# WORKSTATION TOOLS FOR FEATURE EXTRACTION AND CLASSIFICATION FOR NONSTATIONARY AND TRANSIENT SIGNALS

July 30, 1992

Final Report on Contract N00014-89-C-0310



Submitted to The Office of Naval Research
Arlington, Virginia



by

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APPROVAL

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NWW 8/28/92

## 1. Introduction

The purpose of this report is to document the software tool developments and research findings made by Harry L. Hurd Assoc. during the period October 1, 1989 through May 31, 1992 under ONR contract N00014-89-C-0310. This contract was a Phase II SBIR award. This contract resulted from theoretical and simulation efforts performed under ONR contract N00014-86-C-0227 and software concept demonstrations provided under an ONR Phase I SBIR award N00014-88-C-0683.

The primary technical topic addressed by this work is that of nonstationary stochastic processes. During the past few years, a deeper understanding has been aquired concerning the nature of certain types of nonstationary processes. Since the number of ways that a process can be nonstationary are countless, particular classes of nonstationary processes have been investigated. Some of the classes are: *harmonizable, periodically correlated (cyclostationary), almost periodically correlated, periodically unitary, almost periodically unitary and evolutionary.* Specific examples of nonstationary processes of interest to the Navy include the radiated acoustic signals produced by cavitating propellers and other rotating machinery, vibration signals from turbines or gearboxes, meteorological processes (exhibiting daily or yearly nonstationary variations) and transient acoustic signals produced by a variety of sources. Nonstationary processes arise in the communications context where signals are often given a cyclostationary structure in order to facilitate the recovery of timing information from the received signal. Another class of nonstationary processes is given by the class of transient signals. It has also been determined that transient signals often produce high levels of spectral coherence, and the mathematical reason for this observation is understood.

This effort was motivated by the recognition that the solution of technical problems involving nonstationary processes will require a body of theory and practical tools that facilitate observing and characterizing various types of nonstationary processes. Experimental tools in today's environment may be taken to mean software tools. Therefore, this effort has addressed (1) continued research on theoretical issues concerning nonstationary processes and (2) the refinement and development of experimental software tools that will facilitate the investigation and characterization of nonstationary (including transient) data.

The remainder of this report contains (Sec. 2) a sketch of the theoretical background that motivates our experimental methodology, a summary (Sec. 3) of new theoretical results funded or partially funded by this contract, a high-level description (Sec. 4) of the workstation software package, called DSCOH, that was developed under this contract. In addition, the use of this software by other Navy labs and projects (Sec. 5), and the plan for commercialization (Sec. 6) will be briefly discussed.

## 2. Theoretical Background for Spectral Coherence

Our general approach to the development of statistical analysis tools and algorithms for nonstationary and transient processes is based on the theory of harmonizable stochastic processes[5]. The primary idea is that the random amplitudes in the Fourier decompositions of stationary processes are uncorrelated or orthogonal. In contrast, the random amplitudes in the Fourier decompositions of harmonizable nonstationary processes are not generally orthogonal in the mean-square sense. It is clear that there may be many ways in which the random amplitudes will not be orthogonal and hence many ways in which a process may be nonstationary and yet have some particular organization in the correlation of these random amplitudes. We now discuss a little more precisely the notion of harmonizable processes.

A second order stochastic process X(t) is called harmonizable [5] if it can be represented as a Fourier integral with respect to a random measure Z; that is

$$X(t) = \int_{-\infty}^{\infty} \exp(i\lambda t)Z(d\lambda) \tag{1}$$

where the integral is in the sense of quadratic mean. Although there are weaker notions of harmonizability, here we refer to the strong notion for which the spectral distribution

$$r_Z([a,b)\times[c,d)) = E\{[Z(b)-Z(a)]\overline{[Z(d)-Z(c)]}\} \tag{2}$$

is of bounded variation on $\mathbb{R}\times\mathbb{R}$. We express this condition by

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |r_Z(d\lambda_1,d\lambda_2)| < \infty \tag{3}$$

and it follows that the covariance function is given by

$$R(s,t) = E\{X(s)\overline{X(t)}\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(is\lambda_1-it\lambda_2)r_Z(d\lambda_1,d\lambda_2). \tag{4}$$

A second order stochastic process X(t) is said to be (wide-sense) stationary when the covariance function depends only on the lag, i.e.,

$$R(s,t) = R(s-t) \quad \text{for all s and t.} \tag{5}$$

Every mean square continuous stationary process has representation (1) where the random measure Z is orthogonally scattered (Z has orthogonal increments) and this will occur if and only if $r_Z$ concentrates on the diagonal. So X(t) being harmonizable but nonstationary means that X(t) has a decomposition in terms of complex exponentials $\exp(i\lambda t)$ but the increments of Z are no longer orthogonal. In this sense harmonizable processes provide a natural framework for the investigation of processes having spectral correlation.

A second order stochastic process X(t) is said to be periodically correlated (PC)

[1],[2],[6] with period $T < \infty$ when

$$R(s,t) = R(s+T,\ t+T) \quad \text{for all s and t.} \tag{6}$$

If X is harmonizable, then X is periodically correlated with period T if and only if the support of $r_Z$ is contained in the set $S = \bigcup_k S_k$ where $S_k = \{(\lambda_1,\lambda_2): \lambda_2 = \lambda_1 - 2\pi k/T\}$, a set of lines parallel to and including the main diagonal as illustrated in Figure 1. (See [6] for a review of the properties of harmonizable PC processes.) Similarly, X is almost periodically correlated if and only if the support of $r_Z$ is contained in $S = \{(\lambda_1,\lambda_2): \lambda_2 = \lambda_1 - \gamma,\ \gamma \in \Lambda\}$, $\Lambda$ a countable set of real numbers [R2].

So we see that a particular organization of the random amplitudes is that associated with periodically correlated and almost periodically correlated processes. To paraphrase the more mathematical statement above, a harmonizable process is periodically correlated if and only if the complex Fourier coefficients having constant difference frequencies have non-zero correlation. The pairs of frequencies having constant differences are specified by a diagonal line in a two-dimensional or bi-frequency plane, as illustrated in Figure 1. The computation of *diagonal* spectral coherence looks for this particular coherence structure between the complex Fourier coefficients (amplitudes) arising from discrete Fourier transforms made on experimental time series.

## 2.1 The Spectral Coherence Computation

We shall now summarize the computation of empirical spectral coherence (see [R1]) used thus far in this work.

As background for our discussion of spectral coherence we review the notion of coherence for complex Gaussian random variables. Suppose $\{U_m, m=1,M\}$ and $\{V_m, m=1,M\}$ are each sequences of independent, complex zero mean Gaussian random variables with $E\{|U_m|^2\}=\sigma_U^2$, $E\{|V_m|^2\}=\sigma_V^2$ and $E\{U_m\overline{V_m}\}=\sigma_{UV}$. If the values of $\sigma_U^2$ and $\sigma_V^2$ are known, then a test for $\sigma_{UV} > 0$ can be constructed from the sample correlation

$$\hat{\sigma}_{UV} = \frac{1}{M}\sum_{m=1}^{M} U_m\overline{V_m}. \tag{7}$$

When $\sigma_U$ and $\sigma_V$ are unknown, the normalized complex correlation, called coherence, $\gamma = \sigma_{UV}/\sigma_U\sigma_V$ may be used. The usual estimator for $\gamma$ (actually $|\gamma|^2$) is given by

$$|\hat{\gamma}|^2 = \frac{\left|\sum_{m=1}^{M} U_m\overline{V_m}\right|^2}{\sum_{m=1}^{M}|U_m|^2 \sum_{m=1}^{M}|V_m|^2}. \tag{8}$$

N. R. Goodman studied the distribution of $|\hat{\gamma}|^2$ among other quantities related to complex Gaussian random variables in his dissertation [3]. Goodman later suggested the application of these notions to testing for nonstationarity [4] by interpreting $U_m$ and $V_m$ to be subsets of the discrete Fourier transform vector $\{\tilde{X}_k, k=0,N\text{-}1\}$ where

$$\tilde{X}_k = \sum_{n=0}^{N-1} X_n \exp(i2\pi kn/N). \tag{9}$$

A test for nonstationarity may be expressed as a test for a significantly large value of $|\hat{\gamma}|^2$; or, in other words, for correlation between the $U_m$ and $V_m$.

Clearly there are many ways to choose the $\{U_m\}$ and $\{V_m\}$ from $\{\tilde{X}_k, k=0, N\text{-}1\}$. We have applied this technique to testing for the presence of the PC property by choosing $\{U_m\}$ and $\{V_m\}$ according to the relationship between frequencies permitted by PC processes. For example, if $\{U_m\}$ is a contiguous set of elements of $\tilde{X}$, say $U_m = \tilde{X}(j+m)$ m=1, M, and we wish to test for a PC property at some difference frequency d, then $\{V_m\}$ is defined by $V_m = \tilde{X}(j+d+m)$ m=1, M.

This algorithm can be clarified by defining a two dimensional discrete frequency periodogram from $\tilde{X}_k$. That is we define

$$g(N,p,q) = \frac{1}{2\pi N}\check{X}_p\overline{\check{X}_q}. \tag{10}$$

If the sequence $X_n$ in (9) is PC with period T (T an integer), then $X_n$ is harmonizable (see [1]) and the support set of the two-dimensional spectral distribution is $\{(\lambda_1,\lambda_2) \in [-\pi,\pi): \lambda_2 = \lambda_1 - 2\pi k/T\}$. Further, the support set $S_k$ corresponding to $\lambda_2 = \lambda_1 - 2\pi k/T$ transforms to $2\pi q/N = 2\pi p/N - 2\pi k/T$ or $p - q = kN/T$. In other words, the two dimensional discrete frequency periodogram should be smoothed over lines of constant difference frequency to search for the presence of the PC property. The spectral coherence calculation of Goodman provides a way of doing this without knowledge of the individual variances.

In order to facilitate this determination for an unknown time series we compute the diagonal spectral coherence at co-ordinate (p,q) according to

$$|\gamma(p,q,M)|^2 = \frac{\left|\sum_{m=0}^{M-1}\check{X}_{p+m}\overline{\check{X}_{q+m}}\right|^2}{\sum_{m=0}^{M-1}|\check{X}_{p+m}|^2\sum_{m=0}^{M-1}|\check{X}_{q+m}|^2}. \tag{11}$$

We compute these values in a square array (over p,q) and plot the values that exceed a threshold. A useful threshold computation is that given by Goodman for the null distribution for $|\gamma|^2$ under the assumption that $\{U_m\}$ and $\{V_m\}$ are complex Gaussian, arise from disjoint frequency sets and the variances $E\{|U_m|^2\} = \sigma_U^2$ and $E\{|V_m|^2\} = \sigma_V^2$ are constant with respect to m. In this case

$Pr[|\gamma|^2 > |\gamma_0|^2] = [1-|\gamma_0|^2]^{M-1}$.

### 3. New Theoretical Results

Theoretical research on non-stationary processes is yielding fundamentally new information about the structure of periodically and almost periodically correlated processes; specifically, we have shown how groups of unitary operators are intimately connected with these processes. These research findings will permit the solution of problems that were difficult when working only with the correlation theory. In addition, these findings provide representations that are extendable to multiple dimensions, that is, to random fields. These findings will also clarify the connection between periodically correlated processes and wavelet representations of processes. The research documented in the following manuscripts has been partly or entirely supported by this contract.


[R1] Hurd, H.L., and Gerr, N., *Graphical Methods for Determining the Presence of Periodic Correlation.* J. Time Series Anal., 12, No. 4, 337-350, 1991.

[R2] Hurd, H.L., *Correlation Theory of Almost Periodically Correlated Processes*, J. Multivariate Anal., 37, No. 1, 24-45, 1991.

[R3] Hurd, H.L., *The Unitary Operator of Periodically Correlated Processes*, Invited paper given at the Hampton Univ. Conference on Nonstationary Stochastic Processes, Aug. 1 and 2, 1991.

[R4] Hurd, H.L., *Almost Periodically Unitary Stochastic Processes*, Tech. Rept. No. 284, Center for Stochastic Processes, Dept. of Statistics, University of NC at Chapel Hill, February 1990. To appear in Stochastic Processes and their Application.

[R5] Houdre', C.H., *On the Spectral Strong Law of Large Numbers and the Pointwise Ergodic Theorem in $L^\alpha$*. To appear in Annals of Probability.

[R6] Hurd, H.L. and Leskow, J., *Strongly Consistent and Aysmptotically Normal Estimation of the Covariance for Almost Periodically Correlated Processes.* Tech. Rept. No. 308, Center for Stochastic Processes, Dept. of Statistics, University of NC at Chapel Hill, Sept. 1990. To appear in Statistics and Decisions.

[R7] Cambanis, S., Houdre, C.H., Hurd, H.L., and Leskow, J., *Laws of Large Numbers for Periodically and Almost Periodically Correlated Processes.* Tech. Rept. No. 334, Center for Stochastic Processes, Dept. of Statistics, University of NC at Chapel Hill, March 1991. Submitted for publication.

[R8] Hurd, H.L. and Leskow, J., *Estimation of the Fourier Coefficient Functions and their Spectral Densities for $\phi$-Mixing Almost Periodically Correlated Processes.* Tech. Rept. No. 330, Center for Stochastic Processes, Dept. of Statistics, University of NC at Chapel Hill, March 1991. To appear in Statistics and Probability Letters.

[R9] Houdre', C.H., *Some Recent Results on the Sampling Theorem.* To appear in 1992.

[R10] Hurd, H.L. and Mandrekar, V., *Spectral Theory of Periodically and Quasi-Periodically Stationary $S\alpha S$ Sequencess*, Tech. Rept. No. 349, Center for Stochastic Processes, Dept. of Statistics, UNC at Chapel Hill, Sept. 1991.

[R11] Hurd, H.L. and Russek, A., *Stepanov Almost Periodically Correlated and Almost Periodically Unitary Processes*. Tech. Rept. No. 368, Center for Stochastic Processes, Dept. of Statistics, UNC at Chapel Hill, August, 1992.

[R12] Hurd, H.L. and Kallianpur, G., *Periodically Correlated and Periodically Unitary Processes and their Relationship to $L_2[0,T]$- Valued Stationary Sequences*. In Nonstationary Stochastic Processes and Their Application, J.C.Hardin and A.G.Miamee Ed., World Scientific Publishing Co., 1992.

[R13] Houdre, C.H., *On the Irregular Sampling Theorem*. In progress.

[R14] Hurd, H.L. and Russek, A., *Almost Periodically Correlated Processes on LCA Groups*, Tech. Rept. No. 369, Center for Stochastic Processes, Dept. of Statistics, UNC at Chapel Hill, 1992.

[R15] Bloomfield, P., Hurd, H.L., and Lund, R., *Periodic Correlation in Meteorlogical Time Series*, Presented at the 5th International Meeting on Statistical Climatology, Toronto, Quebec, June 22-26, 1992.

[R16] Bloomfield, P., Hurd, H.L., and Lund, R., *Periodic Correlation in Stratospheric Ozone*, Tech. Rept. No. 2076, Institute for Statisticss, Dept. of Statistics, UNC at Chapel Hill, June 15,1992.

In addition, the following application oriented reports have been produced.

[A1] Lund, R., *A Sensitivity Study of Goodman's Coherence Statistic Via Simulations*, Technical Report by Harry L. Hurd Associates, on Contract N00014-89-C-0310 , September 15, 1991.

[A2] Lund, R., *The Kernel Method of Density Estimation: A Summary*, Technical Report by Harry L. Hurd Associates, on Contract N00014-89-C-0310 , October 14, 1991.

[A3] Jones, C., *Spectral Coherence of Harmonics*, Consultant's report submitted to Harry L. Hurd Associates, on Contract N00014-89-C-0310 , November 5, 1991.

[A4] Jones, C., *PC Signal Models*, Consultant's report submitted to Harry L. Hurd Associates, on Contract N00014-89-C-0310 , December 9, 1991.

[A5] Jones, C., *Helicopter Data Analysis*, Consultant's report submitted to Harry L. Hurd Associates, on Contract N00014-89-C-0310 , February 3, 1992.

[A6] Jones, C., *DSCOH User's Manual*, Technical Report by Harry L. Hurd Associates, on Contract N00014-89-C-0310 , July 31, 1992.

[A7] Hurd, H. and Jones, C., *DSCOH Technical Reference Manual*, Technical Report by Harry L. Hurd Associates, on Contract N00014-89-C-0310 , July 31, 1992.

[A8] Greineder, S. and Hurd, H., *Investigation of Diagonal Spectral Coherence as a Class Discriminant Function*, NUSC Technical Report(SECRET), in preparation.

## 4. Workstation Software

The developmental part of the effort on contract N00014-89-C-0310 has focused on the development of workstation software to facilitate the examination and characterization of nonstationary data using spectral coherence. We have continued to improve the software functionality and user interface over the span of this project. Early in the project, each basic function was implemented as a stand-alone program. At the next stage, the separate programs were integrated into one master program called TD. Then versions of the program, called TDP, were built that could use array processor cards in both the IBM-PC family or in the MacII family. In the latter part of the project, we have concentrated on the IBM-PC family because of programming ease and because the 80486 chip gives ample speed for the basic functions, even without the array processor. The latest version, called DSCOH, contains a greatly improved user interface, and is considered the final product of the contract.

We shall first summarize the basic functionality of DSCOH, followed by functionality provided by other programs developed under this contract. These other programs include the earlier TD, TDP and TDNEW programs or by more recent stand-alone programs.

### 4.1 DSCOH Functionality

(a) Plotting of time series.

(b) Periodograms. Periodograms are plots of the squared magnotude of sample Fourier transforms.

(c) Spectrograms. Spectrograms are waterfall or gram displays made from successive periodograms using various amplitude *encoding* and *scaling* methods.

(d) Spectral coherence image displays. The SC images are computed according to Eq.(11) and grey-scale encoded. This display combines five visual outputs based on the FFT of a segment of data. These include the SC image, a time series plot of the current data sement, plots of the usual periodogram and the phase spectrum, and two auxilliary plots. The auxilliary plots may be chosen in pairs from either of the following two sets.

(i) Plots supporting the detection of periodic correlation.

These plots both are based on the fact that the support set for a PC process is a collection of straight lines parallel to the main diagonal. The *first* plot in this pair may be motivated by the thought experiment of increasing M to be as large as permitted so that only one value of $|\gamma(p,q,M)|^2$ is determined for each separation d from the main diagonal. In terms of $|\gamma(p,q,M)|^2$ we compute $|\gamma(0,d,N-d)|^2$ where N is the dimension of the Fourier transform vector. The implementation is based on an equivalent but simpler computation, consisting of the DFT of the squares of the input time series. The *second* plot is based on the incoherent averaging of all the values of $|\gamma(p,q,M)|^2$ from a coherence surface for a fixed value of d. Symbolically, we compute the incoherent average according to

$$\delta(d,M) = \frac{1}{L+1} \sum_{p=0}^{L} |\gamma(pM,\ pM+d,\ M)|^2 \tag{12}$$

where $L = \lfloor (N-1-d)/M \rfloor$ and $\lfloor x \rfloor$ denotes the greatest integer in x.

(ii) Plots supporting the detection of transients.

The *first* plot in this pair is a plot of the TR-1 statistic, which is determined by the level of coherence obtained by traversing the spectral coherence images on a diagonal path of difference frequency d. To be more precise, the TR-1 ordinates are given by $|\gamma(k,k+d,M)|$ as a function of k. It is thought that small values of d are appropriate. The *second* plot in this pair is a plot of the TR-2 statistic, which is determined by the ratio of a weighted coherent sum of the Fourier coefficients to the incoherent sum

$$\eta(k,\Delta) = |\sum_{j=0}^{N-1} \tilde{X}_{j+k} \exp(i\Delta j)|^2 / \sum_{j=0}^{N-1} |\tilde{X}_{j+k}|^2 \tag{13}$$

The algorithm is motivated by the observation that if the spectrum of a transient signal has a linear phase function exp(-i$\Delta$j) in the neighborhood of the maximum spectral levels, then the statistic $\eta$(k) will be large (>1) when $\Delta$ is chosen to make the terms in the sum add coherently. A disadvantage of this statistic is that multiple values of $\Delta$ may have to be tried. In contrast, the algorithm TR-1 is much more robust because its output is independent of $\Delta$ for a linear phase function $\phi(j)=\Delta j$.

(e) Correlation and cepstral plots.

Autocorrelation and cepstral plots are provided to assist in determining the presence of multi-path signals.

4.2 Other Functionality

(a) PC-grams. A PC-gram is formed in the same manner as the spectrogram but from successive traces from one of the auxilliary plots supporting detection of PC processes. Either auxilliary plot may be chosen as the source of the PC-gram. The region of the SC image contributing to the incoherent sum may be restricted by the user.

(b) T-grams. T-grams are grams made from successive scans of the TR-1 transient detection statistic.

(c) Bandshifting, filtering and decimation. This basic function permits the investigator to produce data files containing filtered, bandshifted (to center-frequency of zero) and decimated sub-bands of time series from a specified input file.

## 5. Use of Workstation Software bu Navy Labs and Projects

In the course of this project, experimental versions of the software have been installed at various Navy labs. This provided Navy scientists with capabilities for analyzing nonstationary signals using spectral coherence, and also provided feedback from which improvements were based. In particular, versions of the software have been provided to NUWC (formerly NUSC), NCCOSC NRaD (formerly NOSC) NSWC and DTRC. These interactions have lead to contractural support for the investigation of spectral coherence methods in specific applications. Support for this application work is coming from NUWC, NADC and NCCOSC (the DARPA DANTES program). Final versions of the software have been delivered to NOSC and NUSC as called out in the contract.

## 6. Commercialization of the Workstation Software

The DSCOH software package will be offered for public sale in the near future. The capabilities offered by DSCOH will permit an investigator to perform basic analyses of signals that are thought to have diagonal spectral coherence. Further details on the DSCOH program may be found in the *DSCOH User's Manual* and the *DSCOH Technical Reference Manual.*

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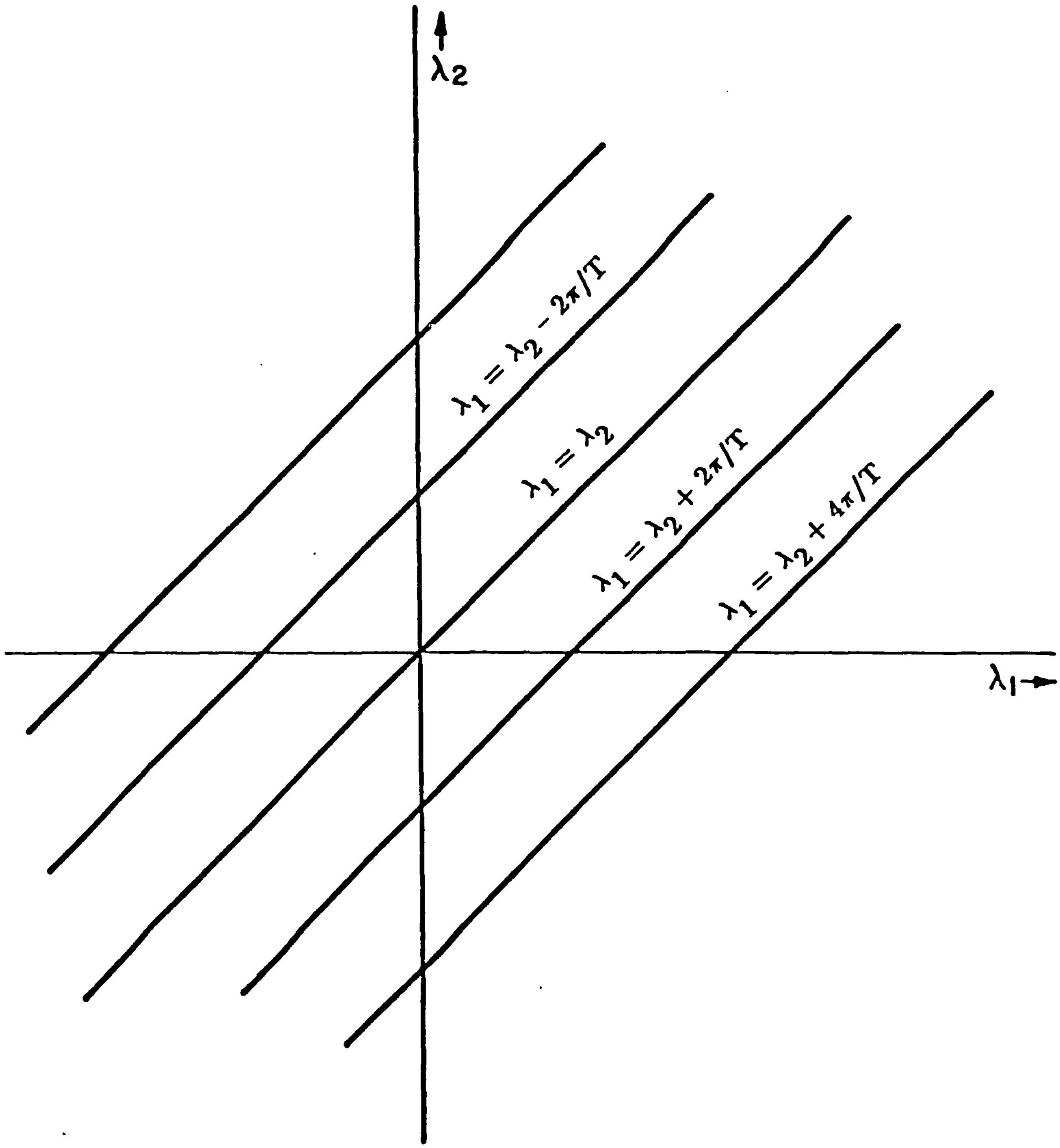


**Figure 1—Spectral Mass Locations for Harmonizable Periodically Correlated Processes**

# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188



| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | 30JUL92 | FINAL 1OCT89 - 31MAY92 |

**4. TITLE AND SUBTITLE**
Workstation Tools for Feature Extraction and Classification for Nonstationary and Transient Signals

**5. FUNDING NUMBERS**
N00014-89-C-0310

**6. AUTHOR(S)**
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**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
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**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
Office of Naval Research, Code 1111
800 N. Quincy Street
Arlington, VA 22217-5000

**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

**11. SUPPLEMENTARY NOTES**

**12a. DISTRIBUTION/AVAILABILITY STATEMENT**

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** (Maximum 200 words)

This work was motivated by the recognition that the solution of technical problems involving nonstationary processes will require a body of theory and practical tools that facilitate observing and characterizing various types of nonstationary processes. This report contains a sketch of the theoretical background that motivates the experimental methodology, a summary of new theoretical results and a high-level description of the workstation software package that was developed under this contract.

**14. SUBJECT TERMS**
Nonstationary, harmonizable, periodically correlated, cyclostationary, almost periodically unitary, stochastic processes.

**15. NUMBER OF PAGES**
15

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | |