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Visual-Auditory Interactions in Sensory-motor processing: Saccades versus Manual responses

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Abstract

Intersensory (visual/auditory) facilitation of reaction times (RTs) was examined using three different response systems: saccadic eye movements, directed manual responses (deflections of a joystick towards the target location) and simple manual responses. The data were examined in the context of race models (in which facilitation is attributed to the minimum of two random variables representing the detection times associated with the visual and auditory targets) versus neural summation [coactivation] models (where the facilitation is attributed to a combination of the activities within the visual and auditory channels prior to detection). The first experiment provides evidence for neural summation [coactivation] in all three response modes. The effects of varying combinations of auditory and visual stimulus intensity were examined in the second experiment. Intensity-dependent mismatches in the auditory and visual RTs had little effect on the magnitude of the redundant targets effect, indicating that visual-auditory integration occurs over temporal intervals of at least 40 msecs. The effects of spatial correspondence (auditory and visual targets presented in spatial register or in opposite hemifields) was examined in the third experiment. Coactivation depends upon the spatial alignment of the targets for directed responses (both saccades and directed manual responses) but not simple manual RTs; evidence of coactivation of simple RTs was found for both in register and out-of-register stimuli.

The character of visual-auditory interactions appears to vary according to the sensory-motor pathways utilized by the task. The results for saccades are consistent with known patterns of auditory-visual convergence within the oculomotor system, and therefore indicate that the redundant target paradigm can provide behavioral correlates of established aspects of the neural architecture of a specific sensorimotor system (eg., Meredith & Stein,1987; Jay & Sparks,1987;1990; Peck, 1987). As there are no known pathways that could mediate sensory convergence of visual information from one hemifield and auditory information from the contralateral hemifield, we suggest that the summation effects observed for simple manual

responses are at least partially attributable to a later stage of information processing (eg., coactivation at the level of response execution). The interference produced by out-of-register stimuli observed for both directed manual responses and saccades is tentatively attributed to processes associated with the selection of the target in addition to a possible contribution from response competition.

Introduction

Recently there has been renewed interest in understanding the manner in which redundant stimuli affect sensory-motor performance (eg., Ashby & Townsend,1986, Miller, 1982; Mordkoff,& Yantis;1991; Raab, 1962; Stein, Meredith, Huneycutt & McDade, 1989; Townsend & Ashby, 1983). In many cases, performance (usually reaction times, RTs) to single stimulus presentations is compared with performance under conditions of dual stimulus presentations. The frequent finding is that reaction times (RTs) to dual stimulus presentations are faster than RTs to either stimulus presented alone (Miller, 1982, 1986; Mordkoff and Yantis, 1991; van der Heijden, La Heij, and Boer, 1983; Raab, 1962). This is referred to as a *redundant signals effect.*

Central to an analysis of the redundant signals effect is the question of whether the facilitation produced by redundant targets is sufficiently robust to rule out the possibility that responses to redundant targets are simply triggered by whichever target is detected first (equivalent to the operation of a logical *OR* gate). Since the detection times associated with each modality are considered to be random variables, some reduction in responses times is expected in a system which applies such an OR operation to otherwise independent sensory channels, an effect known as *probability summation.* (eg., Raab,1962; Miller,1982; Townsend & Ashby, 1983). Probability summation assumes that a separate decision process accumulates information on each afferent channel, and the first channel to detect the target generates the response. For this reason, such models of redundant signal processing are often called "race" models. Miller (1982;1986) pointed out that the magnitude of the redundant targets effect should be greater than that attributable to race models if the activity of several parallel afferent channels were pooled prior to a single decision process. Combined activation would produce RTs which are faster than those predicted by race models (Nozawa, 1989). Thus, facilitation beyond probability summation may be indicative of neural summation ('coactivation') somewhere within the processing system. The pooling of information could occur at the level of a sensory decision (Luce & Green,1972; Wandell and Luce, 1978; Blake, Martens, Garrett & Westendorf, 1980; Fidell, 1970; Fournier & Eriksen, 1990; Rose, Blake and Holper, 1988; Westendorf and Blake, 1988) or at the level of response selection or execution (eg.,Diederich & Colonius, 1987; Eriksen and Schultz, 1977; Fournier & Eriksen, 1990; Miller 1982;1986).

While the sensory channels in many redundant targets experiments may reasonably be regarded as being organized in parallel, the very existence of a redundant targets effect means that information about the individual targets must converge at some point in sensory-motor processing. Recent electrophysiological studies have revealed a specific site of auditory-visual convergence within the oculomotor system of cats and monkeys (Jay & Sparks,1987;1990; Meredith & Stein,1987; Peck, 1987). These findings motivated the present analysis of the redundant targets effect on the latency of saccadic eye movements in humans.

Multimodal Convergence in the Saccadic Control System

Recent electrophysiological studies have revealed a neural mechanism that appears designed to enable multimodal control of saccades: individual neurons within the deeper layers of the superior colliculus (SC) receive *convergent visual and acoustic inputs* (e.g., Jay & Sparks,1987; Peck, 1987; Meredith & Stein, 1987). The coordinates of the visual and auditory receptive fields are usually in spatiotopic register, and spatially aligned bimodal inputs often elicit unit discharges that are substantially greater than responses evoked from either modality alone (Meredith & Stein, 1987). The responses of single cells to bimodal stimuli are sometimes close to the sum of the unimodal responses (Meredith and Stein,1987), a combination rule referred to as superposition of impulse counting (see Fatt & Katz, 1952; Cox, 1962). Often however, bimodal summation effects are much greater than the superposition of the two unimodal responses (Peck ,1987; Stein, Meredith & Wallace, 1991). Thus, the behavioral effects produced by bimodal stimuli might reflect an overadditive combination of each of the unimodal activities (e.g., Stein, Meredith, Honeycutt & McDabe, 1989). Jay and Sparks (1987) report that many of the cells in SC that discharge prior to saccades (pre-saccadic burst (PSB) neurons) also receive convergent visual and acoustic inputs, suggesting that the two modalities might show particularly robust facilitatory interactions in controlling the initiation of saccades. The present experiments examined the degree to which spatially coincident acoustic and visual targets facilitate the latency to initiate saccades relative to the latencies associated with stimuli of either modality presented alone. The experiments thus address the issue of *intersensory facilitation* in the saccadic control system and compare the effects to those obtained with manual responses.

Evaluating Probability Summation

Estimates of the maximal degree of intersensory facilitation attributable to race models is derived from the work of Miller (1982) and is based on the inequality :

$$P(T_{min} \leq t \mid S_1 \ \& \ S_2) \leq P(T_1 \leq t \mid S_1) + P(T_2 \leq t \mid S_2). \qquad (1)$$

where T_1 and T_2 are defined as the random times associated with processing of information in channels 1 and 2 respectively, and T_{min} is the minimum of the two random times, T_1 and T_2 ($T_{min} = \min[T_1, T_2]$). Inequality (1) is also known as Boole's inequality in probability theory (Dudewicz, 1976,p. 15). As (1) indicates an upper limit on performance attributable to race models, it can conveniently be regarded as the "upper boundary" of probability summation (race models). The left side of the inequality represents the cumulative distribution function (CDF) of the redundant targets condition (assuming the race model) and the other terms represent the cumulative distribution functions of the single target conditions. Ignoring the motor-related components of reaction time (i.e., the "base time"), probability summation states that the cumulative distribution function of the redundant targets condition can be expressed as the sum of the two cumulative distribution functions from the single target conditions, minus the joint cumulative distribution function:

$$P(T_{min} \leq t \mid S_1 \ \& \ S_2) = P(T_1 \leq t \mid S_1) + P(T_2 \leq t \mid S_2) - P(T_1 \leq t \text{ and } T_2 \leq t \mid S_1 \ \& S_2). \ (2)$$

The joint CDF, $P(T_1 \leq t$ and $T_2 \leq t \mid S_1 \& S_2)$, can be written as a multiplication of the two single-target CDF's if (1) T_1 and T_2 are stochastically independent, and (2) we assume context independence and selective influence (i.e., the processing times on channel one do not vary with the activity on channel two; cf., Colonius, 1990; Townsend and Ashby, 1983). Thus, the expression

$$P(T_{min} \leq t \mid S_1 \& S_2) = P(T_1 \leq t \mid S_1) + P(T_2 \leq t \mid S_2) - \big(P(T_1 \leq t \mid S_1) \times P(T_2 \leq t \mid S_2)\big) \quad (3)$$

represents the *independent race prediction.* If there is negative dependence between the random times T_1 and T_2, the joint CDF is less than the multiplication of two marginal CDF's. If there is positive dependence between two random times, the joint CDF is greater than the multiplication of two marginal CDF's. Regardless of the dependent structure between T_1 and T_2, inequality (1) holds because $P(T_1 \leq t$ and $T_2 \leq t \mid S_1 \& S_2) \geq 0$. Thus, violations of inequality (1) indicate a redundant targets effect which exceeds the upper limit of probability summation (Miller, 1986; Ulrich and Giray, 1986). We can evaluate applicability of probability summation to obtained redundant targets effects by comparing the CDF obtained with bimodal targets, $P(RT \leq t \mid S_1 \& S_2)$, with the CDFs obtained for unimodal targets, $P(RT \leq t \mid S_1)$ and $P(RT \leq t \mid S_2)$; i.e.,

$$P(RT \leq t \mid S_1 \& S_2) - \{P(RT \leq t \mid S_1) + P(RT \leq t \mid S_2)\}. \quad (3)$$

Notice that this comparison can only be evaluated over values of t such that $P(RT \leq t \mid S_1) + P(RT \leq t \mid S_2) \leq 1$.

Insert Fig. 1 about here

Figure 1 illustrates the relationship between several boundary conditions relevant to reaction time performance in the redundant targets paradigm. The thick solid line illustrates the CDF of RTs based on the independent race model (*independent race prediction*, Eq 3). The thin solid line represents the CDF for the maximum level of performance attributable to any race model, (i.e., the *upper limit of probability summation,* which is the right hand side of the race inequality (Eq. 1). The maximum of the two marginal CDFs (Frechet, 1951) is indicated by the heavy dashed line. The *Frechet boundary* represents the slowest level of performance possible in any model in which responses are determined by the minimum of the completion times (min $[T_1,T_2]$) for two parallel processes (Colonius, 1990). The region between the Frechet boundary and the probability summation limit represents levels of performance that could be produced by race models in which the assumptions of stochastic independence and selective influence are relaxed (see Nozawa, 1989; Ulrich and Giray, 1986). As indicated above, given the assumptions of context independence and selective influence, performance that is faster than the probability summation limit cannot be accounted for by race models, and therefore will be interpreted as evidence of neural summation[1]. One type of neural summation (the superposition model, see Fatt and Katz, 1952; Cox, 1962; Schwarz, 1989) produces the linear sum of the channel inputs. Nozawa (1989) has provided a mathematical proof that the superposition of two neural counting processes will produce faster RTs than the independent race model. To the extent that neural summation is an overadditive combination of channel activities (e.g., Stein et al., in press) predicted performance would be even faster than the superposition model. The degree to which observed redundant targets effects exceed the upper limit of probability summation could be related to the operator which combines the

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[1] While many authors have adopted the term "coactivation" to refer to violations of inequality (1), we refer to such violations as "neural summation" in the present paper because of the parallel relationship between the present results and observations of auditory-visual convergence in the electrophysiological literature. In addition, the set of models which encompass the term coactivation is perhaps larger (e.g., Miller, 1991; Mordkoff & Yantis, 1991) than what might be reasonably included in the type of summation mechanism we consider here. Thus, the term neural summation might be preferable in the context of the effects we consider, since it implies specific patterns of convergence in neural "hardware".

auditory and visual information: a multiplicative operator will produce greater violations of the race inequality (Eq. 1) than superposition.

Intersensory Facilitation as a Function of Task Requirements

Since visual-auditory convergence onto pre-saccadic burst neurons in the superior colliculus is a potentially unique architecture among sensory-motor systems, it seemed desirable to compare the magnitude of bimodal summation observed for saccades with alternative sensory-motor tasks. Thus, we also investigated intersensory facilitation using two types of manual responses. *Directed manual responses* required the subjects to deflect a joystick in the direction of the target as quickly as possible. The second type of manual response was a simple reaction time task. The directed manual responses are similar to saccades in that target position must be encoded before a correct response can be executed. In contrast, *simple manual responses* do not depend upon localization of the target.

General Methods

Apparatus

The basic apparatus consisted of an array of 3 stimulus panels aligned on an arc with a radius of 114 cm. Each stimulus panel contained a red and green light-emitting diode (LED) and a small (4 cm) speaker. Two panels positioned on the horizontal meridian of the left and right visual fields provided the targets. The green LED of the central panel served as a fixation point. Flashes of the peripheral red LEDs (100 ms duration) served as the visual targets. Acoustic signals consisted of white noise bursts (100 ms) delivered through the speakers. Both the amplitude of the acoustic targets and the luminance of the visual targets were controlled by 12 bit D/A converters. Acoustic warning signals (2000 Hz, 300 ms) presented through a centrally located oscillator module preceded the delivery of imperative targets by 1000 ms. In order to prevent echoes which might impair sound localization, the entire apparatus was located in a large (1.54 m. by 1.54 m. by 0.9 m.) enclosure which was lined with a sound-absorbing foam material (Sonex™). The apparatus was located in an isolated, completely darkened room.

Response Recording

Eye position was monitored using a scleral infra-red reflection device (Narco Biosystems™ Model 200 eye tracker). The output of the eye tracker was sampled via a 12 bit A/D converter at 200 Hz, and the digitized records were stored for subsequent off-line data analysis. In addition to measuring saccades, we included sessions in which the observers were required to generate directed and simple manual responses under similar conditions. Directed manual responses were recorded using an inductive-coil joystick. The subjects were simply required to push the joystick in the direction of the eccentric target as quickly as possible. The joystick position was sampled using D/A convertors (200 Hz sampling rate) and the direction and latency of the movements were analyzed in the same way as saccades. In the simple RT condition subjects simply depressed a microswitch in response to the target onset. The microswitch was also sampled at 200 Hz.

Response Detection. Both saccades and directed manual responses (joystick movements) were detected using a velocity criterion. While the detection of both saccades and joystick responses was automated, all records were monitored by an operator in order to insure that misses or false positives were not included within the data set. In general, the velocity criterion for saccades and joystick responses was set to ~ 50 deg. sec^{-1}. However, the criterion was occasionally adjusted in order to maximize the performance of the velocity-based algorithm. This was especially true in the case of joystick responses, which tended to show greater variability in velocity than saccades.

Data Analysis. As indicated in the Introduction, violations of the race inequality (Eq. 1) represent the principle measure of interest. Recent simulations by Miller and Lopes (1991) have shown that fast guesses can bias the results against observing such violations. While the uncorrected (more conservative) data provide clear and robust violations of Eq 1., we did correct for fast guesses using the following procedure, which is derived from the correction suggested by Miller and Lopes (1991).

The CDF for auditory targets, corrected for fast guesses, is given by

$F_{A,c}(t) = (1\text{-}g)\ F_A(t) + g\ G(t)$, where $F_{A,g}(t)$ represents the CDF corrected for fast guesses, $F_A(t)$ represents the uncorrected auditory CDF, g is the probability of a fast guess, and G(t) is the CDF of the fast guesses (which were estimated from the distribution of false alarms (FA) on catch trials). Similarly, we have corrected CDFs for the visual and the bimodal targets; $F_{V,c}(t) = (1\text{-}g)\ F_V(t) + g\ G(t)$ and $F_{A\&V,c}(t) = (1\text{-}g)\ F_{A\&V}(t) + g\ G(t)$, respectively. In this paper, we present many of the results in terms of the difference between the obtained redundant targets CDF and the sum of the unimodal CDFs. Let us refer to this as the magnitude of the race inequality violation, which is given by

$\text{Mag} = F_{A\&V,g}(t) - \{F_{A,g}(t) + F_{V,g}(t)\}$, where the subscript g indicates the obtained CDFs uncorrected for fast guesses.

Substitution yields the corrected magnitude

$\text{Mag}_C = (1\text{-}g)\ F_{A\&V}(t) + g\ G(t) - \{(1\text{-}g)\ F_A(t) + g\ G(t) + (1\text{-}g)\ F_V(t) + g\ G(t)\}$,

and simplification gives

$$\text{Mag}_C = \frac{F_{A\&V}(t) - \{F_A(t) + F_V(t)\} + g \cdot G(t)}{1\text{-}g}$$

Although direction errors occurred, they were not included in the present correction procedure, largely because of the lack of a formal model designed to account for their occurrence. As it turned out, the latencies of the direction errors were often longer than the latencies for correct responses, so they do not appear to be fast guesses per se.

It can be seen that this correction serves to increase the magnitude of the race inequality violations, although the actual difference between the corrected and uncorrected figures was extremely small in most cases. We emphasize however, that all of the reported violations were apparent whether or not this fast guessing correction was applied.

EXPERIMENT I

Preliminary Procedures.

Stimulus intensities that produced equivalent latencies in each observer were identified in a series of preliminary sessions in which we presented unimodal targets of varying intensities. There were 64 trials in each session. Each trial began with a warning tone, followed by either a visual or acoustic target (there were no bimodal stimulus trials in these preliminary sessions). Stimulus intensity, modality and location (left vs. right) varied randomly across trials. At least 4 of these preliminary sessions were run for each response condition (saccades, directed manual responses, simple manual responses). The intensity-RT curves were used to select visual and acoustic intensities that produce comparable response times for use in the formal portion of the experiment.

Data collection was always preceded by 5 min of dark adaptation, during which time the eye tracker was adjusted and eye position calibrated. At the viewing distance of 114 cm., the targets appeared at an eccentricity of 20°. Head movements were minimized using a bite-plate. All subjects were emmetropic (or were appropriately corrected) and had normal hearing. The subjects were paid for their participation.

Experimental Procedures.

When intensities which produced equivalent latencies for the visual and acoustic targets were identified, formal data collection began. Each observer participated in 15 blocks of 60 trials each. Typically, a subject was tested for 2-3 blocks/day. For saccadic and directed manual response sessions, each type of target (auditory, visual and bimodal) occurred with equal frequency in a randomized order. For the simple manual reaction time sessions, bimodal targets were presented on 33.3% of the trials, unimodal targets were presented on 50.0% of the trials (25.0% visual, 25.0 % auditory), and no target (catch trials) was presented on 16.7% of the trials. Targets were presented to the left or the right of fixation with equal frequency in a randomized order. The data reported below are based on at least 100 observations for each of the 9 conditions (acoustic, visual or bimodal stimuli for each response condition) in four naive observers.

Results

Insert Fig. 2 about here

The averaged RTs for acoustic, visual and bimodal targets are illustrated in Fig. 2. The left portion shows the saccade latencies, the middle shows the directed manual responses, and the right portion shows the simple manual response times. It can be seen bimodal stimuli generally produced shorter response times than unimodal stimuli. This was confirmed in a 4-factor analysis of variance (direction (L vs R) x stimulus modality x response mode x subjects) which revealed a significant interaction between response mode and stimulus modality ($F_{4,12} = 7.26, p<.005$). Post hoc analyses of the means contributing to this interaction (Newman-Keuls) showed that bimodal RTs were significantly faster than either of the unimodal RTs for each response condition (all p's < 0.05). The lone exception was that bimodal RTs were not significantly faster than auditory RTs in the simple manual response condition.

Average error rates for each response condition are shown in Table 1. These errors represent anticipations and false alarms in the simple manual RT task, whereas they include anticipations and direction errors for the saccadic and directed manual responses. All RTs less than 125 ms were considered anticipation errors. These error rates were submitted to a 2 factor (response condition × subjects) analysis of variance following an arcsine transformation. This analysis revealed no significant differences in error rates for the three response conditions ($F_{2,6} = 3.25$, p=.11).

Insert Table 1 about here

Evidence for Neural Summation

We compared the obtained redundant target CDFs with the sum of the corresponding unimodal CDFs in order to determine whether the observed intersensory facilitation of RTs might be accounted for by probability summation. All analyses are based on latency histograms with a 10 ms. bin width.

Insert Fig. 3 about here

Figure 3 illustrates the redundant targets effect for a typical observer in the saccade condition. The CDFs from the marginal (unimodal) conditions, the sum of these marginal CDFs (left side of the race inequality (Eq. 1)), and the obtained bimodal CDF are all presented in the top panel. Violations of the upper limit of the race model are indicated whenever the probability associated with the obtained bimodal CDF exceeds the sum of the marginal (unimodal) CDFs. These violations are indicated by the vertical hatching in the top panel (non-violations are represented by the horizontal hatching). The bottom panel represents the difference between the obtained bimodal CDF and the race inequality. As it is much more efficient to present the data in terms of this difference between the obtained and predicted CDF, most of the results will be reported using this difference format.

Figure 4 (a-d) presents of the general pattern of the violations of Inequality 1 in all four observers. The data for saccades, directed manual responses, and simple manual responses are all as indicated in the legend. With one exception, the data in the present report was submitted to the correction for fast guesses outlined above. The only exception to this is the simple RT data from Observer MK. The reason we elected not to use the correction for MK is indicated in Figure 5. The magnitudes of the violations of the race model inequality with and without the fast guessing correction are shown in the bottom panel. It is evident that, using the correction in MK results in violations of the inequality that begin at a latency of 15 msecs. This is entirely the result of the fact that MK committed 12 false alarms on catch trials that have no counterpart in the signal present trials. The CDF for these false alarms (up to 300 msecs., which includes all but one of the false alarms) is shown in the top panel. It seems obvious that, in this particular case, virtually all the evidence for violations of the inequality rely on these 12 False alarms. Given this distortion of the results produced by the fast guessing correction, we elected not to use the procedure in the case of MK's simple RT data.

Insert Figs. 4 and 5 about here

Violations of the race inequality are apparent for each response mode in all four observers. Notice however, that both the proportion of the manual RT data which violates the inequality and the magnitude of the observed violations appear smaller than those associated with saccades. Failures to violate inequality (1) do not necessarily rule out coactivation models. However, if selective influence and context independence holds, violations of inequality (1) can only be realized by neural summation (coactivation). Thus, inequality (1) represents a very conservative test of neural summation models (cf., Miller, 1991; Eriksen,1988). Both superposition and overadditive combinations of individual channel activities will violate inequality (1) (see Townsend and Nozawa, 1992). In the next section, we evaluate the possibility that the liklihood of violations varied with response mode.

Tests of the Response-Dependent Ordering of Neural Summation

The probability of violations of the race inequality was first calculated. The number of violations occurring in the n intervals over which the inequality can be evaluated is distributed as a binomial random variable. Thus the probability of a violation, p_A=P(violation|response mode A) $\sim 1/n_A \, Bin(p_A, n_A)$. The multiplicative factor $1/n_A$ means that we are dealing with the relative frequencies of the violations rather than the actual number of violations observed. As n_A (the number of possible violations) increases, the binomial approximates the normal distribution: $1/n_A \, Bin(p_A, n_A) \sim N(p_A, p_A(1-p_A) / n_A)$. Thus, the probability of observing a violation can be approximated by the normal distribution with the mean of p_A and the variance of $p_A(1\text{-}p_A) / n_A$. Thus, the hypothesis can be expressed as the difference of the two normal random variables, p_A and p_B, the null hypotheses, P(violation|response mode A) ≤ P(violation|response mode B) and P(violation|response mode A) ≤ P(violation|response mode C) are distributed as $N(p_A - p_B, p_A(1\text{-}p_A) / n_A + p_B(1\text{-}p_B) / n_B)$. Since the distribution mean and variance are known, the z-score associated with this comparison can be calculated as follows:

$$z = \frac{p_A - p_B}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} .$$

The results of these calculations are provided in Table 2. This analysis supports the suggestion that the probability of violating the race inequality was greater for saccades than either directed manual or simple manual RTs.

Insert Table 2 about here

Discussion

Response-dependent ordering in the magnitude of neural summation

All three response modes show evidence of neural summation between visual and auditory channels. However, the magnitudes of the violations of the race inequality for saccadic responses appears greater than those observed with either directed manual RTs or simple manual RTs. The magnitudes of the violations of inequality (1) for saccades were quite robust, ranging from 0.23 to 0.40 in the four observers (Fig. 4). By way of comparison, previously reported violations of the probability summation limit generally vary between 0.05 and 0.10 (eg., Diederich & Colonius, 1987; Mordkoff & Yantis, 1991; Miller,1982). The violations in the present manual RT data are generally commensurate with these previous results. Thus, the magnitude of race inequality violations depend upon the response system. Presumably, such differences may relate to the operator which combines the channel activities (eg., addition or multiplication). Alternatively, the number of elements in which the convergence occurs (cf., Kimura and Tamai, 1992) might influence the magnitude of intersensory integration in a manner analogous to manner in which the number of responding elements contributes to the effects of stimulus intensity on RT. We return to the possibility of response-dependent ordering in the magnitude of race inequality violations in the next experiment.

The importance of Central Simultaneity

The response-dependent ordering suggested by these data may also relate to the quality of the matches of the unimodal RTs. Intuitively, one might think that neural summation will be maximized when the activities of each channel are cotemporaneous. Miller (1986) has provided direct evidence in support of this conjecture. Although the procedure is not necessarily infallible, we tried to maximize the likelihood that each channel's activity arrived cotemporaneously at the site of summation by matching the visual and auditory RTs (through manipulations of signal strength). The procedure worked well for both the saccades and the directed manual responses. However, despite a concerted effort, the matches for the simple RTs were not as close as we had hoped (Fig. 2). This mismatch could have produced less robust violations of the race inequality for simple RTs than might otherwise have occurred (eg., Diederich and Colonius, 1987; Miller, 1986). Indeed, the present evidence for neural summation observed for simple manual RTs may actually be a little weaker than in some of the previously published results using similar conditions (eg., Diederich and Colonius, 1987; Miller, 1986). While other paradigmatic differences could play some role in attenuating the level of neural summation relative to some of these earlier reports (for example, interstimulus dependencies [see below] and uncertainty of target location, and the presence of catch trials), the observed violations of inequality (1) may have been more compelling had we succeeded in obtaining better matches between the simple RTs in the single target conditions. The RT matches between the auditory and visual targets were quite good in the saccadic and directed manual response conditions however, and evidence for ordering was still obtained.

It is interesting to note that recent electrophysiological studies indicate that the "temporal window" for bimodal summation in the superior colliculus is actually quite long (for individual neurons, the integration window is often greater than 500 ms, see Stein and Meredith, 1990).

From the ecological perspective, long integration times would appear quite desirable. That is, substantial differences exist between visual and auditory transduction latencies, and the differences in the relative velocities of sound and light would usually not be expected to compensate for the differences in internal processing time. A strict requirement of central simultaneity for neural summation between visual and auditory targets would mean that neural summation could only be expressed with specific combinations of visual and auditory intensities, and the required combination of intensities would further depend on stimulus distance. In order to be generally useful, the summation mechanism must have a long integration time (cf. Stein and Meredith, 1990). A second goal of Experiment II was to investigate the effects of auditory and visual detection asynchrony on the observed violations of the race inequality.

Inter-stimulus Dependencies and Coactivation

Mordkoff and Yantis (1991) have recently provided an interesting analysis of the role of inter-stimulus contingencies and stimulus-response contingencies in experiments on coactivation effects. They point out that in redundant targets experiments, the identity of the stimulus on one channel can confer information[3] concerning stimulus identity to the other second channel. For example, suppose we wish to determine whether the presence of an auditory signal conveys information about the likelihood of a visual signal. If the probability of an auditory and visual target are independent, we have

$$P(A \mid V) = (P(A) \times P(V)) / P(V) = P(A),$$ and no information is conveyed.

However, if

$$P(A \mid V) - P(A) \neq 0,$$

then the occurrence of V conveys information about the likelihood of A (see p 31 of Dudewicz, 1976). Mordkoff and Yantis (1991) refer to this difference between the conditional probability and

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[3] in this context, we do not mean information in the formal sense. Here the term information refers to the idea that presentation of a target on one channel changes the probability that a target is presented on a second channel. This occurs if the probabilities of a signal presentations on channels 1 and 2 are not statistically independent.

the marginal probability as the interstimulus contingency (denoted as: ISC (V -> A)). They observed that previous reports of coactivation were obtained only when ISC > 0. They go on to report the results of several experiments showing no evidence of coactivation (i.e., no violations of inequality (1)) when ISC $= 0$. Table 3 presents the set of interstimulus contingency values from the present experiment.

Insert Table 3 about here

In all cases, targets in one modality convey *negative* information with respect to the probability of a target presentation on the second channel. According to Mordkoff and Yantis (1991), this mitigates against violations of inequality (1). Nonetheless, clear evidence of neural summation was found. To our knowledge, these data represent the first evidence for violations of inequality (1) with negative dependencies between the target stimuli. Mordkoff and Yantis also define the interstimulus contingency benefit, ISCB $(\bar{A})$ as the difference between $P(V|A)$ and $P(V|\bar{A})$. This quantity represents a benefit confered to redundant target trials over visual targets. Similarly, we can define ISCB $(\bar{V})$ as $P(A|V) - P(A|\bar{V})$. Mordkoff and Yantis (1991) use ISCB to analyize redundant target paradigms in which subjects must make one response if either 1 or 2 targets are presented and make a different response if two non-targets are presented. The logical status of this measure is open in the present situation however. In the present case, the only type of non-target was the absence of a stimulus altogether; it is difficult to see how a lack of stimulus energy could facilitate processing in a different modality. In any case, the ISCBs in experiment 1 are also provided in Table 3, and again the values are negative. The interstimulus contingencies indicate that there is no basis for facilitatory cross-talk between the channels in the present experiment. In order to have the ISCBs play a role in the present context, one would have to suggest that the absence of a target on one channel A must slow processing on channel V. By assuming that the interactive race model may include inhibitory cross-talk, it might be suggested that negative ISCBs might slow the unimodal RTs, thereby increasing the likelihood of violations of inequality 1. This suggestion accepts the possibility questionable hypothesis that a lack of activity on one channel can slow processing on another channel. In any case, the ISCBs in Experiment II were 0, and robust violations were still observed.

Note that the interstimulus contingencies were identical for the saccadic and directed manual response conditions. This supports our hypothesis that saccades show a greater degree of intersensory facilitation than the directed manual responses. If anything, the present results may underestimate the strength of neural summation in the saccadic and directed manual conditions. Because we included catch trials in the simple manual RT condition, the negativity of the ISC is much smaller in this condition. Thus, the ISCs in the present experiment actually favor neural summation of simple RTs over the other response conditions. However, the fact that the ISC is (slightly) less than zero might be another factor contributing to the weak evidence of neural summation found for simple RTs relative to earlier reports (the ISCs in many of those reports were positive, eg., Diederich and Colonius, 1987; Miller, 1982,1986; Mordkoff and Yantis,1991). In the next experiment, we attempted to replicate these findings under conditions in which no interstimulus dependencies were operating.

EXPERIMENT II

Introduction

The second experiment was designed to extend these observations by 1) investigating the importance of matching the visual and auditory RTs in determining the magnitudes of the redundant targets effects in the present paradigm and 2) evaluate the redundant targets effects in the absence of the inter-stimulus dependencies that were operative in the first experiment. Thus, the major differences between this experiment and the previous one is that the design includes a factorial combination of high and low stimulus intensities for both the auditory and the visual targets (to produce central asynchrony between auditory and visual detection times) in the context of complete independence between the occurrences of visual and auditory targets.

Preliminary Procedures.

Once again, we attempted to match the RTs to the visual and auditory targets. Two levels of intensity were employed for both the visual and auditory modalities. The intensity levels that produced approximate matches were obtained in preliminary observations. In this experiment the same intensities were used by each observer. The selected intensities of the visual targets were 0.04 cd. m^{-2} and 12.0 cd. m^{-2}. The corresponding auditory intensities were 46 dB_{spl} and 74 dB_{spl}. In order to obtain improve our control over the auditory RTs, we added a constant background of white noise (60 dB_{spl}), which was delivered through an overhead speaker.

The general procedures were the same as in the first experiment. However, the proportions of each trial type was altered to as to eliminate the dependent structure between the stimuli. The proportions of each stimulus condition are provided in Table 4.

Insert Table 4 about here

Experimental Procedures.

Three observers participated in 15 experimental sessions of 384 trials each. Two of the observers were naive with respect to the issues under investigation and were paid for their participation. The third observer was one of the authors. Typically, a subject was tested for 6 blocks of 64 trials per day (two blocks for each response condition/day for 15 days). Each response condition (saccades, directed manual and simple manual responses) were run in accordance with a Latin square to control for possible order effects. Within each block of trials, targets were presented to the left or the right of fixation with equal frequency in a randomized order. The data reported below are based on 90 observations for each of the 9 conditions (acoustic, visual or bimodal stimuli for each response condition) in each observer.

Results

Insert Fig. 6 about here

Mean RTs, averaged across the 3 observers are provided in Figure 6. The intensity effect averaged approximately 40 msecs., which was largely independent of modality and response system. Error rates for each response condition are shown in Table 5. These rates appear quite small, and as indicated previously, the RT distributions were corrected for fast guessing. We therefore did not analyze the errors rates any further.

Insert Table. 5 about here

The results, expressed as differences between the obtained redundant target CDFs and the race inequality are shown in Figure 7.

Insert Fig. 7 about here

Once again, robust violations of the race inequality were observed in all observers and each response condition. There appears to have been little effect of mismatches between the auditory and visual detection times on the size of the redundant targets effect. Large violations are seen among all four combinations of auditory and visual signal strength. In addition, these violations occurred in the context of complete independence between the occurrence of the visual and auditory stimuli, suggesting that interstimulus contingencies appear to make little difference in the present paradigm.

There is again a suggestion that the magnitudes of the observed violations might vary with response mode however. We evaluated this difference using the methods described above, and the results are provided in Table 6.

Insert Table 6 about here

The same trend of response-dependent ordering in the violations of the race inequality seen in Experiment I appeared in Experiment II: saccades were more likely to produce violations than the manual responses.

Discussion

The results of the second experiment replicate and extend the findings of Experiment I. Thus, human saccades show clear evidence of neural summation between the visual and auditory channels. Both directed manual responses and simple manual RTs also show evidence of neural summation, but these latter effects may be less robust. In all three cases, there does not appear to be any strict requirement of simultaneity between the visual and auditory target processing - asynchronies of up to 40 msecs. can easily produce summation effects. Relatively long integration times for visual-auditory summation effects are a necessary component of the processing architecture if the systems under investigation are to have any generally useful ecological validity, as discussed above.

The Site of Neural Summation

Logically, neural summation effects could occur at the level of sensory processing, response selection, or motor execution. Evidence favoring neural summation at each of these levels of sensory-motor processing has been reported (sensory level : Fournier and Eriksen,1990; Mordkoff and Yantis, 1991, response selection : Fournier and Eriksen,1990; Miller, 1982; Mordkoff and Yantis, 1991;Schmidt, Gielen & van den Heuvel, 1984, and motor processing time : Diederich and Colonius, 1987).

Clearly, one major locus of the visual-auditory convergence resides within the oculomotor system. We suggest that the most parsimonious interpretation of the saccade data is that neural summation results from the convergence visual and acoustic afferents onto pre-saccadic burst neurons within the deeper layers of the superior colliculus (e.g. Peck, 1986; Jay & Sparks, 1987; Stein et.al., 1991). These cells share characteristics of sensory neurons as well as motor neurons, and any attempt at a dichotomous classification seems pointless. Thus, the evidence for neural summation (coactivation) in the control of saccades seems best regarded as occurring at the interface between sensory processing and motor execution.

Our interpretation of these findings with respect to saccades would receive additional support if it could be shown that these bimodal summation effects depend on the spatial alignment of the visual and acoustic inputs in a manner similar to that already described for neurons in the superior colliculus (e.g. Meredith and Stein,1987). Inverse relationships between the level of coactivation and target separation have been reported (Fournier and Eriksen,1990; Miller, 1982). An examination of the effects of spatial correspondence in the present paradigm should provide important additional information concerning the mechanisms of neural summation in oculomotor processing. The third experiment represents a preliminary investigation of the role of spatial correspondence in mediating these summation effects.

EXPERIMENT III

Introduction

In the third experiment we sought to provide an analysis of the effects spatial misalignment between the auditory and visual targets on the summation effects reported above. Once again, saccades, directed manual responses, and simple manual RTs were investigated. In this initial analysis, bimodal targets could either occur in spatial register or out-of-register. In the later case, the visual and auditory targets occurred in opposite hemifields. Under such conditions, it would be surprising indeed to see evidence of neural summation, since there are no indications that visual and auditory inputs arising from such disparate locations converge onto common neural elements. Moreover, the out-of-register targets would certainly be expected to produce competing response tendencies, at least in the case of saccades and directed manual responses. The question we wished to address with respect to these directionally specific responses was not so much whether evidence of neural summation would be obtained with these spatially misaligned targets, but rather, could we provide evidence that the occurrence of these summation is specifically dependent on the spatial alignment of the auditory and visual stimuli. The situation for the simple manual responses seemed less clear cut, since it appeared reasonable to hypothesize that auditory-visual summation of simple detection times could occur for both in-register and out-of-register targets.

Methods

In general, the methods and procedures were the same as those used in Experiments I and II. The major differences relate to design changes required by the inclusion of spatially misaligned targets. Thus, there were two basic conditions: one in which the auditory stimulus was designated as the target, and one in which the visual stimulus was designated as the target. In either case, 20.0 % of the trials were catch trials. A target (either visual or auditory, depending on the condition) was presented on the remaining 80.0 % of the trials. Of these target trials, 50 % presented the target stimulus alone, while the remaining 50 % of the target-present trials presented bimodal stimuli. The auditory and visual stimuli were presented in spatial register on half of the bimodal trials and were presented out-of-register (in opposite hemifields) in the remaining half. The distribution of trial types is summarized in Table 7.

Insert Table 7 about here

All stimuli were presented at an eccentricity of 15°. Target location varied between the left and right locations with equal frequency, and the sequence of trial types was randomized.

The 3 subjects from Experiment II again served as observers. Subjects participated in 10 experimental sessions,where each session consisted of 6 blocks of 50 trials/day (two blocks of 50 trials for each of the three response conditions). The order for performing each response condition was counter-balanced across days. Finally, target modality was blocked: subjects participated in 5 sessions with one modality as the target, and then participated in 5 more with the other modality as the target. JZ and JE first responded to auditory targets, while HH first responded to visual targets.

The subjects were told which stimulus they should treat as the target, and to respond to the target modality as quickly as possible. Depending on response mode, these responses were of course to either look at the target, thrust the joystick towards the target, or depress the microswitch. The observers knew that stimuli in the other modality would sometimes appear, and that they may or may not be aligned with the target. They were given no specific instructions concerning how to deal with these ancillary stimuli, other than to realize that responses that were not directed to the designated target would be treated as errors.

The stimulus intensities were taken from experiment II, and were selected on the basis of producing the closest match in unimodal RTs obtained in that experiment. The intensities used for each observers are provided in Table 8. As in experiment II, white noise was present throughout the duration of the session (60 dB_{spl}).

Insert Table 8 about here

As it turned out, these intensities did not produce matched unimodal RTs in Experiment III. Apparently, the change in conditions (only having to respond to a single intensity, only having one modality as the target) altered the decision criteria and caused the resulting mismatches. However, these mismatches in the unimodal RTs proved quite fortuitous in interpreting the results, as we describe below.

Results

Average RTs for the bimodal trials as a function of both spatial correspondence and target modality are illustrated in Figure 8 for each response condition. The corresponding error rates are given in Table 9.

Insert Figure 8 and Table 9 about here

The reaction time data were submitted to a four factor analysis of variance. The factors were spatial correspondence, target modality, response mode and subjects. The analysis revealed significant main effects of correspondence ($p<.02$) and response mode ($p<.05$). In addition, the interaction between spatial correspondence and response mode was significant ($p<.02$). Examination of Figure 7 shows that this interaction is attributable to the effects of spatial correspondence on saccades and directed manual responses; spatial correspondence had little effect on simple manual responses.

In order to evaluate the results in terms of the race inequality, we had to use the unimodal distributions from the two different target conditions (e.g., single auditory stimuli were only presented during the auditory target condition). The only significance of this is that all the data used in the analysis could not be obtained within the same sessions. By the time that these data were collected however, each of the observers had performed these tasks for a long period of time, and were producing data that were quite stable.

Insert Fig. 9 about here

The results, expressed as differences between the obtained bimodal RT distributions and the race inequality, are provided in Figure 9. The data for the corresponding and non-corresponding bimodal stimulus trials are indicated for each of the three response modes. Consider first the simple manual responses. In this case, violations of the race inequality appear equal for both corresponding and non-corresponding bimodal targets. Although the violations observed in

HH were quite small, those observed in JE and JZ were substantial. In all cases however, the spatial alignment of the stimuli appears to have little effect.

This contrasts with the results obtained using directed responses (saccades and directed manual responses). When the bimodal stimuli were presented at non-corresponding spatial locations, performance was substantially below that predicted by the race inequality (negative values indicate performance slower than the inequality). Indeed, there were essentially no violations observed in the entire data set. In contrast, violations of the race inequality did occur when the bimodal targets occurred in corresponding spatial locations, but not under all conditions. Consider for example, observer HH. Violations of the race inequality were observed for both directed manual responses and saccades when the visual stimulus was designated as the target, but not when the auditory stimulus was the target. The pattern of results in observer JE is even more obscure. JE shows robust violations for saccades, but only for visual targets. In contrast, JE's directed manual responses only show violations when the auditory stimulus was designated as the target. An equally obscure pattern is apparent in observer JZ. Thus, the results indicate that evidence of neural summation relies on the spatial correspondence between the auditory and visual targets, but appears to depend on some additional factor(s) as well.

Examination of the actual cumulative distribution functions for the various conditions provides important clues as to what might be the basis for these apparently disorderly results.

Insert Fig. 10 about here

Figure 10 illustrates the unimodal CDFs, the obtained bimodal CDFs, the race inequality, and the independent race prediction for the directed manual and saccade responses in each of the 3 observers. Inspection of the unimodal CDFs show that, in each case, one of the modalities produced faster RTs than the other. When we compare the obtained bimodal distributions for corresponding stimuli to the race inequality, it becomes clear that *the race inequality is only violated when the faster of the two modalities was designated as the target.* There were no exceptions. A process of stimulus selection (based on the designated modality of the target) appears to precede the summation stage. When the target modality is slower than the ancillary modality, subjects apparently withhold their responses until the stimulus in the designated modality has been identified. This precludes the expression of summation effects between the two stimuli. Notice that, when the two stimuli are presented in non-corresponding locations, the RTs approach but never exceed the unimodal CDFs; when the targets appear in corresponding locations, the bimodal CDFs are always faster than the CDF for the target modality presented alone, but exceed the race inequality only when the target modality is the faster of the two. Thus, the observers conform to the requirements of the task: they make their responses contingent on the detection of the designated target. If that target is likely to be detected first, a response can be initiated, but the later arriving ancillary stimulus still is able to facilitate processing if it occurs in the corresponding location. If the slower of the two modalities is the target, subject must await detection of the target in order to avoid errors; they are apparently able to do this with reasonable efficiency.

This pattern of results is also reflected in the error rates (Table 9). Notice that most of the errors occur on non-corresponding trials. The error rates for saccades clearly contrast with those generated by the same observers in Experiment II: in Experiment II, not a single saccadic direction error was made by any of the subjects.

Discussion

The results of the third experiment indicate that summation effects depend upon the spatial alignment of the stimuli if the task requirements make localization of the target essential. If not, as was the case for the simple manual responses, spatial alignment appears unimportant. Moreover, in order to avoid errors when the response is contingent on the location of a designated target, subjects must (and can) efficiently gate the emission of their responses according to target modality. Although this selection process obviously must precede response execution, the selection process does not act as a simple filter which prevents the ancillary stimulus modality from contributing to the activation of a response. If this were the case, then the summation effects produced by spatial correspondence would be prevented (a the pattern of errors would differ as well). Clearly, information from a spatially aligned ancillary target can facilitate processing, but this facilitation can only generate RTs that violate the race inequality if the target modality is likely to be detected first. Otherwise, the subject waits for the target. This waiting slows responses to a level that makes violations of the race inequality difficult to achieve.

The fact that violations of the race inequality were observed for simple RTs for both aligned and misaligned stimuli may suggest that summation can occur at a post-sensory stage of processing. This suggestion is based on the supposition that a specific convergence of visual and auditory information originating from sources separated by 30° (15° on either side of fixation) seems unlikely, especially when they are located in opposing hemifields. At least we are unaware of any evidence that such convergence occurs. Of course, it is entirely possible that both sensory and motor coactivation effects mediate the violations of the race inequality seen for simple RTs. Indeed it seems possible that the summation effects on simple RTs might rely on sensory convergence in the case of corresponding stimuli, and motor coactivation in the case of non-corresponding stimuli.

Interpreting evidence for coactivation in terms of convergence of the sensory pathways seems safer when the effects depend on the spatial alignment of the stimuli. Here too however, coactivation of motor processes appears viable. For example, while the failure to find evidence of neural summation for either directed manual responses or saccades for non-corresponding stimuli might be related to the fact that the sensory pathways that carry the target information are inhibited the addition of by out-of-register stimuli (e.g. Meredith and Stein,1987), it is also possible that the pathways conveying information for such dramatically misaligned stimuli simply do not converge onto any common element that is utilized in the performance of these tasks, but does invoke incompatible response tendencies which must somehow be resolved. What is clear from the present results is that out-of-register stimuli can certainly interfere with the processing of the target, as is shown by the degree to which these responses are actually slower than the corresponding unimodal distribution of response times.

It seems possible that all of these coactivation effects could relate to the actual generation of responses. In the case of saccades, neural circuitry clearly capable of producing all of the effects we report here is know to exist within the superior colliculus. As we suggested above however, these neurons are truly at the interface between sensory and motor processes, so they can be regarded as pre-motor neurons as readily as anything else. Much less is known of sensory convergence in pathways that might be operative during the execution of sensory-motor tasks involving the hands. One can make a case for a transcortical route, and polymodal cortical areas have been identified (eg., Kimura & Tamai, 1992; Neal, Pearson and Powell, 1990; Mistlin & Perrett, 1990; Seltzer & Pandya, 1989). There is little to indicate that they play a pre-motor role that is in any way analogous to the function of the deeper layers of the superior colliculus in initiating saccades however.

In conclusion, these results indicate that saccades show robust evidence of neural summation between auditory and visual inputs that can reasonably be interpreted in terms of known patterns of auditory-visual convergence within an important oculomotor structure: the superior colliculus. Different neural pathways are assumed to mediate auditory-visual summation effects observed for manual responses, and thus there may be either qualitative or quantitative differences; this is, of course, an empirical issue. The suggestions in the present data that the magnitudes of the race inequality violations might differ with different response systems are, in our opinion, promising, and could indicate differences in the mechanisms that produce coactivation effects in different circumstances. Similarly, the importance of spatially aligned targets may reflect important differences in the locus of coactivation effects for different tasks. Future work, involving for example less extreme cases of spatial misalignment, should provide additional insight into the nature and loci of coactivation effects in these systems, and may provide insight relevent to a range of emerging issues in the study of coactivation effects more generally.

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Table 1. Probabilities and Mean RTs for correct responses, false alarms, and direction errors, Experiment I.

	Saccades	Directed Manual resps.	Simple Manual resps
P (false alarm)	--------------------	--------------------	0.0313
Mean RT (false alarm)	--------------------	--------------------	198.0
P (Direction error)	0.004	0.03	--------------------
Mean RT (dir. error)	106	214.4	--------------------
Mean RT (correct resp)	213.9	315.7	267.0

Table 2. Z scores associated with the probability of violations of race inequality: Experiment I

Observer	Saccades vs. Directed. Manual	Saccades vs Simple Manual
GT	7.36 **	1.1
LQ	7.98 **	6.65 **
JA	2.75 **	2.72 **
MK	2.12 *	1.89 *

* $p<0.05$, ** $p<0.01$

Table 3. Dependent structure and the various stimulus probabilities for each response condition in Experiment I.

Saccadic and Directed Manual Response

Probabilities Associated with Various Stimulus Events

$P(A \text{ and } V) = 1/3$ $P(A \text{ and } \bar{V}) = 1/3$ $P(\bar{A} \text{ and } V) = 1/3$ $P(\bar{A} \text{ and } \bar{V}) = 0$
$P(A \mid V) = 1/2$ $P(A \mid \bar{V}) = 1.0$ $P(V \mid \bar{A}) = 1.0$ $P(V \mid A) = 1/2$
$P(A) = 2/3$ $P(V) = 2/3$ $P(\bar{A}) = 1/3$ $P(\bar{V}) = 1/3$

Summary of the dependent structure for saccadic and directed manual responses.

Events	Conditional Probability	Marginal Probability	Difference
$A \mid V$ versus V	1/2	2/3	-0.1666
$V \mid A$ versus A	1/2	2/3	-0.1666
$A \mid \bar{V}$ versus $\bar{V}$	1.0	2/3	0.333
$V \mid \bar{A}$ versus $\bar{A}$	1.0	2/3	0.333
$V \mid A$ versus $V \mid \bar{A}$			-0.5
$A \mid V$ versus $A \mid \bar{V}$			-0.5

Simple Manual Response

Probabilities Associated with Various Stimulus Events

$P(A \text{ and } V) = 1/3$ $P(A \text{ and } \bar{V}) = 1/4$ $P(\bar{A} \text{ and } V) = 1/4$ $P(\bar{A} \text{ and } \bar{V}) = 1/6$
$P(A \mid V) = 4/7$ $P(A \mid \bar{V}) = 3/5$ $P(V \mid \bar{A}) = 3/5$ $P(V \mid A) = 4/7$
$P(A) = 7/12$ $P(V) = 7/12$ $P(\bar{A}) = 5/12$ $P(\bar{V}) = 5/12$

Summary of the dependent structure for simple manual responses.

Events	Conditional Probability	Marginal Probability	Difference
$A \mid V$ versus V	4/7	7/12	-0.0119
$V \mid A$ versus A	4/7	7/12	-0.0119
$A \mid \bar{V}$ versus $\bar{V}$	3/5	5/12	0.183
$V \mid \bar{A}$ versus $\bar{A}$	3/5	5/12	0.183
$V \mid A$ versus $V \mid \bar{A}$			-0.029
$A \mid V$ versus $A \mid \bar{V}$			-0.029

Table 4. Dependent structure and the various stimulus probabilities in Experiment II.

Probabilities Associated with Various Stimulus Events

$P(A \text{ and } V)=0.25$ $P(A \text{ and } \bar{V})=0.25$ $P(\bar{A} \text{ and } V)=0.25$ $P(\bar{A} \text{ and } \bar{V})=0.25$
$P(A \mid V)=0.5$ $P(A \mid \bar{V})=0.5$ $P(V \mid \bar{A})=0.5$ $P(V \mid A)=0.5$
$P(A)=0.5$ $P(V)=0.5$ $P(\bar{A})=0.5$ $P(\bar{V})=0.5$

Summary of the dependent structure for saccadic and directed manual responses.

Events	Conditional Probability	Marginal Probability	Difference
$A \mid V$ versus V	0.5	0.5	0.0
$V \mid A$ versus A	0.5	0.5	0.0
$A \mid \bar{V}$ versus $\bar{V}$	0.5	0.5	0.0
$V \mid \bar{A}$ versus $\bar{A}$	0.5	0.5	0.0
$V \mid A$ versus $V \mid \bar{A}$			0.0
$A \mid V$ versus $A \mid \bar{V}$			0.0

Table 5. Probabilities and Mean RTs for correct responses, false alarms, and direction errors, Experiment II.

	Saccades	Directed Manual resps.	Simple Manual resps
P (false alarm)	0.0167	0.009	0.0715
Mean RT (false alarm)	300.2	233.4	261
P (Direction error)	0.0	0.018	--------------------
Mean RT (dir. error)	--------------------	254.4	--------------------
Mean RT (correct resp)	220.9	287.8	284.8

Table 6. Z scores associated with the probability of violations of race inequality: Experiment II

Observer	Saccades vs. Directed. Manual	Saccades vs Simple Manual
HH	4.88 **	1.7 *
JE	3.84 **	2.67 **
JZ	8.6 **	10.03 **

* $p<0.05$, ** $p<0.01$

Table 7. Dependent structure and the various stimulus probabilities in Experiment III.

Probabilities Associated with Various Stimulus Events: Auditory target condition

P(A)=0.8   P(V)=0.4   P($\overline{A}$)=0.2   P($\overline{V}$)=0.6
P(A and V) =0.4   P(A and $\overline{V}$)=0.4   P($\overline{A}$ and V)=0.0   P($\overline{A}$ and $\overline{V}$)=0.2
P(aligned$\mid$A and V) =0.5   P(misaligned$\mid$A and V)=0.5
P($A \mid V$)=1.0   P($A \mid \overline{V}$)=0.667   P($V \mid \overline{A}$)=0.0   P($V \mid A$)=0.5

Summary of the dependent structure in the auditory target condition.

Events	Conditional Probability	Marginal Probability	Difference
$A \mid V$ versus V	1.0	0.4	0.6
$V \mid A$ versus A	0.5	0.8	-0.3
$A \mid \overline{V}$ versus $\overline{V}$	0.667	0.6	0.067
$V \mid \overline{A}$ versus $\overline{A}$	0.0	0.2	-0.2
$V \mid A$ versus $V \mid \overline{A}$			0.5
$A \mid V$ versus $A \mid \overline{V}$			0.333

Probabilities Associated with Various Stimulus Events: Visual target condition

P(A)=0.4   P(V)=0.8   P($\overline{A}$)=0.6   P($\overline{V}$)=0.2
P(A and V) =0.4   P(A and $\overline{V}$)=0.0   P($\overline{A}$ and V)=0.4   P($\overline{A}$ and $\overline{V}$)=0.2
P(aligned$\mid$A and V) =0.5   P(misaligned$\mid$A and V)=0.5
P($A \mid V$)=0.5   P($A \mid \overline{V}$)=0.0   P($V \mid \overline{A}$)=0..667   P($V \mid A$)=1.0

Summary of the dependent structure in the visual target condition.

Events	Conditional Probability	Marginal Probability	Difference
$A \mid V$ versus V	0.5	0.8	-0.3
$V \mid A$ versus A	1.0	0.4	0.6
$A \mid \overline{V}$ versus $\overline{V}$	0.0	0.2	-0.2
$V \mid \overline{A}$ versus $\overline{A}$	0.667	0.6	0.067
$V \mid A$ versus $V \mid \overline{A}$			0.333
$A \mid V$ versus $A \mid \overline{V}$			0.5

Table 8. Stimulus intensities used in Experiment III

	Saccades	Saccades	Directed Manual	Directed Manual	Simple Manual	Simple Manual
Observer	Auditory Intensity	Visual Intensity	Auditory Intensity	Visual Intensity	Auditory Intensity	Visual Intensity
JE	74 dB	12.0 cd. m^{-2}	74 dB	12.0 cd. m^{-2}	74 dB	12.0 cd. m^{-2}
IZ	74 dB	12.0 cd. m^{-2}	74 dB	12.0 cd. m^{-2}	46 dB	12.0 cd. m^{-2}
HH	46 dB	0.04 cd. m^{-2}	46 dB	0.04 cd. m^{-2}	46 dB	0.04 cd. m^{-2}

Table 9. Probabilities and Mean RTs for correct responses, false alarms, and direction errors, averaged across the three observers. Experiment III.

	Saccades	Directed Manual resps.	Simple Manual resps
P (False Alarm):	0.0233	0.0133	0.0758
Mean RT (False Alarm)	237.1	367.3	243.1
Prob (Direction error)			
All Trials	0.0125	0.04	--------------------
Corresponding Stimuli: Aud. Targets	0.003	0.023	--------------------
Non-Corresponding Stimuli: Aud. Targets	0.073	0.16	--------------------
Corresponding Stimuli: Vis. Targets	0.003	0.013	--------------------
Non-Corresponding Stimuli: Vis. Targets	0.003	0.093	--------------------
Mean RT (Direction. error)	279.2	276	--------------------
Mean RT (correct resp)	227.3	306.1	232.8

Figure Captions

Figure 1. Three boundary conditions for parallel processing systems. See text for details.

Figure 2. Mean latencies for visual, acoustic and bimodal targets for each response condition in Experiment I.

Figure 3. Violation of the race model inequality. The unimodal target CDFs, an obtained bimodal CDF, and the performance limit of the race model inequality (sum of the unimodal CDFs) are as indicated. Violations of the race model inequality are indicated by the vertical hatching. The differences between the obtained bimodal CDF and the inequality are shown in the bottom panel.

Figure 4. Results from experiment I. Positive values indicate the magnitude of the violations of inequality 1. Data for saccades, directed manual responses and simple manual rts are as indicated.

Figure 5. Example of the potential problem associated with the correction for fast guesses. Top panel shows the CDF for false alarms in observer MK. A total of 12 false alarms were recorded in the simple RT task. The earliest of these have no counterpart in the target present trials. Bottom panel illustrates the effects on computations of race inequality violations. Although the changes are small in magnitude, they dramatically affect statistical measures based on the probability of a violation.

Figure 6. Mean RTs as a function of stimulus intensity for the unimodal targets in Experiment II.

Figure 7. Violations of race model inequality using different combinations of auditory and visual stimulus intensity. Format is the same as Figure 4.

Figure 8. Mean RTs to bimodal targets presented at corresponding and non-corresponding locations for saccades, directed manual responses, and simple manual RTs.

Figure 9. Violations of the race inequality for three response modes as a function of the spatial alignment of the targets. Positive values indicate violations. See text for details.

Figure 10. Unimodal CDFs, bimodal CDFs, the independent race prediction, and the limiting case of the race model inequality for each observer in experiment III. Data from trials with corresponding targets versus non-corresponding targets are indicated. Only data for saccades and directed manual responses are shown, since correspondence between the stimuli had no effect on simple manual RTs (Figure 9).

Illustration of Performance boundaries of Race Models

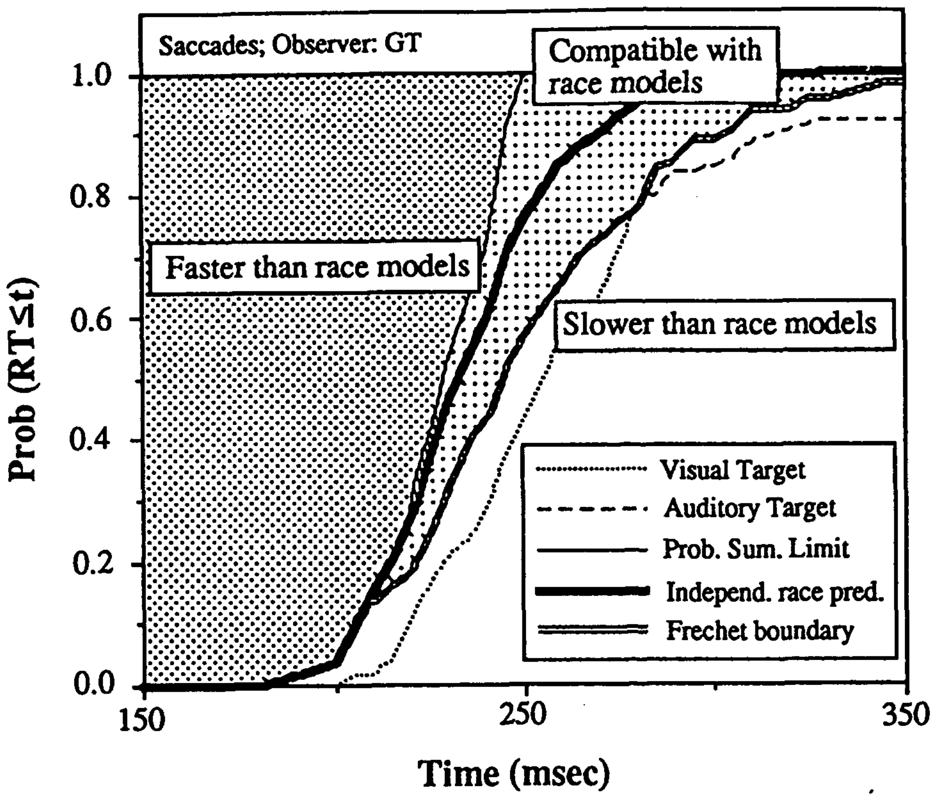

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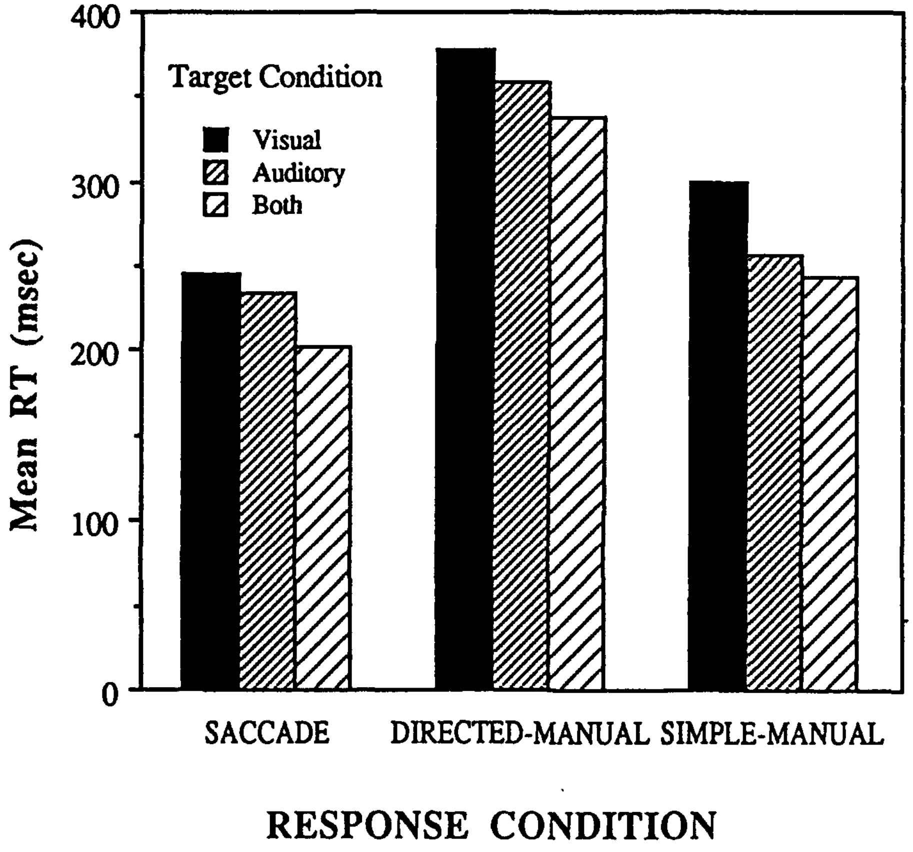
Figure 2.



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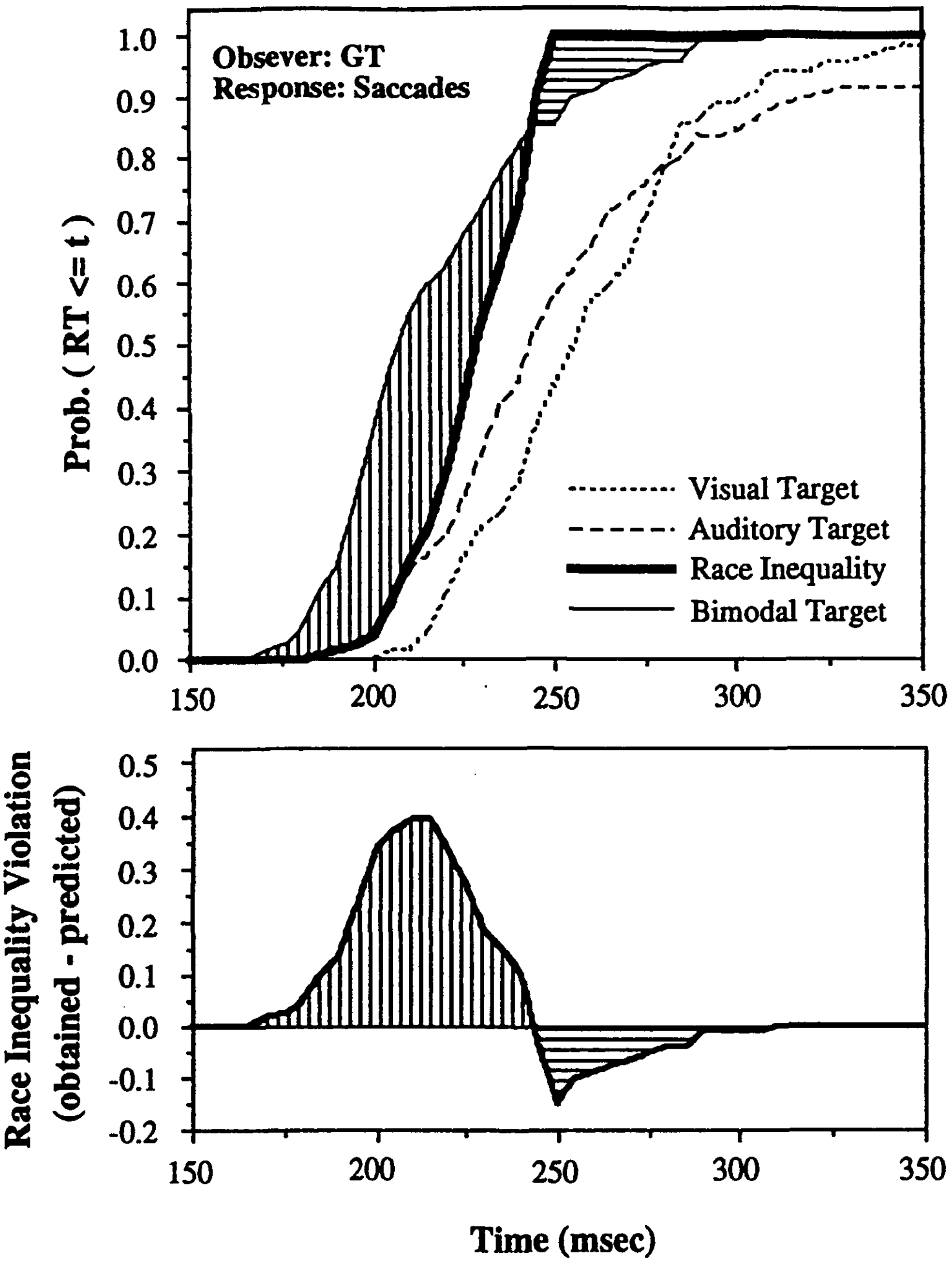

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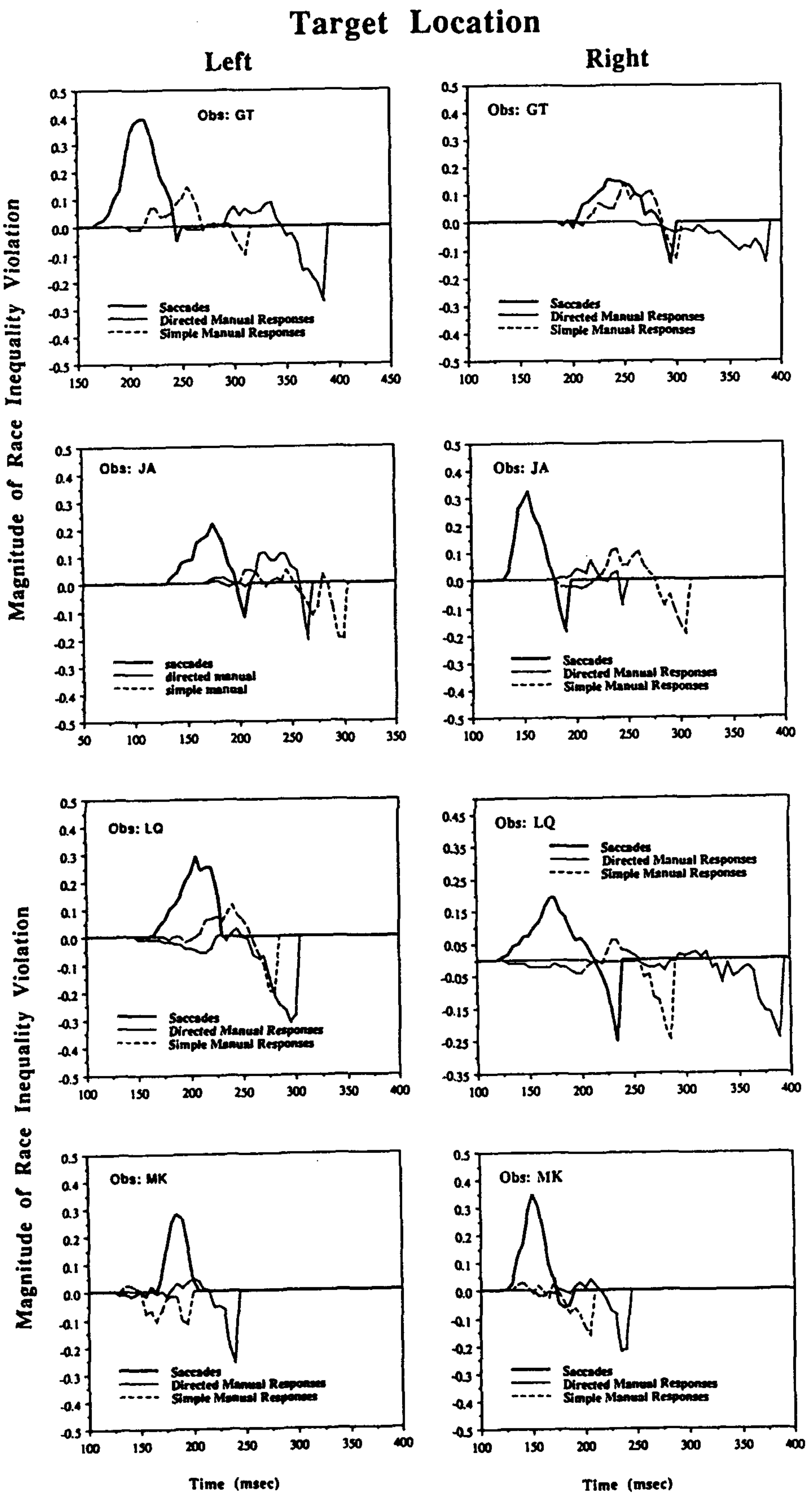

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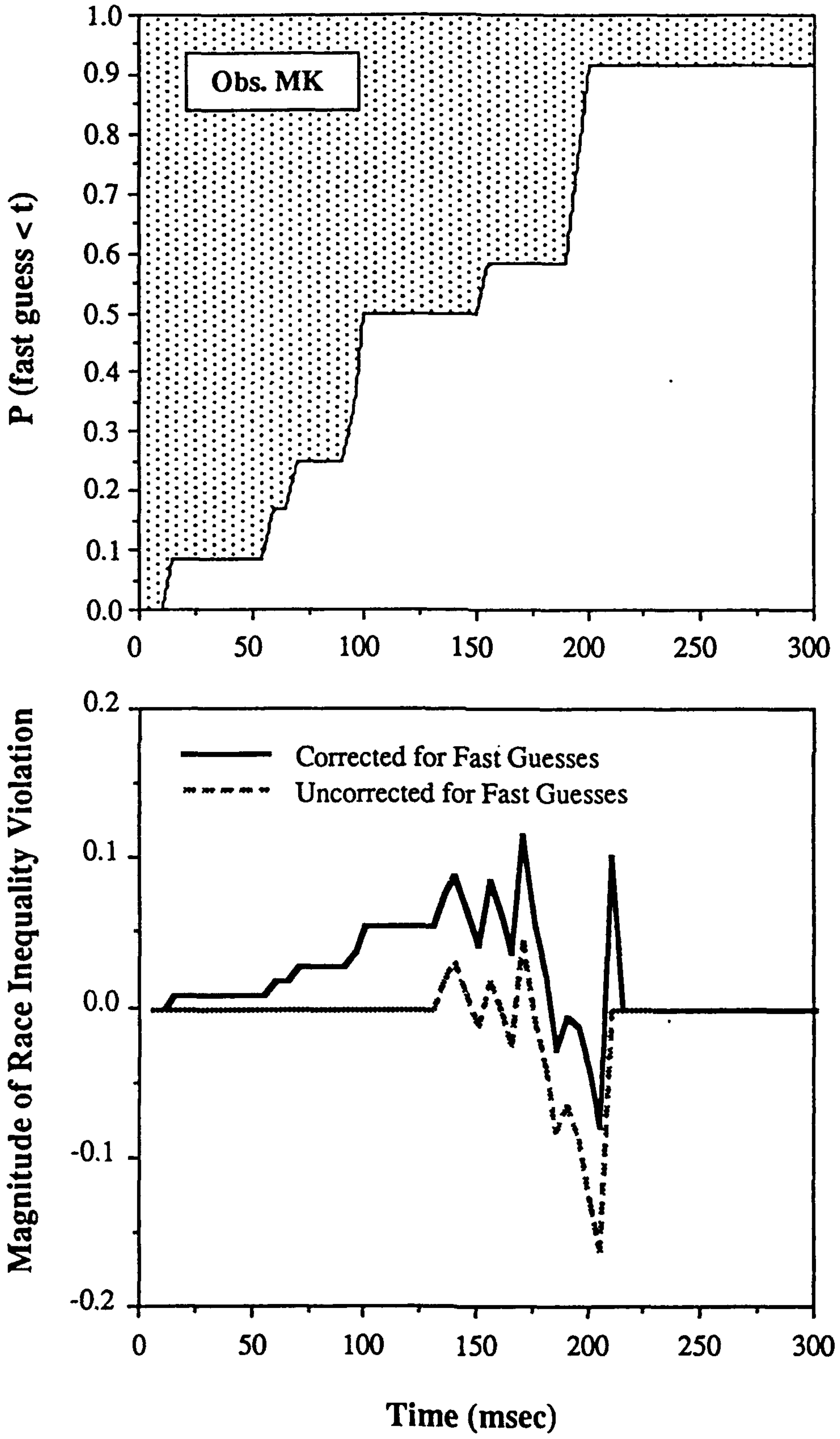
Fig. 4



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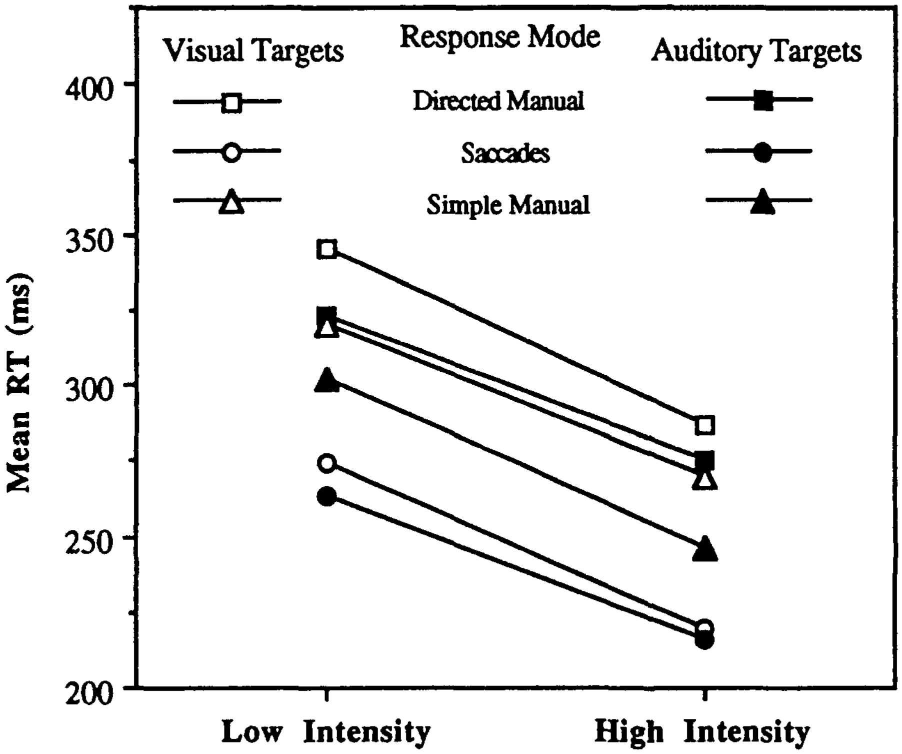

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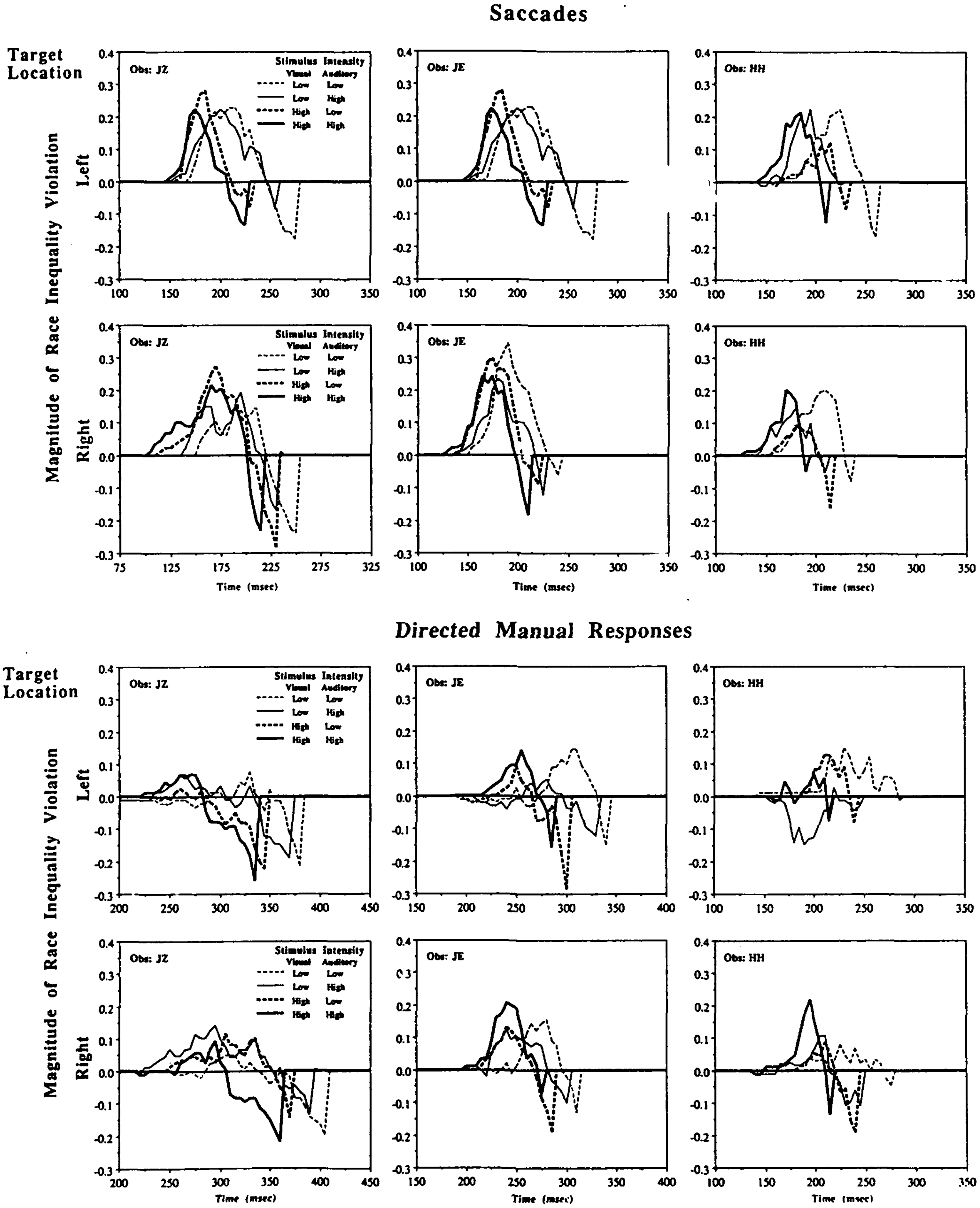

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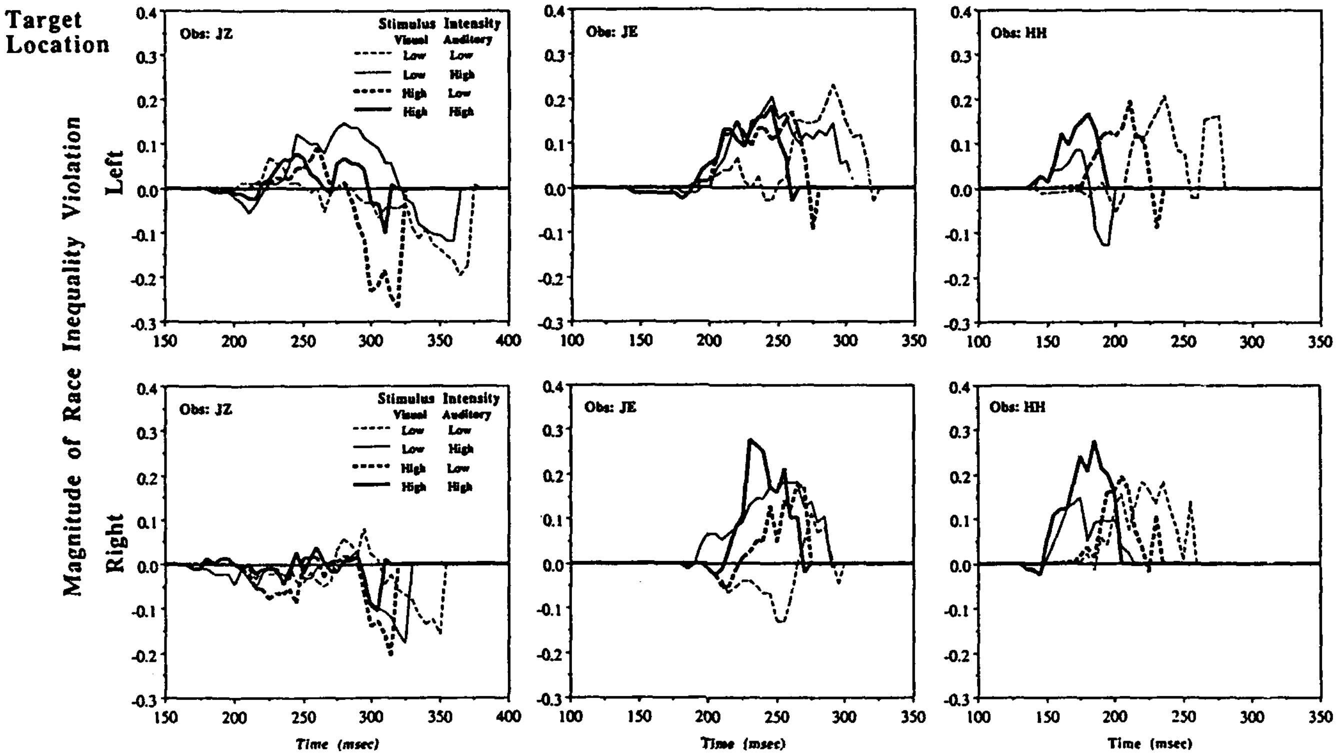
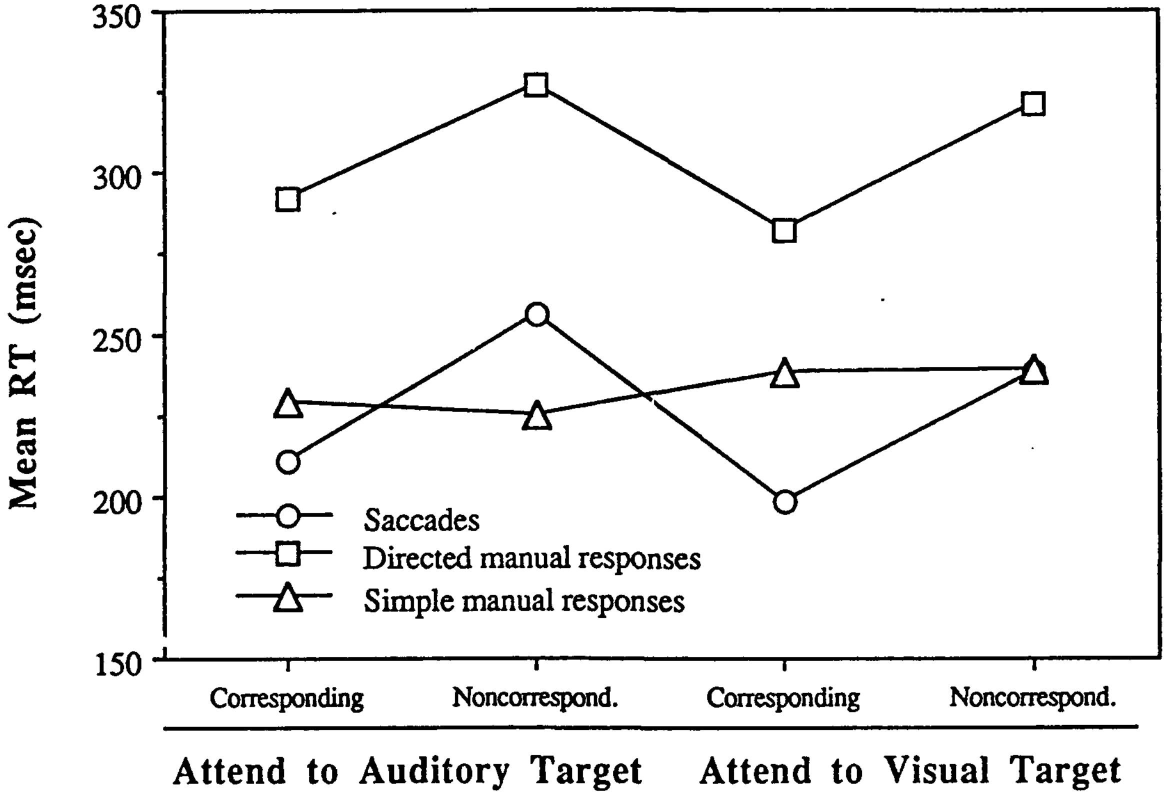
Fig 7.

Simple Manual Responses

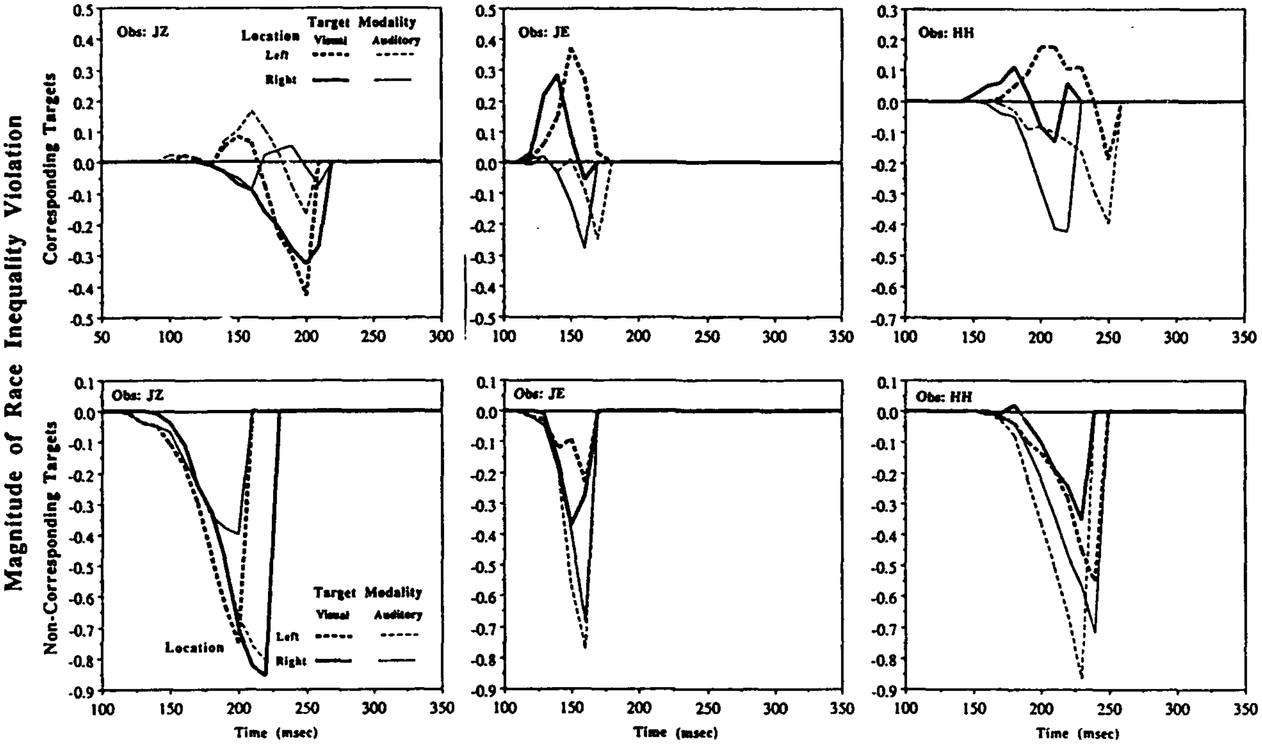


Hughes et.al. Fig 7

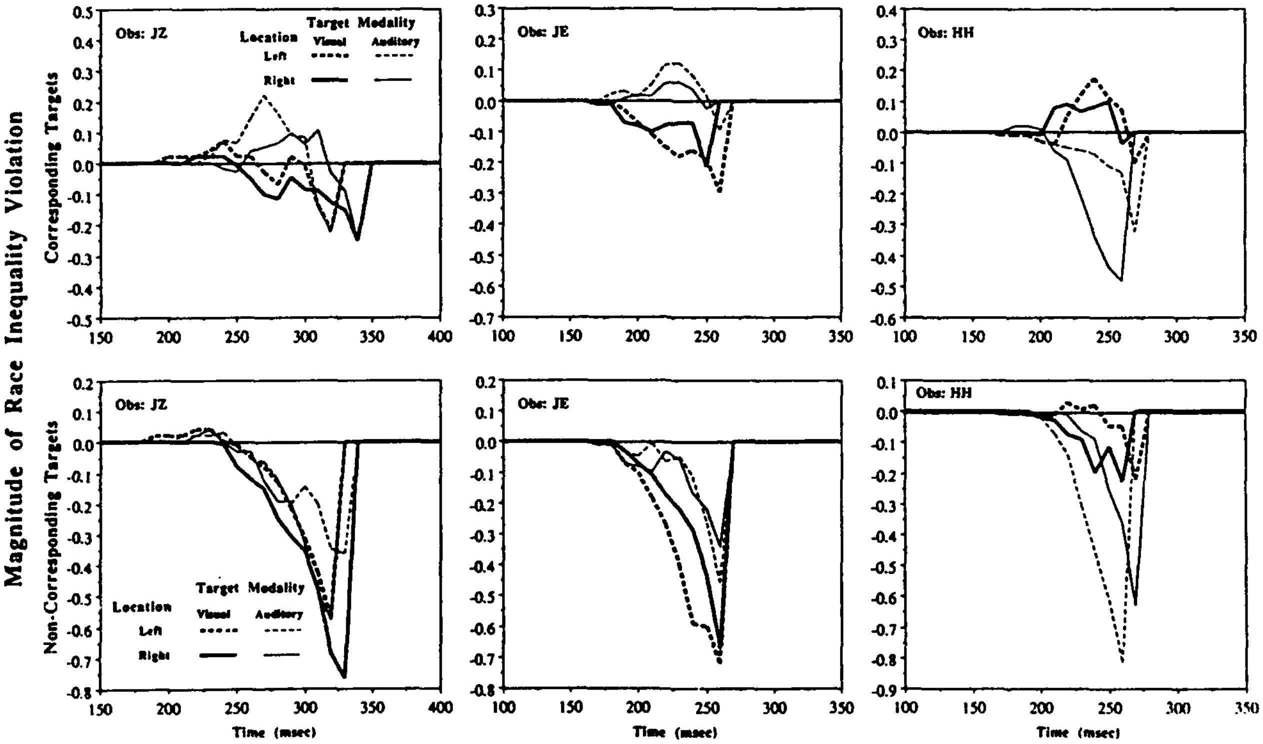


Hughes et.al. Figure 8.

Saccades

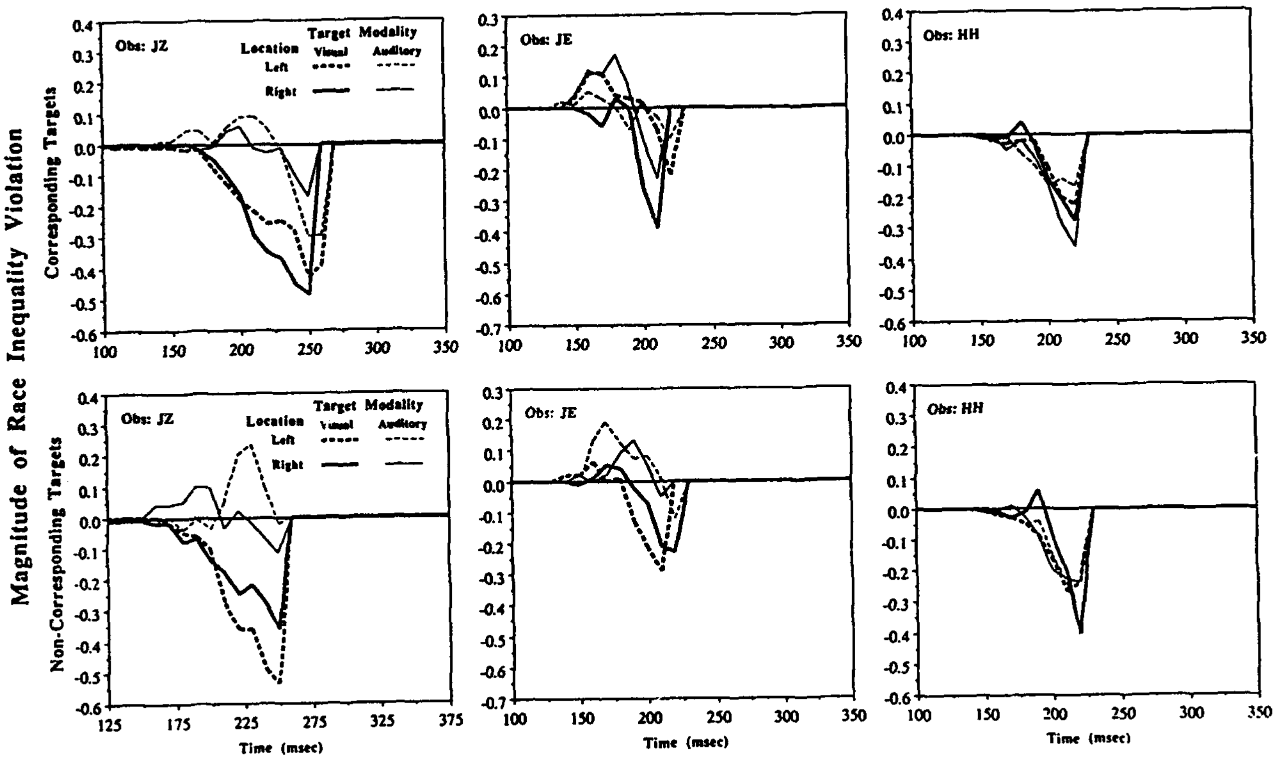


Directed Manual Responses



Hughes et. al. Fig 9.

Simple Manual Responses



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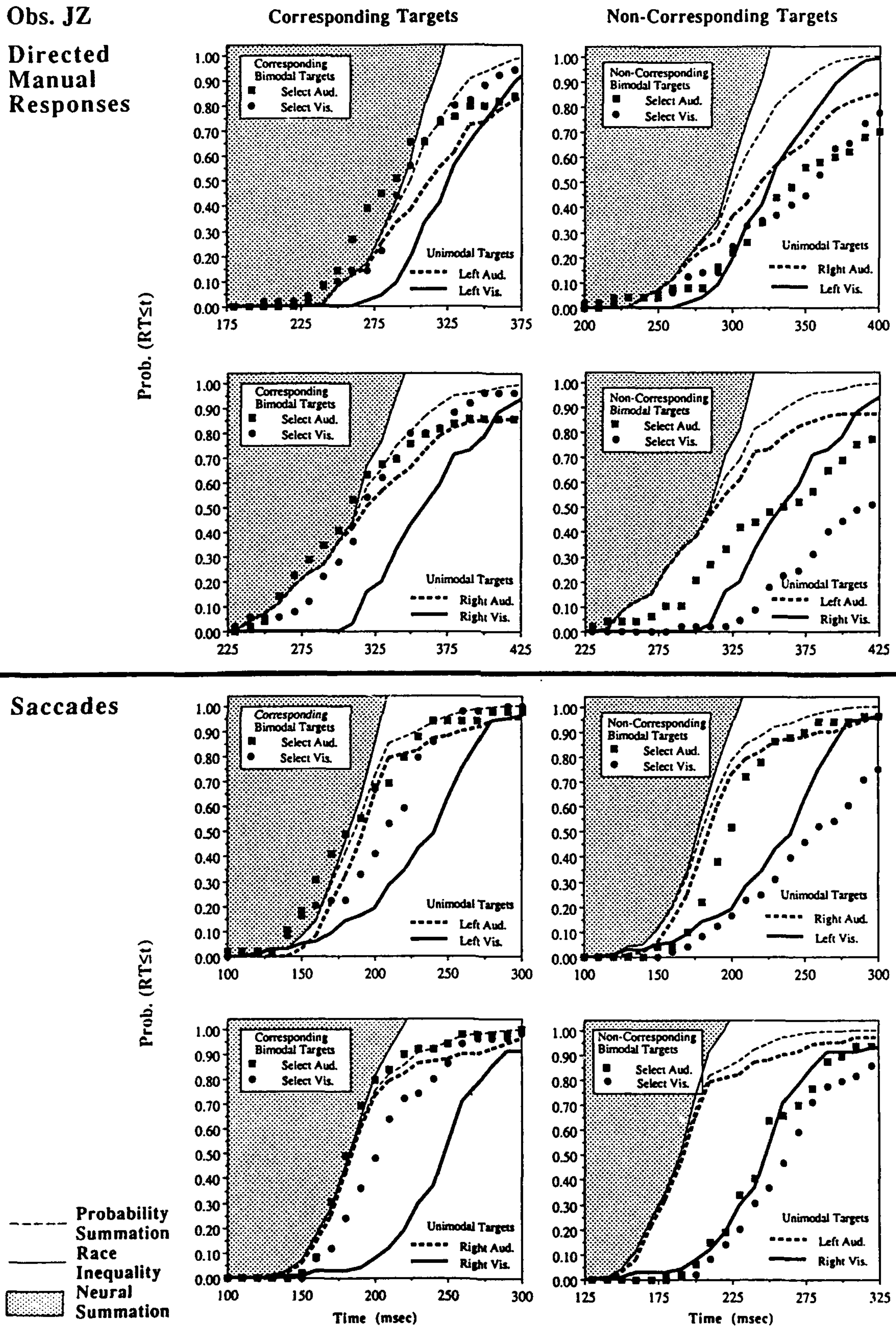
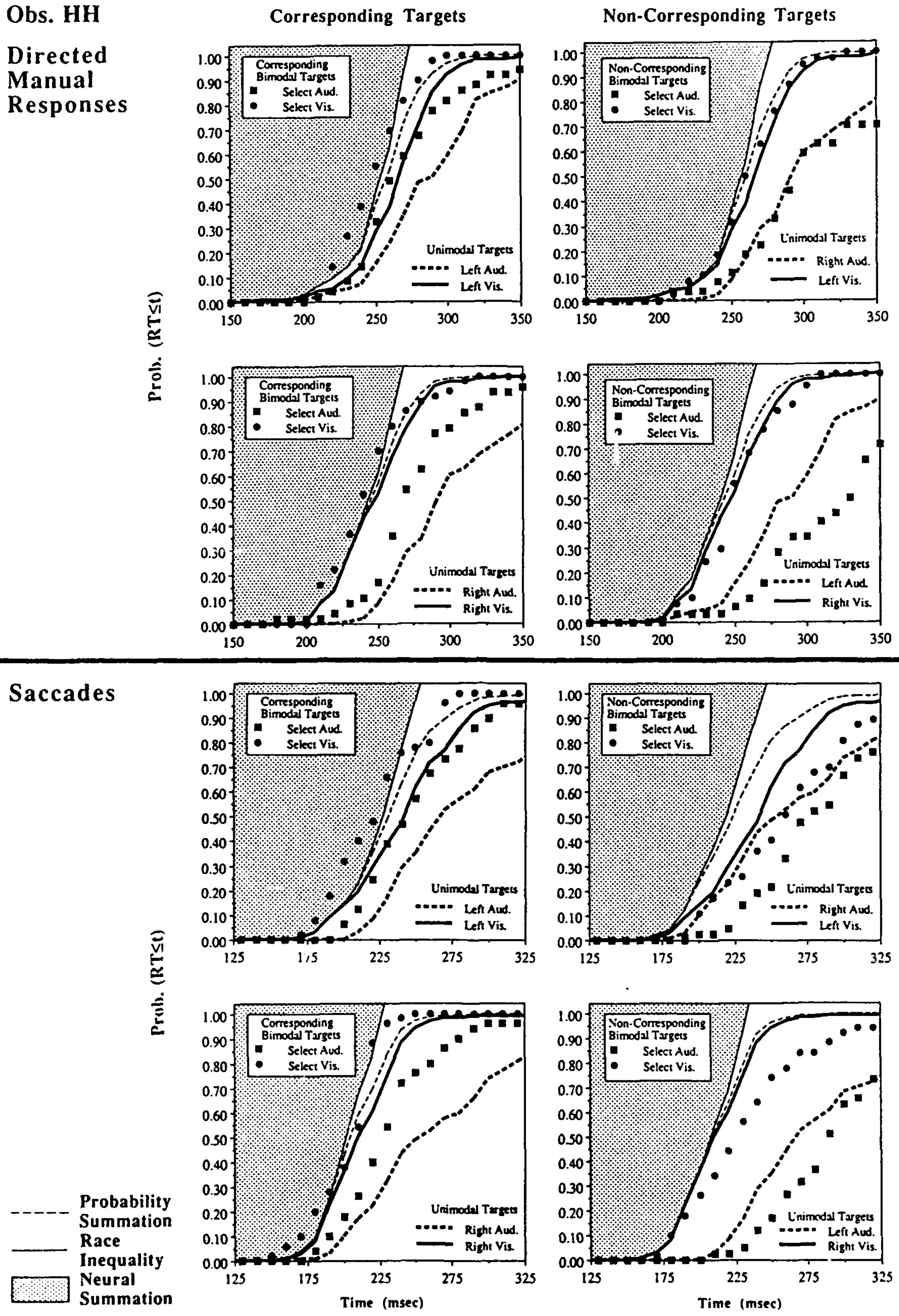
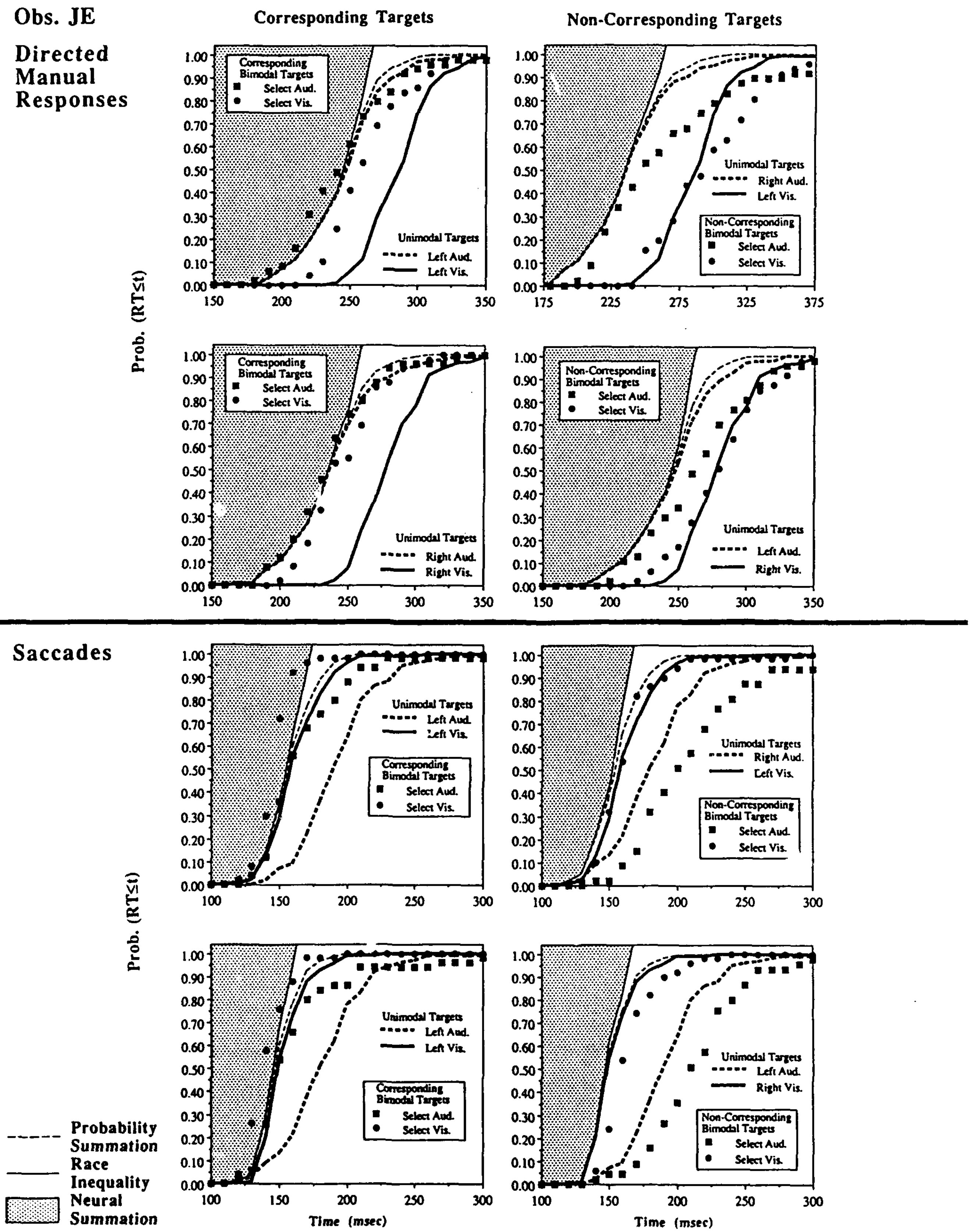


Fig 10 Hughes et. al.



Hughes et.al. Fig 10



Hughes et al. Fig 10.

STOCHASTIC MODELS OF HUMAN INFORMATION PROCESSING

A Thesis

Submitted to the Faculty

in partial fulfilment of the requirements for the

degree of

Doctor of Philosophy

by

Georgie Nozawa

DARTMOUTH COLLEGE
Hanover, New Hampshire
June 1992

Examining Commitee:

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ABSTRACT

The purpose of the study was to investigate human performance in relatively simple information processing tasks which use two input channels of sensory information. Human information processing was viewed as a flexible architecture of component processes whose internal state representations are random variables. The goal was to identify analytically the processing architecture involved in two types of logical tasks: the OR task and the AND task. The experiments used these two tasks in the context of two experimental situations: a supra-threshold reaction time (RT) experiment and a near-threshold signal detection (psychophysical) experiment. The experiment explored various hypotheses of integrating information on two channels. The results are summarized below.

RT Experiment

1. In the OR task superadditivity of the mean and the survivor contrasts were obtained. The model most compatible with the data was the probability summation model.
2. In the AND task subadditivity of the mean and the survivor contrasts were obtained. The model most compatible with the data turned out to be the parallel exhaustive processing model.

Psychophysical Experiment

1.The slope analysis of the standardized ROC curves allowed us to reject the timing hypothesis both in the OR task and the AND task. The counting mechanism was compatible with the data.

2.The experiment provided subadditivity of the mean response contrast in the OR task. Further analysis showed that the results of the OR task was most compatible with the probability summation model.

3.The experiment provided superadditivity of the mean response contrast in the AND task. Further analysis showed that, among the set of models evaluated, the result were most compatible with a channel multiplication model.

The conclusion was reached that there was a correspondence between the logical task requirements and the information processing architecture utilized in the performance of these tasks. The OR task caused convexity of the function which combines the activity within each channel (i.e., the integration function). The AND task caused concavity of the integration function.

Acknowledgment

I wish to thank the following individuals for supporting me in my pursuit to accomplish this research. First of all, I wish to convey my greatest admiration and respect to Professor Howard Hughes. He was very supportive of my generating ideas and the verification of them. Professor Carol Fowler was always generous in spending time to understand my ideas. Professor Robert Fendrich was very supportive in being a good friend of mine. Professor George Wolford was very encouraging. I thank Professor Patricia Reuter-Lorenz for the time she spent to listen to me on my ideas. I thank Hasan Tekman for being critical and being a good friend at the same time.

The original research idea of the experiments presented in the thesis started to develop when I was in Purdue University. My dissertation is a continuation and extension of my master's work at Purdue University. I would like to express my respect toward my former supervisor Professor James Townsend. Without him the ideas presented in my thesis would not have existed. Also my thanks go to Professor Richard Schweickert and Professor Hans Colonius.

I dedicate this work to my mother and my sister.

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INTRODUCTION

An important goal of cognitive psychology is to understand various ways in which humans can process information conveyed over multiple sensory channels. Channels are defined as pathways devoted to the specific dimension of stimulus. In this thesis, the terms channels, subsystems, and processes are used interchangeably.

Humans have a great deal of flexibility in information processing. They can optimize their performance by utilizing a processing strategy suitable for the particular demands of a given task. Given the same set of stimuli, human performance can importantly depend upon the requirements of the task. Moreover, measures of task performance (using reaction times and/or accuracy), can actually reflect the strategy adopted by the observer. One goal of the present research is to explore the extent to which the logical requirements of sensory tasks can determine the processing strategy adopted by an observer. In order to proceed, we must be able first to define what the logical requirements of a task are. We must also define a variety of strategies that might be employed by an observer. Finally, we must be able to develop analytic methods not only capable of generating predicted patterns of performance that would be produced by the various strategies but also capable of testing the predicted patterns of performance against the obtained patterns of performance.

In this thesis, I wish to demonstrate that human performance on relatively simple information processing tasks can be considered in terms of a *flexible architecture of component processes whose internal state representations are random variables*. This approach can be used to identify analytically the processing architecture realized in a variety of tasks.

A second major goal of this thesis is to show how this representation of internal processes in terms of random variables can be used to model sensory processing over a wide range of stimulus strengths. This approach enables to unify the traditionally separate domains of the time-course of suprathreshold detection (as measured with reaction times) and near-threshold detection accuracy (as assessed in the domain of signal detection theory). First I develop the approach in the context of suprathreshold detection performance and the analysis of reaction times.

REACTION TIME

In considering reaction time modeling of an information processing system, there are four major dimensions which distinguish various systems; 1) system architecture, 2) processing completion mode, 3) the mode of factorial influence of experimental factors on the processing times of the various components and 4) stochastic independence/dependence between the processing times of the different components of the system.

The processing architecture can be either serial or parallel (Townsend and Ashby, 1983). In the parallel mode, all subsystems begin processing simultaneously but may finish at different times. In the serial mode, each subsystem operates sequentially without temporal overlap (Sternberg, 1969). More complex network models have also been developed (for example, see Schweickert,1978; Fisher and Goldstein, 1983).

The processing completion mode specifies the rules for terminating information processing. Most generally, we can define two types of completion modes, exhaustive and self-terminating. In the exhaustive processing mode, information processing continues until *all* elements are processed. In the self-terminating mode, processing ends whenever the system has processed enough information to execute a correct response. A special case of the self-terminating mode is called the minimum completion mode (Townsend & Ashby, 1983, p49). For example, if two targets are presented in a detection task, responses could be based on the detection of either target. In this case, only one target needs to be processed; processing of the second target is unnecessary.

There are two aspects of the mode of factorial influence: selective influence versus nonselective influence (Townsend, 1984) and context independence versus context dependence (Colonius, 1990). Given a system composed of subsystems (or channels) S_1 and S_2 with experimental factors X_1 and X_2, selective influence holds if an experimental factor (X_1) affects only a subsystem (S_1) and a second factor (X_2) affects only a different subsystem (S_2).

Nonselective influence therefore occurs when a given subsystem is influenced by more than one factor. Context independence holds when an experimental factor X_1 affects only subsystem S_1 and if another factor X_2 affects only subsystem S_2 regardless of presence or absence of the other factor. Context dependence occurs when addition of another factor (X_2) changes the manner by which the factor (X_1) influences the subsystem say S_1. One example of examination of context independence would be to evaluate the detection probability of a light when it occurs with or without a tone. If the detection probability stays the same, context independence holds.

The stochastic dependence/independence dimension specifies whether the completion time of one subsystem covaries with the completion time of another subsystem. In the case of stochastic independence, the joint processing time can be completely described by a multiplication of the two marginal processing times. For example, independence can be defined as P(coin 1 is head AND coin 2 is head) = P(coin 1 is head) × P(coin 2 is head). In the dependent case, the joint processing time cannot be described by a multiplication of two marginal processing times. Dependence can be defined as P(coin 1 is head AND coin 2 is head) ≠ P(coin 1 is head) × P(coin 2 is head). There are two subcases in the dependent case: a) positive dependence and b) negative dependence.

Donders (1868) was the first to use an analysis of reaction time (RT) to describe the way mental processes are arranged. The analytic method he created is called the subtractive method. Donders assumed that mental processes can be decomposed into a finite number of serially arranged processes that are initiated by an input to the system and culminate with an output. He also assumed that deletion or insertion of an additional process will not alter the nature of other processes engaged by the task. This assumption is called pure insertion (cf., Ashby & Townsend, 1980). Based upon these assumptions, Donders developed an experimental paradigm in which the duration of an elementary psychological process was estimated by subtracting the arithmetic mean RT for a task which does not include the particular mental component from the arithmetic mean RT for another task which does include the component.

Approximately one hundred years after Donders' development of the subtractive method, Sternberg (1969) developed another RT methodology: the additive factor method. As with the Donders' subtraction method, Sternberg assumed seriality of component processes; the stages are executed one by one without temporal overlap (see however, Taylor, 1976). A second assumption in Sternberg's additive factor method is *selective influence*. Selective influence means that a particular experimental factor affects the duration of only one process (Townsend & Ashby, 1983). While the additive factor method allows us to construct an experimental model whose basic components are serially organized stages, it does not allow us to calculate the process duration directly.

Let us consider how the additive factor method is applied to an experimental situation. The method uses the interaction contrast in the *analysis of variance* (ANOVA). The interaction contrast can be defined clearly in the following example. Suppose that we have a 2 × 2 factorial design with experimental factors X and Y with levels 0 and 1 such as

X\Y	0	1
0	(0,0)	(0,1)
1	(1,0)	(1,1)

Table 1. A 2 × 2 factorial design with experimental factors X and Y with levels 0 and 1.

We can define the interaction contrast as follows:

$$\Delta^2_{X,Y}\ \overline{RT}(x,y)=\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1).$$

While the above formula is sometimes referred to as the interaction contrast, I will refer to the quantity $\Delta^2_{X,Y}\ \overline{RT}(x,y)$ as the *mean contrast.*

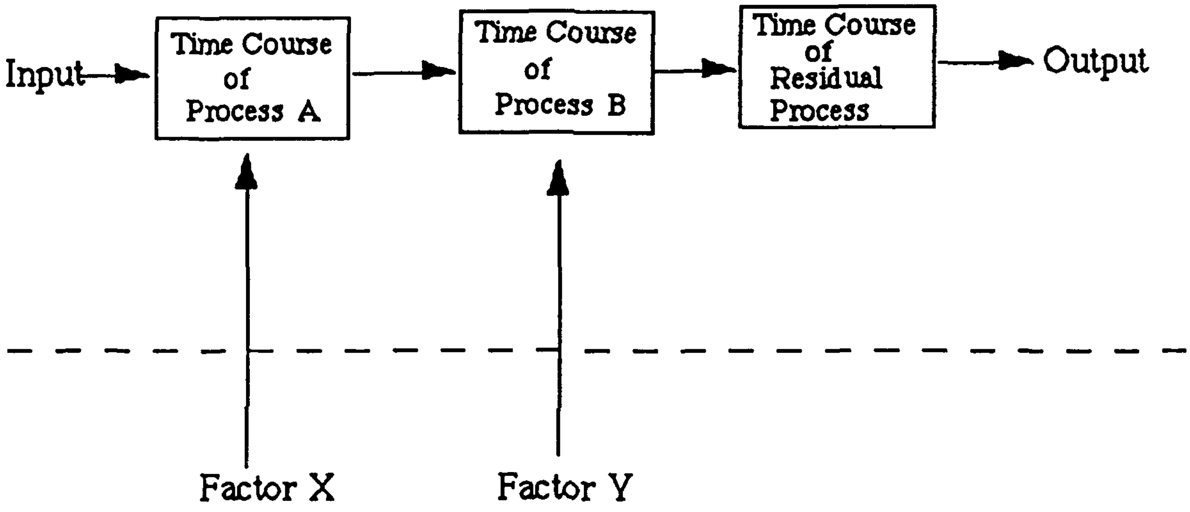


Figure 1. Schematic representation of two serial processes, A and B, selectively influenced by distinct factors, X and Y, respectively. The residual process is defined as non-detection processes in the reaction time chain.

Suppose two processes (A and B) are serially organized and suppose further that the processing completion mode is exhaustive. Assume that Factor X selectively influences the duration of process A and Factor Y selectively influences process B (that is, selective influence holds). This condition implies additivity in the mean RT contrast (that is, $\Delta^2_{X,Y}\ \overline{RT}(x,y)=0$; see Sternberg, 1969; Townsend, 1984). Define $\overline{T}(0,-)$ as the mean duration of process A when factor X is at level 0. Similarly, we define $\overline{T}(-,1)$ as the mean duration of process B when factor Y is at level 1. The mean durations, $\overline{T}(1,-)$ and $\overline{T}(-,0)$ are analogously defined. Since we assume process A and B are serially organized, the exhaustive processing time is simply the sum of the two component processing times if selective influence holds (Townsend, 1984). For example, $\overline{RT}(0,0) = \overline{T}(0,-) + \overline{T}(-,0)$. Thus, the mean contrast can therefore be written as follows:

$$\Delta^{2}_{X,Y}\ \overline{RT}(x,y)=\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1)$$

$$=\overline{T}(0,-) + \overline{T}(-,0) - \overline{T}(0,-) - \overline{T}(-,1)$$

$$- \overline{T}(1,-) - \overline{T}(-,0) + \overline{T}(1,-) + \overline{T}(-,1)$$

$$= 0.$$

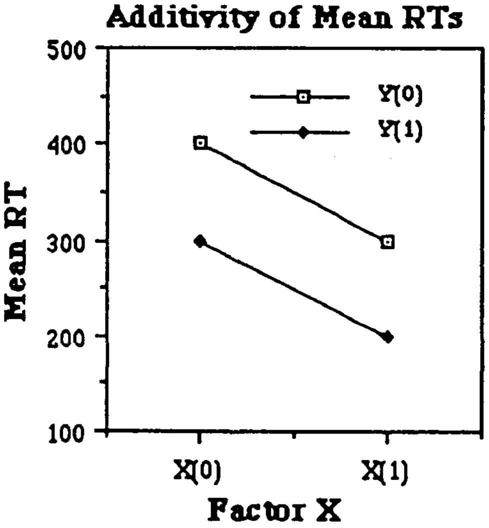


Figure 2. Graph showing additivity of mean contrast.

Serial processing can also operate in the self-terminating mode. In the serial self-terminating model, processing occurs on only one channel at a time. The system processes information on one channel and switches to another channel only if no target is detected on the first channel. The mean contrast of the serial self-terminating model is proven to be additive (see Appendix A for the proof). Thus, while both the exhaustive and self-terminating serial models predict additivity of the mean contrast, the serial self-terminating model predicts faster RTs than the serial exhaustive model with the same accuracy level. This is because the exhaustive serial model always processes the activity on all channels. Other diagnostics for discriminating among the various serial processing architectures are available, but are beyond the focus of the present work.

In a real experimental situation, we first assume that some experimental factor X (e.g., stimulus intensity) selectively influences process A (which we might call "stimulus encoding"). The problem is to determine whether factor Y influences process A (the stimulus encoding stage) or process B (some other, serially organized mental component of the task).

responses decreases monotonically as the number of the targets increases (Grice, Canham, & Boroughs, 1984; Miller,1982, 1986; Nickerson, 1973; Raab, 1962; Ulrich & Giray, 1986). Parallel processing in the minimum completion mode predicts decreased RTs to redundant targets (e.g. Raab, 1962; p80, p248 of Townsend & Ashby, 1983). Although these models are sometimes referred to as horse race models, I will refer to these models as *probability summation models*. The architecture of the probability summation model is parallel because the processing of all targets begins simultaneously. Given two targets minimum completion time is the processing mode, because a response can be produced as soon as the processing of either target is completed. The model usually assumes independence of the processing times of each target. Under this assumption, the model can be classified as an independent model. Finally, the model assumes selective influence as a fundamental assumption.

The probability summation model can be defined in terms of the cumulative distribution function as follows. Let us define the cumulative distribution function of a random variable T as $F(t) = P(T \leq t)$. Probability summation states that the probability that the RT is less than t = the probability that either channel 1 OR channel 2 has been processed by time t = the probability of the processing time of channel 1 is less than t OR the processing time of the channel 2 is less than t. The above statements can be put in the form of equation in the following way:

$$P(T_1 \leq t \text{ OR } T_2 \leq t) = P(T_1 \leq t) + P(T_2 \leq t) - P(T_1 \leq t \text{ AND } T_2 \leq t)$$

$$= P(T_1 \leq t) + P(T_2 \leq t) - P(T_1 \leq t) \times P(T_2 \leq t).$$

The last step holds if and only if independence holds between two random times T_1 and T_2. The symbol " × " denotes a multiplicative operation. Thus, the cumulative RT distribution function according to the independent probability summation is given by:

$$F_1(t) + F_2(t) - F_1(t) \times F_2(t).$$

Mean Contrast Superadditivity of the Probability Summation Model

Let us derive the prediction from the probability summation model's mean RT by way of the additive-factor logic. Before deriving the prediction, we need to define the terminology used in the prediction below. Define X_1 and X_2 as experimental factors influencing the random times T_1 and T_2 respectively such that if we differentiate the survivor function of T_1 with respect to X_1 and differentiate the survivor function of T_2 with respect to X_2 we get negative values. The survivor function of a random variable T is defined as $\overline{F}(t)=P(T>t)$. The survivor function simply tells us the proportion of trials in which the subject has not responded by time t. Thus, the partial derivative of T_1 and T_2 with respect to the factors, X_1 and X_2, can be expressed as follows.

$$\frac{\partial \overline{F}(t_1;X_1)}{\partial X_1} < 0 \text{ and } \frac{\partial \overline{F}(t_2;X_2)}{\partial X_2} < 0.$$

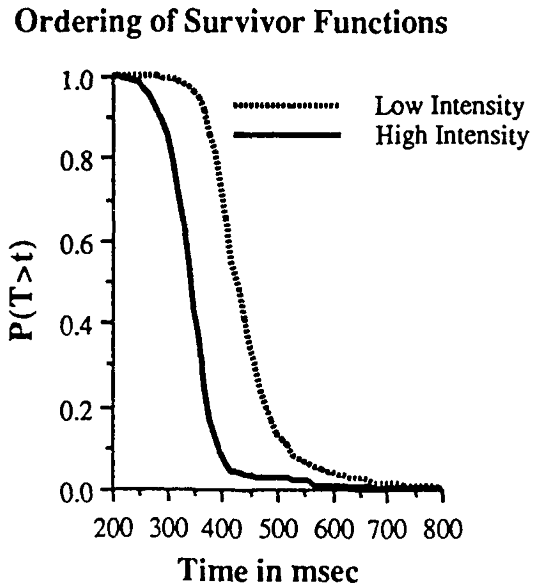


Figure 4. The ordering of survivor functions.

This means that increasing the factor level of X_1 and X_2 will yield smaller survivor probabilities for all values of T_1 and T_2 respectively as can be seen in Figure 4. Notice that greater survivor probability associates with higher intensity. This implies that increasing the factor level reduces the mean RTs. If we manipulate X_1 and X_2 orthogonally in a two-by-two factorial design with factor levels being 0 and 1 for each factor, we have the following four conditions: 1: (0,0), 2: (0,1) , 3: (1,0), and 4: (1,1). The mean RTs of the four conditions can be represented as $\overline{RT}(0,0)$, $\overline{RT}(0,1)$, $\overline{RT}(1,0)$, and $\overline{RT}(1,1)$.

Since we know that the mean of a positive random variable can be calculated by integrating its survivor function, $\overline{F}(t)=P(T>t)$, from zero to infinity (cf., McGill, 1963), we can express the statistical contrast (or mean contrast) in terms of the survivor functions as follows.

$$\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1)$$
$$= \int_0^\infty \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t)\, dt$$

Assuming the probability summation model, we can write the above survivor functions as being composed of two random times T_1 and T_2. That is, for example, we can write $\overline{F}_{(0,0)}(t) = \overline{F}_{1(0)}(t) \times \overline{F}_{2(0)}(t)$ because of the independence assumption in the probability summation model. The symbol, $\overline{F}_{1(0)}(t)$ denotes the survivor function of the random variable T_1 when the experimental factor X_1 is at the level 0 and $\overline{F}_{2(0)}(t)$ represents the survivor function of the random variable T_2 when the experimental factor X_2 is at the level 0. Therefore, the above expression can be expressed as

$$= \int_0^\infty \overline{F}_{1(0)}(t) \times \overline{F}_{2(0)}(t) - \overline{F}_{1(0)}(t) \times \overline{F}_{2(1)}(t) - \overline{F}_{1(1)}(t) \times \overline{F}_{2(0)}(t) + \overline{F}_{1(1)}(t) \times \overline{F}_{2(1)}(t)\, dt$$

$$= \int_0^\infty (\overline{F}_{1(0)}(t) - \overline{F}_{1(1)}(t)) \times (\overline{F}_{2(0)}(t) - \overline{F}_{2(1)}(t))\, dt.$$

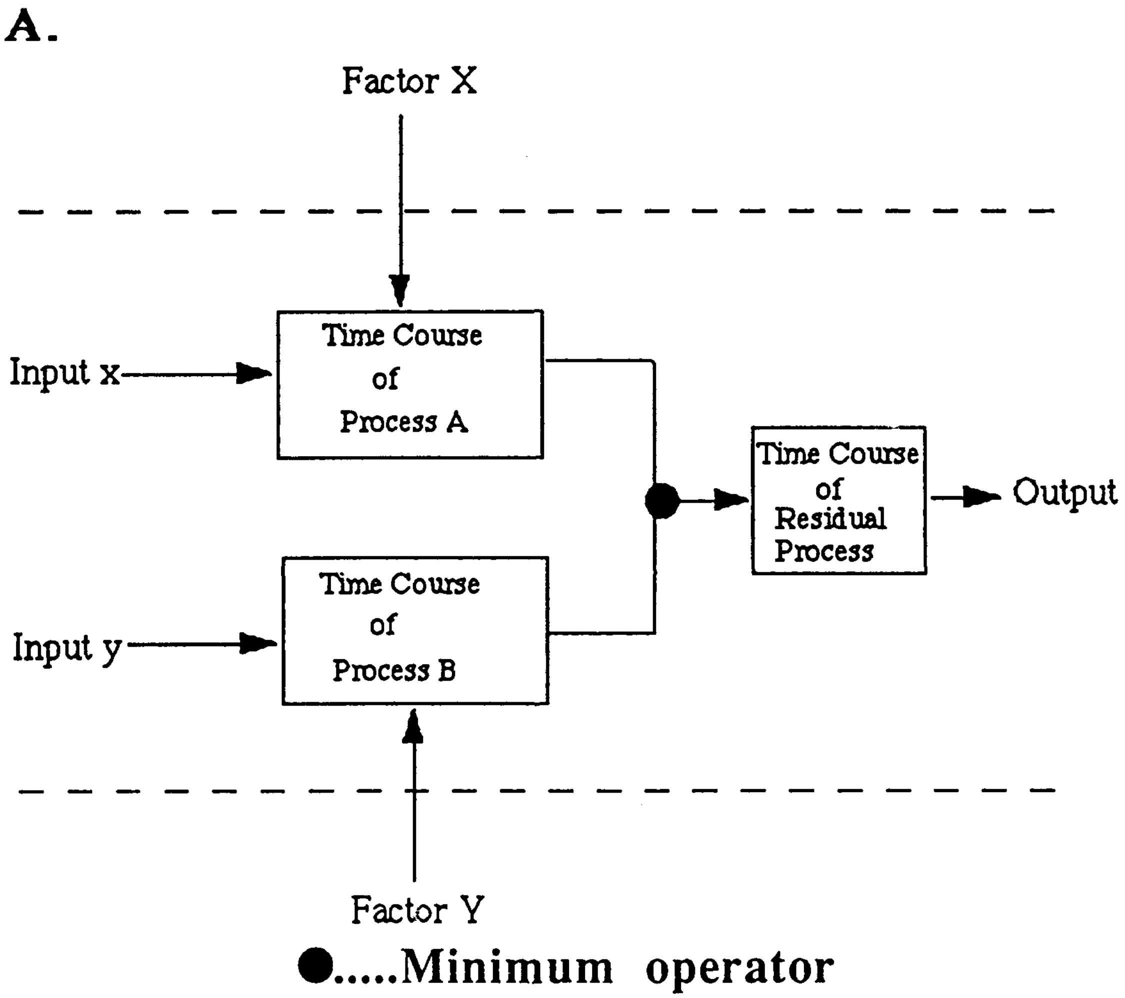
The above quantity is greater than zero, that is the interaction is superadditive, if we have the survivor function ordering on the random times T_1 and T_2. The survivor function ordering means that $\overline{F}_{1(0)}(t) > \overline{F}_{1(1)}(t)$ for T_1 and $\overline{F}_{2(0)}(t) > \overline{F}_{2(1)}(t)$ for T_2. Intuitively and with less rigor, the ordering of survivor functions with respect to intensity can be interpreted as higher intensity giving rise to a reduction in processing time. Thus, if increasing the level of the experimental factors can be shown to produce faster RTs, the probability summation model predicts superadditivity of the mean RT contrast. Notice that the expression of the mean contrast can be reduced to that of the survivor contrast if we take the integration out from the above expression. That is,

$$\overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t)$$
$$= (\overline{F}_{1(0)}(t) - \overline{F}_{1(1)}(t)) \times (\overline{F}_{2(0)}(t) - \overline{F}_{2(1)}(t)).$$

If the survivor function ordering is satisfied for both random times T_1 and T_2, then we have survivor contrast superadditivity. The test in terms of the survivor contrast constitues a stronger test of various models than the test in terms of the mean contrast (see Townsend and Nozawa, 1992).

Channel Summation Model

However, there is another type of model which is capable of explaining the general results. This model is called channel summation model or its special case is sometimes called Poisson superposition model (Schwarz, 1989). It can be shown that the channel summation model also predicts superadditivity of the mean contrast (see Nozawa, 1989 and Appendix B). However, this model can be distinguished from the probability summation model, all other things being equal, because the channel summation model produces faster RTs than probability summation. Thus, although both models predict superadditivity of the mean contrast, it is still possible to distinguish between channel summation and probability summation using other measures, such as the survivor contrast (see Townsend and Nozawa, 1992) or the capacity coefficient (Nozawa, 1989; Townsend and Nozawa, 1992). The channel summation model sums the level of activity in relevant internal random processes to generate an output (see Figure 5 for the schematic representation of the probability summation model and the channel summation model).



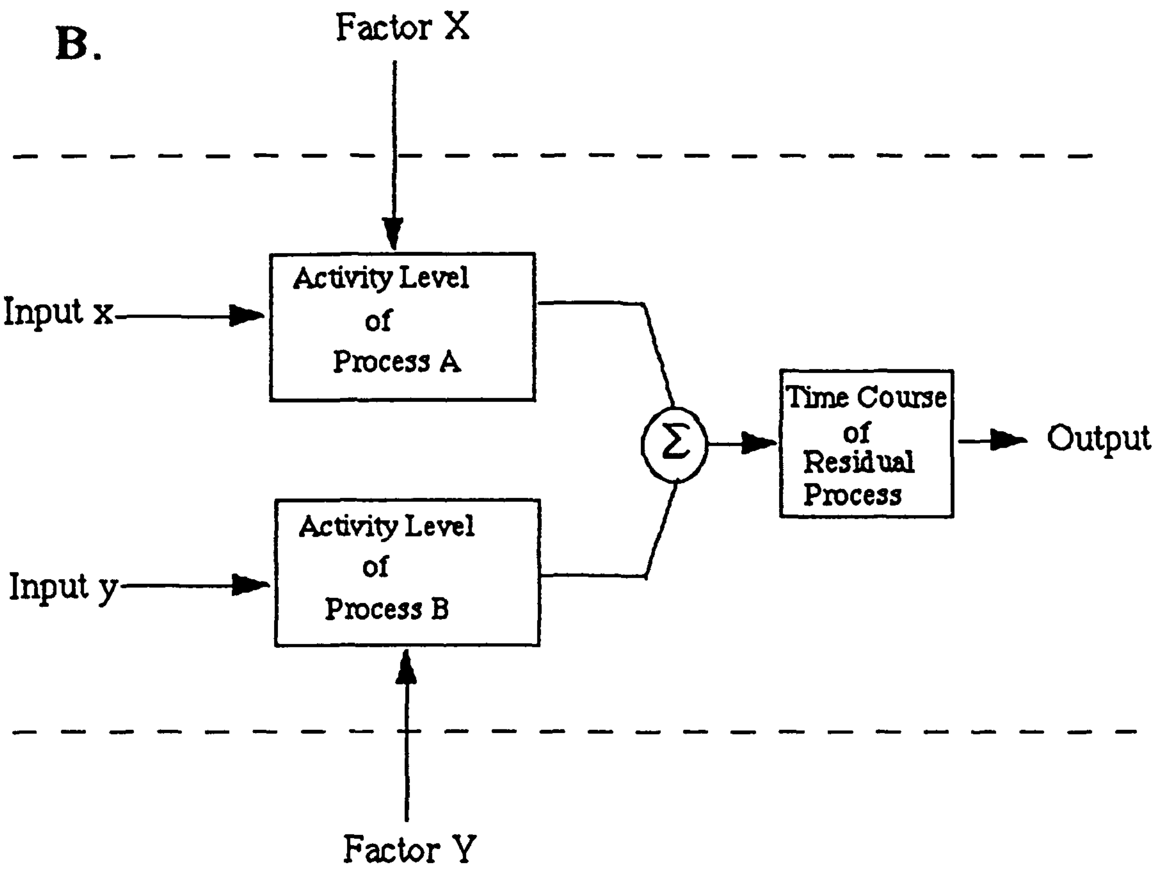


Figure 5. Schematic representations of the probability summation model (A) and the channel summation model (B).

The activity of the channels is represented by stochastic counting processes (see Cox, 1962). The output of the model is also represented as a stochastic counting process. In order to produce the time at which the response is made, the model is equipped with a threshold. When the threshold is reached, the system generates a response. Notice that both the probability summation model and the channel summation model represent parallel processing architectures. The major distinction concerns the manner in which the activity within each process is combined.

While probability summation might represent a useful architecture for stimulus detection in the redundant target paradigm, this architecture is clearly inadequate for performing other types of task. For example, suppose an observer is instructed to respond only if a target is presented on two (or more) input channels. In this case, correct responding depends on detecting the occurrence of a target on both channels: that is, the system must operate in the exhaustive processing mode. We now turn to a consideration of behavior of such systems.

Exhaustive Processing Paradigm

In the exhaustive processing paradigm, the subject is required to respond "yes" only when two targets are presented, otherwise the subject is required to respond "no". I call such a task the AND task. In such a task the optimal strategy is to process information exhaustively on both channels. If the subject applies a self-terminating strategy rather than exhaustive strategy, the RTs will be shorter but the accuracy will be lowered. Let us consider the behavior of a simple parallel architecture operating in the exhaustive processing mode. In the next section, I will develop the proof that parallel exhaustive processing predicts subadditivity of the mean contrast (see also Townsend and Ashby, 1983, p373).

Parallel Exhaustive Processing Model's Mean Contrast Subadditivity

Consider an AND task in which there are two levels of intensity associated with each of the targets. There are four conditions in which the subject is required to respond "yes". Define T_1 as the random variable representing the processing time on channel 1, and T_2 as the random variable for the processing time on channel 2. If the two targets are processed in parallel exhaustively, and the processing of the targets is stochastically independent, then the total processing time is simply the maximum of the two random variables T_1 and T_2 (see Figure 6 for a schematic representation of the parallel exhaustive processing model).

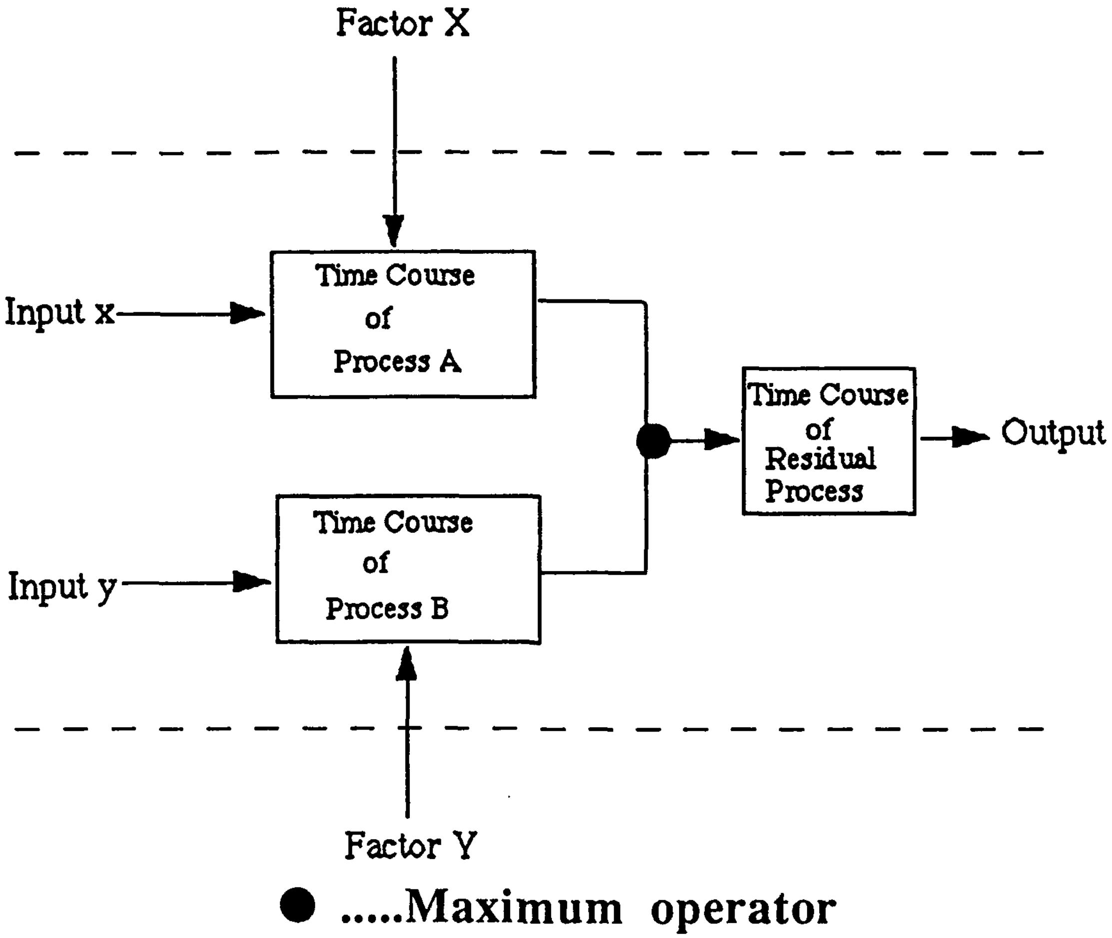


Figure 6. A schematic representation of the parallel exhaustive processing model.

Thus, we can express the model as follows:

$T_{P,Exh}=\max\{T_1,T_2\}$. In terms of the cumulative distribution function, we can write the expression for this model under the double stimulus condition as follows: $F_{\max\{T_1,T_2\}}(t) = F_{T_1}(t) \times F_{T_2}(t)$. That is, given the assumptions of selective influence and stochastic independence, the cumulative probability that both targets have been processed by time t is the product of the cumulative probabilities that each channel has detected its target by time t. The mean contrast can be expressed as follows:

$$\underset{X,Y}{\Delta^2}\ \overline{RT}\,(x,y)= \overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1)$$

$$= \int_0^\infty \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t)\, dt,$$

since integration of the survivor function yields the distribution mean.

Because the survivor function is the complement of the cumulative distribution function, that is, $\overline{F}(t) = 1 - F(t)$, we have

$$= \int_0^\infty 1 - F_{(0,0)}(t) - 1 + F_{(0,1)}(t) - 1 + F_{(1,0)}(t) + 1 - F_{(1,1)}(t)\, dt$$

$$= \int_0^\infty F_{(0,1)}(t) - F_{(0,0)}(t) + F_{(1,0)}(t) - F_{(1,1)}(t)\, dt$$

$$= \int_0^\infty F_{1(0)}(t) \times F_{2(1)}(t) - F_{1(0)}(t) \times F_{2(0)}(t) + F_{1(1)}(t) \times F_{2(0)}(t) - F_{1(1)}(t) \times F_{2(1)}(t)\, dt$$

$$= \int_0^\infty (F_{1(0)}(t) - F_{1(1)}(t)) \times (F_{2(1)}(t) - F_{2(0)}(t))\, dt.$$

The above quantity is less than zero (subadditivity) if we have the survivor function ordering on the random times T_1 and T_2. The survivor function ordering means that $\overline{F}_{1(0)}(t) > \overline{F}_{1(1)}(t)$ for T_1 and $\overline{F}_{2(0)}(t) > \overline{F}_{2(1)}(t)$ for T_2. The survivor function ordering, for example, $\overline{F}_{1(0)}(t) > \overline{F}_{1(1)}(t)$ is equivalent to the cumulative distribution function ordering $F_{1(0)}(t) < F_{1(1)}(t)$. This ends the proof of subadditivity of the mean RT contrast of the parallel independent exhaustive model.

The experiments described below represent situations conducive to parallel processing architectures. Performance on an OR task is evaluated in an attempt to determine whether processing conforms to the predictions of the parallel self-terminating model. Performance on an AND task is evaluated in terms of parallel processing in the exhaustive mode. The paradigms are summarized in Tables 2 and 3.

Redundant Target Paradigm: the OR task

Condition	Event in Top Channel	Event in Bottom Channel	Response
Double Target Condition	○	○	"Yes"
Top Single Target Condition	○	None	"Yes"
Bottom Single Target Condition	None	○	"Yes"
Noise Condition	None	None	"No"

Table 2. The stimulus-response matrix of the redundant target paradigm.

Exhaustive Processing Paradigm: the AND task

Condition	Event in Top Channel	Event in Bottom Channel	Response
Double Target Condition	○	○	"Yes"
Top Single Target Condition	○	None	"No"
Bottom Single Target Condition	None	○	"No"
Noise Condition	None	None	"No"

Table 3. The stimulus-response matrix of the exhaustive processing paradigm.

Hypotheses Tested in the RT Experiments

The hypotheses tested in the redundant target paradigm (OR-task) RT experiment are stated as follows:

H_0: $\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) = 0$: serial hypothesis

H_1: $\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) > 0$: parallel self-terminating hypothesis

The hypotheses tested in the exhaustive processing paradigm (AND-task) RT experiment are stated as follows:

H_0: $\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) = 0$: serial hypothesis

H_1: $\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) < 0$: parallel exhaustive hypothesis

EXPERIMENT 1

Methods

Design

Each subject participated in both the OR task and the AND task, and the order of testing was counter balanced between subjects. The stimulus conditions were identical in both tasks. In each task condition, there were two sessions of practice and four experimental sessions. Practice sessions consisted of 240 trials, whereas experimental sessions consisted of 480 trials. There were nine possible display conditions: four brightness conditions for the double target condition, two brightness conditions for each of the single target conditions, and one noise condition in which no target was presented. All the experimental variables were within-subject factors.

Subjects

Four students from a psychology course at Dartmouth College and two graduate students, including the author, served as subjects. All the subjects had normal or corrected normal vision.

Apparatus and Stimuli

A microcomputer controlled the stimulus presentation and recorded reaction times. Two red light-emitting diodes (LEDs) served as the targets and a green LED served as the fixation point. The targets were located on the vertical meridian, equally spaced above and below the fixation point at an elevation of $\pm 1^\circ$. Subjects viewed the stimuli binocularly at a distance of 57 cm. There were two levels of target luminance (0.19 and 9.98 $\mathrm{cd.m}^{-2}$). Luminances were chosen to insure a robust effect on the RTs, since the predictions described above rely on ordering of the RT distributions with signal strength. The LEDs subtended 0.5° of visual angle. The fixation point remained illuminated throughout the session. Each trial began with the presentation of a warning tone (500 msec. duration). Targets appeared immediately after the offset of the warning tone. Target durations were 500 ms. The subjects were instructed to respond to the onset of the target as quickly as possible. Response sampling began with the onset of the warning tone and continued for 2000 ms following target onset. The inter-trial-interval was 1.0 sec. On each trial there were four possible events: top and bottom targets together, top target alone, bottom target alone, and no target. Subjects pressed one of two response buttons to indicate their response: one for "yes" responses and the other for "no" responses. The stimulus-response mapping for each task was described in the Introduction (see Tables 2 and 3). The probability of presenting both targets was 1/6, the probability of presenting the top target alone was 1/6, the probability of presenting the bottom target alone was 1/6, and the probability of presenting no target was 1/2. The probabilities associated with each target luminance were also 1/2.

RESULTS 1

ANOVA Results

Mean reaction times of all of the subjects were analyzed by ANOVA separately for each of the tasks (see Table 4 for the results of ANOVA for the AND task and the OR task). The mean reaction times, standard deviations and error probabilities for each condition are presented in Appendix C. In terms of means, the null hypothesis and the alternative hypothesis of the OR task tested by ANOVA can be expressed as follows:

$$H_0: \underset{B,T}{\Delta^2} \overline{RT} = \overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) = 0 : \text{serial hypothesis}$$

$$H_1: \underset{B,T}{\Delta^2} \overline{RT} = \overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) > 0 : \text{parallel hypothesis}$$

The null hypothesis and the alternative hypothesis of AND task tested by ANOVA can be expressed as follows:

$$H_0: \underset{B,T}{\Delta^2} \overline{RT} = \overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) = 0 : \text{serial hypothesis}$$

$$H_1: \underset{B,T}{\Delta^2} \overline{RT} = \overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1) < 0 : \text{parallel hypothesis}$$

The results of the ANOVA revealed main effects of target intensity and an interaction between intensity and RT in the double target conditions for both tasks (see Table 4 for the details of the ANOVA). These interactions justify rejection of the null hypothesis in both tasks.

ANOVA Results of the OR Task

Source	df	F	Pr > F
Bottom intensity(B)	1,5	98.56	.0004
Top intensity(T)	1,5	75.99	.0006
B x T	1,5	43.48	.0016

ANOVA Results of the AND Task

Source	df	F	Pr > F
Bottom intensity(B)	1,5	16.99	.0097
Top intensity(T)	1,5	12.15	.0179
B x T	1,5	43.90	.0016

Table 4. ANOVA results of the AND and the OR tasks.

In the OR task, the mean contrasts were positive for all subjects. In the AND task, the mean contrasts were all negative. The mean reaction times for the double stimulus conditions are graphed in Fig.7.

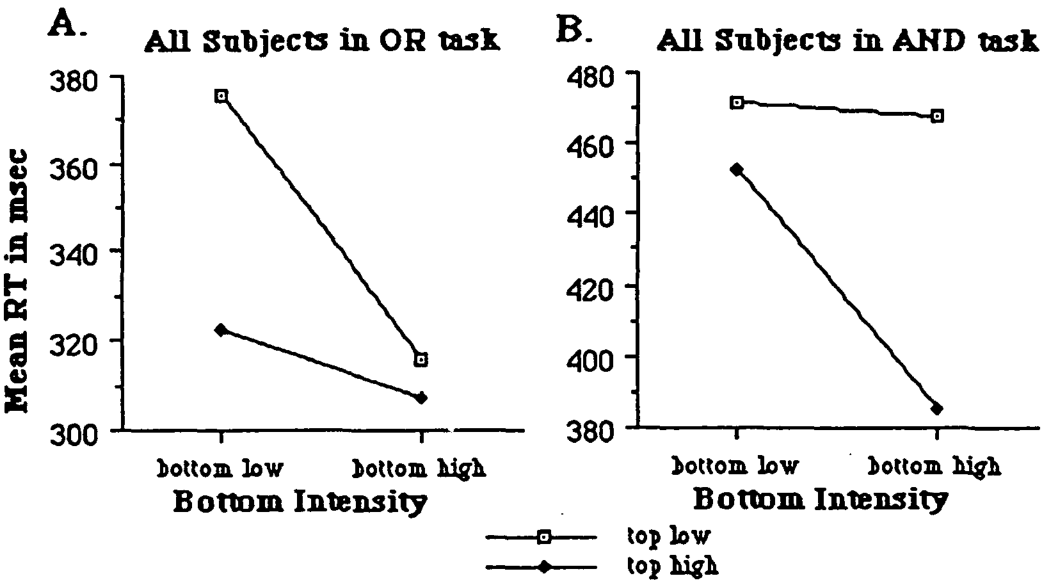


Fig.7. Mean reaction times for the double stimulus conditions in the OR task and the AND task.

Results of Mean Analyses

For each subject, the mean contrast and its associated z-score was calculated and is presented in Table 5. Let us define $s^2[RT(0,0)]$ as the RT variance estimate of the condition (0,0) and n(0,0) as the number of the data in the condition (0,0). The z-scores were calculated as follows:

$$z = \Delta^2_{B,T}\ \overline{RT}/\{\text{standard error}\},$$

where standard error can be expressed as

$$s^2[RT(0,0)]/n(0,0) + s^2[RT(0,1)]/n(0,1) + s^2[RT(1,0)]/n(1,0) + s^2[RT(1,1)]/n(1,1).$$

Mean contrasts of the AND task showed subadditivity implying the possibility of the parallel exhaustive processing model (see Table 5). This result allows us to reject such models as the serial exhaustive processing model, the serial self-terminating model, the probability summation model and the channel summation model in the AND task. Mean contrast of the OR task showed superadditivity implying the probability summation model and the channel summation model as possible candidates to explain the results (see Table 5).

OR task **AND task**

Subject	OR Mean Contrast	z-score	AND Mean Contrast	z-score
BJ	43.66	4.69	-61.44	-7.09
BK	64.61	5.47	-42.58	-2.11
GN	36.19	4.69	-62.30	-3.55
JZ	64.89	4.06	-59.60	-1.61
SK	32.99	2.89	-105.87	-9.47
ZC	26.20	1.52	-45.74	-2.61

Table 5. Mean contrasts and their associated z-statistics in the OR task and in the AND task.

The individual mean RTs, the variance of RTs and error probabilities are tabulated in Appendix C. The graphs of individual mean RTs are listed in Appendix D. There was an interesting trend in the ordering of the mean RTs to double targets in the AND-task double stimulus condition for two of the six subjects; this is the non-monotonicity of mean RTs with respect to two experimental factors (obervers BK and SK, see Appendices C and D). Monotonicity of mean RTs with respect to one experimental factor is equivalent to the concept of the mean RT ordering with respect to the intensity factor: mean RTs for brighter stimuli are faster than mean RTs for dimmer stimuli. In order to satisfy monotonicity the double stimulus conditions, the mean RT of (0,0) condition must be slower than the mean RTs of (0,1) and (1,0) conditions and the mean RT of (1,1) condition must be faster than the mean RTs of (0,1) and (1,0) conditions. The interpretation of these violations of monotonicity will be considered in the General Discussion.

Results of Survivor Contrast Analyses

For each subject, survivor functions were calculated to analyze survivor contrast. The hypotheses tested in the OR task were as follows:

$$H_0: \underset{B,T}{\Delta}^2 \overline{F}(t) = \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t) = 0 : \text{serial hypothesis}$$

$$H_1: \underset{B,T}{\Delta}^2 \overline{F}(t) = \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t) > 0 : \text{parallel hypothesis.}$$

The individual survivor contrasts of the OR task are presented in Appendix E. Since all of the survivor contrasts were substantially greater than zero for all t, the data are clearly incompatible with the serial self-terminating model.

For the AND task, if the crossing point between the two density functions for the processing times for each target is denoted by t*, the hypotheses tested in the AND task can be expressed as follows:

$$H_0: \underset{B,T}{\Delta}^2 \overline{F}(t) = \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t) > 0 \text{ for } t > t^* : \text{serial hypothesis}$$

$H_1: \underset{B,T}{\Delta}^2 \overline{F}(t) = \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t) \leq 0$ for $t > t^*$: parallel hypothesis.

In testing the above hypotheses, we have to take into account the crossing point t* because the survivor contrast of the serial exhaustive model is biphasic:

$\underset{B,T}{\Delta}^2 \overline{F}(t) < 0$ for $t \leq t^*$ and $\underset{B,T}{\Delta}^2 \overline{F}(t) > 0$ for $t > t^*$ (for an elaboration of these ideas, see Townsend and Nozawa, 1992). Since none of the subjects generated a biphasic survivor contrast (see Appendix E), this analysis also confirms that based on the mean contrast. Thus, analyses both of the survivor contrast and of the mean contrast serve to reject serial models for both the AND and OR tasks.

The results clearly show the complementary nature of the AND and OR tasks. That is, the survivor contrasts for the AND task showed subadditivity whereas those for the OR task showed superadditivity (see Figure 8). Mathematically, the results obtained in the survivor contrast imply the results of the mean contrast. Therefore, the results of these analyses are thus more conservative, lending further support to the idea that performance on the OR task was supported by parallel processing in the minimum completion mode, while performance on the AND task was supported by exhaustive parallel processing.

However, a closer look at the pattern of the survivor contrasts obtained in the OR task revealed an interesting trend. Two out of six subjects showed negative blips at the earlier portion of their survivor contrasts (see Appendix E). As noted by Townsend and Nozawa (1992), this can be taken as evidence of the channel summation model. I will put off a detailed treatment of this matter until the discussion section.

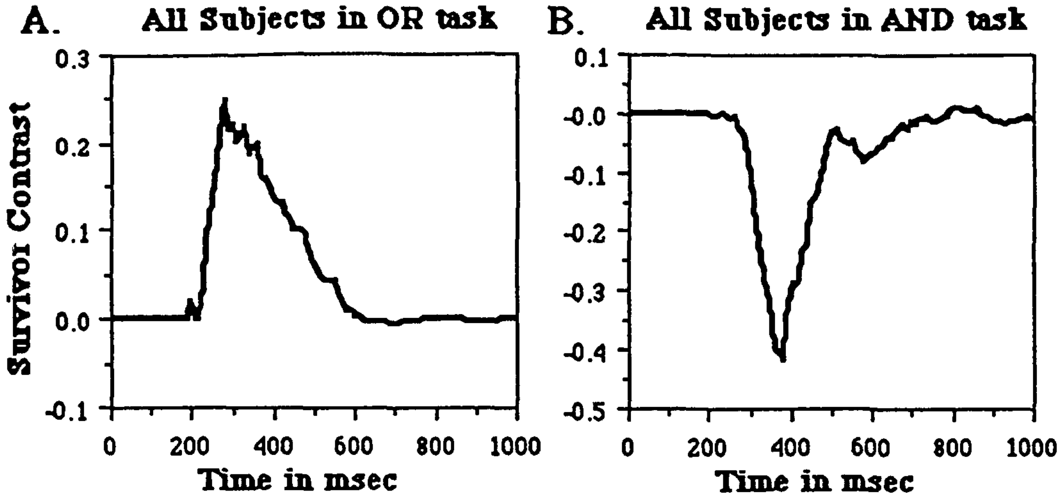


Fig.8. Survivor contrasts in the OR task and the AND task.

Results Of Error Probability Analyses

Overall error probabilities in the double stimulus condition in the AND task and the OR task were 0.099 and 0.0155, respectively. The error probability in the AND-task was greater than that in the OR task and the difference in error probabilities was statistically significant at $p<.00001$. Since the parallel hypotheses were supported in the reaction time analyses, it is natural to expect that the error probability might also reveal the parallel processing architecture. This error analysis is put off until the General Discussion section, since the error analysis techniques are introduced in the next part of the thesis.

Discussion: Experiment 1

The results of experiment 1 can be summarized as follows. In the OR task the mean contrasts and the survivor contrasts showed superadditivity. In the AND task the mean contrasts and the survivor contrasts showed subadditivity. The results allowed us to reject the serial hypotheses in both the OR task and the AND task. The next stage of analysis is the detailed characterization of the obtained results in terms of the plausible parallel processing models. The question can be put in the following way. In the OR task we can ask how fast the obtained results compared to the prediction from the probability summation model. In the AND task we can ask how fast the obtained results were compared to the prediction from parallel exhaustive processing model. In order to accomplish this goal, I am going to use two inequalities and one identity in both the OR task and the AND task. Let us consider the case of the OR task.

In the OR task, as was put forward by Miller (1982), we can use Boole's inequality to evaluate the probability summation model (see for example, Dudewicz, 1976). In this thesis Boole's inequality is called Miller's inequality, since Miller was the first to apply the inequality in experimental psychology and it is written as follows:

$F_{B\&T}(t) \leq F_B(t) + F_T(t)$ for any probability summation model.

The notation, $F_{B\&T}(t)$, is the obtained cumulative distribution function of the double stimulus condition, $F_B(t)$ is the obtained cumulative distribution function of the bottom only condition, and $F_T(t)$ is the obtained cumulative distribution function of the top only condition. Of course, in order for the inequality to be valid we need the assumption of context independence. That is, the effect of the stimulus is unchanged whether another stimulus is present or absent. If this assumption is satisfied, then the right-hand side of this inequality gives us the estimates of the fastest reaction times attainable by the probability summation model. Therefore, the violation of this inequality allows us to reject all the members of the family of the probability summation models: not only the independent probability summation model but also the dependent probability summation models. Since I have presented the inequality to evaluate the obtained distribution at the fastest regions of the distribution, next I am going to present an inequality which establishes the slowest regions for the family of the probability summation models.

The slowest boundary of the probability summation model is called Frechet boundary (cf. Colonius, 1990) and is written as follows:

$F_{B\&T}(t) \geq \mathrm{Max}\{F_B(t), F_T(t)\}$ for any probability summation model.

The assumption of context independence is needed in order for this inequality to be valid. If this inequality is violated under the assumption of context independence, no probability summation model can explain the results. Let us next consider the exact prediction of the independent probability summation model.

The exact prediction of the independent probability summation model in terms of cumulative distribution functions can be written in the following way:

$F_{B\&T}(t) = F_B(t) + F_T(t) - F_B(t) \times F_T(t).$

However, in order for this identity to be satisfied, we need to satisfy two assumptions: context independence and the base time variance being zero. The base time is defined as the time taken by non-detection processes in the reaction time chain. There exist estimates of the motor time variance (see for example, Ulrich and Stampf, 1984). Even if the variance is relatively small, there exists a non-zero variance and therefore we tend to have a bias in the above identity (Colonius, 1990). There is no way to estimate the size of the bias in the context of the present experiment. We need to be concerned about the direction of the possible bias. The direction of the bias is toward underestimating the cumulative distribution of the double stimulus condition. That is, at the level of the mean the estimation is biased toward a larger value. Put in another way the estimates will be slower than the true independent probability summation model's mean. In interpreting the analyses below, the direction of the bias in this identity should be kept in mind.

In the following, I will use the survivor representation of the above inequalities and identity. They can now be expressed as follows:

Miller's inequality

$\bar{F}_{B\&T}(t) \geq \bar{F}_B(t) + \bar{F}_T(t) - 1$

Frechet boundary

$\bar{F}_{B\&T}(t) \geq \mathrm{Min}\{\bar{F}_B(t), \bar{F}_T(t)\}$

and

$\bar{F}_{B\&T}(t) = \bar{F}_B(t) \times \bar{F}_T(t).$

The fit of the above inequalities and the identity are presented in Figure 9. The fit of the above inequalities and the identity are defined as follows:

Miller's inequality fit

$\bar{F}_{B\&T}(t) - (\bar{F}_B(t) + \bar{F}_T(t) - 1)$

Frechet boundary fit

$\bar{F}_{B\&T}(t) - \mathrm{Min}\{\bar{F}_B(t), \bar{F}_T(t)\}$

Probability Summation identity fit

$\bar{F}_{B\&T}(t) - \bar{F}_B(t) \times \bar{F}_T(t).$

The independent probability summation model predicts exactly zero in the probability summation identity fit. In interpreting the fit of the probability summation identity, we should notice that there is a positive bias coming from the base time. Majority of the data points fall above the zero line of the Miller's inequality fit and fall below the zero line of the Frechet inequality. This means that the majority of the data can be explained by the probability summation models.

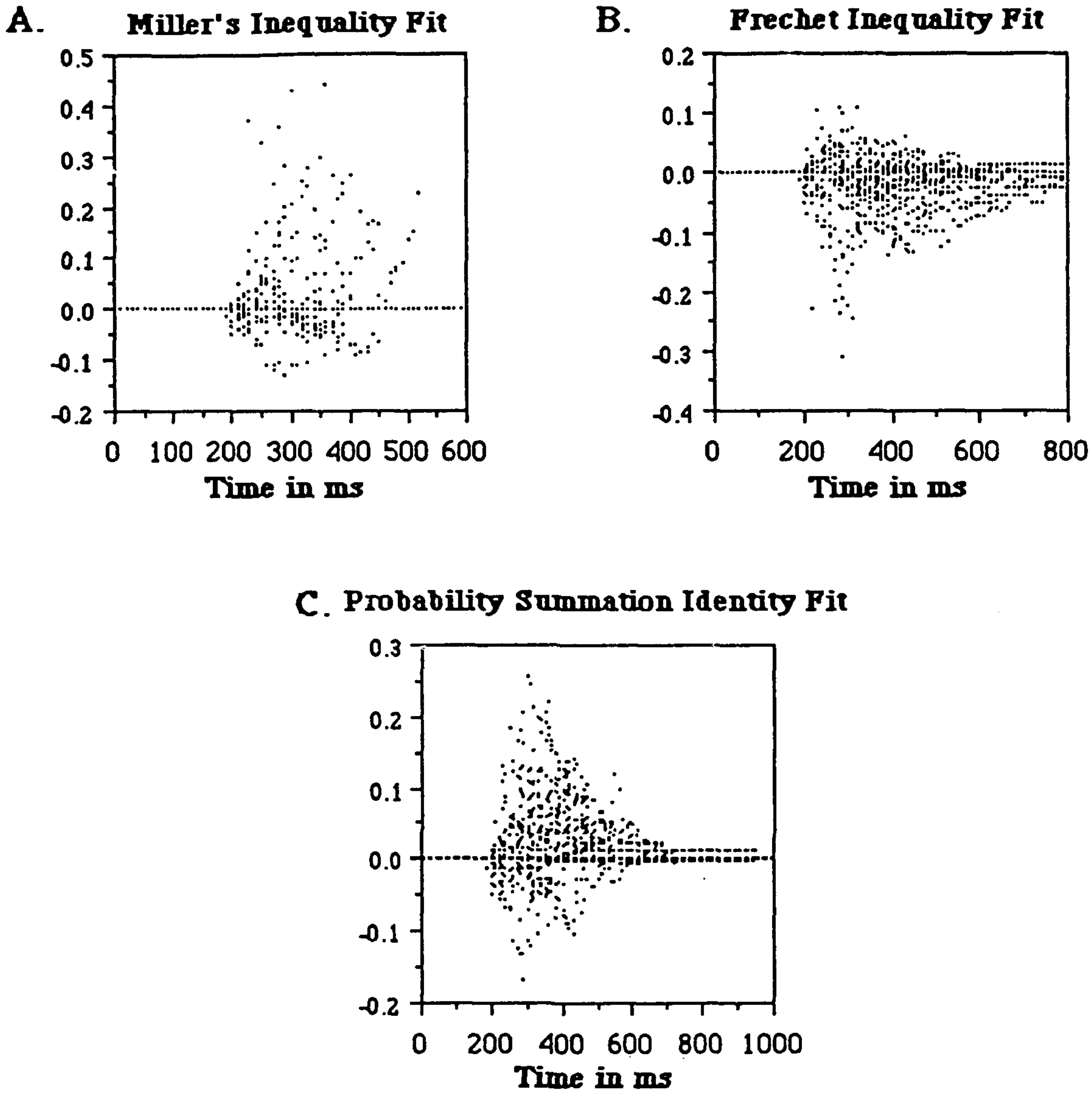


Figure 9. Fit of various boundaries.

One convenient fact about using this survivor representation of the inequalities and the identity is that they directly associate with the means. If we integrate the survivor function of a positive random variable from zero to infinity, we know that this yields the expected value of the random variable (McGill, 1963). Therefore, we can evaluate the fit of the inequalities and the identity at the level of the mean.

The results of the fit of the inequalities and the identity at the level of means in terms of z-scores are presented in Table 6.

Fit of Frechet Boundary

Subject /Condition	(0,0)	(0,1)	(1,0)	(1,1)
BJ	-0.74	-2.18	0.96	0.17
BK	0.45	-0.10	-1.45	-0.18
GN	-3.88	-0.16	0.87	-2.35
JZ	-2.93	-0.57	-4.24	-4.62
SK	-2.04	-0.13	-1.07	-2.32
ZC	-0.95	1.24	-0.03	-2.78

Fit of Miller's Inequality

Subject /Condition	(0,0)	(0,1)	(1,0)	(1,1)
BJ	4.95	0.89	2.64	4.72
BK	5.11	3.04	1.5	4.36
GN	0.58	0.22	1.68	2.68
JZ	2.01	2.28	0.42	4.5
SK	2.23	3.11	0.59	4.04
ZC	3.76	3.22	2.23	3.47

Fit of Probability Summation Identity

Subject /Condition	(0,0)	(0,1)	(1,0)	(1,1)
BJ	2.83	-0.68	1.52	2.91
BK	3.82	1.7	0.01	3.15
GN	-0.91	-0.08	1.11	1.05
JZ	0.36	1.10	-1.51	2.22
SK	0.49	1.62	-0.46	2.05
ZC	2.23	2.34	1.11	1.79

Table 6. Fits of two Inequalities and an identity at the mean level in terms of z-scores.

First, we notice that there were no significant violations in the positive direction in the fit of Frechet boundary at the mean RT level. Therefore, the level of the performance in the experiment can be explained by the probability summation model or models with faster redundant target response than the probability summation model. One of the models which predicts faster redundant target response is the channel summation model. Second, there were no significant violations in the negative direction in the fit of Miller's inequality at the mean RT level. Non-violation of this inequality allows us to reject the channel summation model (however, see the discussion on the subadditive channel summation below). This leaves us with a large class of probability summation models: positively dependent, negatively dependent or independent. Evaluating the results with respect to the probability summation identity, we notice that the results can be summarized reasonably by the positively dependent, rather than the negatively dependent probability summation model. There was only one significant violation of the identity in the negative direction out of 24 fits. Subject JZ in (1,0) condition showed $\underline{z} = -1.51$. The overall results can be restated in terms of the concept of capacity as limited capacity if we use stochastic independence as a pivotal assumption (Nozawa, 1989; Townsend and Nozawa, 1992).

Although the concept of *capacity* was *not* introduced as one of the major dimensions of information processing models, the concept of capacity is one of the important dimensions that can be used to describe the parallel systems. Given the double stimulus condition's survivor function, $\overline{F}_{1\&2}(t)$, and two single stimulus conditions' survivor functions, $\overline{F}_1(t)$ and $\overline{F}_2(t)$, Nozawa (1989) introduced an estimate of the capacity of the system in the following expression:

$$C(t) = \frac{\ln \overline{F}_{1\&2}(t)}{\ln \{\overline{F}_1(t) \times \overline{F}_2(t)\}} .$$

If we assume stochastic independence, selective influence and context independence, in such a situation the capacity coefficient can describe the rate of the processing of the system. For the limited capacity system the capacity coefficient is less than one. For the unlimited capacity system the capacity coefficient is one. For the supercapacity system the capacity coefficient is greater than one. The scatter plot of the capacity coefficients across conditions and subjects is presented below. The line was drawn at the level of C(t)=1, the unlimited capacity level, to facilitate the visual evaluation of the capacity.

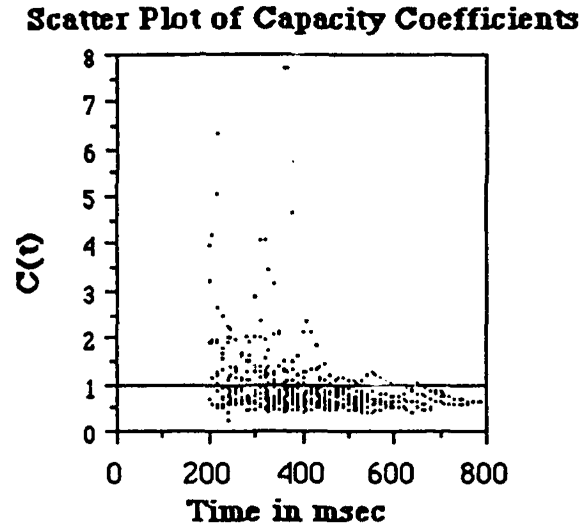


Figure 10. Scatter Plot of the Capacity Coefficients in the OR task. The horizontal line C(t) = 1 is the level of the capacity coefficient equivalent to the unlimited capacity system.

To summarize the results of the capacity analysis, geometric means of the estimated capacity coefficients were calculated and presented in Table 7.

Subject	(0,0)	(0,1)	(1,0)	(1,1)
BJ	0.70	1.25	1.09	0.79
BK	0.54	0.77	0.94	0.57
GN	1.09	1.22	0.87	0.74
JZ	0.95	0.79	1.23	0.76
SK	0.95	0.85	1.07	0.78
ZC	0.62	0.63	0.70	0.78
Average	0.78	0.89	0.97	0.73

Table 7. Geometric means of the capacity coefficients in the double stimulus conditions.

The grand geometric mean of the capacity coefficient was found to be 0.84. Thus, the processing system is likely to be of limited capacity. However, there is a caution in interpreting the results. As was pointed out by Townsend and Nozawa (1992), the capacity coefficient has a tendency of underestimation because of the bias coming from the base time component of the reaction time.

As was mentioned in the results section, there are two out of six subjects who showed negative blips at the earlier portion of the survivor contrast. As noted by Townsend and Nozawa (1992), this can be taken as evidence of the channel summation model. Since the channel summation model was rejected at the level of mean RTs, these negative blips cannot be interpreted by the regular channel summation model. In order for the channel summation model to explain at least the results of the two subjects, either the model must sum the input in a subadditive way or there was a violation of context independence. The channel summation model can be written in general as follows:

$N_{sum}(t) = w_1 \times N_1(t) + w_2 \times N_2(t)$,

where $N_1(t)$ is the counting process of the channel 1, $N_2(t)$ is the counting process of the channel 2, w_1 is the weight for the counting process of the channel 1 and w_2 is the weight for the counting process of the channel 2.

A subadditive summation occurs when the weights, w_1 and w_2, are less when there are two stimuli than when there is only one stimulus. For example, if we have the following relationship,

$$w_1 = \frac{K_1}{E[N_1(t)] + E[N_2(t)]} \text{ and } w_2 = \frac{K_2}{E[N_1(t)] + E[N_2(t)]},$$

where K_1 and K_2 are some constants. It can potentially explain the subadditive summation, if $K_1 < E[N_1(t)] + E[N_2(t)]$ and $K_2 < E[N_1(t)] + E[N_2(t)]$. Psychologically, the constants, K_1 and K_2, can be thought of as representing the capacity limitations of channel 1 and channel 2. This is an example of violation of the context independence assumption.

Let us next consider the case of the AND task. In the AND task we have two inequalities and one identity. They are the Frechet boundary, Boole's inequality and the exact prediction of the independent parallel exhaustive model and can be written as follows:

$F_{B\&T}(t) \geq F_B(t) + F_T(t) - 1,$

$F_{B\&T}(t) \leq \mathrm{Min}\{F_B(t), F_T(t)\},$

and

$F_{B\&T}(t) = F_B(t) \times F_T(t).$

Since we do not observe the RTs of the single stimulus condition with "yes" responses in the AND task, in order for us to evaluate the fits of the inequalities we have to utilize RTs of the single stimulus conditions from the OR task. In order to justify this utilization of the single stimulus condition from the OR task in the AND task , I evaluate the following assumption. This assumption, I call, *marginal task independence*. This concept is very similar to the concept of pure insertion proposed by Donders (1868). The assumption of the marginal task independence states that a component of the mental process network associated with one task does not change even if the component process now is a part of another mental process. In order to test this assumption I created the following test.

Since mean RT contrast of the AND task can be explained by the parallel exhaustive processing model and that of the OR task can be explained by the probability summation model, we can write the survivor functions of the double stimulus conditions in the AND task and the OR task as follows.

We ignore the base time for a while.

For the AND task

$\overline{F}^{AND}{}_{B\&T}(t) = \overline{F}_{T_B}(t) + \overline{F}_{T_T}(t) - \overline{F}_{\min\{T_B, T_T\}}(t)$

For the OR task

$\overline{F}^{OR}{}_{B\&T}(t) = \overline{F}_{\min\{T_B, T_T\}}(t).$

Then combine the above two survivor functions to yield the following:

$$\overline{F}^{AND}_{B\&T}(t) + \overline{F}^{OR}_{B\&T}(t) = \overline{F}_{T_B}(t) + \overline{F}_{T_T}(t).$$

This is the sum of the two marginal survivor functions. The marginal survivor functions can be defined as the survivor function of a single random time. Next the problem is to determine whether the sum of the two marginal survivor functions from the OR task is statistically not significantly different from the sum of the estimated marginal survivor functions. This test of marginal task independence can be performed at the level of mean RTs as follows. Now introducing the base time in the expression, we have

$$\overline{RT}^{AND}_{B\&T} + \overline{RT}^{OR}_{B\&T} = \overline{T}_{T_B} + \overline{T}_b + \overline{T}_{T_T} + \overline{T}_b.$$

If we subtract the mean RTs of the single stimulus conditions from the OR task, we have

$$\overline{RT}^{AND}_{B\&T} + \overline{RT}^{OR}_{B\&T} - \overline{RT}^{OR}_{B} - \overline{RT}^{OR}_{T}$$

$$= \overline{T}_{T_B} + \overline{T}_b + \overline{T}_{T_T} + \overline{T}_b - \overline{T}^{OR}_{T_B} + \overline{T}_b + \overline{T}^{OR}_{T_T} + \overline{T}_b$$

If we have $\overline{T}_{T_B} = \overline{T}^{OR}_{T_B}$ *and* $\overline{T}_{T_T} = \overline{T}^{OR}_{T_T}$ *, then the above quantity should be equal to zero.*

The above mentioned test was conducted using *z*-statistics on all six subjects' mean RTs and presented in Table 8.

Subject	Statistics	(0,0)	(0,1)	(1,0)	(1,1)
BJ	Difference	37.68	51.74	40.94	37.24
	z-score	3.71	7.31	4.49	5.99
BK	Difference	52.95	44.77	63.92	77.77
	z-score	3.18	3.65	4.27	8.39
GN	Difference	36.40	58.60	58.60	54.70
	z-score	3.20	4.72	4.83	7.78
JZ	Difference	147	139	94.01	91.40
	z-score	5.34	6.04	3.26	5.89
SK	Difference	-65.87	5.13	-28.54	-31.00
	z-score	-5.73	0.47	-3.13	-3.19
ZC	Difference	54.19	-17.37	57.98	58.58
	z-score	3.14	-0.99	3.86	4.20

Table 8. Test of marginal task independence between OR task and AND task.

For five out of six subjects the test of marginal task independence yielded mostly significantly positive z-scores. Only one subject showed an opposite trend. Since the assumption of marginal task independence does not seem to be valid, further tests on the mean RTs of the AND task using mean RT's of the OR task are not going to be conducted. The conclusion of the RT experiments can be summarized into the following items:

1. Superadditivity of the mean and the survivor contrast was obtained in the OR task. The serial exhaustive and the serial self-terminating models were rejected.
2. Subadditivity of the mean and the survivor contrast was obtained in the AND task. The serial exhaustive and the serial self-terminating models were rejected.
3. Using inequalities, the data in the OR task were further examined. The channel summation model was rejected as a result of the analysis. However, there were two subjects who showed negative blips at the beginning of the survivor contrast indicating the possibility of the subadditive channel summation.

The final point of discussion is concerned with the dependent structure which is present in the stimulus presentation. In the present experiment the probability of both targets being presented at the same time was 1/6, the probability of the top target being presented alone was 1/6, the probability of the bottom target being presented was 1/6, and the probability of no target being presented was 1/2. These stimulus probabilities produce a positive dependent structure between the targets. That is, the probability of the joint occurrence of both targets is greater than the product of the probabilities associated with each single target:

$P(B\&T) > P(B)\,P(T)$,

where B&T represents the joint occurrence of the bottom and the top targets, B represents the union of all events in which the bottom target occurred and T is defined as the union of all events in which the top target occurred. Positive dependency means that the occurrence of either single target conveys statistical information for the likelihood of a target occurring in the other location. For example, the above inequality shows that both the top target and the bottom target carry positive information. That is, knowing that there is the bottom target increases the likelihood of having the top target. In the present experiments, the *absence* of a target also carried positive information on the likelihood of the presence of a target on the opposite side. A recent report by Mordkoff and Yantis (1991) showed that the dependent structure of the stimuli can influence performance in the OR task. They suggested that the dependent structure of the stimulus presentation probabilities may cause a between-channel interaction to produce a dependent structure in the processing times between two channels. The direction of the dependence between stimuli, according to Mordkoff and Yantis (1991), should be revealed as that of the dependence between two processing times. They defined three types of interstimulus contingencies, the interstimulus contingency benefit and the nontarget-response contingency benefit as follows:

$ISC(N_i \Rightarrow T) = P(T^{(a)} | N_i^{(b)}) - P(T^{(a)})$

$ISC(T \Rightarrow T) = P(T^{(a)} | T^{(b)}) - P(T^{(a)})$

$ISCB(N_i) = P(T^{(a)} | T^{(b)}) - P(T^{(a)} | N_i^{(b)})$

$NRCB(N_i) = P(+) - P(+ | N_i)$,

where ISC represents the interstimulus contingency, ISCB denotes the interstimulus contingency benefit, NRCB represents the nontarget-response contingency benefit, N_i represents a specific nontarget element, $T^{(a)}$ represents the target in channel "a", $N_i^{(b)}$ represents nontarget in channel "b" and "+" represents "Go" trials. If $ISC(N_i \Rightarrow T)$ is greater than 0, there is a positive dependence between nontarget and a target. If $ISC(T \Rightarrow T)$ is negative, presence of a target carries information with regard to absence of the other target. In the present experiment, $ISC(T \Rightarrow T)$ was positive, $ISC(N_i \Rightarrow T)$ was negative, $ISCB(N_i)$ was positive and finally $NRCB(N_i)$ turned out to be positive. According to Mordkoff and Yantis (1991), a positive ISCB and a positive NRCB should have caused speeding up of the redundant target processing times compared to the processing times of the single target conditions. This would have lead to negativity of fit with respect to the probability summation identity since their "interactive race model" cannot predict any violation of Miller's inequality without violating the assumption of context independence. However, as we saw in the results, the data tend not to support their claim as far as this experiment is concerned. Future investigation is needed both in terms of its analytical specification of the interactive race model and the empirical research.

ACCURACY MEASURES

The Additive Factors-type of Method in a Psychophysical Experiment

Let us consider the application of the additive factor-type method to a psychophysical experiment in which the main dependent variable is *response accuracy*. In this domain of theorizing and experimentation, the most important contribution was provided by Shaw and colleagues (eg., Shaw, 1980; Mulligan and Shaw, 1980). Mulligan and Shaw (1980) showed that the probability summation prediction holds in a bimodal detection experiment. The experiment involved four different presentation conditions: an auditory signal and a visual signal occurred at the same time, the auditory signal only was presented, presentation of visual signal only was presented, or no signal at all was presented. Since the subjects were required to respond "yes" whenever they detected the auditory OR visual stimulus and were required to respond "no" otherwise, the logical task requirement is equivalent to that of the redundant target paradigm described above. Mulligan and Shaw (1980) analyzed the probabilities of "no" responses in each of the various stimulus conditions using the following formula, which I call the "no" response probability contrast. In the following formula I use the notation of my experiment.

$$P[no|(\emptyset,\emptyset)] - P[no|(\emptyset,B)] - P[no|(T,\emptyset)] + P[no|(T,B)]$$

The symbol, $P[no|(\emptyset,\emptyset)]$, represents the probability of "no" response given no signal at all. The symbol, $P[no|(T,B)]$, represents the "no" response probability given the top and the bottom signals.

The results showed that the "no" response probability contrast was superadditive. Actual derivation and testing were performed on the logarithmically transformed probabilities. Shaw utilized the fact that the logarithm of the multiplication of two quantities can be expressed as the sum of the logarithm of individual quantities. This fact allowed her to test the independent parallel exhaustive processing model.

Following the analysis by Shaw (1980), I will now show that the parallel exhaustive processing model for the "no" response probability contrast is superadditive. In deriving predictions from various models, it is necessary to realize that the four major dimensions of information processing models presented in the previous section are as important in constructing models in terms of the accuracy measures as they were in the RT measures. In the following derivation, the crucial assumptions are 1) stochastic independence, 2) selective influence and 3) context independence. Under these assumptions the "no" response probability contrast of the parallel exhaustive processing is written as follows:

$P[no|(\emptyset,\emptyset)] - P[no|(\emptyset,B)] - P[no|(T,\emptyset)] + P[no|(T,B)]$

=P[no to top noise AND no to bottom noise]

-P[no to top noise AND no to bottom signal]

-P[no to top signal AND no to bottom noise]

+P[no to top signal AND no to bottom signal]

Assuming stochastic independence, we have

$= P[\text{no}|\text{top noise}] \times P[\text{no}|\text{bottom noise}]$

$- P[\text{no}|\text{top noise}] \times P[\text{no}|\text{bottom signal}]$

$- P[\text{no}|\text{top signal}] \times P[\text{no}|\text{bottom noise}]$

$+ P[\text{no}|\text{top signal}] \times P[\text{no}|\text{bottom signal}]$.

Assuming context independence, we can express the above probabilities in terms of internal counting processes (which represent the activity within each sensory channel). The accumulated count is compared with a criterion (θ), yielding the expression

$= P[N_T(t)<\theta \,|\text{top noise}] \times P[N_B(t)<\theta \,|\text{bottom noise}]$

$- P[N_T(t)<\theta \,|\text{top noise}] \times P[N_B(t)<\theta \,|\text{bottom signal}]$

$- P[N_T(t)<\theta \,|\text{top signal}] \times P[N_B(t)<\theta \,|\text{bottom noise}]$

$+ P[N_T(t)<\theta \,|\text{top signal}] \times P[N_B(t)<\theta \,|\text{bottom signal}]$. Factoring yields

$= \{P[N_T(t)<\theta \,|\text{top noise}] - P[N_T(t)<\theta \,|\text{top signal}]\}$

$\times \{P[N_B(t)<\theta \,|\text{bottom noise}] - P[N_B(t)<\theta \,|\text{bottom signal}]\}$

In the above expression I used $N_T(t)<\theta$ instead of "no on the top channel" and $N_B(t)<\theta$ instead of "no on the bottom channel." It simply means that a particular internal variable has not exceeded the threshold value, θ, by time t. If the subject produces "no" responses more often in the noise condition than in the signal condition (that is the rate associated with each counting process is increased by the presentation of a signal), we then have the following:

$P[N_T(t)<\theta|\text{top noise}] > P[N_T(t)<\theta|\text{top signal}]$

and

$P[N_B(t)<\theta|\text{bottom noise}] > P[N_B(t)<\theta|\text{bottom signal}]$.

Thus, the ordering of the counting process with stimulus presentation insures that

$\{P[N_T(t)<\theta\ |\text{top noise}] - P[N_T(t)<\theta\ |\text{top signal}]\}$ and

$\{P[N_B(t)<\theta\ |\text{bottom noise}] - P[N_B(t)<\theta\ |\text{bottom signal}]\}$ are positive. The multiplication of these terms is also positive, so the "no" response probability contrast is superadditive.

Shaw did not prove nor empircally demonstrate subadditivity of the response probability contrast of the "yes" response. She just mentioned that predictions of the "no" response are less cumbersome than those for probability of the "yes" response (Shaw, 1982). The "yes" response probability contrast can be defined as follows:

$P[\text{yes}|(\varnothing,\varnothing)] - P[\text{yes}|(\varnothing,B)] - P[\text{yes}|(T,\varnothing)] + P[\text{yes}|(T,B)]$.

Probability of a "yes" response given a bottom signal being presented = Probability of the bottom signal exceeding the criterion OR noise in the top channel exceeding the criterion $= P[N_B(t)>\theta] + P[N_{Tn}(t)>\theta] - P[N_B(t)>\theta] \times P[N_{Tn}(t)>\theta]$, where $N_{Tn}(t)$ is the counting process of the top channel given noise only, $N_B(t)$ is the counting process of the bottom channel given signal plus noise. Probability of "yes" response given top signal being presented = Probability of top signal exceeding the criterion OR noise in the bottom channel exceeding the criterion =

$P[N_T(t)>\theta] + P[N_{Bn}(t)>\theta] - P[N_T(t)>\theta] \times P[N_{Bn}(t)>\theta]$. The probability of a "yes" response given bottom signal and top signal being presented = Probability of bottom signal exceeding the criterion OR the top signal exceeding the criterion =

$P[N_B(t)>\theta] + P[N_T(t)>\theta] - P[N_B(t)>\theta] \times P[N_T(t)>\theta]$. Finally, the probability of a"yes" response given only noise being presented = Probability of noise in the bottom channel exceeding the criterion OR noise in the top system exceeding the criterion = $P[N_{Bn}(t)>\theta] + P[N_{Tn}(t)>\theta] - P[N_{Bn}(t)>\theta] \times P[N_{Tn}(t)>\theta]$. Let us derive the prediction from the probability summation model in terms of the "yes" response probability contrast. Again, stochastic independence, context independence and selective influence are assumed.

$P[yes|(\varnothing,\varnothing)] - P[yes|(B,\varnothing)] - P[yes|(\varnothing,T)] + P[yes|(B,T)]$, which by substitution yields

$$= P[N_{Bn}(t)>\theta] + P[N_{Tn}(t)>\theta] - P[N_{Bn}(t)>\theta] \times P[N_{Tn}(t)>\theta]$$

$$- P[N_B(t)>\theta] + P[N_{Tn}(t)>\theta] + P[N_B(t)>\theta] \times P[N_{Tn}(t)>\theta]$$

$$- P[N_T(t)>\theta] + P[N_{Bn}(t)>\theta] + P[N_T(t)>\theta] \times P[N_{Bn}(t)>\theta]$$

$+ P[N_B(t)>\theta] + P[N_T(t)>\theta] - P[N_B(t)>\theta] \times P[N_T(t)>\theta]$. Factoring and rearranging terms gives

$$= P[N_T(t)>\theta] \times P[N_{Bn}(t)>\theta] + P[N_B(t)>\theta] \times P[N_{Tn}(t)>\theta]$$

$$- P[N_T(t)>\theta] \times P[N_B(t)>\theta] - P[N_{Bn}(t)>\theta] \times P[N_{Tn}(t)>\theta]$$

$$= \{P[N_T(t)>\theta] - P[N_{Tn}(t)>\theta]\} \times \{P[N_{Bn}(t)>\theta] - P[N_B(t)>\theta]\}$$

Since $P[N_T(t)>\theta] - P[N_{Tn}(t)>\theta]$ is greater than zero and $P[N_{Bn}(t)>\theta] - P[N_T(t)>\theta]$ is less than zero, the whole expression is negative. This ends the proof of the probability summation model's subadditivity with respect to the "yes" response probability. A subadditivity prediction at the level of the "yes" response probability contrast implies subadditivity at the level of mean. I call the contrast at the level of the mean the *mean response contrast.* The mean response contrast is written as follows:

$$E[N_{(Bn,Tn)}(t)] - E[N_{(Bn,T)}(t)] - E[N_{(B,Tn)}(t)] + E[N_{(B,T)}(t)].$$

In order to explain the concept of the mean response contrast, we need to understand the concept of the decision variable in the signal detection theory (see Egan, 1975, p 19; Treisman, 1976, p248). The concept of the decision variable is similar to what I call the "internal random variable", which generates RTs and psychophysical judgments by way of certain transformations. One of the transformations I used in order to transform the internal random variable to RT in the previous section was the renewal theorem. According to Treisman (1976) the decision variable (or axis), E, can be considered the result of a certain transformation of a physical variable, I. In the case of a weak signal we only need a small range of operation and therefore the following expression may be a good approximation for a short range of stimulus intensity (Treisman, 1976).

$$E = aI + b$$

However, McGill and Goldberg (1968) assumed the form

$$E = aI^{b}.$$

Green (1976) assumed the following form

$$E = a(I + \Delta I)^{b}.$$

Although I recognize that it is important to ask questions regarding the transformation of the internal random variable or the decision variable into the values of confidence ratings, this problem is not within the scope of this thesis. If we use a single observation interval with a simple Yes-No response, random variation of the decision variable cannot be observed. However, if the subjects are allowed to generate their confidence ratings in addition to a Yes-No response, it is possible to observe the variation of the decision variable and to make an inference about the nature of the internal random variables.

Let us consider an example of a psychophysical experiment in which a subject is presented with either a visual signal, an auditory signal, both a visual and an auditory signal, or no signal at all (the noise condition). One of the interesting questions we can ask in the context of the detection of multiple signals is: Can we identify the function by which multiple inputs are transformed? In order to identify the function, we need to observe outputs which contain information on the decision variable. This can be done by way of confidence ratings. When a stimulus is presented, the physical stimulus is transformed into a physiological signal. This physiological signal (which we assume is a random variable) is mapped onto the decision axis by some transformation. At this point we call the quantity a decision variable. The decision axis is categorized in terms of confidence ratings ranging from the highest confidence on a "no" response to the highest confidence on a "yes" response. When the decision variable's value on the decision axis falls within a particular interval, the subject responds with the category associated with that particular interval. These ideas are portrayed in Figure 11.

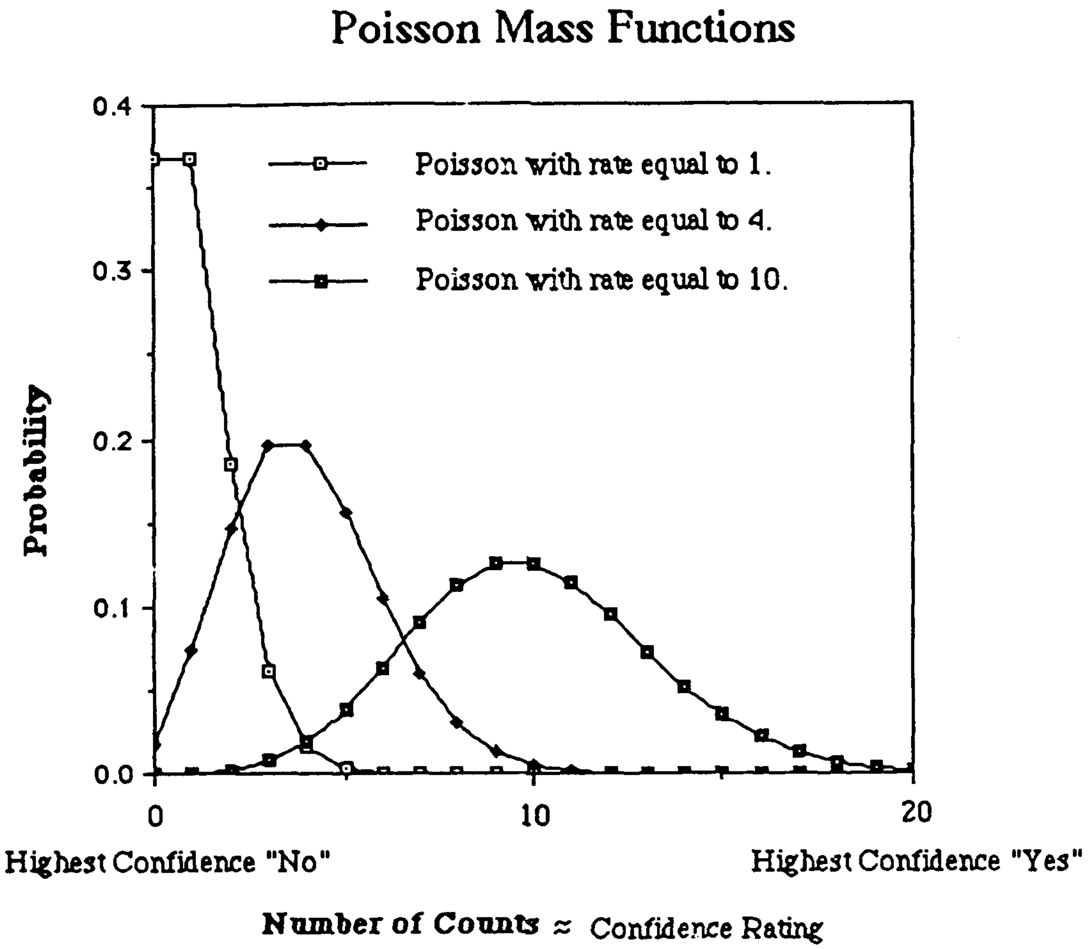


Figure 11. A graph showing the relationship between the number of counts and the confidence rating scale.

In the above figure, the Poisson mass function with rate equal to 1 represents the noise distribution and the Poisson mass functions with rates equal to 4 and 10 can be considered as the signal plus noise distributions. Notice that a fewer number of counts associate with a higher confidence rating on the "no" response and a greater number of counts associate with a higher confidence rating on the "yes" response. Assuming that a mechanism which accomplishes a mapping of the internal stochastic process into the confidence rating scale, it is possible to identify the form of the function by which the two signals are transformed.

If the input is "a" for one channel and "b" for another channel, and the transformations which convert "a" and "b", real numbers, to the activity levels of the channels are "g" and "h", respectively, then the function which maps each activity level to the integrated activity level, the output "y", can be expressed as follows:

$$y = f(\ g(a),h(b)\ ).$$

For example, the signal detection system can be represented in the following flow diagram.

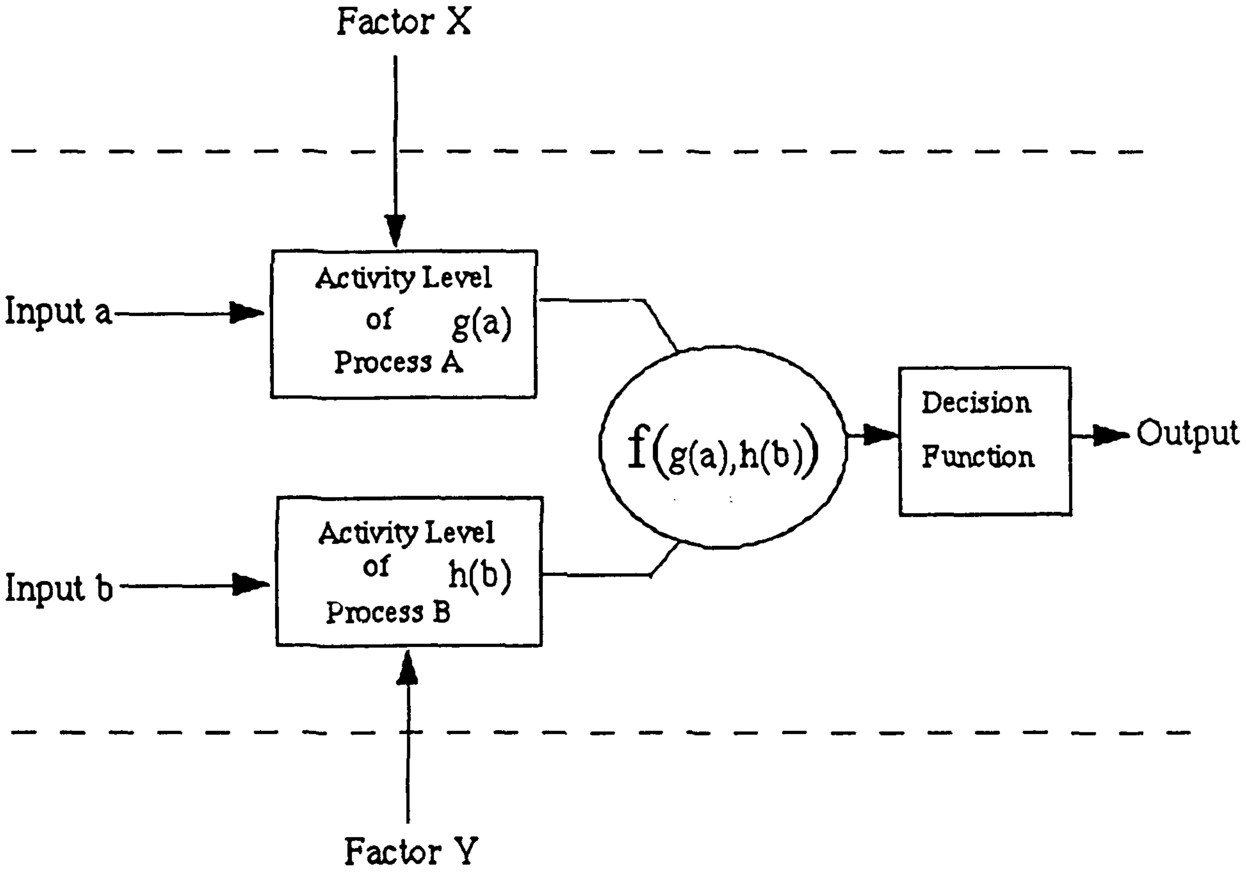


Figure 12. A general representation of parallel processing system with two inputs.

What we are interested in is to identify the form of the function f(g(a), h(b)). One informative analysis is the evaluation of the concavity and convexity of the function. Let us represent g(a) by x_1 and h(b) by x_2 for the sake of simplicity. Let x'_1 and x'_2 be real values representing g(a') and h(b'), where a' and b' are another set of inputs. Let β be a real number between zero and one. Then the function f(., .) is said to be convex if the following inequality is satisfied.

$$f(\beta\cdot(x_1,x_2) + (1-\beta)\cdot(x'_1,x'_2)) \leq \beta\cdot f(x_1,x_2) + (1-\beta)\cdot f(x'_1,x'_2)$$

For example, the function $f(x_1, x_2) = \min(x_1, x_2)$ is concave, the function $f(x_1, x_2) = \max(x_1, x_2)$ is convex, the function $f(x_1, x_2) = x_1 \times x_2$ is concave, and the function $f(x_1, x_2) = x_1 + x_2$ is both concave and convex. In order to perform this analysis in detail we need to know the form of the functions, g(.) and h(.).

Another way to identify the form of the function is to use the mean response contrast, which was defined earlier. The mean response contrast can discriminate the form of various functions. For example, the function $f(x_1, x_2) = \min(x_1, x_2)$ is superadditive, the function $f(x_1, x_2) = \max(x_1, x_2)$ is subadditive, the function $f(x_1, x_2) = x_1 \times x_2$ is superadditive, and the function $f(x_1, x_2) = x_1 + x_2$ is additive in terms of mean response contrast. However, if the quantity turns out to be superadditive, there are at least two possible functions, $\min(x_1, x_2)$ and $x_1 \times x_2$. In this case we still can distinguish between these two functions by predicting the response mean of the double stimulus condition from two single stimulus conditions. Thus, the response means are very informative in distinguishing among various models of information processing. However, there are questions regarding the internal statistics for the activity of a single sensory channel. Various proposed statistics (neural codes) have been reviewed by Perkel and Bullock (1967). Luce and Green (1972) proposed counting and timing codes as the statistics describing the activity of a single channel. In the present work, I will concentrate on these two codes. The counting code is related to the decision variable in a monotonically increasing way. For example, as the expected value of the counting statistics increases, the expected value of the decision variable also increases. However, the timing code is related to the decision variable in a monotonically decreasing way because the average time interval of noise is longer than the average time interval of signal and therefore is mapped into smaller confidence rating values (see Luce and Green, 1972; Egan, 1975).

It is possible to distinguish the counting models from the timing models using the signal detection theory (Egan, 1975). The counting model counts the number of events by fixing the interval, whereas the timing model waits for a specific number of events. Both models deal with the same underlying stochastic process, the renewal process. The difference between the counting and the timing models is the sampling mechanism by which the observation is made. If the sampling mechanism keeps track of counts within a fixed interval, it is called the counting model, whereas if it keeps track of time at which a fixed number of counts are made, it is called the timing model. The question in terms of psychology is: Can we distinguish these two models? According to Luce (1977), slopes of standardized ROC curves are less than one for the counting model and greater than one for the timing model. Standardized ROC curves are the ROC curves plotted on the normal vs. normal coordinate system. Therefore, two coordinates are 1) z-scores of the false alarm probability on the x-axis and 2) z-scores of the hit probability on the y-axis. Now define z(s) as the z-score associated with the hit probability, z(n) as the z-score associated with the false alarm probability, μ as the mean of the Poisson process generating signal plus noise, ν as the mean of the Poisson process generating noise and Δ as the amount of time spent by the subject in observation. As expressed in Green and Luce (1973), the functional relationship between these two z-scores for the counting model is

$$z(s) \cong (\nu/\mu)^{1/2}\, z(n) + \Delta^{1/2}\, [(\mu - \nu)/\mu^{1/2}].$$

Assuming that the decision mechanism accumulates k counts from a single channel, the functional relationship between these two z-scores for the timing model is written

$$z(s) \cong \mu / \nu \;\; z(n) + k^{1/2}\, [\mu / \nu \; - 1].$$

Since we have $\mu .> \nu$ the slope for the counting model is less than one and the slope for the timing model is greater than one. The same analysis was also used by Wandell and Luce (1978) to distinguish the timing model from the counting model. This analysis may potentially lead to a situation in which all the timing models or all the counting models can be rejected. Therefore, this test will be performed at the beginning of the data analysis of the present experiment.

As was the case in the RT experiment, two types of tasks are involved in the psychophysical experiment . One task is the OR-task, in which the subject is asked to respond "yes" to the occurrence of any target; otherwise a "no" response is required. Another task is the AND task in which the subject is asked to respond "yes" only if two targets are presented; otherwise the subject is required to respond "no". Let us consider the predictions from the parallel models in the OR task and the AND task. First, I am going to present the predictions from all of the counting models except the serial models. The reason for this exclusion is that there is no representation of the serial counting models of which I am aware. After introducing various counting models I am going to introduce the timing models including serial timing models. As before, the crucial assumptions are 1) stochastic independence, 2) selective influence and 3) context independence.

Counting Models

The Probability Summation Model

Probability summation assumes that decisions are based on the maximum of the counting processes for both channels. This model predicts subadditivity with respect to the mean response contrast.

$E[N_{(Bn,Tn)}(t)] - E[N_{(Bn,T)}(t)] - E[N_{(B,Tn)}(t)] + E[N_{(B,T)}(t)].$

$=E[\mathrm{Max}\{N_{Bn}(t),N_{Tn}(t)\}] - E[\mathrm{Max}\{N_{Bn}(t),N_{T}(t)\}]$

$- E[\mathrm{Max}\{N_{B}(t),N_{Tn}(t)\}] + E[\mathrm{Max}\{N_{B}(t),N_{T}(t)\}]$

Since, for example,

$E[\mathrm{Max}\{N_{Bn}(t),N_{Tn}(t)\}] = P[N_{Bn}(t)>k] + P[N_{Tn}(t)>k] - (P[N_{Bn}(t)>k] \times P[N_{Tn}(t)>k]),$

by substitution and factoring, we see that the mean response contrast

$$=\sum_{k=0}^{\theta} \{P[N_T(t)>k] - P[N_{Tn}(t)>k]\} \times \{P[N_{Bn}(t)>k] - P[N_B(t)>k]\} < 0.$$

The Parallel Exhaustive Model

This model predicts superadditivity with respect to the mean response contrast.

$E[N_{(Bn,Tn)}(t)] - E[N_{(Bn,T)}(t)] - E[N_{(B,Tn)}(t)] + E[N_{(B,T)}(t)].$

$=E[\mathrm{Min}\{N_{Bn}(t),N_{Tn}(t)\}] - E[\mathrm{Min}\{N_{Bn}(t),N_{T}(t)\}]$

$- E[\mathrm{Min}\{N_{B}(t),N_{Tn}(t)\}] + E[\mathrm{Min}\{N_{B}(t),N_{T}(t)\}]$

$$=\sum_{k=0}^{\theta} \{P[N_T(t)>k] - P[N_{Tn}(t)>k]\} \times \{P[N_B(t)>k] - P[N_{Bn}(t)>k]\} > 0.$$

The Channel Summation Model

This model predicts additivity with respect to the mean response contrast.

$E[N_{(Bn,Tn)}(t)] - E[N_{(Bn,T)}(t)] - E[N_{(B,Tn)}(t)] + E[N_{(B,T)}(t)].$

$=E[N_{Bn}(t)+N_{Tn}(t)] - E[N_{Bn}(t)+N_{T}(t)]$

$- E[N_{B}(t)+N_{Tn}(t)] + E[N_{B}(t)+N_{T}(t)]$

$=E[N_{Bn}(t)]+E[N_{Tn}(t)] - E[N_{Bn}(t)]-E[N_{T}(t)]$

$- E[N_{B}(t)]-E[N_{Tn}(t)] + E[N_{B}(t)]+E[N_{T}(t)] = 0.$

The Channel Multiplication Model

This model predicts superadditivity with respect to the mean response contrast.

$E[N_{(Bn,Tn)}(t)] - E[N_{(Bn,T)}(t)] - E[N_{(B,Tn)}(t)] + E[N_{(B,T)}(t)].$

$=E[N_{Bn}(t)\times N_{Tn}(t)] - E[N_{Bn}(t)\times N_{T}(t)]$

$- E[N_{B}(t)\times N_{Tn}(t)] + E[N_{B}(t)\times N_{T}(t)]$

$=E[N_{Bn}(t)]\times E[N_{Tn}(t)] - E[N_{Bn}(t)]\times E[N_{T}(t)]$

$- E[N_{B}(t)]\times E[N_{Tn}(t)] + E[N_{B}(t)]\times E[N_{T}(t)]$

$=(E[N_{Bn}(t)] - E[N_{B}(t)]) \times (E[N_{Tn}(t)] - E[N_{T}(t)]) > 0$

Timing Models

The Probability Summation Model

This model predicts superadditivity with respect to the mean RT contrast. However, since the decision variable is transformed by a monotonically decreasing function, this model predicts subadditivity with respect to the mean response contrast.

Parallel Exhaustive Model

This model predicts subadditivity with respect to mean RT contrast. However, since the decision variable is transformed by a monotonically decreasing function, this model predicts superadditivity with respect to mean response contrast.

Channel Summation Model

This model predicts superadditivity with respect to mean RT contrast. Because of the monotonically decreasing transformation from the internal random variable to the decision variable, the model generates a prediction of subadditivity.

Let us turn our attention to the serial models. There are two types of serial models to consider: they are the serial self-terminating model and the serial exhaustive model. The self-terminating mode is most appropriate for the OR task, while the exhaustive mode is most appropriate for the AND task. A serial model most naturally generates time domain decision variables. In the derivation below I use $E[T_{(Bn,Tn)}]$ to represent the expected value of the timing observation variable under noise only condition. The symbol $E[T_{Bn}+T_{Tn}]$ means the expected value of the sum of the two timing observation variables, T_{Bn} and T_{Tn}.

Serial Self-terminating Timing Model

The mean response contrast is given by

$E[T_{(Bn,Tn)}] - E[T_{(Bn,T)}] - E[T_{(B,Tn)}] + E[T_{(B,T)}]$. Assume that the observer selects one channel (eg., the bottom channel) for processing with a probability p. The probability that the other (top) channel is selected for processing is therefore (1-p). If the first channel selected for processing does not have a target, then the model assumes that processing is switched to the second channel. In this case, time is devoted to processing noise on channel 1 before processing is transfered to the second channel. Thus, the expected value of the timing process for the serial model on those trials in which the first channel selected does not contain a target is the processing time devoted the exhaustive processing of noise on channel, plus the time associated with target detection on channel 2, which is given by $E[T_{Bn}+T_T]$. Ofcourse, this expected value must be weighted by the probability that a particular channel is selected first. Therefore, in order to exhaust all possibilities of selecting a channel given all the possible stimulus alternatives, we have

$$= E[T_{Bn}+T_{Tn}] - p\times E[T_{Bn}+T_T] -(1-p)\times E[T_T] - (1-p)\times E[T_B+T_{Tn}] - p\times E[T_B]+ p\times E[T_B]+(1-p)\times E[T_T]$$

$$= E[T_{Bn}+T_{Tn}]- p\times E[T_{Bn}+T_T] - (1-p)\times E[T_B+T_{Tn}]$$

$$= (1-p)\times\{E[T_{Bn}] -E[T_B]\}+p\times\{E[T_{Tn}] - E[T_T]\}$$

The above quantity is positive if we assume $E[T_{Bn}] > E[T_B]$ and $E[T_{Tn}] - E[T_{Tn}]$.

Serial Exhaustive Timing Model

$E[T_{(Bn,Tn)}] - E[T_{(Bn,T)}] - E[T_{(B,Tn)}] + E[T_{(B,T)}]$.

$= E[T_{Bn}]+E[T_{Tn}] - E[T_{Bn}] -E[T_T] - E[T_B] - E[T_{Tn}]+ E[T_B]+E[T_T]$

$=0$

Additivity holds for the serial exhaustive timing model.

In order to analyze the signal detection "Yes-No" data in detail, I invented a new type of ROC analysis technique which utilizes more information than does the traditional way of analysis. Appendix F gives a full details of the analysis technique. This analysis technique can be used in the situation in which confidence ratings are obtained. This technique allows us to generate stable estimates of the slope by being able to provide more points than the regular "yes" ROC curves. The major difference between this technique and the regular ROC analysis is that my technique uses all the data points, including "yes" responses and "no" responses, to generate ROC curves, whereas the regular one only utilizes the data points which are associated with "yes" responses.

Since I have presented the model predictions in terms of mean response contrast, let us turn our attention to the hypotheses which are tested in the experiment. As I noted earlier, at first I am going to perform the test of slopes of the standardized ROC curve's.

Hypotheses Tested in the Psychophysical Experiments

The first hypothesis states that the slope of the standardized ROC curve should be less than one.

The counting hypothesis: the slope of the z-score ROC curve < 1

The second hypothesis states that the slope of the standardized ROC curve should be greater than one.

The timing hypothesis: the slope of the standardized ROC curve > 1

The second test which I am going to perform is the test on the mean response contrast. The results of the test on the mean response contrast can be classified into three categories: subadditive, additive and superadditive. From the results of the RT experiments, I chose plausible models: the probability summation model in the OR task and the parallel exhaustive processing model in the AND task. From the above presentation of the models, we see that the probability summation model predicts subadditivity and the parallel exhaustive processing model predicts superadditivity. To put it in the form of hypotheses for the OR task, we have

H_1: mean response contrast < 0

and

H_2: mean response contrast ≥ 0.

For the AND task we have

H_1: mean response contrast > 0

and

H_2: mean response contrast < 0.

I felt that the description at the mean level was more general, and less microscopic and still distinguishes various models. Therefore, I preferred the analyses in terms of the means. Unless it is necessary to analyze the data in further detail, the analyses will be performed at the level of means.

EXPERIMENT 2

Methods

Design

Each subject participated in two types of tasks: the OR task and the AND task. Subjects were dark-adapted for at least five minutes before the experiment. Before the experiment proceeded to the data collecting phase, the intensities of targets used in the data collecting trials were determined in two stages. At the first stage the subject performed a variation of the method of limits. After obtaining a rough estimate of threshold values the experimenter introduced the Yes-No detection task. The intensities used in the experiment satisfied the following criterion: detection performance on a single target were greater than 50% and less than 80% in terms of hit proportion. When the intensities satisfied this condition, the experimenter instructed the subject for the experiment. In the OR task subjects were required to press a "yes" button whenever one of the targets, either bottom target or top target, appeared; otherwise, when there was no target presented they were required to respond with a "no" button. In the AND task subjects were required to press a "yes" response only when two targets appeared; otherwise when either only one target was presented or no target was presented, they were required to respond with a "no" response. In both tasks the stimulus conditions were identical. All subjects performed both tasks in a counterbalanced order. There were four possible display conditions: the two-target condition, the top only condition, the bottom only condition, and one noise condition in which no target was presented. All the experimental variables were within-subject factors.

Subjects

Five males, including the author, and one female served as subjects. All of the subjects had normal or corrected normal vision.

Apparatus and Stimuli

A microcomputer controlled the stimulus presentation and recorded the responses. The target LEDs and the fixation LED were fixed in a display stand which stood on an optical bench 57 cm away from the subject's eyes. The top target (red LED) was presented on the vertical meridian with an elevation of +1.0 ° from the center of the fixation point (green LED) and the bottom target (red LED) was also presented on the vertical meridian at an elevation of - 1.0 ° from the center of the fixation point. The diameter of LEDs was 0.5 °. The average luminance of the target was 0.034 cd/m^2. Subjects viewed the display binocularly. The time course of a trial was as follows. A warning tone sounded for a 100 ms. After a fixed foreperiod (100 ms) the stimuli were presented for 100 ms. After a fixed interval of 800 ms a click sounded and subjects were given 3000 ms to respond "yes" or "no" by pressing one of the buttons. Following this 3000 ms. interval, a double click sounded, indicating that a confidence judgement was then required. Subjects were given 4 secs. to make their confidence judgements, which ranged from 1 to 5 (5 being "most confident" that their previous response had been correct). These responses were coded via four different response buttons (pressing no button during the 4 sec. interval indicated the lowest level of confidence). The inter-trial-interval was 1.5 sec. In each trial there were four possible events: top and bottom targets together, only top target, only bottom target, and no target at all. The probability that both targets were presented at the same time was 1/6, the probability that the top target was presented alone was 1/6, the probability that the bottom target was presented alone was 1/6, and the probability that no target was presented was 1/2.

RESULTS EXPERIMENT 2

Results of Analysis on slopes of standardized ROC curves

As explained above, the first hypotheses to test were as follows:

The counting hypothesis: the slope of the standardized ROC curve < 1

and

The timing hypothesis: the slope of the standardized ROC curve > 1.

The following ROC curves are a couple of typical ROC curves: one from the OR task and the other from the AND task.

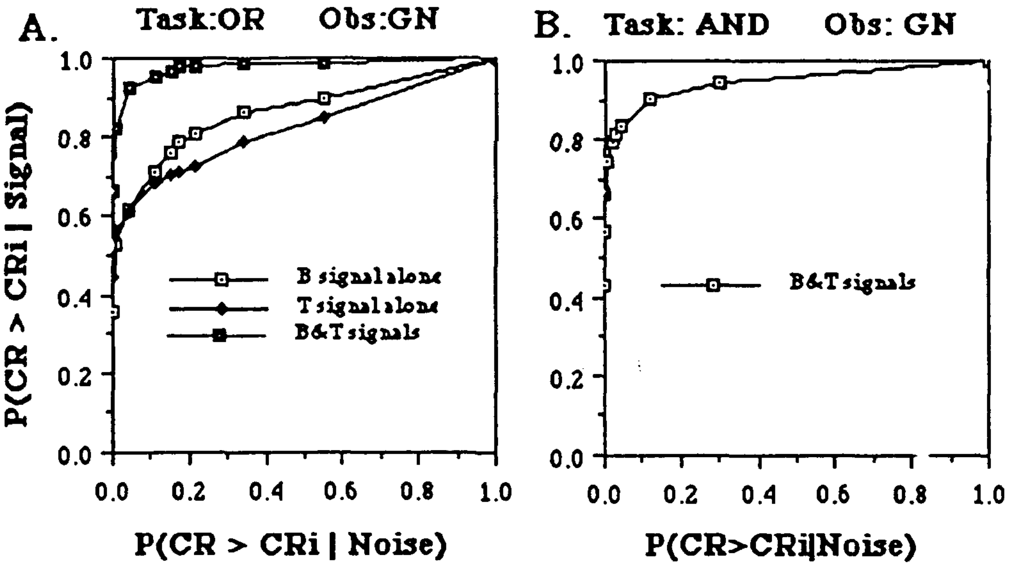


Figure 13. Typical ROC curves from the OR task and the AND task. The notation P(CR>CRi|Signal) represents the probability of the confidence ratings greater than a particular

point "i" on the confidence rating scale given signal presence and P(CR>CRi|Noise) represents that probability given signal absence.

In the OR task there were three signal plus noise conditions and one noise condition. Therefore, there were three ROC curves obtained for each subject. The results of the slope analysis showed all slopes were less than one in the OR task.

Subject/Condition	Slope	Intercept	R^2
GN OR B	0.543	-1.263	0.982
GN OR T	0.36	-0.918	0.977
GN OR B&T	0.613	-2.386	0.927
GT OR B	0.654	-0.957	0.996
GT OR T	0.574	-1.125	0.984
GT OR B&T	0.531	-1.291	0.99
HH OR B	0.341	-0.849	0.801
HH OR T	0.392	-1.344	0.874
HH OR B&T	0.381	-1.8	0.907
JM OR B	0.622	-2.093	0.835
JM OR T	0.595	-1.729	0.868
JM OR B&T	0.325	-1.851	0.784
MM OR B	0.503	-2.165	0.948
MM OR T	0.612	-2.165	0.948
MM OR B&T	0.943	-4.324	0.944
OO OR B	0.603	-1.14	0.992
OO OR T	0.531	-1.055	0.996
OO OR B&T	0.417	-1.38	0.963

Subject/Condition	Slope	Intercept	R^2
GN AND B&T	0.569	-1.953	0.961
GT AND B&T	0.68	-1.559	0.816
HH AND B&T	0.705	-1.508	0.99
JM AND B&T	1.028	-3.492	0.849
MM AND B&T	0.842	-3.612	0.923
OO AND B&T	0.479	-0.996	0.983

Table 9. Slope, intercept and R^2 of linear regression analysis of standardized ROC curves' in the OR task and in the AND task.

The R^2s of the regression lines on the standardized of ROC curves ranged from 0.784 to 0.996. In the AND task there was only one signal plus noise condition. Therefore, there were six slopes calculated, one from each subject. Five out of six slopes were less than one. Only one slope turned out to be greater than one: 1.028. The R^2s of the regression lines on the standardized ROC curves ranged from 0.849 to 0.996. All the obtained slopes were pooled together to compute the mean of the slopes and a z-test was conducted on the mean of the slopes. The pooled mean of the slopes was 0.577 and the z-score associated with it turned out to be -2.376. The hypothesis that the mean of the slopes is greater or equal to unity was clearly rejected at $p < 0.0089$. From the results of the above analyses, the counting models were considered to be more plausible than the timing models. This result leads to the rejection of all the timing models, including all varieties of parallel and serial timing models. Since there is no known representation of the serial counting model, I concentrate in the next section on the evaluation of various parallel counting models. This evaluation was done by the following analysis on the mean response contrast. Recall that the mean response contrast is derived from the confidence ratings (including both "yes" and "no" responses) obtained in each of the four stimulus conditions (see Accuracy Measures section for definitions).

Results of Mean Response Contrast Analysis

The hypotheses tested are in order.

For the OR task we have

H_1: mean response contrast < 0

and

H_2: mean response contrast ≥ 0.

For the AND task we have

H_1: mean response contrast > 0

and

H_2: mean response contrast < 0.

The results of the calculation of the means and variances of the responses in the OR task and the AND task are presented in Appendix G. The mean response contrast for each subject in the OR task and in the AND task are presented in Table 10.

Subject/Condition	Mean Response Contrast	z-scores
GN OR	-2.56	-5.73
GT OR	-2.40	-5.48
HH OR	-1.79	-4.69
JM OR	-4.69	-13.30
MM OR	-3.13	-10.76
OO OR	-1.74	-5.10

Subject/Condition	Mean Response Contrast	z-scores
GN AND	3.08	8.50
GT AND	2.12	6.62
HH AND	0.37	1.03
JM AND	3.26	12.80
MM AND	3.32	14.19
OO AND	1.61	6.30

Table 10. Mean response contrast and its associated z-score for each subject in the OR task and in the AND task.

In the OR task the z-values of the mean response contrasts ranged from -4.68 to -13.29, demonstrating substantial subadditivity. The probability summation model's prediction was confirmed. In the AND task the z-values ranged from 1.03 to 12.8 The smallest z-score, 1.03, came from one subject. The other five subjects showed large positivity in the z-values, ranging from 6.62 to 12.8. In the AND task, therefore, except for one subject, the prediction from the parallel exhaustive processing model held at the level of the mean response contrast. However, notice that the channel multiplication model also predicts superadditivity with respect to the mean response contrast. Next I evaluate the adequacy of various models in predicting the mean responses of the double stimulus condition from the other three conditions: the noise condition, the bottom only condition and the top only condition.

The method of exact predictions from various models was developed. The method for exact prediction from the probability summation model, the channel summation model, the channel multiplication model and the parallel exhaustive model with respect to the mean, which is presented in Appendix H. The method yields an estimate of the mean response measure of the double stimulus condition by utilizing two single stimulus conditions and the noise condition. The obtained means of the double stimulus condition and the mean predictions from various models for each subject are presented in Table 11. The z-scores of the difference between the prediction from various models and the obtained means are reported in Table 12.

Subject Condition	Double Stimulus Condition	Channel Summation Prediction	Channel Multiplication Prediction	Probability Summation Prediction	Parallel Exhaustive Prediction(A)
GN OR	9.26	11.82	20.01	8.87	5.44
GT OR	7.18	9.58	13.61	7.79	4.48
HH OR	8.70	10.49	12.9	8.68	5.79
JM OR	9.06	13.75	27.36	9.24	6.88
MM OR	9.16	12.29	36.91	7.92	5.61
OO OR	6.23	7.98	10.80	6.48	4.00

Subject Condition	Double Stimulus Condition	Channel Summation Prediction	Channel Multiplication Prediction	Probability Summation Prediction	Parallel Exhaustive Prediction(A)
GN AND	7.70	4.62	6.20	4.09	1.92
GT AND	6.49	4.36	4.61	4.18	2.15
HH AND	6.74	6.37	7.49	5.53	3.44
JM AND	8.24	4.97	6.77	4.03	2.43
MM AND	8.07	4.75	7.04	3.79	2.21
OO AND	5.02	4.63	6.20	3.31	2.23

Table 11. Obtained means of the double stimulus condition and the mean predictions from various models for each subject in the OR task and in the AND task.

Subject Condition	Channel Summation	Channel Multiplication	Probability Summation	Parallel Exhaustive(A)
GN OR	-18.94	-79.53	2.85	28.27
GT OR	-10.38	-27.76	-2.63	11.65
HH OR	-11.93	-28.01	0.11	19.4
JM OR	-23.93	-93.38	-0.92	11.17
MM OR	-32.49	-288.16	12.92	36.98
OO OR	-9.57	-25.06	-1.34	12.25

Subject Condition	Channel Summation	Channel Multiplication	Probability Summation	Parallel Exhaustive(A)
GN AND	12.62	6.15	14.82	23.79
GT AND	9.09	8.01	9.88	18.63
HH AND	1.61	-3.24	5.23	14.25
JM AND	19.02	8.57	24.51	33.98
MM AND	25.13	7.81	32.41	44.39
OO AND	1.99	-6.02	8.72	14.23

Table 12. The z-scores between the prediction from various models and the obtained means in the OR task and in the AND task.

All the obtained mean response measures of the double stimulus condition in the OR task were greater in magnitude than those of the prediction from the parallel exhaustive model and less in magnitude than those of the predictions from both the channel multiplication model and the channel summation model. The performance of three subjects was not significantly different from that of the probability summation model. The mean response of two subjects exceeded the prediction from the probability summation model. However, their means were significantly less than the channel summation model's prediction. The weight of the evidence thus favors the probability summation model of performance in the OR task. In the AND task, the parallel exhaustive model's prediction, superadditivity, was confirmed in the above analysis. However, the analysis of the mean response measure of the double stimulus condition did not support the parallel exhaustive processing model. The exact prediction from the parallel exhaustive model was not possible to calculate. Therefore, I presented the approximate predictions instead in Table 11. In terms of ordering of the means, the parallel exhaustive processing model's exact prediction on the mean must be less than the mean of the exact prediction from the probability summation model. The means obtained in the AND task were all significantly greater than the mean predicted by the probability summation model. Therefore the exhaustive parallel processing model was clearly rejected. The reason is straightforward. We know that $E[\max(X,Y)] \geq E[\min(X,Y)]$ for arbitrary positive random variables X and Y. Now if the obtained mean is greater than $E[\max(X,Y)]$, then the obtained mean must be greater than $E[\min(X,Y)]$. The z-scores of the difference between the approximated predictions from the parallel exhaustive model and the obtained means were reported in Table 12. All six subjects demonstrated response means significantly exceeding that of the channel summation model. Five out of six subjects demonstrated response means exceeding even the prediction from the channel multiplication model. In general, the data conform most closely to predictions based on the channel multiplication model.

Summary of the Results

1.The results of the slope analysis of the standardized ROC curves allowed us to clearly reject the timing hypothesis both in the OR task and the AND task. The counting mechanism was found to be compatible with the obtained data in both tasks.

2.The experiment provided subadditivity of the mean response contrast in the OR task. Further analysis showed that the results of the OR task were most compatible with the probability summation model at the level of mean response.

3.The experiment provided superadditivity of the mean response contrast in the AND task. Further analysis showed that, rather than supporting the parallel exhaustive processing model, the results were most consistent with the channel multiplication model.

GENERAL DISCUSSION

Let us begin by summarizing the results of the RT and psychophysical experiments.

RT Experiment

1. In the OR task, superadditivity of the mean and the survivor contrasts was obtained. The serial exhaustive and the serial self-terminating models were rejected. The viable alternatives to the serial models were the parallel models: the probability summation model and the channel summation model.

2. In the AND task subadditivity of the mean and the survivor contrast was obtained. The serial exhaustive and the serial self-terminating models were rejected. The most compatible model with the obtained data turned out to be the parallel exhaustive processing model.

3. Using inequalities, the data in the OR task were further examined. The channel summation model was rejected as a result of the analysis. The model most compatible with the obtained data turned out to be the probability summation model. There were two subjects who showed negative blips at the beginning of the survivor contrast indicating the possibility of the subadditive channel summation.

Psychophysical Experiment

1.The slope analysis of the standardized ROC curves was conducted. The results were incompatible with the timing hypothesis both in the OR task and the AND task. The counting mechanism was found to be compatible with the obtained data in both tasks.

2.In the OR task subadditivity of the mean response contrast was obtained. Further analysis showed that the results of the OR task were compatible with the probability summation model.

3.In the AND task superadditivity of the mean response contrast was obtained. Further analysis showed that the channel multiplication model was rather compatible with the results obtained.

Discussion

In this section I wish to evaluate further the data obtained in the RT and psychophysical experiments. First, I am going to analyze the error response probabilities in the RT experiment to see if the error probabilities are compatible with the processing models accepted in the course of the hypothesis testing of the mean contrast and the survivor contrast analyses. Second, I will point out non-monotonicity in the obtained data in the AND task RT experiment and present two ways of interpreting the non-monotonic mean RTs. Third, I wish to present a model of speed-accuracy trade-offs which can incorporate not only "yes" response RTs but also can incorporate "no" response RTs. Fourth, I wish to extract some common aspects between the results of RT and psychophysical experiments with regard to the convexity versus concavity of the functions.

Error Analyses

First, I am going to present analysis of the error probabilities in the RT experiment. There are a couple of types of errors which we can identify in the RT experiment. One of them comes from fast guessing and the other comes from genuine statistical error of the decision process. Ollman (1966) and Yellot (1967) developed the Fast Guess model to account for speed-accuracy trade-offs. There are two modes of behavior represented in the model: a fast guessing mode and a stimulus controlled response mode. In the fast guessing mode the subject guesses and responds as fast as he can in a detection task. In this mode the subject's correct response probability is at chance level. In the stimulus controlled response mode the subject's response is controlled by the signal. In this mode the subject's correct response probability may not be perfect but reflects the level of performance of the detection process. Let us define the mean reaction time of this type of trials as μ_S. Let us denote the mean RT of the correct responses as MRT_C, the probability of correct response as P(C), the mean RT of the incorrect responses as MRT_e and the probability of incorrect response as P(E). If we have the signal presentation probability of 0.5, there is a convenient relationship which can be tested easily. That is,

$P(C)\,MRT_c - P(E)\,MRT_e = \mu_s\,(P(C) - P(E))$.

Since in the OR task we satisfy the requirement of the signal presentation probability of 0.5, we can apply the analysis. However, in the OR task the probabilities of correct responses in every subject were too high and there was a very small range of variation: the percentage of errors range from 0.6 % to 5.7 %. Since there is not enough variation in the data, the linear analysis cannot be done meaningfully. Therefore, I decided to analyze the data to evaluate the statistical error of the decision process. In the following analysis I am going to concentrate on the error probabilities of the double stimulus conditions. In order to present an account for the error probabilities, we need to establish a model which can explain decision processes of both "yes" and "no" responses. The model assumes a time criterion t_c before which a "yes" response is allowed and after which only a "no" response is allowed. If there is only one signal to consider rather than two signals, the probability of a "no" response can be expressed as follows:

$P(\text{No} \mid \text{signal}) = P(W_k > t_c \mid \text{signal})$.

This means that a miss response occurs as a result of truncation of the waiting time for the signal detection. For the double stimulus condition in the OR task, the probability of a "no" response can be expressed as follows:

$P(\text{No} \mid \text{Bottom signal and Top signal})$

$= P(\text{No on Bottom and No on Top} \mid \text{Bottom signal and Top signal})$

assuming independence, we have

$= P(\text{No} \mid \text{Bottom signal}) \times P(\text{No} \mid \text{Top signal})$.

Let the time criterion for the bottom channel be t_{cB} and the time criterion for the top channel be t_{cT} then we have

$= P(W_B > t_{cB}) \times P(W_T > t_{cT})$.

For the double stimulus condition in the AND task, the probability of a "no" response can be expressed as follows:

$P(\text{No} \mid \text{Bottom signal AND Top signal})$

$= P(\text{No on Bottom OR No on Top} \mid \text{Bottom signal AND Top signal})$

assuming independence, we have

$= P(\text{No} \mid \text{Bottom signal}) + P(\text{No} \mid \text{Top signal}) - P(\text{No} \mid \text{Bottom signal}) \times P(\text{No} \mid \text{Top signal})$

$= P(W_B > t_{cB}).+ P(W_T > t_{cT}) - P(W_B > t_{cB}).\times P(W_T > t_{cT})$

Next let us define the error response probability contrast as follows:

$P(\text{No} \mid B(l),T(l)) - P(\text{No} \mid B(l),T(h)) - P(\text{No} \mid B(h),T(l)) + P(\text{No} \mid B(h),T(h))$, where $P(\text{No} \mid B(l),T(h))$ means the probability of a "no" response given the bottom signal is at the low level and the top signal is at the high level.

The above defined error response probability contrast for the OR task assuming the parallel exhaustive processing model for a "no" response is derived as follows:

$P(\text{No} \mid B(l),T(l)) - P(\text{No} \mid B(l),T(h)) - P(\text{No} \mid B(h),T(l)) + P(\text{No} \mid B(h),T(h))$

$= P(\text{No} \mid B(l)) \times P(\text{No} \mid T(l)) - P(\text{No} \mid B(l)) \times P(\text{No} \mid T(h))$

$- P(\text{No} \mid B(h)) \times P(\text{No} \mid T(l)) + P(\text{No} \mid B(h)) \times P(\text{No} \mid T(h))$

$= P(W_{B(l)} > t_{cB}) \times P(W_{T(l)} > t_{cT}) - P(W_{B(l)} > t_{cB}) \times P(W_{T(h)} > t_{cT})$

$- P(W_{B(h)} > t_{cB}) \times P(W_{T(l)} > t_{cT}) + P(W_{B(h)} > t_{cB}) \times P(W_{T(h)} > t_{cT})$

$= \{P(W_{B(l)} > t_{cB}) - P(W_{B(h)} > t_{cB})\} \times \{P(W_{T(l)} > t_{cT}) - P(W_{T(h)} > t_{cT})\}$

Since both $P(W_{B(l)} > t_{cB}) - P(W_{B(h)} > t_{cB})$ and $P(W_{T(l)} > t_{cT}) - P(W_{T(h)} > t_{cT})$ are assumed to be positive, the error response probability contrast is superadditive.

Assuming the probability summation model, the error response probability contrast for the "no" response in the AND task is derived as follows:

$P(\text{No} \mid B(l),T(l)) - P(\text{No} \mid B(l),T(h)) - P(\text{No} \mid B(h),T(l)) + P(\text{No} \mid B(h),T(h))$

$= P(\text{No} \mid B(l)) + P(\text{No} \mid T(l)) - P(\text{No} \mid B(l)) \times P(\text{No} \mid T(l))$

$- P(\text{No} \mid B(l)) - P(\text{No} \mid T(h)) + P(\text{No} \mid B(l)) \times P(\text{No} \mid T(h))$

$$- P(No \mid B(h)) - P(No \mid T(l)) + P(No \mid B(h)) \times P(No \mid T(l))$$

$$+ P(No \mid B(h)) + P(No \mid T(h)) - P(No \mid B(h)) \times P(No \mid T(h))$$

$$= - P(No \mid B(l)) \times P(No \mid T(l)) + P(No \mid B(l)) \times P(No \mid T(h))$$

$$+ P(No \mid B(h)) \times P(No \mid T(l)) - P(No \mid B(h)) \times P(No \mid T(h))$$

$$= - P(W_{B(l)} > t_{cB}) \times P(W_{T(l)} > t_{cT}) + P(W_{B(l)} > t_{cB}) \times P(W_{T(h)} > t_{cT})$$

$$+ P(W_{B(h)} > t_{cB}) \times P(W_{T(l)} > t_{cT}) - P(W_{B(h)} > t_{cB}) \times P(W_{T(h)} > t_{cT})$$

$$= - \{P(W_{B(l)} > t_{cB}) - P(W_{B(h)} > t_{cB})\} \times \{P(W_{T(l)} > t_{cT}) - P(W_{T(h)} > t_{cT})\}$$

Since both $P(W_{B(l)} > t_{cB}) - P(W_{B(h)} > t_{cB})$ and $P(W_{T(l)} > t_{cT}) - P(W_{T(h)} > t_{cT})$ are assumed to be positive, the error response probability contrast is subadditive.

The error response probability for the OR task and the AND task with all error response probabilities across subjects were pooled to obtain four components of the error response probability contrast. For the OR task the error response probability contrast was found to be superadditive with $z = 1.41$. For the AND task the error response probability contrast was found to be subadditive with $z = -1.69$. The z-score in the OR task was not significant, only at $p < .2$. However, the z-score in the AND task was significant at $p < 0.05$. Even though one of the statistical tests was not significant, these results on the error response probability contrasts seem to support the interpretation of the reaction time results given before.

Non-Monotonicity in mean RTs

Next, I will conduct an analysis of the obtained data in the RT experiments in order to examine the possibly dependent nature of the processing times. As I have pointed out in the AND task RT experiment's results, there were two subjects out of six subjects who showed non-monotonicity of mean RTs with respect to two experimental factors. Only one of them turned out to show significant violations of monotonicity with respect to the mean RTs. Subject BK showed the reverse order of mean RTs between (0,0) and (1,0) conditions. However, the difference, $\overline{RT}(0,0) - \overline{RT}(1,0) = -13$ msec, was not significant: $z = -0.78$. Subject SK also showed the reverse order of mean RTs between (0,0) and (0,1) in addition to the reverse order of mean RTs between (0,0) and (1,0). In this case both violations, $\overline{RT}(0,0)$ - $\overline{RT}(0,1)$ = -17 msec and $\overline{RT}(0,0)$ - $\overline{RT}(1,0)$ = -30 msec, turned out to be significant: z=-2.05 for the difference between (0,0) and (0,1) and z=-3.75 for the difference between (0,0) and (1,0). The significant violations of the monotonicity in terms of mean RTs cannot be easily reconciled with the independent parallel exhaustive processing model unless we assume violations of some other assumptions such as selective influence. However, if we are willing to relax the assumption of stochastic independence, we may be able to explain the non-monotonic results from the subject SK in the AND task. Townsend and Thomas (1991) showed that negative dependence between two total completion times results in a reversal of the factorial effect (see Townsend and Ashby, 1983, p 49 for a definition of total completion time). A reversal of the factorial effect does not happen when two total completion times are positively dependent. Next, I am going to present another way to look at the reversal effect.

My attempt to explain the reversal of the factorial effect can be formulated as follows. Let us define two channels *operating simultaneously with the* processing completion mode being exhaustive: let the actual processing time of the first channel be T_1 and that of the second channel be T_2. If $T_1 < T_2$, then the second stage of processing is devoted to the processing of the second channel. If $T_1 \geq T_2$, then the second stage of processing is devoted to the first channel. Let us suppose that there is a certain amount of interference time (T_i) added to the total processing time depending upon the amount of time difference between actual processing times, $|T_1 - T_2|$. Let us suppose if $|T_1 - T_2|$ is less than t_d, then the integration of the information from two channels will not take an additional amount of time. However, if $|T_1 - T_2|$ is greater or equal to t_d, the integration of the information from two channels will take an additional amount of time, T_i. Then the non-monotonic mean RTs in subject SK may be explained. In order to verify that the model is compatible with the data, I conducted two types of analyses: one utilizes a Thurstonian distance measure between the reaction times of the single stimulus conditions and the reaction times of the double stimulus conditions and the other utilizes the mean reaction time estimates of the heterogeneous intensity condition via two homogeneous intensity conditions. The heterogeneous intensity condition is defined as the stimulus condition under which two levels of different intensities are used: (1,0) and (0,1). The homogeneous intensity condition is defined as the stimulus condition under which two signals are presented with two low intensities or two high intensities. The measures d(S,D), the magnitude of inhibition in (0,1) condition: $\overline{RT}(0,1) - \widehat{RT}$ and the magnitude of inhibition in (1,0) condition: $\overline{RT}(1,0) - \widehat{RT}$ are defined as follows: $d(S,D) = \{ d(N,(0,\emptyset)) + d(N,(1,\emptyset)) + d(N,(\emptyset,0)) + d(N,(\emptyset,1)) \} / 4 - \{ d(N,(0,0)) + d(N,(1,1)) \} / 2$, where $d(N,(0,\emptyset)) = \{\overline{RT}(0,\emptyset) - \overline{RT}(\emptyset,\emptyset)\} / \{Var[RT_N]\}^{1/2}$ and $d(N,(\emptyset,1)) = \{\overline{RT}(\emptyset,1) - \overline{RT}(\emptyset,\emptyset)\} / \{Var[RT_N]\}^{1/2}$. The symbol d(S,D) represents the distance function between the single stimulus conditions and the double stimulus conditions. In order to define $\overline{RT}(0,1) - \widehat{RT}$ and $\overline{RT}(1,0) - \widehat{RT}$, we only need to define $\widehat{RT}$, we write $\widehat{RT} = \int_0^\infty 1 - (F_{(0,0)}(t) F_{(1,1)}(t))^{1/2} dt$. There was no statistical test conducted, however, the results of both analyses seem to suggest that the performance of both subjects BK and SK is distinguishable from other subjects' performance. The results of the analyses seem to be, at least, consistent with the model. That is, non-monotonic mean RTs occurred in the subjects whose values of d(S,D) are smaller in the absolute values and whose magnitudes of inhibition are larger. Further research is necessary for the clarification of this matter.

A Model of Speed-Accuracy Trade-offs

Next I wish to present a model of speed-accuracy trade-offs which can incorporate not only "yes" response RTs but also can incorporate "no" response RTs. This model was already partially presented in the first section of the general discussion in which the error response probability contrast was analyzed. In this section I will first present the model in terms of its defining parameters with its behavior, and next I will apply some pertinent analysis to the obtained data from the point of view of the model. In order to present the model which is capable of explaining both "yes" and "no" responses, I will first consider a part of the model which explains "yes" responses with a single input channel with an infinite observation time. In the case of a "yes" response, there are two important components of the model: one of the components is the counting stochastic process, N(t), to generate the timing responses and the other is the threshold or criterion, k. In terms of speed, as the expected value of the counting stochastic process, E[N(t)], increases the mean RT will decrease. The mean RT increases with an increasing threshold, k. In terms of accuracy, increasing E[N(t)] results in higher accuracy in the signal detection situation. Accuracy also becomes higher if k increases. In the special case, a gamma observer's performance with respect to accuracy (by way of detectability) increases linearly with the rate parameter, E[N(t)], and increases with a square root of the threshold value, k (see Egan, 1975, p213). Next, let us introduce another parameter in the model so that the model can predict "no" responses. This parameter can be characterized as the time criterion, t_c. This time criterion parameter forces the system to respond "yes" within a certain restricted amount of observation time, otherwise the system responds with a "no" response. For the speed of responses, increasing this parameter results in slowing down of the mean RT of "no" responses and also results in slowing down of the mean RT of "yes" responses. If the time criterion parameter decreases, the mean RTs of both "yes" and "no" responses speed up. In terms of accuracy of the responses, decreasing t_c causes a decrease in hit and false alarm probabilities and may cause an introduction of a larger proportion of fast guess responses. Increasing t_c causes both hit and false alarm probabilities to increase. Therefore, the manipulation of t_c from 0 to some finite number can control the detectability of signal from chance level performance to performance associated with the limit of sensory integration. Testing procedures for the above model in the empirical data should be developed in the future.

Convex/Concave Analysis

Next, I wish to extract some common aspects between the results of the RT and psychophysical experiments. In the OR task, both the RT and psychophysical experiments supported the probability summation model at the level of mean response. In the AND task, RT experiment favored the parallel exhaustive processing hypothesis. However, in the psychophysical experiment the channel multiplication model was more appropriate than the parallel exhaustive processing model. Apparently, these two models have different processing mechanisms; the parallel exhaustive processing model takes a minimum of two counting processes to generate its output, whereas the channel multiplication model multiplies two counting processes to yield its output. However, there is a common aspect between the parallel exhaustive processing model and the channel multiplication model. That is both functions in terms of counting process, the minimum of two counting processes and the multiplication of two counting processes, are concave rather than convex (see Dudewicz, 1976, p 196). In contrast to the concavity of the functions in the AND task, that is the minimum function and the multiplication, the processing mode of the OR task seems to be convex. The probability summation model can be represented in the counting domain as the maximum function. The maximum function is convex.

An interesting question to ask is whether convexity (or concavity) of the functions revealed by the subjects' performance is necessitated by the task. My conjecture is that the convexity (or concavity) of the functions revealed by the subjects' performance is caused by the task. For now, I would like to mention the following relations between Boolean variables and some algebraic functions. Given two Boolean variables, X and Y, the logical OR operating on X and Y can be represented as the maximum of X and Y and the logical AND operating on X and Y can be represented as the minimum of X and Y (Schneeweiss, 1989, p134). Another equivalent expression is that the logical OR operating on X and Y can be represented by $X + Y - X Y$ (Boole, 1952, p 91) and the logical AND operating on X and Y can be represented by $X Y$ (Boole, 1854, 1958, p 29). For the logical OR operator we have $f(X,Y)= \max(X,Y)$ and $f(X,Y)= X + Y - X Y$, which are convex and for the logical AND operator we have $g(X,Y) = \min(X,Y)$ and $g(X,Y) = X Y$, which are concave (see Figure 14).

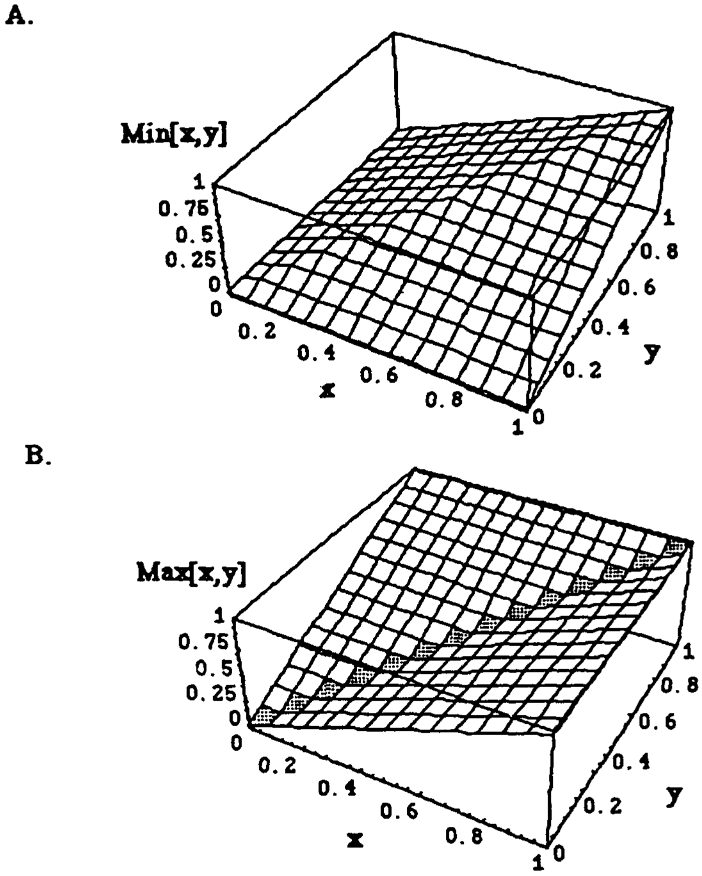


Figure 14. Examples of convex and concave functions: A. Min[x,y] is convex and B. Max[x,y] is concave.

Suppose one is given an exclusive OR (XOR) operator operating on X and Y. That can be represented by X (1-Y) + (1-X) Y = X + Y - 2 X Y. It turns out that it is locally concave in the region where X>Y or X<Y hold. This way of representing the logical operator using algebraic operators can lead to predictions from a large number of logical operations.

It seems to me that the coincidence between the convexity of the OR task and the convexity of the obtained performance in the OR task and that between the concavity of the AND task and the concavity of the obtained performance of the AND task occurred not merely because of chance but because of the adaptability of human behavior to a given problem situation.

Conclusion

RT Experiment

1. In the OR task superadditivity of the mean and the survivor contrasts were obtained. The most compatible model to explain the data was the probability summation model.

2. In the AND task subadditivity of the mean and the survivor contrasts were obtained. The most compatible model to explain the obtained data turned out to be the parallel exhaustive processing model.

Psychophysical Experiment

1.The slope analysis of the standardized ROC curves was conducted. The results clearly reject the timing hypothesis both in the OR task and the AND task. The counting mechanism was found to be compatible with the obtained data in both tasks.

2.In the OR task subadditivity of the mean response contrast was obtained. Further analysis showed that the results of the OR task were compatible with the probability summation model. However, there were two subjects who indicated the possibility of the subadditive channel summation.

3.In the AND task superadditivity of the mean response contrast was obtained. Further analysis showed that the channel multiplication model is rather compatible with the results obtained.

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APPENDIX A

Mean Contrast Additivity of Serial Self-terminating Model

Let us derive mean contrast additivity of the serial self-terminating model. In the derivation, the constant p is defined as the probability of the system picking up channel 1 to process the information. The symbol $f_{1(0)}(t)$ represents the density function of the processing time of channel 1 when the luminance of the target is low. The symbol $f_{2(1)}(t)$ represents the density function of the processing time of channel 2 when the luminance of the target is high. The density representation of the serial self-terminating model in the redundant target condition when both X_1 and X_2 are in the level 0 can be expressed as

$$f_{ST}(t \mid (0,0)) = p \times f_{1(0)}(t) + (1-p) \times f_{2(0)}(t).$$

Similarly for (0,1) condition,

$$f_{ST}(t \mid (0,1)) = p \times f_{1(0)}(t) + (1-p) \times f_{2(1)}(t).$$

For (1,0) condition,

$$f_{ST}(t \mid (1,0)) = p \times f_{1(1)}(t) + (1-p) \times f_{2(0)}(t).$$

And for (1,1) condition,

$$f_{ST}(t \mid (1,1)) = p \times f_{1(1)}(t) + (1-p) \times f_{2(1)}(t).$$

The constant (1-p) is the probability that channel 2 is processed first. The subscript ST represents that the processing completion mode is self-terminating. Putting all of the above expressions together in the mean contrast expression, we have

$$\int_0^\infty t \times \{f_{ST}(t \mid (0,0)) - f_{ST}(t \mid (0,1)) - f_{ST}(t \mid (1,0)) + f_{ST}(t \mid (1,1))\} \, dt$$

$$= \int_0^\infty t \times \{p \times f_{1(0)}(t) + (1\text{-}p) \times f_{2(0)}(t) - p \times f_{1(0)}(t) - (1\text{-}p) \times f_{2(1)}(t)$$

$$- p \times f_{1(1)}(t) - (1\text{-}p) \times f_{2(0)}(t) + p \times f_{1(1)}(t) + (1\text{-}p) \times f_{2(1)}(t) \, \} dt$$

$$=0$$

Additivity holds. Notice that in the serial self-terminating model, we even have additivity at the level of density function. This ends the proof of mean contrast additivity of the serial self-terminating model.

APPENDIX B

Mean Contrast Superadditivity of the Channel Summation model

Let us derive the mean contrast superadditivity of the channel summation model. Define $N_1(t)$ as the neural counting stochastic process for channel 1. And define $N_2(t)$ similarly as the neural counting stochastic process for channel 2. The channel summation model can be defined as follows:

$$N_{sum}(t) = N_1(t) + N_2(t).$$

In order to derive the RT prediction of the channel summation model, we have to relate the two domains of stochastic processes, the counting domain and the waiting time domain. This can be accomplished by a theorem known as the Renewal Theorem (Karling and Taylor, 1975). The renewal theorem relates the processes of counting stochastic events with time, and this is the source of its importance to models of reaction times. We define a renewal process as a sequence of mutually independent random variables which assume nonnegative values (Prabu, 1965). This theorem relates the process of counting stochastic events (i.e., the activity of a channel) to time as follows:

$$P(N(t) \geq k) = P(W_k \leq t).$$

The symbol, $N(t)$, represents a counting process and the symbol, W_k, represents the waiting time process. Intuitively, $N(t) \geq k$ represents the event in which the counting criterion is reached by time t. In terms of time of the response generated, the response has been already generated by time t. A counting process is defined as the stochastic process that results when the time of observation is fixed and the number of counts is studied (p77, Townsend & Ashby, 1983). A waiting time is defined as the time taken in order to observe a fixed amount of counts (p77, Townsend & Ashby, 1983). Thus, if we envision the detection process as accumulating the activity over an input channel, the relationship between the sensory activity and the time needed to detect the signal can be expressed in terms of the renewal theorem. The identity, $P(N(t) \geq k) = P(W_k \leq t)$, tells us that for any renewal stochastic process the probability of counting being equal to or greater than an integer k when the observation time is fixed to "t" is the same as the probability of the k-th event being observed before "t". The importance of this identity lies in the fact that it allows us to derive predictions in terms of a RT measure by knowing the form of the expression in the counting domain. As will become clear later in the thesis, this identity not only allows us to derive RT predictions but also allows us to derive accuracy predictions.

A counting mass function, $P(N(t) = n) = g(n,t)$, tells us the probability of getting n events during an observation interval of duration t. It is defined for the number of counts from zero to infinity. Since it is a probability mass function, the sum across all n from zero to infinity is one. Let $P(N(t) \leq n) = G(n,t)$ be the counting cumulative distribution function, and let X be a parameter of G(n,t) having two levels, 0 and 1, such that $G(n,t;X=0) - G(n,t;X=1) > 0$ for all $t \geq 0$. This means that the activity of the counting process is greater when the counting process is driven by a high intensity stimulus, X=1, than when the counting process is driven by a low intensity stimulus, X=0. Now we can express the mean contrast of the channel summation model as follows.

$$\overline{RT}(0,0) - \overline{RT}(0,1) - \overline{RT}(1,0) + \overline{RT}(1,1)$$

$$= \int_0^\infty \overline{F}_{(0,0)}(t) - \overline{F}_{(0,1)}(t) - \overline{F}_{(1,0)}(t) + \overline{F}_{(1,1)}(t)\, dt$$

$$= \int_0^\infty P(W_{sum,k|(0,0)} > t) - P(W_{sum,k|(0,1)} > t) - P(W_{sum,k|(1,0)} > t) + P(W_{sum,k|(1,1)} > t)\, dt$$

By applying the renewal theorem, we have

$$= \int_0^\infty P(N_{sum|(0,0)}(t) < k) - P(N_{sum|(0,1)}(t) < k) - P(N_{sum|(1,0)}(t) < k) + P(N_{sum|(1,1)}(t) < k)\, dt.$$

$$= \int_0^\infty P(N_{1(0)}(t) + N_{2(0)}(t) < k) - P(N_{1(0)}(t) + N_{2(1)}(t) < k) - P(N_{1(1)}(t) + N_{2(0)}(t) < k)$$

$$+ P(N_{1(1)}(t) + N_{2(1)}(t) < k)\ dt$$

This expression was proven to be positive for the special case of the Poisson channel summation model.

In order to prove the superadditivity of the Poisson channel summation model, first let us define λ_{ll} as the rate of the Poisson process associated with the condition in which a target (target 1) is dim and the other target (target 2) is dim, define λ_{lh} as the rate of the Poisson process associated with the condition in which target 1 is dim and the other target target 2 is bright, define λ_{hl} as the rate of the Poisson process associated with the condition in which target 1 is bright and target 2 is dim, and define λ_{hh} as the rate of the Poisson process associated with the condition in which targets 1 and 2 are bright. Since we assume selective influence and stochastic independence, we can express the redundant target rates as the sum of individual processing rates. We have

$\lambda_{ll}=\lambda_{l-}+\lambda_{-l}$,

$\lambda_{lh}=\lambda_{l-}+\lambda_{-h}$,

$\lambda_{hl}=\lambda_{h-}+\lambda_{-l}$,

and

$\lambda_{hh}=\lambda_{h-}+\lambda_{-h}$.

Then we can express the mean contrast of the Poisson channel summation model ,assuming a fixed criterion k for all four conditions, as follows.

$$\Delta^2\ \overline{RT} = k/\lambda_{ll} - k/\lambda_{lh} - k/\lambda_{hl} + k/\lambda_{hh}$$

$$= k/(\lambda_{l-}+\lambda_{-l}) - k/(\lambda_{l-}+\lambda_{-h}) - k/(\lambda_{h-}+\lambda_{-l}) + k/(\lambda_{h-}+\lambda_{-h})$$

$$= k\{(\lambda_{-h}-\lambda_{-l})/((\lambda_{l-}+\lambda_{-l})\ (\lambda_{l-}+\lambda_{-h})) - (\lambda_{-h}-\lambda_{-l})/((\lambda_{h-}+\lambda_{-l})\ (\lambda_{h-}+\lambda_{-h}))\}$$

$$= k(\lambda_{-h}-\lambda_{-l})\{[(\lambda_{h-}+\lambda_{-l})\,(\lambda_{h-}+\lambda_{-h})-(\lambda_{l-}+\lambda_{-l})\,(\lambda_{l-}+\lambda_{-h})]\,/\,[(\lambda_{l-}+\lambda_{-l})\,(\lambda_{l-}+\lambda_{-h})\,(\lambda_{h-}+\lambda_{-l})\,(\lambda_{h-}+\lambda_{-h})]\}$$

We have $\lambda_{-h} > \lambda_{-l}$ and $\lambda_{h-} > \lambda_{l-}$ as a result of survivor ordering assumption. Therefore, we have $(\lambda_{-h}-\lambda_{-l})\{[(\lambda_{h-}+\lambda_{-l})\,(\lambda_{h-}+\lambda_{-h})-(\lambda_{l-}+\lambda_{-l})\,(\lambda_{l-}+\lambda_{-h}))] > 0$.

This proves superadditivity of the Poisson channel summation model.

APPENDIX C

Mean RTs, variance RTs and error probabilities of various conditions in RT experiment.

Mean reaction times, variance reaction times, and error percentages in conditions under which a bottom dim dot and a top dim dot are presented (0,0), a dim at the bottom and a bright dot on the top are presented (0,1), a bright dot at the bottom and a dim dot on the top are presented (1,0), a bright dot at the bottom and a bright dot on the bottom are presented (1,1), only a dim dot at the bottom is presented (0,∅), only bright dot at the bottom is presented (1,∅), only a dim dot on the top is presented (∅,0), only a bright dot on the top is presented (∅,1), and nothing is presented (∅,∅).

OR task

Subject 1

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	336	1945	0.065
	(0,1)	290	1253	0.025
	(1,0)	286	1253	0.027
	(1,1)	284	1082	0.013
Bottom Only	(0,∅)	340	2162	0.109
	(1,∅)	283	870	0.013
Top Only	(∅,0)	365	3564	0.373
	(∅,1)	369	1048	0.007
Noise	(∅,∅)	379	1340	0.019

Subject 2

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	335	6123	0.083
	(0,1)	271	1849	0.000
	(1,0)	265	778	0.000
	(1,1)	266	1901	0.000
Bottom Only	(0,∅)	337	6191	0.091
	(1,∅)	270	1356	0.013
Top Only	(∅,0)	335	4476	0.151
	(∅,1)	272	1610	0.006
Noise	(∅,∅)	406	5667	0.020

Subject 3

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	284	1204	0.044
	(0,1)	237	303	0.000
	(1,0)	242	729	0.023
	(1,1)	231	335	0.000
Bottom Only	(0,∅)	307	2098	0.057
	(1,∅)	238	543	0.001
Top Only	(∅,0)	319	1806	0.078
	(∅,1)	238	313	0.000
Noise	(∅,∅)	441	3329	0.008

Subject 4

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	501	7208	0.000
	(0,1)	425	5914	0.000
	(1,0)	418	4409	0.000
	(1,1)	407	2134	0.000
Bottom Only	(0,∅)	533	15775	0.113
	(1,∅)	450	13924	0.000
Top Only	(∅,0)	580	14957	0.171
	(∅,1)	430	8336	0.000
Noise	(∅,∅)	517	16205	0.016

Subject 5

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	420	2190	0.000
	(0,1)	381	2642	0.000
	(1,0)	367	1986	0.031
	(1,1)	361	1489	0.032
Bottom Only	(0,Ø)	433	3654	0.023
	(1,Ø)	373	2237	0.008
Top Only	(Ø,0)	475	4182	0.031
	(Ø,1)	382	2934	0.008
Noise	(Ø,Ø)	465	5242	0.013

Subject 6

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	378	6131	0.000
	(0,1)	332	8742	0.000
	(1,0)	317	4998	0.000
	(1,1)	297	2992	0.000
Bottom Only	(0,Ø)	389	7106	0.068
	(1,Ø)	317	4422	0.000
Top Only	(Ø,0)	410	5329	0.013
	(Ø,1)	319	3795	0.007
Noise	(Ø,Ø)	795	42341	0.008

AND task

Subject 1

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	406	1531	0.150
	(0,1)	400	903	0.041
	(1,0)	402	1809	0.147
	(1,1)	334	969	0.000
Bottom Only	(0,∅)	440	2663	0.020
	(1,∅)	411	973	0.007
Top Only	(∅,0)	394	1815	0.007
	(∅,1)	400	692	0.000
Noise	(∅,∅)	369	1187	0.000

Subject 2

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	390	4204	0.526
	(0,1)	382	3793	0.350
	(1,0)	403	9497	0.286
	(1,1)	353	2777	0.158
Bottom Only	(0,∅)	337	6191	0.091
	(1,∅)	270	1356	0.013
Top Only	(∅,0)	335	4476	0.151
	(∅,1)	272	1610	0.006
Noise	(∅,∅)	406	5667	0.019

Subject 3

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	378	2294	0.045
	(0,1)	366	4984	0.067
	(1,0)	374	3869	0.156
	(1,1)	300	1414	0.022
Bottom Only	(0,∅)	512	5314	0.000
	(1,∅)	473	4886	0.022
Top Only	(∅,0)	489	5670	0.000
	(∅,1)	463	4173	0.011
Noise	(∅,∅)	419	3317	0.008

Subject 4

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	758	31364	0.053
	(0,1)	677	20535	0.056
	(1,0)	706	40885	0.056
	(1,1)	565	5791	0.000
Bottom Only	(0,∅)	898	50625	0.000
	(1,∅)	774	45454	0.007
Top Only	(∅,0)	828	44521	0.007
	(∅,1)	790	54149	0.000
Noise	(∅,∅)	653	50310	0.000

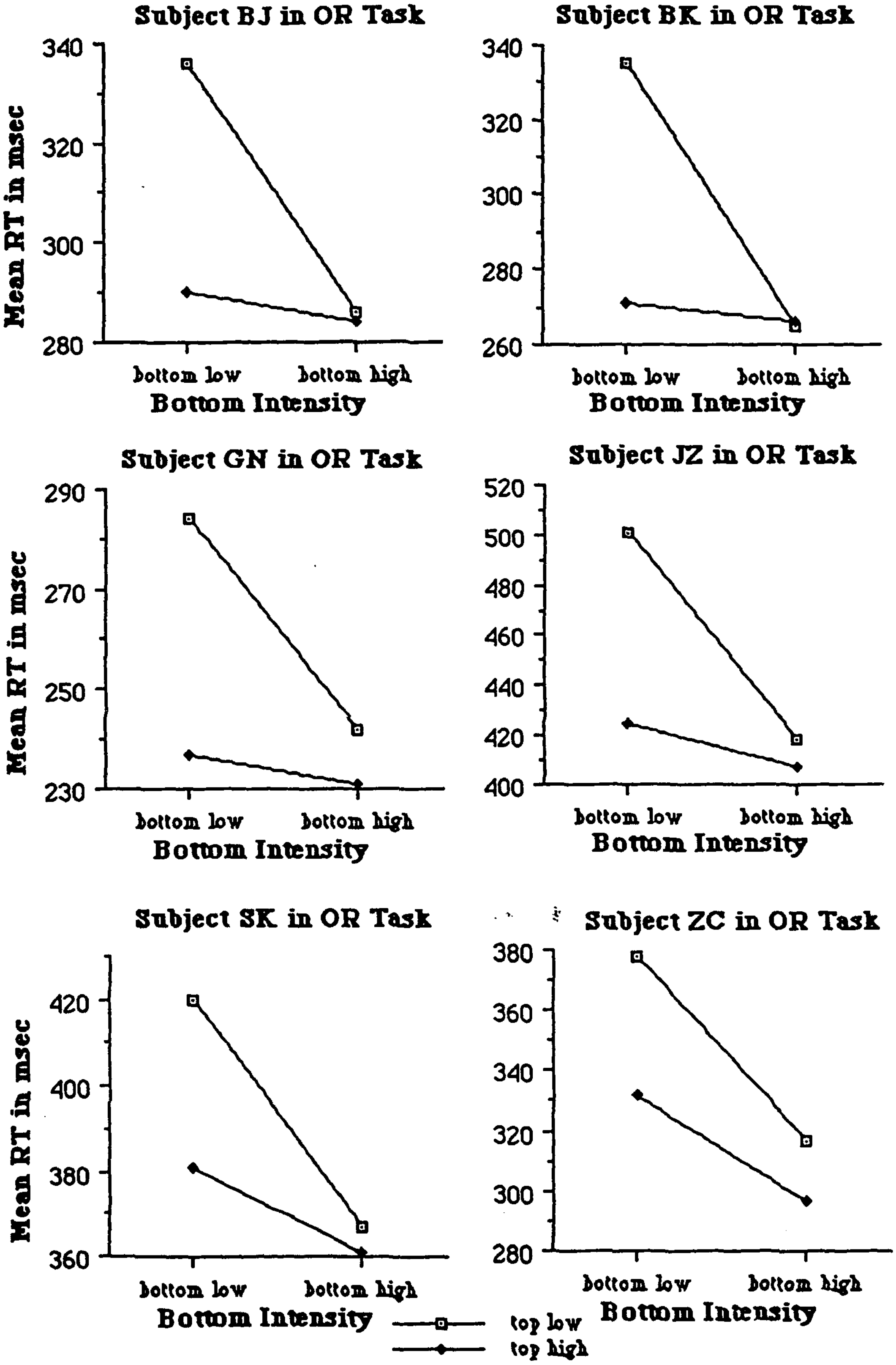
Subject 5

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	422	2304	0.082
	(0,1)	439	2391	0.013
	(1,0)	453	2209	0.053
	(1,1)	363	2190	0.000
Bottom Only	(0,∅)	458	8046	0.002
	(1,∅)	476	2862	0.008
Top Only	(∅,0)	435	3036	0.000
	(∅,1)	315	412	0.000
Noise	(∅,∅)	390	2478	0.002

Subject 6

	Condition	Mean RT	Variance RT	Error Probability
Double	(0,0)	476	7448	0.000
	(0,1)	450	6068	0.038
	(1,0)	468	4610	0.074
	(1,1)	397	4610	0.000
Bottom Only	(0,∅)	542	9120	0.000
	(1,∅)	519	5027	0.028
Top Only	(∅,0)	542	9235	0.000
	(∅,1)	520	6839	0.010
Noise	(∅,∅)	548	15972	0.025

APPENDIX D



Subject BJ in OR Task

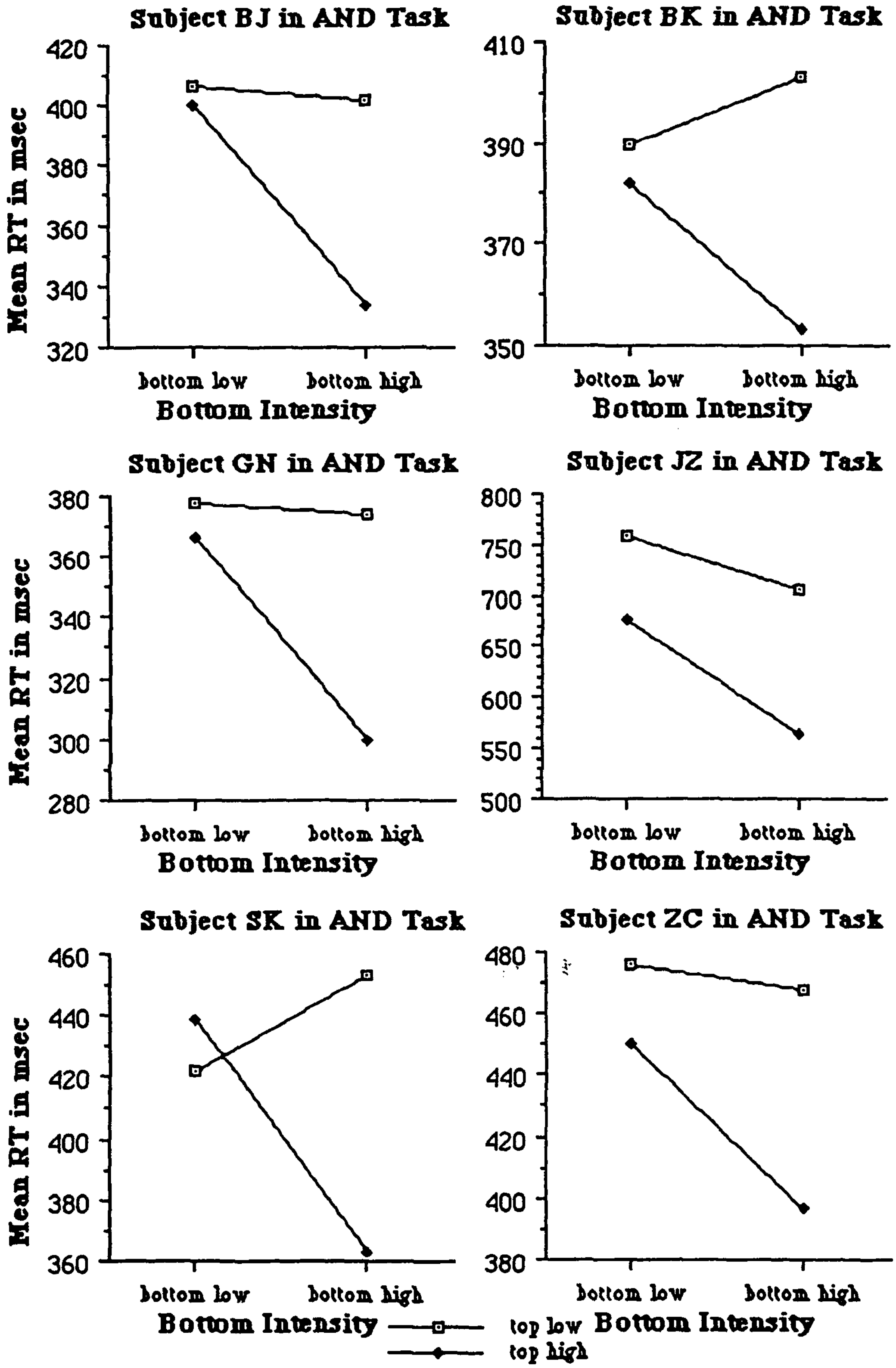
Subject BK in OR Task

Subject GN in OR Task

Subject JZ in OR Task

Subject SK in OR Task

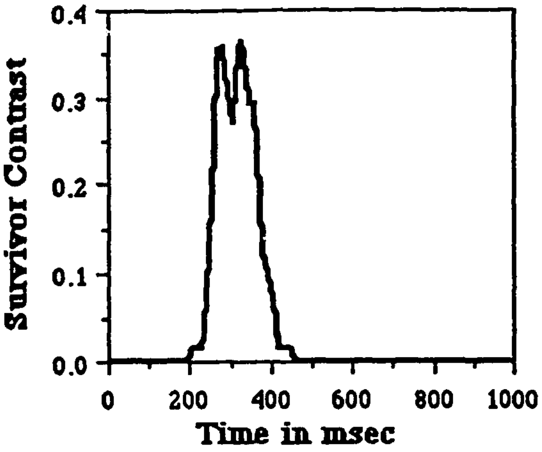
Subject ZC in OR Task



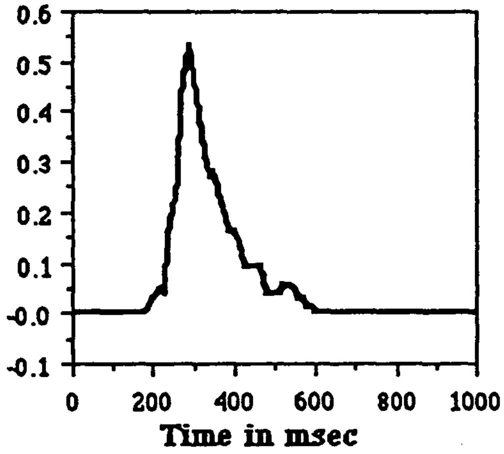
Mean reaction times of the double stimulus conditions in the OR task and the AND task.

APPENDIX E

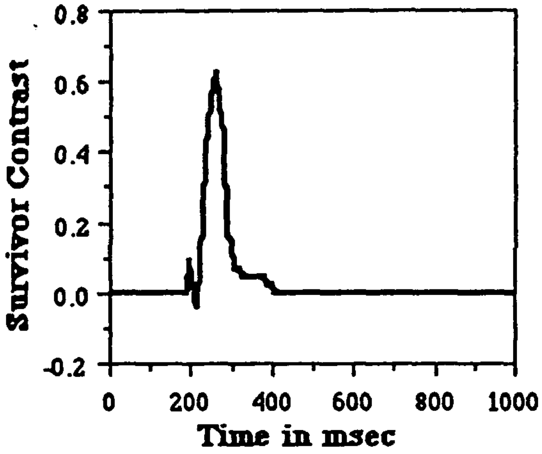
Subject BJ in OR Task



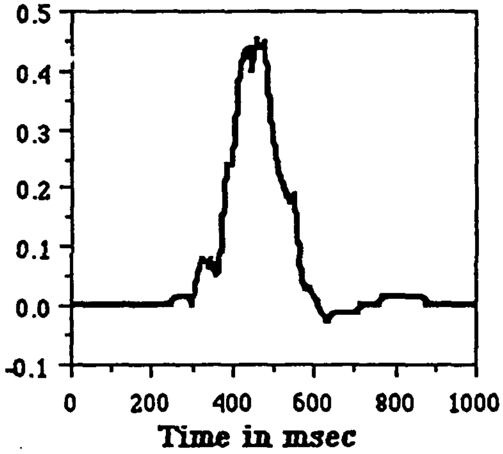
Subject BK in OR Task



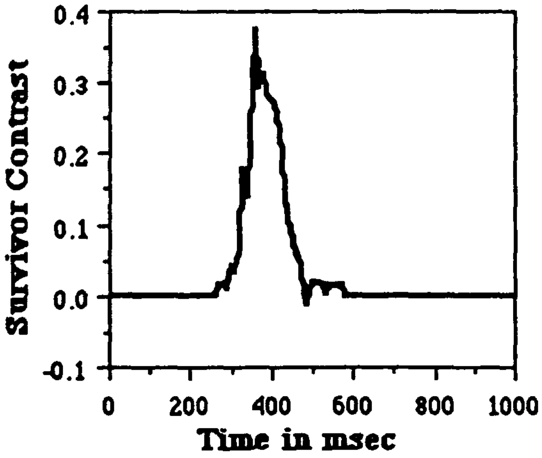
Subject GN in OR Task



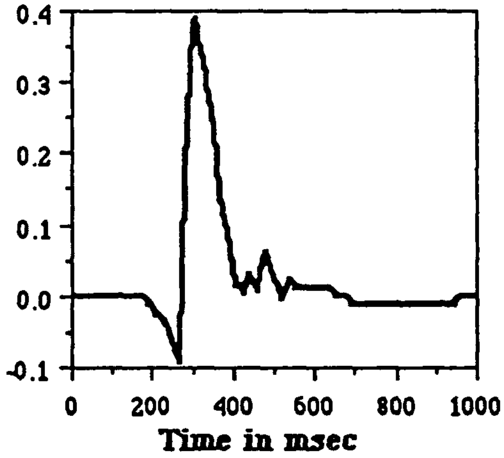
Subject JZ in OR Task

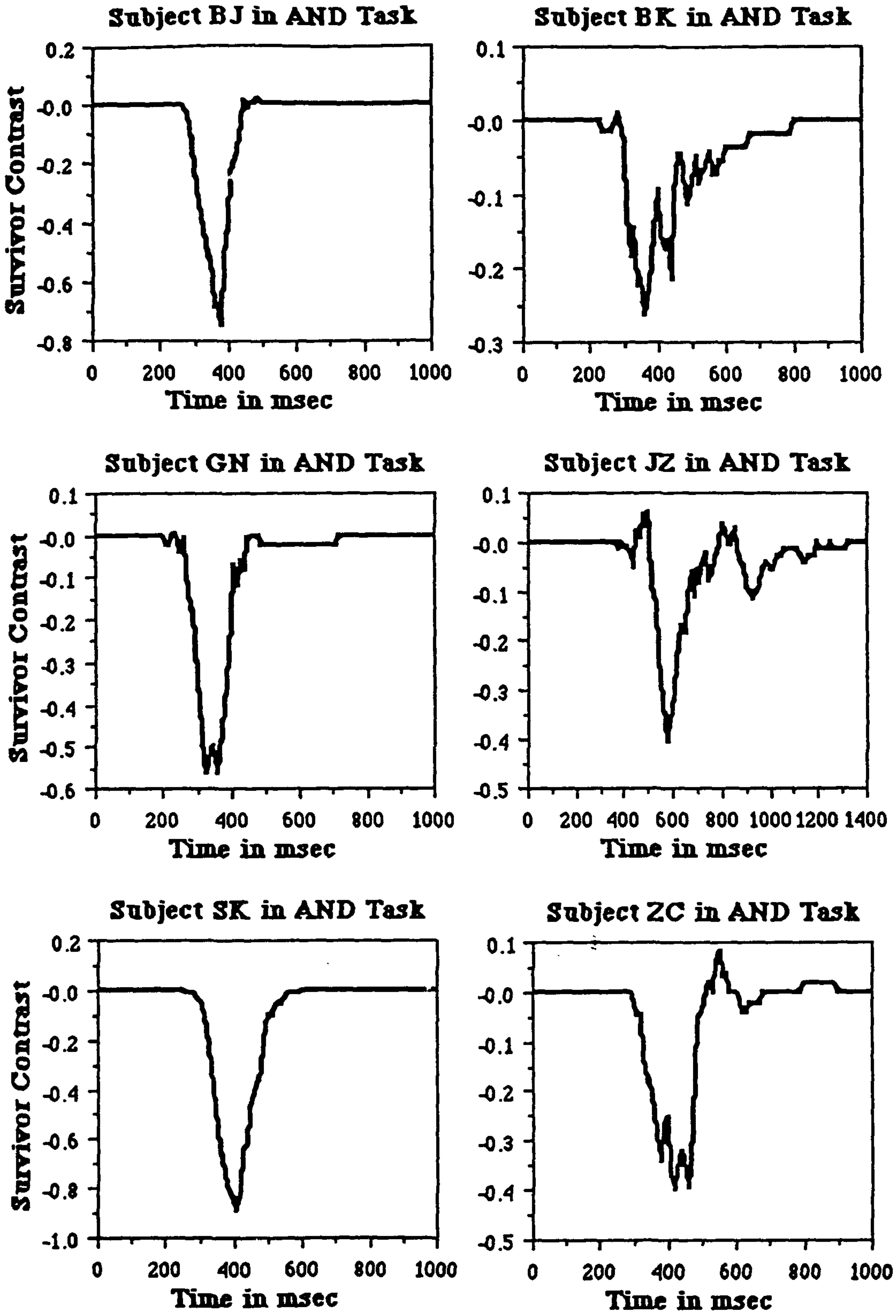


Subject SK in OR Task



Subject ZC in OR Task





Survivor contrasts of the double stimulus conditions in the OR task and the AND task.

APPENDIX F

The ROC curve constructing technique used in the signal detection experiment.

In this appendix, I am going to explain the ROC curve construction technique used in the signal detection experiment. The raw data come in the form of confidence ratings distributed along the confidence rating scale. First, define histograms from the noise condition as P(CR=cr(i)|(Ø,Ø)), the bottom target only condition as P(CR=cr(i)|(Ø,B)), the top target only condition as P(CR=cr(i)|(T,Ø)) and the double target condition as P(CR=cr(i)|(T,B)), where cr(i) is the i-th point on the confidence scale. Since there were 5 confidence rating intervals, from the most certain to the most uncertain, for each response in the experiment, there were total of 10 intervals on the confidence rating scale. Let us assign the most certain "no" response category the first interval of the confidence rating scale and the most uncertain "no" response category the fifth interval. Similarly, we can assign the most uncertain "yes" response category the number "6" and the most certain "yes" response category the number "10." Therefore, cr(i), runs from 1 to 10 on the confidence rating scale.

The next step in constructing an ROC curve is to sum the histogram along the confidence rating scale. After this operation, we have what we call the cumulative distribution function: P(CR≤cr(i)). The next step is to calculate P(CR>cr(i)), the survivor function, which is one minus the quantity of the cumulative distribution function. Notice that there are four types of survivor functions associated with four different stimulus conditions in the present experiment. Plot the survivor function of the noise condition on the x-axis and that of the signal-plus-noise condition on the y-axis. This produces the ROC curve.

Traditionally, the ROC curve was created using the "yes" responses alone on both the noise condition and the signal-plus-noise condition. However, the procedure of utilizing the "yes" responses only can potentially lead to a situation in which not enough false alarm responses are observed. This can cause a problem to estimate an ROC curve because the estimate of the curve does not span zero to one completely. However, if we utilize the confidence rating of all of the points on the scale, we may be able to increase the number of points. Therefore, the procedure of utilizing all of the points tend to generate relatively stable estimates of the ROC curves.

APPENDIX G

Subject/Condition	Mean	Variance
GN OR noise	2.59	4.60
GN OR B	7.34	9.62
GN OR T	7.08	12.70
GN OR B&T	9.26	2.36
GT OR noise	2.81	2.97
GT OR B	5.77	8.60
GT OR T	6.62	8.84
GT OR B&T	7.18	8.05
HH OR noise	4.11	4.60
HH OR B	6.79	8.53
HH OR T	7.81	5.42
HH OR B&T	8.70	3.11
JM OR noise	2.37	2.51
JM OR B	8.37	4.32
JM OR T	7.75	6.34
JM OR B&T	9.06	4.45
MM OR noise	1.23	0.61
MM OR B	7.02	4.63
MM OR T	6.51	5.54
MM OR B&T	9.16	1.24
OO OR noise	2.58	1.33
OO OR B	5.37	5.52
OO OR T	5.19	5.01
OO OR B&T	6.23	4.78

Subject/Condition	Mean	Variance
GN AND noise	1.51	1.11
GN AND B	3.25	5.16
GN AND T	2.89	4.27
GN AND B&T	7.70	8.45
GT AND noise	2.17	1.47
GT AND B	2.45	1.71
GT AND T	4.08	3.76
GT AND B&T	6.49	7.31
HH AND noise	2.81	2.93
HH AND B	4.76	4.57
HH AND T	4.41	3.59
HH AND B&T	6.74	6.36
JM AND noise	1.56	1.06
JM AND B	3.59	2.57
JM AND T	2.95	1.87
JM AND B&T	8.23	3.74
MM AND noise	1.30	0.63
MM AND B	2.93	1.89
MM AND T	3.12	3.12
MM AND B&T	8.07	2.17
OO AND noise	2.39	1.00
OO AND B	3.01	1.71
OO AND T	2.79	1.38
OO AND B&T	5.02	5.65

Means and variances of the responses in the OR task and the AND task of the signal detectability experiments.

APPENDIX H

The method for exact prediction of the probability summation model, the channel summation model, the multiplication model and the parallel exhaustive model with respect to the mean.

Before turning to the exact prediction of various models, I will first present an approximate prediction of the probability summation model and the parallel exhaustive processing model. The first application of the probability summation model can be traced back to Pirenne (1943). The approximate prediction of the probability summation model in the bimodal detection was predicted from the unimodal conditions by, for example, Morton (1967) and Sanford (1970) as follows:

$$p[Bim] = p[Aud] + p[Vis] - p[Aud]\, p[Vis],$$

where p[Bim] represents the probability of detection under the bimodal condition, p[Aud] represents the probability of detection under the auditory condition and p[Vis] represents the probability of detection under the visual condition. The above formulation neglects the effect of the noise in estimating the single modality's real detection probability. To put it in another way, the above formulation equates the detection probability produced by ,for example, the auditory channel with the detection probability obtained under the auditory signal plus visual noise condition. This will overestimate the probability of detection by the single channel, therefore, yielding underestimated values for the bimodal detection probability. The exact prediction of the probability summation model in the context of my experiment is in order. In the following derivation, three important assumptions are made: 1) stochastic independence, 2) selective influence and 3) context independence. Under these assumptions, the probability of a miss in the double stimulus condition can be written as

$$\begin{aligned} P[\text{Miss}| \text{ Bottom and Top Signals}] &= P[\text{Miss}| \text{ Bottom Signal plus Top Noise}] \\ &\times P[\text{Miss}|\text{Top Signal plus Bottom Noise}] \\ &/ \ P[\text{Miss}|\text{Top Noise plus Bottom Noise}]. \end{aligned}$$

Generalizing to an arbitrary point on a confidence rating scale, we have

$$\begin{aligned} P[CR < cr(i)| \text{ Bottom and Top Signals}] &= P[CR < cr(i)| \text{ Bottom Signal plus Top Noise}] \\ &\times P[CR < cr(i)|\text{Top Signal plus Bottom Noise}] \\ &/ \ P[CR < cr(i)|\text{Top Noise plus Bottom Noise}]. \end{aligned}$$

Therefore, we have

$$P[CR > cr\ (i),\text{probability summation}|(B,T)]$$
$$= 1 - (P[CR < cr\ (i)|(B\varnothing)] \times P[CR < cr\ (i)|(\varnothing,T)]) \ / P[CR < cr\ (i)|(\varnothing,\varnothing)] \ .$$

The division by the term, $P[CR < cr\ (i)|(\varnothing,\varnothing)]$, nullifies the overestimation. The notation, $CR > cr\ (i)$, means that the obtained confidence rating is greater than the i-th confidence value.

Next, I present an approximate prediction of the parallel exhaustive processing model and the exact prediction of the same model.

An approximate prediction:

$$P[CR > cr\ (i),\text{parallel exhaustive processing}|(B,T)]$$
$$= P[CR > cr\ (i)|(B,\varnothing)] \times P[CR > cr\ (i)|(\varnothing,T)]$$

The exact prediction of the parallel exhaustive processing model assuming stochastic independence is written

$$P[CR > cr\ (i),\text{parallel exhaustive processing}|(B,T)] =$$
$$P[CR > cr\ (i)|(B,\varnothing)] \times P[CR > cr\ (i)|(\varnothing,T)] \ ) \ / \ P[CR > cr\ (i)|(\varnothing,\varnothing)]$$

In order to derive the prediction at the mean level, the above survivor functions should be summed across all the confidence rating values. In the case of the parallel exhaustive processing model, the bias of the approximate prediction is toward underestimation of the probability that the confidence rating exceeds a certain value on the confidence rating scale. For the exact prediction of the probability summation model, the expected value is written

E[CR, probability summation|(B,T)]

$$= \sum_{i=1}^{\infty} P[CR > cr\ (i), \text{probability summation}|(B,T)]$$

$$= \sum_{i=1}^{\infty} \left\{ 1 - (P[CR < cr\ (i)|(B\varnothing)] \times P[CR < cr\ (i)|(\varnothing,T)]) \ / \ P[CR < cr\ (i)|(\varnothing,\varnothing)] \right\}$$

For the exact prediction of the parallel exhaustive processing model, the expected value is written

E[CR, parallel exhaustive processing|(B,T)]

$$= \sum_{i=1}^{\infty} P[CR > cr\ (i), \text{parallel exhaustive processing}|(B,T)]$$

$$\sum_{i=1}^{\infty} \ P[CR > cr\ (i)|(B,\varnothing)] \times P[CR > cr\ (i)|(\varnothing,T)]\ ) \ / \ P[CR > cr\ (i)|(\varnothing,\varnothing)]$$

The expected values of the approximate predictions can be derived in a similar way.

Let us turn our attention to the exact prediction of the channel summation model and the channel multiplication model in terms of the expected value. The expected value of the channel summation model can be written as

E[CR, channel summation|(B,T)] = E[CR, channel summation|(B,∅)] + E[CR, channel summation|(∅,T)] - E[CR, channel summation|(∅,∅)].

The expected value of the channel multiplication model can be written as

E[CR, channel multiplication|(B,T)] = E[CR, channel multiplication|(B,∅)] × E[CR, channel multiplication|(∅,T)] / E[CR, channel multiplication|(∅,∅)].