	TION PAGE	Form Approved OME No. 0704-0188
	TARE I NOW BEY ISTROME WITHOUT IN THE INTERNATION OF THE INTERNATION OF INTERNATION STREET, ST	n igr remained militations, watching conting data spurces, is regarding the Butden extimate or any other marcs of the are for mild matide Uperations and Accort, 1/15 (Prevan
	E 3. REPORT TYPE	AND DATES COVERED
HILE AND SUBTILE	IFINAL	15JULS 1-14 SEPTI
Innovative Methods in the Dynami Structures	ics of Large Space $\int u$	) F49620-87-C-0000
AUTHOR(S)		
Satya N. Atluri	nTIC	
PERFORMING ORGANIZATION NAME(S) AND ADDE	RESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Computational Mechanics Center	AUG 19 1996	CMC-GIT-92-3
Atlanta, GA 30332-0356		AFPSR-TR- ST 😒 - 1907 (18
SPONSORING / MONITORING AGENCY NAMELS) AN	D AODRESSIES)	10. SPONSORING / MONITORING
AFOSR/NA	4	AGENCY REPORT NUMBER
Bullding 410 Bolling Air Force Base DC 2033	$\lambda \mu$	
DOTTING ALL FOLCE DASE, DO 2003		1
boring An Force base, bc 2003		
. SUPPLEMENTARY NOTES		
SUPPLEMENTARY NOTES	ival papers	
Also includes a list of 14 archi	ival papers	
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di	ival papers istribution is unlimite	d.
Also includes a list of 14 archi	ival papers	ed.
Also includes a list of 14 archi	ival papers	d.
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di Approved for public release.	ival papers istribution is unlimite	d.
Also includes a list of 14 archi a. OISTRIBUTION : AVAILABIL: 1Y STEMENT Approved for public release. Di This report presents a summary dealing with the issues of nonli	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit	ed. ed. ed. ed. i 2b. DISTRIBUTION CODE ed. ed. i 2b. DISTRIBUTION CODE ed. ed. ed. ed. ed. ed. ed. ed.
Also includes a list of 14 archi a. OISTRIBUTION AVAILABIL!!Y STITEMENT Approved for public release. Di This report presents a summary dealing with the issues of nonlininized time finite element method	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational	ed. 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE 12b. DISTRIBUTION CODE
Also includes a list of 14 archi a. OISTRIBUTION: AVAILABIL: IY STITEMENT Approved for public release. Di This report presents a summary lealing with the issues of nonli lixed time finite element method nultibody systems, and wave prop	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. ed. ed. id. narchival literature, ely deformed space structures motions of constrained fuctures.
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di This report presents a summary lealing with the issues of nonlinities and the finite element method nultibody systems, and wave prop	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ed in archival literature, tely deformed space structures motions of constrained fuctures.
Also includes a list of 14 archi <b>a. OISTRIBUTION</b> A VALLABILITY STEMENT Approved for public release. Di <b>ABSTRACT</b> (Maximum 200 words) This report presents a summary dealing with the issues of nonlinities and the finite element method nultibody systems, and wave prop	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ed in archival literature, rely deformed space structures motions of constrained ructures.
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di Approved for public release. Di This report presents a summary dealing with the issues of nonlinities and the finite element method multibody systems, and wave prop	ival papers istribution is unlimite v of 14 papers publishe inear dynamics of finit ds for large rotational bagation in lattice str	ed. 12b. DISTRIBUTION CODE ad in archival literature, rely deformed space structures motions of constrained ructures.
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di Approved for public release. Di Massing with the issues of nonlinities of the finite element method nultibody systems, and wave prop	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. ed. ed. ed. in archival literature, rely deformed space structures motions of constrained fuctures. 02_22052
Also includes a list of 14 archi <b>A. OISTRIBUTION</b> AVAILABILITY STREAMENT Approved for public release. Di <b>A. ABSTRACT</b> (Maximum 200 words) This report presents a summary dealing with the issues of nonlinities and wave propond nultibody systems, and wave prop	ival papers istribution is unlimite near dynamics of finit for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE ed in archival literature, rely deformed space structure, motions of constrained fuctures. 92-23052
Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di Approved for public release. Di Asstract (Maximum 200 words) This report presents a summary dealing with the issues of nonlinities and the finite element method multibody systems, and wave prop	ival papers istribution is unlimite v of 14 papers publishe inear dynamics of finit is for large rotational bagation in lattice str	ed. 12b. DISTRIBUTION CODE ad in archival literature, sely deformed space structures motions of constrained uctures. 92-23052
SUPPLEMENTARY NOTES Also includes a list of 14 archi Approved for public release. Di Approved for public release. Di Approved for public release of nonli nis report presents a summary dealing with the issues of nonli nixed time finite element method nultibody systems, and wave prop	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ed in archival literature, sely deformed space structures motions of constrained uctures. 92-23052 10 PM:
SUPPLEMENTARY NOTES Also includes a list of 14 archi a distract (Maximum 200 words) This report presents a summary dealing with the issues of nonlininized time finite element method nultibody systems, and wave prop	ival papers istribution is unlimite of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. D
SUPPLEMENTARY NOTES Also includes a list of 14 archi a. OISTRIBUTION: AVAILABILITY STATEMENT Approved for public release. Di Distribution 200 words: This report presents a summary dealing with the issues of nonlininized time finite element method nultibody systems, and wave prop 92 8 18 095 Subject TERMS Dynamics; Control of Flexible St odeling: Space Structures	ival papers istribution is unlimite of 14 papers publishe inear dynamics of finit is for large rotational pagation in lattice str	ed. 12b. DISTRIBUTION CODE ad in archival literature, rely deformed space structure, motions of constrained uctures. 92-23052 15. NUMBER OF PAGES r
SUPPLEMENTARY NOTES Also includes a list of 14 archi Also includes a list of 14 archi Approved for public release. Di ABSTRACT (Maximum 200 words) This report presents a summary dealing with the issues of nonlinities and wave proposed multibody systems, and wave proposed SUBJECT TERMS Dynamics; Control of Flexible St fodeling; Space Structures.	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit ds for large rotational bagation in lattice str UQQC	ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE ed. 12b. DISTRIBUTION CODE 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. DISTRIBUTION 12b. D
<b>92</b> 8 18 095 <b>92</b> 8 18 095 <b>92</b> 8 18 095 <b>1</b> SUPPLEMENTARY NOTES <b>ABSTRACT</b> (Maximum 200 words) This report presents a summary dealing with the issues of nonlininized time finite element method nultibody systems, and wave prop <b>92</b> 8 18 095 <b>5</b> SUBJECT TERMS Dynamics; Control of Flexible St fodeling; Space Structures. <b>7</b> SECURITY CLASSIFICATION 18. S	ival papers istribution is unlimite y of 14 papers publishe inear dynamics of finit is for large rotational bagation in lattice str dagation in lattice str :ructures; Reduced Orde	ed. 12b. DISTRIBUTION CODE ed in archival literature, ely deformed space structures motions of constrained uctures. 92-23052 15. NUMBER OF PAGES r 15. NUMBER OF PAGES r 15. NUMBER OF PAGES 16. PRICE CODE SSIFICATION 20. LIMITATION OF ABSTRACT

an an an an ann an an ann an Arabert ann an an an an an Arabert an an an an an an an Arabert an an an an an an

. **.**.

. . . .

t

## INNOVATIVE METHODS IN THE DYNAMICS OF LARGE SPACE STRUCTURES

ŧ

Final Technical Report on AFOSR Contract No. F49620-87-C-0064

Prepared by:

Satya N. Atluri

Institute Professor & Regents' Professor of Aerospace Engineering & Mechanics

Computational Mechanics Center Georgia Institute of Technology Atlanta, GA 30332-0356

July 1992

In this final report, a descriptive summary of research accomplished under AFOSR Contract No. F49620-87-C-0064 is presented. Technical details of research, mathematical formulations, their computational implementations, and their verifications are presented in the archival papers listed at the end of the report.

In [1] the problem of transient dynamics of highly flexible three-dimensional space-curved beams, undergoing large rotations and stretches, is treated. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is considered. Using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangian description of motion, the mass-matrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotation parameters and attendant accelerations; the stiffness matrix depends nonlinearly on the deformation; and an 'apparent' damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semi-discrete finite element equations in time. Several examples of transient dynamic response of highly flexible beam-like structures, including those in free flight, are presented to illustrate the validity of the theoretical methodology developed in this paper.

The following topics are discussed in [2]: (i) some of the recent advances in formulating finite deformation (large rotations as well as stretches) plate and shell theories, and attendant mixed finite element formulations based on symmetric variational statements; (ii) finic. element/boundary element formulations based on unsymmetric variational statements, Petrov-Galerkin methods, and the use of fundamental solutions in infinite space, for the highest-order differential operator of the problem, as test functions in solving nonlinear plate and shell problems; and (iii) algorithms for solving the problems of control of nonlinear dynamic motion of plates and shells.

Article [3] deals with nonlinearities that arise in the study of dynamics and control of highly flexible large-space-structures. Broadly speaking, these nonlinearities have various origins: (i) <u>geometrical</u>: due to large deformations and large rotations of these structures and their members; (ii) <u>inertia</u>: depending on the coordinate systems used in characterizing the overall dynamic motion as well as elastic deformations; (iii) <u>damping</u>: due to nonlinear hysteresis in flexible joints; viscoelastic coatings, etc., and (iv) <u>material</u>: due to the nonlinear behavior of the structural material. The geometrical and material nonlinearities affect the "tangent stiffness operator" of the structure; the inertia nonlinearities affect the "tangent inertia operator".

To study the nonlinear transient dynamic response and control of flexible space-structures, one may think of: (i) semi-discrete approximation methods, and (ii) space-time methods. In the former class of methods, an appropriate spatial discretization is employed through weak-formulations (finite-element and field/boundary element) in space, and thus a set of coupled nonlinear ordinary differential equations (O. D. E.) is derived. These O. D. E.s are solved often through temporal integration techniques of the finite difference-type. The semi-discrete methods are not ideally suited for traveling-wave type propagating disturbances. The second category of methods, viz., the space-time methods, wherein weak formulations in both space and time are employed, are somewhat better suited for wave-propagation type problems. In this article, attention is primarily focused on semi-discrete methods.

Depending on the scale of the response that is required to be studied, a large-space-structure may either be modeled as an equivalent continuum, or as a lattice structure with the details of each member being accounted for. The spatial discretization in either case is required to be of the least-order possible so that the control algorithms may be meaningfully implemented. The reduced-order-modeling of the "tangent stiffness" operator of either a continuum model, or a lattice model of a space-structure is treated in some detail in this paper, for structures undergoing large dynamic deformations.

The control of dynamic motion of space-structures is currently envisaged to be through either active processes, passive processes, or some combinations there of. One of the concepts of active control that is considered in detail in [3], and by other authors, is the use of piezo-ceramic actuators that are bonded to the truss and frame members of the space-structure in various locations. The controlling shear stress transmitted by the actuator to the truss by frame member depends on the axial force, transverse shear forces, and bending and twisting moments, in the member itself, as well as the excitation voltage applied to the piezo-actuator. This problem of mechanical coupling between the structural member and actuators is discussed in some detail in this work.

The problem of control of nonlinear dynamic motion is addressed in [3]. The problem is posed in the form of determining the feed-back gain matrix and the attendant control force vector, such that the response as predicted by a semi-discrete system of coupled nonlinear ordinary differential equations, subject to a set of arbitrary initial conditions, is damped out in a pre-set time.

In the first part of article [3], continuum models of spacestructures are analysed. These include models of the space-beam type as well as the shallow shell type. In the case of space-beams, the problem of nonlinear dynamic response, when the beam undergoes large overall rigid as well as elastic motion, is discussed. The beam is assumed to undergo large rotations as well as stretches. A simple finite element algorithm to predict the response is presented. When a shallow-shell type continuum model is used, a field-boundary element approach based on nonlinear integral equations is presented as a means to create a reduced-order dynamic model of the semi-discrete type. A simple algorithm to control the response predicted by these nonlinear semi-discrete equations is discussed.

In the second part of [3], detailed models of the lattice type space-structures are discussed. Each member of the structural lattice is assumed to be either a "truss member", or as a "frame member." The "truss member" is assumed to carry only an axial load, and has three displacement degrees of freedom at each node. The "frame member" is assumed to carry an axial force, transverse shear forces, bending moments, and a twisting moment; and is assumed to have three displacement and three rotational degrees of freedom at each node. Explicit expressions for the tangent stiffness matrices of both "truss" type and "frame" type members, which undergo arbitrarily large displacements, arbitrarily large overall rigid rotations, and moderate local (relative) rotations, are derived. In all cases, each member (truss or frame type) is modeled by a single finite element, in the entire range of large deformations. Several examples are presented to illustrate the efficiency and cn-board computational feasibility of these reduced-order models for lattice structures. In each instance, remarks on needs for future research are made.

transient dynamics of highly-flexible 3-The problem of dimensional space-beams, undergoing large rotations and stretches, is treated in [4]. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. Based on the present governing equations, a general mixed variational principle for the static problem is presented. Furthermore, using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the dynamic problem of a flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangian description of motion, the massmatrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotational parameters and attendant angular accelerations; the stiffness matrix depends nonlinearly on the deformation; and an "apparent" damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semidiscrete finite element equations in time. An example of transient dynamic response of highly flexible beam-like structures in freeflight is presented to illustrate the validity of the theoretical methodology developed in this paper.

A scheme for active control of nonlinear vibration of spacestructures, wherein each member is modeled as a beam-column, is presented in [5]. The expressions for shear stresses transmitted to the structural member by the distributed segmented piezo-electric actuators, which are bonded on the surfaces of the member, are derived in the general case in which the structural member is subjected to moments, transverse shear forces and an axial force. Based on the weak form of the governing equations, and a complementary energy approach based on assumed stress fields, the viability of active control of nonlinear dynamic response of lattice-type space structures, using piezo-electric actuators, is studied. Four examples are given to demonstrate the feasibility of the approaches presented in this paper.

The deformation of a beam-column, the upper and lower surfaces of which are bonded in segments with piezo-ceramic liners, is studied in [6] for the purpose of obtaining appropriate expressions for the force transferred to the structural member by the piezoactuator. This concept may be employed for the control of large dynamic deformations of a lattice-type flexible space-structure. The present model, which is based upon a static analysis, accounts for the effects of transverse shear and axial forces in addition to a bending moment on the beam in formulating the governing equilibrium equations. The present model provides more complete expressions for the force transmitted to the structural member than a model reported earlier in literature, in which the shear and axial forces are neglected.

The dynamic response of frame-type structures with hysteretic damping at the structural joints, resulting from slipping and nonlinear flexible connections, is investigated in [7]. The slipping at a structural joint is represented by the modified Coulomb joint model. The behavior of a nonlinear flexible connection is modeled by the Ramberg-Osgood function. A simple computational model for the dynamic analysis of frames with hysteretic damping is presented here. Several numerical examples are included to illustrate the usefulness of the approach in analyzing large space structures.

The prediction of transient response of structures, in the form of traveling waves, is very important for controlling the dynamic behavior of structures. It is well known that the standard semi-discrete form of the finite element method is not suitable for predicting the wave propagation, due to the inherent dispersion involved. In [8], an application of space-time finite element method to the wave propagation problem is discussed. The main concerns in such problems consist of developing a consistent and stable scheme and also of capturing a shock wave, without wiggling. At first, a weak form of the wave propagation problem is discussed, taking into account the jump condition associated with velocity and stress. A mixed finite element formulation plays an important role in evaluating the velocity explicitly. The application of present formulation to the linear wave equation shows that the present numerical results at the discontinuity give the mean values of jump. In the case of flexural wave propagations in a Timoshenko beam, the present method captures the wave front easily, as opposed to the semi-discrete method.

A novel theory and its computational implementation are presented in [9] for the analysis of strongly nonlinear dynamic response of highly-flexible space-beams that undergo large overall motions as well as elastic motions with arbitrarily large rotations and stretches. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. A symmetric tangent stiffness matrix is derived at all times even if the distributed external moments exist. An example of transient dynamic response of the beam is presented to illustrate the validity of the theoretical methodology.

Currently, there is renewed interest in the study of multibody-dynamics and its application in many fields of engineering. The mathematical model of a rigid body is useful whenever the overall motion, involving large rigid rotation, is of interest. The nonlinear dynamic equations of motion, in their explicit form, appear quite complex due to the expression for the absolute accelerations. In [10], weak formulations of linear and angular momentum balance laws of a rigid body undergoing large overall motion are stated a priori. Holonomic as well as nonholonomic constraints, that may exist on the motion of the rigid body, are introduced into this weak form in a fundamentally novel fashion here. Comments are made on the incremental form (and consistent linearization) of the weak formulation (with constraints), and the time-finite-element solutions thereof.

The paper referred to in [11] presents general variational formulations for dynamic problems, which are easily implemented numerically. The development presents the relationship between the very general weak formulation arising from linear and angular momentum balance considerations, and well known variational principles. Two and three field mixed forms are developed from the general weak form. The variational principles governing large rotational motions are linearized and implemented in a time finite element framework, with appropriate expressions for the relevent "tangent" operators being derived. In order to demonstrate the validity of the various formulations, the special case of free rigid body motion is considered. The primal formulation is shown to have unstable numerical behavior, while the mixed formulation exhibits physically stable behavior. The formulations into constrained dynamical systems and multi-rigid-body systems.

Constrained equations arise in the dynamics of mechanical systems whenever there is the need to restrict kinematically possible motions of the system. In practical applications, constraint equations can be used to simulate complex, connected systems. If the simulation must be carried out numerically, it is useful to look for a formulation that leads straightforwardly to a numerical approximation. In [12], the methodology of previous work is extended to incorporate the dynamics of holonomically and nonholonomically constrained systems. The constraint equations are cast in a variational form, which may be included easily in the time finite element framework. The development of the weak constraint equations and their associated "tangent" operators is presented. This approach to constraint equations may be employed to develop time finite elements using a quaternion parametrization of finite rotation.

Weak formulations in analytical dynamics are developed, paralleling the variational methods in elastostatics, and including a fundamental yet novel approach for treating constraints (both holonomic and nonholonomic). A general three-field approach is presented, in which the momentum balance conditions, the compatibility conditions between displacement and velocity, the constitutive relations and the displacement and momentum boundary conditions are all enforced in weak form. A primal, or kinematic, formulation is developed from the general form by enforcing the compatibility conditions and displacement boundary conditions a priori. The conditional stability of the kinematic formulation is the counterpart of the locking phenomenon in elastostatics and may be avoided, either by reduced order integration or by utilizing a mixed formulation. Toward this end, a two-field mixed formulation is presented, which follows from the general form, when the constitutive relations are satisfied a priori. A general set of the constraint equations is introduced into the kinematic and mixed formulations, using a specific choice of multipliers, which results in modified variational principles. Several simple examples concerning rigid body dynamics are presented [13].

The article in [14] deals with the effect of non-linearly hysteretic joints on the static and dynamic response of space frames. It is shown that a complementary energy approach based on a weak form of the compatibility condition as a whole of a frame member, and of the joint equilibrium conditions for the frame, is best suited for the analysis of flexibly jointed frames. The present methodology represents an extention of the authors' earlier work on rigidly connected frames. In the present case also, an explicit expression for the tangent stiffness matrix is given when (i) each frame member, along with the flexible connections at its ends, is represented by a single finite element, (ii) each member can undergo arbitrarily large rigid rotations and only moderate relative rotations and (iii) the non-linear bending-stretching coupling is accounted for in each member. Several examples, with both quasi-static and dynamic loading, are included to illustrate the accuracy and efficiency of the developed methodology.

## DTIC QUALITY INSPECTED 5

Acces	ssion For	1
NTIS	GRA&I	
DTIC	TAB	
Unarg	lownoed	<u>п</u> .
Just'	is stion	
By		
Dist:	to the fork	
Avai	letiing r	t lag
	Avail and	/or
Dist	Special	
	l I.	[
<b>N</b>		
r i		

## **References:**

- M. Iura and S. N. Atluri, "Dynamic Analysis of Finitely Stretched and Rotated Three-Dimensional Space-Curved Beams," Computers & Structures, Vol. 29, No. 5, pp. 875-889, 1989.
- 2. S. N. Atluri, J-D. Zhang, and P. E. O'Donoghue, "Analysis and Control of Finite Deformations of Plates and Shells: Formulations and Interior/Boundary Element Algorithms," in <u>Finite Element Analysis of Plates and Shells</u>, Eds. T. R. J. Hughes and E. Hinton, Pineridge Press, Swansea, 1989.
- 3. S. N. Atluri and M. Iura, "Nonlinearities in the Dynamics and Control of Space Structures: Some Issues for Computational Studies," in <u>Dynamics and Control of Large Space Structures</u>, Chapter 3, (Eds. S. N. Atluri and A. K. Amos), Springer-Verlag, 1988.
- 4. M. Iura and S. N. Atluri, "Dynamics of 3-D Space-Curved Beams Undergoing Finite Rotations and Finite Strains: A Variational Theory and Numerical Studies," in Proc. of 21st Annual Meeting of the Society of Engineering Science, pp. 210-221, 1988.
- 5. G. Shi and S. N. Atluri, "Active Control of Nonlinear Dynamic Response of Space-Frames Using Piezo-Electric Actuators," Computers & Structures, Vol. 34, No. 4, pp. 549-564, 1990.
- 6. S. Im and S. N. Atluri, "Effects of a Piezo-Actuator on a Finitely Deformed Beam Subjected to General Loading," AIAA Journal, Vol. 27, No. 12, pp. 1801-1807, 1989.
- 7. G. Shi and S. N. Atluri, "Nonlinear Dynamic Response of Frame-Type Structures with Hysteretic Damping at the Joints," Int. J. Num. Meth. in Engg., Vol. 28, pp. 2635-2650, 1989.
- 8. M.Iura, M. Borri, and S. N. Atluri, "Analysis of Traveling Wave Responses of Structures," in <u>Computational Mechanics '88</u> (Eds. S. N. Atluri and G. Yagawa), Springer-Verlag, paper 44v, 1988.
- 9. M. Iura and S. N. Atluri, "Dynamic Analysis of Finitely Stretched and Rotated 3-D Space-Curved Beams," in <u>Computational</u> <u>Mechanics '88</u>, (Eds. S. N. Atluri and G. Yagawa), Springer-Verlag, pape: 40ii, 1988.
- 10. M. Borri and S. N. Atluri, "Time-Finite Element Method for the Constrained Dynamics of a Rigid Body," in <u>Computational</u> <u>Mechanics '88</u>, (Eds. S. N. Atluri and G. Yagawa), Springer-Verlag, paper 41i, 1988.
- 11. M. Borri, F. Mello, and S. N. Atluri, "Variational Approaches for Dynamics and Time-Finite-Elements: Numerical Studies," Computational Mechanics, Vol. 7, pp. 49-76, 1990.
- 12. M. Borri, F. Mello, and S. N. Atluri, "Primal and Mixed Forms

of Hamilton's Principle for Constrained Rigid Body Systems: Numerical Studies," Computational Mechanics, Vol. 7, pp. 205-220, 1991.

13. F. J. Mello, M. Borri, and S. N. Atluri, "Time Finite Element Methods for Large Rotational Dynamics of Multibody Systems," Computers & Structures, Vol. 37, No. 2, pp. 231-240, 1990.

1 \*

14. G. Shi and S. N. Atluri, "Static and Dynamic Analysis of Space Frames with Non-linear Flexible Connections," Int. J. Num. Meth. Engg., Vol. 28, 1989.