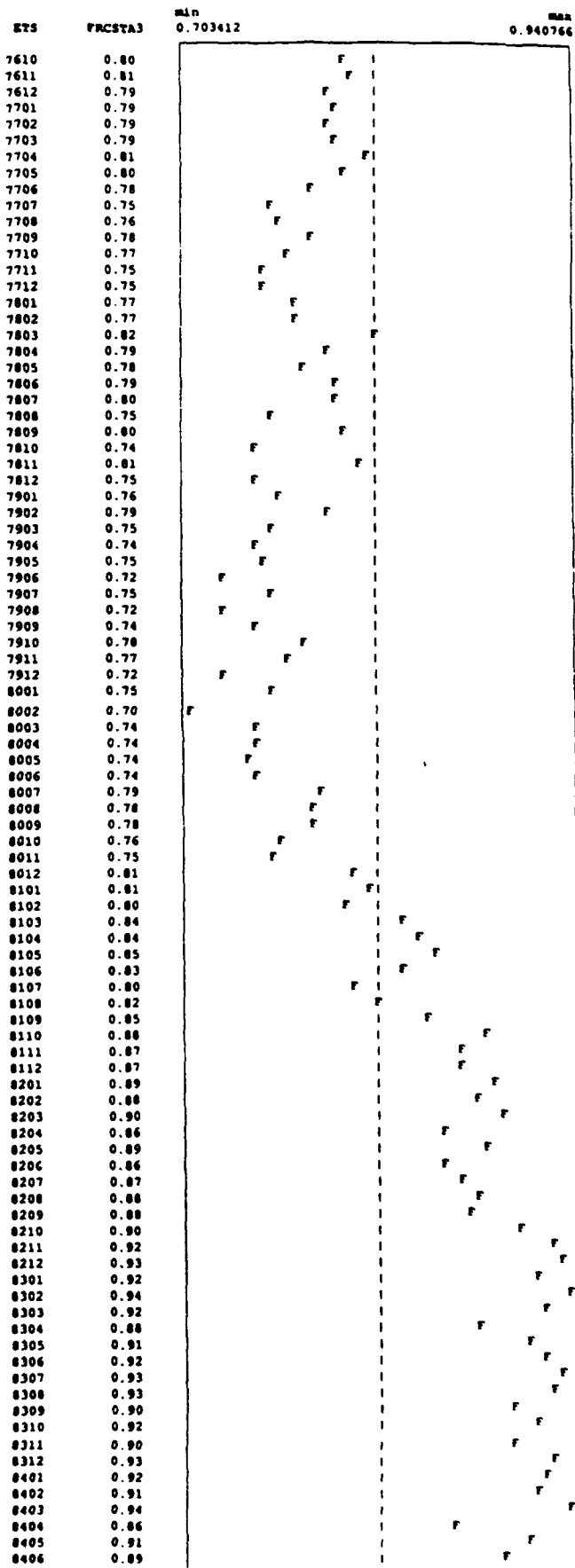


Fig. 9—Time plot of the fraction of the ETS cohort who were present at ETS - 3 months and were still in the force at ETS - 0

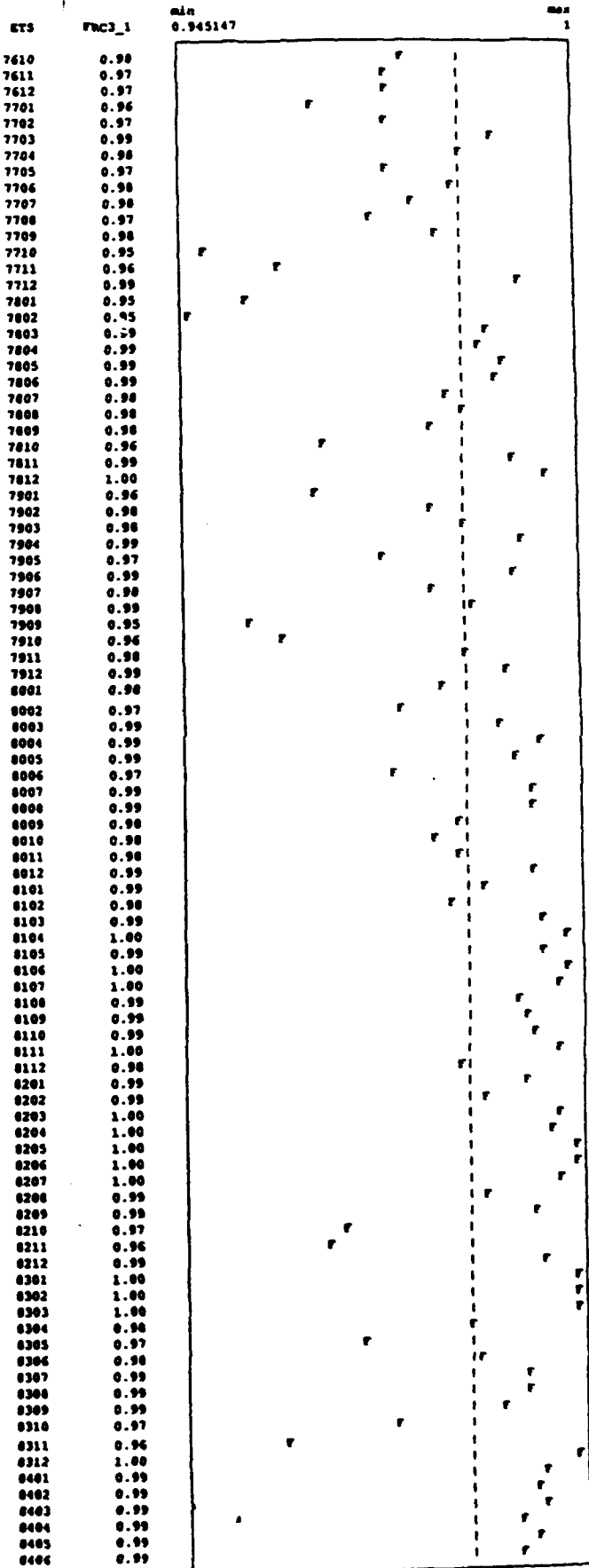


Fig. 10—Time plot of the fraction of the ETS cohort who were present at ETS - 3 months and were still in the force at ETS - 1

**ADDITION OF GRADE DIMENSION TO SECOND-TERM
ETS LOSS MODELS**

The middle-term aggregate loss model (Carter et al., 1987) makes it possible to calculate the marginal effects of grade on loss rates. The coefficients of pertinent variables in the middle term aggregate's second-term ETS model are given below:

YOS = 6	.082
YOS = 7	.063
YOS = 8	.023
YOS = 9	.000
GRADE = E-5	-.077
GRADE ≥ E-6	-.151

With these coefficients, the appropriate cells in Table 18 can be calculated. Letting a, b, c, and d be the relative weights for an airman in grade E-4 with YOS = 6, 7, 8, and 9 respectively, the matrix in Table 18 displays the weights for each pertinent grade/YOS combination in terms of a, b, c, and d. Solving the equations in Table 18 for a, b, c, and d produces the relative weights shown in Table 19. To use Table 19 at any time, we need the number of people in each cell. When running the models, we will know these numbers. Assume that they have the values shown in Table 20.

Table 18
WEIGHTS FOR EACH GRADE/YOS COMBINATION
FOR SECOND-TERM ETS LOSSES
(In terms of a, b, c, d)

Grade	YOS				Marginal Probabilities
	6	7	8	9	
E-4	a	b	c	d	
E-5	a-.077	b-.077	c-.077	d-.077	-.077
E-6+	a-.151	b-.151	c-.151	d-.151	-.151
Marginal Probabilities	.082	.063	.023	.000	

Thus,

$$d = a - .082 = a - .082$$

$$c = a + .063 - .082 = a - .019$$

$$b = a + .023 - .082 = a - .059$$

Table 19

RELATIVE WEIGHTS FOR EACH GRADE/YOS
COMBINATION FOR SECOND-TERM
ETS LOSSES

Grade	YOS			
	6	7	8	9
E-4	a	a-.059	a-.019	a-.082
E-5	a-.077	a-.136	a-.096	a-.159
E-6+	a-.151	a-.210	a-.170	a-.233

Table 20

VARIABLES REPRESENTING NUMBER OF AIRMEN
IN EACH GRADE/YOS CELL

Grade	YOS			
	7	8	9	10
E-4	w ₁	w ₂	w ₃	w ₄
E-5	w ₅	w ₆	w ₇	w ₈
E-6+	w ₉	w ₁₀	w ₁₁	w ₁₂

where w_k = number airmen in each cell k.
 $(a + x_k)$ = value, from Table 19 cell k, expressed in terms of a.
 $w_k (a + x_k)$ = predicted number lost in cell k.

Hence, the overall loss rate is

$$p = \sum [w_k(a + x_k)] / \sum w_k$$

In this equation, the only unknown is a; p is known from the short-term loss model's prediction (see first part of Sec. VI), and w_k is known as stated above.

The fraction lost would be calculated by

$$F_k = w_k(a + x_k) / \sum (w_k).$$

To apply this fraction lost to the output from the loss models described in the first part of Sec. VI we need to "normalize."

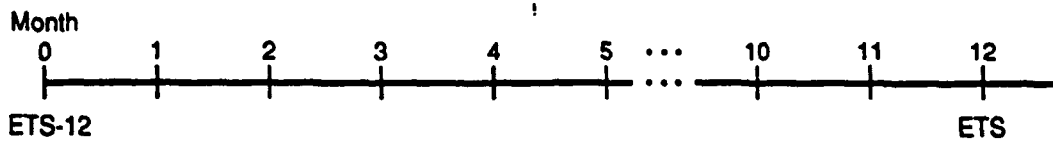
If z_k = proportion lost in grade/YOS cell k, then

$$z_k = p(F_k / \sum (F_k)).$$

Thus, $\sum_k z_k = p$.

SECOND TERM REENLISTMENT MODELS

Of all reenlistments at the end of the second term 96 percent occur in the ETS year. Hence, we modeled only reenlistments in the ETS year. The variable being predicted is the cumulative fraction of airmen still present in month m who will have reenlisted by the end of month m ($freup(m,0)$). Table 21 summarizes the resulting models. The models for all months except month 1 of the ETS year are first-order autoregressive. The model for month 1 is a constant rate model.



To use the models, the following variables must be defined:

Table 21

SECOND TERM REENLISTMENT MODELS

Month	Parameters of Model	
	a1	constant
1	—	.0317
2	.5482	.0268
3	.6598	.0280
4	.7630	.0233
5	.8870	.0106
6	.9313	.0067
7	.7726	.0359
8	.8537	.0248
9	.9607	.0070
10	.9823	.0036
11	.9827	.0043
12	.7400	.9760

- freup(m,0):** cumulative fraction of airmen present in month m and time now who will have reenlisted by the end of month m.
- freup(m,1):** cumulative fraction of airmen present in month m and time now - 1 who will have reenlisted by the end of month m.
- nreup(m):** number of reenlists in month m and time now.
- inv(m,0):** inventory in month m and time now.

All first-order autoregressive models use the fraction of people in the same positions last month (freup(m,1)) to predict the number this month (freup(m,0)). Thus the prediction formula for the cumulative fraction of reenlistments is

$$\text{freup}(m,0) = \text{freup}(m,1)(a1 + c).$$

Calculating the number of reenlistments requires knowing the inventory this month (inv(m,0)) and the fraction expected to reenlist this month. The formula for the predicted number of reenlistments is

$$\text{nreup}(m) = [\text{freup}(m,0) - \text{freup}(m - 1,1)] \text{inv}(m,0).$$

ADDITION OF GRADE DIMENSION TO SECOND-TERM REENLISTMENT MODELS

Predicting the number of reenlistments by grade requires two steps. First, multiply the inventory by the predicted fraction of reenlistments to obtain the number of reenlistments in each month of the ETS year. Then multiply this predicted number by the fraction in Table 22.

Table 22

SECOND-TERM GRADE DISTRIBUTION BY METS

Grade	METS												
	12	11	10	9	8	7	6	5	4	3	2	1	0
E-4	.32	.44	.19	.44	.41	.39	.20	.24	.19	.17	.16	.09	.09
E-5	.68	.56	.81	.56	.59	.61	.80	.76	.81	.80	.82	.86	.86
E-6	.00	.00	.00	.00	.00	.00	.00	.00	.00	.02	.02	.04	.05

SOURCE: Data in YAR3 file for period June 1978 through September 1983.

VII. CAREER TERM ATTRITION MODELS

Our analysis of losses during the career term of enlistment revealed that almost all ETS losses occurred in the last month of the ETS year. Thus the attrition loss model covers the entire term, excluding only the ETS month.

From the research conducted while fitting the middle-term loss models we gleaned valuable information about career attrition. Losses in this category are small and stable over time because of the relative homogeneity of the career force. For modeling purposes, we created a monthly loss rate file using the YAR data file for the time frame 7407-8006. This file included all career airmen with TOE = 4, 5, or 6 years. In addition we restricted the airmen to have more than nine years of service and less than 240 months of service at their ETS point. Regression models similar to the middle-term loss models were fitted to the career term loss rates. Then we made plots of

1. Actual monthly losses.
2. Monthly losses predicted from new regressions.
3. (Middle-term predicted losses)/12.

Predictions from the two regression models closely fitted the actual losses. For simplicity we decided it was best to use the (middle-term predicted losses)/12 to predict monthly career-term attrition losses. The models for the career grades are given below:

loss rate grade E-4 =

$$\begin{aligned} & ([.0038 + .5589 \exp(-YOS/2)] && + \\ & [-.0012 \pm .2369 \exp(-YOS/2)] YOETS && + \\ & [.0015 + .1201 \exp(-YOS/2)] TOE && + \\ & [.0319 \pm .4357 \exp(-YOS/2)]/12 && + .0027/12 \end{aligned}$$

loss rate grade E-5 =

$$\begin{aligned} & ([.0038 + .5589 * \exp(-YOS/2)] && + \\ & [-.0012 \pm .2369 * \exp(-YOS/2)] YOETS && + \\ & [.0015 + .1201 * \exp(-YOS/2)] TOE)/12 && + .0027/12 \end{aligned}$$

loss rate grade E-6, E-7, E-8, or E-9 =

$$\begin{aligned} & ([.0038 + .5589 * \exp(- YOS/2)] && + \\ & [-.0012 \pm .2369 * \exp(- YOS/2)] YOETS && + \\ & [.0015 + .1201 * \exp(- YOS/2)] TOE && + \\ & [-.0046 + .0691 * \exp(- YOS/2)]/12 && + .0027/12 \end{aligned}$$

where YOETS = number of years to original ETS.

VIII. CAREER-TERM ETS MODELS

CAREER TERM ETS LOSS MODELS

The original analysis of career-term loss rates in the ETS period was performed by Chaiken and Adams (unpublished RAND research), but they did not specify reenlistment models, nor did their models predict ETS losses by grade. The models described here are substantially different from their original models for two reasons. First, the models as originally documented required knowledge of terms of enlistment other than TOE = 4 and TOE = 6. Thus, implementation would have required much more "bookkeeping" for the IPM. Second, our analysis indicated that knowledge of YOS was sufficient for good loss prediction.

The data for the new career ETS models came from a 30 percent sample of all airmen in the force from 7306 to 8406. Only career airmen with more than nine but fewer than 20 years of service at ETS were selected. The model was actually fitted to data from 7610-8306, so that the last year of data could be used to verify the accuracy of the forecasts.

The career ETS models are time-series models for predicting how many careerists present at the end of the calendar month preceding their ETS will still be present at the end of the next calendar month (after their ETS has passed). The variable OETSYOS is the number of years of service that the airman will have completed by the end of the month of ETS; it is an integer less than or equal to 19. TOE is the term of enlistment.

The prediction is for

$$\text{FRAC} = (\# \text{ present at ETS}) / (\# \text{ present at ETS} - 1 \text{ month})$$

There are 11 career-term ETS loss models (one for each of the values of OETSYOS from 9 to 19). Four of them were determined to be white noise, so we used a constant loss rate model is used. The remaining 7 are third-order autoregressive models. The white noise models are presented in Table 23. The autoregressive models are presented in Table 24.

Table 23

WHITE NOISE CAREER-TERM
ETS MODELS

OETSYOS	Constant
≤ 9	.9287
17, 18, 19	.9982

Table 24

THIRD-ORDER AUTOREGRESSIVE CAREER-TERM ETS MODELS

OETSYOS	Constant	a1	a2	a3	Mean
10	.1859	.2023	.4167	.1827	.9369
11	.3463	.3401	.2162	.0801	.9523
12	.2390	.1721	.1586	.4237	.9726
13-16	.5284	.2125	.0910	.1642	.9927

ADDITION OF GRADE DIMENSION TO CAREER TERM ETS LOSS MODELS

Using the middle-term aggregate loss model (Carter et al., 1987), we can calculate the effects of grade on loss rates. We tried to use the middle-term model whenever possible, because the goal is to have the short-term model and the middle-term model producing consistent estimates. Losses in the career term are so stable from month to month that we are able to use the yearly projections from the middle-term model successfully.

To calculate the proportion lost in each grade (E-5 through E-9), it is necessary to first calculate the variables A, R6, R79, L5, L6, L7, L8, L9, and TOTLOS using the following formulas:

$$\begin{aligned}
 A = & [.0094 + 163.1210 \times \exp(-YOS/2)] + \\
 & [-.0001 + 2.8924 \times \exp(-YOS/2)] \times .3098 + \\
 & [.0001 + -1.3101 \times \exp(-YOS/2)] \times .2882 + \\
 & [.0001 + -1.1924 \times \exp(-YOS/2)] \times .1175 + \\
 & [-.0002 + 5.1970 \times \exp(-YOS/2)] \times .0000 + \\
 & [-.0126 + -2.1246 \times \exp(-YOS/2)] \times .4700 + \\
 & [-.0133 + -4.9089 \times \exp(-YOS/2)] \times .1583 +
 \end{aligned}$$

$$\begin{aligned} & [.0001 + -12.5183 \times \exp(- YOS/2) \times \log(.4999)] + \\ & [.0018 + -40.0881 \times \exp(- YOS/2) \times \log(.4999)] \end{aligned}$$

$$R6 = -.0126 + -2.1246 \times \exp(- YOS/2)$$

$$R79 = -.0133 + -4.9089 \times \exp(- YOS/2)$$

$$L5 = (A)$$

$$L6 = (A - R6)$$

$$L7 = (A - R79)$$

$$L8 = (A - R79)$$

$$L9 = (A - R79)$$

$$TOTLOS = L5 + L6 + L7 + L8 + L9$$

The following formulas are then used to distribute the losses by grade.

$$E-5 \text{ loss} = (A)/TOTLOS$$

$$E-6 \text{ loss} = (A - R6)/TOTLOS$$

$$E-7 \text{ loss} = (A - R79)/TOTLOS$$

$$E-8 \text{ loss} = (A - R79)/TOTLOS$$

$$E-9 \text{ loss} = (A - R79)/TOTLOS$$

CAREER TERM REENLISTMENT MODELS

It was originally argued that a career-term reenlistment model was not necessary because if an airman was in his career term and not lost, he remained in the career term until retirement eligible. The short-term IPM, however, needs to predict the proportion of the force that will reenlist and the proportion that will go on extension. Thus, a career-term reenlistment model is necessary. Data on airmen with ETS between 7906 and 8406 were used to fit the following models.

Decision Group I: OETSYOS \geq 10 and TOE = 4, 5, or 6

The probability of reenlisting is a first order autoregressive model with

$$\text{mean} = 0.7363$$

$$\text{ar1} = 0.5303$$

$$\text{ar2} = 0.0697$$

$$\text{constant} = 0.2946$$

Decision Group II: OETSYOS \geq 10 and TOE 2 or 3

The only reenlistments that occurred in this group were those with OETSYOS = 19 (with the exception of one or two who had OETSYOS = 18). Several models were explored. The average value for the last five years predicted as well as any other model.

probability reenlistment (OETSYOS < 19) = 0

probability reenlistment (OETSYOS = 19) = .5433

Decision Group III: OETSYOS < 10

There are very few people in this subgroup after 7906. After looking at the data by OETSYOS and OETS and trying several models, we found that the average value predicted as well as any other model.

probability reenlistment =

total reenlistments 7906-8406 / total number of airmen reaching ETS = .6813

ADDITION OF GRADE DIMENSION TO CAREER-TERM REENLISTMENT MODELS

After studying the pattern of reenlistments by grade and decision group, it was decided to use the actual grade distributions to apportion career term ETS losses by grade. The distributions are in Table 25 and were based on data from airmen who reached ETS between 7906 and 8406.

Table 25

GRADE DISTRIBUTIONS FOR CAREER-TERM REENLISTMENT MODELS

Decision Group	Grade				
	E-4	E-5	E-6	E-7	E-8+
I	.00	.28	.49	.21	.02
II	.00	.03	.61	.30	.06
III	.08	.86	.06	.00	.00

IX. EXTENSION SEPARATION MODELS

For an airman to go on extension he must first sign an extension contract specifying a new date of separation (DOS). The Air Force defines a person to be on extension when this contract is signed. If an airman decides to sign such a contract before his ETS and then reenlists at his ETS, no distinction can be made between this airman and one who has not signed an extension contract. For this reason, in the EFMS we have defined an airman to be on extension only if he remains in the service in a given term after his original ETS. We also do not consider any airmen with YOS \geq 20 who are eligible for retirement to be on extension.

Modeling extension separations requires nine models. Each model predicts the proportion of the population on extension status that will separate during a given month. The nine extension separation models are:

	First term	Second term	Career term
Attrition losses	1A	2A	3A
Nonattrition losses	1N	2N	3N
Reenlistments	1R	2R	3R

MODELING CONSIDERATIONS

In each month of an extension, an airman has three choices: (1) leave the Air Force, (2) reenlist into another term, or (3) stay on extension (this includes re-extending). There is no limit to the number of extensions that an airman can request, and he can reenlist at any time.

An airman on extension is constrained by the following:

- First term airmen may not have an extension, or combination of extensions, in excess of 23 months.

- Second-term and career-term airmen¹ may not have an extension, or combination of extensions, in excess of 48 months.
- Airmen may extend only for reasons that are in the best interest of the Air Force (to complete technical school training, to get an overseas assignment, promotion eligibility).²

In most cases an airman cannot leave the Air Force before his DOS, which is the date an extension contract ends. For this reason, the proportion of airmen separating should vary as a function of the number of months to DOS. However, the DOS variable presents considerable prediction and implementation problems.

Airmen can extend as many times as they wish, provided the combined lengths of the extension are within acceptable limits. In addition to predicting losses and reenlistments, the months to DOS variable would require the prediction of DOS for those airmen who had not reached extension status at the start of a run of SAM, and predictions for second, third, and higher extensions along with their associated DOSs. Also, including a "Months to DOS" variable would expand the dimensionality of the database by a factor of 23 for first-term airmen and a factor of 48 for all other terms.

The variable Months to Expiration of Term of Service (METS) was found to be a sufficient substitute for DOS in predicting losses and reenlistments for airmen on extension. Airmen on extension status have negative values of METS, since they have passed their ETS (METS = 0 is equivalent to the ETS month). METS values range from -1 (first month on extension) to -23 for the first term, and from -1 to -48 for all other categories of enlistment.

The middle-term model for losses from extension (Carter et al. 1987) divides extendees into two groups:

- a. Nondecisionmakers (those whose DOS occurs after the end of the current fiscal year). We call the loss models for these airmen "attrition loss models."
- b. Decisionmakers (those whose DOS occurs within the current fiscal year). We call the loss models for these airmen "nonattrition loss models."

¹Career-term airmen are defined to be airmen who are in their third or higher term of service but are not yet eligible to retire.

²Before April 1982, first- and second-term airmen were also allowed to extend their enlistments for 3-23 months for "personal reasons."

These same groupings can be used in the short-term modeling. The conjecture is that the behavior of airmen with a decision point coming up during the projection period of the short-term model will be different from that of airmen in the nondecision group.

To capture the explanatory power of the above defined decision groups and to group airmen with somewhat similar reasons for extension, a variable indicating length of the first extension, EXTLEN, is used in the short-term extension models.

EXTLEN = 0 if the length of the first extension is 11 months or less (short extension).
 = 1 otherwise (long extension).

The EXTLEN variable is a proxy for "reason for extension," which we believe is an important predictor of loss behavior.³ For example, airmen who extend to enter technical school are required to extend for 23 months. Similarly, airmen who extend to go overseas generally extend for a long period of time. These airmen can reenlist at any time, but if they choose to leave the Air Force they are required to complete their extension contract. However, airmen who extend for retirement eligibility, promotion eligibility, or (in the past) personal reasons tended to do so for short periods of time (less than a year).

ATTRITION LOSS MODELS

Table 26 identifies the attrition loss models for airmen on extension for each term. It shows the average monthly inventory, losses, and loss rate, as well as the model used for prediction in each term. The number of attrition losses and the attrition loss rates are

Table 26

DATA FOR ATTRITION FROM EXTENSION MODELS

Term	METS Interval	Average Monthly Inventory	Average Monthly Losses	Average Loss Rate (%)	Prediction Model
1	-1,-22	14350.25	59.83	0.41	6-mth running avg.
2	-1,-47	6000.13	12.55	0.21	Constant 0.21%
3	-1,-47	4854.80	8.47	0.17	Constant 0.17%

³"Reason for extension" is not on the YAR file.

small. These data were taken from the YAR file using information from July 1979 through June 1984.

Second-term and career-term loss rates were uniform over time. The rates for first-term airmen exhibited a slight upward trend. A six-month straight line running average model was the model fitted for the first-term attrition loss model. Constant attrition rate models were fitted for the second-term and career-term attrition loss models.

NONATTRITION LOSS MODELS

Each of the three loss models for nonattrition losses are made up of submodels. Table 27 identifies the 13 submodels of the nonattrition loss models. The table also lists the average monthly inventory, average monthly losses, average monthly loss rate, and the prediction model for each of them.

First Term Submodels

Eight submodels make up the first-term nonattrition loss models. Submodels 1N.1 and 1N.2 apply to first-term airmen with short first extensions. These airmen are considered decisionmakers. The submodels were formed based on similar behavior and

Table 27

DATA FOR NONATTRITION FROM EXTENSION SUBMODELS

Sub-model	EXTLEN	METS Interval	Average Monthly Inventory	Average Monthly Losses	Average Loss Rate (%)	Type of Prediction Model
1N.1	0	-1,-6	2561.64	212.29	8.29	12mth running avg. ^a
1N.2	0	-7,-11	814.28	88.33	10.85	12mth running avg. ^a
1N.3	0	-12,-22	534.84	19.05	3.56	Constant = 3.56
1N.4	0	-23	17.33	8.28	47.79	Constant = 47.79
1N.5	1	-1,-11	6448.58	11.65	0.18	Constant = 0.18
1N.6	1	-12	460.17	39.94	8.68	12mth running avg. ^a
1N.7	1	-13,-22	2608.87	75.18	2.88	12mth running avg. ^a
1N.8	1	-23	118.46	70.25	59.30	Constant = 59.30
2N.1	NA	-1,-11	3400.25	58.54	1.77	12mth running avg. ^a
2N.2	NA	-12,-47	2275.37	32.71	1.44	12mth running avg. ^a
2N.3	NA	-48	2.44	0.10	4.84	Constant = 4.84
3N.1	NA	-1,-47	4784.16	28.64	0.60	AR1 ^b
3N.2	NA	-48	1.62	0.08	4.95	Constant = 4.95

^aAdjustment factors for each calendar month are in Table 27.

^bFirst-order autoregressive model: mean = 0.51, a1 = 0.38, constant = 0.32.

similar average monthly nonattrition loss rates by METS. The behavior had a seasonal pattern. The monthly adjustment factors are listed in Table 28. Submodel 1N.3 applies to first term airmen with a short first extension who have re-extended at least one time. Their behavior exhibited no seasonal pattern.

Submodel 1N.5 applies to first term airmen with a long first extension (12 months or more). Airmen in this submodel are considered nondecisionmakers. This submodel was needed because the average monthly nonattrition loss rates are very small.

Submodel 1N.6 applies to first-term airmen with a long first extension at the point METS = -12. A first extension of one year is common in first-term airmen. The average monthly nonattrition loss rate is substantially higher at METS = -12 than at any subsequent value of METS. Submodel 1N.7 includes first-term airmen with a long first extension who are past the METS = -12 point. The loss behavior of airmen in these last two groups exhibited a seasonal pattern. The monthly adjustment factors are listed in Table 28.

The mandatory decision point for first-term airmen is METS = -23. Submodel 1N.4 reflects this decision point for airmen with a short first extension and 1N.8 for a long first extension.

Table 28

MONTHLY FACTORS FOR FIRST- AND SECOND-TERM
RUNNING AVERAGE LOSS FROM EXTENSION MODELS

Month	Submodel					
	1N.1	1N.2	1N.6	1N.7	2N.1	2N.2
Jan	.97	.96	.73	1.21	.76	.82
Feb	.79	.69	.96	.60	.83	.92
Mar	.91	.98	.67	.78	.77	.91
Apr	.84	.87	1.12	1.08	.87	.65
May	1.02	1.07	1.40	1.22	1.09	.96
Jun	1.16	1.09	1.60	1.20	1.21	1.20
Jul	1.25	1.37	1.24	1.37	1.20	1.35
Aug	1.31	1.18	.86	1.08	1.54	1.25
Sep	1.02	.94	1.16	1.04	1.09	1.13
Oct	.85	.79	.74	.72	1.01	1.01
Nov	.88	.92	.61	.70	.73	.76
Dec	1.04	1.05	.84	1.06	.90	.84

Second-Term Submodels

Three submodels make up the second-term nonattrition loss model. These submodels were formed based on similar behavior and similar average monthly losses by METS. Submodel 2N.3 accounts for the mandatory decision point at METS = -48. The monthly adjustment factors for submodels 2N.1 and 2N.2 are given in Table 28.

Career Term Submodels

Two submodels make up the career-term nonattrition loss model. Career-term nonattrition loss rates are small and uniform over time, except at the mandatory decision point (METS = -48). Airmen at the mandatory decision point constitute one submodel (3.N.2), and the remaining airmen in the career term on extension make up the last submodel (3N.1).

REENLISTMENT MODELS

Like the nonattrition models, each of the three extension reenlistment models is made up of submodels. Table 29 identifies the 12 submodels of the reenlistment models

Table 29

DATA FOR REENLISTMENT FROM EXTENSION MODELS

Sub-model	EXT-LEN	METS Interval	Average Monthly Inventory	Average Monthly Reups	Average Reup Rate %	(a)	Prediction Model Parameter Values			
							ar1	ar2	ar3	constant
1R.1	0	-1,11	3375.92	156.32	4.64	AR1	.48	-	-	2.72
1R.2	0	-12,-22	617.94	46.37	7.92	AR3	.22	-.01	.52	1.77
1R.3	0	-23	17.33	9.05	52.21	CON	-	-	-	52.21
1R.4	1	-1,-11	6454.32	211.31	3.27	AR1	.47	-	-	1.65
1R.5	1	-12,-22	3069.04	193.81	6.32	AR1	.50	-	-	2.97
1R.6	1	-23	118.46	48.21	40.70	CON	-	-	-	40.71
2R.1	NA	-1,-11	3400.25	157.15	4.62	AR1	.43	-	-	2.82
2R.2	NA	-12,-47	2275.37	144.55	6.35	AR1	.30	-	-	4.76
2R.3	NA	-48	2.44	2.32	95.16	CON	-	-	-	95.16
3R.1	NA	-1,-12	2952.48	197.03	6.67	CON	-	-	-	6.67
3R.2	NA	-13,-47	1724.40	123.85	7.18	AR1	.39	-	-	4.66
3R.3	NA	-48	1.62	1.54	95.05	CON	-	-	-	95.05

^aAR1 - first order autoregressive.
 AR3 - third order autoregressive.
 CON - constant rate.

and shows the average monthly inventory, average monthly reenlistments, and average monthly reenlistment rate for each of them.

The groupings for the reenlistment submodels were determined in the same manner as for the nonattrition submodels. (See the examples above illustrating how we determined the groupings for the nonattrition submodels.)

X. RETIREMENT SEPARATION MODELS

The purpose of the short-term retirement loss model is to provide loss rates that will allow SAM to project monthly retirements for 1-12 months into the future. This section describes the Air Force's retirement system, past research within the EFMP on retirement prediction, the analysis leading to this retirement model, and the resultant model.

Enlisted persons become eligible to retire when they reach 20 years of service (YOS). The job of the short-term aggregate retirement model is to predict monthly retirements by grade and MOS for up to 12 months into the future. Other separation models for retirement-eligible airmen include an attrition loss model and a reenlistment model.

The data used in this analysis was taken from a 30 percent sample from the YAR file.¹

POPULATION-AT-RISK AND GROUPING

The number of monthly retirements varies within any given year as well as from year to year. Figure 11 shows actual monthly retirements for a 30 percent sample of the enlisted force from June 1976 to May 1982.

The within-year variation shows a seasonal effect. Airmen prefer to retire during the summer.

One explanation of the year-to-year variation in retirements is that the population at risk changes. Figure 12 shows the number of people eligible to retire, or the population at risk each month, from June 1972 to May 1982 for the 30 percent sample. The population at risk declined fairly steadily over the time period from 1977 to 1982.

A second explanation for the year-to-year variation is that the mix of airmen is changing over time. The retirement-eligible population consists of airmen between 20 years of service (240 months of service) and 30 years of service (360 months of service). Institutional policies change the individual's propensity to retire as a function of that individual's month of service.

¹For a description of the data on the file, see Murray et al., 1989.

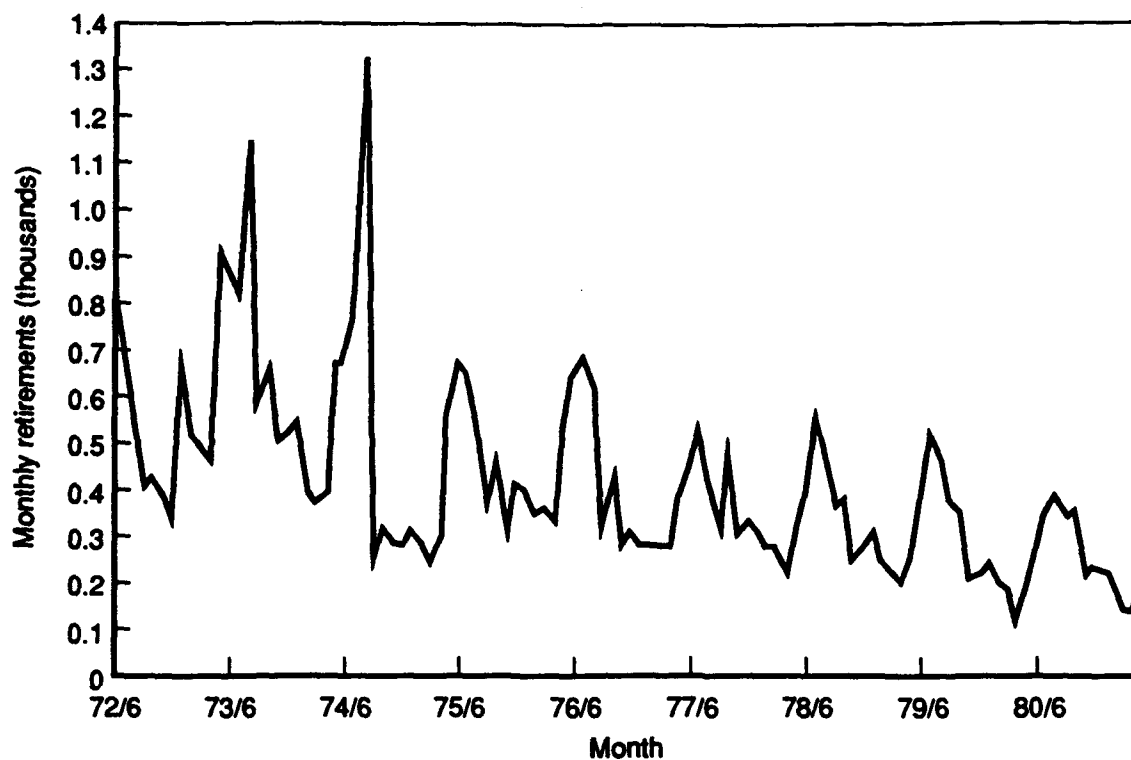


Fig. 11—Monthly retirements (June 1972 to May 1982)

Table 30 shows the 14 MOS groups used in the model. These groups define airmen with the same MOS range. The "Explanation" column of Table 30 gives the reason each MOS distinction is required. For example, group 4 consists of all retirement eligibles with exactly 22 years (264 months) of service. This group exists to account for the fact that, under the current retirement system, waiting for the "pay bogie" month allows airmen to retire with a higher base pay.

The propensity for airmen within the same MOS group to retire varies as a function of grade. Therefore, for modeling purposes we divided retirement eligibles into 35 decision groups and determined a separate retirement rate for each. These 35 groups are simply the MOS groups subdivided by applicable grades. Table 31 identifies the 35 decision groups (defined by MOS group and Grade) and shows the average monthly retirement rate for each group. The table also shows, for each group, the average monthly population at risk and the average monthly retirements based on the 30 percent sample used in the analysis.

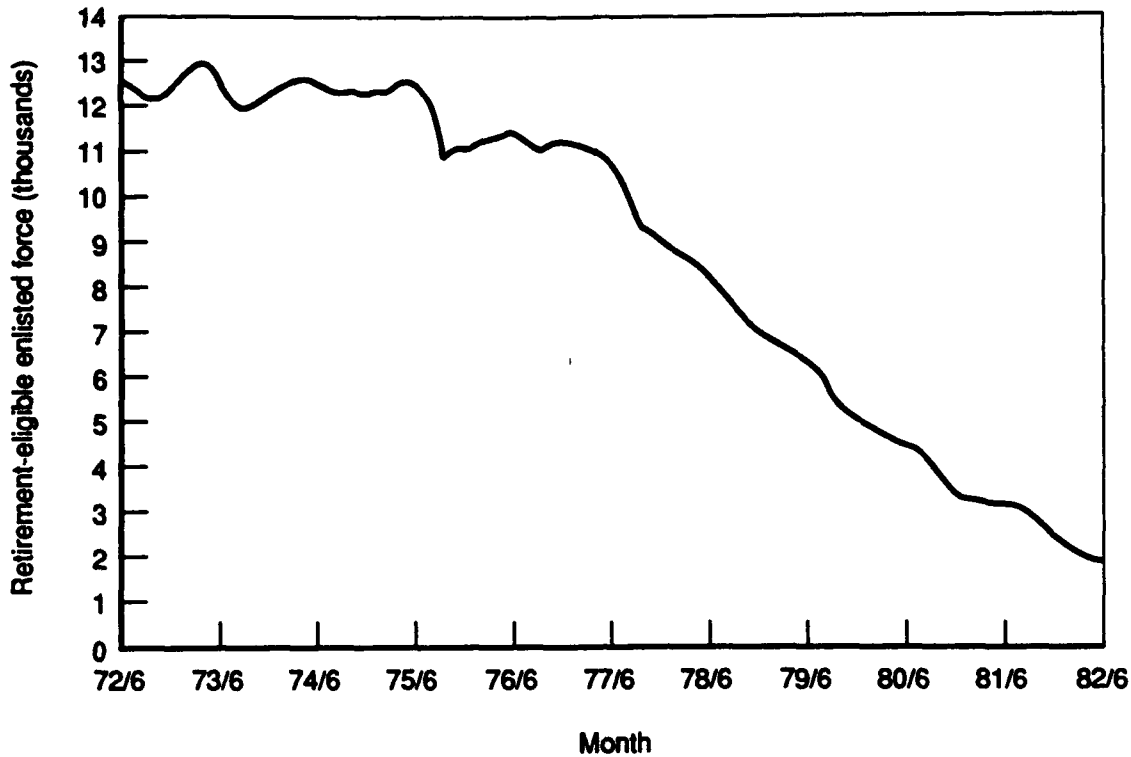


Fig. 12—Retirement-eligible enlisted force (June 1972 to May 1982)

Retirement rates vary considerably by decision group. For example, airmen who are in Grade E-5 at their first opportunity to retire have a 65 percent monthly retirement rate, while airmen who are neither at their first opportunity point nor at a high-year-of-tenure (HYT) point tend to have monthly retirement rates between 2 and 5 percent.

TREND

Table 32 presents yearly retirement rates by decision group for four different 12-month periods. It shows that trends exist in the time series of retirement rates. There are differing rates within groups across the four one-year periods and also between groups. The aggregate rate shown at the bottom of the table is the ratio of the average monthly retirements (for all groups) to the average population at risk for the given year.

Figure 13 shows the retirement rate from June 1972 to May 1982. Although the increased rate during the months June, July, and August is prevalent throughout the ten years of data, the graph shows no recognizable upward or downward trend for these

Table 30

MOS GROUPS FOR RETIREMENT SEPARATION MODELS

MOS Group	MOS Range	Explanation
1	240	First opportunity, E-5 HYT
2	241-243	Three months after first opportunity
3	244-263	Remainder of two-year period (except final month)
4	264	Pay increase
5	265-275	Rest of time to E-6 HYT
6	265-287	Rest of time to pay increase
7	276	E-6 HYT
8	288	Pay increase
9	289-311	Two-year period to pay increase and E-7 HYT
10	312	Pay increase, E-7 HYT
11	313-335	Two-year period to E-8 HYT
12	336	E-8 HYT
13	313-359	Rest of time to E-9 HYT
14	360	E-9 HYT

aggregate rates through the first six years of the time period. However, there is a slight upward trend in the overall retirement rate over the last few years of the time period.

SEASONALITY

A monthly factor, defined as the ratio of the average retirement rate for a month to the overall average retirement rate, was calculated for each of the 12 months. Three sets of factors (shown in Table 33) were calculated:

- All-group factors, which were computed using aggregate (across all groups) rates.
- Inflexible-group factors, which were computed using the rates obtained by pooling across all first-opportunity and HYT decision groups. (Airmen in

Table 31

RETIREMENT SEPARATION MODEL DECISION GROUPS
(June 1976 to May 1982)

MOS Group	Grade	Average Population at Risk	Average Monthly Retirements	Monthly Retirement Rate (%)
1	E-5	38.3	24.9	65.0
1	E-6	150.0	71.2	47.5
1	E-7	125.6	37.8	30.1
1	E-8	21.4	4.0	18.6
1	E-9	3.8	0.6	14.7
2	E-6	218.3	17.0	7.8
2	E-7	258.3	13.5	5.2
2	E-8	54.4	1.8	3.3
2	E-9	10.1	0.3	3.1
3	E-8	854.2	37.7	4.4
3	E-7	1385.5	44.4	3.2
3	E-8	403.7	9.5	2.4
3	E-9	115.4	2.1	1.8
4	E-6	237.8	12.0	5.1
4	E-7	579.6	21.6	3.7
4	E-8	230.6	6.4	2.8
4	E-9	111.5	2.1	1.9
5	E-6	211.5	10.8	5.1
6	E-7	948.3	27.1	2.9
6	E-8	409.8	9.3	2.3
6	E-9	221.4	3.7	1.7
7	E-6	11.1	7.1	63.7
8	E-7	30.2	1.0	3.4
8	E-8	16.3	0.5	2.9
8	E-9	12.9	0.1	1.1
9	E-7	545.6	11.1	2.0
9	E-8	364.7	5.6	1.5
9	E-9	346.4	3.2	0.9
10	E-7	18.1	12.9	71.5
10	E-8	33.7	1.9	5.5
10	E-9	17.3	2.3	13.4
11	E-8	222.1	9.4	4.2
12	E-8	4.6	2.7	58.6
13	E-9	480.8	13.9	2.9
14	E-9	5.1	3.1	59.7
All groups		8698.5	432.8	5.0

Table 32

YEARLY RETIREMENT RATES BY DECISION GROUP
(Percent)

Decision Group	12-Month Period			
	Jun 72 May 73	Jun 75 May 76	Jun 78 May 79	Jun 81 May 82
1.5	52.6	71.8	83.5	76.4
1.6	45.4	38.7	58.7	67.9
1.7	28.4	22.5	31.1	45.6
1.8	16.9	14.5	18.8	26.7
1.9	13.3	13.9	26.3	9.1
2.6	8.7	6.2	6.4	16.6
2.7	5.9	4.3	4.3	9.5
2.8	3.4	2.4	2.7	5.7
2.9	3.0	4.2	2.4	4.7
3.6	4.5	3.8	5.1	9.2
3.7	3.1	2.5	3.6	8.1
3.8	2.1	2.1	2.4	5.9
3.9	1.7	1.9	1.7	4.2
4.6	3.2	5.9	5.5	6.7
4.7	3.1	3.0	4.5	11.1
4.8	2.2	2.7	3.7	6.1
4.9	1.8	1.8	2.8	6.9
5.6	3.0	6.2	5.7	7.3
6.7	2.2	2.4	3.4	9.0
6.8	1.7	2.1	3.0	6.7
6.9	1.6	1.5	2.0	5.4
7.6	3.4	69.3	86.7	94.7
8.7	2.4	2.6	3.9	12.7
8.8	1.3	1.2	4.8	12.1
8.9	0.9	0.0	0.6	8.3
9.7	1.6	1.6	2.4	3.7
9.8	1.5	1.1	2.0	5.4
9.9	1.1	0.9	0.8	2.5
10.7	14.3	81.6	92.4	86.7
10.8	5.4	4.5	5.6	9.8
10.9	8.2	12.5	10.3	33.3
11.8	2.8	6.5	3.7	5.7
12.8	11.9	76.2	83.8	56.7
13.9	2.0	3.2	2.5	5.3
14.9	17.8	82.5	87.7	66.7
All groups	4.7	4.1	5.0	10.6

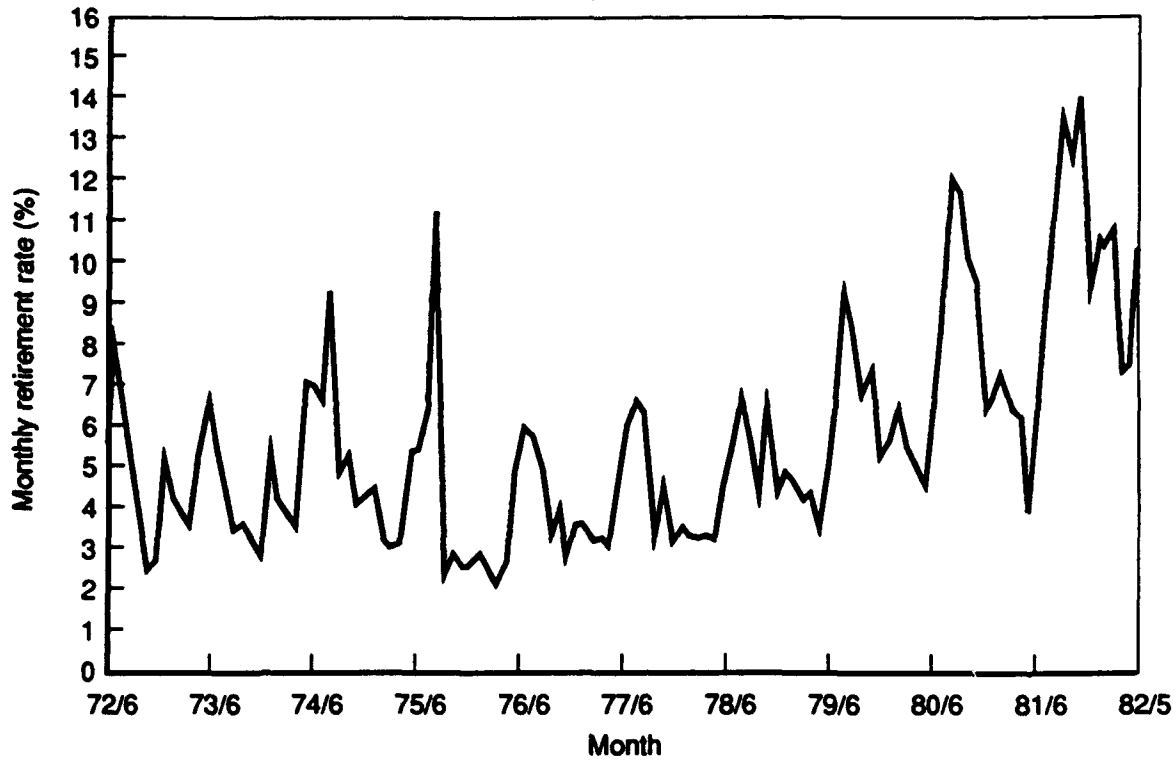


Fig. 13—Monthly retirement rate (%) (June 1972 to May 1982)

these groups have generally decided when they want to retire or have had the decision made for them.)

- Flexible-group factors, which were computed using the rates obtained by pooling across all non-HYT and all non-first-opportunity decision groups.

The model applies the inflexible factors to the inflexible decision groups (1.5, 1.6, 1.7, 1.8, 1.9, 7.6, 10.7, 12.8, and 14.9). The flexible factors are applied to all the other decision groups.

Table 34 shows the differing degrees of monthly variation within each of the three sets of factors. There is little spread in the distribution of the inflexible-group factors. The flexible-group factors exhibit a much wider range and larger variance. Figures 13 and 14 graphically depict the differences in the characteristics between the flexible-group factors and the nonflexible group factors. Figure 14 shows that airmen in the flexible decision groups are about twice as likely to retire during the summer months as they are

Table 33

MONTHLY FACTORS FOR RETIREMENT SEPARATION MODEL^a

Month	Type of Decision Group ^b		
	Flexible	Inflexible	All
Oct	0.949	1.015	1.003
Nov	0.633	0.981	0.737
Dec	0.783	0.984	0.774
Jan	0.835	0.970	0.935
Feb	0.696	0.953	0.799
Mar	0.659	0.925	0.741
Apr	0.643	0.929	0.698
May	1.241	1.069	1.110
Jun	1.356	1.023	1.257
Jul	1.594	1.043	1.425
Aug	1.692	1.098	1.509
Sep	0.818	0.979	0.927

^aA monthly factor is the ratio of the average retirement rate for that month to the overall average retirement rate.

^bFirst-opportunity and high-year-of-tenure decision groups (i.e., 1.5, 1.6, 1.7, 1.8, 1.9, 7.6, 10.7, 12.8, 14.9) are "inflexible," all others are "flexible."

Table 34

CHARACTERISTICS OF MONTHLY FACTORS FOR RETIREMENT SEPARATION MODEL^a

Month	Type of Decision Group ^b		
	Flexible	Inflexible	All
Maximum	1.692	1.098	1.509
Minimum	0.633	0.925	0.698
Std. Dev.	0.365	0.051	0.265

^aA monthly factor is the ratio of the average retirement rate for that month to the overall average retirement rate.

^bFirst-opportunity and high-year-of-tenure decision groups (i.e., 1.5, 1.6, 1.7, 1.8, 1.9, 7.6, 10.7, 12.8, 14.9) are "inflexible," all others are "flexible."

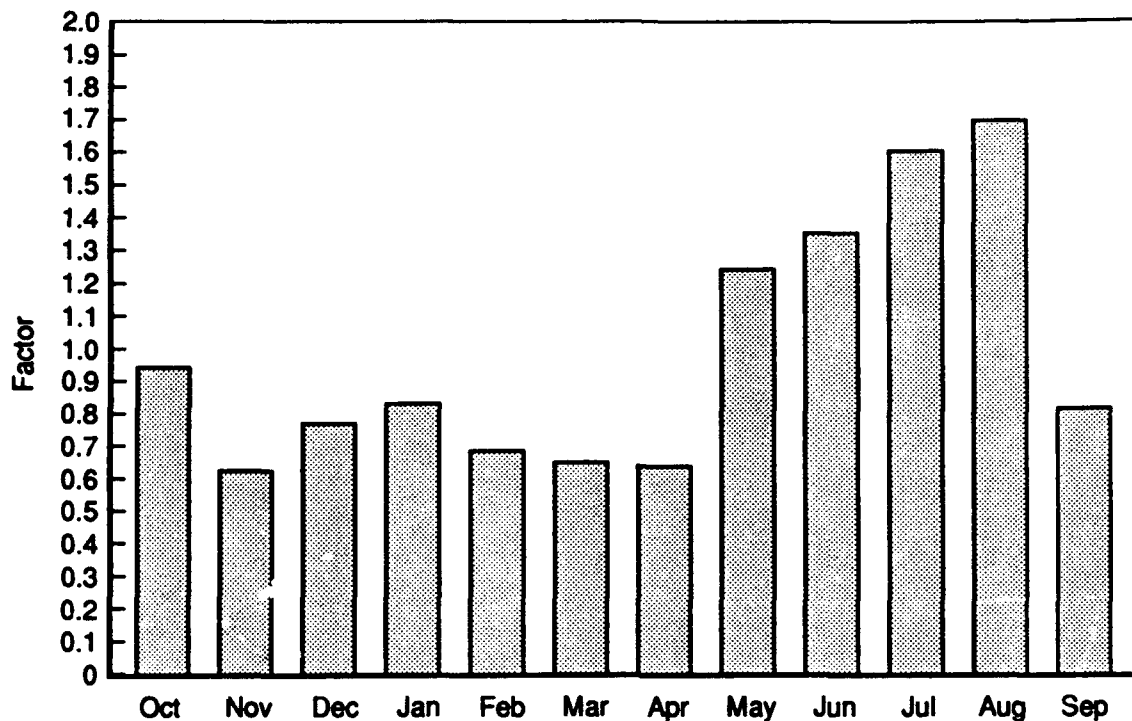


Fig. 14—Monthly factors for flexible decision groups

during the winter months. However, Fig. 15 shows that airmen in the inflexible decision groups have only a slightly higher propensity to leave in the summer months than in the winter months.

THE RETIREMENT SEPARATION MODEL

The short-term aggregate retirement model multiplies the number of currently retirement-eligible enlisted persons (population at risk) by the average retirement rate over the previous 12 months (to adjust for trend) and then by a monthly factor (to adjust for seasonality). This is done separately for each of 35 decision groups, which are defined by MOS range (14 categories) and grade.

The model is as follows:

$$r(t,i) = P(t,i) S(t,i) \left(\sum_{s=-11}^0 r(s,i) / \sum_{s=-11}^0 P(s,i) \right)$$

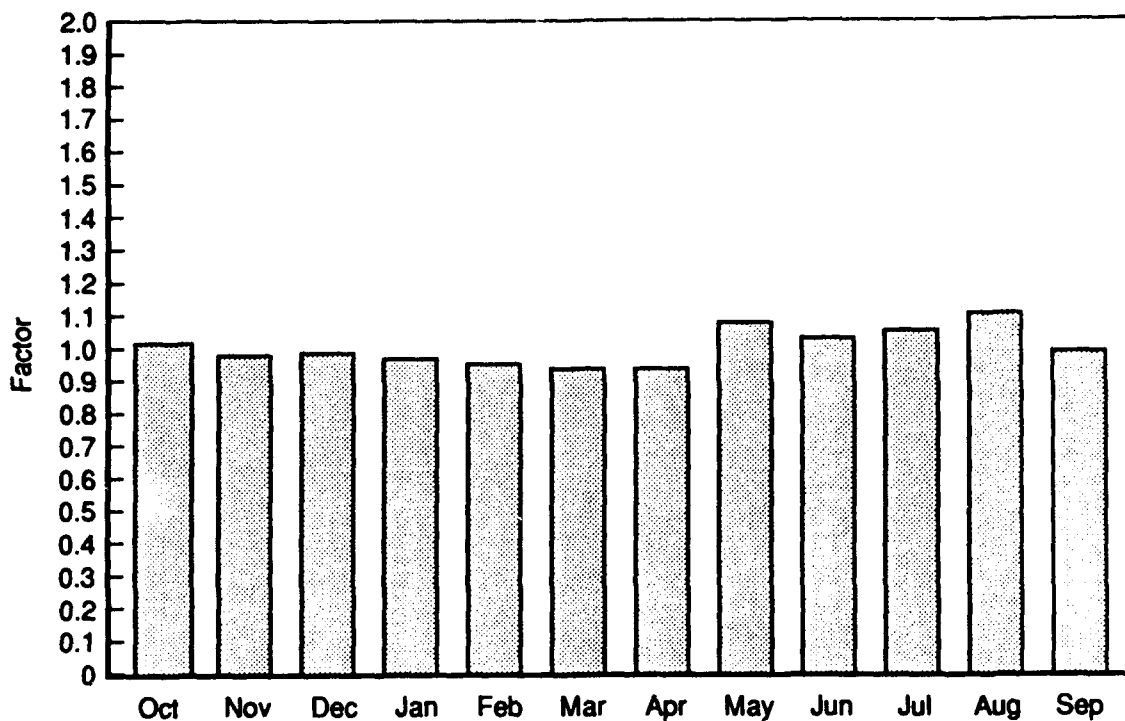


Fig. 15—Monthly factors for inflexible decision groups

where

s, t identify months since the start of the projection period, so $s = -11$ for the start of the year before the projection period, $s = 0$ for the month before the start of the projection period, and $t = 1$ for the first month to be projected.

i is the decision group (there are 35 decision groups);

$r(t,i)$ is the predicted number of retirements in projection month t for group i ;

$r(t,i)$ is the actual number of retirements in month t for group i ;

$P(t,i)$ is the (actual or projected) population at risk at the beginning of month t for group i ; and

$S(t,i)$ is the appropriate monthly factor to be applied to group i for projection month t .

Table 35 is an example of predicting retirements for June 1982. Column 1 identifies the decision group. The first digit is the MOS category and the second digit is

Table 35

EXAMPLE PREDICTION OF RETIREMENTS
FOR JUNE 1982

Decision Group i	P(1,i)	S(1,i)	Sum r	Sum P	r
1.5	6	1.023	68	89	4.7
1.6	53	1.023	417	614	36.2
1.7	91	1.023	417	914	42.5
1.8	11	1.023	40	150	3.0
1.9	2	1.023	2	22	0.2
2.6	30	1.356	83	501	6.7
2.7	79	1.356	134	1415	10.1
2.8	13	1.356	19	334	1.0
2.9	1	1.356	3	64	0.1
3.6	109	1.356	155	1691	13.5
3.7	383	1.356	482	5972	41.9
3.8	120	1.356	102	1729	9.6
3.9	22	1.356	17	408	1.2
4.6	37	1.356	27	406	3.3
4.7	112	1.356	152	1372	16.8
4.8	36	1.356	30	489	3.0
4.9	15	1.356	15	216	1.4
5.6	35	1.356	26	358	3.4
6.7	137	1.356	179	1991	16.7
6.8	53	1.356	61	910	4.8
6.9	20	1.356	16	294	1.5
7.6	3	1.023	18	19	2.9
8.7	3	1.356	8	63	0.5
8.8	2	1.356	4	33	0.3
8.9	0	1.356	1	12	0.0
9.7	134	1.356	91	2429	6.8
9.8	37	1.356	45	841	2.7
9.9	38	1.356	22	882	1.3
10.7	7	1.023	98	113	6.2
10.8	9	1.356	17	173	1.2
10.9	6	1.356	23	69	2.7
11.8	105	1.356	105	1839	8.1
12.8	2	1.023	17	30	1.2
13.9	136	1.356	136	2573	9.7
14.9	7	1.023	50	75	4.8
Total	1854				271.0

the grade. Columns 2, 4, and 5 correspond to the population at risk for June 1982, sum of retirements from June 1981 to May 1982, and sum of population at risk from June 1981 to May 1982, respectively. Column 3 (monthly factor) comes from Table 33.

RETIREMENT-ELIGIBLE ATTRITION MODEL

Attrition is quite small during the retirement-eligible years, with the majority of attrition attributable to death. This model provides an unbiased estimate of retirement-eligible attrition that is similar in structure to the retirement model presented above.

The model to predict attrition within the retirement-eligible years is a 12-month straight line running average model. This model is required to allow SAM to report all of career attrition, which includes attrition from the retirement-eligible years.

The attrition model for retirement-eligible airmen is:

$$a(t,i) = P(t,i) \left\{ \sum_{s=-11}^0 a(s,i) / \sum_{s=-11}^0 P(s,i) \right\}, t = 1, 2, \dots$$

where

$a(t,i)$ is the predicted attrition for retirement group i in month t ,

$a(t,i)$ is the actual attrition for retirement group i in month t ,

$P(t,i)$ is the (actual or projected) population at risk in group i at the start of month t ,

s, t identify months since the start of the projection period ($t = 0$ for last month; $t = 1$ for this month, the first projection month; $t = 2$ for the second projection month, etc.),

i is the retirement decision group (as defined in Table 31).

RETIREMENT-ELIGIBLE REENLISTMENT MODEL

Reenlistments occur as a matter of course during the retirement-eligible years. Reenlistments are highly correlated with retirements, since enlisted members must reenlist to get a new DOS if they elect not to retire at their current DOS. This model provides an unbiased estimate of retirement-eligible reenlistments that is similar in structure to the retirement model presented above.

The model to predict reenlistments within the retirement-eligible years is a 12-month straight line running average model. This model is required to allow SAM to report all career reenlistments, which include reenlistments from the retirement-eligible years.

The reenlistment model for retirement-eligible airmen is:

$$u(t,i) = [P(t,i) - r(t,i)] \left\{ \sum_{s=-11}^0 u(s,i) / \sum_{s=-11}^0 [P(s,i) - r(s,i)] \right\}, t = 1, 2, \dots$$

where

$u(t,i)$ are the predicted reenlistments for retirement group i month t ,

$P(t,i)$ is the (actual or projected) population at risk in group i at the start of month t ,

$r(t,i)$ are the (actual or projected) retirements for group i in month t ,

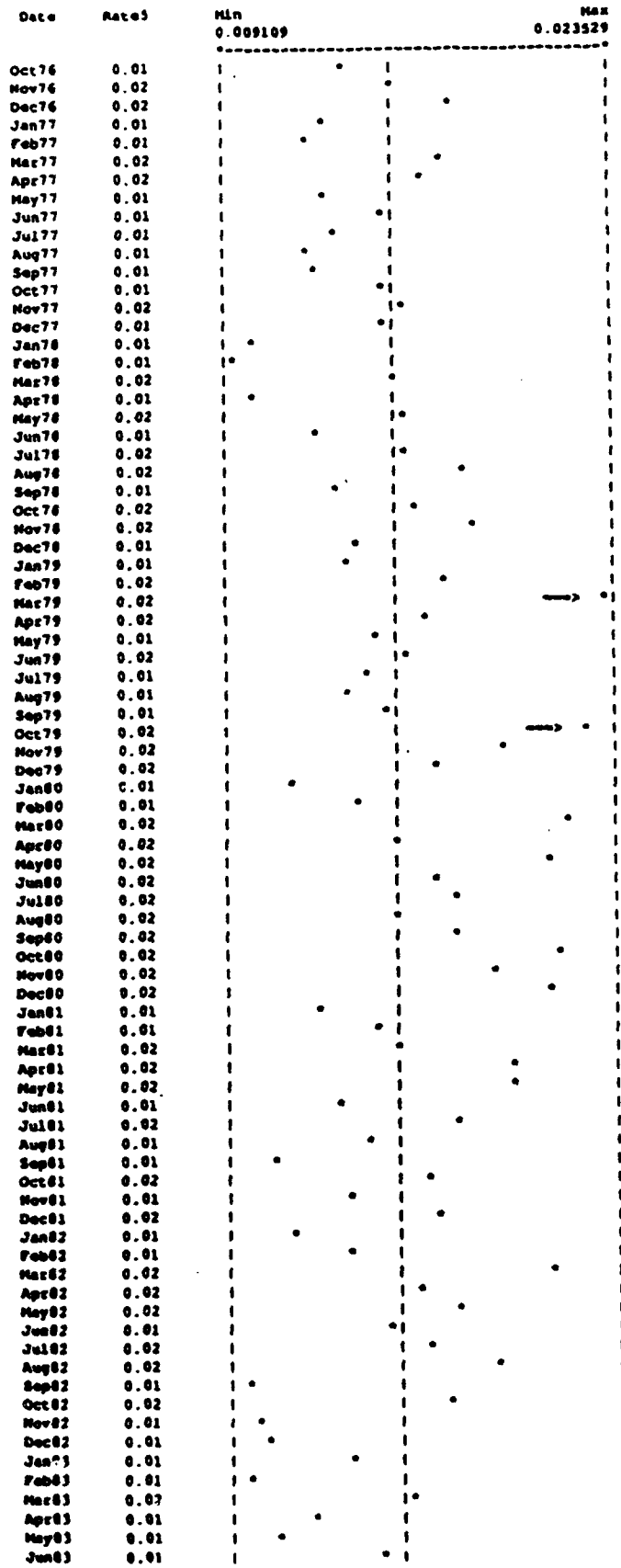
s, t identify months since the start of the projection period ($t = 0$ for last month; $t = 1$ for this month, the first projection month; $t = 2$ for the second projection month, etc.),

i is the retirement decision group (as defined in Table 31).

Appendix A
Attrition rates for high school
graduates, MOS = 1

date	rate	min	max
		0.001392	0.027618
Oct 76	0.01		
Nov 76	0.01		
Dec 76	0.03		
Jan 77	0.01		
Feb 77	0.01		
Mar 77	0.02		
Apr 77	0.01		
May 77	0.02		
Jun 77	0.01		
Jul 77	0.02		
Aug 77	0.02		
Sep 77	0.02		
Oct 77	0.01		
Nov 77	0.01		
Dec 77	0.03		
Jan 78	0.02		
Feb 78	0.01		
Mar 78	0.02		
Apr 78	0.02		
May 78	0.01		
Jun 78	0.01		
Jul 78	0.01		
Aug 78	0.01		
Sep 78	0.01		
Oct 78	0.01		
Nov 78	0.01		
Dec 78	0.02		
Jan 79	0.02		
Feb 79	0.01		
Mar 79	0.01		
Apr 79	0.02		
May 79	0.01		
Jun 79	0.01		
Jul 79	0.01		
Aug 79	0.01		
Sep 79	0.01		
Oct 79	0.01		
Nov 79	0.01		
Dec 79	0.01		
Jan 80	0.01		
Feb 80	0.01		
Mar 80	0.01		
Apr 80	0.01		
May 80	0.01		
Jun 80	0.01		
Jul 80	0.01		
Aug 80	0.01		
Sep 80	0.01		
Oct 80	0.01		
Nov 80	0.00		
Dec 80	0.01		
Jan 81	0.01		
Feb 81	0.01		
Mar 81	0.01		
Apr 81	0.00		
May 81	0.01		
Jun 81	0.01		
Jul 81	0.01		
Aug 81	0.01		
Sep 81	0.01		
Oct 81	0.01		
Nov 81	0.00		
Dec 81	0.00		
Jan 82	0.01		
Feb 82	0.01		
Mar 82	0.02		
Apr 82	0.01		
May 82	0.02		
Jun 82	0.02		
Jul 82	0.02		
Aug 82	0.01		
Sep 82	0.01		
Oct 82	0.01		
Nov 82	0.00		
Dec 82	0.01		
Jan 83	0.01		
Feb 83	0.00		
Mar 83	0.01		
Apr 83	0.01		
May 83	0.01		
Jun 83	0.01		

Appendix B
Attrition rates for non-high school
graduates, MOS = 13-24

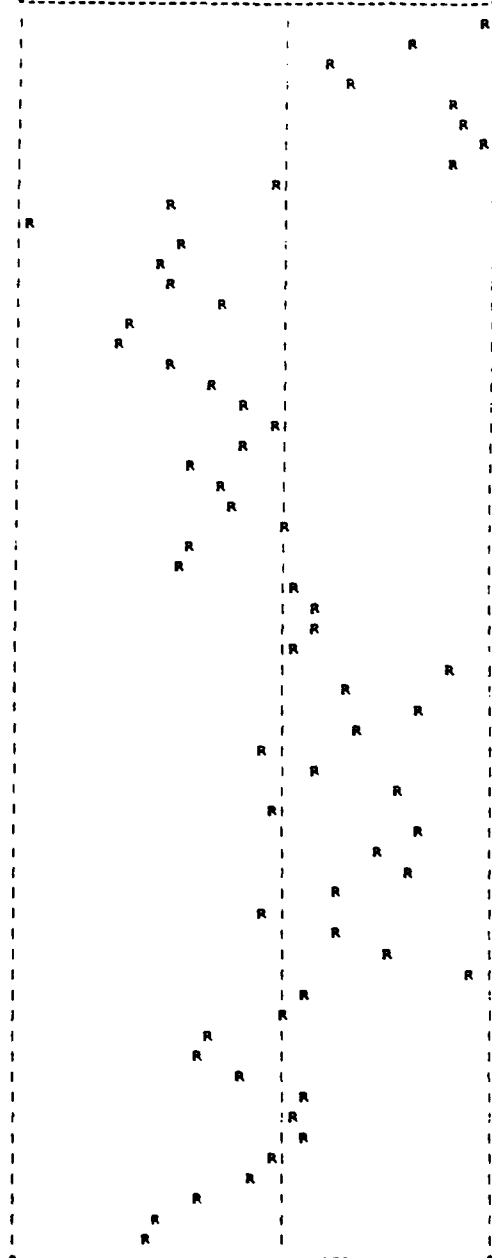


Appendix C
 Fraction staying past end of ETS
 decision period, high school
 graduates, TOE = 4

ETS	fraction	min 0.195895	max 0.687306
7207	0.23	S	
7208	0.21	S	
7209	0.21	S	
7210	0.24	S	
7211	0.22	S	
7212	0.23	S	
7301	0.26	S	
7302	0.20	S	
7303	0.22	S	
7304	0.24	S	
7305	0.22	S	
7306	0.23	S	
7307	0.32	S	
7308	0.29	S	
7309	0.28	S	
7310	0.30	S	
7311	0.29	S	
7312	0.34	S	
7401	0.32	S	
7402	0.36	S	
7403	0.31	S	
7404	0.31	S	
7405	0.32	S	
7406	0.38	S	
7407	0.42	S	
7408	0.31	S	
7409	0.32	S	
7410	0.34	S	
7411	0.32	S	
7412	0.35	S	
7501	0.37	S	
7502	0.34	S	
7503	0.34	S	
7504	0.39	S	
7505	0.40	S	
7506	0.45	S	
7507	0.43	S	
7508	0.40	S	
7509	0.40	S	
7510	0.39	S	
7511	0.39	S	
7512	0.42	S	
7601	0.44	S	
7602	0.48	S	
7603	0.43	S	
7604	0.44	S	
7605	0.45	S	
7606	0.43	S	
7607	0.51	S	
7608	0.41	S	
7609	0.44	S	
7610	0.47	S	
7611	0.43	S	
7612	0.38	S	
7701	0.40	S	
7702	0.45	S	
7703	0.44	S	
7704	0.45	S	
7705	0.48	S	
7706	0.46	S	
7707	0.47	S	
7708	0.48	S	
7709	0.46	S	
7710	0.39	S	
7711	0.54	S	
7712	0.51	S	
7801	0.47	S	
7802	0.49	S	
7803	0.47	S	
7804	0.52	S	
7805	0.47	S	
7806	0.52	S	
7807	0.50	S	
7808	0.47	S	
7809	0.49	S	
7810	0.52	S	
7811	0.53	S	
7812	0.53	S	
7901	0.49	S	
7902	0.45	S	
7903	0.40	S	
7904	0.37	S	
7905	0.42	S	
7906	0.43	S	
7907	0.43	S	
7908	0.43	S	
7909	0.42	S	
7910	0.41	S	
7911	0.42	S	
7912	0.44	S	
8001	0.45	S	
8002	0.44	S	

Appendix D
Residuals for first term ETS
loss model, METS = 12,
high school graduates, TOE = 4

date	residuals	min	max
		-0.09025	0.071602
7806	0.07		
7807	0.04		
7808	0.01		
7809	0.02		
7810	0.06		
7811	0.06		
7812	0.07		
7901	0.06		
7902	-0.00		
7903	-0.04		
7904	-0.09		
7905	-0.03		
7906	-0.04		
7907	-0.04		
7908	-0.02		
7909	-0.05		
7910	-0.06		
7911	-0.04		
7912	-0.03		
8001	-0.01		
8002	-0.00		
8003	-0.02		
8004	-0.03		
8005	-0.02		
8006	-0.02		
8007	-0.00		
8008	-0.03		
8009	-0.04		
8010	0.00		
8011	0.01		
8012	0.01		
8101	0.00		
8102	0.06		
8103	0.02		
8104	0.05		
8105	0.02		
8106	-0.01		
8107	0.01		
8108	0.04		
8109	-0.00		
8110	0.05		
8111	0.03		
8112	0.04		
8201	0.02		
8202	-0.01		
8203	0.02		
8204	0.04		
8205	0.07		
8206	0.01		
8207	-0.00		
8208	-0.03		
8209	-0.03		
8210	-0.01		
8211	0.01		
8212	0.00		
8301	0.01		
8302	-0.00		
8303	-0.01		
8304	-0.03		
8305	-0.04		
8306	-0.05		



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