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Integrated Design Analysis and Optimisation of Aircraft Structures

(L'Analyse Intégrale de la Conception et l'Optimisation des Structures des Aéronefs)

This material in this publication was assembled to support a Lecture Series under the sponsorship of the Structures and Materials Panel of AGARD and the Consultant and Exchange Programme of AGARD presented on 8th—9th June 1992 in Pasadena, CA, United States, 22nd—23rd June 1992 in Lisbon, Portugal and 25th—26th June 1992 in London, United Kingdom.

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Abstract

There is a lack of precise information on the effectiveness of specific methods in generating optimum designs for realistic aircraft structures. In this situation it is difficult for designers to make decisions on which systems to employ for a given design problem and which developments to pursue. Thus it is necessary for designers to be aware of the relative merits of the different methods currently used for the design optimisation of advanced aircraft.

This Lecture Series covers the methods available for the computer based design analysis and design optimisation of aircraft structures. The Lecture Series deals with the principles and practices adopted to integrate the various factors which are considered in the design of advanced aircraft. These factors include: structural shape, aerodynamics, active control technology and aircraft performance. Realistic case studies are used to illustrate the methods used for different design problems.

The following topics are covered in detail:

- Overview of integrated design analysis, background, methods, objectives and requirements.
- Optimisation in design (CAE/CAD).
- A system approach to aircraft optimisation.
- Case studies for different design problems.

This Lecture Series, sponsored by the Structures and Materials Panel of AGARD, has been implemented by the Consultant and Exchange Programme.

Abrégé

Il y a un manque d'informations précises sur l'efficacité des méthodes spécifiques qui ont été élaborées pour l'optimisation des études en vue de la réalisation de structures d'aeronefs. Dans cette situation il est difficile pour les concepteurs d'avion de décider des systèmes à employer pour résoudre tel ou tel problème de conception et d'identifier les développements intéressants. Il importe donc, de sensibiliser les concepteurs sur la valeur relative des différentes méthodes employées pour l'optimisation de la conception des aéronefs.

Ce cycle de conférences couvre les méthodes disponibles pour l'analyse de la conception assistée par ordinateur et l'optimisation de la conception des structures d'aéronefs. Il examine les principes et les pratiques adoptés pour l'intégration des différents facteurs pris en compte lors de la conception des aéronefs. Ces facteurs comprennent: la forme structurelle, l'aérodynamique, la technologie des commandes actives et les performances. Des études de cas réelles sont utilisées pour illustrer les méthodes employées pour résoudre divers problèmes de conception.

Les questions suivantes sont examinées dans le détail:

- panorama de l'analyse intégrée de la conception, historique, méthodes, objectifs et besoins
- l'optimisation de la conception (IAO/CAO)
- une approche "systèmes" à l'optimisation des aéronefs
- des études de cas pour des problèmes de conception.

Ce cycle de conférences est présenté par le Panel AGARD des structures et matériaux; et organisé dans le cadre du programme des Consultants et des Echanges.

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FUNDAMENTALS OF STRUCTURAL OPTIMISATION

by

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1. Introduction

Structural Optimisation is concerned with the computerised automatic design of structures which are optimum with respect to some major design parameter. In the aircraft industry this parameter has usually been structural weight, though cost, performance or other factors are now being considered. The parameter being optimised is referred to as the objective function and the variables which can be changed to achieve the desired optimum are referred to as design variables. Mathematically this can be characterised by saying that the problem is;

$$\text{minimise (or maximise)} \qquad f(x) \quad x \in R^n$$

$$\text{subject to the constraints} \qquad g_j(x) \geq 0 \qquad j = 1 \ldots\ldots m$$

$$h_k(x) = 0 \qquad k = 1 \ldots\ldots p$$

where the design variables $x \in R^n$ are positive and the range of x for which the constraints are not violated constitute the feasible region. If the objective function f(x) is structural weight the design variables are size parameters such as bar cross-sections, plate thicknesses and, in certain cases, shape parameters which vary the geometrical configuration of the structure. Current researches are seeking to extend the scope of structural optimisation to cover more extensive objective functions which include factors such as performance, cost, etc. Indeed, certain commercially available systems already cover non-weight objective functions. The constraints on the optimum will include behavioural parameters so that the terms $g_j(x)$ could include, stress, displacement, flutter speed, vibration limits or any other relevant parameters. In addition, to behavioural aspects these constraints also cover physical limits imposed by practical manufacturing considerations such as gauge limits. Whilst equality constraints are not common in minimum weight optimisation they can occur where design codes are employed or where components can be selected from a specific range (i.e. stock items).

In all this variety two aspects remain constant in all current structural optimisation applications. First, the general problem which is characterised by (1.1) remains unchanged so that the basic nature of the optimisation problem is the same for all applications. Thus, the theory described in this Lecture Series can be used in all design applications. Secondly, the structural behaviour of the optimisation problem is always characterised by the Finite Element Method. In many cases this has lead to the development of optimisation modules which form an integral part of many commercially supported F.E. packages i.e. NASTRAN, SAMCEF, IDEAS, ELFINI, ANSYS, etc. In addition, independent structural optimisation systems have been developed, such as the DRA/SCICON STARS, MBB Lagrange, systems, which can be attached to any existing FE system. These developments have resulted in Structural Optimisation Methods being routinely available to users of modern CAD systems.

The use and application of these methods in a safe and effective manner requires some understanding of the underlying mathematical principles. As in the case of the Finite Element Method the basic mathematics provides a 'tool-kit' which is repeatedly used to develop solution methods. It is shown in later sections that this process of developing solution methods use the optimisation criteria as the basis for creating the up-date formulae which are the solution algorithm drivers. Thus, this first part of the Lecture Series, describes the optimality criteria, the associated duality theory and the algorithms themselves.

2. A Basic Algorithm

The computer based numerical solution process for the problem defined at (1.1) is in essence, simple. It requires that a repetitive formulae is used which starts with an initial estimate of the design variables $x^{(0)} \in R^n$ and systematically changes them until a set $x^{(\ell)}$ is generated after ℓ iterations which satisfy (1.1).

The process is best demonstrated by considering an optimisation problem which has no constraints, thus we seek to solve the problem

$$\text{minimise} \quad f(x) \qquad x \in R^n$$

The solution can be found using the following solution algorithm:

Basic Algorithm.

```
Step 1   Select starting values $x^{(0)}$ and choose a value for $\epsilon$
Step 2   Set $k = 0$
Step 3   Set $k = k + 1$
Step 4   Set $x^{(k+1)} = A(x^{(k)})$
             $f^{(k+1)} = f(x^{(k+1)})$
```

Step 5 If $B(x^{(k+1)}) \leq \epsilon$ go to Step 6;

Otherwise go to Step 3

Step 6 Set $x^{*} = x^{(k+1)}$, $f^{*} = f(x^{(k+1)})$;
STOP.

Thus, step 4 generates a new version of the design variables $x^{(k+1)}$ from the earlier values $x^{(k)}$ and the formula for doing this $A(x^{(k)})$ is known as the up-date formula. Because we are moving from one position $x^{(k)}$ in R^n to another position $x^{(k+1)}$ in R^n $A(x^{(k)})$ this constitutes a move of specific length along a given direction. Hence

$$A(x^{(k)}) = \underset{\sim}{x}^{(k)} + \alpha\, \underset{\sim}{\mu} = \underset{\sim}{x}^{(k+1)}$$

where $\underset{\sim}{\mu}$ is a direction, in R^n, from $x^{(k)}$ and α is a value giving the distance to be moved in this direction. Two questions now need to be answered:

- what should be used for $\underset{\sim}{\mu}$
- how far to move along $\underset{\sim}{\mu}$ -i.e. what is the value for α?

In answering the first of these questions we must select a direction which points towards the optimum and one, very effective, method for achieving this is to enforce the satisfaction of the optimality criterion. Once a direction has been selected the value of α is found by seeking the minimum value of f(x) along the direction .

EXAMPLE (Newton's Method)

Suppose that the function to be minimised f(x) has first and second derivatives available so that it can be approximated by a second order Taylor expansion about a point $x^{(k)} \in R^n$

$$f(\underset{\sim}{x}^{(k)} + \delta\underset{\sim}{x}) = f(\underset{\sim}{x}^{(k)}) + \underset{\sim}{\nabla} f^{T}(\underset{\sim}{x}^{(k)}).\ \delta\underset{\sim}{x} \\ + \frac{1}{2}\,\delta\underset{\sim}{x}^{T}.\ \underset{\sim}{H}(\underset{\sim}{x}^{(k)})\delta\underset{\sim}{x} \qquad (1.2)$$

where $\delta\underset{\sim}{x}$ is an increment $x^{(k+1)} - x^{(k)}$; $\Delta f(x^{(k)})$, $H(x^{(k)})$ are the first derivative and the Hessian respectively of $f^{(r)}$ at $x^{(k)} \in R^n$.

In order to generate the up-date formula we note that the optimising condition for f(x) $x \in R^n$ is:

$$\nabla f(x) = 0$$

If this is enforced on (1.2) with $\delta\underset{\sim}{x}$ as the free variables (since the start point $x^{(k)}$ is fixed) then

$$\underset{\sim}{\nabla} f(x) = \underset{\sim}{\nabla} f(x^{(k)}) + \underset{\sim}{H}(\underset{\sim}{x}^{(k)}).\delta\underset{\sim}{x} = 0$$

or

$$\delta\underset{\sim}{x} = - \underset{\sim}{H}^{-1}(x^{(k)}).\underset{\sim}{\nabla} f(x^{k})$$

and thus we choose $$\underset{\sim}{\mu} = -\underset{\sim}{H}^{-1}(x^{(k)}).\underset{\sim}{\nabla} f(x^{(k)}).$$

If $f(x)$ is of higher order than a quadratic in $x \epsilon R^{n}$ then α is found by minimising

$$f(\underset{\sim}{x}^{(k)} + \alpha\underset{\sim}{\mu}) \text{ along } \underset{\sim}{\mu}$$

Having discussed the generation of the up-date formula $A(x^{(k)})$ we now return to the second unexplained term in the algorithm, namely $B(x^{(k)})$. This is simply a stopping criteria! Because the algorithm is a computational process the optimum point is only located to a specified level of accuracy. The term B is, therefore, an accuracy measure and can be represented by the change in objective function during an iteration, or the design variables. As the next section shows a very effective measure is given by noting the difference between the feasible value of the objective function and the associated dual.

Although the algorithm described above is simple in concept it is applicable to all optimisation problems, the difference in applying it to a range of problems lies in the changes associated with $A(x^{(k)})$. In the next section the optimality criteria for (1.1) is introduced and algorithms developed from it.

3. Optimality Criteria and Duality

In order to generate the up-date formula in the previous section the optimality criteria was used to generate the solution algorithm. However, this applied only for the case of an unconstrained optimisation problem which does not represent the situation described in (1.1). Generating the optimality conditions for the constrained problem is done by sequentially moving from an unconstrained problem to an equality constrained one then, finally, to the full inequality constrained optimisation which is the heart of the structural optimisation design problem. This is not a complex process but is too lengthy for inclusion here and is fully described in reference [1].

Because the inequality constrained problem is the most common form for structural optimisation we shall consider a reduced form for (1.1) and, for simplicity take the problem to be a minimisation. Thus (1.1) becomes:

$$\begin{aligned} &\text{minimise } f(x) \\ &\text{subject to } g_{j}(x) \geq 0 \quad j = 1 \ldots m \end{aligned} \qquad (1.3)$$

and the optimality criteria for this problem are known as the Kuhn-Tucker conditions. These state that $x^{*} \epsilon R^{n}$ is a local optimum for (1.3) if these exist $\lambda \in E^m$ such that:

$$\begin{aligned} \nabla_{\sim} f(x^*) - \underset{\sim}{\lambda}^T \nabla_{\sim} g(x^*) &= 0 \\ \underset{\sim}{\lambda}^T \underset{\sim}{g}(x^*) &= 0 \\ \underset{\sim}{\lambda} \geq 0 \qquad \underset{\sim}{g}(x^*) &\geq 0 \end{aligned} \tag{1.4}$$

The first part of (1.5) are the constrained derivatives of the objective function. That is, the gradient of the objective function projected onto the linearised form of the constraints. In essence this projection returns us to an unconstrained optimisation problem so that the algorithm developed in section 2 once more applies.

An alternative form for (1.3) uses the Lagrangian which is defined by

$$L(x,\lambda) = f(x) - \underset{\sim}{\lambda}^T . \underset{\sim}{g}(x)$$

in which case (1.4) can be re-written as

$$\begin{aligned} \nabla_x L(x^*, \lambda) &= 0 \\ \underset{\sim}{\lambda}^T . \underset{\sim}{g}(x^*) &= 0 \\ \underset{\sim}{\lambda} \geq 0 \quad \underset{\sim}{g}(x^*) &\geq 0 \end{aligned} \tag{1.5}$$

where $\nabla x = \left\{ \frac{\partial}{\delta x_i} \right\}$

The standard problem defined by (1.3) is clearly a minimisation problem which is often called the 'primal problem'. Associated with this is maximisation problem known as the 'dual problem' where a new function is maximised subject to a new set of constraints. These two problems are connected by a saddle point so that the minimum value which represents the solution of the primal problem is also the value which is the maximum value for the dual problem. The dual has, therefore, two uses both of which have been exploited by the developers of structural optimisation programmes. First, the dual formulation provides an alternative description of the optimisation problem which can be used to create solution algorithms. Secondly, it has been employed as a method for generating bounds on the optimum which can play the rôle of the accuracy check $B(x^{(k)})$.

Many forms for the dual can be developed and are discussed elsewhere, ref.[2]. The one usually employed for structural optimisation states that the dual associated with the primal problem (1.3) requires that, for f(x) convex and g(x) concave, we

$$\begin{aligned}&\text{maximise} && L(x,\lambda)\\ &\text{subject to} && \nabla_x L(x,\lambda) = 0 \\ & && \underset{\sim}{\lambda} \geq 0\end{aligned} \qquad (1.6)$$

Although not obvious from this formulation, for many structural problems, the dual constraints can be solved to yield $x(\lambda)$ giving rise to an unconstrained optimisation:

$$\text{maximise} \quad L(x(\lambda),\lambda)$$

with $x(\lambda)$ the solution of $\nabla_x L(x,\lambda) = 0$. As we shall see later this allows the creation of powerfull dual solution algorithms. These have formed the basis of the class of dual algorithms successfully employed in the SAMCEF system for several years.

4. Structural Optimisation Algorithms

4.1 STRESS RATIOING

The Kuhn-Tucker conditions (1.4) or (1.5) provide the optimality criterion which are used to generate the up-date formulae employed in the modern automated design systems. But a very simple up-date formula has been effectively employed both as a hand calculation and as a computerised optimisation method. This assumes that the optimum is a vertex solution in constrained design space. It required that the number of constraints in (1.3) is equal to or exceeds the number of design variables i.e. $m \geq n$. In this situation the optimum is found by solving n equality constraint equations

$$g_k(x) = 0 \qquad k = 1 \ldots\ldots n$$

and, used iteratively, this produces the classical stress ratioing algorithm.

For the minimum weight design of statically determinate structures subject to stress constraints only, this method will find the optimum in a single iteration. For indeterminate structures there is no guarantee that an up-date formula based on enforcing vertex solution will locate the optimum design. This is because the solution process takes no account of the desire to minimise the structural weight. For problems involving constraints other than stress the approach is highly inappropriate. However, it is robust and does not require the calculation of any derivatives so is effective in the initial stages of any solution involving a structural optimisation problem in which stress constraints play a rôle. For this reason the stress ratio algorithm is available in all systems used for the design of minimum weight aircraft structures.

4.2 OPTIMALITY CRITERION ALGORITHMS

In the past the optimality criterion method was a term applied to a set of algorithms devised by Venkayya, Khott and Berke [3] in the United States and Kerr [4] in the U.K. However, these methods are a special case of the general process of using the Kuhn-Tucker conditions as the basis of the up-date formulae. The term Optimality Criterion methods is, more properly, applied to a general class of solution methods. In this situation the unifying factor is that the constraints are always approximated by a first order Taylor expansion. (Though more recent work has attempted to employ second order approximations for the constraints, but this is not considered in the present paper.) The differentiating factor between the various methods in this class is the order of the Taylor expansion used to approximate the objective function. A common factor to all the methods is the need to select from the total number of constraints a sub-set which are considered to be active. This means that, at each iteration, the solution process must establish which constraints are good candidates for being strict equality, as opposed to inequality constraints, at the optimum point.

4.2.1. *Linear Approximation.* The first optimality criterion approach assumes that both the objective function and the constraints are approximated by first order Taylor expansions. Thus, (1.3) now becomes

$$\begin{array}{ll} \text{minimise} & f(x) + \nabla \underset{\sim}{f}.\delta \underset{\sim}{x} \\ \text{subject to} & g(x) + \underset{\sim}{N}.\delta \underset{\sim}{x} \geq 0 \end{array}$$

where N is the matrix of constraint gradients $\nabla g(x)$ taken with respect to the design variables. These gradients can be computed in a variety of ways but the finite element method lends itself to analytic derivatives for a range of element types. For complex problems recourse may be made to semi-analytic derivatives or, if absolutely necessary, to finite difference schemes. The generation of gradient derivatives is not discussed here as it is a well documented procedure available in standard texts.

The Lagrangian associated with the linearised problem is:

$$L(x,\lambda) = f(x) + \underset{\sim}{\nabla}^T.\delta \underset{\sim}{x} - \underset{\sim}{\lambda}^T.(\underset{\sim}{g}(x) + \underset{\sim}{N}.\delta \underset{\sim}{x})$$

The differential part of the Kuhn-Tucker condition are then

$$\nabla_x L(x,\lambda) = \nabla \underset{\sim}{f} - \underset{\sim}{N}^T.\underset{\sim}{\lambda} = 0$$

The lagrangian multipliers can now be extracted:

$$\underset{\sim}{\lambda} = (\underset{\sim}{N}.\underset{\sim}{N}^T)^{-1}.\underset{\sim}{N}.\nabla \underset{\sim}{f}$$

and can be used to generate the constrained derivative.

$$\nabla \underset{\sim}{f}_c = \left\{\underset{\sim}{I} - (\underset{\sim}{N}.\underset{\sim}{N}^T)^{-1}.\underset{\sim}{N}\right\}.\nabla \underset{\sim}{f}$$

The up-date formula can now be constructed on the basis that the optimum can be located along the direction of steepest descent. Thus, the algorithm uses $(-\nabla f_c)_{\max}$ as the direction $\utilde{\mu}$ and a line search can then be conducted. If a single constraint (displacement) only is active, the equations transform into those used by Knott/Venkayya/Berke and Kerr for the original optimality criterion method. This formulation can also be used to generate a solution algorithm based on the premise that the optimum can be found from a sequence of linear programmes.

4.2.1. *Quadratic/Linear Application.* The next level in the hierarchy of solution methods assumes that the objective function is approximated by a second order Taylor expansion and the constraints, as at 4.2.1, by a first order expansion. Thus, the problem (1.3) becomes:

$$\text{minimise} \quad f(x) + \nabla \utilde{f}^T.\delta \utilde{x} + \frac{1}{2}\, \delta \utilde{x}.\utilde{H}.\delta \utilde{x}$$

$$\text{subject to} \quad \utilde{g}(x) + \utilde{N}.\delta \utilde{x} \geq 0$$

where $\utilde{H}$ is the Hessian of the objective function. The Lagrangian associated with this problem is:

$$L(x,\lambda) = f(x) + \nabla \utilde{f}^T.\delta \utilde{x} + \frac{1}{2}\delta \utilde{x}.\utilde{H}.\delta \utilde{x} - \utilde{\lambda}^T(\utilde{g}(x) + \utilde{N}\delta \utilde{x}$$

Thus, for optimality:

$$\nabla_x L(x,\lambda) = \nabla \utilde{f} + \utilde{H}.\delta \utilde{x} - \utilde{N}^T.\utilde{\lambda} = 0$$

giving

$$\delta \utilde{x} = -\utilde{H}^{-1}.(\nabla \utilde{f} - \utilde{N}^T.\utilde{\lambda})$$

Noting that the constraints involved in this formula are the active sub-set then:

$$\utilde{g}(x) + \utilde{N}.\delta \utilde{x} = 0$$

or

$$\begin{aligned} \utilde{N}.\delta \utilde{x} &= -\utilde{g}(x) \\ &= -\utilde{N}.\utilde{H}^{-1}.(\nabla \utilde{f} - \utilde{N}^T.\utilde{\lambda}) \end{aligned}$$

Thus

$$\utilde{\lambda} = (\utilde{N}.\utilde{H}^{-1}.\utilde{N}^T)^{-1}.(\utilde{N}.\utilde{H}^{-1}.\nabla \utilde{f} - \utilde{g}(x))$$

and substituting back into the expression for $\delta \utilde{x}$ gives

$$\delta \utilde{x} = -\utilde{H}^{-1}.\left[\nabla \utilde{f} - \utilde{N}^T.\left\{(\utilde{N}.\utilde{H}^{-1}.\utilde{N}^T)^{-1}.(\utilde{N}.\utilde{H}^{-1}.\nabla \utilde{f} - \utilde{g}(x))\right\}\right]$$

or

$$\delta \utilde{x} = -\utilde{H}^{-1}.\utilde{N}^T.(\utilde{N}^T.\utilde{H}^{-1}.\utilde{N}^T)^{-1}\utilde{g}(x) - \left\{\utilde{I} - \utilde{N}^T.(\utilde{N}.\utilde{H}^{-1}.\utilde{N}^T)^{-1}.\utilde{N}.\utilde{H}^{-1}\right\}.\utilde{H}^{-1}.\nabla \utilde{f}$$

As in section 2 this is a Newton step and gives the direction $\utilde{\mu}$. It has two components, the first

$$- \underset{\sim}{H}^{-1}.\underset{\sim}{N}^{T}.(\underset{\sim}{N}.\underset{\sim}{H}^{-1}.\underset{\sim}{N}^{T})^{-1}g(x)$$

steps onto the active constraints, and the second

$$- \left\{\underset{\sim}{I} - \underset{\sim}{N}^{T}.(\underset{\sim}{N}.\underset{\sim}{H}^{-1}.\underset{\sim}{N}^{T})^{-1}.\underset{\sim}{N}.\underset{\sim}{H}^{-1}\right\}.\underset{\sim}{H}^{-1}.\nabla \underset{\sim}{f}$$

projects the Newton direction onto the plane (linearised) of the active constraints.

The Quadratic/Linear approximation is a very popular algorithm and has found application in a variety of systems including STARS, OPTISEN, OPTI, ASTROS. It is used with direct design variables, inverse variables or asymptotic variables. Also the precise implementation of the algorithm may vary from system to system and a range of generalised quadratic programming methods have been employed.

4.2.1. *Active Set Strategies.* As indicated above the algorithms described rely on the fact that the set of constraints being used at each iteration of the algorithm are a sub-set of the total. These control the space available for the up-date search direction to sweep. In principle this reduced set of constraints generate a local feasible direction. The projection vectors developed in 4.2.2 restrict the search direction to lie along this reduced set which are actively controlling the up-date process. These constraints are, therefore, called the active set of constraints and the procedure for deciding on which of the total constraints are active, at any iteration, is known as the Active Set Strategy.

There are two parts to this strategy; one part deciding which constraints are to be included, the other deciding those to be dropped from the active set. The process of including a new constraint is straightforward; at each iteration an analysis must be performed and should any constraint be seen to be violated it must be included in the active set. Dropping constraints is a little more complicated. As shown in the Kuhn-Tucker optimality conditions the Lagrange multipliers must be positive. Thus, any constraint at any iteration which has a negative Lagrange multiplier should not be in the active set. The formulae in 4.2.2 which calculate the Lagrange multipliers can be used to identify these constraints with negative multipliers. Such constraints can then be dropped before the next iteration is performed.

The exact process of implementation is a little more complicated than the outline given above and anti zig-zag rules need to be imposed to provide a degree of smoothness to the operation of the strategy. Nevertheless, the basic principles of most active set strategies are those given here.

5. Exploiting the Dual

In this section the power of the dual formulation is demonstrated both as the basis for a solution algorithm and as a bounding procedure. It is convenient to be a little more specific in formulating the problem and, to this end, a linear weight function is taken as the objective function. However, to assist in the process of linearising the constraints this is transformed into a non-linear form by the use of inverse sizing variables. The generic structural minimum weight design problem becomes

$$\text{minimise} \quad \sum_{i=1}^{n} \frac{w_i}{x_i}$$

$$\text{subject to} \quad \sum_{j=1}^{m} \left(\sum_{i=1}^{n} c_{ij}\, x_i - b_j \right) \le 0 \qquad (5.1)$$

where w_i is the specific weight(mass) associated with the i th design variable.

5.1 DUAL BOUNDING

The dual problem associated with the primal optimisation problem (5.1) is

$$\text{maximise } L(x,\lambda) = \sum_{i=1}^{n} \frac{w_i}{x_i} + \sum_{j=1}^{m} \lambda_j \left(\sum_{i=1}^{n} c_{ij}\, x_i - b_j \right)$$

$$\text{subject to} \quad -\frac{w_i}{x_i^2} + \sum_{j=1}^{m} \lambda_j\, c_{ij} = 0 \qquad i = 1 \ldots\ldots n \qquad (5.2)$$

Multiply each of the constraint equations by x_i and sum $i = 1 \ldots\ldots n$ gives

$$-\sum_{i=1}^{n} \frac{w_i}{x_i} + \sum_{j=1}^{m} \lambda_j \sum_{i=1}^{n} c_{ij}\, x_i = 0$$

and substituting this into the dual problem gives a new dual:

$$\text{maximise} \quad \sum_{j=1}^{m} \lambda_j\, b_j$$

$$\text{subject to} \quad \sum_{j=1}^{m} \lambda_j\, c_{ij} = \frac{w_i}{x_i^2} \qquad i = 1 \ldots\ldots n$$

If we explicitly take into account the positivity of the design variables this problem becomes:

$$\begin{aligned}&\text{minimise} \quad \sum_{j=1}^{m} \lambda_j b_j \\ &\text{subject to} \quad \sum_{j=1}^{m} \lambda_j c_{ij} \geq \frac{w_i}{x_i^2} \quad i = 1 \ldots\ldots n\end{aligned} \tag{5.3}$$

Thus (5.3) is the linearised dual to the primal problem (5.1) and can be used to provide a pseudo-dual bound on the optimum.

In order to demonstrate the procedure we assume that at the end of the kth iteration of any of the algorithms described in section 4 we have the current estimate of the "optimising" inverse design variables $x_i^{(k)}$. These may now be fed into (5.3) to provide a 'dual' problem

$$\begin{aligned}&\text{minimise} \quad \sum_{j=1}^{m} \lambda_j b_j \\ &\text{subject to} \quad \sum_{j=1}^{m} \lambda_j c_{ij} \geq \frac{w_i}{(x_i^{(k)})^2} \qquad i = 1 \ldots\ldots n\end{aligned}$$

and because the variables $x_i^{(k)}$ $i = 1 \ldots\ldots n$ are fixed the above is a linear programming problem which can be solved to produce a set of dual variables $\lambda_j^{(k)}$ $j = 1 \ldots\ldots m$. These can be used to compute a value of the Lagrangian $L(x^{(k)}, \lambda^{(k)})$ which can be compared with

$$\text{weight} = \sum_{i=1}^{n} \frac{w_i}{x_i^{(k)}}$$

to give a bound on the optimum. The gap between these two values is known as the duality gap and can be used as the function $B(x^{(k)})$ used as a stopping criteria in the basic algorithm of section 2.

This approach of using a linear programming routine to solve a linearised dual to provide a bound on the optimum is used in many structural optimisation codes including STARS and OPTISEN.

5.2 A DUAL ALGORITHM

In order to create a dual based algorithm we note that the dual problem can be developed to remove any dependence on the design variables $x_i^{(k)}$ $i = 1 \ldots\ldots n$. This is done by explicitly solving the dual constraints to obtain

$$x_i^2 = \frac{w_i}{\left(\sum_{j=1}^{m} \lambda_j c_{ij}\right)} \qquad i = 1 \ldots\ldots n$$

This expression for the design variables can be substituted into the dual objective function so that (5.2) becomes, simply;

$$\text{maximise } L(\lambda)$$

which is an unconstrained maximisation problem for which the optimality criteria is

$$\underset{\sim}{\nabla} L(\lambda) = 0$$

Assuming a 2nd order Taylor expansion for L(λ) gives a new unconstrained maximisation problem

$$\text{maximise} \quad L(\lambda) + \underset{\sim}{\nabla} L(\lambda)\delta\underset{\sim}{\lambda} + \frac{1}{2}\,\delta\underset{\sim}{\lambda}\, \underset{\sim}{H}(\lambda)\, \delta\underset{\sim}{\lambda}$$

for which the optimum is given by

$$\underset{\sim}{\nabla} L(\lambda) = \underset{\sim}{\Delta} L(\lambda) + \underset{\sim}{H}(\lambda)\, \delta\underset{\sim}{\lambda} = 0$$

or

$$\delta\underset{\sim}{\lambda} = -\underset{\sim}{H}^{-1}\underset{\sim}{\nabla} L$$

This provides the up-date formulae $A(\lambda^{(k)})$ in terms of the Lagrange multipliers:

$$\underset{\sim}{\lambda}^{(k+1)} = \underset{\sim}{\lambda}^{(k)} - \underset{\sim}{H}^{-1}\underset{\sim}{\nabla} L \tag{5.4}$$

We note that:

$$\nabla L \text{ has terms } \frac{\partial L}{\partial \lambda_j} = \sum_{i=1} c_{ij}\, x_i - b_j = g_j(x)$$

$$\text{and } H \text{ has terms } \frac{\partial^2 L}{\partial \lambda_\ell\, \partial \lambda_j} = \frac{\partial}{\partial \lambda_\ell}(g_j)$$

$$= \sum_{i=1}^{n} \frac{\partial g_i}{\partial x_i}\, \frac{dx_i}{d\lambda_\ell}$$

$$= -\sum_{i=1}^{n} \frac{c_{ij}\, c_{i\ell}}{2w_i\, x_i^3}$$

The up-date formulae (5.4) is used in the module OPTI in the SAMCEF system and was created by Fleuxy, [1]. However, it was used as the basis of Kerr's up-date formulae within his 'optimality-criterion' code which still forms the core of the B.Ae Warton code ECLYPSE.

6. Conclusion

The above sections show that fundamental principles upon which the modern structural optimisation codes are based are sound. The theory with respect to sizing variables is well established and the following chapters exploit it for a wide range of applications. It also forms a secure platform on which current research can build to develop new applications. The use of shape variables represents one such development and the inclusion of performance characteristics is another. Thus alternative design variables and objective functions are being introduced but the basis of algorithm development still remains the same as that given above.

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PRACTICAL ARCHITECTURE OF DESIGN OPTIMISATION SOFTWARE FOR AIRCRAFT STRUCTURES TAKING THE MBB-LAGRANGE CODE AS AN EXAMPLE

by

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Summary

The structural optimisation system MBB-Lagrange allows the optimisation of homogeneous isotropic, orthotropic or anisotropic structures as well as fiber reinforced materials. With the simultaneous consideration of different requirements in the design of aircraft structures it is possible to reduce the number of iteration steps between design, analysis and manufacturing.

Based on finite element methods for structures and panel methods for aerodynamics, the analysis with sensitivity includes modules for static, buckling, dynamic, static aeroelastic and flutter calculations.

The optimisation algorithms consists of mathematical programming methods and an optimality criteria procedure.

The important link between optimisation and analysis/sensitivity is the optimisation model which leads to a very modular architecture.

Typical application examples show the power and generality of the approach.

1. INTRODUCTION

Modern aircrafts are complex systems whose performance depends on the interaction of many different disciplines and parts. The complexity of the problems, that means the coupling among a very large set of governing equations, is dealed traditionally by solving only a subset of the system, such as aerodynamics, structures, flightmechanics, controls, etc. (Fig.1.1 [1]). For these individual disciplines great advances were made due to theoretical, computational and methodology break throughs. However, these more sophisticted methods of ten result in a decrease in the awarenes of the influence of the specialist's decisions in his area on other disciplines. On the other hand it became more and more troublesome to account strictly for all those couplings between these subsets only by parametric studies. In such investigations a relatively small number of principle parameters were varied, to find out their effects on the design requirements - which were themselves often contradictory - and to improve the design.

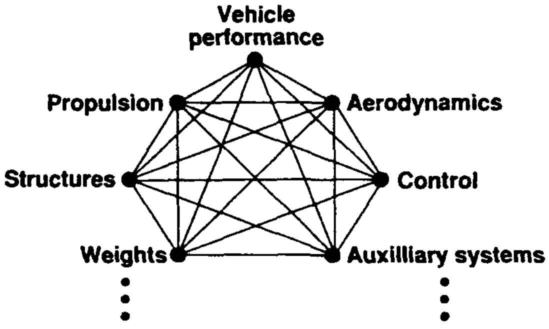


Fig. 1.1: The network of influences

With this approach, working in a limited design space, the engineer may achieve better results, but more often it leads to a penalty on the design objectives to make the initial concept feasible.

A more efficient way to integrate the different disciplines and to balance their distribution in the early design phases, is the multidisciplinary design optimisation approach (MDO). Mathematical optimisation algorithms together with reliable analysis programmes and the so-called optimisation model build up a basis for MDO-calculations with a high rate of generality and efficiency. This concept makes it possible to

- find designs which meet all specified requirements simultaneously
- achieve an optimal objective (or a combination of different goals)

without time consuming manual and more or less intuitive search for modifications of the initial design.

Looking at a typical data flow in the structural design phases of an aircraft (Fig. 1.2) the integrating effects of a general structural optimisation program can be seen. The program MBB-Lagrange is such a procedure which has been developed by MBB and several university institutes since 1984 [2].

In this lecture the different parts and the basic procedure of the structural optimisation process are described.

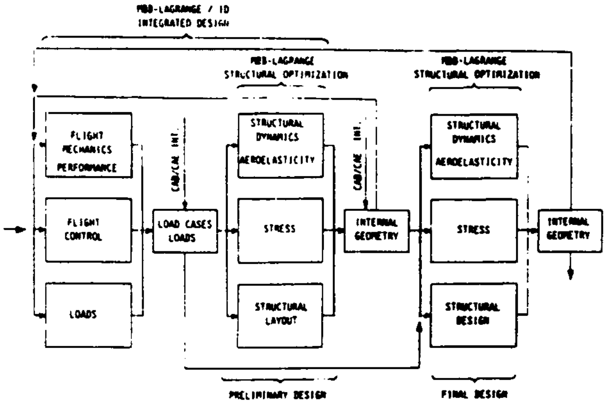

MBB-LAGRANGE / ID
INTEGRATED DESIGN
MBB-LAGRANGE
STRUCTURAL OPTIMIZATION
MBB-LAGRANGE
STRUCTURAL OPTIMIZATION
FLIGHT MECHANICS PERFORMANCE
FLIGHT CONTROL
LOADS
CAD/CAE INT.
LOAD CASES LOADS
STRUCTURAL DYNAMICS
AEROELASTICITY
STRESS
STRUCTURAL LAYOUT
CAD/CAE INT.
INTERNAL GEOMETRY
STRUCTURAL DYNAMICS
AEROELASTICITY
STRESS
STRUCTURAL DESIGN
INTERNAL GEOMETRY
PRELIMINARY DESIGN
FINAL DESIGN

Fig. 1.2: General data flow

2. PRACTICAL ARCHITECTURE OF MULTIDISCIPLINARY DESIGN OPTIMISATION SOFTWARE

For the treatment of optimisation problems the "Three-Columns-Concept" [3] defines the practical architecture of an optimisation program. In the case of structural design these three columns are

- Structural model
- Optimisation algorithm
- Optimisation model

The structural model is the mathematical description of the physical behaviour of the structure, i.e. the necessary analysis procedures for calculating state quantities. They are often based on finite element methods (FEM) for the structural part and on panel methods for the aerodynamic calculations but other analysis methods can also be applied (e.g. transfer matrix procedures for special shell structures).

The optimisation algorithm is a mathematical method for solving the general nonlinear problem (NLP).

minimize $f(\mathbf{x})$ (objective function) (2.1)

subject to $\mathbf{g}(\mathbf{x}) \geq \mathbf{0}$ (m_g in-equality constraints)
$\mathbf{h}(\mathbf{x}) = \mathbf{0}$ (m_h equality constraints)
$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u$ (lower and upper bounds for the designvaribles $\mathbf{x}$).

The relationship between the structural model and the optimisation algorithm is defined in the optimisation model, which is devided in the design model and the evaluation model. The design model contains the transformation between the mathematical quantities - the design variables - which are processed by the optimisation algorithm and the physical parameters - the structural variables - of which the optimal values have to be determined.

In the evaluation model, the values for the objective function and for the constraints are computed from the response quantities of the structural analysis.

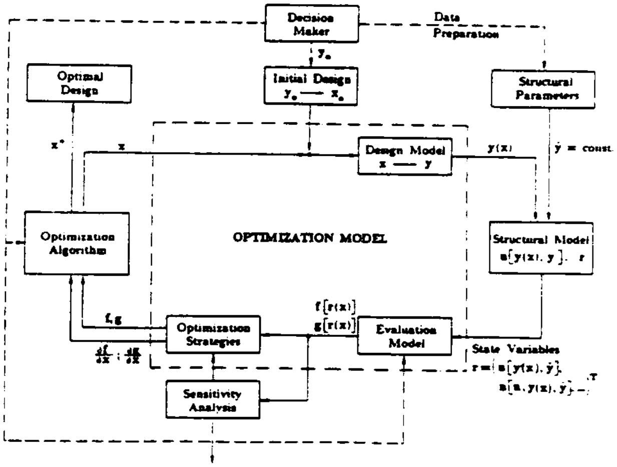

Decision Maker
Data Preparation
y_o
Optimal Design
Initial Design $y_o \longrightarrow x_o$
Structural Parameters
x^*
x
Design Model $x \longrightarrow y$
$y(x)$
$\bar{y}$ = const.
Optimization Algorithm
OPTIMIZATION MODEL
Structural Model $u[y(x), \bar{y}]$, r
f, g
$f[r(x)]$
$g[r(x)]$
Optimization Strategies
Evaluation Model
$\frac{df}{dx}$; $\frac{dg}{dx}$
State Variables
$r = \{u[y(x), \bar{y}], \sigma[u, y(x), \bar{y}], \dots\}^T$
Sensitivity Analysis

Fig. 2.1: Optimization loop

Fig. 2.1 shows the interaction of the three columns in the optimisation process [4]. First, the decision maker has to describe the structural and optimisation model for the special design problem. Based upon an initial design y_o for the structural variables, the corresponding initial x_o for the design variables are determined. The design model then yields the variable subset of the structural parameters to be optimised. These, together with the constant structural parameters (material constants, non-variable structural parameters), are taken to define a special design for which the state variables are calculated by the structural analysis. By means of the evaluation model the objective function and constraint values are calculated as one part of the input values for the optimisation algorithm. In addition to the functional values, most optimisation algorithms require the gradients of the behaviour functions with respect to the design variables which are evaluated by the sensitivity analysis. If a special optimisation strategy is applied, for example a strategy for solving a multicriteria optimisation problem, the behaviour functions and their derivatives are transformed into corresponding substitute values. Otherwise, they are directly transferred to the optimisation algorithm. Using this information, the optimisation algorithm calculates a new design variable vector and, thereby, one obtains a closed optimisation loop. If the optimal design is achieved, which is indicated in the optimisation algorithm by breaking-off criteria, the optimisation process is terminated.

3. Optimisation MODEL

Design Model

The design model describes the relationship between the structural variables and the design variables determined by the optimisation alorithm. If the finite element method is used for structural analysis the following structural variables are possible.

- Sizing Problems
  - Cross sectional area of elements
  - Thickness of elements
  - Laminte thickness of composite elements

  - Mass of concentrated masses
- Geometric problems
  - Fibre orientation angles for composite structures
  - Coordinates of nodes
  - Control parameters of parametric curves (e.g. B-Spline, Bezier, polynome, ...)
  - Pseudo loads
- Topological problems
  - Arrangement of elements (e.g. ribs, spars)

For integrated design problems, with additional discipline analysis techniques, such as linear aerodynamics and flight performance and flightmechanics, new types of variables must be considered. Aerodynamic variables can be:

- Wing shape
  - Surface area
  - Aspect ratio
  - Taper ratio
  - Sweep angle
  - Profile shape
- Wing topology
  - Arrangement of rudders and flaps
  - Hingelines

Looking at the flight performance, flightmechanics and control possible variables are

- Weight
  - Gross weight
  - Fuel weight
  - Payload
- Wing load
- Mission parameters
  - Range
  - Block times
- Thrust parameters
- Fin volume
- Control parameters

All these different types of physical variables, used in the discipline analysis codes, are often not very suited for a general mathematical optimisation algorithm. In order to avoid numerical difficulties and - especially for the finite element analysis - to reduce the number of design variables, a normalization and "Linking" of variables is performed.

As an example, equation (3.1) shows a linear transformation between structural sizing and fiber orientation variables and the corresponding design variables:

$$\begin{bmatrix} t \\ \alpha \end{bmatrix} = \begin{bmatrix} A_t & 0 \\ 0 & A_\alpha \end{bmatrix} x + \begin{bmatrix} t_0 \\ \alpha_0 \end{bmatrix} \quad ; \quad (3.1)$$

with

t	vector of the layer thicknesses of all finite elements,
α	vector of the layer angles of all finite elements,
A_t, A_α	linking matrices of layer thicknesses and angles,
x	design variable vector,
t_0, α_0	constant portions of the layer thicknesses and angles.

Arbitrary design models can be defined by the arrangement of the linking matrices A_t and A_α and the vectors of constants t_0 and α_0. It is also possible to link one common design variable with the structural variables of several elements in order to carry out a so-called "variable linking". On the other hand, one structural variable depends at most on one design variable which means that each row of the linking matrices contains at most one coefficient different from zero. The coefficients of the linking matrices and the vectors of constants are chosen in such a way that the design variables take on the dimension "1" in the design space and are precisely "1" in the initial design.

Evaluation Model

The evaluation model describes the requirements on the structure to be optimised. The special behaviour, which should attain a minimal or maximal value in the optimal design, is chosen as the objective function. With aircraft design it is primarily important to find a design with a minimal structural weight. However, any other state quantity (e.g. costs, fuel consumption etc.) can be considered as objective as well if there are several objective functions, the problem has to be solved by multicriteria optimisation strategies, which are discussed in [5].

All nonobjective requirements on the structural behaviour are formulated as constraints which are normally upper and/or lower bounds on the corresponding state variables. For the design of aircraft structures many different types of design requirements have to be considered and, there still is quite a lot of work in order to combine all of the necessary analysis and sensitivity analysis modules and optimisation modules within an multidisciplinary optimisation system. The following list contains constraint types for the different discipline analyses:

- Structural analysis,
  - Thermal stresses,
  - Strength (failure safety factor),
  - Displacements,
  - Stability (local buckling) [6, 7],
  - Dynamic quantities (eigenvalues, eigenvectors, transient- and frequency-response) [8, 9]
  - Manufactoring aspects
- Including steady and unsteady aerodynamic analysis methods, additional constraints are for example:
  - Aeroelastic efficiencies [10, 11]
  - Flutter speeds and damping
  - (elastic) Polar quantities (lift/drag ratios)
  - (elastic) Derivatives

- From flight performance, flightmechanics and -control the following constraints may arise:
  - Manoeuvre quantities
    - > Roll- and turn rates
    - > Start- and landing performance
  - Hinge moments
  - Stability margins
  - Handling qualities

The typical requirements for all these types of constraints is, that the considered response quantity must be less than an allowable value (e.g. stress, cost) or greater than a certain limit (e.g. flutter velocity, roll rate). In the case of the optimisation of a static structural model, the number of constraints can become very high (e.g. failure criteria in a large multilayer FRP-structure with a lot of critical load cases may lead to 100.000 and more constraints). It is clear, that the treatment of such a kind of problems is much different to optimisation tasks with a few constraints only (e.g. the maximum turn rates of an aircraft or the frequencies of rigid or/and elastic aircraft vibrations).

As an example for the mathematical formulation of the constraints, this latter mentioned frequency requirement is described in the following:

$$g_j(x) \ := f_{max} - f(x) \geq 0 \qquad (3.2)$$

or in a normalized form:

$$g_j(x) \ := 1 - \frac{f(x)}{f_{max}} \geq 0$$

This normalized representation has the great advantage of the independence of the physical value of the response quantity and guarantees a similar magnitude for all types of constraints. Thus an improvement of the convergence behaviour of the mathematical optimisation algorithm can be reached.

By means of all these above mentioned constraints on the state variables, many of the most important requirements for the design of aircrafts can be formulated. All the different types of constraints and the objective function - which can be defined by one or more of these constraints - form the evaluation model, and together with the design model they completely describe the optimisation model and the design task.

For practical applications it can not be expected and even it is not desired that there will be one computer program only for the optimal design of aircraft. A much better solution for the multidisciplinary design optimisation of a complex, internally coupled system behaviour is the separate evaluation of the individual discipline analysis and the partial sensitivity analysis with a well organized exchange of input and output data. But most important is an efficient method for calculating the coupled system sensitivities.

Such a formulation is presented in [13] and shortly described in the second part of this lecture by applying it to the integrated design of a fin.

4. THE OPTIMISATION ALGORITHM

Another column in the "Three-Column-Concept" represents the optimisation algorithms. In the previous chapter the different types of problems in the multidisciplinary design optimisation process were shown. Many practical applications in the last decade have proved, that it is necessary to provide several different optimisation strategies and algorithm to get reliable solutions, because there is no known single method which is adapted to every type of problem.

To understand the solution process for the NLP-problem formulated in equation (2.1), it is necessary to formulate the required optimality conditions (Kuhn-Tucker-Conditions):

$$\begin{aligned} &\nabla_x L(x^*, \lambda^*) = 0 \iff \nabla_x f(x^*) = \Sigma \lambda_j^* \nabla_x g_j(x^*) \\ &\lambda^*_j \cdot g_j(x^*) = 0 \qquad (4.1) \\ &\lambda^*_j \geq 0 \end{aligned}$$

where	$L = f - \lambda^T g$	is the Lagrangian function
	x^*	the optimal solution vector
	λ^*	the Langrangian multiplier in the optimum
and	∇_x	the gradient with respect to x.

That means, that in the optimum the gradient of the objective function is a non-negative linear combination of the gradient of the so-called "active constraints". The determination of these active constraints is one of the main problems of all optimisation procedures. The "less active" constraints have less influence on the current design change and are therefore temporarily neglected. Suitable deletion of these constraints accelerates the optimisation process, but it is not easy to manage.

To find the optimal solution vector x^*, most of the mathematical proramming algorithms uses the following iterative formulation:

$$x^{v+1} := x^v + \alpha^v s^v \qquad (4.2)$$

where s^v is the downhill search direction and α^v the step size. α^v is a positive scalar, which minimizes a function F in the direction of s^vusing a one-dimensional line search, that means:

$$F(x^{v+1}) = \min_{\alpha} [F(x^v + \alpha^v s^v)] \qquad (4.3)$$

The formulation of F depends on the optimisation method and is explained somewhat later. The calculation of the step size α^v is a relatively simple matter, which however requires the evaluation of the structural model and must therefore be carried out very effeciently.

Without going too much into detail, a classification of mathematical programming methods is given below [14]:

- Transformation methods
  - Penalty functions
  - Barrier functions
  - Method of multipliers

For these strategies, the function F is the original objective function f (**x**), augmented by a weighted penalty term, which summarizes the constraint values **g** (**x**).

For the inverse barrier method (IBF) for example, the transformed problem can be written as

$$F\,(\mathbf{x}^{v}) = f\,(\mathbf{x}^{v}) + r^{v}\,\Sigma\; g_j^{-1}\,(\mathbf{x}^{v}) \qquad (4.4)$$

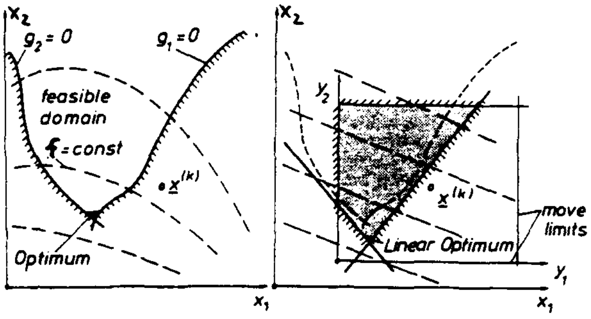
where the penalty parameter r^{v} is updated after each iteration step by

$$r^{v+1} = r^{v}\,c, \qquad 0 < c < 1 \qquad (4.5)$$

These strategies are very reliable, but they need a lot of function and gradient evaluations and are very "expensive".

- Primal methods
  - Indirect methods
  - Sequential linear programming (SLP)
  - Sequential quadratic programming (SQP)

These methods solve a sequence of linearized or quadratic subproblems. In the sequential linear programming methods the nonlinear functions f(**x**) and g(**x**) are expanded in Taylor series considering only the linear terms (Fig. 4.1). This linearized problem can then be efficiently solved by using the simplex algorithm. For problems with many active constraints this method works very efficiently. (Normally less than 10 iterations are needed!). But if there are only a few active constraints, convergence can worsen. In the case of highly nonlinear problems (e.g. buckling, structural dynamics), the method tends to fail because of the rough approximation (= linearization) of the original problem.



Original Nonlinear Problem *Linearized Substitute Problem*

Fig. 4.1: Sequential linearization SLP

For the SQP-methods, the quadratic subproblems result from a second order approximation of the Lagrangian function L(**x**,λ) and a linearization of the constraint functions **g**(**x**). The search direction **s** of equation (4.2) is then found by solving the following quadratic subproblem [15]:

$$\min.\,[\,\tfrac{1}{2}\,\mathbf{s}^{T}\,\mathbf{B}_{v}\mathbf{s} + \nabla_{x} f\,(\mathbf{x}^{v})^{T}\mathbf{s}\,] \qquad (4.6)$$

$$\nabla_{x} g_j\,(\mathbf{x}^{v})^{T}\,\mathbf{s} + g_j\,(\mathbf{x}^{v}) \geq 0$$

where $\mathbf{B}_{v}$ is an approximation of the Hessian matrix of the Lagrangian function for the v-th iteration step.

These methods are very general and robust. They can be used for a wide range of problems, independent of the ratio of active constraints and design variables. The accuracy of the optimal result is very good. If the starting point is far from the optimal solution, it might happen, that a lot of iteration steps (= function and gradient evaluations) are needed (more than 20). For this reason, the best efficiency is shown for medium size problems.

- Direct methods
  - Gradient projection method (GPM)
  - Generalized reduced gradients (GRG)
  - Method of feasible directions (MFD)

One principle idea of the GRG-method is the transformation of the m_g inequality constraints g(**x**) of the original problem (2.1) into equality constraints by introducing additional variables. By this means the optimisation process is working in the feasible domain. This leads to the following modified optimisation problem:

$$\begin{array}{lll} \min.\ f\,(\mathbf{x}) & & \\ \text{s. t. } \mathbf{h}\,(\mathbf{x}) = \mathbf{0} & & \\ \quad x_{l,i} \leq x_i \leq x_{u,i} & ; & i = 1 \ldots n \\ \quad 0 \leq x_i \leq \infty & ; & i = n+1, n+m_g \end{array} \qquad (4.7)$$

This system of (m_g + m_n) equations with (n + m_g) unknowns gives a solution for m_g + m_n so-called basis variables which depends on (n - m_h) nonbasis variables. A clever separation technique for these two types of variables and a linearization of the constraint functions **h**(**x**) results in a linearized objective function f_R(**x**), with a reduced set of variables, which depend only from the (n - m_h) non-basis variables x. The search direction s^{v} for this smaller problem can be found for example by using the negative reduced gradient of the objective function

$$s_R^{\,v} = -\,\frac{df_R\,(\mathbf{x})}{dx} \qquad (4.8)$$

or a modified direction, which take into account second order informations.

An important improvement of the efficiency of the GRG-method is the use of a SQP-search direction that means the solution of a quadratic subproblem [16]. This hybrid SQP-GRG-Algorithm can reduce considerably the number of function calcs and gradient evaluations.

- Dual concepts

The principles of the dual formulation are summarized in the following [17]: The solution of the primal problem (equation 2.1) can be obtained by a "Min-max" two phase procedure:

$$\begin{array}{l} \text{maximize } \; l\,(\lambda) \\ \text{subject to } \lambda_j \geq 0 \end{array}$$

where the dual function l(λ) which depends only on the Langrangian multipliers, result from minimizing the Lagrangian L (**x**, λ) over the allowable primal variables:

$$l(\lambda) = \min_{\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u} L(\mathbf{x},\lambda) \qquad (4.9)$$

Very important for the practical application of this method is the approximation concept of the objective and the constraint functions. A so-called "convex linearization" [18] for example, leads to a sequence of convex and separable subproblems with a likewise separable Lagrangian function, which can be solved easily by one-dimensional minimizing methods:

$$\min L_i(x_i) = a_i x_i + \frac{b_i}{x_i} \quad \text{s.t.} \quad x_{l,i} \le x_i \le x_{u,i} \tag{4.10}$$

where the coefficients

$$a_i = \sum_j c_{ij} \lambda_i + f_i$$

and $$b_i = \sum_j d_{ij} \lambda_j$$

depend only upon the dual variables λ_j. The coefficients f_i, d_{ij} and c_{ij} results from a mixed approximation of the objective and the constraint functions, where the f_i represents the first derivatives of the objective function and the d_{ij} denote the first derivatives of the constraint functions with respect to the design variables x_j (i.e the components of the gradients). The c_{ij} are the first derivatives of the constraint functions with respect to the reciprocal variable $z_i = 1/x_i$. (The type of the constraint approximation (direct or reciprocal) can be decided by the sign of the derivatives for example).

Practical applications of the CONLIN-algorithm for structural optimisation problems have shown a very efficient convergence behaviour for sizing as well as for shape design tasks.

Besides these mathemtical programming methods there exists another approach to solve the structural optimisation problem - the optimality criteria procedure [19].

For this formulation the stationary conditions of the Lagrangian function in the optimum (Kuhn-Tucker-conditions, eqs. 4.1).

$$\frac{\delta f}{\delta x} = \sum_j \lambda_j \frac{\delta g}{\delta x}$$

is written in the form

$$\sum_j e_{ij} \lambda_j = 1 \tag{4.11}$$

where e_{ij} is the ratio of the first derivatives of the constraints and the objective function. This set of equations can be solved easily for the unknown Lagrangian multiplies λ_j with a kind of separability assumption for the active constraints, which leads to an estimate of the Lagrangian multipliers:

$$\lambda_j = \frac{1}{e_{ij}} \tag{4.12}$$

Now an iterative resizing algorithm can be derived by multiplying both sides of equation (4.11) by x_i^α and taking the α-th root:

$$x_i^{v+1} = x_i^{v} \left[\Sigma\, e_{ij} \lambda_j \right]^{1/\alpha} \tag{4.13}$$

where α is relaxation parameter and can be seen as a step size parameter (e.g. $\alpha = 2$ assures a reasonable rate of convergence). The optimality criteria approach lead with a reltive small amount of computing effort to a solution, almost regardless of the number of variables. This is in general however, not an optimal design, especially if there exists not only one dominant type of constraint in the optimum, but a variety of different constraint types, which is often the case in mutlidisciplinary design optimization problems. Therefore it is necessary to have a very good understanding of the physics of the problem to decide, if an optimality criteria method can be used.

5. DISCIPLINE ANALYSIS AND SENSITIVITY ANALYSIS

The task of discipline analysis is to calculate the state quantities of the structure required to determine the constraint and objective values defined in the evaluation model. As mentioned before, most optimization algorithms do not only require the functional values of the behaviour functions but also their sensitivities with respect to the design variables. The calculation of these sensitivities can be carried out analytically as well as numerically by means of simple differential quotients. Since in the aircraft design one has often large scale design problems with sometimes several hundreds of design variables, it is necessary, for the sake of calculation effort and economy, to determine the sensitivities by the analytical differentation of all descriptive equations as far as possible.

In the following a brief survey of different type of analysis and sensitivity calculations is given.

Structural analysis

The structural analysis is based on the finite element method. This is a well-known reliable and very general way of modelling both the static and the dynamic behaviour of structures. It is possible to treat homogeneous materials with isothropic, orthotropic and anisotropic properties as well as composite materials. (For special types of structures it can make sense to use other - often very efficient - methods to describe the response quantities, e.g. Kirchhoff plate theory for thin wing structures [20] or transfer matrix procedures for cylindric shell structures [21].

Static problems

For linear elastic structures with static loads, the fundamental stiffness equation describes the structural response:

$$\mathbf{K}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x}) = \mathbf{p}(\mathbf{u}, \mathbf{x}) \tag{5.1}$$

with

$\mathbf{K}(\mathbf{x})$ stiffness matrix

$\mathbf{u}\,(\mathbf{x})$	displacement vector
$\mathbf{p}\,(\mathbf{u},\mathbf{x})$	load vector

With the displacement vector $\mathbf{u}\,(\mathbf{x})$ and the design variables $\mathbf{x}$, the stress and the strains in the structure can be calculated.

Besides these strength quantities, for lightweight aerospace structures exists the important problem of local and global instability, than means large deformations of the structure. Two main concepts for solving the buckling mechanism are shortly described in the following (Fig. 5.1).

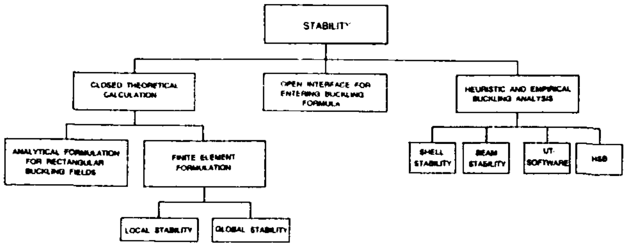

STABILITY
CLOSED THEORETICAL CALCULATION
OPEN INTERFACE FOR ENTERING BUCKLING FORMULA
HEURISTIC AND EMPIRICAL BUCKLING ANALYSIS
ANALYTICAL FORMULATION FOR RECTANGULAR BUCKLING FIELDS
FINITE ELEMENT FORMULATION
LOCAL STABILITY
GLOBAL STABILITY
SHELL STABILITY
BEAM STABILITY
UT-SOFTWARE
HSB

Fig. 5.1: Concept for stability calculation

One possibility is the formulation of an eigenvalue probelm:

$$[\mathbf{K}\,(\mathbf{x}) + \lambda \cdot \mathbf{K}_g\,(\mathbf{x})] = \mathbf{0} \qquad (5.2)$$

where

$\mathbf{K}_g\,(\mathbf{x})$	is the geometrical stiffness matrix the lowest eigenvalue, which defines the critical load by $p_{cr} = \lambda \cdot p$

and

$\mathbf{u}\,(\mathbf{x})$	the eigenvector, which represents the buckling shape.

For local instability problems (buckling of bars and shells) the critical loads and stresses can often be calculated with special stability equations (e.g. the well-known Euler equations for bars) [6,7].

In the case of a two-dimensional loading, considering tension/compression and shear forces with the following formula the margin of safety can be defined:

$$\frac{|\sigma\,(\mathbf{x})|}{\sigma_{cr}} + \frac{\tau\,(\mathbf{x})^2}{\tau_{cr}} \leq 1 \qquad (5.3)$$

The calculation of the critical stresses σ_{cr}, τ_{cr} can be done by solving analytically the differential equations of the plate-theory with special material and geometrical assumptions.

In the case of aeroelastic problems, the load vector $\mathbf{p}(\mathbf{u},\mathbf{x})$ depends on the deformation of the structure. The load vector is composed of a part that depends on the solution and one that is independent of it. Thus equation (5.1) can be written as:

$$\mathbf{K}\,(\mathbf{x}) \cdot \mathbf{u}\,(\mathbf{x}) = \mathbf{p}_o + \mathbf{p}\,(\mathbf{u}) \qquad (5.4)$$

Assuming linear aerodynamics the load can be expressed as

$$\mathbf{p}_o + \mathbf{p}\,(\mathbf{u}) = q\mathbf{T}_{SL}\,\mathbf{F} \cdot \mathbf{A}\,(\mathbf{w}_o + \mathbf{w}_e) + \mathbf{m}_s \qquad (5.5)$$

with

$q = \frac{\rho}{2}\,v^2$	Dynamic pressure
ρ	Air density
v	Air speed
$\mathbf{T}_{SL}$	Transformation of aerodynamic panel forces into finite element mode forces
$\mathbf{F}$	Aerodynamic panel surfaces
$\mathbf{A}$	Aerodynamic influence matrix
$\mathbf{w}_o$	Angle of attack of the panels for the rigid structure
$\mathbf{w}_e$	Angle of attack of the panels due to elastic deformations of the structure
$\mathbf{m}_s$	Mass loads

Expressing the elastic part by the finite element node displacements, the aerodynamic forces acting at the nodal points of the *FE-mesh become*

$$\mathbf{p}_A = q\,\mathbf{T}_{SL}\,\mathbf{F}\,\mathbf{A}\,\mathbf{w}_o + q\,\mathbf{T}_{SL}\,\mathbf{F}\,\mathbf{A}\,\mathbf{T}_{LC}\,\mathbf{T}_{SC}{}^{T}\mathbf{u}\,(\mathbf{x}) \qquad (5.6)$$

where the transformation

$\mathbf{T}_{LC}$	relates the panel corner displacements with the equivalent panel angle of attack

and

$\mathbf{T}_{SC}$	the panel corner displacements with the finite element node displacement.

With the abbrevation

$$\mathbf{C} = q\,\mathbf{T}_{SL}\,\mathbf{F}\;\mathbf{A}\,\mathbf{T}_{LC}\,\mathbf{T}_{SC}{}^{T}$$

the aeroelastic equilibrium equation can be written in the general form

$$(\mathbf{K} - \mathbf{C})\,\mathbf{u}\,(\mathbf{x}) = \mathbf{p}_o \qquad (5.7)$$

If the difference $(\mathbf{K} - \mathbf{C})$ is regular, that means non-singular, this equation can be solved. Because of the non-symmetry of the aerodynamic influence matrix, which is part of the matrix $\mathbf{C}$, the numerical effort for a direct solution of the equation (5.7) became very high, already for minor problems.

For that reason an iterative solution procedure was developed, where an additional relaxation process is introduced to improve convergence [10]:

$$\mathbf{K}\,\mathbf{u}^{(i+1)} = \omega\,\mathbf{C}\,\mathbf{u}^{(i)} + (1-\omega)\,\mathbf{K}\,\mathbf{u}^{(i)} + \omega\,\mathbf{p}_0 \qquad (5.8)$$

The convergence of the iteration strongly depends on the dominant eigenvalues λ_t of the corresponding eigenvalue problem:

$$(\omega\,\mathbf{C} + (1-\omega)\,\mathbf{K} - \lambda_t(\omega)\,\mathbf{K})\,\mathbf{w} = \mathbf{0} \quad , \qquad (5.9)$$

with the eigenvalue transformation

$$\lambda_t(\omega) = \lambda\omega - \omega + 1 \qquad (5.10)$$

where λ is either the maximal or the minimal eigenvalue of the original problem (eq. 5.7)

$$(\mathbf{C} - \lambda\,\mathbf{K})\,\mathbf{w} \qquad (5.11)$$

which can be computed by a simple v. Mises-Iteration:

$$\Delta\,\mathbf{x}^{(i+1)} = \mathbf{K}^{-1}\,\mathbf{C}\,\Delta\,\mathbf{x}^{(i)} \qquad (5.12)$$

An approach for the optimal relaxation parameter ω can than be found by the mean of the minimal and maximal eigenvalues from the following simple geometric relation, which is also indicated in Fig. 5.2:

$$\omega_{opt} = \frac{2}{2 - \lambda_{min} - \lambda_{max}} \qquad (5.13)$$

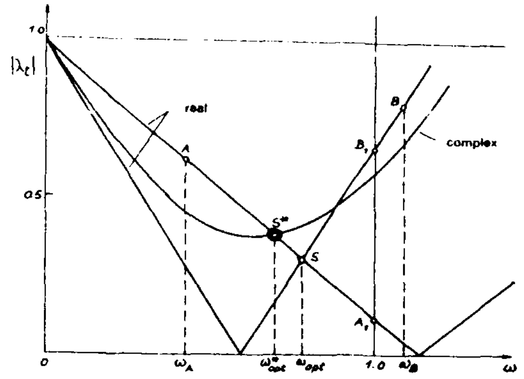


Fig. 5.2: Dominant Eigenvalues

In most cases **K** represents the symmetric and banded stiffness matrix of the finite element model of the structure (e.g. a wing) which can be decomposed by a Cholesky facturization as follows:

$$\mathbf{K} = \mathbf{L} \cdot \mathbf{L}^T \qquad (5.14)$$

where **L** is a lower triangular matrix. This fact is very important for the efficiency of the method, because one iteraion step, that means a better approximation of the solution vector $\mathbf{u}^{(i+1)}$, requires only one forward and one backward substitution with the right side of equation (5.8) using the vector $\mathbf{u}^{(i)}$ of the proceeding iteration step. Practical applications have shown a good convergence behaviour of this solution process. With the optimal value of the relaxation parameter ω, the displacement vector $\mathbf{u}(\mathbf{x})$, due to aerodynamic and mass forces can be determined. The solution can then be used for the computation of a so-called static aeroelastic efficiency of a structure. These factors describe the influence of the elastic structure on the aerodynamic forces and moments an are usually expressed in the form

$$\eta = \frac{\text{total load}}{\text{rigid load}} \qquad (5.15)$$

This ratio is normally less than one and depends on the dynamic pressure q acting on the structure. From equations (5.4 and 5.7) the aeroelastic efficiency can be obtained

$$\eta = \frac{\mathbf{s}^T\,[\mathbf{p}_0 + \mathbf{C}\,\mathbf{u}(\mathbf{x})]}{\mathbf{s}^T\,\mathbf{p}_0} \qquad (5.16)$$

with s as a vector for summing up the forces or moments of those aerodynamic panels which contributes to the efficiency (e.g. the panels on a control surface).

By using only unit cases for the angle of attack of the panels i.g. $\alpha_{uc} = 1$, it is possible to compute all α-dependent derivatives of the elastic complete aircraft (e.g. $C_{L\alpha}$, $C_{m\alpha}$ ..) and by the same way the derivatives with respect to the side-slip angle β (e.g. $C_{Y\beta}$, $C_{n\beta}$...) can be calculated, too.

These method is also applicable to the determination of the derivatives which depend on the rotational degrees of freedom of the airplane, i.e. roll velocity p, pitch velocity q and yaw velocity r. For these cases the distribution of angle of attack α and sideslip β depend on the distance of the panels to the corresponding axis, respectively to the center of gravity and on the flight velocing v. With that, the angle of attack α_i of a panel i due to pitching for example, can be written as:

$$\alpha_i = q\,x_i / v \qquad (5.16)$$

with x_i as distance from the pitch axis.

To calculate these elastic derivatives of the complete aircraft, the decomposition of the stiffness matrix has to be done (eq. 5.14), which is only possible if the matrix **K** is positiv definit. That means, that the airplane either has to be supported or the rigid body degrees of freedom must be eliminated by special expensive and time consuming transformations [22]. It can be shown [23], that a statical determined support of the aircraft (e.g. close by the center of gravity) gives correct results for total aircraft loads using the unit case method, if trim conditions are considered. That means, that the total sum of forces and moments due to aerodynamics and masses, which act on the airplane, has to be zero. These trim conditions can be written in the following short form [12], [23].

$$\mathbf{Z}^T\,\mathbf{r} = \mathbf{Z}^T\,\mathbf{H}\,\Psi + n_g\,\mathbf{Z}^T\,\mathbf{h}_m \qquad (5.17)$$

with

$\mathbf{r}$	total sum of external loads
$\mathbf{H}$	unit aerodynamic loads
$\mathbf{h}_m$	mass loads
n_g	load factor
$\mathbf{Z}$	marix, which contains the trim conditions (= sum of forces and moments)

and

ψ	the vector of the unknown factors for the trim parameters (e.g. rudder deflection, angle of attack for a steady two degree of freedom longitudinal case).

The total elastic deformation $\mathbf{u}(\mathbf{x})$ is finally achieved by multiplying the deformations $\mathbf{u}_{uc}(\mathbf{x})$ resulting from the unit cases with the scaling factors ψ:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_{uc}(\mathbf{x})\, \Psi(\mathbf{x}) + n_g\, \mathbf{u}_m(\mathbf{x}) \tag{5.18}$$

where $\mathbf{u}_m(\mathbf{x})$ is the deformation due to mass load for load factor one.

Using the displacements determined by the global static structural analysis, the strains and stresses can now be calculated. Especially for fiber compostite structures the safety against material failure is usually checked by means of various failure criterias, e.g. according to Tsai-Wu, Tsi-Hill, Hoffmann and others [24].

All these in the foregoing sections explained state variables, which will be generally denoted by the vector $\mathbf{r}$ in the following, depend on design variables with an explicit dependency on the equation parameters and on implicit dependency on the structural deformation $\mathbf{n}$. Therefore the corresponding constraints are formulated as

$$\mathbf{g} = \mathbf{g}\,[\mathbf{r}(\mathbf{x},\mathbf{u})] \tag{5.19}$$

and the derivatives of the constraint vector $\mathbf{g}$ with respect to the design variables as it is needed for the optimisation algorithm can then be achieved by using the chain rule:

$$\frac{\delta \mathbf{g}}{\delta \mathbf{x}} = \frac{\delta \mathbf{g}}{\delta \mathbf{r}}\left(\frac{\delta \mathbf{r}}{\delta \mathbf{x}} + \frac{\delta \mathbf{r}}{\delta \mathbf{u}}\frac{\delta \mathbf{u}}{\delta \mathbf{x}}\right) \tag{5.20}$$

The derivative of the constraint vector $\mathbf{g}$ with respect to the state variables $\mathbf{r}$ depends only on the applied discipline analysis and is determined by the evaluation model. In the following the solution method is shortly explained for the state variable aeroelastic efficiency η, where

$$g = \frac{\eta}{\eta_{min}} - 1 \geq 0 \tag{5.21}$$

Differentation with respect to design variable x_i leads to

$$\frac{\delta g}{\delta x_i} = \frac{1}{\eta_{min}}\frac{\delta \eta}{\delta \mathbf{u}}\frac{\delta \mathbf{u}}{\delta x_i} \tag{5.22}$$

With equations (5.7,5.15)

$$\frac{\delta \eta}{\delta \mathbf{u}} = \mathbf{h}^T$$

and

$$\frac{\delta \mathbf{u}}{\delta x_i} = -(\mathbf{K} - \mathbf{C})^{-1}\frac{\delta \mathbf{K}}{\delta x_i}\mathbf{u} \tag{5.23}$$

and using an auxiliary vector $\mathbf{d}$, which can be calculated by the same iterative solution procedure as shown in equation (5.8)

$$\mathbf{h}^T(\mathbf{K} - \mathbf{C})^{-1} = \mathbf{d} \tag{5.24}$$

the derivative can be calculated finally by

$$\frac{\delta g}{\delta x_i} = -\frac{1}{\eta_{min}}\mathbf{d}^T\frac{\delta \mathbf{K}}{\delta x_i}\mathbf{u} \tag{5.25}$$

The derivtive of the stiffness marix $\mathbf{K}$ can be calculated analytically for thickness and fiber orientation as design variables by using the finite element formulation [25]. For general geometry variables the sensitivity can be achieved numerically:

$$\frac{\delta \mathbf{K}}{\delta x_i} = \frac{\mathbf{K}(\mathbf{x} + \varepsilon x_i \mathbf{e}_i) - \mathbf{K}(\mathbf{x})}{\varepsilon\, x_i} \tag{5.26}$$

with

$\mathbf{e}_i$	unit vector
ε	small real number

The computing effort for the additional stiffness matrices can be essentially reduced, if only the terms effected by x_i are calculated anew.

Dynamic Problems

The general equation of motion describing time-dependent system deformations reads as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}(t) \tag{5.27}$$

with

$\mathbf{M}$	Mass matrix
$\mathbf{D}$	Damping matrix
t	Time

In the case of undamped eigenvibrations, equation (5.27) can be transformed into a real eigenvalue problem using a harmonic approach for the displacements

$$(\mathbf{K} - \omega_j \mathbf{M})\,\mathbf{y}_j = \mathbf{0} \tag{5.28}$$

where

ω_j is the j-th natural frequency

and $\mathbf{y}_j$ the j-th eigenvector

The constraints on natural frequencies usually consist in imposing lower or upper limits. With the normalized formulation (eq. 3.2, 5.19) the sensitivity of problems with frequency constraint is given by

$$\frac{\delta g_j}{\delta x_i} = \frac{\delta g_j}{\delta \omega_j}\frac{1}{2\omega_j} \cdot y_j^T\left(\frac{\delta \mathbf{K}}{\delta x_i} - \omega^T \frac{\delta \mathbf{M}}{\delta x_i}\right) y_j \tag{5.29}$$

The derivative of the mass matrix **M** can be calculated in the same way as described earlier for the stiffness matrix **K**.

A more advanced type of constraints from dynamics is given by frequency or transient response problems. These may occur when a structure is loaded harmonically or by a time dependent load and it is required that the displacement, velocity or acceleration at certain points of the structure must not exceed prescribed values.

To get the structural response quantities in the time-domaine the second order differential equation (5.27) has to be solved. This can be done for example by the method of Newmark.

The displacement vector of a harmonically loaded structure (excitation frequency Ω) is computed by

$$(-\Omega^2\mathbf{M} + i\Omega\mathbf{D} + \mathbf{K})\,\mathbf{u} = \mathbf{p}(\Omega) \tag{5.30}$$

The solution of these equations (5.27, 5.30) is quite expensive. To reduce the computational effort, normally a transformation is introduced:

$$\mathbf{u} = \mathbf{T}\,\mathbf{q} \tag{5.31}$$

The transformation matrix **T** is a n x m - Matrix (m <<n) which reduces the original system drastically. If **T** contains eigenvectors of the structure (normally the lowest one), equation (5.31) represents the transformation to modal coordinates. In recent years the transformation to Lanczos coordinates have been proposed in structural dynamics. This method was shown to be especially promising because it is a load-depenent transformtion which approximates the influence of higher modes as well.

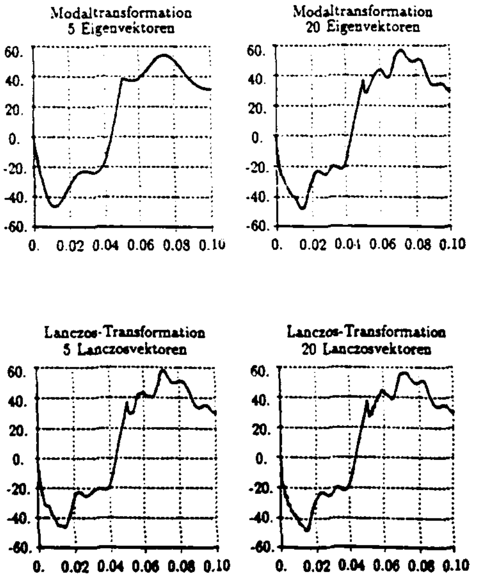


Fig. 5.3: Comparison of the acceleration of a cantilever plate

Fig. 5.3 shows the results for the acceleration of a point of a cantilever plate in the time domaine. In this example five and twenty transformation vectors for the modal respectively the Lanczos transformation are used

It can be seen that only five vectors on the Lanczos case are necessary to get good results [8]. Formulation of constraints on transient and frequency response quantities and the corresponding sensitivity analysis is quite complicated and out of scope of this lecture and well described in [8,9].

A special kind of harmonically loading, however, must be mentioned in this context. The phenomena of self-exciting vibrations of elastic structures in a flow field, which is a dynamic stability problem and designated as flutter. Due to the interaction of the aerodynamic forces, the elastic forces and the inertia forces with the structural deformation, there is an exchange of kinetic energy of the air flow with the elastic and kinetic energy of the structure

At the boundary between damping and excitation there is no energy exchange, which means that small disturbances lead to harmonic vibrations. Depending on the stiffness and mass distribution of a structure, such a critical case occurs when certain combinations of flow velocity and Mach number are given. The corresponding critical flow velocity is called flutter speed. Since no flutter case can be admitted in the whole mission range of an aircraft it must be required that the smallest flutter speed does not fall short of a certain limit given by the maximal flight speed plus safety increase (15% safety increase for military aircrafts, 20% for civial aircrafts). The maximal flight speed can be taken from the so-called flight envelope, which depicts the mission range of an aircraft.

According to (2.1) the flutter constraint can be formulated as

$$g = {}^{v}F / v_{max} - 1 \tag{5.32}$$

with v_F flutter speed
v_{max} maximum flight speed

For the determination of the flutter point, that means the calculation of the critical flow velocity v_F, harmonic aerodynamic forces which depend on the harmonic deflection **u** of the structure are introduced:

$$\mathbf{p}(t,\omega) = \mathbf{C}(\omega,\mathrm{Ma})\,\mathbf{u}\,e^{i\omega t} \tag{5.33}$$

where **C** contains the complex aerodynamic influence matrix and the transformation of loads from the aerodynamic into the finite element mesh similar as described for the static aeroelastic (eq. 5.6) and the dynamic pressure.

The aerodynamic influence matrix is a fully occupied, non-symmetric complex matrix depending on the Mach number and the reduced frequency k. For constant altitudes the dependency on the Mach Number can be transformed into a dependency on the airspeed. With (5.27, 5.33) and by neglecting material damping the flutter analysis equation can be written as:

$$[K - C(v,k) + \lambda^2(v,k)\,M]\;q(v,k) \tag{5.34}$$

This equation contains a system reduction according to

(5.31) to reduce the numerical effort,with

$K = T^T K T$		Generalized stiffness matrix
$M = T^T M T$		Generalized mass matrix
$C = T^T C T$		Generalized aerodynamic load matrix

and

q	generalized coordinates, right hand eigenvector

The complex eigenvalue problem of equation (5.34) can be solved for example with a QR-algorithm or - if an initial solution is known - with the very efficient perturbation method by Wittmeyer [26].

The resulting complex eigenvalues of the flutter equation depend on the reduced frequency k and the velocity v:

$$\lambda_E(v,k) = \lambda_E'(v,k) + i\,\lambda_E''(v,k) \qquad (5.35)$$

where

λ_E	is the real part or the damping
λ_E	the imaginary part or the eigenfrequency.

With the notation of (5.35) the definition equation of the reduced frequency k of the aerodynamic matrix, is given by

$$k = \frac{\lambda_A''\,l}{v} \qquad (5.36)$$

or

$$\lambda_A''(v,k) = \frac{v\,k}{l}$$

That means, that valid points for a flutter curve are only those, where the imaginary part of a solution λ_E, i.g. the frequency λ_E'', corresponds to the frequency λ_A'' of the oscillating airload. This requirement can be formulated by the following intersection condition

$$\lambda'' = \lambda_A'' = \lambda_E'' = \lambda''[v(k)] \qquad (5.37)$$

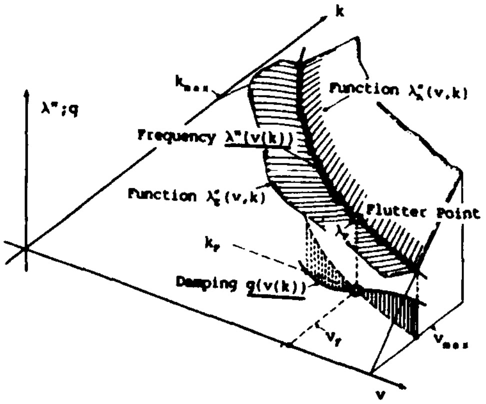


Fig. 5.4: Evaluation of flutter curves

A flutter point, finally, is found for a velocity v_F, in which the real part of the eigenvalue λ_E', vanishes, that means an undamped, harmonic motion takes places (Fig. 5.4) with the frequency λ_F''.

Using this flutter velocity in the constraint equation (5.32), the derivative of the flutter constraint with respect to design variables, can be achieved.

At first the differentation of the flutter equation (5.34) leads to the sensitivity of the eigenvalue in the flutter point:

$$\frac{\delta\lambda_F}{\delta x_i} = -\frac{1}{2\lambda_F}\,p^T\left(\frac{\delta K}{\delta x_i} - \frac{\delta C}{\delta x_i} + \lambda^2\frac{\delta M}{\delta x_i}\right)q \qquad (5.38)$$

with

p	left hand eigenvector

and the normalization

$$p^T M q = 1 \;.$$

The sensitivity equation of the flutter speed itself is obtained by re-arrangement and differentation of the definition equation of the reduced frequency:

$$\frac{\delta v_F}{\delta x_i} = \frac{1}{k}\frac{\delta\lambda_F}{\delta x_i} - \frac{1\,\lambda_F}{k^2}\frac{\delta k}{\delta x_i} \qquad (5.39)$$

With these basic equations (5.38, 5.39), and the additional condition, that in the flutter point, the real part of the eigenvalue, i.g. the damping vanishes and finally taking into account the dependency of the aerodynamic influence matrix from the reduced frequency k and the Mach number Ma, the derivation of the flutter constraint can be obtained.

It should be mentioned, that the differentiation of the transformed matrices *K*, *M* and *C* with respect to the design variables includes terms, which depend on the derivative of the transformation matrix T. In the case of a modal transformation, T contains a number of eigenvectors of the undamped, homogeneous eigenvalue problem (5.28), which have been chosen for the system reduction. The calculation of these derivatives requires a high numerical effort and it has to be investigated if their influences can be neglected. It can be shown, that if the transformation T contains all possible eigenvectors, that means no system reduction is achieved (m = n), these terms vanishes exactly.

	15 Normal Modes		20 Normal Modes	
$\frac{\partial g}{\partial x_i}$	V_1 = 426.5 m/s analyt.	num.	V_1 = 419.6 m/s analyt.	num.
1	-6.084-3	-7.350-3	-8.117-3	-8.668-3
2	7.971-4	-6.534-4	-5.751-4	-2.127-3
3	1.593-4	+2.742-4	-2.117-4	-4.145-4
4	1.457-1	1.677-1	1.602-1	1.637-1

Fig. 5.5: Comparison of analytical/numerical flutter gradients

In Fig. 5.5, the results for the derivation of the flutter constraints for the fin - example described in [27] are depicted. The comparison between the numerical and the analytical flutter gradients shows, that with an increasing number of modes, the quality of the analytical derivation becomes much better.

6. THE SOFTWARE SYSTEM MBB-LAGRANGE

Corresponding to the Three-Columns-Concept described in section 2, the software system MBB-Lagrange is divided into the main modules structural and sensitivity analysis, optimization modeland optimization algorithms (see Fig. 6.1). The proram contains design models for cross-sections or wall-thicknesses of isotropic elements for layer thicknesses and layer angles of fiber composite materials, and for concentrated masses as well.

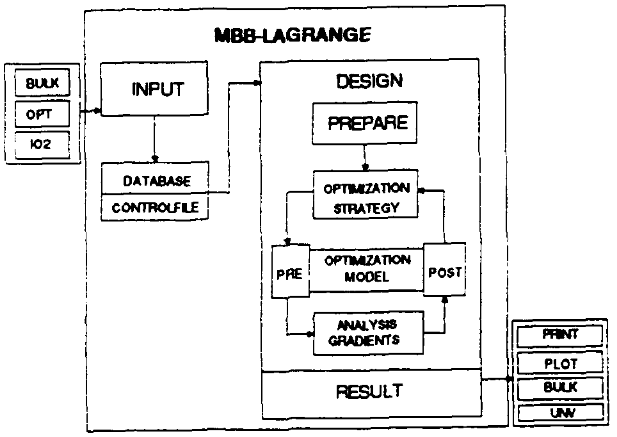


Fig. 6.1: General program architecture

In order to choose the most suitable optimization algorithm for a specific problem, the following algorithms are supplied:

1. IBF : Inverse Barrier Function,
2. MOM : Method of Multipliers,
3. SLP : Sequential Linear Programming,
4. SRM : Stress Ratio Method,
5. RQP1 : Recursive Quadratic Programming (Schittkowski),
6. RQP2 : Recursive Quadratic Programming (Powell),
7. GRG : Generalized Reduced Gradient,
8. CONLIN : Convex Linearization,
9. QPRLT : Quadratic Programming with Reduced Line Search Technique (SQP-GRG-Combination)

The structural and sensitivity analysis consists of the procedures for determining the various state variables and their gradients, to characterize the static, dynamic, aeroelastic and stability behaviour.

The input data for the optimisation are divided into a control part, a part for the description of the FE-model and one describing the optimization model. The FE-description is done in form of a NASTRAN-Bulk-Data-Deck. For an aeroelastic analysis the aerodynamic influence matrices must be supplied additionally.

The optimisation results are documented by the following data:

- a file for describing the optimization history,
- plotfiles for the graphic illustration of the optimisation history,
- re- and warmstart files for continuing an optimisation,
- a NASTRAN-Bulk-Data-Deck of the optimised structure,
- an IDEAS-Universal file of initial and final design for the graphic illustration of the structural parameters, displacements, stresses, strains, values of the safety factor etc.

Besides the IDEAS-interface there is also an interface to the pre- and post-processing-system PATRAN.

Since many different optimisation routines are available, a user must either define them "by hand" or he requires a selection made by the system. In this case a rule-based subprocess [28] will send some questions to the terminal and depending on the answers and the information on the design available so far, a heuristic proposal is made. A user may accept the proposed method and parameters or he may choose another code. The following table shows an example for the so-called safety factors, which indicates if a strategy will be more or less successful: 0 means not possible, 100 means it is the best.

IBF	MOM	SLP	SRM	RQP1	RQP2	GRG	CONLIN
0	63	64	0	70	70	14	75

If some results are available obtained from a previous run with the same algorithm, it is possible to perform a warm start, i.e., continuation of the iteration which was interrupted before by exceeding the maximum number of iterations. Otherwise a cold start may be activated starting from the last computed iterate or alternatively, a new optimisation cycle is initiated starting from the originally given design variables.

MBB-Lagrange possesses a very flexible failure sytem and it is out of the scope of this report to explain all of its features. Severe failures interrupting the optimisation, are written to a output file and are sent to the terminal. By activating the failure analysis, a user will see the same failure information again. Subsequently a rule-based, heuristic proposal of a suitable remedy is displayed and the user may accept the proposed action or not.

7. EXAMPLES

The following sample of examples gives a good idea of the capabilities of MBB-Lagrange.

Airbus A300/600 Support Beam

Beside the stringers and flanges, the cabin floor influences the mechanical behaviour of the fuselage of a passenger or cargo aircraft. Important parts of the cabin floor are a large number of support beams. The beams are connected to the fuselage at both ends and are supported by struts.

The support beam presented here has a shape of an U-profile. Because of the symmetry only half of the structure is considered for the FE-model, which gives a total size of 1068 elements, 6409 degrees of freedom and one load case (Fig. 7.1).

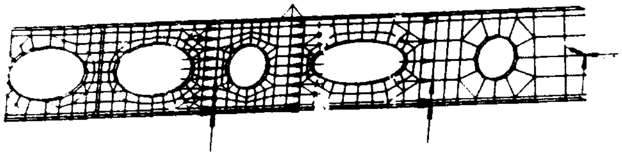


Fig. 7.1: A300/600 Support Beam (Finite Element Model)

The beam is manufactured by milling, which allows a very fine discretisation of the wall thickness. For that reason 221 design variables could be defined. The stress requirements are assured by von Mises constraints on each element.

Additional special empirical functions for compression (crippling allowables), according to the german aircraft industry's design manual, are defined for the flanges and stiffening holes, to include stability constraints.

The optimisation is carried out by the SLP-algorithm in 5 to 13 iteration steps and the weight is reduced by about 30 percent depending on the loading condition.

Frame of a Combat Aircraft Fuselage

This frame is located in the inlet for the engine of a combat aircraft. It is a typical example for a sizing problem of a light weight structure made of an aluminium alloy. For the formulation of the optimisation problem it is important to know that a milling machine will be used to realize a variable thickness distribution. So large number of design variables can be defined in order to calculate the optimal thickness of the frame.

The finite element model is shown in Fig. 7.2. It involves 975 degrees of freedom, 930 elements and 97 load cases.

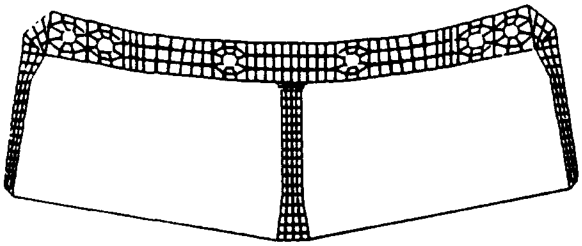


Fig. 7.2: Finite Element Model of Frame

The objective function is the weight. The constraints ensure static requirements. For each element the feasibility of the stresses have to be ensured. Stability constraints are taken into account to prevent buckling and lateral instability of webs. Since these constraints have to be satisfied for each single load case we get a very large number of inequality constraints ($m_g = 92829$). Together with the thickness design variables ($n = 187$) it is a really large scale optimisation problem.

The initial design has infeasible buckling loads and stresses. The optimisation algorithm SLP needs 20 iterations to achieve convergence. Fig. 7.3 shows the optimisatoin history of the buckling constraints, where respectively the most critical constraint of the corresponding iteration is taken. The optimal design fulfills all static requirements of all 97 load cases and achieves a weight reduction of about 25 percent.

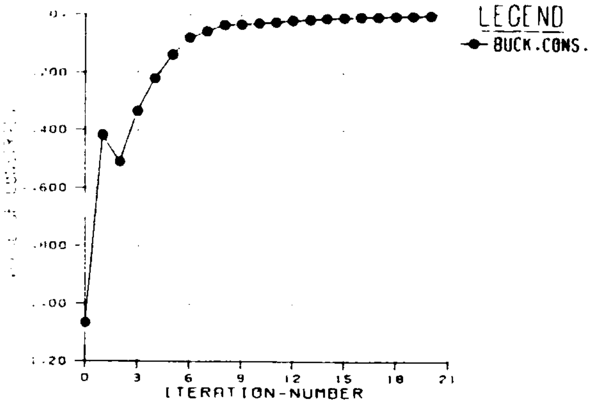


Fig. 7.3: History of most critical buckling constraint

Horizontal Stabilizer of a Helicopter [7]

The structure consists of an airfoil section like an airplane wing and endplates which act as vertical stabilizers. The upper and lower panels of the airfoil section are sandwich plates with a honeycomb core and aramid fiber face sheets. The spar is an I-shaped bar with sraps made from unidirectional carbon fiber reinforced material and a shear web which is a honeycomb sandwich with CFRP face sheets.

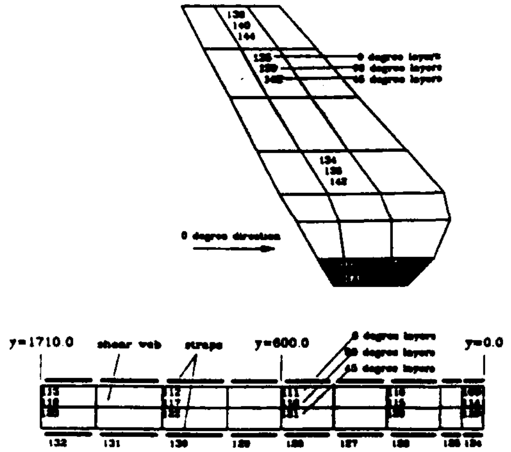


Fig. 7.4: Design variables of the spar and the endplate

The endplates are sandwich plates of constant thickness consisting of aramid fiber reinforced face sheets and a honeycomb core. They are fixed to the airfoil section by screws.

Fig. 7.4 shows the design variables of the span and the endplate.

Three load cases define the loading of the stabilizer. The structure shall withstand these loadings with a factor of safety larger than 1.5. Sandwich wrinkling has to be considered as well as a composite failure criterion such as that from Tsai-Wu. Additionally to these constraints, a lower bound for the first eigenfrequency is given. The results are shown in Fig. 7.5.

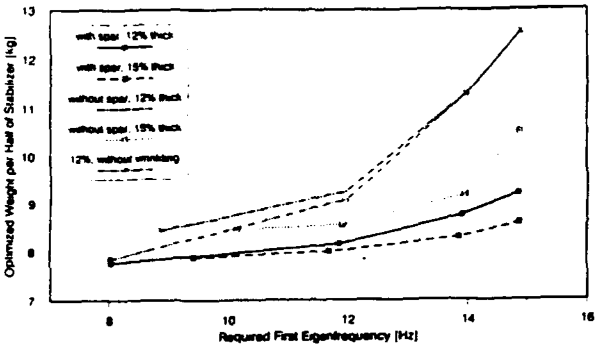


Fig. 7.5: Minimum weight of the horizontal stabilizer versus the first eigenfrequency

The original design with spar has a structural weight of 10.6 kg (one half of the airfoil section plus one endplate) and a first eigenfrequency of 14 Hz. Considering only strength restrictions, the weight can be reduced by nearly 3 kg, but in this case the first eigenfrequency drops significantly. If the first eigenfrequency is held constant, then a weight reduction of about 2 kg is possible.

From Fig. 7.4 it can be clearly seen that the design with a spar is far better than a design without a special spar. Concerning the thickness of the airfoil section it can be stated that for low stiffness (e.g. a low first eigenfrequency) there is nearly no difference in the weight between a thin (12%) and a thicker (15%) profile. Only if a high stiffness is required (a high first eigenfrequency) a thicker aerodynamic profile is useful.

Composite Fin

Fig. 7.6 shows the structural model of the well known MBB-Fin. The cover skins of the fin are made of carbon fiber laminate with four different fiber orientations in the stabilizer and three in the rudder. The inner supporting structure is realized by an aluminium honeycomb core. The fin is supported at the connection points to the fuselage and the stiffness of the fuselage is modeled with a general stiffness element. As static load cases the aerodynamic forces of five different flight conditions (different sideslips and rudder deflections; subsonic and supersonic) are chosen.

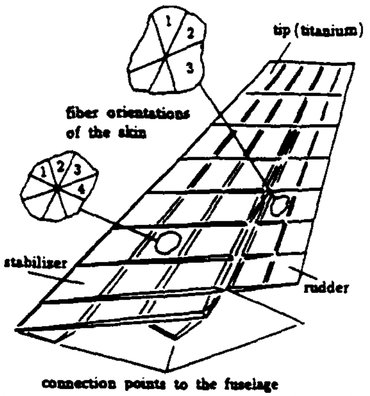

1
2
3
tip (titanium)
fiber orientations of the skin
1
2
3
4
stabilizer
rudder
connection points to the fuselage

Fig. 7.6: Structural model of the fin

For this concept study 1862 constraints were defined:

- Stress limitation (isotropic elements 119/load case)
- Limitation of failure safety (FRC. 252/load case)
- Aeroelastic efficiencies - Fin (0.8)
  (Ma = 1.8 (750 kts) - Rudder (0.5)
- Flutter speed - 530 m/s

The problem consists of 102 sizing-design variables (one independent design variable for every layer in every element).

The sizing optimum results in a weight of 42.3 kg (= 100%) for the variable skin weight.

By introducing the layer angles as additional design variables it is possible to define a lot of other optimisation models.

A model with sizing plus 7 layer angle variables (one design variable is assigned to each layer of the stabilizer and of the rudder) leads to an optimal weight of 34.6 kg. An optimal weiht of 25.3 kg is achieved for an optimisation model with additional 84 layer angle variables, Fig. 7.7 (one design variable for every angle in each element and a linking of the first and the third layer). This weight is the theoretical lower limit and it will be not manufacturable but it shows the high potential on weight saving possibilities including fibre orientation as design variables. But it is also obvious, that manufacturing requirements has to be considered.

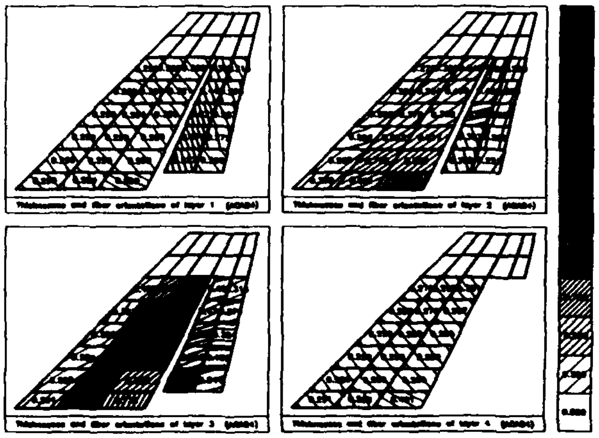


Fig. 7.7: Optimal thickness distribution for the design model with 84 layer angle variables

For this reason a development was initiated which includes these manufacturing informations as additional constraints into the optimisation model. By this it will be possible in the near future, to have the optimisation as the "driving part" in the complete composite design process.

Integrated Fin Design

This example, using the same structural and aerodynamic model, is an approach to an integrated design analysis with not only structural sizing variables t but also three additional aerodynamic design variables:

- taper ratio λ
- aspect ratio Λ
- surface area S

The interesting response quantity of this study is the unit side load p as basic flight mechanic design requirement for a vertical fin. It depends on the aerodynamic derivative C_β on the surface area S and the aeroelastic efficiency η. The state variable equations for this multidiscipline problem can be formulated in a generalised form as:

$$p = c_\beta \, \eta \, S \qquad \text{(Flight mechanics)}$$

$$C_\beta = f_A (\lambda, \Lambda) \qquad \text{(Aerodynamic)}$$

$$\eta = f_s (\lambda, \Lambda, S, t) \qquad \text{(Structure/Aeroelastics)}$$

The internal coupling of the system is given by the first equation. The system sensitivity equations can be formulated, using the method proposed by [13]. The partial derivatives of the state variables, which will be provided by the individual disciplines are on the right hand side of the system sensitivity equations.

$$\begin{bmatrix} I & -\frac{\partial p}{\partial c_\beta} & -\frac{\partial p}{\partial \eta} \\ -\frac{\partial c_\beta}{\partial p} & I & -\frac{\partial c_\beta}{\partial \eta} \\ -\frac{\partial \eta}{\partial p} & -\frac{\partial \eta}{\partial c_\beta} & I \end{bmatrix} \begin{bmatrix} \frac{dp}{d\lambda} & \frac{dp}{d\Lambda} & \frac{dp}{dS} & \frac{dp}{d\underline{t}} \\ \frac{dc_\beta}{d\lambda} & \frac{dc_\beta}{d\Lambda} & \frac{dc_\beta}{dS} & \frac{dc_\beta}{d\underline{t}} \\ \frac{d\eta}{d\lambda} & \frac{d\eta}{d\Lambda} & \frac{d\eta}{dS} & \frac{d\eta}{d\underline{t}} \end{bmatrix} = \begin{bmatrix} \frac{\partial p}{\partial \lambda} & \frac{\partial p}{\partial \Lambda} & \frac{\partial p}{\partial S} & \frac{\partial p}{\partial \underline{t}} \\ \frac{\partial c_\beta}{\partial \lambda} & \frac{\partial c_\beta}{\partial \Lambda} & \frac{\partial c_\beta}{\partial S} & \frac{\partial c_\beta}{\partial \underline{t}} \\ \frac{\partial \eta}{\partial \lambda} & \frac{\partial \eta}{\partial \Lambda} & \frac{\partial \eta}{\partial S} & \frac{\partial \eta}{\partial \underline{t}} \end{bmatrix}$$

$$\boxed{\text{DESIGN VARIABLES} \qquad \lambda \qquad \Lambda \qquad S \qquad \underline{t}}$$

Fig. 7.8 System sensitivity equations

The derivatives with respect to the aerodynamic design variables are done by using the finite difference method with a 10% perturbation magnitude (Fig. 7.9).

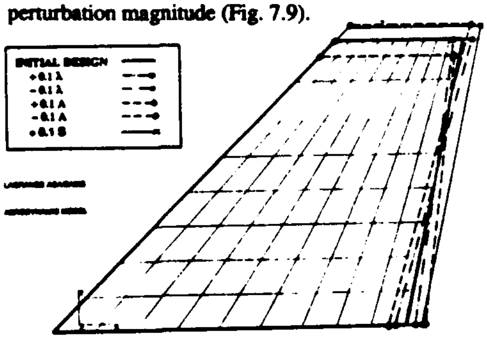


Fig. 7.9: Aerodynamic shape differencies

The results of this design study are shown in Fig. 7.9). The state variable p is plotted for all finite difference sensitivities of the design variable λ, Λ, S and for the optimised element thicknesses, with an aeroelastic efficiency fin requirement of 80 percent. (The stress and strain constraints coming from five static load cases are in the optimal design also fulfilled.) The best integrated design solution is got with a 10 percent reduction of aspect ratio. In this case the lateral unit load will be slightly increased and the weight is reduced by 7.5 percent.

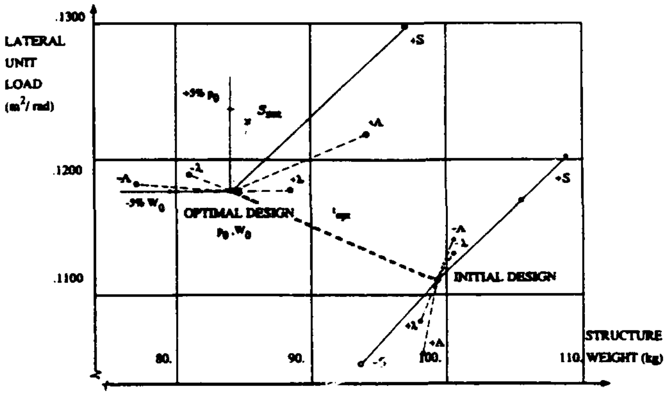


Fig. 7.10 Summary of partial sensitivities

8. CONCLUSION

This paper presents a way of solving design tasks in the aircraft development process using structural optimisation methods. As design criteria, requirements on the static, dynamic and aeroelastic behaviour of aircrafts are considered. The analysis procedures for the various state variables describing this behaviour are based on the Finite-Element-Method. The application of this program demonstrates that the design process can be supported very efficiently by the structural optimisation method. Another important advantage is the fact that the structural optimisation enables to achieve technically optimal design.

In order to optimise real-life structures many important procedures and methods are combined in the optimisation system MBB-Lagrange. Many further developments, however, must follow. Since an optimal design has to fulfill all demands on the structure simultaneously, suitable completions and extensions of the structural and sensitivity methods as well as the optimisation models (local and global stability, heat transfer, acoustics, thermal stresses, flight mechanics, and control, manufacturing) are furthermore required.

For fiber composite materials in particular the characteristic possibilities and requirements of manufacturing must be included in the optimisation process in order to guarantee that optimal designs can be produced efficiently by fully utilising the design potential.

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STRUCTURAL OPTIMIZATION OF AIRCRAFT

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ABSTRACT

A general survey of Dassault experience and knowledge on Aircraft Design with Optimization Methods is depicted.

This survey results from compiling the developments and the results already worked out and already presented in several papers by C. PETIAU and Al.

Part I gives a detailed description of the methodology. The special features of optimization with composite materials are shown. The organization of design resulting from use of optimization techniques is described and techniques neighbouring optimization as model adjustment are reviewed, as well as further developments.

Part II illustrates this methodology by an actual case study of an aircraft design by Dassault-Aviation with relevant examples of structural and aeroelastic optimization on carbon structures of a wing and a fin.

PART I - METHODOLOGY OF STRUCTURAL OPTIMIZATION

1 - INTRODUCTION

The structural optimization technique has been a routine process at Dassault since the late 1970s. It has been applied for all projects from the Mirage 2000 to the Rafale.

In the past, the design of a structure was achieved by the "fully stressed design" process (FSD), which consists of iterations of drawing and analyses, with reinforcement where the structure is not sufficiently strong and lightening where there are strength margins. However, where the only constraints on a metallic structure are those relating to strength of material, it has been demonstrated (see Ref. 1) that this approach is neither optimal (maximization of stresses is not equivalent to weight minimization) nor efficient for the design process. In practise, the designer is completely unable to predict intuitively any solution when constraints relating to flexibility (such as eigenfrequencies, aerodistorsion and flutter) or to the ply arrangement of composite materials are involved.

Therefore we consider today that the use of mathematical optimization is compulsory for the design of aircraft.

We have built the structural optimization tool around the Dassault softwares CATIA and ELFINI which include.

(1) the well-known CAD tool CATIA, which gives us geometry and mesh generation,
(2) static finite element analysis for linear and nonlinear problems,
(3) static aeroelasticity, calculation and management of loads,
(4) linear dynamics : calculation of eigenmodes, harmonic and transient responses,
(5) nonlinear dynamics : impact and crash analysis, landing gear and aircraft interaction,
(6) unsteady aeroelasticity, flutter, coupling with flight control system,
(7) fatigue and crack propagation analyses,
(8) heat transfer and thermo-elastic coupling,
(9) acoustic and elastoacoustic coupling.

The optimization monitor covers most of these branches.

The system works on request, in either an interactive or a batch mode, and uses a common data base managed automatically. Some of the main common characteristics of the branches are :

- topological dialogue for mesh and all data generation. All properties as connectors between nodes and elements (geometry connection with CATIA surface element characteristics, etc.) are described by blocks of constant properties in a space of indices referring to node and element. The process leads to very clean meshes for all types of structure from the whole aircraft meshes to tridimensional analyses of fitting details,
- a wide range of possibilities for visualization of inputs and outputs, many of "wire frame" and "pixel" types of pictures for displacement stresses, failure criteria and for optimization design variables, active constraints and safety margin plots,
- advanced mathematical solution : the solution of linear problems is run by a very powerful variant of the Frontal Gauss method, which minimizes the computer time for classical linear problems.

For large three-dimensional problems the use of the conjugate gradient technique enables the same level of performance to be maintained, taking into account the contact nonlinearities.

For geometric nonlinear problems (membrane effects, post-buckling, snap through, etc.) an original algorithm called "preconditioned BFGS with exact line search" has been developed. This algorithm benefits directly from the biquadratic character of total potential. It can handle the most severe snap-through conditions which shows calculation of post-buckling of a curved stiffened panel in carbon epoxy material).

We must underline the strong practical interest of the post-buckling analysis, which enables the design of thin composite skin, which buckles before ultimate loading.

We are going to present a more detailed view of :

- the optimization technique which is mainly used to set the general dimensions of the structure. It is supported by FE models of the whole aircraft, which are elaborated only from the rough definition of external shape and internal architecture, the result of this optimization being the starting point of detail drawing,
- the checking analysis which comes with detail drawings,
- the organization of drawing and analysis which are a necessity of composite design, and are present possibilities of computer tools.

2 - THE OPTIMIZATION METHOD

We present the operational tool as it was used for the design of the Rafale, the organization is iterative, and the flow-chart is shown in Fig. 1 :

2.1 - Cost function

The current goal in optimization is weight minimization. Nevertheless, in some cases, weight can be taken as a constraint, the objective being maximization of the safety margin see table 1).

2.2 - Design variables

The characterization of the optimisation design variables is made on groups of finite elements (FE). The choice of these variables partly takes into account manufacturing constraints and tooling rules for metallic material.

For a composite material, the design variables are the number of plies in each direction for each group.

The number of design variables often reaches 500, which can act simultaneously over several analysis models.

2.3 - Constraints

Constraints inequalities come from the different analysis branches of ELFINI. We can consider simultaneously :

(1) various failure criteria (including composite materials), computed from static stresses for all the dimensioning cases of loads,
(2) local buckling criteria,
(3) limited displacements,
(4) aeroelastic variation of aerodynamic derivatives,
(5) dynamic natural frequencies,
(6) flutter speed and aeroelastic dynamic damping,
(7) various technological constraints (such as minimum values of design variables, and limitations of the thickness variation between adjacent design variables).

The constraints considered during the same optimization can come from several analysis models (e.g. symmetric and antisymmetric FE aircraft model, local buckling analysis by the Rayleigh-Ritz method, local refined FE analysis, different external store configurations for dynamics and flutter, variation of shape because of control surface deflections, etc.).

2.4 - Sensitivities

We define "sensitivities" as the derivatives of constraints in the function of design variables. The principle of ELFINI optimization is to compute these derivatives by a correct mathematical process. It can easily be demonstrated (see Table 2 and Refs 1 and 2) that the computation of derivatives of static stresses, displacements, and aeroelastic coefficients is equivalent to solutions with a "dummy" case of loads.

The number of loads in this dummy case is :

(a) number of loading cases x number of design variables if formula (1) of Table 1 is used :
(b) number of constraints if formula (2) is used.

For practical problems the number of loads in the dummy case currently reaches several thousands, and their solution makes up the main part of the computer cost of optimization.

For nonlinear constraints relative to the static displacements (equivalent stresses, failure criteria...). The operator $[\partial\sigma/\partial X]$ is linearized near X.

When constraints are eigenvalue or are directly related to eigenvalue (E.g. eigenfrequency, linear buckling load, divergence or flutter speed, aeroelasticity damping) the cost of their derivation is negligible (see Tables 3,4,5,6 and 7). However, we must underline that these derivations need a far more accurate calculation of eigenvector than those needed for eigenvalue analysis. Also, we have found that it is very difficult to compute with reasonable accuracy derivations of solution of problems treated with the classical modal basis reduction (e.g. dynamic response, aeroelasticity), in practise it would be necessary to compute the correct mathematical derivative of the basic vectors. This is mainly why we have developed a static aeroelasticity approach without the basic truncation effect (see Ref. 1), as it leads to a mathematically exact and low-cost calculation of derivatives.

2.5 - Mathematical optimization

Starting of the analysis and derivation of constraints, we use an explicit nonlinear approximation of the constraints in terms of the design variables, mainly the formulation in inverse variables. Taking as new variables the inverses of design variables, leads to minimization of homographic function (weight) subject to linear inequalities. This problem is easily solved by a projected conjugate gradient algorithm (see TABLE 8)

Sub-iteration process

The right results and the good convergence of our algorithm in static optimization are mainly due to the explicit approximation of constraints in $1/\lambda i$.

But for other types of constraints as natural frequencies, flutter, speed, dynamic responses, this explicit form in $1/\lambda i$ has no theoretical basis and on some cases we could have a bad convergence.

Since the cost of the calculation of these dynamic constraints is raised relatively low compared to the analysis cost, it's interesting to carry out a sub-iteration process in order to improve convergence.

The convergence of the sub-iteration process in ensured by move limits and a relaxation on admissible value of the constraints making possible the detection of unfeasible approximate problems.

The cost of the mathematical optimization step is low. The mathematical optimization step gives a prediction of the optimum, from which we start new iterations. The number of iterations needed to obtain global convergence ranges from three to five . The cost of all of the iterations of optimization ranges from about eight to 15 times the cost of the analysis.

2.6 - Final touches

Generally, the theoretical optimum obtained from the optimization algorithm needs some modification, as it often does not represent a realistic design. Starting from the table of constraint derivatives, the final touches consist in examining interactively the effect of small modifications, made directly by the designer during the drawing. The program instantaneously shows the new safety margin and any violated constraints.

We can also interactively rerun the mathematical optimization step after changing the assigned values of constraints.

3 - SPECIAL FEATURES OF OPTIMIZATION WITH COMPOSITE MATERIAL

The organization described above is well suited for a composite material, with the addition of the following specifities.

3.1 - Failure criteria analysis and derivation

Inside the optimization loop we use failure criteria of the "Tsaï-Hill" family :

$$C = \sqrt{\left(\frac{\sigma_x^2}{\sigma_{xad}^2} + S_1^* \frac{\sigma_y^2}{\sigma_{yad}^2} + S_2^* \frac{\tau_{xy}^2}{\tau_{xyad}^2} - S_3^* \frac{\sigma_x \sigma_y}{\sigma_{xad}^2}\right)}$$

where σ_x, σ_y and τ_{xy} are stress tensor components,and $\sigma_{xad}, \sigma_{yad}, \tau_{xyad}$ and S1 = 0 or 1 are criteria parameters.

The arguments of the criteria are adapted to each situation (e.g. tension, compression, bending, holed panel, etc.), by calibration with more sophisticated criteria and test results.

Because, at a given point, the final failure mode is not known beforehand, it is necessary to handle constraints on all potential failure modes simultaneously. This is achieved at a relatively low cost if the derivation is performed in two steps :

(1) the strain tensor and its derivative are computed by formula (1) of Table 2 (three components common to all plies with membrane assumption),
(2) starting from the strain tensor and Hooke's law for the material, the failure criteria and their derivatives are calculated ply by ply.

3.2 - Local buckling criteria

Even if optimization can handle global buckling directly, for management and cost-effectiveness it is generally preferable to calculate and derive local buckling criteria with the following post-processing analysis :

(1) using the general FE model, stress flows of structural meshes are calculated and derived,
(2) local buckling load factors and their derivatives are calculated by a Rayleigh-Ritz method (see Table 4).

Sizes of meshes for local buckling analyses are independent of their representation in the global FE model, and they can be tuned to suit the actual stiffening.

In the optimization loop, stacking sequences are not taken into account (it is assumed that the material is homogeneous through the panel thickness), for the sake of algorithm simplicity, and because of difficulties in expressing the drawing constraints due to restrictions of cutting and stacking the fiber layers.

The order of buckling modes can change between iterations, this can cause a non-convergence of iterations if all potential buckling modes are not controlled simultaneously (see Ref. 2).

3.3 - Design constraints

These constraints express the fact that the results of optimization must correspond to a real drawing of a composite panel, which must be made of stacked layers with special rules for easy manufacture. Design constraints are handled at two levels :

(1) inside the optimization loop, by placing constraints on the minimum number or a given minimum proportion of plies in each direction, or on a maximum slope of thickness (these constraints correspond to linear inequalities in design variables),
(2) after mathematical convergence, automatic rounding of thicknesses is used to obtain a whole number of plies, and a special half-interactive program transforms the stacking of plies by area, which are the rough output of optimization, into a proper lay-up.

4 - MULTIMODEL OPTIMIZATION

Optimization has to provide the single physical characteristics of a structure and must take all the sizing considerations into account. So many F.E. models (or other type) are necessary, depending on the studied phenomena. So optimization must insure :

- identification between design variables defined on several models,
- data transfers between models (characteristics, boundary conditions, loads...),
- management of calculation and derivation of constraints defined on several models,
- the linking of all the design variables and constraints (values and derivatives) in the single explicit optimization step giving the optimum.

The organization (see TABLE 9) of the models has needed some software investment and is able to manage several FE meshes with several boundary conditions, mass configurations (modal and flutter analysis), Mach number (aeroelasticity and flutter analysis). Other models are used for panel buckling analysis.

5 - CHECKING ANALYSIS

It must be understood that, if an optimization tool is essential to achieve a good general drawing rationally, the result must be justified in detail, using more complex analyses than those which can be handled inside the optimization loop. The most typical of these checking analyses are the following :

(1) effect of local loads (e.g. fuel tank pressures, vibration, thermal load, etc.),
(2) local fatigue analysis.
(3) damage tolerance analysis,
(4) detailed local analysis of holed composite panel (e.g. point stress analysis),
(5) post-buckling analysis (see Refs 3 and 4).

Design constraints corresponding to these detail-checking analyses have been simplified to be handled by general optimization. These simplified assumptions must be validated by local checking analysis.

Effects of calibration of these constraints can be examined with a Lagrange multiplier of active constraints (handled interactively by "final touches" modules) or by re-running the mathematical optimization step.

6 - ORGANIZATION OF DESIGN PROCESS

We now have the following organization for the design of composite structures, from the preliminary project to the delivery of manufacturing drawings :

(1) start from a CATIA drawing of the external shape only and a brief definition of the internal architecture,
(2) elaboration, by CATIA-MESH, of a first simple general FE mesh of the whole aircraft (10-30000 dof) with approximate cross-sections and thicknesses (see Fig. 1). The model is adjusted with simple cases of load,
(3) analysis of static aeroelasticity and loads, which give the envelope cases of loads and show the latent problems of aeroelasticity,
(4) examination of internal load fields and stresses for selection of "strength of material" constraints in the optimization.
(5) computation of dynamic modes with the various external store configurations : flutter problem recognition,
(6) first run of optimization,
(7) drawings of the structure supported by :
    (a) an interactive test of authoritative modifications of optimization results to make drawing easier, together with use of the "final touches" module :
    (b) changes and additions of constraints,
    (c) critical examination of "cost of requirements", directly obtained from "Lagrange multipliers" of optimization. This allows appreciation of the real influence of the safety margin of uncertain criteria (composite materials),
    (d) Detail-checking analyses supported by methods described above in Section 5. These are performed taking proper boundary conditions in the FE model for the whole aircraft via a super-element technique. Detail-checking analyses must validate the simplified criteria used for mathematical optimization, otherwise, optimization must be re-run with calibrated criteria.

Although a single optimization run lasts only one night, the optimization job can remain inside the computer for more than 6 months, for examination of the detail analysis effects, the influence of the choice of constraints and alternative designs.

7 - TECHNIQUES NEIGHBOURING OPTIMIZATION, IDENTIFICATION AND COMPUTATION WITH UNCERTAIN DATA

The solution of these problems can be considered because of the possibilities of derivative elaboration.

7.1 Model adjustment

Generally, this involves adjustment of the FE dynamic model to measured natural modes, the unknowns are design variables of local thickness and mass, modal deformation and frequencies. The modal equation appears as an equality constraint, and the objective is to minimize the "distance" between the measured and the computed modes. The method does not require knowledge of the connection between computed and measured modes, some results of this technique applied to the Mirage III NG are shown in Fig. 2.

For general cases of model adjustment, we use a simpler technique. The objective function is to minimize the "distance" between a design variable and its theoretical value, we take as constraints the fact that the computations must give measurements with a given approximation (which can be objectively estimated from the accuracy of measurements).

The advantage of this technique over the classical mean-square method is that underdetermination is not possible, if a design variable, or a combination of design variables, is not "observable" by measurement the process gives the theoretical values automatically. In Ref. 5 a good example of this process for flight identification of aerodynamic loads was given.

7.2 - Computation with uncertain data

Sometimes, at the start of a problem, the data are imprecisely known, the idea of computation with uncertain data is to find the "worst" point in the uncertain design variable space. The problem is solved by two approaches :

(1) find the "worst" possible point by minimization of a safety margin function inside the authorized space of design variable variations, and

(2) if there exists a possibility of failure, compute the probability of failure, starting from the probability density of the design variables.

We have now started to apply these ideas on flutter and vibro-acoustic analysis of preliminary projects.

8 - FURTHER LEVELS OF OPTIMIZATION

The general tendency is to introduce progressively all the "arguments" of structural design in the optimization loop. The next steps of development are as follows.

8.1 - Optimization with "bending" design variables

This does not give rise to any theoretical difficulties, the relative complication comes from the nonlinear dependence of stiffness, neutral surface and constraints on design variables, which complicates program writing.

8.2 - Optimization with post-buckling analysis

This is one of the most important needs of the present operational optimization. The difficulty is avoided, generally by an empirical adjustment of the load level of linear buckling, and the results of optimization are checked by post-buckling analysis.

The correct solution is not much more intricate than that of the bending case, it can easily be demonstrated that the derivation cost is almost that of the linear problem ("dummy" cases of load at the final equilibrium state) (see table 10).

8.3 - Shape optimization

This is needed in many pratical problems of varying difficulty (shape of stiffeners, pressurised vessels, fitting, etc.). The main difficulty is to express design variables and "topological" constraints. For such problems, many workers and ourselves have elaborated specimen programs for scholastic cases, but for a really operational tool, it is necessary to introduce geometrical design variables and the associated "topological" constraints at the level of a CAD system, which requires considerable investment.

8.4 - Optimization in heat transfer problems

One of the necessities of the Hermes project has been to achieve the same level of sophistication for thermal analysis as for structural analysis, we have met the need for a thermal optimization tool. The general arrangement of thermal optimization is the same as in structural optimization. The complications are in the transient and highly non-linear character of thermal problems.

Fortunately, it can be demonstrated that temperature derivation needs the solution of the same differential linear equation system for all design variables, and, as it is integrated at the same time as the analysis, it does not need additional factorization. The cost of derivatives is therefore relatively lower than that of solution of the static elasticity problem.

We have developed a joint heat transfer identification process with computation with uncertain data, which is needed particularly because of the random or badly known character of many data.

8.5 - Multidisciplinary interactions

For a combat aircraft, the idea should be to optimize simultaneously the structure, dimensions of control surfaces, actuators and hydraulic power, the parameters of the electrical flight control system, and the aerodynamic shape.

This state of grace has not yet been reached, the tendency is to apply optimization to each discipline and to proceed in relation to the other matters by a "fixed point method" or by simplification of the interactions. Thus, starting from Lagrange multipliers obtained from the optimization of each discipline, it is possible to "condense" their interactions, for instance, as far as structure is concerned, we can easily give the weight cost of requirements of other disciplines (exchange rate between structure weight and roll speed, profile relative thickness, etc.).

9 - CONCLUSION

The tendency to include increasingly detailed analyses in the mathematical optimization loop is hindered by the difficulties of the task. The tool described above represents achievement of the first level of structural optimization, where geometry is given and mass and stiffness matrices are linear functions of design variables. significant progress is not easy. It corresponds to including in the optimization.

(1) "bending" design variables,

(2) nonlinear and post-buckling analysis, rules of effective width,

(3) stacking order of plies and constraints due to cutting of layers of composite material,

(4) shape optimization, which is also implicitly necessary in the above functionalities.

Apart from their theoretical difficulty, these developments need a higher level of integration of FE optimization with CAD, in particular, the architecture of the CAD system must support the description of design variables and of drawing constraints.

Another promising field of research is to use expert systems to pilot the design, this seems to be one way to manage optimization with discontinuous evolution of design variables. At present, we have started the development of this technique at the level of size check of carbon fiber panels. It rests on a knowledge base composed of rules, relating to technological constraints and calculation methods.

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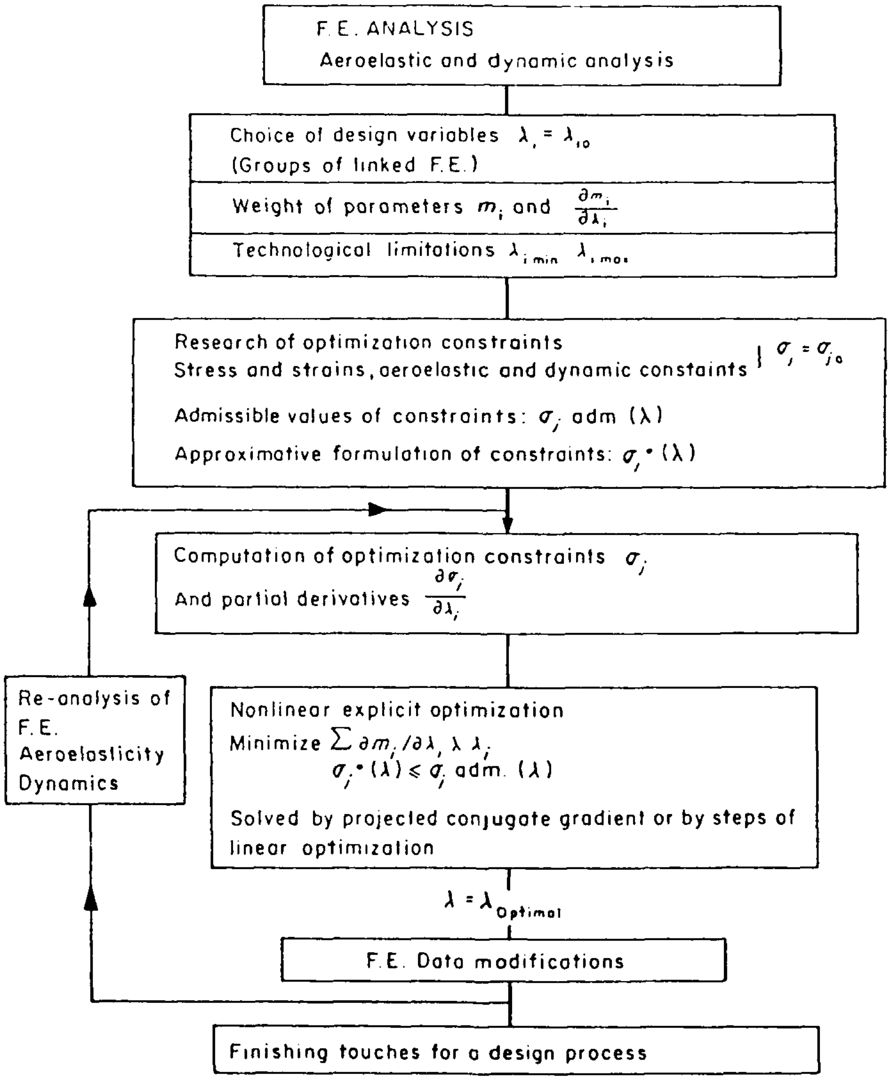
F.E. ANALYSIS
Aeroelastic and dynamic analysis
Choice of design variables $\lambda_i = \lambda_{io}$
(Groups of linked F.E.)
Weight of parameters m_i and $\frac{\partial m_i}{\partial \lambda_i}$
Technological limitations $\lambda_{i\,min}$ $\lambda_{i\,max}$
Research of optimization constraints
Stress and strains, aeroelastic and dynamic constaints
$\sigma_j = \sigma_{jo}$
Admissible values of constraints: σ_j adm (λ)
Approximative formulation of constraints: $\sigma_j^*(\lambda)$
Computation of optimization constraints σ_j
And partial derivatives $\frac{\partial \sigma_j}{\partial \lambda_i}$
Re-analysis of F.E. Aeroelasticity Dynamics
Nonlinear explicit optimization
Minimize $\sum \partial m_i/\partial \lambda_i \; \lambda \; \lambda_i$
$\sigma_j^*(\lambda) \leq \sigma_j$ adm. (λ)
Solved by projected conjugate gradient or by steps of linear optimization
$\lambda = \lambda_{Optimal}$
F.E. Data modifications
Finishing touches for a design process

Fig. 1 Optimization method flow-chart

TABLE 1

MAXIMIZATION OF MARGINS

Minimization of the weight is not the only aim of the optimization of structures. Indeed in projects, by example, a global mass is allocated to a part of a structure (fuselage, wing, fin...) and it's interesting to design the safest structure for a limited mass.

We have to maximize the margin for a limited mass

Margin : $\frac{\overline{\sigma}_j - \sigma_j}{\sigma_j} = \frac{\overline{\sigma}_j}{\sigma_j} - 1$

with σ_j value of the j constraint

$\overline{\sigma}_j$ admissible value

The algorithm has to minimize the $\mu_j = \frac{\sigma_j}{\overline{\sigma}_j}$ with a constraint on the mass : $\Sigma_{mi} \; \lambda_j \leq M_o$

To minimize the μ_j we define $\mu = \underset{j}{\mathrm{Max}} \, (\mu_j)$ and the problem has the following form :

$$\begin{cases} \mathrm{Min}\ \mu \\ \sigma_j \leq \mu \overline{\sigma}_j & \text{for constraints concerned by the margin} \\ \sigma_k \leq \overline{\sigma}_K & \text{for other constraints} \\ & \text{Mass constraint} \end{cases}$$

In reciprocal variables $\alpha_i = 1/\lambda_i$ we have :

$$\begin{cases} \text{Minimize} \quad \mu \\ \sigma_{oj} + \sum_i \frac{\partial \sigma_j}{\partial \alpha_i} (\alpha_i - \alpha_{io}) \leq \mu \, \overline{\sigma}_j \\ \sigma_{ok} + \sum_i \frac{\partial \sigma_k}{\partial \alpha_i} (\alpha_i - \alpha_{io}) \leq \overline{\sigma}_k \\ \Sigma \frac{a_i}{\alpha_i} \leq M_o \\ 0 \leq \mu \leq 1 \end{cases}$$

This optimization problem, with a linear objective function and nonlinear constraints (mainly mass constraint) is solved by a sequence of linear programming using moove limits.

TABLE 2

Derivation of FE Static Solution

Finite element analysis

Displacement computation:

$$X = [K]^{-1} F$$

Strength. stress computation:

$$\sigma = \left[\frac{\partial \sigma}{\partial X}\right] X$$

Optimization constraint derivation

Displacement derivation:

$$\Delta X = -[K]^{-1} [[\Delta K] X - \Delta F]$$

Strength. stress derivation:

$$\Delta \sigma = -\left[\frac{\partial \sigma}{\partial X}\right] [K]^{-1} [[\Delta K] X - \Delta F] \qquad (1)$$

$$\Delta \sigma = -\left[[K]^{-1} \left[\frac{\partial \sigma}{\partial X}\right]^{t} \right] [[\Delta K] X - \Delta F] \qquad (2)$$

(1) number of resolutions = number of load cases

(2) number of resolutions = number of constraint operators

TABLE 3

Derivation of Eigenvalues

Analysis:

eigenmodes: V_i

eigenvalues: ω_i

$$[[K] - \omega_i^2 [M]] V_i = 0$$

Sensivity analysis of eigenvalues:

$$\Delta [V_i^t [[K] - \omega_i^2 [M]] V_i] = 0$$

$$2 V_i^t [[K] - \omega_i^2 [M]] \Delta V_i + V_i^t [[\Delta K] - \omega_i^2 [\Delta M]] + \Delta \omega_i^2 V_i^t [M] V_i = 0$$

$$\Delta \omega_i = \frac{V_i^t [[\Delta K] - \omega_i^2 [\Delta M]] V_i}{2 \omega_i V_i^t [M] V_i}$$

TABLE 4

Local Buckling Analysis by Rayleigh-Ritz Method

Rayleigh-Ritz model

External load fluxes: $\phi = \begin{bmatrix} \phi_{x0} \\ \phi_{y0} \\ \phi_{xy0} \end{bmatrix} = \rho \phi_0$

Normal deflection:

$w = \Sigma a_{mn} x^i \cdot x^j \cdot L(x,y) \qquad V = \begin{bmatrix} a_{11} \\ \vdots \\ a_{mn} \end{bmatrix}$

Buckling factors

Buckling initiation: $\boxed{W_1 = W_2}$

W_1 = bending elastic energy
W_2 = membrane work of external loads ($W_2 = \rho U_2$)

$$W_1 = \rho U_2$$

$$\min \rho = W_1/U_2 \langle == \rangle \partial W_1/\partial V - \rho \partial U_2/\partial V = 0$$
$$|K - \rho G| V = 0$$

Derivation of buckling factors

$$\rho = \frac{V^t \cdot K(\lambda) \cdot V}{V^t \cdot G(\phi) \cdot V}$$

$$\frac{\partial \rho}{\partial \lambda} = - \frac{V^t \cdot \partial K/\partial \lambda \cdot V}{V^t \cdot G(\phi) \cdot V} + \rho \frac{V^t \cdot \partial G/\partial \phi \cdot \partial \phi/\partial \lambda \cdot V}{V^t \cdot G(\phi) \cdot V}$$

TABLE 5

- Derivacives of extrema in transient response

Mechanical equation in F.E. basis :

$$[M] \ddot{X} + [K] X = F(t) \qquad ①$$

is solved in the reduced basis $X = [V] x$

by integration of $[m] \ddot{x} + [k] x = f(t)$

with $[m] = V_t [M] V \quad [k] = V_t [K] V \quad f(t) = V_t F(t)$

The dynamic stress σ can be written :

$$\sigma = [\partial\sigma/\partial X] X = [\partial\sigma/\partial X][V] x = [\partial\sigma/\partial x] x$$

Derivation of equation 6 gives

$$[M] \Delta\ddot{X} + [K] \Delta X = -[\Delta M] \ddot{X} - [\Delta K] X$$

which is similar to equation ① except the excitation. If this dummy excitation can be expressed in the reduced basis, we obtain :

$$[m] \Delta\ddot{x} + [k] \Delta x = -[\Delta m] \ddot{x} - [\Delta k] x$$

$$[\Delta m] = V_t [\Delta M] V \quad [\Delta k] = V_t [\Delta K] V$$

On the extremum

$$\Delta \sigma(t) = [\partial\sigma/\partial x] \Delta x_t$$

$$d\sigma(\lambda,t) = [\Delta\sigma/\Delta\lambda] d\lambda + \cancel{(\partial\sigma/\partial t)} dt$$

TABLE 6

STATIC AEROELASTIC COEFFICIENTS ANALYSIS AND DERIVATIVES

1 - Basic equations of static aeroelasticity with finite elements

- Discrete pressure field

$$Kp = [\partial Kp/\partial qr]\, qr + [\partial Kp/\partial qs]\, qs$$

q_r = rigid aerodynamic effect (incidence, control surface setting, etc.)
q_s = deformation of lifting surface expressed in a monomial base.

- Flight equation

$$F\ C.D.G. = \tfrac{1}{2} \rho V^2 ([Cr]\, qr + [Cs]\, qs)$$

[Cr], [Cs] torseur resulting from $[\partial Kp/\partial qr]$, $[\partial Kp/\partial qs]$ (aerodynamic coefficients).

- Loads on F.E.

$$F = \tfrac{1}{2} \rho V^2 [R]\, Kp \qquad \begin{matrix} [\partial F/\partial qr] = [R]\, [\partial Kp/\partial qr] \\ [\partial F/\partial qs] = [R]\, [\partial Kp/\partial qs] \end{matrix}$$

- F.E. deformed

$$X = [K]^{-1} F \qquad \begin{matrix} [\partial X/\partial qr] = [K]^{-1}\, [\partial F/\partial qr] \\ [\partial X/\partial qs] = [K]^{-1}\, [\partial F/\partial qs] \end{matrix}$$

- Smoothed transition F.E. $qs = [L]X$

$$① \quad qs = \tfrac{1}{2} \rho V^2 ([A1]qr + [A2]qs) \qquad \begin{matrix} [A]1 = [L]\, [\partial X/\partial qr] \\ [A2] = [L]\, [\partial X/\partial qs] \end{matrix}$$

F.E. = finite elements

- Flexible effect elimination

$$② \quad qs = \tfrac{1}{2} \rho V^2 [\mu]\, qr\ ,\ [\mu] = [D]^{-1} [A1]\ , \quad [D] = \left[I - \tfrac{1}{2} \rho V^2 [A2]\right]$$

- Flexible aerodynamic coefficients

$$③ \quad [c] = [Cr] + \tfrac{1}{2} \rho V^2 [Cs]\, [\mu]$$

2 - Derivative relative to structural parameters

Differentiation of 1 knowing that

$$\Delta ([K]^{-1} F) = - [K]^{-1} [\Delta K]\, X$$

$$\Delta qs = - \tfrac{1}{2} \rho V^2 [L]\, [K]^{-1} [\Delta K] ([\partial X/\partial qr]\, qr + [\partial X/\partial qs]\, qs) + \tfrac{1}{2} \rho V^2 [A2]\, \Delta qs$$

By eliminating q_s from 2

$$\Delta qs = - \tfrac{1}{2} \rho V^2 [D]^{-1} [L]\, [K]^{-1} [\Delta K] ([\partial X/\partial qr] + [\partial X/\partial qs]\, [\mu])\, qr = [\Delta\mu]\, qr$$

Differentiation of 3 $[\Delta C] = \tfrac{1}{2} \rho V^2 [Cs]\, [\Delta\mu]$

$$[\Delta C] = \tfrac{1}{2} \rho V^2 [Cs]\, [D]^{-1} [L]\, [K]^{-1} [-\Delta K] \left[[\partial X/\partial qr] + [\partial X/\partial qs]\, [\mu]\right]$$

Preferable in the form

$$[\Delta C] = \tfrac{1}{2} \rho V^2 (([K]^{-1} ([Cs]\, [D]^{-1} [L])t)_t [-\Delta K] \left[[\partial X/\partial qr] + [\partial X/\partial qs]\, [\mu]\right]$$

which means solving the equilibrium equation for a single load for each aerodynamic coefficient at a given Mach and dynamic pressure

$$[\Delta C] = \tfrac{1}{2} \rho V^2 [Cs]\, [D]^{-1} ([K]^{-1} [L]_t)_t [-\Delta K] \left[[\partial X/\partial qr] + [\partial X/\partial qs]\, [\mu]\right]$$

with q_s solutions for all Mach numbers and dynamic pressures.

TABLE 7

DERIVATION OF FLUTTER SPEED (J.P. BREVAN'S METHOD)

1 - Analysis

- We attempt to solve the flutter equation at a given Mach number

$$[K(\lambda) - \omega^2 (1+ig)^2 M(\lambda) - \rho V^2 A(\tfrac{\omega}{V})]\, q = 0$$

[K] = reduced stiffness matrix
[M] = reduced mass matrix
ω = frequency of the solution
g = damping (g = 0 ⟹ flutter)
V = velocity
[A] = matrix of aerodynamic forces
P = left solution
q = right solution

- In the simplified form

① $[D(\lambda, \omega, g, V)]\, q = 0$

② $Pt\, [D(\lambda, \omega, g, V)] = 0$

with g = 0 (or given).

2 - Differentiation

① $\Longrightarrow \Delta([D]q) = [\Delta D]q + [D]\, \Delta q = 0$

by multiplmying with Pt

$Pt\, [\Delta D]q + \cancel{Pt D \Delta q} = 0$

we obtain ② $Pt\, [\Delta D]\, q = 0$

If we fix the damping g

③ $Pt\, [\partial D / \partial \lambda]q + Pt\, [\partial D / \partial \omega]q + Pt\, [\partial D / \partial V]q = 0$

This complex equation gives the derivatives of the frequency of the solution and the flutter speed.

TABLE 8

EXPLICIT OPTIMIZATION

The main idea is to replace the exact formulation $\sigma(\lambda)$ of the constraints, which is only implicitly known by a F.E. analysis, by an explicit approximation $\sigma^*(\lambda)$.

$\sigma^*(\lambda)$ is selected so that :

- at $\lambda = \lambda$ analysis $\sigma^*(\lambda) = \sigma$ E.F. and
  $\partial\sigma^*(\lambda) / \partial\lambda = \partial\sigma$ E.F. $/ \partial\lambda$
- $\sigma^*(\lambda)$ must be exact for statically determinate structure
- $\sigma^*(\lambda)$ must have a good form when $\lambda \longrightarrow 0$ and $\lambda \longrightarrow \infty$

At the moment we think that the most efficient explicit approximation is in reciprocal variables.

with

$\alpha i = 1/\lambda i \qquad aij = -1/\alpha i^2\ \partial\sigma j\ \text{E.F.}/\partial\lambda i$

$$\sigma_j{}^*(\alpha) = aoj + \sum_i aij\, \alpha i$$

$$aoj = \sigma\ \text{E.F.} + \sum_i aij\, (\alpha i - \alpha io)$$

The optimization problem becomes :

$$\Sigma\, mi/\alpha i \text{ minimal}$$

Subject to
$$\begin{cases} a_{oj} + \sum_i aij\, \alpha i \leqslant \sigma j\ ad\ (\alpha) \\ \alpha \leqslant 1/\lambda i \text{ mini} \end{cases}$$

With constraints on local buckling criteria, the admissible values are not constant but function of parameters.

The explicit optimization problem is solved with a conjuguate projected gradient method improved by an efficient normalisation of the tangent Hessien.

TABLE 9

MULTI-MODEL OPTIMISATION

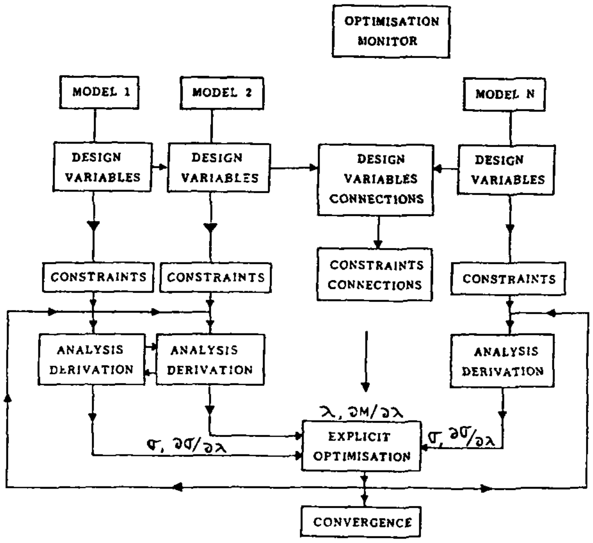
OPTIMISATION MONITOR
MODEL 1
MODEL 2
MODEL N
DESIGN VARIABLES
DESIGN VARIABLES
DESIGN VARIABLES CONNECTIONS
DESIGN VARIABLES
CONSTRAINTS
CONSTRAINTS
CONSTRAINTS CONNECTIONS
CONSTRAINTS
ANALYSIS DERIVATION
ANALYSIS DERIVATION
ANALYSIS DERIVATION
λ, ∂M/∂λ
σ, ∂σ/∂λ
EXPLICIT OPTIMISATION
σ, ∂σ/∂λ
CONVERGENCE

TABLE 10

OPTIMIZATION OF NONLINEAR STRUCTURES

Post-buckling analysis of composite structures is one of the most significant advance of the last years (Ref. 2). The next challenge is the optimization of non-linear structure including post-buckling behavior.

Most of the problems can be solved with a similar algorithm to those of linear structures with a sequence of analysis and partial derivatives. Derivatives computation are relatively less expensive than in linear structures :

$$\mathrm{Fint} - \mathrm{Fext} = 0 \qquad \sigma = L.X$$

$$\frac{d\mathrm{Fint}}{d\lambda} = \frac{\partial \mathrm{Fint}}{\partial \lambda}.\ X + \mathrm{Ktg}.\ \frac{\partial X}{\partial \lambda} = 0 \qquad \left(\frac{\partial \mathrm{Fint}}{\partial X} = \mathrm{Ktg}.\right)$$

$$\frac{\partial X}{\partial \lambda} = -\ \mathrm{Ktg}^{-1}\ \frac{\partial \mathrm{Fint}}{\partial \lambda}.X\ ,\ \frac{\partial \sigma}{\partial \lambda} = L.\ \frac{\partial X}{\partial \lambda}$$

But this type of algorithm could lead to bad convergence on post-buckled structures with snap-through behavior.

So a simultaneous solution of the analysis and optimization problems can be considered.

$$\begin{cases} \mathrm{Min.}\ M\ (\ \lambda\ ) \\ \sigma\ (X,\ \lambda\ )\ <\ \sigma\ \mathrm{ad} \\ G_{RAD}\ W_{tot} = 0 \end{cases}$$

λ = design variables
X = displacements
W_{tot} = non-linear potential

Recent advances in minimization method based upon preconditionned matrices and explicit line-search (Ref. 2) will be intensively used.

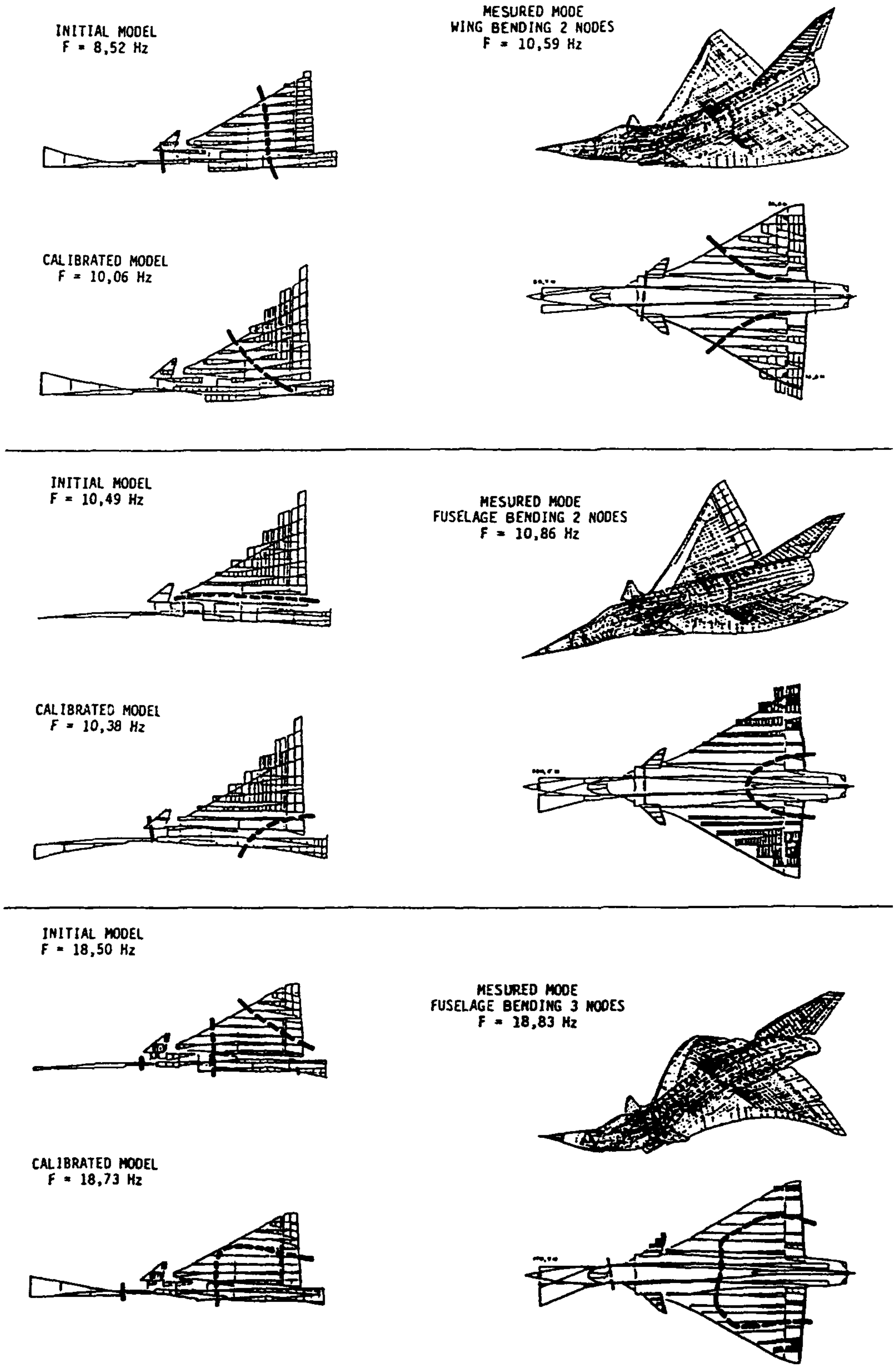
INITIAL MODEL
F = 8,52 Hz
MESURED MODE
WING BENDING 2 NODES
F = 10,59 Hz
CALIBRATED MODEL
F = 10,06 Hz
INITIAL MODEL
F = 10,49 Hz
MESURED MODE
FUSELAGE BENDING 2 NODES
F = 10,86 Hz
CALIBRATED MODEL
F = 10,38 Hz
INITIAL MODEL
F = 18,50 Hz
MESURED MODE
FUSELAGE BENDING 3 NODES
F = 18,83 Hz
CALIBRATED MODEL
F = 18,73 Hz

Fig. 2 Automatic calibration of dynamic FE model of MIRAGE III NG

PART II - ACTUAL CASE STUDY OF AN AIRCRAFT DESIGNED BY DASSAULT AVIATION

EXAMPLE OF APPLICATION OF OPTIMIZATION OF CARBON EPOXY STRUCTURE

We present two examples of optimization calculations for carbon epoxy parts for a combat aircraft.

1 - Optimization of a combat aircraft wing

We summarize here the configuration of the optimization of a carbon epoxy delta wing box, corresponding to the mesh presented in Fig. 1, with the design variable patch of Fig. 4.

We have used two analysis models for static and aeroelasticity with a survey of flutter on three external load configurations (see Table 1).

In Fig. 5 we present the history of convergence in weight. Drawing constraints and flutter constraints have been successively introduced later, to study their influence. The optimum values of design variables are presented in Fig. 4 and the corresponding lay-up of plies is shown in Fig. 7 (obtained automatically).

In Table 2 the weight sensitivities of wing panels to a typical project hypothesis obtained by optimization are shown.

TABLE 1

Characterization of Optimization of a Carbon Epoxy Wing

	Model 1	*Model 2*
FE models	Wing model with a representation of other parts of the aircraft by super-element technique (3544 dof): symmetric and antisymmetric analysis	Complete plane 13 003 dof: symmetric and antisymmetric analysis
Design variable	476 design variables (number of plies in four directions)	
Static cases of loads	24 cases of loads combined from symmetric and antisymmetric analysis	0
Failure criteria	476 failure criteria equivalent Tsaï-Hill criteria	
Buckling criteria	144 critical buckling factors. obtained from 77 local buckling analyses of composite plates by Rayleigh-Ritz method	0
Static aeroelastic constraint	0	7 control surface efficiencies and minimal roll speed
Flutter		5 flutter speeds and 60 aeroelastic dampings corresponding to three external load configuration
Technological constraint	374 constraints on composite lay-up (thickness shape. maximum and minimum ratio between each ply direction)	

TABLE 2

Influence of Design Assumptions

	Design hypothesis	*Weight (ratio)*
1	Composite material Strength of material constraints only, rough from computer optimization	1·0
2	+ Aeroelasticity constraint	1·19
3	+ Aeroelasticity + technological constraints	1·25
4	Weight from final detailed drawing (review by checking analyses)	1·36
5	Aluminium alloys solution Strength of material + aeroelasticity (comparable with line 3)	2·10

2 - Optimum design of a vertical fin

The layout of the box and the rudder in carbon epoxy are optimized considering the static load for two rudder deflection cases, and constraints on rudder aeroelastic efficiencies and dynamic frequencies (see Fig. 6). The exact configuration of this optimization is shown in Table 3.

TABLE 3

Characteristics of Optimization of a Vertical Fin

	Model 1	*Model 2*
FE models	Fin model (1800 dof) with a super-element of the whole aircraft	Fin model with a deflected rudder
Design variables	237 design variables (number of fiber layers of panels. cross-sections of flanges and thicknesses of webs for spars and ribs)	
Static load cases	3	1
Failure criteria	190 failure criteria on composite materials with holes	190
Buckling criteria	98 buckling criteria computed from Rayleigh-Ritz models for panel buckling analysis	82
Displacements	1 displacement on the step between box and rudder	0
Aeroelasticity	8 constraints on fin and rudder yaw efficiencies for two Mach numbers	0
Dynamic	Frequencies of first flexion mode and rudder mode	0
Technology	107 constraints on ply distributions and on minimum distance between lay-up interruptions	

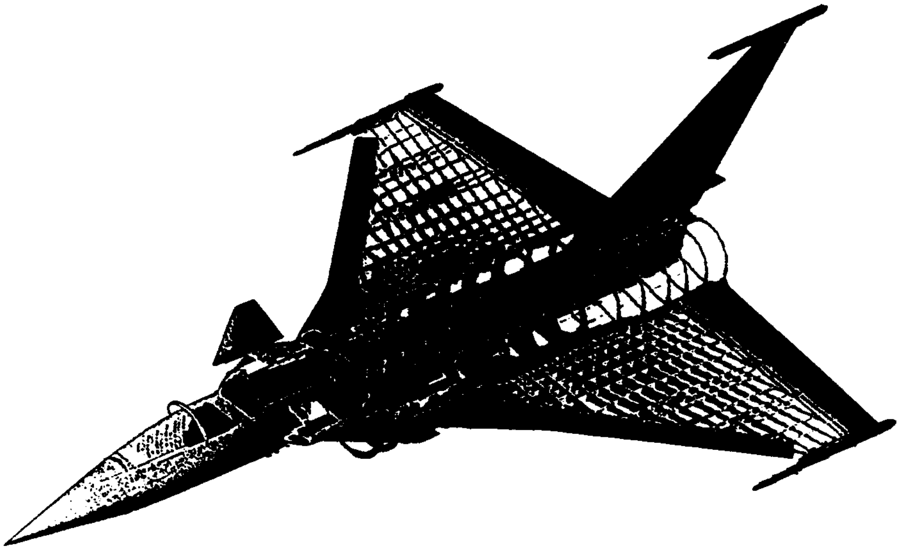

Fig. 1 General mesh of combat aircraft

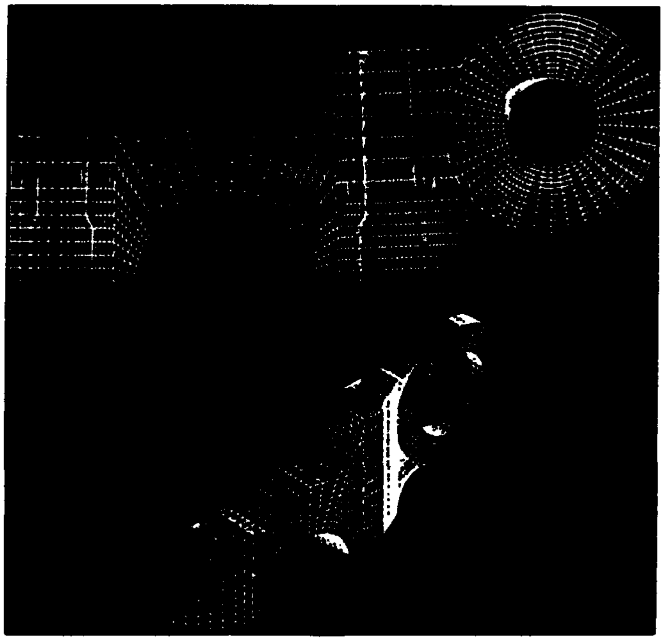

Fig. 2 Landing gear fitting analysis

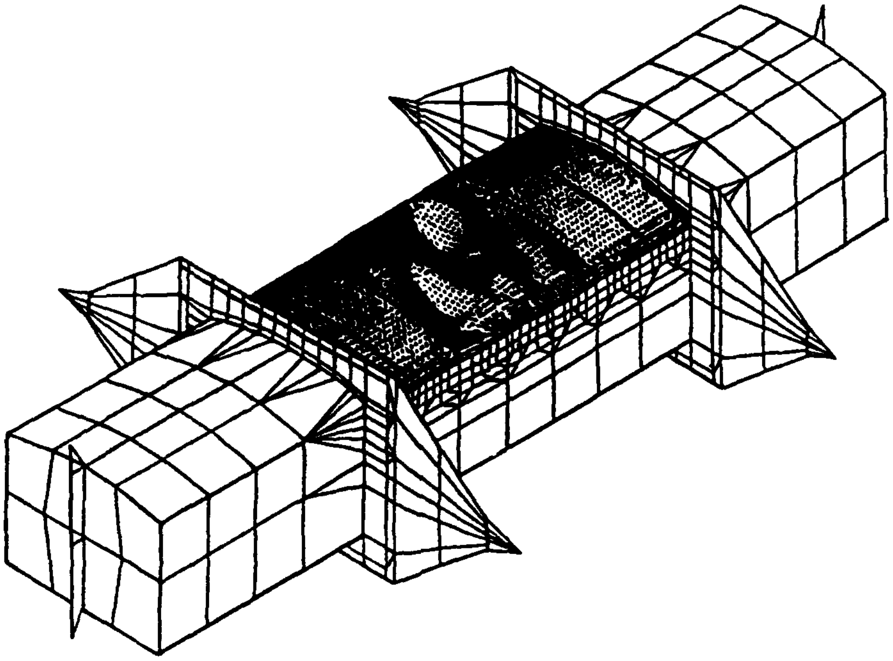

Fig. 3 Post-buckling analysis of a curved carbon epoxy panel (test on fuselage panel of combat aircraft)

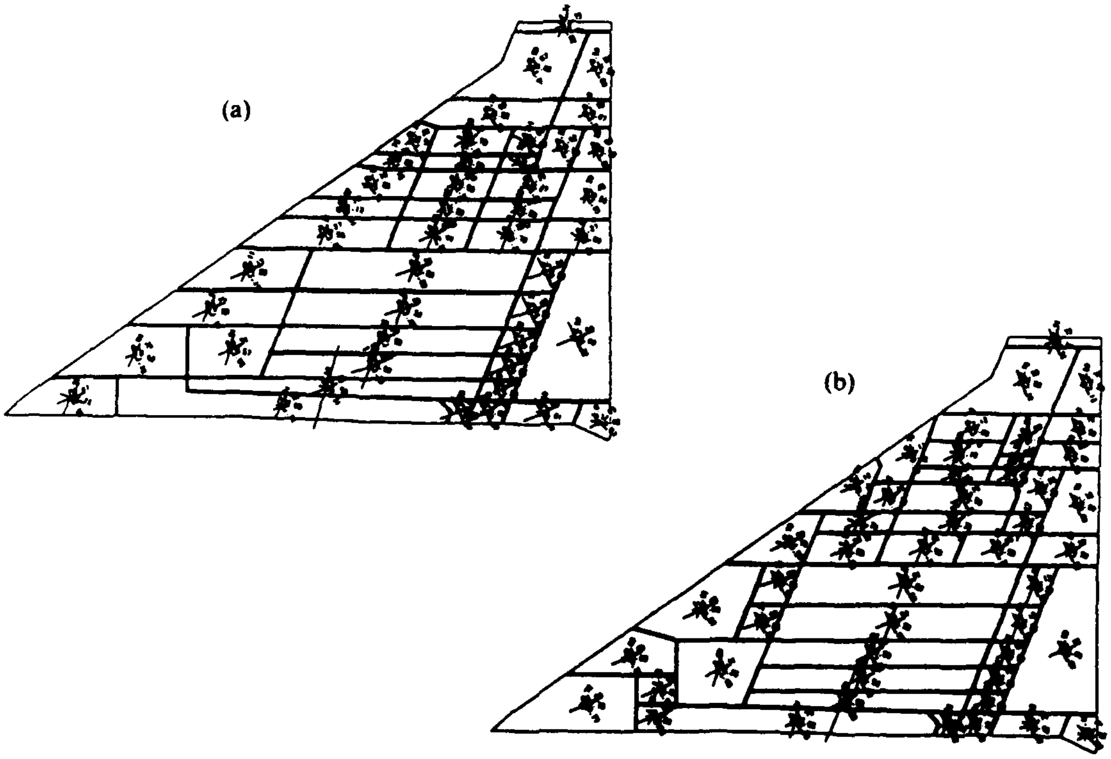

Fig. 4 Optimization of a carbon epoxy wing : (a) upper and (b) lower panel optimum lay-up

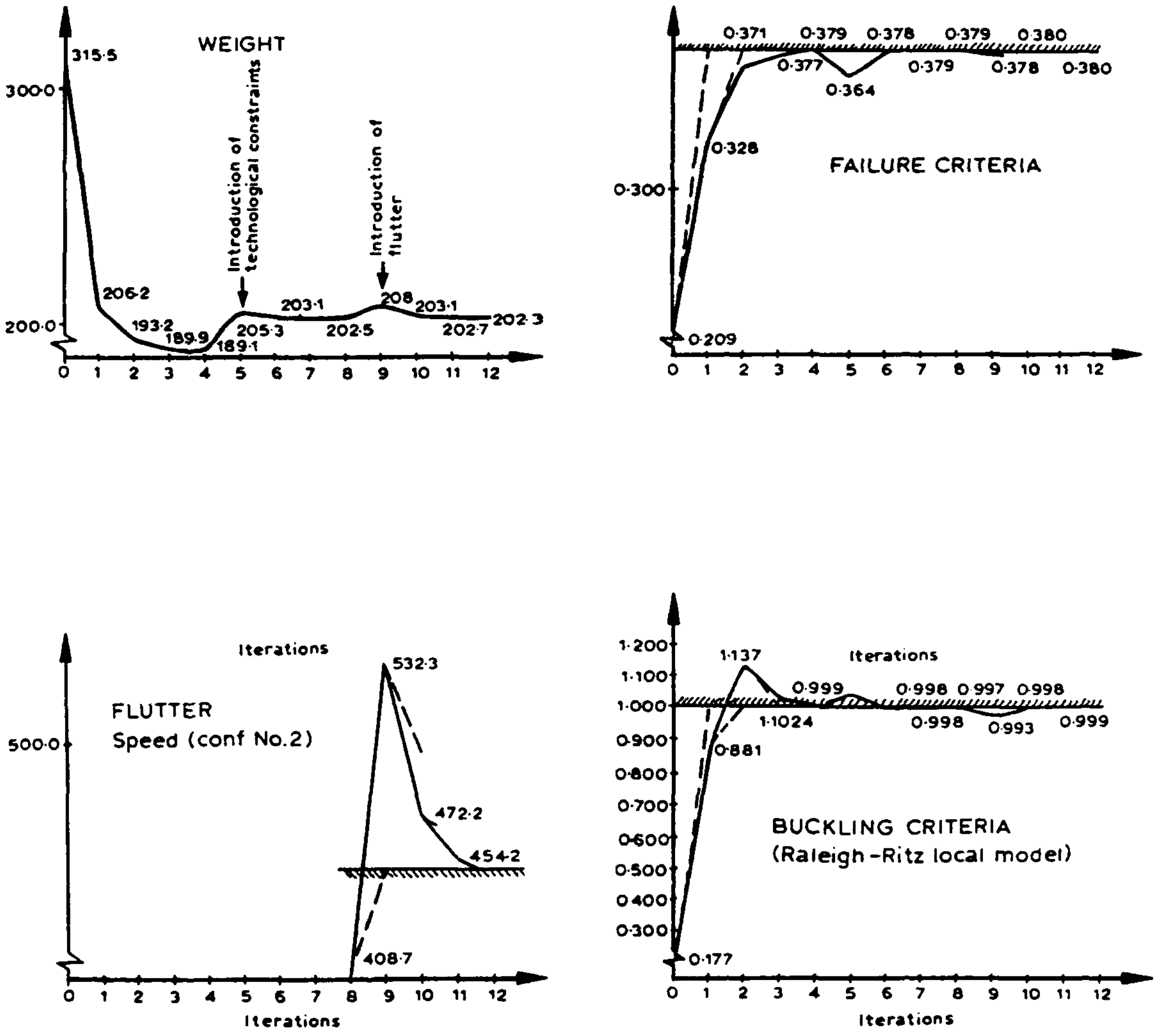

Fig. 5 Optimization of carbon epoxy wing : history of convergence

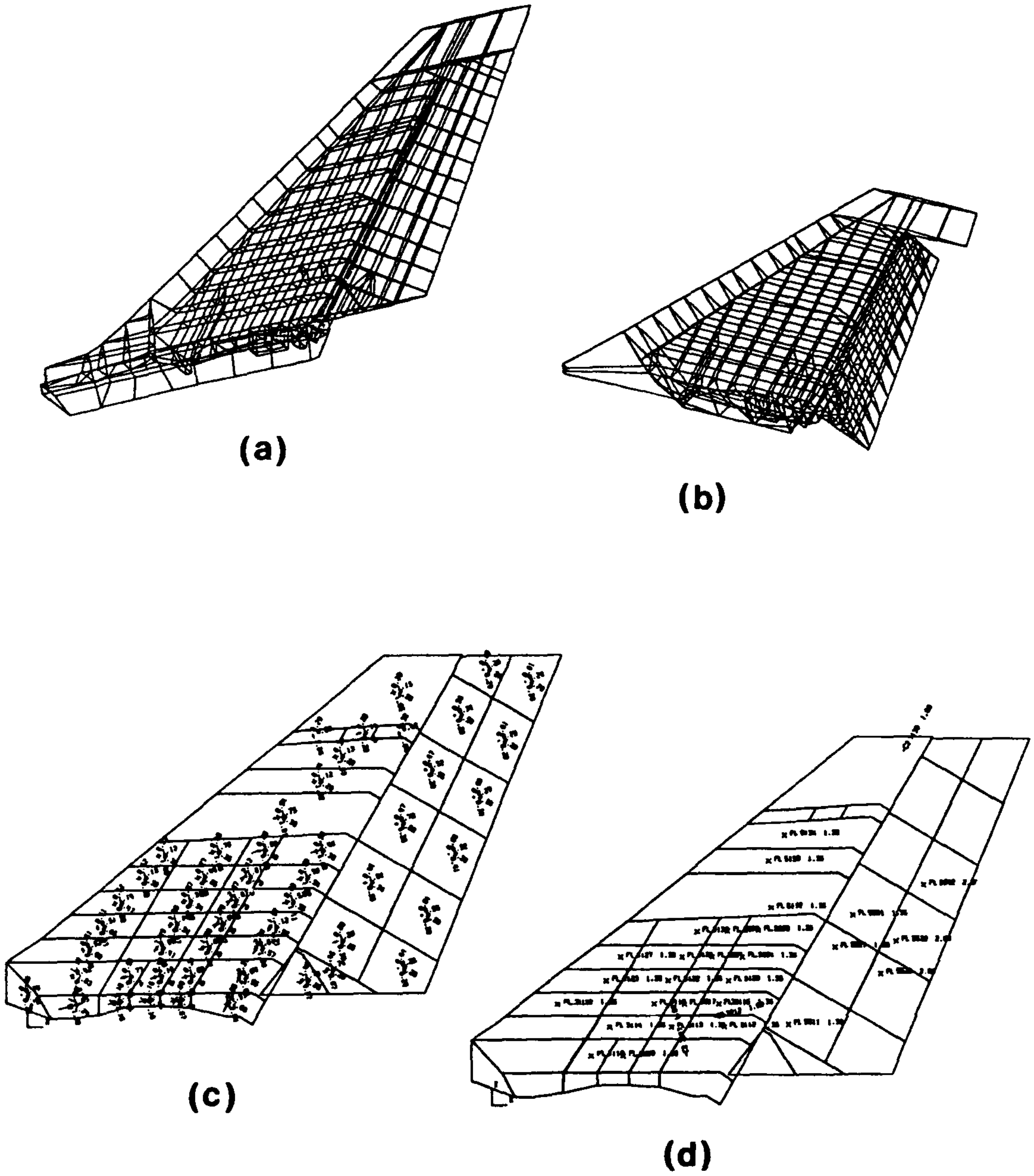

Fig. 6 Optimization of a carbon epoxy fin. (a) Model 1 : no rudder deflection.
(b) Model 2 : with rudder deflection. (c) Optimal lay-up. (d) Active constraints at optimum.

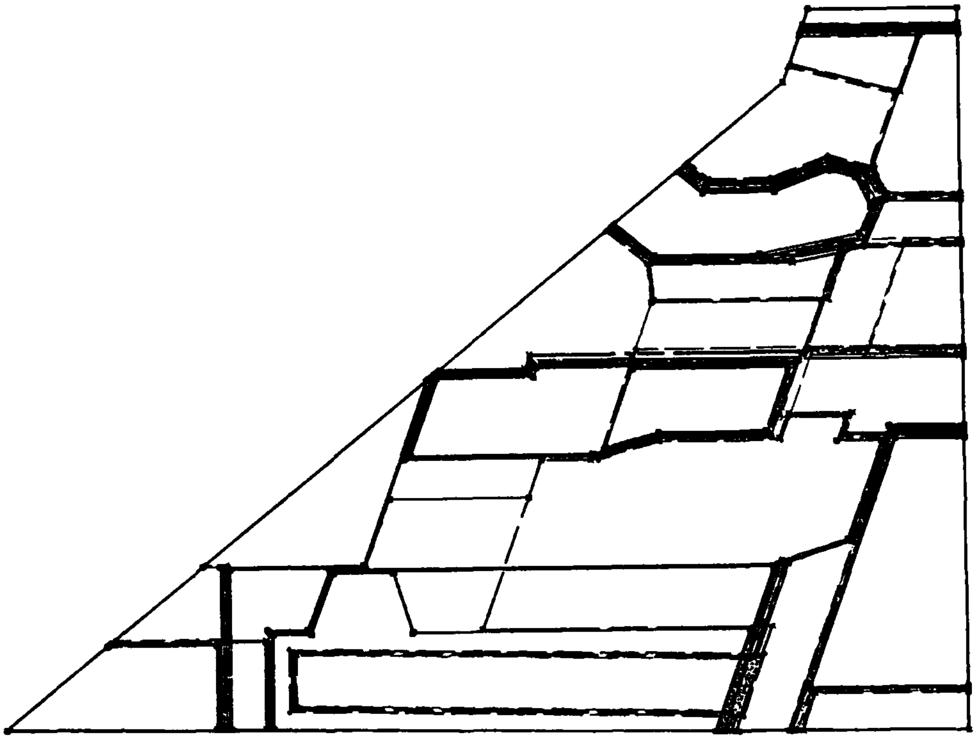

Fig. 7 Interactive lay-up design using optimum pattern

MULTIDISCIPLINARY DESIGN AND OPTIMIZATION[1]

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SUMMARY

Mutual couplings among the mathematical models of physical phenomena and parts of a system such as an aircraft complicate the design process because each contemplated design change may have a far reaching consequences throughout the system. This paper outlines techniques for computing these influences as system design derivatives useful for both judgmental and formal optimization purposes. The techniques facilitate decomposition of the design process into smaller, more manageable tasks and they form a methodology that can easily fit into existing engineering organizations and incorporate their design tools.

1. INTRODUCTION

The engineering design process is a two-sided activity as illustrated in Fig. 1. It has a qualitative side dominated by human inventiveness, creativity, and intuition. The other side is quantitative, concerned with generating numerical answers to the questions that arise on the qualitative side. The process goes forward by a continual question-answer iteration between the two sides. To support that process one needs a computational infrastructure capable of answering the above questions expeditiously and accurately. For development of such an infrastructure, the idea of "push button design" ought to be discarded in favor of a realistic recognition of the role of human mind as the leading force in the design process and of the role of mathematics and computers as the indispensable tools. It is clear that while conceiving different design concepts is a function of human mind, the evaluation and choice among competing, discretely different concepts, e.g., classical configuration vs. a forward swept wing and a canard configuration, requires that each concept be optimized to reveal its full potential. This approach is consistent with the creative characteristics of the human mind and the efficiency, precision, and infallible memory of the computer.

The computational infrastructure for support of the design process entails data management, graphics, and numerics. The first two embodied in CAD/CAM systems are well-known and are taken for granted as a framework for the numerics. The purpose of this paper is to introduce some new techniques which may be regarded as a subset of the latter. Included in the discussion are the system behavior derivatives with respect to design variables, their use for both judgmental and mathematical optimization purposes, formal decomposition of a system into its components, and ramifications of that decomposition for system sensitivity analysis and optimization, all illustrated by aircraft application examples. The impact on the design process of a methodology formed by these techniques is also examined.

2. EFFECT OF DESIGN VARIABLE CHANGE IN A COMPLEX SYSTEM

An aircraft is a complex system of interacting parts and physical phenomena whose behavior may be influenced by assigning values to the design variables. Since the design process is, generally, concerned with an aircraft that does not yet exist, one works with its surrogate—a system of mathematical models that correspond, roughly, to the engineering disciplines, and to physical parts of the vehicle. These mathematical models send data to each other as depicted in the center of Fig. 2, and they also accept design variable values as inputs from the designers. To know how to change these design variables, designers must know the answers to "what if" questions, such as "what will be the effect on the system behavior if the design variables X, Y, Z will be changed to $X+\Delta X$, $Y+\Delta Y$, $Z+\Delta Z$?", implied by the loop in Fig. 2.

An example of a hypersonic aircraft in Fig. 3 illustrates how difficult it may be to answer an "what if" question for even a single variable change in a complex system in which everything influences everything else. Consider a structural cross-sectional thickness t in the forebody of a hypersonic aircraft shown in the upper half of Fig. 3 as a design variable that is to be changed. The lower half of the figure depicts a complex chain of influences triggered by the change of t and, ultimately, affecting the vehicle performance. The change of t influences the position of the bow shock wave relative to the inlet in two ways: through the nose deflection, and through the weight and the center of gravity position both of which affect the trimmed angle of attack. The shock wave position relative to the inlet is a strong factor in the propulsive efficiency of the engine that, in turn, combines with the weight to influence the aircraft performance. Additional influence on performance is through the angle of attack whose change alters the vehicle aerodynamic lift and drag. The resultant modifications of the performance may require resizing of the vehicle which, of course, may be a sufficient reason to change t again, and so on, until the iteration represented by the feedback loop in Fig. 3 converges.

The above iteration engages a number of mathematical models such as structures, aerodynamics, propulsion, and vehicle performance. For the purposes of this discussion, each such model may be regarded as a black box converting input to output and, consistent with the black box concept, the inner workings of the model will be left outside of the scope of the discussion. While it may not be too difficult to evaluate the input-on-output effect for each single black box taken separately, evaluation of the resultant change for the entire system of such black boxes may be exceedingly difficult, especially when iterations are involved. In general, the resultant may be a small difference of large numbers, so even its sign may be impossible to predict without a precise reanalysis of the entire system.

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[1] Originally presented under the title "A System Approach to Aircraft Optimization" as Paper No. 2 at the AGARD Workshop on Integrated Design Analysis and Optimization of Aircraft Structures, 1–2 May 1991, Bath, United Kingdom.

To generalize from the above example, let X and Y denote the system input and output, respectively, e.g., the structural cover thickness t and a measure of performance such as the aircraft range. Then, the derivative dY/dX is a measure of the influence of X on Y and its value answers quantitatively the associated "what if" question. More precisely, the derivative value informs only about the rate of change of Y at the value of X for which the derivative was obtained. Determination of the increment of Y for a given finite increment of X, if $Y(X)$ is nonlinear, can be done approximately by a linear extrapolation

$$Y_{\text{new}} = Y_{\text{old}} + \frac{dY}{dX}\Delta X \tag{1}$$

Capability to extrapolate as above for many different X and Y variables, enables one to decide, either judgmentally or by means of an optimization program, which variables X to change and by how much, in order to improve the design in some way. However, that capability is predicated on availability of the derivatives dY/dX termed the system design derivatives (SDD). For large system analysis, especially if the analysis is iterative, its is advantageous to avoid the brute force method of finite differencing on the entire system analysis in computation of these derivatives.

2.1 System Design Derivatives

Remembering that the mathematical model of an engineering system may be an assemblage of a large number of mathematical models representing its components and the governing physical phenomena, it is convenient to limit the discussion to three such black box models since that number is small enough to foster comprehension and, yet, large enough to develop a general solution pattern. Ascribing a vector function representation to each black box, the set of equations representing the system of the black boxes α, β, γ exchanging data as illustrated in Fig. 4 is

$$\begin{aligned} Y_\alpha &= Y_\alpha(X, Y_\beta, Y_\gamma) \\ Y_\beta &= Y_\beta(X, Y_\alpha, Y_\gamma) \\ Y_\gamma &= Y_\gamma(X, Y_\alpha, Y_\beta) \end{aligned} \tag{2}$$

The Y and X variables in the above are vectors entered in the black boxes selectively, e.g., some, but not necessarily all, elements of the vectors X and Y_α enter the black box β as inputs. Regarding $Y_\beta(X, Y_\alpha, Y_\gamma)$ as an example of a black box, the arguments, X, Y_α, Y_γ, are the inputs and Y_β is an output. The functions in eq. 2 are coupled by their outputs appearing as inputs, hence they form a set of simultaneous equations that can be solved for Y for given X. The act of obtaining such a solution is referred to as the system analysis (SA). In the presence of nonlinearities, SA is usually iterative.

For each function in eq. 2, one can calculate derivatives of output with respect to any particular input variable, assuming that other variables are fixed. From the entire system perspective, these derivatives are partial derivatives since they measure only the local input-on-output effect, as opposed to SDD which are total derivatives because they include the effect of the couplings. To prepare for further discussion, the partial derivatives corresponding to the Y-inputs are collected in the Jacobian matrices designated by a pair of subscripts identifying the origins of the output and input, respectively. For example,

$$J_{\gamma\alpha} = [\partial Y_\gamma/\partial Y_\alpha] \tag{3}$$

is a matrix whose j-th column is made of the partial derivatives $\partial Y_{\gamma i}/\partial Y_{\alpha j}$. Assuming the length of Y_γ as N_γ and the length of Y_α as N_α, the dimensions of matrix $J_{\gamma\alpha}$ are $N_\gamma \times N_\alpha$. It will be mnemonic to refer to the partial derivatives in the Jacobian matrices as the cross-derivatives.

The remaining partial derivatives corresponding to the X-inputs are collected in vectors, one vector per each of the NX elements of the vector of design variables X, e.g,

$$\{\partial Y_\alpha/\partial X_k\}' = [\partial Y_\alpha/\partial X_k], \quad k = 1, \ldots, NX; \tag{4}$$

is a vector of the length N_α ($'$ denotes transposition).

Calculation of the above partial derivatives may be accomplished by any means available for a particular black box at hand, and may range from finite differencing to quasi-analytical methods (ref. 1, and 2).

It was shown in ref. 3 that differentiation of the functions in eq. 2 as composite functions and application of the implicit function theorem leads to a set of simultaneous, linear, algebraic equations, referred to as the Global Sensitivity Equations (GSE), in which the above partial derivatives appear as coefficients and the SDD are the unknowns. For the system of eq. 2, the GSE are

$$\begin{bmatrix} I & -J_{\alpha\beta} & -J_{\alpha\gamma} \\ -J_{\beta\alpha} & I & -J_{\beta\gamma} \\ -J_{\gamma\alpha} & -J_{\gamma\beta} & I \end{bmatrix} \begin{Bmatrix} dY_\alpha/dX_k \\ dY_\beta/dX_k \\ dY_\gamma/dX_k \end{Bmatrix} = \begin{Bmatrix} \partial Y_\alpha/\partial X_k \\ \partial Y_\beta/\partial X_k \\ \partial Y_\gamma/\partial X_k \end{Bmatrix} \tag{5}$$

These equations may be formed only after the SA was performed for a particular X, a particular point in the design space because the computation of the partial derivatives requires that all the X and Y values be known. For a given X, the matrix of coefficients depends only on the system couplings and is not affected by the choice of X for the right hand side. Hence that matrix may be factored once and reused in a backsubstitution operation to compute as many sets of SDD's as many different X_k variables are represented in the set of multiple right-hand-side vectors.

As recommended in ref. 3, numerical solution of eq. 5 and interpretation of the SDD values will be facilitated by normalization of the coefficients in the matrix and in the right hand sides by the values of Y_o and X_o of the Y and X variables for which the partial derivatives were calculated. The normalized coefficients take on the following form, illustrated by a few examples from i-th row in the β partition in eq. 5

$$(6) \qquad -\frac{\partial Y_{\beta i}}{\partial Y_{\alpha j}} q_{\beta\alpha ij}; \quad -\frac{\partial Y_{\beta i}}{\partial Y_{\gamma j}} q_{\beta\gamma ij}; \quad \frac{dY_{\beta i}}{dX_k} q_{\beta X ik}$$

where the normalization coefficients q are

$$q_{\beta\alpha ij} = \frac{Y_{\alpha jo}}{Y_{\beta io}}; \quad q_{\beta\gamma ij} = \frac{Y_{\gamma jo}}{Y_{\beta io}}; \quad q_{\beta X ik} = \frac{X_{ko}}{Y_{\beta io}}$$

Solution of the normalized eq. 5 yields normalized values of the SDD's from which the unnormalized values may always be recovered given the above definitions.

Formation of the GSE and their solution for a set of SDD's will be referred to as the System Sensitivity Analysis (SSA).

2.2 Utility of the System Design Derivatives

The SDD carry the trend information that under a conventional approach would be sought by resorting to statistical data or to the parametric studies. The former have the merit of capturing a vast precedent knowledge but may turn out to be ineffective if the vehicle at hand is advanced far beyond the existing experience. The latter provide an insight into the entire interval of interest but only for a few variables at a time, and that insight tends to be quickly lost if there are many design variables, in which case the computational cost of the parametric studies also may become an impediment.

In contrast, the SDD information is strictly local but it reflects the influences of all the design variables on all aspects of the system behavior. Therefore, the SSA should not be regarded as a replacement of the above two approaches but as their logical complement whose results are useful in at least two ways.

2.2.1 *Ranking design variables for effectiveness*

A full set of SDD for a system with NY variables in Y and NX variables in X is a matrix $NY \times NX$. The j-th column of the matrix describes the degree of influence of variable X_j on the behavior variables Y. Conversely, the i-th row shows the strength of influence of all the design variables X on the i-th behavior variable Y_i. For normalized SDD's, comparison of these strengths of influence becomes meaningful and may be used to rank the design variables by the degree of their influence on the particular behavior variable. This ranking may be used as a basis for judgmentally changing the design variable values and for deciding which design variables to use in a formal optimization.

An example of such ranking is illustrated for the wing of a general aviation aircraft shown in Fig. 5. The design variables are thicknesses t of the panels in the upper cover of the wing box and the behavior variable is the aircraft range R. The chain of influences leading from a panel thickness to the range calculated by means of the Breguet formula is depicted on the left side in Fig. 6. In the Breguet formula, W_e denotes the zero-fuel weight and W_p stands for the fuel weight. Increasing t in one of the panels increases the weight W_e and, in general, reduces the drag of a flexible wing by stiffening its structure. Consequently, the range is influenced in conflicting ways that would make prediction by judgment difficult. However, the corresponding SSA yields the SDD's for the upper row of the wing cover panels illustrated by the heights of the vertical bars over the upper wing cover panels in Fig. 6. The bars show that among all the wing cover panels, increasing t in the extreme outboard panel would increase range the most.

2.2.2 *Gradient-guided formal optimization*

Most of the formal optimization methods applicable in large engineering problems use the first derivative information to guide the search for a better design. Since the SDD values provide such information for all the Y and X variables of interest, the SSA may be incorporated, together with SA, in a system optimization procedure (SOP) based on the well-known piecewise approximate analysis approach (e.g., ref. 4). The SOP flowchart is depicted in Fig. 7. An important benefit of the SOP organization is the opportunity for parallel processing seen in the flowchart operation immediately following the SA. In that operation, one computes concurrently the partial derivatives of input with respect to output for all the system black boxes, in order to form the Jacobian matrices (eq. 3) and the right-hand-side vectors (eq. 4) needed to form the GSE (eq. 5) whose solution yields the SDD's. In a conventional approach, these SDD's would be computed by finite differencing on SA. The SDD values are subsequently used in Approximate Analysis (extrapolation formulas) that supplies the optimizer (a design space search algorithm) with information on the system behavior for every change of the design variables generated by that optimizer, and does it at a cost negligible in comparison with the cost of SA.

A generic hypersonic aircraft similar to the one that was discussed in Fig. 3 was used as a test for the above optimization. The geometrical design variables for the case are shown in Fig. 8. Additional design variables were the deflections of the control surfaces, and the cross-sectional structural dimensions of the forebody. The propulsive efficiency measured by the I_{sp} index, defined as the thrust minus drag divided by the fuel mass flow rate, was chosen as the objective function to be maximized. The aircraft take-off gross weight (TOGW) for a given mission is very sensitive to that index, thus maximization of the index effectively minimizes TOGW. For the reasons discussed in conjunction with Fig. 3, the problem requires consideration of a system composed of aerodynamics, propulsion, performance analysis, and structures. The optimization included constraints on the aircraft as a whole and on behavior in the above disciplines. Results are shown in Table 1 in terms of the initial and final values of the design variables (cross-sectional dimensions omitted) and of the objective function, all normalized by the initial values. Considering that the initial values resulted from an extensive design effort using a conventional approach, the nearly 13% improvement in the propulsive efficiency was regarded as very significant indeed.

Another example of the SOP application is the case of a hypersonic interceptor (Fig. 9a) reported in ref. 5. The optimization objective was the minimum of TOGW for the mission profile illustrated in Fig. 9b. The system comprised the modules of the configuration geometry, configuration mass properties, mission performance analysis, aerodynamics, and propulsion as depicted in Fig. 10, and the design variables were the wing area, scale factor for the turbojet engine, scale factor for the ramjet engine, and the fuselage length. The constraint list included a limit on the time needed to reach the combat zone, the take-off velocity, and the fuel available mass being at least equal to the one required (the fuel balance constraint). It should be noted that in a conventional approach to aircraft design, satisfaction of the latter constraint is one of the principal goals in development of a baseline configuration whose improvement is subsequently sought by parametric studies in which the design variables are varied while always striving to hold the fuel balance constraint satisfied. In contrast to that practice, the optimization reported in ref. 5 allowed the fuel balance constraint to be violated in the baseline configuration and achieved satisfaction of that constraint in the course of the optimization process. This demonstrated that an optimization procedure may do more than just improve on an initial, feasible configuration; it can actually synthesize an optimal configuration starting with one that is not even capable of performing a required mission.

The optimization results are illustrated by a vertical bar chart in Fig. 11 that shows the changes of the design variables and of a significant (13%) improvement of the objective function. The figure shows also that the initially violated constraints of time to intercept and take-off velocity were brought to satisfaction in the optimal configuration. The SOP converged in only 4 to 5 repetitions of SA and SSA.

3. MERITS AND DEMERITS

Before discussion of the ramifications of the above sensitivity-based optimization in a system design process, it may be useful to examine briefly the merits and demerits of the proposed approach relative to the conventional technique of generating SSD by finite differencing on the entire SA.

3.1 Accuracy and Concurrent Computing

The SSA based on eq. 5 has two unique advantages. First, the accuracy of SDD is intrinsically superior to that obtainable from finite differencing whose precision depends on the step length in a manner that is difficult to predict. As pointed out in ref. 6 it is particularly true in the case of an iterative SA whose result often depends on an arbitrary, "practical" convergence criterion. Second, there is an opportunity for concurrent computing in the generation of the partial derivatives which exploits the technology of parallel processing offered by multiprocessor computers and computer networks. Concurrent computing also enables the engineering workload to be distributed among the specialty groups in an engineering organization to compress the project execution time.

3.2 Computational Cost

Experience indicates that in large engineering applications, most of the optimization computational cost is generated by the finite difference operations. Therefore, relative reduction of the cost of these operations translates into nearly the same relative reduction of the cost of the entire optimization procedure.

The computational cost of the SSA based on eq. 5, designated C_1, may be reduced, *in most cases very decisively*, below that of finite differencing on the entire SA, denoted by C_2, but to achieve that reduction the analyst should be aware of the principal factors involved. To define these factors, let the computational cost of one SA be denoted by CSA while CBA_i will stand for the computational cost of one analysis of the i-th black box in the system composed of NB black boxes. The i-th black box receives an input of NX_i design variables X, and NY_i variables Y from the other black boxes in the system. Assuming for both alternatives the simplest one step finite difference algorithm that requires one reference analysis and one perturbed analysis for each input variable, the costs C_1 and C_2 may be estimated as

$$C_1 = \sum_i (1 + NX_i + NY_i)CBA_i; \qquad C_2 = (1 + NX)CSA \tag{7}$$

Even though one may expect $CBA_i < CSA$, a sufficiently large NY_i may generate $C_1 > C_2$ and render SSA based on eq. 5 unattractive compared to finite differencing on the entire SA. This points to NY_i, termed the interaction bandwidth, as the critical factor whose magnitude should be reduced as much as possible. Reducing the interaction bandwidth requires judgment as illustrated by an example of an elastic, high aspect ratio wing treated as a system whose aeroelastic behavior is modeled by interaction of aerodynamics and structures, represented by an CFD analysis and Finite Element analysis codes, respectively. If one let the full output from each of these black boxes be transmitted to the other, there might be hundreds of pressure coefficients entering the structural analysis and thousands of deformations sent to the aerodynamic analysis. With the NY_i values in the hundreds and thousands, respectively, it would be quite likely that $C_1 > C_2$. However, one may condense the information flowing between the two black boxes by taking advantage of the high aspect ratio wing slenderness. For a slender wing it is reasonable to represent the entire aerodynamic load by, say, a set of 5 concentrated forces at each of 10 separate chords, and to reduce the elastic deformation data to, say, elastic twist angles at 7 separate chords. This condensation reduces the NY_i values to 50 for structures and 7 for aerodynamics. In the finite element code, that implies 50 additional loading cases all of which can be computed very efficiently by the multiple loading case option—a standard feature in finite element codes. The CFD code would have to be executed only 7 additional times. Thus, the advantage of the interaction bandwidth condensation is evident. In general, a condensation such as the one described above for a particular example may be accomplished by the reduced basis methods, among which the Ritz functions approach is, perhaps, the best known one.

3.3 Potential Singularity

One should be aware when using SSA based on eq. 5 that, in some cases, the matrix of coefficients in these equations may be singular. In geometrical terms, a solution in SA may be interpreted geometrically as a vertex of hyperplanes on which the residuals of the governing equations for the black boxes involved are zero. As pointed in ref. 3, eq. 5 are well-conditioned if these hyperplanes intersect at large angles, *ideally* when they are mutually orthogonal. For two functions of two variables the zero-residual hyperplanes reduce to the zero-residual contours, and an example of a nearly-orthogonal solution intersection is shown in Fig. 12a. In some cases, the intersection angles may tend to be very acute, in the limit they may be zero in which case a solution exist by virtue of tangency of two curved contours as illustrated in Fig. 12b. It is shown in ref. 3 that eq. 5 imply local linearization of these contours in the vicinity of the intersection point so that the solution point is interpreted as an intersection of the tangents. Consequently, in the situation depicted in Fig. 12b the tangents coincide and the matrix of eq. 5 becomes singular. In such a case, eq. 5 should be replaced by an alternative formulation of the system sensitivity equations in ref. 3 based on residuals.

There were no cases of singularity reported so far in any applications probably because the system solutions of the type illustrated in Fig. 12b characterize an ill-posed system analysis usually avoided in practice.

3.4 Discrete Variables

Neither the reference technique nor the SSA based on eq. 5 can accommodate truly discrete design variables. Truly discrete design variables are defined for the purposes of this discussion as those with respect to which SA is not differentiable. These are distinct from quasi-discrete variables with respect to which SA is differentiable but which may only be physically realizable in a set of discrete values. An example of the former is an engine location on the aircraft: either under the wing or at the aft end of the fuselage. An example of the latter is sheet metal thickness available in a set of commercial gages.

In the case of truly discrete design variables, different combinations of such variables define different design concepts (alternatives) and each concept may be optimized in its own design space of the remaining continuous variables, in order to bring it up to its true potential. Then, one may choose from among the optimal alternatives. Occasionally, a continuous transformation might be possible between two concepts that seem to be discretely different. For example, a baseline aircraft with a canard, a wing, and a conventional tail may be reshaped into any configuration featuring all, or only some of these three lifting surfaces. This is so because a sensitivity-guided SOP may eliminate a particular feature, if a design variable is reserved for that feature and if the feature is present in the initial design (however, a feature initially absent cannot, in general, be created).

3.5 Non-utilization of Disciplinary Optimization

Organization of the SOP discussed above may be described as "decomposition for sensitivity analysis followed by optimization of the entire, undecomposed system". It may be regarded as a shortcoming that the procedure leaves no clear place for the use of the vast expertize of optimization available in the individual black boxes representing engineering disciplines. Examples of such local, disciplinary optimization techniques are the optimality criteria for minimum weight in structures, and shaping for minimum drag for a constant lift in aerodynamics. It appears that combining these local, disciplinary optimization techniques with the overall system optimization should benefit the latter. Indeed, one way in which these techniques may be used without changing anything in the SOP organization described above is in the SOP initialization. Obviously, starting SOP from a baseline system composed of the black boxes already preoptimized for minimum weight, minimum drag, maximum propulsive efficiency, etc. should accelerate the SOP convergence and improve the end result. Such local optimizations could be accomplished separately for each black box, assuming X and guessing at the Y inputs.

Beyond that, the issue of incorporating the local, disciplinary optimization in SOP remains to be a challenge for further development. Some solutions were proposed in ref. 7 and 8 but their effectiveness is yet to be proven in practice. This issue will be taken up again in the later discussion in conjunction with the special case of a hierarchic system decomposition which does accommodate the local optimizations.

4. FORMAL DECOMPOSITION

When the system at hand contains a large number of black boxes and, especially, if there is little or no experience with its solution, it is useful to apply a formal technique to determine the data flow among the black boxes. The data flow information is useful because it characterizes the system as non-hierarchic, hierarchic, or hybrid, and this, in turn, helps to choose an optimization approach and to establish an efficient organization of computing. Such formal techniques are available in Operations Research and some of them were adapted for the system analysis and optimization purposes, e.g., ref. 9.

4.1 N-square Matrix

A brief introduction to one such technique begins with a formalization of a black box (a module) in the system as one that receives inputs through the top and bottom horizontal sides and sends the output through the left and right vertical sides as as shown in Fig. 13. Using that formalism, one can represent a four-module system example depicted by the diagram (known as the graph-theoretic format) in Fig. 14a in a different format shown in Fig. 14b. That format is known as the N-square Matrix format because N modules placed along the diagonal form an N^2 table. The N-square Matrix format assumes that the modules are executed in order from upper left to lower right (although, if possible, concurrent executions are allowed). If the execution order is not yet known, the order along the diagonal may be arbitrary. Referring to Fig. 13, each module may, potentially, send data horizontally, left and right, and receive vertically from above and from below. The actual data transmissions from and to i-th module are determined by *comparing the module input list to the predecessor module* output lists while moving upward in column i. Wherever a needed input item is found on the output list from module j, a dot is placed at the intersection of the i-th column and j-th row as a data junction indicating transmission of output from module j to input of module i. After the predecessor module search gets to the first module, it switches to module $i + 1$ and continues downward through all the successor modules to module N. If more than one source is found for a particular input item, a unique, single source must be judgmentally selected. However, an output item may be used by several receiver modules and may also be sent to the outside. The input items that could not be found in the vertical search are designated primary inputs to be obtained from the outside of the system. The above search is readily implementable on a computer.

When the above search procedure is completed for all the modules, the result is an N-square Matrix as in Fig. 14b that conveys the same information as the diagram in Fig. 14a but is amenable to computerized manipulation. To see what such manipulation may achieve, observe that each dot in the upper triangle of the N-square Matrix denotes an instance of the data feedforward, and each dot in the lower triangle notes an instance of the data feedback. Of course, every instance of a feedback implies an iteration loop required by the assumed diagonal order of the modules. However, that order may be changed at will by a code that may be instructed to switch the modules around, with the associated permutations of the rows and columns to preserve the data junction information, in order to eliminate as many instances of feedback as possible. If all of them are eliminated the system admits a sequential module execution, and may offer opportunities for concurrent executions of some modules. If a complete elimination of the feedbacks is not possible, they are reduced in number and clustered. An example of a fairly large N-square Matrix in the initial, arbitrary order is shown in Fig. 15a while its clustered state is shown in Fig. 15b. In the clustered state the system is hybrid—partially hierarchic and partially non-hierarchic. A software tool that is available to make the above transformation is described in ref. 9. All the modules in one of the clusters in Fig. 15b may be regarded as a new supermodule, and the system diagram may be drawn in terms of these supermodules as shown in Fig. 16. This diagram defines a hierarchic decomposition of a system because the data flow from the top of the pyramidal hierarchy to the bottom, without reversing the flow and without lateral flow, while inside of each cluster there is a system whose modules define a non-hierarchic decomposition.

The N-square Matrix structure has a reflection in the structure of the matrix of coefficients in eq. 5: each feedforward instance in the former gives rise to a Jacobian matrix located below the diagonal in the latter and each feedback is reflected in a Jacobian above the diagonal. Hence, a sequential system without feedbacks has a matrix of coefficients populated only below the diagonal so that eq. 5 may be solved by backsubstitution of the right hand sides without factoring of the matrix of coefficients.

4.2 SOP Adapted to Hierarchic System

When a decomposed system has a hierarchic structure, its SOP may be reorganized to include separate optimizations in each black box. This SOP version was introduced in ref. 10 and called an optimization by linear decomposition. It has found a number of applications, for example, it was the basis for an algorithm for multilevel structural optimization by substructuring in ref. 11, and its use in multidisciplinary applications was reported in ref. 12 for control-structure interaction and in ref. 13 for optimization of a transport aircraft.

Multilevel optimization of a hierarchic system by a linear decomposition exploits the top-down flow of the analysis information. At the bottom level, the inputs obtained from analysis at the next higher level and the appropriate design variables are regarded as constants in optimization of each, bottom-level black box. Derivatives of each such optimization are computed with respect to these input constants by means of an algorithm described in ref. 14 and are used in linear extrapolations (hence the name of the technique) to approximate the effect of the input constants on the optimization results. Optimizations in the black boxes at the next higher level approximate their influence on the lower level optimization by means of these extrapolations. Thus, the top black box optimization is performed taking an approximate account of the effect of its variables (the system level variables) on all the black boxes in the hierarchic pyramid. As mentioned in the foregoing, the advantages of the SOP exploiting the hierarchic structure of the system is a separation of the bottom level detailed optimizations from the top level system optimization, and breaking the large system optimization problem into a number of smaller optimization problems, in contrast to the non-hierarchic system SOP (Fig. 7) in which optimization is performed for the system as a whole. However, if any of these black boxes in a hierarchic system contains a cluster (see discussion of Fig. 16) of black boxes forming a non-hierarchic system, the non-hierachic system SOP (Fig. 7) may be used to optimize it locally. Hence, both methods for system optimization described above, the one based on the linear decomposition (ref. 10) as well as the SOP based on Fig. 7 flowchart have their place in optimization of a general case of a hybrid engineering system that exhibits both the hierarchic and non-hierarchic structures depicted in Fig. 16.

As reported in ref. 13, the linear decomposition method was used to optimize the variables of configuration geometry and cross-sectional structural dimensions of a transport aircraft illustrated in Fig. 17a for minimum fuel burned in a prescribed mission, under constraints drawn from the disciplines of aerodynamics, performance and structures. The analysis was relatively deep, e.g., a CFD code in aerodynamics, and a finite element model of the built-up structure of the airframe structures. The number of design variables was over 1300, and the number of constraints was also in thousands. Optimization was conducted decomposing the problem into a three-level hierarchic system shown in Fig. 17b. A sample of results is depicted in Fig. 18 showing a smooth convergence of the fuel mass and the structural weight in only 4 to 6 cycles (one cycle comprised the top-down analysis and the bottom-up optimizations), for both feasible and infeasible initial design.

5. GENERALIZATION TO ENTIRE VEHICLE DESIGN PROCESS

The approach to the system sensitivity and optimization discussed in the foregoing may be generalized to serve the entire design process as shown in ref. 15 using as an example a definition of that process given in ref. 16. The process defined in ref. 16 is a conventional, sequential process illustrated in Fig. 19. As suggested in the upper right corner of the flowchart, any change in a major design variable such as the wing or engine size requires reentry into the sequence and repetition of all the operations in the chain. However, the black boxes forming the sequence are also forming a coupled system whose diagram is depicted in Fig. 20. The arrows in the diagram represent the data flow among the black boxes, examples of the data being defined in Table 2. Application of the SSA based on eq. 5 to the system in Fig. 20 leads to GSE in the format shown in Fig. 21. In the abbreviated notation used in that figure, Y_{ij} stands for a Jacobian matrix J_{ij} defined in eq. 3. Solution of the equations shown in Fig. 21 yields the SDD values that answer the "what if" questions implied in the upper right corner of the flowchart in Fig. 19, and does it for all the variables of interest simultaneously and without repeating the entire chain for every question. The SDD values may then be used to support judgmental design decisions and/or to guide a formal optimization according to the SOP in Fig. 7.

6. CONCLUDING REMARKS

Design of an engineering system, such as an aircraft, is a formidable task involving a myriad of cross-influences among the engineering disciplines and parts of the system. The time-honored approach to that task is to decompose it into smaller, more manageable tasks. The paper outlines some recently developed techniques that support such an approach by building an engineering system optimization on a modular basis, that comprises engineering specialty groups and their black box tools and allows engineers to retain responsibility for their domains while working concurrently on manageable tasks and communicating with each other by means of sensitivity data. The modularity and concurrence of operations map onto the familiar structure of the engineering organizations and are compatible with the emerging computer technology of multiprocessor computers and distributed computing. The only major new requirement is the generation of derivatives of output with respect to input in each specialty domain.

The use of sensitivity data as the communication medium is the distinguishing feature of the proposed approach and represent a major improvement over the present practice because it adds the trend information to the function value information. Both types of information enhance the human judgment and intuition while being readily usable in guiding the formal optimization procedures.

Acknowledgment: Contribution of the NASP configuration optimization example (Fig. 8 and Table 1) by Dr. F. Abdi and Mr. J. Tulinius of Rockwell International—North American is gratefully acknowledged.

7. REFERENCES

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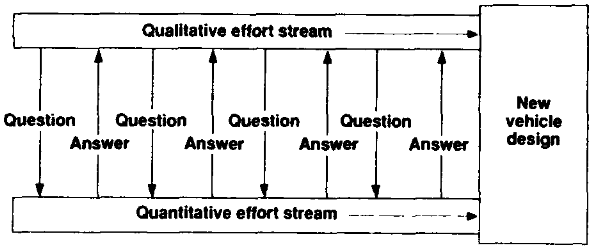
Table 1
Hypersonic aircraft optimization results

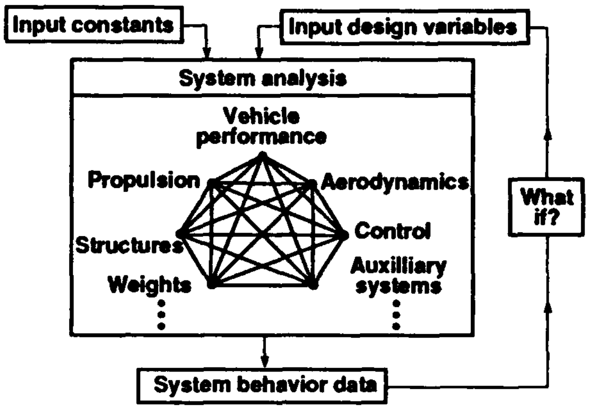
Optimization parameter	Baseline value	Optimization results
Design variable		
1. Forebody length	1.000	1.0209
2. Cone angle	1.000	0.9693
3. Upper surface height	1.000	1.0029
4. Geometric transition length	1.000	1.0760
5. Elevon deflection	1.000	0.8620
6. Bodyflap deflection	1.000	1.0320
Objective		
Effective trimmed Isp	1.000	1.1259

Table 2
Coupling data in aircraft system

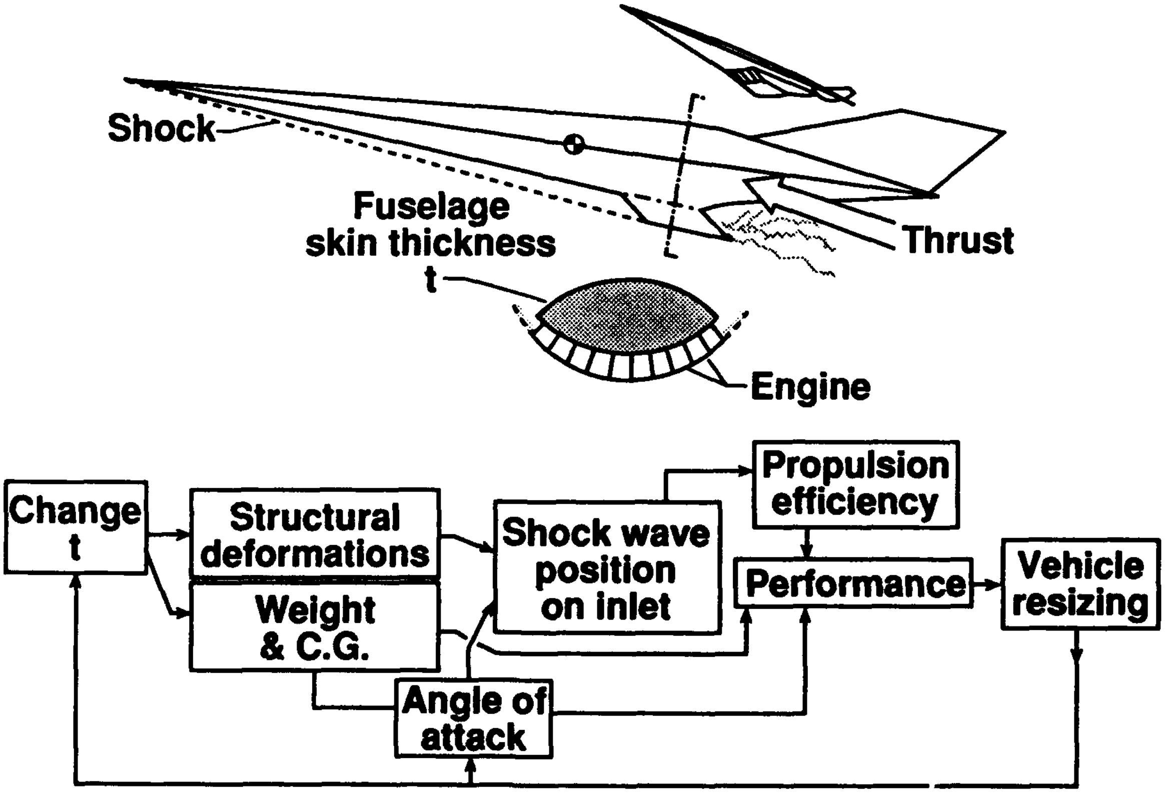
Vector Y	Content examples
1	See the box labeled INPUT
2	Wing area, aspect ratio, taper, sweep angle, airfoil geometry data. Engine thrust.
3	Fuel tank locations and assumed volumes.
4	Wing structural weight and internal volume.
5	Take-off Gross Weight.
6	See box 6.
7	Landing gear weight and location, in stowed and extended position. Take-off field length.


Qualitative effort stream
Question
Answer
Question
Answer
Question
Answer
Question
Answer
New vehicle design
Quantitative effort stream

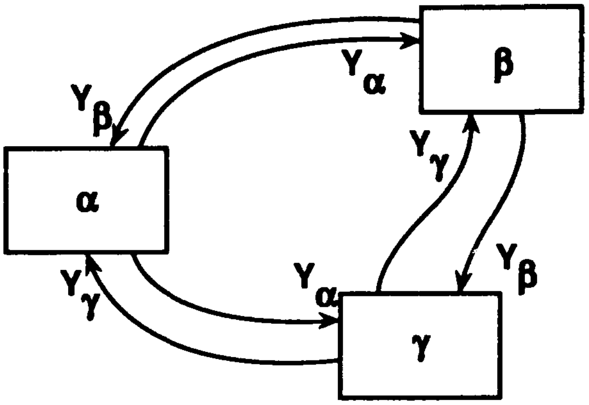
1. Qualitative and quantitative sides of a design process.


Input constants
Input design variables
System analysis
Vehicle performance
Propulsion
Aerodynamics
Structures
Control
Weights
Auxiliary systems
What if?
System behavior data

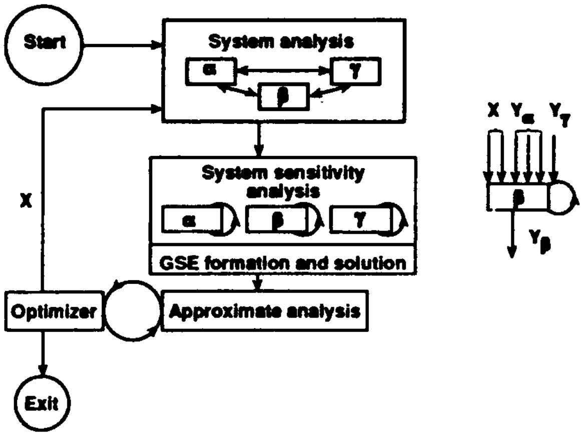
2. Interactions in a system analysis and "What if" questions.


Shock
Fuselage skin thickness
t
Thrust
Engine
Change t
Structural deformations
Weight & C.G.
Shock wave position on inlet
Propulsion efficiency
Performance
Vehicle resizing
Angle of attack

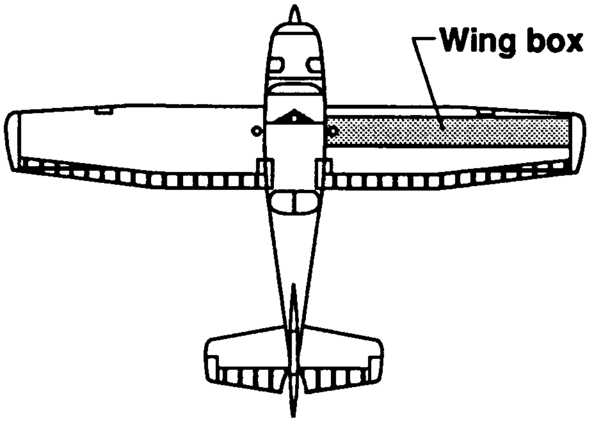
3. A design change triggering a complex chain of effects.


Y_α
β
Y_β
α
Y_γ
Y_γ
Y_α
Y_β
Y_γ
γ

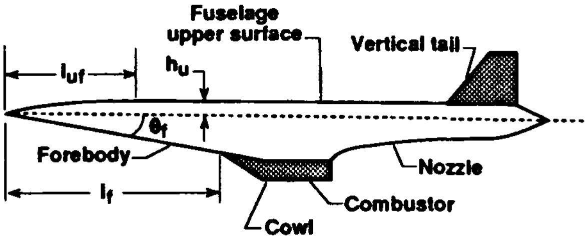
4. Example of a three component system.


Start
System analysis
α
γ
β
X Y_α Y_γ
β
Y_β
System sensitivity analysis
α
β
γ
x
GSE formation and solution
Optimizer
Approximate analysis
Exit

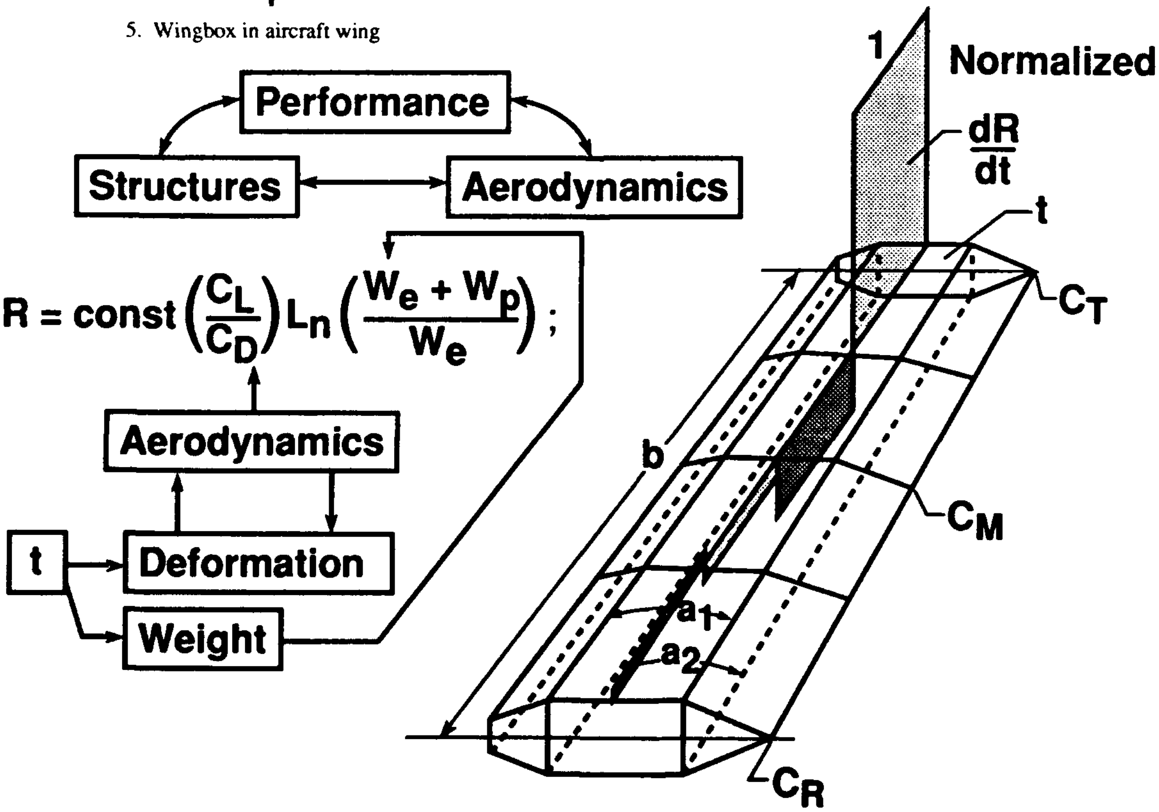
7. Flowchart of the System Optimization procedure (SOP).



5. Wingbox in aircraft wing

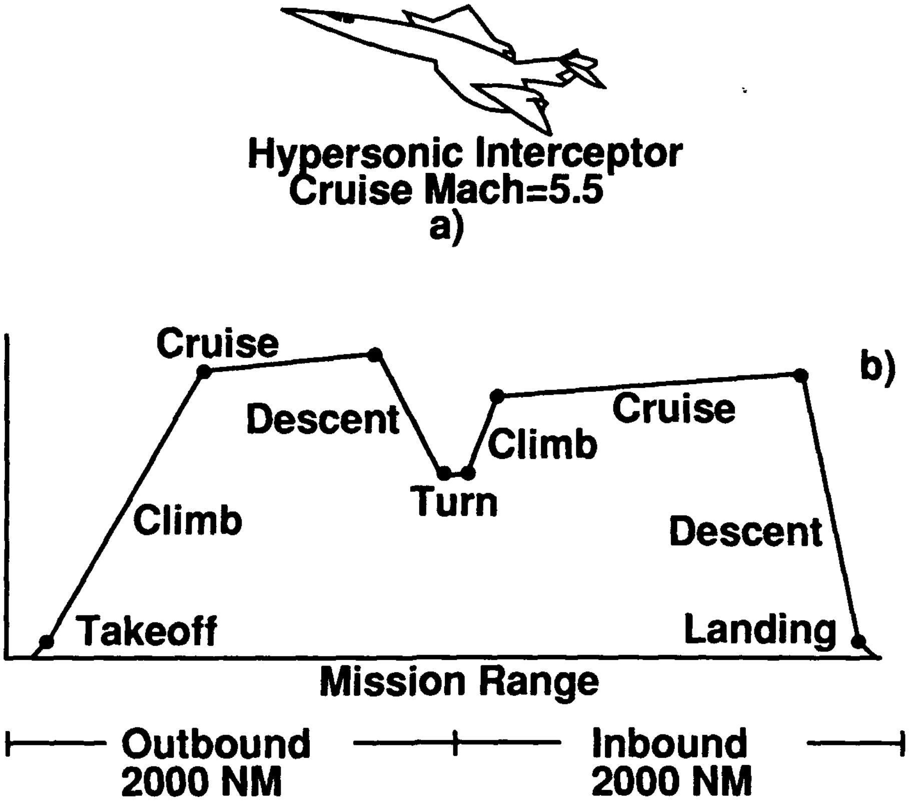

Fuselage upper surface
Vertical tail
l_uf
h_u
θ_f
Forebody
Nozzle
l_f
Combustor
Cowl

8. Hypersonic aircraft; some of the configuration design variables.

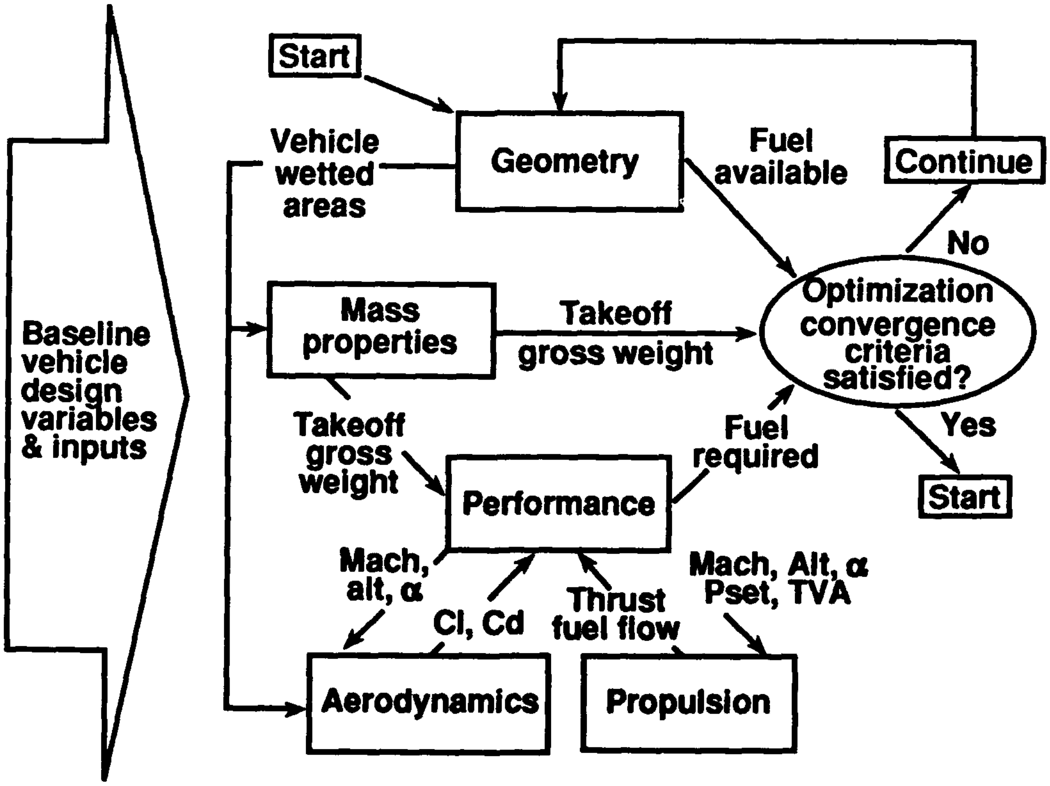

Performance
Structures
Aerodynamics
R = const (C_L/C_D) L_n ((W_e + W_p)/W_e) ;
Aerodynamics
t
Deformation
Weight
1
Normalized
dR/dt
t
C_T
b
C_M
a_1
a_2
C_R

6. a) System of mathematical models, the Breguet formula, and the channels of influence for the wing cover thickness;

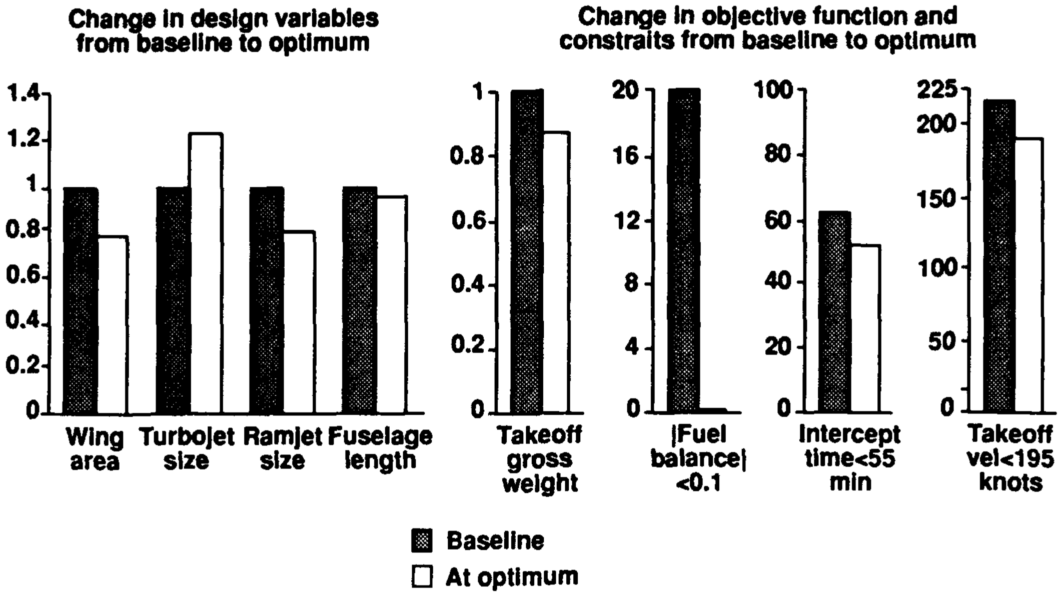
b) Vertical bars illustrate magnitude of derivatives of range with respect to thickness.


Hypersonic Interceptor
Cruise Mach=5.5
a)
b)
Cruise
Descent
Climb
Turn
Climb
Cruise
Descent
Takeoff
Landing
Mission Range
Outbound
2000 NM
Inbound
2000 NM

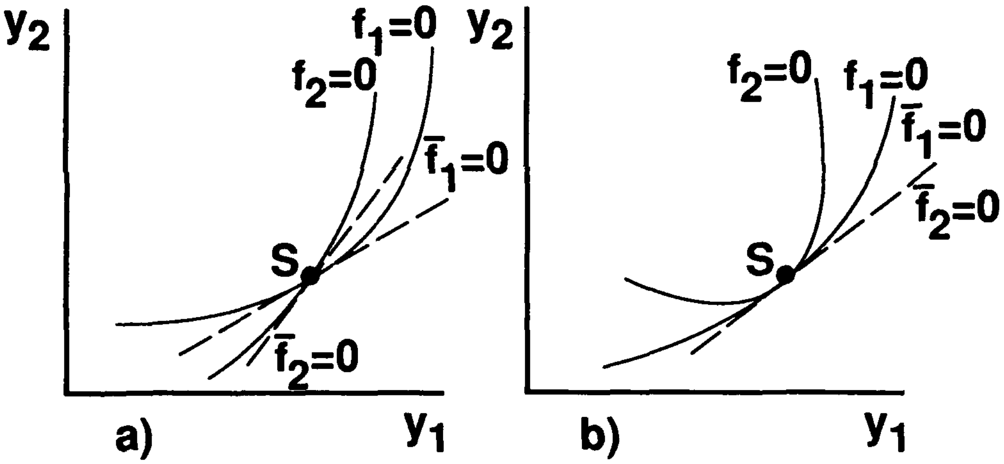
9. a) Hypersonic interceptor, b) Mission profile.


Baseline vehicle design variables & inputs
Start
Geometry
Vehicle wetted areas
Fuel available
Continue
No
Mass properties
Takeoff gross weight
Optimization convergence criteria satisfied?
Yes
Start
Takeoff gross weight
Fuel required
Performance
Mach, alt, α
Cl, Cd
Thrust fuel flow
Mach, Alt, α Pset, TVA
Aerodynamics
Propulsion

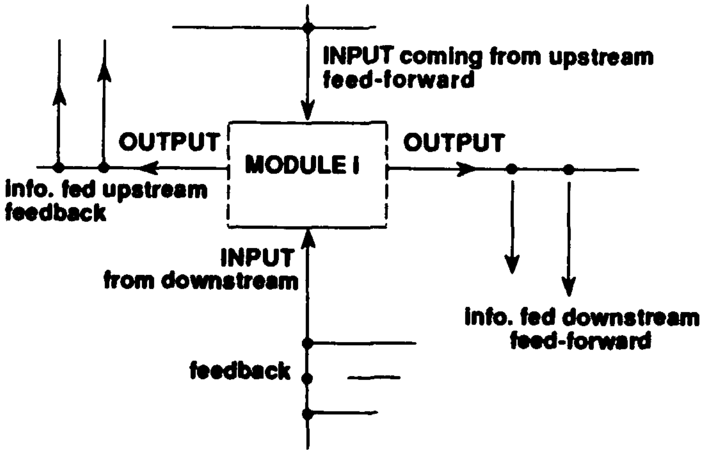
10. System of mathematical models for hypersonic interceptor optimization.



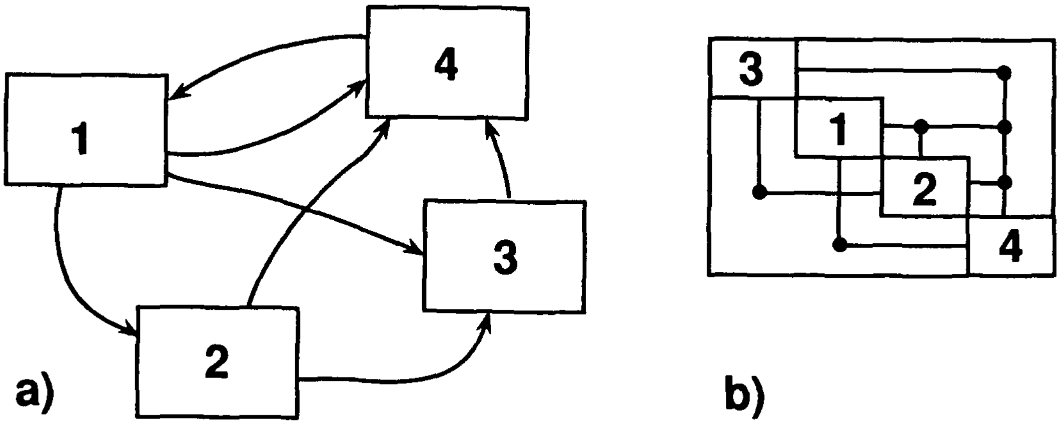
11. Sample results from hypersonic interceptor optimization.


y_2
$f_1=0$
$f_2=0$
$\bar{f}_1=0$
S
$\bar{f}_2=0$
a)
y_1
y_2
$f_2=0$
$f_1=0$
$\bar{f}_1=0$
$\bar{f}_2=0$
S
b)
y_1

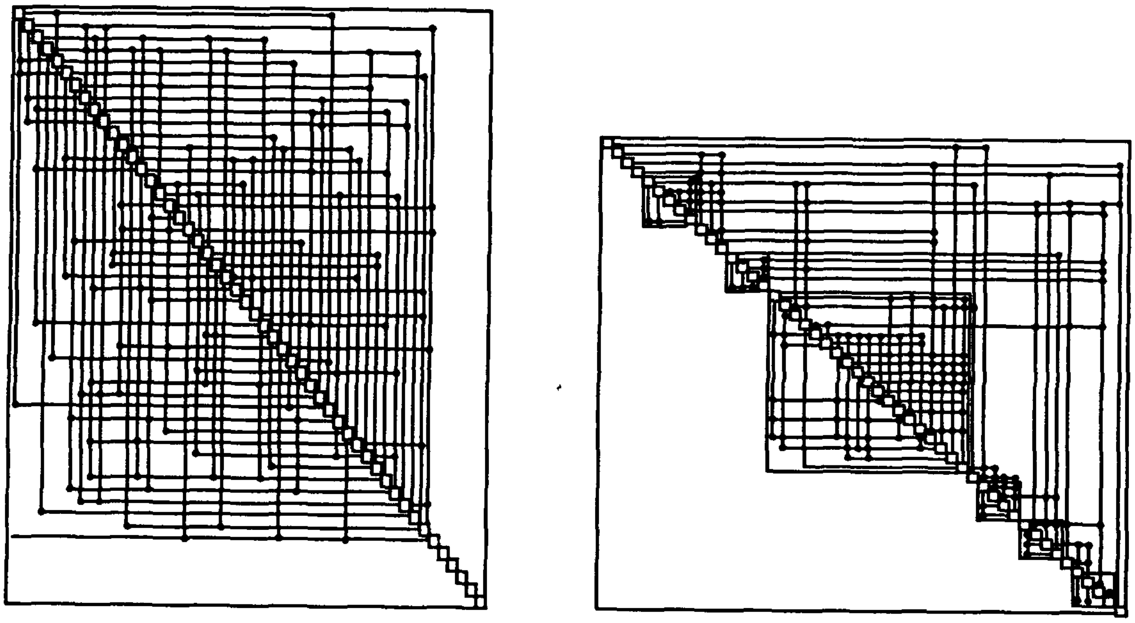
12. System solution: a) Intersection point; b) Tangency point.


INPUT coming from upstream feed-forward
OUTPUT
MODULE i
OUTPUT
Info. fed upstream feedback
INPUT from downstream
Info. fed downstream feed-forward
feedback

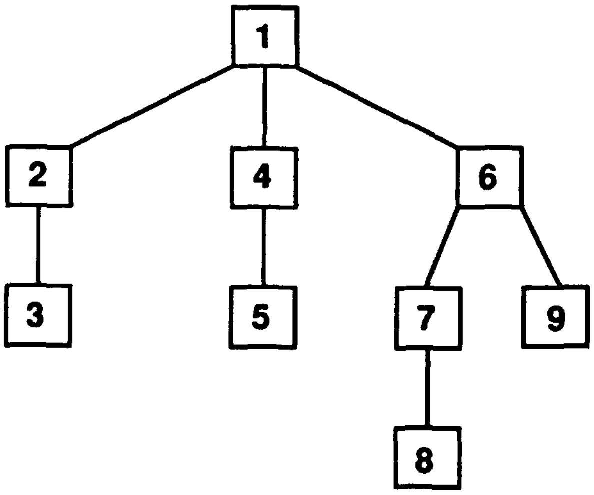
13. Schematic definition of a module.



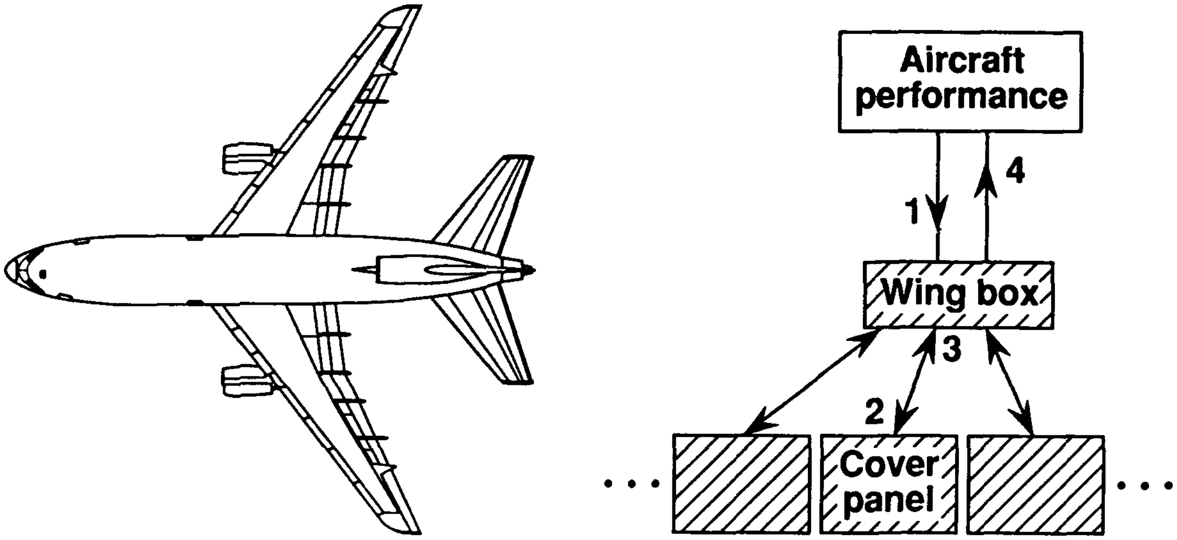
14. Example of a system: a) Graph format; b) N-square Matrix format.



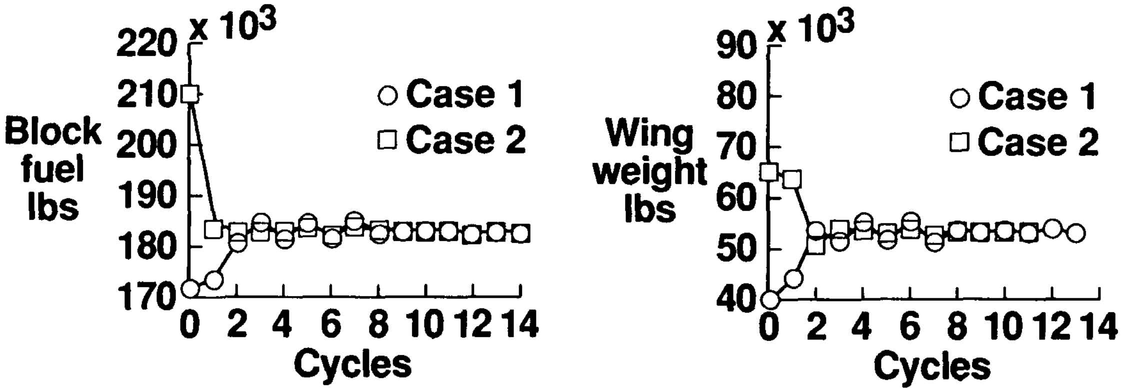
15. System N-square Matrix: a) Random execution order, b) Execution order rearranged to reduce and cluster the feedbacks.



16. Hierarchic structure of clusters in a system.

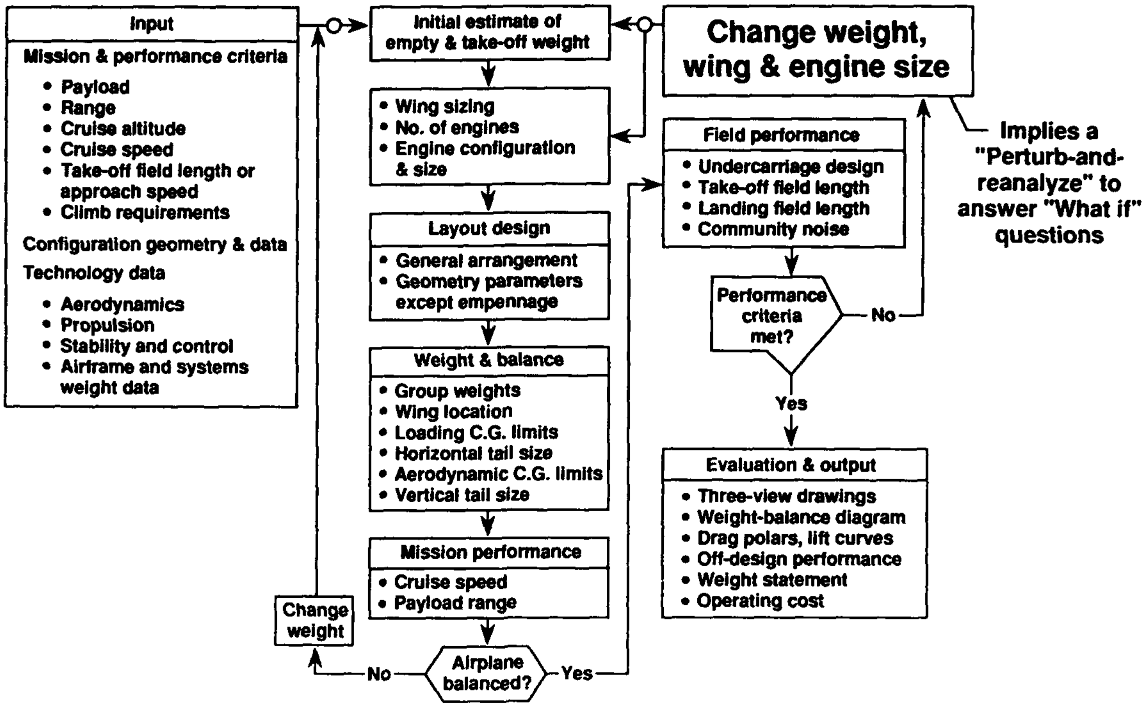

Aircraft performance
1
4
Wing box
3
2
Cover panel
• • •
• • •

17. Optimization of a transport aircraft: a) Configuration; b) Hierarchic system of modules.



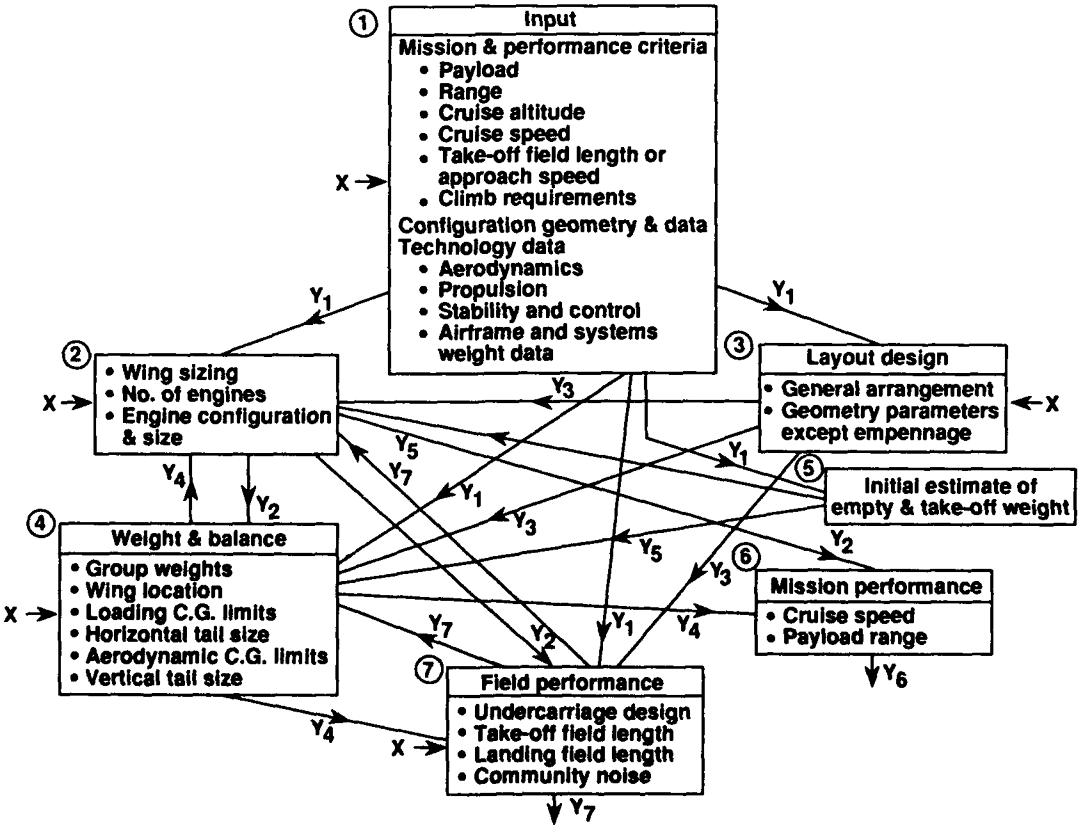
18. Sample of results from transport aircraft optimization.

- **Standard sequential design process from a textbook**


Input
Mission & performance criteria
• Payload
• Range
• Cruise altitude
• Cruise speed
• Take-off field length or approach speed
• Climb requirements
Configuration geometry & data
Technology data
• Aerodynamics
• Propulsion
• Stability and control
• Airframe and systems weight data
Initial estimate of empty & take-off weight
• Wing sizing
• No. of engines
• Engine configuration & size
Layout design
• General arrangement
• Geometry parameters except empennage
Weight & balance
• Group weights
• Wing location
• Loading C.G. limits
• Horizontal tail size
• Aerodynamic C.G. limits
• Vertical tail size
Mission performance
• Cruise speed
• Payload range
Change weight
No
Airplane balanced?
Yes
Change weight, wing & engine size
Field performance
• Undercarriage design
• Take-off field length
• Landing field length
• Community noise
Performance criteria met?
No
Yes
Evaluation & output
• Three-view drawings
• Weight-balance diagram
• Drag polars, lift curves
• Off-design performance
• Weight statement
• Operating cost
Implies a "Perturb-and-reanalyze" to answer "What if" questions

19. A conventional, sequential design process for aircraft.

- **Design represented as coupled system**


①
Input
Mission & performance criteria
• Payload
• Range
• Cruise altitude
• Cruise speed
• Take-off field length or approach speed
• Climb requirements
Configuration geometry & data
Technology data
• Aerodynamics
• Propulsion
• Stability and control
• Airframe and systems weight data
X
Y_1
Y_1
②
• Wing sizing
• No. of engines
• Engine configuration & size
X
③
Layout design
• General arrangement
• Geometry parameters except empennage
X
Y_3
Y_5
Y_7
Y_1
Y_4
Y_2
Y_1
⑤
Initial estimate of empty & take-off weight
Y_3
Y_5
Y_2
④
Weight & balance
• Group weights
• Wing location
• Loading C.G. limits
• Horizontal tail size
• Aerodynamic C.G. limits
• Vertical tail size
X
Y_3
⑥
Mission performance
• Cruise speed
• Payload range
Y_6
Y_7
Y_2
Y_1
Y_4
⑦
Field performance
• Undercarriage design
• Take-off field length
• Landing field length
• Community noise
Y_4
X
Y_7

20. Black boxes from Fig. 19 forming a system.

- **System sensitivity equations of design represented as coupled system**

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -Y_{21} & 1 & -Y_{23} & -Y_{24} & -Y_{25} & 0 & -Y_{27} \\ -Y_{31} & 0 & 1 & 0 & 0 & 0 & 0 \\ -Y_{41} & -Y_{42} & -Y_{43} & 1 & -Y_{45} & 0 & -Y_{47} \\ -Y_{51} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -Y_{62} & 0 & 0 & -Y_{65} & 1 & 0 \\ -Y_{71} & -Y_{72} & -Y_{73} & -Y_{74} & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{dY_1}{dX_k} \\ \frac{dY_2}{dX_k} \\ \frac{dY_3}{dX_k} \\ \frac{dY_4}{dX_k} \\ \frac{dY_5}{dX_k} \\ \frac{dY_6}{dX_k} \\ \frac{dY_7}{dX_k} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial Y_1}{\partial X_k} \\ \vdots \\ \vdots \\ \frac{\partial Y_i}{\partial X_k} \\ \vdots \\ \vdots \\ \vdots \end{Bmatrix} \cdots \begin{Bmatrix} \frac{\partial Y_1}{\partial X_L} \\ \vdots \\ \vdots \\ \frac{\partial Y_i}{\partial X_L} \\ \vdots \\ \vdots \\ \vdots \end{Bmatrix} \cdot\cdot$$

- **These system derivatives answer "What if" questions regarding these variables without reanalyzing the system**

21. GSE matrix for the system of Fig. 20.

MATHEMATICAL OPTIMIZATION
A POWERFUL TOOL FOR AIRCRAFT DESIGN

by

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Abstract

Formal mathematical optimization methods have been developed during the past 10 to 15 years for the structural design of aircraft. Together with reliable analysis programs like finite element methods they provide powerful tools for the structural design. They are efficient in at least two ways:

- producing designs that meet all specified requirements at minimum weight in one step;
- relieving the engineer from a time consuming search for modifications that give better results, they allow more creative design modifications.

MBB has developed a powerful optimization code called MBB-Lagrange which uses mathematical programming and gradients to fulfill different constraints simultaneously [1].

Some examples depicting the successful application of the MBB-LAGRANGE code are presented. Also results of other optimization codes are shown.

The paper closes with an outlook how the optimization problem could be enlarged to include also shape and size of airplanes.

Introduction

To improve or modify a design, a process, a procedure, or any given task into a "better" direction, is referred to as "optimization". This is often done by experience, parametric investigations, iterative procedures, by experimental testing and modifications, or based on empirical data. Such an approach usually leads to better results but nobody can tell how far away the optimum still is or even where it is. A more efficient way to perform this task is provided by a special branch of applied mathematics, called optimization. This kind of optimization changes the chosen variables in a design problem in a way to achieve the best value for an objective while not violating defined constraints that represent the boundaries of the design space.

This formal optimization was rather early introduced in economics or chemical engineering due to the linearity of the problems, as described by Ashley in an excellent overview paper on the aeronautical use of optimization [2]. In order to use the potential of mathematical optimization, it is necessary to describe the physical nature of the problem in a way that allows the use of optimization algorithms.

In structural design, finite element methods together with modern computers have provided tools that allow to analyse complex structures with high accuracy. These were main essentials to initiate development and application of optimization programs for structural design in 1970. Approximately at the same time, composite materials were introduced in aerospace design. They offer an infinite variety to combine their highly anisotropic elastic properties for any specific combination of design requirements. For a more efficient use of these materials, optimization programs are required to handle the complexity of the problem, especially if additional requirements besides strength are involved in the problem [3]. During the last decade considerable effort has been spent to develop modern structural optimization procedures, using efficient mathematical optimization algorithms as well as optimality criteria which satisfy all requirements simultanously and find optimal values of the design variables by direct computation. The increasing emphasis of aeroelastic considerations is shown in Fig. 1, which was taken from [4].

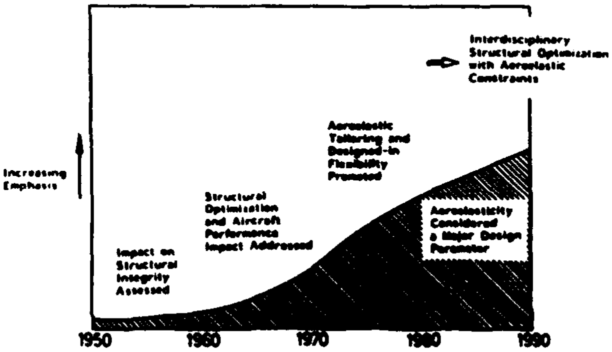


Fig. 1 EVOLUTION OF AEROELASTIC CONSIDERATIONS IN FIGHTER AIRCRAFT DESIGN

STRUCTURAL OPTIMIZATION IN THE GENERAL DATA FLOW

The use of structural optimization tools during the preliminary design stage of an advanced aircraft gives the following potential improvements:

- satisfies the requirements of new aircrafts
- minimizes the objective (weight)
- increases the quality of products
- shortens the development phase
- increases chances of the company in competition.

In order to do this, an appropriate mathematical programming procedure has to be embedded in the general data flow, which is depicted in Fig. 2 and Fig. 3 taken from [5].

For the general data flow I refer to [1].

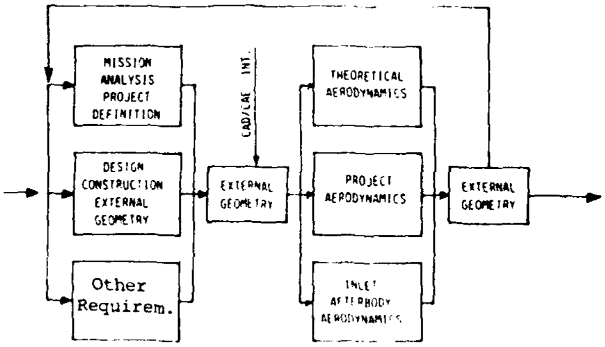

MISSION ANALYSIS PROJECT DEFINITION
DESIGN CONSTRUCTION EXTERNAL GEOMETRY
Other Requirem.
CAD/CAE INT.
EXTERNAL GEOMETRY
THEORETICAL AERODYNAMICS
PROJECT AERODYNAMICS
INLET AFTERBODY AERODYNAMICS
EXTERNAL GEOMETRY

Fig. 2 EXTERNAL GEOMETRY IN DATA FLOW

These figures show a typical flow of geometric, aerodynamic, structural and other data which are used during the design phase of an aircraft. The improved productivity is a result of the integrating effects of the structural optimization. Shorter time of development is realised and fewer data transfers go wrong.

At the present time the development of new airplanes is influenced by new techniques, such as flutter suppression,, CCV-configuration, gust load alleviation etc. (Fig. 3). In addition to stress, displacement, aeroelastic and dynamic constraints an integrated design involves all these techniques and the optimization procedures must be extended for these new constraints. A reliable optimization code is the basis, which allows parametric investigations and weight penalties to be evaluated properly.

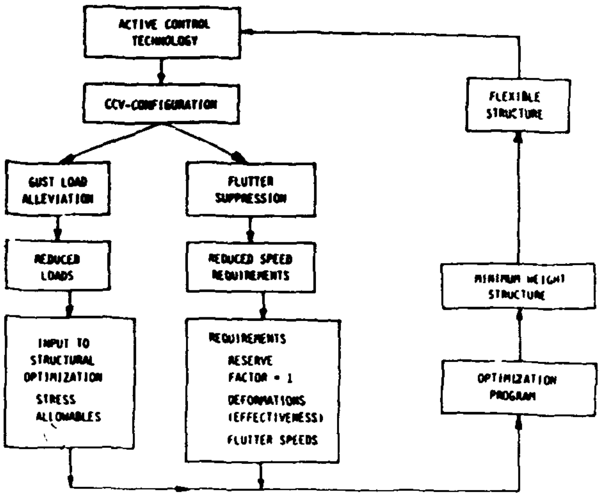

ACTIVE CONTROL TECHNOLOGY
CCV-CONFIGURATION
FLEXIBLE STRUCTURE
GUST LOAD ALLEVIATION
FLUTTER SUPPRESSION
REDUCED LOADS
REDUCED SPEED REQUIREMENTS
MINIMUM WEIGHT STRUCTURE
INPUT TO STRUCTURAL OPTIMIZATION STRESS ALLOWABLES
REQUIREMENTS RESERVE FACTOR = 1 DEFORMATIONS (EFFECTIVENESS) FLUTTER SPEEDS
OPTIMIZATION PROGRAM

Fig. 3 NEW TECHNOLOGIES OF RECENT AIRCRAFT

STRUCTURAL OPTIMIZATION AT MBB

MBB has developed its own structural optimization system called

MBB-LAGRANGE

The performance and requirements/constraints of this new program system are shown below:

- **Requirements**
  Finite Element Structure
- **Structure Variables**
  - Skin Thickness
  - Balance Masses
  - Fibre Directions
  - Grid Point Coordinates
- **Constraints**
  - Min./Max. - Variable
    Stresses
  - Strains
  - Deformations
  - Flutter Speed
  - Divergence Speed
  - Aeroelastic Efficiencies
  - Eigen Frequencies
  - Element Stability
  - Dynamic Response
  - Weight

For more information reference is made to [1].

- **Multiobjective Function** $f(x) \Rightarrow$ **Min.**

  Vector Optimization = "Trade Off" Studies of Convex Combination of Objectives

The program architecture is organized according to the concept of H. A. Eschenauer [6] with the main parts optimization algorithm, optimization model and structural analysis including sensitivity analysis.

The corresponding optimization models are based on the general nonlinear programming problem according to [6]. The design variables $\underline{x}$ are cross sectional areas of trusses and beams, wall thicknesses of membrane and shell elements, laminate thicknesses for every single layer in composite elements or nodal coordinates for geometry optimization problems. The constraints in form of inequalities may be any combination of displacements, stresses, strains, buckling loads, aeroelastic efficiencies, flutter speed, divergence speed, natural frequencies, dynamic response and design variables [7].

In the case of scalar optimization, the objective function $f(\underline{x})$ often includes the structural weight or another linear combination of the design variables. However, it is also possible to define one of the constraint functions as objective and to introduce the weight as constraint at the same time. If vector optimization problems are under consideration, then optimization strategies $p[\underline{f}(\underline{x})]$ according to [6] ensure the transformation to scalar substitute problems.

It is necessary to provide several different optimization algorithms, because there is no known single algorithm which is adapted to every type of problem. Some of the algorithms which are implemented in LAGRANGE are shown below:

- IBF : Inverse Barrier Function,
- MOM : Method of Multipliers,
- SLP : Sequential Linear Programming,
- SRM : Stress Ratio Method,
- RQP1, RQP2 : Recursive Quadratic Programming
- GRG : Generalized Reduced Gradients

A more extensive explanation is given in [1].

Early Investigations with FASTOP

In 1979 the structural optimization program FASTOP (Flutter and Strength Optimization Program) was acquired by MBB. The capability of the program was extended to be able to analyse also the static aeroelastic behaviour of structures. Main features of the program were

- a fully stressed design for static requirements that usually goes near optimal results.
- a flutter redesign for a defined minimum speed.

The program was extensively used for wing design studies [8]. An example for the flutter redesign of a stress designed (optimised) wing is shown in Fig. 4. Essential elements that are sensitive for flutter are marked for the upper and lower cover skin. At that time the program could not handle the different layers of composite materials individually as design variables. The design space was limited by predetermined fibre orientations and the proportion of the individual layer thickness in the total laminate.

Fig. 5 demonstrates the iteration steps to achieve the desired flutter speed of 900 kts starting at 700 kts for the initial fully stressed design. Results of flutter calculations for both designs are plotted in Fig. 6. It should be mentioned also that FASTOP could not solve the optimization for strength and flutter simultaneously.

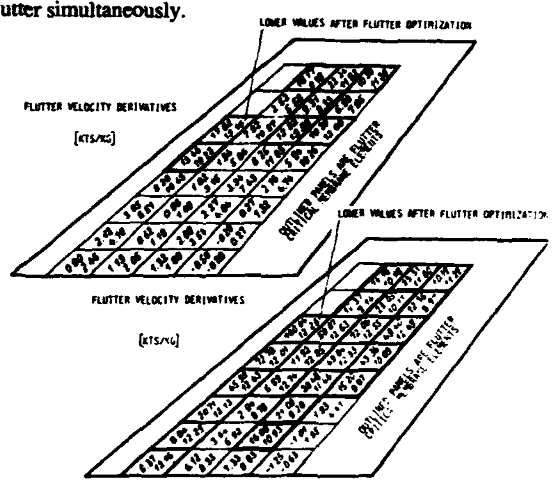

LOWER VALUES AFTER FLUTTER OPTIMIZATION
FLUTTER VELOCITY DERIVATIVES
[KTS/KG]
OUTLINED PANELS ARE FLUTTER CRITICAL STRUCTURAL ELEMENTS
LOWER VALUES AFTER FLUTTER OPTIMIZATION
FLUTTER VELOCITY DERIVATIVES
[KTS/KG]
OUTLINED PANELS ARE FLUTTER CRITICAL STRUCTURAL ELEMENTS

Fig. 4 FLUTTER SENSITIVITIES FOR WING COVER SKIN

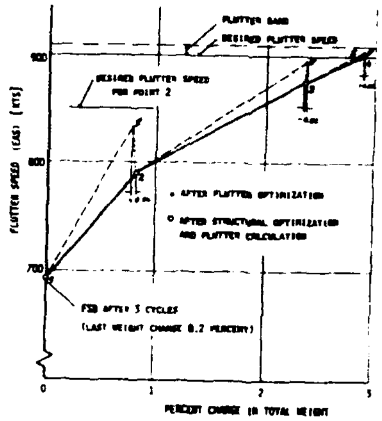


Fig. 5 RESULTS OF REDESIGN STUDY

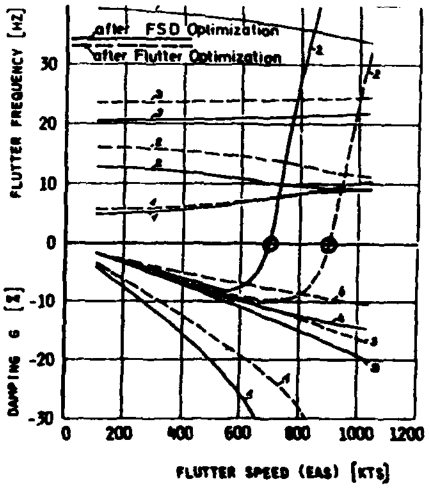


Fig. 6 FLUTTER ANALYSES

AEROELASTIC PROBLEMS AND STRUCTURAL DESIGN OF A TAILLESS CFC-SAILPLNE [9]

Tailless planes, sometimes also called "Flying Wings", have always been a challenge in airplane design. They offer a great potential in performance compared to conventional designs because of less surface (parasite drag), less weight, and less trim drag. Although this has been known for a long time, tailless planes never have experienced the success one might expect. Many carefully designed tailless gliders had to be redesigned or designed completely new after first flight tests because of strange instabilities, which were often not understood or misinterpreted.

It was an interesting task for the "Akademische Fliegergruppe Braunschweig" to launch a tailless ailplane project for the 15 meters standard class in 1983. During flight test with a 1/3 scaled remotely piloted model a severe instability occured at very low speed. Flutter calculations using data from a ground resonance test showed that coupling of the rigid body short period mode with the first elastic mode caused the phenomenon. Solving this problem is a multidisciplinary task. MBB offered assistance to redesign the wing with the help of modern optimization programs.

By applying these codes the flutter speed could be increased to an acceptable level with small modifications of the wing root geometry, a new design of the main spar, and by the use of a new high modulus fiber type. With a small weight penalty-compared to the initial design - the flutter speed could be doubled.

Sailplanes have achieved a very high technological level during the last 20 years, mainly due to fiber composite structures and improved aerodynamics.

Further improvements can be expected only from small detail modification or expensive projects like variable wing geometry. For this reason an unconventional design concept like the tailless wing is a challenge for designers. It offers several advantages like

- less parasite drag
- less weight
- less construction effort

due to the missing rear fuselage and the tail. If the airfoil is designed carefully for a small pitching moment, the flying wing will not have a higher profile drag compared to conventional planes. If the vertical tail is integrated in winglets, the advantage of less induced drag can be explored without additional weight penalties.

The SB 13 project, Fig. 7, shows performance improvements of up to 10% compared to existing competitors in the 15-meters standard class, as indicated in Fig. 8 for the velocity polar. Table 1 gives some main design parameters.

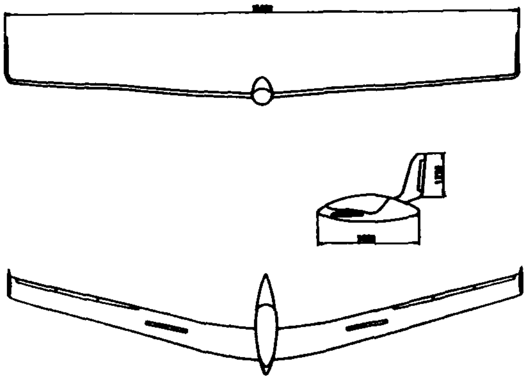


Fig. 7 3-SIDE VIEW OF THE TAILLESS SAILPLANE PROJECT SB13

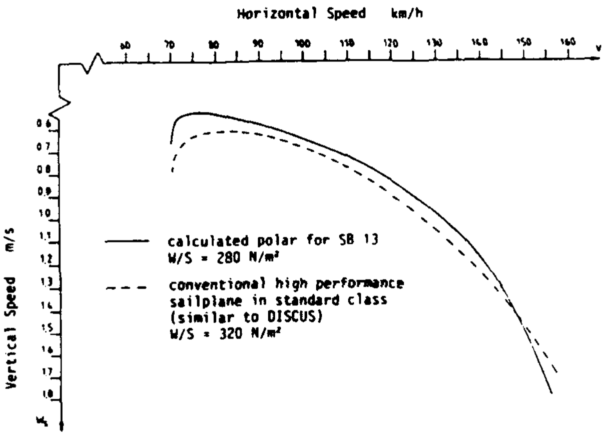


Fig. 8 CALCULATED SPEED POLAR FOR SB13

Wing	
Span	15 m
Area	11.6 m2
Aspect Ratio	19.4
Dihedral	+ 4°
Twist	- 1.5° outboard
Wing Section	HQ 34 N/14.83 inboard HQ 36 N/15.12 outboard
Winglets	
Length	1.25 m
Area	0.675 m^2
Aspect Ratio	2.31
Profile	FX-71-L 150/30
Fuselage	
Length	3.02 m
Width	0.66 m
Height	0.84 m
Landing gear	2 retractable wheels, spring-suspended

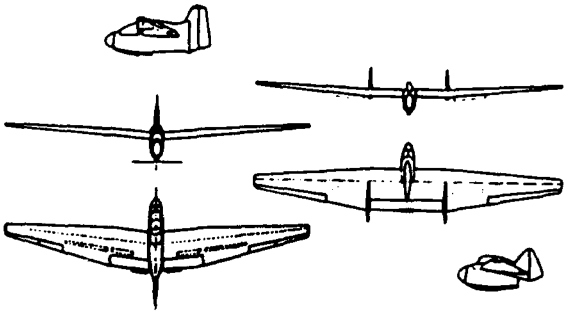
Weights	
Weight empty	2240 N
Payload	700 - 1100 N
Water ballast	max. 1330 N
Gross weight	2940 -4270 N
Wing Loading	248 - 360 N/m^2
Performance	
V_{min}	70 km/h
V_{max}	210 km/h
min. sink speed	0.53 m/s
max. L/D-ratio	43.5 : 1

TABLE 1 TECHNICAL DATA FOR THE SAIL PLANE SB13

Although tailless aircraft have been studied almost since the beginning of aviation they have never experienced the success one might expect. One reason for the lack of success is described in [10] and [11] as the extreme difficulty of achieving satisfactory unaugmented handling qualities, control and dynamic stability.

Probably the most experienced designer of tailless planes was A. Lippisch with numerous powered and unpowered designs [12]. He reported about several difficulties and crashes, caused among others by longitudinal oscillations or "unsatisfactory handling qualities"- The Horten brothers also designed, constructed, and tested various tailless planes between 1936 and 1960 [13].

Among the few successfull tailless sailplane were the single-seat AV-36 and the twin-seat AV-22 by Charles Fauvel [14], Fig. 9, and the very similar looking designs from J. Marske [15].

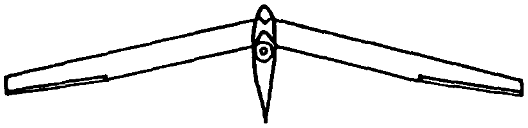


Fauvel AV-22 (1956) Fauvel AV-36 (1951)

Fig. 9 FAUVEL TAILLESS SAILPLANES AV-22 AND AV-36

To study stability and handling qualities of the SB 13, a remotely piloted 1/3 scale model was built and flown. The handling qualities showed no problems, but an unexpected instability in the longitudinal motion occured at very low speeds. When a ground resonance test and a flutter calculation was performed, it could be shown that the reason for the instability was the coupling between the rigid body short period mode and the first symmetric structural mode.

After the problem was solved analytically by the means of aeroelastic tailoring and the application of a new carbon fiber, a paper about a very similar design study at Cranfield was published [16]. This project, called "Ricochet" showed the same aeroelastic behaviour as the SB 13. Fig. 10 shows the design and gives some design parameters. Because the flutter problem could not be solved in this case, the project was finally given up. But this study is the first one known to the authors which identified the problem correctly.



The Ricochet

Parameter	The Ricochet
Material of construction	Aluminium alloy (6061-T6)
Span	15m
Wing area	10.26 m^2
Aspect ratio	22.93
Wing root chord	0.73 m
Wing tip chord	0.50 m
Sweep angle	13°
Mass of each wing	50 Kg
Fuselage mass with equipments	65 Kg

Fig. 10 RICOCHET SAILPLANE PROJECT

Flutter Calculations for the RPV-Model

To investigate the flutter behaviour more thoroughly, a ground resonance test was performed at the DFVLR, Institute for Aeroelasticity in Göttingen. Fig. 11 shows the test installation, Table 2 gives the main results for two configurations, where configuration II contains additional fuselage mass for non-structural items. The first bending mode for both configurations is shown in Fig. 12. Several flutter calculations were performed using the described data. If the rigid body modes are ignored, the first structural mode shows divergence in the flutter calculation as indicated in Fig. 13 for free-free boundary condition.

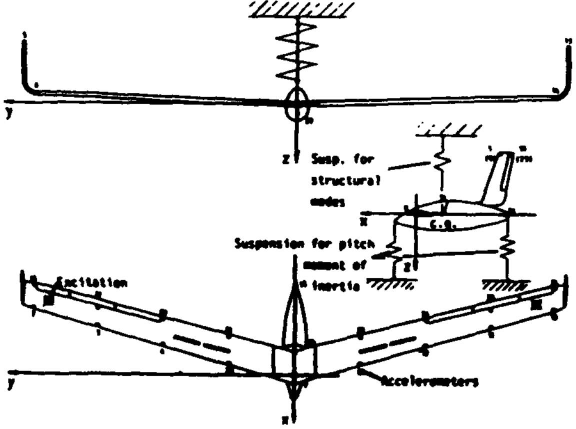

y
z
Susp. for structural modes
x
c.g.
Suspension for pitch moment of inertia
z
Excitation
y
Accelerometers
x

Fig. 11 GROUND RESONANCE TEST EQUIPMENT FOR SCALED RPV MODEL

Symmetric Modes

Configuration I		
Type	Frequency [Hz]	gen. mass [kg cm^2]
S1	3,71	1,231
S2	11,76	1,108
ST	20,75	0,162

Configuration II		
Type	Frequency [Hz]	gen. Mass [kg cm^2]
S1	2,82	2,755
S2	11,21	0,788
ST	19,96	0,029

Antisymmetric Modes

Configuration I		
Type	Frequency [Hz]	gen. mass [kg cm^2]
A1	7,52	1,996
A2	20,74	0,445
AT	17,87	0,122

Configuration II		
Type	Frequency [Hz]	gen. mass [kg cm^2]
A1	7,44	2,075
A2	20,17	0,664
AT	17,64	0,171

Total Mass and Pitch Moment of Inertia

Configuration I	
m_{tot} [kg]	7,2
Θ_y [kgm^2]	0,683

Configuration II	
m_{tot} [kg]	12,6
Θ_y [kgm^2]	0,884

TABLE 2 TEST RESULTS FOR SB13-RPV MODEL

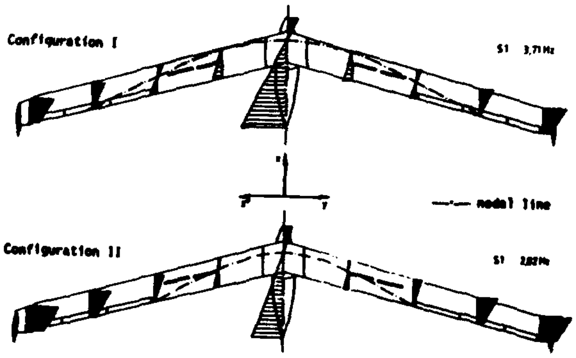

Configuration I
S1 3,71 Hz
— · — nodal line
Configuration II
S1 2,82 Hz

Fig. 12 STRUCTURAL MODE SHAPE FOR FIRST SYMMETRIC MODE

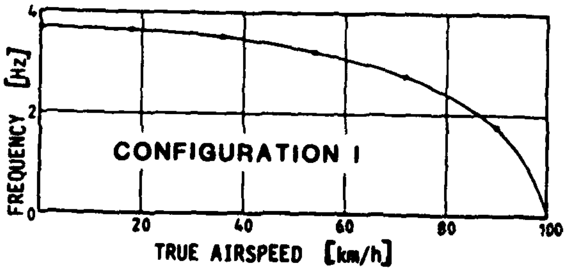


Fig. 13 RESULT OF FLUTTER CALCULATION FOR FIRST STRUCTURAL MODE

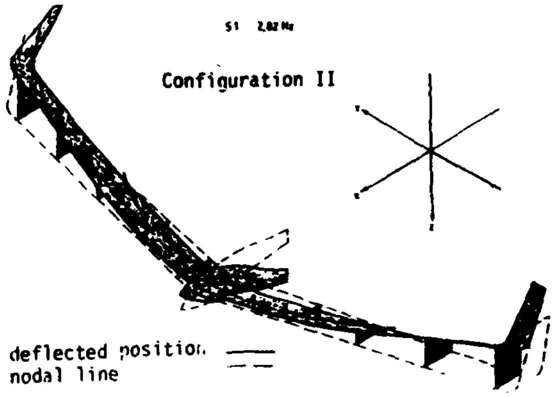

S1 2,82 Hz
Configuration II
deflected position
nodal line

Fig. 14 ISOMETRIC VIEW OF MODE 1

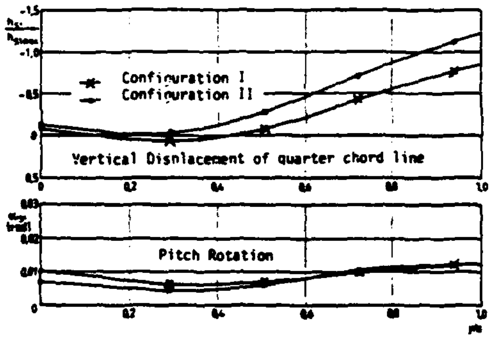


Fig. 15 VERTICAL DISPLACEMENT AND PITCH ROTATION FOR MODE 1 AT QUARTER CHORD LINE

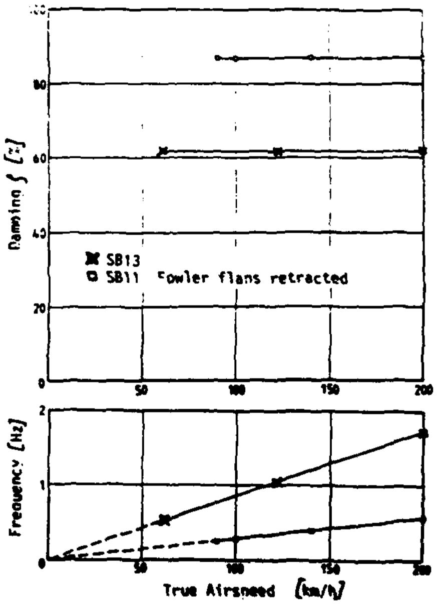


Fig. 16 COMPARISON OF RIGID BODY SHORT PERIOD MODES FOR SB13 AND CONVENTIONAL SAILPLANE

Fig. 14 gives a isometric view of mode 1 and Fig. 15 shows more clearly the local deformations at the quarter chord line for both configuration.

In Fig. 16 the eigenvalues of the short period are plotted vs. airspeed for the full scale SB13 and the conventional SB11. The frequency of the SB13 is almost three times that of the SB11 while the damping is smaller. This difference is mainly due to the small pitch moment of inertia of the SB13. Table 3 gives a comparison of important parameters for the longitudinal motion of the model, the SB13 and the SB11.

The equations for the short period mode are

$$\omega_{0,\alpha} = \sqrt{-\left(\frac{V_0}{i_y}\right)^2 \cdot \frac{C_{L\alpha}}{\mu_L} \cdot \left(\frac{C_M}{C_L} + \frac{C_{Mq}}{\mu_L}\right)} \tag{1}$$

for the frequency, and

$$\sigma_\alpha = -\frac{V_0}{2\mu_L \cdot \bar{c}} \cdot \left[C_{L\alpha} - \left(\frac{\bar{c}}{i_y}\right)^2 \cdot \left(C_{Mq} + C_{M\dot{\alpha}}\right)\right] \tag{2}$$

for the damping. Here $C_L\alpha$ and C_{Mq} are functions of geometry only, $C_{M\alpha}$ also depends on the c.g. location. $\partial C_M/\partial C_L$ must be identical for the model and the full scale version, if geometrical proportions are similar and the static longitudinal stability is equivalent. The last parameter for comparison is the relative mass density.

$$\mu_L = \frac{2m}{\varrho \cdot S \cdot \bar{c}} \tag{3}$$

As table 3 shows, these terms are identical. The construction of the model is similar to the one of a modern sailplane. Therefore it can be expected that there is also elastic similarity (replica).

With the results from the ground resonance test the rigid body coupling with the elastic mode results in a flutter speed of 53 km/h for configuration I, Fig. 17, and due to the smaller f_1 only 44 km/h for configuration II, Fig. 18. If we assume linearity between flutter speed and first elastic mode frequency, we get

$$\frac{V_L}{V_M} = \frac{\bar{c}_L}{\bar{c}_M} \cdot \frac{f_L}{f_\mu} \tag{4}$$

(L = large scale, M 0 model) for the velocity scale. With the length scale $\lambda = \frac{\bar{c}_L}{\bar{c}_M} = 2.71$ we got

$$V_L = f_L \cdot 42.3 \; \left[{}^{km/h}/_{Hz}\right] \tag{5}$$

for the SB13. If we demand a flutter speed equal to the maximum velocity V_D = 283 km/h for the SB13, this would require a first structural mode frequency of 6.7 Hz.

	SB 13-Model Configuration II	SB 13	SB 11 - 18 m Fowlerflaps in
m [kg]	12,6	280	465
Θ_y [kgm²]	0,884	113	835
i_y [m]	0,265	0,635	1,340
S [m²]	1,33	11,0	13,32
$\bar{c}$ [m]	0,271	0,735	0,772
μ_L	57,07	56,54	73,83
$c_{L\alpha}$	5,72	5,72	5,71
c_{Mq}	-2,749	-2,749	-20,49
$c_{M\dot{\alpha}}$	0,194	0,194	0,649
$\frac{\partial c_M}{\partial c_L}$	-0,10	-0,10	-0.10

Table 3 LONGITUDINAL MOTION PARAMETERS FOR SB13, AND A COMPARABLE EXISTING SAILPLANE (SB11)

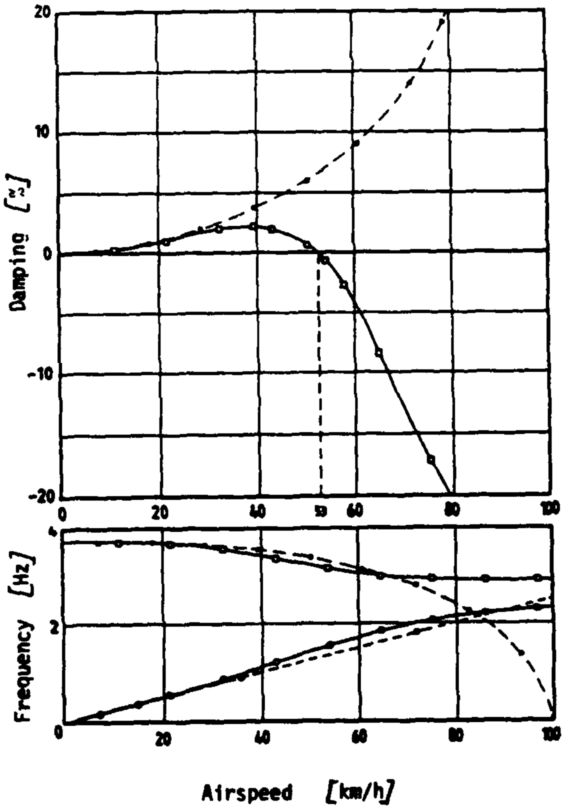


Fig. 17 FLUTTER CALCULATION WITH RIGID BODY DEGREES OF FREEDOM FOR MODEL CONFIGURATION I

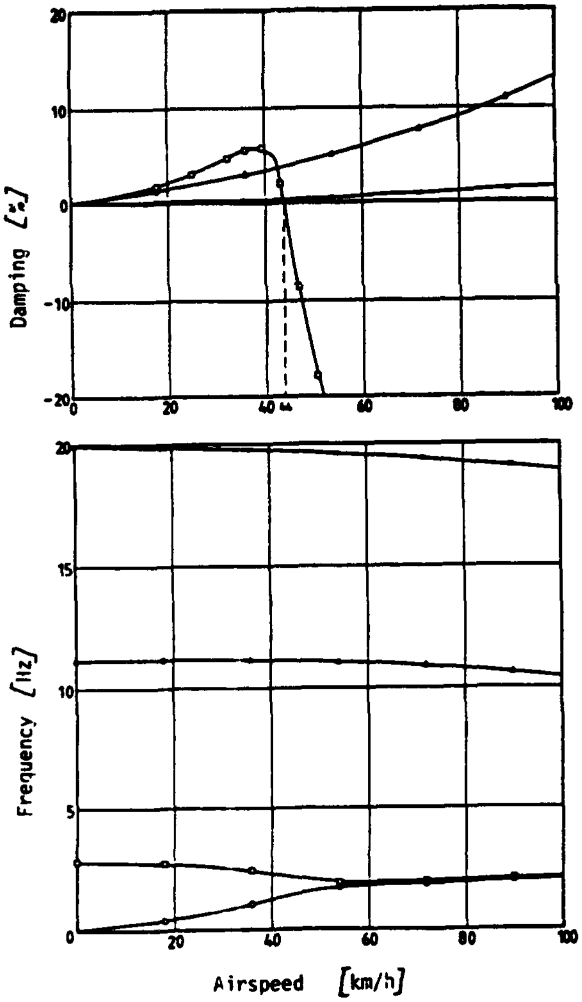


Fig. 18 FLUTTER CALCULATION FOR CONFIGURATION II INCLUDING ADDITIONAL MASS FOR THE PILOT

Possible Solutions to Increase the Flutter Speed

As described before, the flutter problem is caused by the coupling of structural bending mode (B1) and rigid body short period mode (S1). Obviously a separation of the two frequencies would be favourable.

- To reduce mode S1 frequency one must increase the pitch moment of inertia largely which is not possible with a tailless configuration. As also the Ricochet study showed, this is the only important parameter for the short period mode. Static longitudinal stability, or wing sweep angle do not improve the rigid body motion. Fig. 19 shows the change in flutter speed with the pitch moment of inertia for the Ricochet sail plane.
- Changing the configuration in a way that the first elastic mode shape (S1) frequency shows no reduction with increasing airspeed would require a completely new design. Because a large effort had already been invested in the aerodynamic layout, this solution was not desirable.
- Active Control Technology is a good method to extend the flight envelope. Wykes [17] proposed a CSAS system for the U.S. forward swept wing fighter project, where the same coupling between rigid body and elastic structure occurs. Although tested for several military and commercial aircraft projects sucessfully, ACT is not a feasible solution for sailplanes. It would require power supply and a complicated sensor-, control-, and actuation system.
- To change the aeroelastic behaviour using mass balance by addition of lumped masses does not improve the situation with feasible arrangements.
- Decoupling the pilot from the fuselage to change the critical mode shape/frequency would result in an unfavourable sensing system for the pilot. In addition all spring systems for this purpose would have large amplitudes (non linearities) under load, or would require too much volume (air bag).
- The only practicable solution is a combination of a structural redesign (with small modifications in the wing root geometry), using high elastic modulus carbon fibers to increase the first elastic frequency and tailor the wing for a different aeroelastic behaviour (exploiting the anisotropic material properties to change mode shapes). This procedure - finally selected - will be decribed in the next chapter.

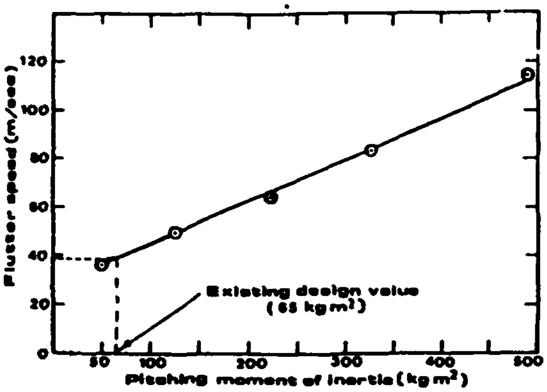


Fig. 19 FLUTTER SPEED VS. PITCH MOMENT OF INERTIA FOR THE RICOCHET PROJECT

Structural Redesign for Increased Flutter Speed

The first handicap in the application of TSO and FASTOP for "aeroelastic tailoring" the wing was caused by the lack of rigid body modes in both computer programs. Therefore we had to choose substitution systems to describe the critical flight mechanical mode. This can be achieved by defining soft springs between the structure and an earthed point. The softness of these springs is limited by numerical problems in the stiffness matrix. Unfortunately this system caused other problems. The very flexible wing attachment did not allow to use the great advantage of TSO, the simultaneous optimization for different objectives and constraints. The soft attachment caused too high deflections under load for the strength design.

Only limited potential of aeroelastic tailoring sailplane-wings is available, constrained by the extremely high aspect ratio and slenderness of these wings. The main fiber direction can be swept only within small limits. Additionally there is only a small number of + 45 ° - plies which makes it impossible to use them to change the elastic behaviour.

Usually, modern sailplane wings are designed as shown in Fig. 20. There is one main spar with the flange fabricated from unidirectional rovings. The torsional forces are carried by ± 45° plies in a sandwich shell construction. Alternatively a shell construction with coupled bending and torsion plies was investigated first. Due to the small number of required plies with a still very small box chord, this design was given up later, because it showed no improvements and is also very difficult to make manually.

The final solution has a two web main spar (with 0°-plies) and an uncoupled torsion shell (± 45° -plies), described in detail later on.

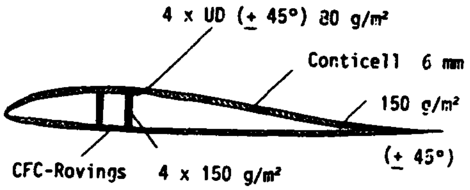


Fig. 20 TYPICAL WING SECTION FOR MODERN SAILPLANES

TSO-Calculations:

This program describes the wing structure as a plate model. Therefore, check calculations were necessary to see if the plate theory can be used for slender wings. For a constant chord, constant thickness beam with an aspect ratio of 20, the TSO results could be confirmed with analytical beam theory. Fig. 21 shows the idealization of the wing box within the planform geometry. This plot also shows typical thickness contours for the bending layers.

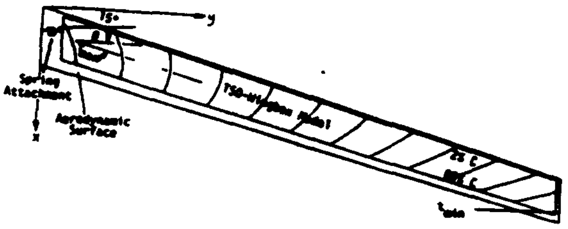


Fig. 21 WING IDEALISATION FOR TSO PROGRAM

As mentioned before, the possibilities for aeroelastic tailoring were limited. Due to geometrical constraints, a main fiber sweep angle of 3° forward was the maximum. Because of these limitations, several different materials and material combinations were used very early. Table 4 gives some typical results from these optimization runs.

	Glass Fiber	Carbon Fibers	
		High Tension	High Modulus
Fiber Volume Ratio	0.4	0.55	0.52
E_{11} [N/mm²]	30948	131835	195103
E_{22} [N/mm²]	6481	7669	6089
G [N/mm²]	2120	3220	3434
ν_{12}	0.310	0.3115	0.272
ρ [g/cm³]	1.696	1.510	1.469
ε_{11}	0.0129	0.00455	0.001287
ε_{22}	0.00347	0.0022	0.00266
ε_{12}	0.01886	0.01242	0.01165

Table 5 MATERIAL PROPERTIES USED IN THIS STUDY

Run-No.	Description	Main Fiber Sweep Angle (positive fwd.)	0°-plies	±45°-plies	f_1 [Hz]	V_F [km/h]	m_{wing} [kg] per side
1	initial Design	0°	HT	HT	2.42	74.1	60.2
2	increased bending stiffness with HM-Fibers	0°	HM	HT	3.14	99.0	61.1
3	additional 0°-plies	0°	HM	HT	4.31	140.8	79.7
4	swept 0°-plies skin thickness as no. 2	+ 2.5°	HM	HT	3.40	111.5	61.1
5	swept, optimized lay up	+ 2.5°	HM	HT	3.76	115.0	64.5
6	+ 3.0°, linear thickness distribution	+ 3.0°	HM	HT	4.76	162.2	79.6
7	free thickness distribution less weight, higher V_F than 6	+ 3.0°	HM	HT	4.85	178.5	79.2
8	+ 5.0, glass fiber torsion plies, sweep angle not practicalbe	+ 5.0°	HM	glass fibers	4.26	187.6	79.8

Table 4 TSO CALCULATIONS FOR SB13

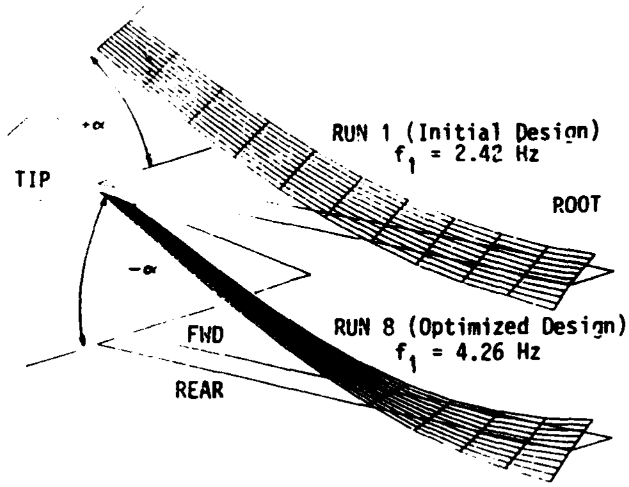


Fig. 22 TSO MODE SHAPE 1 FOR INITIAL AND OPTIMIZED DESIGN

Rather soon, it became obvious that the flutter problem could not be solved with presently used materials.

The advent of high modulus carbon fibers provides a Young's modulus 50% higher than in presently used high tension fiber. Table 5 gives a comparison of material properties for unidirectional layers. Fig. 22 shows the first elastic mode shape for the initial design and for the high modulus fiber with swept 0°-direction.

In the flutter calculation results (Fig. 23) for the optimized design No. 8 the improved flutter behaviour is shown. It should also be mentioned that the rigid body damping (mode 1) is predicted wrongly.

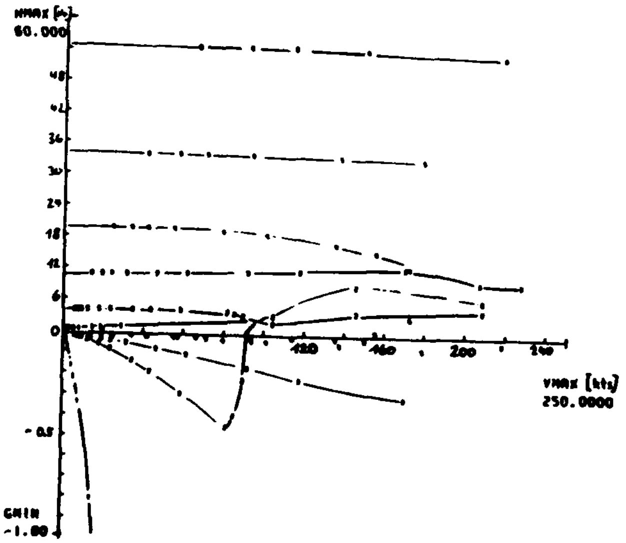


Fig. 23 TSO FLUTTER CALCULATION RESULTS FOR OPTIMIZED DESIGN

FASTOP-Calculations:

Fig. 24 shows the finite element idealization for the FASTOP model. The skin is described by membrane elements whereas the ribs and spars are modelled with shear panels. Although FASTOP can consider free-free conditions in the vibration analysis, it is not capable, to handle rigid body modes separately. They can only be superposed to the elastic modes. For this reason, it was necessary to simulate this mode with soft spring attachments and cantilever conditions in the vibration analysis.

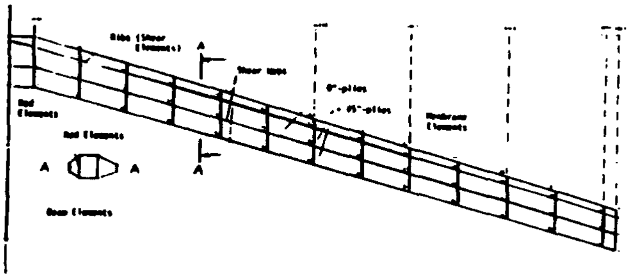


Fig. 24 FIRST STRUCTURAL MODEL FOR FASTOP CALCULATIONS

Instead of sweeping the fibers within the spar and manufacturing the spar with prepregs, a new model with a swept spar inside the wing, fabricated conventionally from rovings, showed better results in the flutter behaviour. Fig. 25 shows the new idealization. To allow a higher sweep angle for the spar, the wing planform was modified in the inboard section. The leading edge sweep angle is reduced with three kinks, the trailing edge sweep angle is reduced with two kinks. The intention was to bring the main spar closer to the pilot's mass without increasing the wing area.

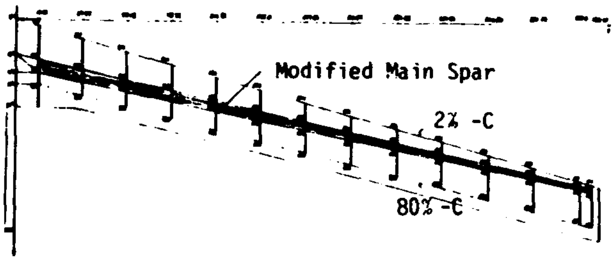


Fig. 25 MODIFIED STRUCTURAL MODEL

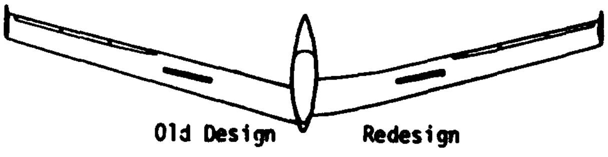


Fig. 26 WING GEOMETRY MODIFICATIONS FOR IMPROVED FLUTTER BEHAVIOUR

Fig. 26 shows these modifications.

Using HM-instead of HT-fibers for the unswept spar increased the flutter speed from 110 km/h to 210 km/h (+ 90%), sweeping the spar 3 degrees gives 122 km/h for HT-fibers (+ 11%), and 237 km/h for HM-fibers. If HT-carbon fibers are used for the torsion layers instead of glass fibers, the flutter speed is 3.5 % higher. Table 6 gives a summary of these results.

Fig. 27 shows the mode shapes and flutter calculation results for the best design. This wing has a weight of 67.7 kg compared to 60.0 kg for the initial design (+ 12.8 %) but the flutter speed is 114 % higher ! Further calculations were necessary for different water tank positions in the wing. Fig. 28 shows two possible solutions which do not decrease the flutter speed. Because the water is positioned very close to the nodal line of mode 1, the first frequency does not drop more than 6% while the short period mode is more than 10% smaller due to the higher moment of inertia. The flutter calculation for water ballast configuration II (No. 7 in Table 6) is shown in Fig. 29.

Flutter calculations for antisymmetric modes were also performed. Because the first mode is higher than 6 Hz in this case, there is no coupling with low frequency modes. Higher modes are separated without tendency to couple up to 400 km/h.

The final configuration was analysed using a different approach whereby the rigid body mode frequencies could be described more accurately (0.01 Hz for the z-translation and 0.05 Hz for the rigid rotation at 0 airspeed). This influence improved the flutter speed considerably. Table 6 gives also the results using this method.

From these results it could be expected that the flutter speed will be sufficiently high to clear the full flight envelope up to the maximum speed V_{NE} = 210 km/h (including a safety margin). The predictions were verified during flight test which happened in the year of 1985. The airplane has been flown ever since and has never shown any structural instability.

In summary it can be said that most tailless sailplanes seem to have the great disadvantage of high frequency short period modes compared to conventional constructions. To prevent flutter because of coupling with the first elastic mode, the wing has to be fabricated from extremely stiff materials. New fibers with a very high elastic modulus could provide the necessary stiffness for the SB13 wing.

To overcome these difficulties easier in other tailless aircraft designs, it could be possible that sweeping the wing forward and thus having the pilot in front of the wing, might change the first elastic mode in a way that the coupling with the short period mode will be delayed to higher speeds. J. J. Marske gathered a lot of experience in the design and construction of several tailless sailplane [15]. His final solution was a swept forward configuration which had no stability problems and showed good handling and performance characteristics. For the SB13 design, a swept forward solution was not possible because of the slender fuselage with a carry-through main spar. And it is not possible if the winglets are used as vertical tails.

Run-No.	Description	Main Fiber Direction (pos. fwd.)	Materials		f_1 [Hz]		m_{wing} [kg] per side	free/free divergence speed [km/h]	Flutter Speed [km/h	
			0°-plies	± 45°-plies	free/free	spring attachm.			FASTOP	Check-pgm.
1	Initial Design	0°	HT	HT	2.59	2.74	60.0	186.0	110.6	120.0
2	HM-fibers	0°	HM	HT	5.27	5.19	69.5	342.0	210.2	236.0
3	modified main spar HT-fibers	3°	HT	HT	2.85	2.93	60.0	207.0	122.4	140.0
4	modified spar HM + glass fibers	3°	HM	glass fibers	5.64	5.46	73.4	390.0	229.0	260.0
5	HM + HT fibers	3′	HM	HT	5.78	5.60	67.7	>400	237.1	273.0
6	structure 5 & water ballast I	3°	HM	HT	5.58	5.46	67.7	>400	240.2	275.0
7	structure 5 & water ballast II	3°	HM	HT	5.33	5.24	67.7	>400	238.7	290.0
8	structure 5 antisymmetric modes	3°	HM	HT	7.0	-	67.7	-	-	-

Table 6 FASTOP CALCULATION RESULTS FOR SB13

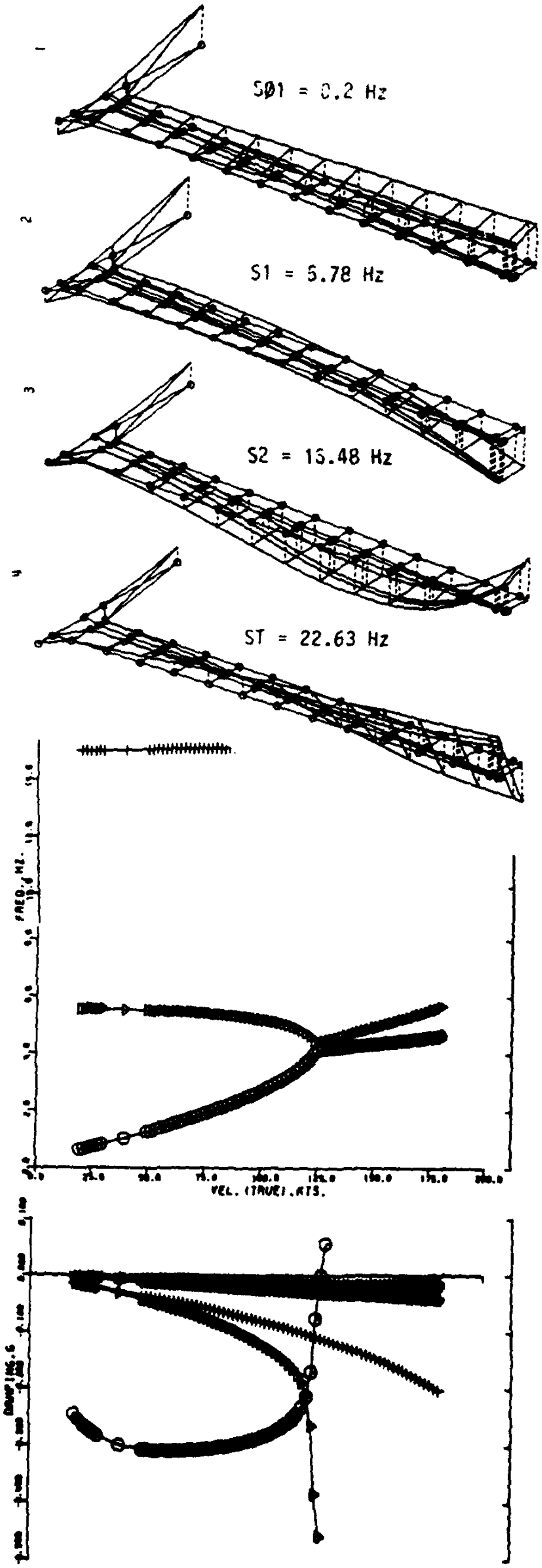


Fig. 27 MODE SHAPES AND FLUTTER CALCULATION RESULTS FOR THE FINAL FASTOP DESIGN

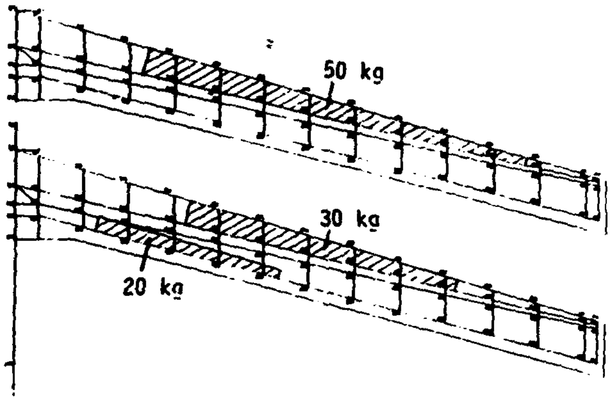


Fig. 28 TWO POSSIBLE WATER BALLAST SOLUTIONS

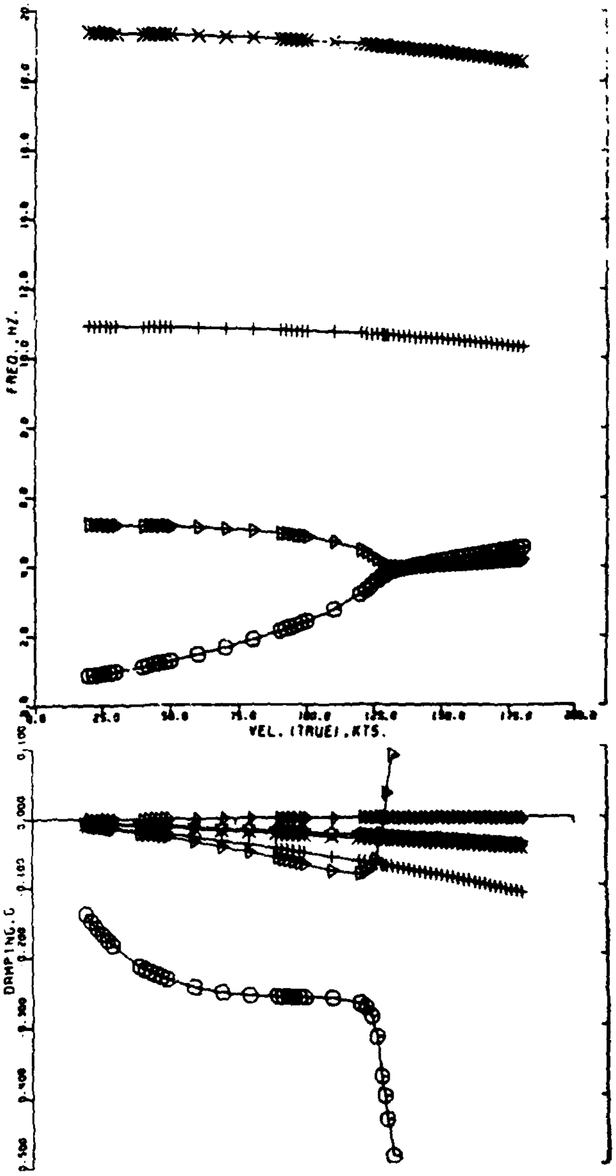


Fig. 29 FLUTTER CALCULATION RESULT FOR WATER BALLAST CONFIGURATION II

There was also a shortcoming in the US-swept forward wing aircraft X29 which was solely optimized to achieve a high divergence speed. With a combination of aeroelastic tailoring and active control [17] high structural weight savings can be achieved if such a problem exists. Since CCV is quite common now for fighter airplanes flight certification for such a flight control system could be received. Most important is that the lay out is done in the design stage and not as a repair solutions.

It should also be mentioned here that the interaction between rigid body and elastic structure shows the necessity to incorporate flight mechanics in modern design- and optimization programs. A twin-engine prototype from Partenavia was lost in a fatal accident because of the coupling between a horizontal tail tab-mode and the short period motion [18].

The use of aeroelastic tailoring makes it even more important to study aeroelasticity parallel to other disciplines (in the design) and not in series as it was done in the past.

Compared to the initial design, the more than 110% increase in flutter speed with a small weight increase shows the potential of new carbon fibers and the use of aeroelastic tailoring.

THE PRELIMINARY DESIGN OF LIFTING SURFACES WITH TSO

Since 1982 the TSO program (Aeroelastic Tailoring and Structural Optmization [19]) is in use at MBB as a preliminary design tool for aerodynamic surfaces. In this program the structure of the surface is represented as a continuous plate with variable thickness.Design variables are coefficients, describing the thickness distribution of different composite layers, the fiber orientation, and, if necessary, variable concentrated masses for flutter optimization. Due to a wide range of aeroelastic constraints such as frequency, flutter speed, deformations, aeroelastic effectiveness and divergence speed, the program is very suitable for aeroelastic tailoring.

In 1986, TSO was used for a design study on a light combat aircraft wing shown in Fig. 30. In this case, three static load cases were used for the pure strength design with the main objective: minimum weight.

The flexible wing roll rate was not a constraint in the beginning.The wing flap hinge moments, however, are often a critical design criterium that can also limit the roll rate of the aircraft. It could be demonstrated in this study, that by defining an aileron roll moment effectiveness constraint, the wing cover skin thickness and fiber orientations could be designed for higher roll rates and a considerable reduction in hinge moment with a small weight increase.

The relations between structural weight, roll effectiveness and flap hinge moment can be seen in Fig. 31. With very little increase in weight the hinge moment for the required roll rate of 180° at maximum dynamic pressure and Mach 1.1 can be reduced to 30% of the original value. This sensitivity analysis also indicates the region where an additional increase in structural mass can not improve the performance. Without the use of an optimization program, this kind of trend studies would be impossible, especially if composite materials or aeroelastic requirements are involved. Fig. 32 gives a comparison of the wing cover skin designs for strength requirements only and for an additional flexible aileron roll effectiveness requirement.The importance of the seletion of design variables is demonstrated in Fig. 33 for four different approaches:

- fixed fiber orientation and balanced -45° and -45° plies	W = 100%
- fixed fiber orientation, unbalanced	W = 76%
- free fiber orientation, balanced	W = 67%
- free fiber orientation, unbalanced	W = 54%

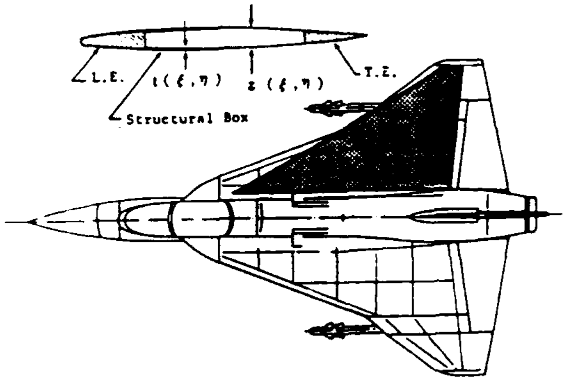


Fig. 30 LIGHT COMBAT AIRCRAFT WING PLATE MODEL

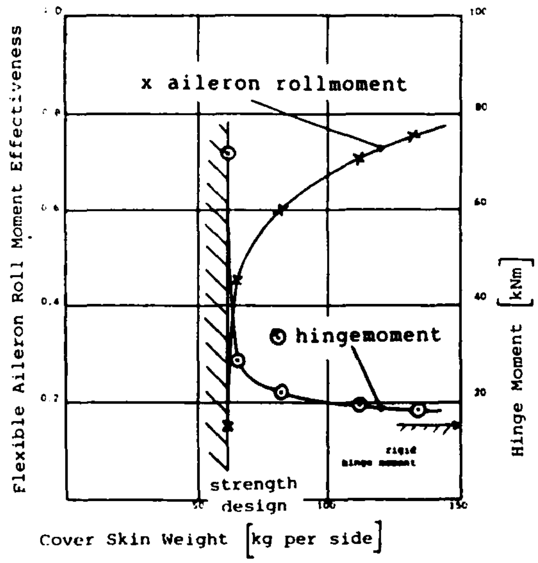


Fig. 31 OPTIMIZATION WING COVER SKIN WEIGHT FOR DIFFERENT ROLL EFFECTIVENESS CONSTRAINTS AND REQUIRED FLAP HINGE MOMENT FOR 180 DEG/SEC ROLL RATE AT MA. 1.1, SEA LEVEL

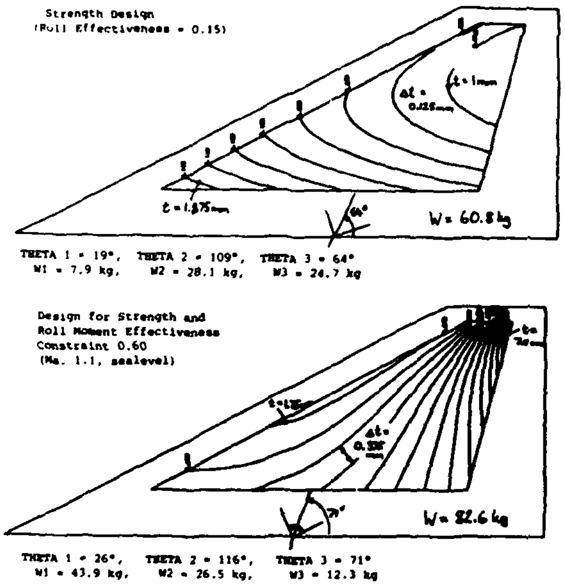
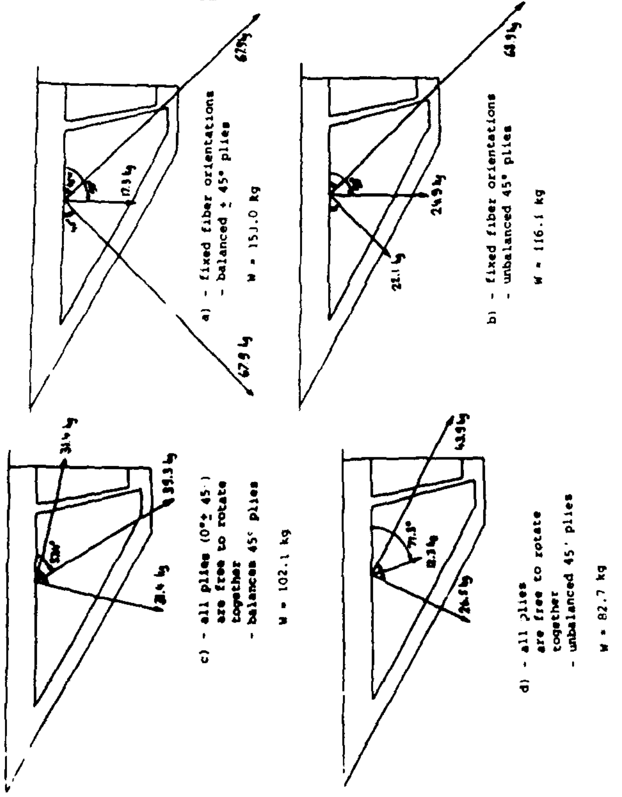

Strength Design
(Roll Effectiveness = 0.15)
t = 1mm
Δt = 0.125mm
t = 1.875mm
64°
W = 60.8 kg
THETA 1 = 19°, THETA 2 = 109°, THETA 3 = 64°
W1 = 7.9 kg, W2 = 28.1 kg, W3 = 24.7 kg
Design for Strength and
Roll Moment Effectiveness
Constraint 0.60
(Ma. 1.1, sealevel)
Δt = 0.125mm
71°
W = 82.6 kg
THETA 1 = 26°, THETA 2 = 116°, THETA 3 = 71°
W1 = 43.9 kg, W2 = 26.5 kg, W3 = 12.3 kg

Fig. 32 OPTIMIZED WING SKIN THICKNESS DISRIBUTIONS FOR DESIGNS WITH AND WITHOUT FLAP ROLL MOMENT EFFECTIVENESS CONSTRAINT


a) - fixed fiber orientations - balanced ± 45° plies W = 153.0 kg

b) - fixed fiber orientations - unbalanced 45° plies W = 116.1 kg

c) - all plies (0°± 45°) are free to rotate together - balanced 45° plies W = 102.1 kg

d) - all plies are free to rotate together - unbalanced 45° plies W = 82.7 kg

Fig. 33 FOUR DIFFERENT DESIGNS FOR STRENGTH AND ROLL EFFECTIVENESS CONSTRAINT (0.60). ARROWS INDICATE AMOUNT OF PLIES IN EACH DIRECTION

ADVANCED AIRCRAFT DESIGN WITH MBB-LAGRANGE

A large number of other studies and applications of LAGRANGE to current projects have been performed already and presented in several publications [20], [21], [22].

A typical example for the application of LAGRANGE is the composite wing structure for the experimental aircraft X-31A. In this case optimization was beneficial for two main objectives of the program: a low cost approach and a very short time for development and design. Besides a design for minimum weight another requirement was a high flutter margin to reduce efforts and costs for flutter wind tunnel and flight tests. Although flutter did not effect the design, it could be surveyed simultaneously during optimization. Static aeroelastic effectiveness was also investigated during the design process.

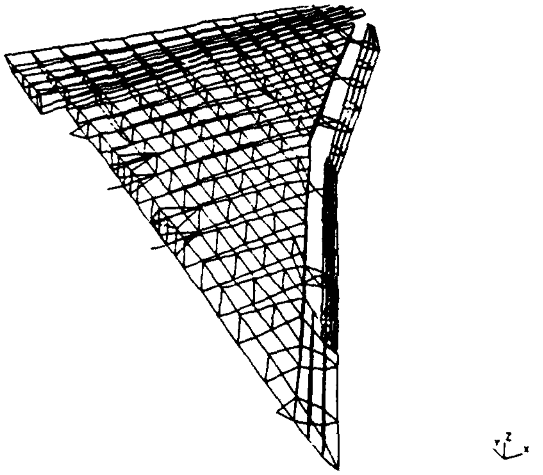


Fig. 34 FINITE ELEMENT MODEL OF THE X-31 WING

A finite element model of the wing is depicted in Fig. 34. It has 1764 elements and 1871 degrees of freedom. The optimized skin thicknesses were then translated into design drawings with small modifications. As an example, the upper wing skin weight of 53 kg from an initial design (preoptimized with another program) could be reduced to 44 kg in the FEM which resulted in 45 kg in the actual design. The final design meets the target weight and has a margin of 100% in airspeed for flutter.

INTEGRATED DESIGN CONCEPT

The experience obtained from various designs with design requirements coming from different disciplines has shown the need for integrated design concepts and programs.

The interactions between aerodynamics, performance, flight control, structural loads, dynamics, aeroelasticity, strength and materials, and finally the design have always existed.

Due to increasing aircraft performance requirements, like payload, fuel efficiency, or maneuvering capability, these interactions have become stronger and more important.

The mass is very important for an aircraft. The aerodynamic lift or drag from different sources like surface area, distribution of cross section areas, lift-induced drag and airfoil shape are also main design parameters. The influence of geometric parameters of an aircraft on lift and drag characteristics has been studied since the earliest days of aviation.

The importance of aspect ratio on lift induced drag, of wing taper ratio on lift distribution and drag of wing taper ratio on lift distribution and drag, of wing thickness ratio and sweep angle on drag increase with Mach number is well known. But how is the influence of these parameters on the structural mass and on the loads that cause the mass?

To demonstrate these relations, parametric studies have been performed for typical fighter aircraft wings, using the TSO proram. Some of these wings are depicted in Fig. 35.

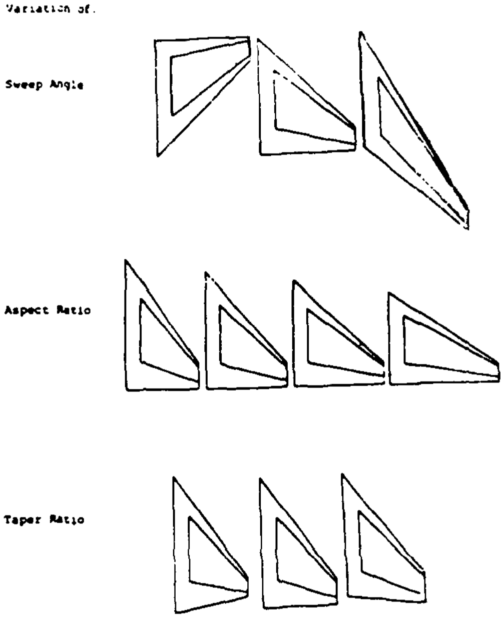


Fig. 35 WING GEOMETRY PARAMETERS FOR OPTIMIZATION STUDIES

For all these wings the same basic design requirements have been defined, using identical total weight and the same wing area:

- a 9 g static load case with aeroelastically trimmed load distribution
- a maximum roll rate at high dynamic pressure for the aileron effectiveness
- a minimum flutter speed of 1000 kts

The carbon fiber wing cover skins of these wings were then optimized for the above load cases separately and simultaneously.

Fig. 36 depicts the influence of the aspect ratio on the skin weight for the different design requirements. Of course, a change in requirements, a different flap geometry or a different mass distribution will give different trends for the geometry parameters and for the additional weight required to meet each individual requirement on top of others.

Therefore, similar studies should be performed in the preliminary stage of a new aircraft design.

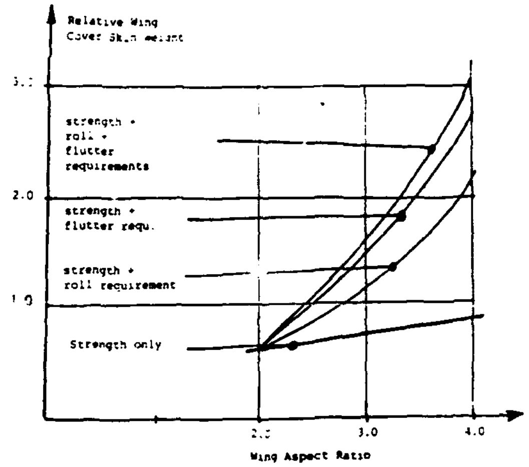


Fig. 36 OPTIMIZED WING COVER SKIN WEIGHT VS ASPECT RATIO

AEROELASTIC TAILORING OF A FIN MADE OF COMPOSITE MATERIAL [23]

An aircraft fin has to fullfil quite different design requirements with a similar priority and the final design requires the evaluation of many off-design point studies.

The design of aerodynamic surfaces such as wing, fin, foreplane and tailplane needs two major design steps:

First, the aerodynamic design to define the overall geometry like area, span, aspect ratio, taper ratio and profil.

Second, the structural design to develop the internal structural arrangement of skin, ribs, stringers, spars, rudder support, rudder actuation, attachments, equipment systems.

The final design must fulfill the following design requirements with minimum weight:

- Static strength to withstand design loads
- Aeroelastic efficiencies for performance
- No flutter inside of the flight envelope
- Manufacturing constraints, min. and max. gauges

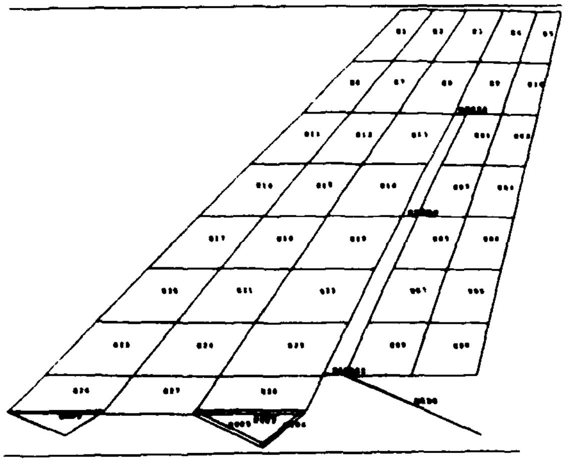


Fig. 37 FIN STRUCTURAL MODEL WITH SKIN ELEMENT NUMBERS

It is quite clear that such a design requires an interactive coupling of the above mentioned two design steps. A structural model of the investigated fin is shown in Fig. 37.. A comparison of initial design analysis results and design constraints is given in the following table:

	DESIGN CONSTRAINT	INITIAL DESIGN	VIOLATION
STRENGTH	Strain allowables Tension ≤ .0037 Comparison ≤-.0028	Load case 2 Element 18 Element 22	 -.123 -.228
AEROELASTICS Ma 1.8 750KTS	Efficiencies FIN .8 RUDDER .5	 .753 .441	 -.059 -.118
FLUTTER Ma 1.2/S.L.	Flutterspeed VF = 530 m/sec	495 m/sec	-.066

The frequency versus speed behaviour for the optimized/initial structure is given in Fig. 38. The corresponding damping is plotted in Fig. 39. The results of the optimization procedure are shown in Table 8.

The flutter speed is increased to 530 m/sec. and aeroelastic efficiencies are increased 8% for the fin and for the rudder by 13%. The structural weight is reduced by 7%.

Skin thicknesses for the different carbon fibre layers of the optimum structural design are presented in Fig. 40-43.

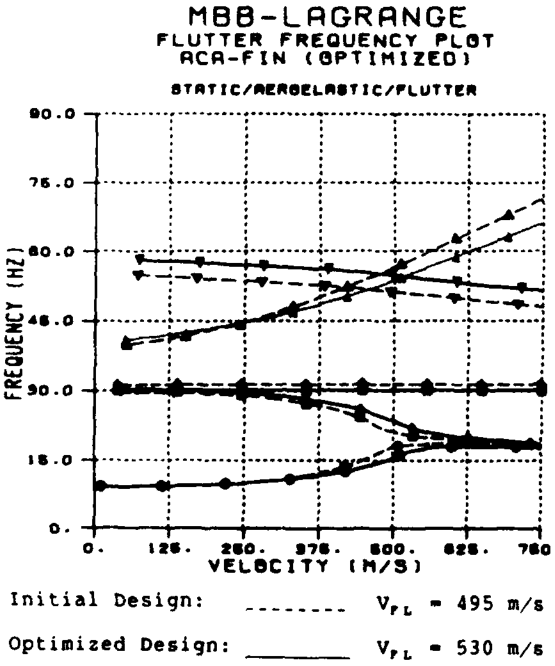


Fig. 38 FLUTTER ANALYSIS FREQUENCY PLOT FOR INITIAL AND OPTIMIZED DESIGN

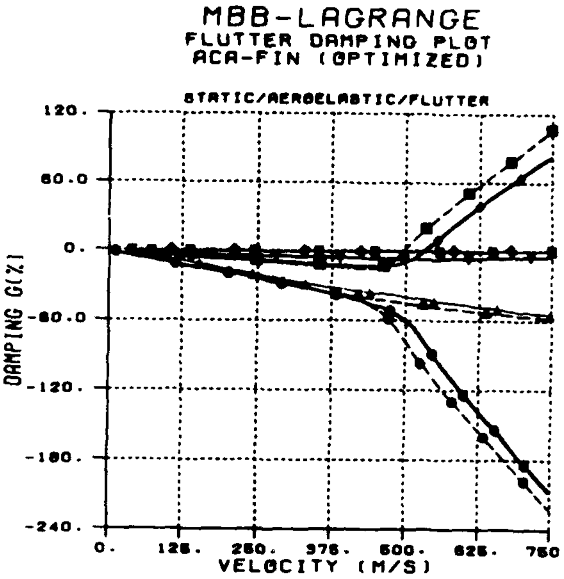


Fig. 39 FLUTTER ANALYSIS DAMPING PLOT FOR INITIAL AND OPTIMIZED DESIGN

DESIGN	INITIAL CONSTRAINT	VIOLATED DESIGN	OPTIMAL
WEIGHT [kg]			
Structure	99.4		92.9
Non Structure	53.6		53.6
Total	153.		146.5
STRENGTH	Loadcase 2 Element 18 - .123 22 - .224	 -.123 -.228	
EIGEN-FREQUENCY [Hz]	f1 = 8.90 f2 = 29.83 f3 = 31.16* f4 = 39.97 f5 = 54.86		9.20 30.21(x=1.) 30.61 41.08 58.31
FLUTTER FREQUENCY	fF = 21.22		22.0
SPEED [m/s]	VF = 495.	-.066	530.
AERO-ELASTICS FIN RUDDER	 .753 .441	 -.059 -.118	 .814 .500

* (x=1.)

Table 8 OPTIMIZATION RESULTS

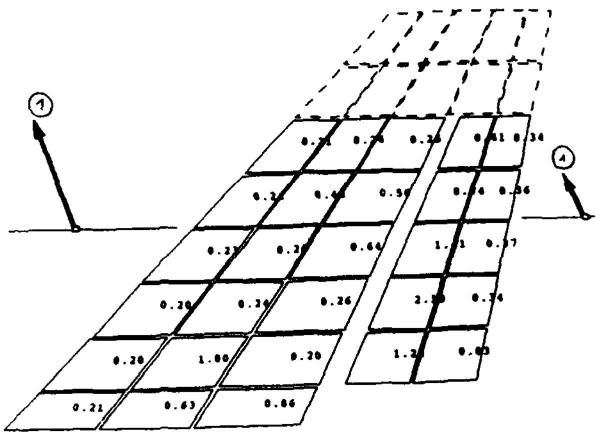


Fig. 40 AEROELASTIC DESIGN: SKIN THICKNESS FOR LAYER 1

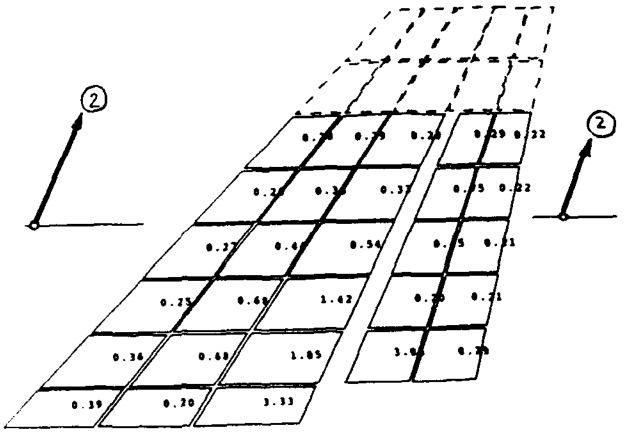


Fig. 41 AEROELASTIC DESIGN: SKIN THICKNESS FOR LAYER 2

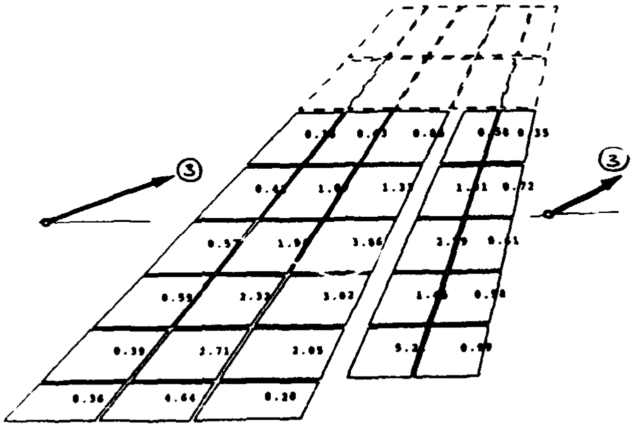


Fig. 42 AEROELASTIC DESIGN: SKIN THICKNESS FOR LAYER 3

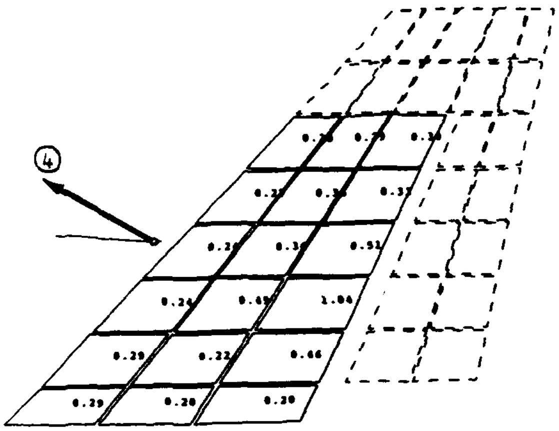


Fig. 43 AEROELASTIC DESIGN: SKIN THICKNESS FOR LAYER 4

CURRENT ACTIVITIES AT MBB MUNICH IN THE FIELD OF AEROELASTIC TAILORING

Shape Optimization:

For aeroelastic applications, the variation of the external geometry is most important. The problem in dealing with grid point coordinates as design variables is the connection with the aerodynamic model.

The variation of the aerodynamic model and its elements has to be combined with the structural model. This synthesis is currently being investigated.

Topology Optimization

As a higher level aspect of structural optimization problems (after sizing and shape optimization), topology optimization of the internal and external structural geometry is promissing great potentials in aircraft performance and structural design. Based on experience gained during a study which has been made to find the optimal attachment coordinates for wing-flap connections and with basic analytical tools developed for general investigations of topological aspects, an extended version of this program is currently planned for aircraft structures.

Smart Actuators

A new type of actuators with multi-signal input capability is currently under development. This actuator will offer several advantages in aeroelasticity: it will improve the aeroservoelastic stability of the system, it could be used to replace buzz dampers, and it is capable to cover active control technology for aeroelastic aspects without additional work load for the flight control computer.

AIRCRAFT OPTIMIZATION PROGRAM

Based on MBB-LAGRANGE, an optimization program for a total aircraft is currently being developed. In a first step, it will integrate the external loads in the optimization process. Because the complete aircraft structure is used, trimmed free-

free conditions will be simulated to describe the correct load distribution, inlcuding static aeroelastic deformations. Other aeroelastic problems like flutter and gust response will also be analysed and included in the optimization process for the complete configuration from the beginning of the design process.

The program will then be extended to cover aerodynamic aspects like drag, L/D, $C_{L\alpha}$, optimum twist and camber or design lift conditions.

For this purpose, the aerodynamic model of the aircraft will be used to provide design variables in addition to the structural variables. This combination will also allow to cover other important aspects like stability, performance and control and it will provide a basis for the integration of active control techniques into the optimization loop.

The challenge on aeroelastic tailoring is depending on new flight performance requirements, demanding new configurations as well as new technologies and new materials.

At the present time the development of new airplanes is influenced by new techniques, such as flutter suppression, CCV-configuration, gust load alleviation etc. In addition to stress, displacement, aeroelastic and dynamic constraints an integrated design involves all these techniques and the optimization procedures must be extended for these new constraints.

Especially the combination of new developments in aerodynamic shape optimization and the well experienced active control technologies with structural optimization routines will necessarily enter into a multidisciplinary optimization process.

It will take a further period of development even if the progress in computer power and new mathematical optimization methods are enormous. Existing technologies have to be refined, new developments like shape optimization still have to be completed and experiences gained with this tools, and, last not least, specialists of different disciplines have to be convinced that the new opportunities are worth the effort taking into the bargain a highly increased complexity of the design process. To account for the increased complexity, adapted intelligent user interfaces and checking routines for the generation of reliable inputs, for the check of interim solutions and for the interpretation of the output as well as improved integrated expert systems to support the selection of appropiate algorithm are required.

Up to this status it can not be expected that there will be one program only for the optimal design during different stages of aircraft projects.

An initial preliminary design should in fact include as many desciplines as possibles. But at the same time, this task must remain in a not too detailed and complex level to allow the investigation of a great number of designs and to answer questions concerning essential changes of design requirements in a relatively short period of time. After this, the individual disciplines should use their own programs and methods to find the optimum in a more detailed model, without forgetting the neighbour areas.

The preliminary design program could in parallel serve as a tool to integrate the results from detail designs.

Large efforts will be required to reduce the enormous computational costs by the development of efficient methods for cross sensitivity calculations and for approximate optimization procedures.

An extensive description of the use of optimization for concurrent engineering is given in [24].

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14. Abstract

There is a lack of precise information on the effectiveness of specific methods in generating optimum designs for realistic aircraft structures. In this situation it is difficult for designers to make decisions on which systems to employ for a given design problem and which developments to pursue. Thus it is necessary for designers to be aware of the relative merits of the different methods currently used for the design optimisation of advanced aircraft.

This Lecture Series covers the methods available for the computer based design analysis and design optimisation of aircraft structures. The Lecture Series deals with the principles and practices adopted to integrate the various factors which are considered in the design of advanced aircraft. These factors include: structural shape, aerodynamics, active control technology and aircraft performance. Realistic case studies are used to illustrate the methods used for different design problems.

The following topics are covered in detail:

- Overview of integrated design analysis, background, methods, objectives and requirements.
- Optimisation in design (CAE/CAD).
- A system approach to aircraft optimisation.
- Case studies for different design problems.

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