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Specifications for Integrating Expert Systems With Equipment Maintenance Management Systems To Optimize Equipment Investment Decisions

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Improved management of public works, mobile construction equipment, and other publicly and privately owned fleets can yield significant cost savings through reduced operating expenses, smaller fleet size, and increased availability of these resources. To gain these benefits, the Army has funded research into combining the data contained in modern equipment maintenance management systems with optimization techniques, expert systems, and managerial judgment. This study outlines the underlying concepts and specifications for a model that will consider downtime and its effects, enable equipment trade-off analyses, and suggest fleet sizes and replacement policies, based on consideration of operating environment, demand patterns, and expected maintenance costs.

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Improved management of public works, mobile construction equipment, and other publicly and privately owned fleets can yield significant cost savings through reduced operating expenses, smaller fleet size, and increased availability of these resources. To gain these benefits, the Army has funded research into combining the data contained in modern equipment maintenance management systems with optimization techniques, expert systems, and managerial judgment. This study outlines the underlying concepts and specifications for a model that will consider downtime and its effects, enable equipment trade-off analyses, and suggest fleet sizes and replacement policies, based on consideration of operating environment, demand patterns, and expected maintenance costs.

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FOREWORD

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COL Daniel Waldo, Jr., is Commander and Director of USACERL, and Dr. L.R. Shaffer is Technical Director.

2

CONTENTS

FIGURE

TABLES

SPECIFICATIONS FOR INTEGRATING EXPERT SYSTEMS
WITH EQUIPMENT MAINTENANCE MANAGEMENT SYSTEMS
TO OPTIMIZE EQUIPMENT INVESTMENT DECISIONS

1 INTRODUCTION

Background

U.S. Army Directorate of Engineering and Housing (DEH) equipment managers are charged with the responsibility to make effective decisions concerning their equipment fleets, including:

1. How many of each type of equipment to own

2. When to replace units of each type of equipment

3. Whether to replace old units with like or different types of equipment

4. What form of transaction replacements should take (e.g., purchase or rental)

5. For each type of equipment, which units to replace.

Managing equipment fleets involves complex decisions that include many variables. In times of tight budgets, the total capital investment in even a modest-size fleet is large, and the annual expenditures for equipment acquisition, replacement, operation, and maintenance must be optimized.

An equipment upgrade may increase that unit's availability (which may in turn reduce the number of units required) and may reduce operating, repair, and maintenance costs. However, total funding available for equipment replacement is often much less than the amount apparently needed to upgrade marginal and substandard units. Two basic problems complicate the process: (1) the difficulty of deciding which units to replace, and (2) the difficulty of preparing and presenting documentary support for the decision to replace.

In the past, equipment managers have based operating policies and replacement decisions on a limited awareness of previously collected information and data—often on the memory recall of the manager, mechanics, and equipment operators. Such simplified approaches can create basic planning problems. Often, what intuitively appears to be the best solution for the current period results in far greater costs than other solutions evaluated for a longer period of time. In a long-term context, decisions made in a single year's time are complex enough so that a manager needs computer assistance to choose the best alternative.

Modern 386/486-based personal computers (PCs) with large hard disk storage capacity can manipulate large amounts of data with relative ease. This low-cost computational power has fostered commercial development of PC-based equipment maintenance management systems (EMMSs). More sophisticated systems can maintain parts inventories, automate purchase order generation, manage labor, generate work requests, track warranties, produce many predefined reports, and create user-designed reports through an integrated or supplemental report writer.

Although the better EMMSs derive output to help make life cycle cost decisions, they do not explicitly incorporate such output into procedures for making optimal life cycle cost decisions, such as (1) when to replace old or outworn units, (2) when to choose ownership over rental, or (3) how to gauge the effects of reducing maintenance budgets.

Objectives

The objectives of this study were to develop and outline the underlying concepts and specifications for a computer-based expert system with the ability to develop and optimize recommendations to acquire or replace equipment units over a planning horizon of several years.

Approach

Preliminary information was gathered in several simultaneous investigations: (1) a review of the capabilities of commercially available equipment maintenance management software systems, (2) a literature review of analytic models and economic principles that can contribute to life cycle equipment decisions, and (3) meetings and telephone conversations with equipment fleet management professionals. This information was further developed into the system concepts described in this report jointly by the U.S Army Construction Engineering Research Laboratories (USACERL) and Prototype Inc., the developer and vendor of *EMS/PC*, a computer-based equipment management system currently in use at several Army installation DEHs.

Mode of Technology Transfer

In their final form, the algorithms and procedures described in this report will be incorporated into a computer program that will operate with any commercially available EMMS offering a database that includes certain specific cost and equipment information. This system prototype will be designed to operate with Prototype, Inc.'s *EMS/PC*, and will be commercially available through Prototype Inc.

Equation Notation

The analysis in this report uses many equations. This section explains a few of the conventions employed.

The letters "g, h, i, k, j, and d" most frequently appear as subscripts. Not all are always needed, but when one or more appear, they tend to do so in the order shown.

- gh - gth planning period of an h period lease
- i - planning period
- k - planning subperiod
- j - usage bracket group j
- d - a demand period, or a demand period requiring d demand units.

Other subscripts, such as min, max and other characters or letters, occasionally appear, and are explained as needed. The most common superscripts tend to be:

- a - all
- u - used or utilized
- v - available.

Summations are almost always over subscripts, and not the subscripted variable. Thus, the expression "Σd_d" represents a summation over the range of the subscript "d."

2 CONCEPTS FOR THE SYSTEM

Overview

This system should be able to specify the best combination of units (in terms of age, accumulated usage, and user-defined desirability) for each equipment fleet during any year of a specified planning period, given the mix in the preceding year, using data from EMMSs and from expert systems (to define operating environment and optimization models) supplemented by management judgement. The system will guide equipment managers in creating an analytical model that will help manage the purchase, lease, resale/salvage, and usage of one or more "functionally homogeneous" sets of equipment items (fleets) over a planning horizon of multiple planning periods. (Functionally homogeneous items are those that can be used interchangeably for a common set of tasks.) When fully implemented in a specific organization, the system will analyze and review repair and usage history. The model will minimize the present value of the cost of fleet operations over the planning horizon. By this definition, most Army installations own many fleets of equipment, e.g., sedans and bulldozers. The term multi-fleet refers to more than one fleet.

This approach will provide managers much freedom and flexibility in describing the operating environment in terms of:

- Anticipating seasonal demands

- Anticipating stochastic (random fluctuations in) demand

- Selecting which of the available units to use

- Guaranteeing minimum availability of equipment, which may differ among time periods

- Guaranteeing various minimum availabilities of items for planning time periods and/or demand time periods

- Achieving desired probabilities of meeting demand levels in high demand time periods

- Meeting all demand levels some desired fraction of the time

- Anticipating the disposition of users to favor new or otherwise desirable items

- Planning for varying lease periods

- Specifying minimum age (measured in time or accumulated usage) after which an item may normally be sold or salvaged

- Specifying maximum age (measured in time or accumulated usage) after which an item should not be kept in service

- Balancing capital expenditure and operating budgetary constraints.

This system will help the manager establish the demand patterns and appropriate demand, use, operating and availability constraints; and in addressing issues not yet explicitly considered.

The fundamental acquisition/replacement issue addressed here is how to define the mix of items (in terms of age, accumulated usage, and user-defined desirability) for each equipment fleet during any year of a planning period, given the mix in the preceding year.

Equipment replacement issues are often approached "from the bottom up." The manager determines the items to replace in a given period. This approach masks a critical assumption—that the equipment owned (or possibly leased) is correct for the services provided. This assumption must be tested, since the single most important determinant of equipment ownership and operating costs is whether the equipment actually owned is appropriate to its function.

For this purpose, the manager must establish "usage bracket groups" for each fleet of equipment. Fleet items are assigned to usage bracket groups depending on the item's total accumulated usage at the beginning of each planning period, on its age, and/or its desirability. An alternate concept to usage bracket groups is that of "cohorts," or groupings of units by operational and other characteristics.

The system starts by determining an optimum equipment complement. (A "complement" is the collection, for each fleet, of items in each age use bracket, operated or leased in each year of the planning horizon.) The manager defines the constraints that limit the range of possible complements, including:

1. The length of the planning period

2. The budget amounts for both equipment operations and equipment replacement for each planning period

3. The minimum number of items required of each equipment fleet, without regard to the demand for items of that fleet (for example, fire trucks or snow plows)

4. The maximum amount of demand a single item of each fleet in each usage bracket group can satisfy in some time period, considering the requirements of preventive maintenance programs and the realities of equipment failures.

Within these constraints, the system seeks to optimize (minimize) the present value of the cost of operating all equipment over the planning horizon. A single planning period (year) view would incorrectly be biased by the presumption that it is less costly (in the current period only) to continue to repair and maintain than it is to replace an item.

The system generates an equipment complement for each usage bracket group in each equipment fleet for each planning period. The differences between the complement for a planning period and that for the previous planning period determine how many items in the usage bracket group should be acquired or disposed of. This information will rank appropriate candidates for repair or replacement, and will help document the reasoning that supports the manager's equipment decisions.

Fundamental Units and Time Periods

Planning Periods

The planning horizon consists of one or more planning periods, typically a year, which generally correspond to an organization's budgetary cycle. The planning horizon should extend well beyond the last planning period for which the manager is seeking useful recommendations from the model, to insulate the recommendations from distortions that might be introduced by proximity to the end of the horizon

(e.g., not purchasing a new item near the end of the planning horizon). The number of planning periods in the horizon, and the length of each planning period does not vary among fleets. By assumption, any fleet item disposal or purchase will occur at one or more designated points in a planning period, typically the beginning.

If the size or nature of the demand that a fleet satisfies varies significantly during the planning period (e.g., seasonal variations, or annual busy seasons), then the planning period should be divided into planning subperiods. For any fleet:

1. Two to four planning subperiods should suffice

2. Planning subperiods need not be of equal length

3. The division of planning periods into planning subperiods does not change between planning periods.

However, the length of planning subperiods can differ between fleets. For equipment used only or almost exclusively during one portion of a year, the planning period could be a "year" equal to the number of workdays the items are used.

Usage Units and Usage Brackets

"Usage units" are typically meter units such as miles (for road vehicles) or hours of engine use (for heavy equipment), although some other measure, such as loads lifted, could be used.

"Usage bracket units" extend the concept of usage units, and are a function of age, meter units, and/or possibly other equipment-specific measures of use (e.g., number or weight of loads carried or lifted). The manager defines a set of usage bracket groups for each fleet, which are used to predict a planning period's repair and operating costs per usage or usage bracket unit, and, when appropriate, availability and/or desirability for equipment items in the fleet.

The term "usage bracket group" actually represents the following more comprehensive interpretation: any group of items that differs sufficiently from other groups of items in one of the following categories, deserves separate consideration:

1. Anticipated maintenance, operating, repair and downtime costs (MORD costs) per unit of usage in a planning period, or

2. Anticipated demand satisfied in a planning period (i.e., since some items may have higher productivities, although productivity differences may best be handled as different fleets).

While differences in repair costs result primarily from age or accumulated usage bracket units, differences in expected demand satisfied can result from condition, reliability, operating characteristics, installed optional equipment, or perceived user desirability. Although the term usage bracket group will be used, a better term might be "usage bracket/desirability group." In any case, the concept embraces differentiations such as:

• Identical sedans with radios or air conditioners versus those without, which, although the same age or older, may experience different usage levels

10

- Items purchased used with limited warranties, as opposed to similar items of the same usage bracket group purchased new, but now with expired warranties, so that they may have different effective repair cost characteristics

- Items leased for multiyear periods that have different cost patterns from comparable, owned items.

Usage brackets provide an alternative to age in years to help establish performance expectations for individual units in a fleet. They are useful since many categories of operating, maintenance, and repair cost per usage or usage bracket unit correlate more closely with the number of usage or usage bracket units the equipment has experienced than to the chronological age of the equipment.

Other assumptions concerning usage bracket groups include:

1. Equipment items in a fleet are assigned to usage bracket groups at the start of each planning period depending on the accumulated usage measured for them as of the beginning of the planning period.

2. Items remain in a single usage bracket group for an entire planning period, regardless of the number of usage units accumulated during the planning period.

3. An item assigned to a usage bracket group at the start of a planning period is assigned to a usage bracket group for the next planning period based on the number of usage units it is operated during the preceding planning period.

4. The estimates of MORD costs per unit of usage for an item in any planning period depend only upon its assigned usage bracket group at the start of the planning period.

5. The capital value of an item at the start of any planning period depends uniquely on its assigned usage bracket group at the start of the planning period.

6. From (5), it follows that the loss in capital value experienced by an equipment item during a planning period depends on its assigned usage bracket group at the start of that period and the item's usage bracket group at the start of the next period.

Usage bracket groups need not span equal ranges of usage or usage bracket units, and both management judgment and statistical confirmation can temper their creation. Since usage bracket groups at least differentiate among items having, in a planning period, different anticipated MORD costs, approaches to defining usage bracket groups include:

- Using linear combination of age and usage meter reading, possibly in half-year, full-year or 2-year increments.*

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* Unfortunately, high correlations often exist among chronological age, meter usage (and any other item-specific measure). For example, for a fleet of British Army vehicles, Mahon and Bailey ("A Proposed Replacement Policy for Army Vehicles," *Operational Research Quarterly* [1975], Vol 26, No. 3, pp 477-494) found only slightly better correlation of repair costs with the following combination of age and use: (0.5 [age + (life-to-date use)/(expected annual meter)] as compared with age or meter alone, although using the linear combination led to wider acceptance by the project sponsors. Their paper suggests that homogeneous average annual use of individual equipment items (i.e., high correlation between life-to-date meter and age) probably caused their results. Therefore, any usage bracket measure should undergo confirmation of its statistical significance as a superior predictor (over age or usage units) of maintenance costs and availability. Thus, the term usage bracket, when used in the remainder of this document, in practice often reduces to usage units or chronological age. From now on, the term usage unit should be interpreted as usage or usage bracket unit.

11

- Defining usage bracket periods that bracket either the expected value or (most likely) the mode of accumulated life-to-date usage at which major repairs occur for the fleet.

- Statistically determining usage bracket periods that result in the most significant differences in costs per usage unit for the fleet.

Cohorts

Since items are assigned to usage bracket groups based on usage units (from [3] above), during application of the model, members of each usage bracket group progress together during successive planning periods to increasing, but not necessary consecutive usage bracket groups.

An alternate concept to usage bracket group requires the slightly more restrictive assumption that items are grouped according to their operating and MORD characteristics for the rest of their lives, and that such groups advance from planning period to planning period together as a "cohort." This assumes that:

1. During their lives, items in a cohort advance through different "usage bracket intervals," each interval having a different MORD cost per unit

2. The initial accumulated life-to-date usage units for all items in the cohort is the average of the values of the individual cohort members,

3. All items in a cohort experience the same usage in a planning period.

A cohort's occupying a planning period in more than one usage bracket interval is not a conceptual problem. Items purchased new constitute a new cohort, while items purchased used can either start a new cohort or join an existing cohort.

The terms "cohort" and "usage bracket groups" will largely be used synonymously in this report; the difference between the terms will only be emphasized when necessary to discuss certain techniques.

Figure 1 shows the relationship between the equipment owned and operated, fleets (one for each type of equipment owned), usage bracket groups (for bulldozers owned and for sedans leased), and cohorts (for sedans owned).

Demand Unit Modeling

Although usage is measured by usage or usage bracket units, often an organization thinks in terms of "demand periods," and the "demand units" to be satisfied during a demand period. Demand periods are typically hours, shifts, days, weeks, or months; demand units are either item-periods (e.g., item-days), usage bracket units, or some well defined task (e.g., deliveries, service calls). The "period" of the item-period must be the same (the usual case) or shorter than the demand period. For example, each day (the demand period) a construction company may need some number of item-days of bulldozers. A delivery service on a given day or shift or half-day (the demand period) may have to complete some number of deliveries (the demand unit). A municipal bus service may need to provide, based on its route configuration and desired schedule, for each shift (which may be from 2 to 8 hours), a certain number of buses or bus-shifts (the demand unit).
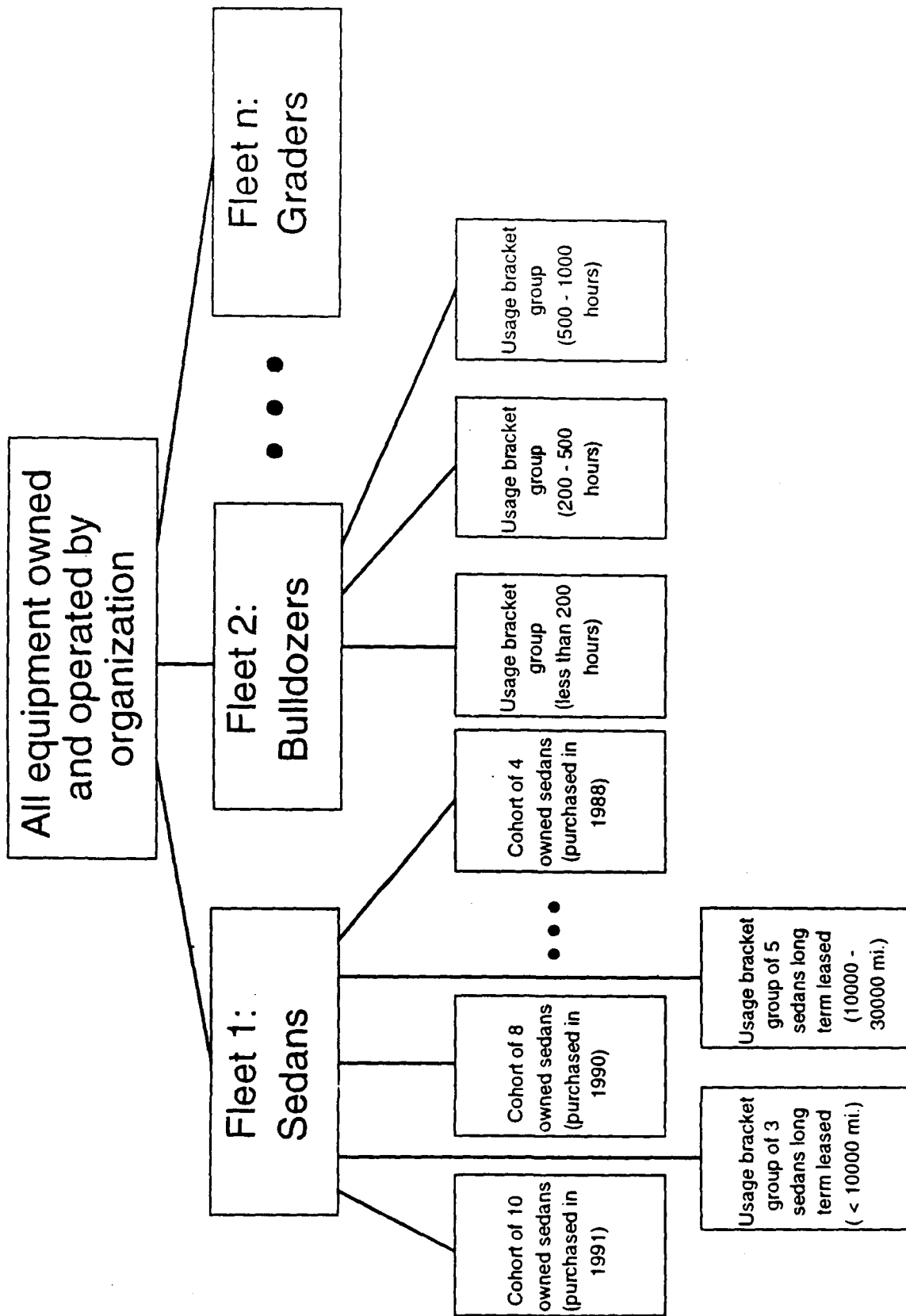
Figure 1. Fleets, Age-Use Brackets, and Cohorts.

13

The term "demand unit modeling" designates the use of demand periods and demand units. The available historical data base will often determine the viability of using demand unit modeling. A substantial amount of detailed information about usage assignments and recorded usage on each assignment for equipment items in a fleet is required to support analysis using demand unit modeling, and many equipment managers may not have access to adequate data.

If items are leased on a short term basis to fulfill demand peaks, the minimal lease length (hour, day or week) is often a good choice for the demand period. For instance, bulldozers' usage bracket group is measured in engine hours, but they may be assigned to jobs on a daily or weekly basis. If appropriate, a planning period or subperiod can consist of a single demand period.

The manager or analyst can determine whether to allow item leasing for one or more planning periods, subperiods, and/or, if demand unit modeling is used, demand periods.

Demand Patterns

Principles

The manager must select one of the following descriptions of demand pattern during a planning period or planning subperiod:

1. The aggregate usage or "demand units" required during the planning subperiod or planning period. This requires a constraint on the maximum number of usage or demand units an item can supply during the planning period or subperiod. Expressing aggregate demands by demand units also requires an estimate of the number of usage units an item experiences per demand unit. (For example, the average number of metered engine hours for a day of bulldozer use.)

2. A probability density function for the number of demand units "required" in a demand period. "Required" means that failure to satisfy that number of demand units causes either a measurable economic penalty (as a function of the number requested and/or the number actually provided), or an organizational penalty (e.g., not satisfying customers' service expectations). Note that assuming a uniform density for demand for a planning period or subperiod is not the same as assuming a deterministic number of usage or demand units for a planning period or subperiod.

3. A predictable cycle of demand units needed in a sequence of demand periods (e.g., a municipal bus service). This often will be analytically handled analogously to a probability density function.

When demand units are some sort of task, or use a period (e.g., item-*days*) smaller than the demand period (e.g., *month*), methods (2) and (3) require a constraint limiting the demand units per planning period or subperiod an item of each usage bracket group can provide.

Considerations in Selecting the Demand Pattern Description

Expressing the demand pattern as a probability density function or cycle helps to:

1. Assess penalty costs resulting from item shortages (as a function of demand required or of shortfall from that demand)

2. Accurately distribute the demand satisfied by various usage bracket groups during a cyclic demand

3. Distribute demand by user preference.

Specifying an aggregate demand for a planning period or subperiod becomes preferable when:

1. There is no great concern with day-to-day fluctuations in demand requirements

2. The manager can adjust or change schedules as needed without economic or organizational inconvenience.

For example, tree-trimming equipment can be diverted from routine use when emergency dictates.

For any description of demand pattern, the model supports a definition of a minimum fleet size required to guarantee some minimum level of service or availability. The minimum size constraint can be expressed in terms of equipment items or, as explained below, item equivalents. Such a constraint should often supplement demand patterns expressed as aggregate values, to satisfy peak service requirements not explicitly modeled or specified, or for which detailed data are not available.

Using an aggregate demand to represent a regular, cyclical demand pattern risks that an analytical model may assign so much demand to economically more desireable items that they could only satisfy that demand by operating unnecessarily during periods of low actual demand. The underlying problem relates to the lack of sensitivity of aggregate demand to the number of items required at any given time.[1]

Availability

Any model for planning fleet size or distributing demand among items comprising a fleet must represent the pattern of "item availability," and the maximum number of usage units an item can provide in any time period. These depend upon:

1. The preventive maintenance (PM) program required to maintain the item in good operating condition

2. Item failures, defined as conditions that prevent an item's further operation

3. Delays in scheduling repairs and the timeliness in procuring parts.

Estimating Availability Through Simulation

Each organization conceptualizes "failure" according to its own maintenance policies and safety regulations. Failures can even include PM if removal from service for PM causes some economic penalty associated with an item's unavailability. Revisions in these organizational factors justify re-evaluation of a fleet's availability representations.

The most accurate representation of availability requires empirically fitted probability densities for:

1. The number of demand periods between failures, assumed to be dependent only upon the usage bracket group to which a unit currently belongs. The chosen probability density determines how the

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[1] For example, Simms, et al. ("Optimal Buy, Operate and Sell Policies for Fleets of Vehicles," *European Journal of Operational Research* [1984], Vol 15, pp 183-195) modeled a municipal bus fleet, setting both aggregate mileage and minimum fleet size in each planning period (year), but did not specifically discuss whether this lack of sensitivity problem occurred.

failure rate changes with time since the preceding failure. For example, a negative exponential density implies a constant failure rate, and a normal density implies an increasing failure rate.[2] In contrast, a Weibull density can have either a decreasing, constant, or increasing failure rate, depending on its parameters.[3]

2. Demand periods until repair, which can depend upon:

    a. The usage bracket group of the failed item, since older items may require more extensive repairs

    b. The number of items needed at the time when the failure occurs

    c. The number of items available at the time when the failure occurs

    d. The position in a cycle for regular cyclic demands

    e. The nature of the planning subperiod in which the failure occurs (one of high demand or low demand).

These parameters can capture differences due to usage bracket group and an organization's ability to speed repairs when needed. Of course, any such dependencies require statistical confirmation. These probability densities, in combination with a cycle or probability density of demand period requirements, support a simulation of the fleet operation that can include, if desired:

1. A policy to assign work to either the least used in the period/subperiod (to enforce equalization of use), or to an item available from the most desired usage bracket group (within a group selecting the item with the least usage)

2. Rules for permitting overtime

3. Customer tendencies to use the most desirable item available.

Through such simulations, a manager can accumulate, for each usage bracket group, statistics on usage (both regular and overtime) and unavailability for each combination of items needed and items short.

Estimating Availability Through Availability Factors

There is an alternative, simpler method to describe availability. Each item in a usage bracket group at the beginning of a planning period has an estimated availability factor (AF), defined as:

$$AF = \frac{\text{time actually used}}{\text{time actually used } + \text{ time unusable due to failures}} \qquad [\text{Eq 1}]$$

Time unusable due to failures refers to that spent undergoing or awaiting repairs needed for an item to operate. Under demand unit modeling, "time" means demand periods. Otherwise, any convenient measure can be used (days, shifts, etc.). For example, for items that are assigned work by days:

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[2] A.K.S. Jardine, *Maintenance Replacement and Reliability* (Pitman Publishers, 1973), ch 2.6.

[3] Irwin Miller and John E. Freund, *Probability and Statistics for Engineers*, 2d ed. (Prentice-Hall, 1977), p 455.

$$AF = \frac{\text{days actually used}}{\text{days actually used} + \text{days unusable due to failures}} \qquad \text{[Eq 2]}$$

The AF is not a net value; rather, it expresses the ratio between two values; as a ratio, it is time-independent.

An AF indicates an item's expected maximum availability. Thus, an item with an AF of 0.95 can provide at most:

0.95 item-equivalents per month, or

176 x 0.95 hours per month (if a month has 176 working hours), or

22 x 0.95 shifts per month (if a month has 22 shifts), or

0.95 shifts per day (assuming 1 day per shift).

For example, a sedan from a motor pool when demand unit modeling is not being used may have records indicating that:

1. Sedans are driven 10 miles per hour while checked out

2. A month has 176 hours

3. Three-year-old sedans spend 1 day (8 hours) in the shop for each 1500 miles driven (150 hours at 10 miles per hour average).

These records show that 3-year-old sedans spend 8 hours in the shop per 150 hours driven, for an availability factor of

$$AF = \frac{150}{(150+8)} = 0.95 \qquad \text{[Eq 3]}$$

The sedan's maximum use in a month is either 176(0.95) hours or 1760(0.95) miles.

The virtues, limitations, and implications of using availability factors require explanation. Availability factors establish accurate upper bounds for the expected aggregate demand or "item equivalents" that an item can satisfy in a planning period or subperiod. Availability factors can be used in two ways:

1. Assume that an item produces exactly AF times the number of item-periods available in a demand period. This certainly is appropriate when using aggregate demands for a demand period, but miscalculates expected shortages when using probability densities for required demand in a demand period. Chapter 3 (p 25) includes an example of this use of availability factors.

2. When using probability densities for demand in a demand period, assume that an item's probability of availability on any given day equals AF.

In either case, availability factors become a joint surrogate for the densities of time between failure and of time for repair when either (1) those densities cannot be derived, or (2) the computational burden of

17

simulating various usage bracket configurations for a fleet, while seeking an optimum, is deemed excessive. Using availability factors as this surrogate implies a constant downtime per unit of usage no matter how long since the last failure. This artificially lowers availabilities.

In practice, however, an organization may place low priority on repairs to a fleet in periods of known nonpeak demand, but prepare for and give high priority to repairs during high-demand seasons. In such cases, increasing availability factors (above the overall average AF for the usage bracket group) for demand periods with higher requirements can compensate for the inherent bias to underestimate availability. Such demand-related adjustments can capture, for instance, the need for dump trucks fitted with plows to be ready for snow removal work. In contrast, a municipal bus service may not require such adjustments.

Demand Satisfied by a Mix of Available Items

Assuming that each item can satisfy up to some number of demand units in a demand period (which may differ among bracket usage groups), for certain fleets, the amount of "required" demand actually satisfied, and the resulting usage units, can depend nonlinearly upon:

1. The demand units "required" in the demand period

2. The position in a cycle of demands of the demand period

3. The number, or possibly the usage bracket mix, of items available to do them

4. The assigned task (note that this adds an additional possible dimension to the model, which is beyond the scope of this work).

For instance, municipal buses may traverse a route fewer times and thus accumulate less mileage per shift late at night than during rush hours. A taxi service, for example, may expect 40 calls a shift (i.e., one demand period). A single taxi may on the average handle only 10 calls. Additional calls are lost to competitors. Adding a second and third taxi may result in decreased "empty" traveled distance between calls. (With more taxis, a nearby empty one is more likely assignable.) Thus capability may increase to 22 and 36 calls per shift, respectively. Usage units (taxi miles) might be estimated by the average distance traveled empty between calls and the average distance per trip (probably independent of the number of taxis available). Also, a shortage of items might inspire additional work from operators. The possible complexity of transforming demand "required" (calls, in the taxi example) to number of items used, may encourage many managers to express demand units directly as number of items.

Distributing Demand Among Usage Bracket Groups
and Calculating the Resulting Operating Costs

The fleet manager designates how to distribute the demand for equipment items in each planning period or planning subperiod among the usage bracket groups. Within a usage bracket group, for planning purposes, all items are assumed to receive the same amount of use. (Note: When the demand units do not equal the usage units, a distribution of demand among items of course implies a corresponding distribution of usage units among the items. Occasionally the terms "usage distribution" and "demand distribution" will be used interchangeably.)

The nature of the fleet, management policy, and the methods chosen for describing demand and availability may influence selection of one of the following methods of distributing the satisfied demand (which, if insufficient items are available, may be less than the "required" demand) to the usage bracket groups:

1. Distributing equally among fleet items. Management may impose such a policy if the aggregate demand is sufficiently less than the fleet capacity, or if the nature of the fleet may inherently produce such a demand distribution (e.g., a fleet of lawn mowers or other items checked out to users on a first in, first out basis).

2. Distributing proportionally to items' AF. Natural events will tend to generate this method of distribution; it would be inefficient for management to impose method (1) by policy decision. The model user must decide whether failure to meet this constraint constitutes an invalid fleet configuration, or whether the demand or usage as distributed is adjusted to meet the constraint.

3. Distributing according to user preference. An item is chosen for use from the highest usage bracket/desirability group having an item available for use. Pooled items often have demand distributed in this manner.

4. Distributing to minimize the cost operations in the fleet over some planning period or subperiod.

Any of these methods can be subject to constraints on maximum and minimum usage (to justify retention of items) or on the number of demand units to be provided in a planning period, planning subperiod, or demand period.

When using the methods to minimize costs (1), the model distributes demands that guarantee compliance with the applicable constraints. However, under the other three methods, the manager must designate whether violation of the constraints implies an invalid fleet configuration, or whether the demand distribution can be adjusted to correct the violation.

If using aggregate demands for a planning period or subperiod, the model attempts to satisfy the "required" demand with the owned usage bracket mix of items. If this is possible, the model calculates (1) the demand (and hence usage) assigned to each usage bracket group, and (2) the operating cost of the assignment.

Instead, if using probability density functions for the "required" demand in a demand period, given a realization of the demand "required" and of the usage bracket mix of items available from those owned, the model attempts to satisfy the "required" demand with the available items. If this is possible, the model calculates:

1. How many of the available items from each usage bracket group to use

2. How much demand (and hence usage) to assign collectively to the items chosen from each usage bracket group

3. The operating cost of the assignment.

In either case, if the available items cannot satisfy the "required" demand, all available items are assumed used, and the model determines, as allowed by the operating environment and management policy and judgement, the combination of overtime and/or short term rental that best satisfies the objective of meeting as much of the demand as possible at a minimum cost, and the resulting usage assignments and

19

operating costs. The expert system portion of the model would have to elicit the options for and constraints affecting short term rentals and overtime from the manager.

Components of Operating Costs

Depending on the defined operating environment, the operating costs in a planning period, planning subperiod, or demand period can have several components:

1. Marginal cost per usage unit of vehicle-related operating costs, which:

   a. Includes fuel and other fluids, repairs, and preventive maintenance

   b. Does not change between regular and overtime operations

   c. Is predicted from historical records and management judgement.

2. Fixed cost of ownership (i.e., costs that accrue just from owning an item, whether used or not). These might typically include insurance, and minimum maintenance levels

3. Operator costs during nonovertime periods are predicted from projections of time operated (hours, shifts, etc.).

4. Short term cost to rent equipment to meet "required" demands not met with owned equipment, consisting of one or more of:

   a. Fixed cost per time period item rented

   b. Cost per usage unit (rental charges, fuel)

   c. Nonovertime and overtime operator charges (may differ between in-house vs out-of-house).

5. Overtime in-house operator costs are predicted from overtime time operated.

6. Consequential costs of field failure.

For each usage bracket group, the estimated number of field failures per usage unit is derived from historical records, and the estimated consequential cost of field failures is derived from management judgement. Note that capital depreciation and fixed costs of ownership are not included in operating costs.

Model Outputs

For each planning period from the present to the planning horizon, the model determines, for each usage bracket group within each fleet:

1. The number of items to eliminate (i.e., sell/salvage) from the usage bracket group at the start of the planning period

2. The number of items to add to the usage bracket group at the start of the planning period (items acquired new fall into the first usage bracket group, items acquired used fall into an existing usage bracket group or into a newly created group)

3. The number of demand units (also used to derive the usage units) that items in the usage bracket group should or will experience during the planning period

4. As applicable, the number of items to lease for planning periods, planning subperiods, or demand periods.

The model assumes that, within a single usage bracket group, items can be either eliminated or added during a single planning period, but not both. Consequently, for each planning period, one or both of the number of items to eliminate, or to add, will be zero.

The model also assumes that, after implementing the decisions to sell/salvage or add items to a usage bracket group at the start of a planning period, all items in the usage bracket group survive to the end of the planning period. Many organizations operate in this manner. Corollaries to this assumption are:

1. Newly acquired items come with warranties that cover any repairs required to guarantee survival to the end of the first planning period of ownership

2. Accidents and other casualties are handled by insurance or by including such costs in the repair cost densities.

Additional policies may already exist:

1. To not sell or salvage items before they reach some usage bracket group (not always an economically sound policy)

2. To dispose of items that reach a certain age (a policy most applicable to multiyear lease items when their leases have expired)

3. To not purchase used items that have already progressed beyond a certain usage bracket group.

3   DISTRIBUTING A SINGLE PLANNING PERIOD/SUBPERIOD'S DEMAND FOR A HOMOGENEOUS FLEET OF KNOWN CONFIGURATION

This chapter explains how to distribute usage among the various usage bracket groups and to evaluate the cost of operations for the period over a single planning period (or subperiod), for a homogeneous set of items of known usage bracket mix. Three combinations of model demand requirements and item availability are considered:

1. Aggregate demand for a planning period or subperiod, and having availability factors represent expected availability

2. Demand densities for demand periods, and having availability factors represent expected availability

3. Demand densities for demand periods, and having availability factors represent the probability of an item's being available.

Four methods are applied to distribute demand to the individual items in each usage bracket group for each combination:

1. Equal distribution among fleet items

2. Proportional distribution according to availability

3. Distribution according to user preferences

4. Distribution to minimize operating costs.

These scenarios turn out to be subproblems that need repeated solution to reach multiple period and seasonal decision. Chapter 6 (p 53) addresses how to approach these more general decisions.

A hypothetical fleet of bulldozers illustrates the details of many of these concepts and modeling techniques. The many calculations to be described generally will not concern the model user. Notation will be developed and assumptions added as needed for the successive approaches.

Aggregate Demands for a Planning Period or Subperiod With Availability Factors Representing Expected Availability

The following notation is necessary to begin the example and illustrate demand distribution under aggregate demands:

n_j = Number of owned items in usage bracket group j, usage bracket increasing with j, j=1,...,J.

NP = Number of demand periods per planning period

d_j = Demand, in chosen demand units, distributed to an item in the jth usage bracket group

d_j^a = Demand, in chosen demand units, distributed to all items in the jth usage bracket group

AF_j, $AF_j(d)$ = Availability factor for an item in usage bracket group j. The (d) appears when the availability factor depends upon the demand required in a demand period

P_d = The probability that a demand period requires d demand units within some planning period or subperiod

D,U = Demand and usage units, when modeled as an aggregate requirement for a planning period or subperiod.

The following assumptions hold for this example and will remain true for all models discussed in this chapter:

1. The bulldozers in this fleet fall into three different usage bracket groups, denoted here by subscripts 1, 2, and 3

2. Bulldozers are assigned by the day (demand period)

3. The planning period has NP = 200 days

4. The organization's mix of work typically produces 5 engine operating hours for each day of bulldozer use.

Assumption (2) is especially important; it implies that in our example demand units are item-days.

From past records, assume the average values for AF in each usage bracket group to be those shown in Table 1. Thus, on average, the two items in usage bracket group 3 are unavailable due to failure 1 day for every 4 days operated. This might mean that the average time between failures is 8 days and mean time for repair is 2 days. The repair time can reflect factors such as scheduling convenience, priorities, personnel or bay availability, and parts delays, as well as the actual time used to complete the repairs.

The first and second columns of Table 2 display a hypothetical demand pattern for the number of bulldozers an organization might "need" on any day (the demand period). The third column displays the probability that at least n are needed on any day. Thus, the probability of needing 8 bulldozer days or 40 bulldozer engine hours on a single day is 0.25 (from Table 1), the probability of needing 9 bulldozer days or 45 bulldozer engine hours on a single day is 0.30, and so on.

Table 1

Usage Bracket Groups in the Fleet

Usage Bracket Group (J)	Number of Items in Group (n_j)	AF_j
1	4	0.95
2	6	0.90
3	2	0.80

Table 2

Example Demand Pattern

Prob of Needing *Exactly* n Items in a Demand Period (P_d)	d	Prob of Needing *at Least* n Items in the Period
0.20	8	1.00
0.30	9	0.80
0.15	10	0.50
0.25	11	0.35
0.10	12	0.10
Expected value	9.75	

This demand pattern is relevant to all our subsequent models, but this section considers aggregate demands, the use of which implies that any unsatisfied demand on a given day can be satisfied, without penalty on a later day. For the moment, only the following aggregate demands for the planning period are of interest:

- $D = (9.75)(200) = 1950$ item-days, or
- $U = 1950(5) = 9750$ item-hours.

Expressing demand as an aggregate value for a planning period or subperiod:

1. Precludes considering overtime costs and shortage penalties

2. Precludes modeling availability by probability densities for both time between failure and time for repair (such modeling provides no benefit)

3. Suggests use of a fixed availability factor for each usage bracket group, used both to constrain maximum demand an item can satisfy in the planning period and to set a minimum requirement for item equivalents.

Table 3 works out the example. The top two rows of the table give the average availability factors and the number of items for the three usage bracket groups. In rows A through D, each with two subrows, the upper subrow displays item days, and the lower subrow displays item hours (five times the item days). Each usage bracket group's column is divided into a "Per Group" subcolumn and a "Per Item" subcolumn, whose values differ by a factor of n_j. Row A shows the maximum demand that items in the usage bracket group can satisfy in the 200 work-day planning period, while rows B through D

24

Table 3

Demand Distribution Among Items and Usage Bracket Groups Under Aggregate Demand

Distribution Type		Measures					
AF_j for usage bracket group		0.95		0.90		0.80	
n_j (# of items in usage bracket group)		4		6		2	
		Per Group	Per Item	Per Group	Per Item	Per Group	Per Item
A.	Maximum per planning period	760.00	190.00	1030.00	1·).00	320.00	160.00
		3800.00	950.00	5400.00	900.00	1600.00	800.00
B.	Equal demand distribution	650.00	162.50	975.00	162.50	325.00	162.50
		250.00	812.50	4875.00	812.50	1625.00	812.50
C.	Proportional to availability distribution	686.21	171.55	975.00	162.50	288.89	144.40
		3431.03	857.76	4°⁻	812.47	1444.44	722.22
D.	User preference distribution	760.00	190.00	1080.00	180.00	90.00	45.00
		3800.00	950.00	5400.00	900.00	450.00	225.00

disı ɔute the demand among the usage bracket groups according to the indicated methods. The formula to calculate the per group item days entries are:

Maximum per planning period:

$$200\ n_j\ AF_j \qquad \text{[Eq 4]}$$

Equal demand or usage distribution:

$$d_j^a = D\ \frac{n_j}{\sum_j n_j} \qquad \text{[Eq 5]}$$

Proportional to availability distribution:

$$d_j^a = D\ \frac{n_j\ AF_j}{\sum_j (n_j\ AF_j)} \qquad \text{[Eq 6]}$$

User preference distribution:

$$d_j^a = D \min\{\max[0, 1 - \sum_{j' < j} (n_{j'} \, AF_{j'})], \; n_j \, AF_j\} \qquad \text{[Eq 7]}$$

Distributing demand to minimize costs (not shown in the table) requires an optimization of the following form:

$$\min \sum_j n_j C_j(d_j)$$
$$\text{subject to} \qquad \qquad \text{[Eq 8]}$$
$$\sum_j (n_j d_j) \geq D$$
$$D_{p,min} \leq d_j \leq D_{p,max} = 200 \, AF_j$$

where $D_{p,max}$ and $D_{p,\,min}$ are the maximum demand units an item can, and the minimum demand units an item must supply to justify its retention. Since this example does not include a cost structure, this last distribution method is not further illustrated.

Note that equal demand distribution violates the maximum demand for usage bracket group 3. (The values on the lines for Equal Demand Distribution exceed the maximum values on the lines for Maximum per Planning Period.) From this violation, either:

1. The proposed fleet configuration is declared infeasible for the proposed demand distribution method, or

2. The violating group is distributed its maximum allowed demand, with the remaining demand distributed to the other groups by the selected distribution method. Of course equal usage probably would not be applied to a fleet of bulldozers.

Under user preference distribution, usage bracket group 3 receives very little use. Specifying an aggregate demand for the whole planning period implies that no great concern exists with day to day fluctuations in demand requirements, and that schedules can generally be juggled as needed without economic or organizational inconvenience. If these implied conditions are true, and if items are used only when all the available more desired ones have already been assigned, the fleet may be too large. (However, it is not necessarily true that the least desirable units are the ones which should be excessed.) If a minimum usage for retention (D_{min}) had been specified, again usage might either require redistribution, or the fleet mix might be declared infeasible.

If the usage bracket mix of items could not satisfy the demand, the model could allow overtime or rentals (in some chunk of demand unit), to satisfy the excess demand.

The cost structure for the models in this section has, for each usage bracket, a cost per unit usage bracket unit. When using the cohort concept, this fixed cost changes as a cohort advances through a usage bracket interval.

Demand Densities With Availability Factors Representing Expected Availability

Additional Assumptions

This section explicitly considers the random pattern of demands in a demand period. Thus the analysis focuses on demand periods, rather than the whole planning period. In this analysis, the assumption that missed demand can be made up later without penalty, no longer holds. An explicit penalty function is presented. Preparations can be made for anticipated peak demand periods. Hence, higher availability factors can be used for such peak periods. Availability factors indicate the expected demand an item can satisfy in a demand period (rather than in a planning period as in the previous section). Each item used contributes equally to satisfying demand. (The equations shown will not require this, but the example calculations will.)

Expressing demand as a density or distribution: (1) implies the use of demand unit modeling, rather than aggregate demand for a planning period or subperiod; (2) allows modeling of cyclic demands without shortage costs; and (3) justifies demand dependent availability factors for each usage bracket group to model peak demand periods. The fixed availability factors should still be used to constrain the maximum demand an item can satisfy during a planning period or subperiod, and to set a minimum requirement for item equivalents.

Table 4 adds several columns to Table 2 for the demand dependent availability factors. Note the use of the average availability factors of Table 1 for the lower demand values. (If demands were expressed in units other than item-days, in practice a manager might first convert such units to item days before adjusting the availability factors.)

Additional Notation

The following terms are used to develop the demand distribution proportional to availability:

n_{jd}^v, n_{jd}^u = The number of items in usage bracket group j available for service, and used in service in a demand period requiring d units. Obviously, $n_{jd}^u \leq n_{jd}^v \leq n_j$. Also, each n_{jd}^u depends on the J values on n_{jd}^v, as well as on d.

Table 4

Demand Density and Usage Bracket Group Demand Dependent Availability Factors

Prob of Needing *Exactly* d Items in a Demand Period (P_d)	d	Prob of Needing *at Least* d Items in the Period	$AF_1(d)$	$AF_2(d)$	$AF_3(d)$
0.20	8	1.00	0.95	0.90	0.80
0.30	9	0.80	0.95	0.90	0.80
0.15	10	0.50	0.95	0.90	0.80
0.25	11	0.35	0.98	0.95	0.85
0.10	12	0.10	0.98	0.95	0.90
Expected value	9.75				

d_{jd}, u_{jd} = The demand and usage units distributed to individual items in usage bracket group j in a demand period having a demand of d.

d_{jd}^a, u_{jd}^a = The demand and usage units collectively distributed to all the items of usage bracket group j in a demand period requiring a demand of d. The superscript "a" denotes "all" items of usage bracket group j. Note that $d_{jd}^a = n_j d_{jd}$ and $u_{jd}^a = n_j u_{jd}$.

$d_{j.}, u_{j.}$ = The demand and usage units collectively distributed to each item in usage bracket group j in a demand period. Usually, $d_{j.} = \Sigma P_d d_{jd}$ and $u_{j.} = \Sigma P_d u_{jd}$, but this can be adjusted to meet usage constraints on individual items.

$d_{j.}^a, u_{j.}^a$ = The demand and usage units collectively distributed to the items of usage bracket group j in a demand period. The superscript "a" denotes "all" items of usage bracket group j. Note that $d_{j.}^a = n_j d_{j.}$ and $u_{j.}^a = n_j u_{j.}$.

$d_{\{.\}}^a(), u_{\{.\}}^a()$ = The number of demand units collectively satisfied and usage units collectively generated in a demand period by some subset of the usage bracket groups. The subsets of most interest are {J} for all J usage bracket groups, {J-j} for all usage bracket groups except j, and {j} for the 1st through jth usage bracket groups.

d_j, u_j = The demand units and usage collectively distributed to the items of age-group j in a planning period or subperiod. If using aggregate demand requirements in a planning period or subperiod, d_j and u_j are calculated directly. The relations $d_j = (NP)d_{j.}^a$ and $u_j = (NP)u_{j.}^a$ always hold. The quantities d_j and u_j might be adjusted to satisfy a minimum or maximum usage constraint for items during a planning period or subperiod.

d_j^a, u_j^a = The demand units and usage collectively distributed to the items of age-group j in a planning period or subperiod. Note $d_j^a = n_j d_j$ and $u_j^a = n_j u_j$.

While the above definitions let availability factors represent the expected availabilities, their definitions of the various super- and subscripted n's, and d's denote expected values of the indicated quantities. In a later section, when the availability factors represent probability densities for availability, these various d's, u's and n's will denote random variables of the described quantities.

$D_{x,max}, U_{x,max}$ = Maximum demand or usage units a single item can provide if run continuously during a planning period (x=p), planning subperiod (x=s), or demand period (x=d).

$D_{x,min}, U_{x,min}$ = Minimum demand or usage units an item must experience in a planning period (x=p), planning subperiod (x=s), or demand period (x=d) to justify its retention.

$C_j(\sum_d d_{jd})$ or $C_j(d_j)$ = Cost of operating n_j^u items during the planning period or subperiod.

This item could be expressed in terms of usage units. It includes operating costs, expected costs/penalties associated with field failures, and, if measured on a per unit use basis, capital depreciation costs. It does not include the penalty costs for unavailability. The arguments to C_j can be u's rather than d's.

$C_s(d,x)$ = The penalty when n units are required in a demand period and x are supplied. Typically, zero for d<x. This applies only for demand unit modeling, and never when using aggregate demands in a period or subperiod as an aggregate. For our example, the economic penalty on any day for having a shortage of x units is $0 for x equal zero or 1,

28

and $100x for x>1. Thus, rescheduling, although inconvenient, is organizationally possible for a shortage of 1. However, a shortage of 2 or more causes measurable economic penalties due to resulting delay of related tasks, or resulting idleness of other resources.

Demand Distribution Proportional to Availability

Demand distribution proportional to availability occurs via the equation:

$$d_{jd}^{a} = \frac{n_j\ AF_j(d)}{\sum_j (n_j\ AF_j(d))}\ d_{(J)}^{a}(d, n_j\ AF_j(d), j=1..J) \qquad \text{[Eq 9]}$$

For our example, since demand units equal the number of items, demand distribution becomes:

$$d_{jd}^{a} = n_{jd}^{u} = \frac{n_j\ AF_j(d)}{\sum_j (n_j\ AF_j(d))}\ \min[d, \sum_j (n_j\ AF_j(d))] \qquad \text{[Eq 10]}$$

The minimization in this equation is between the number required (d) and the expected number available.

In this example, using the fixed availability factors of 0.95, 0.90 and 0.80 yields demand and usage distribution shown in Table 5. Table 6 uses the demand dependent availability factors instead.

Table 5

Demand Distribution by User Preference Using Demand Densities, and Fixed Availability Factors for Expected Availabilities

No. Needed	P_d	Demand Units Assigned per Day to Each Usage Bracket Group						Total
		$AF_1(d)$	d_{1d}^{a}	$AF_2(d)$	d_{2d}^{a}	$AF_3(d)$	d_{3d}^{a}	
8	0.20	0.95	2.81	0.90	4.00	0.80	1.19	8.00
9	0.30	0.95	3.17	0.90	4.50	0.80	1.33	9.00
10	0.15	0.95	3.52	0.90	5.00	0.80	1.48	10.00
11	0.25	0.95	3.80	0.90	5.40	0.80	1.60	10.80
12	0.10	0.95	3.80	0.90	5.40	0.80	1.60	10.80
$d_{j.}^{a}$ = Expected Daily Item-Days for Group			3.37		4.79		1.42	9.58
d_{j}^{a} = Expected Annual Item-Days for Group			674.75		958.00		283.85	1916.00
d_j = Expected Annual Item-Days per Item			168.54		159.67		141.93	
u_j = Expected Annual Engine hrs per Item			842.69		798.33		709.63	

29

Table 6

**Demand Distribution Proportional Using Demand Densities,
and Demand Dependent Availability Factors for Expected Availabilities**

No. Needed	P_n	$AF_1(n)$	d^a_{1n}	$AF_2(n)$	d^a_{2n}	$AF_3(n)$	d^a_{3n}	Total
8	0.20	0.95	2.81	0.90	4.00	0.80	1.19	8.00
9	0.30	0.95	3.17	0.90	4.50	0.80	1.33	9.00
10	0.15	0.95	3.52	0.90	5.00	0.80	1.48	10.00
11	0.25	0.98	3.81	0.95	5.54	0.85	1.65	11.00
12	0.10	0.98	3.92	0.95	5.70	0.90	1.80	11.42
$d^a_{j.}$ = Expected Daily Item-Days for Group			3.39		4.85		1.45	9.69
d^a_j = Expected Annual Item-Days for Group			677.01		970.94		290.45	1938.40
d_j = Expected Annual Item-Days per Item			169.25		161.82		145.22	
u_j = Expected Annual Engine hrs per Item			846.26		809.12		726.12	

Demand Distribution in Order of User Preference

The general procedure for distributing demand to items in order of user preference is to calculate for $j=1..J$

$$n^u_{jd} = \begin{cases} 0, & \text{if } d^a_{()}(n_j / AF_{j'}(d), j' < j) > d \\ n_j AF_j(d), & \text{if } d > d^a_{()}(n_j / AF_{j'}(d), j' < j) \\ \min_{x = 0..n_j AF_j(d)} x \text{ s.t. } d > d^a_{()}(x, n_j / AF_{j'}(d), j' < j), & \text{otherwise} \end{cases} \qquad [\text{Eq 11}]$$

and then

$$d^a_{jd} = \frac{n^u_{jd}}{\sum_j n^u_{jd}} \; d^a_{()}(n^u_{jd}, j=1..J) \qquad [\text{Eq 12}]$$

This last equation could be revised to have d^a_{jd} depend upon different usage bracket group's relative productivity.

30

For our example, if the number of units are the demand units, the preceding two equations reduce to:

$$d_{jd}^a = \min\{\max[0, d - \sum_{j' < j} (n_{j'} \, AF_{j'}(d)], n_j \, AF_j(d)\}$$ [Eq 13]

Table 7 shows the resulting demand and usage distribution for the fixed availability factors of 0.95, 0.90 and 0.80. Table 8 uses the demand dependent availability factors instead.

Demand Distribution to Minimize Operating Costs

To distribute demand to items to minimize operating costs, and/or to more accurately model repeating cyclical demands, an optimization of the following form can be solved. Let:

D_l, D_u = The lower and upper bounds of the possible number of demand units required in a demand period

D_0 = D_l-1,D_1,...,D_{r-1},D_r=D_u divide the interval (D_l, D_u) into r user defined intervals. The subscript q shall range from 1 to r.

d_{jq} = The demand an individual item supplies during demand periods having demands between D_{q-1}+1 and D_q. The d_{jq} are the only decision variables in the formulation.

Table 7

Demand Distribution by User Preference Using Demand Densities, and Fixed Availability Factors for Expected Availabilities

No. Needed	P_d	AF$_1$(d)	d_{1d}^a	AF$_2$(d)	d_{2d}^a	AF$_3$(d)	d_{3d}^a	Total
8	0.20	0.95	3.80	0.90	4.20	0.80	0.00	8.00
9	0.30	0.95	3.80	0.90	5.20	0.80	0.00	9.00
10	0.15	0.95	3.80	0.90	5.40	0.80	0.80	10.00
11	0.25	0.95	3.80	0.90	5.40	0.80	1.60	10.80
12	0.10	0.95	3.80	0.90	5.40	0.80	1.60	10.80
d_j^a = Expected Daily Item-Days for Group			3.80		5.10		0.68	9.58
d_j^a = Expected Annual Item-Days for Group			760.00		1020.00		136.00	1916.00
d_j = Expected Annual Item-Days per Item			190.00		170.00		68.00	
u_j = Expected Annual Engine hrs per Item			950.00		850.00		340.00	

31

Table 8

Demand Distribution by User Preference Using Demand Densities,
and Demand Dependent Availability Factors for Expected Availability

No. Needed	P_d	Demand Units Assigned per Day to Each Usage Bracket Group						Total
		$AF_1(d)$	d^a_{1d}	$AF_2(d)$	d^a_{2d}	$AF_3(d)$	d^a_{3d}	
8	0.20	0.95	3.80	0.90	4.20	0.80	0.00	8.00
9	0.30	0.95	3.80	0.90	5.20	0.80	0.00	9.00
10	0.15	0.95	3.80	0.90	5.40	0.80	0.80	10.00
11	0.25	0.98	3.92	0.95	5.70	0.85	1.38	11.00
12	0.10	0.98	3.92	0.95	5.70	0.90	1.80	11.42
d^a_j = Expected Daily Item-Days for Group			3.84		5.21		0.65	9.69
d^a_j = Expected Annual Item-Days for Group			768.40		1041.00		129.00	1938.40
d_j = Expected Annual Item-Days per Item			192.10		173.50		64.50	
u_j = Expected Annual Engine hrs per Item			960.50		867.50		322.50	

N_d = The expected number of demand periods in a planning period or planning subperiod requiring d demand units.

m_q = Aggregate shortage of demand units during the planning period over all demand periods requiring between $D_{q-1}+1$ and D_q demand units.

M_q = Cost per demand unit shortage during a demand period requiring between $D_{q-1}+1$ and D_q demand units.

$AF_j(q)$ = An average availability for items of age use j during a demand period requiring between $D_{q-1}+1$ and D_q demand units.

Other symbols remain as previously defined.

The objective function is

$$\min \sum_j n_j \, C_j \left(\sum_q d_{jq} \right) + \sum_q M_q m_q \left(\sum_{d=D_{q-1}+1}^{D_q} dP_d \right) \qquad \text{[Eq 14]}$$

Note how M_q and m_q represent the shortage costs, which were defined earlier as $C_s(n,x)$.

The applicable constraints are:

1. Satisfaction of demand levels

$$\sum_j n_j \, d_{jq} + m_q \geq \sum_{d=D_{q-1}+1}^{D_q} d \, N_d \quad \text{for} \quad q=1,\ldots,r \qquad \text{[Eq 15]}$$

2. A maximum number of items that any usage bracket group can supply in a demand interval

$$n_j d_{jq} \leq AF_j(q) \, D_{d,max} \sum_{d=D_{q-1}+1}^{D_q} N_d \quad \text{for} \quad q=1,\ldots,r; \quad j=1,\ldots,J \qquad \text{[Eq 16]}$$

3. If desired, minimum usage in a demand period (or subperiod) to justify retention of items

$$D_{p,min} \leq \sum_{q=1}^{r} d_{jq} \quad \text{for} \quad j=1,\ldots,J \qquad \text{[Eq 17]}$$

One might use this model by aggregating the lower demand values (for which demand shortages, and therefore associated penalties, are not expected, i.e., $M_q=0$) and have separate demand constraints for the higher demand levels for which shortage penalties occur.

Observations

The next step is to assign usage based on user preference. In contrast to assignments proportional to availability, the aggregate availability is unchanged, but the mix of usage among the usage bracket groups favors the lower (assumed to be more desirable) usage bracket groups. In fact, in our example (but necessarily in general), the lowest usage bracket group operates at 100 percent of net availability before any usage is transferred to higher groups.

In this light, the principal of assignment by user preference is clear. The most desirable (often newest) items in each fleet receive the most usage, while older items remain idle until demand rises sufficiently to press them into service. Usage is not spread evenly among fleet items. Over a period of years, each time a new usage bracket group is added to a fleet (through purchase of new items), usage immediately shifts to it, at the expense of all older usage bracket groups. The following observations can be made:

1. For usage proportional to availability and according to user preference, the demand-dependent availability model allows adjustments for peak demands and hence comes closer to meeting the aggregate demand requirement, as summarized in Table 9.

2. Analogous to the aggregate demand model discussion, if usage assignment exceeds the maximum or does not meet the minimum requirements for an usage bracket group, the fleet mix can be considered infeasible, or the required minimum or maximum can be assigned to the deficient group, and reassignment

33

made to the other groups. For user preference demand distribution and demand dependent availability factors (Table 8), violations of the maximum usage for items in usage bracket groups 1 (768.4 vs. 720) and 2 (1041 vs. 1020) occur, which suggests the need for a small shift of usage from these groups to group 3, during nonpeak periods. Also, the low usage for group 3, might, de >ending on local policy, produce a larger shift.

3. If the demand for a demand period (e.g., a day) exceeds the number of items available, overtime work can be allowed. For example, Table 7 shows demand distribution based on user preference and fixed availability factors, and provides a maximum of 10.8 available items during a demand period even if more are required. Supposing that overtime is permitted up to a maximum of 20 percent of a day's work, the overtime allocations would work as follows:

> When the demand is for 11 items, the number of items provided from usage bracket group 1 would increase from 3.8 to 4.0, bringing the total number of items provided up from 10.8 to 11.

> When 12 items are needed, overtime work must provide the equivalent of 1.2 additional items. The number of items provided from usage bracket group 1 would increase from 3.8 to 4.56 (the maximum increase is 20 percent of 3.8, or 0.76). The remaining 0.44 items would be provided from usage bracket group 2, when the number of items provided would rise from 5.4 to 5.84.

Note that if repairs are required during regular work hours resulting from failures attributable to the additional work performed on overtime, the limit on the maximum demand that an item can provide decreases by an amount equal to $(1 - AF_j)$ times the number of overtime hours worked.

4. The methods discussed above estimate, for each demand level, the expected number of available items (which is constant for fixed availability factors), but say nothing about the distribution of shortfalls. The manager can compute the cost of fleet shortfalls assuming this includes the costs of rentals and/or overtime) as follows:

> a. For a shortage of one item in any demand period (say, one day), the economic penalty is $0.00.

> b. For a shortage of more than one item in any demand period, the economic penalty is $100 times the number of items short.

> c. Such a structure may come about when a manager feels that:

> (1) Rescheduling, although inconvenient, is organizationally possible for a shortage of one item.

Table 9

Comparison of Availability Models

Model	Usage
Aggregate demand requirement	1950 item-days
Fixed availability	1916 item-days
Demand-dependent availability	1938 item-days

(2) A shortage of two or more items causes measurable economic penalties resulting from a delay of related work tasks or idleness of other resources (equipment, crews, etc.).

There is a problem in representing the actual cost of shortfalls in this example. Based on Tables 5 through 8, for fixed availability factors, shortfalls occur when demand is for 12 items; and for demand-dependent availability factors, shortfalls never exceed 1 item. In practice, of course, shortfalls of two or more items can occur regardless of the number of items needed on a given day. Accordingly, the example underestimates penalty costs attributable to shortfalls. The next section addresses this problem.

Demand Densities With Availability Factors Representing Availability Probability Densities

Two random events associated with a demand period:

- The demand period's requirement (which may be cyclic rather than random)
- The usage bracket mix of items (i.e., demand units) available in that demand period to satisfy the requirement

and two policy factors:

- The organization's policies concerning short term renting and overtime
- Management's judgement concerning the operating environment

can be combined to calculate for the demand period:

1. The number of items (i.e., demand units) from each usage bracket group actually supplied or used (based on the method of assigning usage)

2. The economic penalty of any shortage

3. The cost of alleviating any shortage that occurs by either enlisting a selected number of items for overtime use, and/or short term renting.

This section describes how to make these calculations when using availability factors to represent the probability density function for availability, and how to choose an operating policy to minimize a single planning period or planning subperiod's expected costs of operations (including costs to alleviate shortages and penalty costs of shortages).

Additional Notation

These calculations require some additional notation. This section uses n_j^v and n_j^u, rather than n_{jd}^v and n_{jd}^u. Note that the value of n_j^v for any j is independent of the value for every other value of j. The previously defined symbols no longer denote expected values, but rather random variables for their respective quantities:

n_{jd}^v and n_{jd}^u = The number of items available and used from usage bracket group j in a demand period "requiring" d demand units

d_{jd}^a = The demand units collectively assigned to items in usage bracket j during a demand period "requiring" d demand units.

$P_{n_j^v}(x|d)$ = The probability that $n_j^v = x$. Assuming that the probability of any item's availability in a demand period is independent of the probability of any other item's availability, the availability of each individual item becomes a Bernoulli process with a probability of success equal to AF(d), and a probability of failure equal to 1-AF(d). Then the coefficient of the xth power of the z in the z-transform:

$$T_j(z,d) = [(1-AF_j(d)) + AF_j(d) \, z]^{n_j} \qquad \text{[Eq 18]}$$

equals $P_{n_j^v}(x|d)$. Note that this depends on d only if AF_j also depends on d. Drake discusses Bernoulli processes and z-transforms in more detail.

$n_{\{\cdot\}}^v$ = The number of items available from some subset of the J usage bracket groups. Subsets of interest for the purposes herein are: (1) the first j usage bracket groups, (2) the number available from such denoted $n_{\{j\}}^v$, and (3) all but the jth usage bracket group (The number available from such denote $n_{\{J-j\}}^v$). Whatever subset is considered, $n_{\{\cdot\}}^v$ equals the sum, over the usage bracket groups in the subset, of the independent n_j^v. Note that $n_{\{j\}}^v$ is independent of $n_{\{J-j\}}^v$.

$P_{n_{\{\cdot\}}^v}(x|d)$ = The probability that $n_{\{\cdot\}}^v = x$ items from some subset of the J usage bracket groups are available in a demand period requiring d demand units. Again the dependence on d exists only if AF_j also depends on d. This probability equals the coefficient of the xth power of z in:

$$T_{\{\cdot\}}(z,d) = \prod_{j \in \{\cdot\}} T_j(z,d) \qquad \text{[Eq 19]}$$

$n_j^u(n_j^v, n_{\{J-j\}}^v, d)$ = The number of items of usage bracket group j used (as opposed to available) during a demand period. The arguments will generally be dropped.

Computing Items Used From and Demand Assigned to Each Usage Bracket Group

The calculation of $n_j^u(n_j^v, n_{\{J-j\}}^v, d)$ and $d_{jd}^a(n_j^v, n_{\{J-j\}}^v, d)$ depends on the method to distribute demand among items:

1. For demand distribution proportional to availability, calculate:

$$y = \left\{ \begin{array}{l} n_{\{J\}}^a, \quad \text{if } d_{\{\}}^a(n_{\{J\}}^a) < d \\ \min_{x = 0..n_{\{J\}}^a} x \ \ \text{s.t. } d_{\{\}}(x) > d, \quad \text{otherwise} \end{array} \right\} \qquad \text{[Eq 20]}$$

$$z = \min(d, d_{\{\}}(n_{\{J\}}^v))$$

and then

$$n_j^u = \frac{n_j^v}{n_{(J)}^v}\, y = \frac{n_j^v}{n_j^v + n_{(J-j)}^v}\, y$$

$$d_{jd}^a = \frac{n_j^v}{n_{(J)}^v}\, z = \frac{n_j^v}{n_j^v + n_{(J-j)}^v}\, z \qquad \text{[Eq 21]}$$

2. For demand distribution based on user preference, calculate successively for $j=1..J$ by:

$$n_j^u = \begin{cases} 0, & \text{if } d_{\{\}}^a(n_{(j-1)}^v) > d \\[4pt] n_j^v, & \text{if } d > d_{\{\}}^a(n_{(j)}^v) \\[4pt] \min_{x=0..n_j^v} x \text{ s.t. } d_{(\ )}^a(x+n_{(j-1)}^v) > d, & \text{otherwise} \end{cases} \qquad \text{[Eq 22]}$$

and then

$$d_{jd}^a = \frac{n_j^u}{\sum_j n_j^u}\, d_{\{\}}^a\!\left(\sum_j n_j^u\right) \qquad \text{[Eq 23]}$$

(These last two equations can be adjusted to reflect varying productivity among units of different usage bracket groups.)

3. For demand distribution to minimize costs, order the usage bracket groups so increasing cost per demand unit supplied implies lesser desirability, and then proceed as for user preference.

The ability to calculate a demand period's n_j^u's and d_{jd}^a's from the realization of d and n_j^v's allows an evaluation of expected values of various functions of the n_j^u's and d_{jd}^a's over a demand period and hence, our ultimate interest, over a planning period. First, the expected values of the n_j^u's must be evaluated to ensure that any policy of minimum usage of items is satisfied. More importantly, the expected cost of the fleet operations can be calculated. Of course, operating costs for the planning period are of primary interest. When available items cannot satisfy a demand period's requirements, the demand period's operating costs might include penalty costs, overtime costs, and short term rental costs. In fact, given a demand requirement and availability mix for a demand period, overtime and short term rental decisions could be determined from a calculation to minimize the resulting operating costs for the demand period, subject to management set constraints.

Thus, if $f(d,n_j^v,j=1..J)$ represents some function of interest for a demand period, remembering that n_j^v, and $n_{(j-1)}^v$ are assumed independent, the expected value of f for a demand period equals:

$$E(n_j^u) = \sum_n \sum_y \sum_x f(x,y,n)\, P_{n_j^a}(x|n)\, P_{n_{(.)}^a}(y|n)\, P_n \qquad \text{[Eq 24]}$$

The {.} may depend on the method of assigning usage: J-j if assigning usage equally or proportional to availability, and j-1 if assigning usage by user preference. The evaluation of the expected values might proceed as follows:

For j=1...J

    For n=1...n_{max}

      For $n^v_{\{.\}} = 0... \sum_{j' \neq j} n_j$

        Calculate $P_{n^v_{\{.\}}}$

        For $n^v_j = 0...n_j$

          Calculate $P_{n^v_j}$

          $E(f) += f(d,n^v_{\{.\}}) \, P_n \, P_{n^v_{\{.\}}} \, P_{n^v_j}$

Techniques to accelerate the computations include:

1. Elimination of the loop over n, if appropriate

2. Using a binomial or Poisson approximations to the sum of Bernoulli variables, when calculating the:

$$P_{n^v_{\{.\}}} \quad \text{and} \quad P_{n^v_j}$$

for sufficiently large fleet sizes.[4] Based on the examples in these references and some additional numerical experiments, 20 items are adequate when dealing with availability factors greater than 0.80.

3. Skipping values that have sufficiently low probabilities.

If the computations still take too long, simulation of values of n and of n^v_j can approximate E(f). These suggestions become more important when calculating expected operating costs.

Modeling Overtime and Short Term Renting

The following four parameters might describe an overtime policy:

1. Item shortage in a demand period to invoke overtime

2. Minimum number of overtime item periods (can be fractional) per demand period an item can operate

3. Maximum number of overtime item periods per demand period an item can operate

4. Maximum number of items that can operate overtime.

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[4] Alvin W. Drake, *Fundamentals of Applied Probability* (McGraw-Hill,1967); Norman I. Johnson and Samuel Kotz, *Discrete Distributions* (John Wiley and Sons, 1969); Norman I. Johnson, and Samuel Kotz, *Continuous Distributions – 2* (John Wiley and Sons, 1970); Jagdish K. Patel and Campbell B. Read, *Handbook of the Normal Distribution* (Marcel Dekker, Inc., 1982).

All of these parameters can depend on the number of items needed and the number available in the demand period. Define:

$o_p(n,n^a_{\{j\}}), p = 1,...,4$ = The four parameters just mentioned. (This term can be abbreviated to O_p.)

no = The number of items operated overtime as a result of shortages during a demand period. Then, if $n-n^a_{\{j\}} \geq o_1$, feasible values of n^o are those that satisfy $o_3 \geq o_1/n^o \geq o_2$ and $n^o \leq o_4$.

n^o_j = The number of usage bracket j items invoked for overtime.

Given a value of n^o:

1. When distributing demand to items according to user preference, then

$$\text{If } n^a_j > 0 \text{ and } n_{\{j-1\}} < n, \ n^o_j = \max(0, n^o - \sum_{j' < j} n^o_{j'}, 0) \qquad [\text{Eq 25}]$$

2. Also, once for some j, $n^o_j = 0$, then $n^o_{j'} = 0$ for all $j' > j$.

3. When distributing demand equally among items or proportional to availability, each available item is used for overtime with probability of $1/n^a_{\{J\}}$.

This scenario can include short term rentals as an additional usage bracket group. Given a realization of d and the n^v_j's for a demand period, computations would select the number of short term rentals that result in the lowest overall operating costs (including penalty and overtime costs). Each possible number of short term rental items would require a computation of the corresponding overtime policy producing minimum resulting operating costs. The previously described expected value computation would use the calculated operating cost.

4 THE MANAGER'S INTERACTION WITH THE MODEL

Introduction

The equipment manager will have two general levels of interaction with the model. At the first level, the model requires a definition of fleets and usage bracket groups and the operating parameters and rules to apply to each. This activity creates the setup information, and will be supported through interactive data entry guided by an expert system shell or other intelligent program. At the second level, the manager will execute the model and review and refine its results and recommendations. Conceptual work on this second level of study is beyond the scope of this report.

Any tool that undertakes complex data analysis requires a user to define the background and parameters for operation; this model is no exception. Setup for the various fleets an organization owns, each rendering different services to equipment users, will be a time-consuming process. However, how useful the model will be is largely determined by the setup process. This chapter presents much of the material in Chapter 2, and also suggests the order (and sometimes the dialogue) that the expert system might use to prompt the manager for a description of the environment.

This chapter also expands some concepts from Chapter 2 to allow assignment of items in a fleet to one or more "services" in increments designated "item-units" (item-miles, item-hours, item-shifts, item-days, item-weeks, etc.), and to measure the required demand for each "service" in terms of demand units. As a result, any service that is assigned an item uses the demand units required in an appropriate period (planning period, planning subperiod, or demand period) and the mix of items available (possibly from multiple fleets) to perform the service, to calculate the number of item-units to use from each available item, and the number of demand units satisfied by each item-unit. Specifying item-units for a fleet identical to the demand units for a service would result in a scenario of a single fleet providing a single service as described in Chapter 2.

General Information

The model requires general information for its operation, including:

1. The length of a planning period

2. The number of planning periods over which the model should execute (identical for all fleets)

3. Annual constraints on capital expenditures for each planning period (the maximum amount that can be expended for equipment acquisitions)

4. Annual constraints on operating expenditures for each planning period (the maximum amount that can be expended on equipment operations, including direct and indirect operating, maintenance and repair costs)

5. Factors to convert cost information from previous periods into constant dollars

6. Discount factors to convert expected costs from future periods into constant dollars and net present value.

Defining and Describing Equipment Fleets

Many EMMSs support tracking of the diversity of equipment an organization might own through some classification scheme that allocates individual items into "classes," "types," or "fleets." For the model, each item must be assigned to a fleet. This classification operates independently of other existing classifications in an EMMS, so that allocation of items for the model will not depend on other, unrelated allocation considerations.

The description of each fleet consists of:

1. The usage bracket groups to which fleet items are assigned (Long term lease items are considered as a special usage bracket group.)

2. Usage units for the fleet

3. The measure, in item-units, used to assign items to services, e.g., item-miles, item-hours, item-days, item-weeks, and/or item-months (Typically, only one such measure applies to a fleet. However, if multiple measures are specified, only one applies to a service/subperiod combination. Also, conversion factors must be given, to allow all item-units used to be expressed in common terms.)

4. Operator costs per item-unit (regular and overtime)

5. Default ages or usages after which items are not procured or are disposed

6. Charge for a field failure (which may be a function of the mix of services to which items are assigned)

7. Minimum/maximum number and/or item-equivalents in possession (leased and owned) for every planning period (If fleet provides only one service, then this corresponds to constraints on minimum and maximum numbers on hand. Note that either a minimum usage requirement or a maximum number of items that can be owned [or owned plus leased] will prevent the model from selecting solutions with more than some [arbitrary] number of older items, even if such solutions are better than others. Both types of constraints are generally unecessary.)

8. Minimum number of item-units and/or usage units to retain an item

9. Method used to assign item-units to the items available from the various usage bracket groups by (1) equal distribution, (2) proportional to availabilty, (3) user preference (i.e., most desirable available), or (4) cost minimization.

The description of each usage bracket group within the fleet consists of:

1. The default number of usage units per time unit served (which may be recalculated by a service that items satisfy)

2. One of the following: either a. or b.; or a. in combination with either b. or c., where:
   a. Average availability
   b. Availability as a function of item-units needed in a "unit" of the item-"units"
   c. Probability densities for usage between failures and time to remedy failures

3. Probability density of or expected operating costs per unit usage

41

4. Number of field failures per usage unit or item-unit assigned

5. Fixed cost of ownership/item, no matter how little used (including insurance, registration fees, minimum maintenance, fixed portions of lease costs, and costs of exercised options to purchase)

6. Usage bracket group-specific constraints on purchasing or disposing of an item (e.g., leased items are initiated in the first year of lease and are not disposed of until last year of lease).

A facility will be included that will enable the manager to easily copy the setup of one fleet for use by another fleet.

In establishing the demand and operating constraints for a fleet, the manager must be careful to avoid setting parameters that inadvertently force the model into an inappropriate solution. For example, many fleet operations currently apply a rule stating a maximum age, beyond which an equipment item must be disposed of. Proper application of the model makes such a rule superfluous, and there should be a good external reason for continuing to apply it.

Establishing Demand Patterns and Operating Constraints for a Service

The expert system can use a dialogue to guide a manager to describe a service that one or more fleets might satisfy. The system may pose this series of questions and prompts (in a logical order) and suggest possible answers with corresponding impacts:

1. "Provide a name for the service. (Name can exactly correspond to a fleet, if that fleet satisfies this service without assistance from any other fleet.)"

2. "In what units is demand measured?"

3. "Do the amount of required demand and any patterns (seasonal or cyclical) in the demand generally repeat from planning period to planning period?"

Often demand patterns are similar from planning period to planning period, either because demand is constant or because it follows a seasonal pattern. If demand patterns vary substantially from planning period to planning period, they will have to be described individually for each planning period. Unless some definite change is anticipated (e.g., expansion of a municipal bus system, or a high number of new construction projects in coming years), managers can and should assume an unvarying pattern in planning periods.

4. "Does the demand pattern vary significantly on a seasonal basis?"

If the demand pattern will vary, the manager is guided through the division of each planning period into the required number of planning subperiods.

The following questions apply to every planning subperiod within a period. (A planning period not divided into subperiods consists of a single subperiod.) The term subperiod will therefore be used.

5. "Which of the following best describes the nature of the demand requirements during the subperiod?"

a. "When items are not available to satisfy peak demand requests, such requests are deferred or rescheduled as needed without economic or organizational penalties"

This implies use of an aggregate demand for the subperiod.

b. "Is there a regular occurring cyclic demand that must be met?"

c. "Is there a randomly occurring demand for which the probability of the demand units 'required' in some time period (i.e., demand period) can be estimated?"

If either b. or c. is chosen, the model guides the manager in specifying a demand period, the number of demand periods in the planning subperiod, and a probability density for demand units "required" in a demand period.

6. "Is there an established minimum number of demand units that available items must absolutely be prepared to satisfy during the planning subperiod"?

7. "Can items be rented for the single planning subperiod?"

If items can be rented, then cost functions must be provided for operating, maintenance, and repair costs for renting items to help satisfy the demand.

The next several questions elicit information concerning the fleets that can satisfy the specified service and define characteristics and parameters that depend upon the combination of fleet and service. Note that some constraint values can be calculated from user-supplied values.

8. "Identify the fleets that satisfy this service. (A single fleet often solely satisfies a service.)"

9. "For each chosen fleet, what is the default number of demand units satisfied per item-unit utilized? "

This number may differ among usage bracket groups. These values and the availability factors can set the maximum demand that can be supplied by items in the planning period of subperiod.

10. "Define the interfleet preferences for distributing the demand."

11. "(If demands are expressed as a probability density over demand periods) Describe the algorithms to calculate the number of demand units that a given mix of available items can supply, given the 'required' demand units."

The simplest, and often best approach, is to specify default demand units per item-unit for each item. This approach is automatically implemented if the item-units and service units are identical.

12. "Define the procedures for choosing penalty costs, overtime assignments and short term rental if the available items can not satisfy the 'required' demand in a planning subperiod or demand period as appropriate."

Chapter 3 presented simple cases of this. Further research is needed to develop more general procedures.

5   MULTI-PERIOD OPTIMIZATION MODEL FOR A SINGLE FLEET

The concepts from previous sections can help define a model to determine what configuration changes should be made in a single fleet over a multiperiod horizon. The first section of this chapter presents an intuitive explanation of the model's operation, and then describes the complete mathematical formulation. The remaining sections describe the mathematics in detail. The optimization problem is expressed as a dynamic programming problem, and as the complete formulation demonstrates, establishing the required notation is the greatest challenge. These models expand on those presented in Simms, et al.

An Intuitive Explanation

The solution procedure starts with the last planning period in the last period in the horizon ($i=1$, denotes this last period), and works backwards toward the initial planning period ($i=I$). The solution for any planning period i uses the solution for the following planning period, the $i+1$st. Thus, solution for the $i-1$st is computed prior to that for the ith planning period. For each planning period, the algorithm solves an outer- an inner-stage problem.

The outer-stage problem for planning period i is defined for each feasible bracket usage group mix of items to be operated (via ownership or previously started long-term lease) at the start of period i, but before making any decisions concerning acquisitions, retirements and initiation of multiplanning period leases made at the start of or during period i. The outer stage problem is described as:

$$\pi_i = \min (\mu_i + \tau_i + \pi_{i-1}) \qquad \text{[Eq 26]}$$

where: π_i = For some fleet configuration at the start of period i, given the prior decisions concerning acquisition, retirement, and initiation of multi-period leases, the minimum present value of operations from the start of period i to the end of the horizon

μ_i = The cost (or revenue) of any purchase and retirement decisions at the start (or during) period i

τ_i = The minimum cost (or calculated cost, depending on the method used to distribute demand) of operations during period i once the purchase and retirement decisions at the start of or during the period have been made. This is the inner stage minimization, which is performed over any allowable decisions concerning demand distribution or short term rentals within the planning period i.

Equation 24 is minimized over all possible fleet configurations feasible at the start of period i before acquisition. Solving the outer stage problem for any fleet configuration requires solving the inner stage problem for all possible fleet combinations resulting from any purchase and retirement decisions at the start of (or during) period i (with a resulting cost or revenue of μ_i).The outer stage problem is subject to one or more of the following constraints:

1. A minimum and maximum value on the number of items needed in a usage bracket group in a planning period.

2. A maximum value for the amount of capital that can be expended on acquisitions during a planning period (across all usage bracket groups). Revenues from retirement liquidations are assumed to re-enter the capital equipment fund.

44

3. A combined maximum value for capital and operating expenses in a planning period (across all usage bracket groups). For this case, choosing acquisitions and retirements dictates the amount left for operating expenses, and this amount becomes the limit on operations during the planning period.

4. Minimum age or accumulated usage before which an item is not a candidate for retirement.

5. Maximum age or accumulated usage of items to be acquired.

6. Maximum age or accumulated usage after which an item must be retired.

Each inner stage minimization is subject to one or more of the following constraints:

1. Minimum or maximum number of items needed (i.e., owned, leased and rented), or of items owned, or of items leased in any planning period, and/or planning subperiods, if used

2. A minimum value for the demand supplied during the planning period or individual planning subperiods

3. A maximum value for the total demand supplied by each item in a usage bracket group during a planning period, planning subperiod, or demand periods; this value can reflect availability factors if AFs are used

4. Restrictions on leasing for entire planning subperiods, if imposed by the manager

5. A requirement that a leased item remain in the fleet for the entire lease term

6. A budget for the planning period's maintenance, operating, and repair (but not downtime) costs. If this constraint cannot be satisfied, the fleet configuration for which the inner stage problem is currently being solved may not be feasible.

The inner stage problem can be simplified computationally by:

1. Prohibiting leasing for planning periods or planning subperiods

2. Eliminating the concept of planning subperiods

3. For demand distribution methods other than the cost-optimizing method, calculating usage values directly from the usage distribution method rather than by optimization

4. If demand patterns do not change from planning period to planning period, and operating budget constraints remain constant, and all costs change the same percentage each planning period, solving the inner stage problem only once for all planning periods in each usage bracket group mix.

Notation

The model deals with usage bracket groups, usage units, and time periods as follows:

- Planning period i refers to the period that is i planning periods prior to the end of the planning horizon; period $i = 1$ is the last planning period of the planning horizon.

45

- _ _ pment usage is measured in usage or demand units u, which may be miles, engine hours, or some combination of these or other measures; each fleet has a definition of usage units.

- Usage bracket group j refers to the usage bracket group assigned an equipment item for a planning period.

Using this terminology, an equipment item belonging to usage bracket group j_i at the start of planning period i and experiencing u usage units during period i can be uniquely assigned to some usage bracket group j_{i-1} at the start of planning period i-1

Multiple year leases are considered as another form of ownership. Hence items in the second year of a 5-year lease constitute a unique usage bracket group. Additional constraints and decisions, not explicitly stated in the formulation must allow leased items to either be disposed of, or to experience a capital charge (i.e., the option to purchase) at the end of the lease. Management may choose to apply all or part of nonvariable (i.e., not usage dependent) costs of long-term leases against capital budget constraints. The expert system portion of the model must allow users to address these possibilities. Also, a value must be placed on multiyear leases that do not expire at the end of the planning horizon.

Variables Related to Planning Periods or Planning Subperiods

Generally, lower case letters represent state, decision, and random variables, while upper case letters represent limits and constraints. U's and u's denote both demand units and usage units, with the context (i.e., the information the user provides to the model) revealing the variable's intended use. The following terms are used in the following analysis:

i(k) = The kth planning subperiod of the ith planning period. In many cases, a planning period will not be divided into planning subperiods; i.e., the length of the sole planning subperiod equals the length of the planning period. All planning periods are the same length. The length of the kth planning subperiod does not change from planning period to planning period.

K = The number of planning subperiods in each planning period

I = The number of planning periods (i.e., the planning horizon)

B_i = Maximum capital expenditure or combined capital and operating expenditure in planning period i

O_i = Operating expenditure in planning period i

U_{ik} = The number of usage units or demand units required in planning period i and planning subperiod i(k). When a planning period is not divided into planning subperiods, k = 1 and the k subscript is omitted.

$N_{i,min}, N_{i,max}, N_{ik,min}, N_{ik,max}$ = The minimum and maximum number of items or item equivalents required in planning period i and planning subperiod i(k). The minimums reflect management policy to always provide some minimum level of service.

$U_{pj,min}, U_{pj,max}, U_{kj,min}, U_{kj,max}$ = The minimum and maximum number of usage units or demand units that an item of usage bracket group j can experience in a planning period and in the kth planning subperiod of a planning period. The minimums reflect a management policy to enforce minimum use for retention of items in the fleet, and will often not vary with j. $U_{kj,min}$ is

46

generally not used. The maximums reflect physical limits of item use in the planning period or planning subperiod, and are dependent on j when using availability factors.

$U_{kjd,max}$ = The maximum use an item of usage bracket group j can experience in a demand period during the kth subperiod of a planning period.

The nature of the fleet being modeled dictates the use of the constraints suggested by the preceding definitions of various U's and N's.

Variables Related to the Number of Items in a Usage Bracket Group

The following notation presents the major variables relating to the composition of a usage bracket group that the optimization model uses to solve the problem:

n_{ij} = The number of items at the start of planning period i (i.e., before any acquisition or retirement actions for that planning period) that belong to usage bracket group j. The n_{ij}'s, for j > 0, are state variables, not decision variables. j = 0 is reserved for new acquisitions.

u_{ij}, u_{ij}^a = The number of usage units or demand units that individual items or all (superscript a) items of usage bracket group j experience in planning period i. Of course, $u_{ij}^a = n_j u_{ij}$. Depending upon the method of distributing usage among items, the u_{ij}'s and u_{ij}^a's are either (1) decision variables, or (2) calculated from the distribution of items among usage bracket groups.

u_{ikj}, u_{ikj}^a = The number of usage units or demand units that items of usage bracket j experience in the kth planning subperiod of planning period i. These become necessary only if planning period i contains planning subperiods. Of course:

$$u_{ij} = \sum_k u_{ikj} \quad \text{and} \quad u_{ij}^a = \sum_k u_{ikj}^a .$$

These also are either decision variables or calculated values.

e_{ij} = The number of units retired at the start of planning period i from usage bracket group j

p_{ij} = The number of items acquired at the start of planning period i and added to usage bracket group j. The p_{ij}'s are decision variables. Note that p_{i0} represents items purchased new.

$a_{i-1}(n_{ij}, p_{ij}, e_{ij}, u_{ij})$ = The function that determines the number of items in usage bracket group j in planning period i-1, given the number of items and individual usage units or demand units of usage bracket group j during planning period i.

Note that items from two different usage bracket groups j_1 and j_2 at planning period i can join the same usage bracket group in period i-1. For such usage bracket groups only, items might be both acquired in and retired from the same usage bracket group (p_{ij}'s and e_{ij}'s might both be nonzero).

This function could change with i-1 to reflect warranty variations, but the difficulty in predicting such variations is beyond the scope of the present calculations.

l^s_{ikj}, u^s_{ikj} = The number of items leased, only for subperiod k in planning period i, and the number of demand units each satisfies. The j subscript differentiates among leased items of the various usage bracket groups. In practice, this distinction is often ignored. (The subscript j is dropped.) The superscript s denotes "within a subperiod."

L^s_{jkjd}, u^s_{jkjd} = The number of items leased for individual demand periods during subperiod k of planning period i. As defined here, an item leased for two demand periods counts as two "items leased." Assume that each item-lease provides u^s_{jkjd} demand units. The j subscript differentiates among leased items of the various usage bracket groups. In practice, this distinction is often ignored. (The subscript j is dropped.) The superscript s denotes "within a subperiod."

Variables Related to Costs

The following notation deals with the expected costs to be incurred under each possible solution considered by the model.

L_{ij} = The fixed cost of owning an item in usage bracket group j for planning period i.

$F^u_{ij}(x \mid .)$, $f^u_{ij}(x \mid .)$ = The probability distribution and density functions of MORD costs per unit of usage (hence the superscript u) in planning period i for an item of usage bracket group j. That is, $F^u_{ij}(x \mid .)$ equals the probability that the MORD cost per unit of usage for an item of usage bracket group j is less than or equal to x. As indicated by the period, these can be conditioned on n_{ij} and e_{ij}.

$F_{ij}(x \mid u_{ij},.)$, $f_{ij}(x \mid u_{ij},.)$ = The probability distribution and density functions of total MORD costs for a single item of usage bracket group j in planning period i, given planned usage of u_{ij} during the planning period. As indicated by the period, these can be conditioned upon n_{ij} and e_{ij}. If historical records allow determination of $F^u_{ij}(x \mid .)$ or $f^u_{ij}(x \mid .)$, then $F_j(x \mid u_{ij},.) = F^u_{ij}(x/u_{ij} \mid .)$ and $f_j(x \mid u_{ij},.) = (1/u_{ij})f^u_{ij}(x/u_{ij} \mid .)$.

$m_{ij}(n_{ij},p_{ij},e_{ij},,u_{ij})$ = The expectation of total MORD costs for a single item of usage bracket group j that is not retired at the start of planning period i. Note the decision or calculated variables that this can depend on.

$m^a_{ij}(n_{ij},p_{ij},e_{ij},u_{ij})$ = The expectation for the total MORD costs for all items of usage bracket group j that are not retired at the start of planning period i.

$C_{ik}(w_{ij},u_{ikj},l^s_{jkj},u^s_{jkj},l^s_{jkjd},u^s_{jkjd})$ = All costs of operations in the kth subperiod of period i, except those included in $m^a_j(n_{ij},e_{ij},p_{ij},u_{ij})$.

In the above definitions of variables related to costs, adding an additional superscript r changes the reference to include only maintenance, operating, and repair costs, but not downtime charges (MOR costs rather than MORD costs).

The following notation deals with capital costs:

A_{ij} = The acquisition cost of an item for usage bracket group j at the time during planning period i when acquisitions occur—typically the start of the period.

S_{ij} = The salvage/resale cost of an item in usage bracket j at the time during planning period i when salvage/resale occurs—typically the start of the period. This cost can be expressed as a function of the repairs the item needs, although we will not do so.

R_{ij} = The present value of the total capital deterioration and MOR (not MORD) costs of an item in usage bracket group j for the rest of its economic life.

In many, perhaps even most, applications, the various densities, expected operating costs of owned items, lease costs, and acquisition and salvage costs may be: (1) independent of i (i.e., constant for all planning periods), except for inflationary adjustments, or (2) independent of k (if all planning subperiods are the same length) or if planning subperiods are not used.

Components of the Objective Function

α = One period discount rate for funds, $0 < \alpha \le 1.0$

π_i or $\pi_i(.)$ = The minimum expected cost from the start of period i through the end of the planning horizon, given one or more of n_{ij}'s, p_{ij}'s, e_{ij}'s, u_{ij}'s and u_{ghi}'s.

τ_i or $\tau_i(.)$ = The minimum expected noncapital costs for a single period i given one or more of p_{ij}'s, e_{ij}'s, u_{ij}'s and u_{ghi}'s. The minimization occurs over variables within a single period i. If required for clarity, the given values appear within the parentheses.

Outer Stage Problem Formulation

The solution procedure starts with planning period i = 1 (the last planning period in the planning horizon), and works backwards towards the current planning period. The solution for each planning period i uses that of planning period i-1. Outer and inner stage problems must be solved for each planning period.

For each feasible combination of items owned (n_{ij}'s) at the start of a planning period (before acquisition and retirement decisions, including those of multiyear leased items), the outer stage problem finds the acquisition decisions (p_{ij}'s) and retirement decisions (e_{ij}'s) that minimize discounted cost from the current planning period (i) to the last planning period (i = 1):

$$\pi_i(n_{ij}) =$$
$$Min_{p_{ij}, e_{ij}} \left\{ \sum_j (p_{ij}A_j - e_{ij}S_j) + \sum_j L_{ij}(n_{ij} + p_{ij} - e_{ij}) + \right.$$
$$\left. \tau_i(n_{ij}, p_{ij}, e_{ij}) + \alpha\, \pi_{i-1}(n_{i-1,j}) \right\} \qquad \text{[Eq 26]}$$

where

$\tau_i(n_{ij}, p_{ij}, e_{ij})$ = The minimum operating costs possible in planning period i, the inner stage problem. For any planning period, this inner stage problem is solved for all feasible combinations of the parameters shown before solving the outer problem. Approaches to solving this inner problem, and special circumstances that can simplify it, are discussed below.

$n_{i-1,j}$ = The results of the function $a(n_{ij} + p_{ij} - e_{ij}, u_{ij})$ to determine the number of items in a usage bracket group during a planning period.

49

is subject to one or more of the following constraints, as desired, to limit the selection of acquisitions (p_{ij}'s) and retirements (e_{ij}'s) for each combination of items owned (n_{ij}'s):

1. A constraint on the number of items needed in a planning period:

$$N_{i,min} \leq \sum_j n_{ij} - \sum_j e_{ij} + \sum_j p_{ij} \leq N_{i,max}$$ [Eq 27]

2. Constraints on expenditures. The first of these [Eq 29] constrains only capital expenditures. The second [Eq 30] represents a combined constraint on capital and operating expenses. In the second case, values chosen for acquisitions (p_{ij}'s) and retirements (e_{ij}'s) dictate the amount remaining for operating expenses (O_i). This remaining amount becomes a limit on operations in the planning period, and is used in equation [Eq 37]. As written, [Eq 31] and [Eq 27], [Eq 32] make revenues from equipment liquidations available for that planning period's capital fund. They could also be written to allow excess from one ﾑﾑﾑﾑning period to be for the next planning period.

$$\sum_j p_{ij}A_j - \sum_j (n_{ij}-e_{ij})S_j \leq B_i$$ [Eq 28]

or

$$\sum_j p_{ij}A_j - \sum_j (n_{ij} - e_{ij})S_j + \sum_j m_j^{ar}(n_{ij},p_{ij},e_{ij},u_{ij}) \leq B_i$$ [Eq 29]

3. A constraint limiting items retired to those owned:

$$e_{ij} \leq n_{ij}$$ [Eq 30]

4. Constraints on the minimum usage bracket group from which items can be retired, the maximum usage bracket group for which items can be acquired, and the maximum usage bracket group that can exist in a fleet (a group that must be wholly retired). The last of these probably should not be used very often:

$$\begin{aligned} e_{ij} &= 0 \quad \text{for } j \leq j_{(min\_usage\_bracket\_to\_retire)} \\ p_{ij} &= 0 \quad \text{for } j \geq j_{(max\_usage\_bracket\_to\_acquire)} \\ e_{ij} &= n_{ij} \quad \text{for } j \geq j_{(usage\_bracket\_by\_which\_must\_retire)} \end{aligned}$$ [Eq 31]

Inner Stage Problem Formulation

The inner stage problem, which minimizes the operating costs within a planning period, takes the following form in its most general case:

$$\tau_i(n_{ij},p_{ij},e_{ij}) = \min_{u_{ij}=\sum u_{ijk}}\left\{\sum_j m_j^a(n_{ij},p_{ij},e_{ij},\sum_k u_{ijk}) + \sum_k C_{ik}(w_{ij},u_{ijk},l_{ijk}^s,u_{ijk}^s,l_{ijkd}^s,u_{ijkd}^s)\right\}$$ [Eq 32]

Chapters 3-4 discuss the considerations in solving this problem, and the next paragraphs summarize some of the conclusions from these earlier chapters.

Any of the following conditions simplify this general problem:

- Prohibiting leasing for planning periods or planning subperiods eliminates appropriate terms and variables.

- Not dividing planning periods into planning subperiods eliminates all variables and summations over k, and allows use of u_{ij}'s rather than their subperiod-by-subperiod components.

- If only the number of items owned after the period's decisions to acquire and sell/salvage, influence MORD costs, then $m_j^a(n_{ij},p_{ij},e_{ij},u_{ij})$ can be replaced by $m_j^a(w_{ij},u_{ij})$, where $w_{ij} = n_{ij} + p_{ij} - e_{ij}$. This implies that $\tau_i(n_{ij},p_{ij},e_{ij})$ simplifies to $\tau_i(w_{ij})$, thus decreasing the number of parameter combinations for which $\tau_i(.)$ must be solved.

- If demand patterns do not change from period to period, and the operating budget constraints remain constant, and all costs change by the same percentage annually, then the inner stage problems need only be solved once for all planning periods.

Each inner stage minimization (Eq 34) is subject to one or more of the following constraints, as desired or required:

1. Constraint on the number of items needed in a planning subperiod. Note that the first four terms are constant for the planning period, and can be moved to the right-hand side of the inequality. For planning periods not divided into subperiods, this constraint becomes unnecessary. If the usage bracket of leased items is not considered, the second summation reduces to the single term L_{ik}^s.

$$N_{ik,max} \geq \sum_j (n_{ij} - e_{ij} + p_{ij}) + \sum_j l_{ikj}^s \geq N_{ik,min} \qquad \text{[Eq 33]}$$

2. A constraint to achieve minimum usage requirements in a planning subperiod. For planning periods not divided into subperiods, $K = 1$ and the third summation are omitted. If the usage bracket group of leased items is not considered, the summations and the superscript k are dropped from the second and third terms between the $\geq$'s. This equation, as written, also assumes that an item leased for a demand period supplies exactly u_{ijkd}^s demand units per demand period. For some of Chapter 3's demand distribution techniques, this constraint will be implicitly handled.

$$\sum_j u_{ikj}(n_{ij}-e_{ij}+p_{ij}) + \sum_j l_{ikj}^s u_{ikj}^s + \sum_j l_{ikjd}^s u_{ikjd}^s \geq U_{ik} \qquad \text{[Eq 34]}$$

3. A constraint on the minimum usage of items during a planning period or planning subperiod. The first applies when planning periods are divided into planning subperiods, and the second applies when there are no planning subperiods. This constraint could be expanded to specify minimum usage for leased items:

$$\sum_k u_{ikj} \geq U_{pj,min} \quad \text{or} \quad u_{ij} \geq U_{pj,min} \qquad \text{[Eq 35]}$$

4. A constraint on the maximum usage of items during a planning period or planning subperiod. The first applies when planning periods are divided into planning subperiods, and the second applies when there are no planning subperiods. These can reflect availability factors if they are used:

$$u_{ij} \leq U_{pj,max} \quad \text{or} \quad u_{ikj} \leq U_{kj,max} \qquad [Eq\ 36]$$

5. A constraint on the expected value of maintenance, operating, repair (but not downtime) costs (MOR, not MORD). If this constraint cannot be satisfied, the acquisition and retirement decisions for which the inner stage problem is currently being solved may not be feasible. The O_i can be what is left over from a combined capital and operating budget, as indicated in the discussion introducing equation [Eq 29]. Lease costs are not shown in either this or the capital budget; local policy would determine whether and how to distribute lease costs between these two constraints. Note that charging the costs of short-term leases that begin and end within a planning period to the capital budget ([Eq 29]) would be difficult in this formulation:

$$\sum_j m_j^{ar}(n_{ij}, e_{ij}, u_{ij}) \leq O_i \qquad [Eq\ 37]$$

6   MULTIPERIOD OPTIMIZATION FOR MULTIPLE FLEETS

Handling multiple fleets with the formulation of the previous chapter becomes unwieldy unless the number in each fleet remains small, fleet items are aggregated,[5] and/or if the number of usage bracket groups remains small.   Multifleet problems can allow items from multiple fleets to satisfy the same demand, and items from the same fleet to serve multiple demands. These constraints further complicate a dynamic programming formulation.

Solving the problem via genetic algorithms[6] involves four steps:

1.   Generating several hundred or more solutions

2.   Evaluating each solution

3.   Selecting a new generation of solutions by:

   a.   Retaining some of the existing with a probability that increases with its evaluation

   b.   Performing random change (mutations) on a small number of solutions from the old generation

   c.   Combining part of one solution with a part of another to form new solutions

4.   Repeating the previous two steps for many successive generations or solutions.

Eventually, only the better solutions remain in a generation.

Some of the less complicated formulations of the multiple fleet problem may also be amenable to linear or integer programming.

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[5] Simms et al.

[6] David F. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning* (Addison Wesley, 1989); Alvin W. Davis, *Fundamentals of Applied Probability* (McGraw-Hill, 1967).

7   CONCLUSION

This study has developed and outlined the underlying concepts and specifications for a computer-based expert system that will develop and optimize recommendations to acquire or replace equipment units over a planning horizon of several years. This computer-based system will use expert-system techniques, historical data on equipment repairs, and optimization techniques to make equipment fleet replacement and sizing decisions that will specify the best combination of units for each equipment fleet during any year of the specified planning period, given the mix of equipment in the preceding year.

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9
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